Free hyperboloidal evolution

From Einstein to infinity

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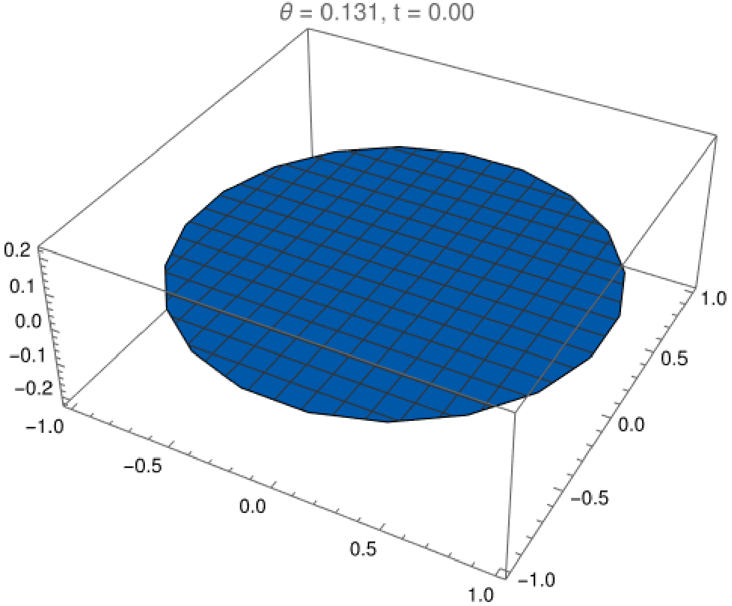
New frontiers in strong gravity - Benasque, 8th July 2022

Hyperboloidal gauge waves in full 3D - ongoing work



General Relativity

↓

Numerical relativity

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Gravitational waveforms

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Waveform models

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Detections and parameter estimation

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Tests of GR | Cosmology | Astrophysics

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State-of-the-art of numerical gravitational wave creation

What we can do

- simulate binary systems of compact objects
- extract their gravitational wave signals
- suitably "propagate" them to infinity
- use good-enough boundary conditions

What we can do better

- make simulations more efficient
- improve the accuracy of the waves
- extract the signals directly at infinity
- do it without any "complicated constructions"

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- extract the signals directly at infinity
- do it without any "complicated constructions"

Suitable method to cover these improvements:

the hyperboloidal approach.

Contents

Outline

1. Introduction
   - Motivation
   - Spacetime foliations
   - Overview of the problem
2. Conformal compactification
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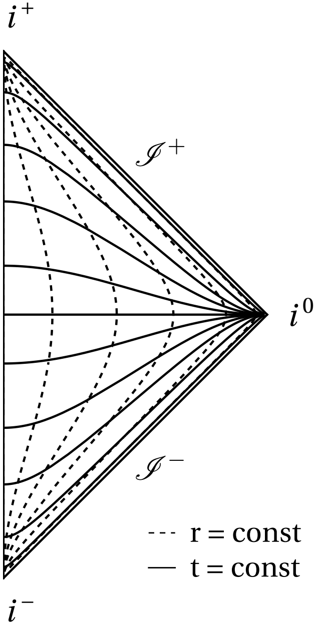
4. Simulations in 3D

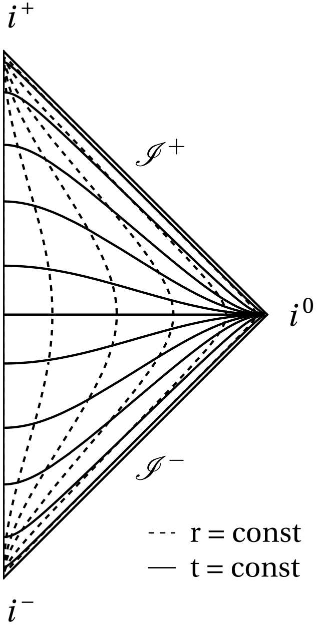
Reaching future lightlike infinity



Future null infinity ($\mathscr{I}^+$) is a region of spacetime of interest

- for the study of global properties of spacetimes and
- for the extraction of gravitational waves (only well described at $\mathscr{I}^+$, where observers are located; GW memory effect).

Reaching future lightlike infinity



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- for the study of global properties of spacetimes and
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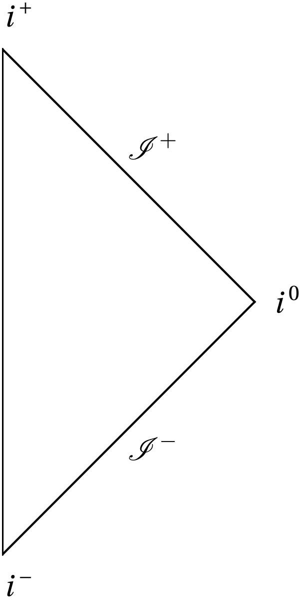
How to include future null infinity in a numerical integration domain?

$\Rightarrow$ Balance the blowup of the coordinates with the decay of the fields.

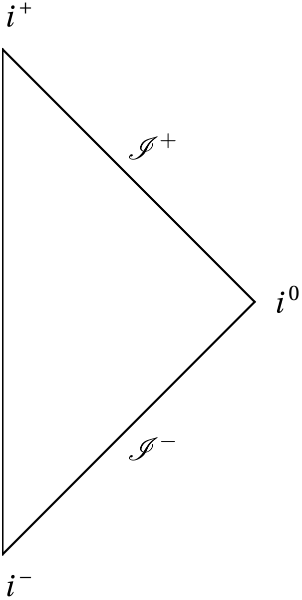
Example:
Penrose's conformal compactification (60s).

Slicing spacetime



Standard slicing options for the initial value formulation of the Einstein equations, to solve them as an evolution in time:

Slicing spacetime



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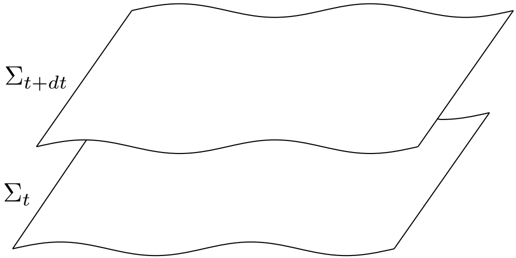

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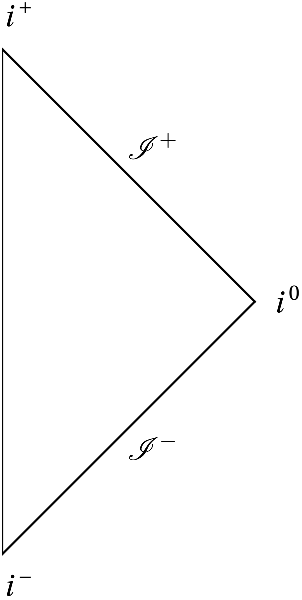

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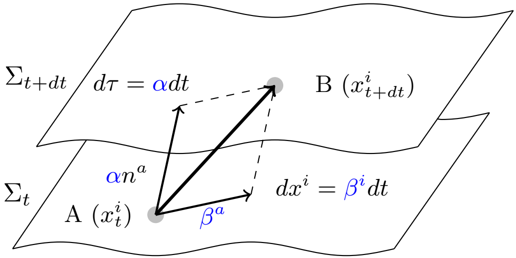

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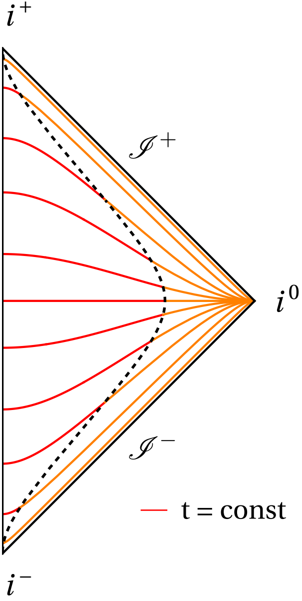
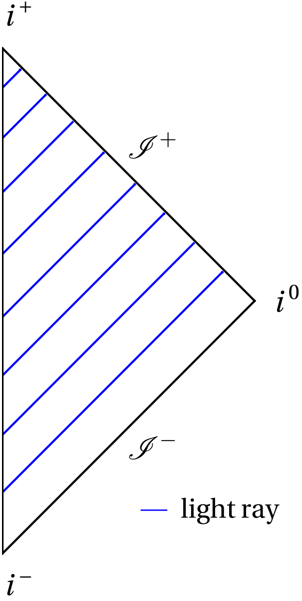


Standard slicing options for the initial value formulation of the Einstein equations, to solve them as an evolution in time:

- Standard Cauchy slices (constant time)

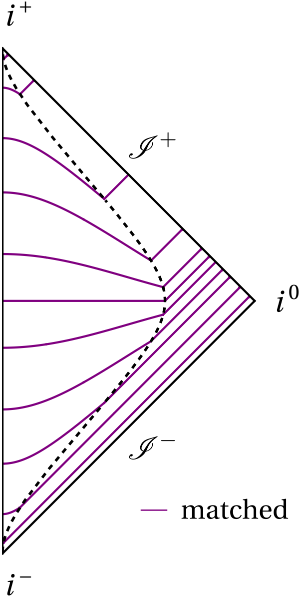
Slicing spacetime



Standard slicing options for the initial value formulation of the Einstein equations, to solve them as an evolution in time:

- Standard Cauchy slices (constant time)
- Null slices (outgoing light rays)

Slicing spacetime



Standard slicing options for the initial value formulation of the Einstein equations, to solve them as an evolution in time:

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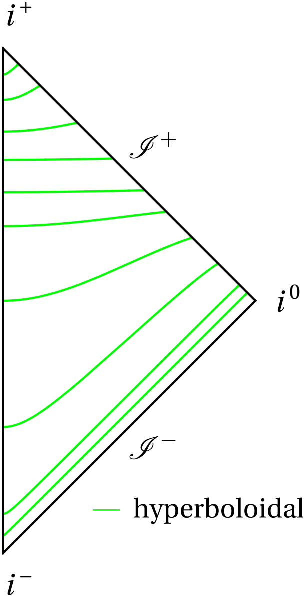
Slicing spacetime



Standard slicing options for the initial value formulation of the Einstein equations, to solve them as an evolution in time:

- Standard Cauchy slices (constant time)
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- Cauchy-Characteristic matching / extraction
- Hyperboloidal slices

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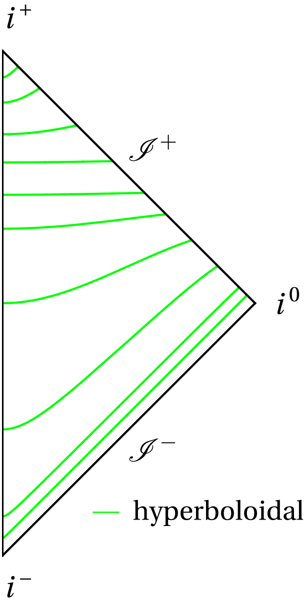

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Advantages of the hyperboloidal approach:

- Slices spacelike & smooth everywhere.
- Extraction at $\mathscr{I}^+$, no approximations.
- More resolution for the central part.

Overview of the hyperboloidal initial value problem

Analytical work by Helmut Friedrich in the 70-80s.

First numerical implementations in the 90s.

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Main difficulties for a numerical implementation:

- Evolution variables diverge at $\mathscr{I}^+$ or formally divergent terms at $\mathscr{I}^+$ appear in the equations.
- $\Rightarrow$ Ensure that cancellations happen; some variables may need rescaling (if $\phi|_{\mathscr{I}} \sim \frac{1}{r}$ $\rightarrow$ use $\Phi = r\phi$ and $\Phi|_{\mathscr{I}} \sim 1$).
- Non-trivial background ($\tilde{K} \neq 0$), unlike with Cauchy slices.

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Present work is the first free evolution hyperboloidal code using a common formulation — BSSN / Z4 (conformal) and generalized harmonic (dual frame).

Outline

Conformal compactification methods

Penrose's conformal compactification of spacetime is a suitable approach for including $\mathscr{I}^+$ in the computational domain. Rescale the physical metric $\tilde{g}_{\mu\nu}$

$$\bar{g}_{\mu\nu} \equiv \Omega^2 \tilde{g}_{\mu\nu} \tag{1}$$

with $\Omega|_{\mathscr{I}^+} = 0$ to keep $\bar{g}_{\mu\nu}$ finite there.

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Expressing them in terms of the rescaled metric $\bar{g}_{ab} = \Omega^2 \tilde{g}_{ab}$ gives

$$G[\bar{g}]_{ab} + \frac{2}{\Omega}(\bar{\nabla}_a \bar{\nabla}_b \Omega - \bar{g}_{ab} \bar{\Box} \Omega) + \frac{3}{\Omega^2} \bar{g}_{ab} (\bar{\nabla}_c \Omega)(\bar{\nabla}^c \Omega) = 8\pi T[\tfrac{\bar{g}}{\Omega^2}]_{ab}.$$

Extra formally divergent terms at $\mathscr{I}$ appear in the equations.

GBSSN and conformal Z4 equations I

Line element: $$d\bar{s}^2 = \left(-\alpha^2 + \beta_i\beta^i\right) dt^2 + 2\beta_i\, dt\, dx^i + \frac{\gamma_{ij}}{\chi} dx^i dx^j$$

$$\begin{aligned}
\partial_\perp \chi &= \frac{2}{3}\alpha\chi(K+2\Theta) + \frac{1}{3}\chi\partial_\perp \ln\gamma \ , \\
\partial_\perp \gamma_{ab} &= -2A_{ab}\alpha + \frac{1}{3}\gamma_{ab}\partial_\perp \ln\gamma \ , \\
\partial_\perp A_{ab} &= \Big[\alpha\chi\left(R[D]_{ab} + 2D_{(a}Z_{b)}\right) - \chi D_a D_b \alpha - D_{(a}\alpha D_{b)}\chi - \frac{\alpha D_a\chi D_b\chi}{4\chi} + \frac{1}{2}\alpha D_a D_b \chi \\
&\quad + 2Z_{(a}\alpha D_{b)}\chi + \frac{2\alpha D_{(a}\chi D_{b)}\Omega}{\Omega} + \frac{2\alpha\chi D_a D_b\Omega}{\Omega} + \frac{4\alpha\chi Z_{(a}D_{b)}\Omega}{\Omega} - 8\pi\alpha\chi S_{ab}\Big]^{TF} \\
&\quad - 2\alpha A^c_a A_{bc} + \alpha A_{ab}[K + 2(1 - C_{Z4c})\Theta] + \frac{2A_{ab}\partial_\perp\Omega}{\Omega} + \frac{1}{3}A_{ab}\partial_\perp \ln\gamma \ , \\
\partial_\perp K &= \alpha\left[A_{ab}A^{ab} + \frac{1}{3}(K+2\Theta)^2 + \frac{\kappa_1(1-\kappa_2)\Theta}{\Omega}\right] - \chi\triangle\alpha + \frac{1}{2}D^a\alpha D_a\chi + 2C_{Z4c}Z^a D_a\alpha \\
&\quad + \frac{3[(\partial_\perp\Omega)^2 - \alpha^2\chi D^a\Omega D_a\Omega]}{\Omega^2\alpha} - \frac{2\alpha Z^a D_a\Omega}{\Omega} + \frac{3\chi D^a\alpha D_a\Omega}{\Omega} - \frac{\alpha D^a\chi D_a\Omega}{2\Omega} + \frac{\alpha\chi\triangle\Omega}{\Omega} \\
&\quad + \frac{[K + 2(1 + C_{Z4c})\Theta]\partial_\perp\Omega}{\Omega} + \frac{3\partial_\perp\alpha\partial_\perp\Omega}{\Omega\alpha^2} - \frac{3\partial_\perp\partial_\perp\Omega}{\Omega\alpha} + 4\pi\alpha(\rho + S) \ ,
\end{aligned}$$

GBSSN and conformal Z4 equations II

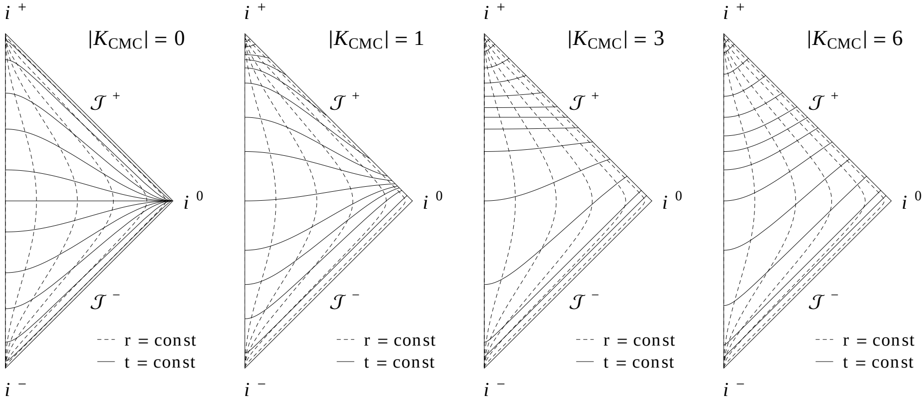
$$\begin{aligned}
\partial_\perp \Lambda^a &= \mathcal{C}^b(D_b\beta^a) + \alpha\left[2A^{bc}\Delta\Gamma^a_{bc} - \frac{4}{3}D^a K - \frac{2}{3}D^a\Theta - \frac{3A^{ab}D_b\chi}{\chi} - \frac{4Z^a(K+2\Theta)}{3\chi} - \frac{2\kappa_1 Z^a}{\Omega\chi}\right] \\
&+ \gamma^{bc}\hat{D}_b\hat{D}_c\beta^a - \gamma^{bc}R[\hat{D}]^a{}_{bcd}\beta^d - 2A^{ab}D_b\alpha - 2C_{Z4c}\Theta D^a\alpha - \frac{4\alpha A^{ab}D_b\Omega}{\Omega} \\
&- \frac{4\alpha(K-\Theta)D^a\Omega}{3\Omega} - \frac{4D^a\partial_\perp\Omega}{\Omega} + \frac{4D^a\alpha\partial_\perp\Omega}{\Omega\alpha} - \frac{4Z^a\partial_\perp\Omega}{\Omega\chi} \\
&- \frac{1}{6}D^a\partial_\perp \ln\gamma - \frac{1}{3}\Delta\Gamma^a\partial_\perp \ln\gamma - \frac{2Z^a\partial_\perp \ln\gamma}{3\chi} - \frac{16\pi J^a\alpha}{\chi} \ , \\
\partial_\perp\Theta &= \frac{\alpha}{2}\left[\chi(R[D] + 2D^a Z_a) - A_{ab}A^{ab} + \frac{2}{3}(K+2\Theta)^2 - 2C_{Z4c}\Theta(K+2\Theta) - \frac{2\kappa_1(2+\kappa_2)\Theta}{\Omega}\right] \\
&+ \alpha \triangle \chi - \frac{5\alpha D^a\chi D_a\chi}{4\chi} - C_{Z4c}Z^a D_a\alpha - \frac{\alpha Z^a D_a\chi}{2\chi} + \frac{2\alpha\chi \triangle \Omega}{\Omega} - \frac{\alpha D^a\chi D_a\Omega}{\Omega} \\
&+ \frac{3[(\partial_\perp\Omega)^2 - \alpha^2\chi D^a\Omega D_a\Omega]}{\Omega^2\alpha} + \frac{2[K+2(1-C_{Z4c})\Theta]\partial_\perp\Omega}{\Omega} - 8\pi\alpha\rho \ .
\end{aligned}$$

$$\begin{aligned}
\mathcal{H} &= \chi R[D] - A_{ab}A^{ab} + \frac{2}{3}(K+2\Theta)^2 + 2\triangle\chi - \frac{5D^a\chi D_a\chi}{2\chi} + \frac{6(\partial_\perp\Omega)^2}{\Omega^2\alpha^2} \\
&- \frac{6\chi D^a\Omega D_a\Omega}{\Omega^2} - \frac{2D^a\chi D_a\Omega}{\Omega} + \frac{4\chi\triangle\Omega}{\Omega} + \frac{4(K+2\Theta)\partial_\perp\Omega}{\Omega\alpha} - 16\pi\rho \ , \\
\mathcal{M}_a &= D_b A_a^b - \frac{2}{3}D_a(K+2\Theta) - \frac{3A_a^b D_b\chi}{2\chi} - \frac{2A_a^b D_b\Omega}{\Omega} - \frac{2(K+2\Theta)D_a\Omega}{3\Omega} - \frac{2D_a\partial_\perp\Omega}{\Omega\alpha} \\
&+ \frac{2D_a\alpha\partial_\perp\Omega}{\Omega\alpha^2} - 8\pi J_a \ , \qquad \mathcal{C}^a = \Lambda^a - \Delta\Gamma^a - \frac{2Z^a}{\chi} \ .
\end{aligned}$$

Flat spacetime on a hyperboloidal foliation

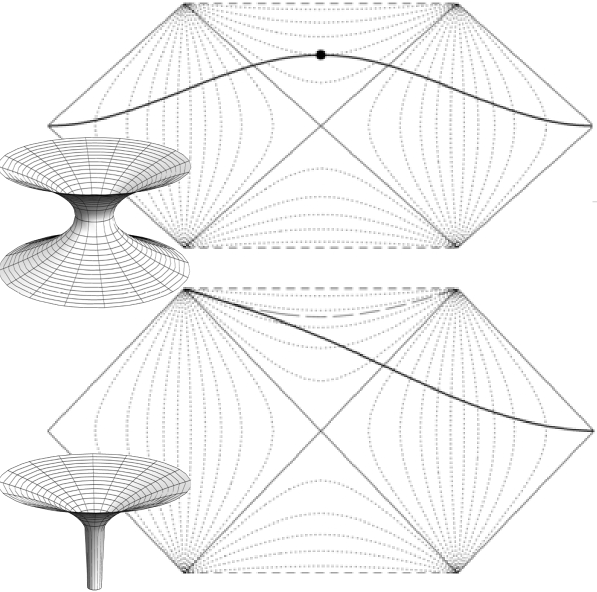
Transform to hyperboloidal time coordinate: $t = \tilde{t} - h(\tilde{r})$.
For flat spacetime CMC slices, the height function can be integrated to

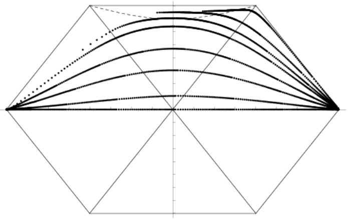
$$h(\tilde{r}) = \sqrt{(3/K_{CMC})^2 + \tilde{r}^2}.$$



The compactification factor is set to $\bar{\Omega} = r/\tilde{r} \equiv \Omega = (-K_{CMC})\frac{r_{\mathscr{I}}^2 - r^2}{6\, r_{\mathscr{I}}}$.

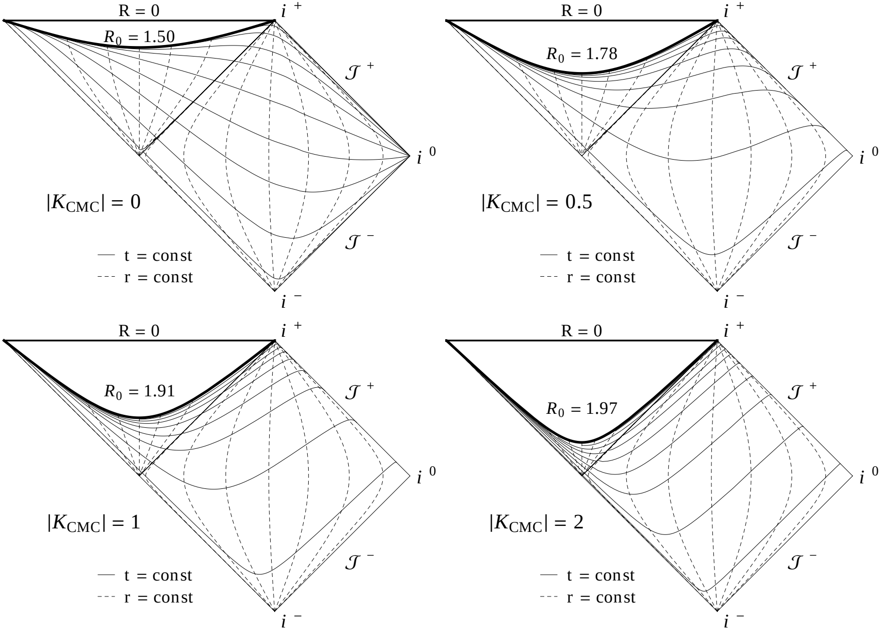
Wormhole to trumpet geometry in puncture evolution





Punctures, (Baumgarte, 2011, Class. Quantum Grav. 28 215003; Hannam et al, 2008, Phys.Rev. D78 064020).

Schwarzschild trumpet CMC slices

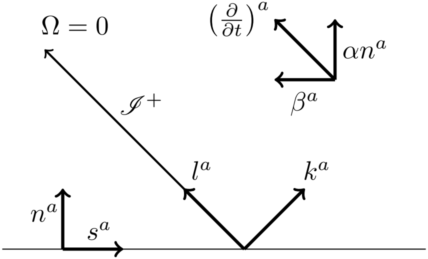


Scri-fixing and hyperbolic gauge conditions

Fix the location of $\mathscr{I}^+ \rightarrow$

make time vector flow along $\mathscr{I}^+$:

$\left(\frac{\partial}{\partial t}\right)^a = \alpha n^a + \beta^a$ is null at $\mathscr{I}^+$.



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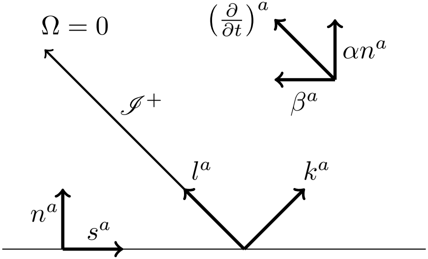


- **Slicing:** Generalized Bona-Massó equation of the form

$$\partial_t \alpha = \beta^r \partial_r \alpha - f(\alpha)\, \alpha^2 \,(K - K_0) + L_0,$$

with freedom to choose the two functions K_0 and L_0.

- **Shift:**
  - Fixed shift throughout the evolution.
  - Gamma-driver shift, with source function and damping:

$$\partial_t \beta^r = \beta^r \partial_r \beta^r + \lambda\, \Lambda^r - \eta\, \beta^r + L_0 - \frac{\xi_{\beta^r}}{\Omega} \beta^r.$$

Basic approach

Formulation:

- Free evolution: BSSN, Z4
- Time-independent $\Omega = |K_{CMC}|\frac{r_{\mathscr{I}}^2 - r^2}{6r_{\mathscr{I}}}$
- Mostly spherical symmetry
- + Massless scalar field

Hyperboloidal initial data:

- Height function approach
- Compactified slice
- Minkowski spacetime
- Schwarzschild trumpet

perturbed by a scalar field.

Hyperbolic gauge conditions:

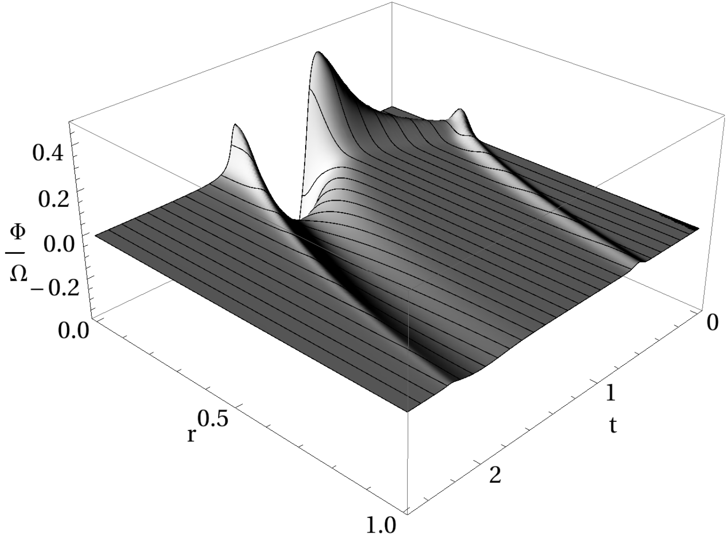
- Slicing (α): adapted Bona-Massó family
- Shift (β^a): evolved (Gamma-driver & similar)
- Preferred conformal gauge

with scri-fixing condition.

Numerical implementation:

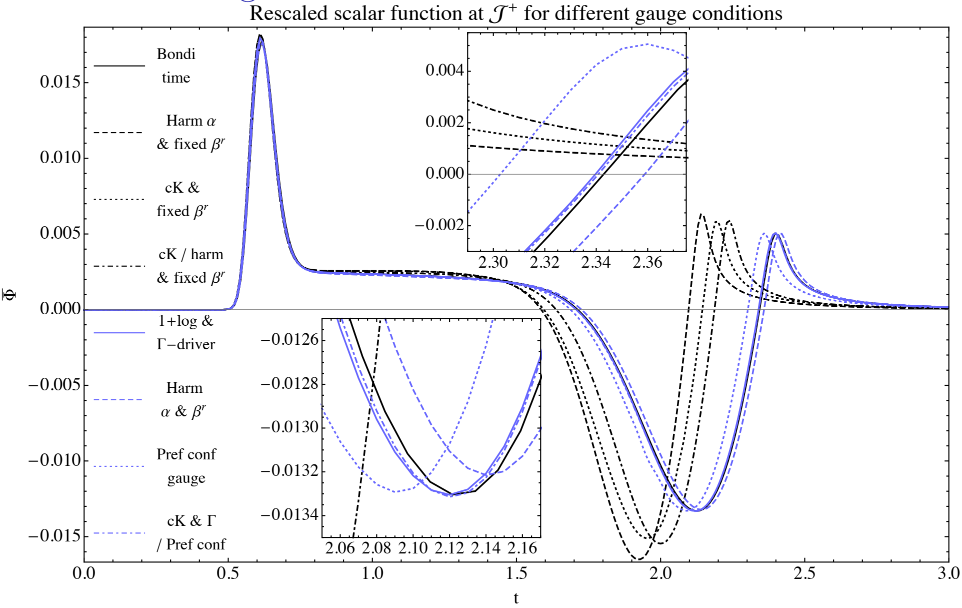
- Method Of Lines
  - Finite differences
  - Runge-Kutta 4th order
- Kreiss-Oliger dissipation
- (Non-)staggered grid.

Scalar field



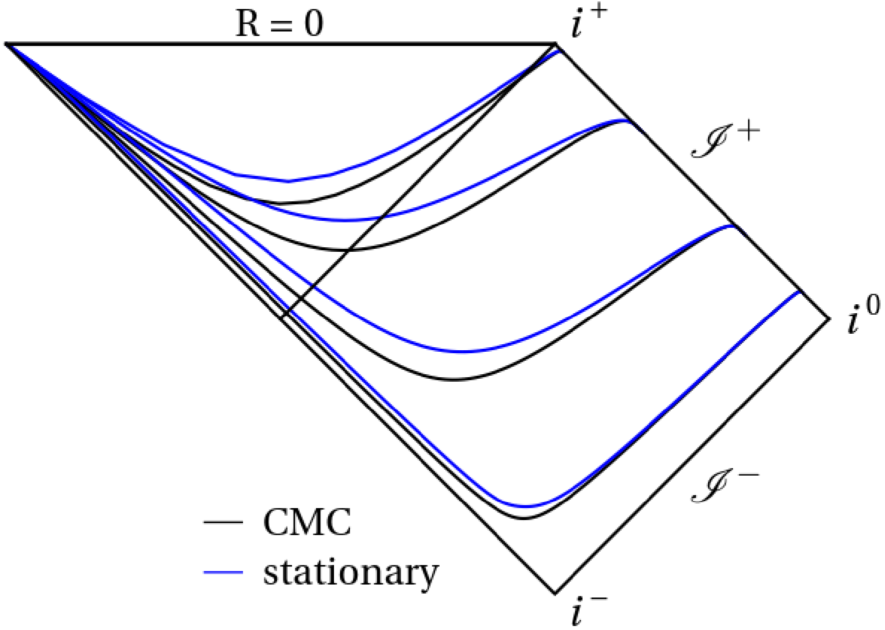
Vañó-Viñuales, Husa and Hilditch. *Class. Quant. Grav.* 32.17 (2015).

Scalar field signal at $\mathscr{I}^+$

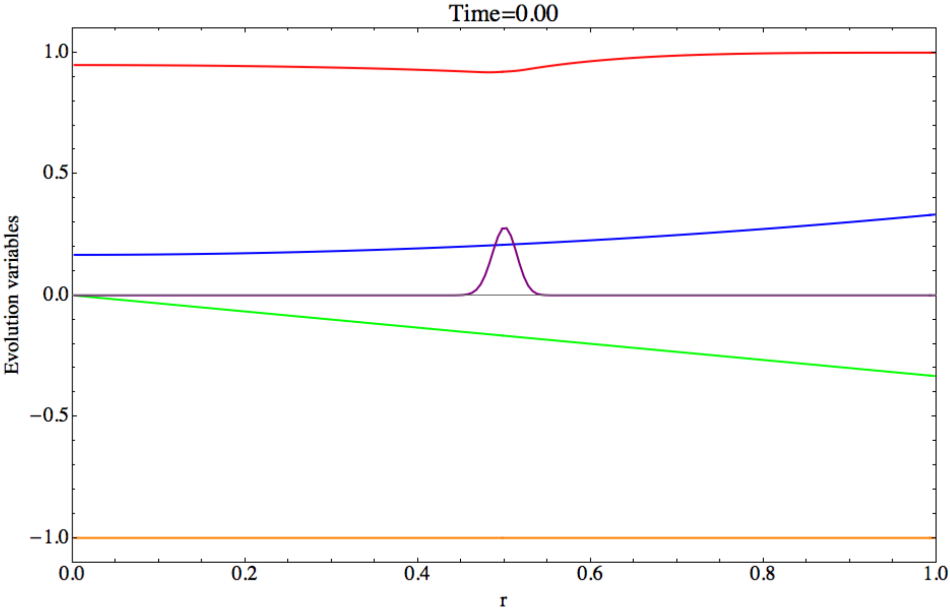


Vañó-Viñuales and Husa. *Class. Quant. Grav.* 35.4 (2018)

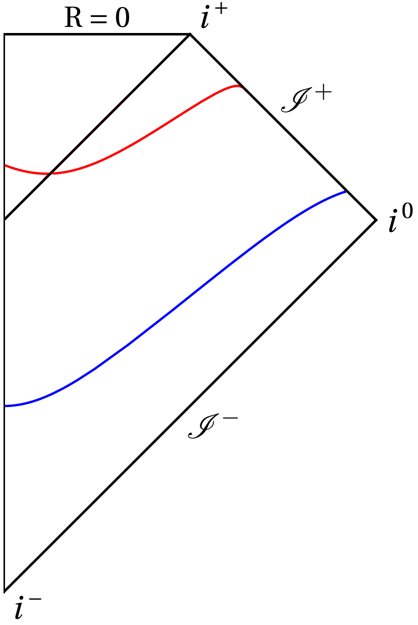
Comparison between CMC and relaxed slices



Evolution: $\chi, \tilde{K}, \alpha, \beta^r, \Phi/\Omega$



Collapse: change in slices - ongoing work



Outline

Dual frame construction

The dual frame approach uses

- one tensor basis $X^{\underline{\mu}}$ to express the equations and ensure their hyperbolicity
- one coordinate system x^{μ} adapted to the geometry of interest

For hyperboloidal problem: aim for regular equations for regular unknowns.

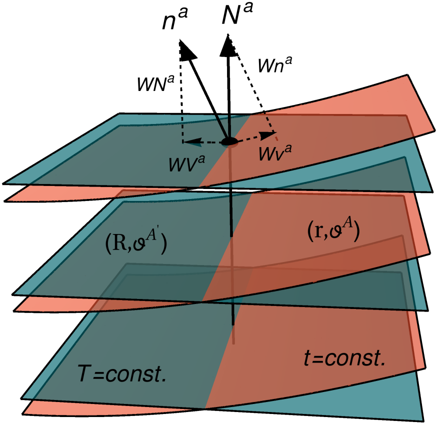


Diagram depicting the bases[a].

[a] Hilditch et al. *Class. Quant. Grav.* 35.5 (2018).

GBU model

Joint work with David Hilditch, Edgar Gasperin and Shalabh Gautam.

The Good-Bad-Ugly (GBU) model[1]

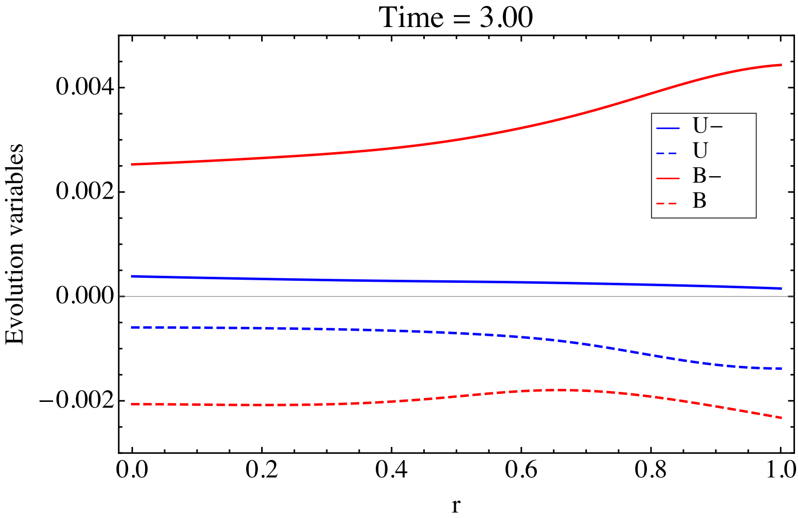
- Asymptotic behaviour of the variables in 1st order formulations of the Einstein equations can be divided into:
- Good fields 'g', Bad fields 'b', Ugly fields 'u'

$$\Box g = 0\,, \quad \Box b = (\partial_T g)^2\,, \quad \Box u = \frac{2}{R}\partial_T u\,.$$

- The decay of the different fields at future null infinity is different: $g \sim 1/R$, $b \sim \ln R/R$, $u \sim 1/R^2$ towards $\mathscr{I}^+$.
- Use the decay information to rescale the fields and make variables and equations as regular as possible.

[1] Gasperin et al. *Class. Quant. Grav.* 37.3 (2020).

Results from the GBU model



Einstein equations in Generalized Harmonic Gauge

In spherical symmetry — with D. Hilditch, S. Gautam, E. Gasperin, M. Duarte, J. Feng, C. Peterson-Bórquez , I. Rainho, ...

$$\text{Metric ansatz:} \qquad \mathbf{g} = \begin{pmatrix} \frac{2e^{\delta}C_+\,C_-}{C_+-C_-} & \frac{e^{\delta}(C_-+C_+)}{C_--C_+} & 0 & 0 \\ \frac{e^{\delta}(C_-+C_+)}{C_--C_+} & \frac{2e^{\delta}}{C_+-C_-} & 0 & 0 \\ 0 & 0 & \mathring{R}^2 & 0 \\ 0 & 0 & 0 & \mathring{R}^2\,\sin^2\theta \end{pmatrix}.$$

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$$\mathbf{g} = \begin{pmatrix} \frac{2e^{\delta}C_+\,C_-}{C_+-C_-} & \frac{e^{\delta}(C_-+C_+)}{C_--C_+} & 0 & 0 \\ \frac{e^{\delta}(C_-+C_+)}{C_--C_+} & \frac{2e^{\delta}}{C_+-C_-} & 0 & 0 \\ 0 & 0 & \mathring{R}^2 & 0 \\ 0 & 0 & 0 & \mathring{R}^2\,\sin^2\theta \end{pmatrix}.$$

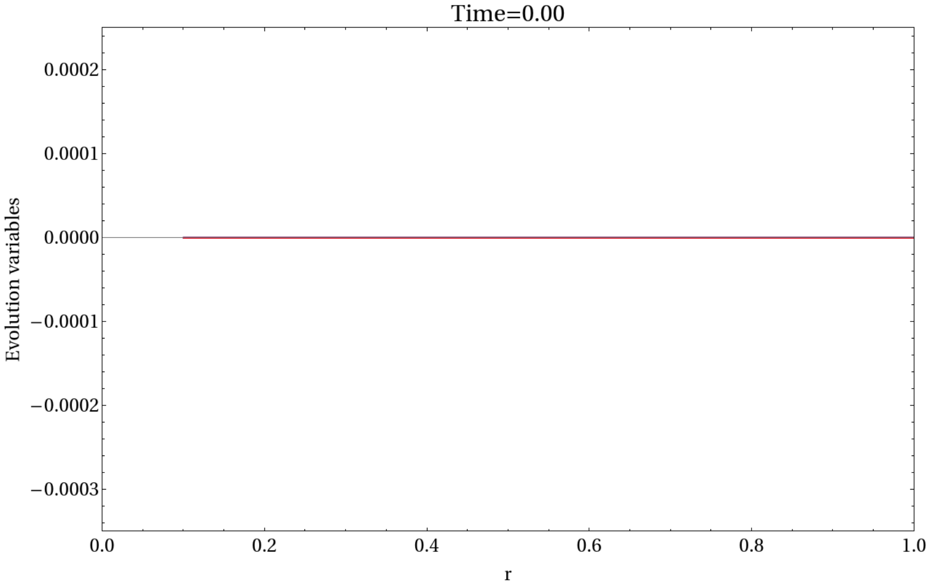
- 'Causal structure' variables $C^R_\pm$:

$$\nabla_\xi\left(\frac{\mathring{R}^2}{e^{\delta}\kappa}\nabla_{\underline{\xi}}C^R_+\right) + \mathring{R}\nabla_\xi H^\eta + \mathring{R}(\nabla_\xi\mathring{R})\nabla_\xi C^R_+ + 8\pi e^{-\delta}\mathring{R}^2 T_{\xi\xi} = 0\,,$$
$$\nabla_{\underline{\xi}}\left(\frac{\mathring{R}^2}{e^{\delta}\kappa}\nabla_{\xi}C^R_-\right) + \mathring{R}\nabla_{\underline{\xi}} H^{\underline{\eta}} + \mathring{R}(\nabla_{\underline{\xi}}\mathring{R})\nabla_{\underline{\xi}} C^R_- + 8\pi e^{-\delta}\mathring{R}^2 T_{\underline{\xi}\underline{\xi}} = 0\,.$$

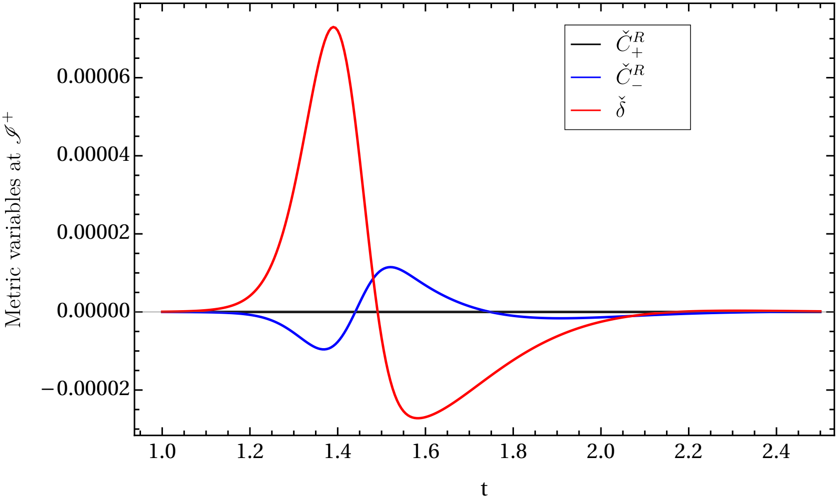
- 'Determinant' variables $(e^{\delta},\mathring{R})$:

$$\nabla_a\left(\frac{\xi^a}{e^{\delta}\kappa}\nabla_{\underline{\xi}}\delta\right) - \mathring{R}\nabla_a\left(\frac{H^a}{\mathring{R}}\right) + \frac{2}{\mathring{R}^2e^{\delta}\kappa}(\nabla_\xi\mathring{R})(\nabla_{\underline{\xi}}\mathring{R})$$
$$+\frac{1}{4e^{\delta}\kappa^3}\left(\nabla_{\underline{\xi}}C^R_-\nabla_\xi C^R_+ - \nabla_{\underline{\xi}}C^R_+\nabla_\xi C^R_-\right) + 16\pi T_T = 0\,,$$
$$\nabla_a\left(\frac{\xi^a}{e^{\delta}\kappa}\nabla_{\underline{\xi}}\mathring{R}^2\right) + 2 = 0\,.$$

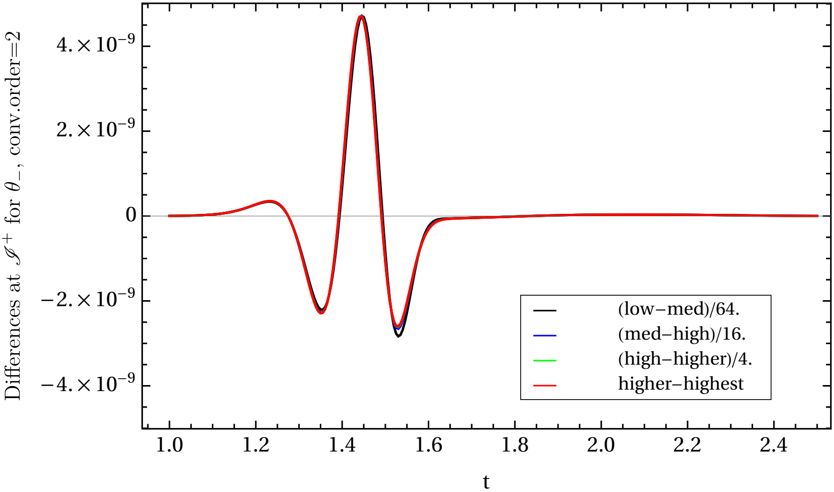
Preliminary results: evolution of metric ($\check{C}_+^R$, $\check{C}_-^R$, $\check{\delta}$)



Preliminary results: fields at $\mathscr{I}^+$



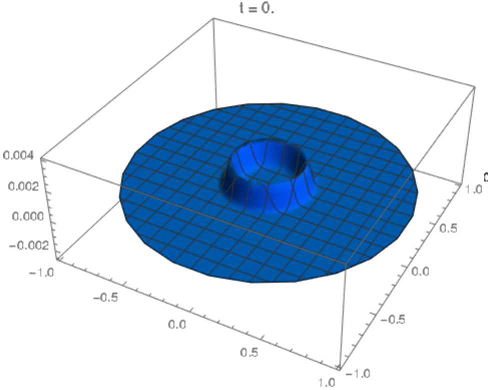
Preliminary results: convergence at $\mathscr{I}^+$

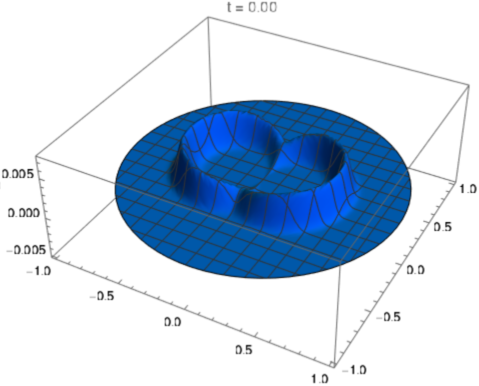


Outline

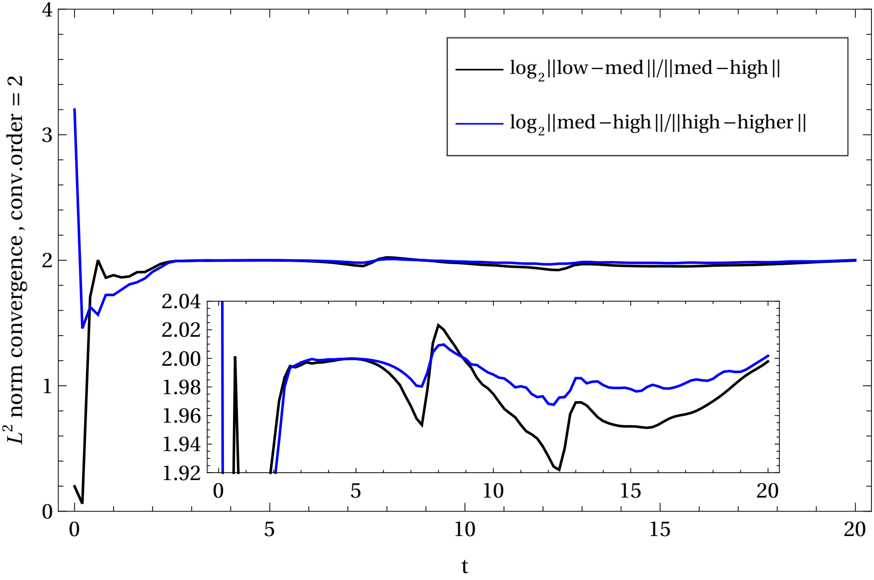
Massless scalar field equation in 3D

Evolution of the massless wave equation on a hyperboloidal slice of Minkowski spacetime using curvilinear coordinates in NRPy+.

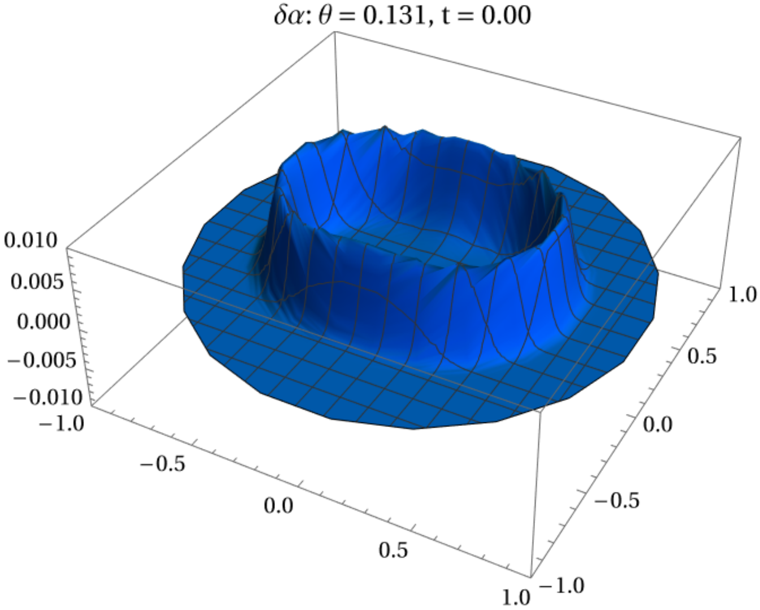




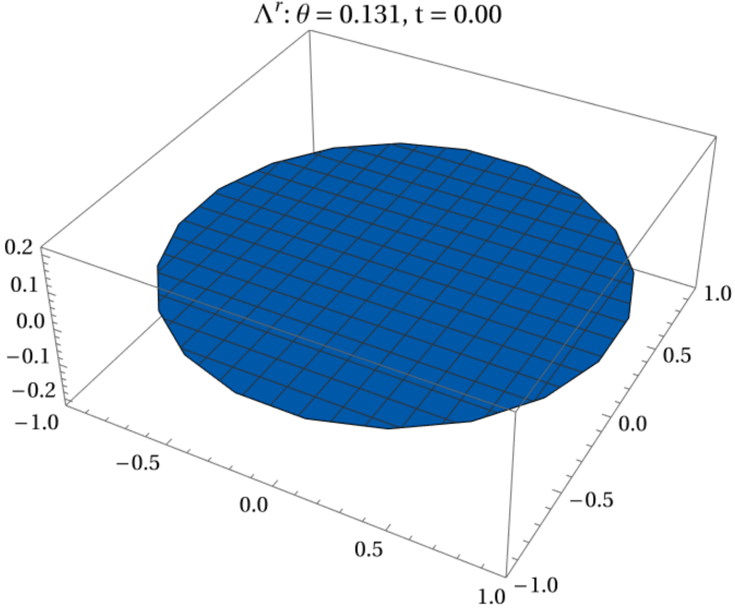
Convergence: axisymmetric scalar field evolution



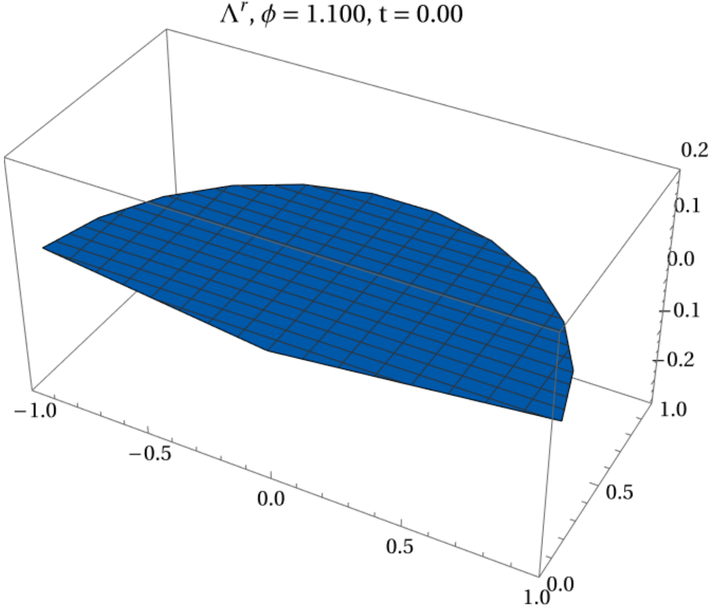
Gauge waves in full 3D: $\delta\alpha$ - ongoing work



Gauge waves in full 3D: Λ^r - ongoing work



Gauge waves in full 3D: Λ^r - ongoing work



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- This will increase the information we can obtain from gravitational wave data, from the observed objects and the underlying theory.

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  - Regular and strong field initial data in stable simulations can be evolved (spherical symmetry, 3D underway).
  - Compatible gauge conditions still to be better understood.

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  - Regular and strong field initial data in stable simulations can be evolved (spherical symmetry, 3D underway).
  - Compatible gauge conditions still to be better understood.
- Dual frame approach:
  - Better mathematical understanding than of the conformal approach, but less mature at the numerical level.
  - Careful understanding of decay at null infinity used to make variables and equations as regular as possible.

Future plans

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Future steps:

- Spherical symmetry:
  - Relation between gauge conditions and BH trumpet geometry.
  - Complete dual frame implementation: add scalar field and use BH initial data (excision).
- 3D:
  - Test robustness and convergence of 3D conformal code.
  - Implement and test regular initial data including gravitational waves.
  - Develop and test initial data including black holes.
  - Dual frame GBU model.
  - Linearised dual frame Einstein equations in GHG form.

Other applications:

- Apply hyperboloidal approach to cosmological scenarios?

Thank you!

Painting:
Infinite LIGO Dreams

Artist:
Penelope Rose Cowley

Exhibited in:
The Rest Frame,
School of Physics and Astronomy,
Cardiff University, UK