# Hadronic Vacuum Polarization: An unblinded window on the g-2 mystery

**C.T.H. Davies, C. DeTar, A.X. El-Khadra, E. Gámiz, Steven Gottlieb*, A.S. Kronfeld, J. Laiho, S. Lahert, M. Lynch, G.P. Lepage, C. McNeile, E.T. Neil, C. Peterson, J.N. Simone, R.S. Van de Water, and A. Vaquero**

**(Fermilab Lattice/HPQCD/MILC Collaborations)**

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# Introduction

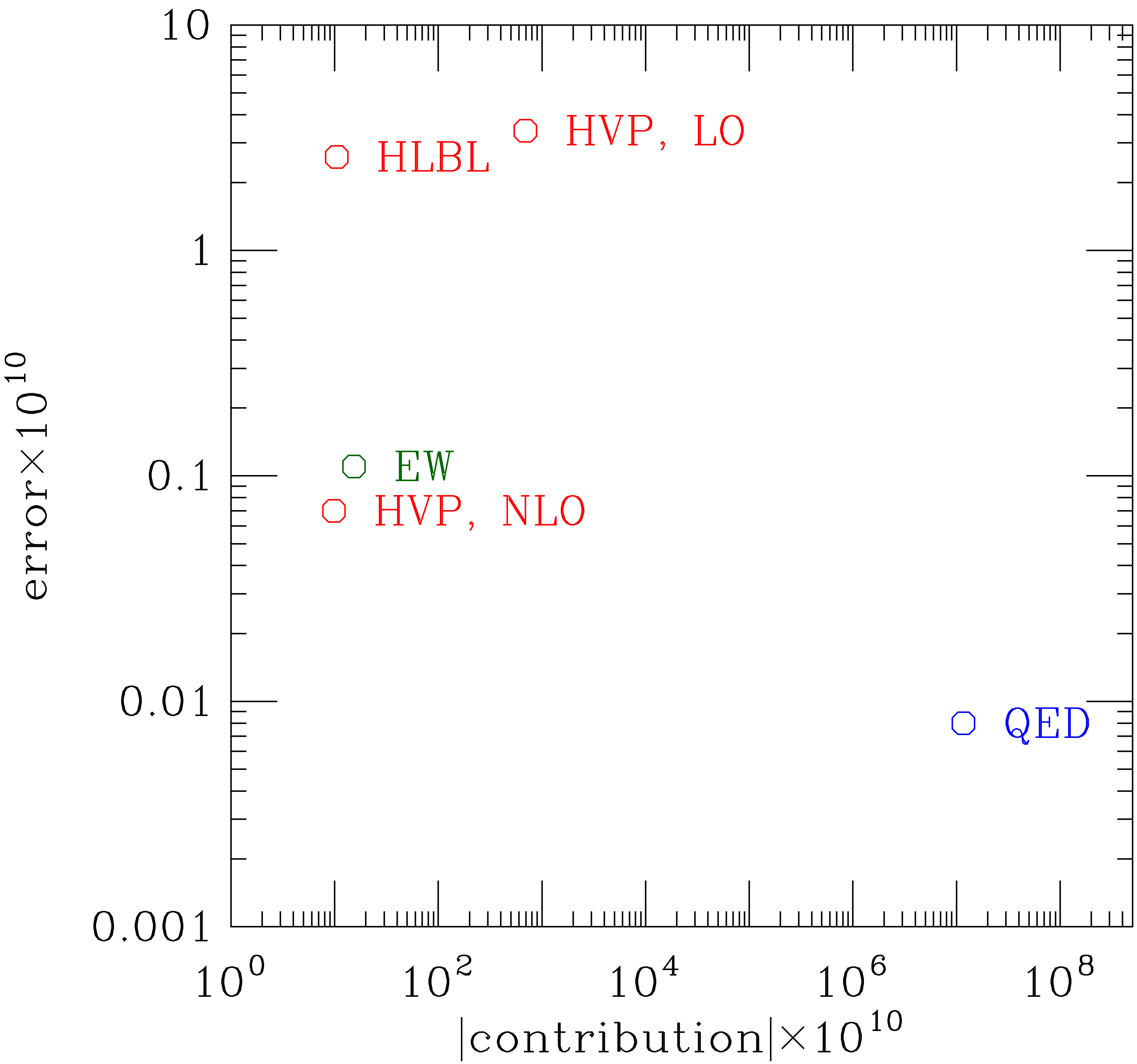
- ✦ Anomalous magnetic moments of electron and muon are two of the most precisely measured quantities in physics
- ✦ E821 at BNL published its final value for the muon in 2006
- ✦ FNAL E989 announced its initial result in April, 2021
  - spectacular agreement with E821
  - continues to run
  - New experiment E34 planned at J-PARC
- ✦ There is $\approx 4.2\ \sigma$ difference between data driven standard model (SM) calculation and experiment
- ✦ BMWc 2021 value lies between SM value and experiment
- ✦ It is important to improve the precision of other lattice QCD calculations

# Theory Overview

- SM contributions come from QED (electron & muon), electroweak contributions, and hadronic contributions that involve quarks
  - all forces save gravity contribute
- Current situation summarized by Muon g-2 Theory Initiative
  - T. Aoyama *et al*., Phys. Rept. 887 (2020), 2006.04822 [hep-ph]
- Next plot shows how the hadronic corrections dominate the error

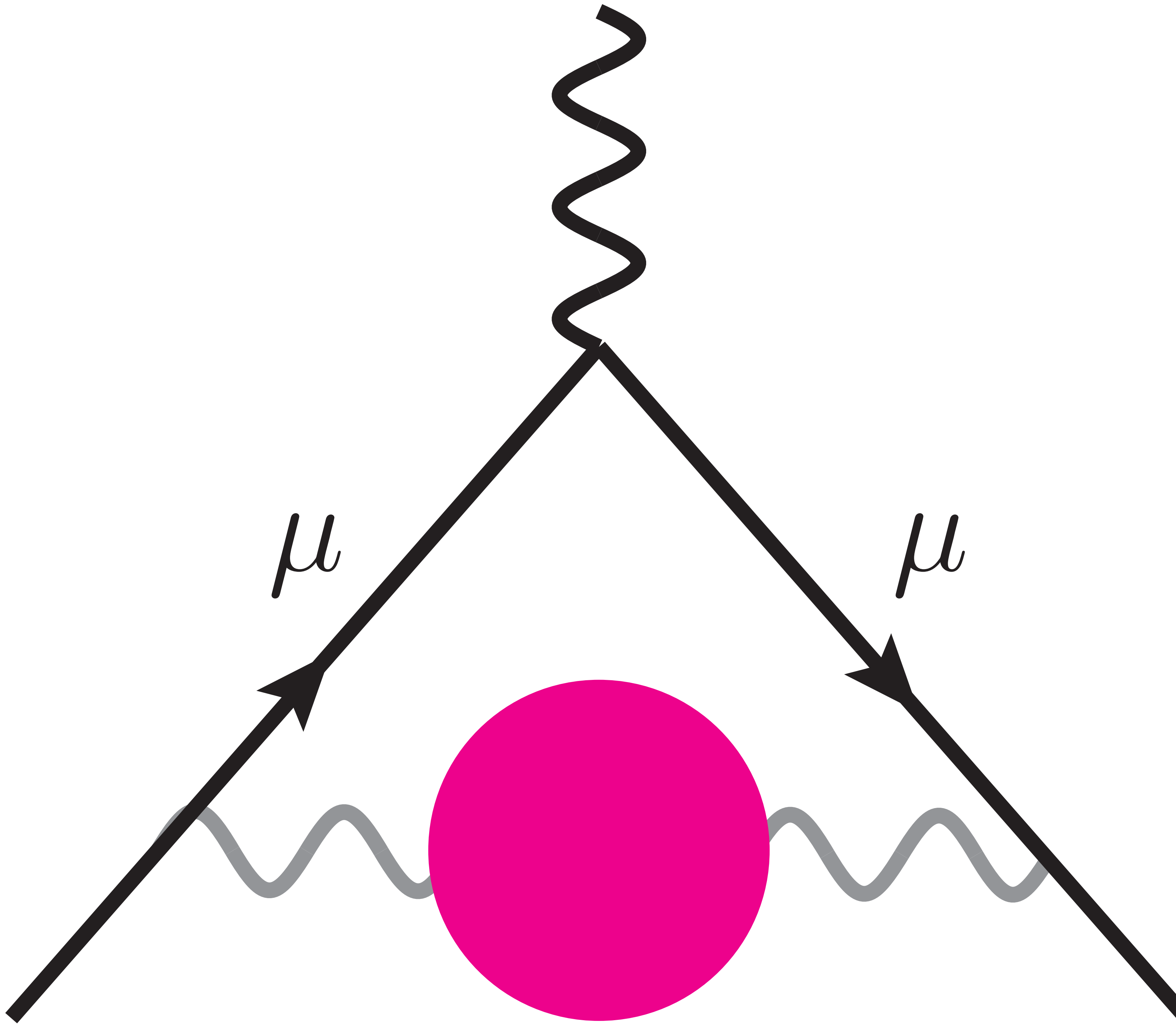
# Error vs. Contribution

- QED in blue has very small error
- Electroweak in green has larger error, but small contribution
- Hadronic contributions are all in red
  - LO Hadronic vacuum polarization largest error and 2nd largest contribution
  - HLBL 2nd largest error
- This talk on LO HVP



# Hadronic Vacuum Polarization

- HVP diagram looks like 2 loop QED diagram, except that red blob includes all QCD corrections to the quark loop, and there are also ‘disconnected’ contributions with two quark loops that can exchange gluons
- Contribution written as integral over 4-momentum-squared



# HVP Calculation

- ✦Hadronic part of the current-current two-point function must be integrated over the loop momentum. Rest of diagram uses known photon and muon propagators.
- ✦Two approaches:
- ✦Using dispersion relations and optical theorem, can convert integral to one involving

$$R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$$

  - This approach relies on careful use of experimental data and is currently the most precise method.
- ✦*Ab initio* lattice QCD non-perturbatively calculates the current-current two-point function for Euclidean time or spacelike $Q^2$.
  - Challenge is to get accurate values at large Euclidean time or small $Q^2$.

# Lowest Order HVP

- ✦ HVP is calculated as sum of several contributions: light quark connected, strange connected, …, light disconnected, …, strong isospin breaking, electromagnetic, etc.
- ✦ $a_\mu^{ll}(\mathrm{conn.})$ light quark connected is biggest contribution, by far
- ✦ FNAL/HPQCD/MILC: PRD **101**, 034512 (2020), 1902.04223 [hep-lat]
    - briefly recap

# Lattice Ensembles

✦ In 2020, we used $N_f$=2+1+1 HISQ ensembles from the MILC collaboration with physical light quark masses

| $\approx a$ (fm) | $am_l^{\rm sea}/am_s^{\rm sea}/am_c^{\rm sea}$ | $w_0/a$ | $M_{\pi_5}$ (MeV) | $(L/a)^3 \times (T/a)$ | $N_{\rm conf.}$ |
|---|---|---|---|---|---|
| 0.15 | 0.00235/0.0647/0.831 | 1.13670(50) | 133.04(70) | $32^3 \times 48$ | 997 |
| 0.15 | 0.002426/0.0673/0.8447 | 1.13215(35) | 134.73(71) | $32^3 \times 48$ | 9362 |
| 0.12 | 0.00184/0.0507/0.628 | 1.41490(60) | 132.73(70) | $48^3 \times 64$ | 998 |
| 0.09 | 0.00120/0.0363/0.432 | 1.95180(70) | 128.34(68) | $64^3 \times 96$ | 1557 |
| 0.06 | 0.0008/0.022/0.260 | 3.0170(23) | 134.95(72) | $96^3 \times 192$ | 1230 |

✦ Have retuned 0.12 fm and added statistics for current analysis. Still adding configurations for 0.06 fm.

| $\approx a$[fm] | $N_s^3 \times N_t$ | $am_l^{\rm sea}/am_s^{\rm sea}/am_c^{\rm sea}$ | $w_0/a$ | $M_{\pi_5}$(MeV) | $N_{\rm conf.}$ | $N_{\rm wall}$ |
|---|---|---|---|---|---|---|
| 0.15 | $32^3 \times 48$ | 0.002426 / 0.0673 / 0.8447 | 1.13215(35) | 134.73(71) | 9362 | 48 |
| 0.12 | $48^3 \times 64$ | 0.001907 / 0.05252 / 0.6382 | 1.41110(59) | 134.86(71) | 9011 | 64 |
| 0.09 | $64^3 \times 96$ | 0.00120 / 0.0363 / 0.432 | 1.95180(70) | 128.34(68) | 5384 | 48 |
| 0.06 | $96^3 \times 128$ | 0.0008 / 0.022 / 0.260 | 3.0170(23) | 134.95(72) | 2621 | 24 |

# Blinding

- ✦ To avoid confirmation bias in analysis, correlators are all blinded by multiplication by an unknown factor.
- ✦ Once all aspects of analysis are completed, the collaboration will decide to unblind and actual result will be available.
- ✦ Collaboration looked at many variations on the analysis and decided on Tuesday, September 13 that we were ready to unblind.
- ✦ This is the first presentation of our unblinded results for the Euclidean time windows $W$ and $W_2$

# Windows Analysis

- ✦ The statistical noise at large Euclidean time is challenging
  - RBC/UKQCD suggested using windows to achieve higher precision and allow better comparison of different calculations
    - PRL 121, 022003 (2018)
  - FNAL/HPQCD/MILC recently advocated one-sided windows with longer time extent than SD defined in PRL above.
    - 2207.04765 [hep-lat] (use such windows as part of this study)
- ✦ We have considered multiple windows and concentrate on just two here

$$\Theta\left(t, t_0, t_1, \Delta\right) = \frac{1}{2}\left[\tanh\left(\frac{t-t_0}{\Delta}\right) - \tanh\left(\frac{t-t_1}{\Delta}\right)\right]$$

# Windows Considered

✦We fix $\Delta = 0.15$ fm.

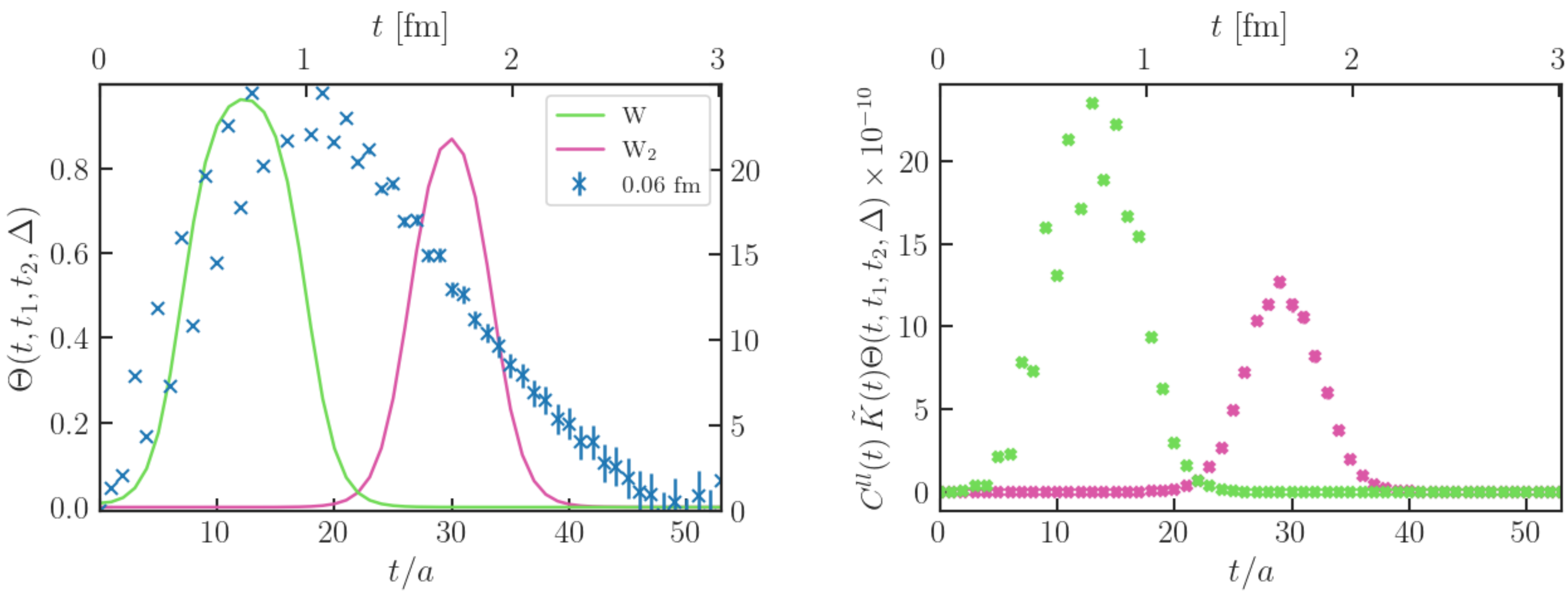
✦For the one-sided (O.S.), $t_1 = 1, 1.5, 2, 3.$

| label | $[t_0, t_1]$ fm |
|---|---|
| $a_\mu^{\mathrm{SD}}$ | $[0, 0.4]$ |
| $a_\mu^{\mathrm{W}}$ | $[0.4, 1]$ |
| $a_\mu^{\mathrm{W_2}}$ | $[1.5, 1.9]$ |
| $a_\mu^{\mathrm{O.S.}}(t_1)$ | $[0, t_1]$ |

✦Here, we only present $W$ and $W_2$ (Aubin *et al*. 2204.12256 [hep-lat])

✦Each window has its own blinding factor, so can unblind independently.

# Effect of Window




- ✦ Left: $a_\mu$ integrand in blue; $W$ window factor in green; $W_2$ in red
- ✦ Right: integrand after multiplication by window factor
  - ✦ note effect of staggering on $W$

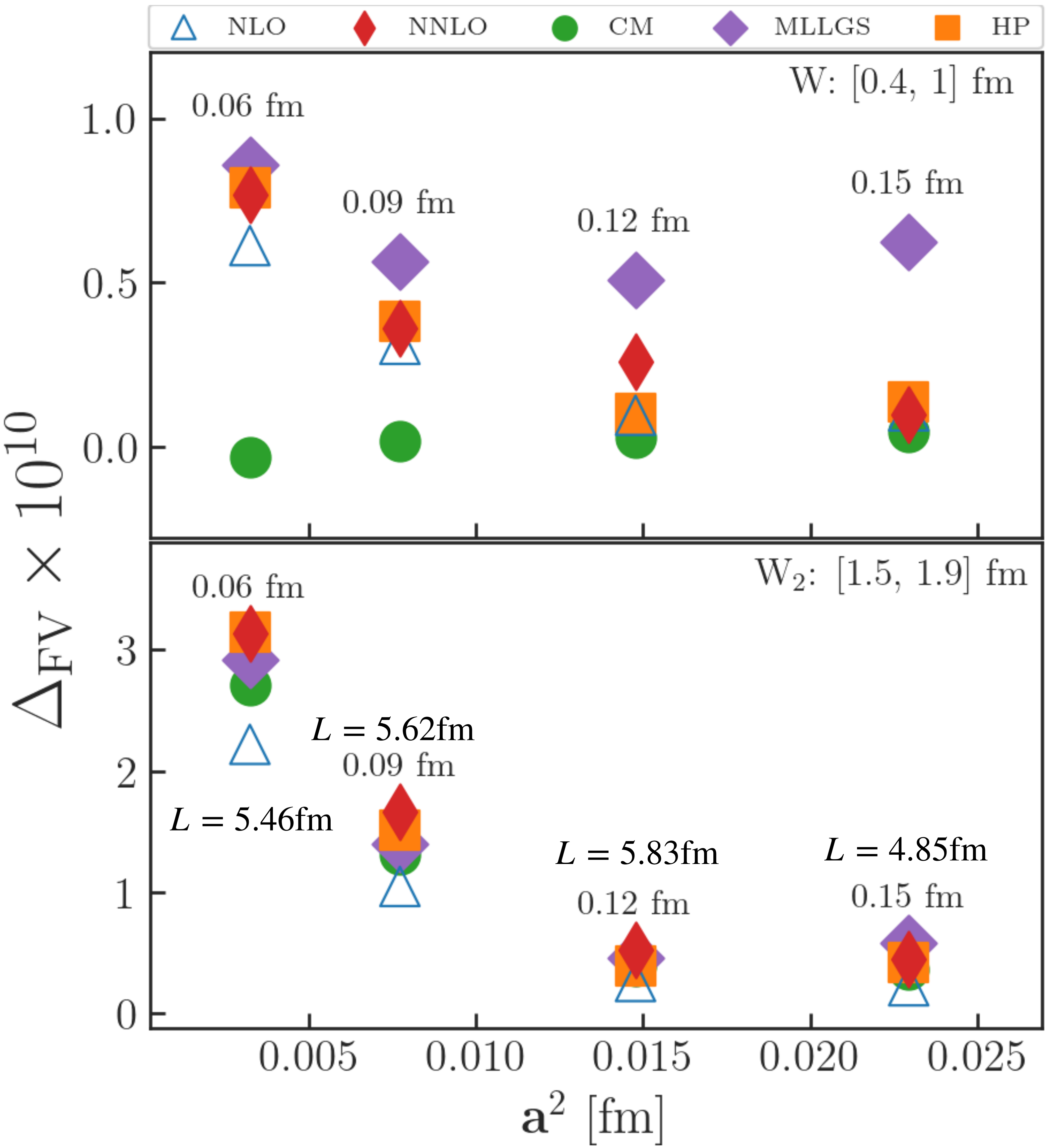
# Corrections

- Three corrections are applied: volume, mass mistuning, and taste breaking. (Latter is optional, see below.)
- $a_\mu(L_\infty, m_{\pi_{phys}}) = a_\mu(L_{latt}, m_{\pi_{latt,\xi_1}}, \cdots, m_{\pi_{latt,\xi_{16}}}) + \Delta_{\rm FV} + \Delta_{m_\pi} + \Delta_{\rm TB}$
- $\Delta_{\rm FV} = a_\mu(L_\infty, m_{\pi_{latt,\xi_1}}, \cdots, m_{\pi_{latt,\xi_{16}}}) - a_\mu(L_{latt}, m_{\pi_{latt,\xi_1}}, \cdots, m_{\pi_{latt,\xi_{16}}})$
- $\Delta_{m_\pi} = a_\mu(L_\infty, m_{\pi_{phys,\xi_1}}, \cdots, m_{\pi_{phys,\xi_{16}}}) - a_\mu(L_\infty, m_{\pi_{latt,\xi_1}}, \cdots, m_{\pi_{latt,\xi_{16}}})$
- $\Delta_{\rm TB} = a_\mu(L_\infty, m_{\pi_{phys}}, \cdots, m_{\pi_{phys}}) - a_\mu(L_\infty, m_{\pi_{phys,\xi_1}}, \cdots, m_{\pi_{phys,\xi_{16}}})$
- Correction terms calculated on each ensemble using several models

# Correction Models

- We consider several models
  - Chiral Perturbation Theory (ChiPT NLO, NNLO)
  - Meyer-Lellouch-Lüscher-Gournaris-Sakurai (MLLGS)
  - Chiral Model (CM, and CM' variation)
  - Hansen and Patella (HP)
    - last is used only for finite volume correction
- We also try neglecting $\Delta_{TB}$ at each lattice spacing and allowing continuum limit to eliminate taste breaking
- Don't need to use the same model for all correction terms.
  - many, many variations

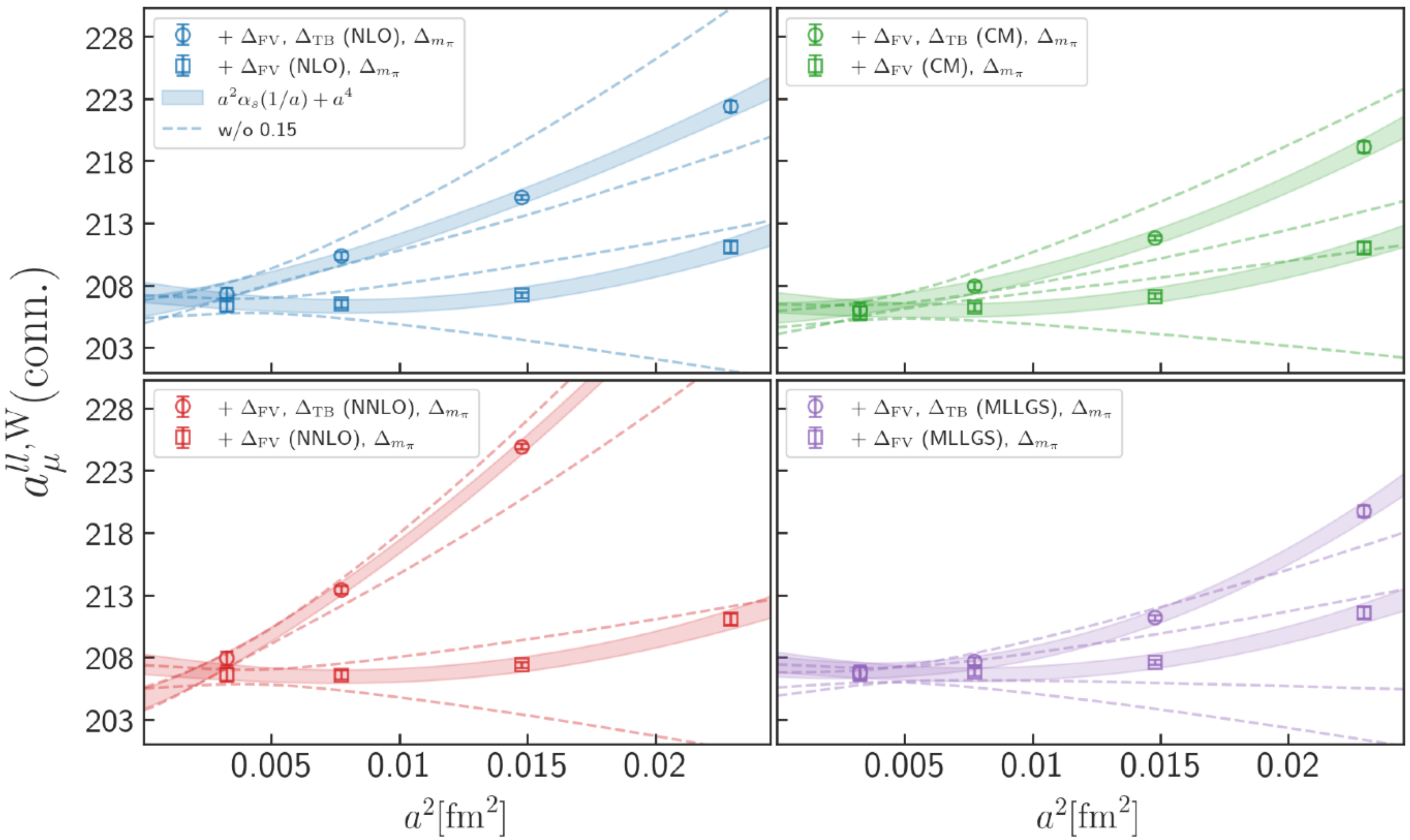
# Finite Volume Correction




- FV correction for $W$ (top) and $W_2$ (bottom) windows, shows much better consistency for the window at larger time advocated by Aubin et al.
- FV correction is so small at smallest volume (coarsest ensemble) because taste breaking is larger there.

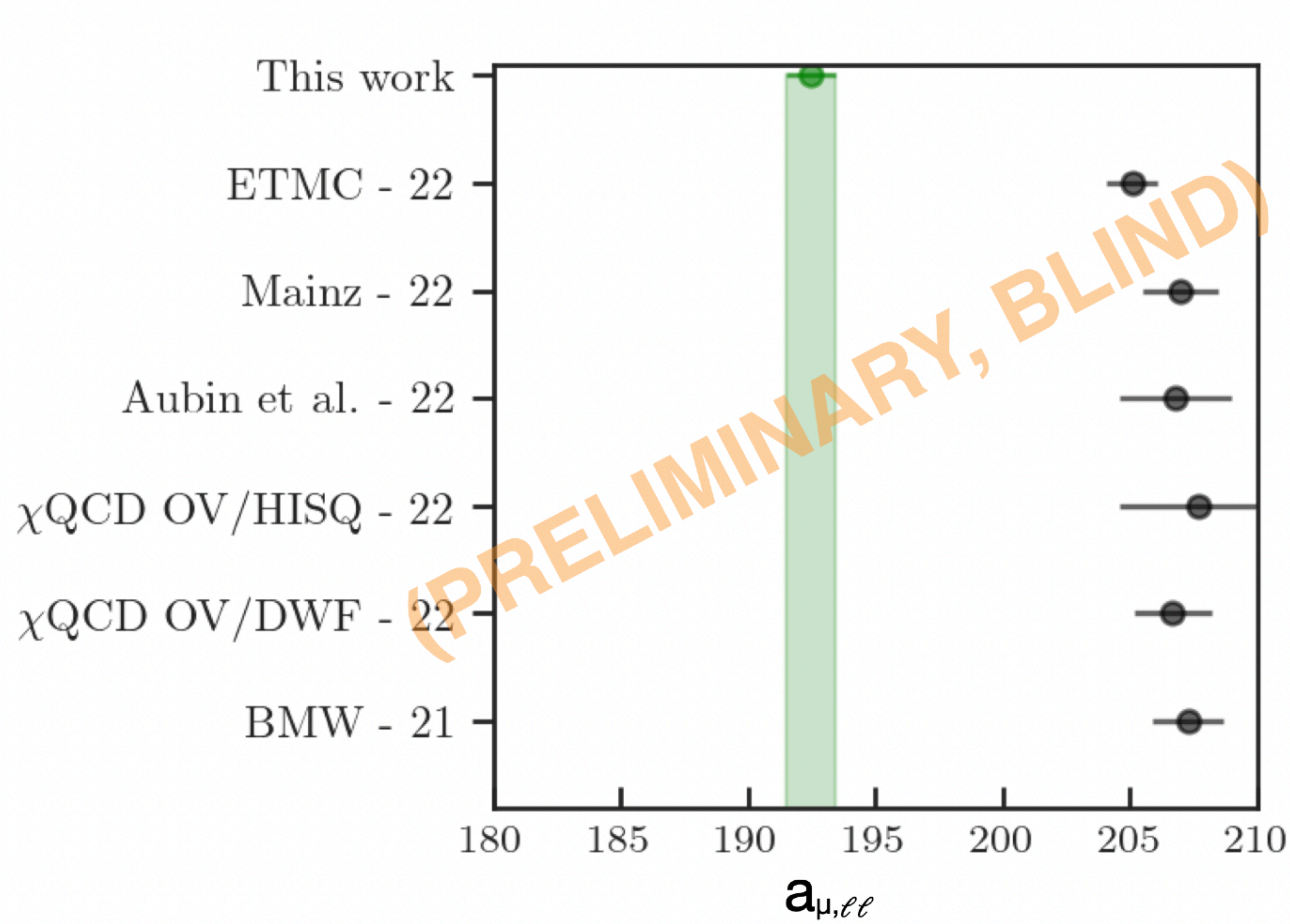
# To Correct TB or Not?

- We can allow continuum limit to remove taste breaking or remove on each ensemble.
- We see some differences as $a \to 0$ depending on model & whether we include coarsest ensemble.
- Blinded result in range 190—195



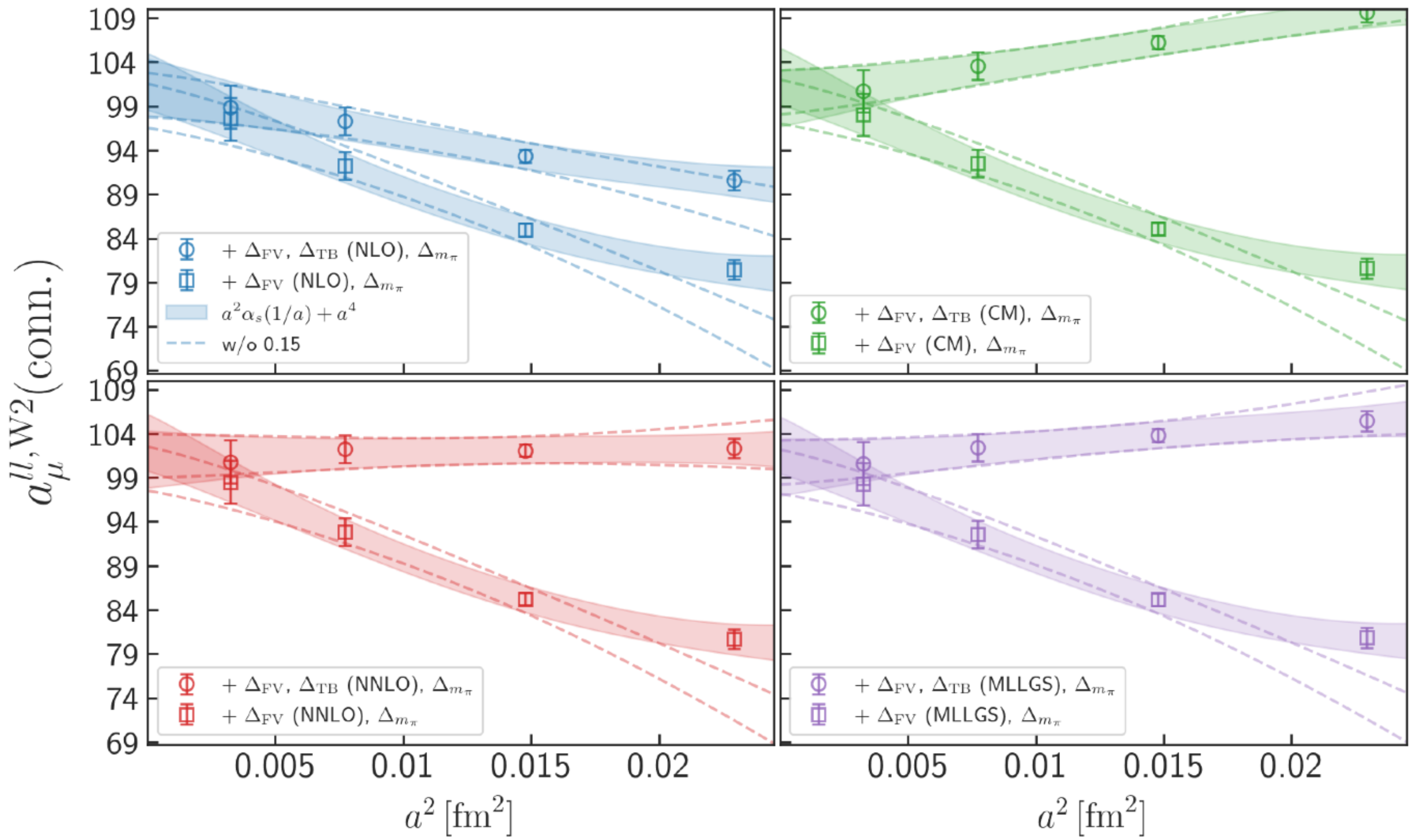

$W$ window

# Blinded Result for $W$



# To Correct TB or Not? II

- Lattice spacing dependence is quite different for window at larger time.
- Model corrections can differ quite a bit, but as $a \to 0$ results are more consistent, than in previous case.
- Error is also larger.




$W_2$ window

# Bayesian Model Averaging

- ✦ Introduced by Jay and Neil, PRD **103**, 114502 (2021).
- ✦ Useful when considering multiple models (or parameter values like $t_{\mathrm{min}}$ in fits).

$$\mathrm{pr}(M \mid D) \equiv \exp\left[-\frac{1}{2}\left(\chi^2_{\mathrm{aug}}\left(\mathbf{a}^\star\right) + 2k + 2N_{\mathrm{cut}}\right)\right]$$

gives the weight of each model in the average.

$$\left\langle a_\mu \right\rangle = \sum_i \left\langle a_\mu \right\rangle_i \mathrm{pr}\left(M_i \mid D\right)$$

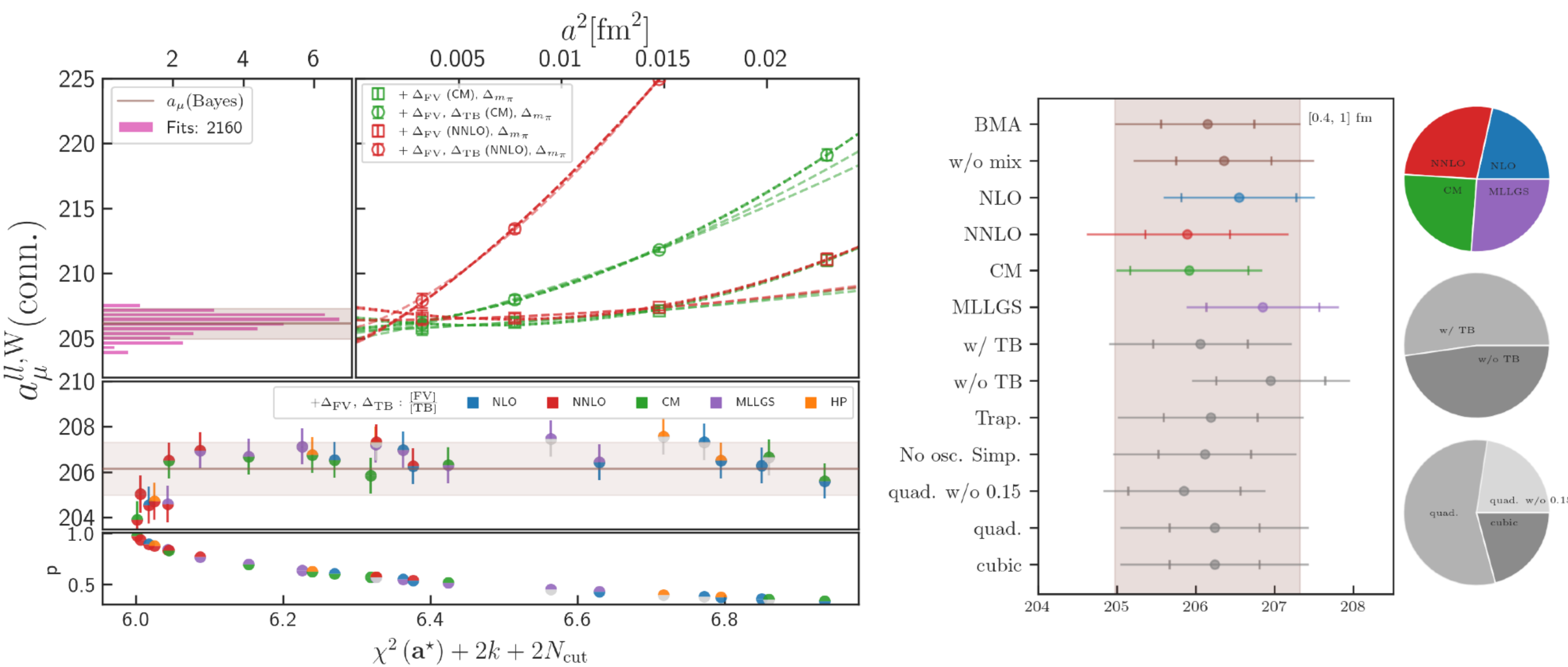
is the average over the models.

# Bayesian Model Averaging II

✦ Many variations in how the fit is done:

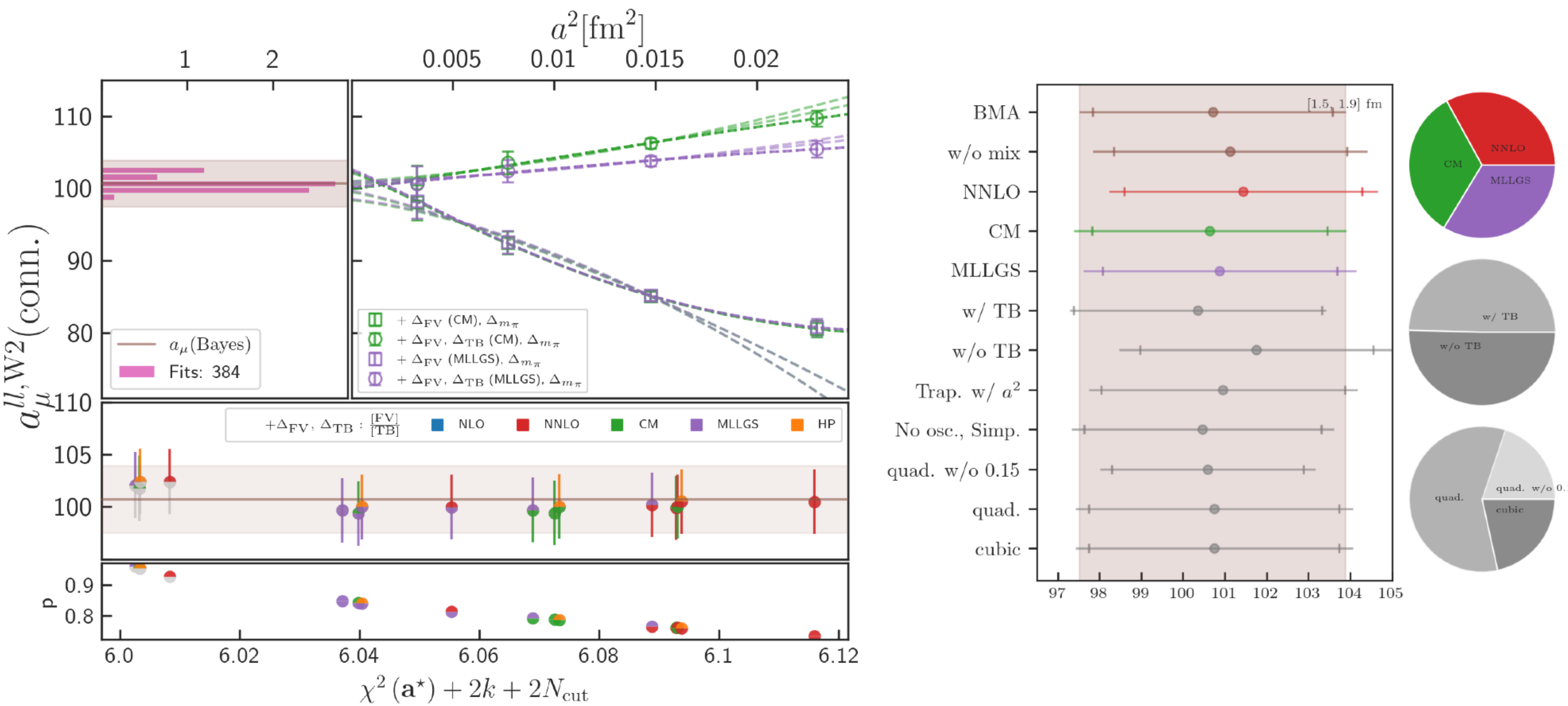
- choice of model for each correction FV, mistuning, TB
  - also no taste breaking correction
- apply corrections to a reduced region of time
- remove opposite parity contributions to vector-correlator that come from using staggered quarks
- dropping coarsest ensemble
- variations in the number of powers of $a^2$ and $\alpha_s$ in continuum fit
- inclusion of sea-quark mistuning term

# BMA for $W$



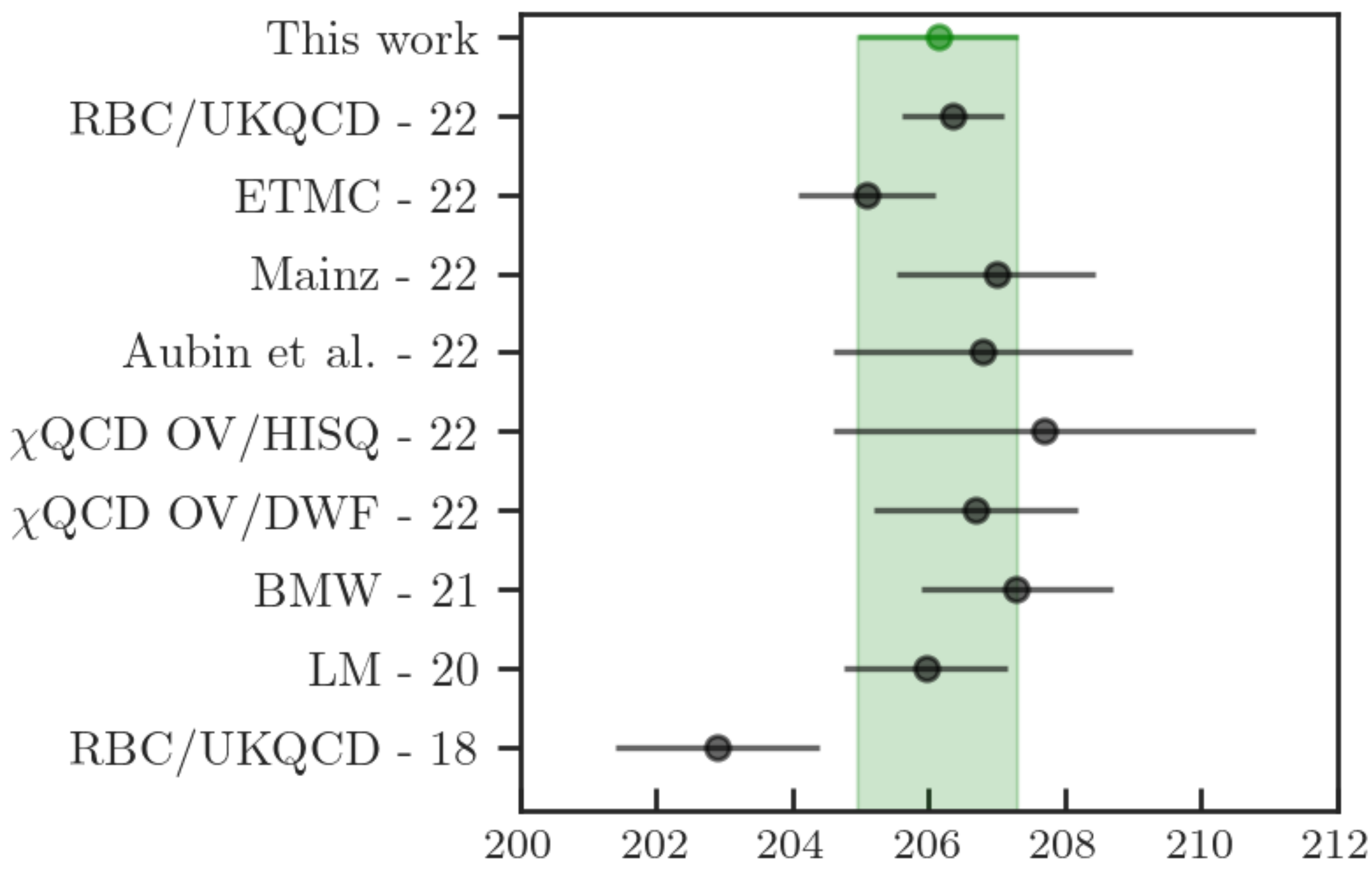

- (L) Four panels show many aspects of the various fits: histogram of 2,160 fits; examples of fits using CM and NLO chiral perturbation theory; 50 best fits; p-value for data contribution to $\chi^2$.
- (R) Model average using only subsets of the models.

# BMA for $W_2$




- Similar to previous slide but for the window suggested by Aubin et al.

# Unblinded Result for $a_{\mu ll}^{W}$(conn.)




- $206.1(1.2) \times 10^{10}$
- Our result is in excellent agreement with recent results.
- Our error is not quite as small as RBC/UKQCD-22, but comparable to best of the rest.

# Error Budget for $W$

- Continuum extrapolation is dominant source of error
- Scale setting, current renormalization, and finite volume are all close in size

| Source | $a_\mu^{ll,\mathrm{W}}$(conn.) % |
|---|---|
| Continuum extrapolation ($a \to 0$, $\Delta_{\mathrm{TB}}$) | 0.37 |
| Scale setting ($w_0$ (fm), $w_0/a$) | 0.24 |
| Current renormalization ($Z_V$) | 0.25 |
| Monte Carlo statistics | 0.17 |
| Finite-volume correction ($\Delta_{\mathrm{FV}}$) | 0.21 |
| Pion-mass adjustment ($\Delta_{m_\pi}$) | 0.09 |
| Total | 0.57% |

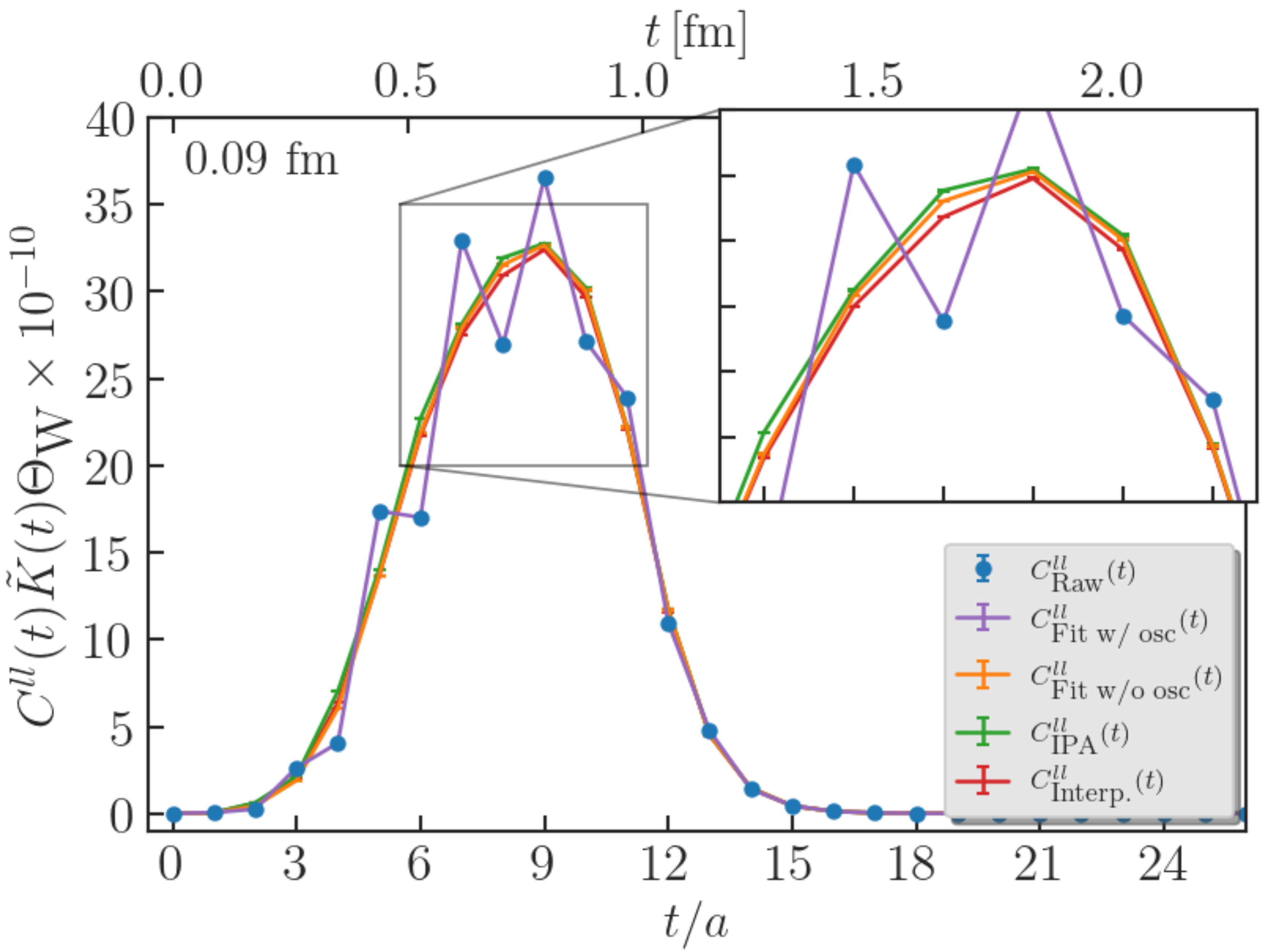
# Error Budget for $W_2$

- ✦ This window is at larger time, so limited statistical precision is dominant source of error.
- ✦ Scale setting, continuum extrapolation, and mass adjustment are also considerable.

| Source | $a_\mu^{ll,\mathrm{W2}}$(conn.) % |
|---|---|
| Monte Carlo statistics | 2.42 |
| Scale setting ($w_0$ (fm), $w_0/a$) | 1.28 |
| Continuum extrapolation ($a \to 0$, $\Delta_{\mathrm{TB}}$) | 1.03 |
| Pion-mass adjustment ($\Delta_{m_\pi}$) | 0.92 |
| Finite-volume correction ($\Delta_{\mathrm{FV}}$) | 0.29 |
| Current renormalization ($Z_V$) | 0.22 |
| Total | 3.16% |

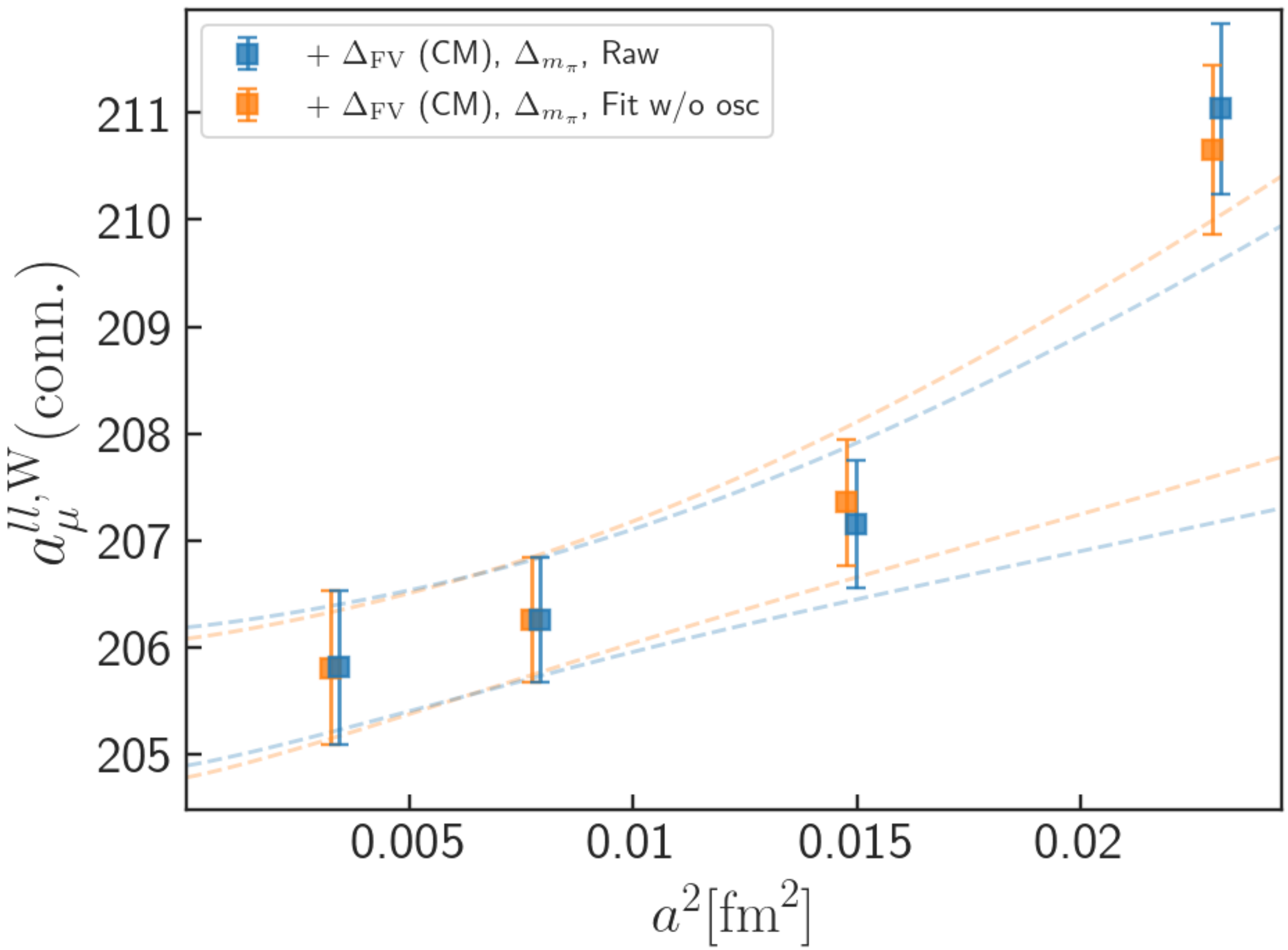
# Staggered Opposite Parity

- Well known that staggered hadronic propagators often couple to opposite parity state resulting in $(-1)^t$ terms
- We have explored some ways to eliminate these terms
- Fit and eliminate the opposite parity part



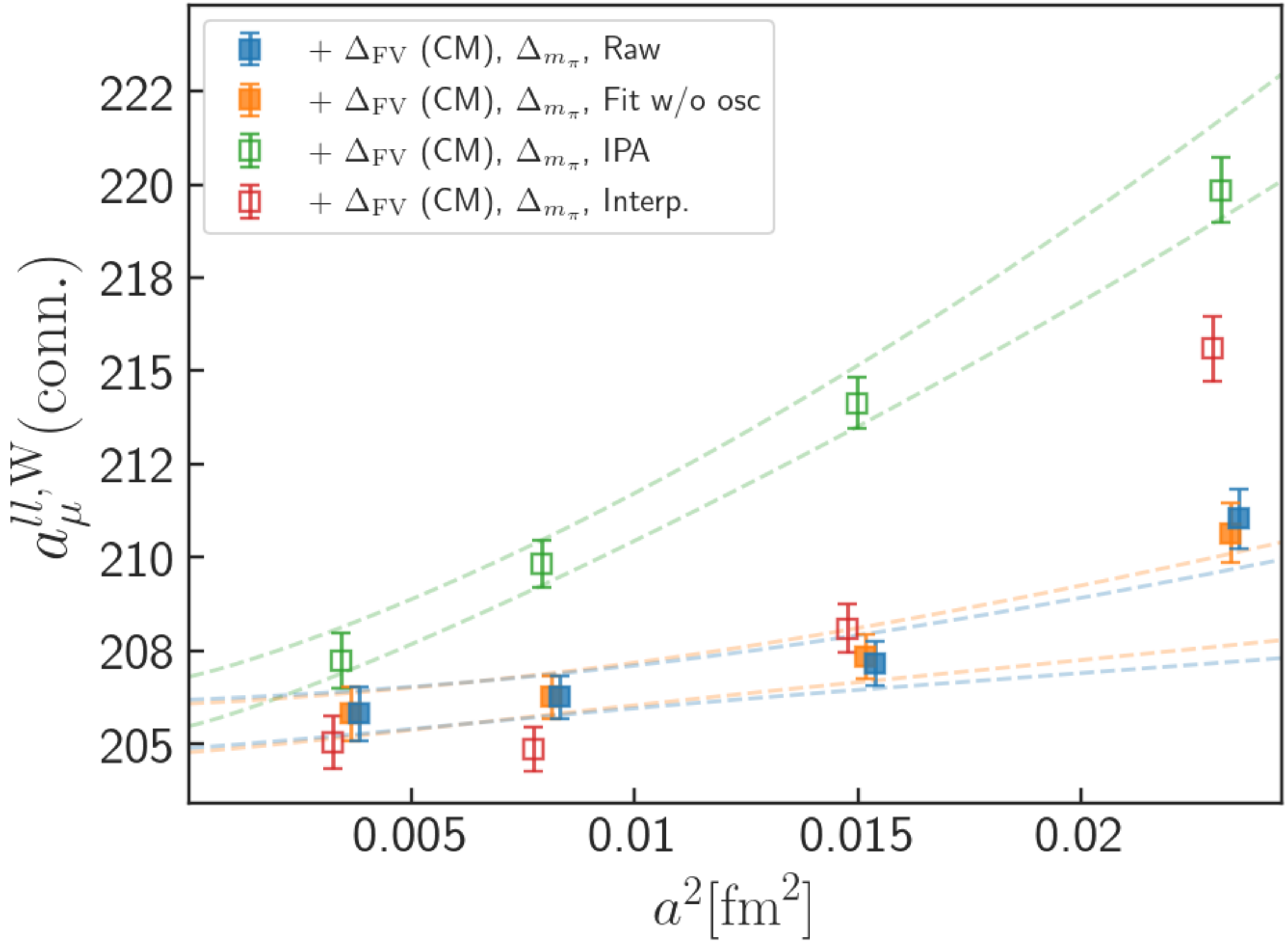
# Staggered Opposite Parity II

- Removing the opposite parity contribution explicitly makes little difference
- Here we use a chiral model for the finite volume correction.



# Staggered Opposite Parity III

- Improved parity averaging (IPA) and interpolation of even only and odd only times are not useful approaches



# Towards a Complete Calculation

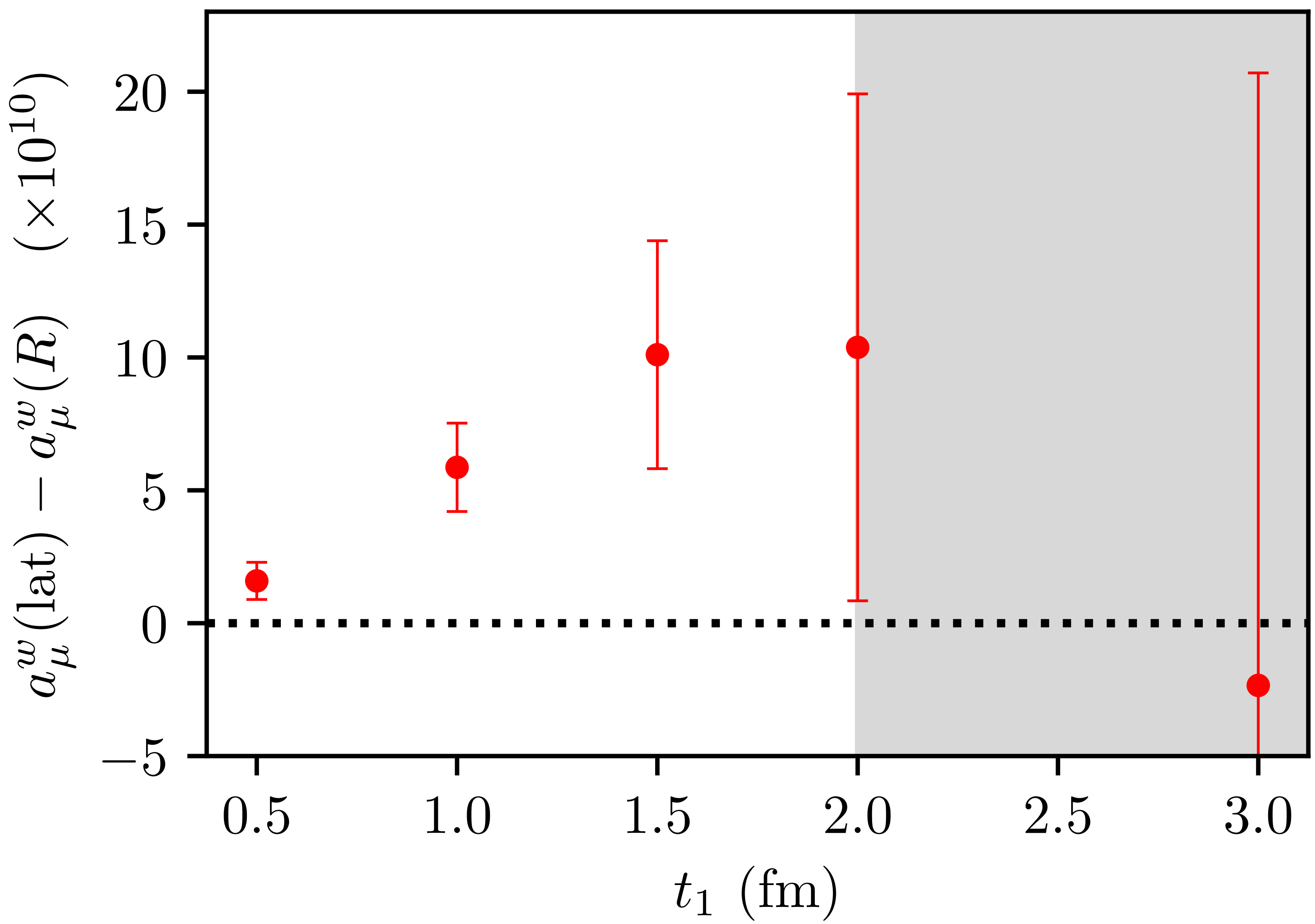
✦ Ultimate goal is $a_\mu$, so we need:

- better scale setting
  - extending range of ensembles with gauge flow data
  - $\Omega$ baryon mass (Yin Lin)
- better statistical accuracy at large time
  - Michael Lynch's Lattice '22 poster on low-mode improvement
  - Shaun Lahert's work on two pion states (not presented here)
    - now analyzing 0.12 fm ensemble
- strong isospin breaking
  - Curtis Peterson's analysis (not presented here)
- electromagnetic corrections
  - Gaurav Ray's work was presented at Lattice 2022

# Conclusions

- Contributions to $a_\mu$ from various windows in Euclidean time provide valuable benchmarks for lattice QCD calculations on the way to complete HVP calculation
- This is our first announced window result
  - We expect a paper on arXiv in a couple of weeks
- The lattice community needs to continue to work hard on the full set of hadronic contributions to $a_\mu$
  - The tension between the data driven (dispersive) approach and lattice QCD is of critical importance and must be resolved or explained

# One sided windows




- Difference between lattice and R-ratio determination for various one-sided windows.
- From 2207.04765, using data from 2020.
- We have analyzed several windows with our updated data set