Reproducing sensory-induced geometric visual hallucinations

(Joint work with Yacine Chitour and Dario Prandi)

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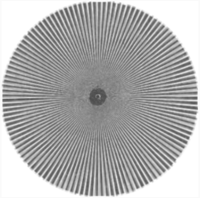
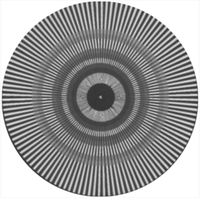
IX Partial differential equations, optimal design and numerics

(Benasque, Espagne)

August 26, 2022

MacKay effects, MacKay (Nature, 1957)

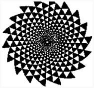
The MacKay effect: the presentation of the stimulus to the left ("MacKay rays") induces the perception of the image (Artist depiction by Isia Léviant) on the right. Adapted from MacKay (Nature, '57) and Zeki *et al* (Bio. Sci., '93).

$\Longrightarrow$ Can we **theoretically describe** these phenomena?

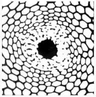
Introduction

Visual hallucination ⇄ perception of an image which does not exist.

We focus on *geometric visual hallucinations* or *form constants* (Klüver, '67).



- Strong pressure on the eyeballs[1]

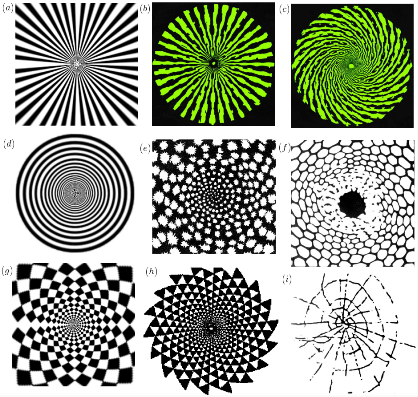


- LSD and marijuana induced hallucinations[2]

- Schizophrenics hallucinations[3]

[1] (Oster '70)
[2] (Oster '70; Siegel '77)
[3] (Siegel '77)

How do form constants look like and where do they originate?

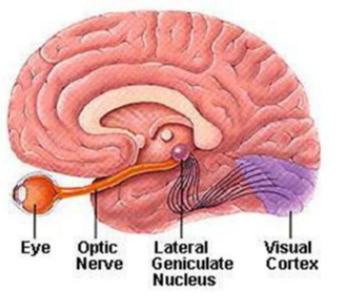


➛ People can constantly hallucinate in total darkness;

➛ Blind people can also hallucinate;

⟹ The origin of form constants is cortical, and seems to be in the primary visual cortex or V1.

The location of V1



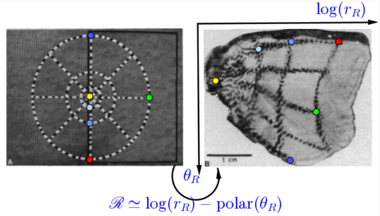
V1 and visual pathway.

Retinotopic structure of V1

Retino-cortical map (Schwartz '77)

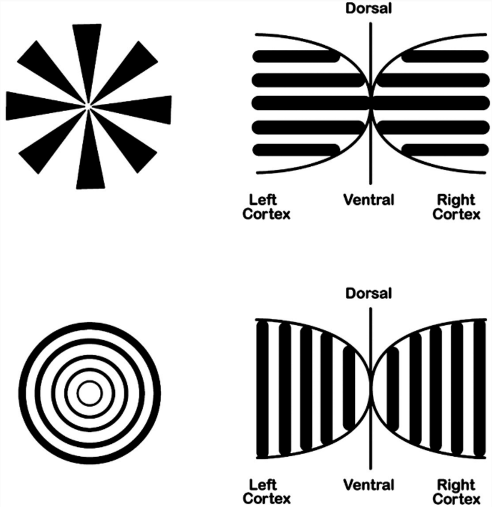
Using polar coordinates on the retinal plane, we have

$$\begin{aligned} \mathscr{R} : \quad (0, +\infty) \times [0, 2\pi) &\longrightarrow \mathbb{R}^2 \\ re^{i\theta} &\longmapsto (x_1, x_2) = (\log(r), \theta). \end{aligned}$$



Adapted from (Tootell *et al*, '82)

Retinotopic structure of V1



Visual illustration of the retino-cortical map. Hallucinatory patterns in the left and cortical patterns in the right. Reproduced from Billock and Tsou (PNAS, '07).

Binary pattern

- We represent hallucinatory and cortical patterns as contrasting white and dark regions;
- For a given pattern P, let B_P denotes its corresponding **binary pattern**. In the regions where $P > 0$ we have $B_P = 1$ (white) whereas, where $P < 0$ we have $B_P = 0$ (dark).

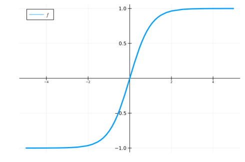
$\Longrightarrow$ Every function $h : \mathbb{R}^2 \to \mathbb{R}$ having the same set of zeroes as P will have the same **binary pattern** B_P.

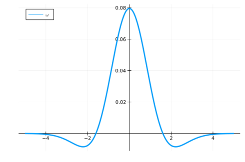
Neuronal Activity in V1

Neuronal field equation

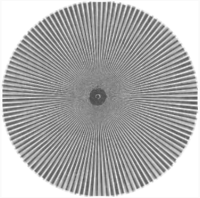
$$\frac{\partial}{\partial t}a(x,t) = -\alpha a(x,t) + \mu \int_{\mathbb{R}^2} \omega(\|x - x'\|) f(a(x',t))dx' + I_{\text{ext}}(x,t). \qquad \text{(NF)}$$

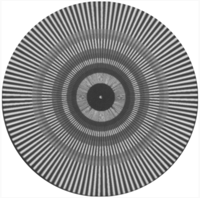
1. $x \in \mathbb{R}^2$ and $t > 0$;
2. $\alpha > 0$ decay rate in absence of stimulation and excitation;
3. f non-linear response function (sigmoid);
4. I_{ext} cortical representation of visual stimulus;
5. ω interaction kernel, typically a DoG;
6. $\mu > 0$ excitability rate of V1.





MacKay effects via controllability of (NF) equation





The MacKay effect: the presentation of the stimulus to the left ("MacKay rays") induces the perception of the image (Artist depiction by Isia Léviant) on the right. Adapted from MacKay (Nature, '57) and Zeki *et al* (Bio. Sci., '93).

$$\begin{cases} \partial_t a(x,t) + \underbrace{\alpha a(x,t) - {\color{red}\mu} \int_{\mathbb{R}^2} \omega(\|x-x'\|) f(a(x',t))dx'}_{\color{blue}\infty\text{-dim, non-local and nonlinear operator}} &= P_F(x) + \varepsilon \mathbb{1}_\Omega u(x), \\ a(x,0) &= a_0(x). \end{cases}$$

Meaning of Controllability

Let the final time $T \gg 1$. Does there a control u such that the solution a of the above Cauchy-problem satisfies $a(T) \approx$ image in the right (cortical one)?

Plan of investigation of the above question

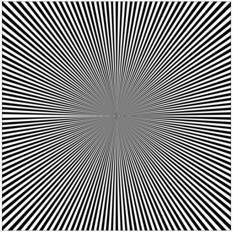
Focus on the funnel pattern

1/ Redundant informations is needed in the P_F for MacKay effects:

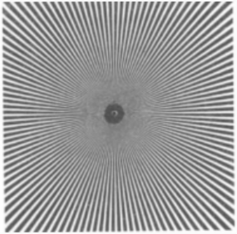
➛ P_F can not induce MacKay effects in the linear and non-linear regime;

2/ $P_F + \varepsilon \mathbb{1}_\Omega u$, $\varepsilon > 0$ induces the MacKay effects

➛ Numerical results in the linear and non-linear regime



P_F.



$P_F + \varepsilon \mathbb{1}_\Omega u$.

MacKay effects with P_F and P_T in the linear regime?

We set

$$\mu_c := \frac{\alpha}{f'(0)\max_{r\geq 0}\widehat{\omega}(r)} \quad \text{and} \quad \mu_0 := \frac{\alpha}{f'(0)\|\omega\|_{L^1(\mathbb{R}^2)}} \leq \mu_c,$$

We let $P_T(x) = \cos(\lambda x_1)$ and $P_F(x) = \cos(\lambda x_2)$, $x = (x_1, x_2) \in \mathbb{R}^2$ and $\lambda > 0$.

Theorem (T, Prandi, Chitour, '22)

Consider the linear equation

$$\begin{cases} \partial_t a(x,t) &= -\alpha a(x,t) + \mu \int_{\mathbb{R}^2} \omega(\|x - x'\|) a(x', t) dx' + \mathrm{I}_{\mathrm{ext}}(x), \\ a(x, 0) &= a_0(x). \end{cases}$$

Let $a_0 \in L^\infty(\mathbb{R}^2)$. If $\mathrm{I}_{\mathrm{ext}} = P_F$ or $\mathrm{I}_{\mathrm{ext}} = P_T$, then the unique solution of the above Cauchy-problem satisfies

$$a(\cdot, t) \xrightarrow[t\to\infty]{} \frac{1}{\alpha}\frac{\mu_c}{\mu_c - \mu}\mathrm{I}_{\mathrm{ext}}(\cdot), \qquad \text{exponentially in} \qquad L^\infty(\mathbb{R}^2),$$

provided that $\mu < \mu_0$.

$\Longrightarrow$ There is no MacKay effects in the linear regime via P_F and P_T

Mackay effects with P_F and P_T in the nonlinear regime?

Theorem (T, Prandi, Chitour, '22)

Consider the (NF) equation

$$\begin{cases} \partial_t a(x,t) & = -\alpha a(x,t) + \mu \int_{\mathbb{R}^2} \omega(\|x - x'\|) f(a(x',t)) dx' + \mathrm{I}_{\mathrm{ext}}(x), \\ a(x,0) & = a_0(x). \end{cases}$$

Let $1 \leq p \leq \infty$, $a_0 \in L^p(\mathbb{R}^2)$ and $\mathrm{I}_{\mathrm{ext}} \in L^p(\mathbb{R}^2)$. If $\mu < \mu_0$, then the solution $a(\cdot, t)$ converges exponentially to the stationary state $a_{\mathrm{I}_{\mathrm{ext}}}(\cdot)$ in $L^p(\mathbb{R}^2)$ when $t \longrightarrow \infty$.

We introduce for every $1 \leq p \leq \infty$, the input-stationary output map

$$\begin{aligned} \Psi : \quad L^p(\mathbb{R}^2) & \longrightarrow L^p(\mathbb{R}^2) \\ \mathrm{I}_{\mathrm{ext}} & \longmapsto \Psi(\mathrm{I}_{\mathrm{ext}}) = \frac{\mu}{\alpha} \int_{\mathbb{R}^2} \omega(\|x - y\|) f\left(\Psi(\mathrm{I}_{\mathrm{ext}})(y)\right) dy + \frac{1}{\alpha} \mathrm{I}_{\mathrm{ext}} . \end{aligned}$$

Mackay effects with P_F and P_T in the nonlinear regime?

Lemma (T, Prandi, Chitour, '22)

Let $1 \leq p \leq \infty$. If $\mu < \mu_0$, then

1. Ψ is well-defined and bi-Lipschitz continuous;
2. Ψ commutes with the plane's rigid motion group $\mathrm{E}(2) := \mathbb{R}^2 \rtimes \mathcal{O}(2)$;
3. If $2 \leq p \leq \infty$, then Ψ belongs to $C^1(L^p(\mathbb{R}^2); L^p(\mathbb{R}^2))$.

Theorem (T, Prandi, Chitour, '22)

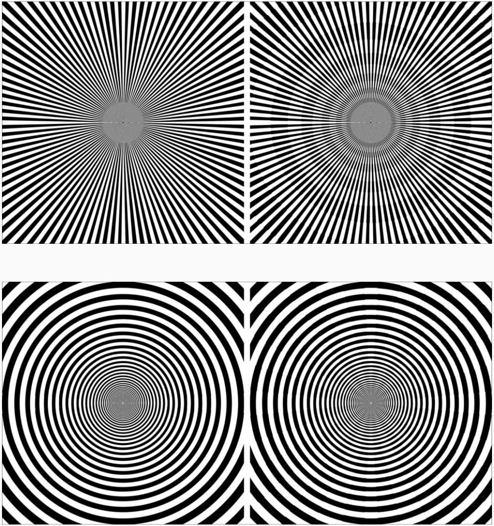
Under the assumption $\mu < \mu_0/2$, the zeroes of P_T (resp. P_F) coincide with those of $\Psi(P_T)$ (resp. $\Psi(P_F)$).

$\Longrightarrow$ There is no MacKay effect in the non-linear regime via P_T and P_F.

It is necessary to break the symmetry of P_F and P_T with a localised control u:

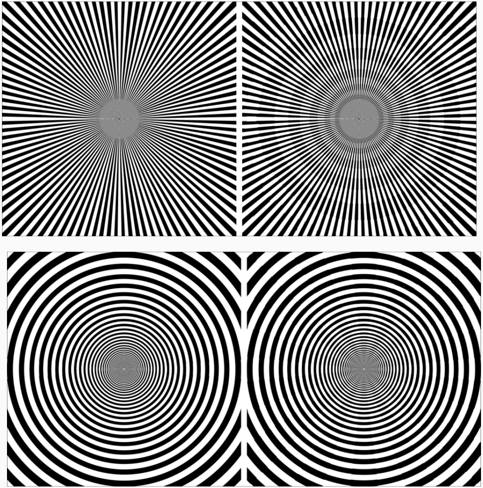
- $P_F + \varepsilon \mathbb{1}_\Omega u$, Ω is a neighbourhood of the fovea and $u \equiv 1$;
- $P_T + \varepsilon \mathbb{1}_{\Omega'} u$, Ω' covers the fovea and the periphery and $u \equiv 1$.

Mackay effects with “MacKay stimulus” in the linear regime, numerical results



Inputs in left and stationary states in the right.

Mackay effects with "MacKay stimulus" in the non-linear regime, numerical results



Inputs in left and stationary states in the right. The non-linear function f is odd, $f(t) = \tanh(t)$.

Conclusion and Ongoing work

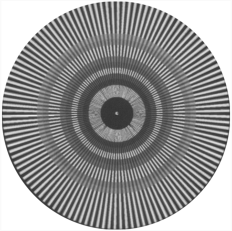
Conclusion :

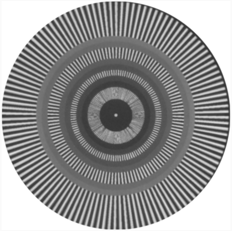
We presented a parsimonious model and mathematical sound framework for the description of sensory induced geometrical visual hallucinations allowing us to reproduce the “static” MacKay effects. These effects result from visual stimuli, with global E(2)-symmetry broken.

QR code for numerical results.

Ongoing work:

What framework can allow to reproduce the “dynamic” version of such effects ?





Thank you for listening!

Tamekue C., Prandi D. & Chitour Y. (February 2022)

Reproducing sensory induced hallucinations via neural fields

Accepted in 2022 IEEE International Conference on Image Processing (ICIP)

Ermentrout B. & Cowan J.

A mathematical theory of visual hallucination patterns

Biological cybernetics, vol. 34, no. 3, pp. 137–150, 1979