INTRODUCTION TO MASS OPTIMIZATION PROBLEMS

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The starting point is the formulation of a *shape optimization problem*

$$\min\Big\{F(\Omega)\ :\ \Omega\in\mathcal{A}\Big\}$$

for which the existence of an optimal domain Ω_{opt} occurs only in particular situations when:

(1) either strong geometrical assumptions are imposed on the class $\mathcal{A}$ of admissible domains, as for instance convexity;

(2) or the shape functional F has particular properties, as for instance the monotonicity with respect to the set inclusion.

In general the existence of an optimal domain fails, and minimizing sequences (Ω_n) tend, in some suitable sense, to *relaxed solutions* that are measures.

In this session the following two main cases will be discussed.

Case I: optimization problems for coefficients.

The relaxed functional becomes

$$F(\mu)=\inf\Big\{\int_D\frac{1}{2}|\nabla u|^2\,d\mu-\langle f,u\rangle-C(\mu)\ :\ u\in C_c^\infty(D)\Big\}$$

where the term $C(\mu)$ takes into account, via Lagrange multipliers, the constraints on the coefficients, D is the given design region, and f is a given datum. The question is to write the cost functional $F(\mu)$ in a more explicit way, and to study the properties of the relaxed solutions μ_{opt}.

Case II: optimization problems for potentials.

Here the relaxed functional takes the form

$$F(\mu)=\inf\Big\{\int_D\frac{1}{2}|\nabla u|^2\,dx+\int_D\frac{1}{2}u^2\,d\mu-\langle f,u\rangle-C(\mu)\ :\ u\in C_c^\infty(D)\Big\}$$

and again the question is to represent $F(\mu)$ is a more explicit way, studying the properties of the relaxed solutions, that in this case belong to the class of *capacitary measures*.

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