



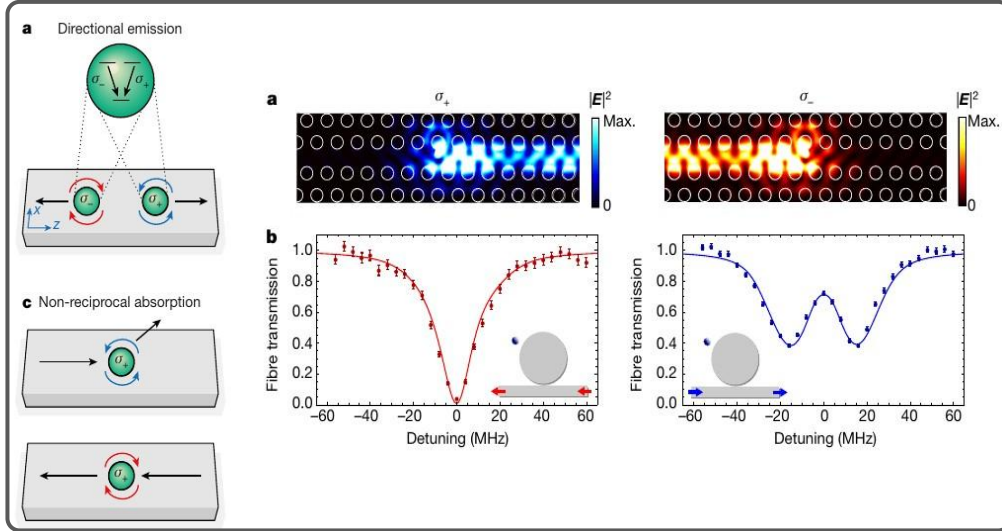
# Spin-momentum locking in chiralitonic metasurfaces

Fernando Lorén

[loren@unizar.es](mailto:loren@unizar.es)

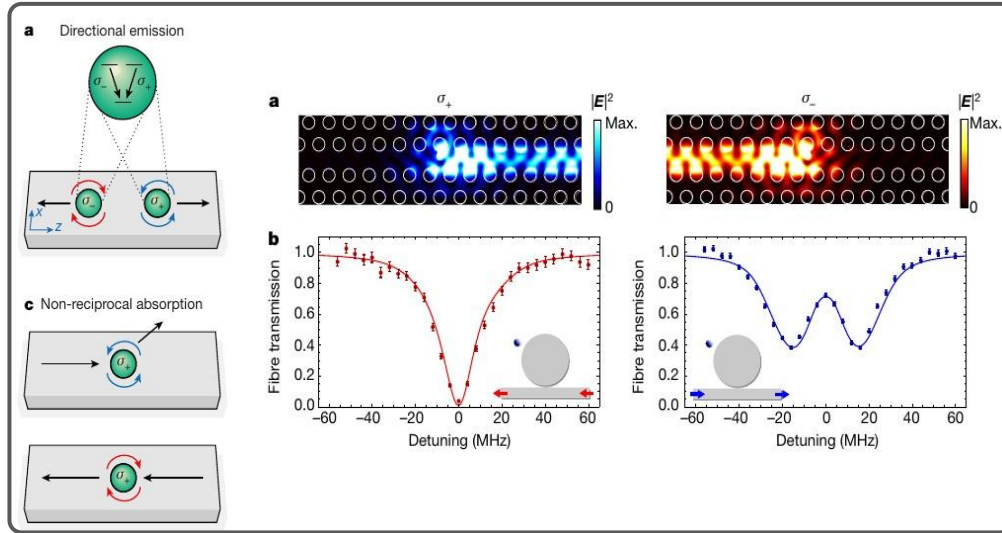
Nanolight 2022, Benasque

# Spin-momentum locking

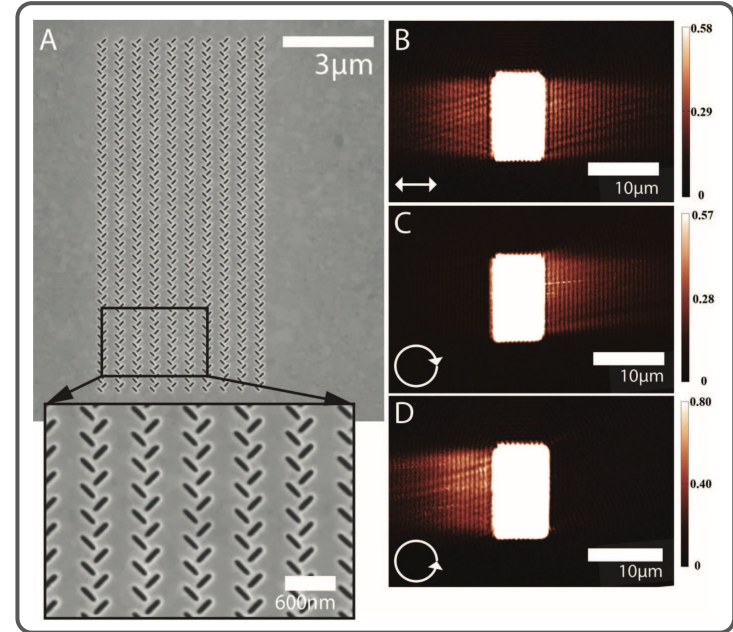


P. Lodahl *et al.*, *Nature* 541, 473-480 (2017)

# Spin-momentum locking

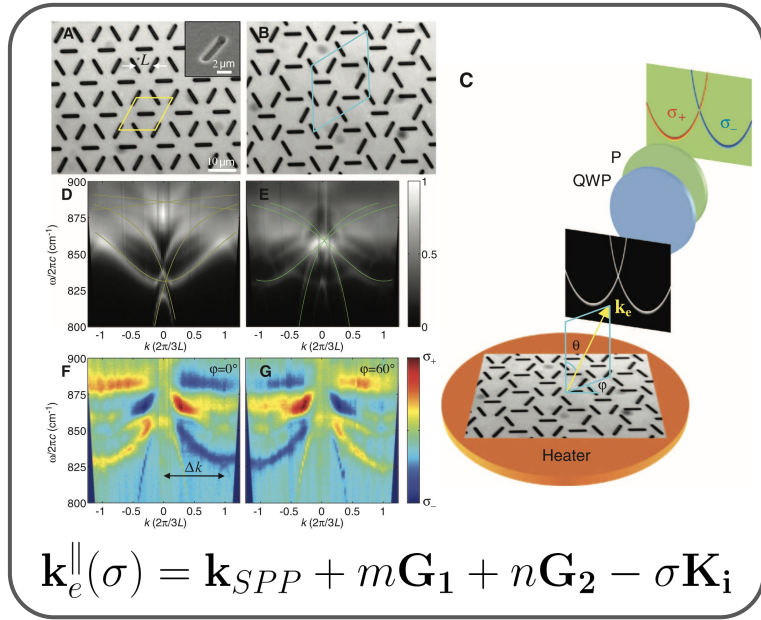


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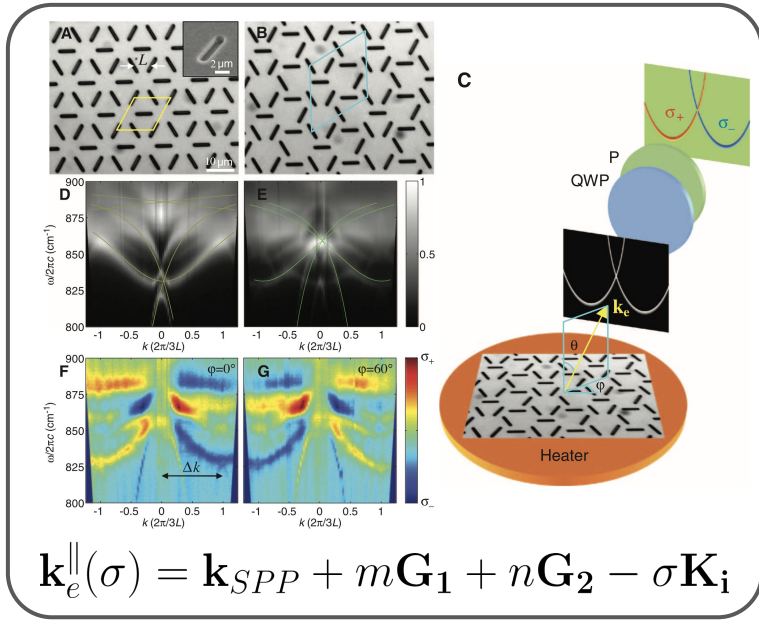
J. Lin *et al.* *Science* 340, 331-3 (2013)

# Motivation

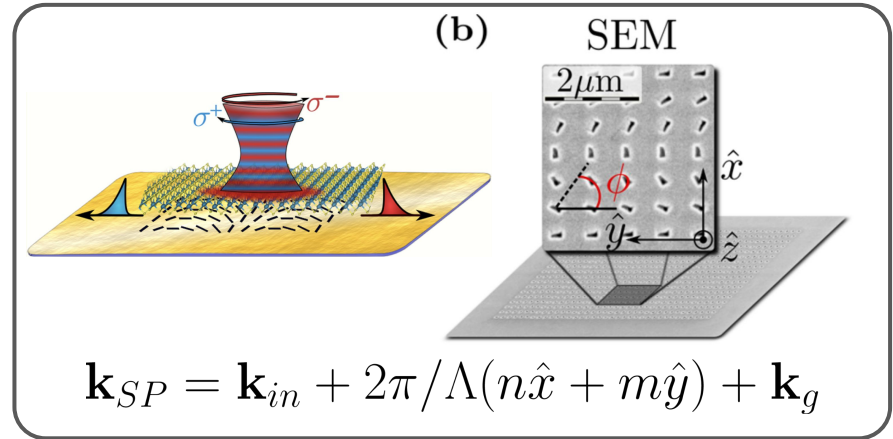


N. Shitrit *et al.*, *Science* 340, 724-6, (2013)

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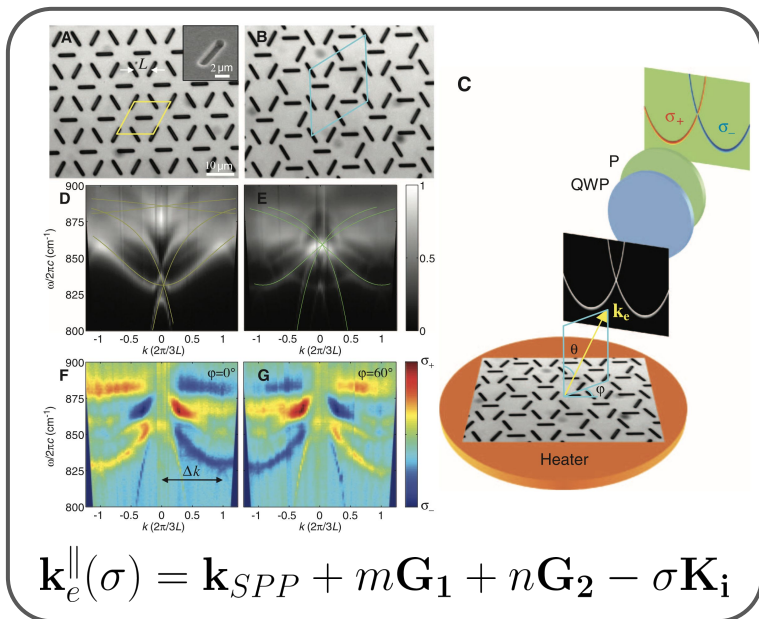


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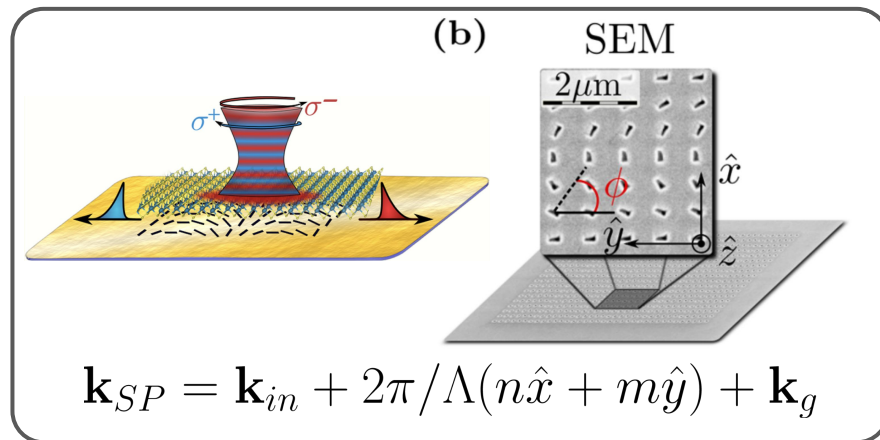


T. Chervy *et al.*, *ACS Photonics* 5, 4, 1281-1287, (2018)

# Motivation



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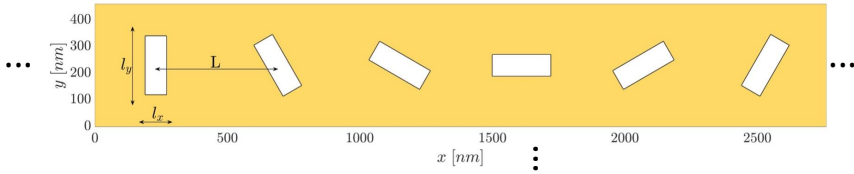


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**What is the origin of the spin-momentum locking in this corrugated surface without the global translation plus rotation symmetry?**

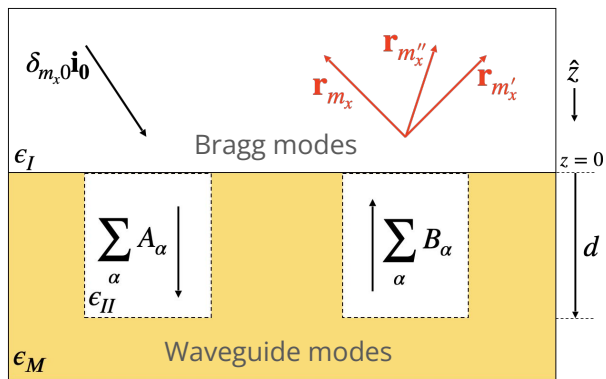
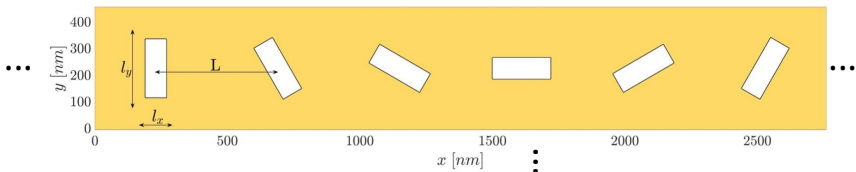
# Theoretical model: Coupled mode method

$$N = 6 \quad n_w = +1/2 \quad \vdots$$



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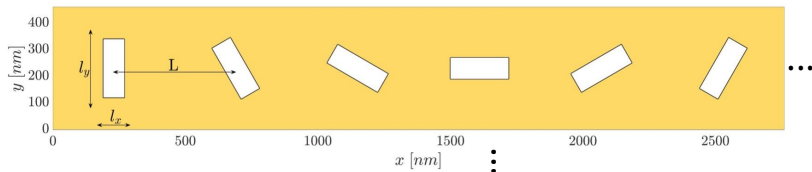


Bragg modes:  $\mathbf{G} = m_x \frac{2\pi}{NL} \hat{u}_x + m_y \frac{2\pi}{L} \hat{u}_y$



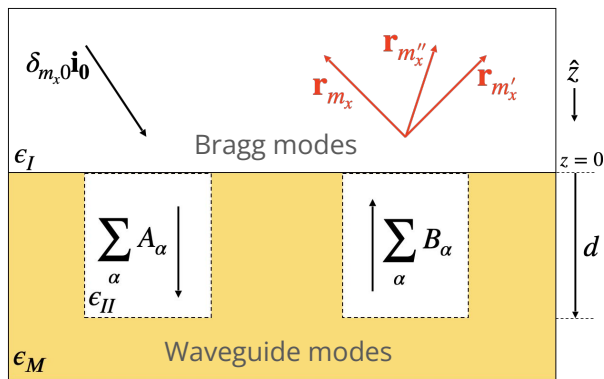
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Far-field equations for the **reflection coefficients**

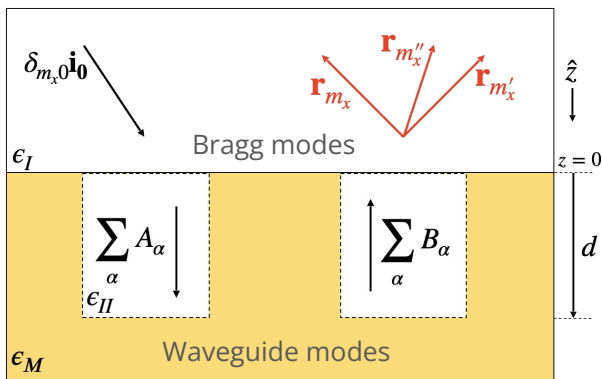
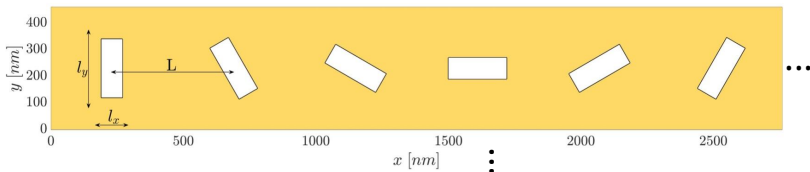
$$\mathbf{r}_{m_x} = 2Z_{m_x 0} Y_0 \delta_{m_x 0} \mathbf{i}_0 - \sum_{m'_x} Z_{m_x m'_x} Y_{m'_x} \mathbf{r}_{m'_x}$$



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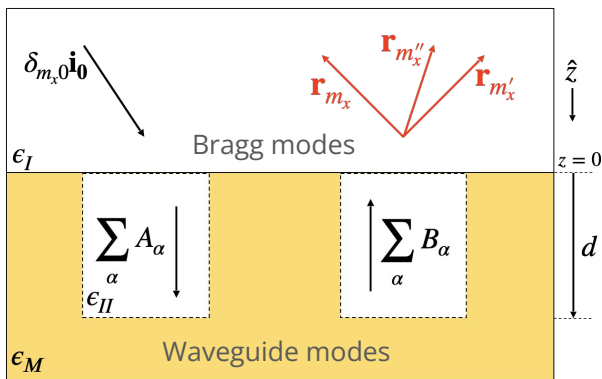
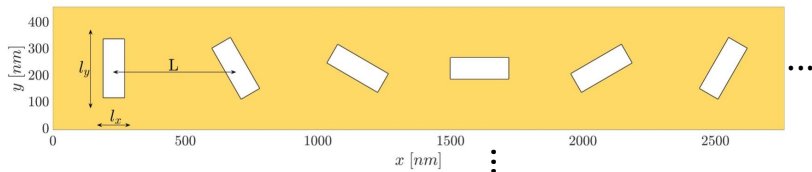
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(Geometric) couplings

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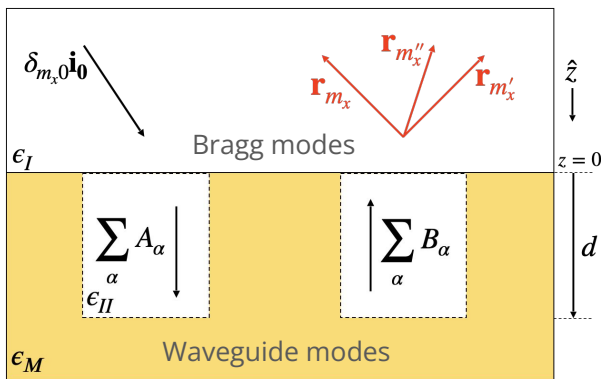
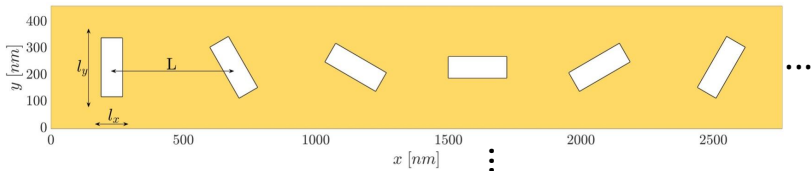
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Modal admittances

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$$\mathbf{i}_0 = \begin{pmatrix} \langle p | \sigma_{in} \rangle \\ \langle s | \sigma_{in} \rangle \end{pmatrix} = \begin{pmatrix} \langle + | \sigma_{in} \rangle \\ \langle - | \sigma_{in} \rangle \end{pmatrix}$$

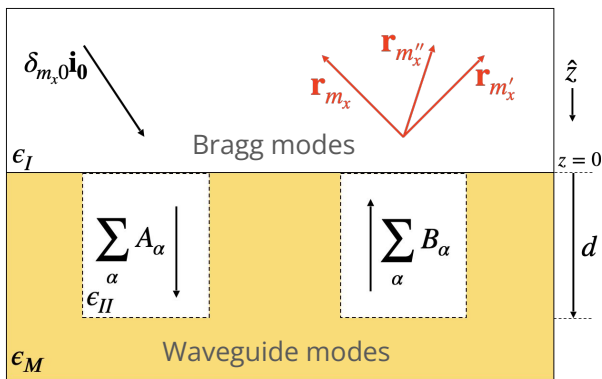
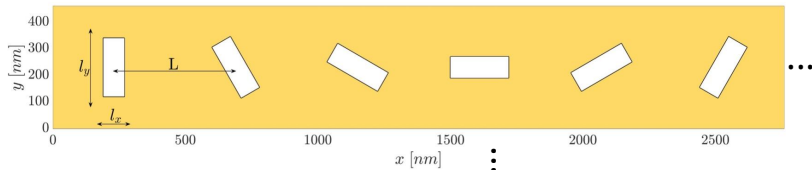
$$\mathbf{r}_{m'_x} = \begin{pmatrix} r_{m'_x,p} \\ r_{m'_x,s} \end{pmatrix} = \begin{pmatrix} r_{m'_x,+} \\ r_{m'_x,-} \end{pmatrix}$$

$$Y_{m'_x} = \begin{pmatrix} Y_{m'_x,p} & 0 \\ 0 & Y_{m'_x,s} \end{pmatrix}_{ps} = \frac{1}{2} \begin{pmatrix} Y_{m'_x,p} + Y_{m'_x,s} & Y_{m'_x,p} - Y_{m'_x,s} \\ Y_{m'_x,p} - Y_{m'_x,s} & Y_{m'_x,p} + Y_{m'_x,s} \end{pmatrix}_{\pm}$$

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$$\begin{array}{l} + = RCP \\ - = LCP \end{array}$$

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# Analysis of the symmetries



$$Z_{m_x m'_x} = ZN \begin{pmatrix} \begin{matrix} + \rightarrow + \\ \delta_{m_x, m'_x + nN} \\ + \rightarrow - \end{matrix} & \begin{matrix} - \rightarrow + \\ \delta_{m_x, m'_x + nN - 2n_w} \\ - \rightarrow - \end{matrix} \\ \begin{matrix} \delta_{m_x, m'_x + nN + 2n_w} \\ + \rightarrow - \end{matrix} & \begin{matrix} \delta_{m_x, m'_x + nN} \\ - \rightarrow - \end{matrix} \end{pmatrix}$$

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$$\delta_{m_x, m'_x + nN}$$

standard Bragg's law

$$k_x^{out} = k_x^{in} + n \frac{2\pi}{L} = k_x^{in} + nG^0$$

It is like if the unit cell was...



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It is like if the unit cell was...



$$\delta_{m_x, m'_x + nN + \sigma_{in} 2n_w}$$

**spin-orbit Bragg's law**

$$k_x^{out} = k_x^{in} + nG^0 + \sigma_{in} 2n_w \underbrace{\frac{2\pi}{NL}}_{k_g}$$

**SPIN-MOMENTUM LOCKING**



# Approximate spin-momentum locking

$$\mathbf{r}_{m_x} = 2Z_{m_x 0} Y_0 \delta_{m_x 0} \mathbf{i}_0 - \sum_{m'_x} Z_{m_x m'_x} Y_{m'_x} \mathbf{r}_{m'_x}$$

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$$\begin{aligned} \mathbf{r}_{m_x} &= (Y_{0,p} + Y_{0,s}) Z_{m_x 0} \delta_{m_x 0} \mathbf{i}_0 - \sum_{m'_x} \frac{1}{2} (Y_{m'_x,p} + Y_{m'_x,s}) Z_{m_x m'_x} \mathbf{r}_{m'_x} \\ &+ (Y_{0,p} - Y_{0,s}) Z_{m_x 0} \sigma^x \delta_{m_x 0} \mathbf{i}_0 - \sum_{m'_x} \frac{1}{2} (Y_{m'_x,p} - Y_{m'_x,s}) Z_{m_x m'_x} \sigma^x \mathbf{r}_{m'_x} \end{aligned}$$

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perfect spin-momentum locking

$$\mathbf{r}_{m_x} = (Y_{0,p} + Y_{0,s}) Z_{m_x 0} \delta_{m_x 0} \mathbf{i}_0 - \sum_{m'_x} \frac{1}{2} (Y_{m'_x,p} + Y_{m'_x,s}) Z_{m_x m'_x} \mathbf{r}_{m'_x} \\ + (Y_{0,p} - Y_{0,s}) Z_{m_x 0} \sigma^x \delta_{m_x 0} \mathbf{i}_0 - \sum_{m'_x} \frac{1}{2} (Y_{m'_x,p} - Y_{m'_x,s}) Z_{m_x m'_x} \sigma^x \mathbf{r}_{m'_x}$$

# Approximate spin-momentum locking

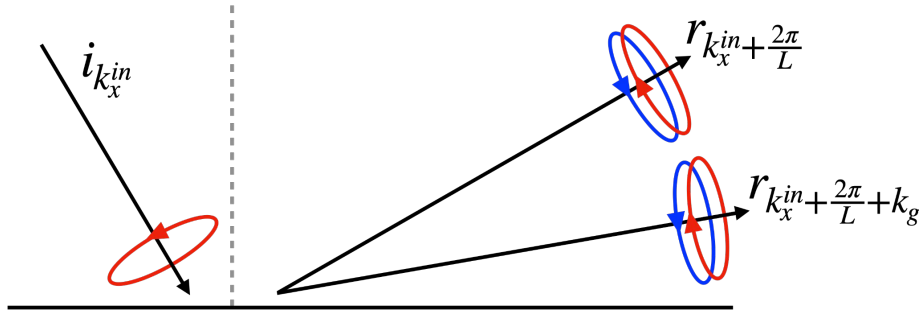
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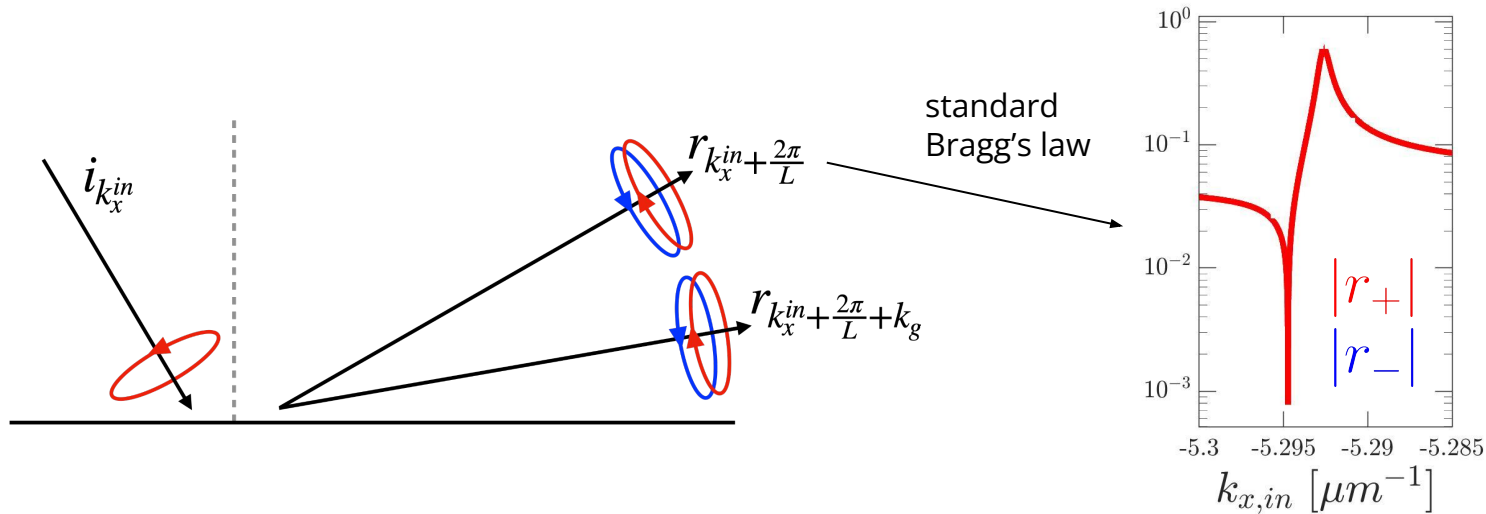
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$$+ (Y_{0,p} - Y_{0,s}) Z_{m_x 0} \sigma^x \delta_{m_x 0} \mathbf{i}_0 - \sum_{m'_x} \frac{1}{2} (Y_{m'_x,p} - Y_{m'_x,s}) Z_{m_x m'_x} \sigma^x \mathbf{r}_{m'_x}$$

spin-momentum locking breakdown

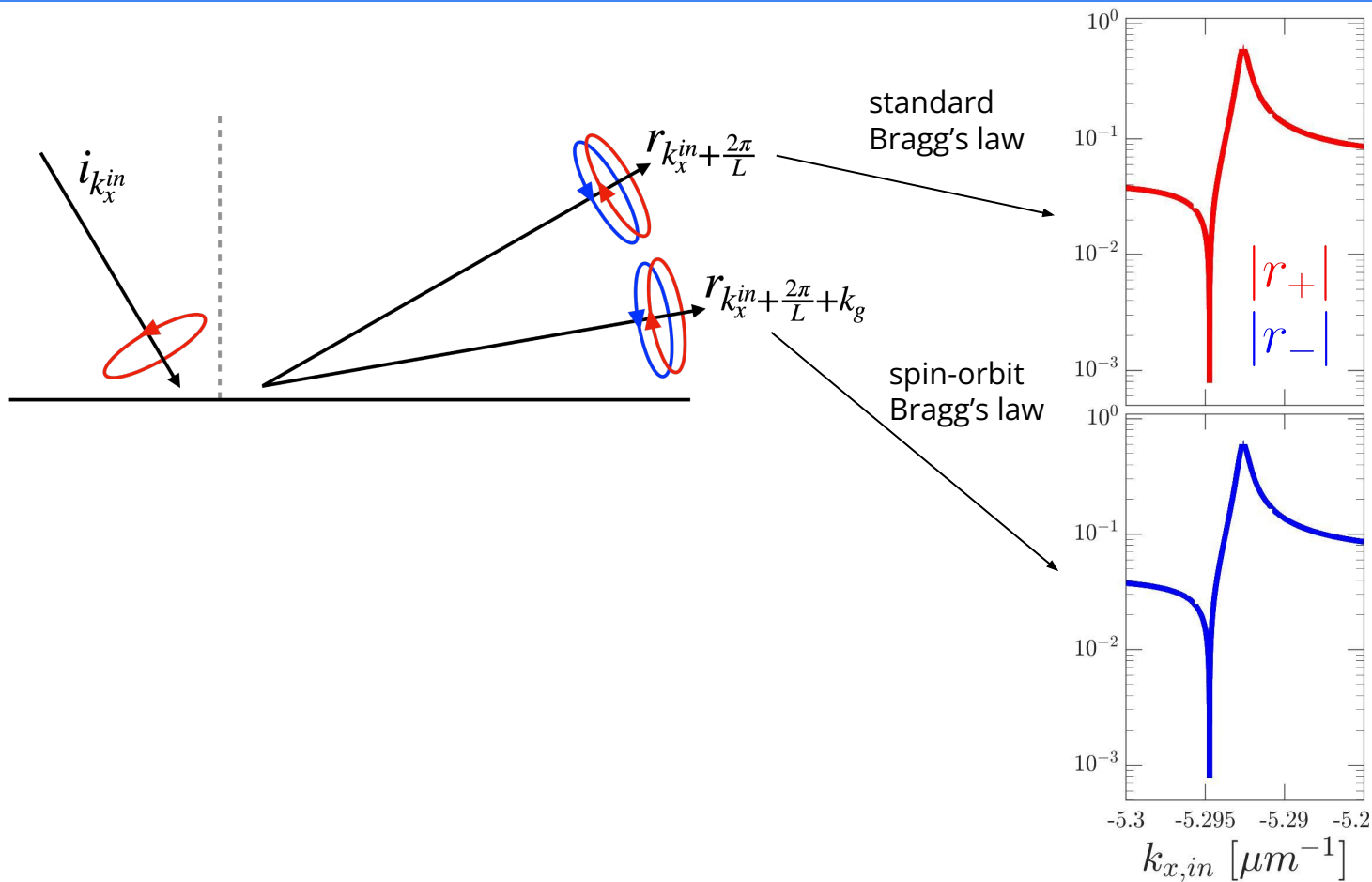
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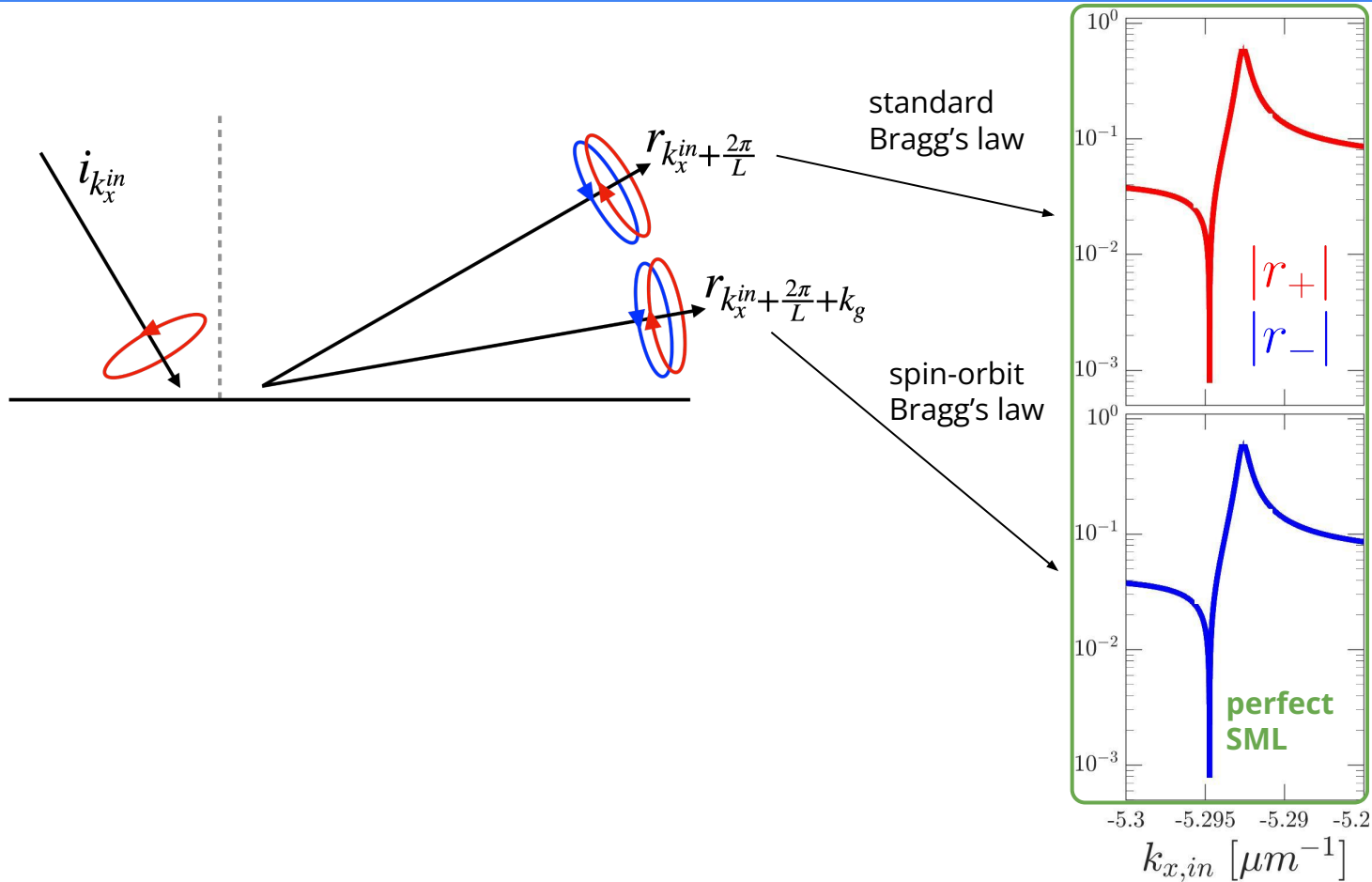
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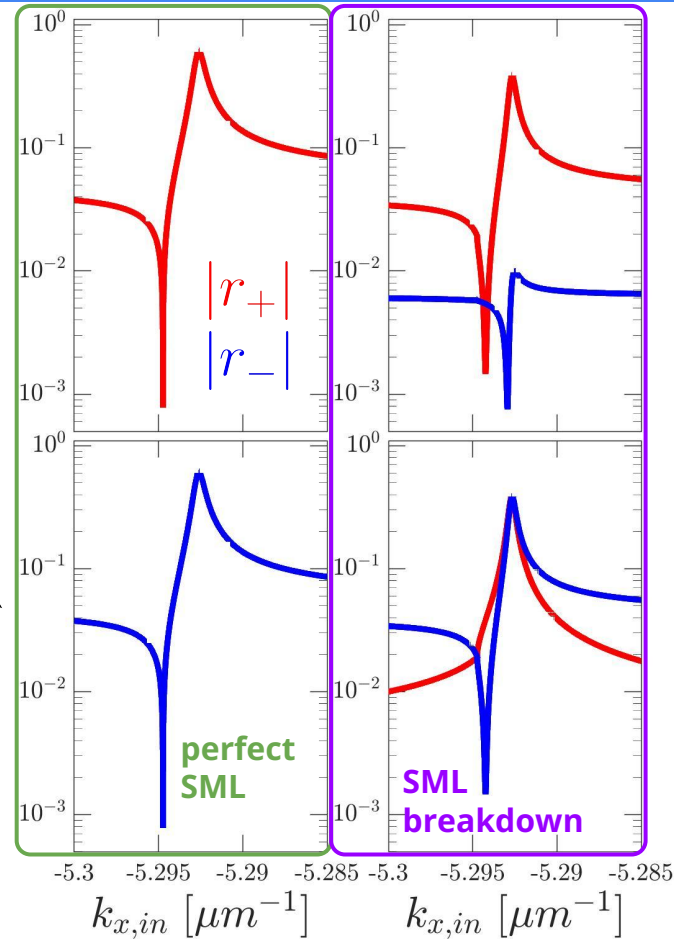
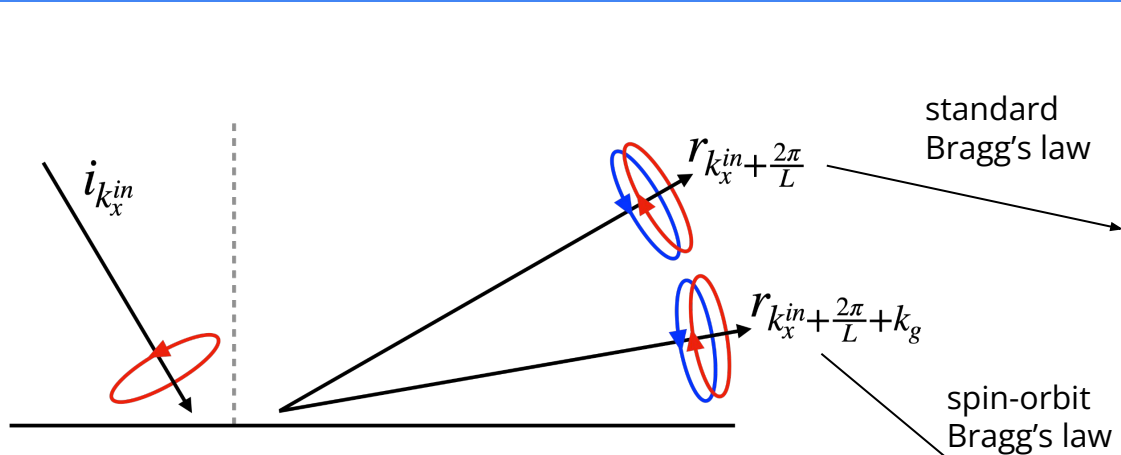


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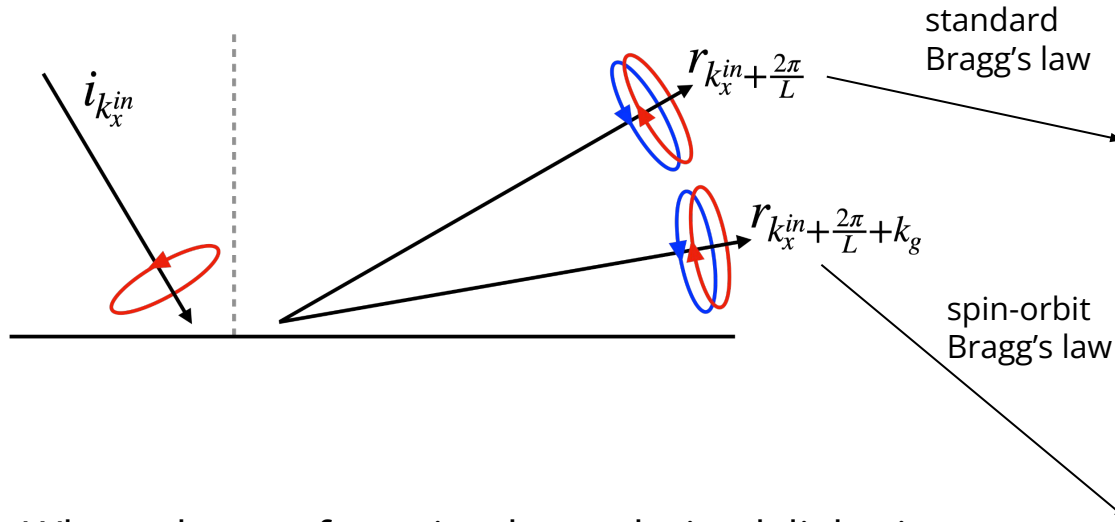




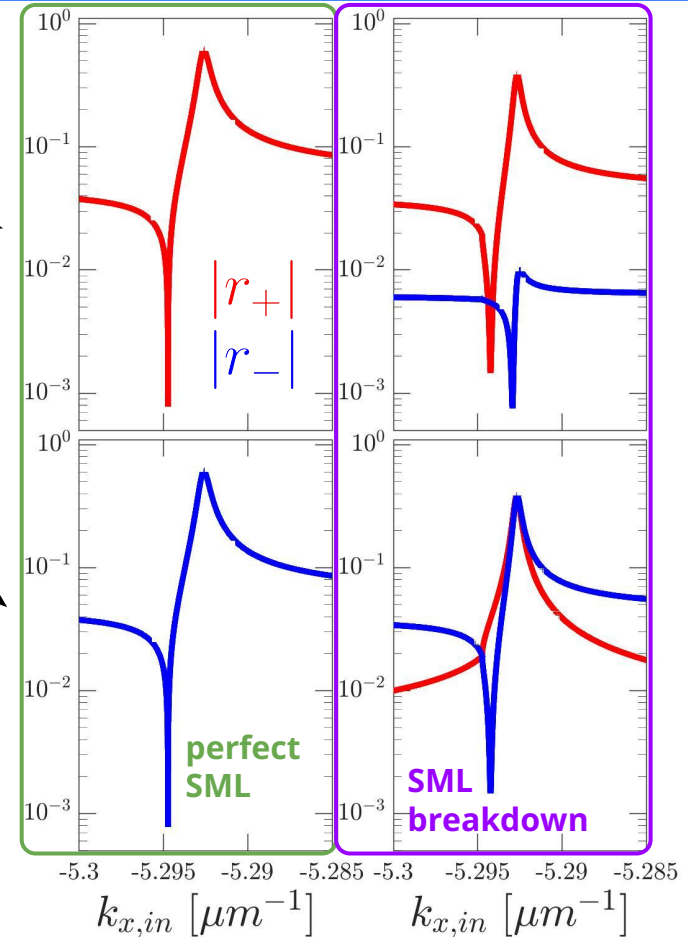
# Approximate spin-momentum locking



# Approximate spin-momentum locking



When the perfect circular polarized light is projected onto the metasurface symmetries, it becomes helical and the **spin-momentum locking is spoiled in spin**

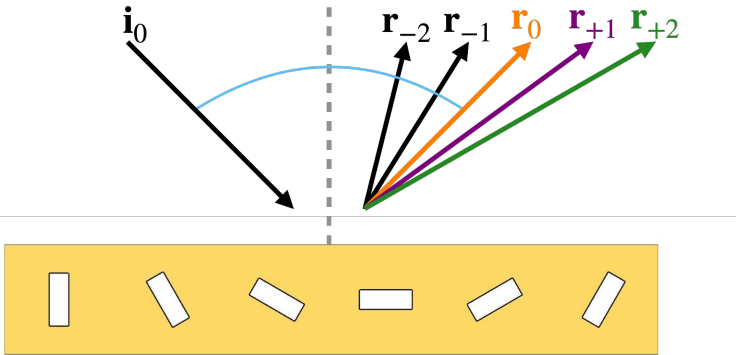


# Mueller polarimetry

$$M_{30} = \frac{S_{out}^3}{S_{in}^0} = \frac{I_{out}^+ - I_{out}^-}{I_{in}^0(\text{unpol.})}$$

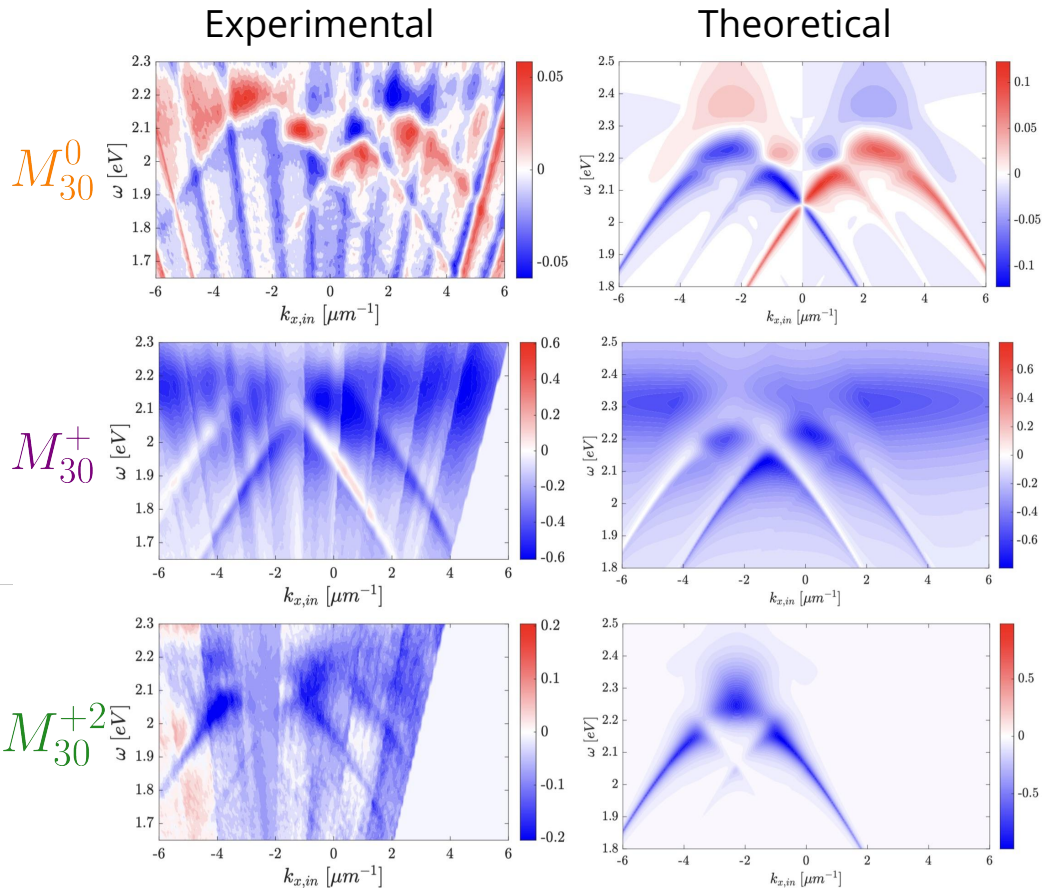
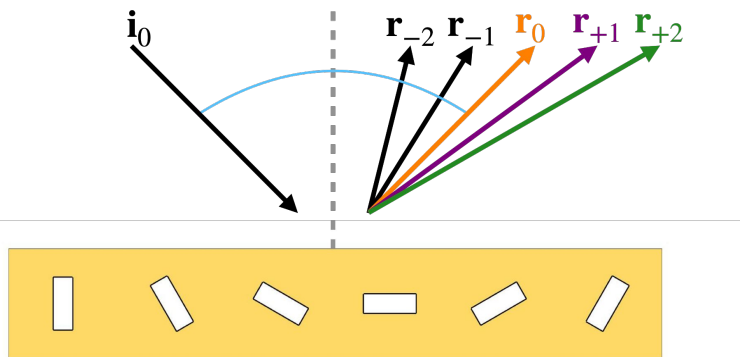
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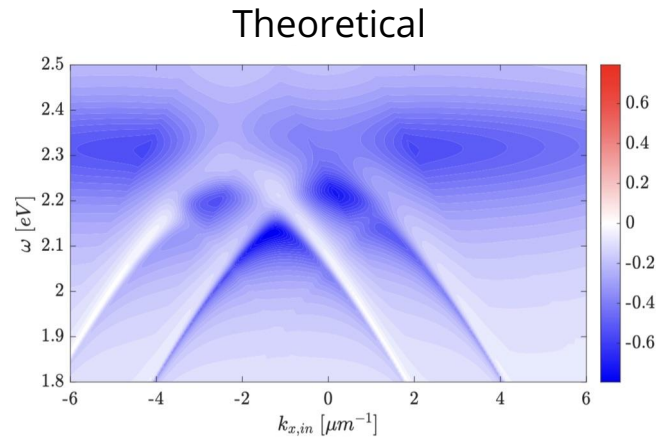
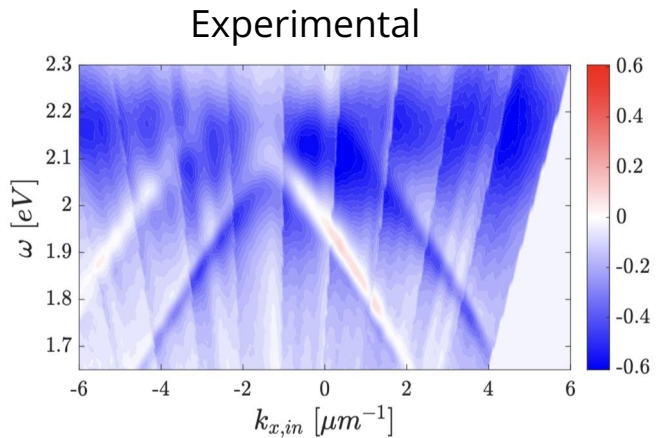
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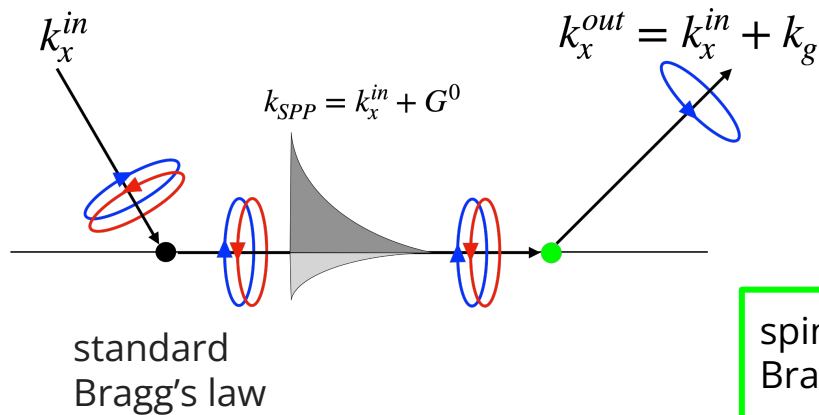
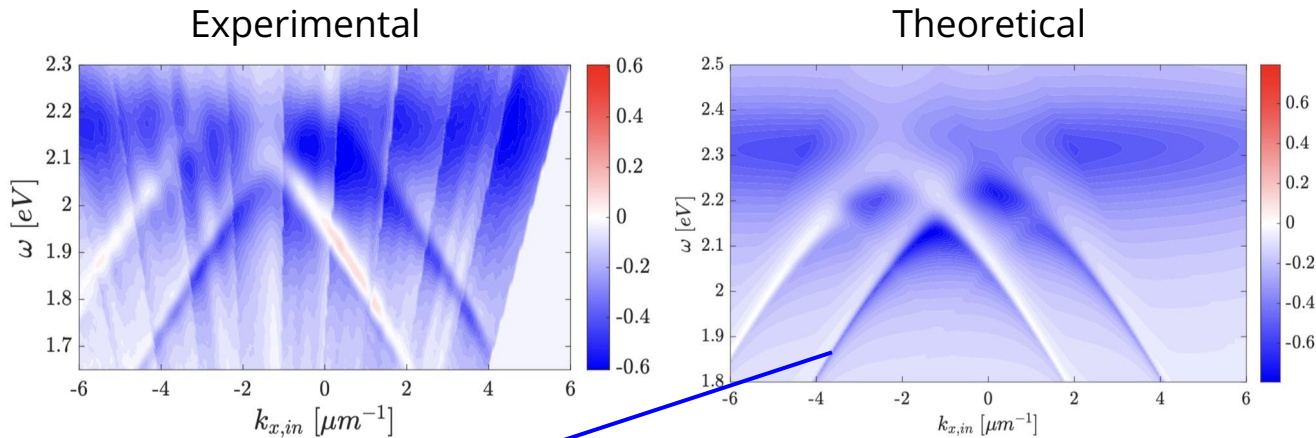
$$k_x^{out} = k_x^{in} + k_g$$



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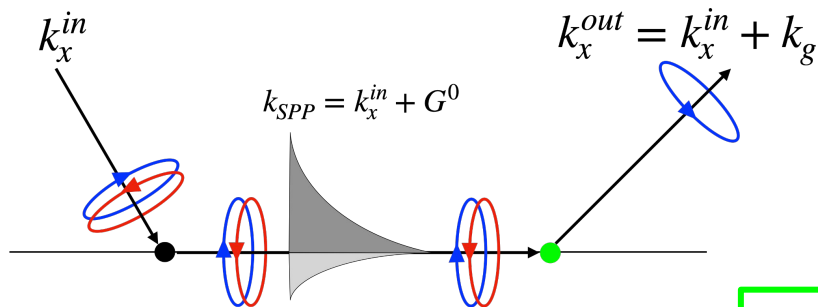
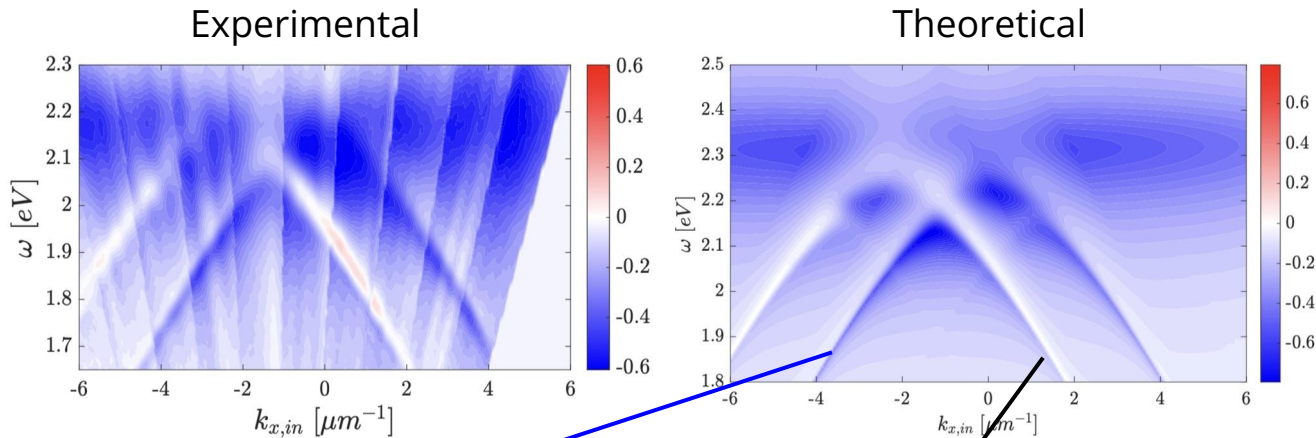
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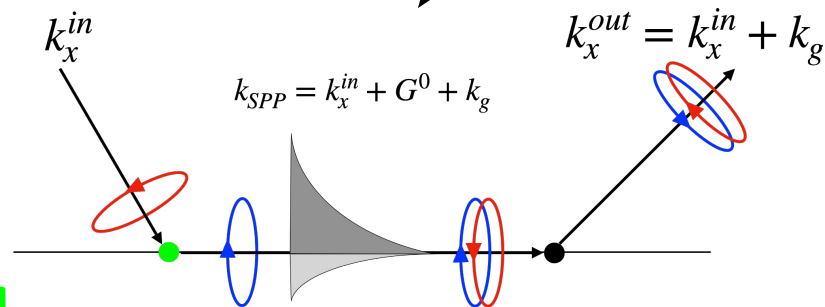
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standard  
Bragg's law

spin-orbit  
Bragg's law



standard  
Bragg's law

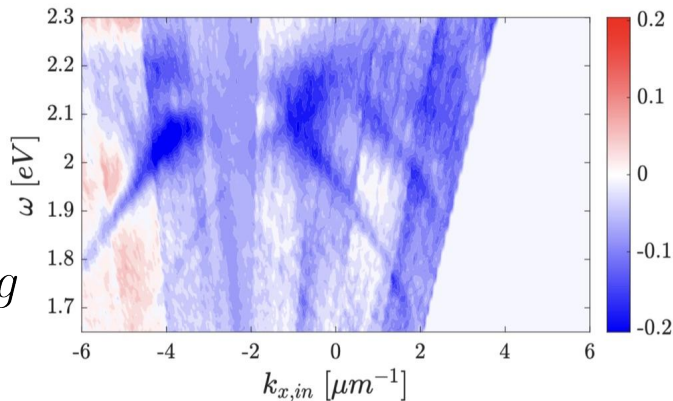


# Mueller polarimetry

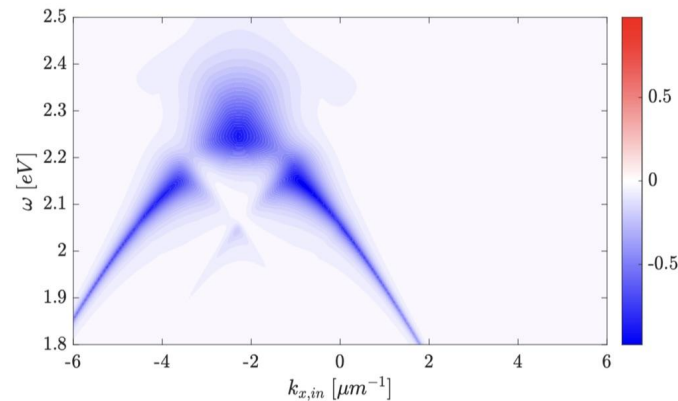
$$M_{30}^{+2}$$

$$k_x^{out} = k_x^{in} + 2k_g$$

Experimental



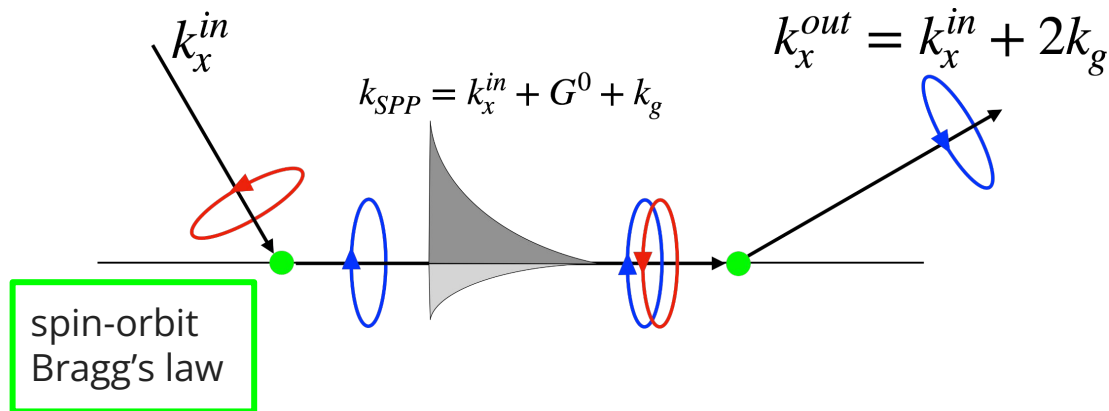
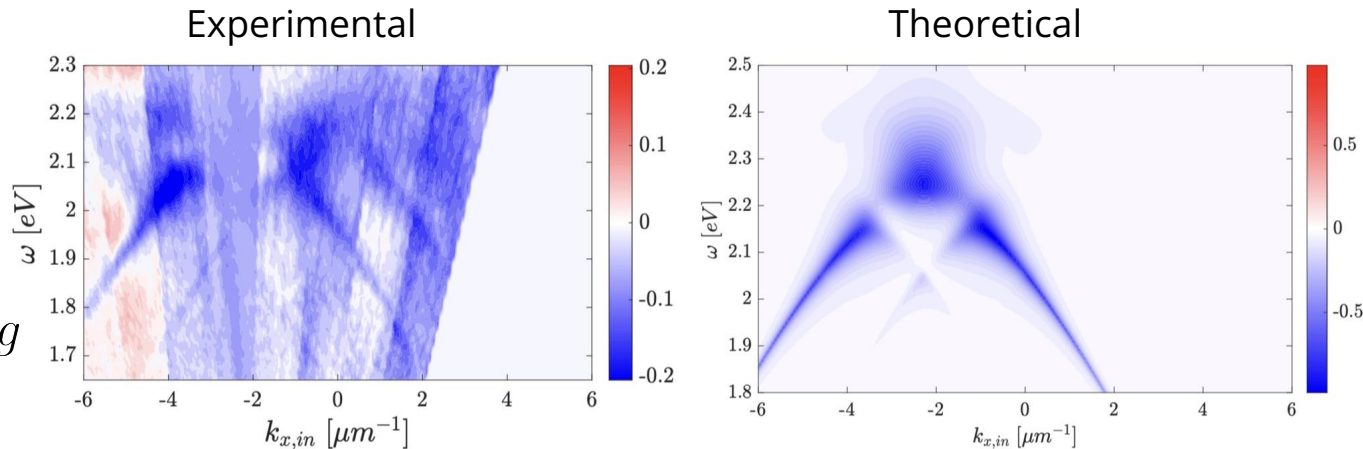
Theoretical



# Mueller polarimetry

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# Summary

- We have demonstrated that the **spin-momentum locking does not require global symmetries** and already appears when elements in the unit cell have a non-zero winding number.

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- We have demonstrated that the **spin-momentum locking does not require global symmetries** and already appears when elements in the unit cell have a non-zero winding number.
- In both cases, **spin-momentum locking is an approximate symmetry**. The reason for this is that circularly polarized light gets an elliptical projection onto the surface away from normal incidence.
- Mueller polarimetry demonstrates that, phenomenologically, we can explain the **plasmonic resonances as two-steps processes**.

# Acknowledgements

## Supervisor



Luis Martín-Moreno

INMA  
CSIC-Universidad de Zaragoza

## Collaborators



Gian Lorenzo  
Paravicini-Bagliani

ISIS  
CNRS-Université de Strasbourg



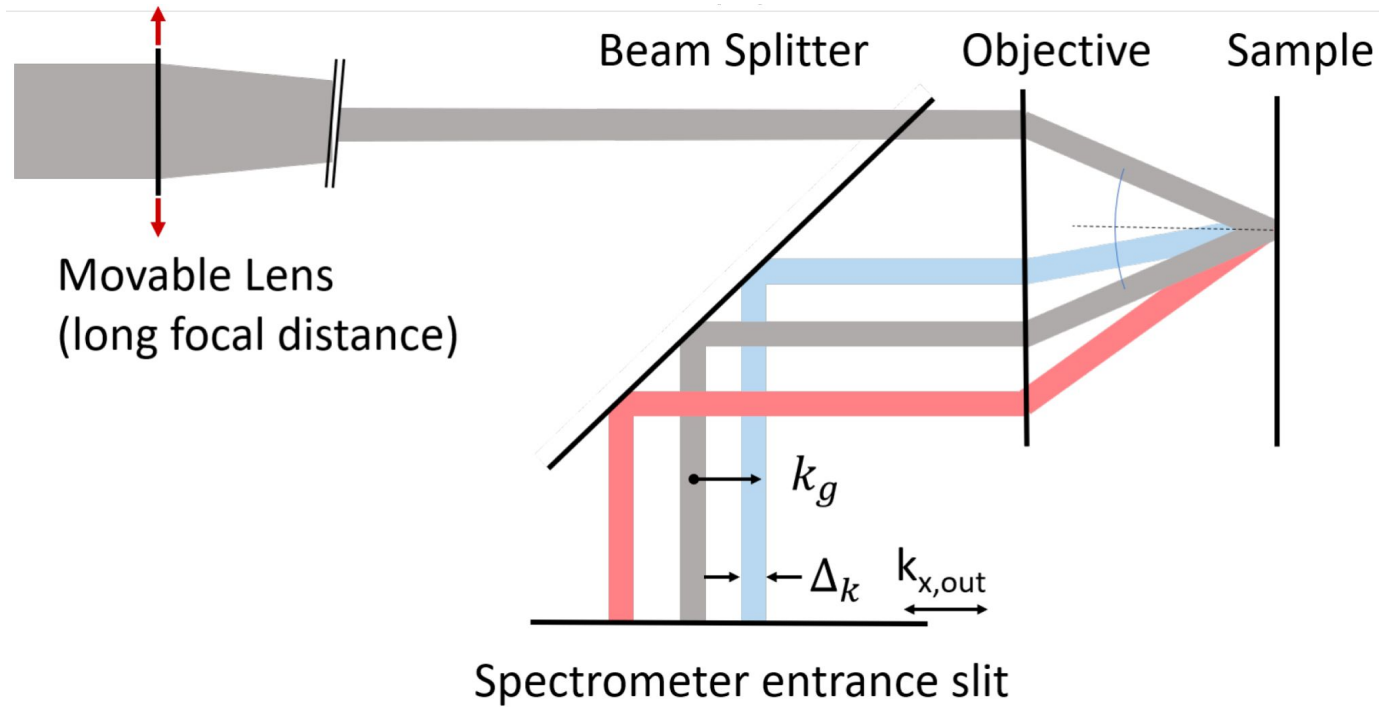
Cyriaque Genet



Sudipta Saha

Thank you for  
your attention!

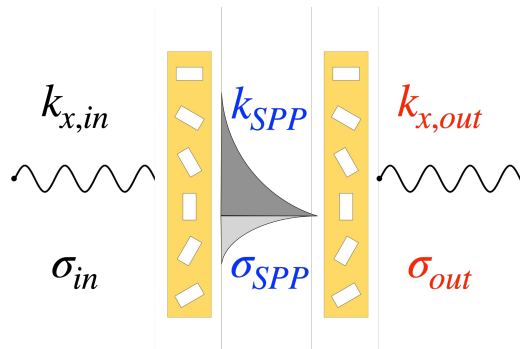
# Experimental set-up



# Photon-plasmon-photon scattering

At the **plasmonic resonance**

Two-steps scattering process  
**photon-plasmon-photon**



$$k_{SPP} = k_{x,in} + s_1 k_g$$

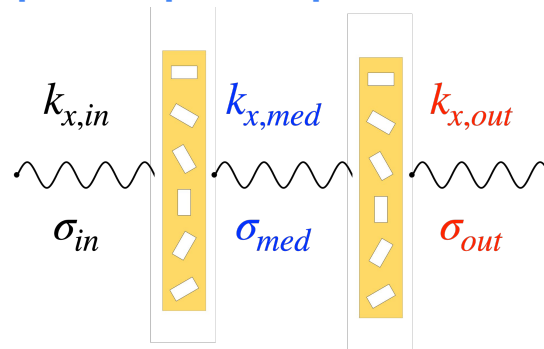
$$|\sigma_{SPP}\rangle = |p\rangle \langle p| \hat{\sigma}^{-s_1} |\sigma_{in}\rangle$$

$$k_{x,out} = k_{SPP} + s_2 k_g = k_{x,in} + (s_1 + s_2) k_g$$

$$|\sigma_{out}\rangle = \hat{\sigma}^{-s_2} |\sigma_{SPP}\rangle = \langle p| \hat{\sigma}^{-s_1} |\sigma_{in}\rangle \hat{\sigma}^{-s_2} |p\rangle$$

**Out of** the plasmonic resonance

Two-steps scattering process  
**photon-photon-photon**



$$k_{x,med} = k_{x,in} + s_1 k_g$$

$$|\sigma_{med}\rangle = \hat{\sigma}^{-s_1} |\sigma_{in}\rangle$$

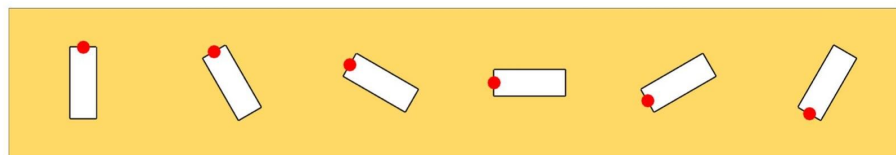
$$k_{x,out} = k_{x,med} + s_2 k_g = k_{x,in} + (s_1 + s_2) k_g$$

$$|\sigma_{out}\rangle = \hat{\sigma}^{-s_2} |\sigma_{med}\rangle = \hat{\sigma}^{-(s_1+s_2)} |\sigma_{in}\rangle$$

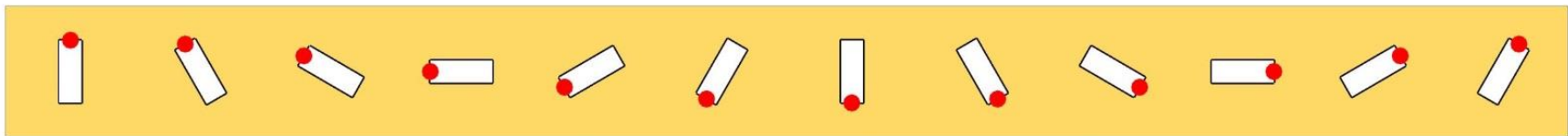


# Winding number

$$N = 6 \quad n_w = +1/2$$

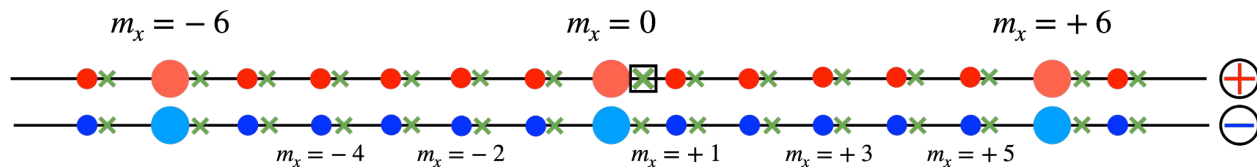


$$N = 12 \quad n_w = +1$$

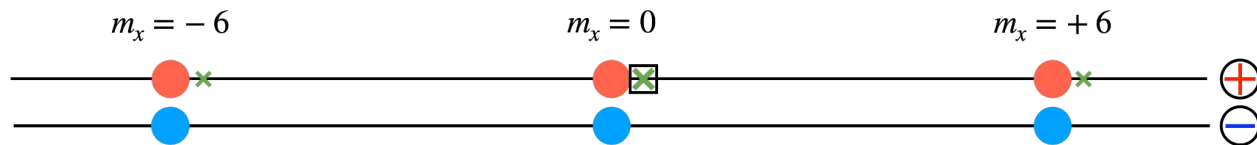
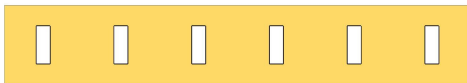


# Fourier representation of the symmetries

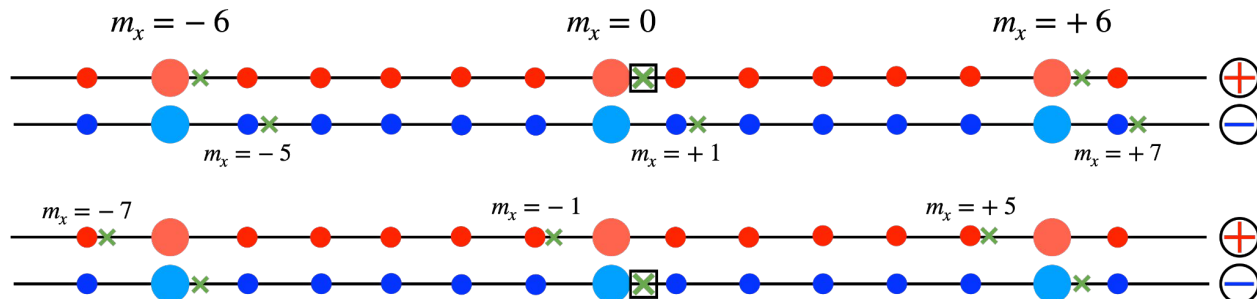
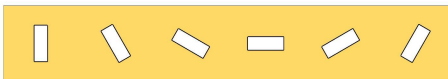
## Disordered unit cell



## Ordered unit cell



## Chiral unit cell

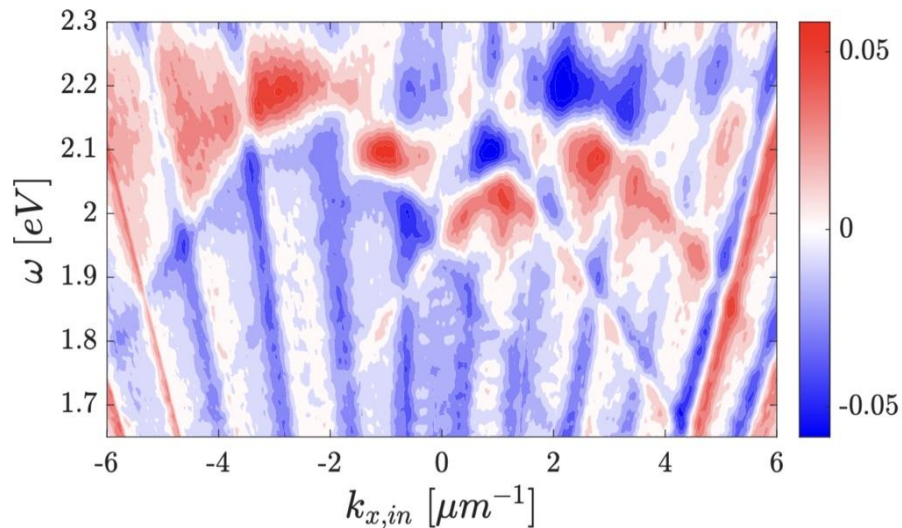


# Mueller polarimetry

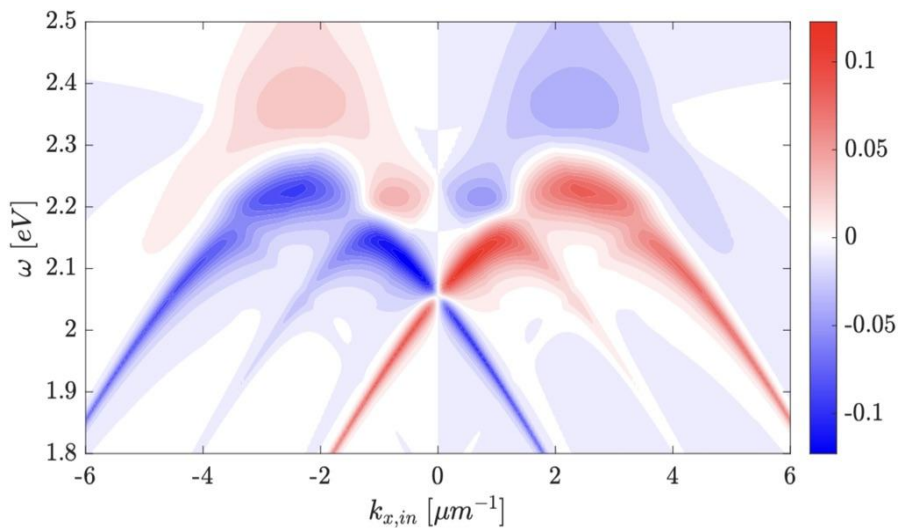
 $M_{30}^0$ 

$$k_{x,out} = k_{x,in}$$

Experimental

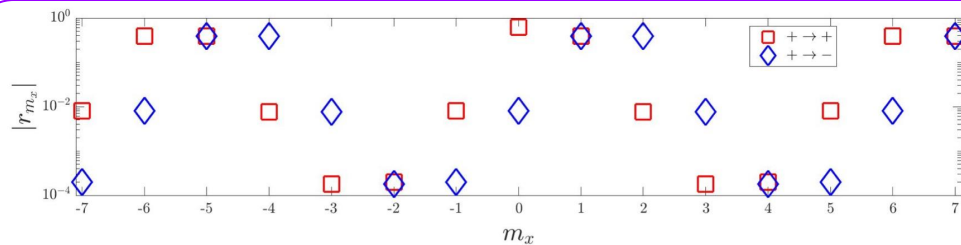


Theoretical

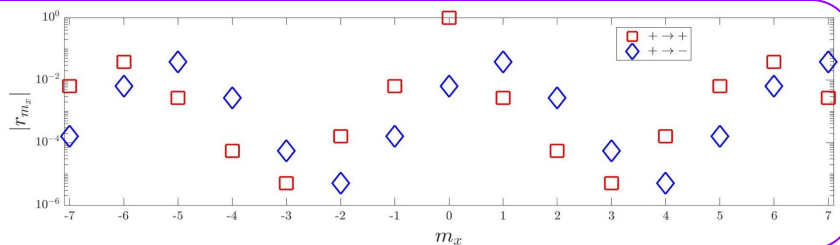


# Breakdown hierarchy

Plasmonic resonance

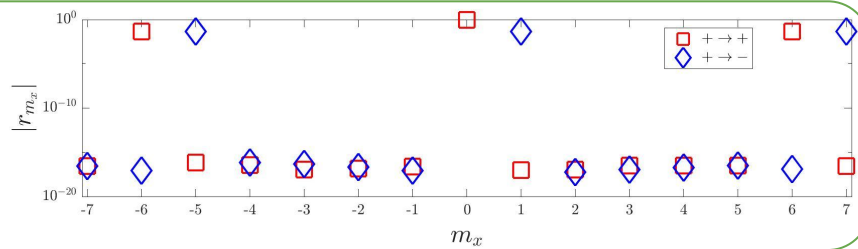
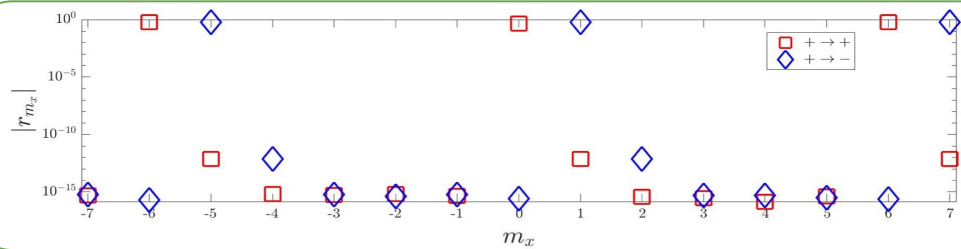


Out of plasmonic resonance



SML breakdown

perfect SML



# Helical light

