Transition Between Nematic and VBS Order With Emergent U(1) Symmetry in SU(4) fermions

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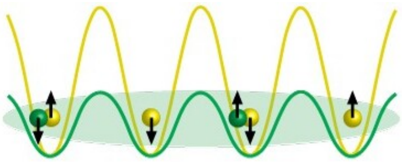
Introduction

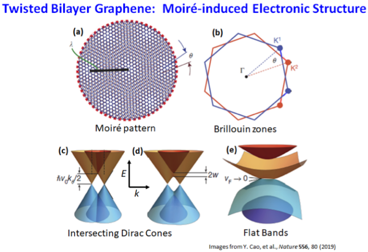
- $SU(N)$ theories and realizations
- Deconfined Quantum Criticality (DQC) and Signatures
- Specific model : $SU(4)/SO(6)$
- Phases, Transition, Connection to DQC
- Comparison to $SO(5)$

SU(N) physics

Used to understand the large-N limit of strongly correlated systems.

Realizations in ultra-cold atoms, spin-orbit coupling and bilayer graphene.





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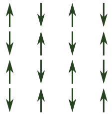
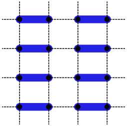
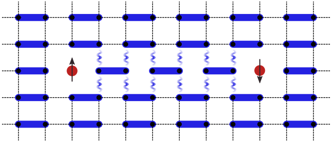
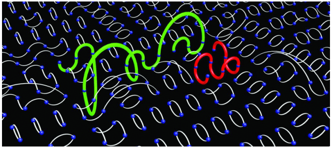
Deconfined Quantum Criticality

Possible explanation for some continuous transitions between two ordered states (not expected from Landau-Ginzburg theory) in (2+1)D.









Decay of discrete-symmetry breaking VBS pattern for deconfinement of spinons.

[1] Senthil, T., Vishwanath, A., Balents, L., Sachdev, S., Fisher, M. P. (2004). Science, 303(5663), 1490-1494.
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Model: Square lattice with two SU(4) fermions on each site.

Possible antisymmetric states:

$$|0\rangle = |AB\rangle - |BA\rangle \qquad |5\rangle = |CD\rangle - |DC\rangle$$
$$|1\rangle = |AC\rangle - |CA\rangle \qquad |4\rangle = |BD\rangle - |DB\rangle$$
$$|2\rangle = |AD\rangle - |DA\rangle \qquad |3\rangle = |BC\rangle - |CB\rangle$$

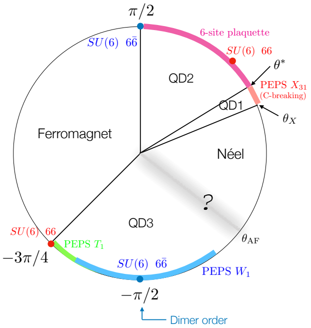
$$J \times \ \mathbf{S}_i \cdot \mathbf{S}_j = \sum_{c,c' \neq \bar{c}} |cc'\rangle\langle c'c| - \sum_{c,s \neq \bar{c}} |c\bar{c}\rangle\langle s\bar{s}|$$

$$K \times \ (\mathbf{S}_i \cdot \mathbf{S}_j)^2 = \sum_{c,s} |c\bar{c}\rangle\langle s\bar{s}| + 1/4$$

System hosts 6-colors on each site:

$$\mathcal{H} = \sum_{\langle i,j \rangle} J \sum_{c,c'} |cc'\rangle\langle c'c|$$
$$+(J-K) \sum_{c,s} |c\bar{c}\rangle\langle s\bar{s}|$$

- $J = \cos(\theta)$ and $K = \sin(\theta)$. QMC compatible for $\theta \in \{-0.75\pi, -0.5\pi\}$.
- Stochastic Series Expansion QMC : Direct sampling of $Tr[e^{-\beta H}]$



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[2] Kim, F. H., Assaad, F. F., Penc, K., & Mila, F. (2019). Physical Review B, 100(8), 085103.
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Definition of Order Parameters:

We use the equivalent of S^z in $SU(2)$ (Cartan operators):

$C_{\alpha=1,2,3} = \sum_c b_\alpha^c |c\rangle\langle c|$

$b_1 = \frac{1}{2}(1, 1, 0, 0, -1, -1)$

$b_2 = \frac{1}{2}(-1, 0, 1, -1, 0, 1)$

$b_3 = \frac{1}{2}(1, -1, 0, 0, 1, -1)$

To detect simple (anti-)ferromagnetic ordering: $C_C = \langle \mathbf{C}_{\mathbf{r}=(0,0)} \cdot \mathbf{C}_{\mathbf{r}=(L/2,L/2)} \rangle$

For nematic ordering: $C_N = \langle Q_{1,\mathbf{r}=(0,0)} Q_{1,\mathbf{r}=(L/2,L/2)} \rangle$, where $Q_1 = C_1 C_1 - \frac{1}{6}$

For dimerization:

$\mathbf{D} = (D_x, D_y)$

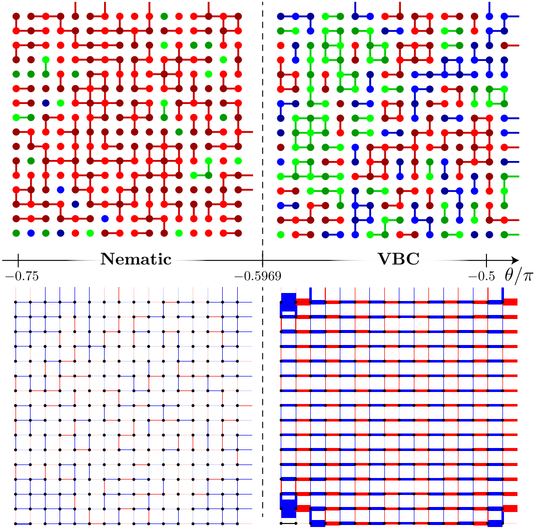
$D_x = \sum_i (-1)^{i_x} \mathbf{C}_{i_x,i_y} \cdot \mathbf{C}_{i_x+1,i_y}$

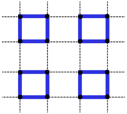
$D_y = \sum_i (-1)^{i_y} \mathbf{C}_{i_x,i_y} \cdot \mathbf{C}_{i_x,i_y+1}$

$D_{\mathrm{corr}}(b) = \langle (\mathbf{C}_{0,0} \cdot \mathbf{C}_{1,0})(\mathbf{C}_{\vec{r}_1^b} \cdot \mathbf{C}_{\vec{r}_2^b}) \rangle$

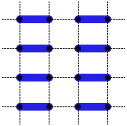
Phase diagram with θ/π

- $|AB\rangle - |BA\rangle$
- $|CD\rangle - |DC\rangle$
- $|AC\rangle - |CA\rangle$
- $|BD\rangle - |DB\rangle$
- $|AD\rangle - |DA\rangle$
- $|BC\rangle - |CB\rangle$





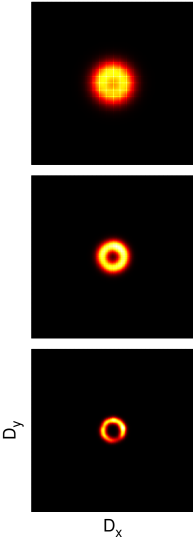
Consistent



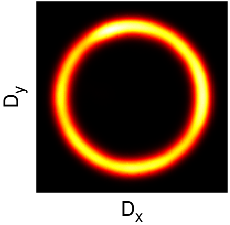
Inconsistent

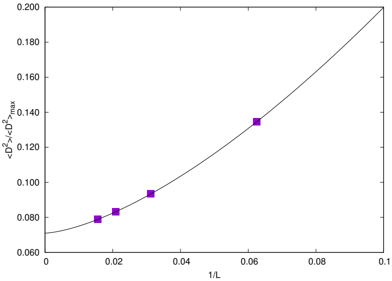
Emergent $U(1)$ symmetry

$\beta = 2L$, with $L = 16, 32, 64$



$\beta = L/8$, with $L = 96$





Mapping to Nematic Basis

Advantageous to use a rotated basis in which nematic behavior is easily identified.

$$
\begin{aligned}
|\mathbb{N}_0\rangle &= \frac{1}{\sqrt{2}}(|0\rangle + |5\rangle),\ |\mathbb{N}_5\rangle = \frac{i}{\sqrt{2}}(|0\rangle - |5\rangle) \\
|\mathbb{N}_1\rangle &= \frac{1}{\sqrt{2}}(|1\rangle + |4\rangle),\ |\mathbb{N}_4\rangle = \frac{i}{\sqrt{2}}(|1\rangle - |4\rangle) \\
|\mathbb{N}_2\rangle &= \frac{1}{\sqrt{2}}(|2\rangle + |3\rangle),\ |\mathbb{N}_3\rangle = \frac{i}{\sqrt{2}}(|2\rangle - |3\rangle).
\end{aligned}
$$

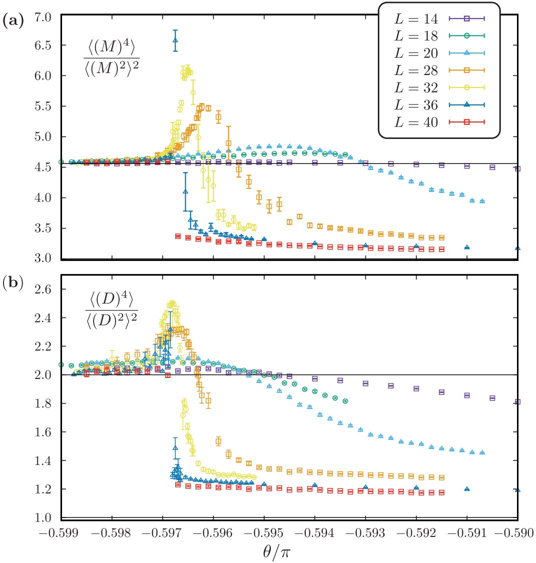
$$
H = \sum_{\langle i,j\rangle} J \sum_{c,c'} |\mathbb{N}_c \mathbb{N}_{c'}\rangle\langle\mathbb{N}_{c'}\mathbb{N}_c| + (J-K)\sum_{c,s}|\mathbb{N}_c\mathbb{N}_c\rangle\langle\mathbb{N}_s\mathbb{N}_s| \tag{1}
$$

Space is invariant under rotations in 6D : $SO(6)$

Explicitly $SO(6)$ symmetric: $((O_i^T)\otimes N)\ H\ ((O_i)\otimes N)$

Nematic ordering in color basis $\implies$ ferromagnetic ordering in nematic basis.

Phase transition



Binder cumulants from expected distribution in respective phases:

Nematic phase:

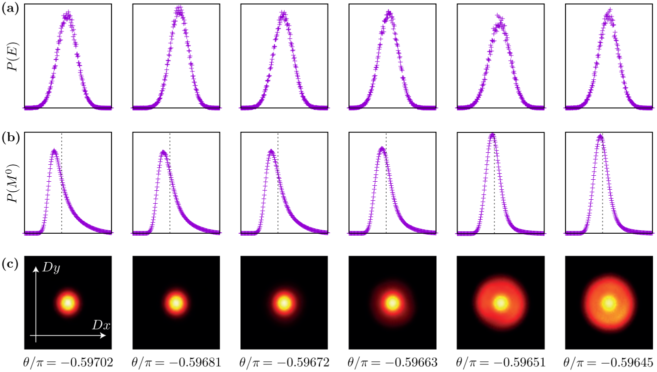
$$\frac{\langle M^4 \rangle}{\langle M^2 \rangle^2} = \frac{114}{25} = 4.56$$

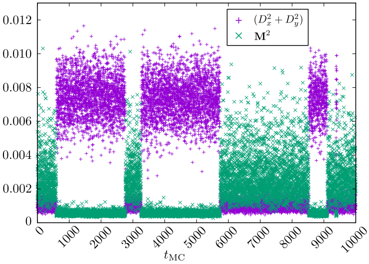
$$\frac{\langle D^4 \rangle}{\langle D^2 \rangle^2} = 2$$

VBS phase:

$$\frac{\langle M^4 \rangle}{\langle M^2 \rangle^2} = 3$$

$$\frac{\langle D^4 \rangle}{\langle D^2 \rangle^2} = 1$$





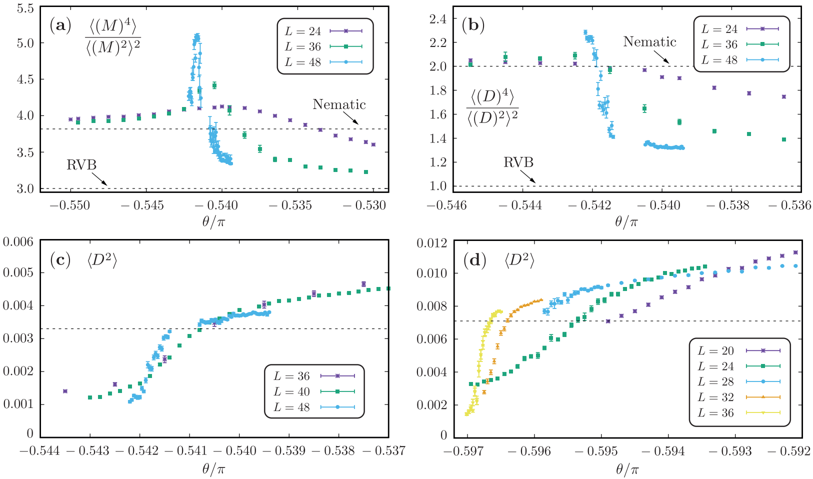
Histograms at $L = 32$ do not conclusively indicate first order behavior.

Evidence of first order behavior for $L \approx 40$.

Weak first order transition.

Comparison with SO(5) symmetry

$$H = \sum_{\langle i,j \rangle} J \sum_{c,c'} |\mathbb{N}_c \mathbb{N}_{c'}\rangle\langle \mathbb{N}_{c'} \mathbb{N}_c| + (J-K) \sum_{c,s} |\mathbb{N}_c \mathbb{N}_c\rangle\langle \mathbb{N}_s \mathbb{N}_s|$$



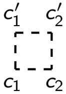
Evidence for weaker first order transition for $SO(5)$: smaller discontinuity in VBS OP (left), compared to $SO(6)$ (right).

Possibility for Continuous N

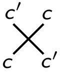
Continuous N can allow for a closer approach to a DQC.

$Tr[e^{-\beta H}] = \sum_n \frac{-\beta^n}{n!} Tr[H^n]$.

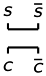
As $H = \sum_i O_i$, many different strings of H^n.

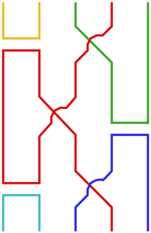


$H_b^1 \quad \hat{=}$



$H_b^2 \quad \hat{=}$





Trace

$\Longrightarrow$ sum over all states

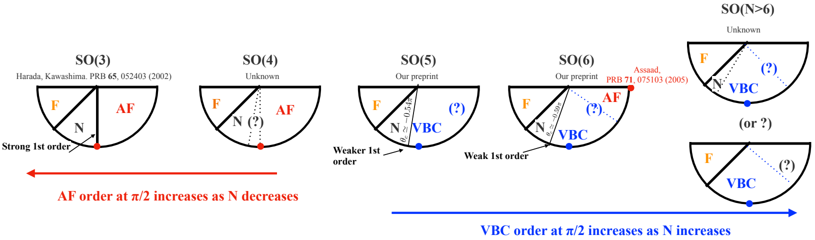
$\Longrightarrow$ sum over all colors in loop diagram

Weight of a loop config

$= J^{n_1}(J-K)^{n_2} N^{n_{loops}}$.

Conclusion and Outlook

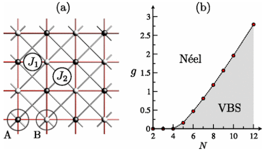
- Realization of nematic phase with $SO(6)$ symmetry.
- Evidence for proximity to a DQC point for $SU(4)$ fermionic system.



Closer approach by using continuous N formalism within QMC ?

Breaking of $U(1)$ symmetry to columnar/plaquette/combination VBS pattern ?

Large-N behavior seen for diagonal J_2.



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