Fractionalization of angular momentum in Atomtronic ring circuits

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Atomtronics

Atomtronics is an emerging field in *quantum technology* seeking to realize atomic circuits exploiting ultra-cold atoms manipulated in micro-magnetic or laser-generated micro-optical traps or circuits.

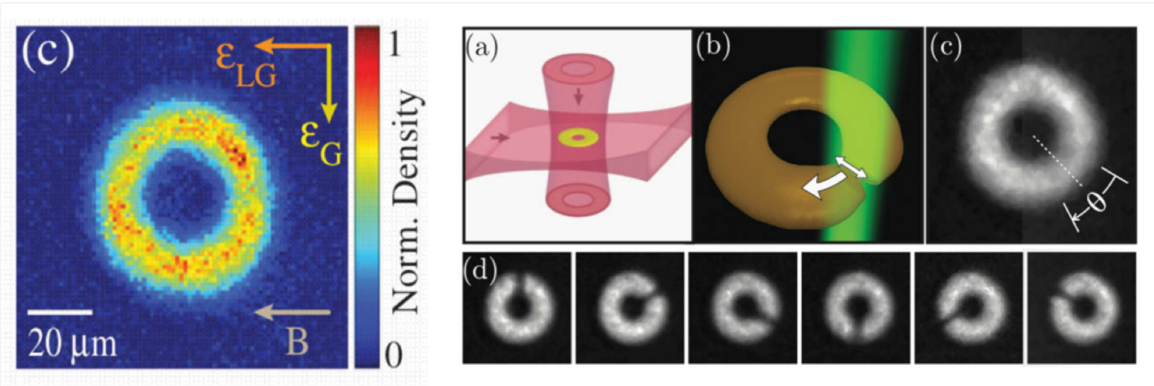
Cold atom circuits: 'Quantum many particles in ring-shaped potentials', Amico, Osterloh, Cataliotti, PRL 2005.
"Atomtronics: Ultracold-atom analogs of electronic devices.", Seaman, Kramer, Anderson , Holland, PRA (2007).

Some goals

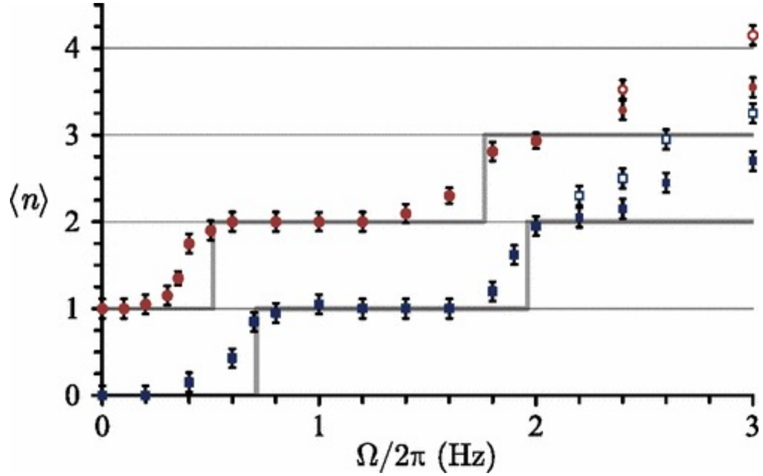
- Enlarge the scope of cold atoms quantum simulators (currents).
- Many-body physics (exotic quantum phases of matter: topological order..)
- Bridging mesoscopic and cold-atoms physics.
- Insights in foundational aspects of quantum science.
- New quantum devices.
- Quantum sensing.
- Hybrid systems.
- ……..

'Atomtronics: from basic research in many-body physics to applications for quantum technologies, Amico, Anderson, Boshier, Brantut, Minguzzi, Kwek, von Klitzing, arXiv:2107.08561
'Roadmap on Atomtronics: state of the art and perspectives', Amico, Boshier, Birkl, Kwek, Miniatura, Minguzzi et al, AVS Quantum Science 2021, arXiv:2008.04439.
'Roadmap on quantum optical systems', Amico, Boshier 'Atomtronics' J.Optics 2016
New J. Phys. Focus on 'Atomtronics enabled quantum technology' 2015, Amico, Birkl, Boshier, Kwek Eds.

Bosons with repulsive interaction



G. Campbell, W. Phillips, C. Clark and co-workers@NIST, (2013–2015)



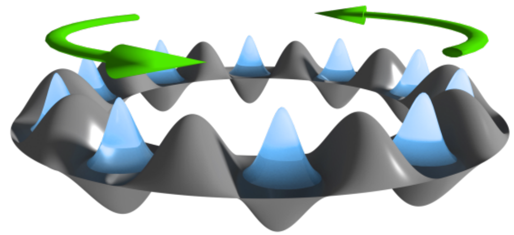
K. Wright et al, PRL 2013

Angular momentum quantized to integer values

Outline

- Bosons with attractive interaction
- Fermions with repulsive interaction
- Fermions with attractive interaction

Energy bands in mesoscopic rings: Leggett Theorem

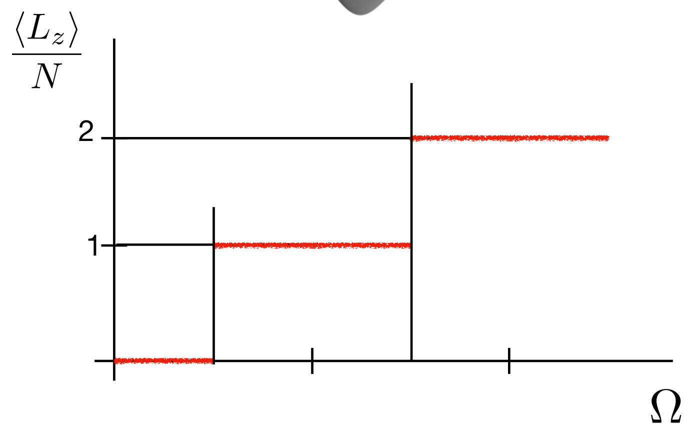


Granular Nanoelectronics, Edited by D.K. Ferry
Plenum Press, New York, 1991

DEPHASING AND NON-DEPHASING COLLISIONS IN NANOSTRUCTURES

A. J. Leggett

$$H(x_1 \ldots x_N) = \sum_i \frac{1}{2m}[p_i - eA(x_i)]^2 + \sum_{i,j} U(x_i - x_j) + \sum_i V(x_i)$$

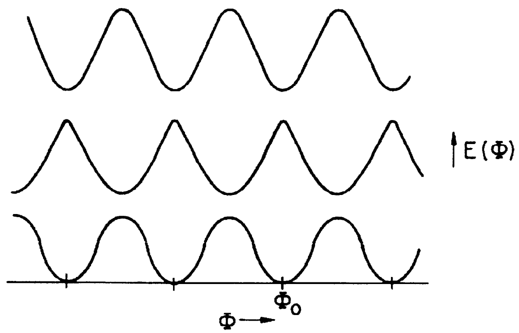


Center of mass and relative coordinate decouple

‘Bloch theorem’:

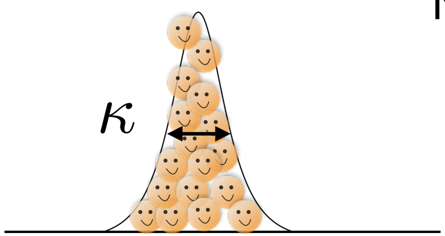
$$E_{GS} = \frac{N}{2M}(\ell - \Omega)^2 + E_{int}$$

$$I \sim \langle L_z \rangle = \frac{\partial E_{GS}}{\partial \Omega}$$



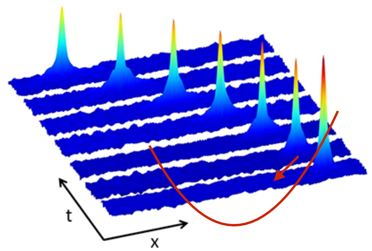
Periodicity of persistent current is fixed by the effective magnetic flux. It does not depend on the interaction/local disorder.

Bosons with attractive interaction



NLSE: Bright solitons

$$i\hbar\psi_t + \frac{\hbar^2}{2m}\psi_{xx} - V(v,t)\psi - g|\psi|^2\psi = 0$$



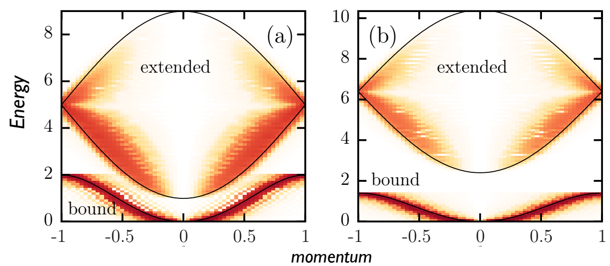
S. Gardiner group (2012), experiment with ultra-cold Rubidium85

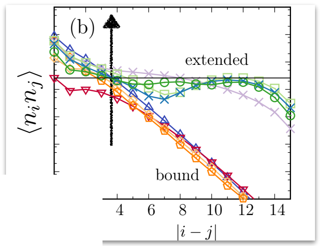
$$\psi(x,t) \sim sech(\frac{x-vt}{\kappa})\exp\left[i\frac{mv}{\hbar}z - \frac{i}{2\hbar}\left(mv^2 - \hbar^2\kappa^2/2\right)t\right]$$

Quantum many-body effects

$$H = -\sum_l |U|n_l(n_l-1) - t(a_l^\dagger a_{l+1} + h.c)$$

$$S(k,\omega) = \sum_{\alpha\neq 0}\sum_r e^{-ikr}|\langle\alpha|n_r|0\rangle|^2\delta(\omega-\omega_\alpha)$$





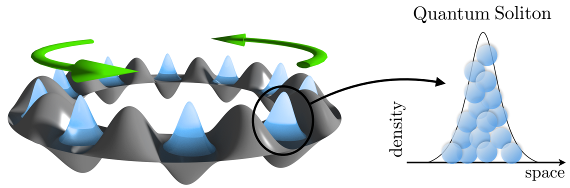
P. Naldesi, J. Polo Gomez, B. Malomed, M. Olshanii, A Minguzzi, and L. Amico, Phys. Rev. Lett. 122, 053001 (2019); P. Naldesi, J. Polo-Gomez, V. Dunjko, M. Olshanii,H. Perrin, L. Amico and A. Minguzzi, arXiv: 1901.09398

Angular momentum fractionalisation for attractive bosons

P. Naldesi, J. Polo-Gomez, V. Dunjko, M. Olshanii, H. Perrin, L. Amico and A. Minguzzi, SciPost 2022

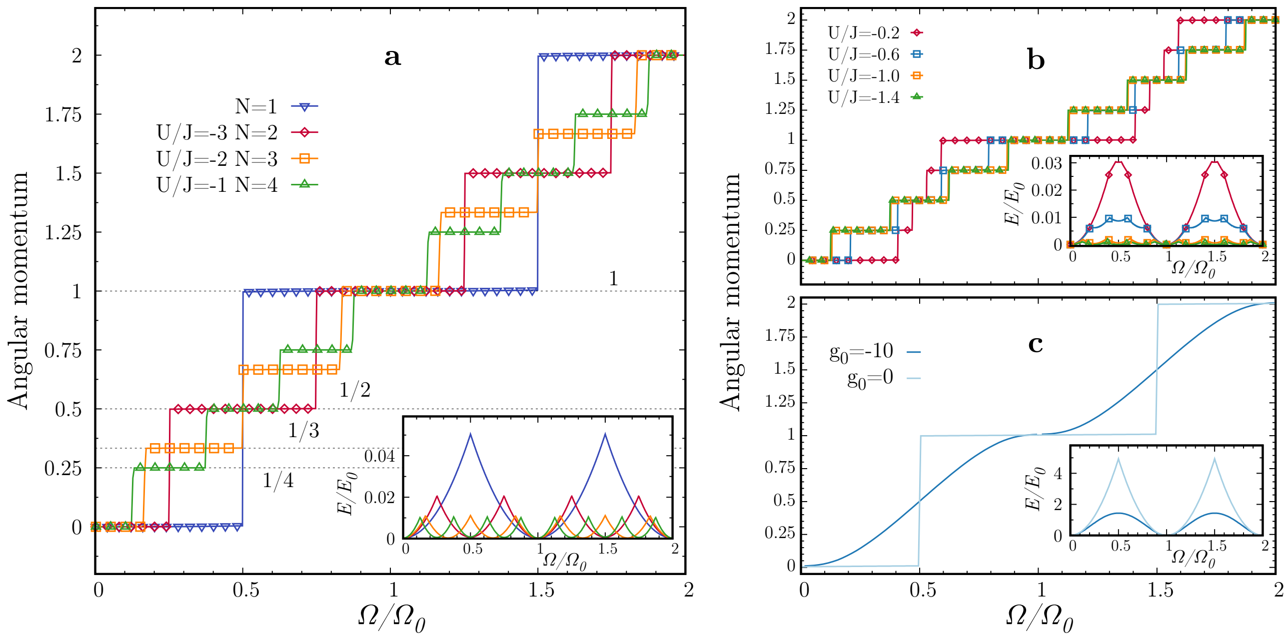
Attracting bosons in ring lattices pierced by synthetic gauge field

$$H = \sum_l U n_l(n_l - 1) - t(e^{i\Phi} a_l^\dagger a_{l+1} + h.c)$$



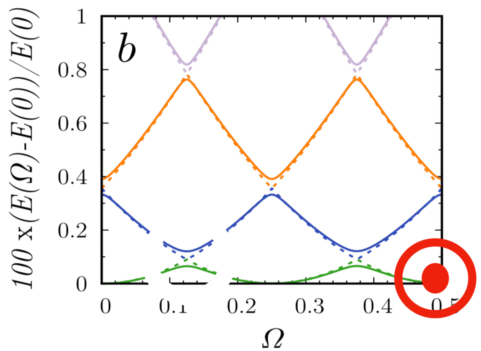
Exact results (Lieb-Lieneger).

$$E_{GS} = \frac{1}{2Nm}(\ell - N\Omega)^2 - \frac{N(N^2-1)g^2}{12}$$

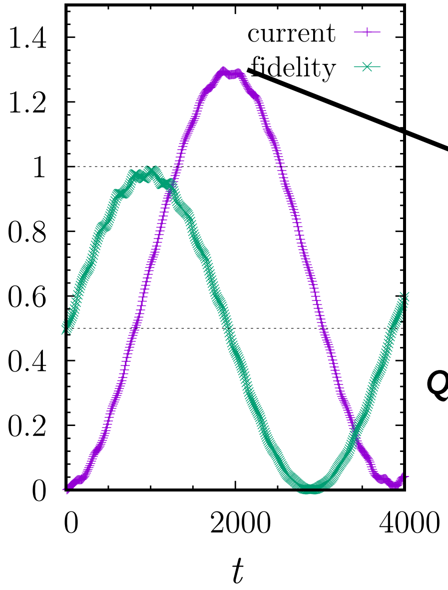


Quantum solitons atomtronic interference device

J Pol.o, P. Naldesi, A. Minguzzi, L.Amico, Quantum Sci. Technol. 7, 015015 (2022)



State preparation: Quench at Ω=1/2



$$|\psi\rangle_{NOON} = \frac{1}{\sqrt{2}}\left(|L_z = 0\rangle + |L_z = N\rangle\right)$$

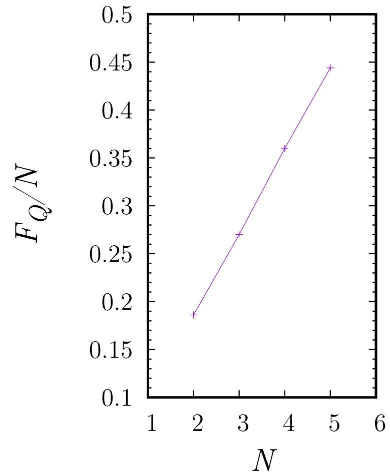
for pure states:

$$F[L_z] = \langle\psi|L_z^2|\rangle\psi - \langle\psi|L_z|\rangle\psi^2$$

Quantum Cramér-Rao bound: $$\delta\Omega \geq \frac{1}{\sqrt{F[L_z]}}$$

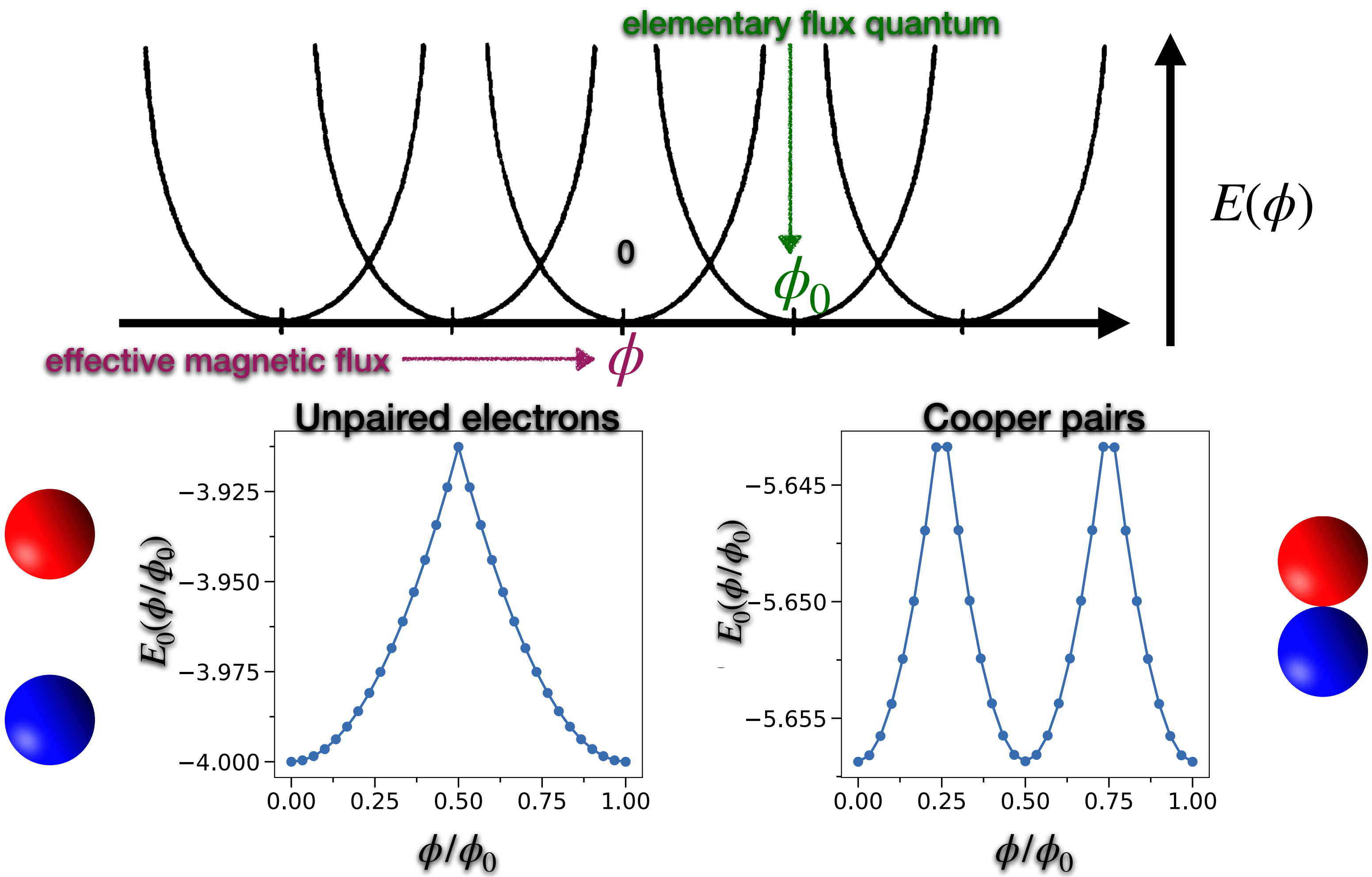
$$F[L_z] \propto \hbar N|0 - N| \quad \longrightarrow \quad \delta\Omega \geq \frac{1}{N}$$

(Standard quantum limit: $F[L_z] \propto N$, $\delta\Omega \geq \frac{1}{\sqrt{N}}$)



[Braunstein and Caves, 1994; Helstrom, 1967]

Leggett Theorem

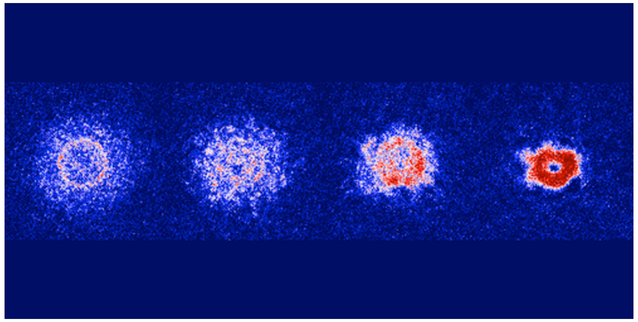


A. Leggett in C.W.J. Beenakker *et al.*, Granular Nanoelectronics (Plenum Press, New York, 1991) p. 359

Byers, Yang, PRL (1961); Onsager, PRL (1961)

Fermions

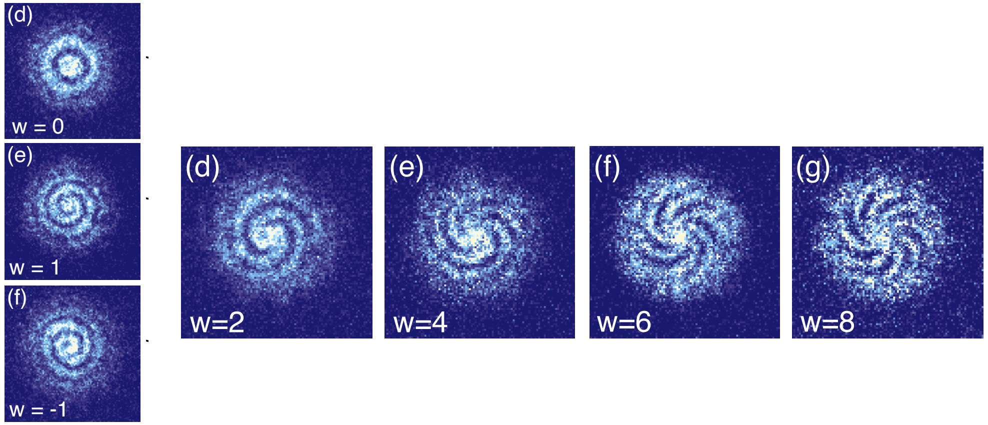
‘Persistent Currents in Rings of Ultracold Fermionic Atoms’



Yanping Cai, Daniel G. Allman, Parth Sabharwal, and Kevin C. Wright

Phys. Rev. Lett. 128, 150401 – Published 12 April 2022

‘Imprinting persistent currents in tunable fermionic rings’



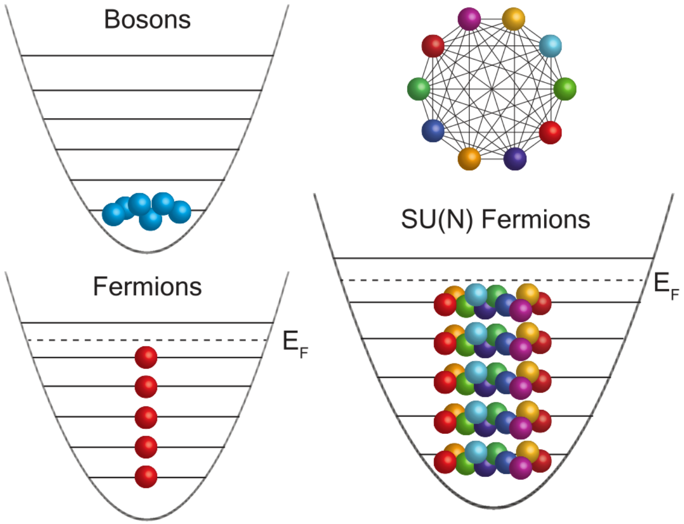
G. Del Pace, K. Xhani, A. Muzi Falconi, M. Fedrizzi, N. Grani, D. Hernandez Rajkov, M. Inguscio, F. Scazza, W. J. Kwon, G. Roati, arXiv:2204.0654,

[Submitted on 13 Apr 2022]

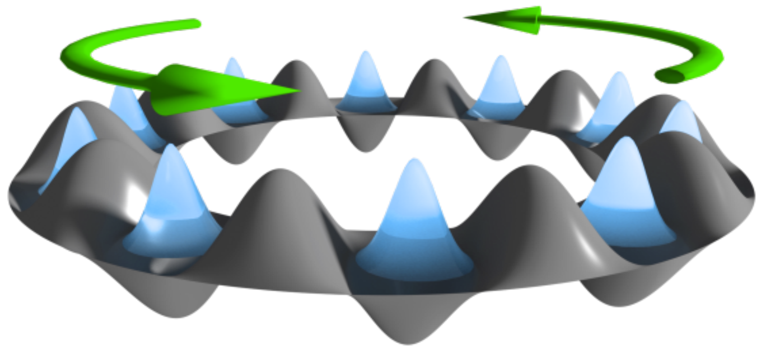
SU(*N*) ring circuits.

“Persistent currents of SU(*N*) fermions”, W. J. Chetcuti, T. Haug, L.-C Kwek and L. Amico, arXiv:2011.00916 (2020)

Unlike indistinguishable fermions, SU(N) fermions can have N particles per level.



“Thermodynamics of a deeply degenerate SU(*N*)-symmetric gas”, Sonderhouse *et al.*, Nature Physics (2020)

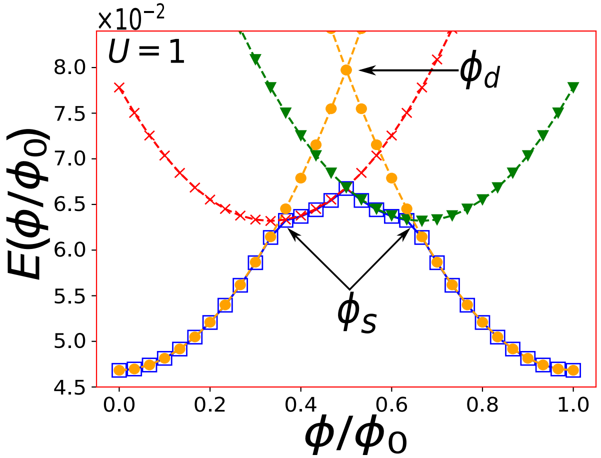


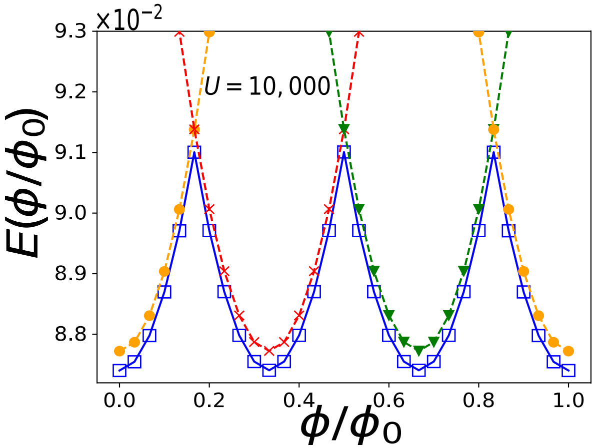
$$H_{\mathrm{SU}(N)} = -t\sum_{j=1}^{L}\sum_{\alpha=1}^{N}\left(e^{\imath\frac{2\pi\phi}{L}}c^{\dagger}_{j,\alpha}c_{j+1,\alpha} + \mathrm{h.c.}\right) + \frac{U_{\alpha,\beta}}{2}\sum_{j}^{L} n_j(n_j-1)$$

$$n_j = \sum_{\alpha} c^{\dagger}_{j,\alpha}c_{j,\alpha}$$ local number operator

SU(N) fermions with repulsive interactions

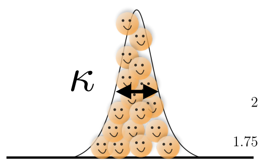
"Persistent currents of SU(*N*) fermions", W. J. Chetcuti, T. Haug, L.-C Kwek and L. Amico, arXiv:2011.00916 (2020), SciPost 2022



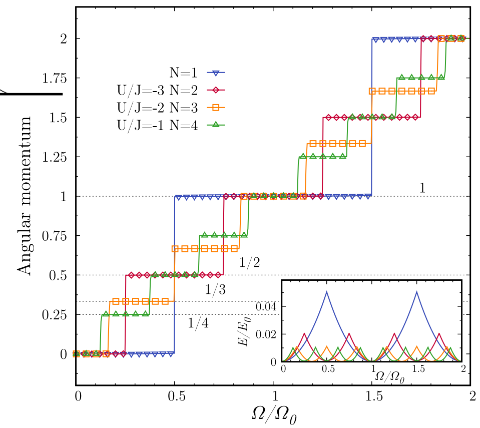


Effective flux quantum is fractionalized: The period is given by $\dfrac{\phi_0}{N_p}$

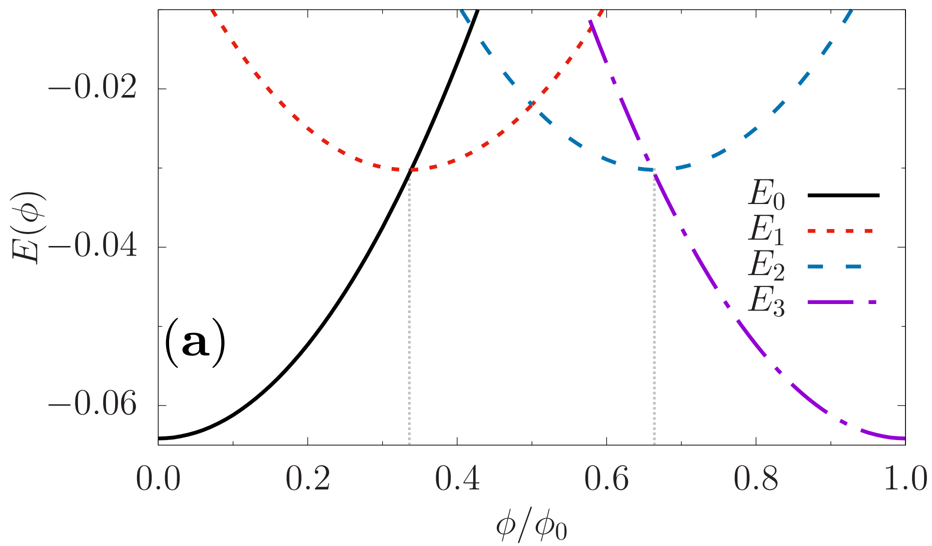
Reduction of the effective flux quantum indicates a form of '*attraction from repulsion*': spin (colour) degrees of freedom are the responsible for such phenomenon.

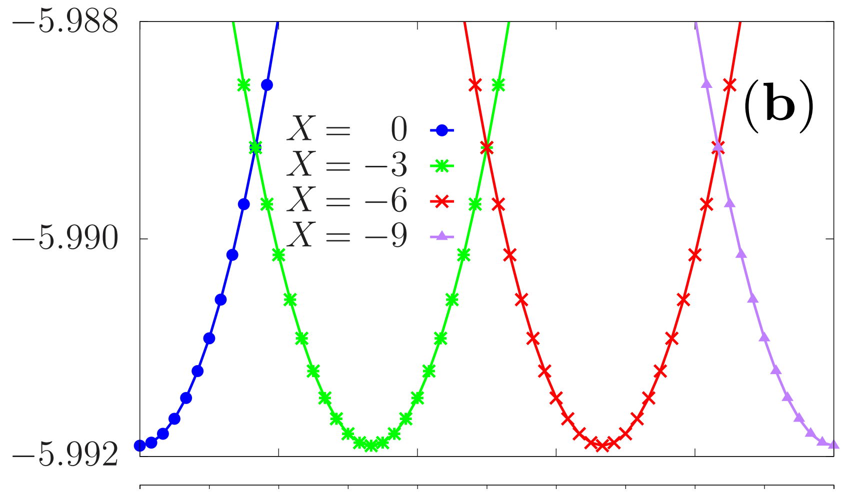


Bosons



SU(N) fermions with attractive interactions





Effective flux quantum is fractionalized: The period is given by $\dfrac{\phi_0}{N}$

See Wayne Chetcuti's talk

Conclusions

- Persistent current can diagnostic basic features of many-body systems.

  - For attracting bosons, the persistent current is periodic with 1/#particles reflecting the formation of 'quantum bright solitons'.

  - For SU(N) repulsive fermions, the persistent current is quantized with 1/#particles reflecting the attraction from repulsion induced by spin correlations

  - For attractive SU(N) repulsive fermions, is quantized with 1/#components

Read-out of angular momentum in fermionic rings: See Juan Polo's talk