

Matter waves in traps, beam-splitters and optical circuits



Teaching old tricks with time-dependent control
to new Things



Dr Seuss, The cat in the hat
<https://www.youtube.com/watch?v=yJFISYQn9kk>

Thing 1
&
Thing 2

M. Sturm, A. Neumann, Reinhold Walser
Collaboration: G. Birkl, M. Schlosser, D. Pfeiffer

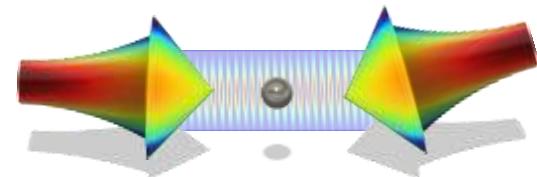


Contents

- **Pulsed Bragg beam-splitters**

Good old Demkov-Kunike $\text{Sech}(t)$
analytical model

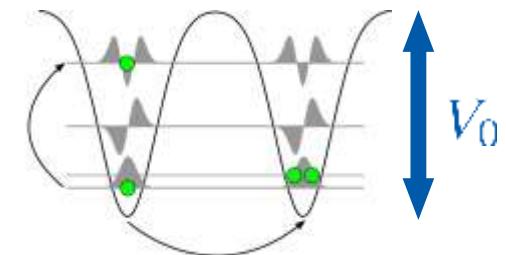
A. Neumann *et al.*, Phys. Rev. A **103**, 043306 (2021)



- **Rapid generation of Mott insulator states**

Good old NMR fast pulse ramps

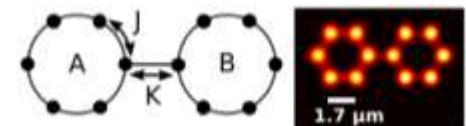
M. Sturm, *et al.*, Phys. Rev. A **97**, 063608 (2018)



- **Exciting multiparticle resonances in Josephson junctions of coupled BH rings**

Good old sudden switching

M. Sturm, *et al.*, Phys. Rev. A **95**, 063625 (2017)



From idealised to ‘realistic’ atomic beamsplitters

Theoretical studies in (3+1)D

Antje Neumann, Reinhold Walser & QUANTUS collaboration



QUANTUM MATTER

GRANADA

1st-5th September 2019



theweathernetwork.com (Susan Cipriano, Creative Commons)

03-May-22



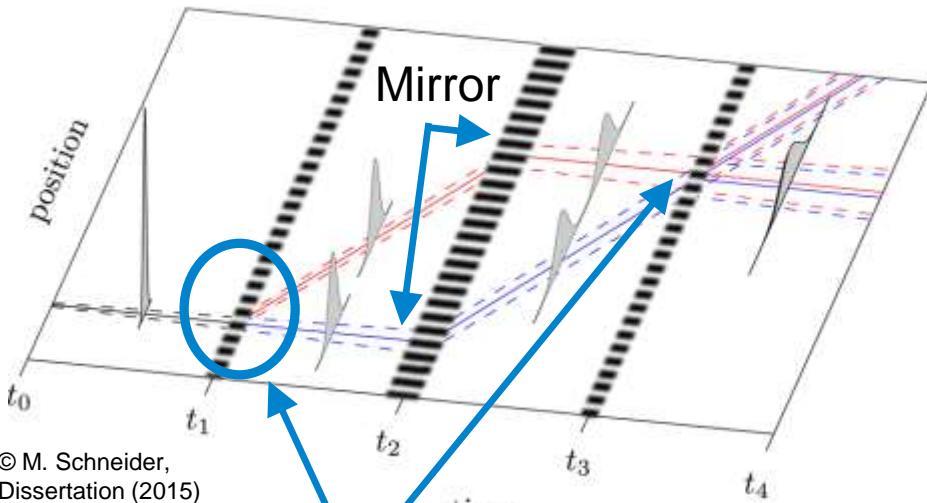
livestockoftheworld.com/Donkeys (Imgur/ButtersTheCat)

Atomtronics

3

Why atomic beamsplitters?

Matter-wave interferometer



© M. Schneider,
Dissertation (2015)

BeamSplitter

coherent superposition of
atomic wave packets
[PRL 82 871 (1999)]

High-precision rotation
& acceleration sensing

$$\text{Sensitivity} \propto T^2, \Delta k$$

Applications:

- Fundamental physics:
e.g. universality
of free fall

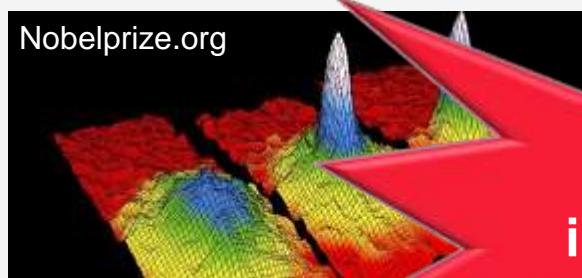


- Inertial navigation, ...

ultracold
Quantum gases

BEC as macroscopic
quantum matter

Nobelprize.org



- Long coherence length
- Slow expansion
- Narrow momentum width

microgravity

Experiments
need reliable
predictions!

QU free fall experiments
[PRL 110 093602 (2013)]

Drop tower:
4.7 sec x 2



Parabolic flights
[Nature comm. 7 13786 (2016)]

AUO first BEC
in space

[Nature 562
391-395 (2018)]

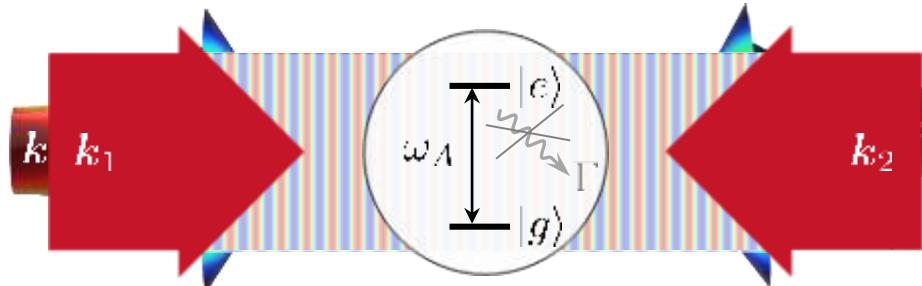
Sounding
rocket:
6 min



Space station:
Cold Atoms Lab
[coldatomlab.jpl.nasa.gov]

Satellites

Bragg diffraction of atoms by an optical standing wave



$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{\hbar\Delta}{2}(|e\rangle\langle e| - |g\rangle\langle g|) + \hat{H}_I$$

$$\hat{H}_I = \frac{\hbar}{2} \Omega_0 t (e^{i\mathbf{k}_1 \cdot \hat{\mathbf{x}}} (e^{i\mathbf{k}_L \cdot \hat{\mathbf{x}}} + e^{-i\mathbf{k}_L \cdot \hat{\mathbf{x}}}) |e\rangle\langle e| + h.c.)$$

absorb / emit 1 photon:

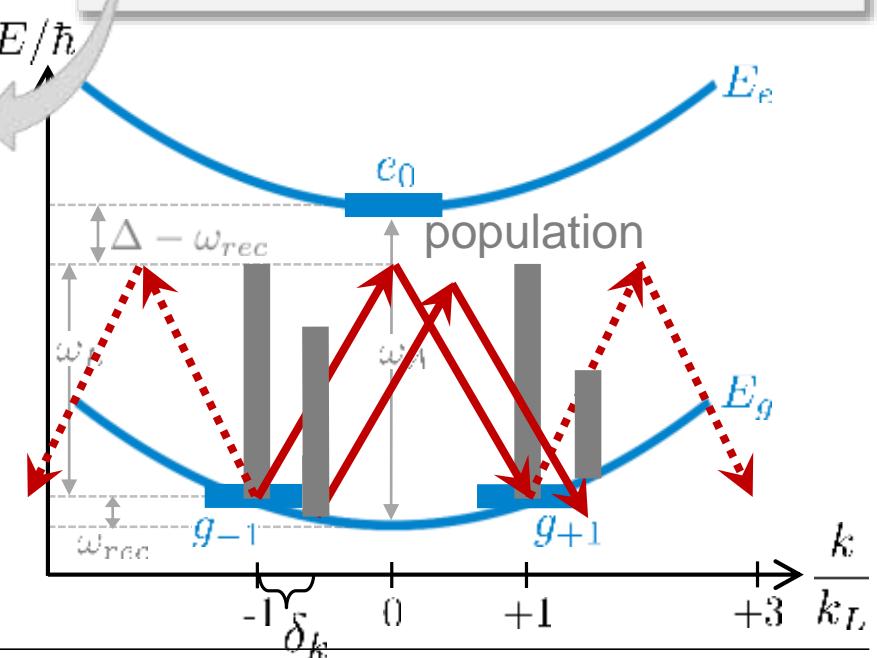
$$e^{\pm i\mathbf{k}_L \cdot \hat{\mathbf{x}}} = \int d^3p |\mathbf{p} \pm \hbar\mathbf{k}_L\rangle\langle \mathbf{p}|$$

coupling:

$$|g, \mathbf{k}\rangle \leftrightarrow |g, \mathbf{k} \pm 2N\mathbf{k}_L\rangle$$

Brattain order

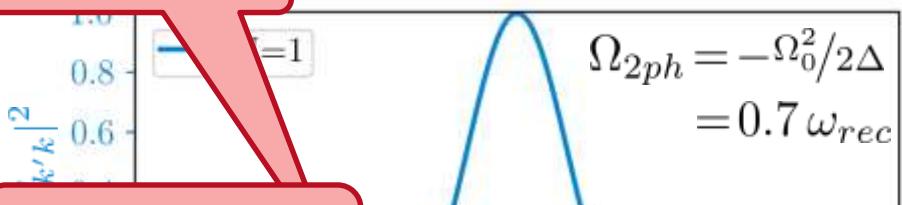
S Frame transformation:
atoms at rest & moving grating
grating at rest & moving atoms
 $\rightarrow |\mathbf{k}_L| = (|\mathbf{k}_1| + |\mathbf{k}_2|)/2$



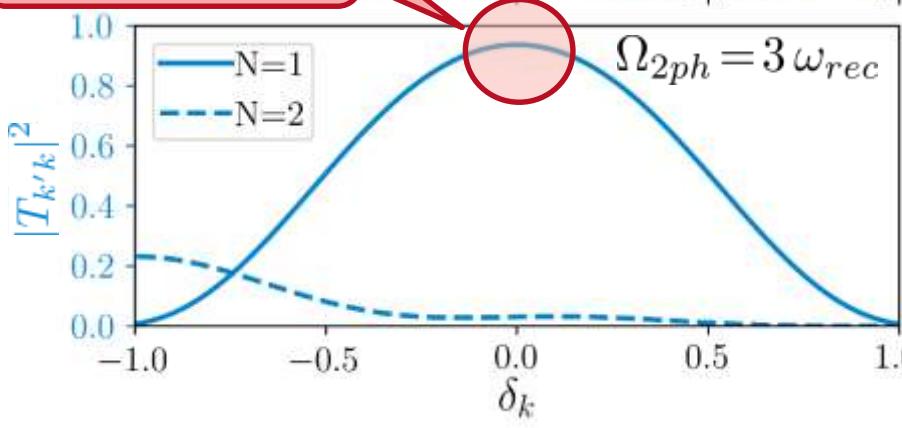
Velocity dispersion for box pulses

$$|\Psi(t)\rangle = \sum_{\delta_k \in (-1,1)} \sum_{m=-N}^N |g, (m+\delta_k)k_L\rangle + |e, (m'+\delta_k)k_L\rangle$$

Side maxima



Population loss
into $N \neq 1$

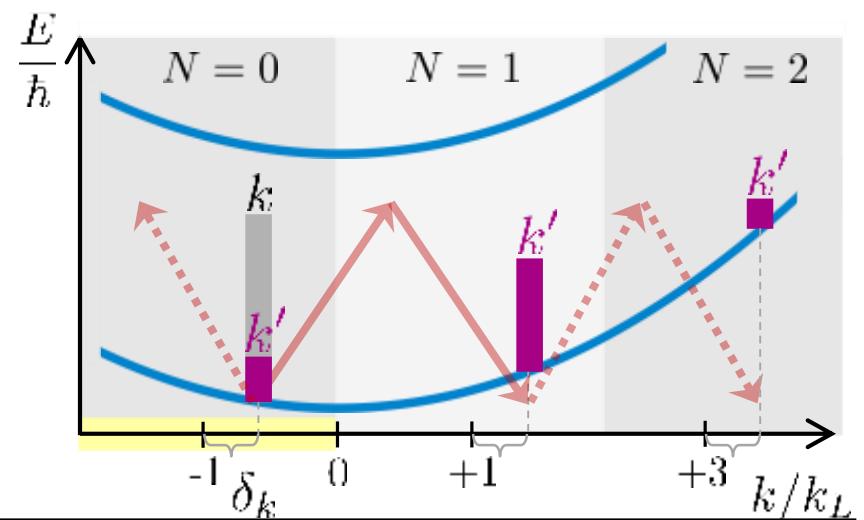


$$|\Psi(t)\rangle = e^{-i/\hbar \hat{H}t} |\Psi(0)\rangle = \hat{U} |\Psi(0)\rangle$$

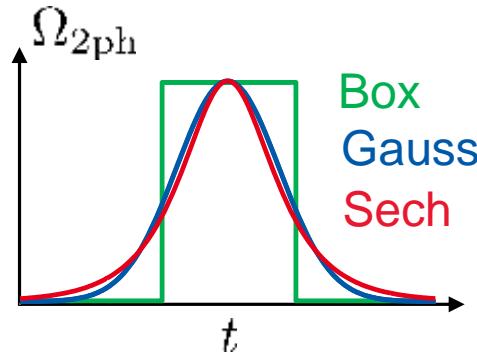
Transition amplitude between initial k and final $k' = k + 2Nk_L$

$$T_{k'k} = \langle k' | \hat{U} | k \rangle$$

→ diffraction efficiency $|T_{k'k}|^2$



Various pulse envelopes, ...

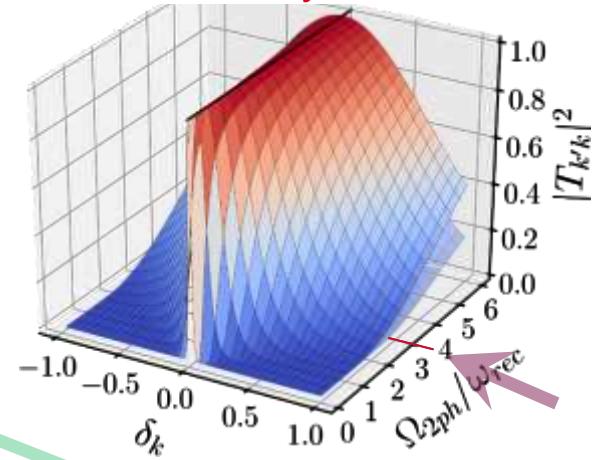


Box: analytical

- suppressed side maxima
- less population loss into higher diffraction orders
- analytic model (DK) for **sech** - pulses



CDK: analytical



Analytic Demkov-Kunike model:

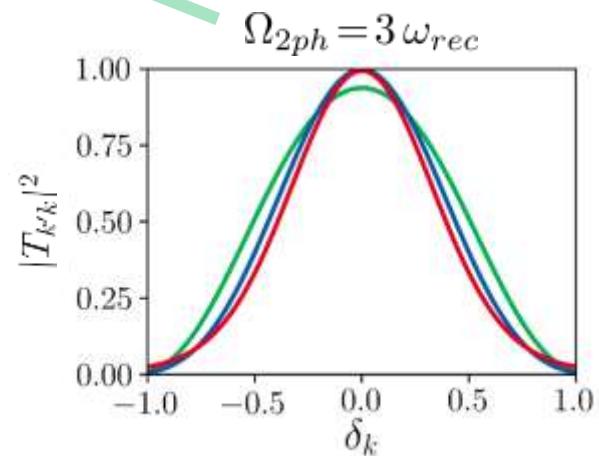
1D, 1st order Bragg diffraction

$$\Omega_{2ph}(t) = \Omega_{2ph} \cdot \text{sec}$$

- suppressed side maxima
- less population loss into higher diffraction orders

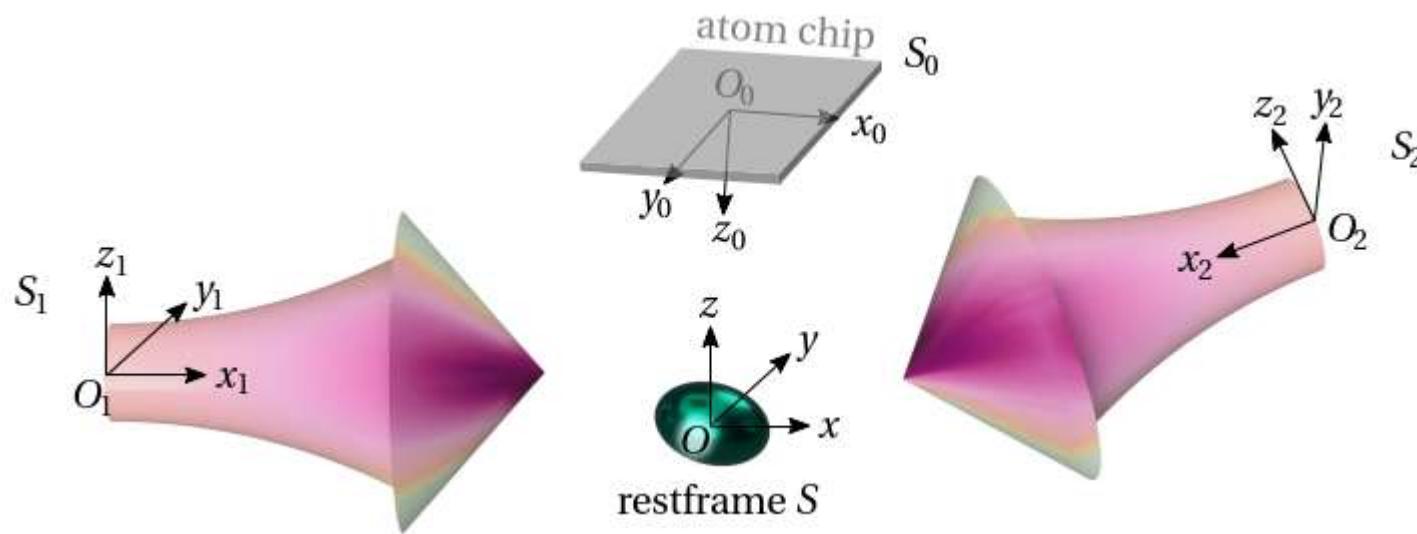
$$T_{k'k}^{N=1} = \frac{\sigma \Omega_{2ph} \sqrt{\text{sec}}}{i\sqrt{2\pi} - 8\omega_{rec}\delta_k\sigma}$$

$$a/b = 1 -/+ \frac{\sigma \Omega_{2ph}}{\sqrt{2\pi}}, \quad c = \frac{3}{2} + 2i\sqrt{\frac{2}{\pi}}\omega_{rec}\delta_k\sigma, \quad z(t) = \frac{1}{2} \left(1 + \tanh \left(\sqrt{\frac{\pi}{2}} \frac{t}{\sigma} \right) \right)$$



Experiment on ground

- Prepare BEC
- Drop with variable time
- Apply Bragg pulse



Comparison with experiment*



TECHNISCHE
UNIVERSITÄT
DARMSTADT

* M. Gebbe (ZARM, University Bremen)

- Laser:
- spatial dependence: ~ plane waves
 - temporal: Gaussian
 - Laser frequency detuned to resonance $\delta_f \rightarrow \delta_k$

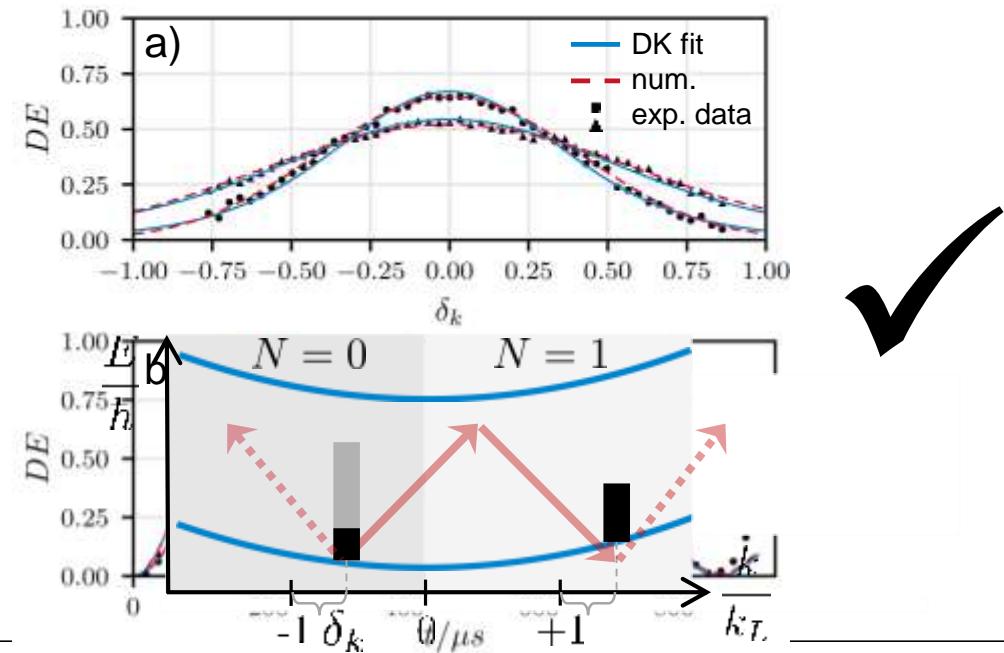
Atoms: BEC @ 50 nK ~ Thomas Fermi, momentum width $\ll k_L$

Frame transformation:
 $\delta_f \rightarrow \delta_k$

Diffraction Efficiency:

$$DE = \frac{P_{+k_L}}{P_{-k_L} + P_{+k_L}}$$

a)	P	(20 ± 2) mW	(30 ± 3) mW
	$\Omega_{2\text{ph}}^{\text{exp}}$	(1.7 ± 0.3) ω_{rec}	(2.6 ± 0.4) ω_{rec}
b)	t	138.75 μs	92.5 μs
	pulse area	0.67π	0.67π
	$\Omega_{2\text{ph}}^{\text{DK}}$	$1.87\omega_{\text{rec}}$	$2.42\omega_{\text{rec}}$
	$\Omega_{2\text{ph}}^{\text{num}}$	$1.83\omega_{\text{rec}}$	$2.40\omega_{\text{rec}}$

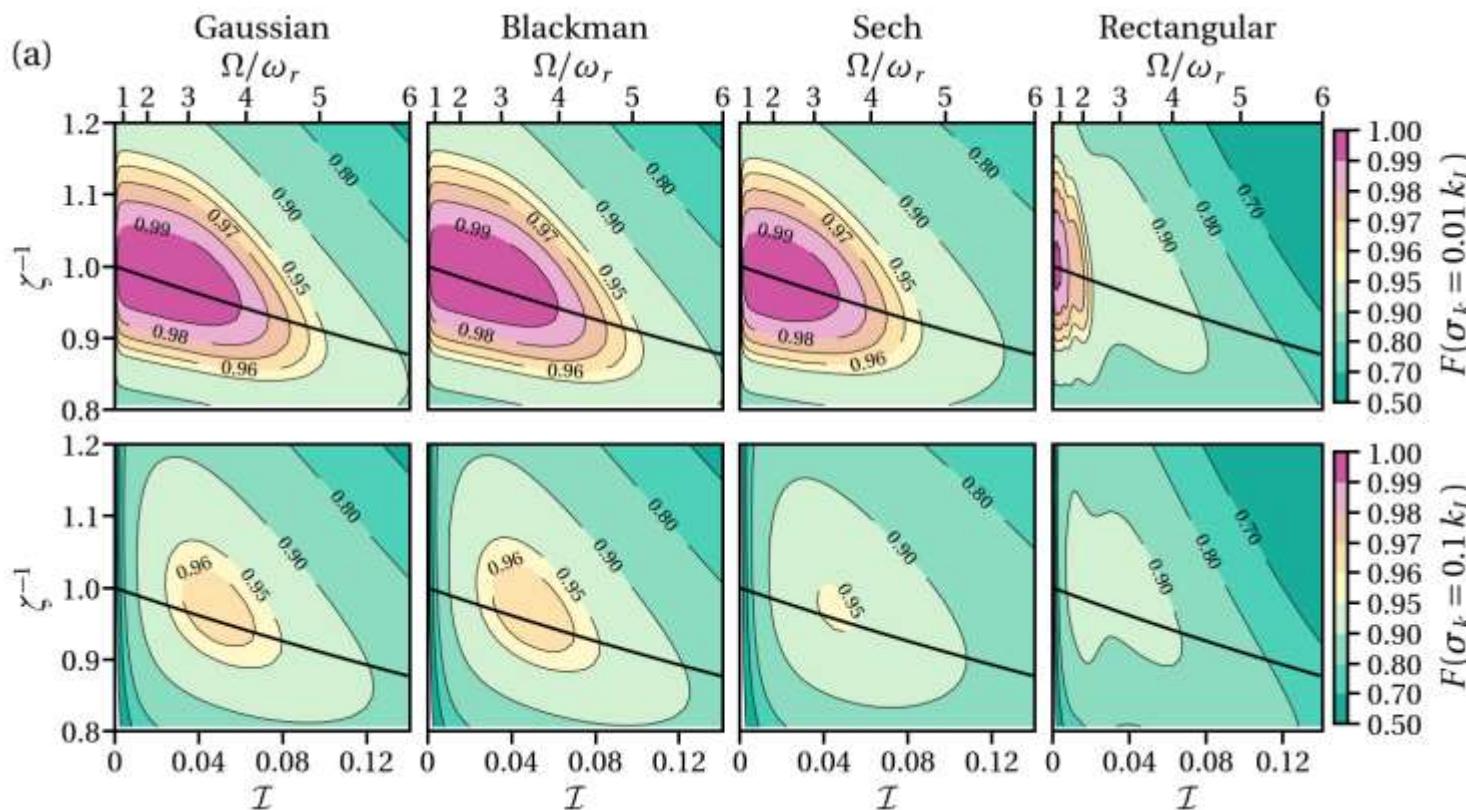


Optimal π -pulse intensities for „bs-mirrors“



- Optimal π -pulse time stretching factor
- Universal rule: Kato deg. perturbation theory

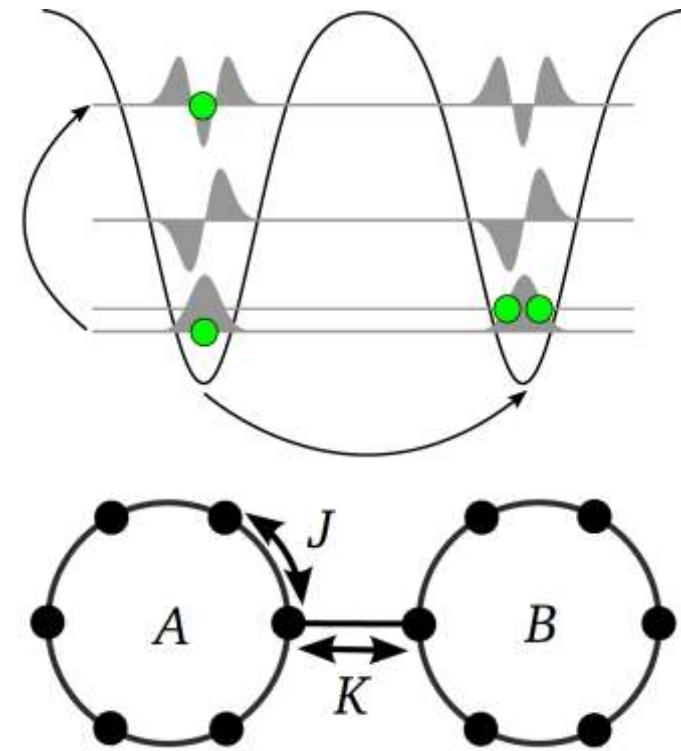
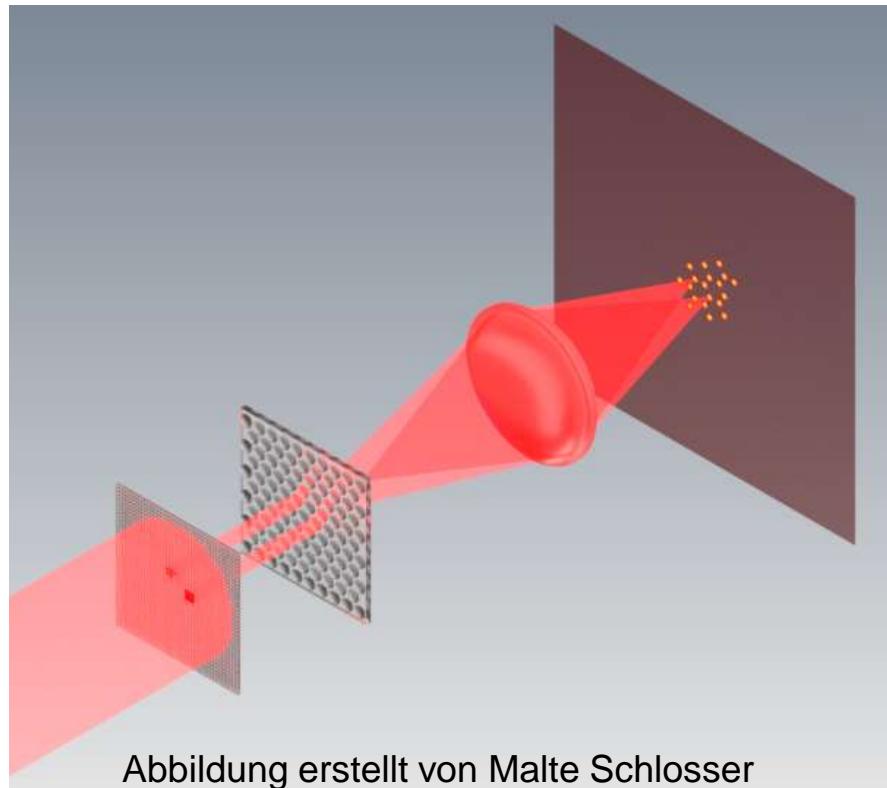
$$\zeta_\pi \equiv \zeta_{\pi}^{\kappa=0} = \frac{1}{1-\mathcal{I}} (\approx 1 + \mathcal{I}).$$



Ultracold atoms in reconfigurable arrays of tunnel coupled traps

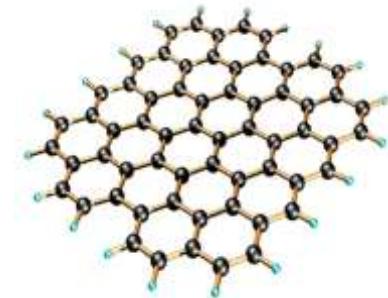
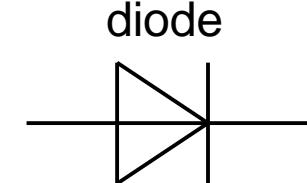
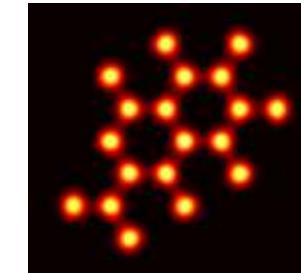
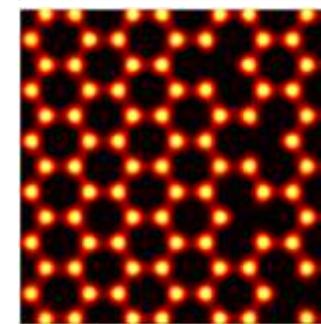
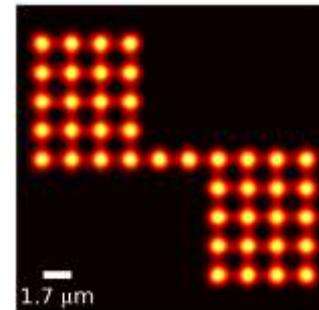
Martin Sturm

Collaboration: Gerhard Birkl, Malte Schlosser

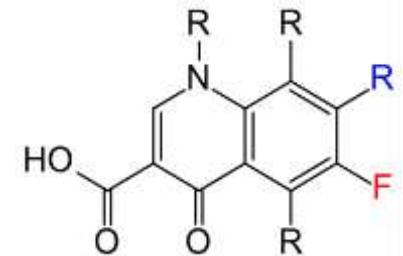


Design of light potential surfaces

- **Pinboard for atomtronics**
implement atomtronic devices
like diodes and transistors
- **Designable lattices**
Exotic lattice geometries
quasi crystals
point/line defects
controllable disorder
- **Molecular structures**
mimick electronic structure
of molecules



Copyright: Chris Ewels



<https://en.wikipedia.org/wiki/Quinolone>

Key parameters

- **Micro lense array**
 - Lense separation: 110 µm
 - Lense diameter: 106 µm
 - # Lenses: 100×100
- **Relais optics**
 - NA=0.68
 - Magnification: 1/65
 - Trap separation: **1.7 µm**
- **Spatial light modulator or DMD's**

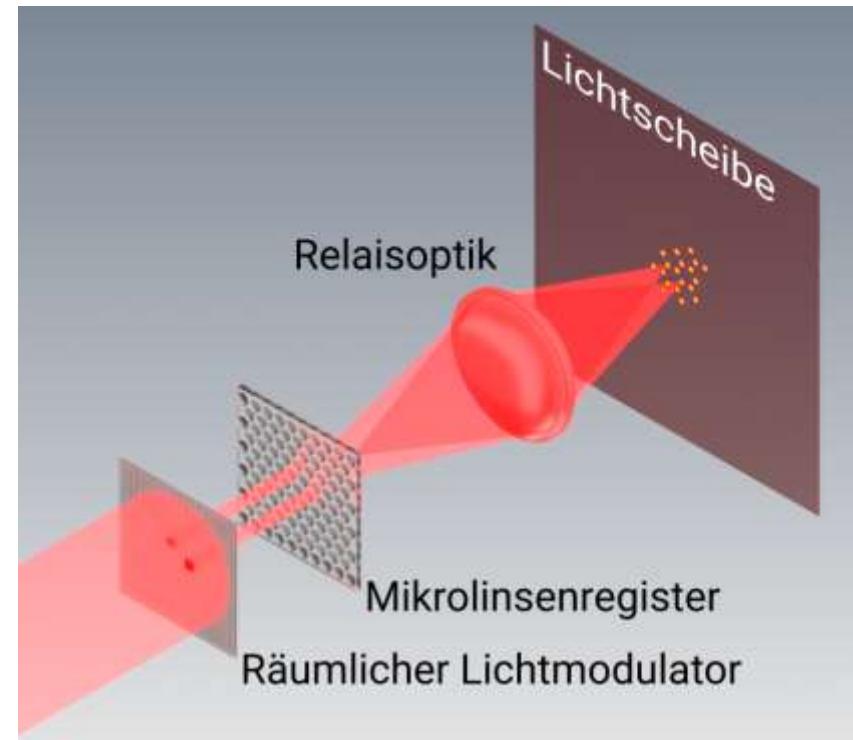


Abbildung erstellt von Malte Schlosser

Intensity distribution/optical potentials

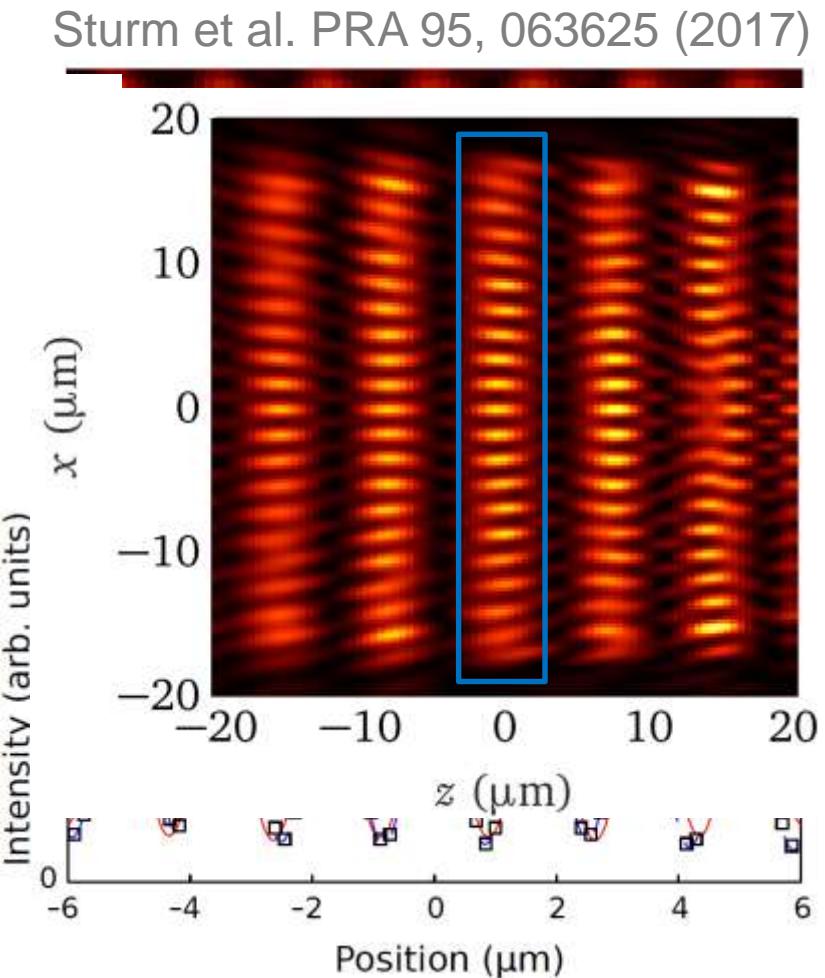
- Register of Gauss-shaped micro traps

$$V_{\perp}(x, y) = \sum_{i=1}^M V_{0\perp}^{(i)} e^{-\frac{(x-X_i)^2+(y-Y_i)^2}{w_{0\perp}^2/2}}$$

$$w_{0\perp} = 0.7 \text{ } \mu\text{m} \quad d = 1.7 \text{ } \mu\text{m}$$

- Talbot-effect (length: 7 μm)
- Select with sheet of light

$$V_{\parallel}(z) = -V_{0\parallel} e^{-\frac{z^2}{w_{0\parallel}^2/2}} \quad w_{0\parallel} = 2.5 \text{ } \mu\text{m}$$

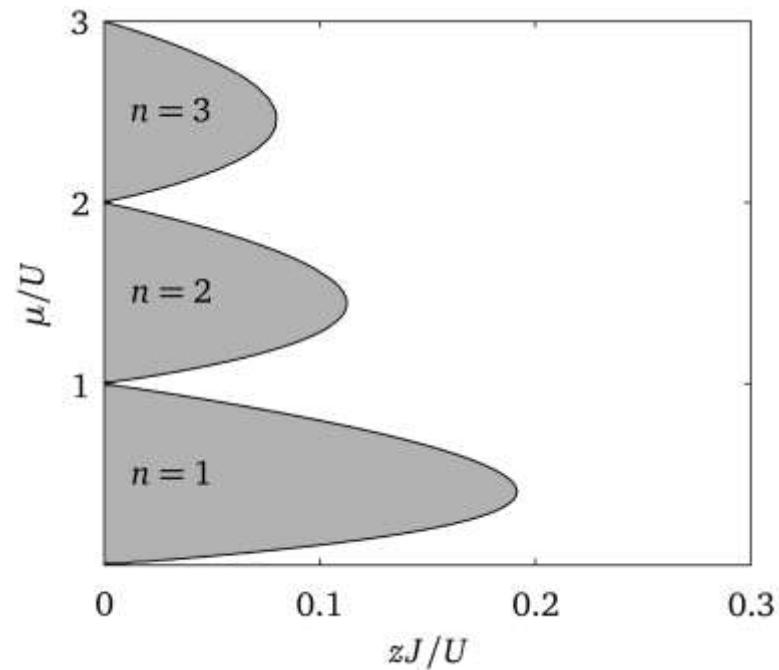
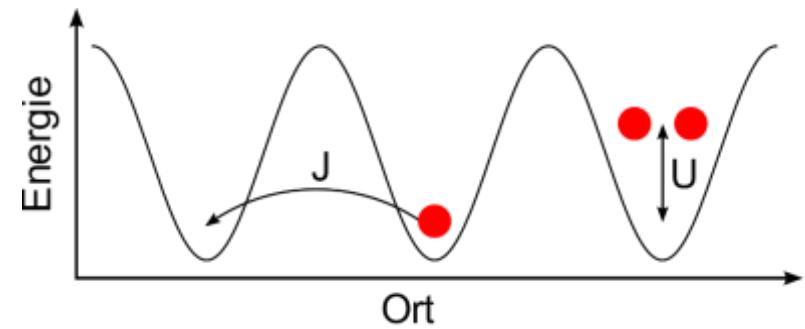
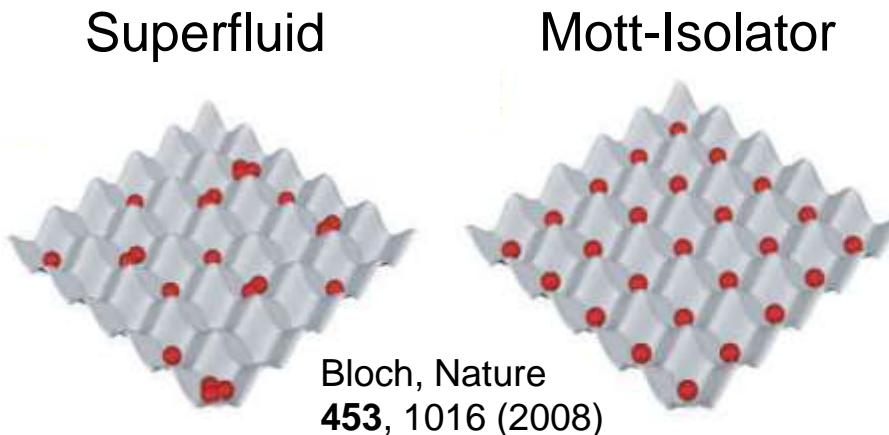


Bose-Hubbard model

- Bosons on a lattice

$$\hat{H} = -J \sum_{\langle ij \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{U}{2} \sum_{i=1}^M \hat{a}_i^\dagger \hat{a}_i^\dagger \hat{a}_i \hat{a}_i$$

- Phase transition



Bose-Hubbard parameters for 1D & 2D lattices

- Periodic potentials $V(\mathbf{r}+\mathbf{R})=V(\mathbf{r})$: Bloch/ Wannier-functions

$$\psi_{\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u_{\mathbf{k}}(\mathbf{r}) \quad u_{\mathbf{k}}(\mathbf{r} + \mathbf{R}) = u_{\mathbf{k}}(\mathbf{r})$$

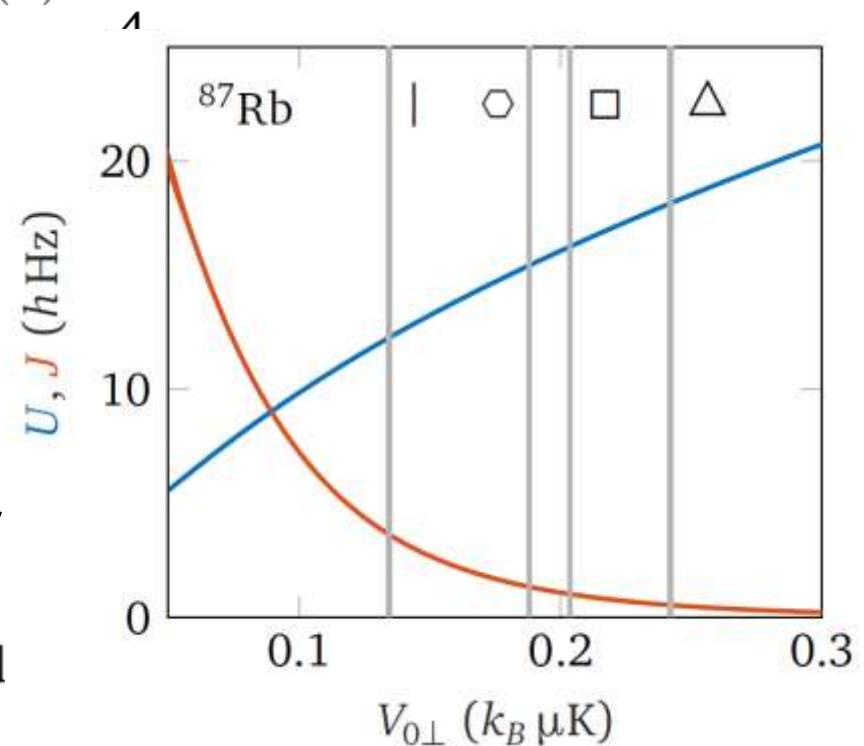
$$w_i(\mathbf{r}) = \frac{\Upsilon}{(2\pi)^D} \int_{BZ} e^{-i\mathbf{k}\cdot\mathbf{R}_i} \psi_{\mathbf{k}}(\mathbf{r}) d^D k$$

- Self-interaction

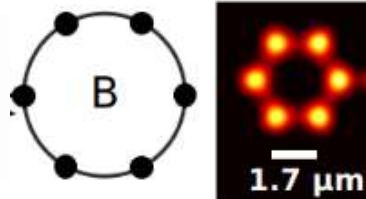
$$U = \frac{4\pi\hbar^2 a_s}{m} \int w_i^4(\mathbf{r}) d^3 r$$

- Hopping strength: nearest neighbour

$$J = \int w_i(\mathbf{r}) \left(-\frac{\hbar^2 \Delta}{2m} + V(\mathbf{r}) \right) w_j(\mathbf{r}) d$$



Loading atoms into the trap arrays



- Loading from BEC
 1. Preparation of an oblate BEC
 2. Adiabatic turn on of trap array
 3. Analogous to experiments in optical lattices

Sherson et al., Nature **467**, 68–72 (2010)

- Loading in the atomic limit
 1. Deterministic preparation of one atom/trap
Endres et al., Science **354**, 1024 (2016) & Barredo et al., Science **354**, 1021 (2016)
 2. Raman-sideband-cooling
Kaufman et al., PRX **2**, 041014 (2012) & Thompson et al., PRL **110**, 133001 (2013)
 3. Adiabatic decrease of trap depth

M. R. Sturm, et al. , Phys. Rev. A 97, 063608 (2018)

Optimal adiabatic parameter ramps $\gamma(t)$

- Conditions for adiabaticity

$$\max_{0 \leq t \leq \tau} \left| \frac{\alpha_{ij}(\gamma, \dot{\gamma})}{\hbar \omega_{ij}^2(\gamma)} \right|^2 \ll 1 \quad \forall j \neq i$$

- Lagrange-error functional

$$\mathcal{L}(\gamma, \dot{\gamma}) = \sum_{j \neq i} \left| \frac{\alpha_{ij}(\gamma, \dot{\gamma})}{\hbar \omega_{ij}^2(\gamma)} \right|^2 = \frac{1}{2} \mathcal{M}(\gamma) \dot{\gamma}^2$$

$$E_\infty[\gamma, \dot{\gamma}] = \max_{0 \leq t \leq \tau} \mathcal{L}(\gamma(t), \dot{\gamma}(t))$$

$$E_1[\gamma, \dot{\gamma}] = \frac{1}{\tau} \int_0^\tau \mathcal{L}(\gamma(t), \dot{\gamma}(t)) dt$$

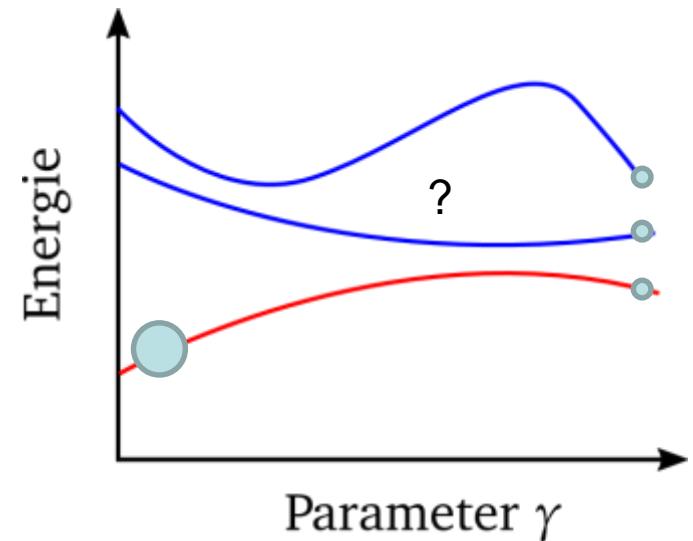
- Euler-Lagrange-equation for optimal ramp

$$\frac{d}{dt} \partial_{\dot{\gamma}} \mathcal{L} = \partial_\gamma \mathcal{L}$$

$$\hat{H}(\gamma) |i(\gamma)\rangle = E_i(\gamma) |i(\gamma)\rangle$$

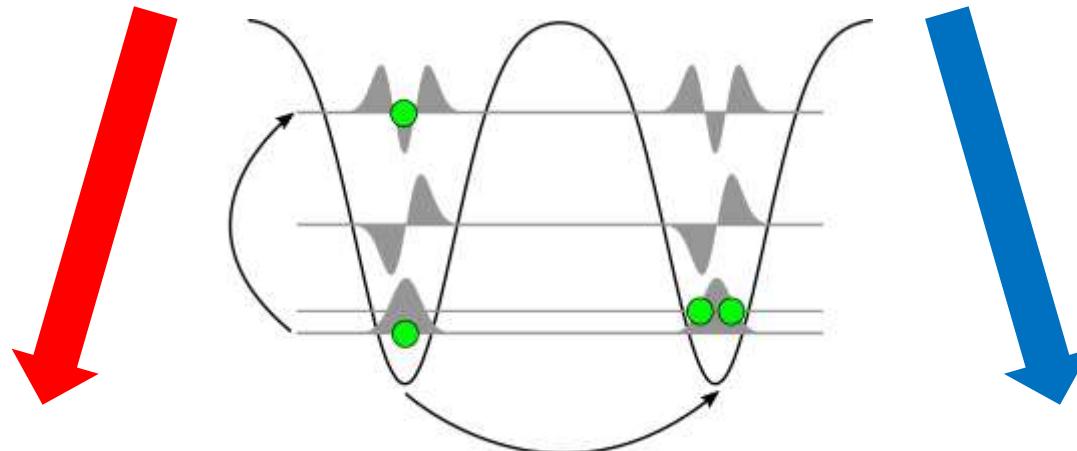
$$\omega_{ij}(\gamma) = \frac{E_j(\gamma) - E_i(\gamma)}{\hbar}$$

$$\alpha_{ij}(\gamma, \dot{\gamma}) = \langle j | \partial_t \hat{H} | i \rangle$$



Multiband-Bose-Hubbard model

$$\hat{H}(\gamma) = \sum_{n,i} \epsilon_i^n(\gamma) \hat{a}_i^{n\dagger} \hat{a}_i^n - \sum_{n,i \neq j} J_{ij}^n(\gamma) \hat{a}_i^{n\dagger} \hat{a}_j^n + \frac{1}{2} \sum_{nopq} \sum_{ijkl} U_{ijkl}^{nopq} \hat{a}_i^{n\dagger} \hat{a}_j^{o\dagger} \hat{a}_k^p \hat{a}_l^q$$



Atomic limit

- Local interband excitations
- Disregard tunneling
- one atom/trap → no self interaction

Mott-insulator

- Intraband excitations
- Disregard higher bands
- Interaction dominated ($U \gg J$)

Atomic limit

- Hamilton operator $\hat{H}_{\text{al}} = \sum_{n,i} \epsilon_i^n(\gamma) \hat{a}_i^{n\dagger} \hat{a}_i^n$
- Harmonic oscillator

$$\gamma = (\Omega_x, \Omega_y, \Omega_z)$$

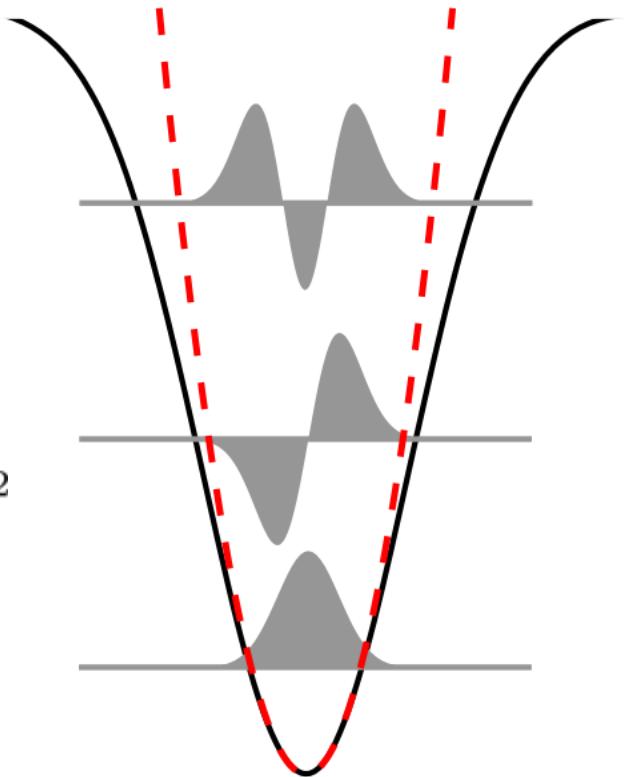
$$\epsilon_i^n(\gamma) = \sum_{l=x,y,z} \hbar \Omega_l \left(n_l + \frac{1}{2} \right)$$

$$w_i^n(\mathbf{r}) = \langle \mathbf{r} - \mathbf{R}_i | n_x n_y n_z \rangle$$

- Lagrange-functional $\mathcal{L}_{\text{al}}(\gamma, \dot{\gamma}) = \sum_{l=1}^3 \frac{1}{2} \frac{M}{(2\gamma_l)^4} \dot{\gamma}_l^2$

- Hyperbolic ramp shape

$$\gamma_l^{-1}(t) = \Omega_{l0}^{-1} + (\Omega_{l\tau}^{-1} - \Omega_{l0}^{-1}) \frac{t}{\tau}$$



Mott-insulator

- Hamilton operator $\hat{H}_{\text{bh}} = \epsilon N + \frac{U}{2} \sum_i \hat{a}_i^\dagger \hat{a}_i^\dagger \hat{a}_i \hat{a}_i - J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j$

- Perturbation theory in J/U

$$|g^0\rangle = \begin{array}{c} \text{wavy line} \\ \bullet \quad \bullet \quad \bullet \quad \bullet \end{array} \quad |e^0\rangle = \begin{array}{c} \text{wavy line} \\ \bullet \quad \bullet \bullet \quad \bullet \end{array}$$

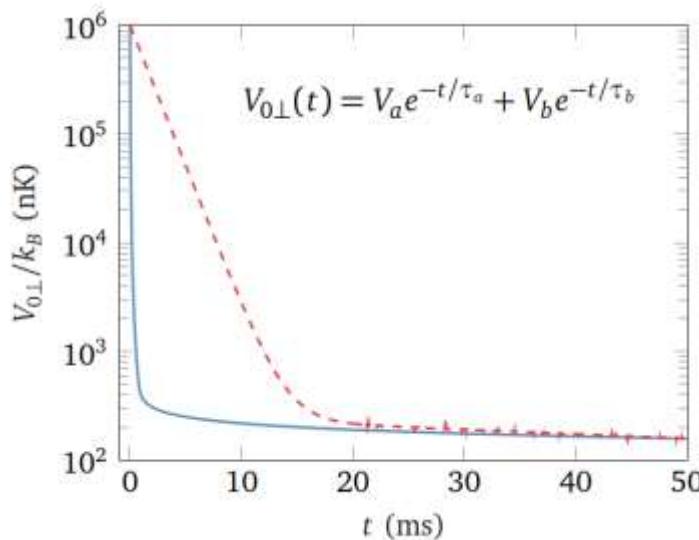
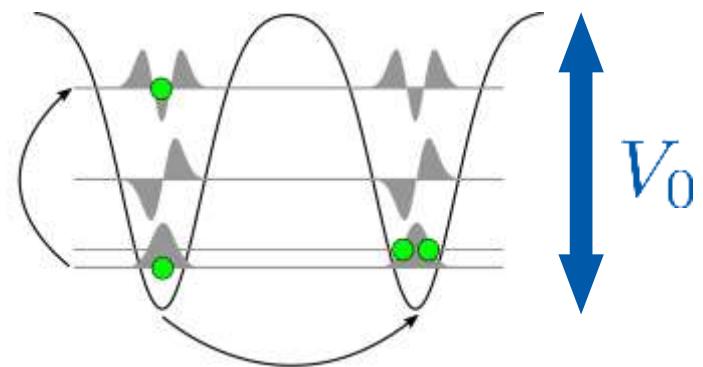
$$E_g^1 = U + \langle g^0 | \hat{H}_{\text{J}} | g^0 \rangle \quad |g^1\rangle = |g^0\rangle + \sum_e \frac{\langle e^0 | \hat{H}_{\text{J}} | g^0 \rangle}{E_g^0 - E_e^0} |e^0\rangle$$

- Lagrange-function $\gamma = (J, U)$

$$\mathcal{L}_{\text{bh}}(\gamma, \dot{\gamma}) = \frac{1}{2} \sum_{k,l=1}^2 \dot{\gamma}_k \mathcal{M}_{kl}(\gamma) \dot{\gamma}_l \quad \mathcal{M}(J, U) = \frac{4Mz\hbar^2}{U^6} \begin{pmatrix} U^2 & -JU \\ -JU & J^2 \end{pmatrix}$$

Optimal adiabatic parameter ramps

- Experimental control-parameter: $I_0 \propto V_0$
 $\Omega_l(V_0) \quad J(V_0) \quad U(V_0)$
- Additivity of errors
 $\mathcal{L}(V_0, \dot{V}_0) = \mathcal{L}_{\text{al}}(V_0, \dot{V}_0) + \mathcal{L}_{\text{bh}}(V_0, \dot{V}_0)$



Größe	Wert
Isotop	^{87}Rb
Fallenabstand	1.7 μm
Fallengröße	0.7 μm
Fallentiefe ($t = 0$)	$1 \text{ k}_B \text{ mK}$
Fallentiefe ($t = \tau$)	$158 \text{ k}_B \text{ nK}$

Constant adiabaticity

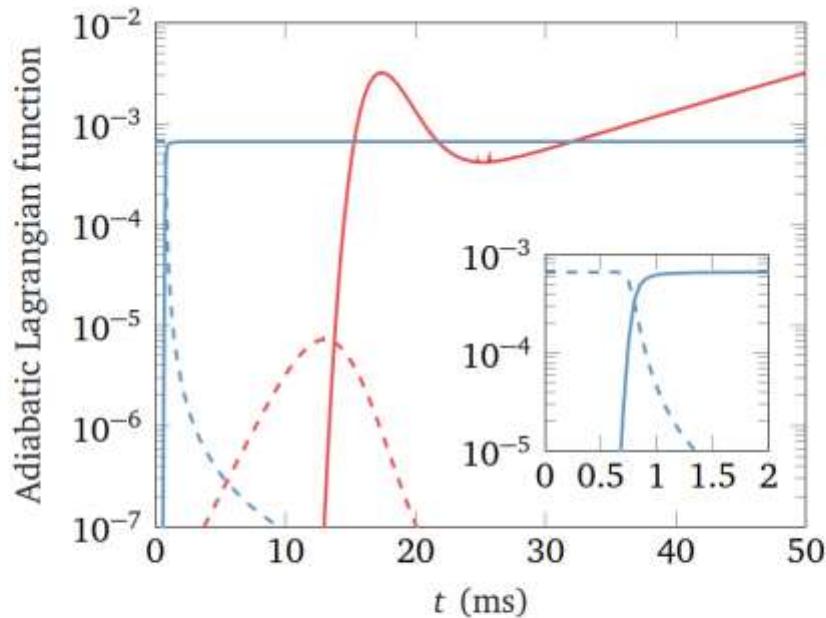


Figure 6.5: The adiabatic Lagrangian functions per site $\mathcal{L}_{\text{al}}/M$ (dashed) and $\mathcal{L}_{\text{bh}}/M$ (solid) are plotted versus time t for a bi-exponential (red) and an optimal adiabatic (blue) transfer sequence of duration $\tau = 50$ ms.

Fidelity and spontaneous light scattering

- Fidelity of time evolved state

$$i\hbar\partial_t|\psi(t)\rangle = \hat{H}_{\text{bh}}(t)|\psi(t)\rangle$$

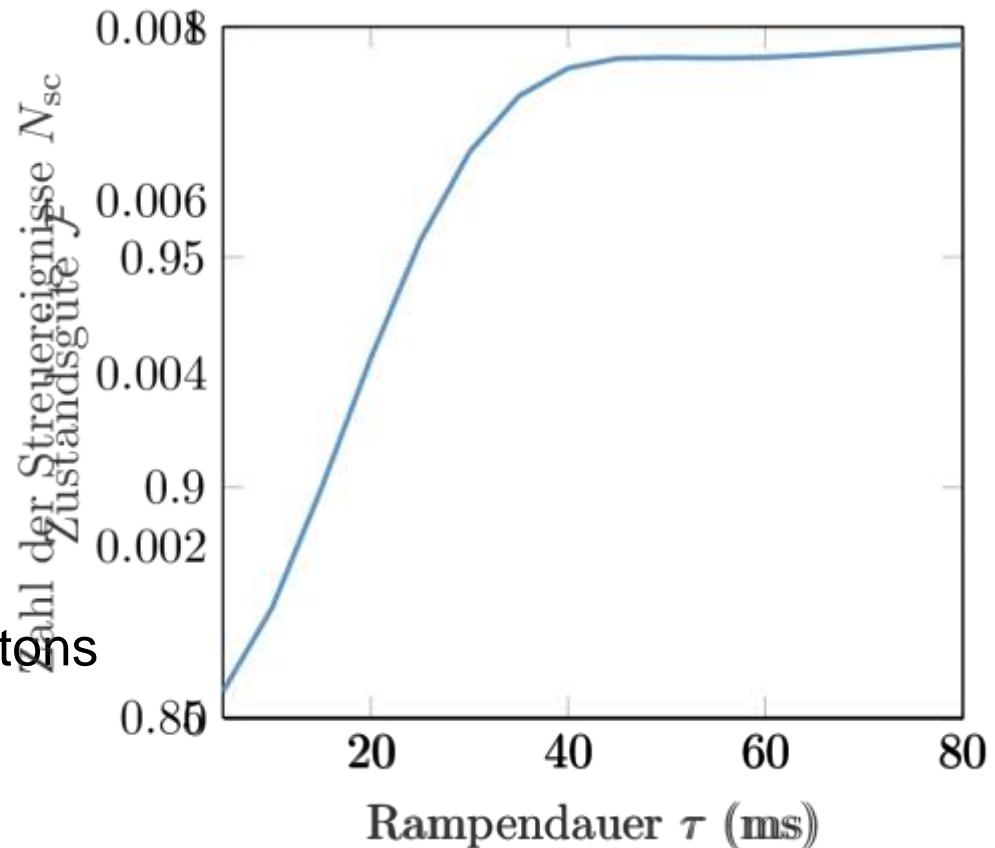
$$|\psi(0)\rangle = \begin{array}{c} \text{wavy line} \\ \bullet \quad \bullet \quad \bullet \quad \bullet \end{array}$$

$$\hat{H}_{\text{bh}}(\tau)|g\rangle = E_g(\tau)|g\rangle$$

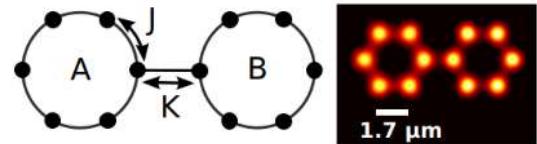
$$\mathcal{F} = |\langle g|\psi(\tau)\rangle|$$

- # spontaneous scattered photons

$$N_{\text{sc}} = \int_0^{\tau} \Gamma_{\text{sc}}(t) \, dt$$



Coupled Josephson rings



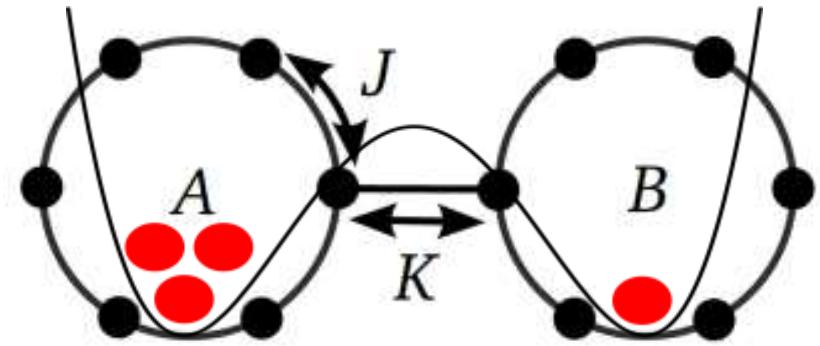
- Extension of double well by rings
cf. q-dots, artificial atoms
- Weak coupling between rings

$$K \ll J$$

- Tunnel dynamics after sudden turn on

$$N_A(0) = N \quad N_B(0) = 0$$

$$\zeta(t) = \frac{N_A(t) - N_B(t)}{N}$$



$$\hat{H} = \hat{H}_A + \hat{H}_B + \hat{H}_{AB}$$

$$\hat{H}_A = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{U}{2} \sum_{i=1}^M \hat{a}_i^\dagger \hat{a}_i^\dagger \hat{a}_i \hat{a}_i$$

$$\hat{H}_B = -J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j + \frac{U}{2} \sum_{i=1}^M \hat{b}_i^\dagger \hat{b}_i^\dagger \hat{b}_i \hat{b}_i$$

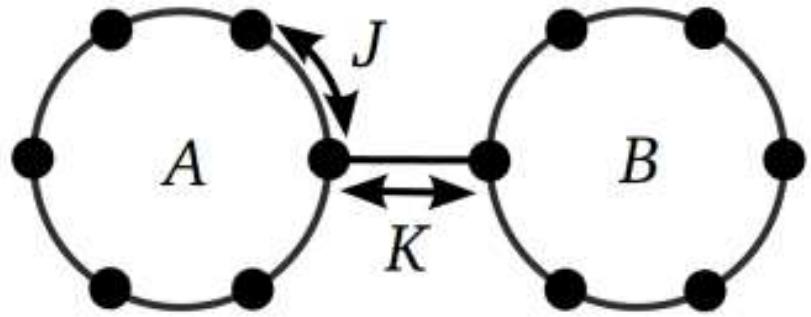
$$\hat{H}_{AB} = -K (\hat{a}_1^\dagger \hat{b}_1 + \hat{b}_1^\dagger \hat{a}_1)$$

Two mode approximation

- Vortex-states quasi momentum k_l
for $K=0$ & $U=0$

$$\hat{\alpha}_\ell = \sum_{m=1}^M \frac{e^{-ik_\ell md}}{\sqrt{M}} \hat{a}_m \quad k_\ell = \frac{2\pi\ell}{Md}$$

$$\hat{\beta}_\ell = \sum_{m=1}^M \frac{e^{-ik_\ell md}}{\sqrt{M}} \hat{b}_m \quad -\frac{M}{2} < \ell \leq \frac{M}{2}$$



- Projection to groundstate manifold

$$\hat{H}_0 = \hat{\mathcal{P}} \hat{H} \hat{\mathcal{P}} = -k(\hat{\alpha}_0^\dagger \hat{\beta}_0 + \hat{\beta}_0^\dagger \hat{\alpha}_0) + \frac{u}{2}(\hat{\alpha}_0^\dagger \hat{\alpha}_0^\dagger \hat{\alpha}_0 \hat{\alpha}_0 + \hat{\beta}_0^\dagger \hat{\beta}_0^\dagger \hat{\beta}_0 \hat{\beta}_0)$$

- Effective double-well for $J \gg U, K$

$$k = K/M$$

$$u = U/M$$

[2018_04_24_PhD_Sturm_eingereicht.pdf - Verknüpfung \(2\).lnk](#)

Josephson-Oscillations: sudden turn on

- No selfinteration ($U=0$) $K(t)$

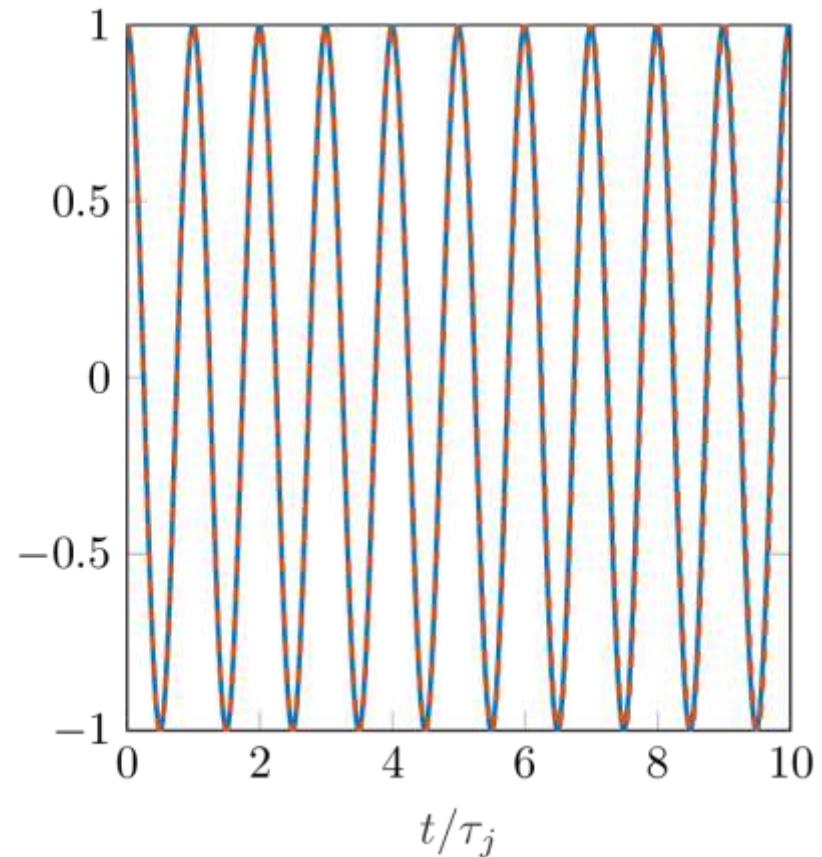
- Collective oscillations

$$\zeta(t) = \cos(2\pi t/\tau_j) \quad \tau_j = \frac{h}{2k}$$

- Parameters

$$K = 0.1 J \quad M = 6 \quad N = 4$$

- See JJ's in Supra conductors



Semifluxons in Superconductivity and Cold Atomic Gases", R. Walser, E. Goldobin, O. Crasser, D. Koelle, R. Kleiner and W. P. Schleich, [N. J. Phys. 10, 45020 \(2008\)](#)

Collapse and revival

- Weak selfinteraction ($U < K$)

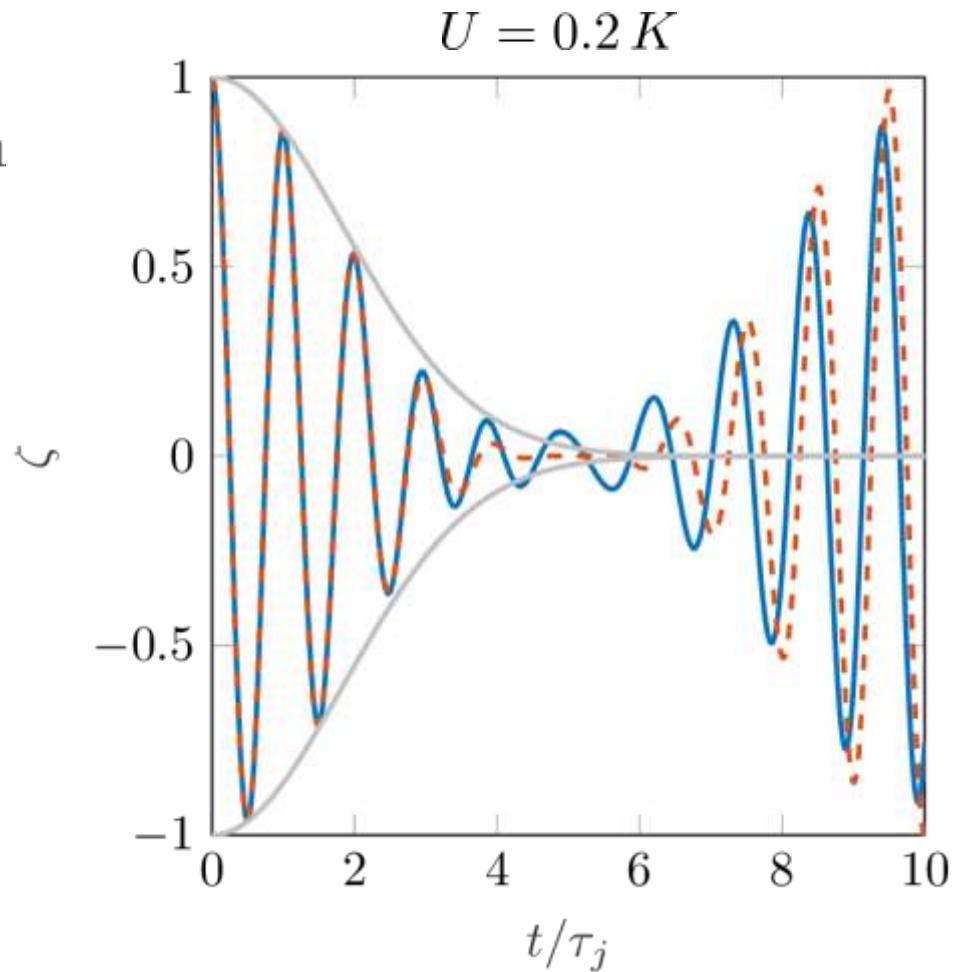
$$\begin{aligned}\zeta(t) &= \cos(2\pi t/\tau_j) \cos(\pi t/\tau_r)^{N-1} \\ &\approx \cos(2\pi t/\tau_j) e^{-t^2/\tau_c^2}\end{aligned}$$

- Gaußscher Zerfall

$$\tau_c = \frac{\hbar}{u} \sqrt{\frac{2}{\pi^2(N-1)}}$$

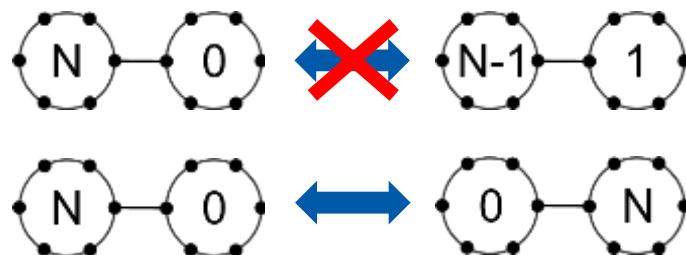
- Wiederkehr

$$\tau_r = \frac{\hbar}{u}$$



Self-Trapping

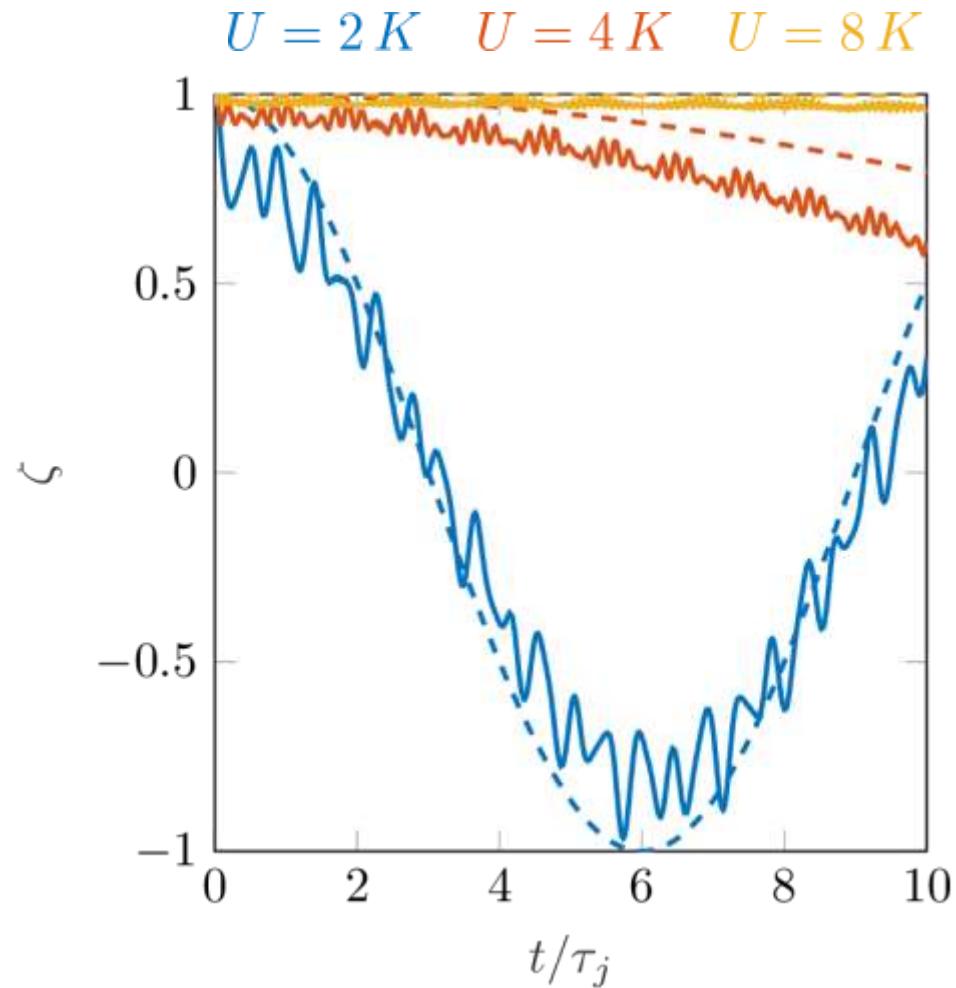
- Strong selfinteraction ($U > K$)



- Adiabatic elimination
effective 2level system

$$\tau_{\text{eff}} = \frac{(N-1)! h u^{N-1}}{2N k^N}$$

- „Self-Trapping“ experimentally
realizable time scales

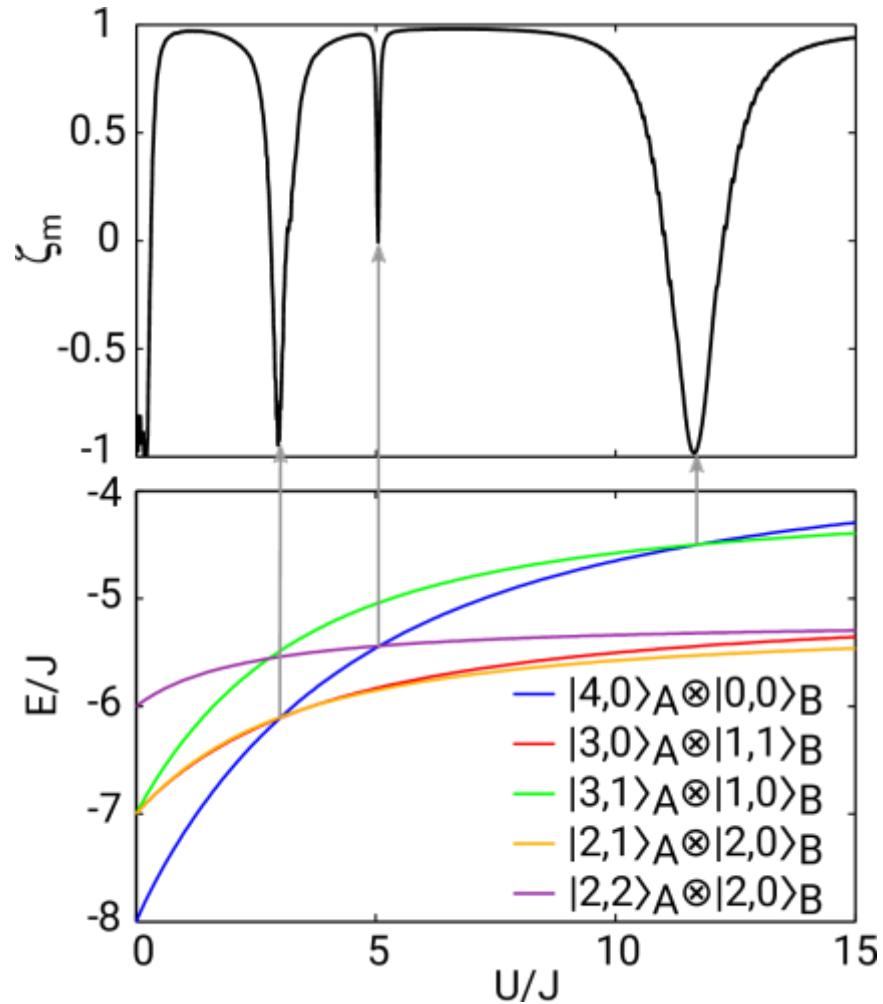


Many-body tunneling resonances

- Amplitude of population oscillations

$$\zeta_m = \min_{0 \leq t \leq 10\tau_j} \zeta(t)$$

- Tunneling resonances in self-trapping regime
- Coupling to higher k -states
- Spectroscopic information

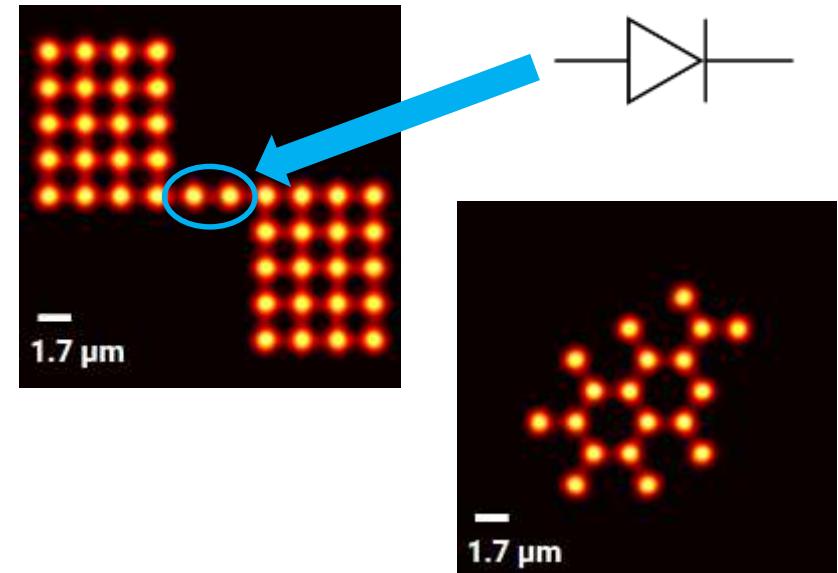


Weitere Anwendungen

- Atomtronics

Seaman et al., PRA **75**, 023615 (2007)

Amico et al., NJP **19**, 020201 (2017)



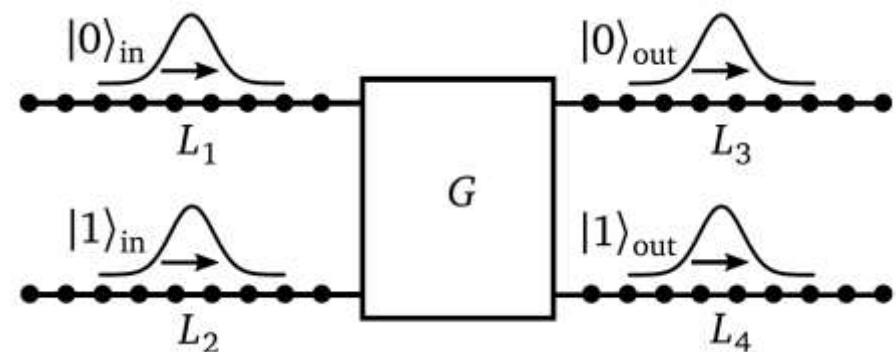
- Moleküle

Lühmann et al., PRX **5**, 031016 (2015)

- Unordnung und Defekte

- Quantum-Walks

Childs et al., Science **339**, 791 (2013)



Summary

- Pulsed Bragg beam-splitters
- Alternative to optical lattices

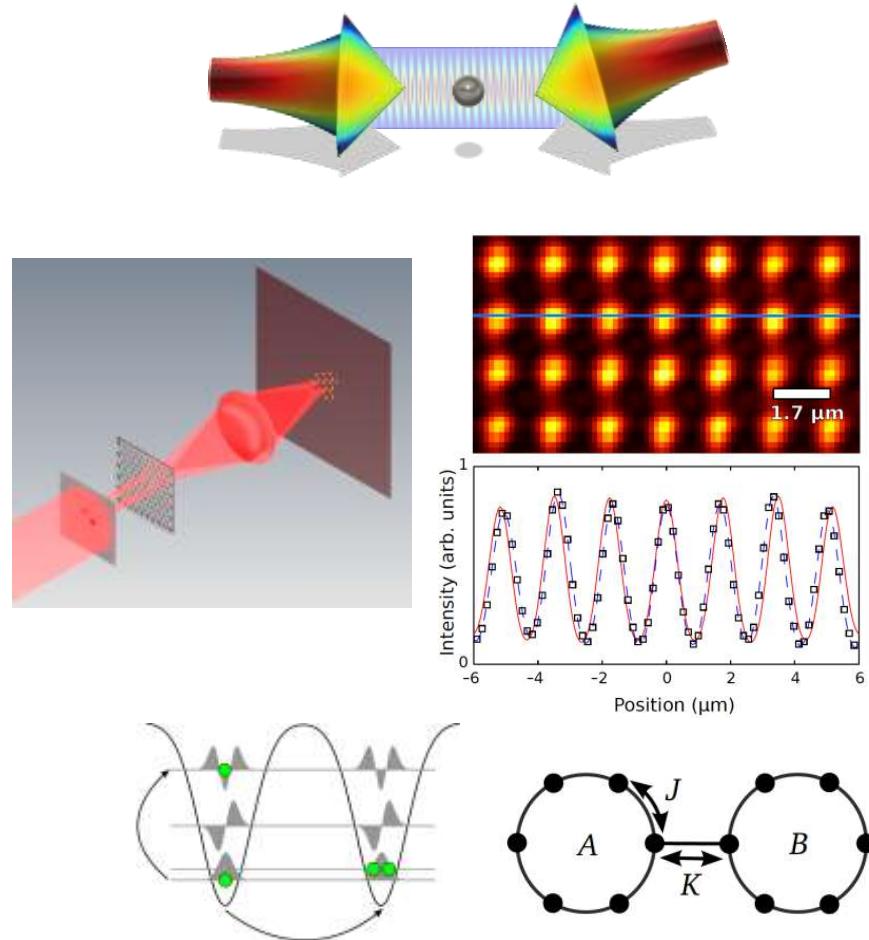
Sturm et al., PRA 95, 603625 (2017)

- Optical potential

- Bose-Hubbard parameter
- Mott state preparation

M. R. Sturm, et al. , Phys. Rev. A 97, 063608 (2018)

- Application: rotation sensing
Josephson-rings



Thank you very much for the attention!



<http://www.iap.tu-darmstadt.de/tqd/>