Magic, majorization and quasi-distributions

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A major challenge is to understand the ways in which quantum computing is fundamentally different from anything possible with classical mechanics. One common approach to studying non-classicality is via a Wigner representation of states and operations in which a simple class of operations (such as Clifford circuits) are represented by classical stochastic maps, while "magic states" that promote these operations to universal quantum computing are negatively represented. Extensive work has been done for magic states, but so far this work ignores hardware considerations, and the distillation bounds are very far from the concrete protocols based on quantum error-correction codes.

Here I will discuss a new approach to this topic that develops a consistent statistical mechanics framework based on the majorization of quasi-probability distributions, and allows us to draw on a range of tools from single-shot resource theories. Specializing first to magic distillation in odd prime dimensions, I will show: that only a strict sub-set of Renyi entropies obey the Data Processing Inequality in the framework, that hardware limitations can be incorporated, that prior results emerge naturally in the framework, and that we obtain bounds on magic distillation that significantly out-perform prior known bounds. I will also describe recent work that extends this framework to qubits, and when applied to the family of CSS-code distillation protocols leads to a surprising new cut-off constraint on the size of CSS code as a function of error rate and success probability. I will finish with open questions and possible applications to the study of non-classicality in other areas of quantum information theory.