NANYANG TECHNOLOGICAL UNIVERSITY

UNIVERSITÉ CÔTE D'AZUR

An Atomtronics Spin Field Effect Transistor

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Centre for Quantum Technologies

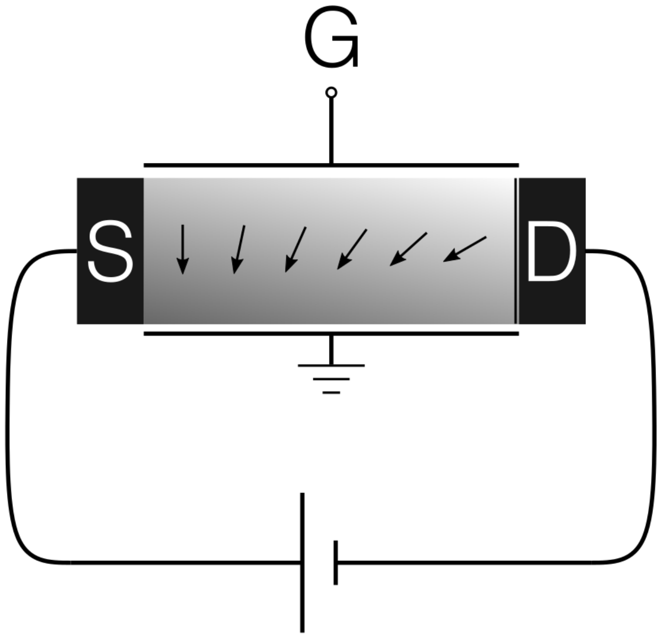
MajuLab

cdpt

Benasque2022

Datta-Das Transistor

S. Datta and B. Das, APL **56**, 665 (1990)



**Issues with
solid-state devices**

- Insufficient spin-orbit coupling
- Insufficient polarisation of the input/output spin injection current
- Depolarization in gate region due to scattering

**Atomtronics
solutions**

Spin-dependent atom-light coupling

Optical pumping

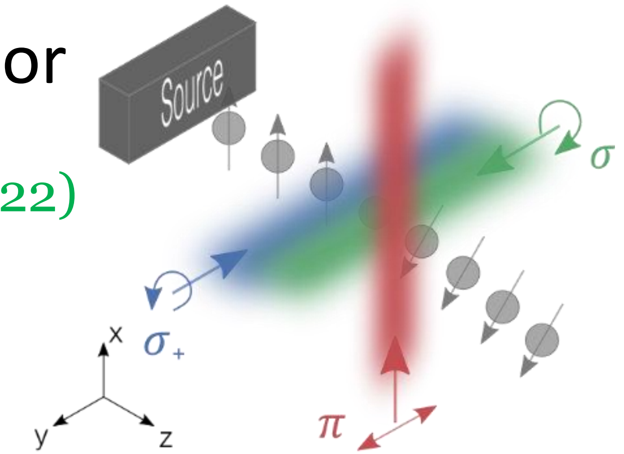
Diluted gases or BEC

Spin-FET expected applications:
High integration density, ultrafast switching and low power consumption

Content

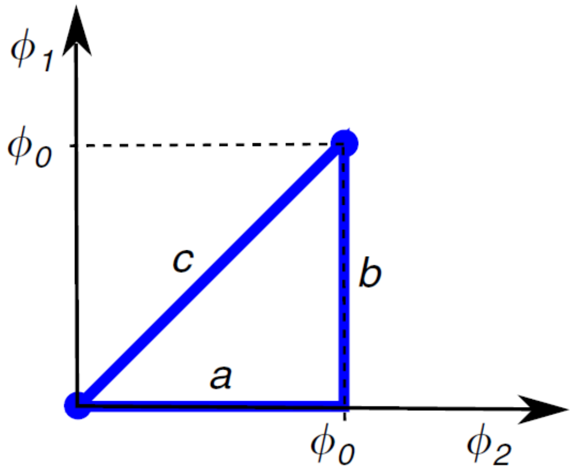
- Atomtronics Datta-Das transistor

C. Madasu et *al*, ArXiv 2203.13360 (2022)



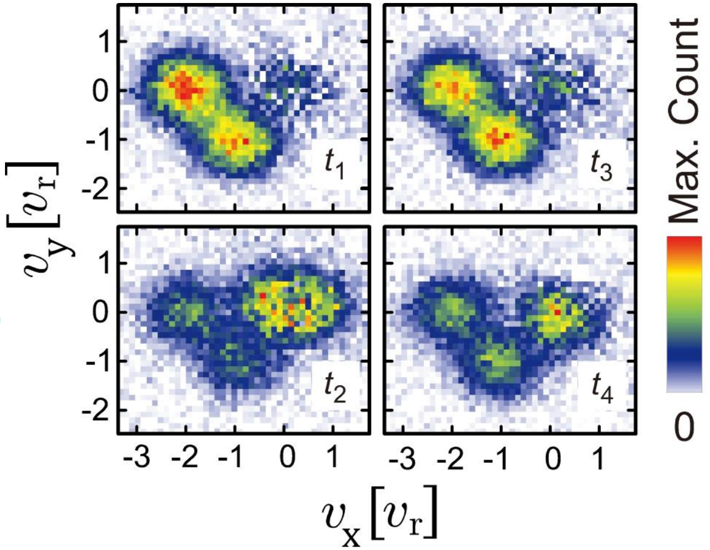
- Non-Abelian transformation

F. Leroux et *al*, Nat. Comm. **9** 7 (2018)



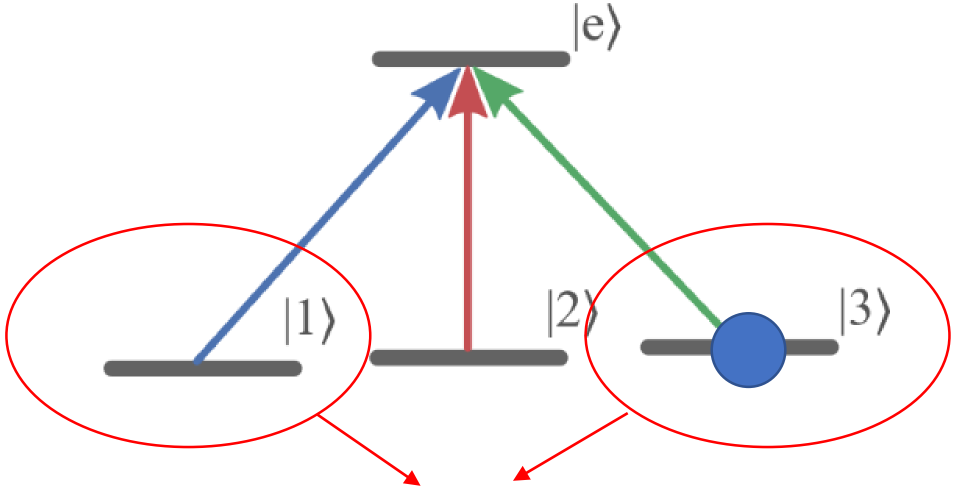
- 2D Zitterbewegung dynamic

M. Hasan et *al*, ArXiv:2201.00885 (2022)

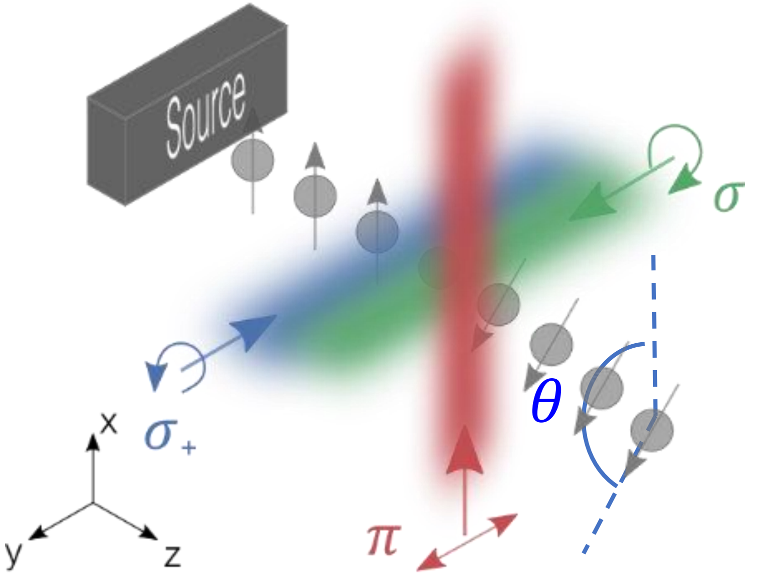


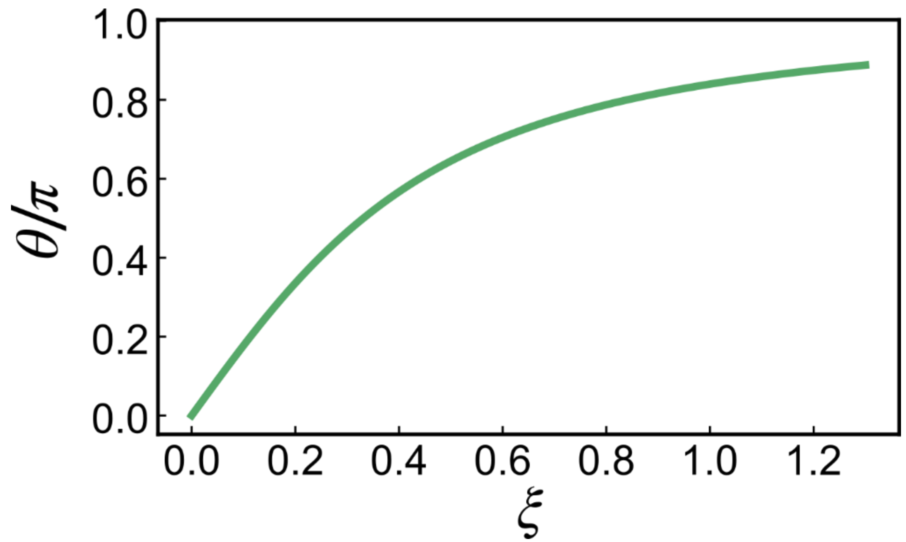
Atomtronics DDT: Basic Idea

J. Y. Vaishnav et *al.*, PRL **101**, 265302 (2008)

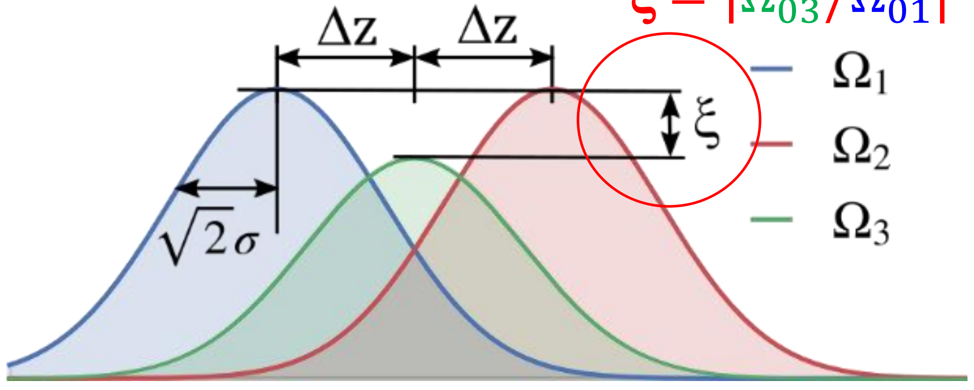


Pseudo-spin basis





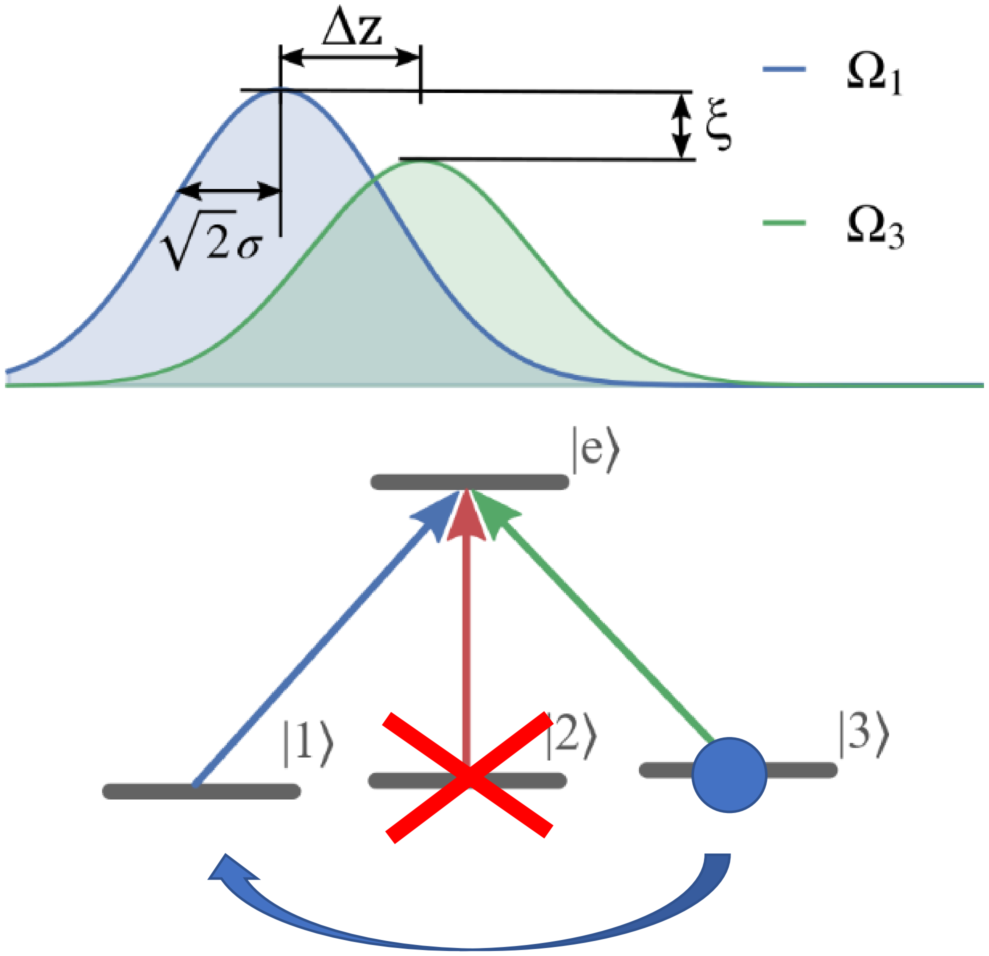
$\xi = |\Omega_{03}/\Omega_{01}|$



That a four-level STIRAP!

STIRAP: Three-State Case

STIRAP: Stimulated Raman Adiabatic Passage

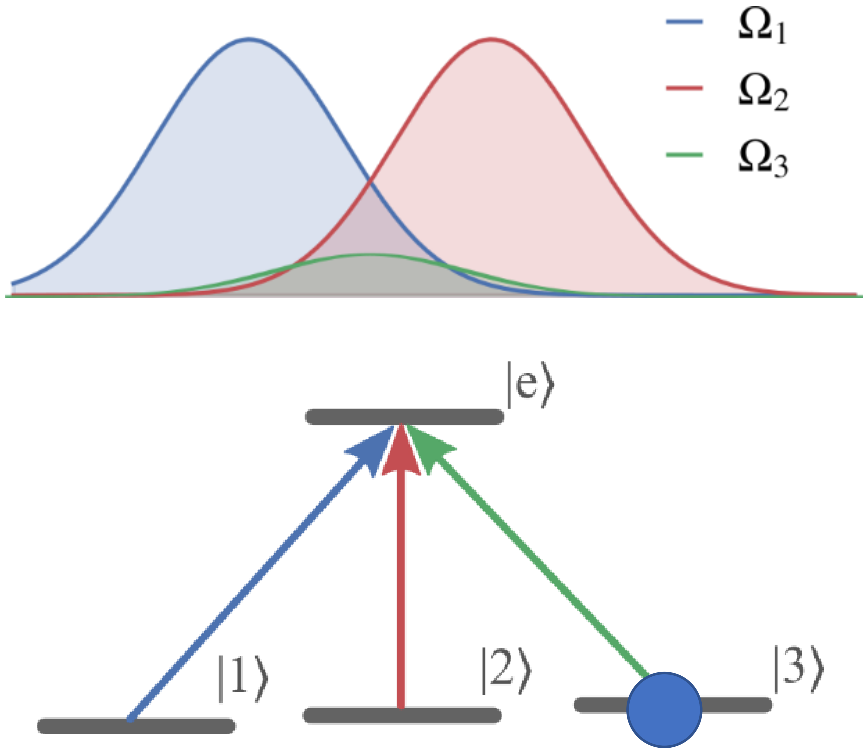


The atom stays in a dark state
i.e. uncoupled to the excited state
Full transfer from $|3\rangle$ (spin up) to $|1\rangle$ (spin down) no matter the value of ξ

Review: N. V. Vitanov, et *al*., RMP **89**, 015006 (2017)

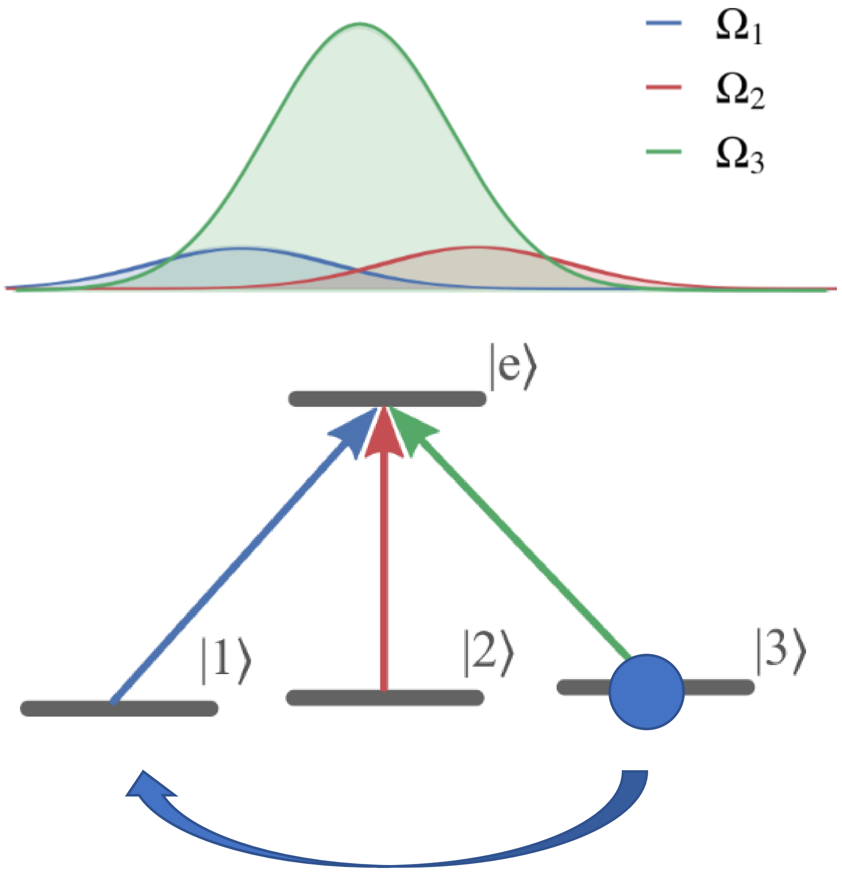
STIRAP: Four-State Limiting Cases

$\xi = |\Omega_{03}/\Omega_{01}| = 0$



The beam "3" is not turned on
No transfer

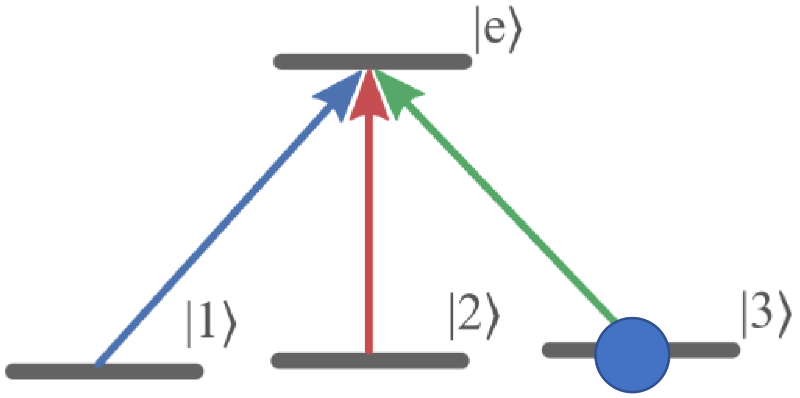
$\xi = |\Omega_{03}/\Omega_{01}| \to +\infty$



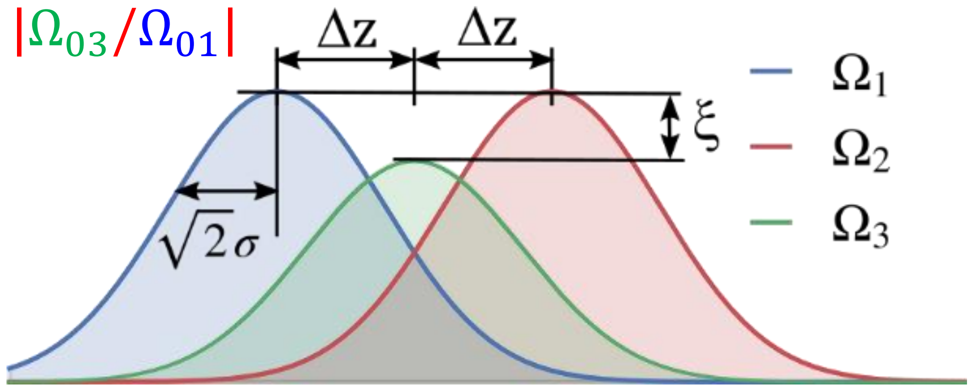
Full transfer similar to the two-level case

STIRAP: Four-State General Case

Allow for full control of the pseudo-spin orientation



$\xi = |\Omega_{03}/\Omega_{01}|$



State $|2\rangle$ is never populated

K. Bergmann’s group, Kaiserslautern (De)

Theory: R. Unanyan et *al*., Optics comm. **155**, 144 (1998) (resolution of the 4-state problem)

J. Y. Vaishnav et *al*., PRL **101**, 265302 (2008) (Synthetic gauge field)

Experiment (Ne* beam): H. Theuer, et *al*., Optics Express **4**, 77 (1999) (Beam splitter)

F. Vewinger, et *al*., PRL **91**, 213001 (2003) (Coherent state superposition)

Review: N. V. Vitanov, et *al*., RMP **89**, 015006 (2017)

Tripod and Degenerate Dark States

$$H_I = \frac{\hbar}{2}\begin{array}{c} \begin{array}{cccc} |1\rangle & |2\rangle & |3\rangle & |4\rangle \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & \Omega_1^* \\ 0 & 0 & 0 & \Omega_2^* \\ 0 & 0 & 0 & \Omega_3^* \\ \Omega_1 & \Omega_2 & \Omega_3 & 0 \end{pmatrix} \end{array}$$ with $\Omega_i = |\Omega_i| e^{i\Phi_i}$

$$|\Omega_1| = \Omega \sin\alpha \cos\beta$$
$$|\Omega_2| = \Omega \sin\alpha \sin\beta$$
$$|\Omega_3| = \Omega \cos\alpha$$

$$\Omega = \sqrt{|\Omega_1|^2 + |\Omega_2|^2 + |\Omega_3|^2}$$

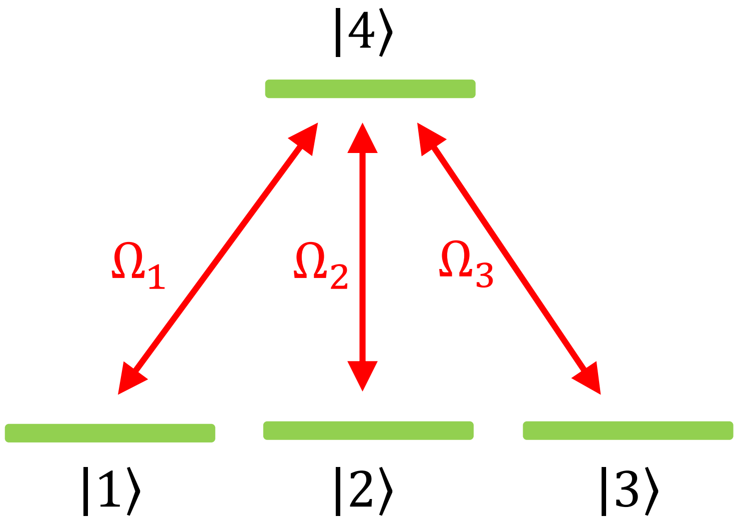
Diagonalization of the Hamiltonian

$$|D_1\rangle = \begin{pmatrix} \sin\beta\, e^{i\Phi_{31}} \\ -\cos\beta\, e^{i\Phi_{32}} \\ 0 \\ \boxed{0} \end{pmatrix} \quad |D_2\rangle = \begin{pmatrix} \cos\alpha\cos\beta\, e^{i\Phi_{31}} \\ \cos\alpha\sin\beta\, e^{i\Phi_{32}} \\ -\sin\alpha \\ \boxed{0} \end{pmatrix}$$

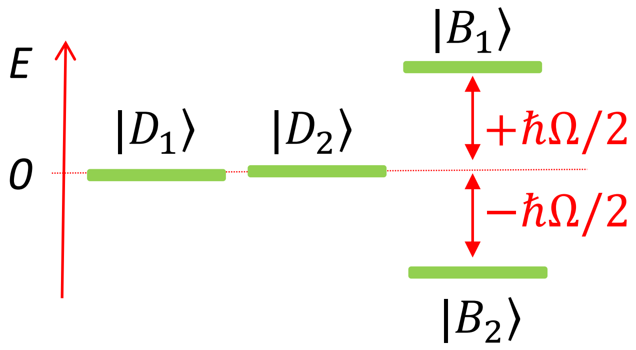
Excited state population is zero

With $\Phi_{ij} = \Phi_i - \Phi_j$

Bare states



Dressed states



J. Dalibard et *al*, RMP **83** 1523 (2011)
J. Ruseckas et *al*, PRL **95** 010404 (2005)

SU(2) Non-Abelian Gauge Field

We perform an adiabatic following in $\{|D_1\rangle, |D_2\rangle\}$ subspace

$$|D_1\rangle = \begin{pmatrix} \sin\beta\, e^{i\Phi_{31}} \\ -\cos\beta\, e^{i\Phi_{32}} \\ 0 \end{pmatrix} \qquad |D_2\rangle = \begin{pmatrix} \cos\alpha\cos\beta\, e^{i\Phi_{31}} \\ \cos\alpha\sin\beta\, e^{i\Phi_{32}} \\ -\sin\alpha \end{pmatrix}$$

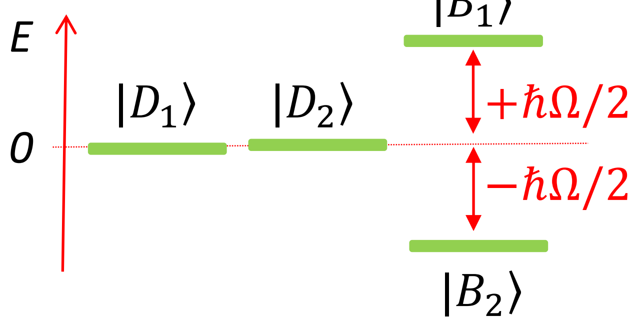
Geometric Gauge field: $\vec{A}_{jk} = i\hbar\langle D_j|\vec{\nabla}D_k\rangle$

We get:

$$\vec{A}_{11} = \hbar\left(\cos^2\beta\vec{\nabla}\Phi_{23} + \sin^2\beta\vec{\nabla}\Phi_{13}\right)$$
$$\vec{A}_{12} = \hbar\cos\alpha\left(\frac{1}{2}\sin(2\beta)\,\vec{\nabla}\Phi_{12} - i\vec{\nabla}\beta\right)$$
$$\vec{A}_{22} = \hbar\cos^2\alpha\left(\cos^2\beta\vec{\nabla}\Phi_{23} + \sin^2\beta\vec{\nabla}\Phi_{13}\right)$$

With $\Phi_{ij} = \Phi_i - \Phi_j$

Dressed states

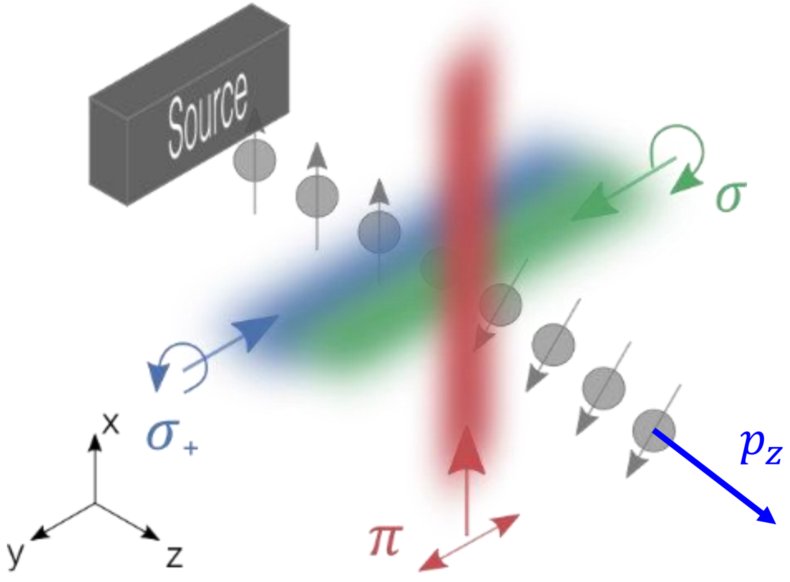


$\vec{A}$ depends on the variation of the laser relative phases Φ_{ij} and one mixing angle $\beta = \tan^{-1}\frac{|\Omega_2|}{|\Omega_1|}$

$$H = \frac{(\vec{p} - \vec{A})^2}{2m} + W$$

Scalar potential: $W_{jk} = \frac{\hbar^2}{2m}\sum_l \langle\nabla D_j|B_l\rangle\langle B_l|\nabla D_k\rangle$

DDT Theory



$$\vec{A}_{11} = \hbar\left(\cos^2\beta\vec{\nabla}\Phi_{23} + \sin^2\beta\vec{\nabla}\Phi_{13}\right)$$
$$\vec{A}_{12} = \hbar\cos\alpha\left(\frac{1}{2}\sin(2\beta)\,\vec{\nabla}\Phi_{12} - i\vec{\nabla}\beta\right)$$
$$\vec{A}_{22} = \hbar\cos^2\alpha\left(\cos^2\beta\vec{\nabla}\Phi_{23} + \sin^2\beta\vec{\nabla}\Phi_{13}\right)$$

For $T << T_r$ and $\langle p\rangle \approx p_z \gg \hbar k$,

We get:

$$\vec{A}_{11} = 0$$
$$\vec{A}_{12} = -i\hbar\cos\alpha\vec{\nabla}\beta$$
$$\vec{A}_{22} = 0$$

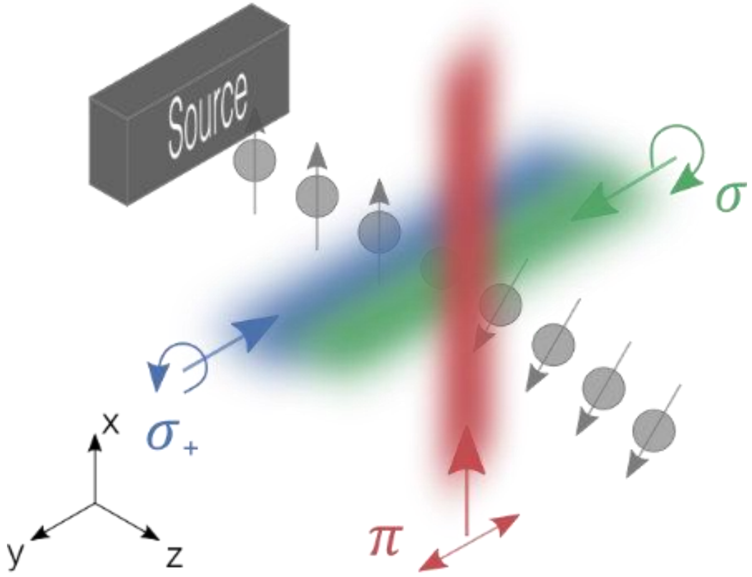
Spin-orbit coupled system

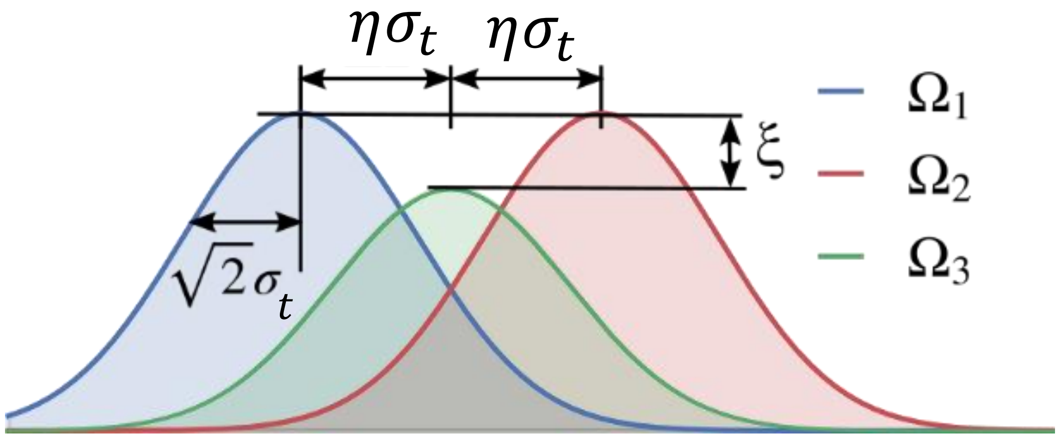
$$H = \frac{\left(\vec{p} - \vec{A}\right)^2}{2m} + W \approx \frac{\vec{p}^2}{2m} - \boxed{\frac{p_z A_z}{m}} \propto p_z\sigma_y$$

Because $p_z \gg \hbar k$: $\left|\left\langle\frac{p_z A_z}{m}\right\rangle\right| \gg \left|\frac{A^2}{2m} + W\right|$

$$A_z = -\hbar\cos\alpha\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\frac{d\beta}{dz}$$

Theory to experiment





For $T << T_r$ and $\langle p \rangle \gg \hbar k$,

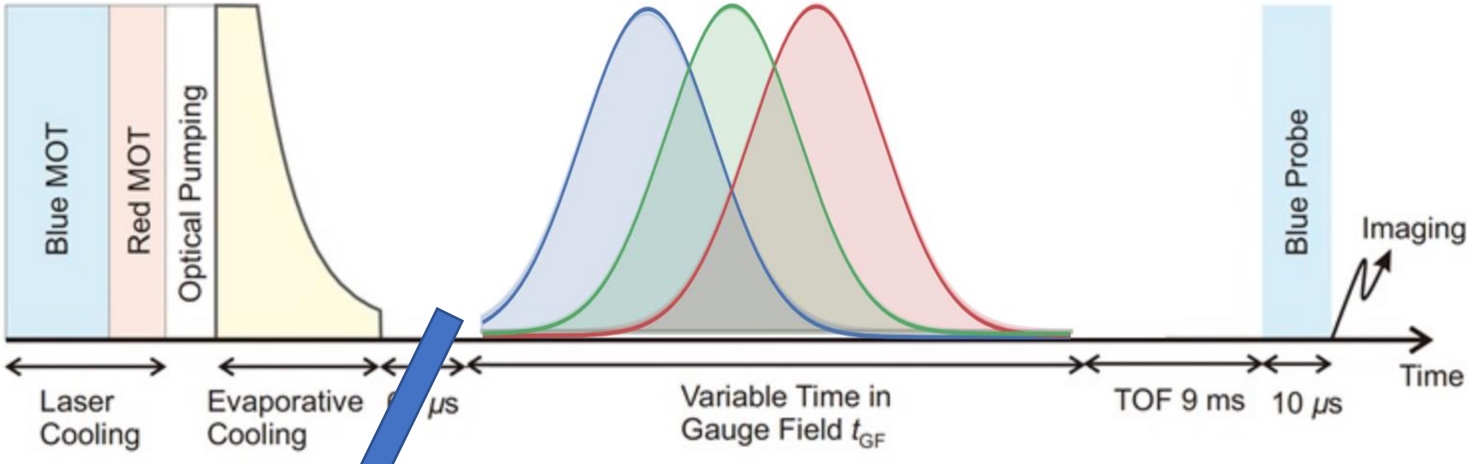
Time-depend approach

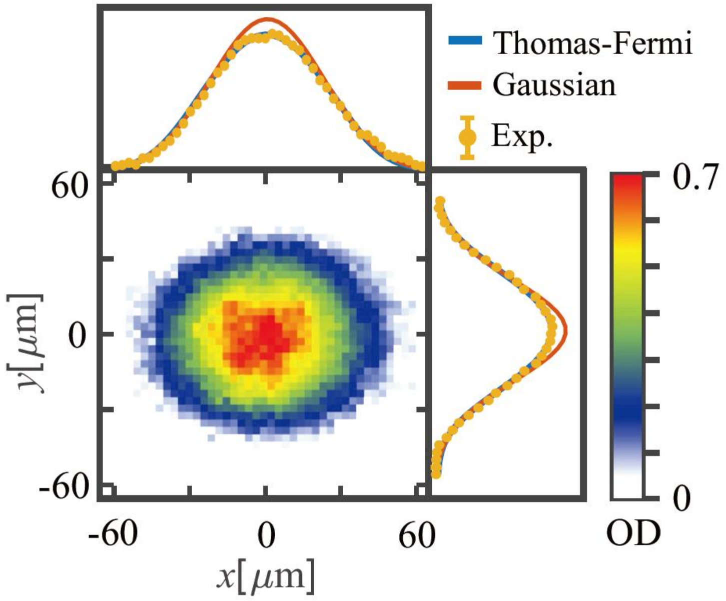
We pulse the laser beams while the atoms remain at rest in the laboratory frame.

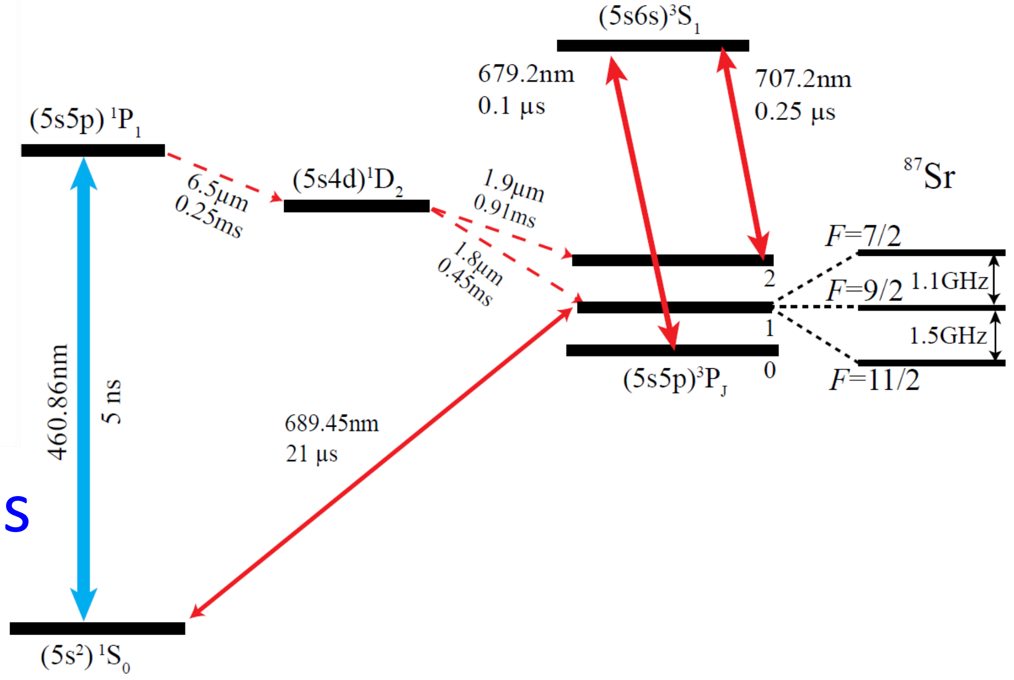
$$i\hbar \frac{\partial \Psi(r,t)}{\partial t} = -\frac{\hat{p}_z \hat{A}_z}{m} \Psi(r,t) \Longleftrightarrow \hat{p}_z \hat{A}_z \equiv -\hbar m \cos \alpha(t) \frac{\partial \beta}{\partial t} \hat{\sigma}_z$$

J. Y. Vaishnav et. al., PRL **101**, 265302(2008)

Experimental Sequence







Strontium ultracold gas

$$T = 30(5)\ \mathrm{nK}$$

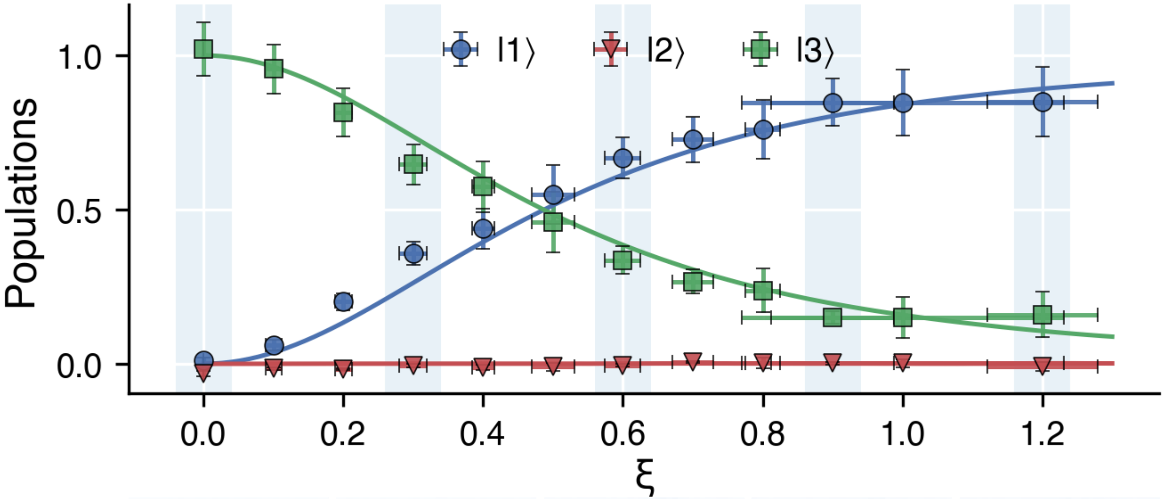
Degenerate Fermi Gas: $\frac{T}{T_\mathrm{F}} = 0.21(4)$

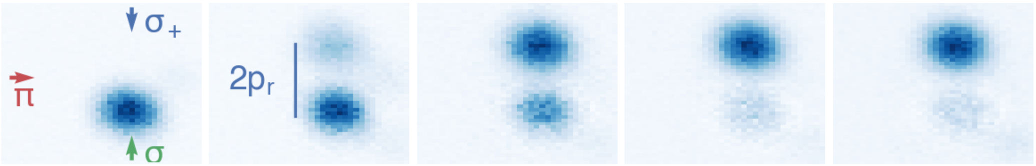
Sub-recoil temperature: $\frac{T}{T_\mathrm{R}} = 0.13(3)$

C. Chalony et *al*, PRL **107** 243002 (2011)
T. Yang, et *al*, EPJD **69**, 226 (2015)

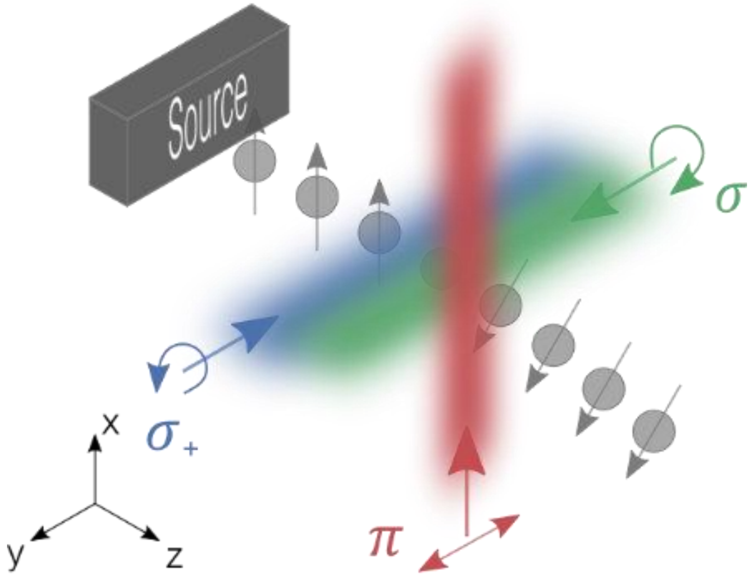
Spin Rotation and Filtering

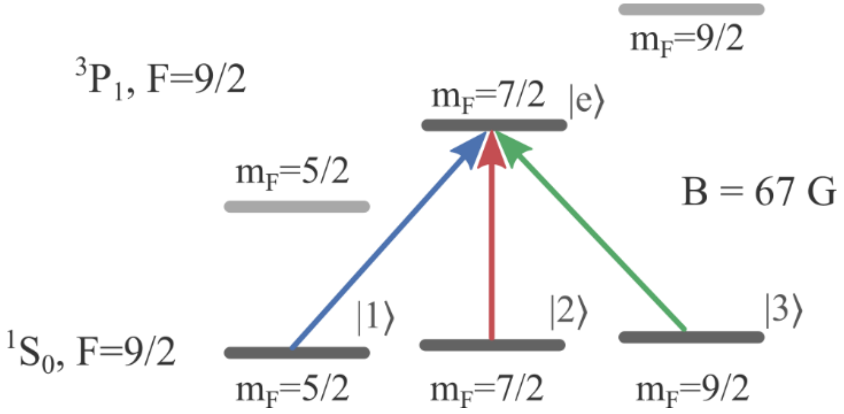
$\xi = |\Omega_{03}/\Omega_{01}|$





TOF = 9 ms



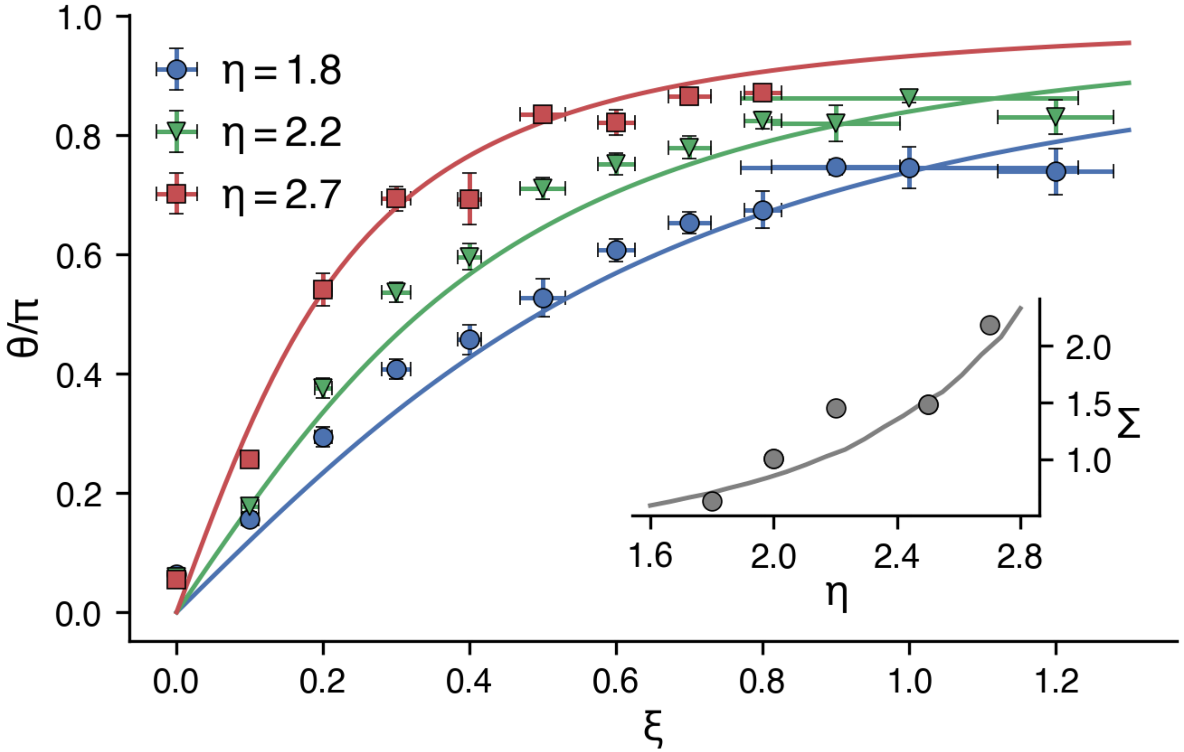


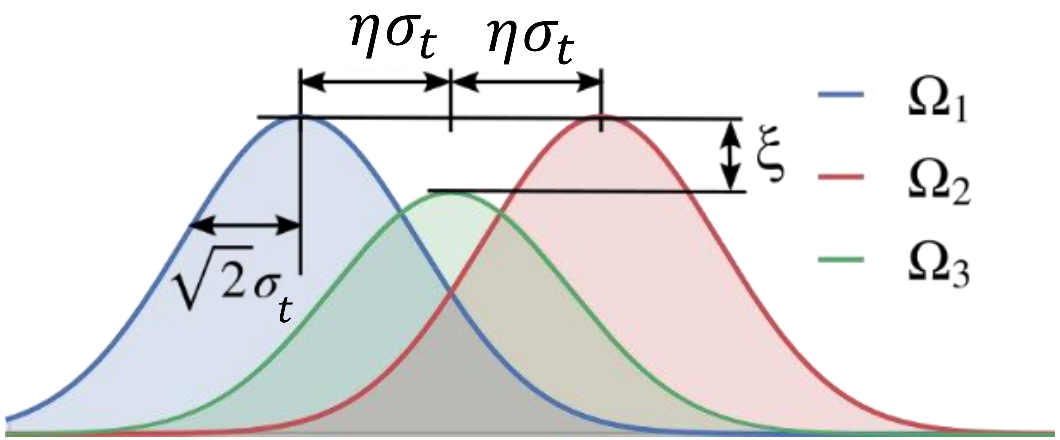
We create a coherent superposition of pseudo-spin states
It separates after a time of flight (TOF)

We swap the “1” beam and the “2” beam

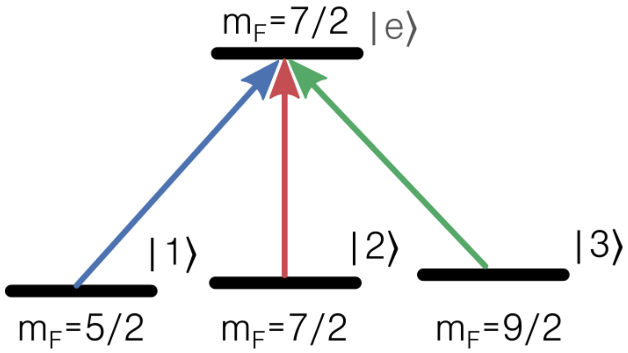
C. Madasu et *al*, ArXiv 2203.13360 (2022)

Sensitivity to Pulse Separation





$$\Sigma = \left.\frac{\Delta\theta}{\Delta\xi}\right|_{\theta=\frac{\pi}{2}}$$

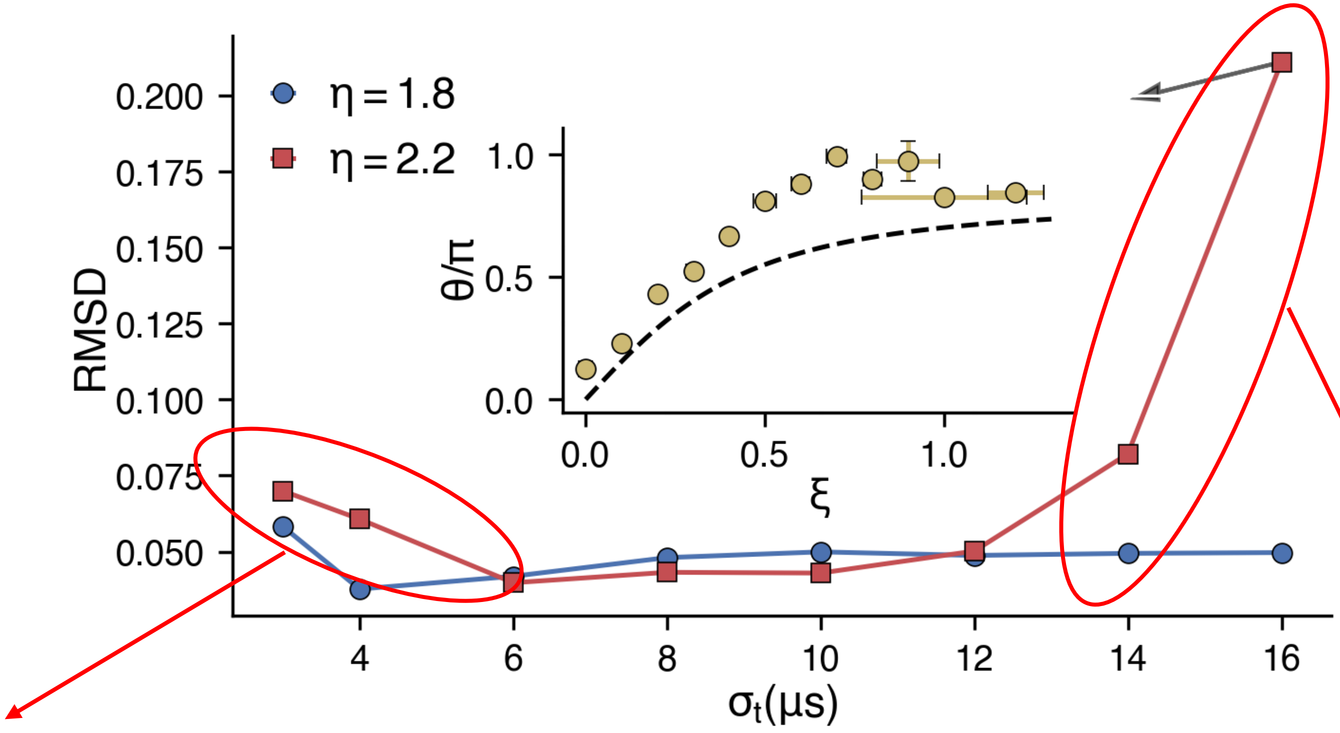


The pulse separation η ⟷ Gate-source voltage in FET

The central pulse amplitude ξ ⟷ Drain-source voltage in FET

The atomtronics devices control *knob* is different from standard FET !

Sensitivity to Atomic Velocity



Root Mean Square Deviation

$$\mathrm{RMSD} = \frac{\sqrt{\left\langle\left(\theta_{\mathrm{exp}} - \theta_{\mathrm{heory}}\right)^2\right\rangle}}{\pi}$$

$$\sigma_t \propto \frac{1}{v_z}$$

Non-adiabaticity

“Low” velocity: Breakdown of $\langle p \rangle \gg \hbar k$

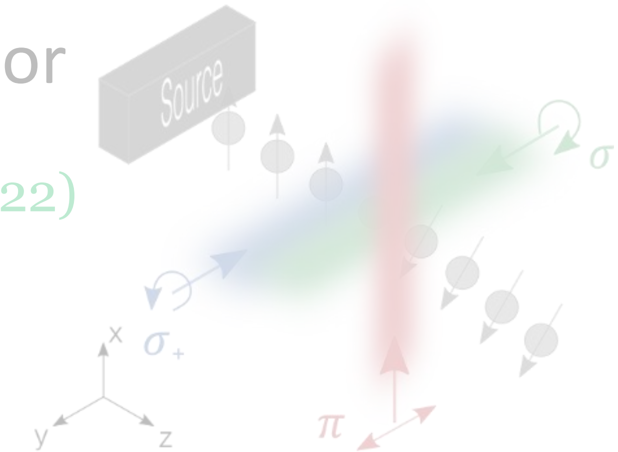
Independent of σ_t, thus of the atomic velocity over a large range (geometric origin)

C. Madasu et *al*, ArXiv 2203.13360 (2022)

Content

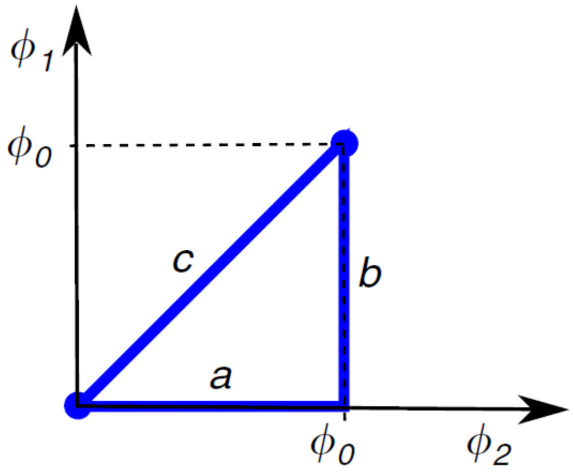
- Atomtronics Datta-Das transistor

C. Madasu et *al*, ArXiv 2203.13360 (2022)



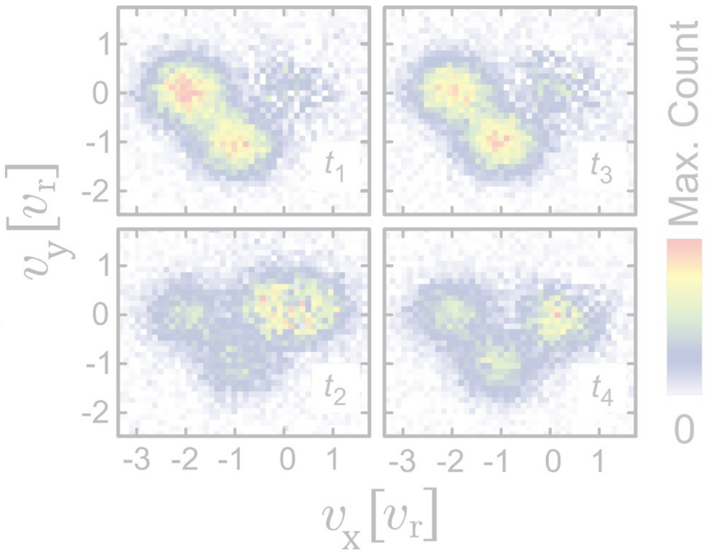
- Non-Abelian transformation

F. Leroux et *al*, Nat. Comm. **9** 7 (2018)



- 2D Zitterbewegung dynamic

M. Hasan et *al*, ArXiv:2201.00885 (2022)



Amplitude to Phase Control

Gauge field (Berry's connection):

$$\vec{A}_{11} = \hbar\left(\cos^2\beta\vec{\nabla}\Phi_{23} + \sin^2\beta\vec{\nabla}\Phi_{13}\right)$$

$$\vec{A}_{12} = \hbar\cos\alpha\left(\frac{1}{2}\sin(2\beta)\,\vec{\nabla}\Phi_{12} - i\vec{\nabla}\beta\right)$$

$$\vec{A}_{22} = \hbar\cos^2\alpha\left(\cos^2\beta\vec{\nabla}\Phi_{23} + \sin^2\beta\vec{\nabla}\Phi_{13}\right)$$

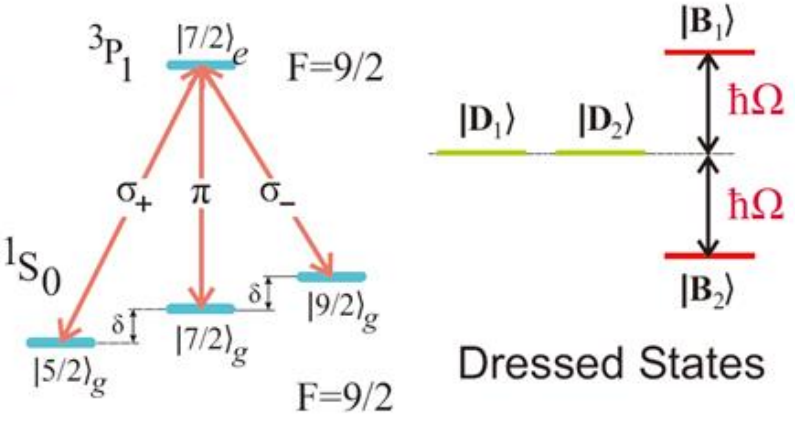
$\vec{A}$ depends on the variation of the laser relative phases Φ_{ij} and one mixing angle $\beta = \tan^{-1}\frac{|\Omega_2|}{|\Omega_1|}$

Mixing angle → one gauge field component → Abelian gauge field

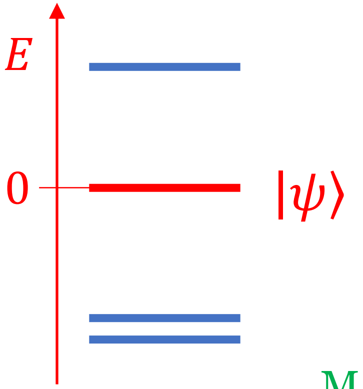
Phases→ all gauge field components → non-Abelian gauge field

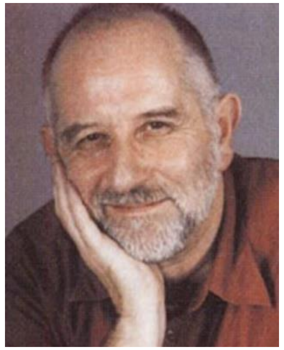
Non-Abelian gauge field → the components of $\vec{A}$ do not commute

Closed loop → Non-Abelian Berry phase (Wilczek-Zee phase)

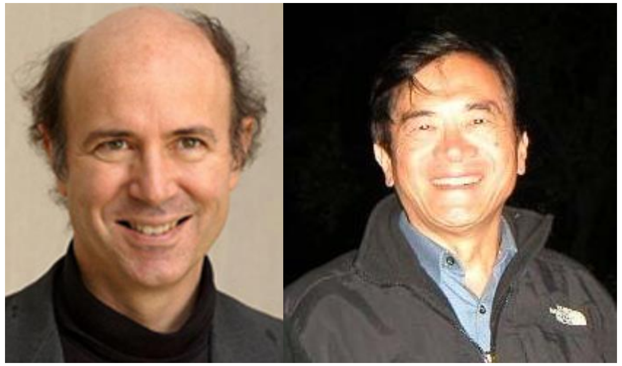


Geometric Transformation: Degenerated or Not

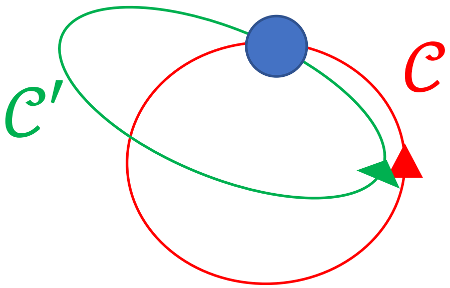




M. V. Berry Proc. R. Soc. Lond. A **392**, 45 (1984)



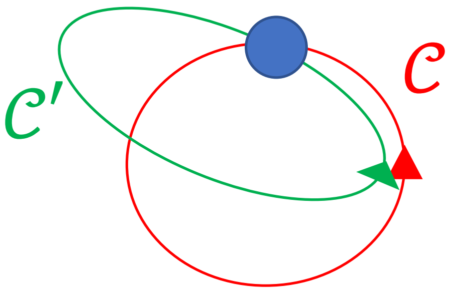
F. Wilczek and A. Zee PRL **52**, 2111 (1984)

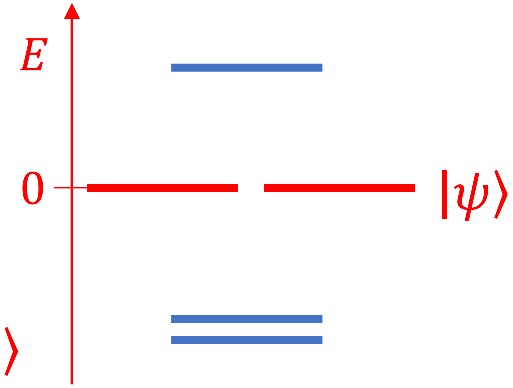


$$|\psi(\mathcal{C})\rangle = e^{i\gamma}|\psi(0)\rangle$$

$$e^{i\gamma}e^{i\gamma'} = e^{i\gamma'}e^{i\gamma}$$

Commute (Abelian transformation)





$$|\psi(\mathcal{C})\rangle = U|\psi(0)\rangle$$

$$UU' \neq U'U$$

Don’t commute (non-Abelian transformation)

Path Ordering Operator

$$U = \wp\exp\left(\frac{i}{\hbar}\oint_C \mathrm{A(t)dt}\right)$$

$$\mathrm{A(t)} = \left[i\hbar\left\langle D_j \middle| \frac{dD_k}{dt}\right\rangle\right]$$

C: close loop

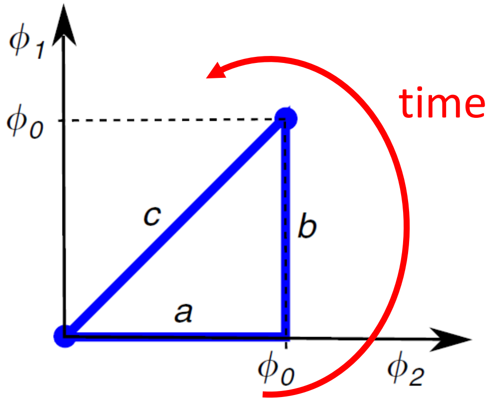
$\wp$: path ordering operator

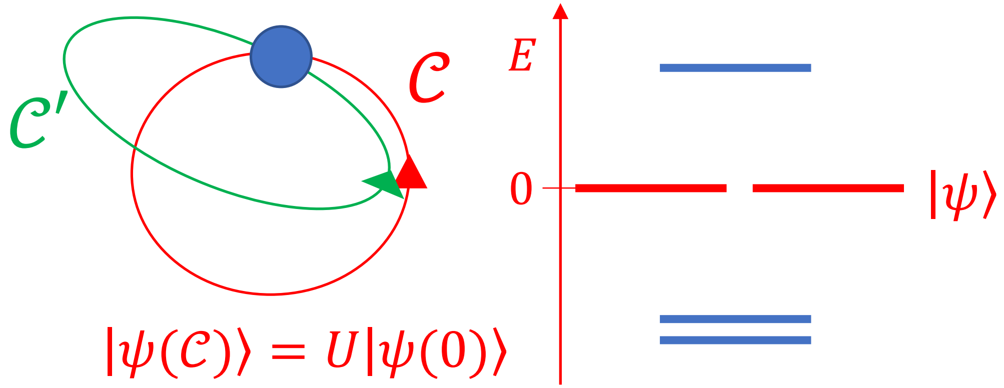
$A(t)$: Berry connection

F. Wilczek and A. Zee PRL **52**, 2111 (1984)

The time dependence means that the parameters are ramped in a defined temporal order

For example:



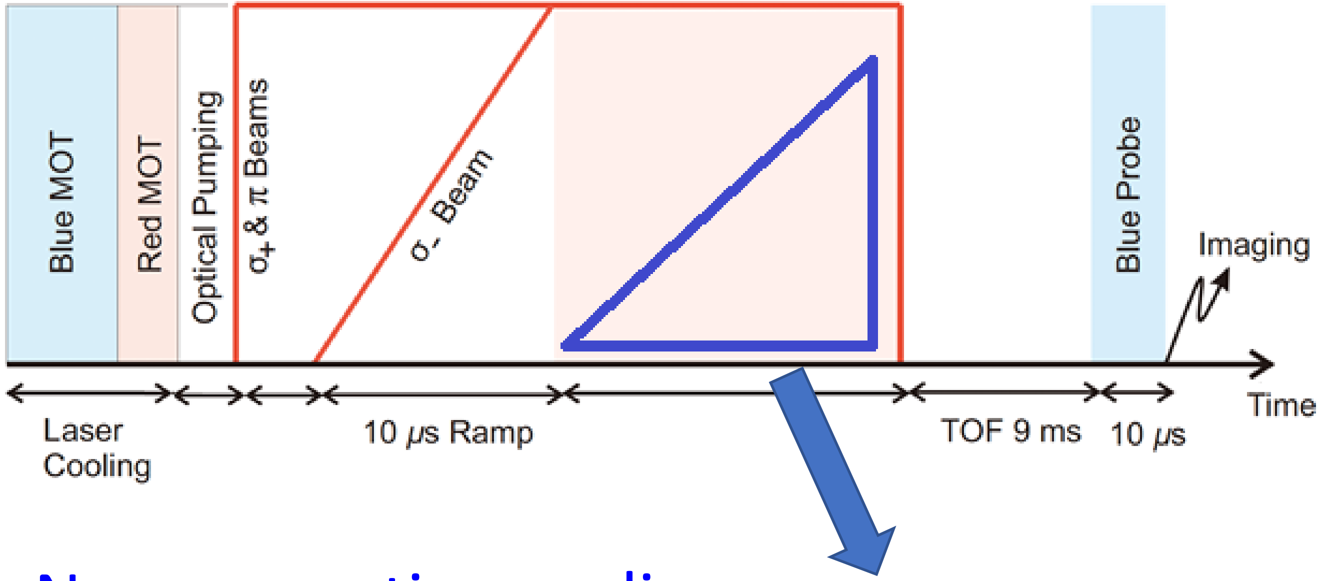


$$|\psi(\mathcal{C})\rangle = U|\psi(0)\rangle$$

$$UU' \neq U'U$$

Don’t commute (non-Abelian transformation)

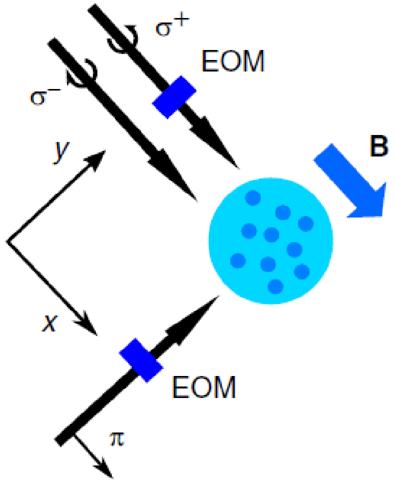
Experimental Sequence



Laser phase ramp

No evaporative cooling

$T = 0.5\ \mu\text{K}$



C. Chalony et *al*, PRL **107** 243002 (2011)
T. Yang, et *al*, EPJD **69**, 226 (2015)

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Geometric Gate: Realization

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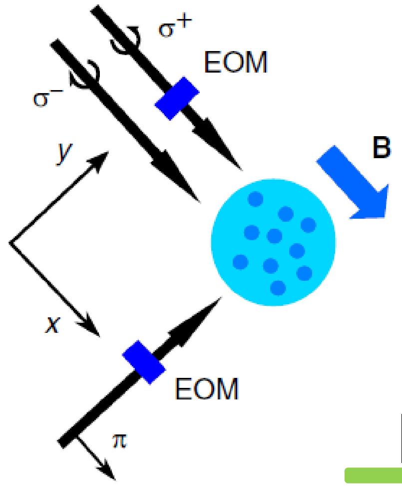
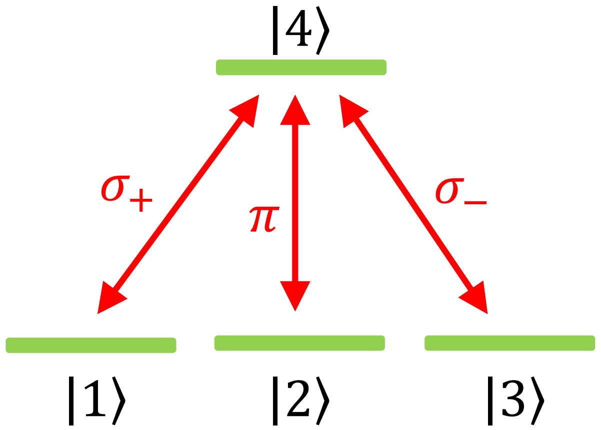
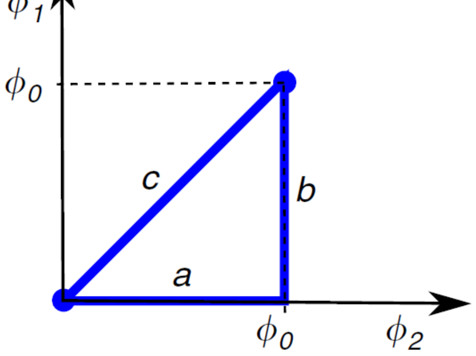
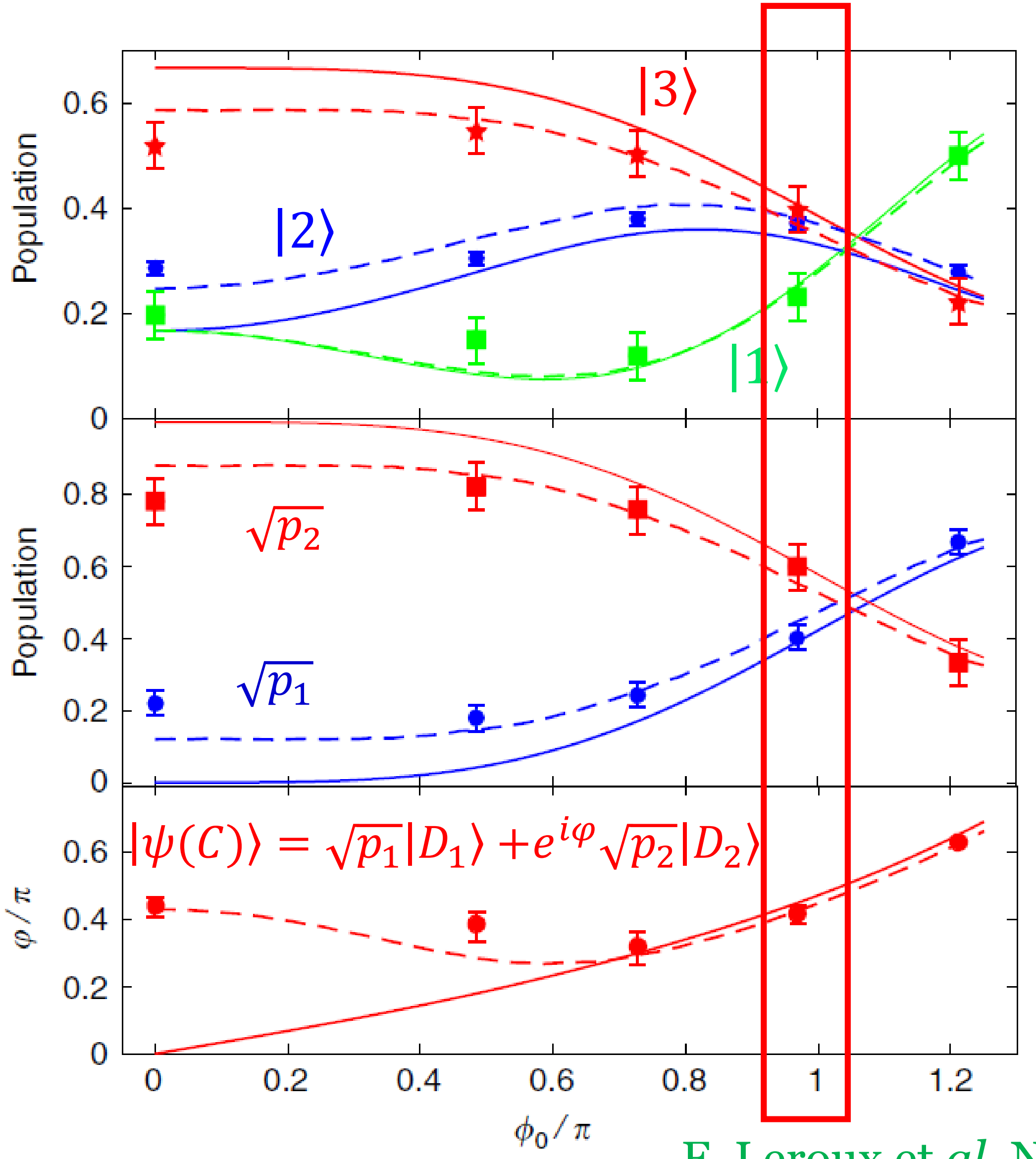


$T = 0.5\ \mu K$







Bare states populations

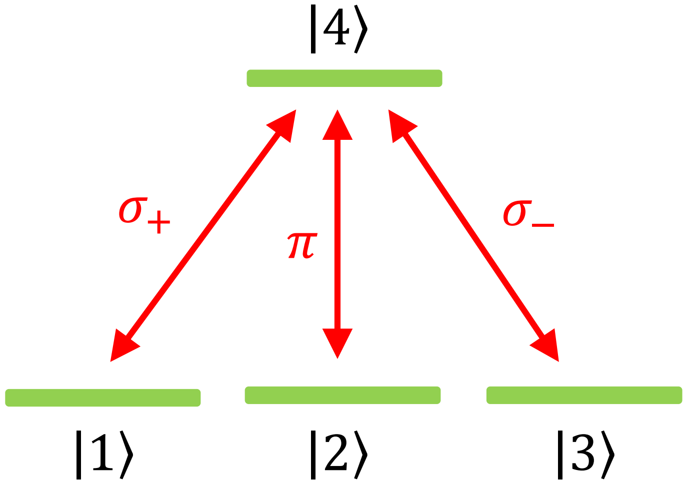
Dark states populations

Dark states azimuthal angle

F. Leroux et *al*, Nat. Comm. **9** 3580 (2018)

Unitary Operator Reconstruction

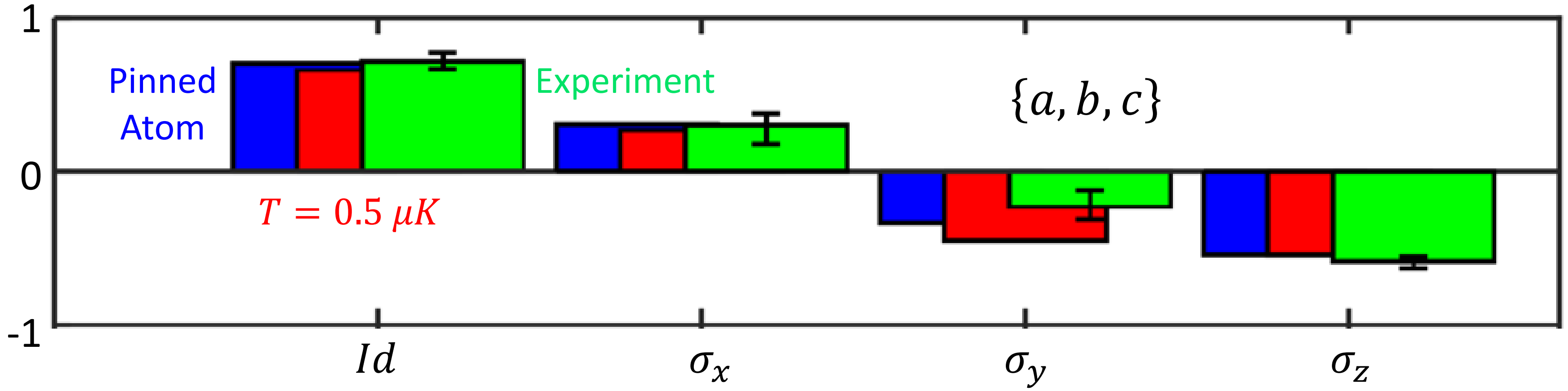
$$|\psi\rangle_f = U|\psi\rangle_i = \left(\sqrt{p_1}|D_1\rangle + \sqrt{p_2}e^{i\varphi}|D_2\rangle\right)e^{i\varphi_g}$$

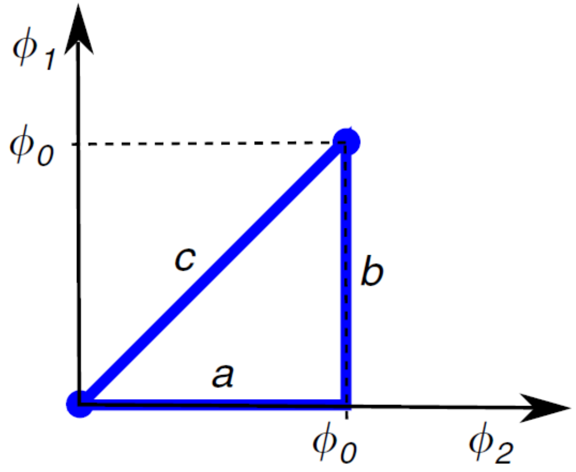


We reconstruct U using two initial (non orthogonal) states

We use the decomposition: $U = \alpha_0 \mathrm{Id} + i\sum_j \alpha_j \sigma_j$

σ_j: Pauli matrices



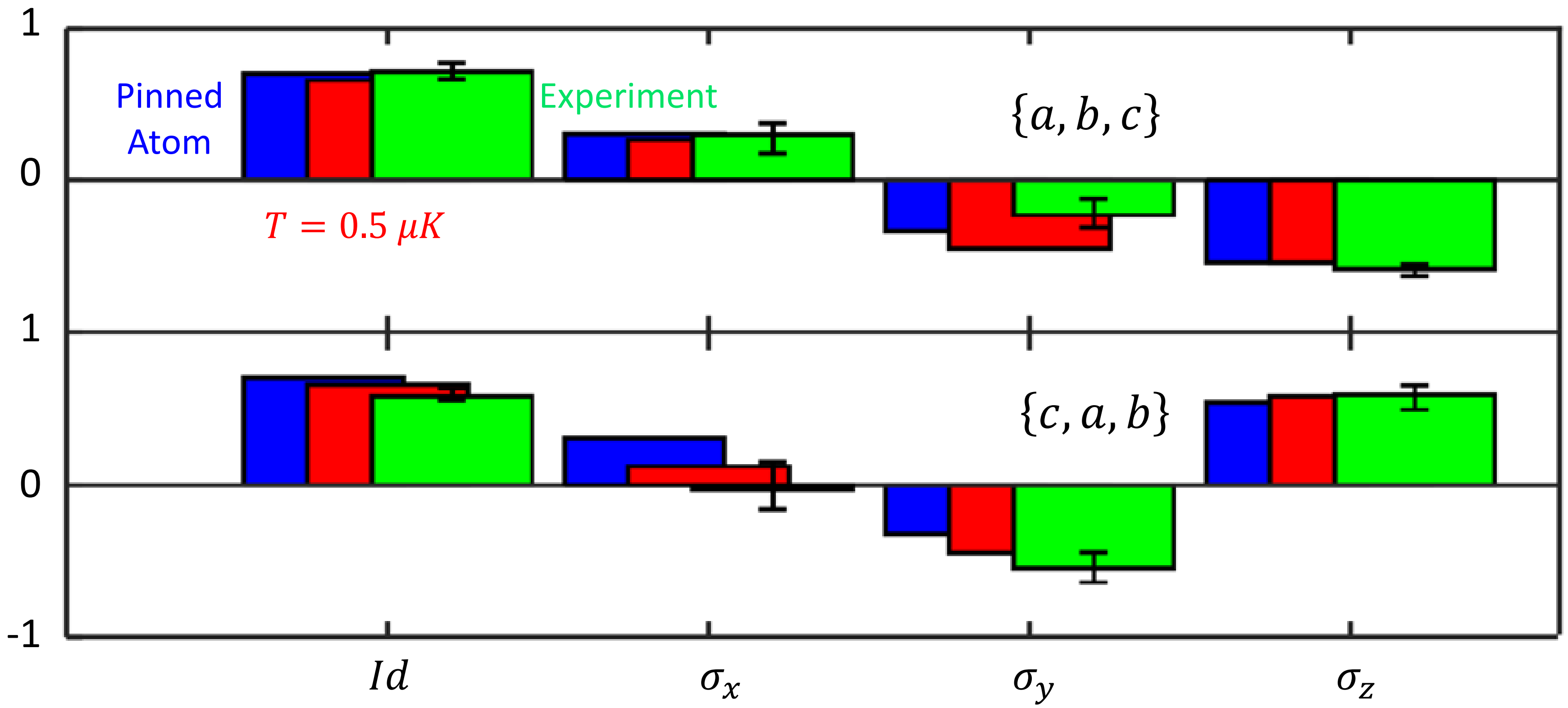


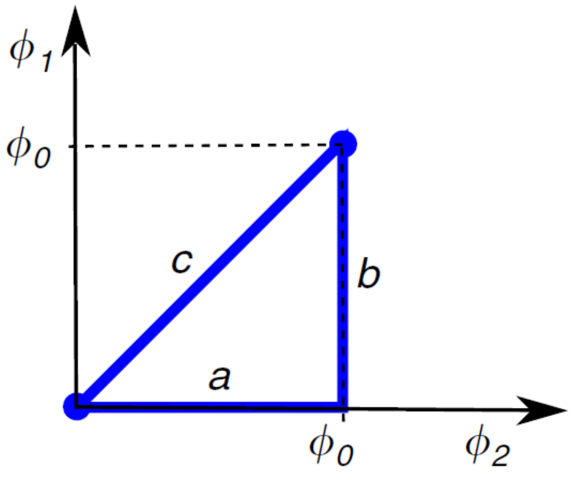
Non-Abelian Transformation: Path-ordering

$$U = U_c U_b U_a \equiv \{a, b, c\}$$

$$U' = \underbrace{U_b U_a}\underbrace{U_c} \equiv \{c, a, b\}$$

$$U - U' \neq 0$$





Forbenius distance:

$$D = \sqrt{2 - |\mathrm{Tr}(U^{\intercal} U')|}$$

$$D = 1.27(25)$$

$$D = 1.09$$

$$D = 1.14$$

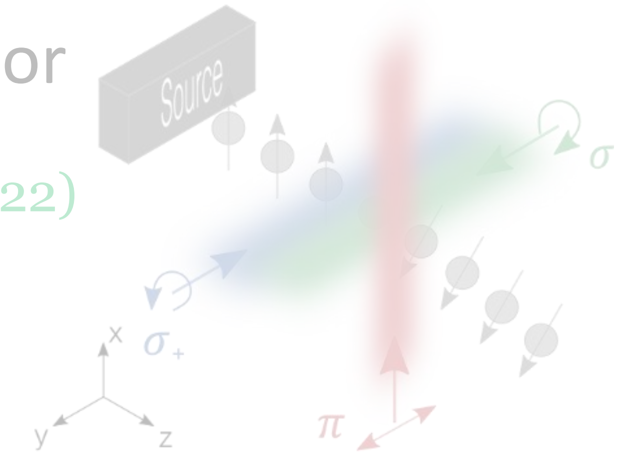
The result depends on the starting point.

F. Leroux et *al*, Nat. Comm. **9** 3580 (2018)

Content

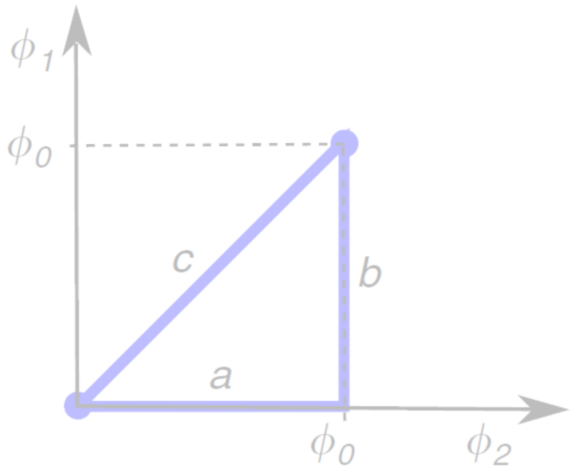
- Atomtronics Datta-Das transistor

C. Madasu et *al*, ArXiv 2203.13360 (2022)



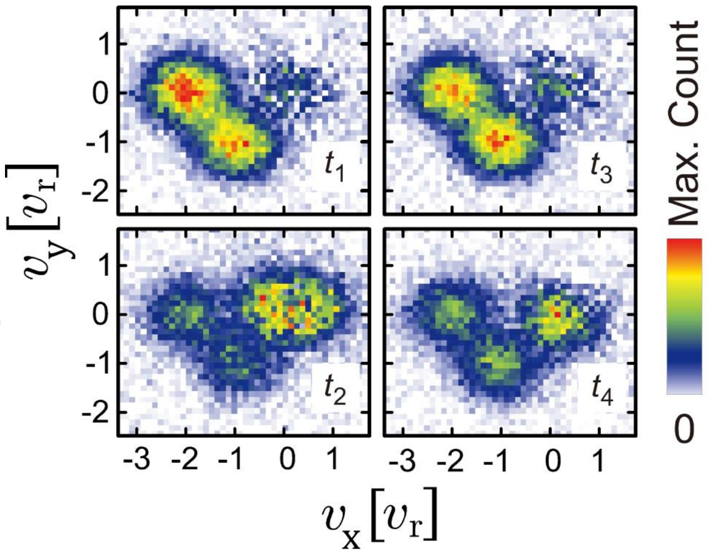
- Non-Abelian transformation

F. Leroux et *al*, Nat. Comm. **9** 7 (2018)



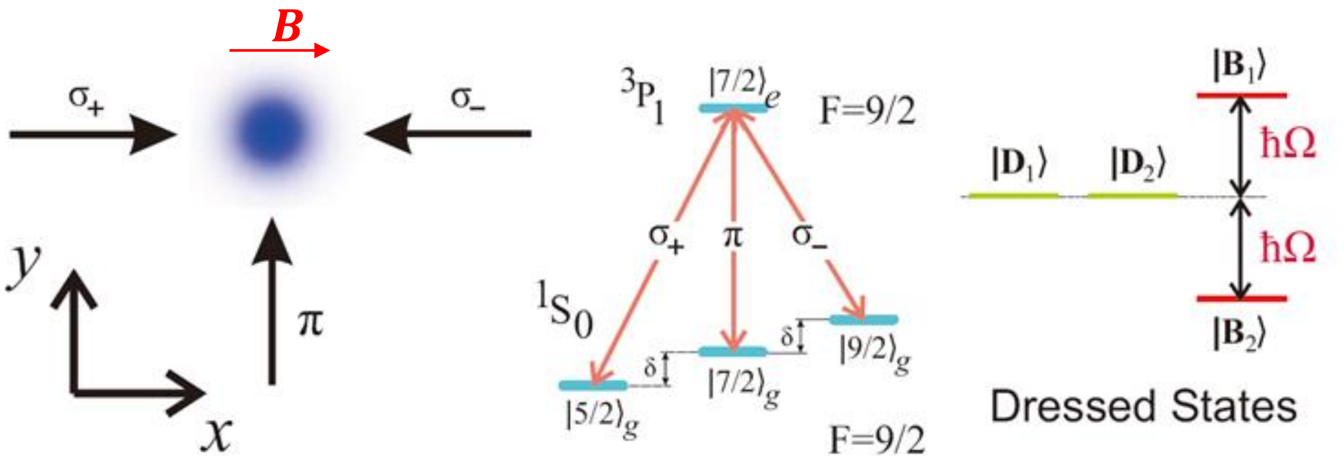
- 2D Zitterbewegung dynamic

M. Hasan et *al*, ArXiv:2201.00885 (2022)



Non-Abelian Gauge Field with Tripod

Tripod scheme on
ultracold fermionic strontium gas



- Hamiltonian (Dark state basis)

$$H = \frac{1}{2m}(\boldsymbol{p} - \boldsymbol{A})^2 + W$$

J. Dalibard et *al*, RMP **83** 1523 (2011)

- Commutation
  - Abelian: $[A_x, A_y] = 0$
  - Non-Abelian: $[A_x, A_y] \neq 0$

Hamiltonian in moving frame at v_0 with laser detuning

$$H = \frac{1}{2m}(\boldsymbol{p} - \boldsymbol{A})^2 + W - \boldsymbol{v_0 A} + w_\delta$$

We choose: $w_\delta = -\frac{\boldsymbol{A^2}}{2m} - W + \boldsymbol{v_0 A}$

We get in the moving frame: $H = \frac{\boldsymbol{p^2}}{2m} - \frac{\boldsymbol{pA}}{m}$

Spin-Orbit Coupling (SOC) Hamiltonian

Dirac Equation: *Zitterbewegung*

Dirac Equation

$$H\psi = i\hbar\frac{\partial}{\partial t}\psi$$

$$H = c\boldsymbol{\alpha}\boldsymbol{p} + \beta mc^2$$

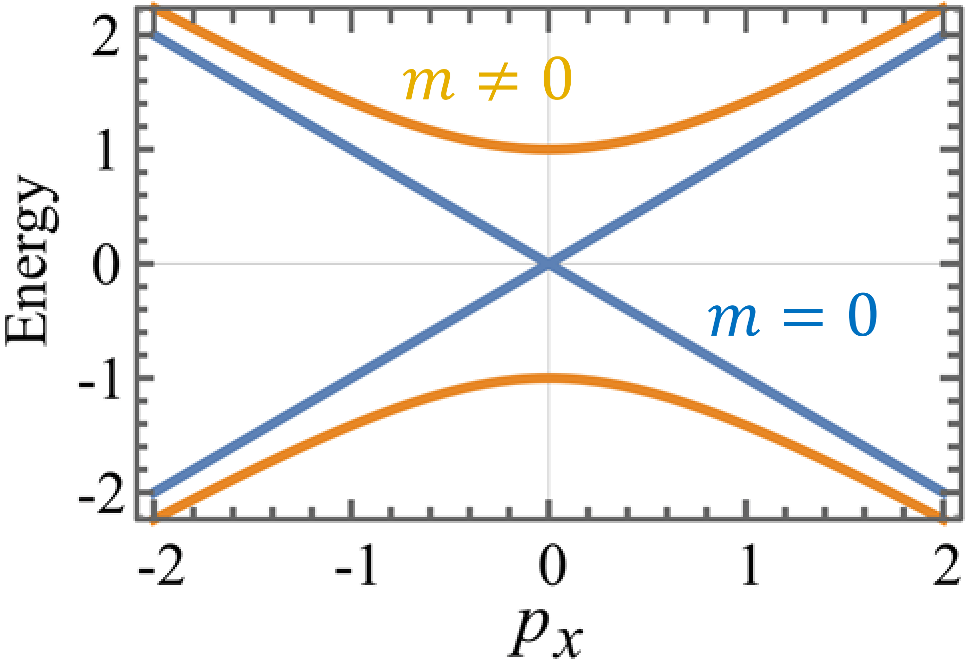
Mass term

Heisenberg Picture

$$\dot{x}_j = \frac{i}{\hbar}[H, x_j]$$

$$\boldsymbol{x} = \boldsymbol{x_0} + \frac{\hbar}{2\text{mc}}\boldsymbol{\alpha}[\sin(\omega t) + i\beta\cos(\omega t)]$$

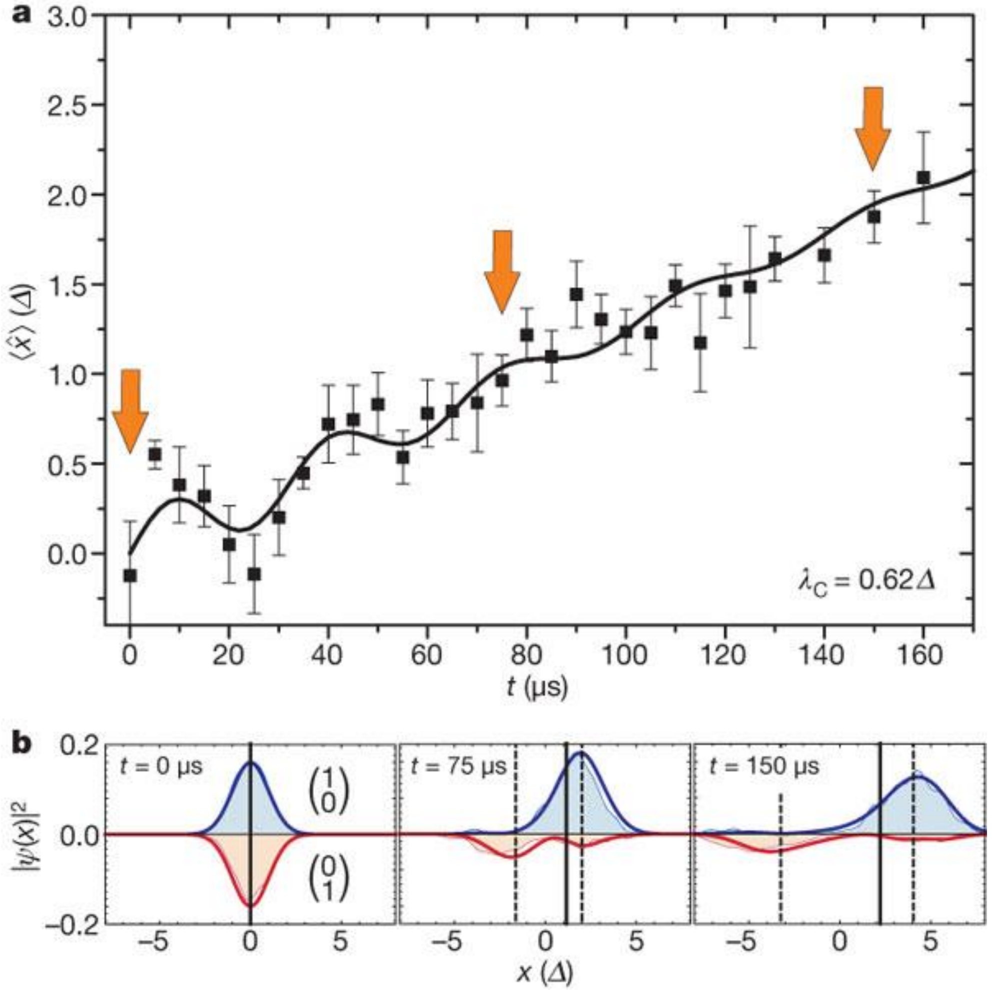
With: $\omega = 2\frac{mc^2}{\hbar}$



Zitterbewegung: Dual nature interference, particle/anti-particle & position

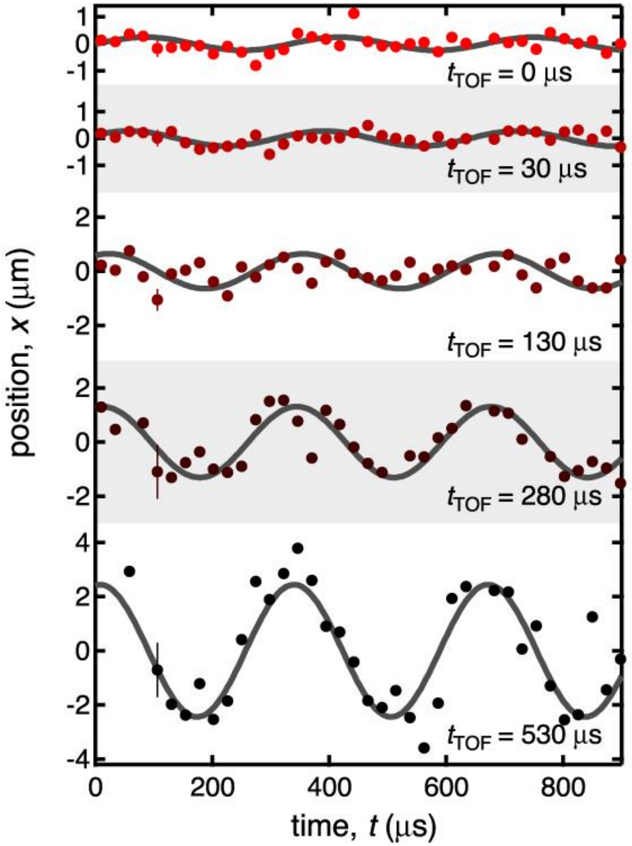
Zitterbewegung in SOC

With trapped ion



R. Gerritsma *et al.* *Nature* **463** 68 (2010)

BEC

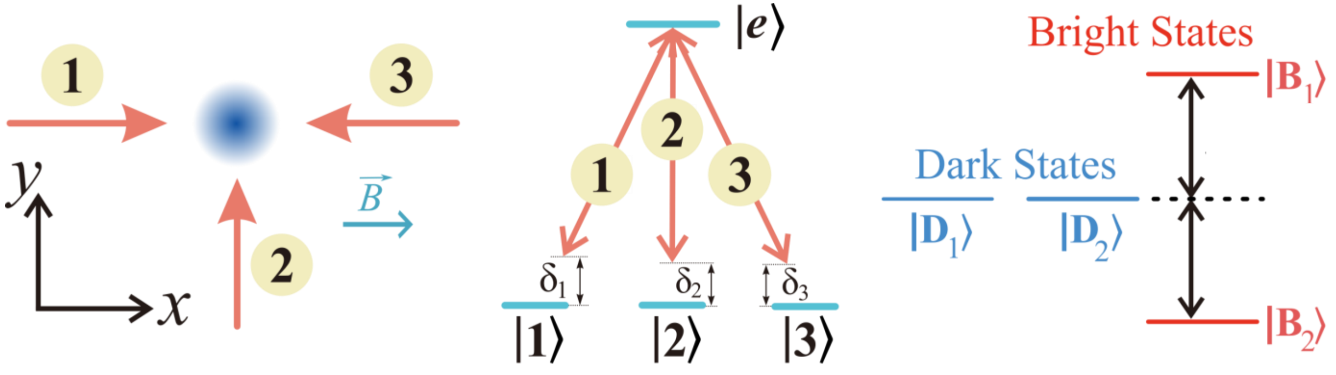


L. J. LeBlanc *et al.* *NJP* **15** 073011 (2013)

Zitterbewegung in 2D Gauge Field

$$H = \frac{\boldsymbol{p}^2}{2m} - \frac{\boldsymbol{pA}}{m} + 0$$

No *mass* term



Velocity operator

$$\boldsymbol{v} = \frac{1}{m}(\boldsymbol{p} - \boldsymbol{A})$$

Heisenberg picture

$$m\dot{\boldsymbol{v}} = \frac{im}{\hbar}[H,\boldsymbol{v}] = \frac{i}{\hbar m}\boldsymbol{p} \times (\boldsymbol{A} \times \boldsymbol{A})$$

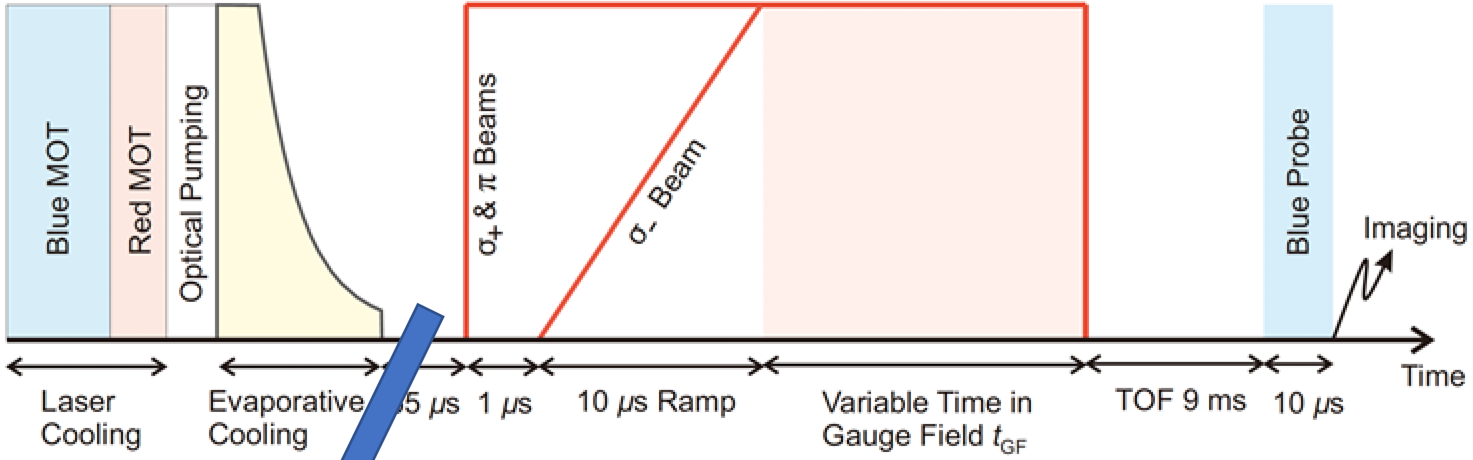
Non-inertial force (non-zero curvature)

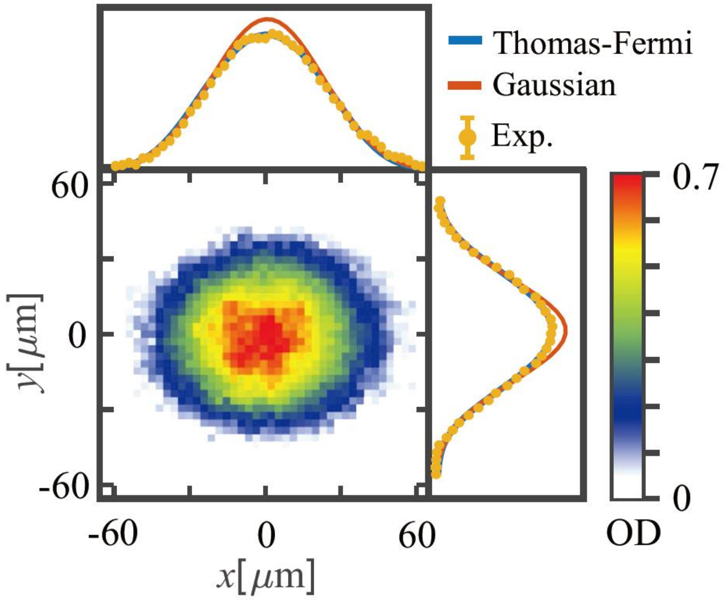
Despite $[H,\boldsymbol{p}] = 0$, $\dot{\boldsymbol{v}} \neq 0$ if $\boldsymbol{p} \neq 0$, and the Gauge field is non-Abelian (even homogenous)

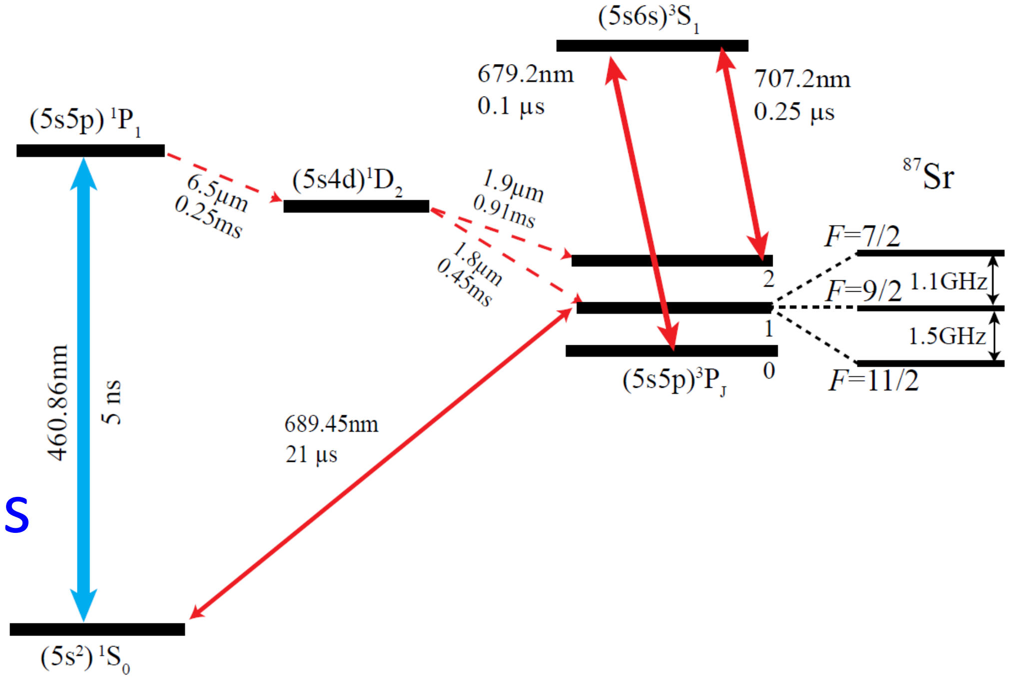
➡ 2D anisotropic *Zitterbewegung* effect

➡ Dynamic transverse to $\boldsymbol{p}$: Spin hall effect

Experimental Sequence







Strontium ultracold gas

$$T = 30(5)\ \mathrm{nK}$$

Degenerate Fermi Gas: $\frac{T}{T_\mathrm{F}} = 0.21(4)$

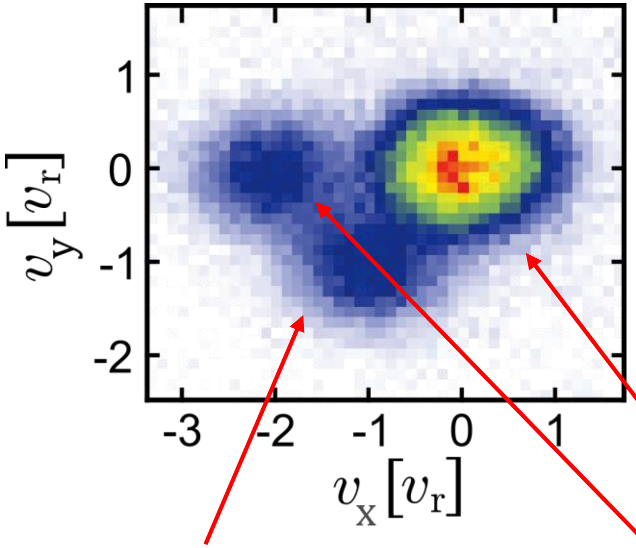
Sub-recoil temperature: $\frac{T}{T_\mathrm{R}} = 0.13(3)$

C. Chalony et *al*, PRL **107** 243002 (2011)
T. Yang, et *al*, EPJD **69**, 226 (2015)

Population and Velocity Detection

Time of flight

$\boldsymbol{p} = 0\ (\delta_j = 0)$

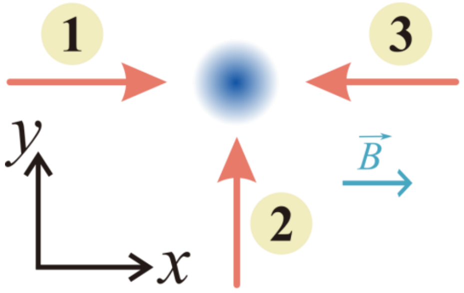


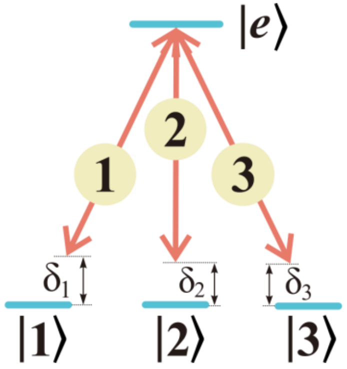
$T = 30\ \mathrm{nK}$

P_3: $|3\rangle$ population

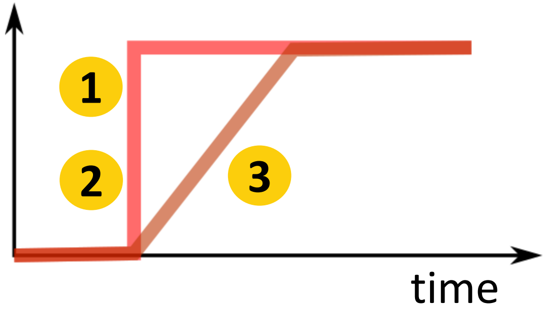
P_1: $|1\rangle$ population

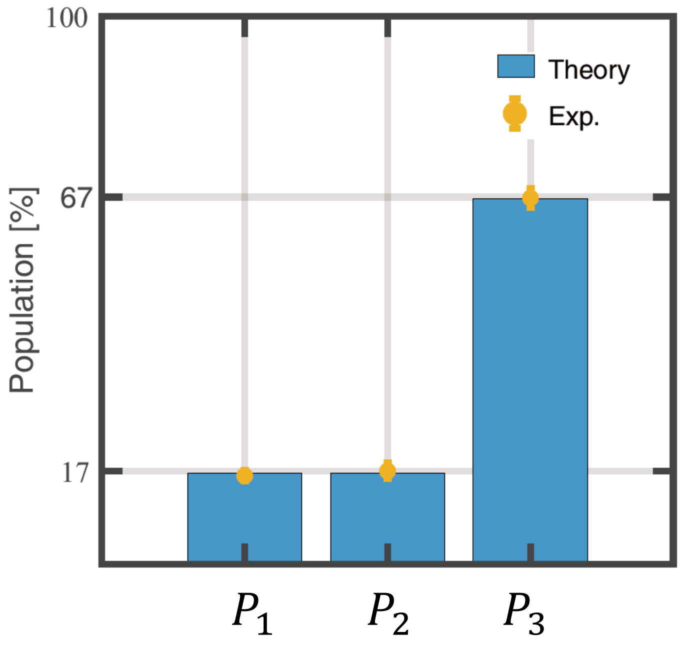
P_2: $|2\rangle$ population





at $t = 0$, atoms are in $|3\rangle$





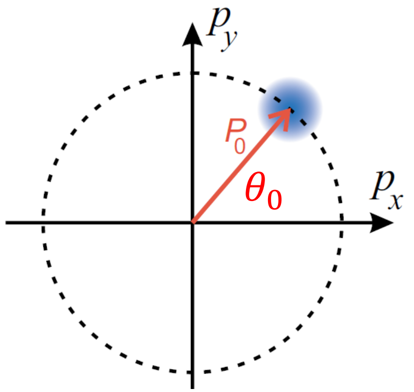
Expectation value of the velocity: $\langle \boldsymbol{v} \rangle / v_r = -(2P_1 + P_2)\hat{x} - P_2\hat{y}$

v_r: recoil velocity

M. Hasan et *al*, ArXiv:2201.00885 (2022)

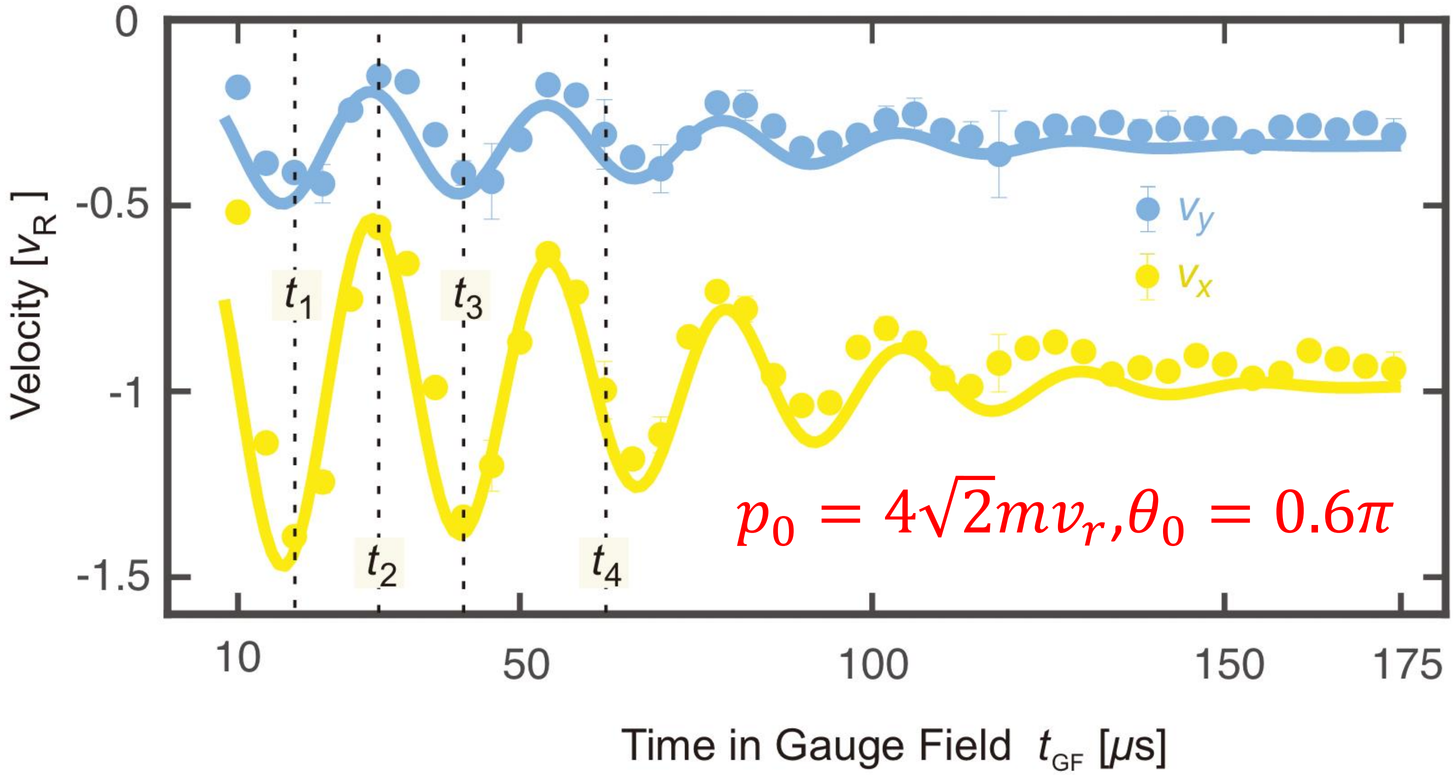
Observation of 2D *Zitterbewegung*

$$m\dot{\boldsymbol{v}} = \frac{im}{\hbar}[H,\boldsymbol{v}] = \frac{i}{\hbar m}\boldsymbol{p}\times(\boldsymbol{A}\times\boldsymbol{A})$$

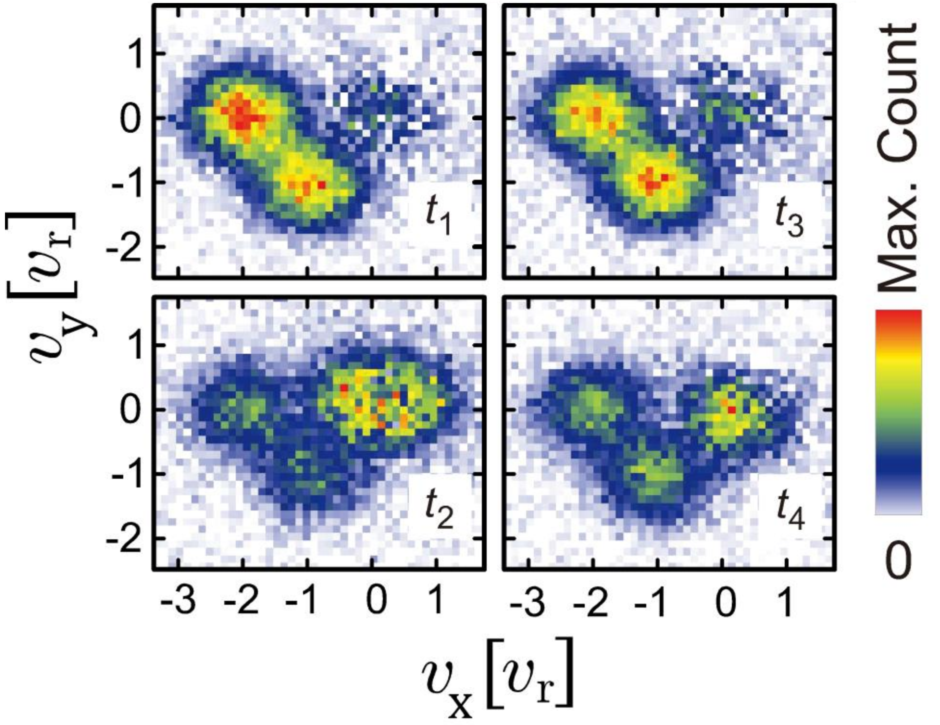


$\langle\boldsymbol{p}\rangle \equiv (p_0,\theta_0) \neq 0$ Mean atomic momentum $(\delta_j \neq 0)$

$\langle\boldsymbol{v}\rangle/v_r = -(2P_1+P_2)\hat{x} - P_2\hat{y}$



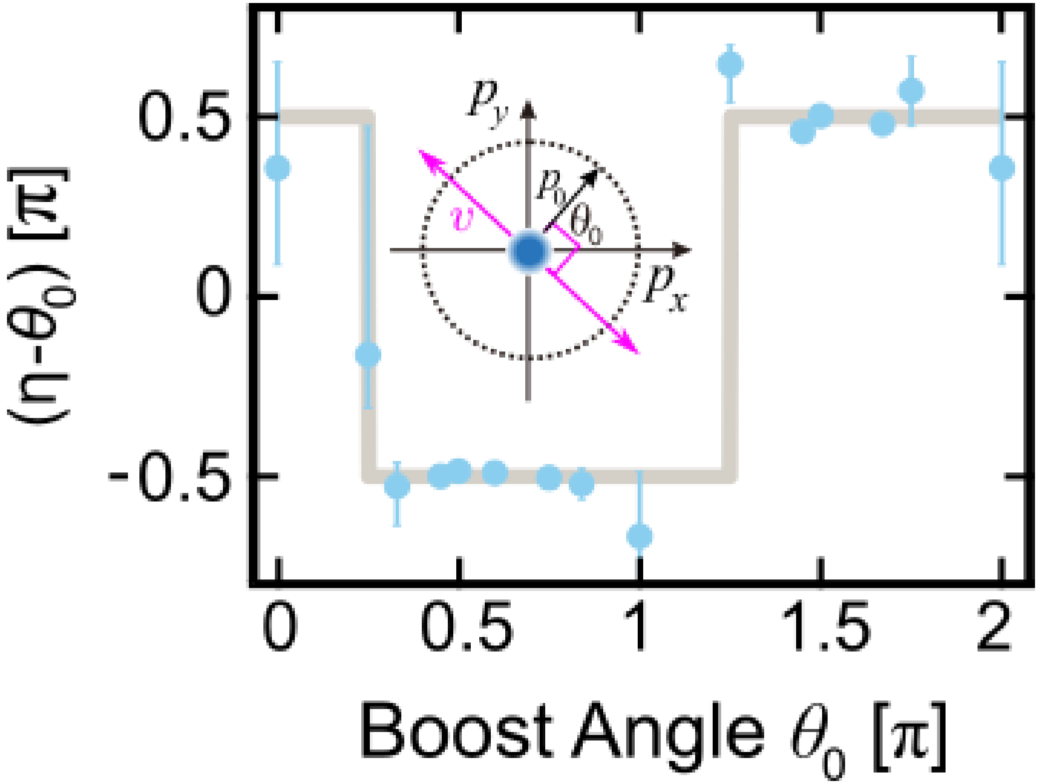
$p_0 = 4\sqrt{2}mv_r, \theta_0 = 0.6\pi$



Spin Hall Effect

$$m\dot{\boldsymbol{v}} = \frac{im}{\hbar}[H, \boldsymbol{v}] = \frac{i}{\hbar m}\boldsymbol{p} \times (\boldsymbol{A} \times \boldsymbol{A})$$

$$p_0 = 4\sqrt{2}mv_r$$



$\boldsymbol{p} \equiv (p_0, \theta_0)$ Mean atomic momentum

$\boldsymbol{v} \equiv (|\boldsymbol{v}|, \eta)$ Mean *Zitterbewegung* velocity

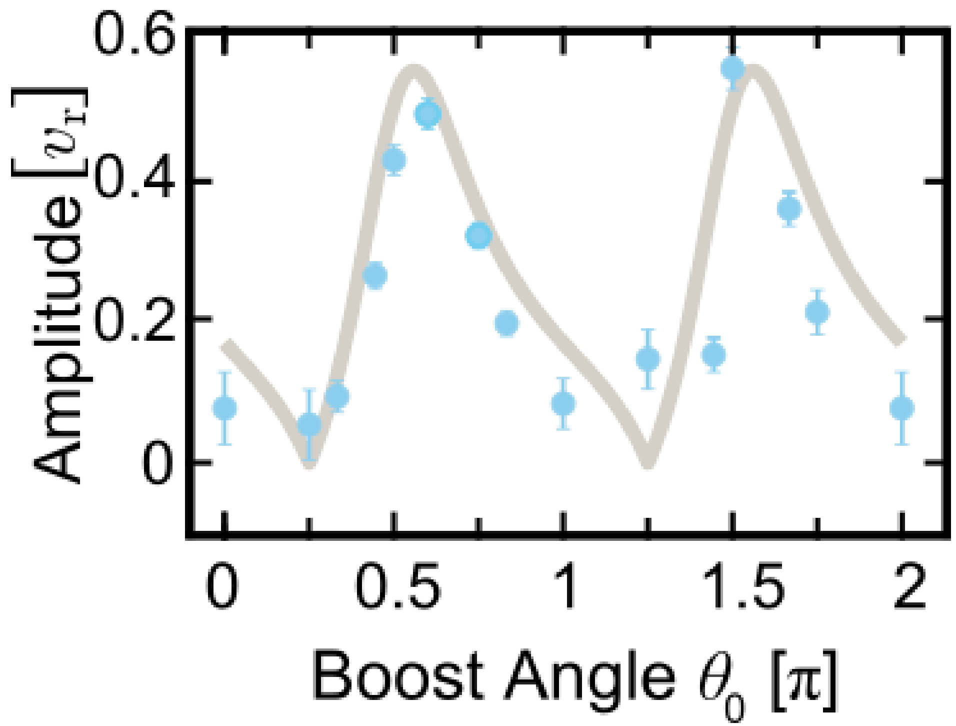
$\boldsymbol{v}$ and $\boldsymbol{p}$ are orthogonal in the *xy*-plane

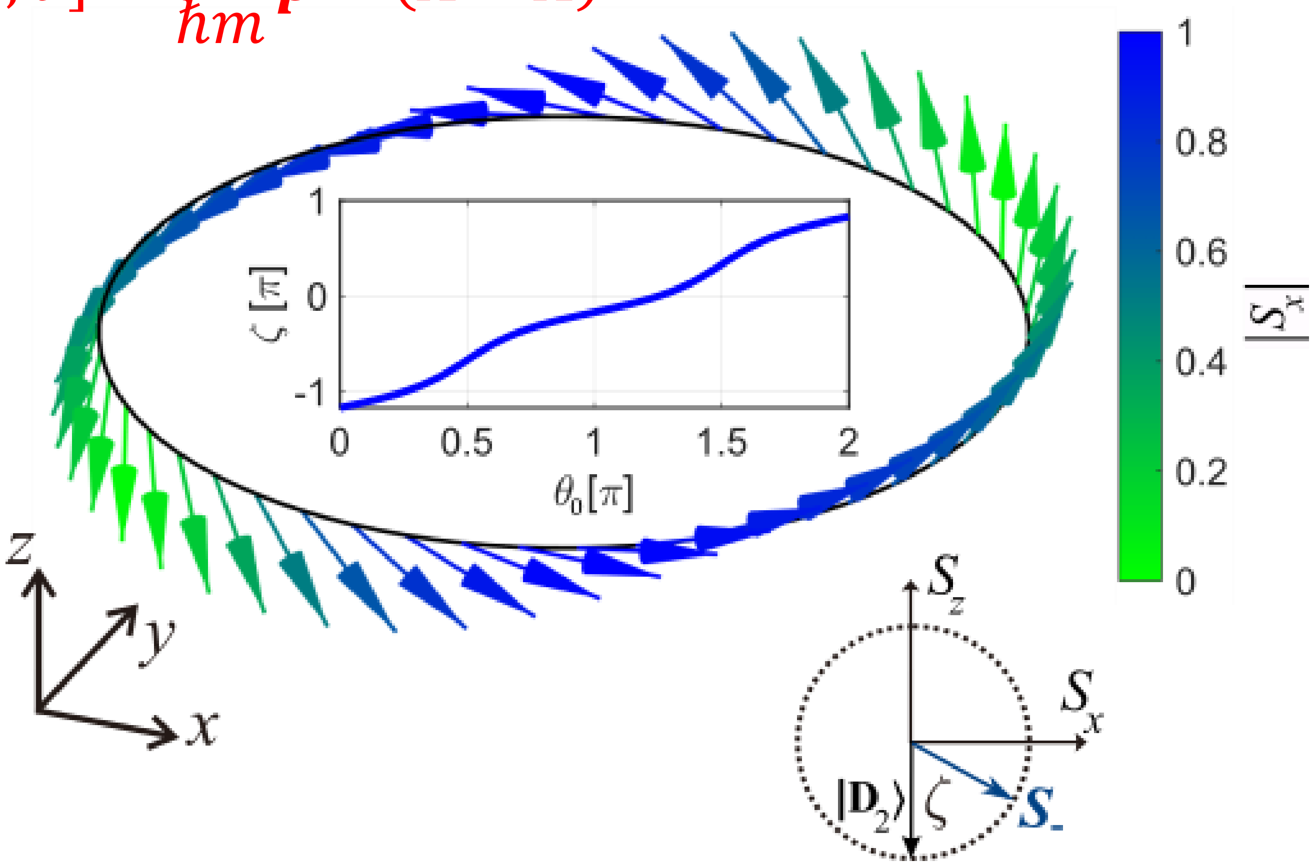
M. Hasan et *al*, ArXiv:2201.00885 (2022)

Amplitude Anisotropy

$$m\dot{\boldsymbol{v}} = \frac{im}{\hbar}[H,\boldsymbol{v}] = \frac{i}{\hbar m}\boldsymbol{p}\times(\boldsymbol{A}\times\boldsymbol{A})$$

$$p_0 = 4\sqrt{2}mv_r$$





The amplitude vanishes when the initial pseudo-spin is collinear to the spin texture (eigenstate)

M. Hasan et *al*, ArXiv:2201.00885 (2022)

Conclusion

- Datta-Das transistor
  - Velocity-independent spin rotation: like a Stren-Gerlach but not a spin Hall effect
  - Two parameters for the amplitude (source-drain voltage) and the sensitivity (gate-source voltage)
  - Key device for future atomtronics circuits

C. Madasu et *al*, ArXiv 2203.13360 (2022)

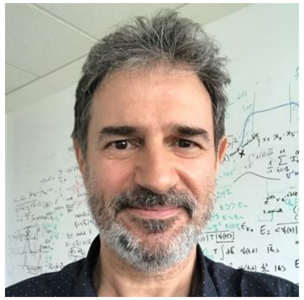
- Path sensitivity on a non-Abelian transformation
  - Transformation depends on the starting point
  - Universal geometric single Qubit gate

F. Leroux et *al*, Nat. Comm. **9** 7 (2018)

- 2D Zitterbewegung dynamic
  - Non-Abelian gauge field, without *mass* (scalar) term
  - Spin Hall effect and amplitude anisotropy

M. Hasan et *al*, ArXiv:2201.00885 (2022)

People & Funding



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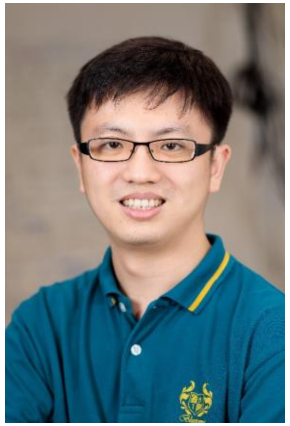

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