

Creating a massive minimal uncertainty wavepacket using Gross-Pitaevskii breathers

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Rice U: De Luo, Yi Jin, Jason H. V. Nguyen,

Sehyun Park, Eva Jin, Ricardo Espinoza

Randall G. Hulet

Atomtronics
2022



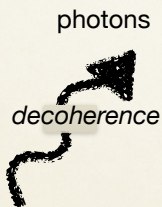
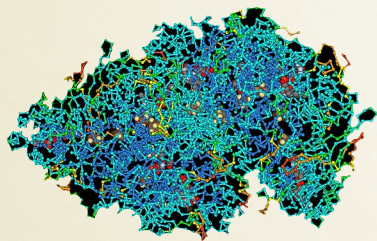




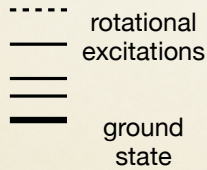
Why solitons
are good for
macroscopic
coherence

Quantum coherence with macroscopic objects

large molecules



$$\text{gap to excitations} \sim \frac{\hbar^2}{I} \sim \frac{\hbar^2 n^{2/3}}{m} \frac{1}{N^{5/3}}$$



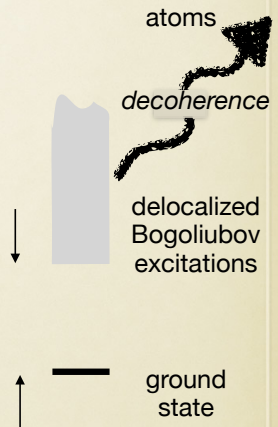
Anton Zeilinger, Markus Arndt, ...

integrable bosonic solitons



$$\text{gap to excitations} \sim \frac{mg}{\hbar^2} N^2 \lesssim \hbar\omega_{\perp}$$

no localized excitations



This proposal

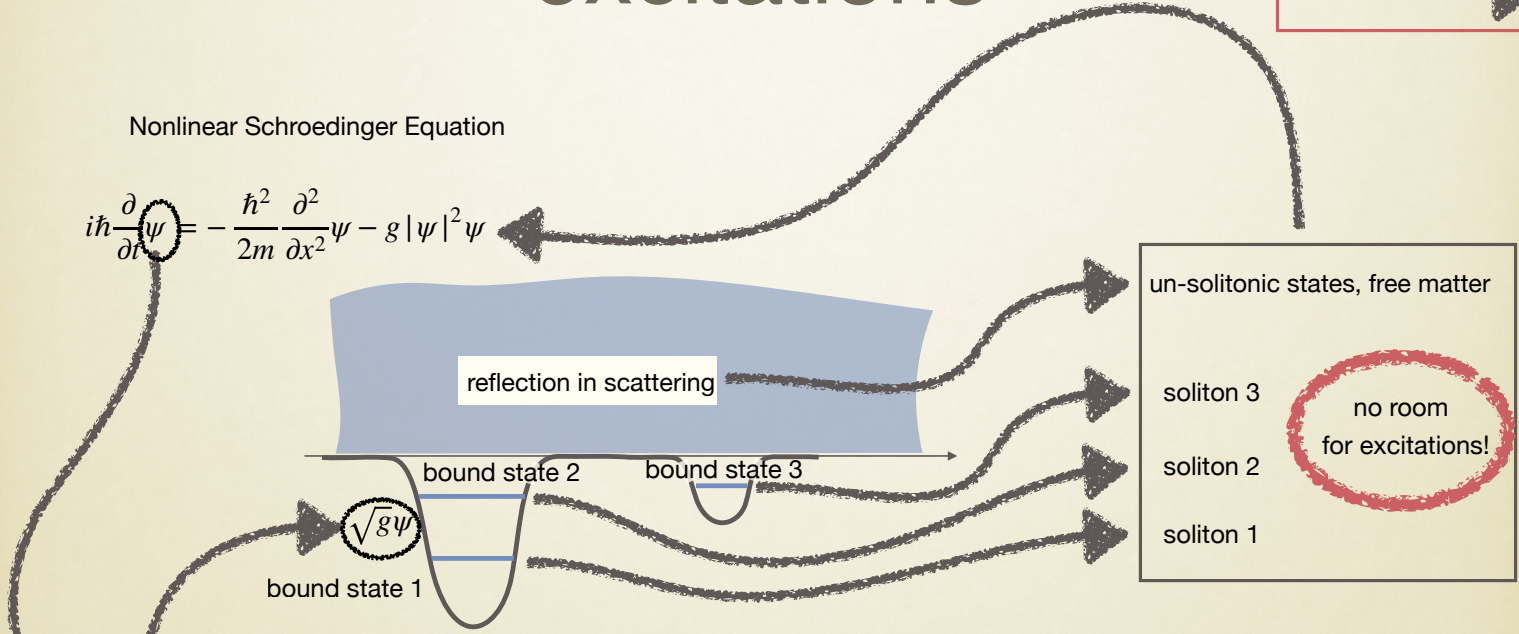
Why don't
solitons have
localized
excitations?

Why solitons have no localized excitations

Inverse Scattering loop
→

Nonlinear Schroedinger Equation

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} - g |\psi|^2 \psi$$



Lax L "potential" $\sqrt{g}\psi$
spectrum is preserved over time
(Lax phenomenon)

Poor taxonomy,
only two types
of states

Q: But the center-of-mass is still hot, isn't it?

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A: True, but we will use the relative motion of two solitons

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Q: Still hot

A: We will create a relatively cold soliton-soliton pair in an implosion

Q: But the center-of-mass is still hot, isn't it?

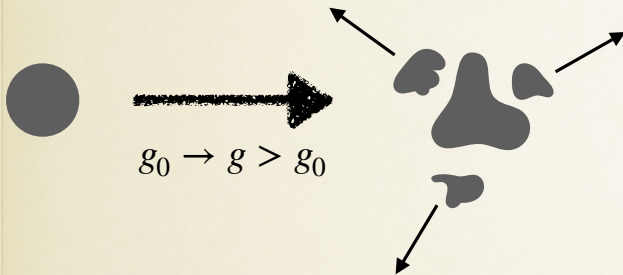
A: True, but we will use the relative motion of two solitons

Q: Still hot

A: We will create a relatively cold soliton-soliton pair in an implosion

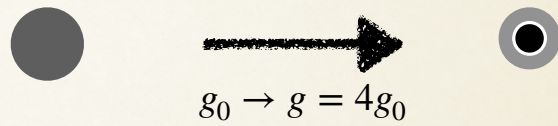
Q: Isn't it dangerous?

generic cluster
classical



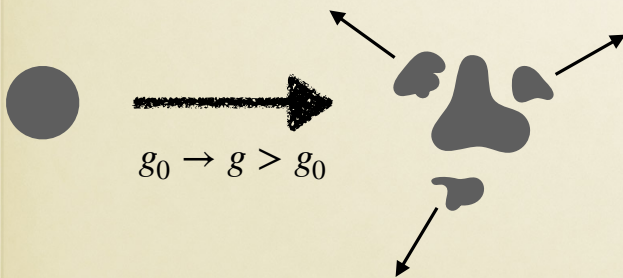
localized excitations;
dissociation with
multiple products

integrable cluster (soliton)
classical



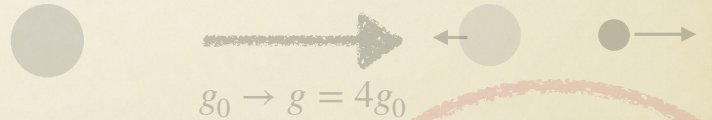
no localized excitations;
dissociation with
two products in the
same location,
with zero relative velocity

generic cluster
quantum



localized excitations;
dissociation with
multiple products,
can't track all \Rightarrow decoherence

integrable cluster (soliton)
quantum



quantum macroscopic
relative shift and drift

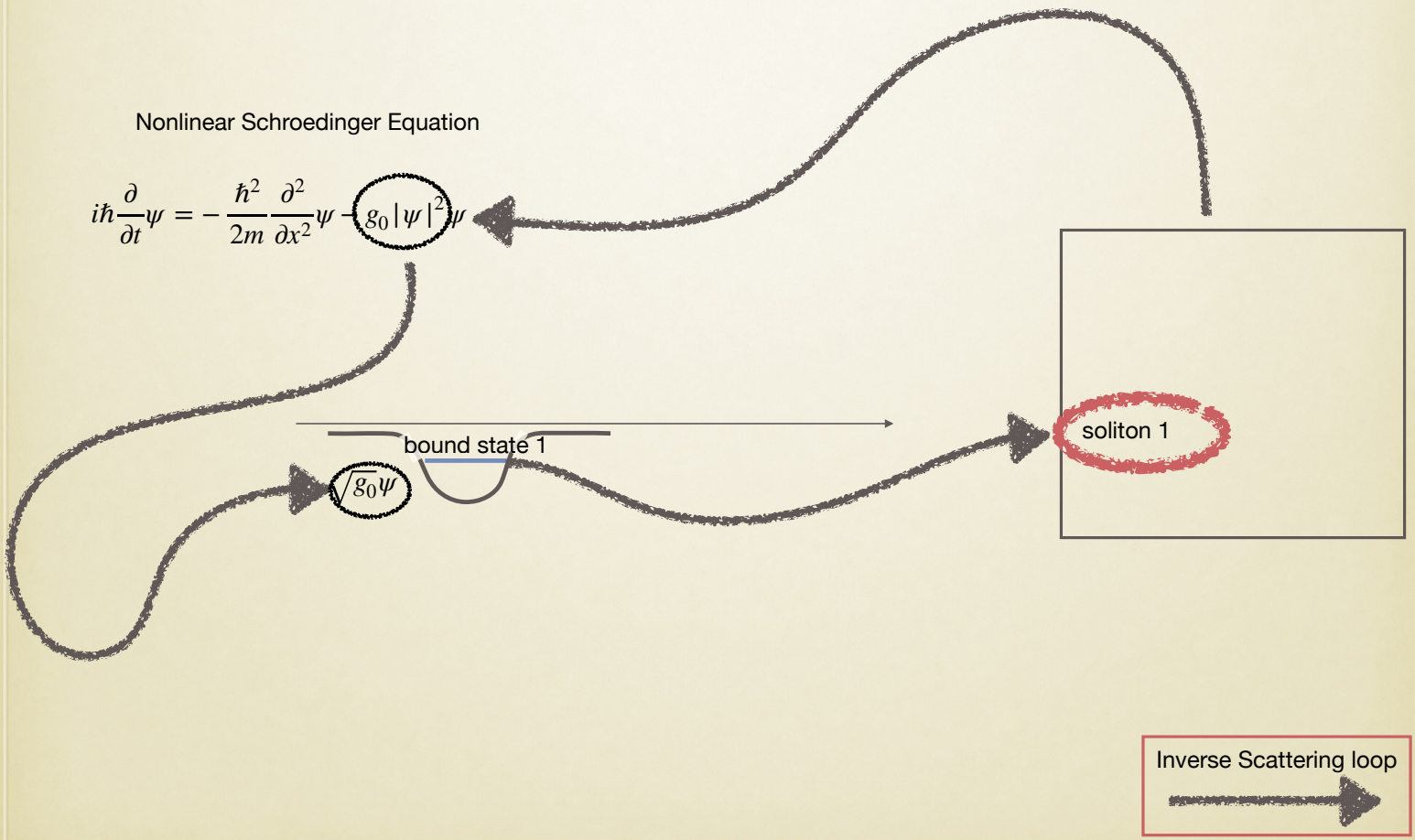
no localized excitations;
dissociation with only
two products,
in the same location,
with zero relative velocity,
on average \rightarrow
no decoherence

Why do solitons
produce a controllable
number of products
in an implosion?

Why solitons produce a controllable number of products in an implosion

Nonlinear Schroedinger Equation

$$i\hbar \frac{\partial}{\partial t} \psi = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi - g_0 |\psi|^2 \psi$$

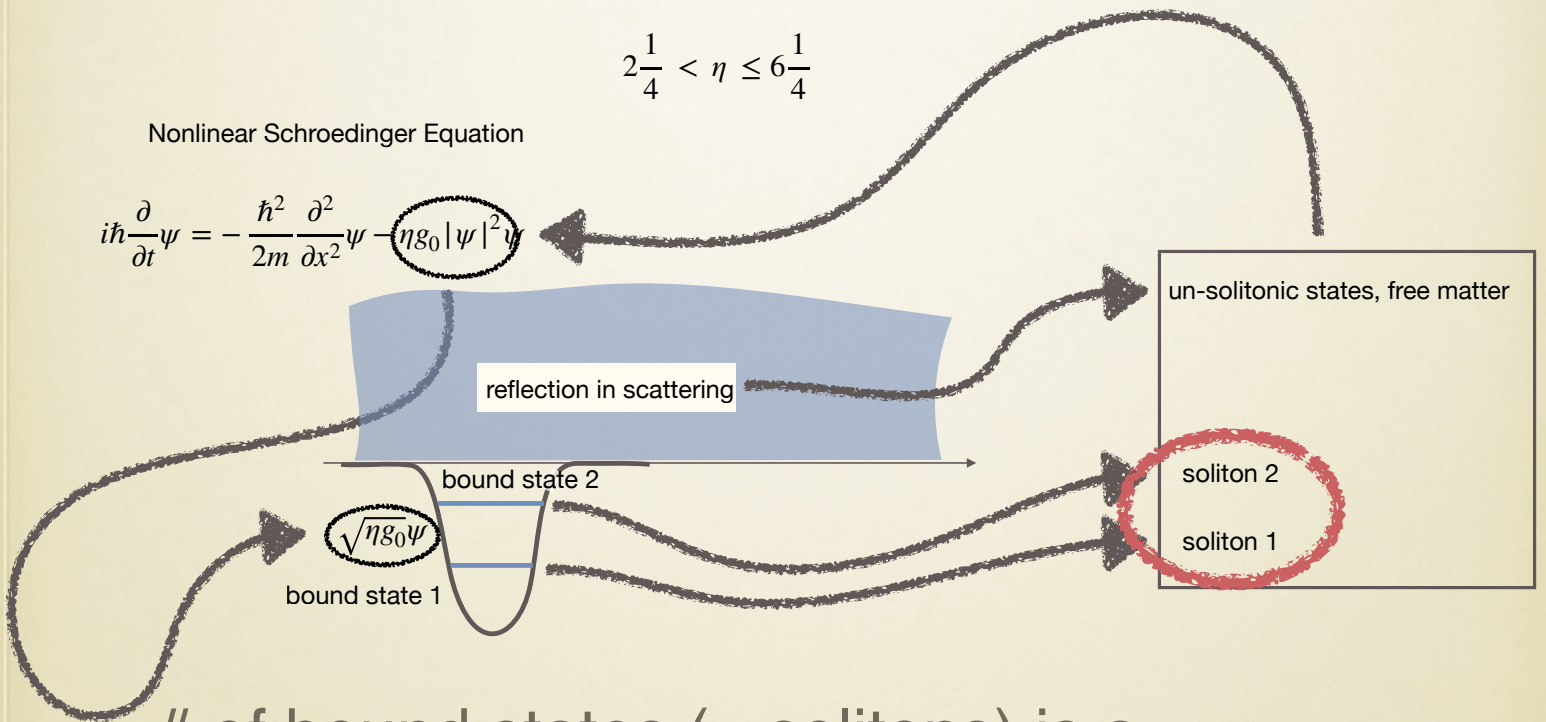


Why solitons produce a controllable number of products in an implosion

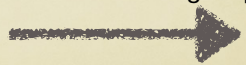
$$2\frac{1}{4} < \eta \leq 6\frac{1}{4}$$

Nonlinear Schroedinger Equation

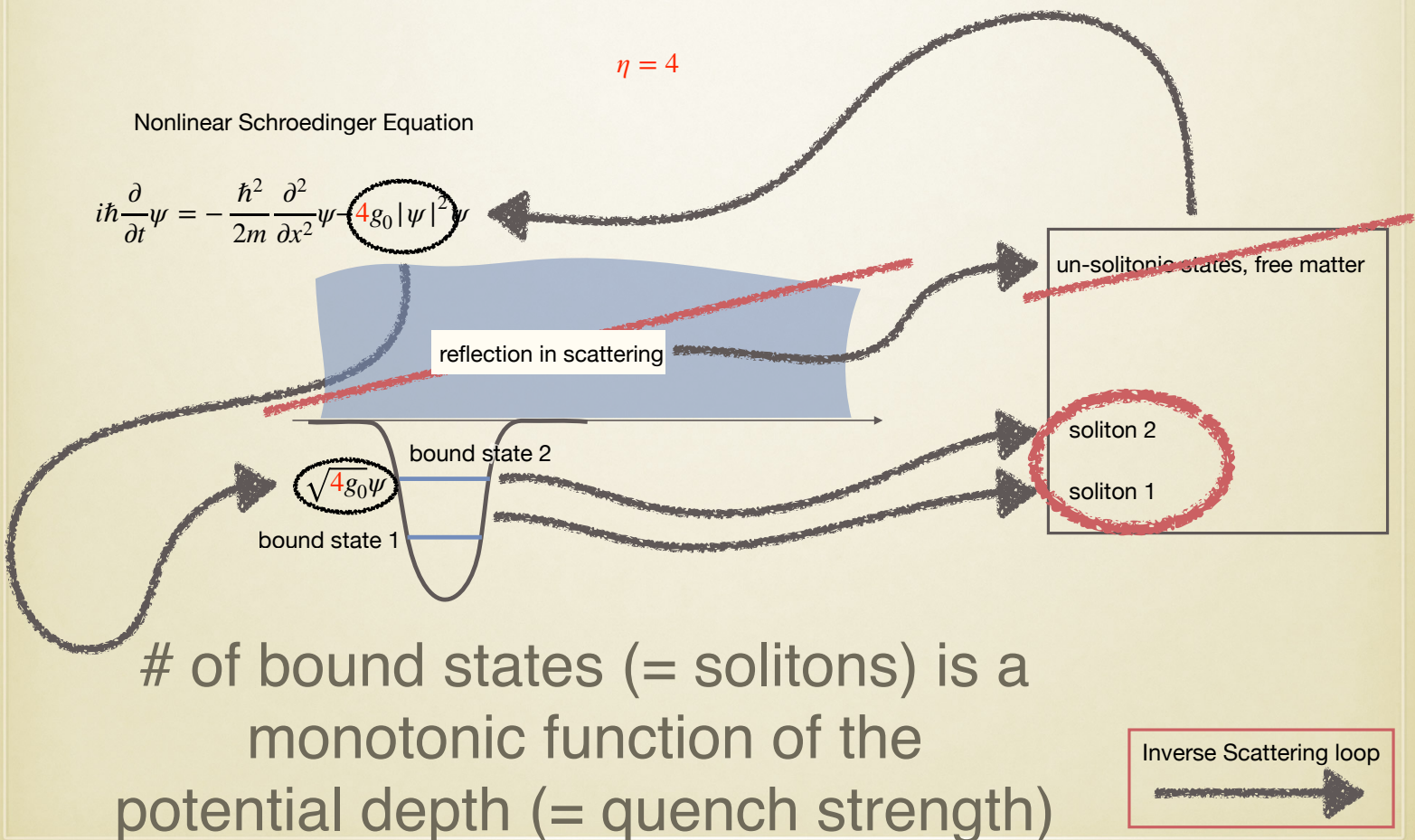
$$i\hbar \frac{\partial}{\partial t} \psi = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi - \eta g_0 |\psi|^2 \psi$$



of bound states (= solitons) is a monotonic function of the potential depth (= quench strength)

Inverse Scattering loop


Why solitons produce a controllable number of products in an implosion

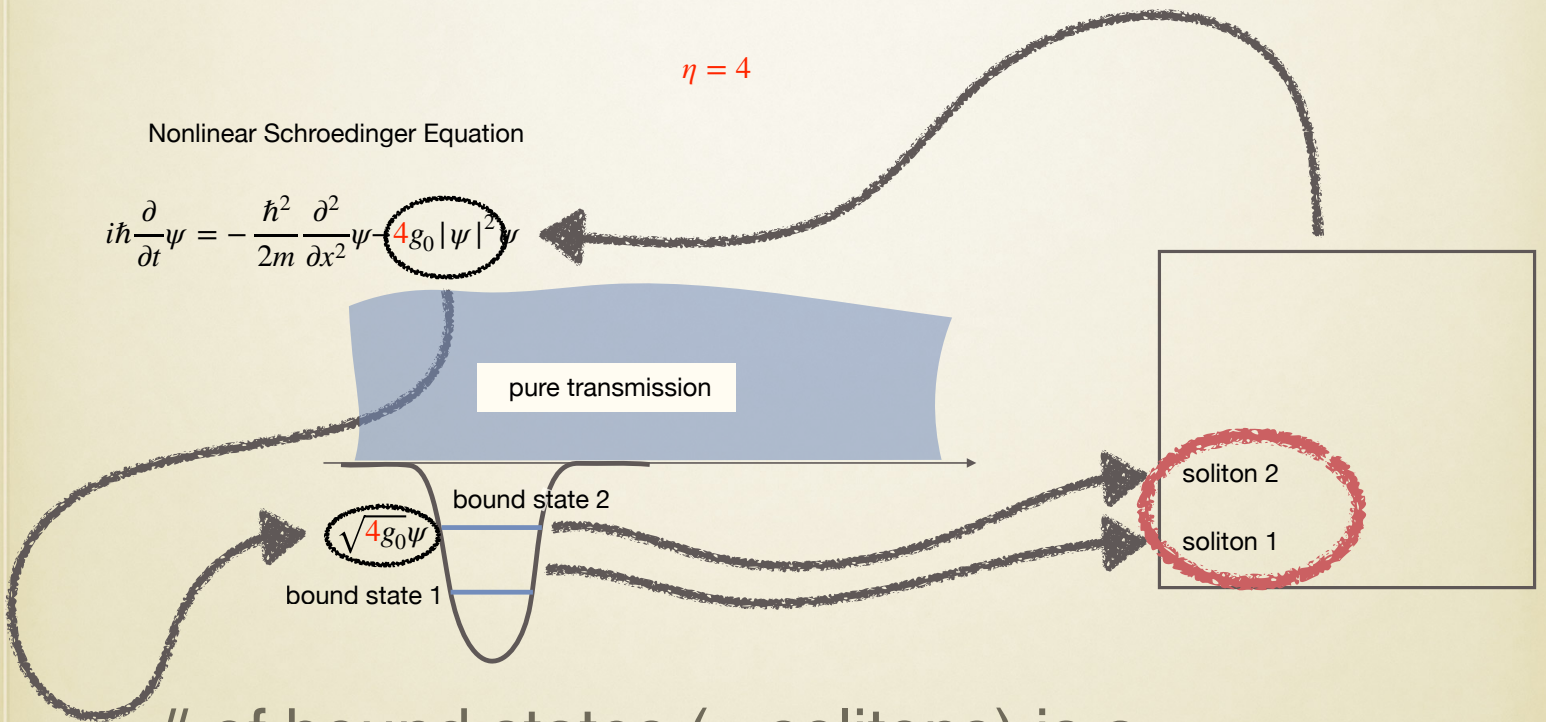


Why solitons produce a controllable number of products in an implosion

$\eta = 4$

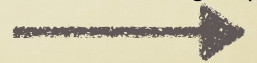
Nonlinear Schroedinger Equation

$$i\hbar \frac{\partial}{\partial t} \psi = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi - 4g_0 |\psi|^2 \psi$$



of bound states (= solitons) is a monotonic function of the potential depth (= quench strength)

Inverse Scattering loop



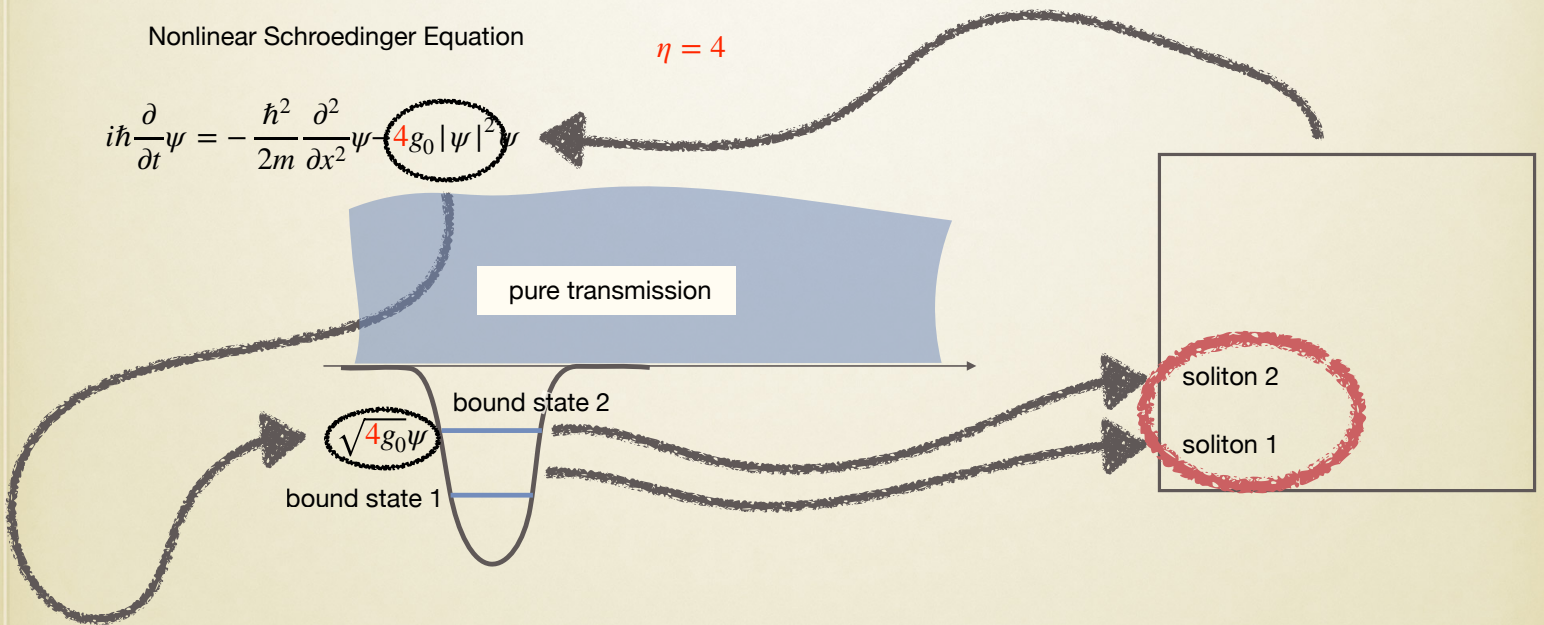
Why don't solitons
produce non-
solitonic states in an
4-fold coupling
quench?

Why solitons do not produce non-solitonic states in an $\eta = 4$ -fold coupling quench

Nonlinear Schroedinger Equation

$$i\hbar \frac{\partial}{\partial t} \psi = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi - 4g_0 |\psi|^2 \psi$$

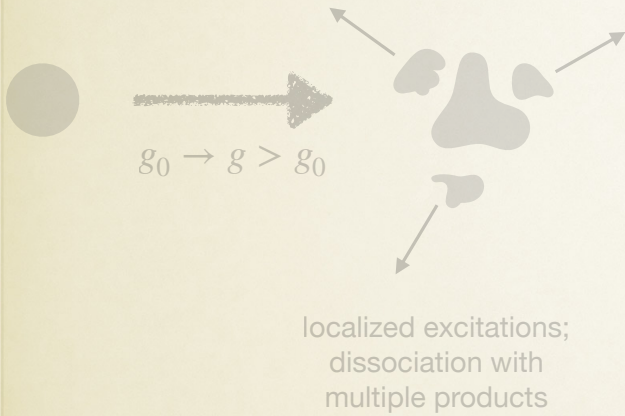
$\eta = 4$



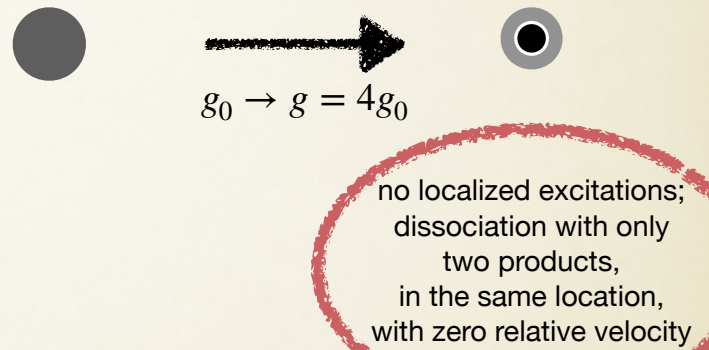
Do not know yet. A QM-SUSY structure may be responsible, Koller and Olshanii (2011)

All in all...

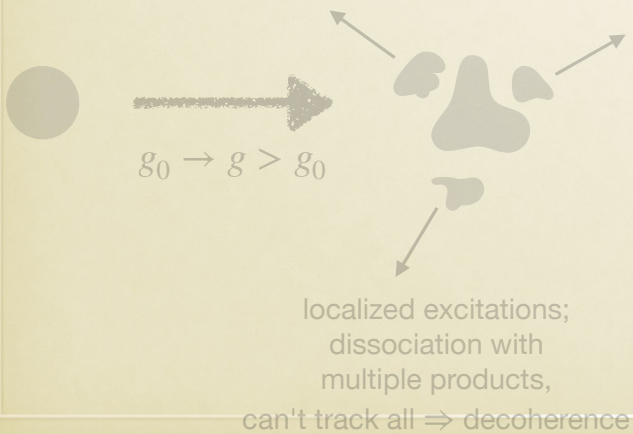
generic cluster
classical



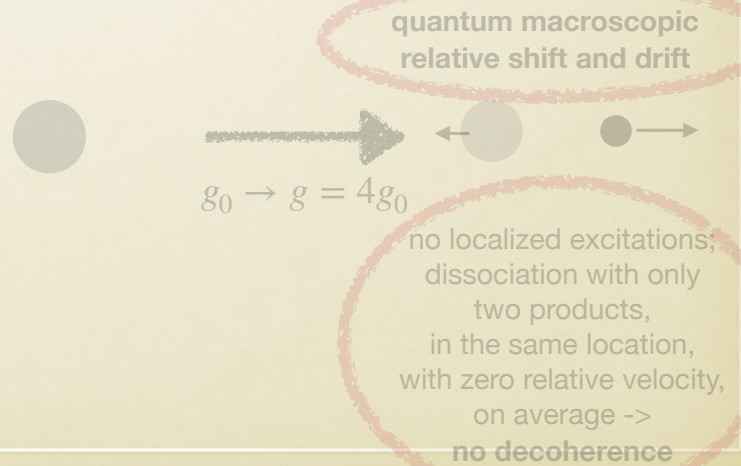
integrable cluster (soliton)
classical



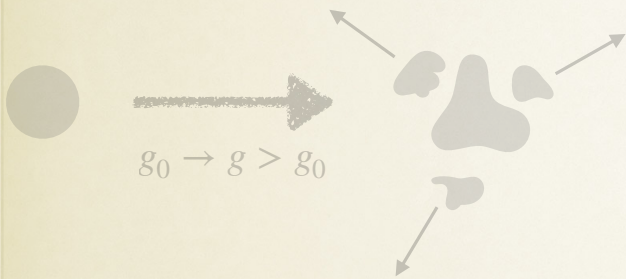
generic cluster
quantum



integrable cluster (soliton)
quantum

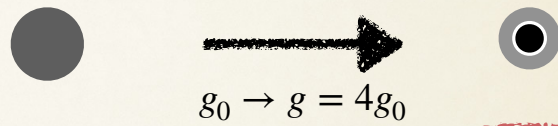


generic cluster
classical



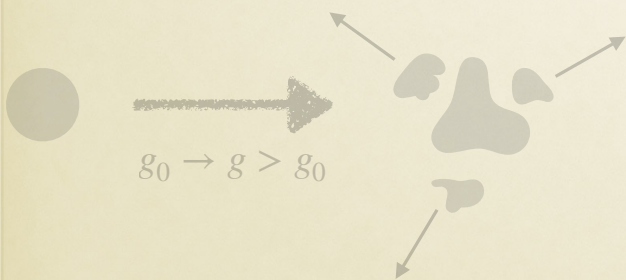
localized excitations;
dissociation with
multiple products

integrable cluster (soliton)
classical



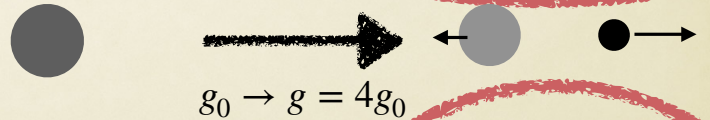
no localized excitations;
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localized excitations;
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can't track all \Rightarrow decoherence

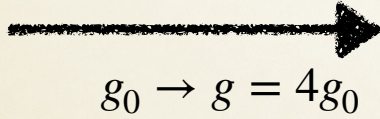
integrable cluster (soliton)
quantum



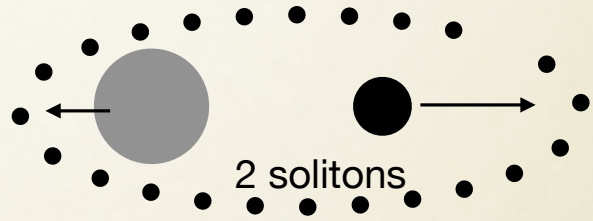
quantum macroscopic
relative shift and drift

no localized excitations,
dissociation with only
two products,
in the same location,
with zero relative velocity,
on average \rightarrow
no decoherence

1 soliton

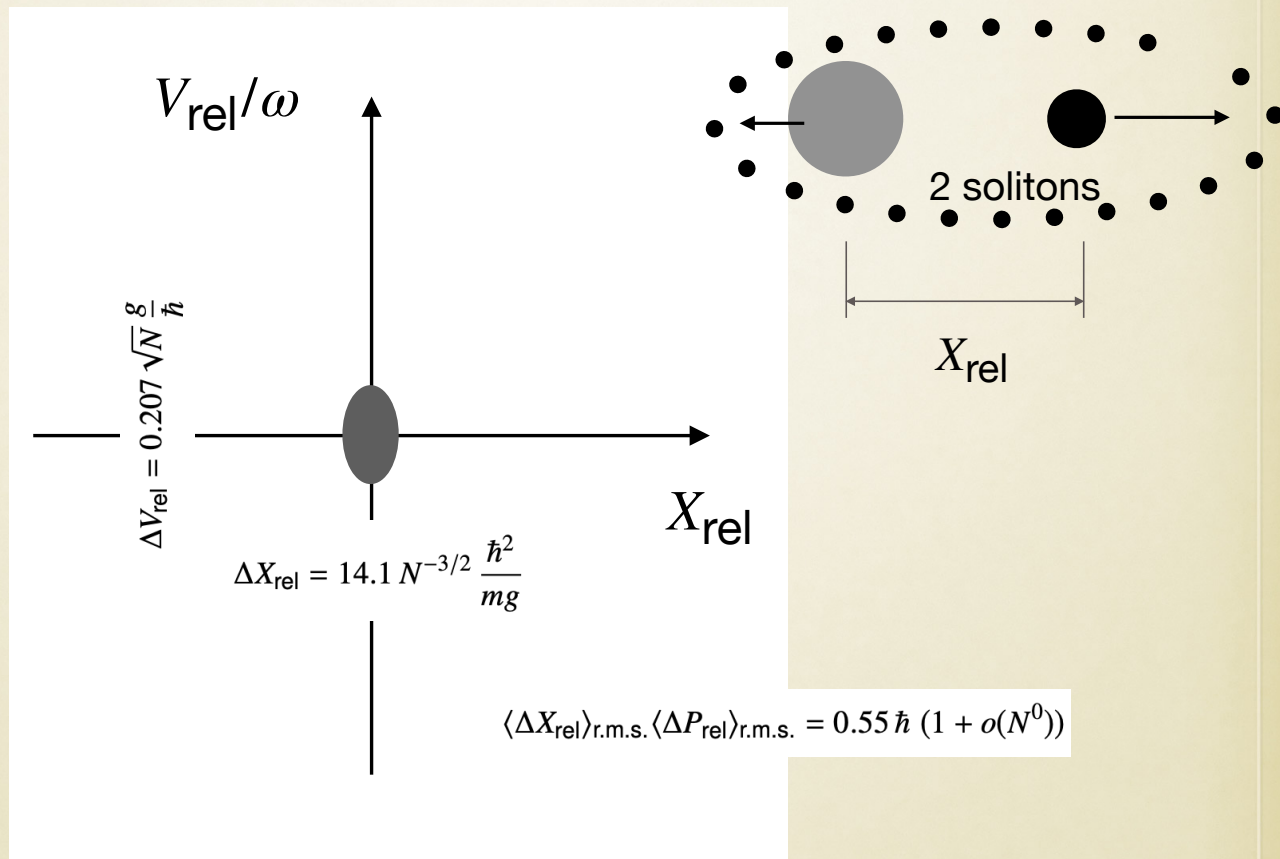


quantum
relative shift and drift =
**quantum macroscopic
minimal-uncertainty
wavepacket**



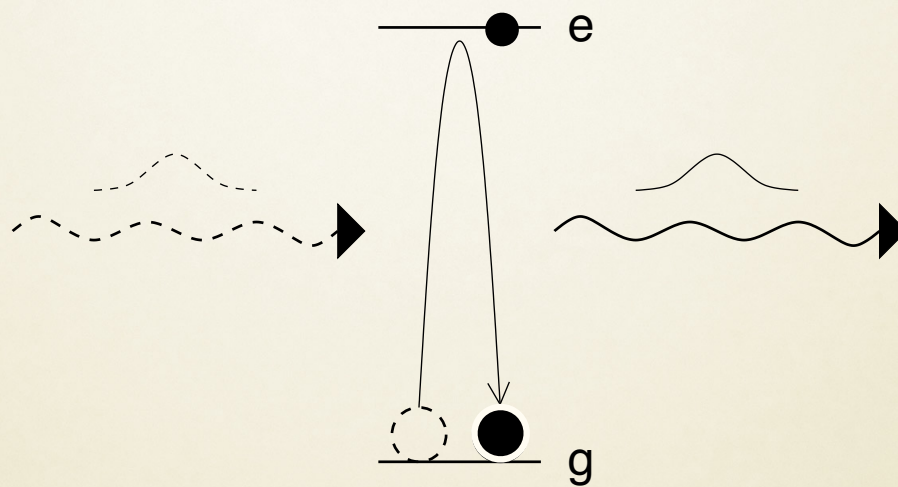
no localized excitations;
dissociation with only
two products,
in the same location,
with zero relative velocity,
on average = **no decoherence**

Predictions

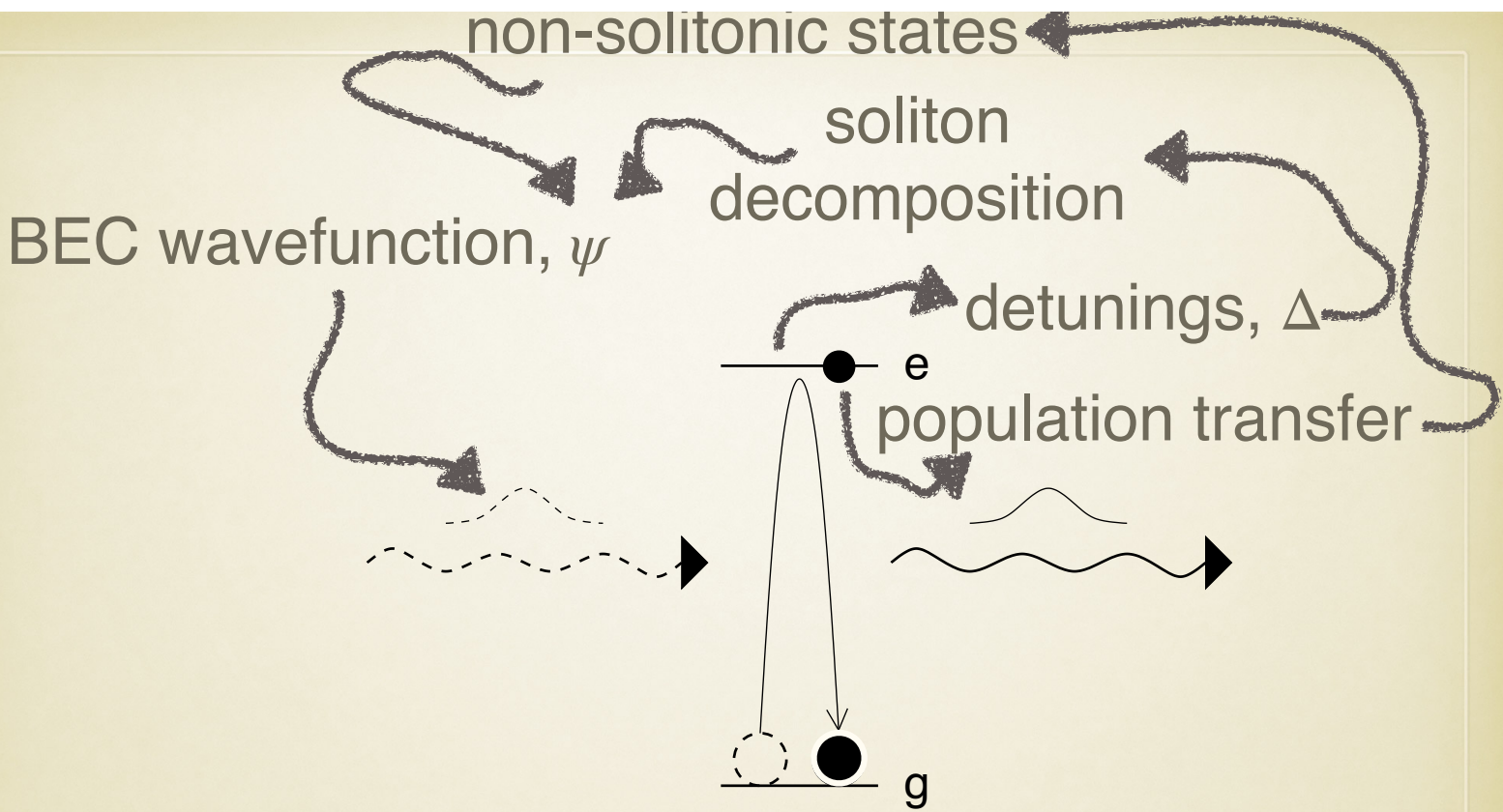


Oleksandr V. Marchukov, Boris A. Malomed, Maxim Olshanii, Vanja Dunjko, Joanna Ruhl, Randall G. Hulet, and Vladimir A. Yurovsky, PRL 125, 050405 (2020); Vladimir A. Yurovsky, Boris A. Malomed, Randall G. Hulet, Maxim Olshanii, PRL 119, 220401 (2017).

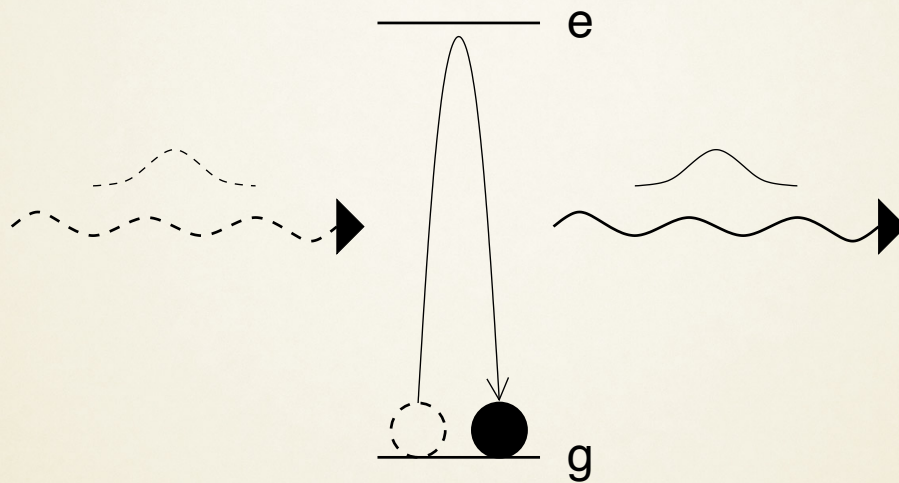
What is this Lax
potential anyways?



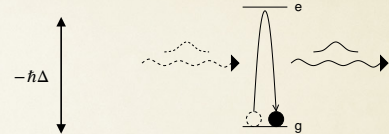
Two-level atom, subject to a laser pulse



Two-level atom, subject to a laser pulse

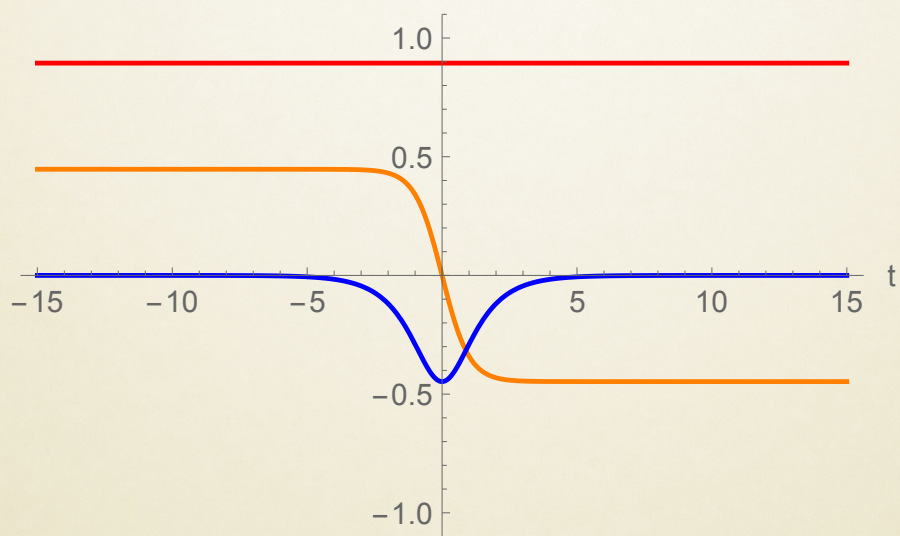


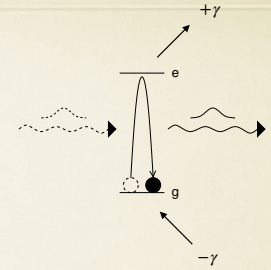
Two-level atom, sech-laser pulse
For a discrete set of amplitudes,
no population transfer for *any* detuning:
thus, pure solitons



Single soliton; scatt. state; $\Delta = -2$; $V_{eg}(t) = -\text{Sech}[t]$

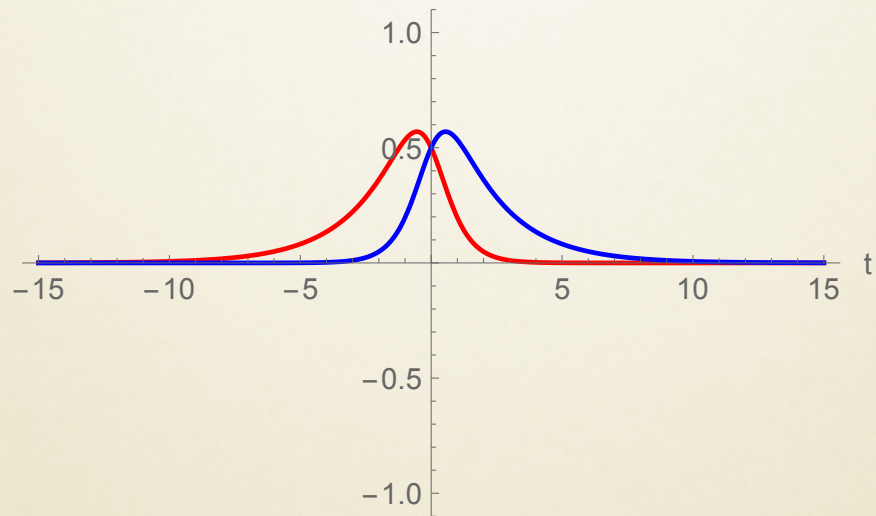
$\text{Re}[\phi_g[t]/e^{-i(\Delta/2)t}]$; $\text{Im}[\phi_g[t]/e^{-i(\Delta/2)t}]$; $\phi_e[t]/e^{-i(\Delta/2)t}$





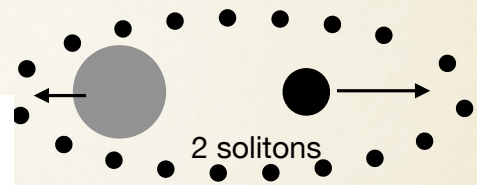
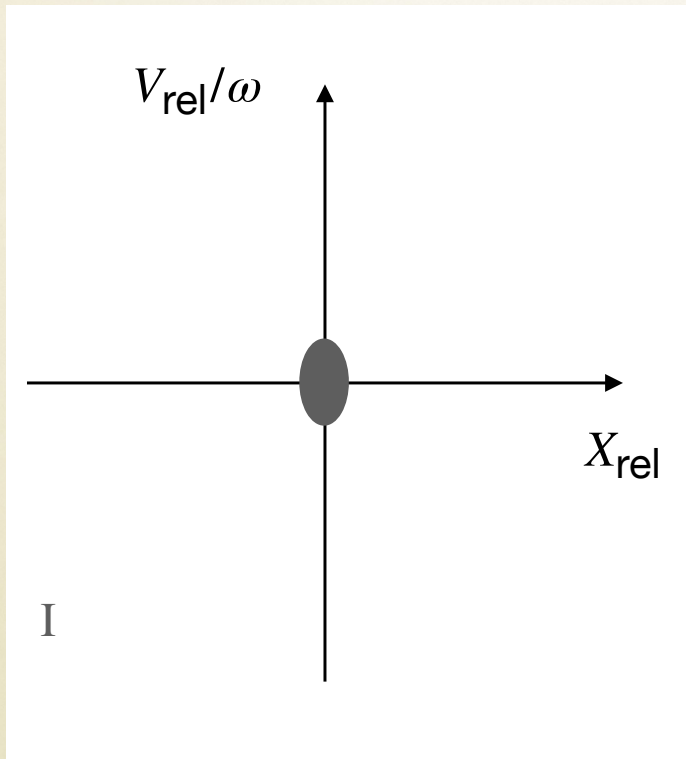
Single soliton; bound state; $\Delta = -1 \times i$; $V_{eg}(t) = -\text{Sech}[t]$

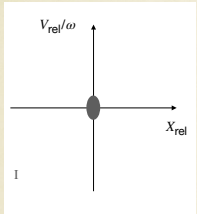
$$i^{+1/2} \phi_g[t]; i^{-1/2} \phi_e[t]$$

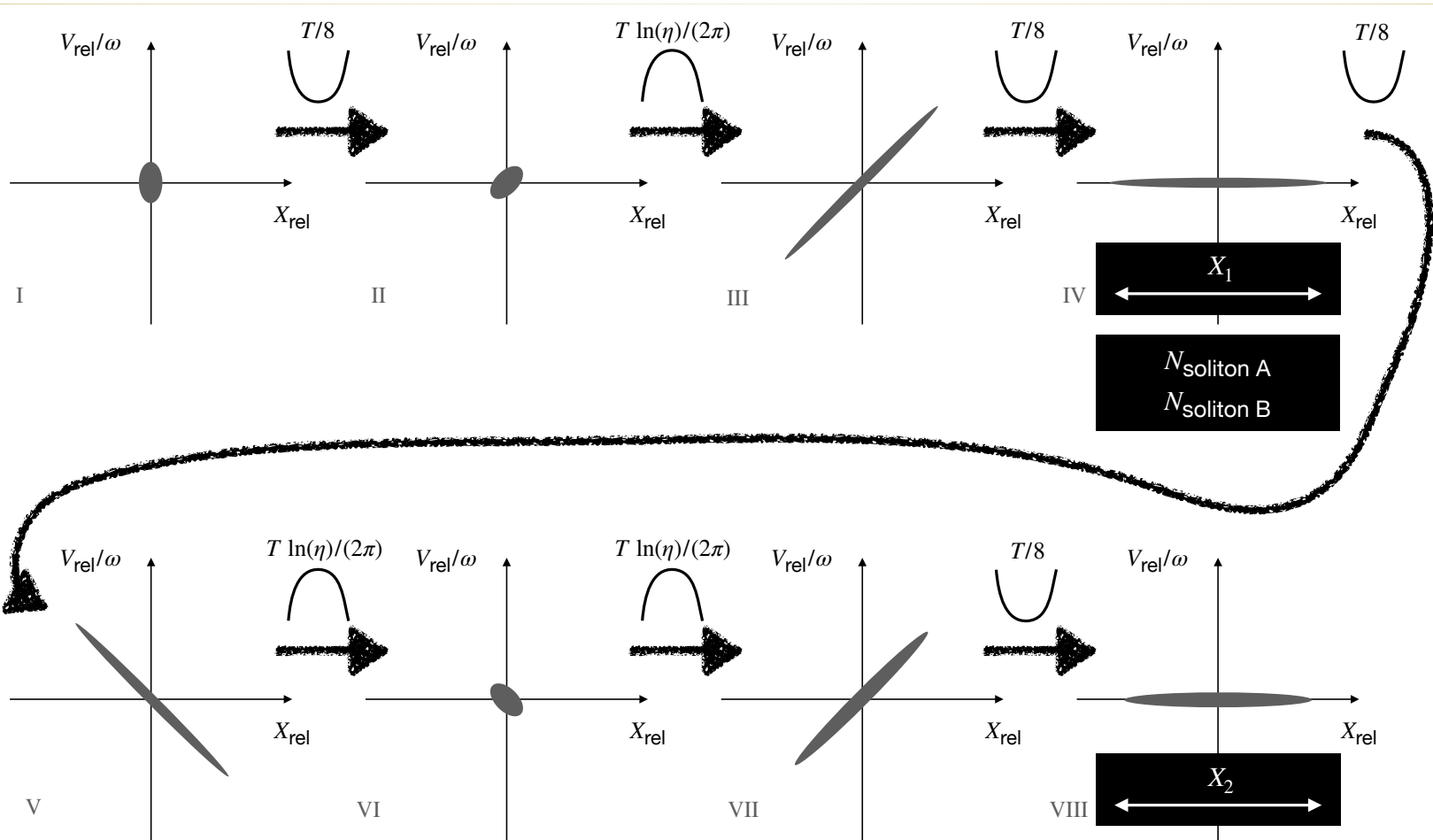


Say we got a
macroscopic quantum
packet: how can we
prove it's quantum?

Our proposal







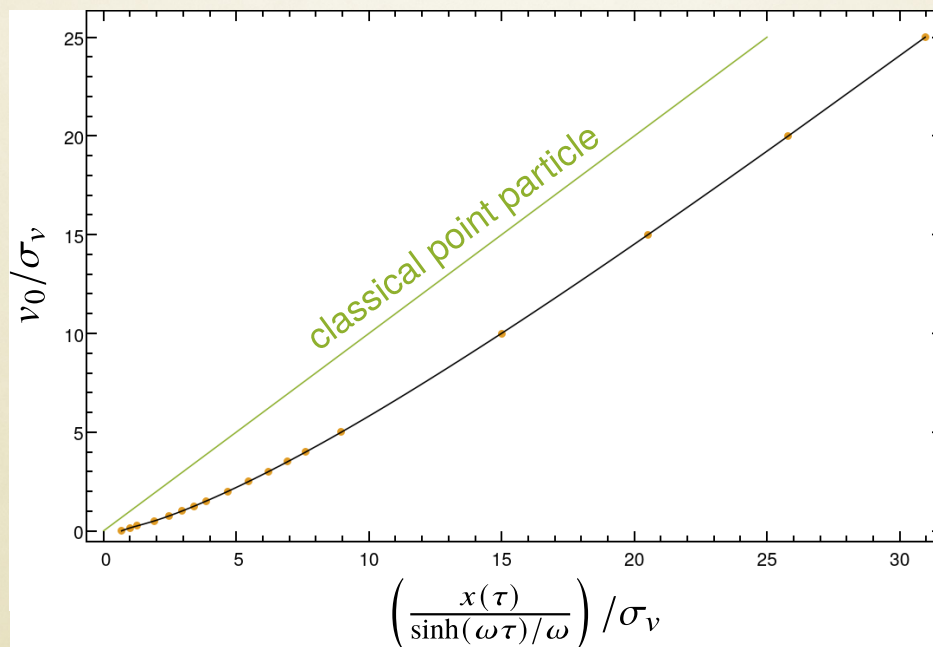
$$m \omega \eta^{-2} \langle (N_{\text{soliton A}} N_{\text{soliton B}} / (N_{\text{soliton A}} + N_{\text{soliton B}})) X_1 \rangle_{\text{r.m.s}} \langle X_2 \rangle_{\text{r.m.s}} \gtrsim \frac{\hbar}{2}$$

Problems:

- soliton-soliton interactions in harmonic potential
- exponential sensitivity to errors on the re-entry to the inverted harmonic potential

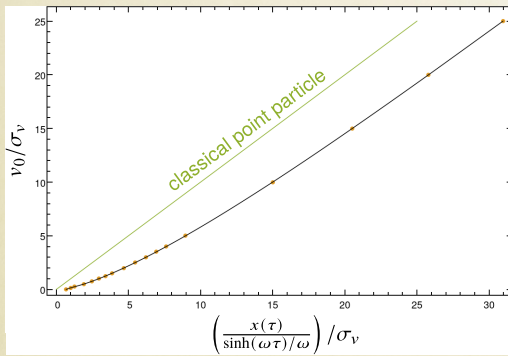
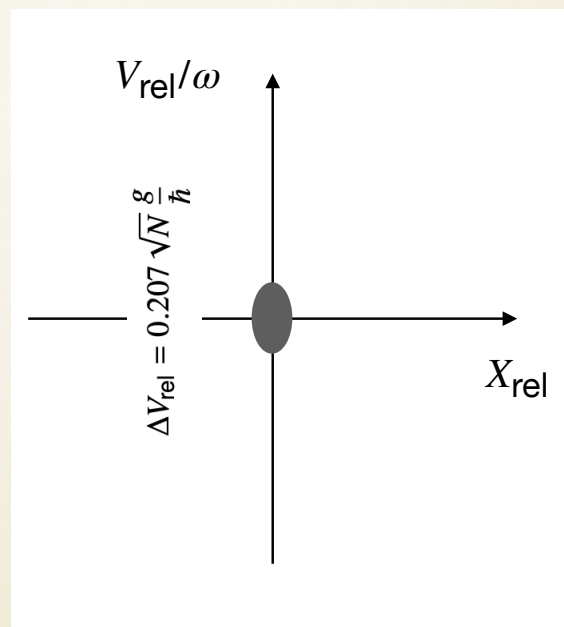
Problem: soliton-soliton interactions in harmonic potential

Solution: tabulation, set the other three fluctuations to zero



Problem: soliton-soliton interactions in harmonic potential

Solution: tabulation



$$\Delta V_{\text{rel}} \Big|_{\text{measured}} = (1.026 \pm 0.017) \Delta V_{\text{rel}} \Big|_{\text{true}}$$

$$\approx 1/\sqrt{2 * \#_of_realizations}$$

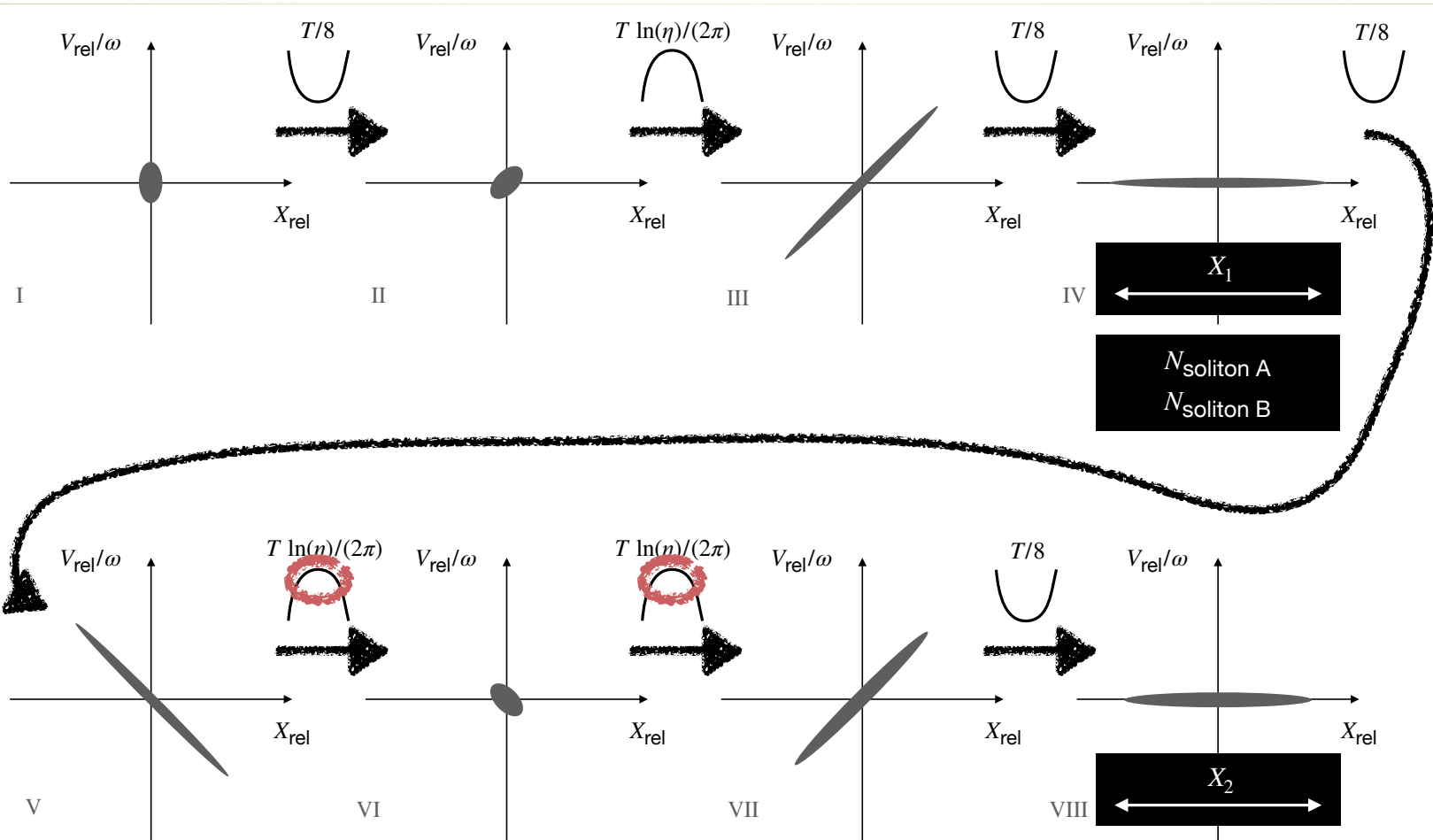
for 1735 realizations

Problem: soliton-soliton interactions in harmonic potential

Solution: tabulation

same for the relative distance

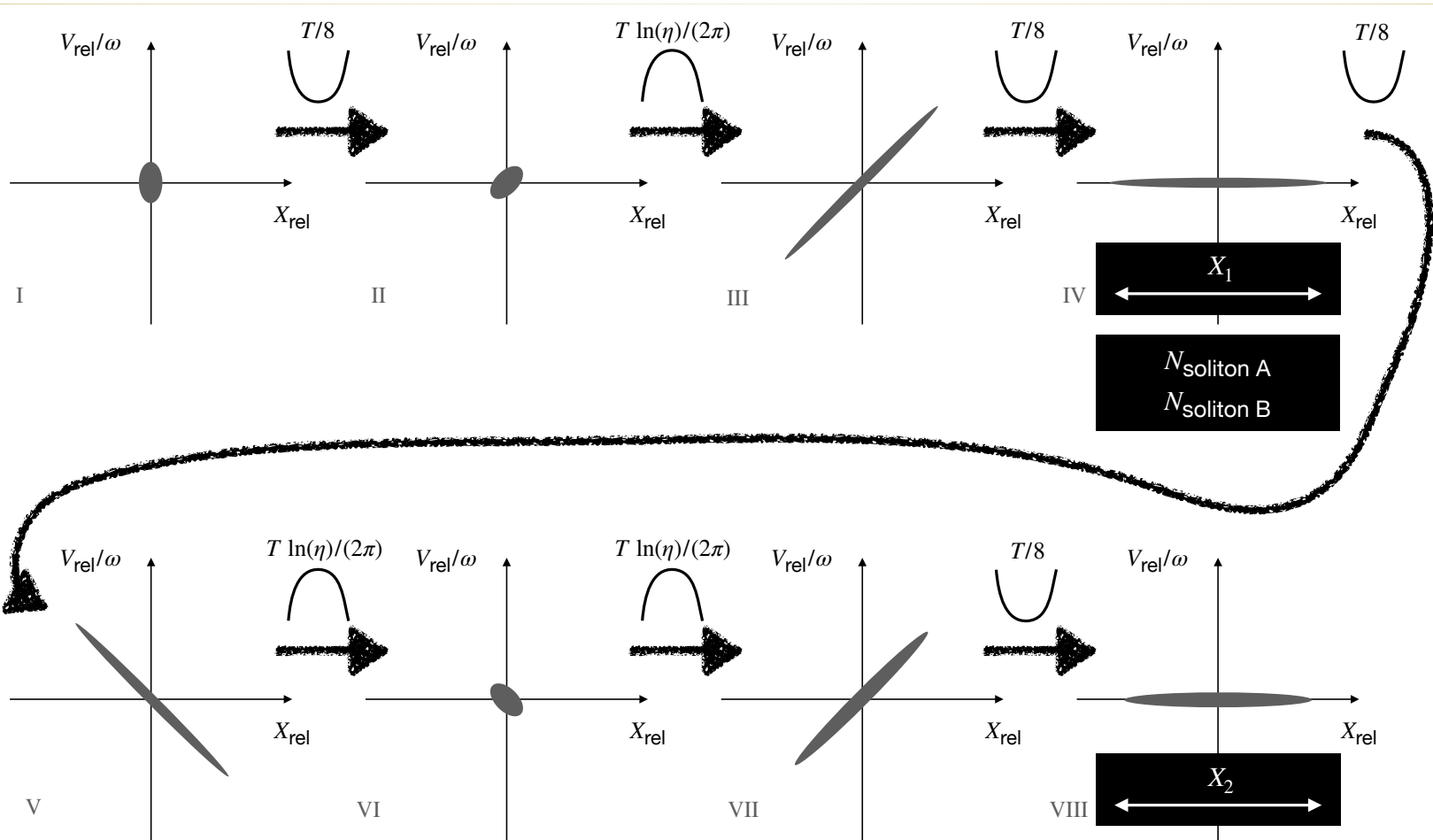
Problem: exponential sensitivity to errors on the re-entry to the inverted harmonic potential



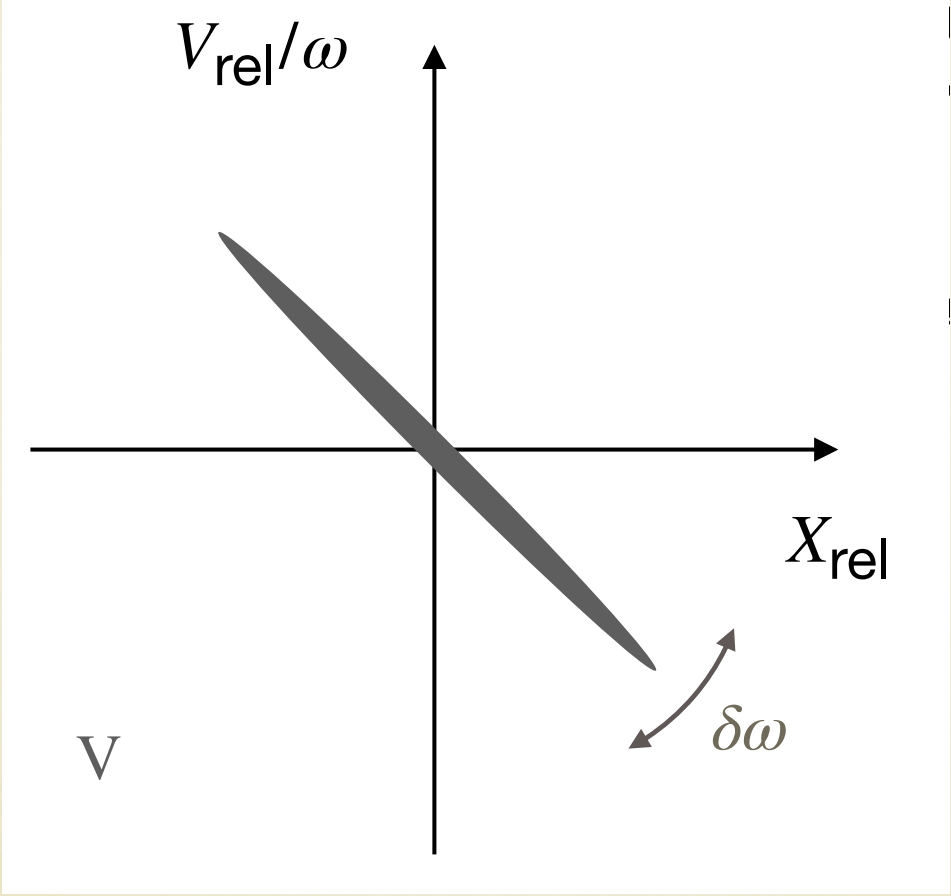
$$m \omega \eta^{-2} \langle (N_{\text{soliton A}} N_{\text{soliton B}} / (N_{\text{soliton A}} + N_{\text{soliton B}})) X_1 \rangle_{\text{r.m.s}} \langle X_2 \rangle_{\text{r.m.s}} \gtrsim \frac{\hbar}{2}$$

Problem: exponential sensitivity to errors on the re-entry to the inverted harmonic potential

One of the possible solutions: "bug to a feature"



$$m \omega \eta^{-2} \langle (N_{\text{soliton A}} N_{\text{soliton B}} / (N_{\text{soliton A}} + N_{\text{soliton B}})) X_1 \rangle_{\text{r.m.s}} \langle X_2 \rangle_{\text{r.m.s}} \gtrsim \frac{\hbar}{2}$$



Prob[intersoliton_distance $\leq 141 \times$ soliton_size]

${}^7\text{Li}$

$N = 54000$

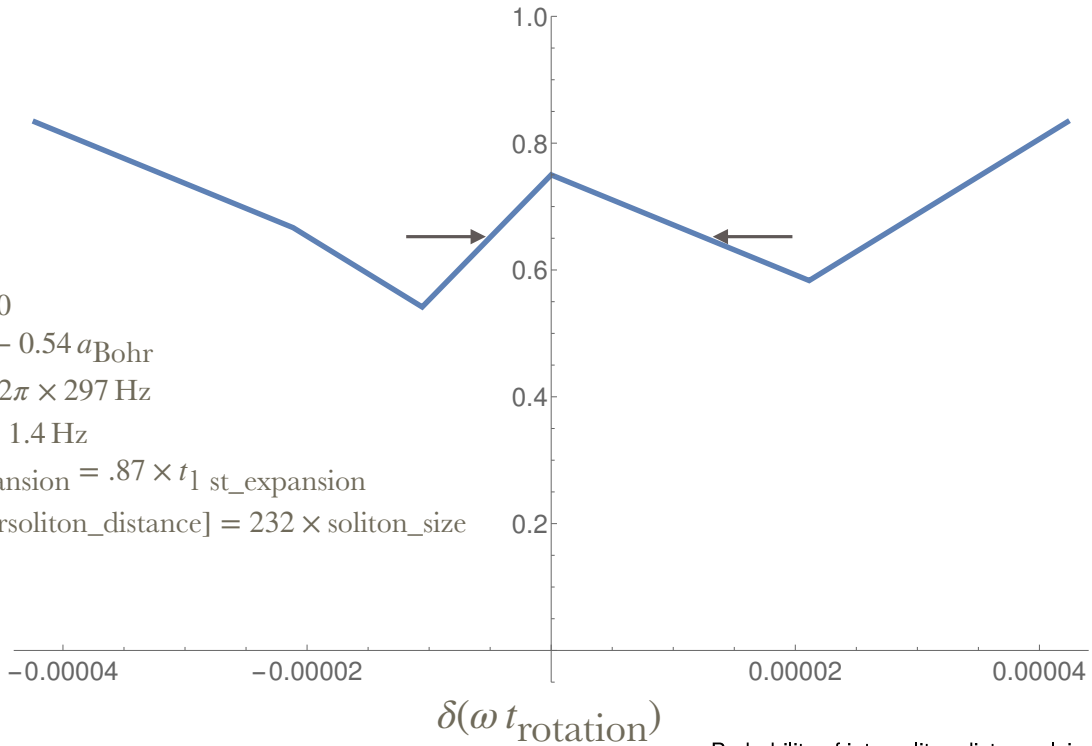
$a_{\text{scatt.}} = -0.54 a_{\text{Bohr}}$

$\omega_{\text{perp.}} = 2\pi \times 297 \text{ Hz}$

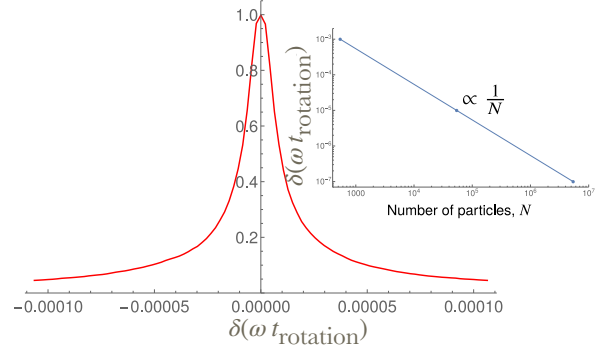
$\omega_z = 2\pi \times 1.4 \text{ Hz}$

$t_{2\text{nd_expansion}} = .87 \times t_{1\text{st_expansion}}$

r.m.s.[intersoliton_distance] = $232 \times$ soliton_size



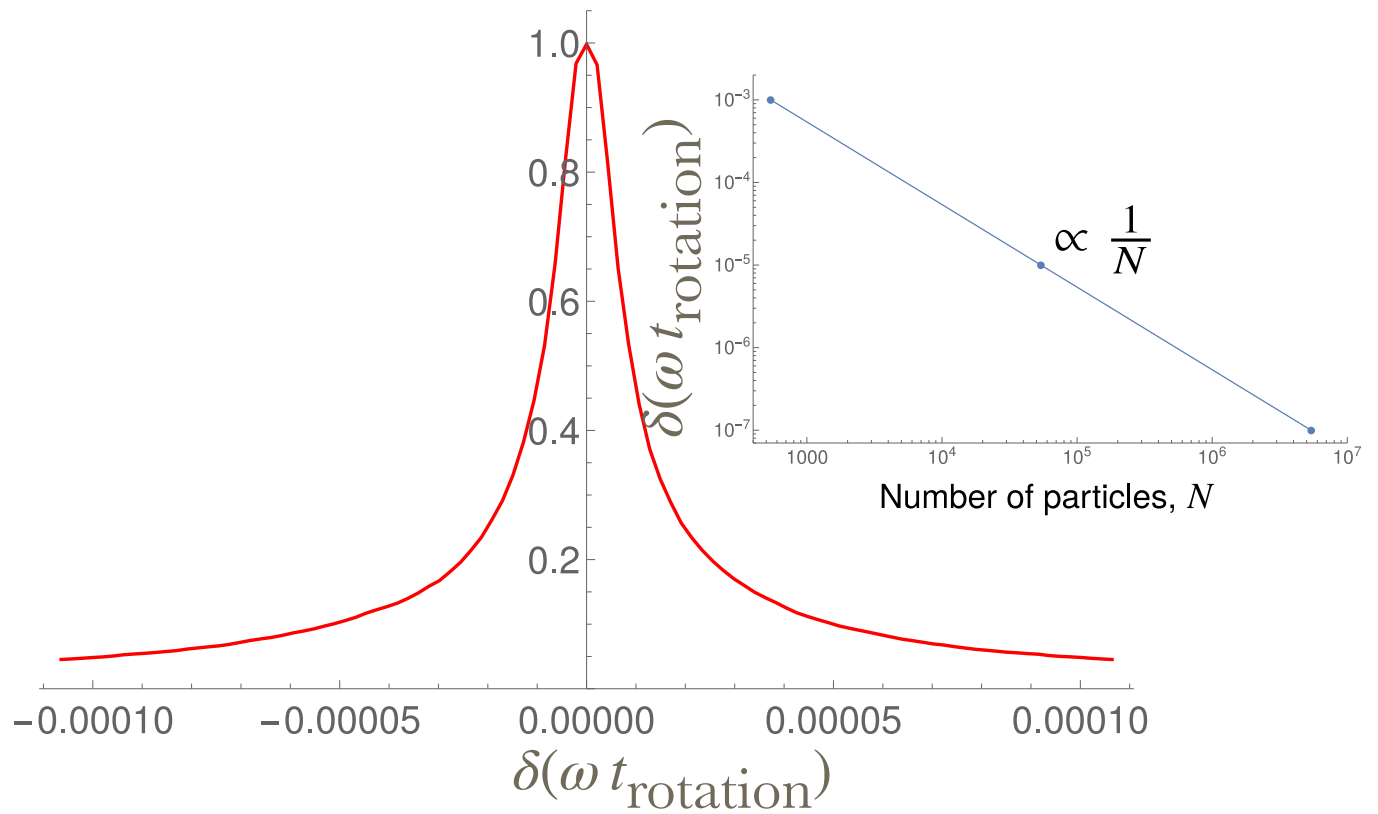
Probability of intersoliton distance lying in $[-3\sigma, 3\sigma]$



$$\delta\omega = 8.9 \times 10^{-6} \times (2\pi) \times \text{Hz}$$

$$\text{Scales as } \frac{\hbar}{N}$$

Probability of intersoliton distance lying in $[-3\sigma, 3\sigma]$



classical estimate

Summary

Inverse scattering map



(*) Solitons do not decohere to localized excitations;

(*) There is a gap to the delocalized excitations, it *grows* with the soliton size;

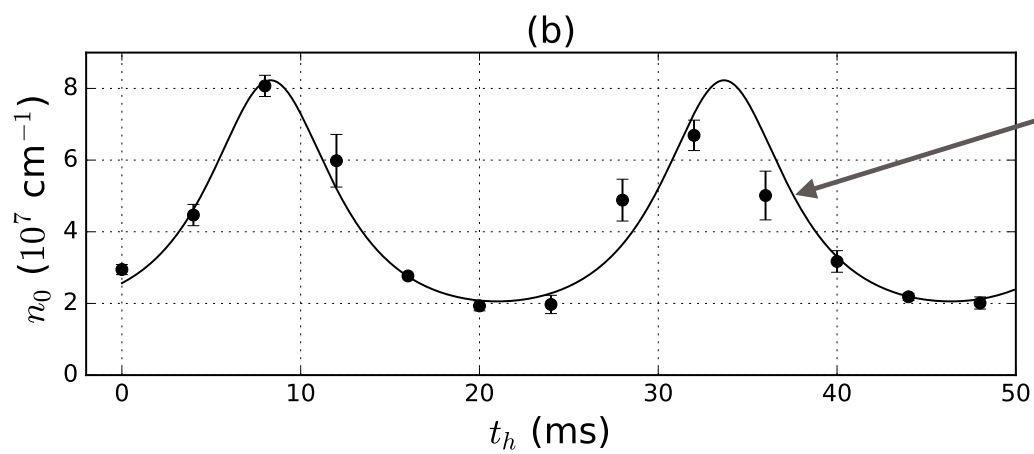
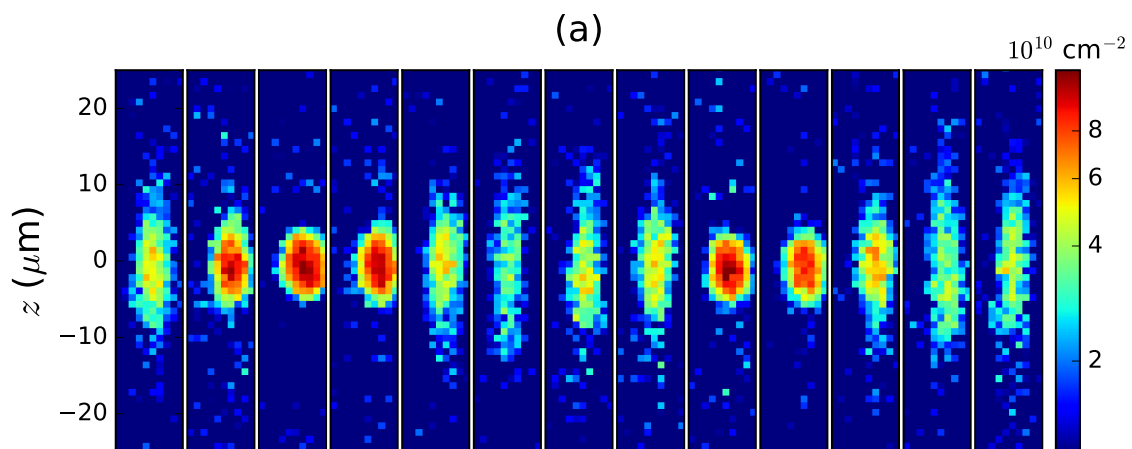
(*) Soliton produces a controllable set of products in an implosion: can get *just two solitons and nothing else*.



Due to the left-right symmetry, the relative motion of the two solitons is a **macroscopic minimal uncertainty wavepacket**. We offer ways of proving the minimal uncertainty.

Publications:
quantum macroscopic coherence
with the relative motion of two solitons

- [6] Sumita Datta, Vanja Dunjko, Maxim Olshanii, **Path Integral Estimates of the Quantum Fluctuations of the Relative Soliton-Soliton Velocity in a Gross-Pitevskii Breather**, *MDPI Physics* 4, 12 (2022). [Macroscopic quantum breather dissociation (Feynman-Kac path integral Monte-Carlo, $1 \lesssim N \lesssim 100$)]
- [5] De Luo, Yi Jin, Jason H. V. Nguyen, Boris A. Malomed, Oleksandr V. Marchukov, Vladimir A. Yurovsky, Vanja Dunjko, Maxim Olshanii, R. G. Hulet, **Creation and Characterization of Matter-Wave Breathers**, *Phys. Rev. Lett.* 125, 183902 (2020). [Experimental creation of a breather ($N = 5.4 \times 10^4$)]
- [4] Oleksandr V. Marchukov, Boris A. Malomed, Maxim Olshanii, Vanja Dunjko, Joanna Ruhl, Randall G. Hulet, and Vladimir A. Yurovsky, **Quantum fluctuations of the center-of-mass and relative parameters of NLS breather**, *Phys. Rev. Lett.* 125, 050405 (2020). [Macroscopic quantum breather dissociation (Bogoliubov, $10 \lesssim N < \infty$)]
- [3] Oleksandr V. Marchukov, Boris A. Malomed, Vladimir A. Yurovsky, Maxim Olshanii, Vanja Dunjko, Randall G. Hulet, **Splitting of nonlinear-Schrödinger breathers by linear and nonlinear localized potentials**, *Phys. Rev. A* 99, 063623 (2019). [How to speed the breather dissociation up, with additional potentials (GPE)]
- [2] Jake Golde, Joanna Ruhl, Sumita Datta, Boris A. Malomed, Maxim Olshanii, Vanja Dunjko, **Metastability versus collapse following a quench in attractive Bose-Einstein condensates**, *Phys. Rev. A* 97, 053604 (2018). [Breather stability against collapse (GPE)]
- [1] Vladimir A. Yurovsky, Boris A. Malomed, Randall G. Hulet, Maxim Olshanii, **Dissociation of one-dimensional matter-wave breathers due to quantum many-body effects**, *Phys. Rev. Lett.* 119, 220401 (2017). [Original proposal for the macroscopic quantum breather dissociation (Bethe Ansatz, $1 \leq N \lesssim 20$)]



Zakharov-Shabat
1972

De Luo, Yi Jin, Jason H. V. Nguyen, Boris A. Malomed, Oleksandr V. Marchukov, Vladimir A. Yurovsky, Vanja Dunjko, Maxim Olshanii, R. G. Hulet, **Creation and Characterization of Matter-Wave Breathers**, PRL 125, 183902 (2020).

Publications:
quantum macroscopic coherence
with the center-of-mass of a single soliton

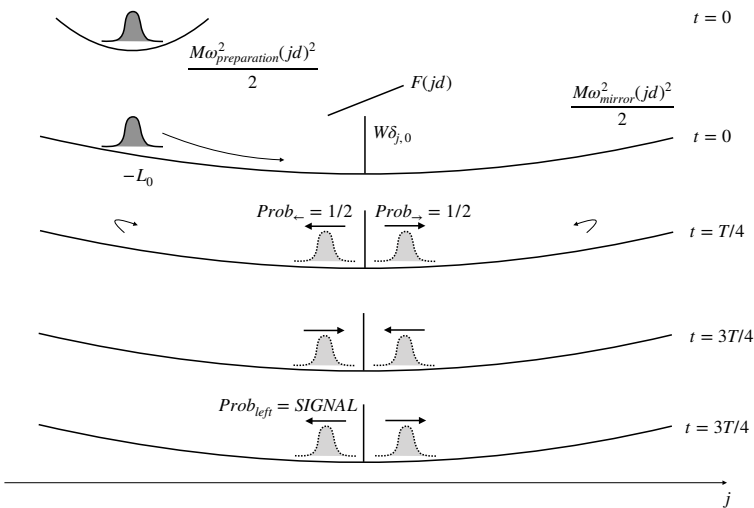
[5] Piero Naldesi, Peter D. Drummond, Vanja Dunjko, Juan Polo, Luigi Amico, Anna Minguzzi, Maxim Olshanii, **Massive particle interferometry with lattice solitons: robustness against ionization**, [arXiv:2201.10479], submitted to SciPost. [[A study of CoM decoherence to delocalized Bogoliubov excitations \(exact diagonalization, \$N = 6\$ \)](#)]

[4] P. Naldesi, J. Polo, S. A. Gardiner, M. Olshanii, A. Minguzzi, L. Amico, **Quantum-enhanced atomtronics with solitons**, chapter in *Roadmap on Atomtronics*, L. Amico, M. Boshier, G. Birkl, A. Minguzzi, C. Miniatura, L.-C. Kwek, eds, AVS Quantum Sci. 3, 039201 (2021) [featured on cover/landing page]. [[Review on soliton interferometry](#)]

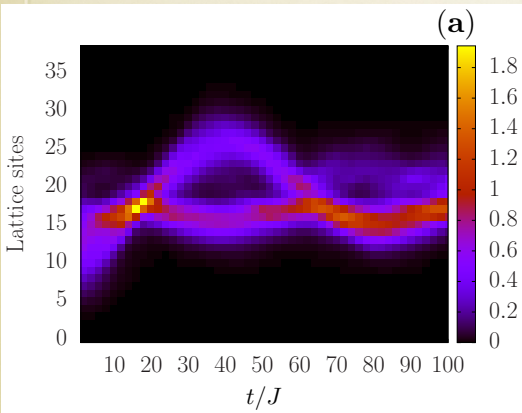
[3] Piero Naldesi, Juan Polo Gomez, Vanja Dunjko, H el ene Perrin, Maxim Olshanii, Luigi Amico, Anna Minguzzi, **Enhancing sensitivity to rotations with quantum solitonic currents**, SciPost Phys. 12, 138 (2022). [[Soliton rotometry \(exact diagonalization, \$N = 4\$ \)](#)]

[2] Maxim Olshanii, Thibault Scoquart, Dmitry Yampolsky, Vanja Dunjko, and Steven Glenn Jackson, **Creating entanglement using integrals of motion**, Phys. Rev. A 97, 013630 (2018). [[Multi-soliton entanglement amplifier \(Bethe Ansatz on CoM of the solitons\)](#)]

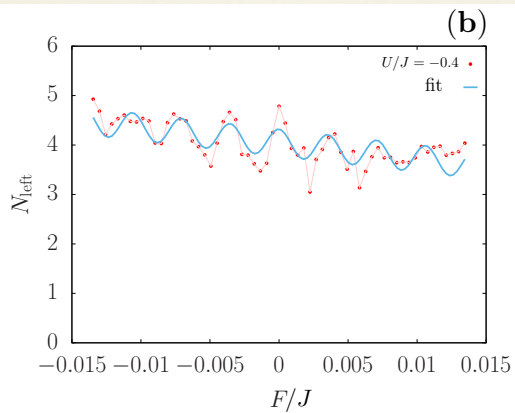
[1] Piero Naldesi, Juan Polo Gomez, Anna Minguzzi, Boris Malomed, Maxim Olshanii, Luigi Amico, **Raise and fall of a bright soliton in an optical lattice**, Phys. Rev. Lett. 122, 053001 (2018). [[Solitons on a lattice \(exact diagonalization, \$N = 5\$ \)](#)]



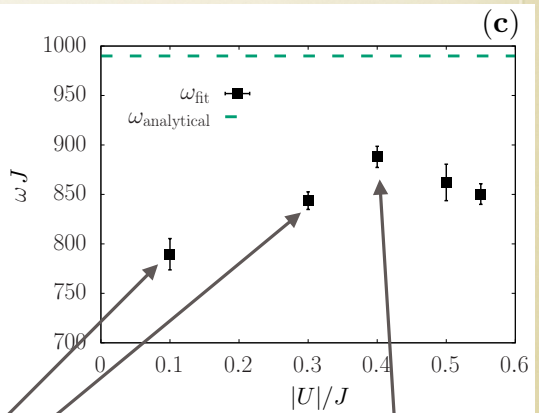
Piero Naldesi, Peter D. Drummond, Vanja Dunjko, Juan Polo, Luigi Amico, Anna Minguzzi, Maxim Olshanii, **Massive particle interferometry with lattice solitons: robustness against ionization**, [arXiv:2201.10479], submitted to SciPost.



Above one-atom ionization threshold



Above one-atom ionization threshold



Above a total ionization threshold

Above one-atom ionization threshold

Support by:



Thank you!