DETERMINISTIC DECOUPLING OF GLOBAL FEATURES AND ITS APPLICATION TO DATA ANALYSIS.

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] IO-CSIC.

MOTIVATION: TEXTURES

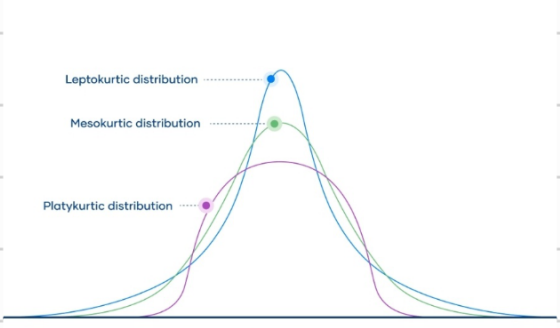
Visual textures in nature

- Portilla-Simoncelli (2000): statistical model for the description of texture images
- Problems: classification, synthesis, transfer, ...

MOMENTS IN STATISTICS:

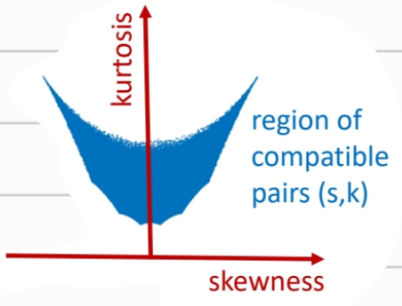
- Mean
- Variance

- Skewness



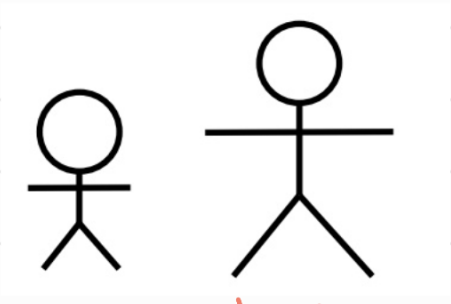
- Kurtosis

heavily coupled!



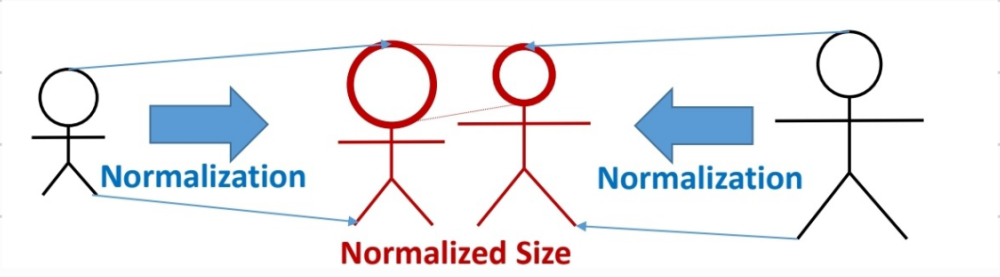
MAIN OBJECTIVE: DECOUPLING

Two measurements: total size, head size



coupled!

New measurement: heads' relative size ↑ decoupled from total size



equalization

Back to (standarized) moments:

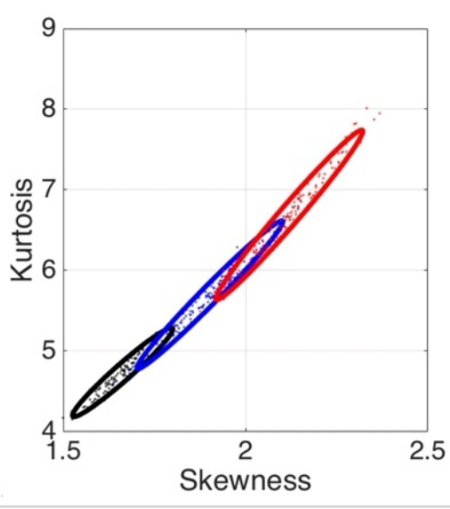
- Mean
- Variance
- Skewness

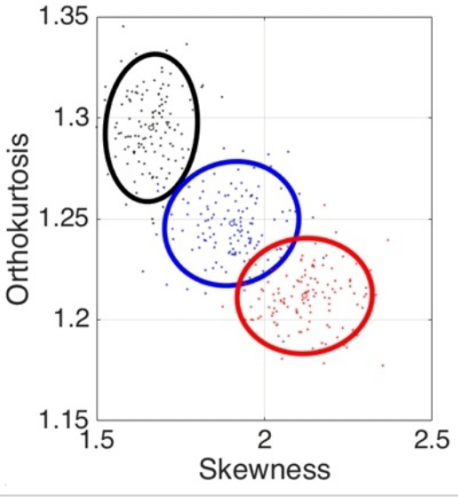
} gradients are orthogonal everywhere!

- Kurtosis ⟶ gradient is not orthogonal

New:

orthokurtosis

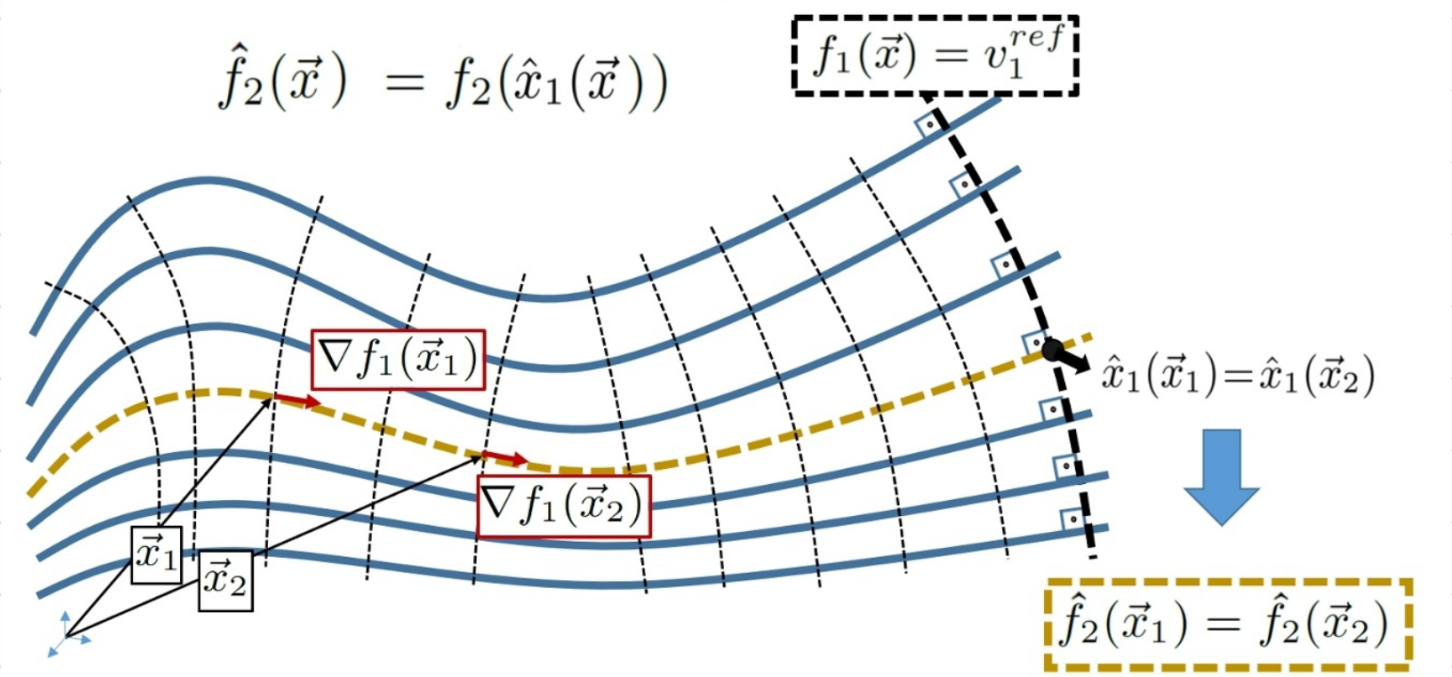




DECOUPLING VIA NORMALIZATION

- feature: $f: \mathbb{R}^N \to \mathbb{R}$
- f_i and f_j are decoupled if $\nabla f_i \perp \nabla f_j$

NAÏVE IDEA: surfing invariance manifolds

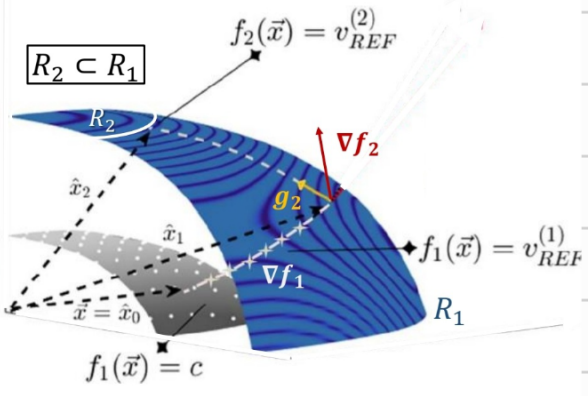


SOME IDEAS OF THE ALGORITHM: $\{f_1, \dots, f_k\}$

- Nested normalization
- Move along the invariance manifold de $\{f_1 \dots f_k\}$ until we cross the reference manifold.
- Frobenius theorem !!
- Algorithm: → Narrow path → approximate decoupling
  → Broad path

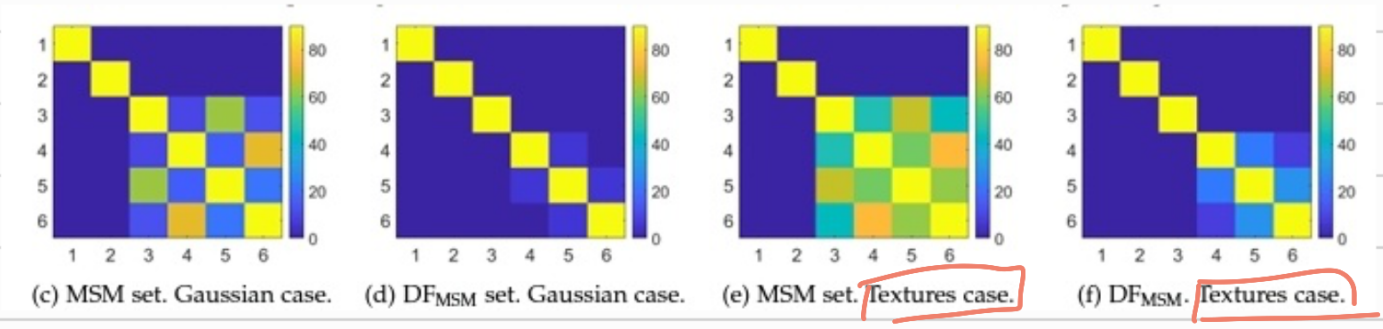
Require: $\mathbf{x} \in \Omega \setminus \Lambda$, $\{f_j, v_j^{ref},\ j = 1, \dots, M\}$

1: **Initialization:** $\hat{f}_1 = f_1$; $\mathcal{R}_0 = \Omega \setminus \Lambda$; $\hat{\mathbf{x}}_0(\mathbf{x}) = \mathbf{x}$
2: **for** $k = 1$ to $k = M - 1$ **do**
3: Compute $\mathbf{g}_k = P_{\mathcal{R}_{k-1}}(\nabla f_k)$
4: $\mathbf{y}_k(0) = \hat{\mathbf{x}}_{k-1}(\mathbf{x})$
5: Follow $\mathbf{g}_k$ until $f_k(\mathbf{y}_k(t)) = v_k^{ref}$
6: $\hat{\mathbf{x}}_k(\mathbf{x}; \mathbf{v}_k^{ref}) = \mathbf{y}_k(t)$
7: $\hat{f}_{k+1}(\mathbf{x}) = f_{k+1}(\hat{\mathbf{x}}_k)$
8: **end for**
9: **return** $\{\hat{f}_j(\mathbf{x}), j = 1, \dots, M\}, \hat{x}_{M-1}(\mathbf{x})$



APPLICATIONS:

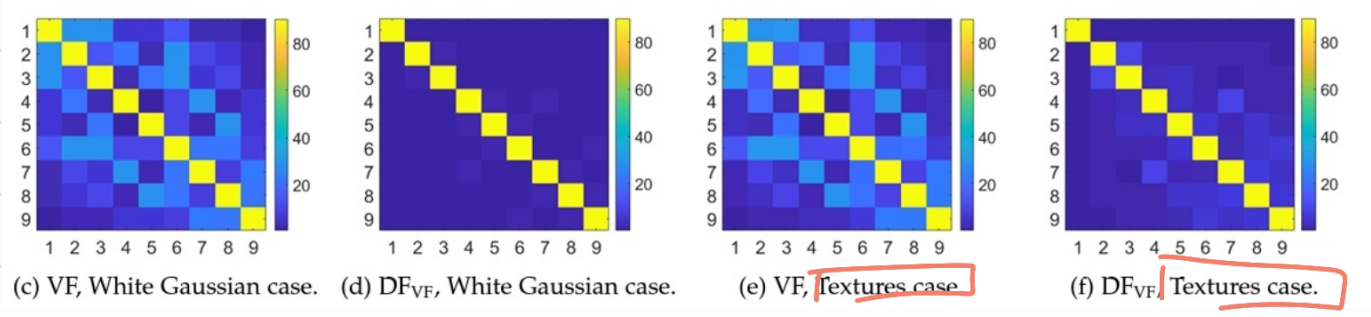
- Kurtosis → orthokurtosis (exact decoupling)
- Higher moments → orthomoments (approximate)



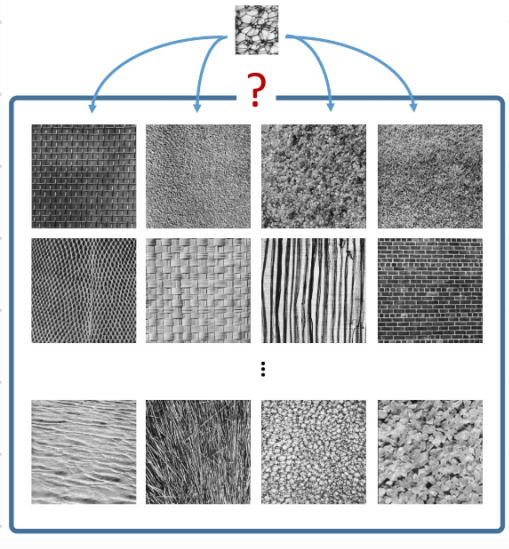
(c) MSM set. Gaussian case. (d) DF_{MSM} set. Gaussian case. (e) MSM set. Textures case. (f) DF_{MSM}. Textures case.

- 2nd order moments at the output of a set of filters

$$f(x) = \frac{1}{N}(x * h)^{\odot 2}$$



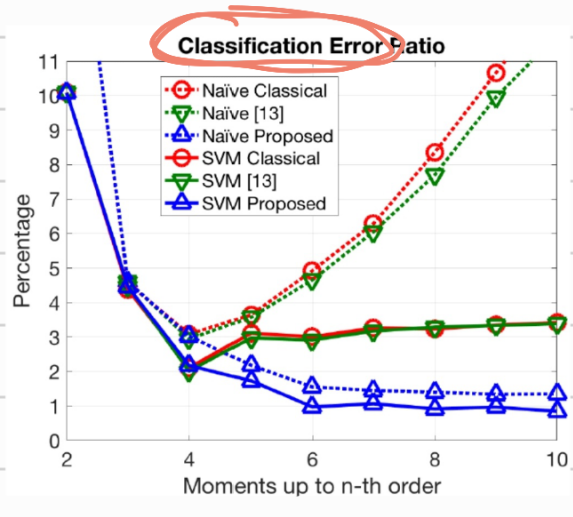
(c) VF, White Gaussian case. (d) DF_{VF}, White Gaussian case. (e) VF, Textures case. (f) DF_{VF}. Textures case.



Texture classification:

Given a small block, decide to which texture it corresponds

Use orthomoments



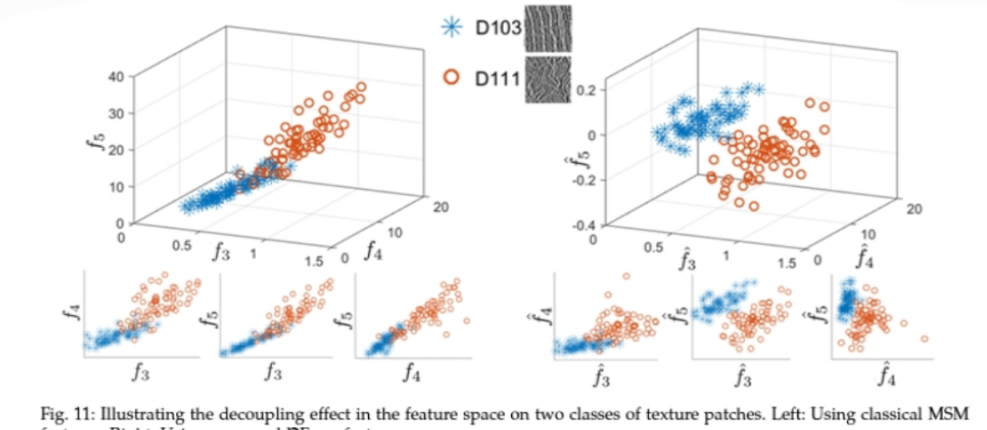


Fig. 11: Illustrating the decoupling effect in the feature space on two classes of texture patches. Left: Using classical MSM features; Right: Using proposed DF_{MSM} features.

APPLICATIONS: Style transfer

- Canham - Martín - Bertalmio - Partilla

  Use higher moments

- Portilla + UV + UJ. Convolutional networks for image recognition.