Ground state manifold in the SU(3) symmetric Heisenberg model on the kagome lattice

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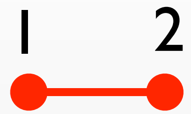
SU(2) vs. SU(3) - two sites



$\mathcal{P}_{12}(|\alpha\beta\rangle - |\beta\alpha\rangle) = -(|\alpha\beta\rangle - |\beta\alpha\rangle)$ E=−1, odd wave function

$\mathcal{H} = \mathcal{P}_{12}$ $\mathcal{P}_{12}(|\alpha\beta\rangle + |\beta\alpha\rangle) = +(|\alpha\beta\rangle + |\beta\alpha\rangle)$ E=+1, even wave function

Addition of two S=½ SU(2) spins:

$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$

using Young diagrams:

$2 \times 2 = 1 + 3$

$\otimes$ $=$ $\oplus$

↑ and ↓ spins

$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$ singlet
odd (anti-symmetrical)

$|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle$ triplet
even (symmetrical)

Addition of two SU(3) spins:

$\mathbf{3} \times \mathbf{3} = \bar{\mathbf{3}} + \mathbf{6}$

$\otimes$ $=$ $\oplus$

$|a\rangle$, $|b\rangle$, and $|c\rangle$.

$|ab\rangle - |ba\rangle$, $|ab\rangle - |ba\rangle$, $|ab\rangle - |ba\rangle$
odd (anti-symmetrical).

$|aa\rangle$, $|bb\rangle$, $|cc\rangle$, $|ab\rangle + |ba\rangle$,
$|ac\rangle + |ca\rangle$, and $|bc\rangle + |cb\rangle$
even (symmetrical)

SU(3) irreps on 3 sites

Addition of three SU(3) spins (27 states):

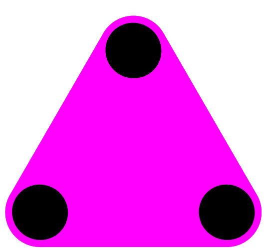
$$\mathbf{3} \times \mathbf{3} \times \mathbf{3} = \mathbf{1} + 2 \times \mathbf{8} + \mathbf{10}$$

$$\ydiagram{1} \otimes \ydiagram{1} \otimes \ydiagram{1} = \ydiagram{1,1,1} \oplus 2\times \ydiagram{2,1} \oplus \ydiagram{3}$$

SU(3) singlet

spins fully antisymmetrized

$$\ydiagram{1,1,1} = |abc\rangle + |bca\rangle + |cab\rangle - |acb\rangle - |acb\rangle - |acb\rangle$$

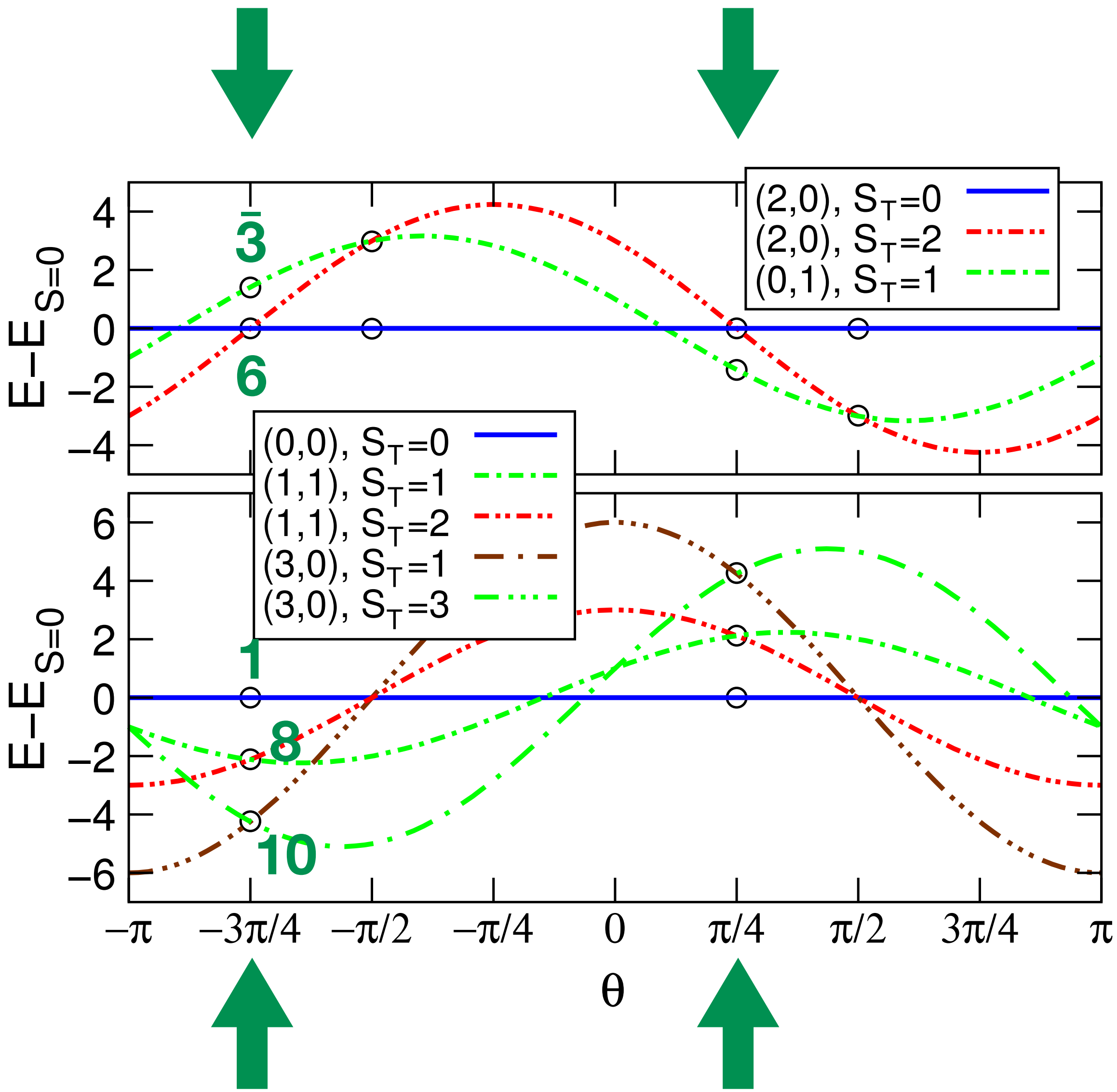


in the SU(3) singlet the spins are fully entangled:
we cannot write it in a product form

SU(3) in S=1 spin model

$$\mathcal{H} = J \sum_{i,j} \left[\cos\vartheta\, \mathbf{S}_i \mathbf{S}_j + \sin\vartheta \left(\mathbf{S}_i \mathbf{S}_j \right)^2 \right]$$

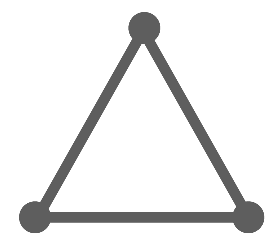
degeneracy as a signature of increased symmetry



S=1

$1 \otimes 1 = 0 \oplus 1 \oplus 2$ SU(2)

$\mathbf{3} \times \mathbf{3} = \bar{\mathbf{3}} + \mathbf{6}$ dim. of SU(3)



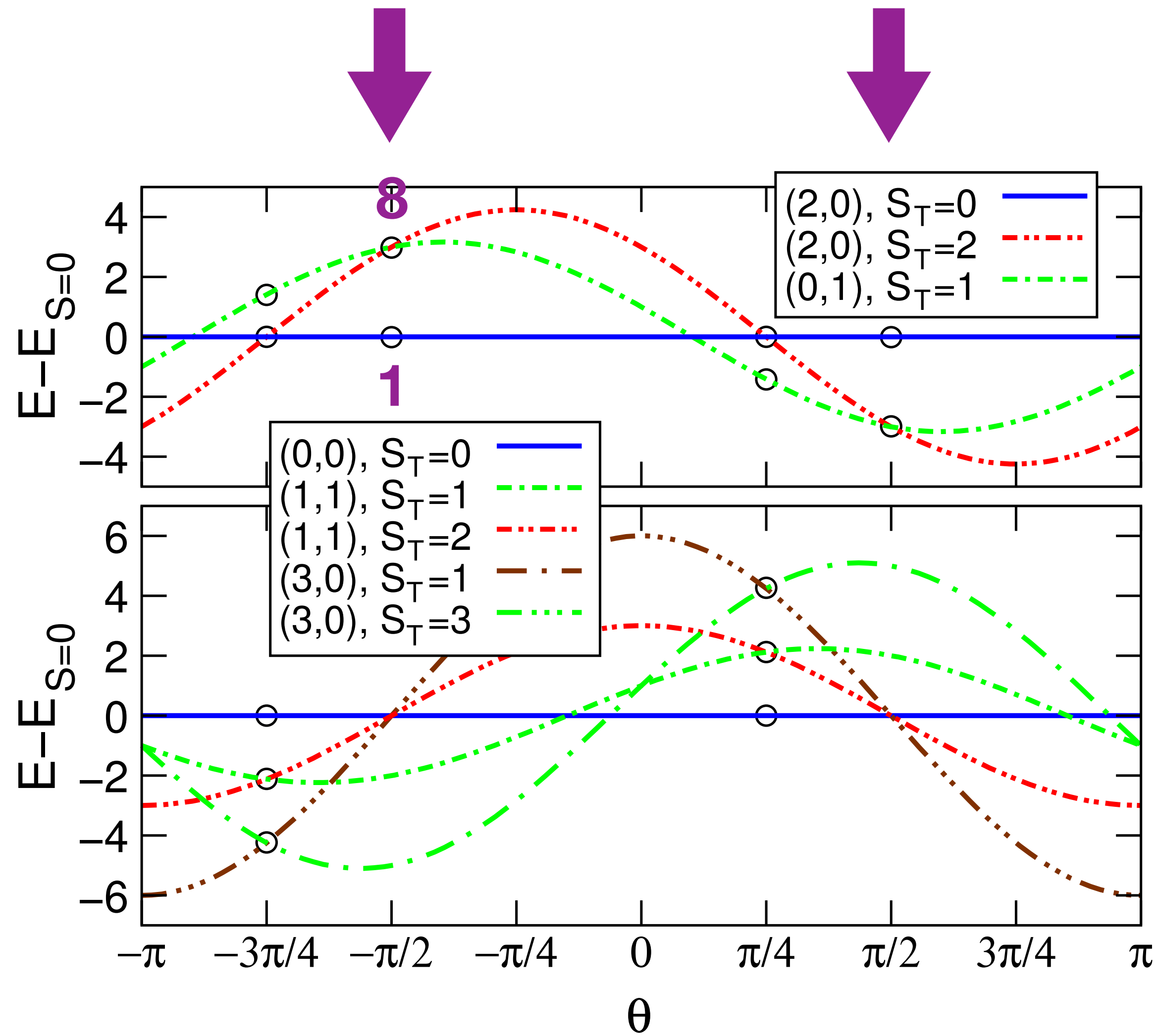
$1 \otimes 1 \otimes 1 = 0 \oplus 1 \oplus 1 \oplus 1 \oplus 2 \oplus 2 \oplus 3$

$\mathbf{3} \times \mathbf{3} \times \mathbf{3} = \mathbf{1} + 2 \times \mathbf{8} + \mathbf{10}$

SU(3) in S=1 spin model

$$\mathcal{H} = J \sum_{i,j} \left[\cos\vartheta \, \mathbf{S}_i \mathbf{S}_j + \sin\vartheta \left(\mathbf{S}_i \mathbf{S}_j \right)^2 \right]$$

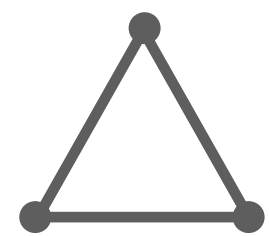
degeneracy as a signature of increased symmetry



S=1

$1\otimes1=0\oplus1\oplus2$ SU(2)

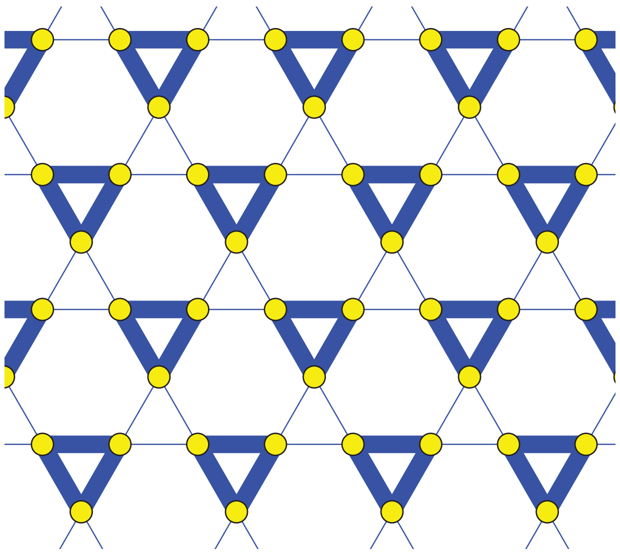
$\mathbf{3} \times \bar{\mathbf{3}} = \mathbf{1} + \mathbf{8}$



$1\otimes1\otimes1=0\oplus1\oplus1\oplus1\oplus2\oplus2\oplus3$

What do we know about SU(3) Kagome ?

The trimerized/simplex solid state/simplex valence-bond crystal for the fundamental **3** irrep model and S=1 Kagome (BLBQ, including the pure Heisenberg point)



D. P. Arovas, Phys. Rev. B **77**, 104404 (2008).

SU(3)
Large-N expansion: Hermele & Gurarie, Phys. Rev. B **84**, 174441 (2011);

iPEPS and ED: Corboz, Penc, Mila, & Läuchli, Phys. Rev. B 86, 041106(R) (2012)

S=1
H. J. Changlani, A. M. Läuchli, Trimerized ground state of the spin-1 Heisenberg antiferromagnet on the kagome lattice, Phys. Rev. B **91**, 100407 (2015)

T. Liu, W. Li, A. Weichselbaum; J von Delft, Jan, G. Su, Simplex valence-bond crystal in the spin-1 kagome Heisenberg antiferromagnet, Phys. Rev. B **91**, 060403(R) (2015)

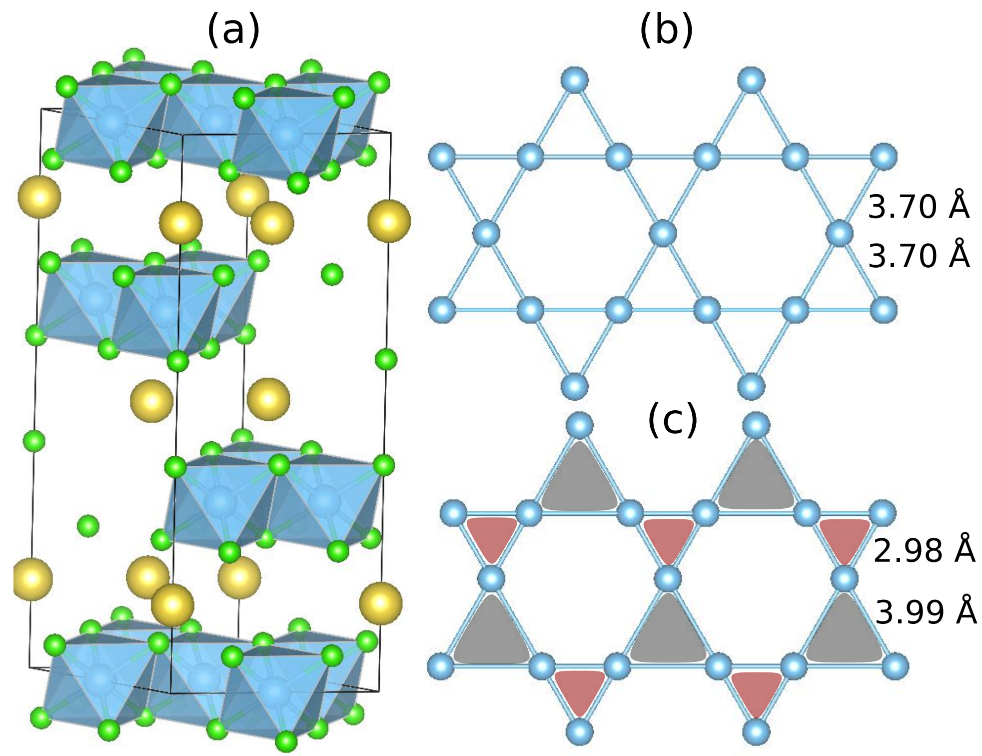
Trimerized phase in the S = 1 Kagome antiferromagnet with ring exchange

Spin–lattice coupling and the emergence of the trimerized phase in the S = 1 Kagome antiferromagnet $Na_2Ti_3Cl_8$

A. Paul, C.-M. Chung, T. Birol, and H. J. Changlani,
Phys. Rev. Lett. 124, 167203 (2020)

layers of edge-sharing $TiCl_6$ octahedra

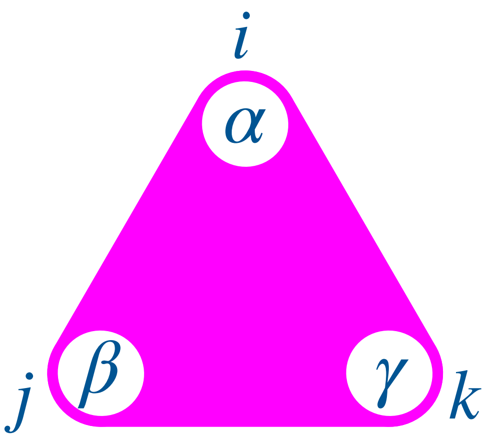
$$\mathcal{H} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_{bq} \sum_{\langle ij \rangle} (\mathbf{S}_i \cdot \mathbf{S}_j)^2$$
$$+ \frac{J_R}{2} \sum_{\Delta = i,j,k} ((\mathbf{S}_i \cdot \mathbf{S}_j)(\mathbf{S}_i \cdot \mathbf{S}_k) + (\mathbf{S}_i \cdot \mathbf{S}_k)(\mathbf{S}_i \cdot \mathbf{S}_j))$$



Simplex solid in SU(3) Kagome

D. P. Arovas, Phys. Rev. B **77**, 104404 (2008).

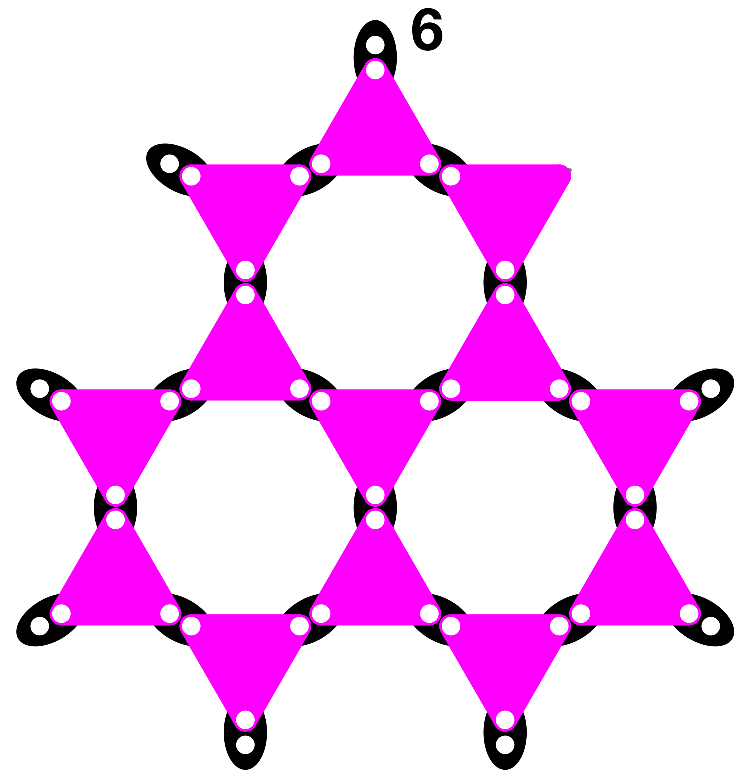
SU(3) singlet on N sites, represented by $b^\dagger_\alpha(i)$ Schwinger bosons:



$$\sum_{\alpha,\beta,\gamma} \varepsilon^{\alpha\beta\gamma} b^\dagger_\alpha(i) b^\dagger_\beta(j) b^\dagger_\gamma(k)\,|0\rangle$$

$$\mathbf{3} \times \mathbf{3} = \bar{\mathbf{3}} + \mathbf{6}$$

Each site hosts the symmetric, 6 dimensional irrep (like in the S=1 AKLT state).



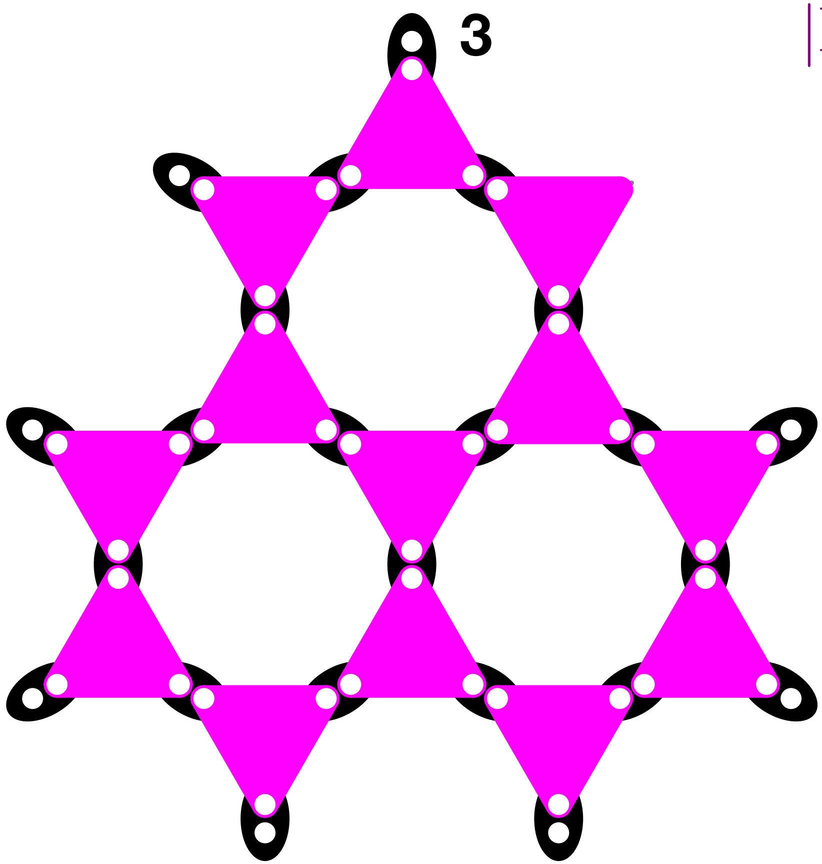
But we can do this with fermions as well !

SU(3) singlet on 3 sites, represented by fermions :

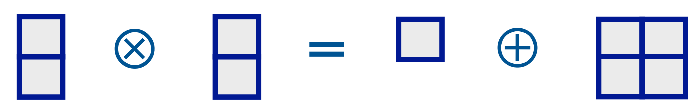
$$|\mathbf{1}(i_1, i_2, i_3)\rangle = \sum_{\alpha,\beta,\gamma} \varepsilon^{\alpha\beta\gamma} f^\dagger_\alpha(i_1) f^\dagger_\beta(i_2) f^\dagger_\gamma(i_3)|0\rangle = \mathcal{F}_{i_1,i_2,i_3}|0\rangle$$

femionic simplex solid wave function:

$$|\mathrm{FSS}\rangle = \prod_{\triangle_i}\prod_{\triangledown_j} \mathcal{F}_{\triangle_i}\mathcal{F}_{\triangledown_j}|0\rangle$$



$\bar{\mathbf{3}} \times \bar{\mathbf{3}} = \mathbf{3} + \bar{\mathbf{6}}$



Each site hosts the antisymmetric, 3 dimensional irrep.

Tensor network for Kagome

I. Kurecic, L. Vanderstraeten, N. Schuch: A gapped SU(3) spin liquid with Z_3 topological order, Phys. Rev. B **99**, 045116 (2019)

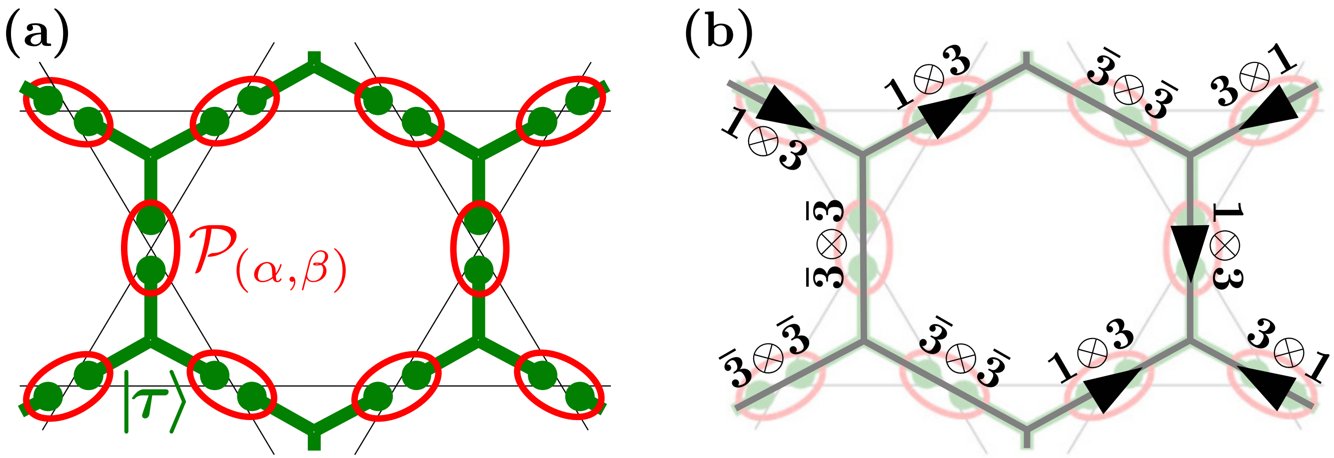


FIG. 1. (a) The model is constructed from trimers $|\tau\rangle$ which are in a singlet state with representation $\mathcal{H}_v \equiv \mathbf{1}\oplus\mathbf{3}\oplus\bar{\mathbf{3}}$ at each site (green dots), to which a map $\mathcal{P}_\bullet$ is applied which selects the physical degrees of freedom from $\mathcal{H}_v \otimes \mathcal{H}_v$. (b) Mapping to a $\mathbb{Z}_3$ topological model: Each site holds a $\mathbb{Z}_3$ degree of freedom: one of two arrows or no arrow. The arrows are pointing towards the **3** representation and satisfy a Gauss law across each vertex due to the fusion rules of the SU(3) irreps.

parent Hamiltonian has 17 (?) sites, not shown in the papers

Do we know the parent Hamiltonian ?

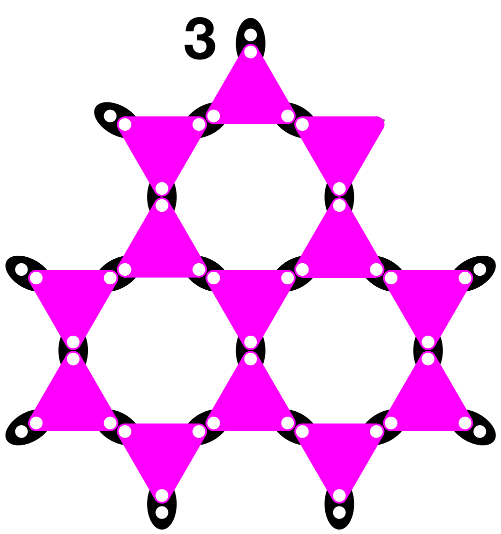
A guess: sum of local projectors, like in the S=1 AKLT model

$$\mathcal{H} = J \sum_{\langle i,j \rangle} \mathcal{P}_{i,j} + K \sum_{\triangle,\bigtriangledown} \left(\mathcal{P}_{i,j,k} + \mathcal{P}_{i,k,j} \right)$$

We may try it on a small cluster: we generate the FSS, and ask whether the condition for being an eigenstate :

$$\langle \mathrm{FSS}|\mathcal{H}^2|\mathrm{FSS}\rangle\langle \mathrm{FSS}|\mathrm{FSS}\rangle = \langle \mathrm{FSS}|\mathcal{H}|\mathrm{FSS}\rangle^2$$

is satisfied with some values of J and K.

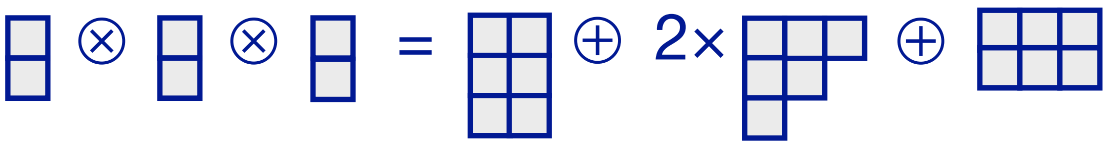


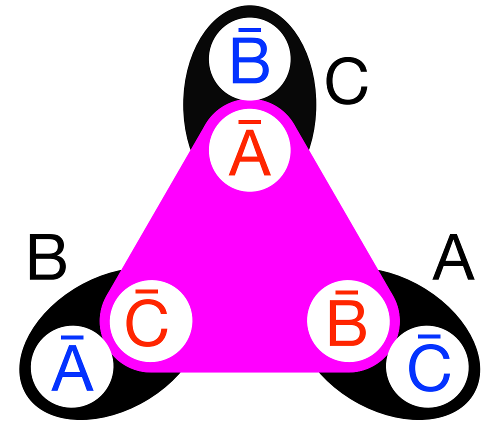
Surprise: it is satisfied for any value of J and K, the FSS is always an eigenstate of H !
(c.f. AKLT in S=1 chain)

But how does this happen?

The irreps in a triangle

$$\bar{\mathbf{3}} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus 2 \times \mathbf{8} \oplus \overline{\mathbf{10}}$$





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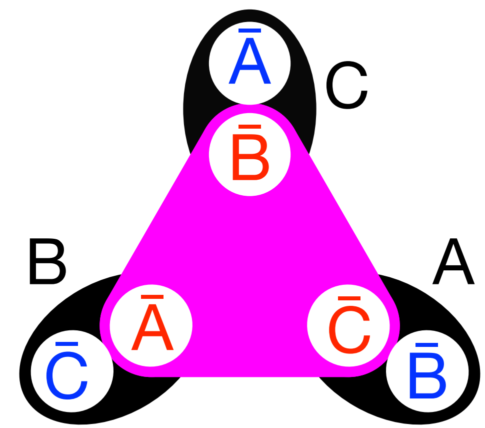
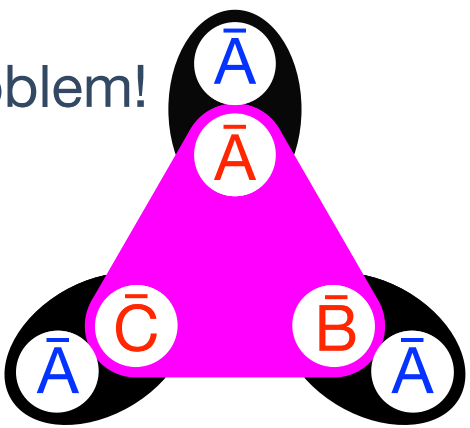
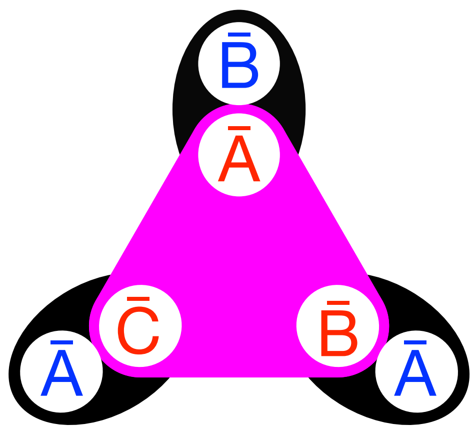


problem!



$\mathbf{1} \odot \mathbf{10} = 0$



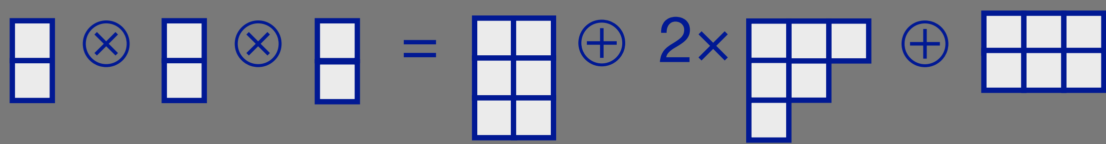
$\mathbf{1} \odot \mathbf{8} = \mathbf{8}$

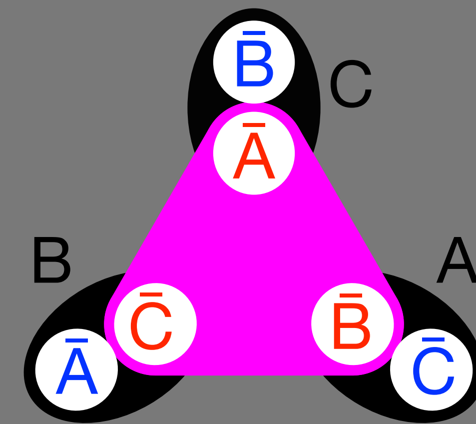
The sum cancels because of odd number of antisymmetrizations: $(-1)^3 = -1$

$\mathbf{1} \odot \mathbf{1} = 0$

The irreps in a triangle

$$\bar{\mathbf{3}} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus 2 \times \mathbf{8} \oplus \overline{\mathbf{10}}$$





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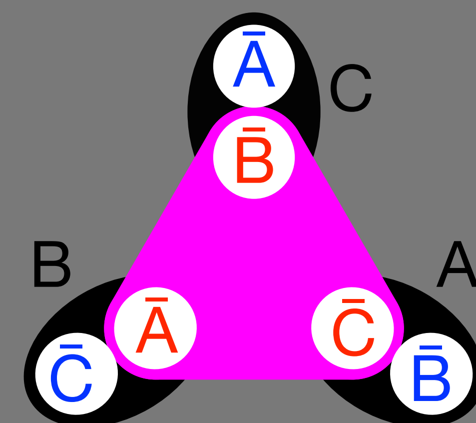
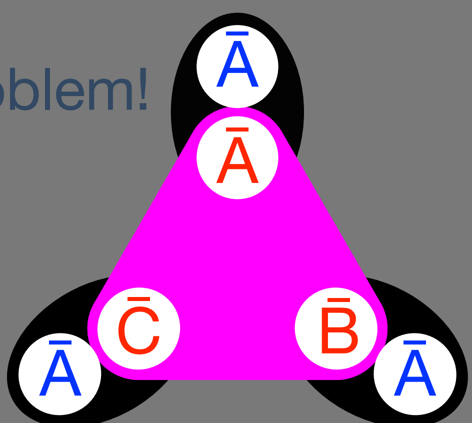
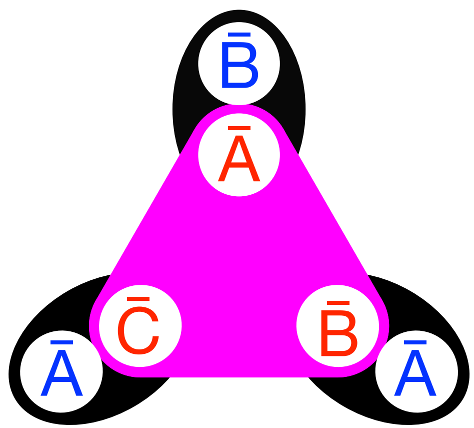


problem!



$\mathbf{1} \odot \mathbf{10} = 0$



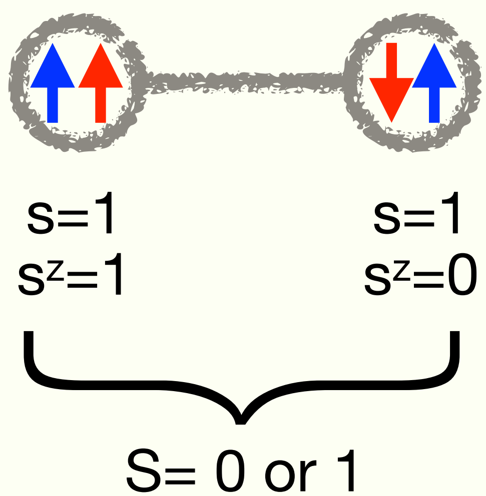
$\mathbf{1} \odot \mathbf{8} = \mathbf{8}$

The sum cancels because of odd number of antisymmetrizations: $(-1)^3 = -1$

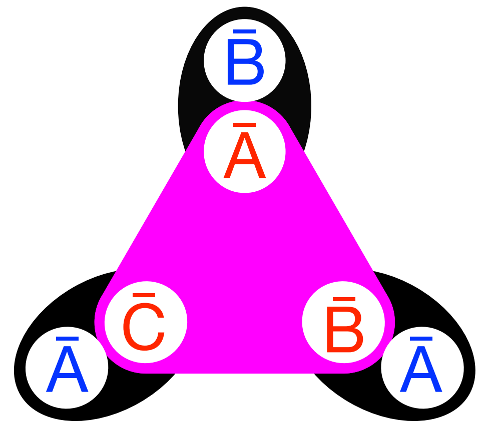
$\mathbf{1} \odot \mathbf{1} = 0$

Comparing the S=1 AKLT chain with FSS

AKLT chain



$$\mathcal{H}^{\mathrm{AKLT}} = \sum_{\mathrm{bonds}} |S=2\rangle\langle S=2|$$



$$\mathbf{1} \odot \mathbf{8} = \mathbf{8}$$

Fermionic simplex solid is an eigenstate of the Hamiltonian

$$\mathcal{H}^{\mathrm{FSS}} = \sum_{\triangle,\triangledown} \left(c_{\mathbf{1}} |\mathbf{1}\rangle\langle\mathbf{1}| + c_{\mathbf{10}} |\mathbf{10}\rangle\langle\mathbf{10}| \right)$$

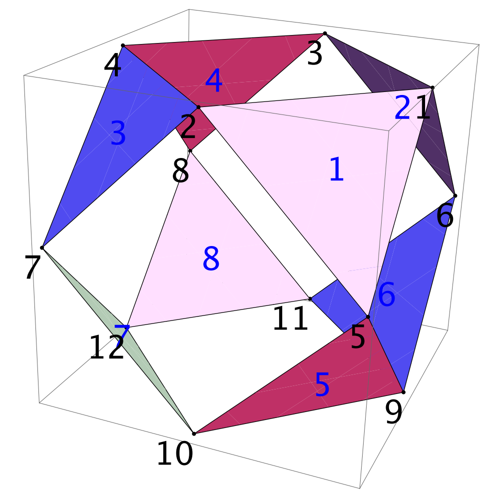
and ground state when $c_{\mathbf{1}} > 0$ and $c_{\mathbf{10}} > 0$.

$$J = \frac{1}{6}\left(c_{\mathbf{10}} - c_{\mathbf{1}}\right), \quad K = \frac{1}{6}\left(c_{\mathbf{10}} + c_{\mathbf{1}}\right)$$

$$\mathcal{H} = J \sum_{\langle i,j \rangle} \mathcal{P}_{i,j} + K \sum_{\triangle,\triangledown} \left(\mathcal{P}_{i,j,k} + \mathcal{P}_{i,k,j} \right)$$

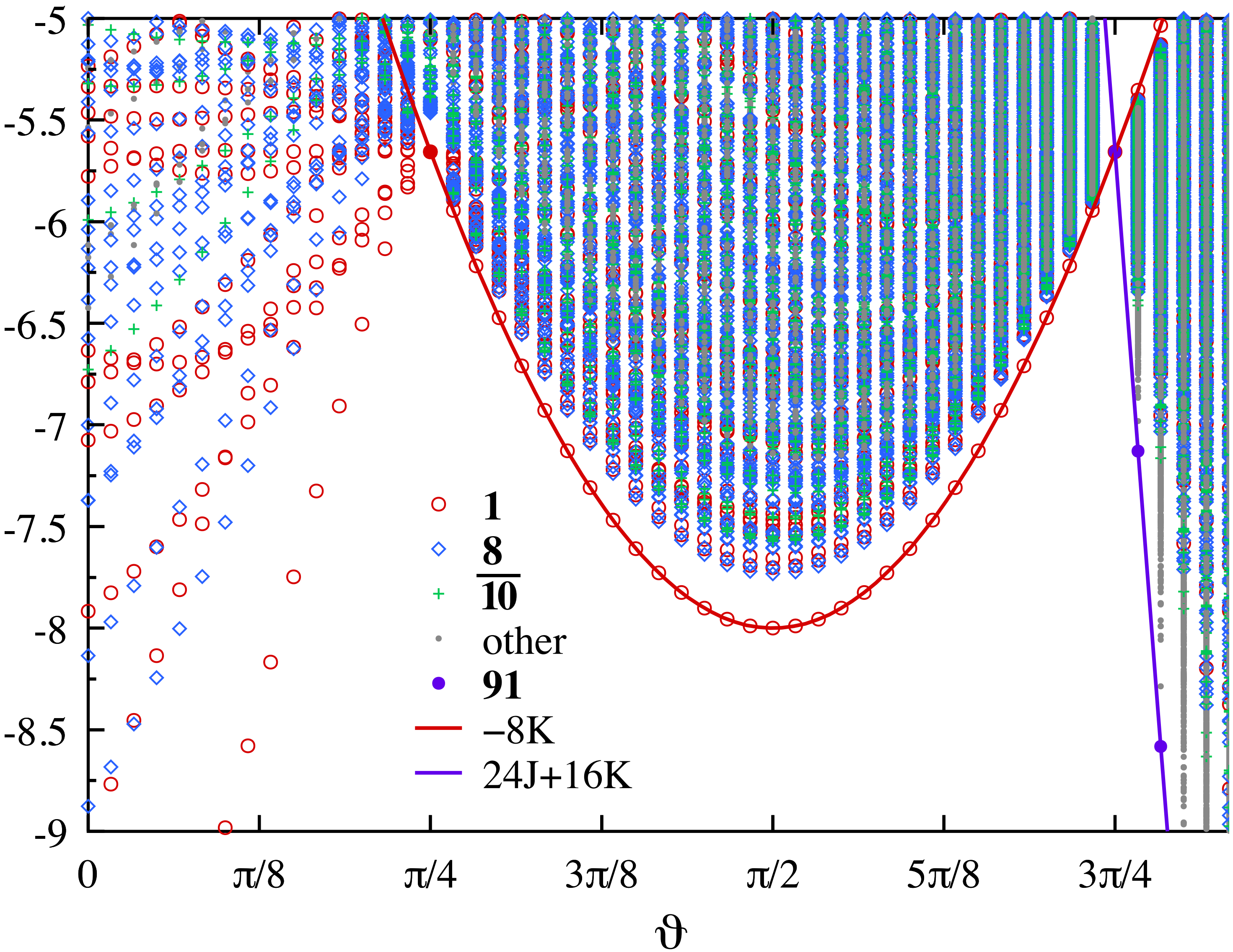
full ED for small system (12 sites)

$$\mathcal{H} = J \sum_{\langle i,j \rangle} \mathcal{P}_{i,j} + K \sum_{\triangle,\triangledown} \left(\mathcal{P}_{i,j,k} + \mathcal{P}_{i,k,j}\right)$$



$J = \cos\vartheta, K = \sin\vartheta$

(a)



34650 states in the $n_A = n_B = n_C$ sector, but the symmetry group is large

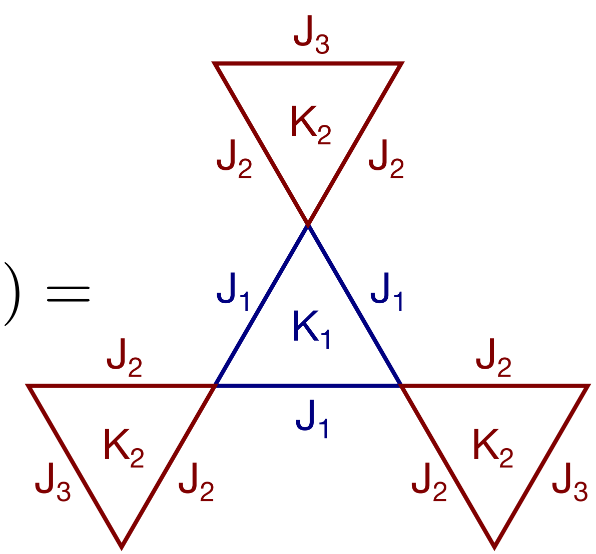
Lower bound on energy

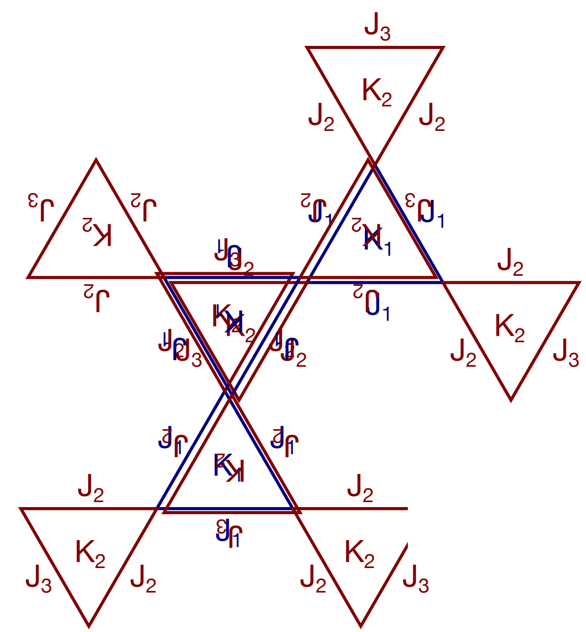
Let us write the lattice Hamiltonian as a sum offer the lattice of a Hamiltonian defined on a (9-site) open cluster:

$$\mathcal{H}(J,K) = \sum_{\text{lattice}} \mathcal{H}_9(J_1, J_2, J_3, K_1, K_2)$$

where

$$\mathcal{H}_9(J_1, J_2, J_3, K_1, K_2) =$$





with a condition

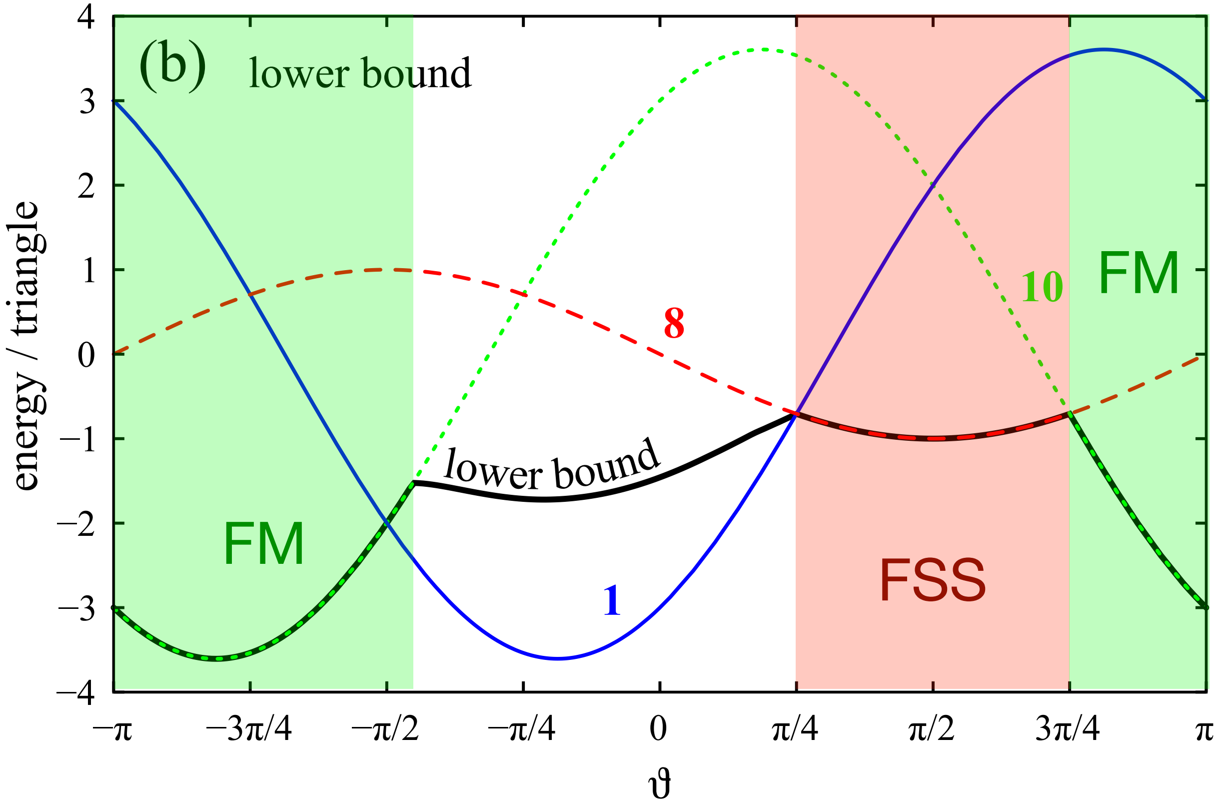
$$J = J_1 + 2J_2 + J_3$$

$$K = K_1 + 3K_2$$

Lower bound on energy

The energy calculated from the ground states of the sub-Hamiltonians will always be lower that the ground state energy of $\mathscr{H}$, as the true ground state of $\mathscr{H}$ can be viewed as a variational wavefunction for $\mathscr{H}_9$

$$E_{\mathrm{LB}} = \max_{\substack{J=J_1+2J_2+J_3 \\ K=K_1+3K_2}} E_{\mathrm{GS}}(J_1, J_2, J_3, K_1, K_2)$$



Actually, the energies of a single triangle gives also a lower bond (per triangle)

$$\varepsilon_{\mathbf{1}} = -3J + 2K$$

$$\varepsilon_{\mathbf{8}} = -K$$

$$\varepsilon_{\mathbf{10}} = 3J + 2K$$

The FM and the FSS saturate the lower bound, they are ground states (beware uniqueness)

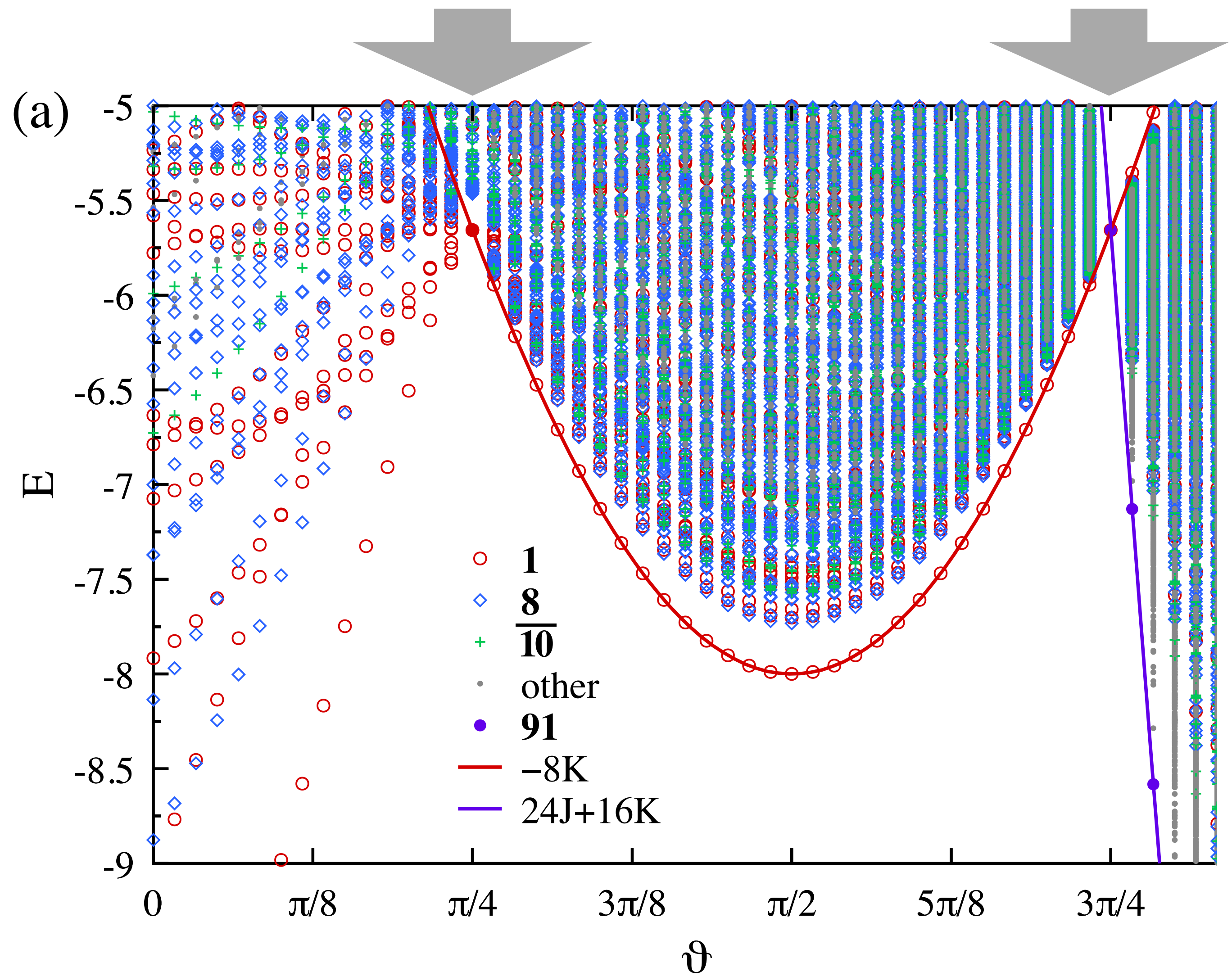
full ED for small system (12 sites)

$$\mathcal{H} = J \sum_{\langle i,j \rangle} \mathcal{P}_{i,j} + K \sum_{\triangle,\triangledown} (\mathcal{P}_{i,j,k} + \mathcal{P}_{i,k,j})$$

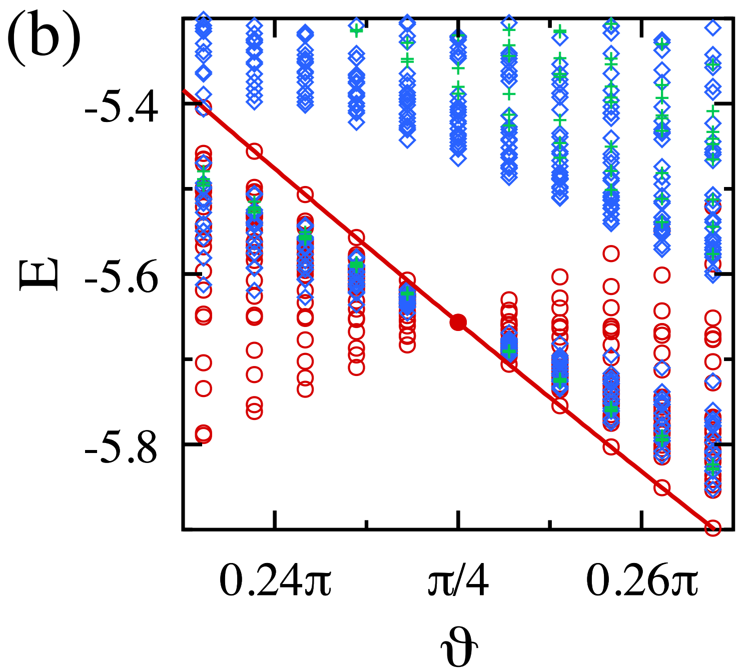
$$\mathcal{H} = \sum_{\triangle,\triangledown} |\mathbf{10}\rangle\langle\mathbf{10}| \qquad \mathcal{H} = \sum_{\triangle,\triangledown} |\mathbf{1}\rangle\langle\mathbf{1}|$$

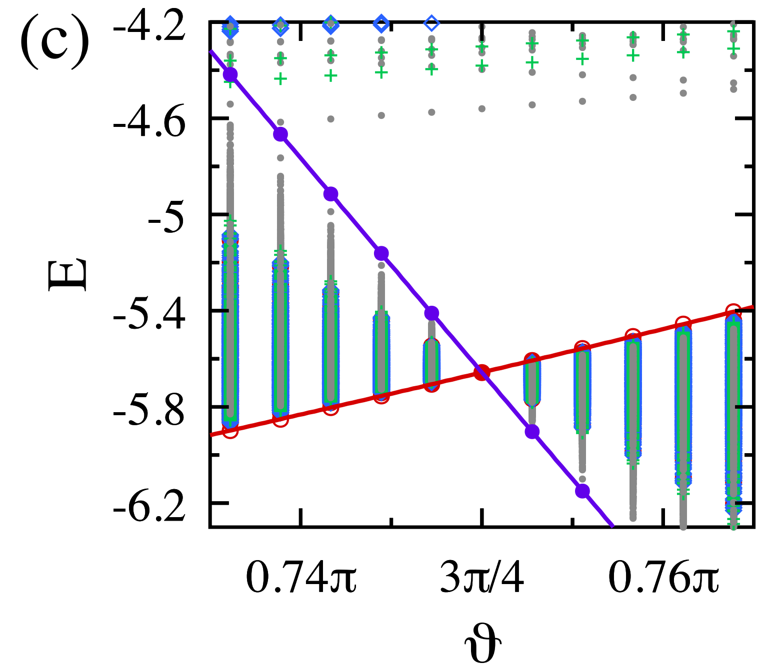
$$c_{\mathbf{1}} = 3(K - J)$$
$$c_{\mathbf{10}} = 3(K + J)$$

$J = \cos\vartheta, K = \sin\vartheta$



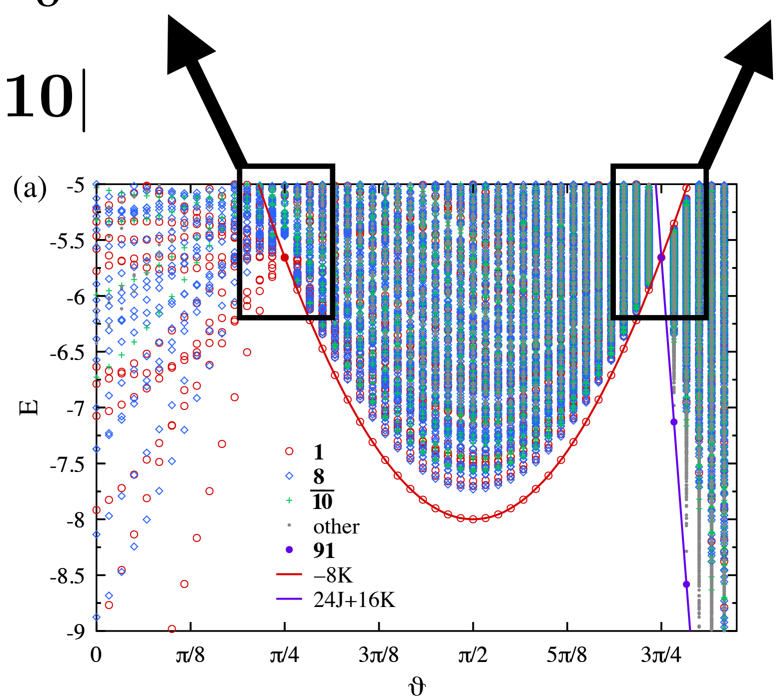
full ED for small system (12 sites) - degenerate GS





$$\mathcal{H} = \sum_{\triangle,\triangledown} |\mathbf{10}\rangle\langle\mathbf{10}|$$

$J = K$



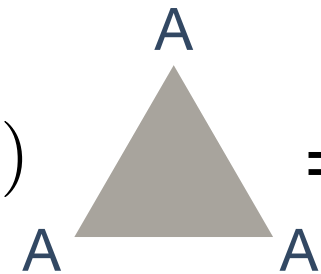
$$\mathcal{H} = \sum_{\triangle,\triangledown} |\mathbf{1}\rangle\langle\mathbf{1}|$$

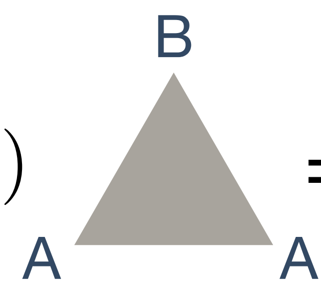
$J = -K$

The ϑ=3π/4 (J = −K) case

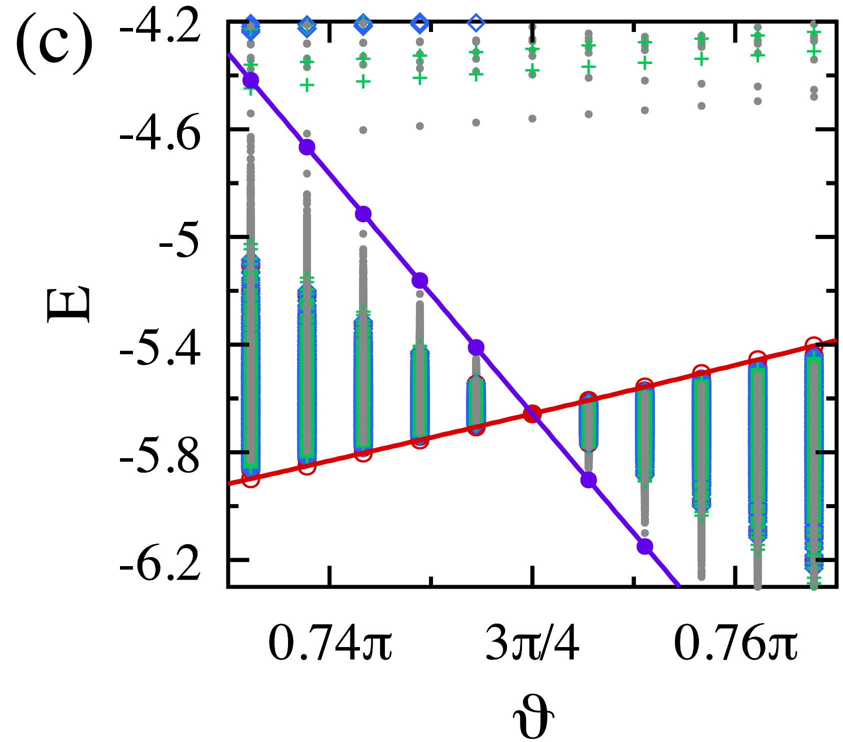
$$\mathcal{H} = \sum_{\triangle,\triangledown} |\mathbf{1}\rangle\langle\mathbf{1}|$$

$$\mathcal{H}_\triangle = \mathcal{P}_{i,j,k} + \mathcal{P}_{i,k,j} - \mathcal{P}_{i,j} - \mathcal{P}_{i,k} - \mathcal{P}_{j,k}$$

$(\mathcal{H}_\triangle + 1)$  = 0

$(\mathcal{H}_\triangle + 1)$  = 0

triangles having no more than two colors are degenerate eigenstates



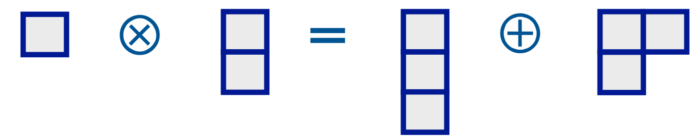
385427 states are degenerate

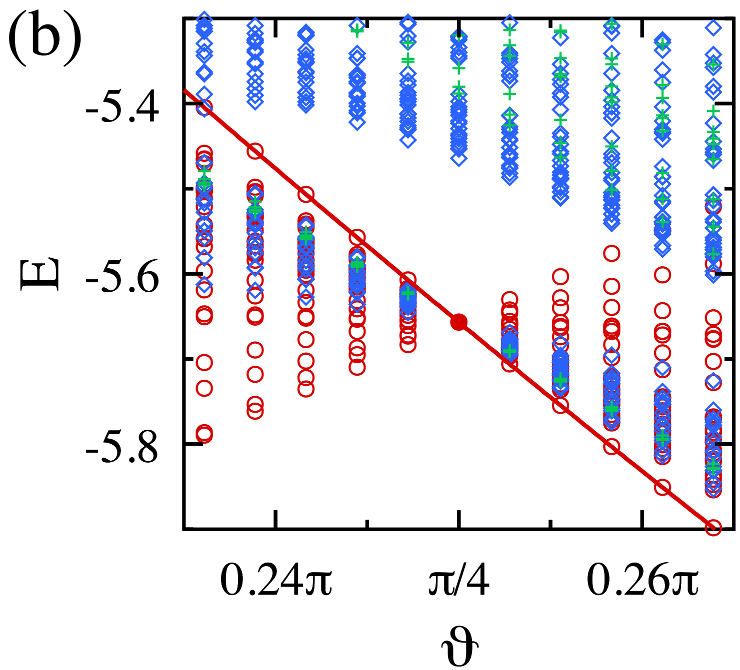
3^{12}=531441 is the total number of states

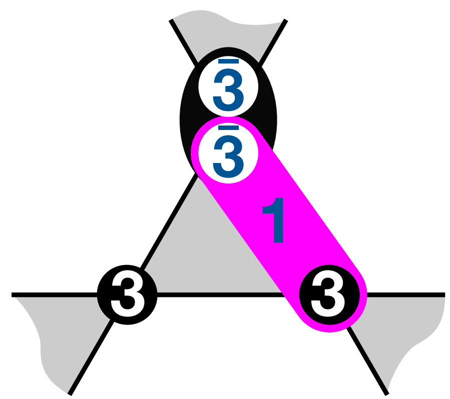
The $\vartheta=\pi/4$ (J = K) case

$$\mathcal{H} = \sum_{\triangle,\triangledown} |\mathbf{10}\rangle\langle\mathbf{10}|$$

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$$



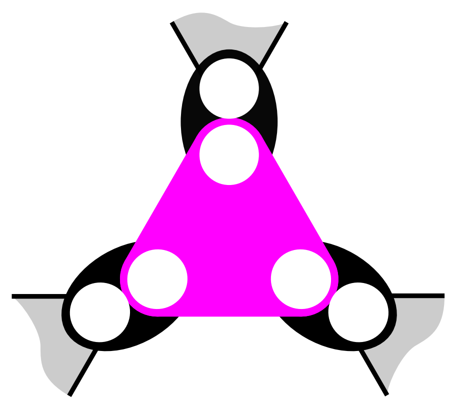


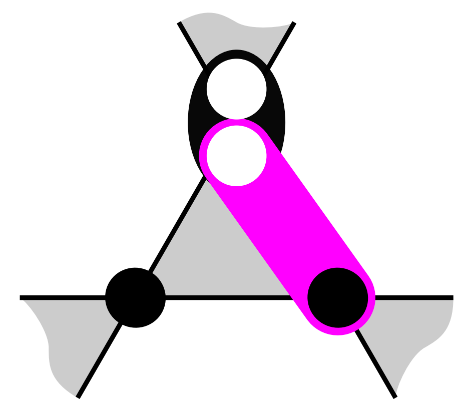


The irepps of 3 spins in the triangle contain **1** and **8, but** no **10.**

cf. I. Kurecic, L. Vanderstraeten, N. Schuch, PRB **99**, 045116 (2019)

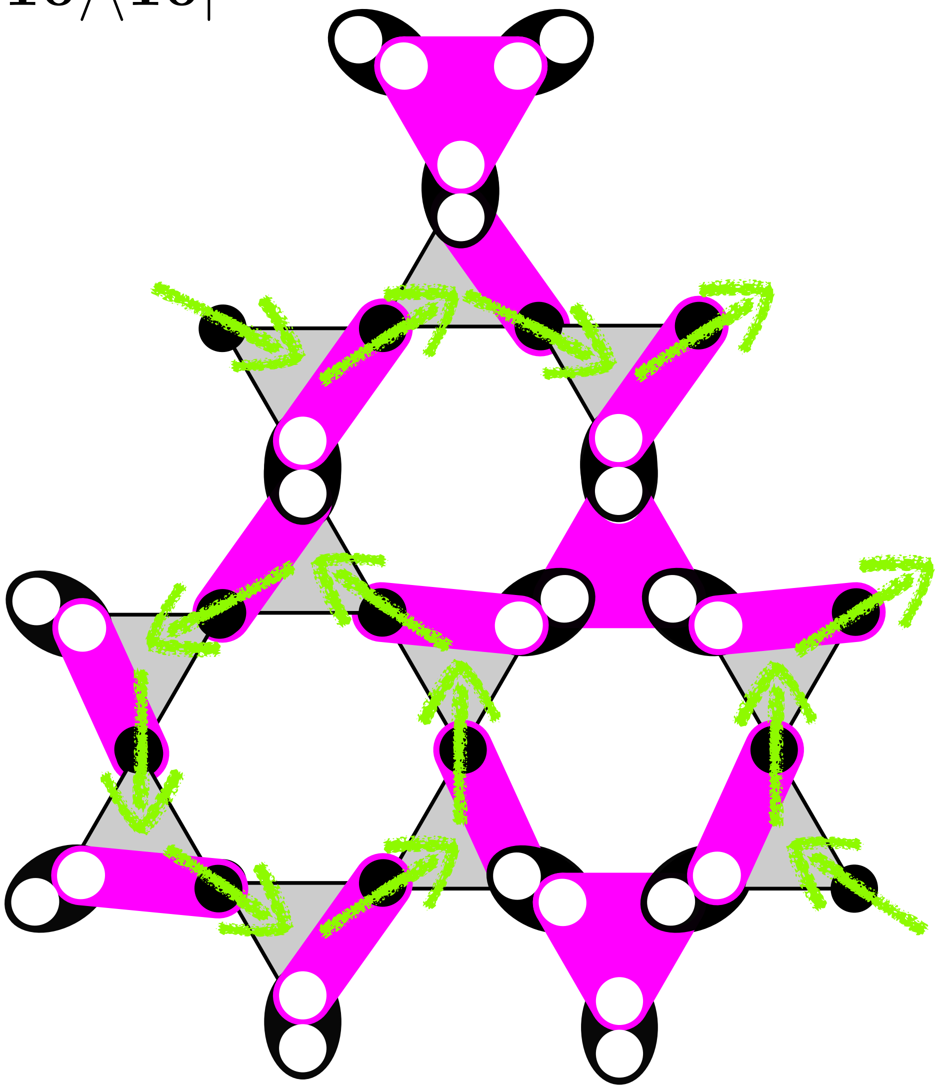
the building blocks are:





The J = K case: Lego time!

$$\mathcal{H} = \sum_{\triangle,\triangledown} |\mathbf{10}\rangle\langle\mathbf{10}|$$



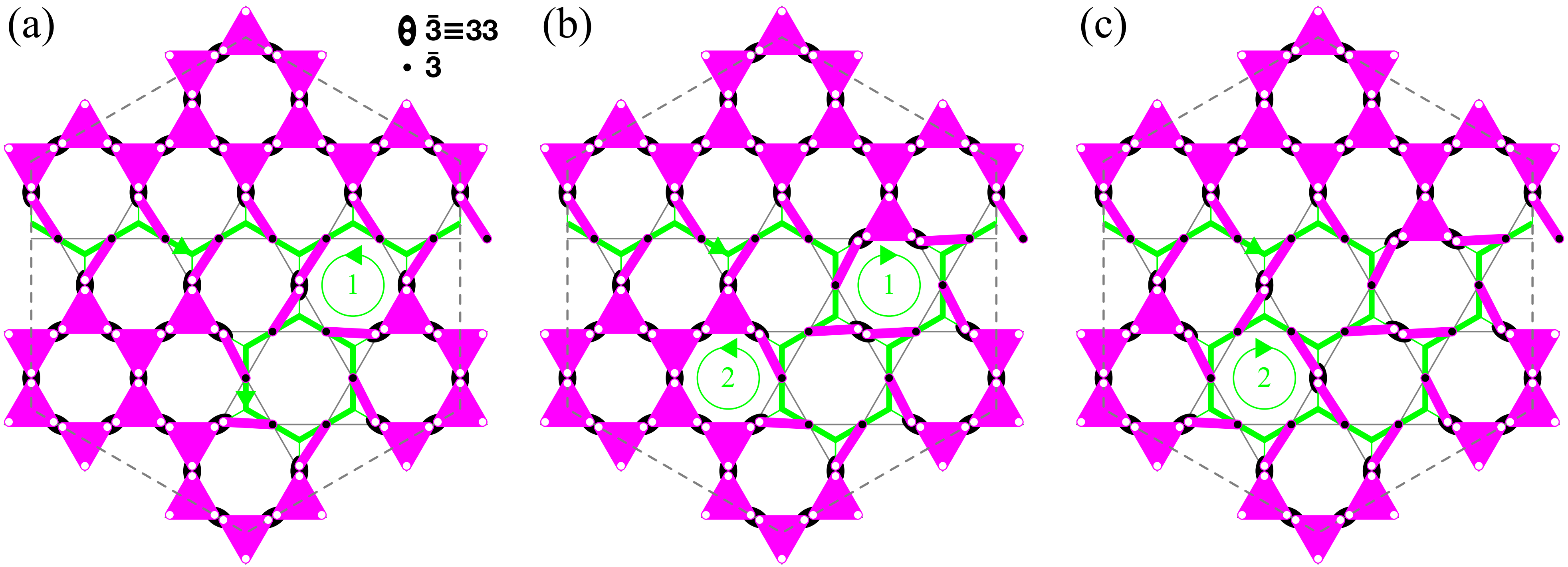
“current conservation” - some kind of a Coulomb liquid ?

On each bond 3 possibilities: 2 directions of arrow and absence of an arrow.

Z3 degrees of freedom

topological sectors (definition not obvious because of overlap and non-orthogonality)

The J = K case: singlet states characterized by directed loops on honeycomb lattice



local moves ⇒

extensive number of loops

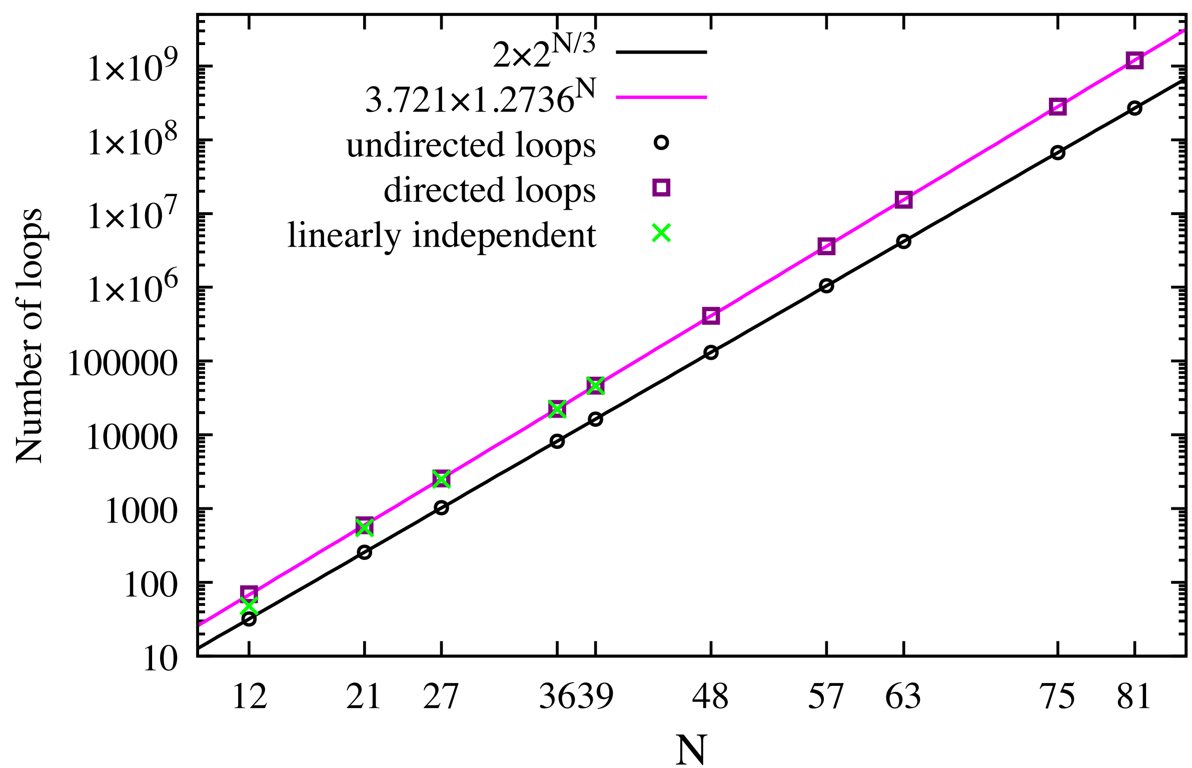
for 12 sites they span the singlet GS manifold

number of undirected loops = $2\times2\times2^{(N_{hex}-1)}$

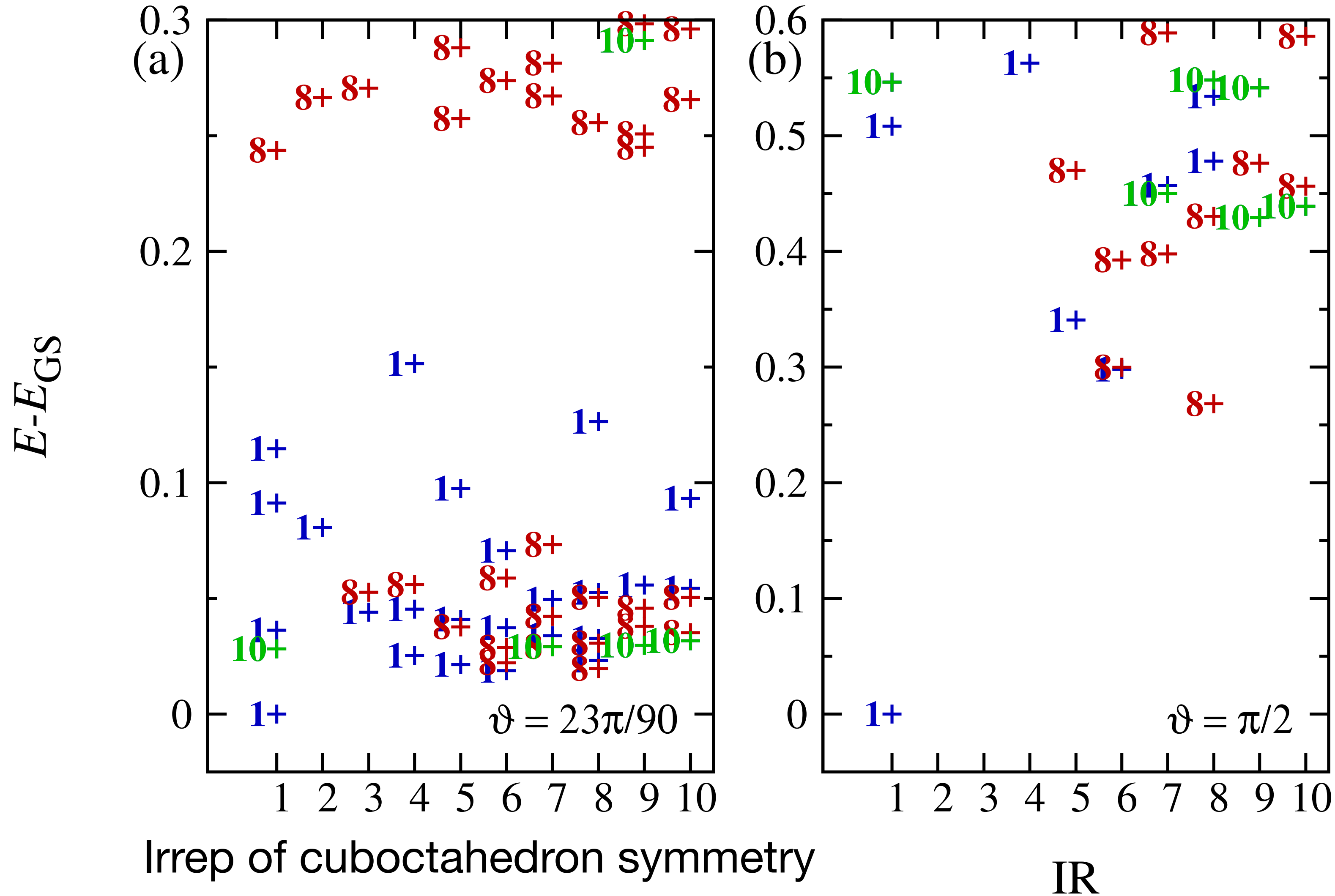
N	undirected	directed	linearly independent
12	32	69	48
27	1024	2551	2485
36	8192	22437	22332

degeneracy of the manifold

N	undirected	directed	linearly independent (GS manifold)	total # of singlets
12	32	69	48	462
21		595		1385670
27	1024	2551	2485	414315330
36	8192	22437	22332	2861142656400
39	16384	46339	46219	57468093927120
48	131072	408665		521086299271824330



The J = K case: other irreps also appear

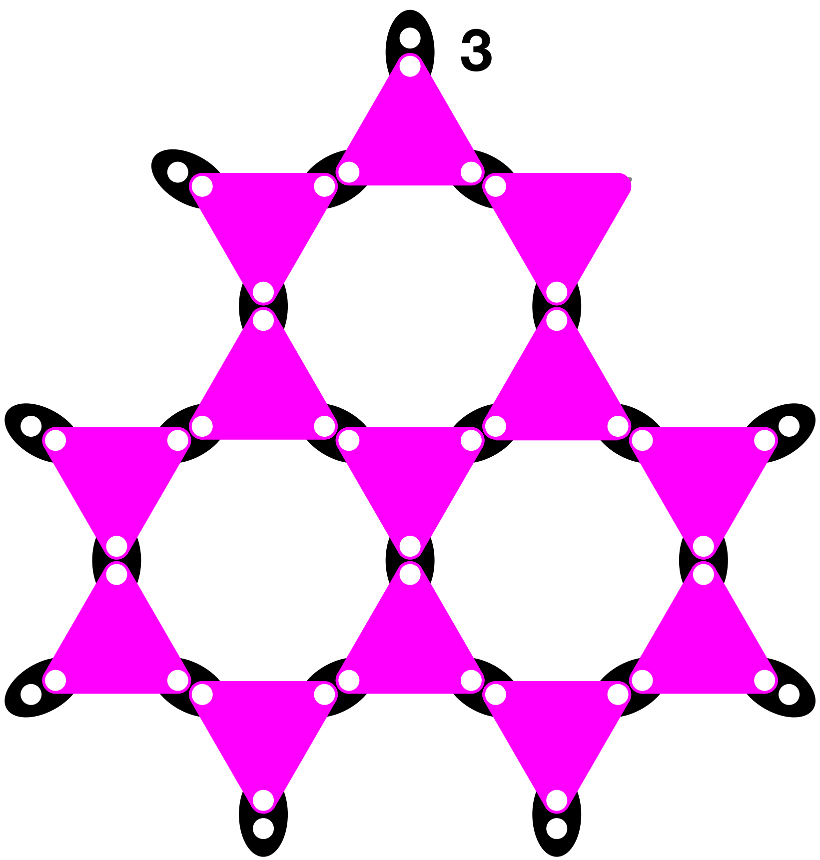


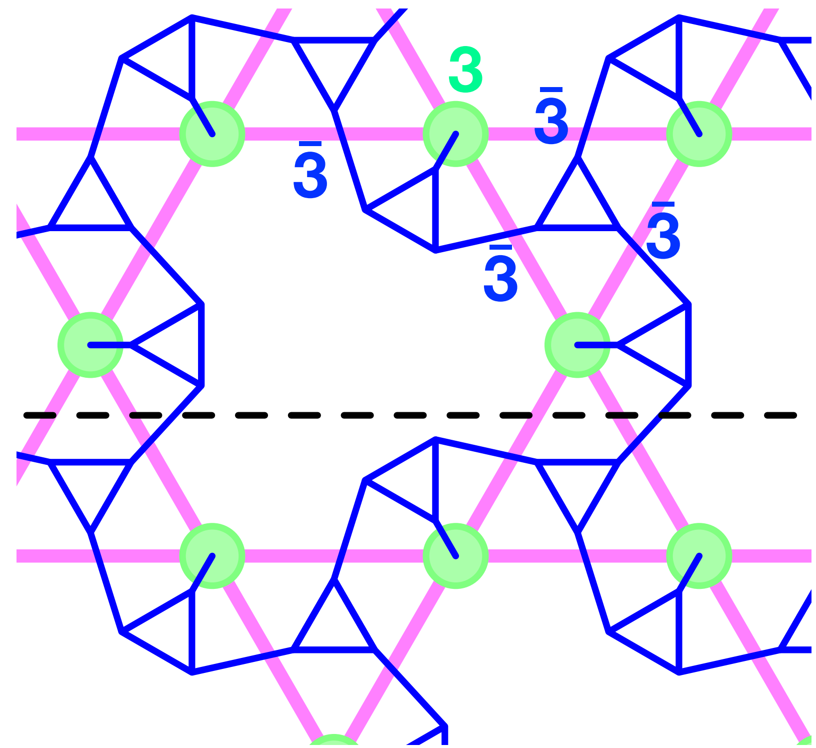
degeneracy at $\vartheta=\pi/4$: 468 = (48)×**1** + (40)×**8** + (10)×$\overline{\mathbf{10}}$

What is the origin of the higher SU(3) irreps ???

Tensor network: the wave function

$$|\mathbf{1}(i_1,i_2,i_3)\rangle = \sum_{\alpha,\beta,\gamma} \varepsilon^{\alpha\beta\gamma} f^\dagger_\alpha(i_1) f^\dagger_\beta(i_2) f^\dagger_\gamma(i_3)|0\rangle$$



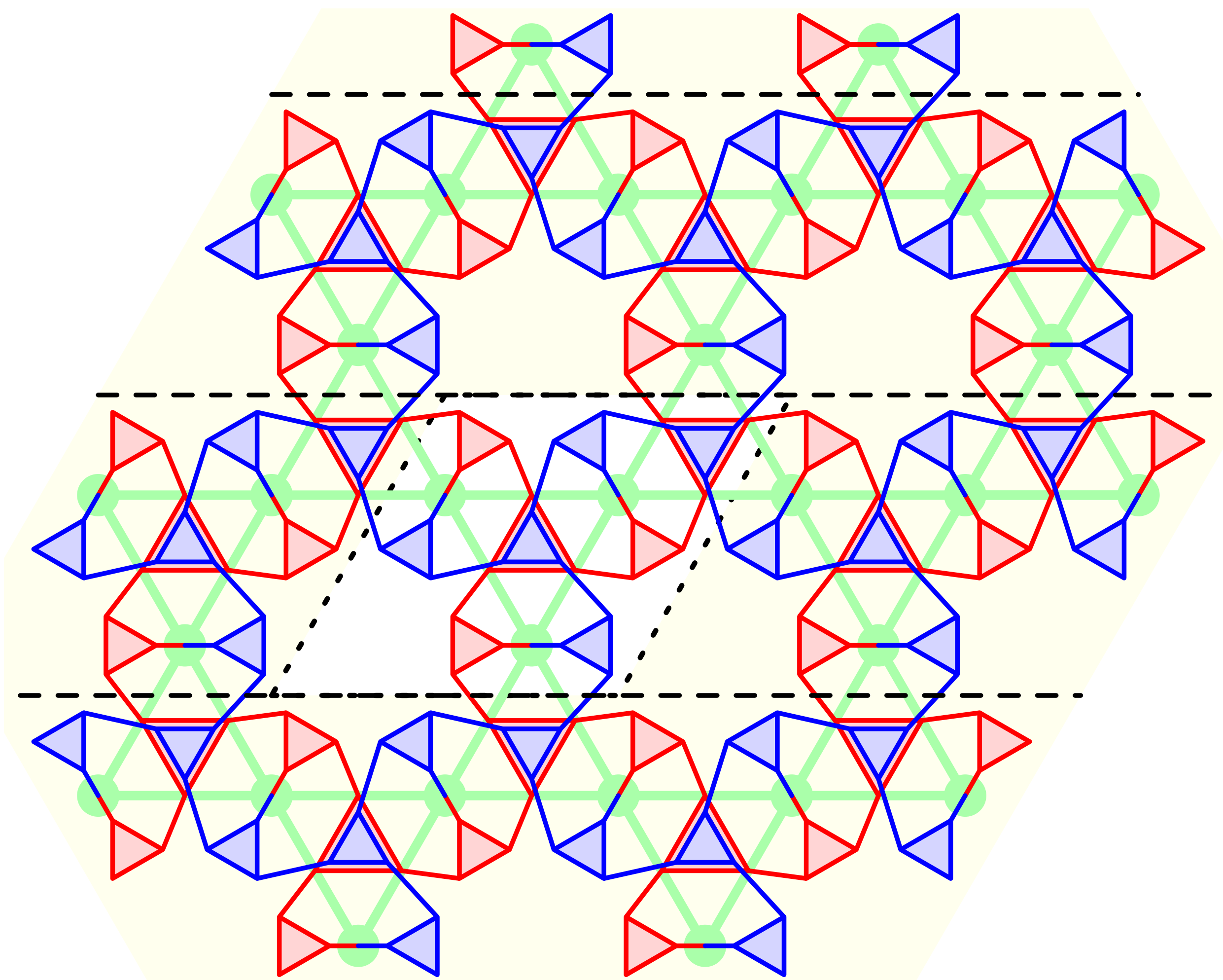


$$\bar{\mathbf{3}} \times \bar{\mathbf{3}} = \mathbf{3} + \bar{\mathbf{6}}$$

each triangle represents the antisymmetrizing Levi-Civita symbol

we antisymmetrize at each **3** site

Tensor network: the overlap



graph of contracted
Levi-Civita symbols

R. Penrose,
Applications of
negative dimensional
tensors, 1971

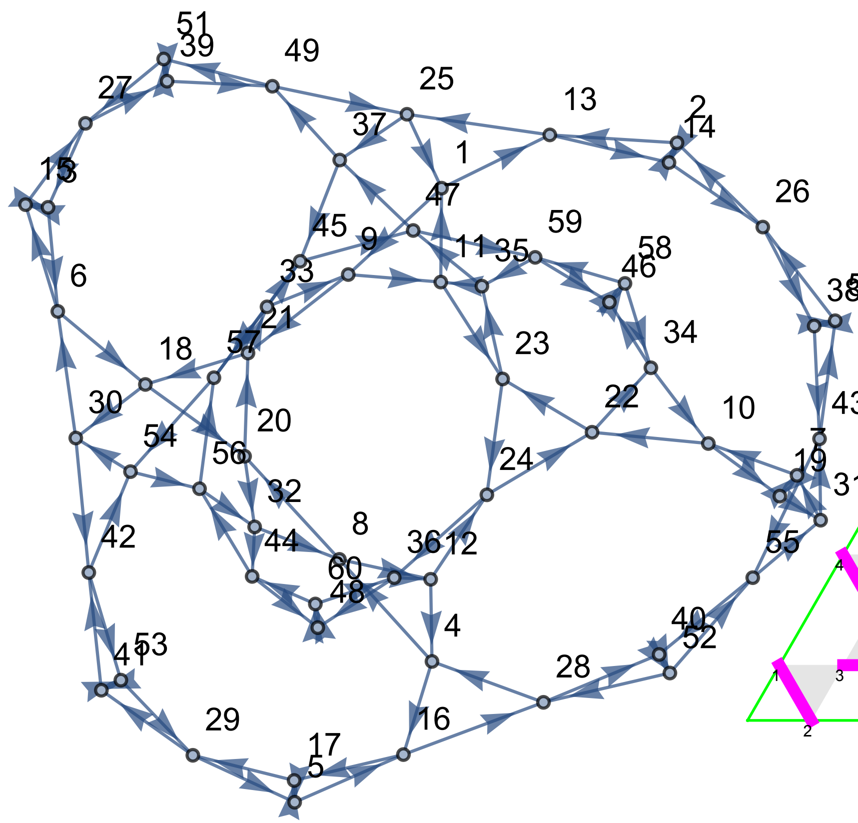
Penrose polynomial,
defined for plane graphs

12: 13392
27: 1828256832
36: 2220531642144

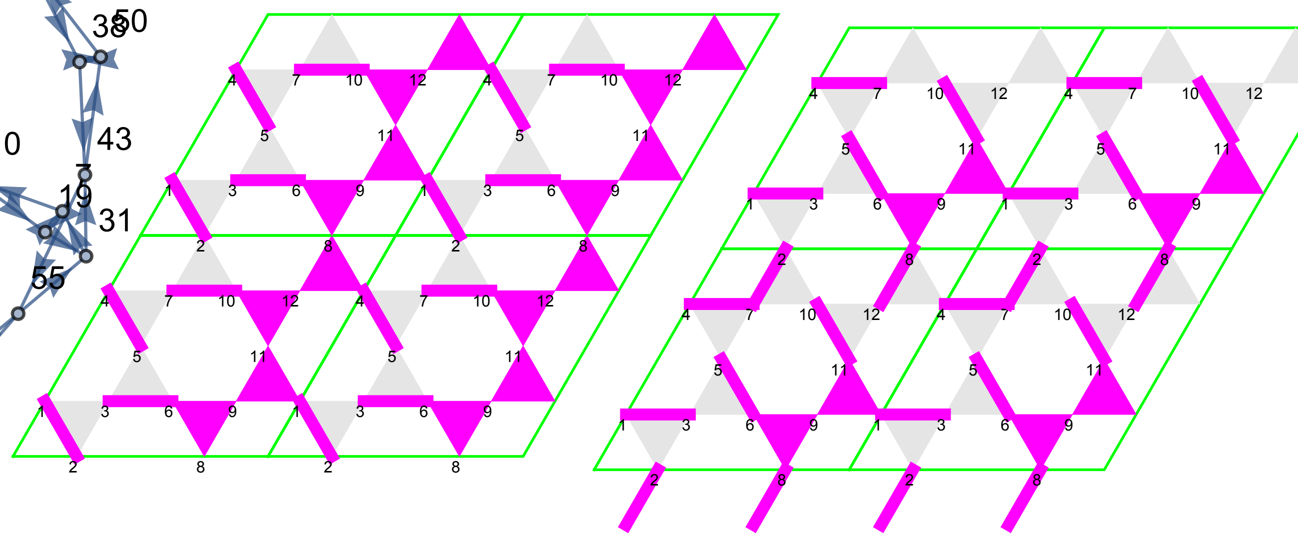
Example for overlap (12 sites)

$$\varepsilon^{1,9,11}\varepsilon^{2,13,14}\varepsilon^{3,6,15}\varepsilon^{4,8,12}\varepsilon^{5,16,17}\varepsilon^{7,10,19}\varepsilon^{18,20,21}\varepsilon^{22,23,24}\varepsilon^{25,37,49}\varepsilon^{26,38,50}$$
$$\varepsilon^{27,39,51}\varepsilon^{28,40,52}\varepsilon^{29,41,53}\varepsilon^{30,42,54}\varepsilon^{31,43,55}\varepsilon^{32,44,56}\varepsilon^{33,45,57}\varepsilon^{34,46,58}\varepsilon^{35,47,59}\varepsilon^{36,48,60}$$
$$\varepsilon_{1,13,25}\varepsilon_{2,14,26}\varepsilon_{3,15,27}\varepsilon_{4,16,28}\varepsilon_{5,17,29}\varepsilon_{6,18,30}\varepsilon_{7,19,31}\varepsilon_{8,20,32}\varepsilon_{9,21,33}\varepsilon_{10,22,34}$$
$$\varepsilon_{11,23,35}\varepsilon_{12,24,36}\varepsilon_{37,45,47}\varepsilon_{38,43,50}\varepsilon_{39,49,51}\varepsilon_{40,52,55}\varepsilon_{41,42,53}\varepsilon_{44,48,60}\varepsilon_{46,58,59}\varepsilon_{54,56,57}$$

$$= 49152$$



The graphs are "bipartite" (median graph for degree 3 regular bipartite graph)



Penrose graph

Evaluating Penrose graphs

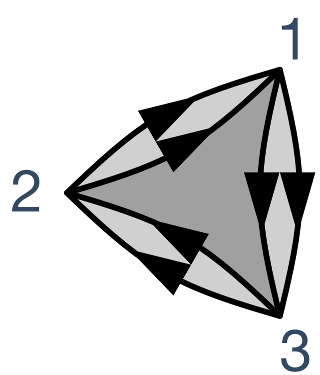
$$\varepsilon_{i,j,k}\,\varepsilon^{i,j,k} = 6$$

implied sum over repeated indices

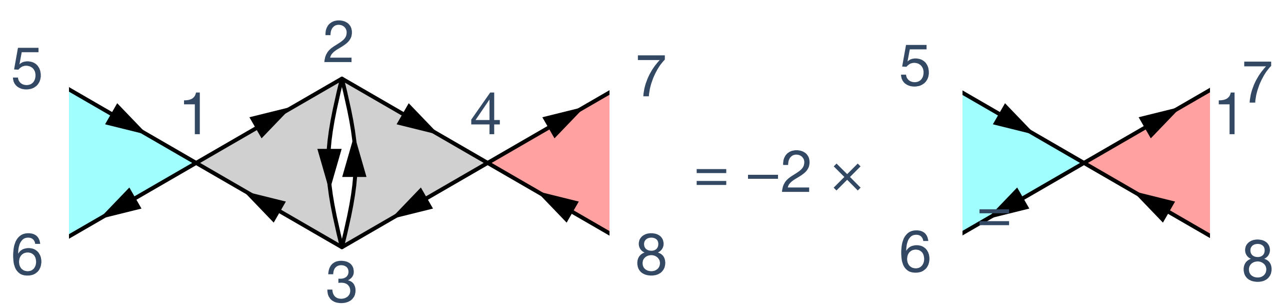
$$\varepsilon_{i,j,k}\,\varepsilon^{i,j,l} = 2\delta_k^l$$

$$\varepsilon_{i,j,k}\,\varepsilon^{i,l,m} = \delta_j^l\delta_k^m - \delta_j^m\delta_k^l$$

We can define a recursive procedure to evaluate the Penrose graph:

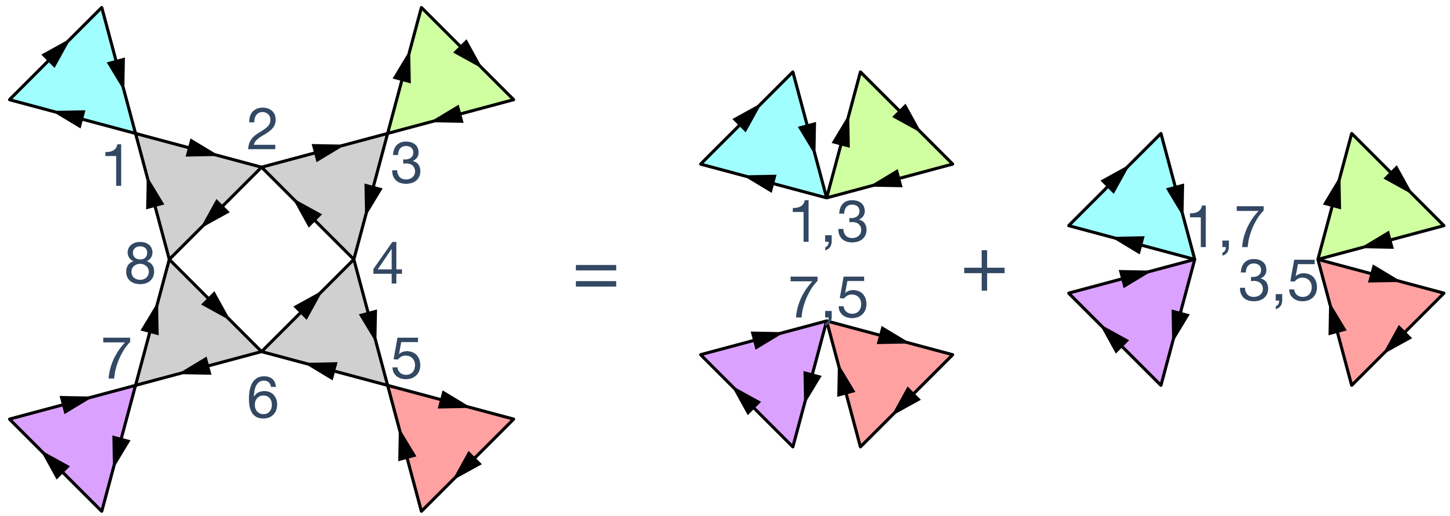


$$\varepsilon_{1,2,3}\,\varepsilon^{1,2,3} = 6$$

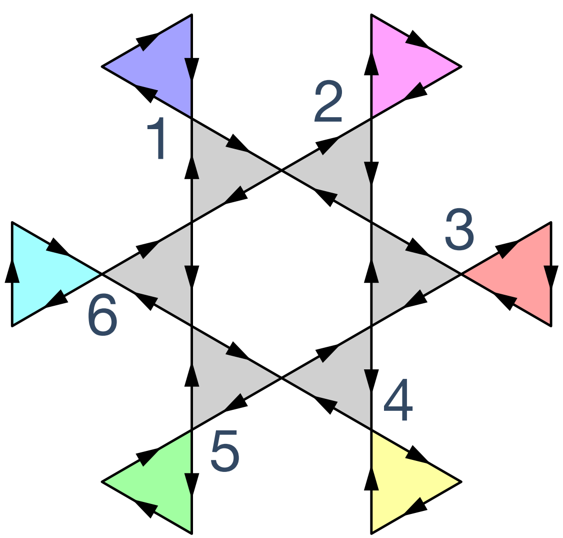


$$\ldots\varepsilon_{5,1,6}\,\varepsilon^{1,2,3}\,\varepsilon_{2,4,3}\,\varepsilon^{4,7,8}\ldots = -2 \times \ldots\varepsilon_{5,1,6}\,\varepsilon^{1,7,8}\ldots$$

Evaluating Penrose graphs



$$\ldots\varepsilon_{1,2,8}\,\varepsilon^{2,3,4}\,\varepsilon_{4,5,6}\,\varepsilon^{6,7,8}\ldots = \delta_1^3\delta_5^7 + \delta_1^7\delta_5^3$$



$$= -\,\delta_1^2\delta_3^6\delta_5^4 - \delta_1^6\delta_3^4\delta_5^2 - \delta_1^4\delta_3^2\delta_5^6 + \delta_1^4\delta_3^6\delta_5^2$$

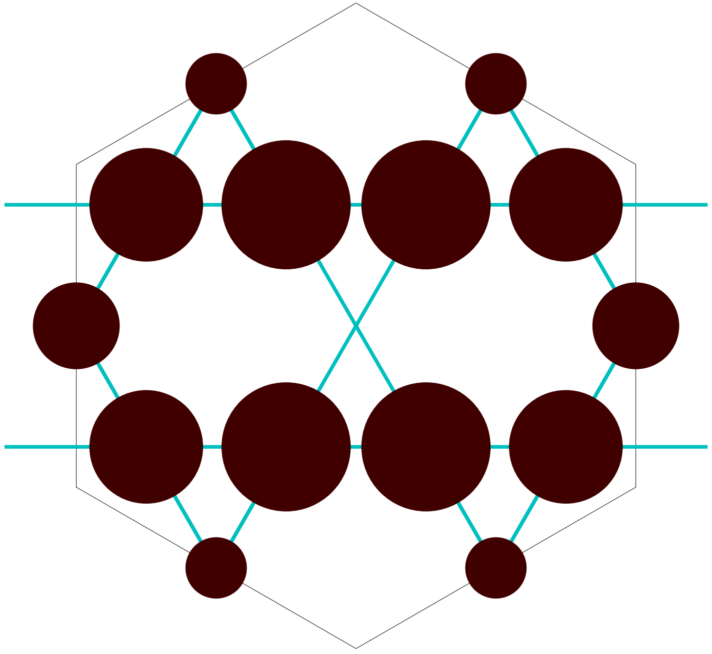
Spin-spin correlation function

calculated using "tensor network"

12 sites

27 sites

48 sites



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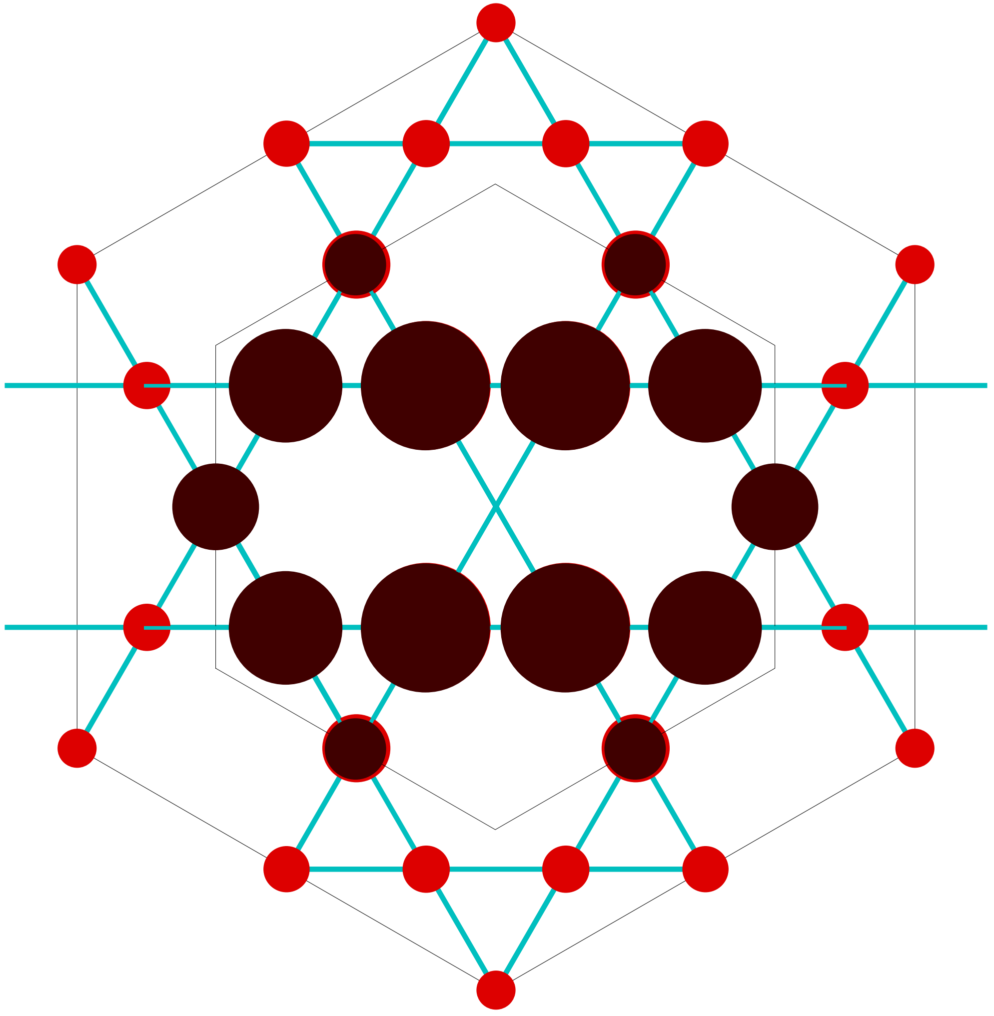

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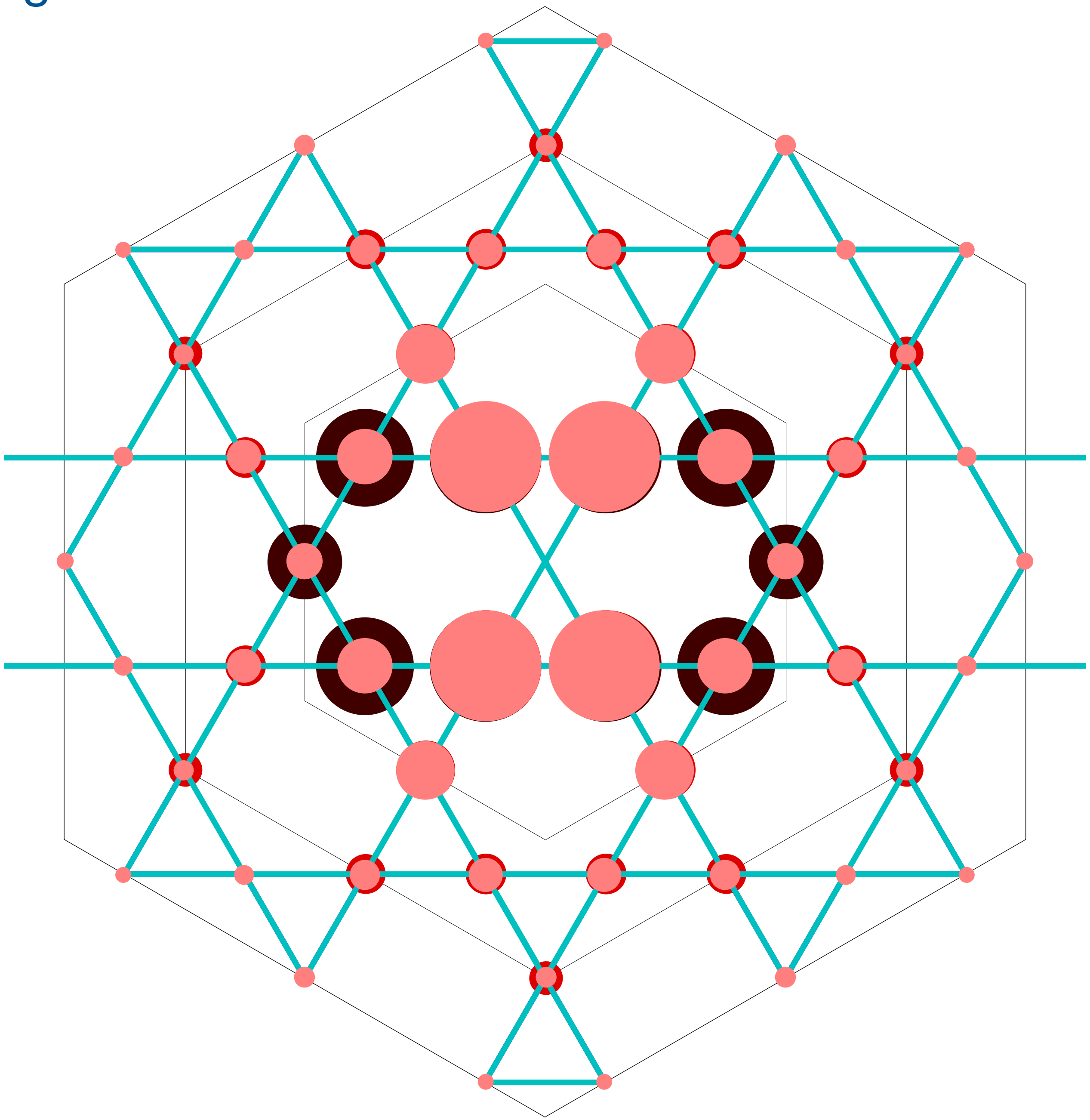
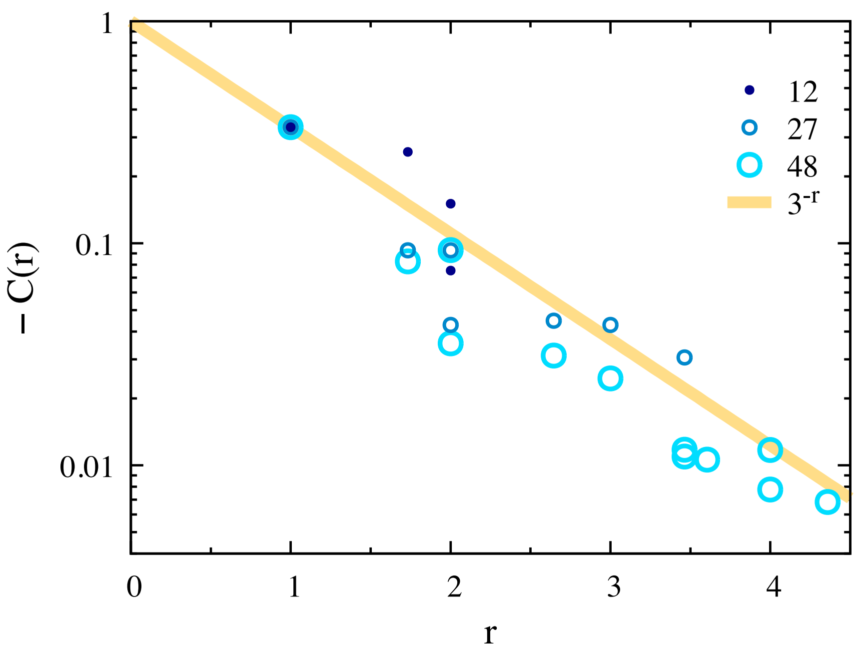
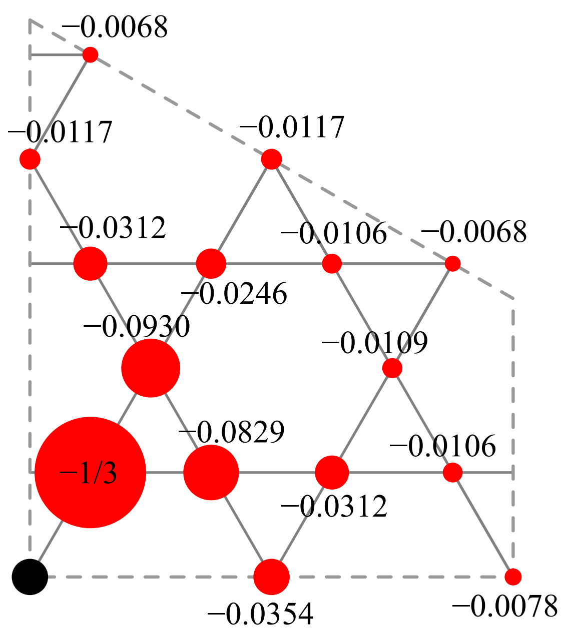


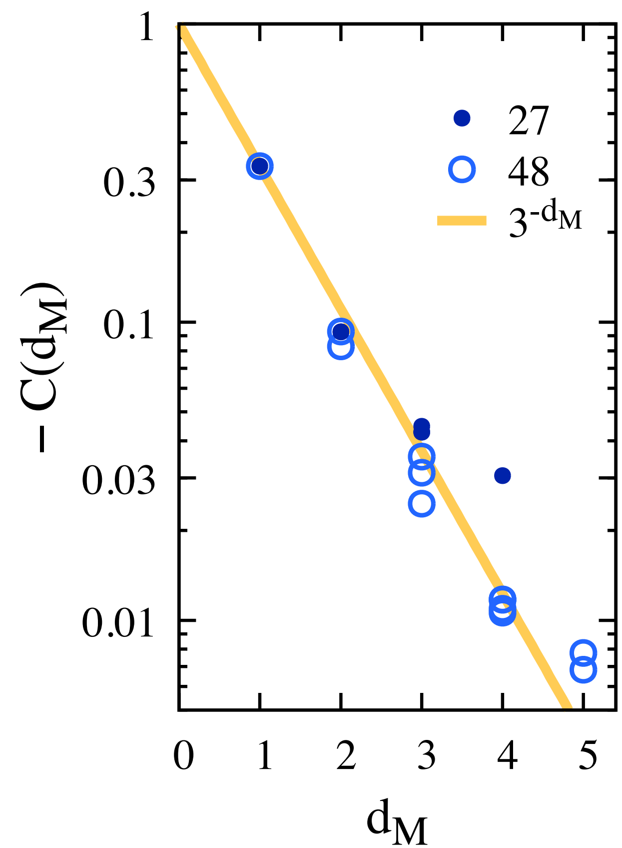
Spin-spin correlation function



decays exponentially,

$$C(r) = \langle \mathrm{FSS}|A^{\mu}A_{\mu}|\mathrm{FSS}\rangle$$
$$= \langle \mathrm{FSS}|\left(P_{0,r} - 1/3\right)|\mathrm{FSS}\rangle$$
$$\approx 3^{-r}$$

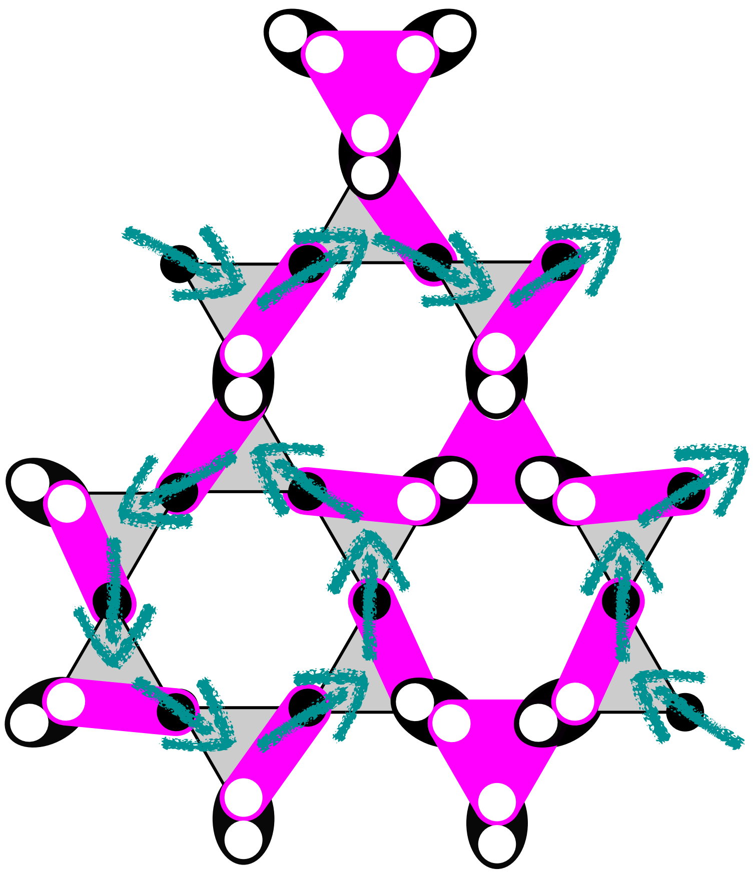




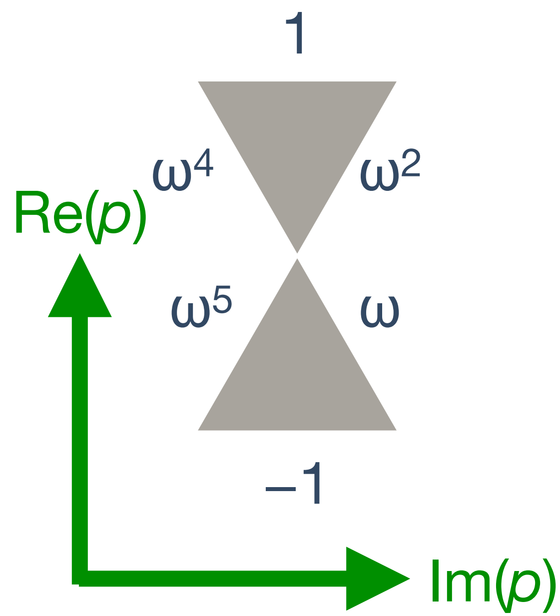
Manhattan distance

Topological sectors (polarizability)

following Bulaevskii, Batista, Mostovoy, and Khomskii,
Phys. Rev. B **78**, 024402 (2008).



$$\omega = \exp\frac{2\pi i}{6}$$

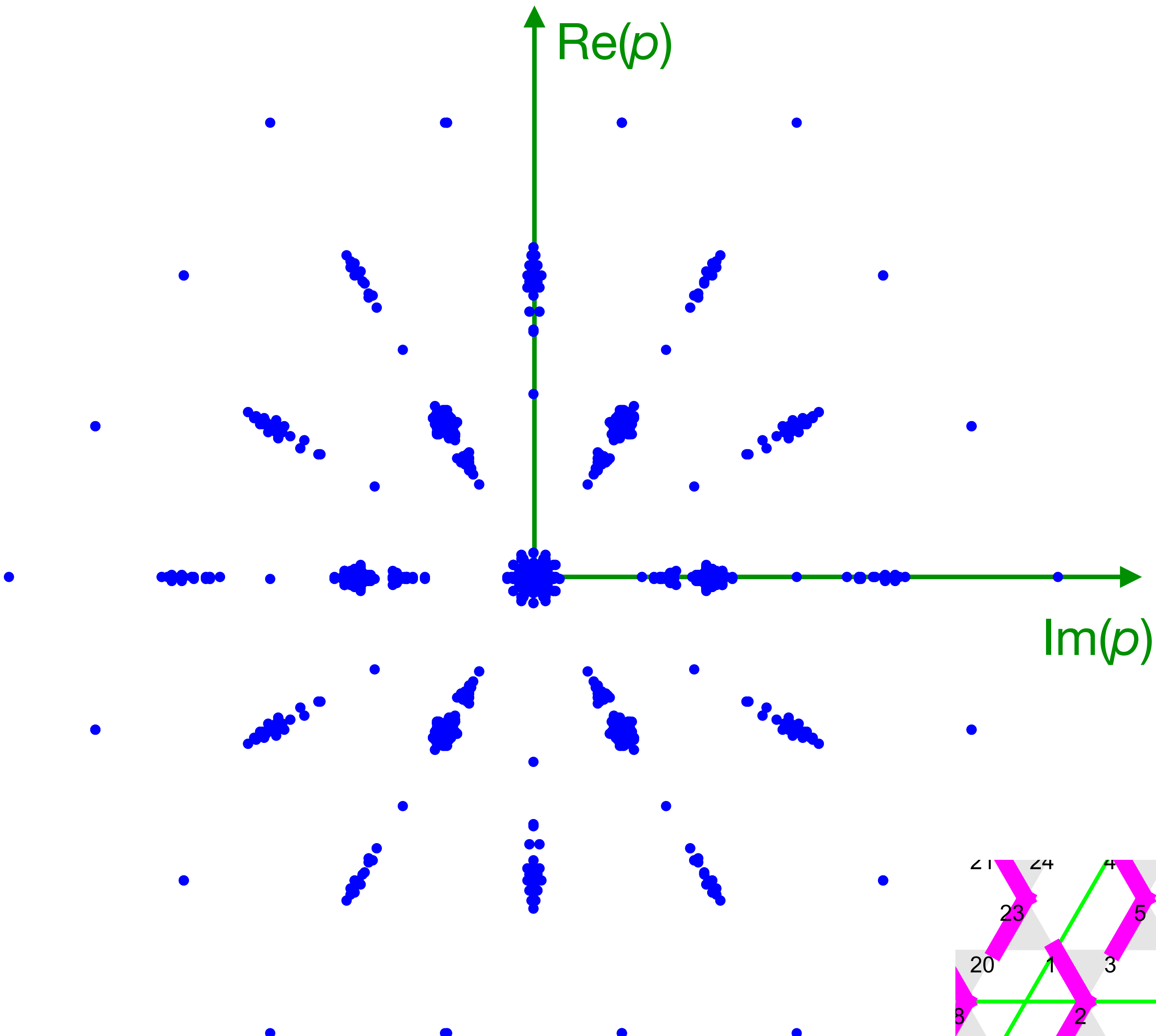


we calculate the eigenvalues of the polarization operator p:

$$p = \sum_{j\in\text{bonds}} \omega^{l(j)}\langle\mathscr{P}_j\rangle$$

where $\langle\mathscr{P}_j\rangle$ is the expectation value of the spin correlation on the bond.

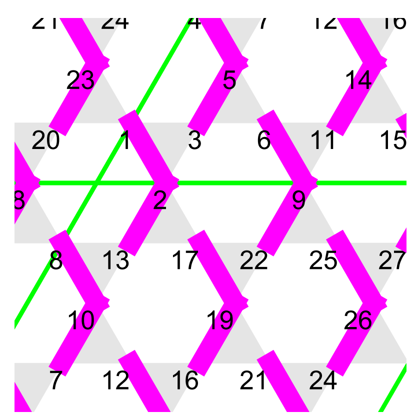
Topological sectors (polarizability)



we calculate the eigenvalues of the polarization operator:

$$p = \sum_{j \in \text{bonds}} \omega^{l(j)} \mathscr{P}_j$$

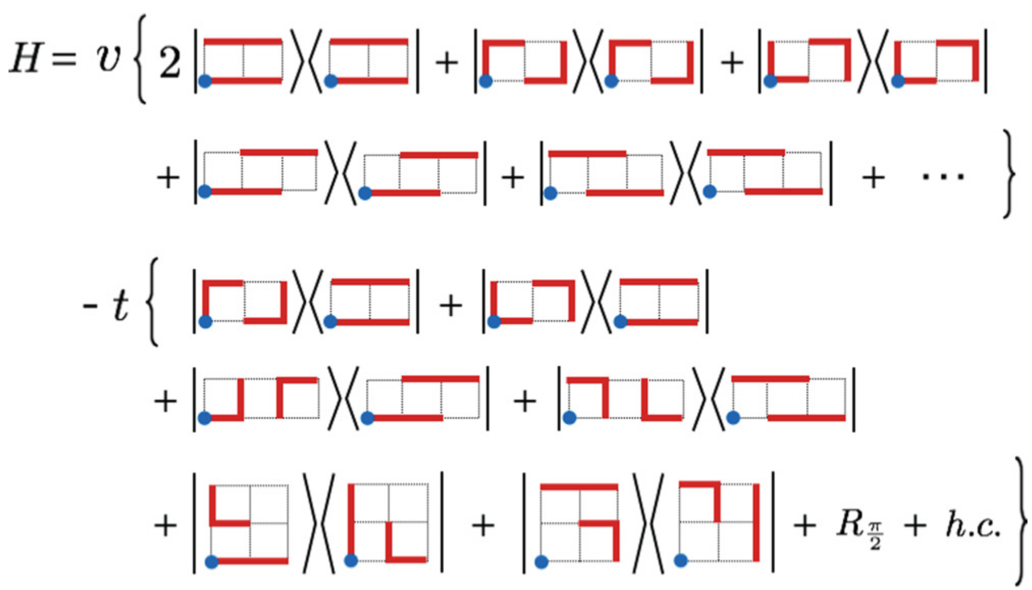
27 sites, 2485 states



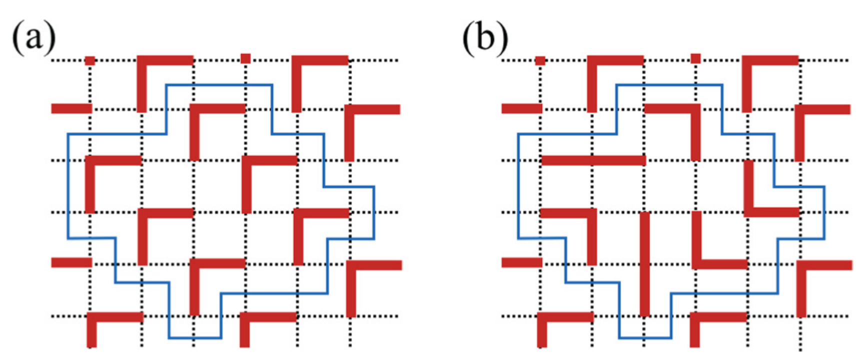
banana states, have maximal polarization

Tensor networks: Z_3 topological order

H. Lee, Y. Oh, J. H. Han, and H. Katsura
Resonating valence bond states with trimer motifs
Phys. Rev. B **95**, 060413(R) (2017)



Trimers are not singlets of an SU(3) models (antisymmetry missing).



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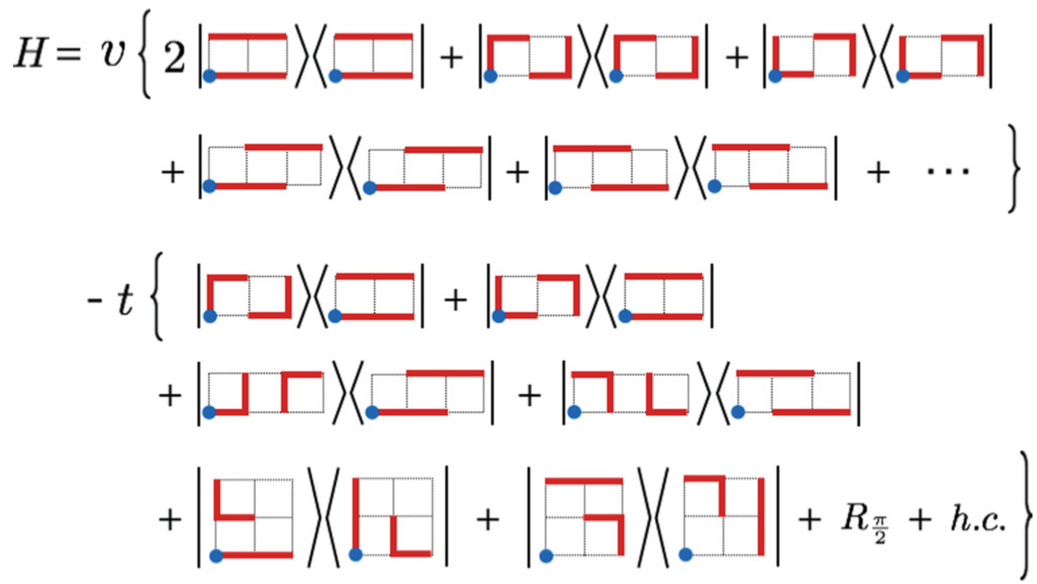
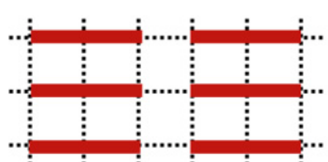
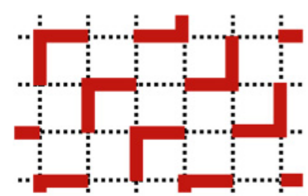
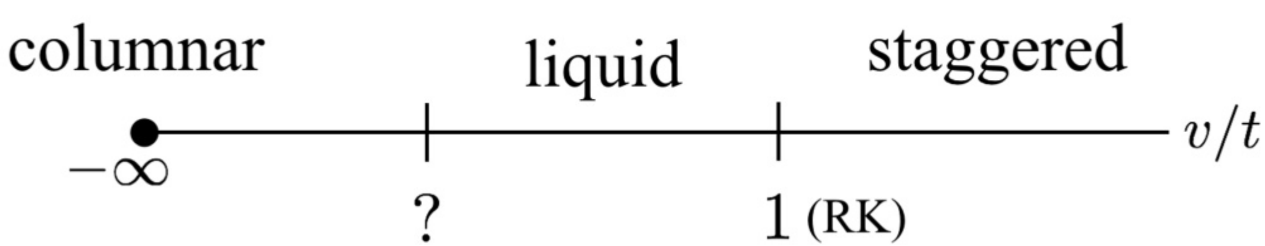


They defined winding numbers, leading to 3 topological sectors along both direction (Z_3 vs. Z_2 for dimer coverings).



ground state degeneracy 9^{genus}





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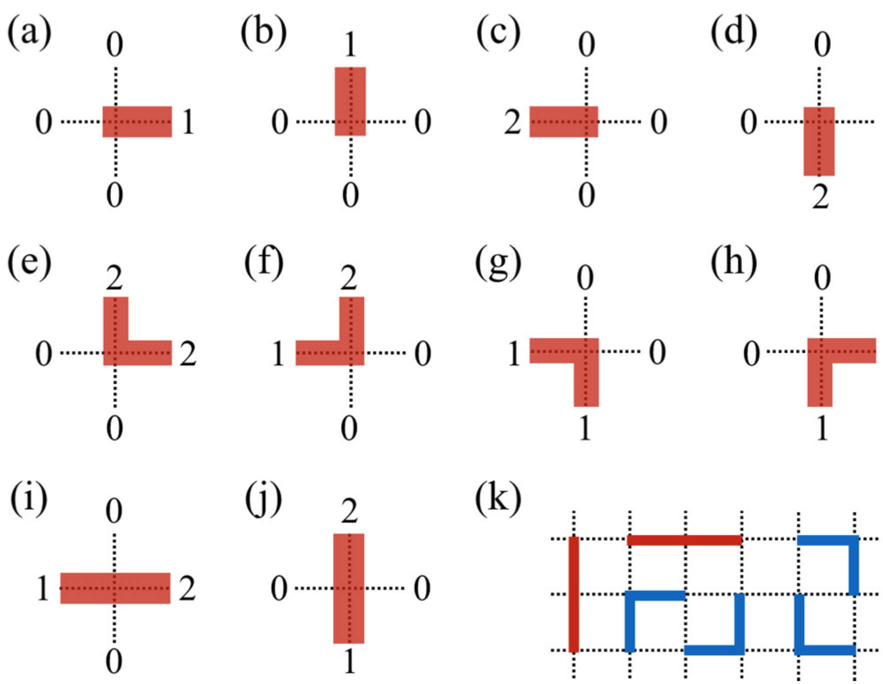


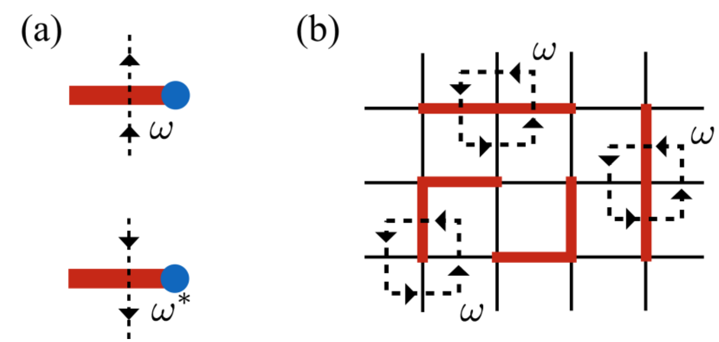


FIG. 2. (a) Assignment of the weight $\omega = e^{2\pi i/3}$ and its conjugate ω^* for the passage through the dual lattice with the center of the trimer (blue dot) on the right and left side of the path, respectively. (b) For any elementary loop surrounding a site, the total weight is always $\Gamma = \omega$. For any loop surrounding a single trimer, the total weight is $\Gamma = \omega^3 = 1$.

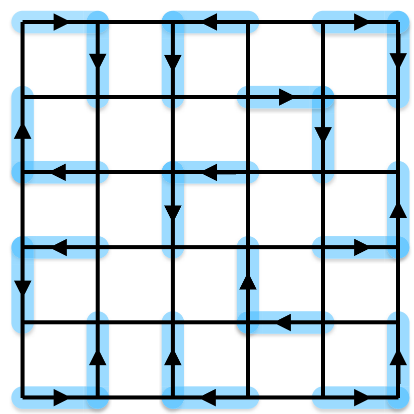
Tensor networks: Z_3 topological order

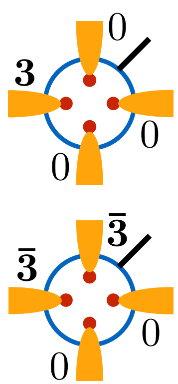
Xiao-Yu Dong, Ji-Yao Chen, Hong-Hao Tu

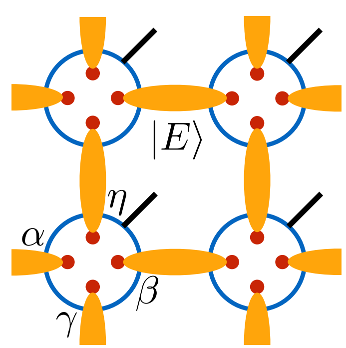
SU(3) trimer resonating-valence-bond state on the square lattice

Phys. Rev. B 98, 205117 (2018).

Trimers are now singlets of an SU(3) models (antisymmetry denoted by arrows).







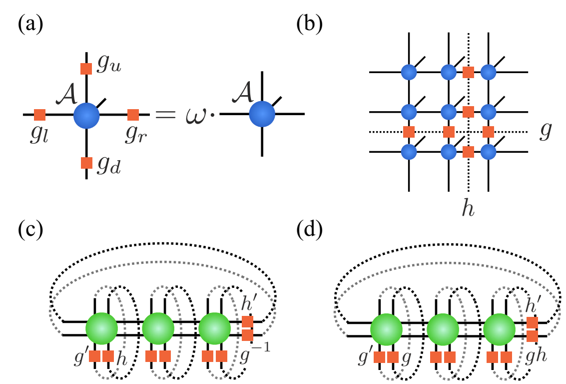


FIG. 5. (a) $\mathbb{Z}_3$ gauge symmetry of the PEPS local tensor. (b) Constructing nine states by inserting gauge flux along two non-contractible loops on a torus. (c) and (d) A 3×1 torus formed by double tensors and the $\mathbb{Z}_3$ gauge symmetry elements is used to compute modular S and T matrices.

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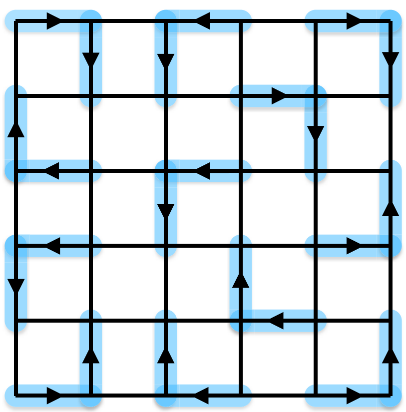

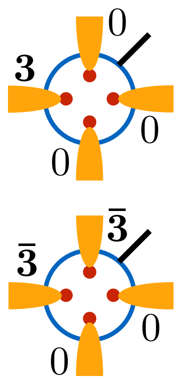

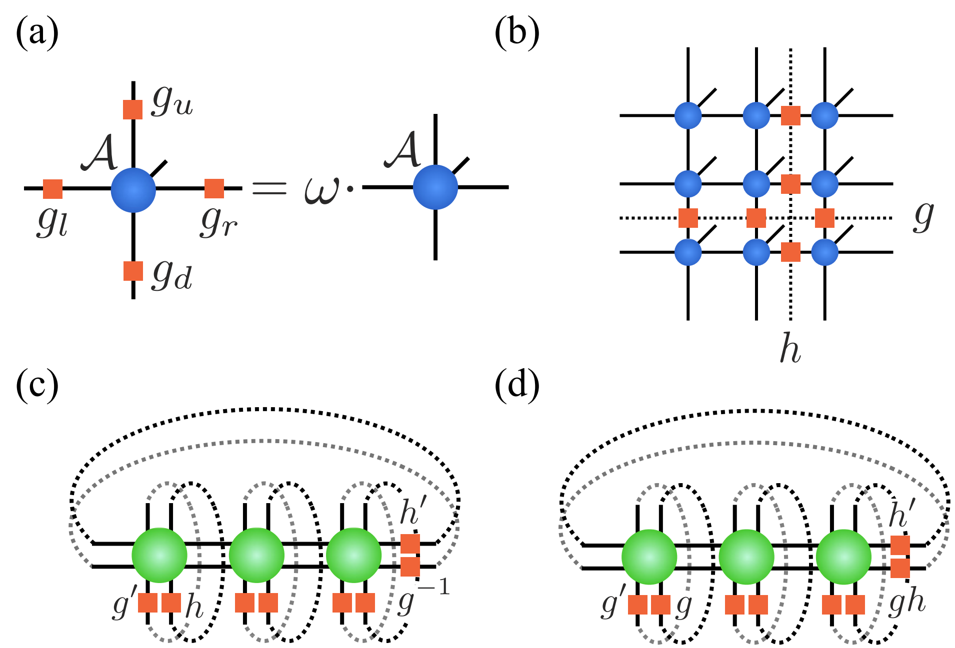

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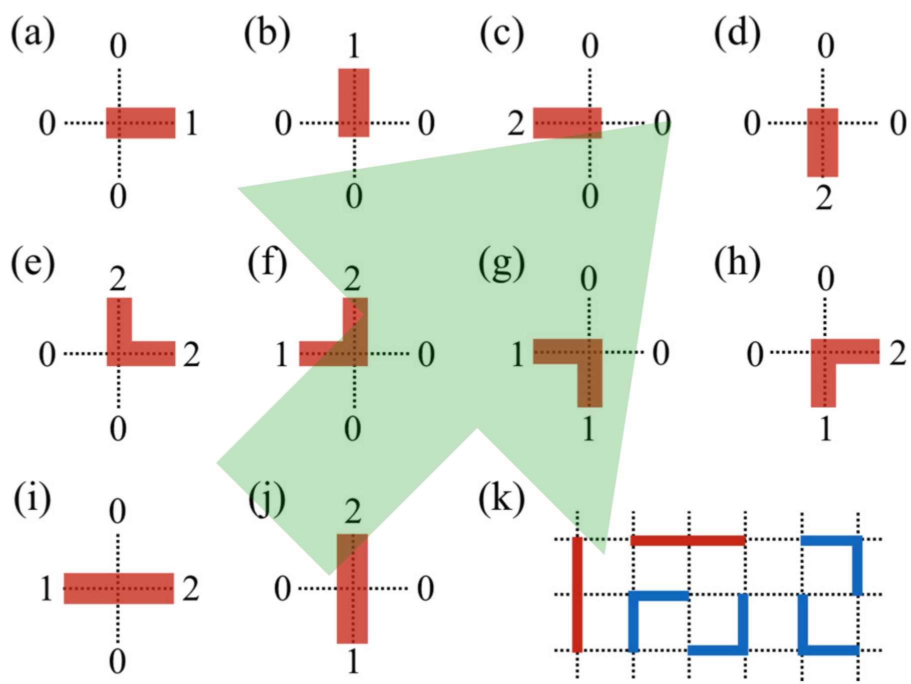
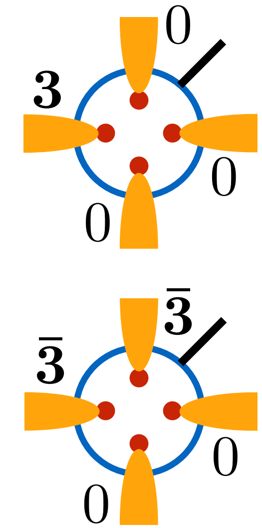
I. Kurecic, L. Vanderstraeten, N. Schuch,
Phys. Rev. B **99**, 045116 (2019)





Tensor networks: Z_3 topological order

I. Kurecic, L. Vanderstraeten, N. Schuch, A gapped SU(3) spin liquid with Z_3 topological order, Phys. Rev. B **99**, 045116 (2019)

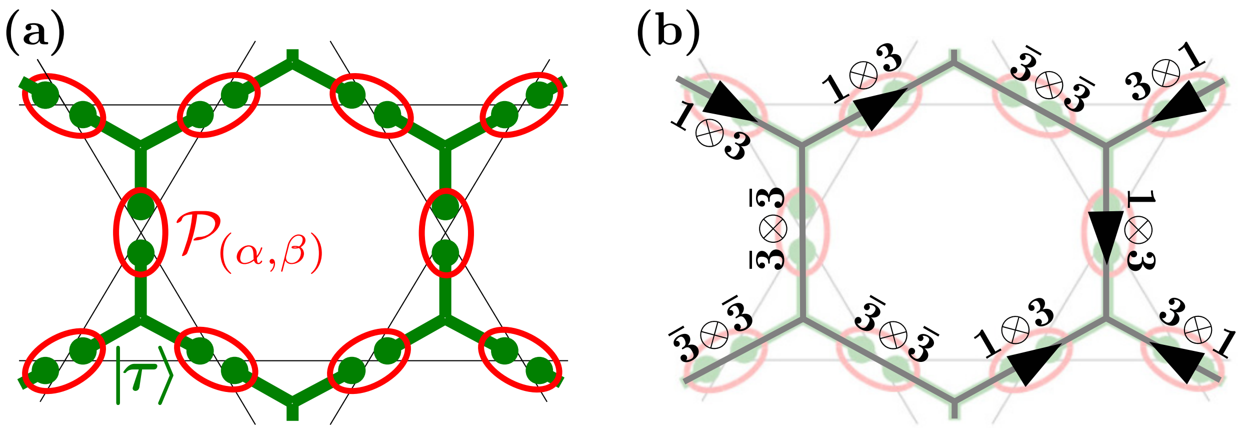
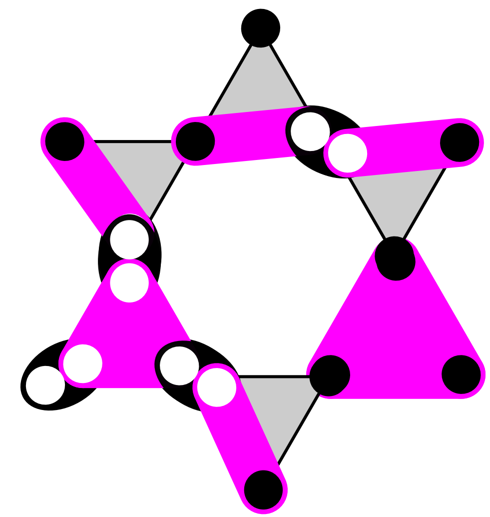
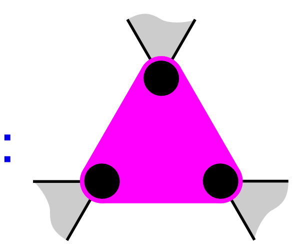


FIG. 1. (a) The model is constructed from trimers $|\tau\rangle$ which are in a singlet state with representation $\mathcal{H}_v \equiv \mathbf{1}\oplus\mathbf{3}\oplus\bar{\mathbf{3}}$ at each site (green dots), to which a map $\mathcal{P}_\bullet$ is applied which selects the physical degrees of freedom from $\mathcal{H}_v\otimes\mathcal{H}_v$. (b) Mapping to a $\mathbb{Z}_3$ topological model: Each site holds a $\mathbb{Z}_3$ degree of freedom: one of two arrows or no arrow. The arrows are pointing towards the $\mathbf{3}$ representation and satisfy a Gauss law across each vertex due to the fusion rules of the SU(3) irreps.



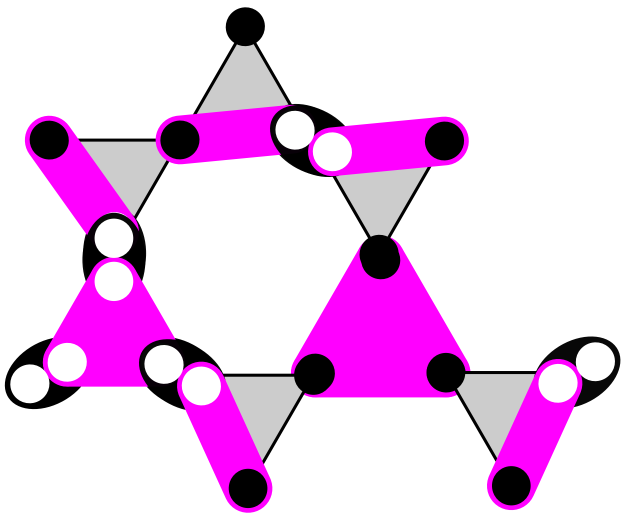
The trimer singlet is new:



parent Hamiltonian has 17 (?) sites, not shown in the papers

Tensor networks: Z_3 topological order

I. Kurecic, L. Vanderstraeten, N. Schuch, A gapped SU(3) spin liquid with Z_3 topological order, Phys. Rev. B **99**, 045116 (2019).



$$N_{\rm sites} = \frac{3}{2} N_{\bar{\mathbf{3}}\bar{\mathbf{3}}\bar{\mathbf{3}}} + 3 N_{\mathbf{333}} + \frac{3}{2} N_{\bar{\mathbf{3}}\mathbf{3}}$$

$$N_{\rm tris} = N_{\bar{\mathbf{3}}\bar{\mathbf{3}}\bar{\mathbf{3}}} + N_{\mathbf{333}} + N_{\bar{\mathbf{3}}\mathbf{3}}$$

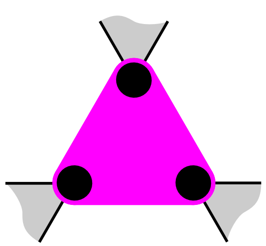
$$3N_{\rm tris} = 2N_{\rm sites}$$

so

$$2N_{\rm sites} = 3N_{\bar{\mathbf{3}}\bar{\mathbf{3}}\bar{\mathbf{3}}} + 6N_{\mathbf{333}} + 3N_{\bar{\mathbf{3}}\mathbf{3}}$$

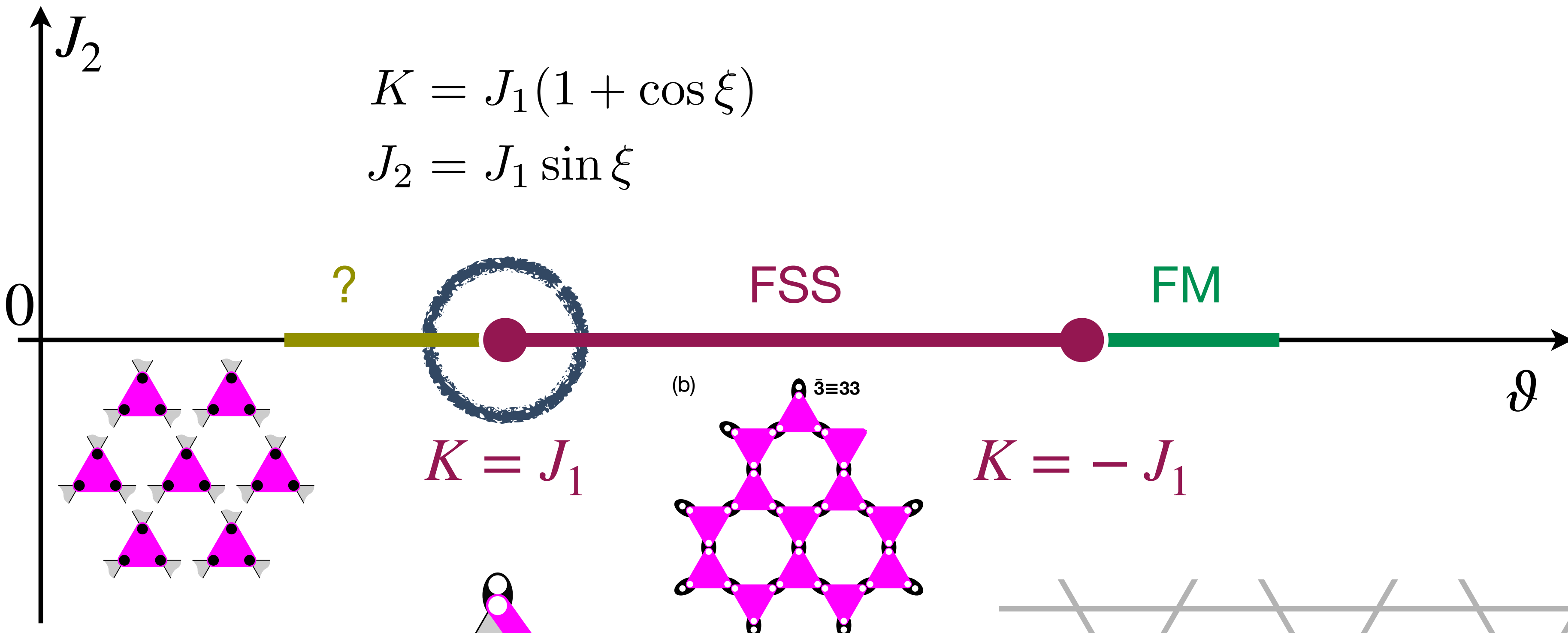
$$3N_{\rm tris} = 3N_{\bar{\mathbf{3}}\bar{\mathbf{3}}\bar{\mathbf{3}}} + 3N_{\mathbf{333}} + 3N_{\bar{\mathbf{3}}\mathbf{3}}$$

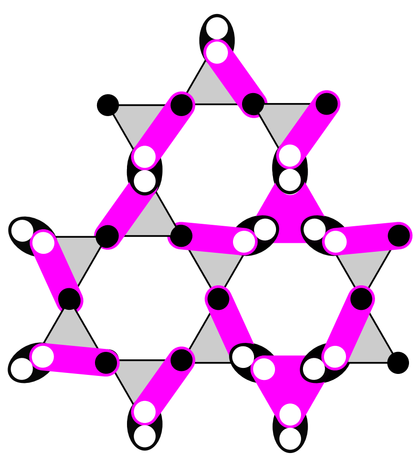
gives that $N_{\mathbf{333}} = 0$.

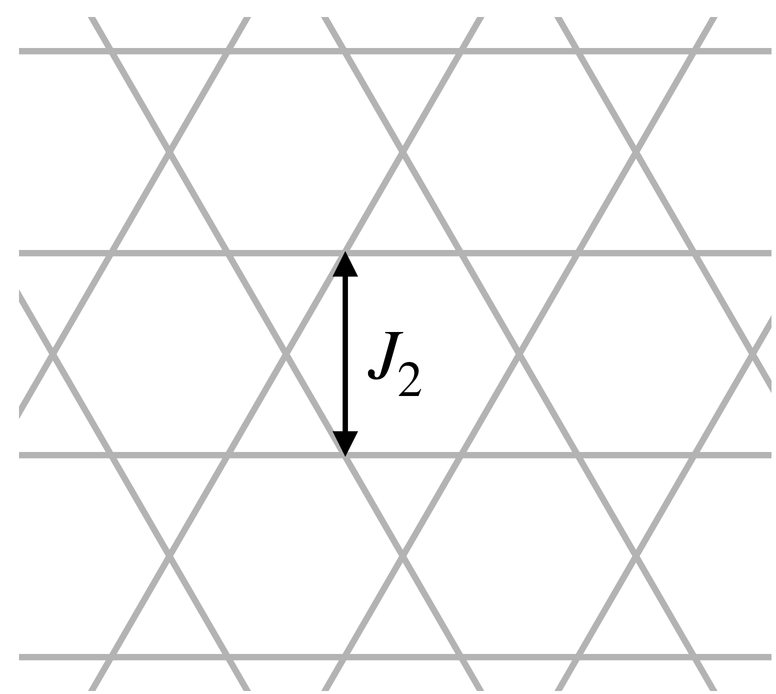


an $N_{\mathbf{333}}$ creates an unhappy triangle somewhere in the lattice (unless saved by non-orthogonality) — they are not part of the ground state manifold

Lifting the degeneracy: $K - J_1 - J_2$ model





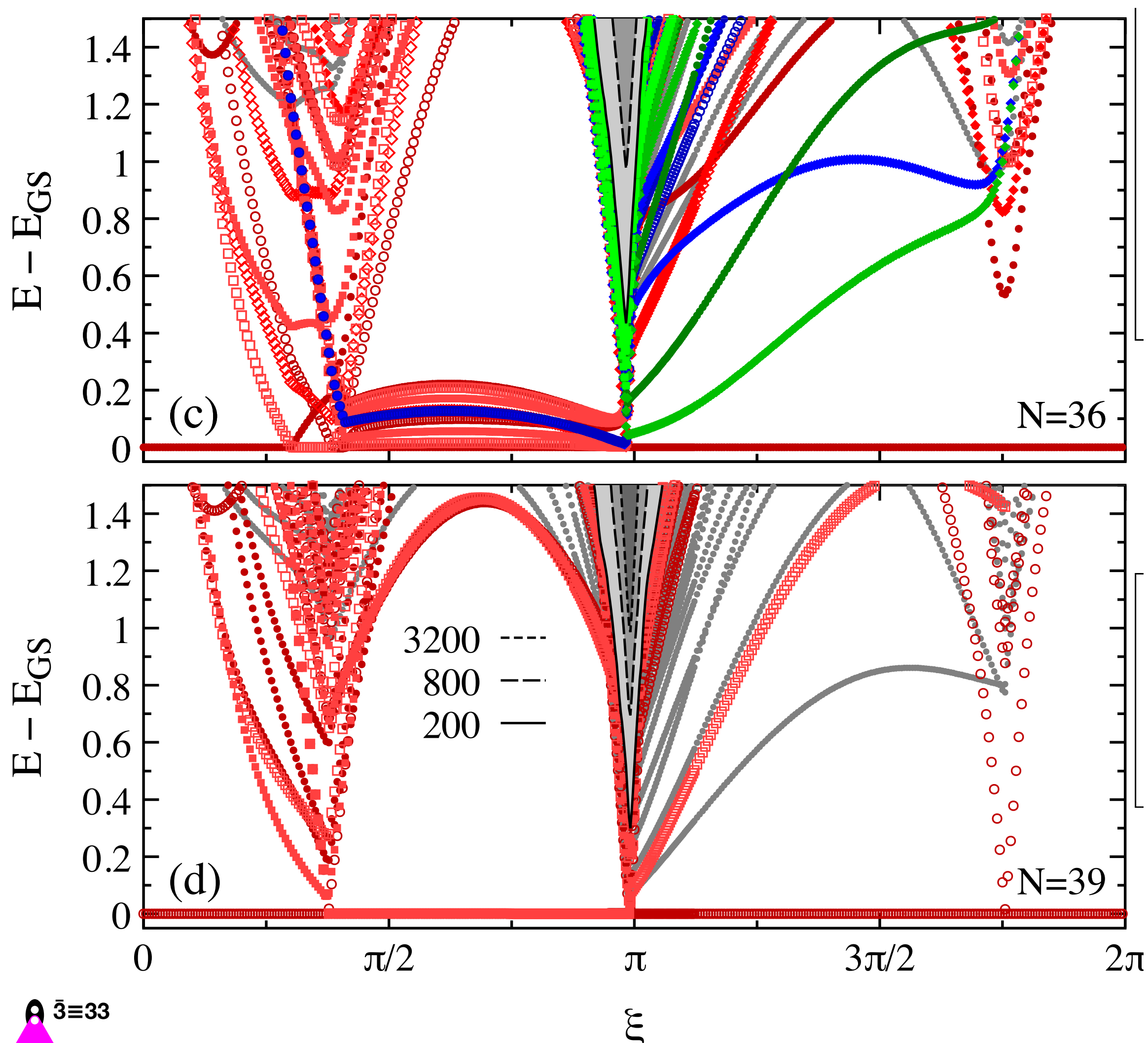


perturbation around the degenerate manifold

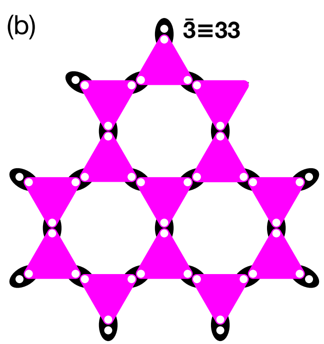
Lifting the degeneracy: $K - J_1 - J_2$ model

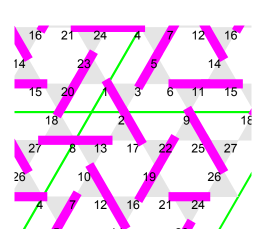
$$K = J_1(1 + \cos\xi)$$

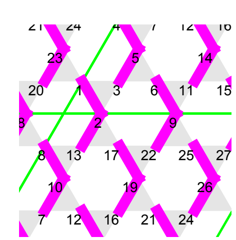
$$J_2 = J_1 \sin\xi$$



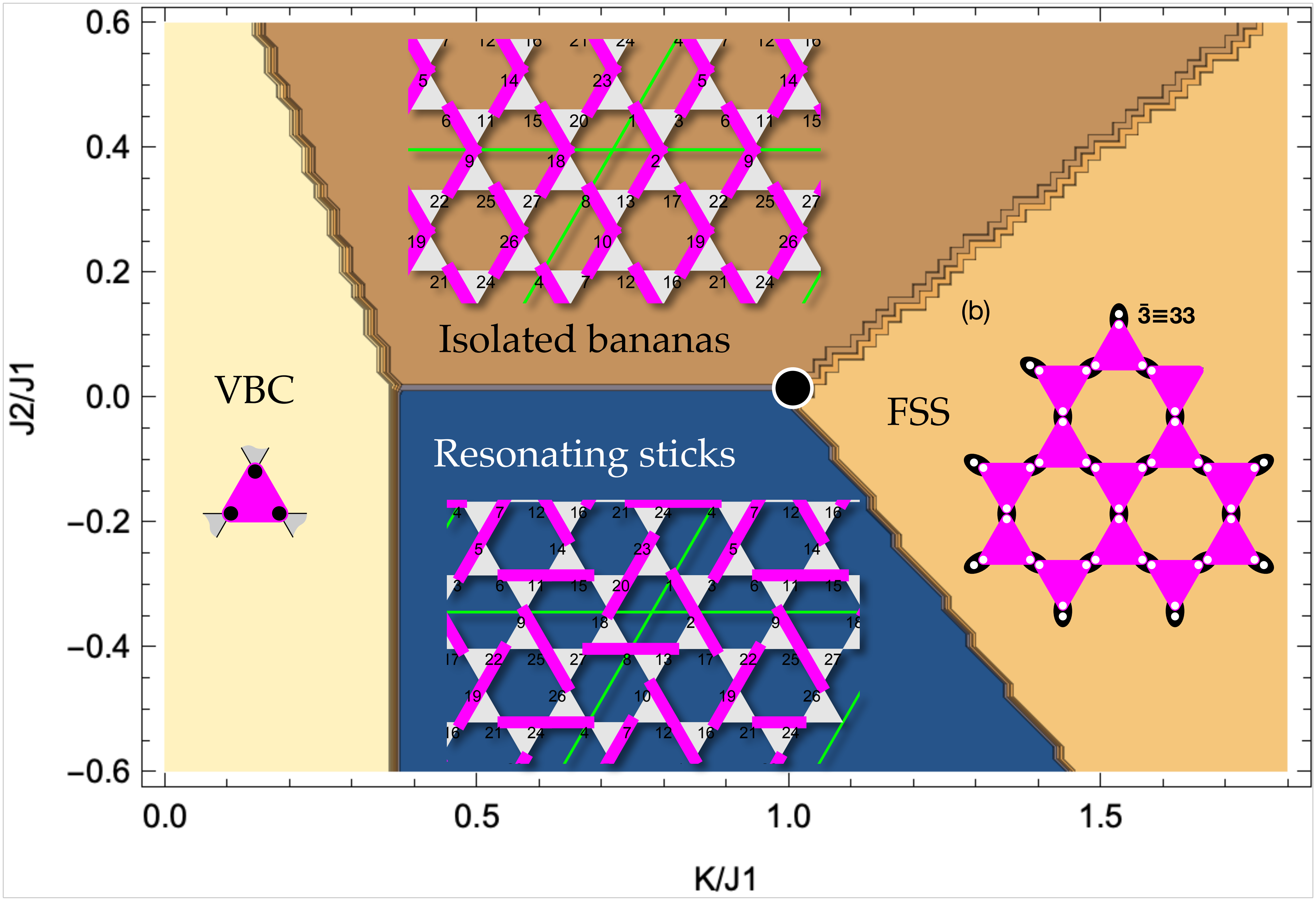
ED in the Hilbert space spanned by singlets





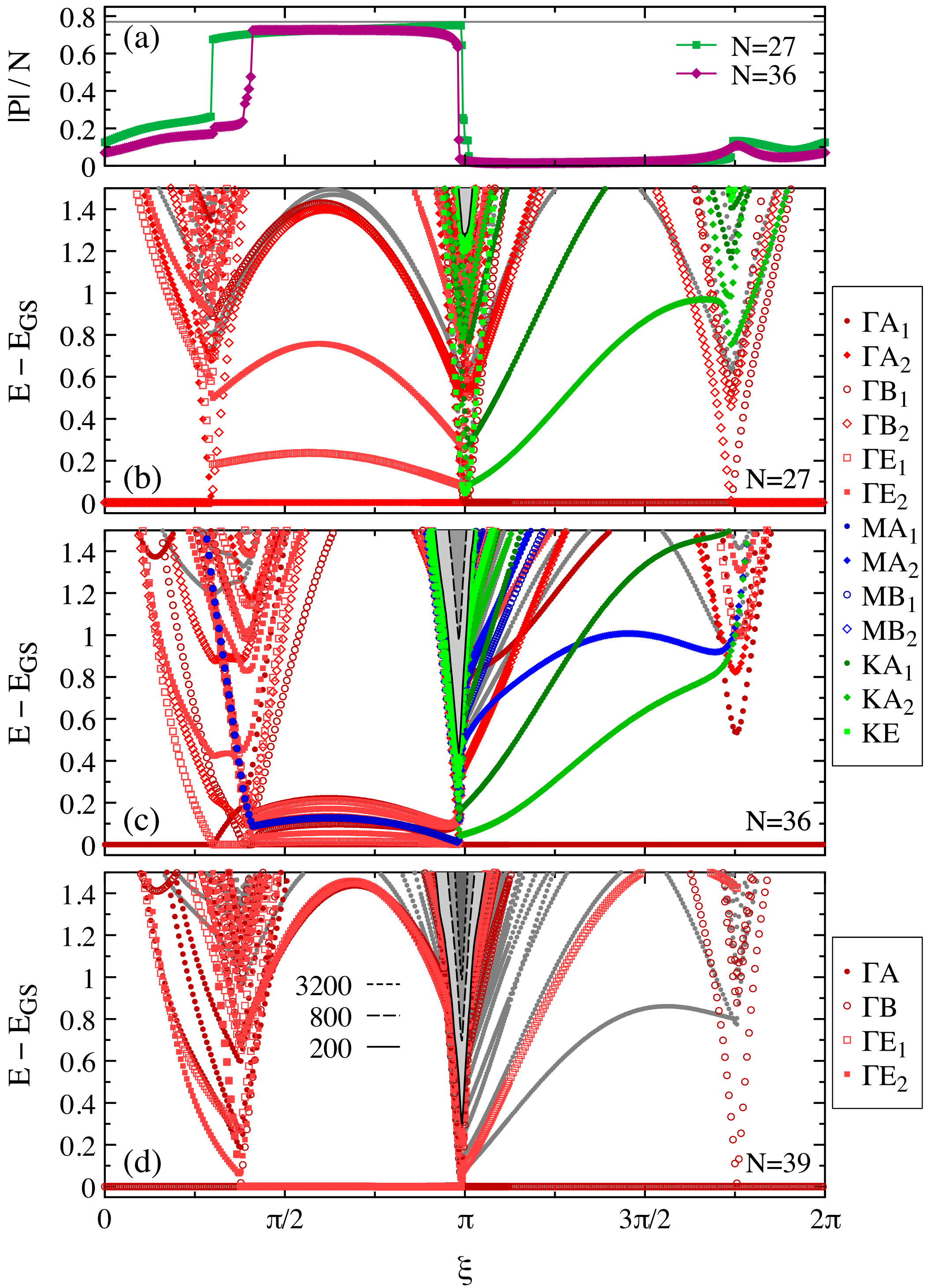


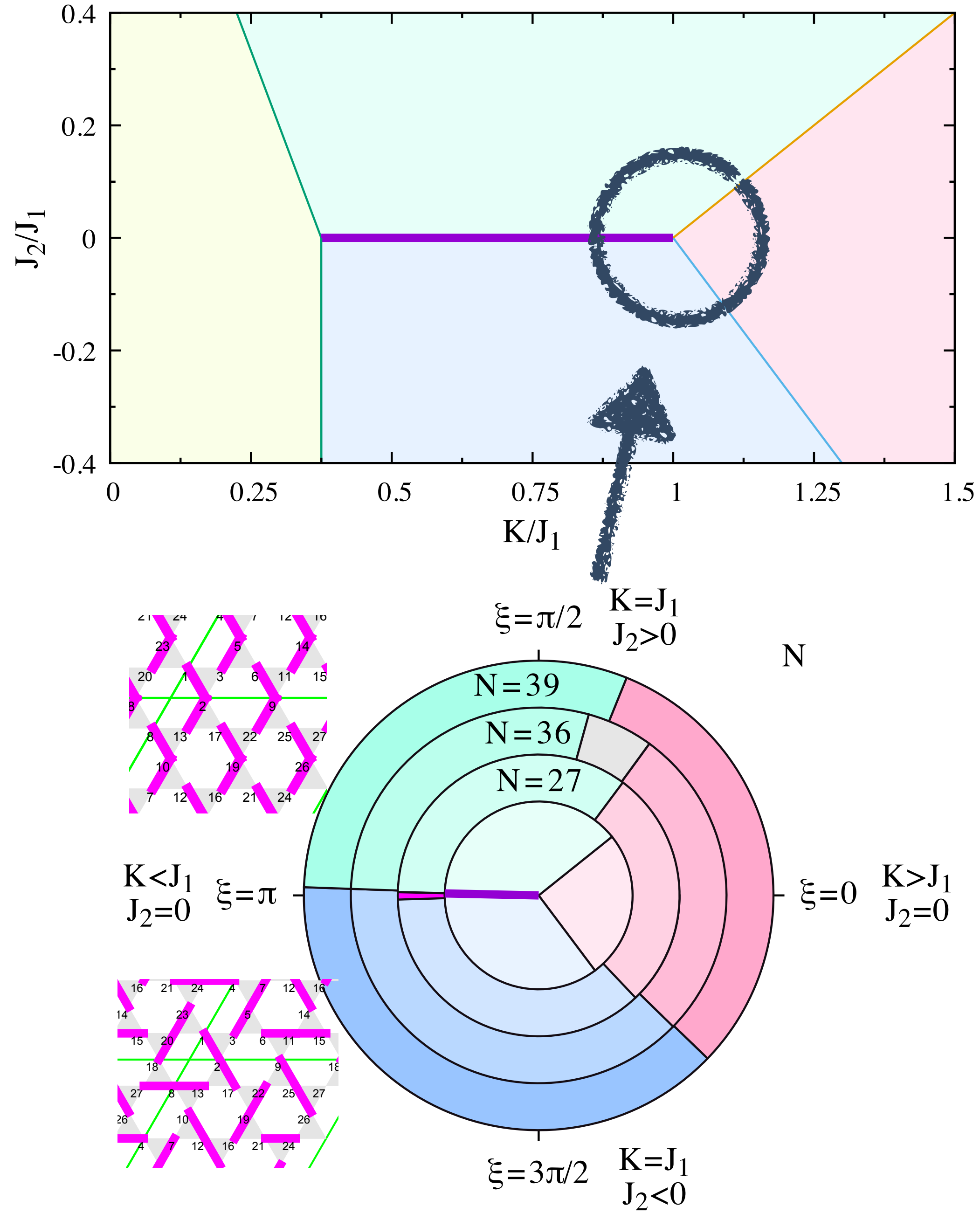
K - J2 model : comparing diagonal energies



ED in the Hilbert space spanned by singlets

$K - J_1 - J_2$ model





Conclusions

- Designed an exact AKLT-like ground state with a simple parent Hamiltonian.
- For special cases, a macroscopically large number of states become degenerate.
- Gauss law, states characterized by topological (?) quantum numbers (sectors)
- Phases emanate from a quantum multicritical point
- many open questions: Coulomb phase, fractional excitations, origin of non-singlet states,…