

QCD

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IGFAE - Santiago de Compostela

TAE (Taller de Altas Energías) - Workshop on High Energy Physics
Benasque September 2022

[Two lectures on selected topics]

Outline

Lecture 1: QCD at colliders - “*gluon multiplication*”

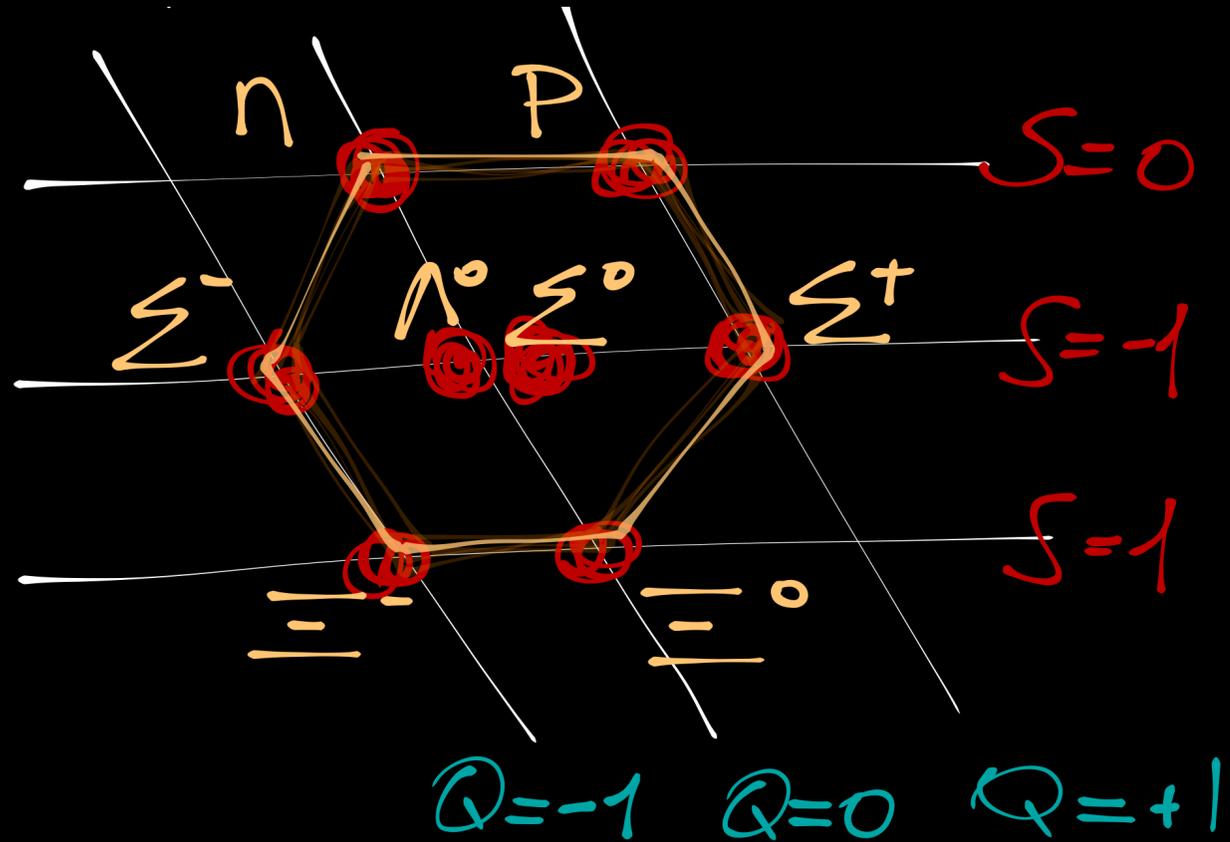
- Jets
- Parton Distribution Functions

Lecture 2: Hot and dense QCD

- The structure of matter in extraordinary conditions of temperature and density

Afternoon - two cases revealing quantum coherence

We are made of ~~stars~~ quarks



Hadrons are composite

A screenshot of the Google Play Store listing for the app "Quantum 3". The app icon shows a stylized atom with a smiling face. The text reads "Quantum 3" and "Match-3 Quantum Physics Fun". There is an "OPEN" button and a search bar at the top.

A screenshot of the "Quantum 3" game interface. At the top, it shows "Level 16" and "0" points, with a "30 BONUS" indicator. The game board is a 6x6 grid of hexagonal tiles containing quark symbols: U (up), D (down), S (strange), and B (bottom). A yellow arrow points from the app listing to the game board. At the bottom, there are three purple icons representing hadrons: a proton (p) with a 0/4 progress indicator, a neutron (n) with a 0/4 progress indicator, and a lambda baryon (Λ^0) with a 0/5 progress indicator. The game board shows a yellow circle around a group of tiles that can be swapped to form these hadrons.

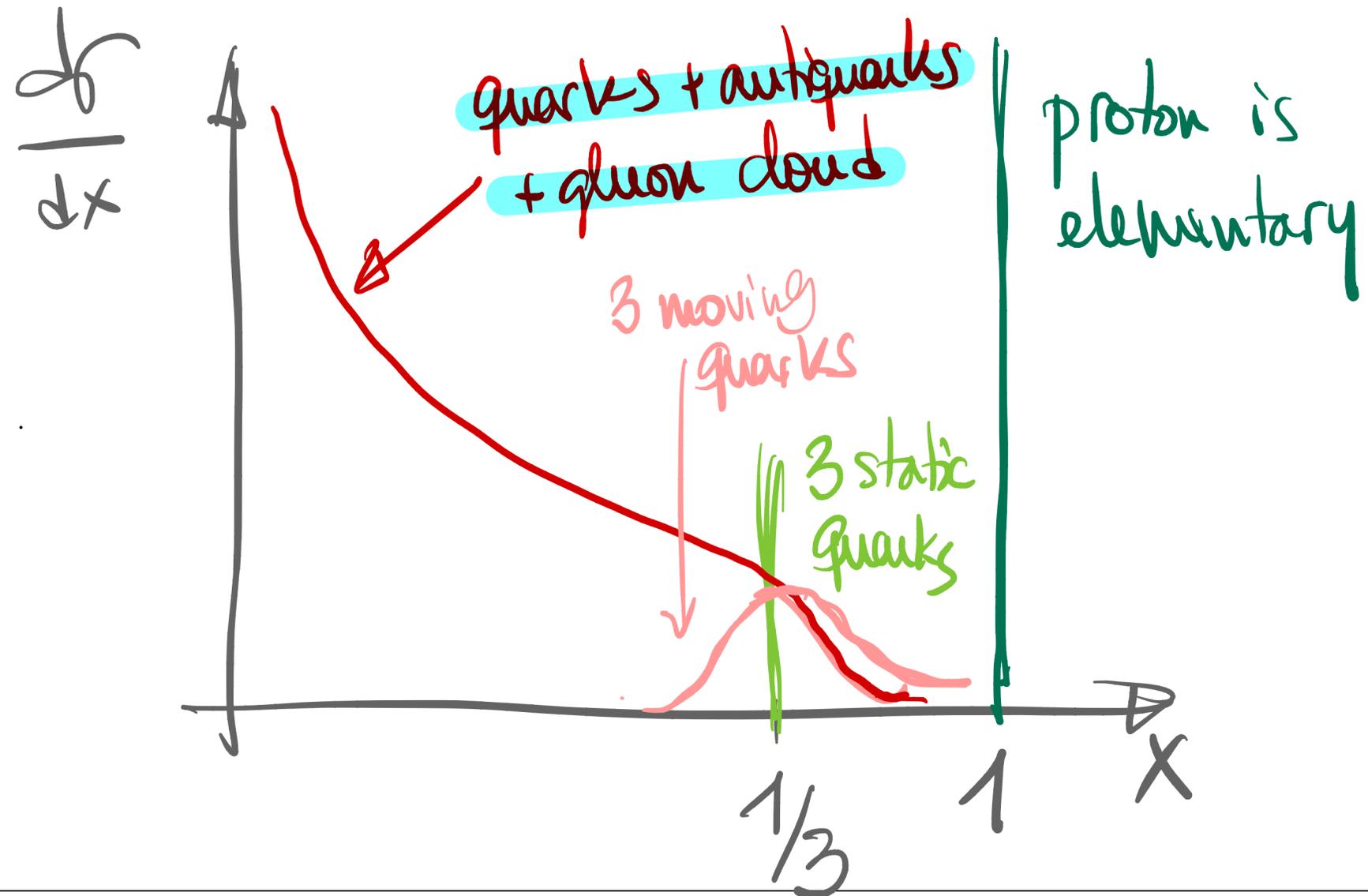
The proton structure

A proton seen in a lepton-proton scattering

Bjorken-x

$$x = \frac{Q^2}{2p \cdot q}$$

Can be written in terms of the lepton kinematics alone
[$x=1$ for elastic scattering]



QCD

QCD is the theory of the strong interaction

□ Describes hadrons and their interactions

- Asymptotic states
- Nuclear matter (us)
- Colorless objects

□ However, quarks and gluons in the Lagrangian

- Fundamental particles
- Color charge

□ To build the Lagrangian proceed as for QED - gauge theory

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$$

the color charge is
a vector

$$\mathcal{L} = \bar{\Psi}_a (i\not{\partial} - m) \Psi_a$$

Invariant under color
"rotations" \rightarrow SU(3)

$$\psi' = e^{i\alpha_a t^a} \psi$$

QCD

Color transformations with the Gell-Mann matrices $t_a = \frac{1}{2}\lambda_a$

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

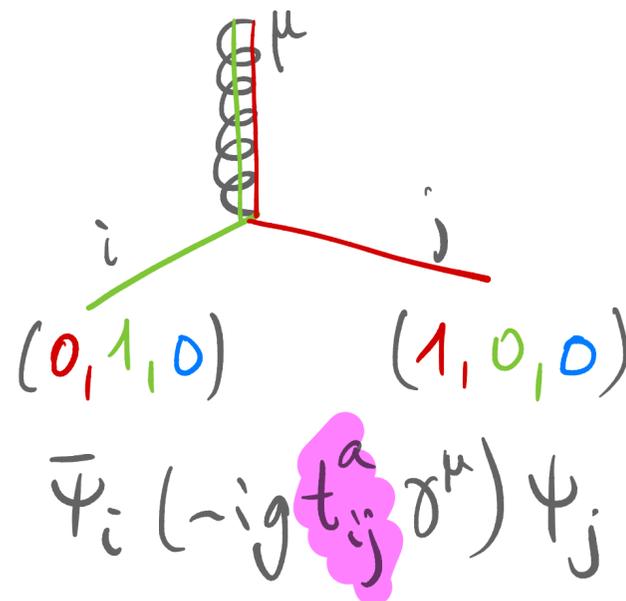
$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda^8 = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{-2}{\sqrt{3}} \end{pmatrix},$$

Quark's color - fundamental representation of SU(3)

$$[t^a, t^b] = if_{abc}t^c$$

Gluons change the color of the quark

[the corresponding vertex in QED does not change the charge of the electron]



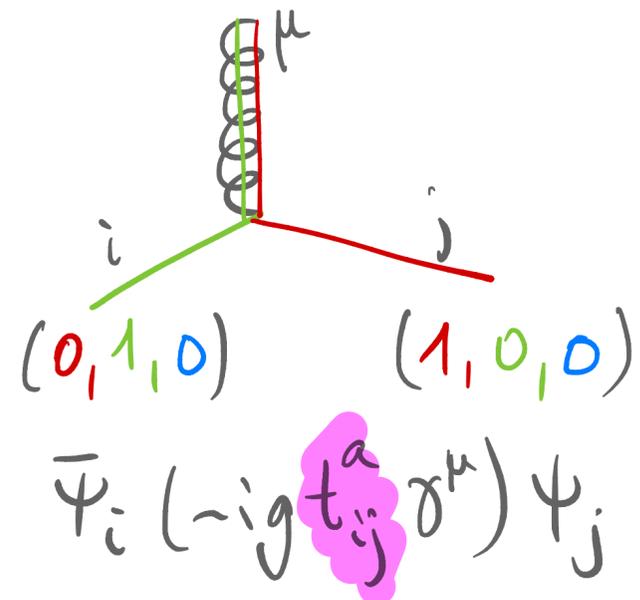
[THIS IS JUST AN EXAMPLE]

$$\underbrace{(0, 1, 0)}_{\Psi_i} \underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{t_{ij}^1} \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}_{\Psi_j}$$

QCD

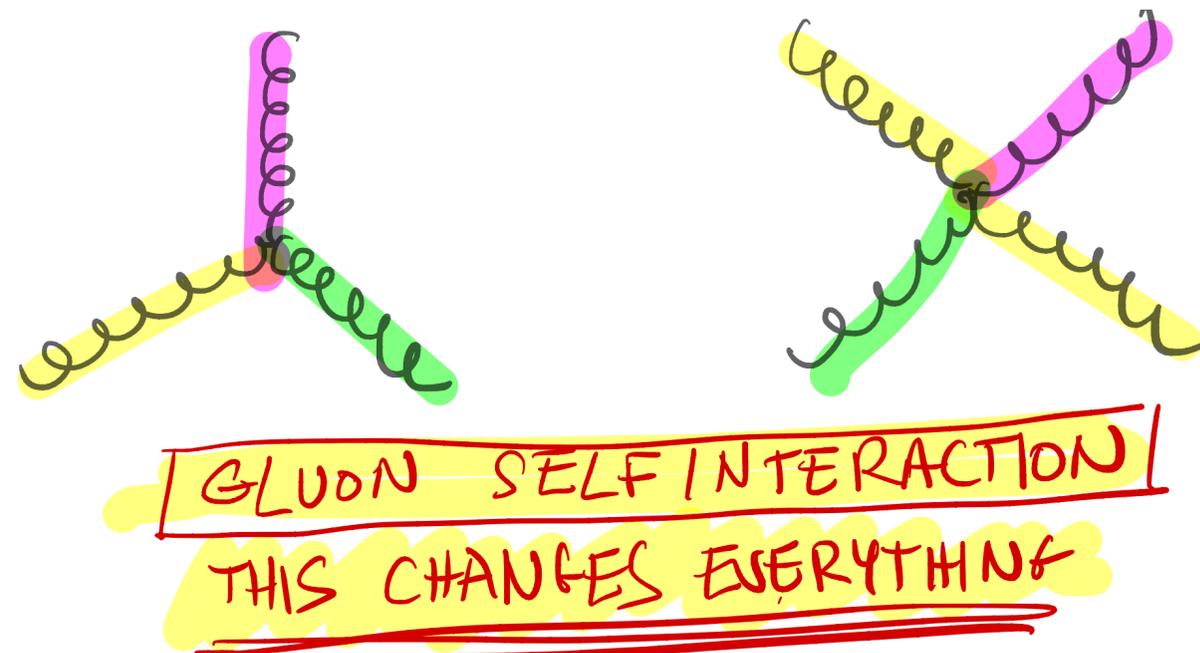
Gluons change the color of the quark ::: **gluons are colored**

[the gluons have the 8 possible colors of the adjoint representation of SU(3)]



A Feynman diagram showing a quark-gluon vertex. A vertical wavy line representing a gluon with index μ is connected to two quark lines. The left quark line is green and labeled with index i and color vector $(0, 1, 0)$. The right quark line is red and labeled with index j and color vector $(1, 0, 0)$. Below the diagram is the mathematical expression for the vertex: $\bar{\Psi}_i (-ig t_{ij}^a \gamma^\mu) \Psi_j$. The term t_{ij}^a is highlighted in pink.

This is similar to QED

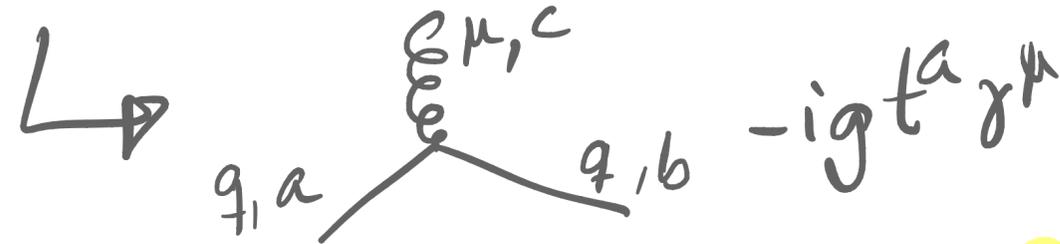


But gluon self-interaction - asymptotic freedom, confinement,...

The QCD Lagrangian

$$\mathcal{L} = \sum_q \bar{\psi}_{q,a} (i\gamma^\mu \partial_\mu \delta_{ab} - g_s \gamma^\mu t_{ab}^C A_\mu^C - m_q \delta_{ab}) \psi_{q,b} - \frac{1}{4} F_{\mu\nu}^A F^{A\mu\nu},$$

quark propag. $i \frac{\not{p}}{p^2 + i\epsilon}$



Use technology to build a gauge theory - with SU(3) group for color

$$F_{\mu\nu}^A = \partial_\mu A_\nu^A - \partial_\nu A_\mu^A - g_s f_{ABC} A_\mu^B A_\nu^C,$$

ABSENT IN QED

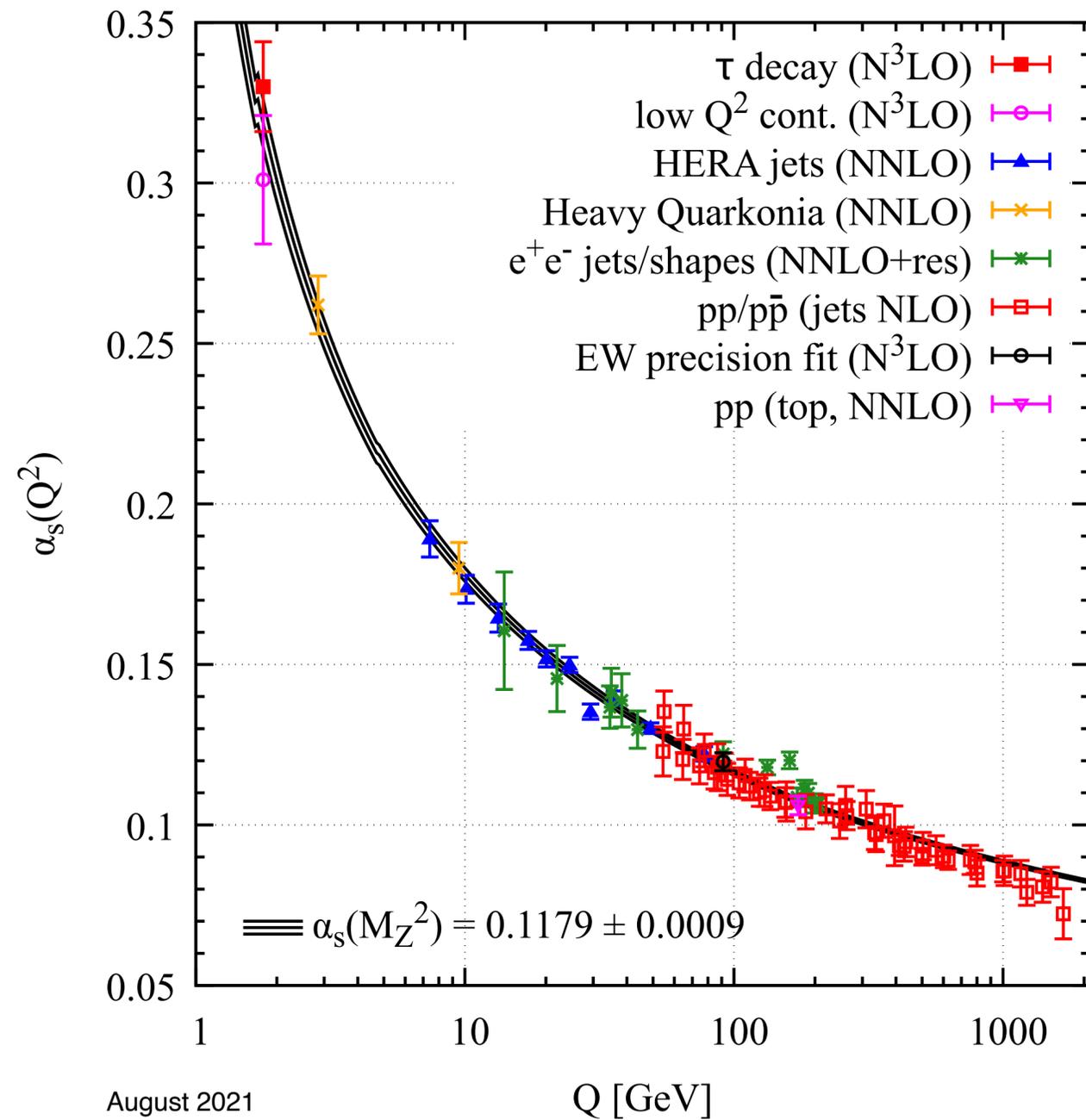
The non-abelian nature of SU(3) - non-linear terms in $F^{\mu\nu}$

$$F_{\mu\nu}^A F^{A\mu\nu}$$

- Gluon kinetic term A^2 - propagator
- 3-gluon vertex A^3
- 4-gluon vertex A^4



Asymptotic freedom



[Particle Data Group - QCD 2021]

$$\mu_R^2 \frac{d\alpha_s}{d\mu_R^2} = \beta(\alpha_s) = -(b_0\alpha_s^2 + b_1\alpha_s^3 + b_2\alpha_s^4 + \dots)$$

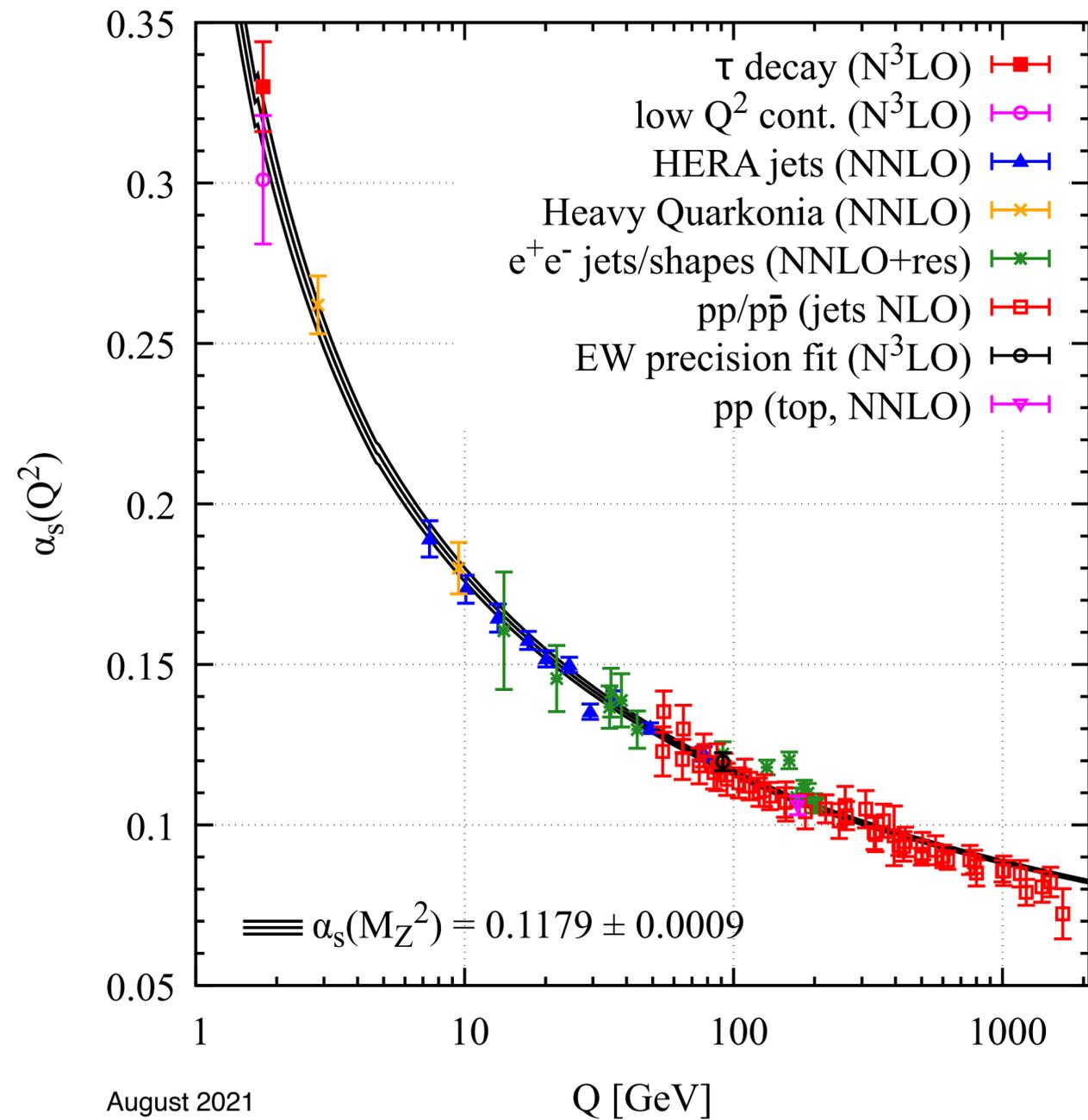
Take $b_1 = b_2 = \dots = 0$

$$\frac{d\alpha_s}{d\log\mu_R} = -b_0\alpha_s$$

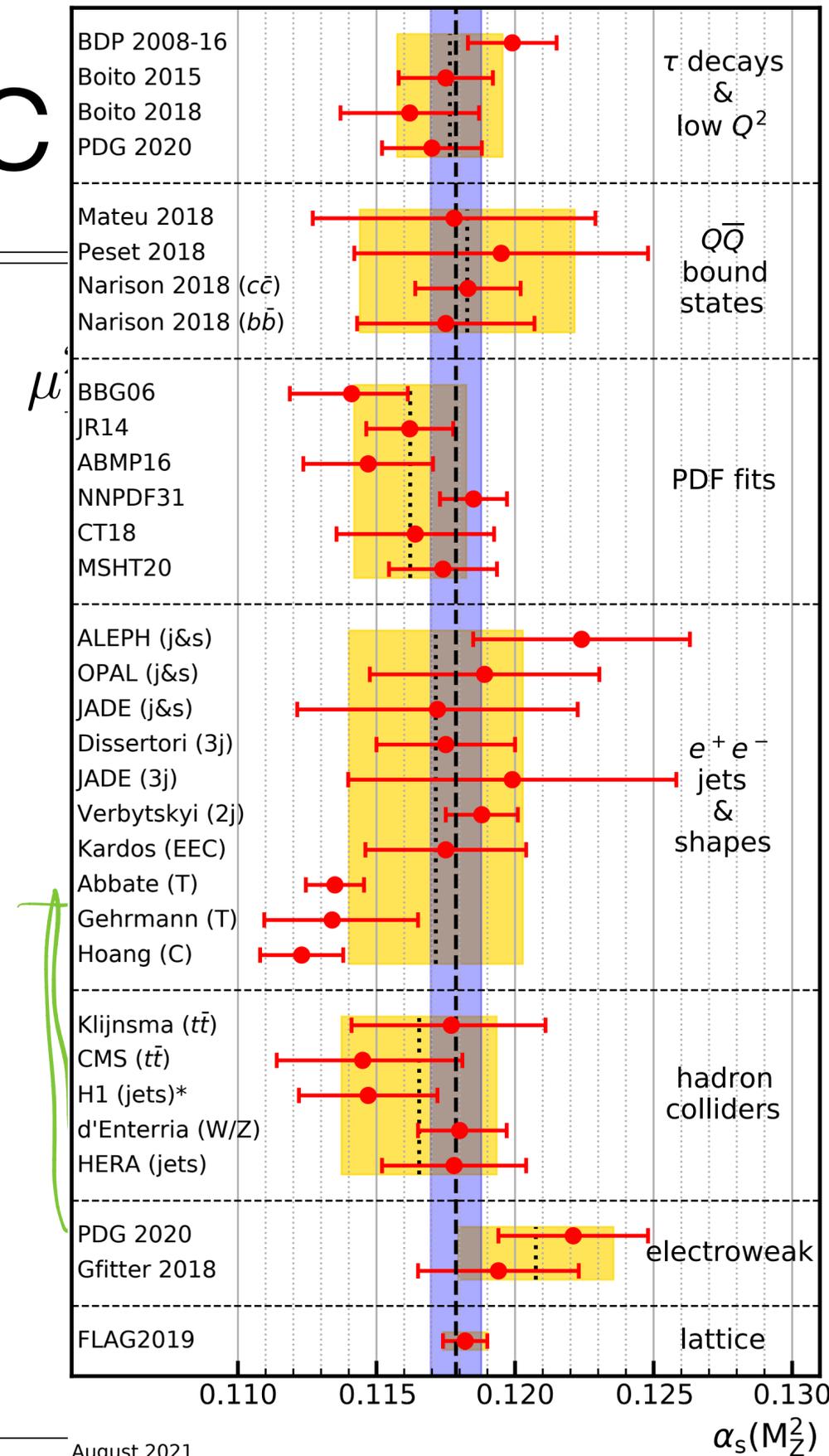
$$\alpha_s(\mu^2) = \frac{\alpha_s(\mu_0^2)}{1 + b_0\alpha_s(\mu_0^2)\log\frac{Q^2}{\mu_0^2}} \equiv \frac{1}{b_0\log\frac{Q^2}{\Lambda_{\text{QCD}}^2}}$$

$$\alpha_s \ll 1 \text{ for } Q^2 \gg \Lambda_{\text{QCD}}^2$$

Asymptotic



[Particle Data Group - QCD 2021]



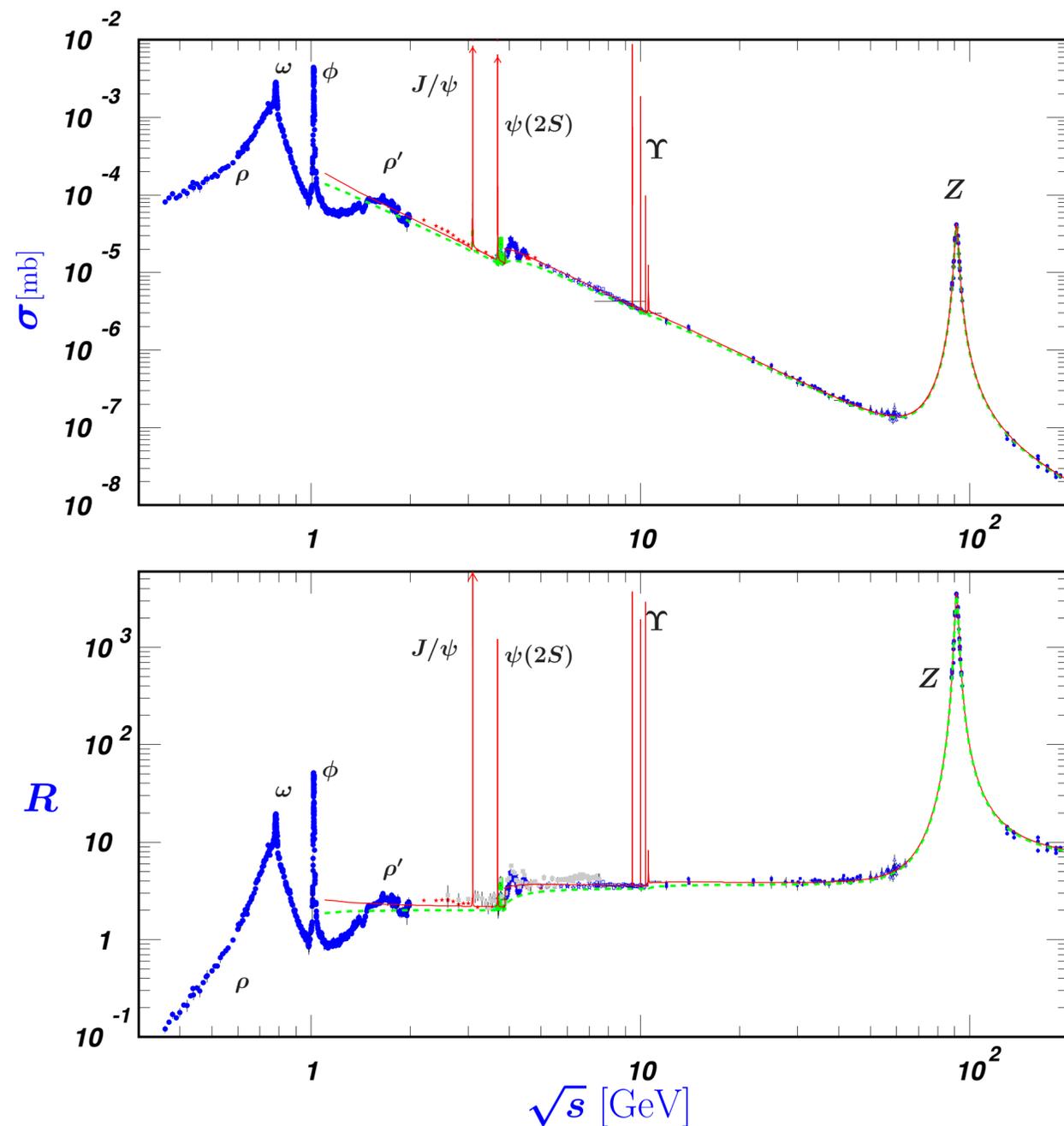
$$\alpha_s^3 + b_2 \alpha_s^4 + \dots$$

1

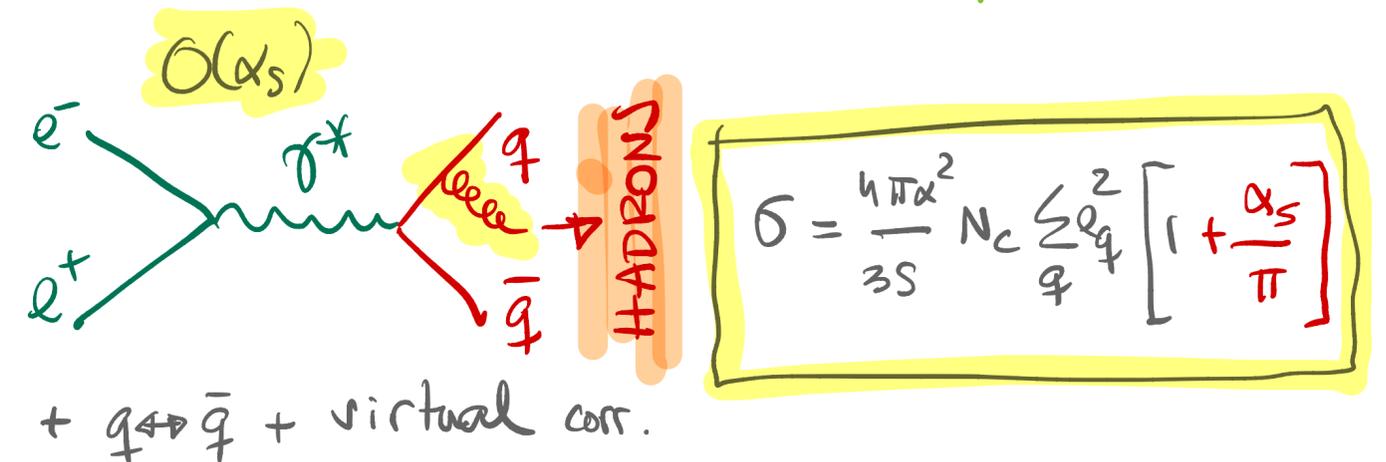
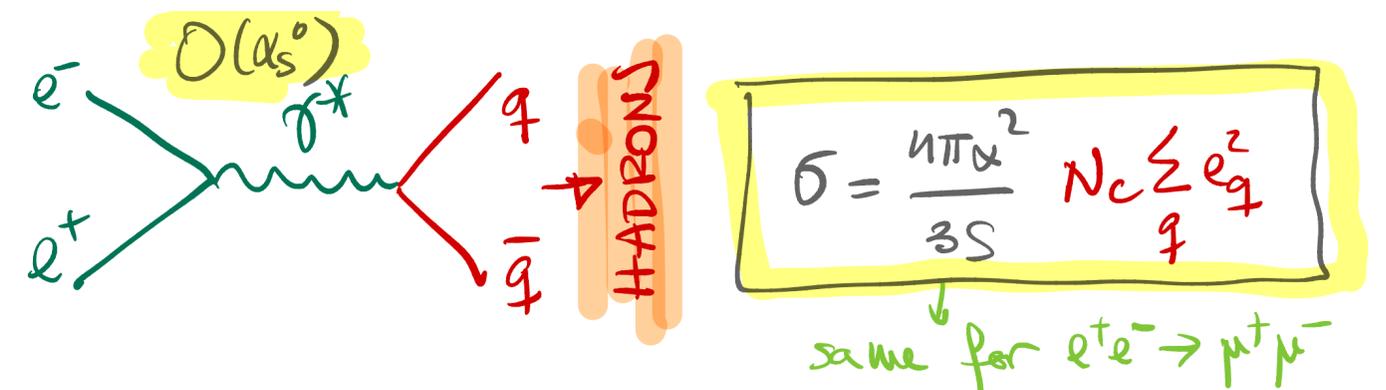
$$b_0 \log \frac{Q^2}{\Lambda_{QCD}^2}$$

2

QCD perturbative at large scales



Example: e^+e^- annihilation into hadrons

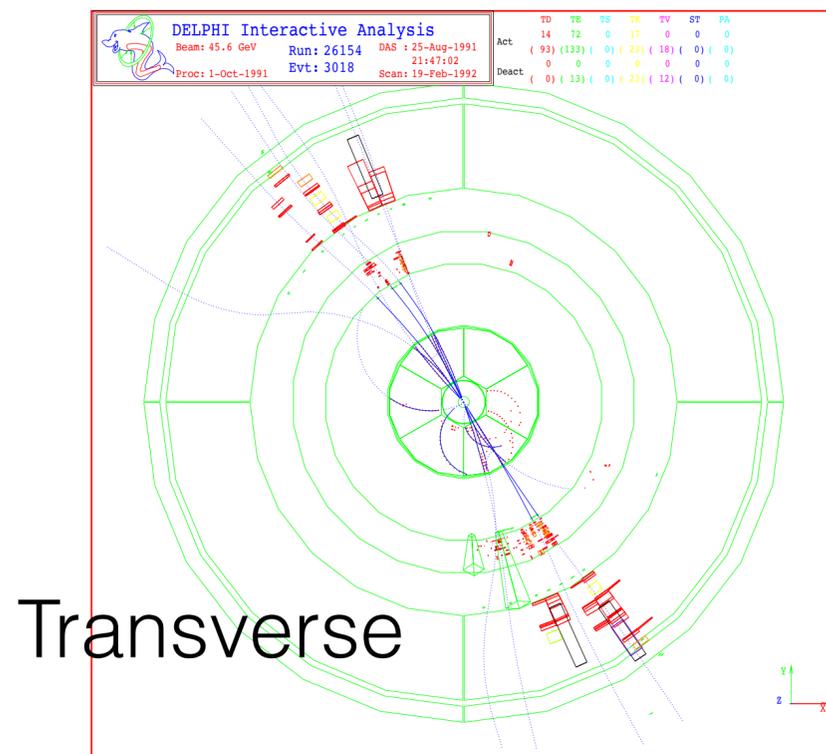


$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = N_c \sum_q e_q^2 \left[1 + \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2)\right]$$

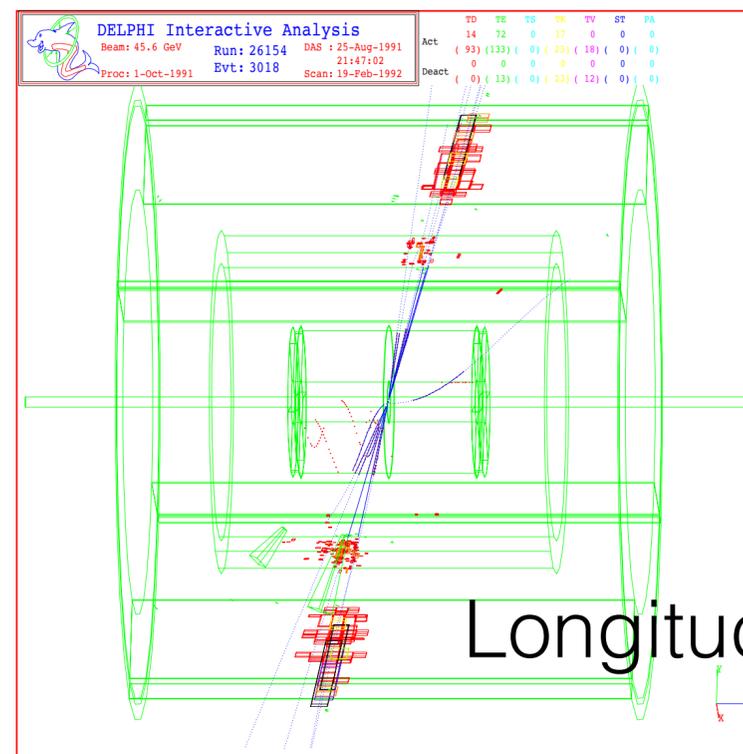
Less inclusive now...

What if, e.g., we want to know the angular distribution? **JETS**

- Hadronization - long distance / non perturbative
- Gloun multiplication**

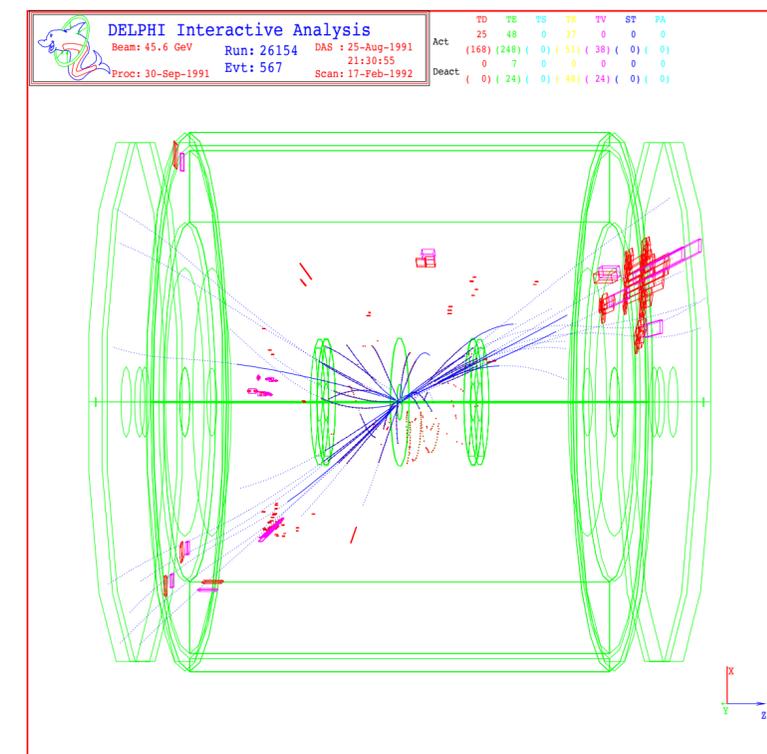


Transverse



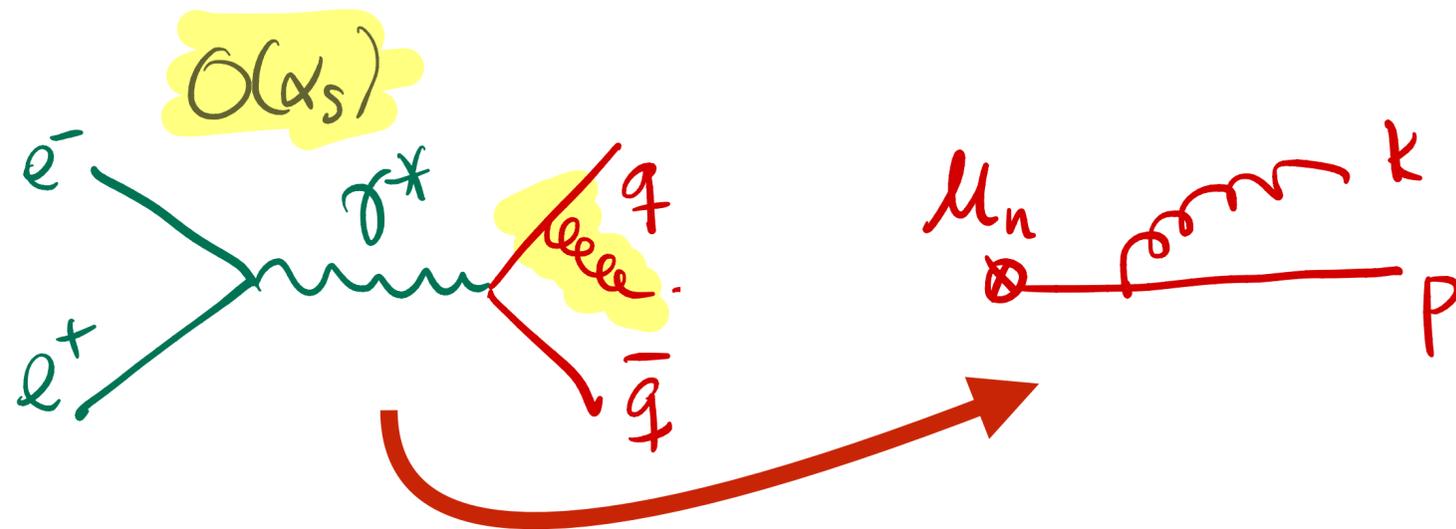
Longitudinal

A 2-jet event at LEP



A 3-jet event at LEP

Gluon (and quark) multiplication



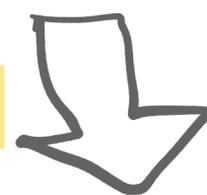
Soft radiation

FOR QUARKS $C_F = \frac{2N_c}{N_c^2 - 1} = \frac{3}{4}$

FOR GLUONS $N_c = 3$

$$\mu_{n+1} = \bar{u}(p) i g t^a \not{\epsilon}(k) \frac{\not{p} + \not{k}}{(p+k)^2} \mu_n$$

[Exercise]



SOFT RADIATION LIMIT

$\omega \ll E$

$$\mu_{n+1} = -g t^a \frac{p \cdot \epsilon}{p \cdot k} \bar{u}(p) \mu_n$$

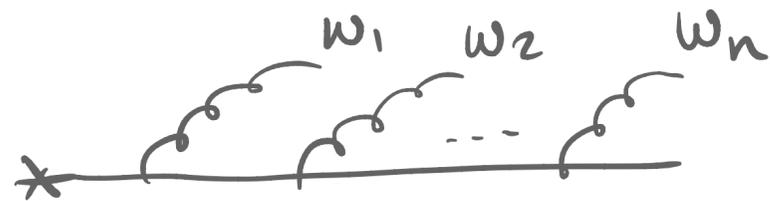
$$\frac{d\sigma^{n+1}/d\Phi^{n+1}}{d\sigma^n/d\Phi^n}$$

$$\equiv \omega \frac{dN}{d\omega d^2k_\perp} = C_F \frac{\alpha_s}{\pi^2} \frac{1}{k_\perp^2}$$

$$\frac{dN}{d\omega d\theta} = \frac{2\alpha_s C_F}{\pi} \frac{1}{\omega} \frac{1}{\theta}$$

Soft and collinear divergent

Gluon (and quark) multiplication

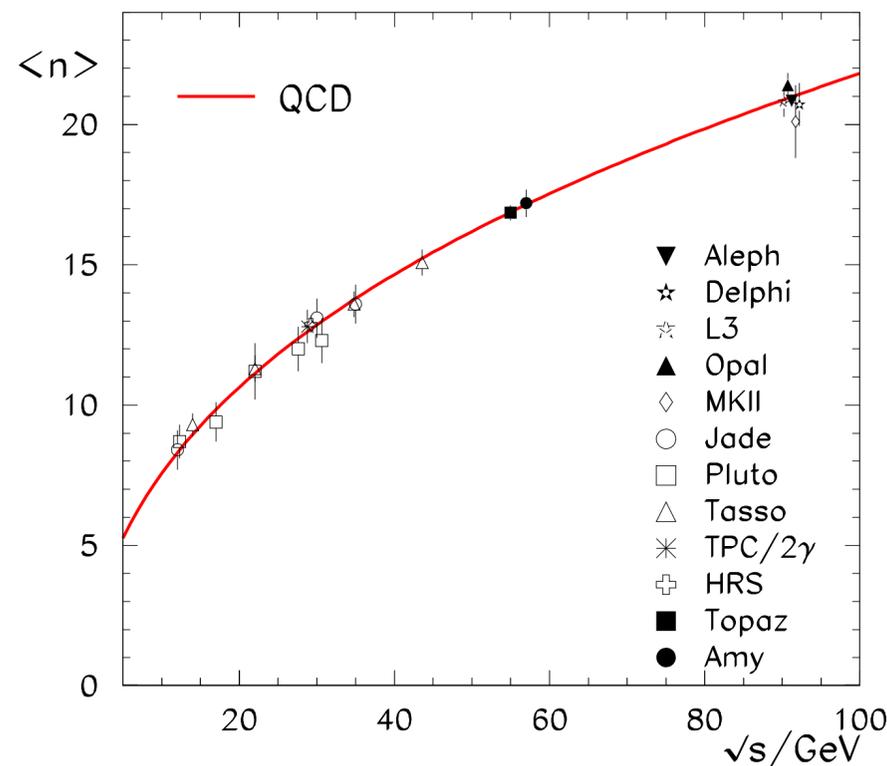


→ small- ω , small- θ
 compensate small- α_s

Counting gluons (naively) $\frac{1}{n!} \int_{Q_0}^E \frac{d\omega_1}{\omega_1} \int_{Q_0}^{\omega_1} \frac{dk_1}{k_1} \times \dots \times \int_{Q_0}^E \frac{d\omega_n}{\omega_n} \int_{Q_0}^{\omega_n} \frac{dk_n}{k_n}$

For large $\alpha_s \log \omega \log \theta$

Exponential growth

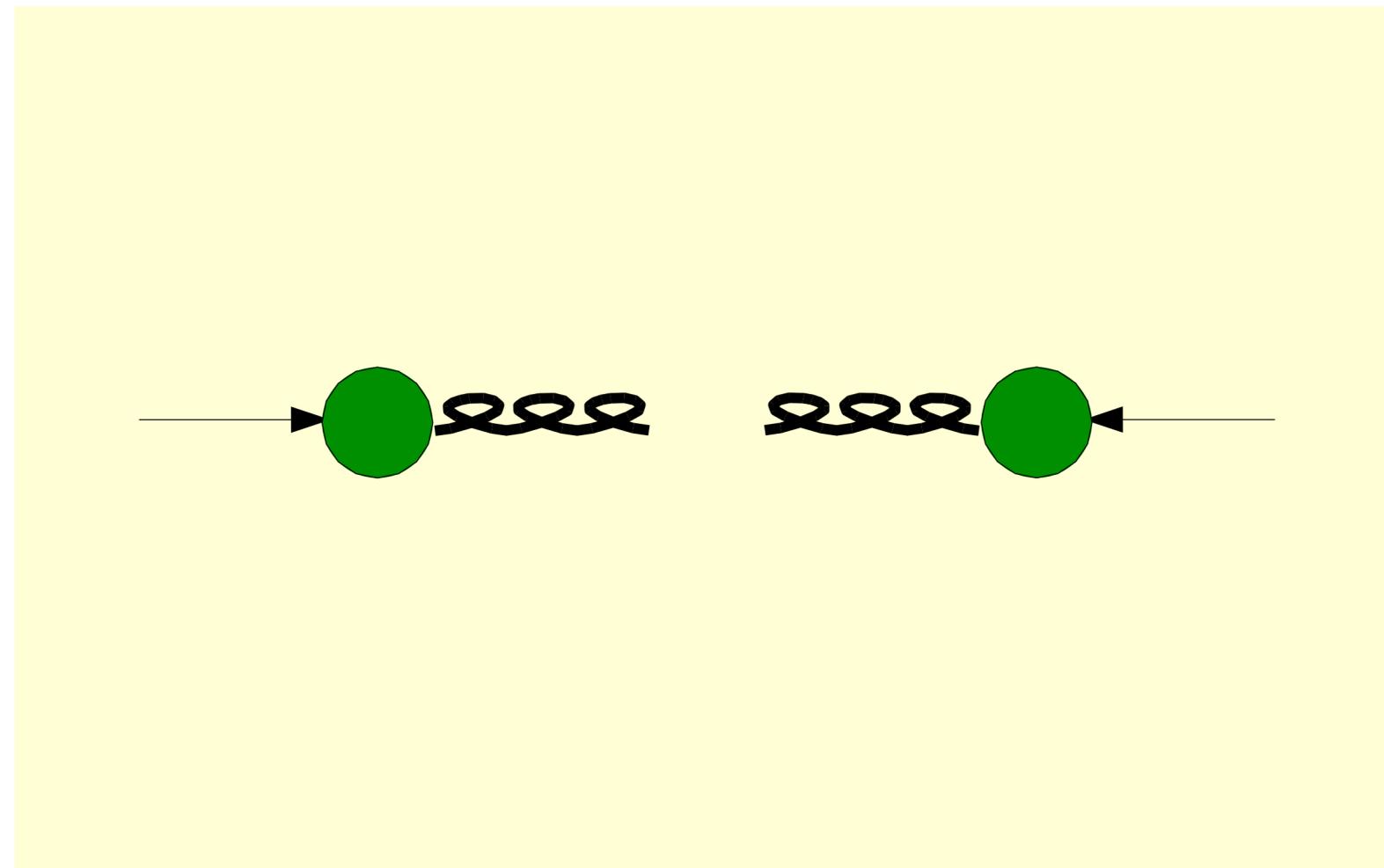


Resummation needed - **gluon multiplication**

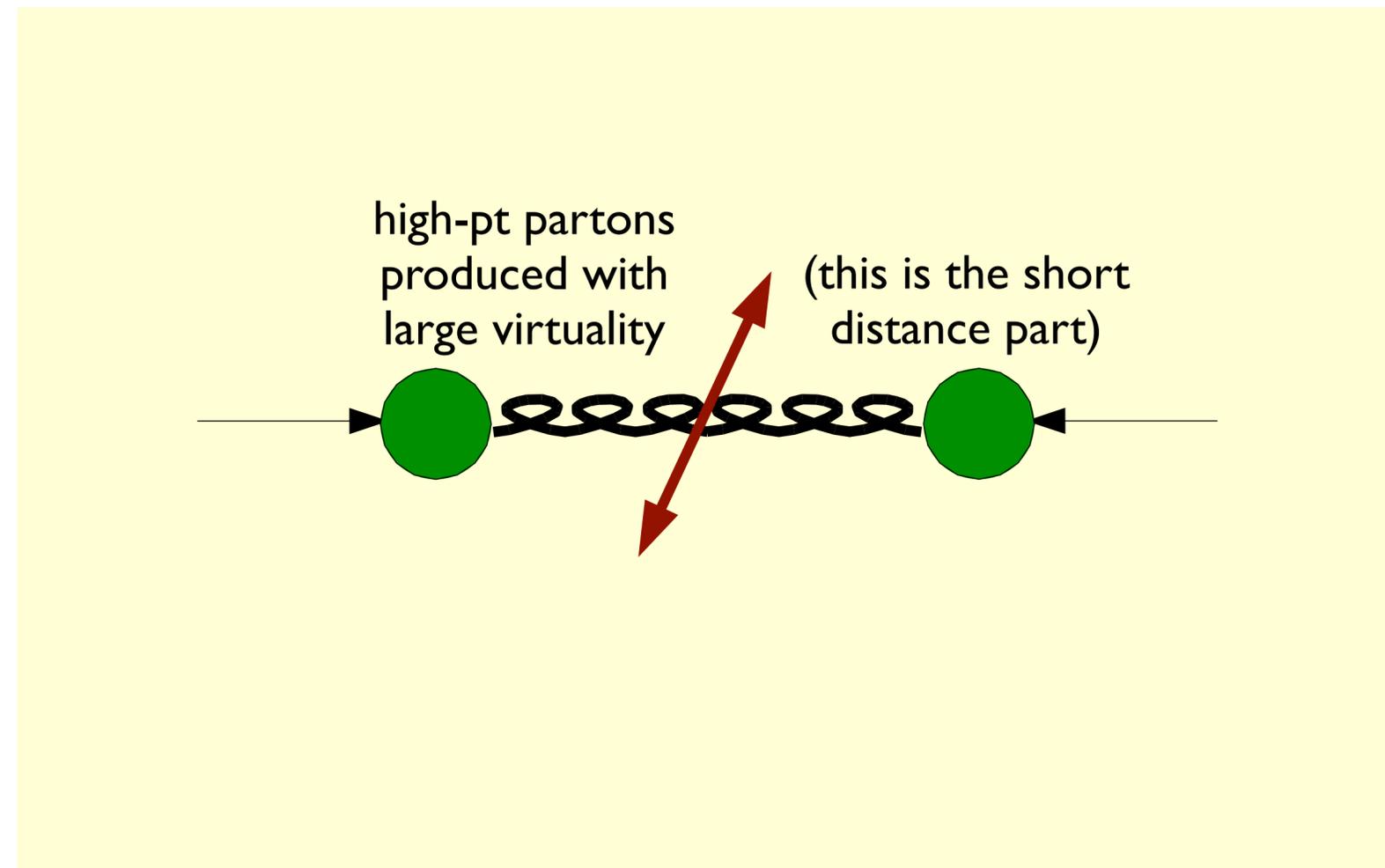
$$\langle N_g \rangle \sim \frac{C_F}{C_A} \sum_{n=1}^{\infty} \frac{1}{(n!)^2} \left(\frac{C_A}{2\pi b^2 \alpha_s} \right)^n \sim \frac{C_F}{C_A} \exp \left(\sqrt{\frac{2C_A}{\pi b^2 \alpha_s(Q)}} \right),$$

G. Salam CERN Yellow Rep. School Proc. 5 (2020) 1-56

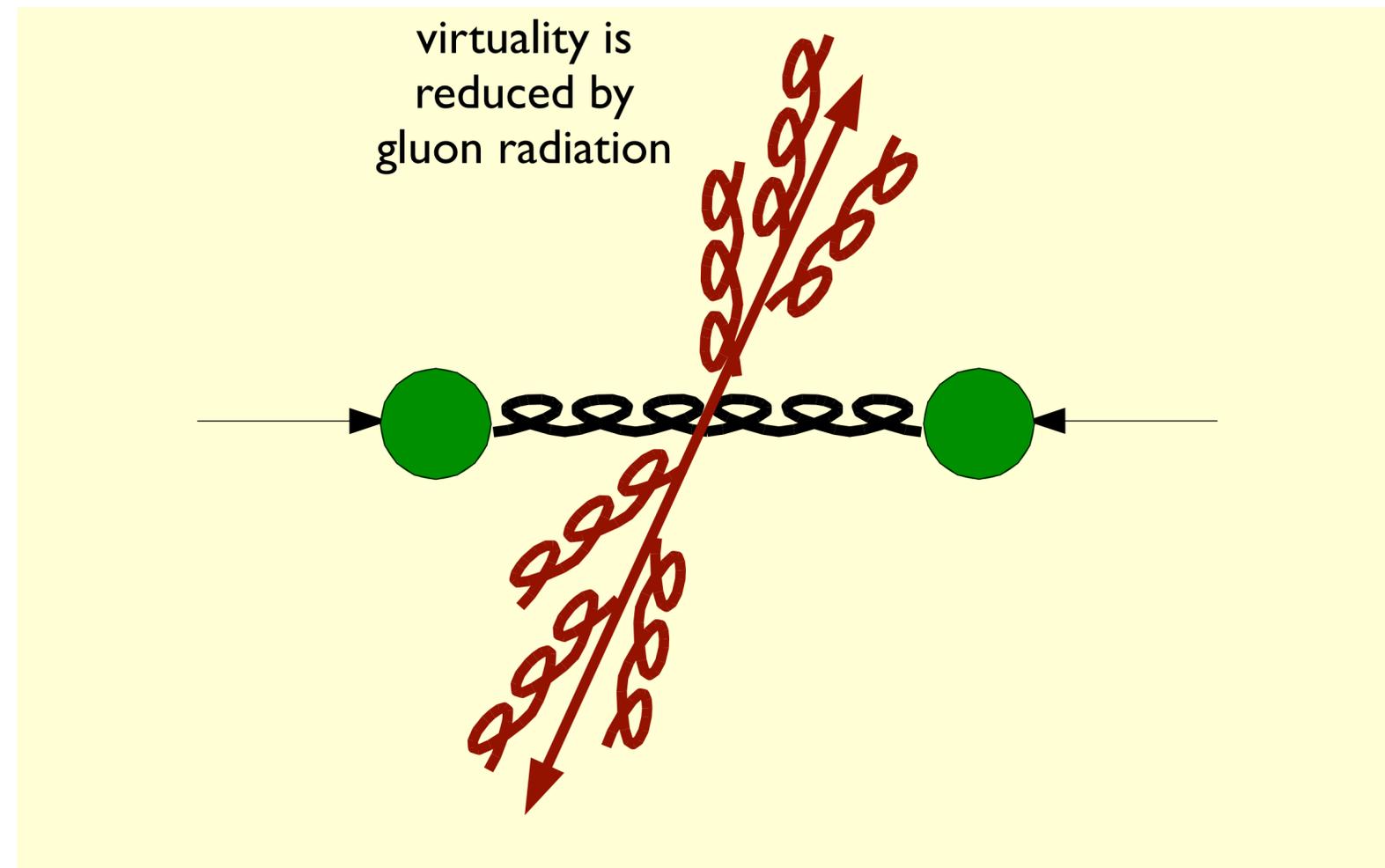
Jets: simple picture



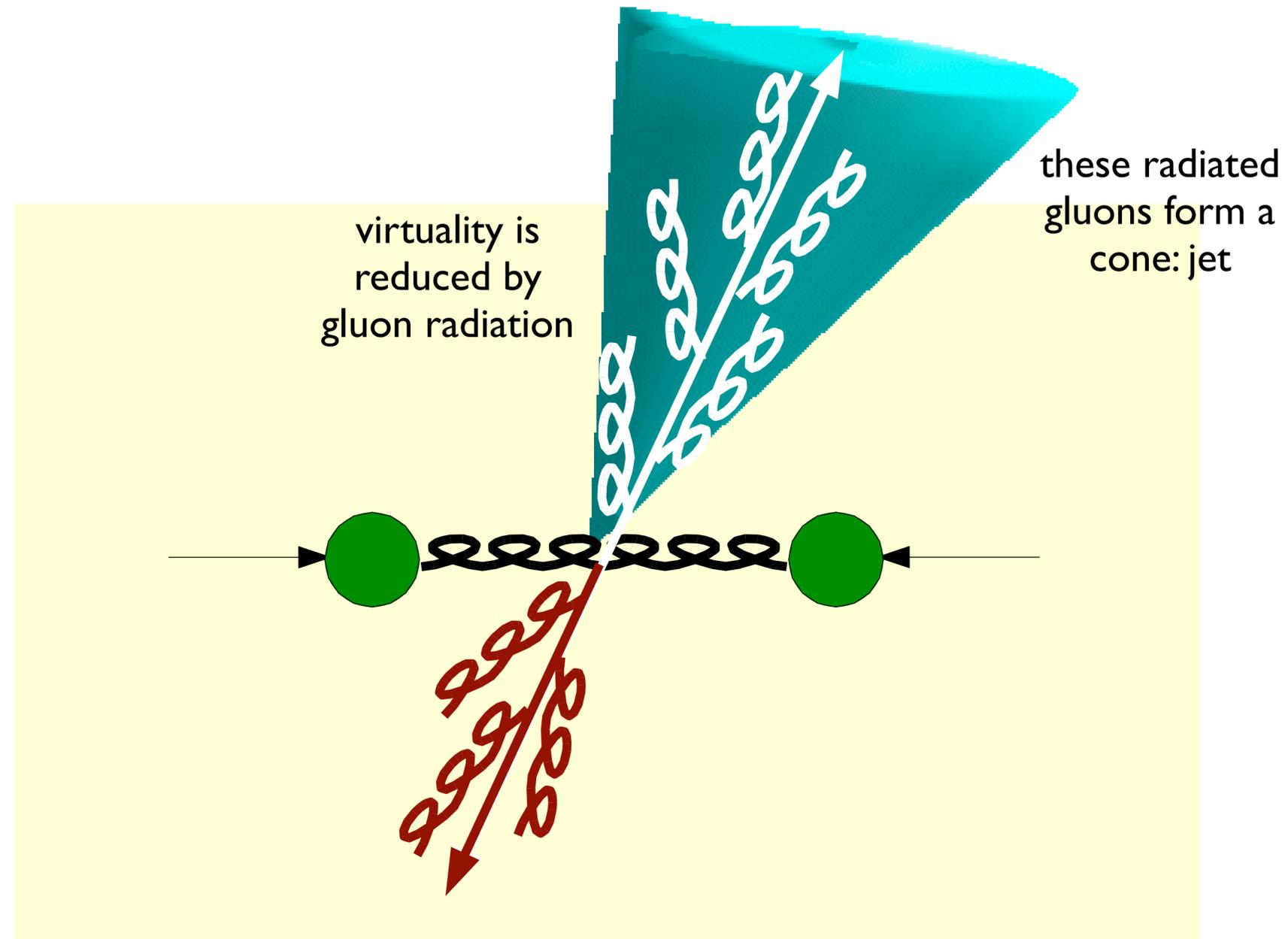
Jets: simple picture



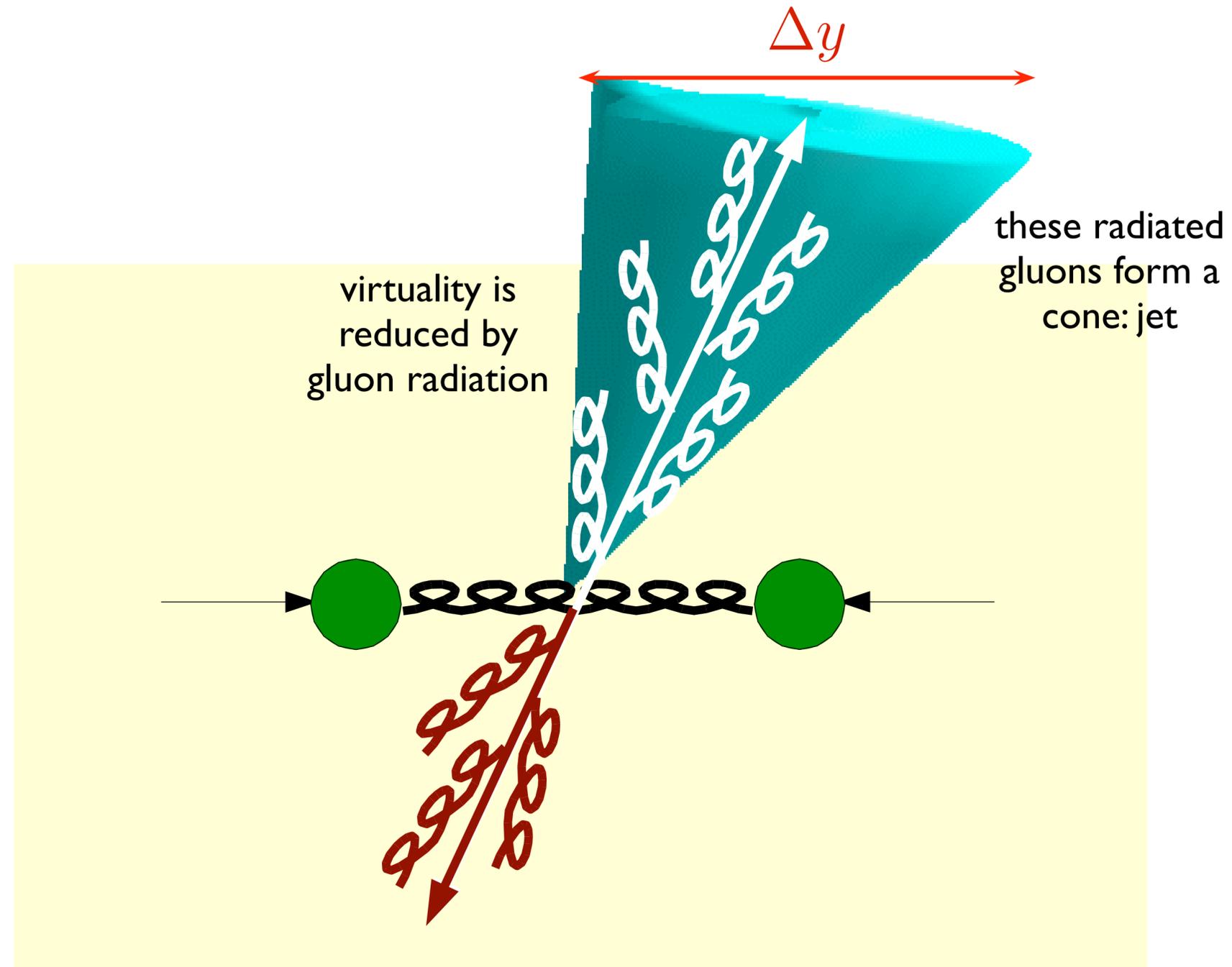
Jets: simple picture



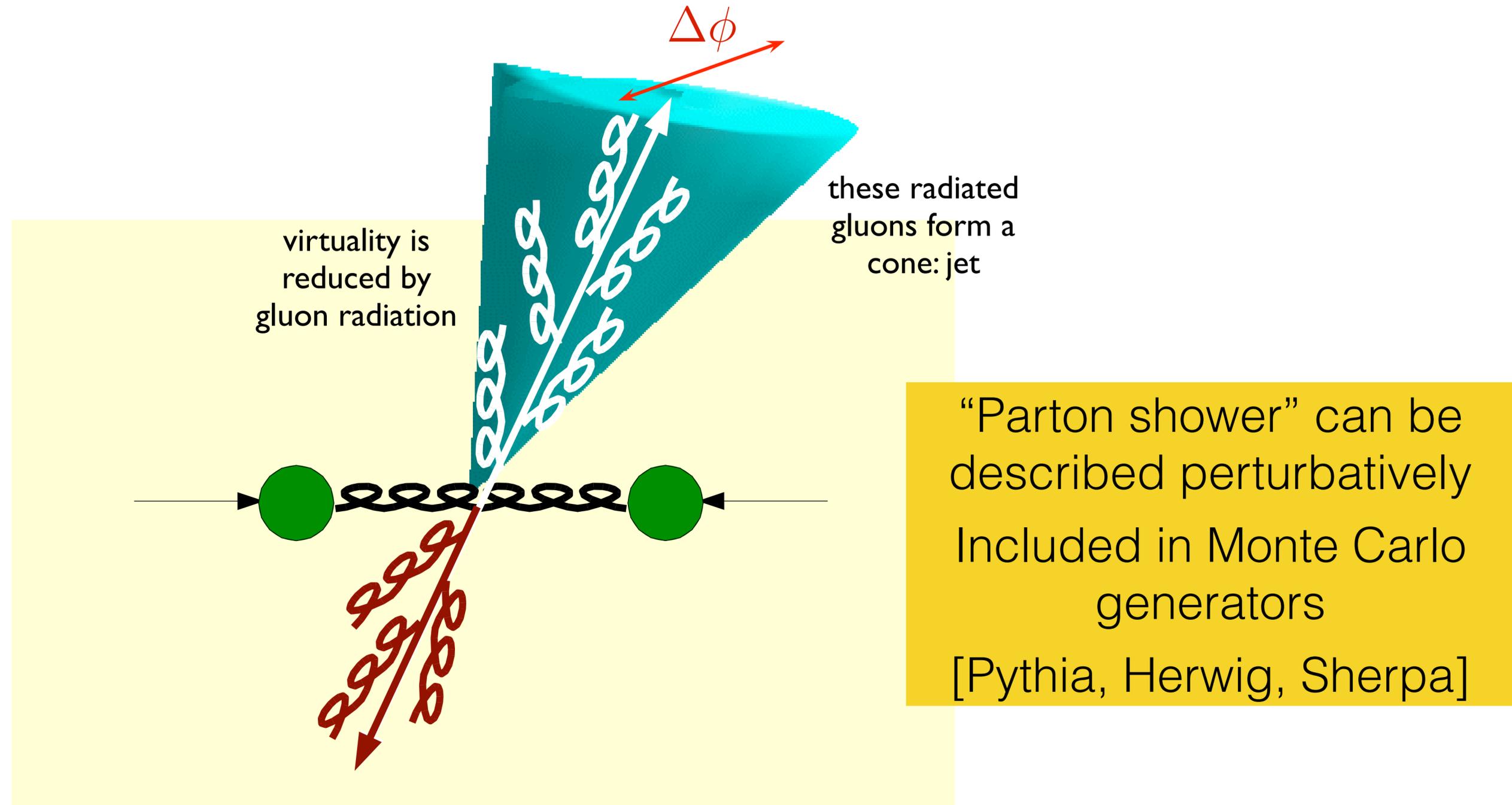
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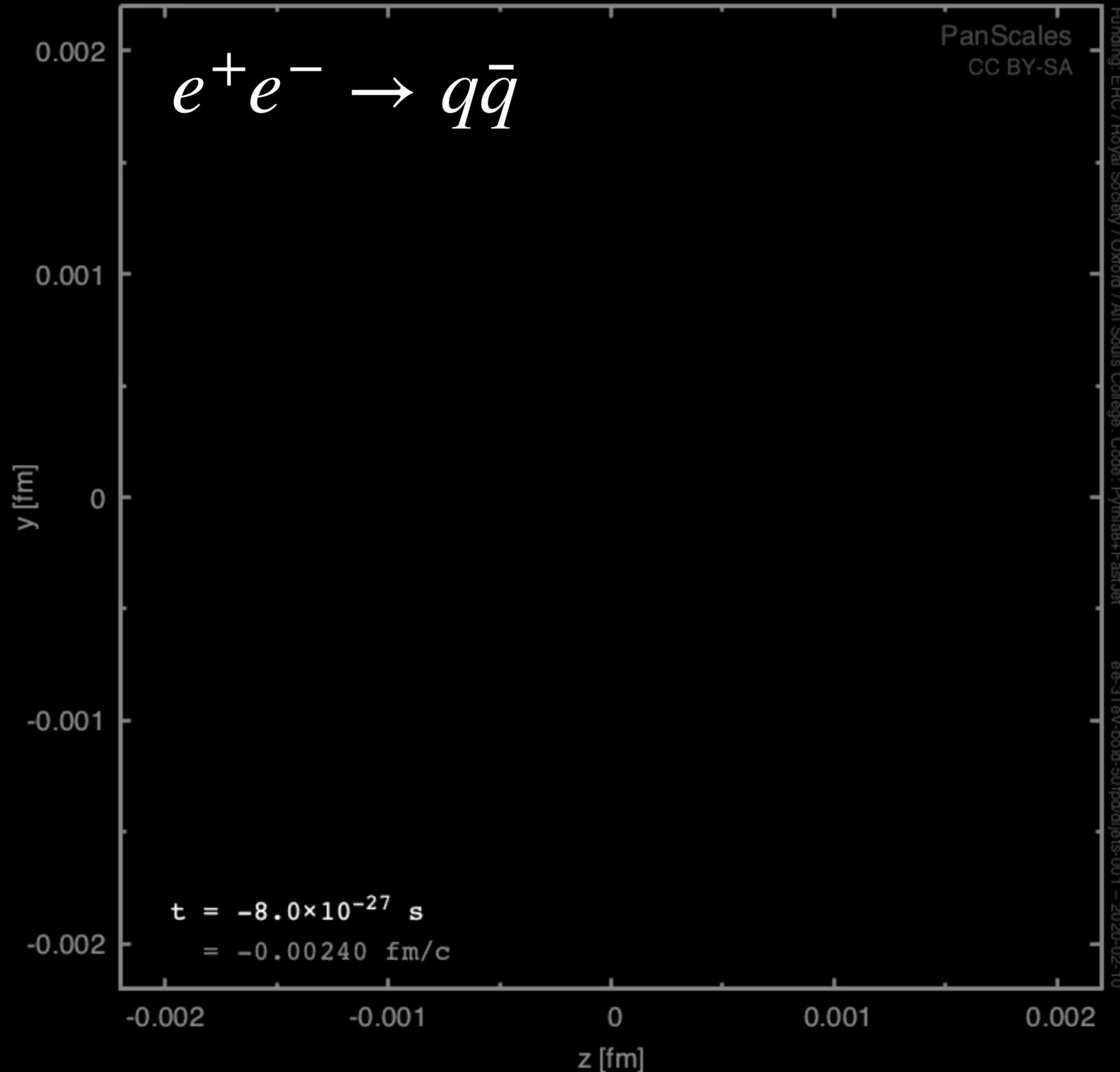


Jets: simple picture



Jets: simple picture

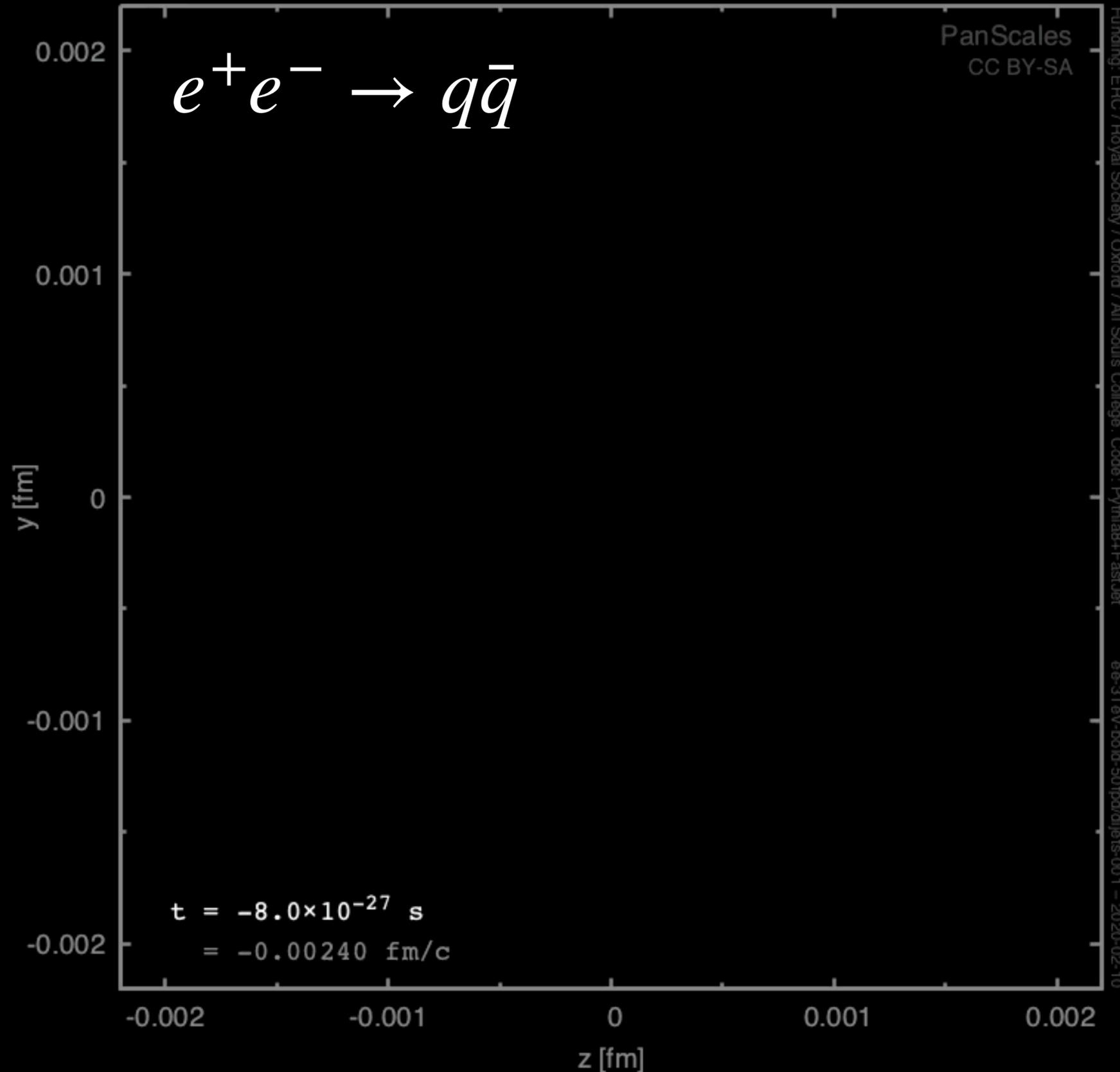




3TeV e^+e^- events

Initial particles in yellow
Intermediate particles in blue
Final particles in red

[Simulation of the events are produced with Pythia 8 times estimated by clustering algorithm - see details in the web page]



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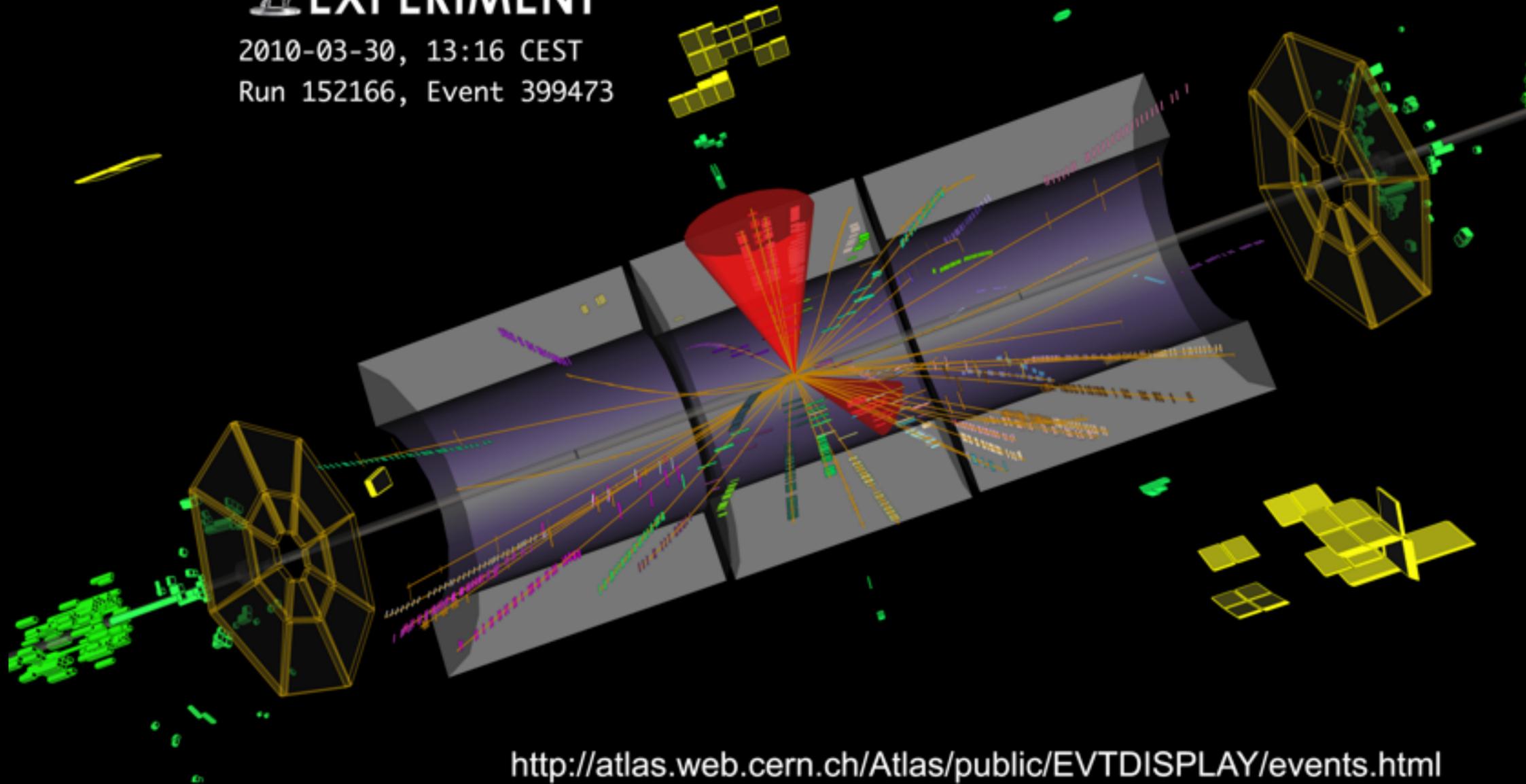
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Jets in hadronic colliders

 **ATLAS**
EXPERIMENT

2010-03-30, 13:16 CEST
Run 152166, Event 399473

2-Jet Collision Event at 7 TeV



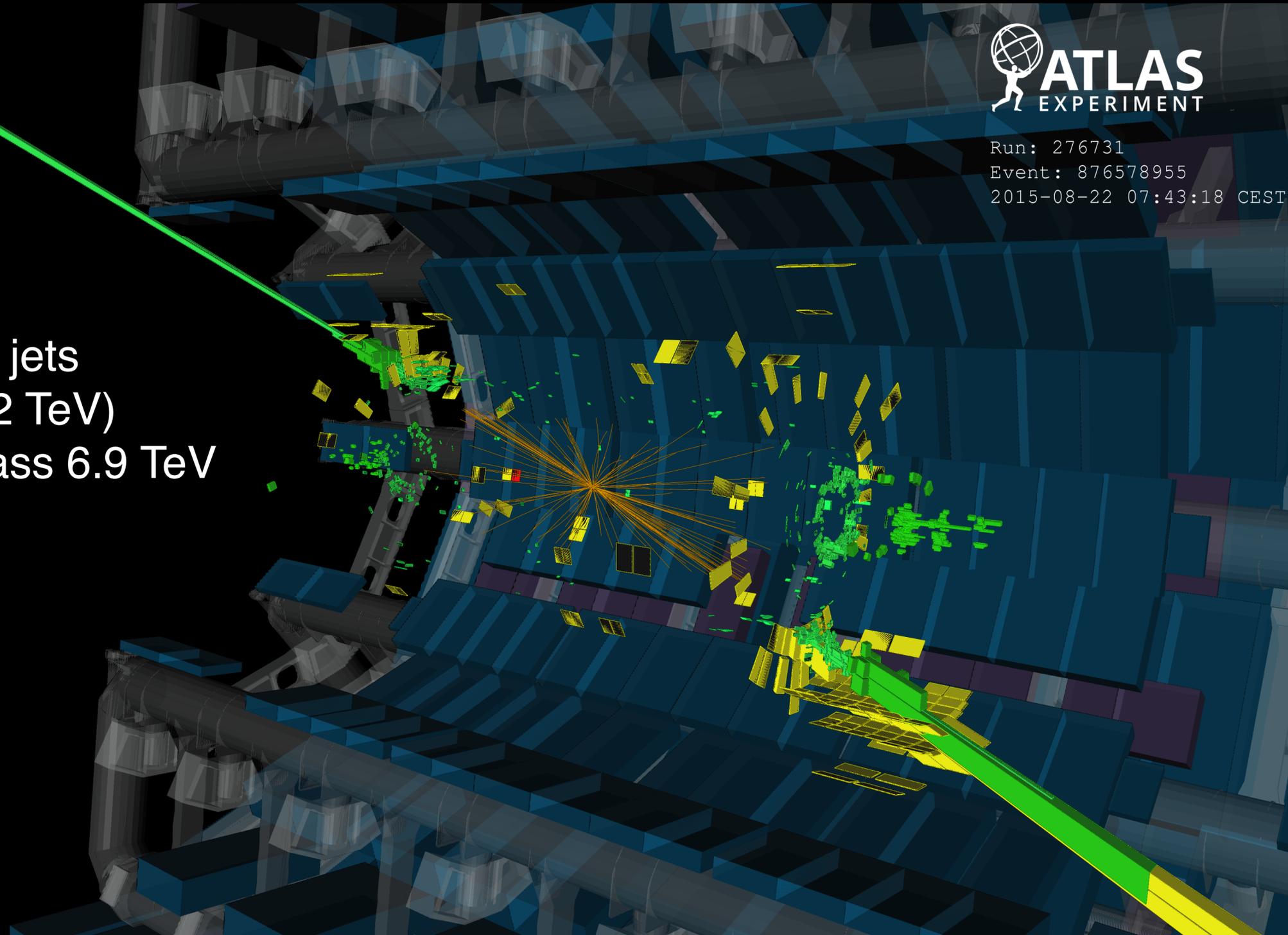
<http://atlas.web.cern.ch/Atlas/public/EVTDISPLAY/events.html>

Jets in hadronic colliders



Run: 276731
Event: 876578955
2015-08-22 07:43:18 CEST

2 high p_T jets
(1.3 and 1.2 TeV)
with invariant mass 6.9 TeV



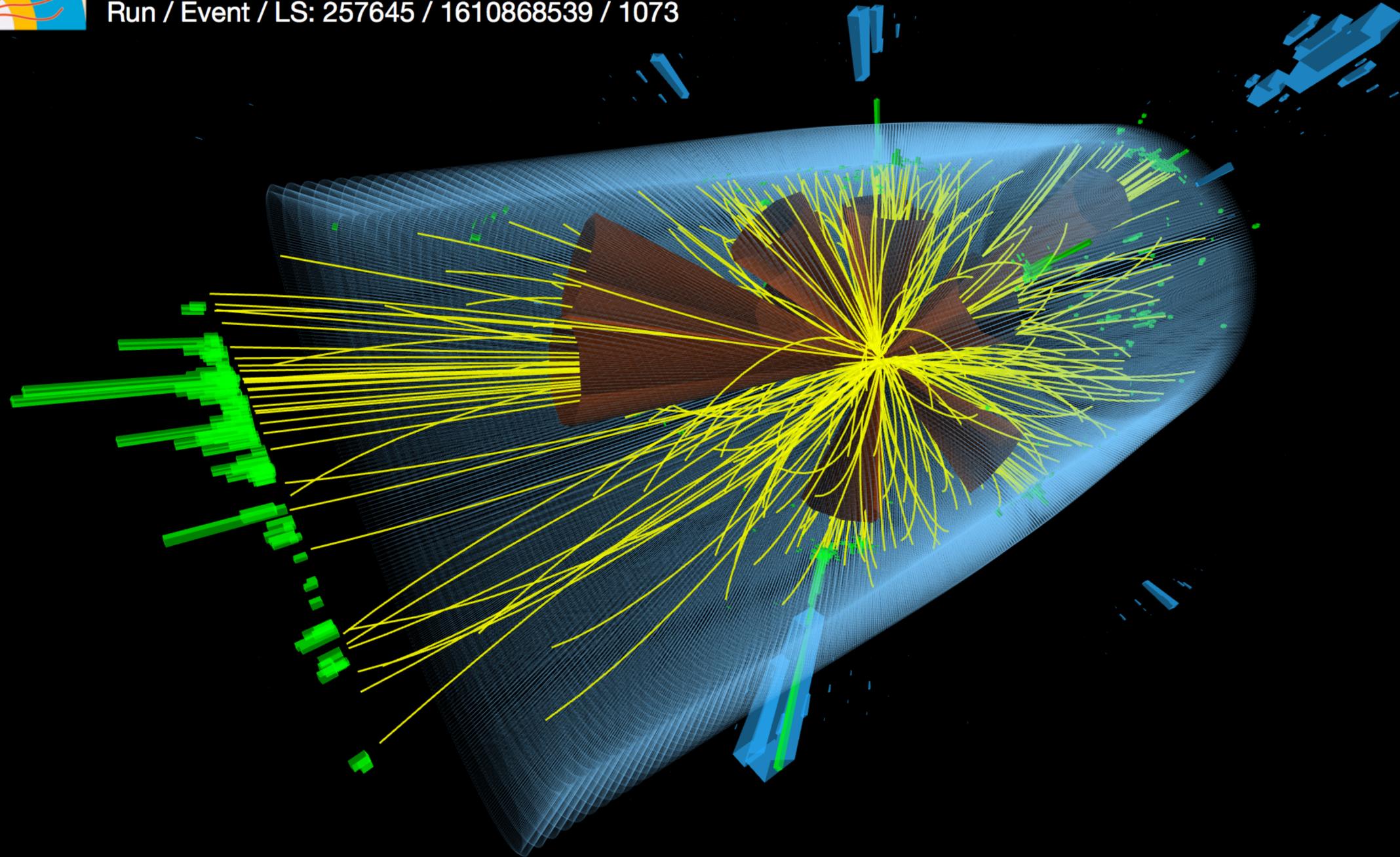
A multijet event at the LHC@13TeV



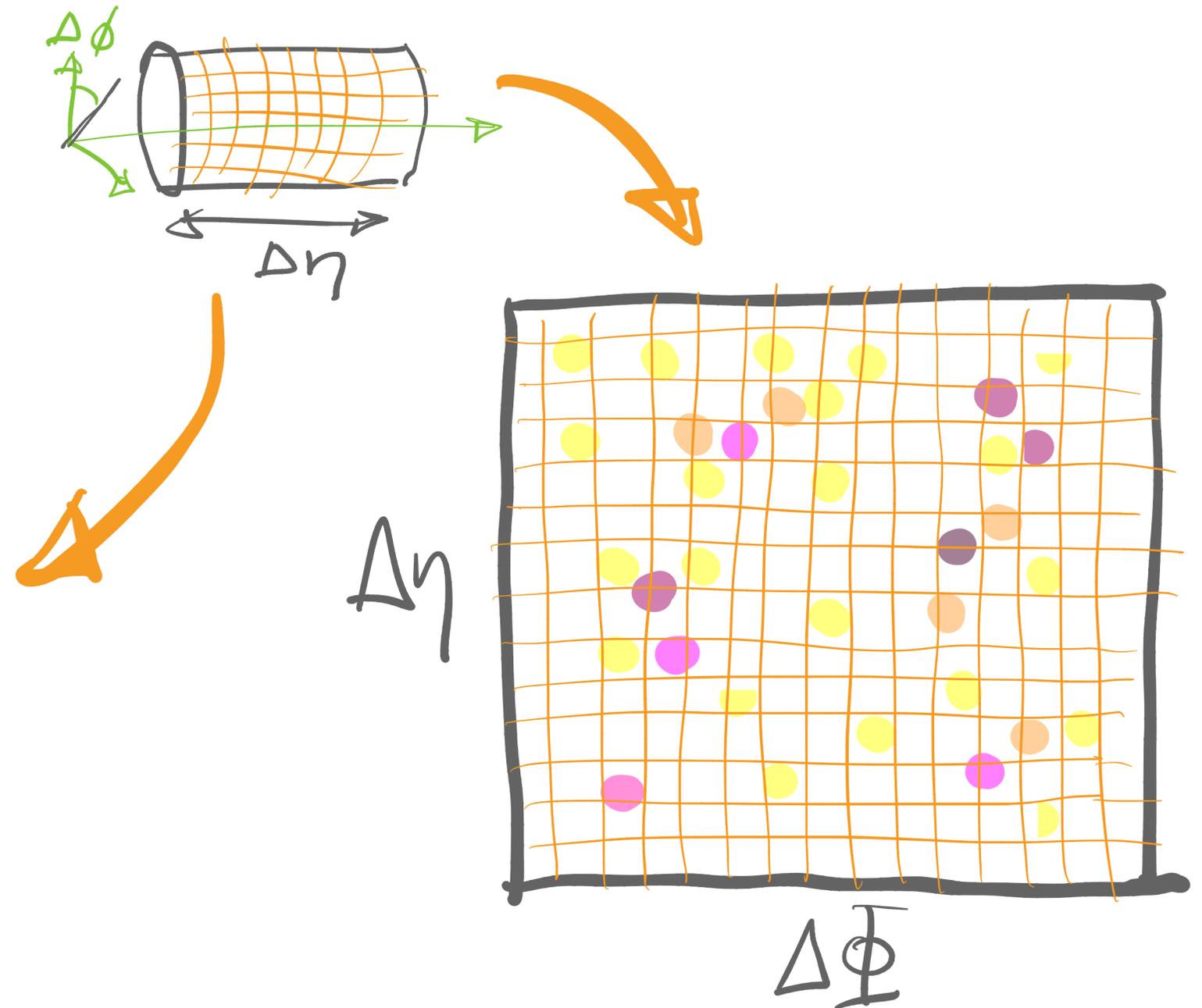
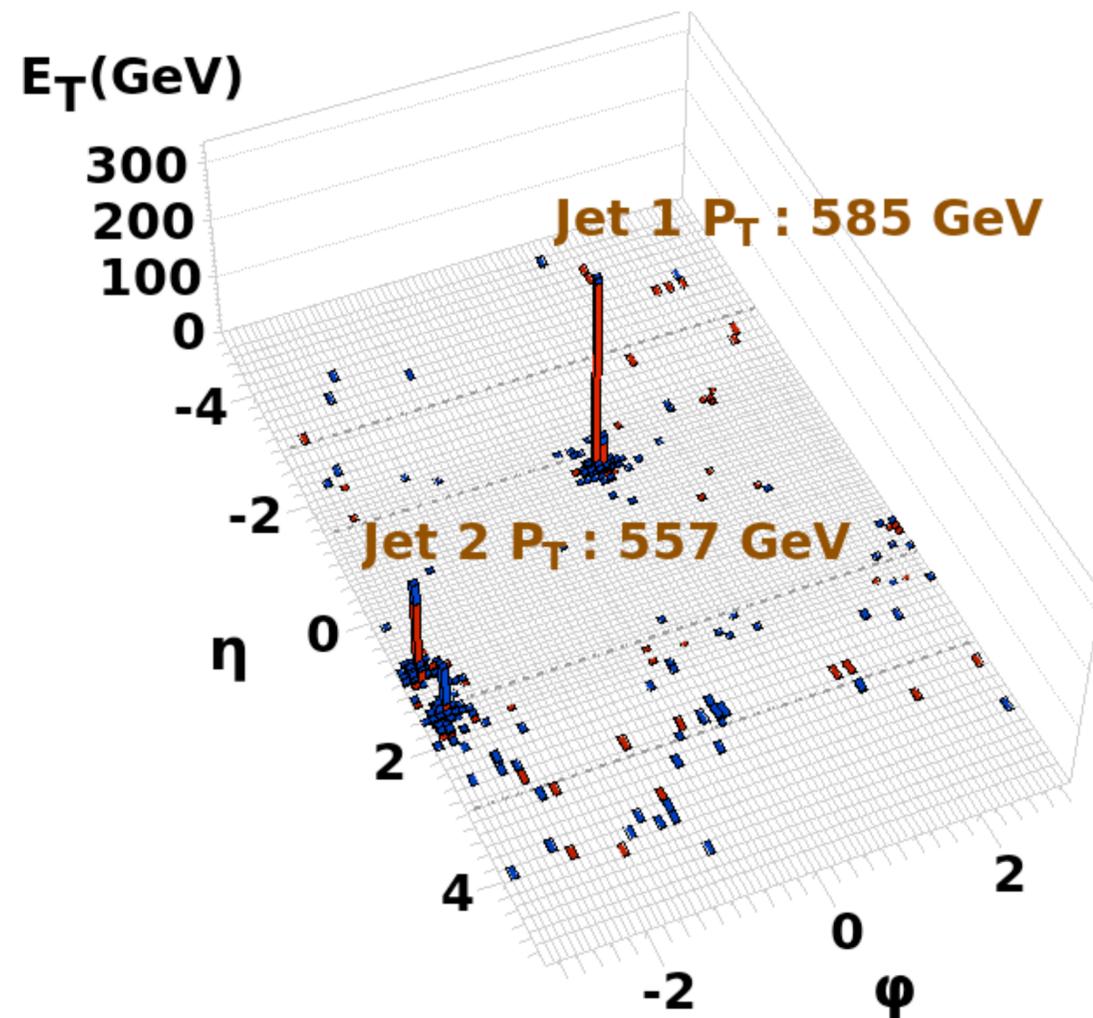
CMS Experiment at the LHC, CERN

Data recorded: 2015-Sep-28 06:09:43.129280 GMT

Run / Event / LS: 257645 / 1610868539 / 1073

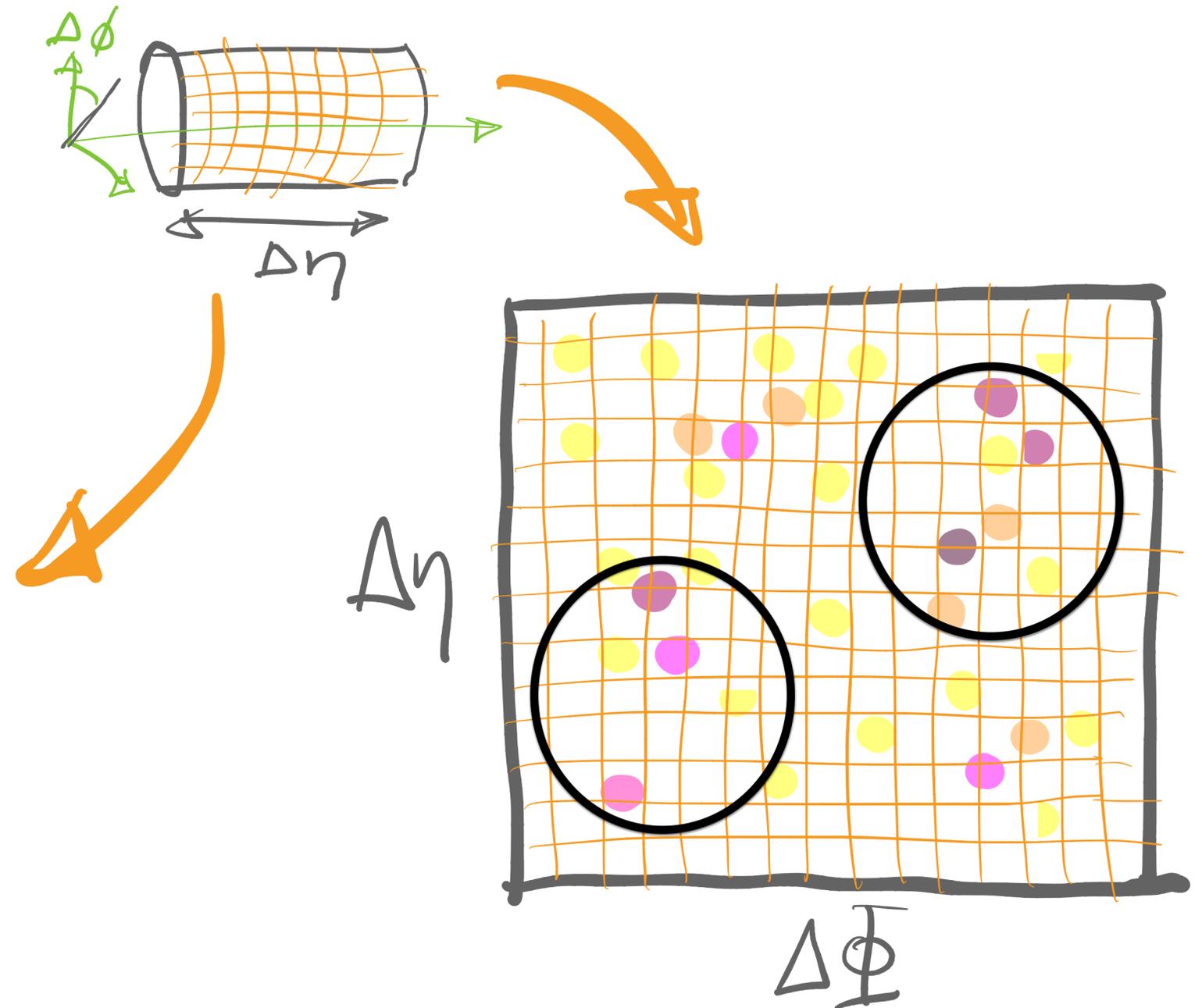
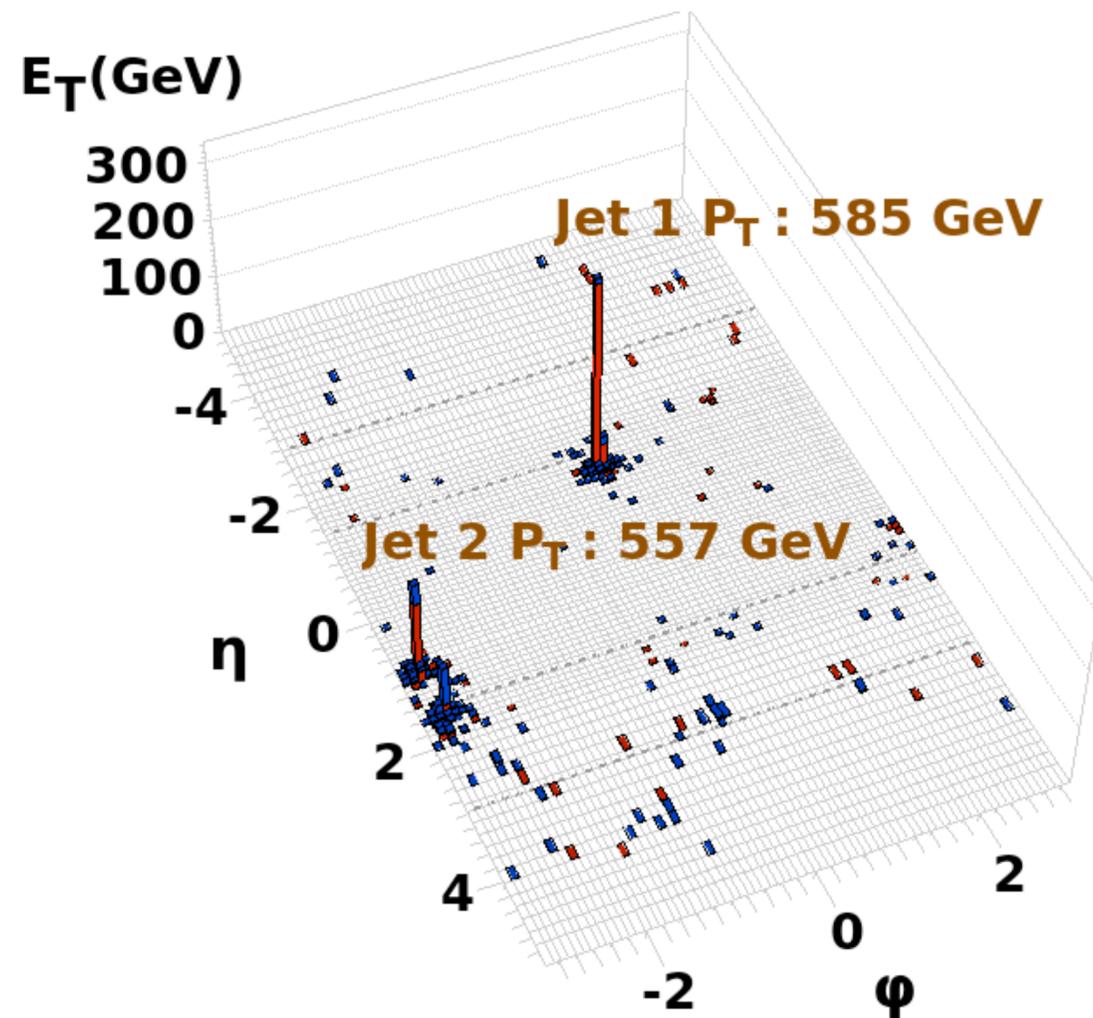


How to identify jets?



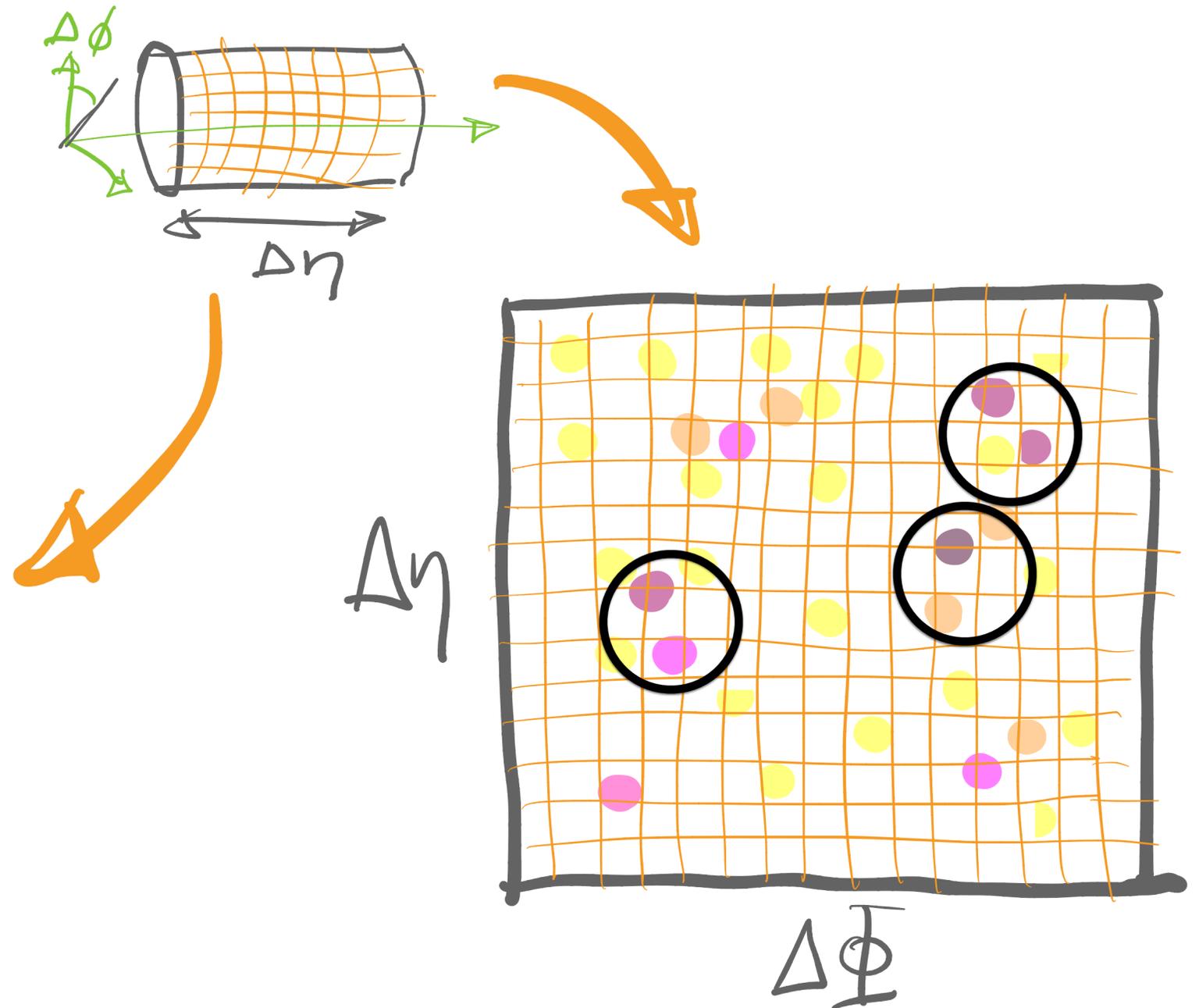
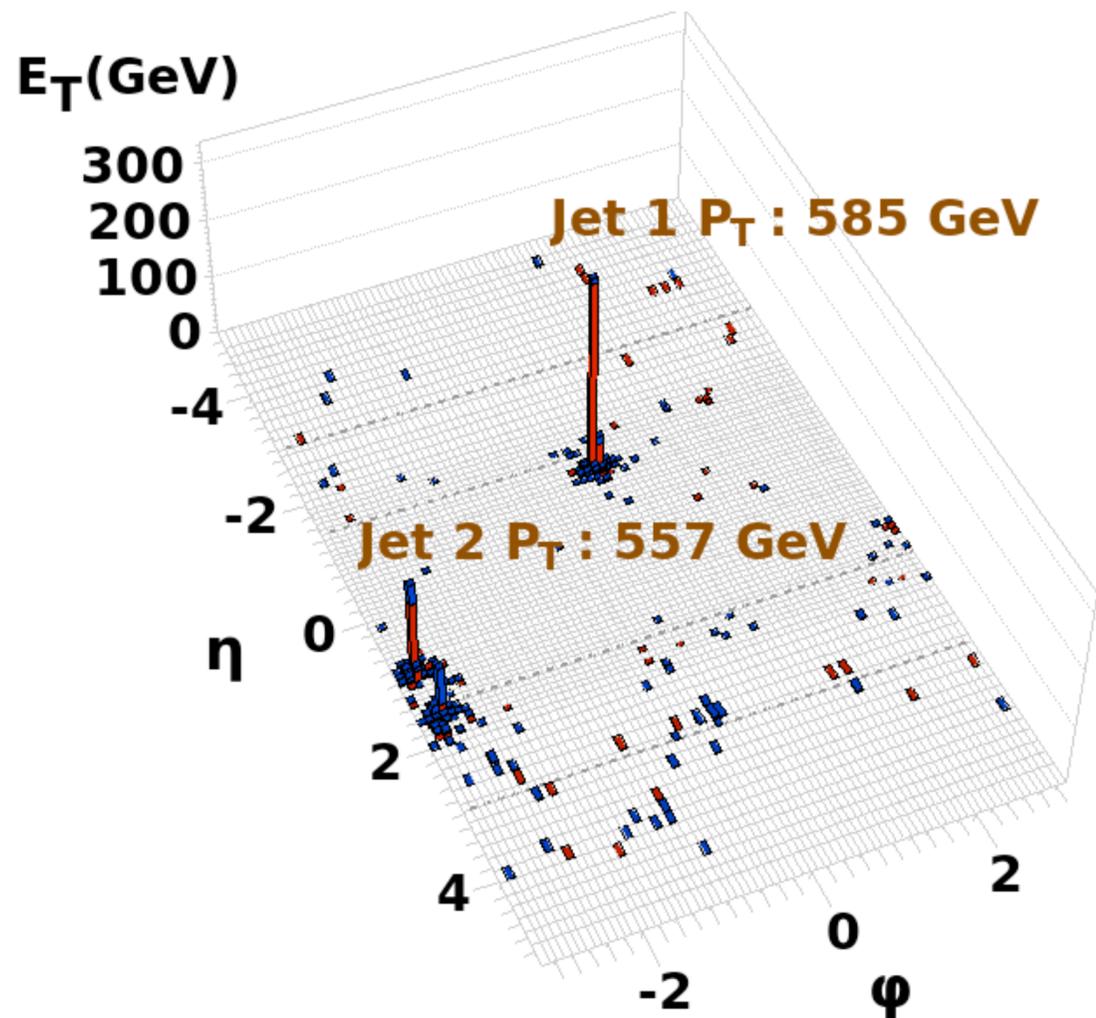
Number of jets depend on the definition - jet finding algorithm

How to identify jets?



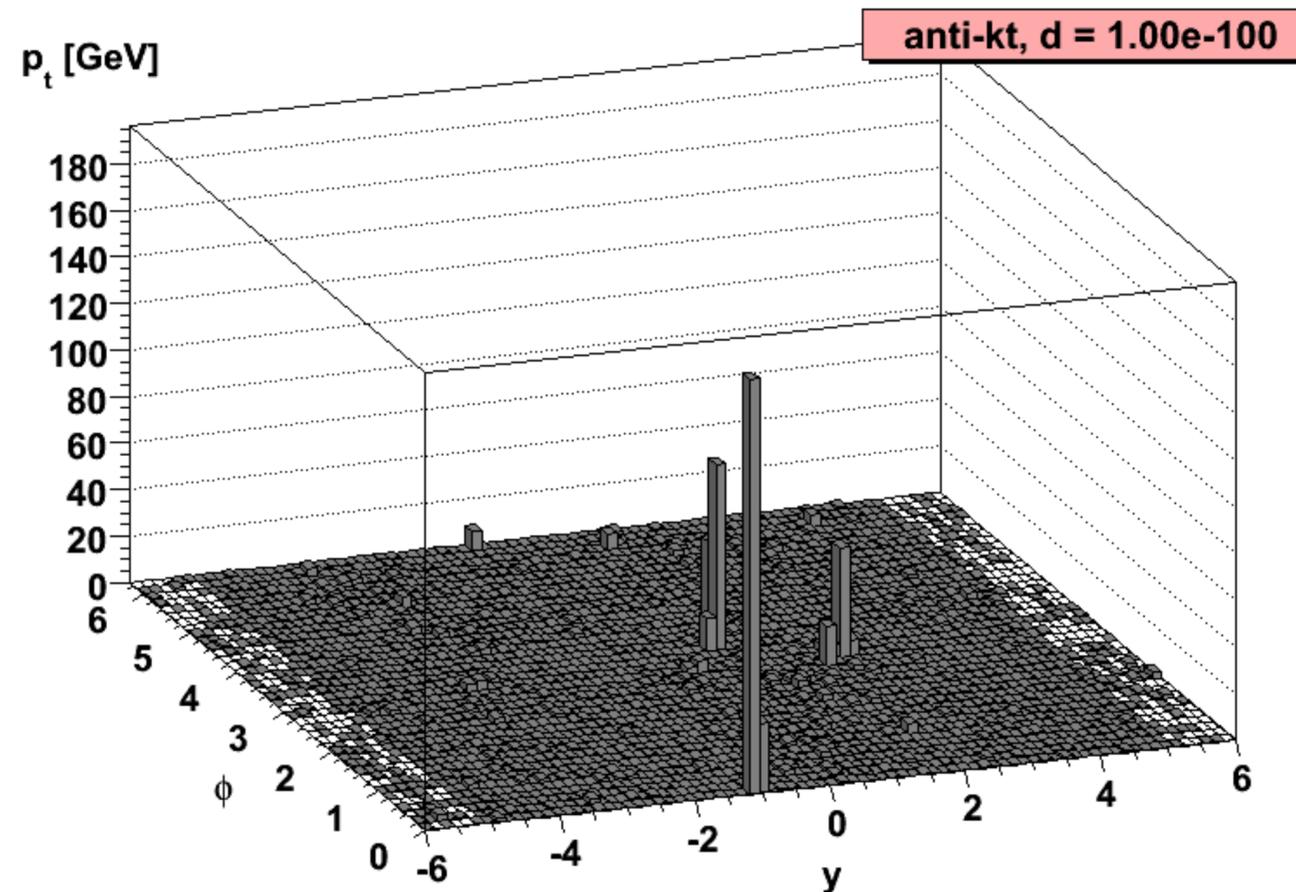
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How to identify jets?



Number of jets depend on the definition - jet finding algorithm

Jet clustering



Anti-kt has become the most widely used jet-finding algorithm at LHC

Sequential recombination. Define:

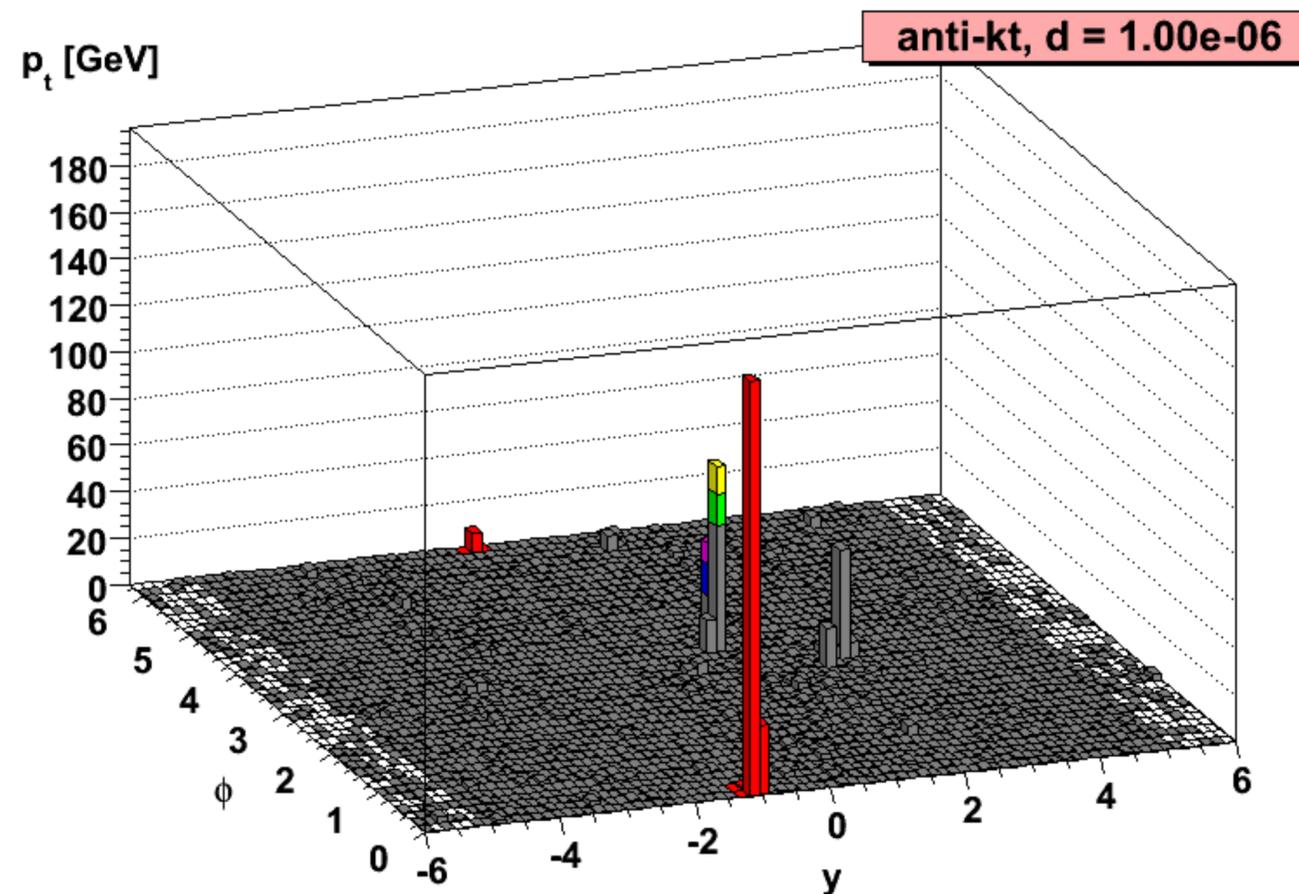
$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \quad d_{iB} = \frac{1}{p_{ti}^2}$$

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

- Find smallest of d_{ij} and d_{iB}
- If ij , recombine them
- If iB , call it a jet and remove from the list
- Repeat until no particles left

[From <https://gsalam.web.cern.ch/gsalam/teaching/PhD-courses.html>]

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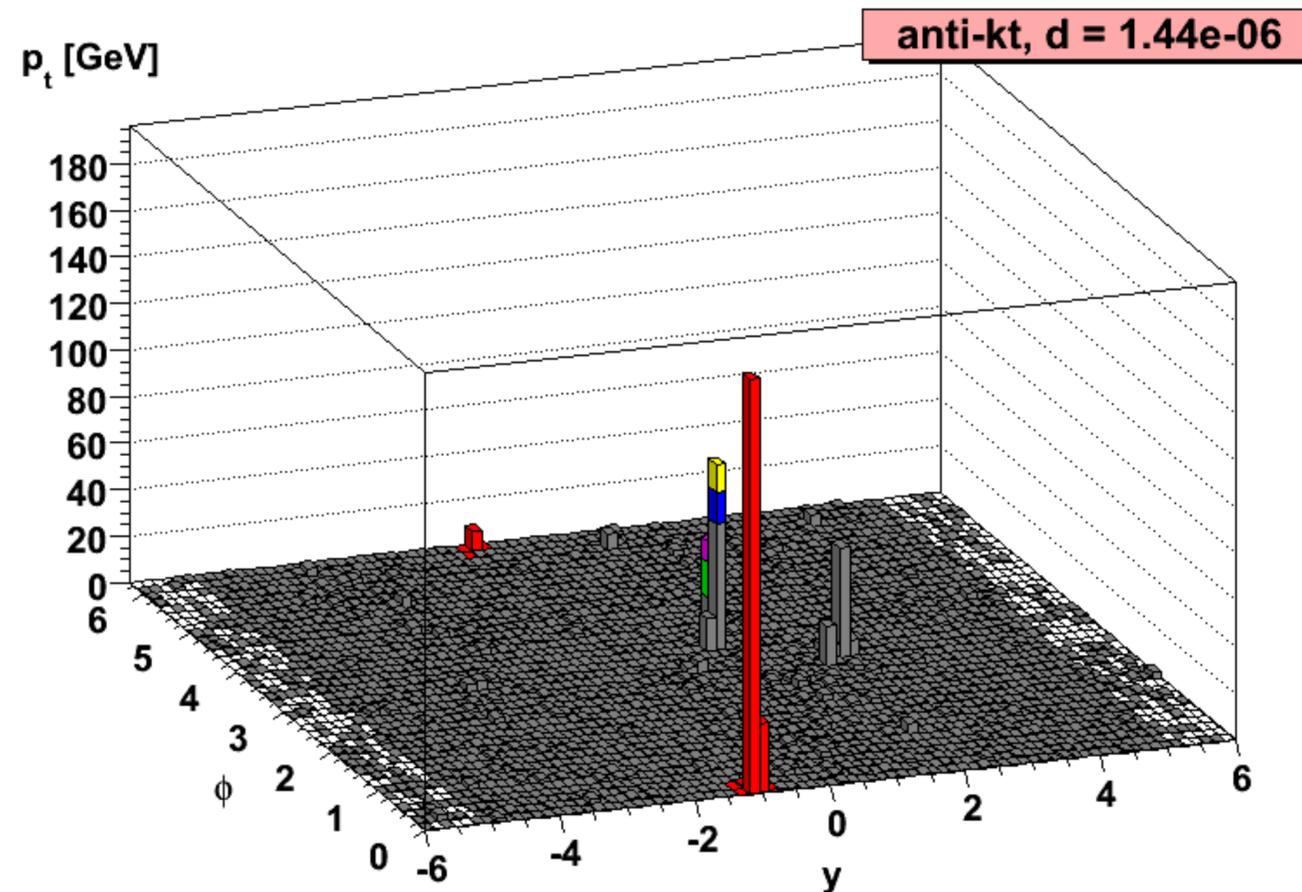
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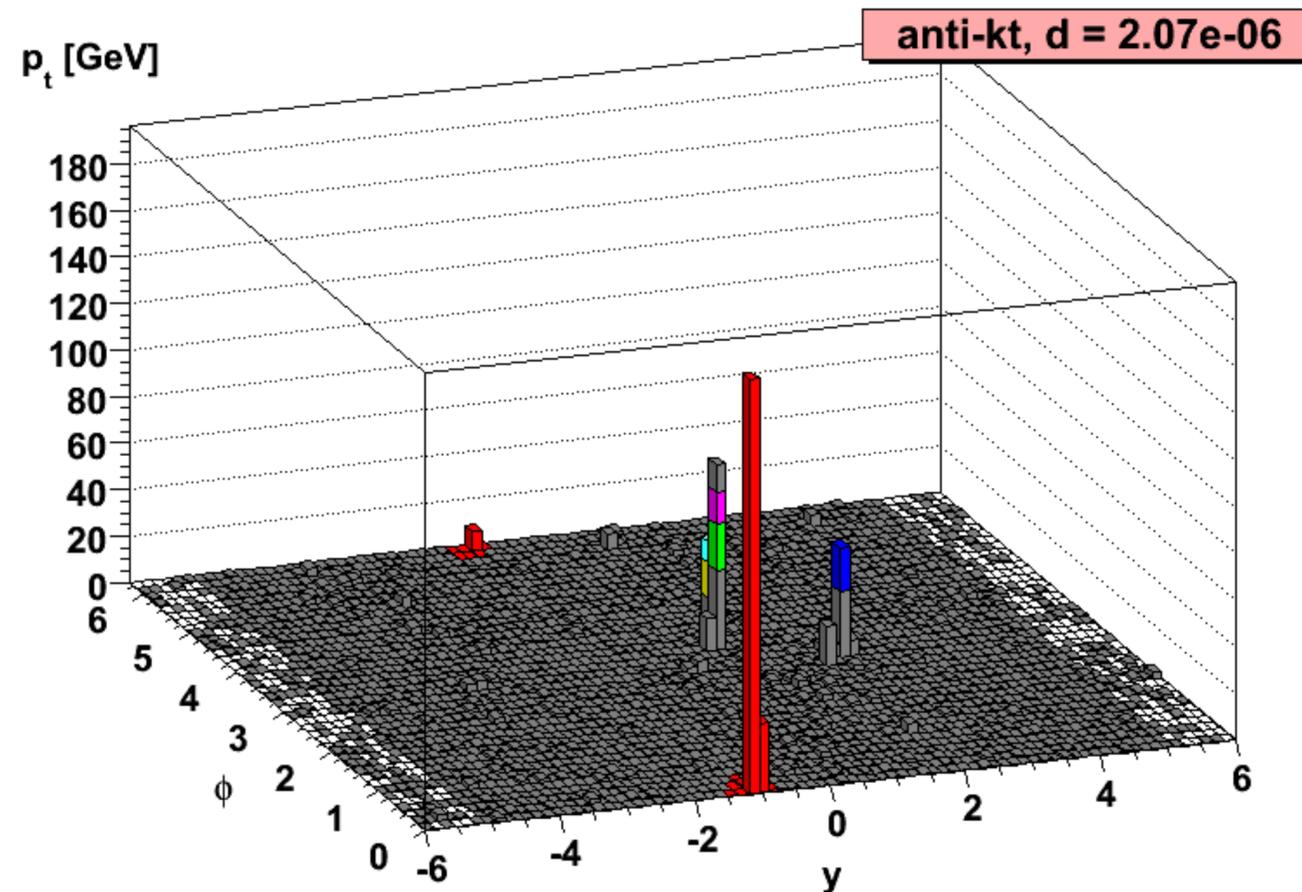
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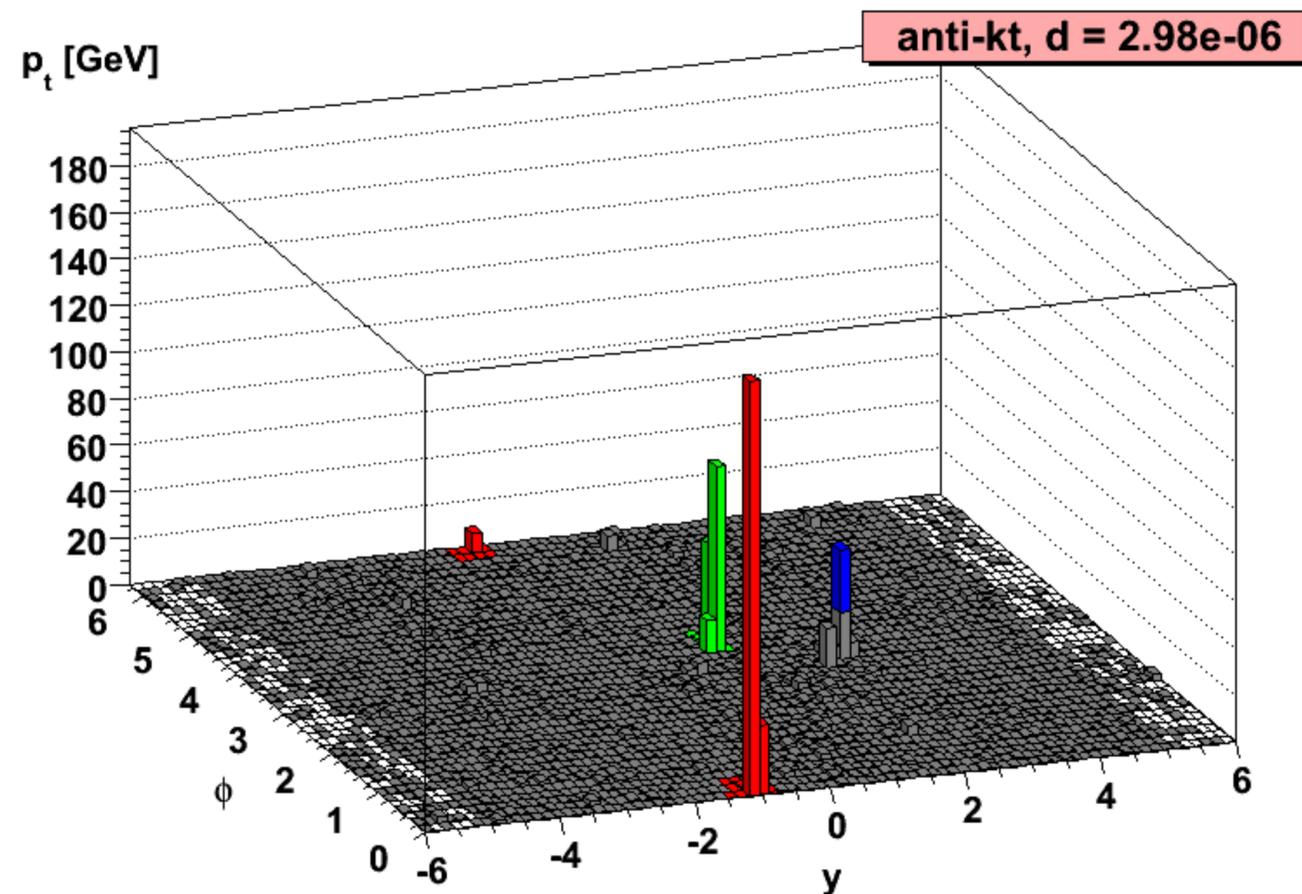
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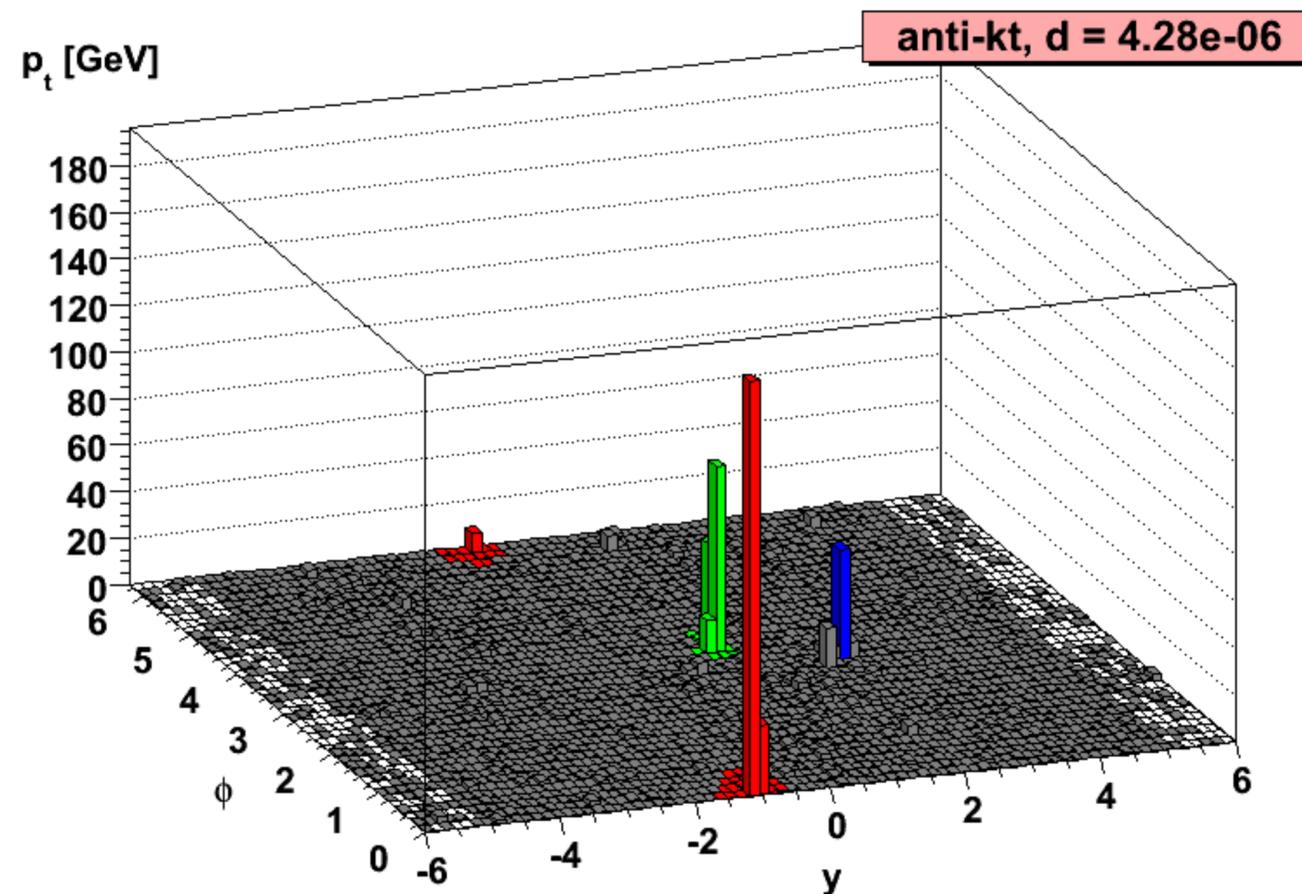
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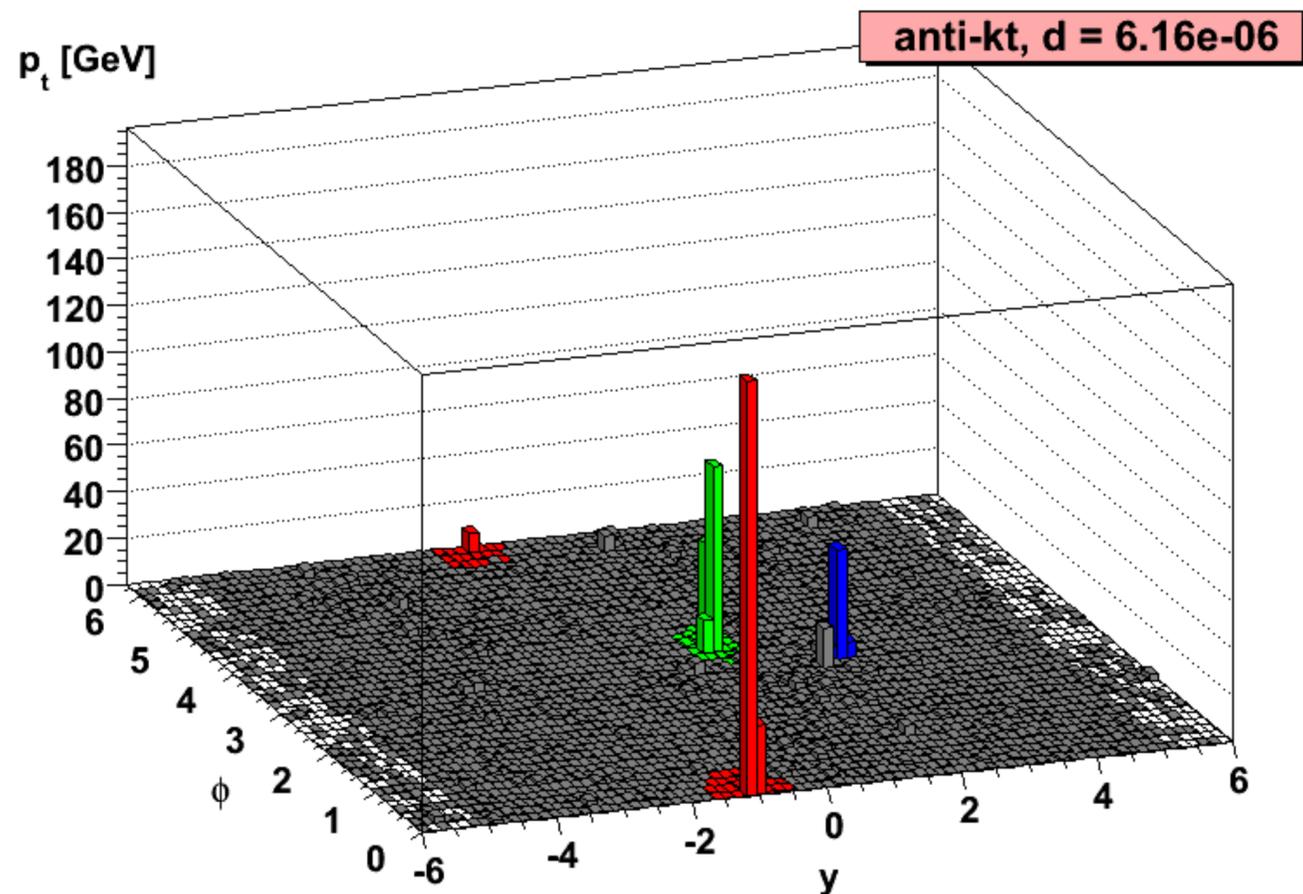
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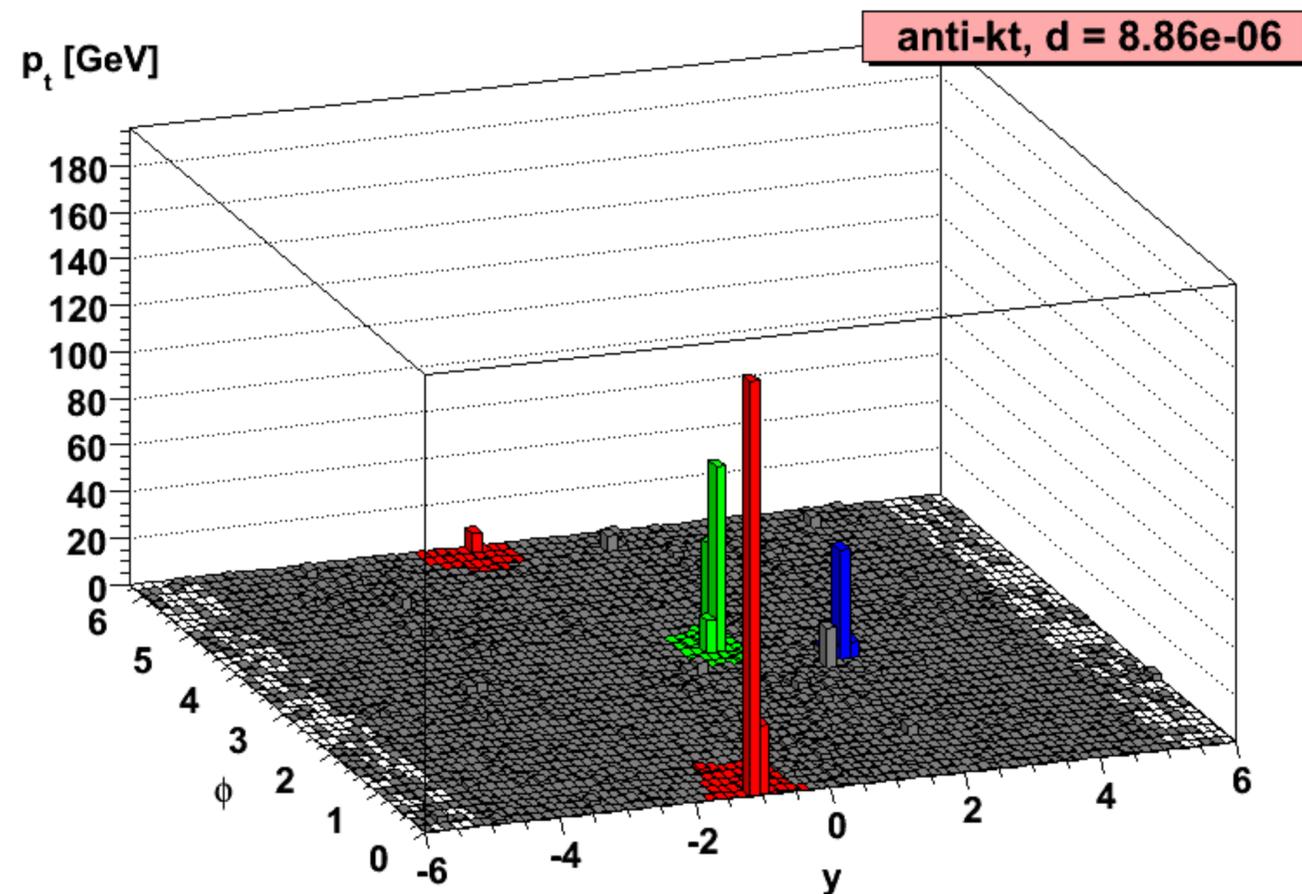
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Jet clustering



Anti-kt has become the most widely used jet-finding algorithm at LHC

Sequential recombination. Define:

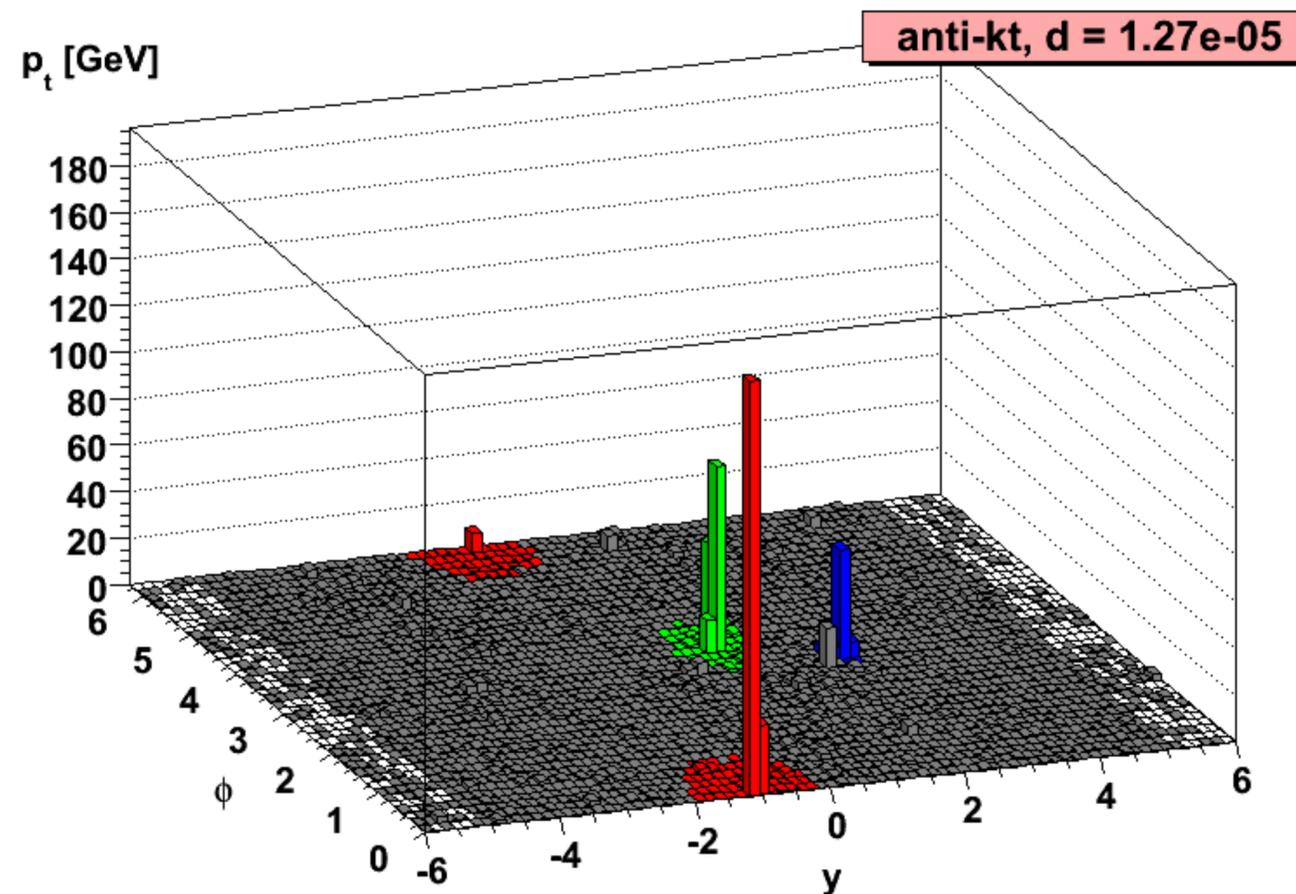
$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \quad d_{iB} = \frac{1}{p_{ti}^2}$$

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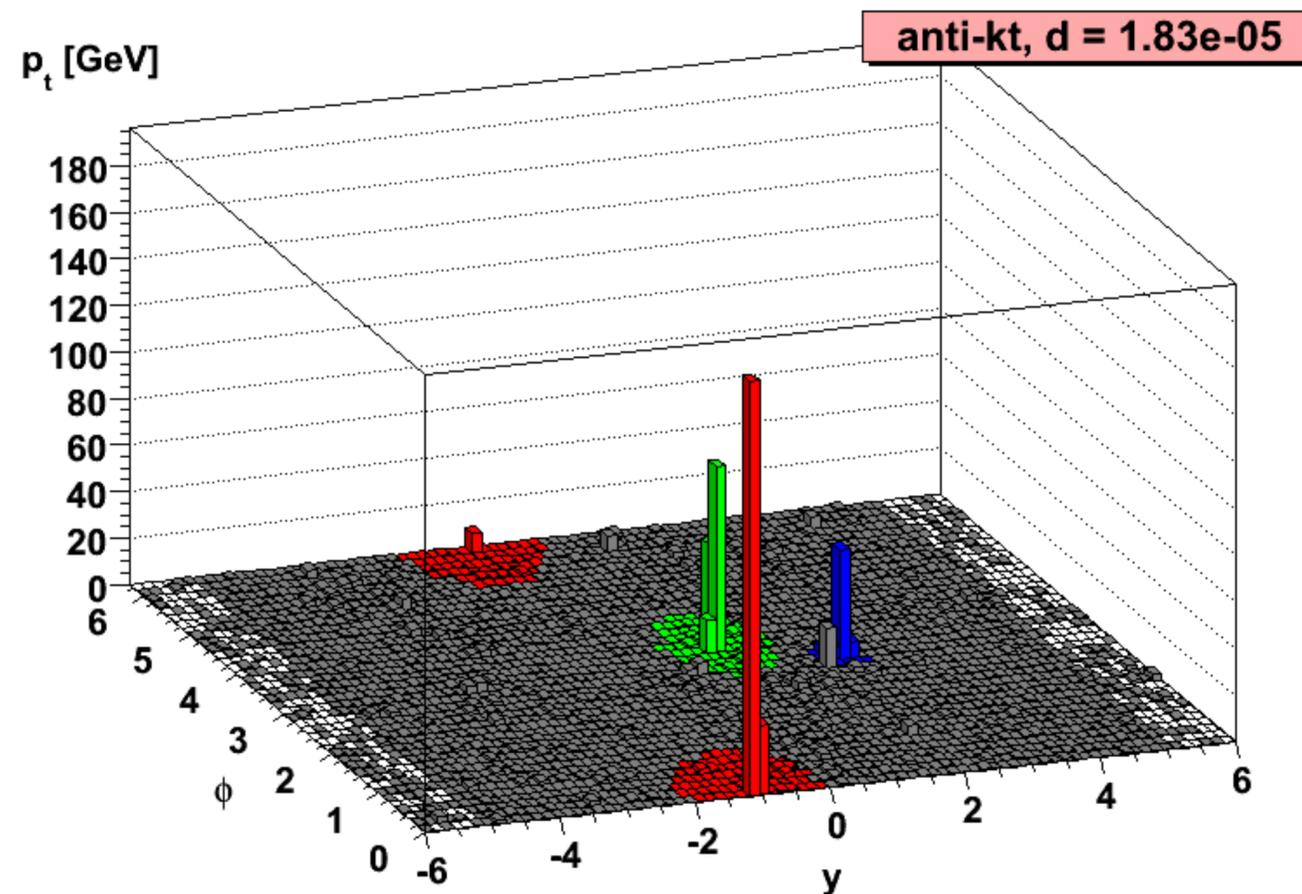
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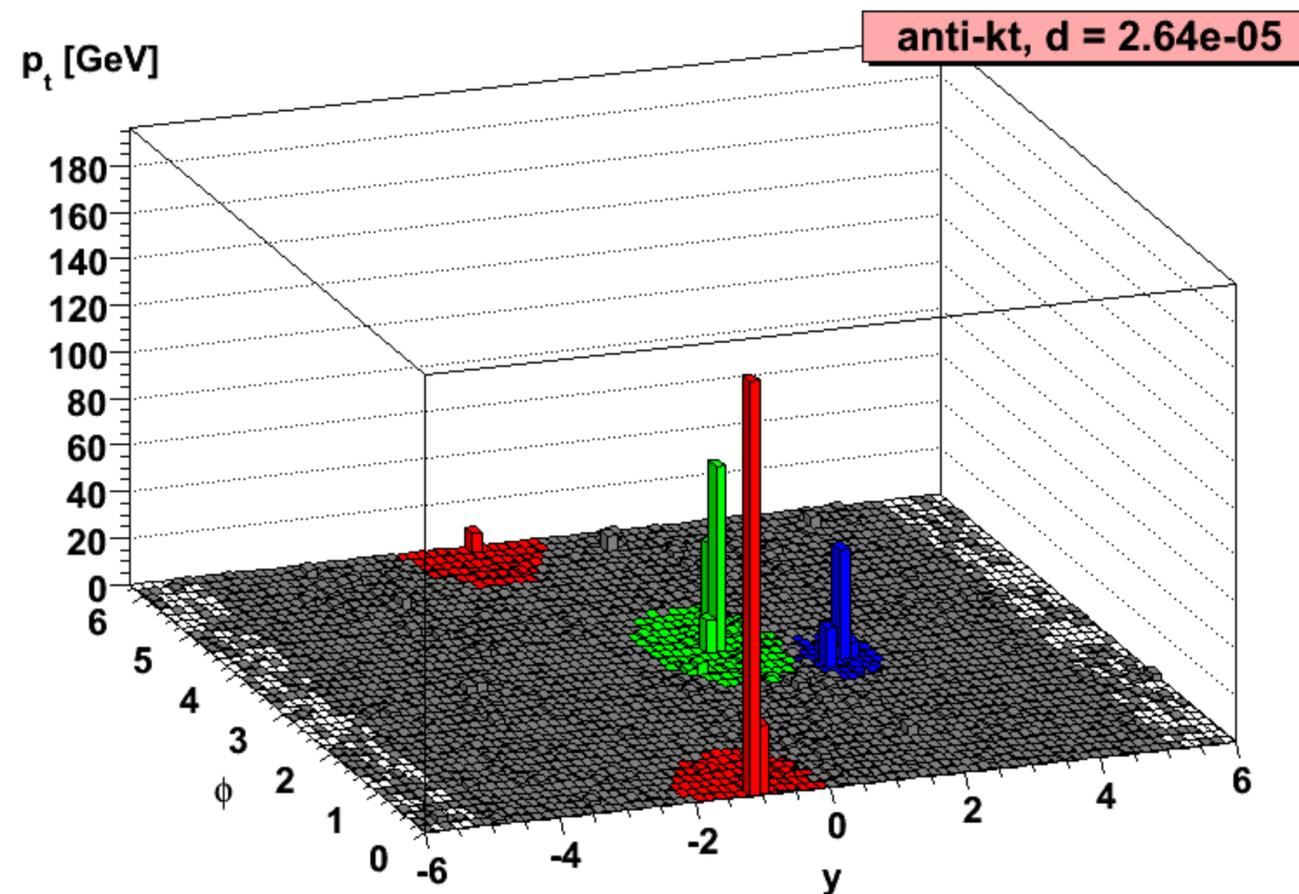
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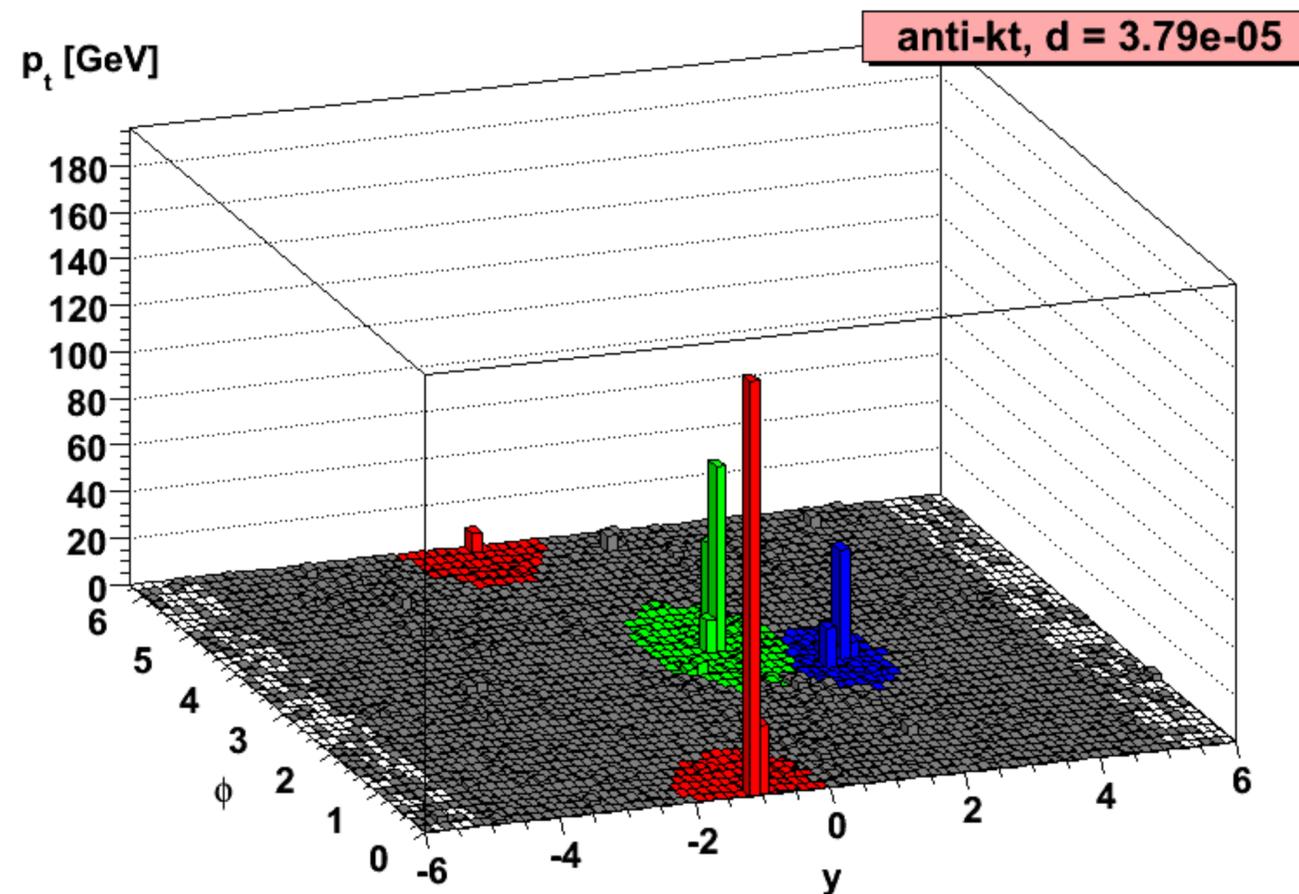
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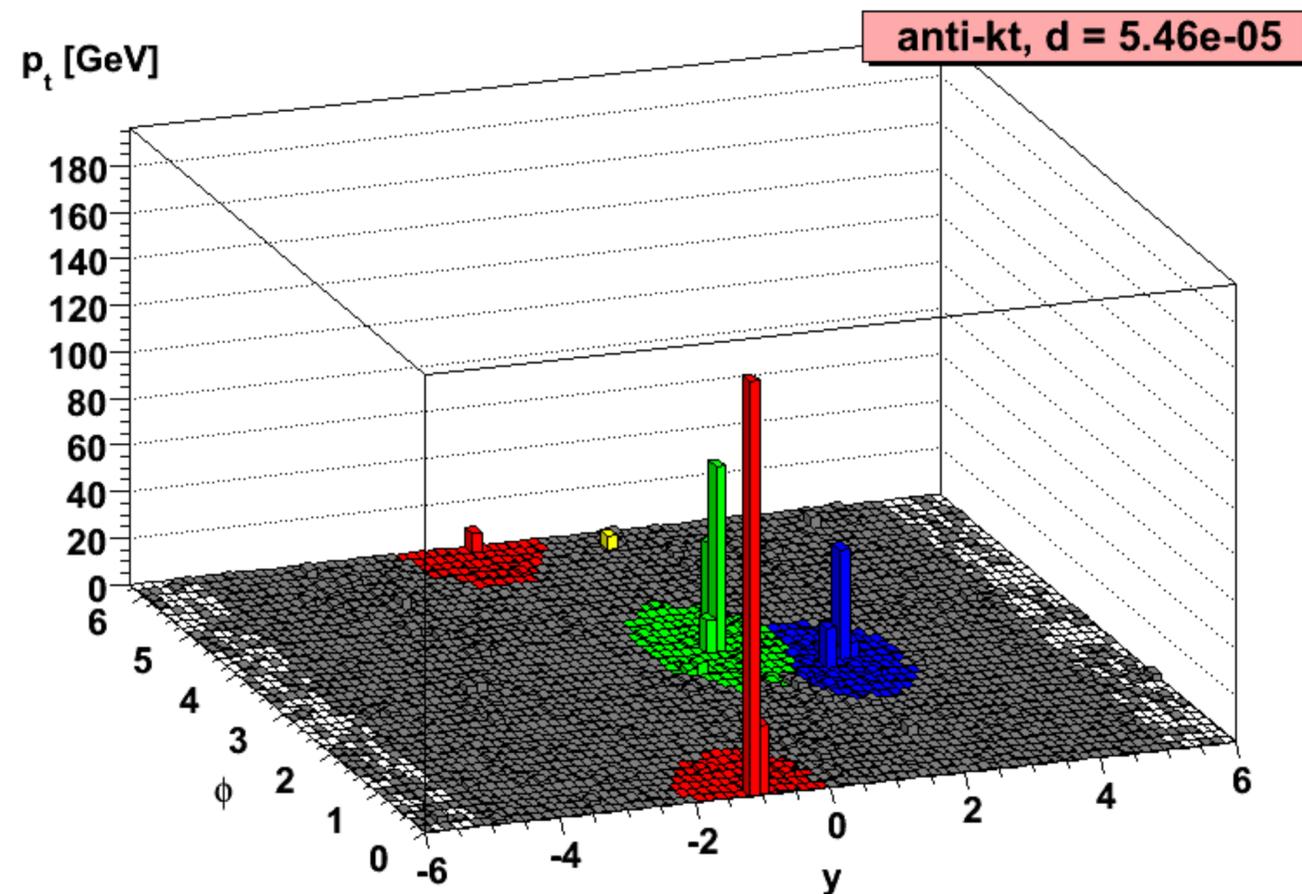
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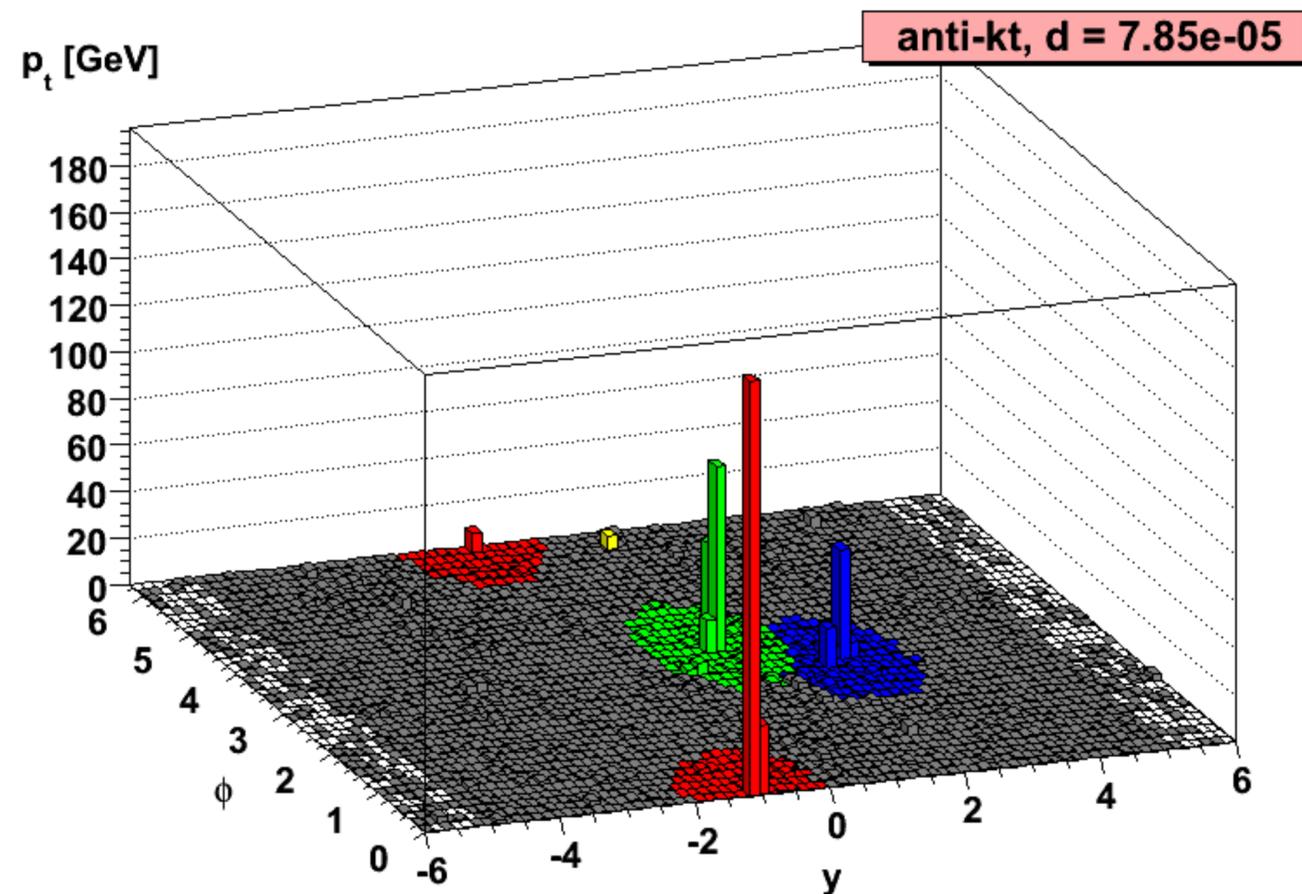
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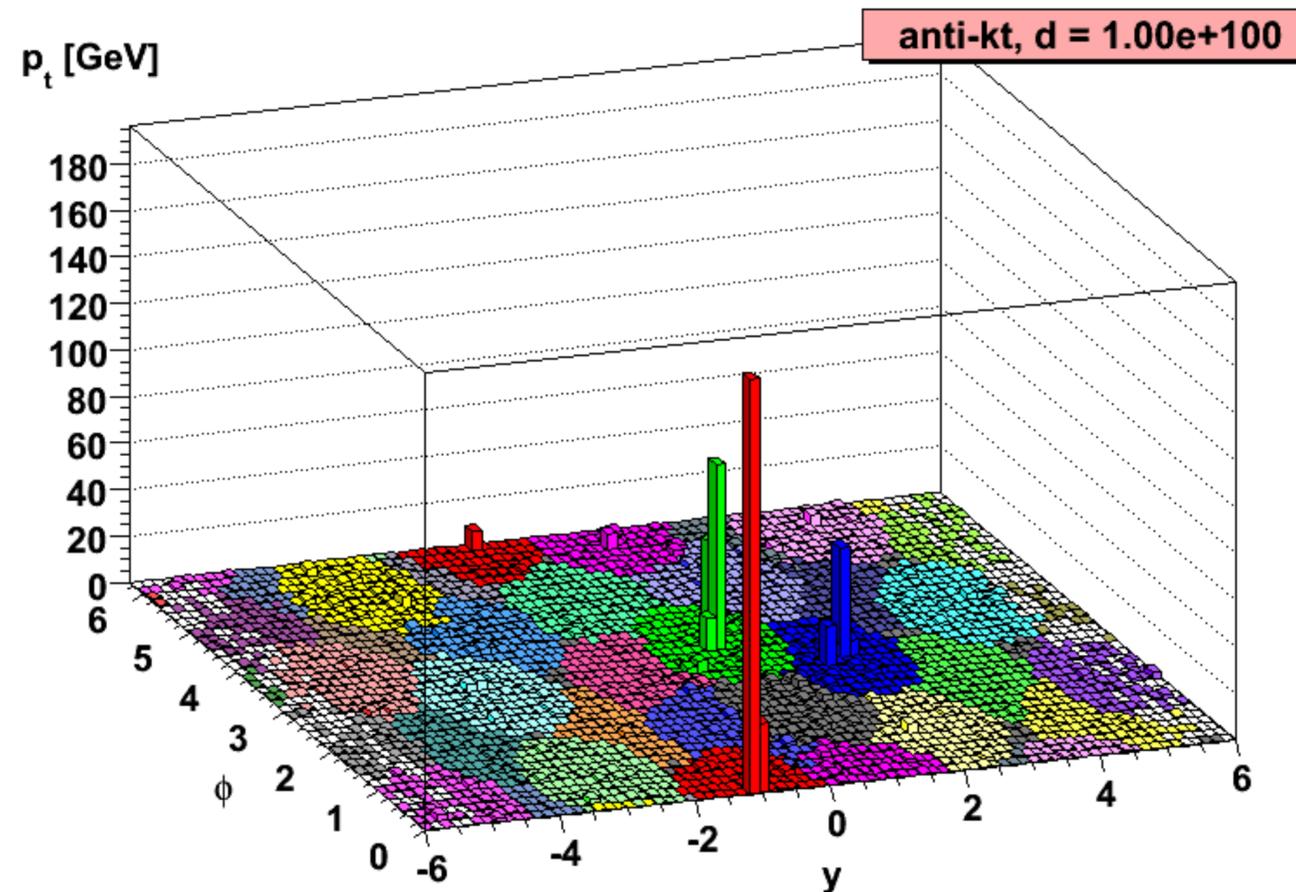
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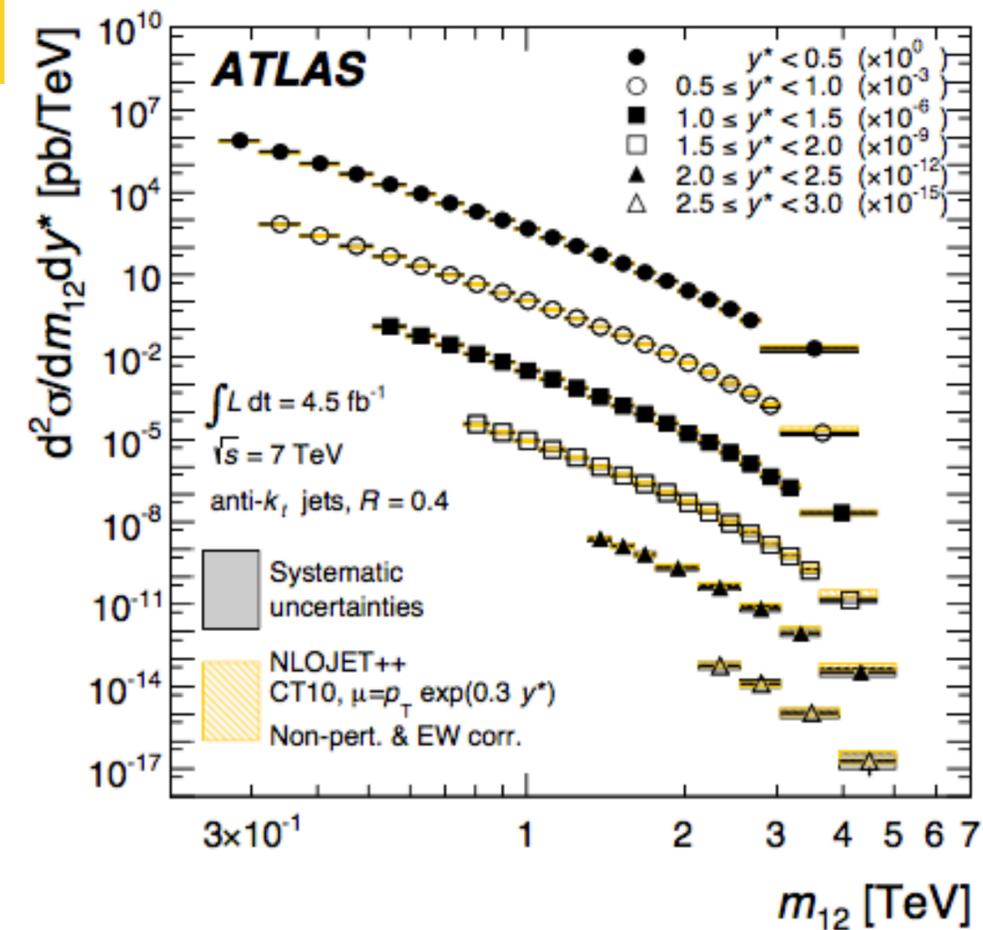
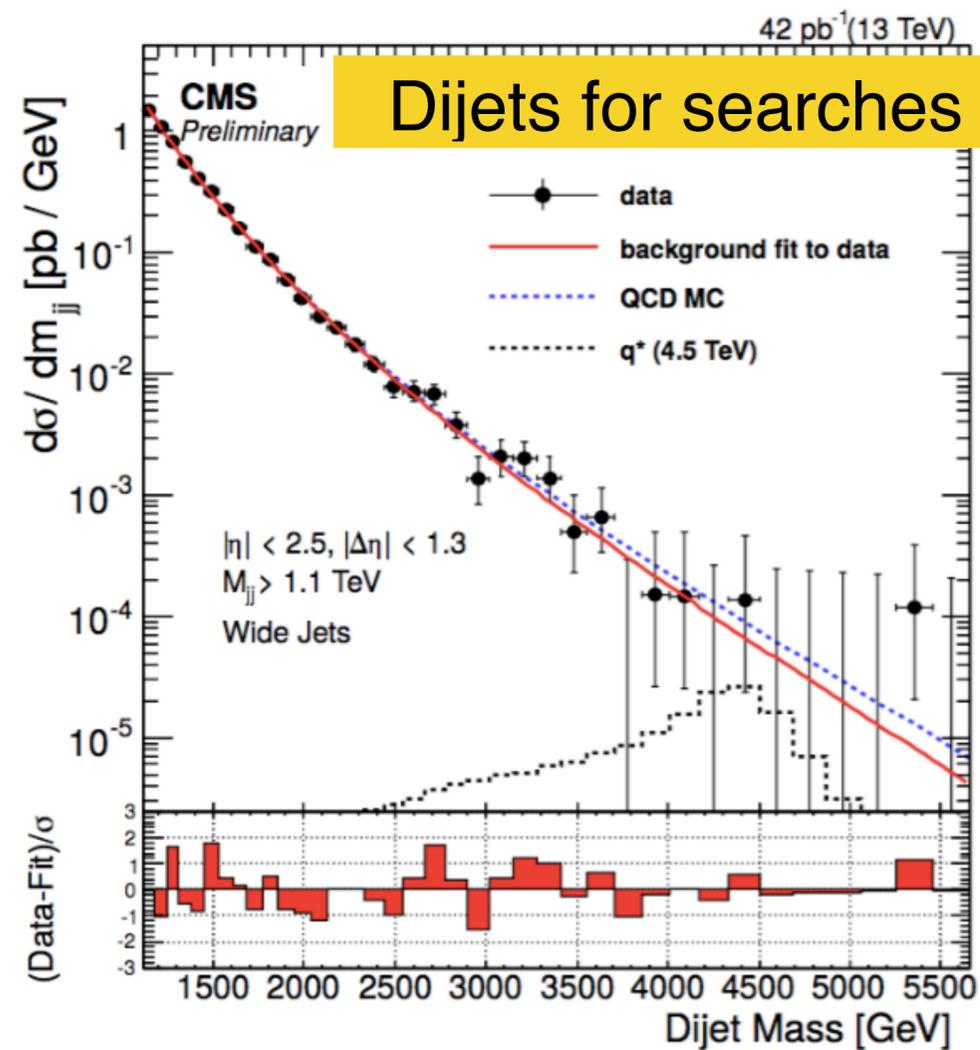
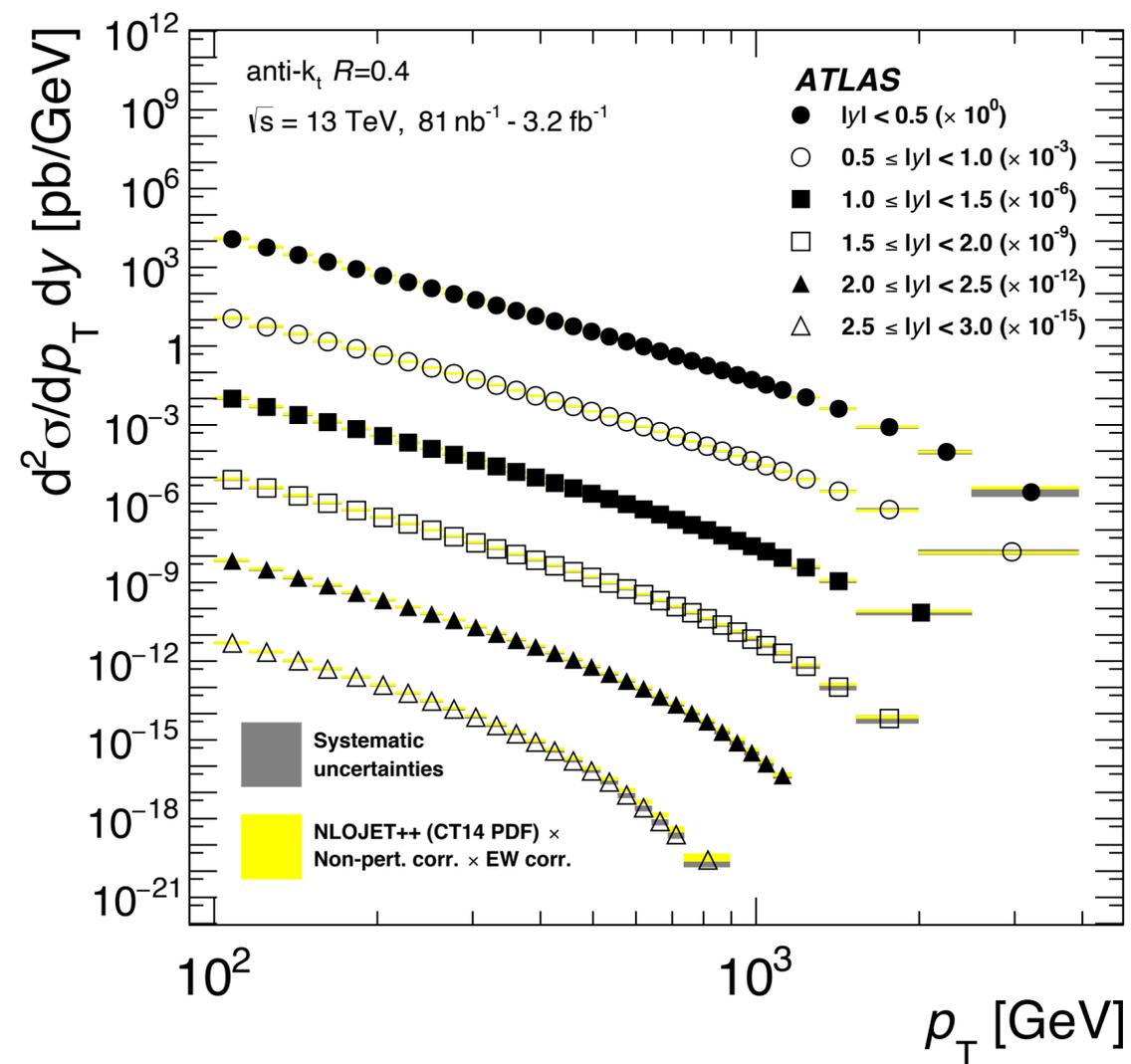
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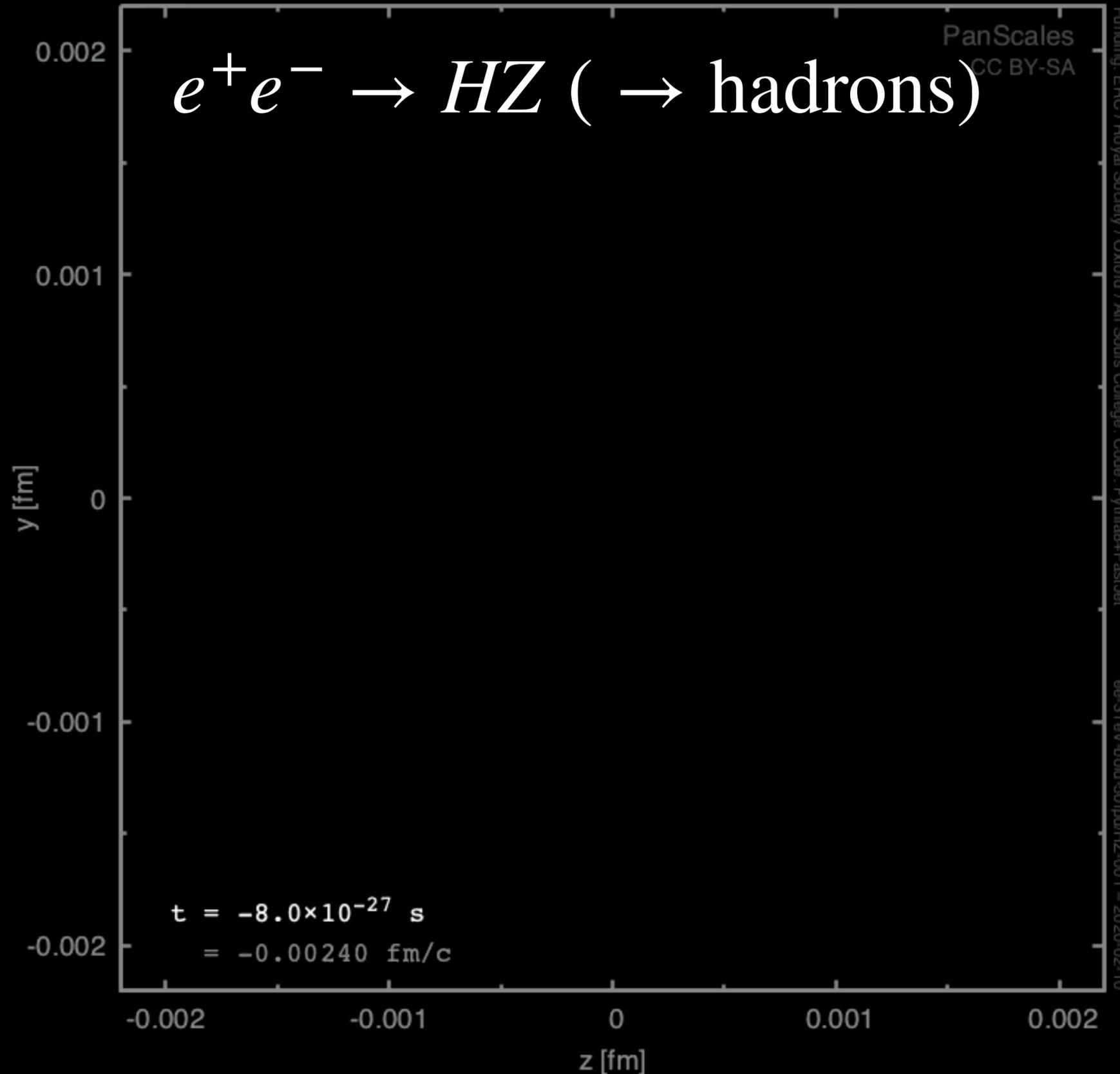
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Some results from LHC



Excellent agreement with QCD - strong constraints to PDFs [later]

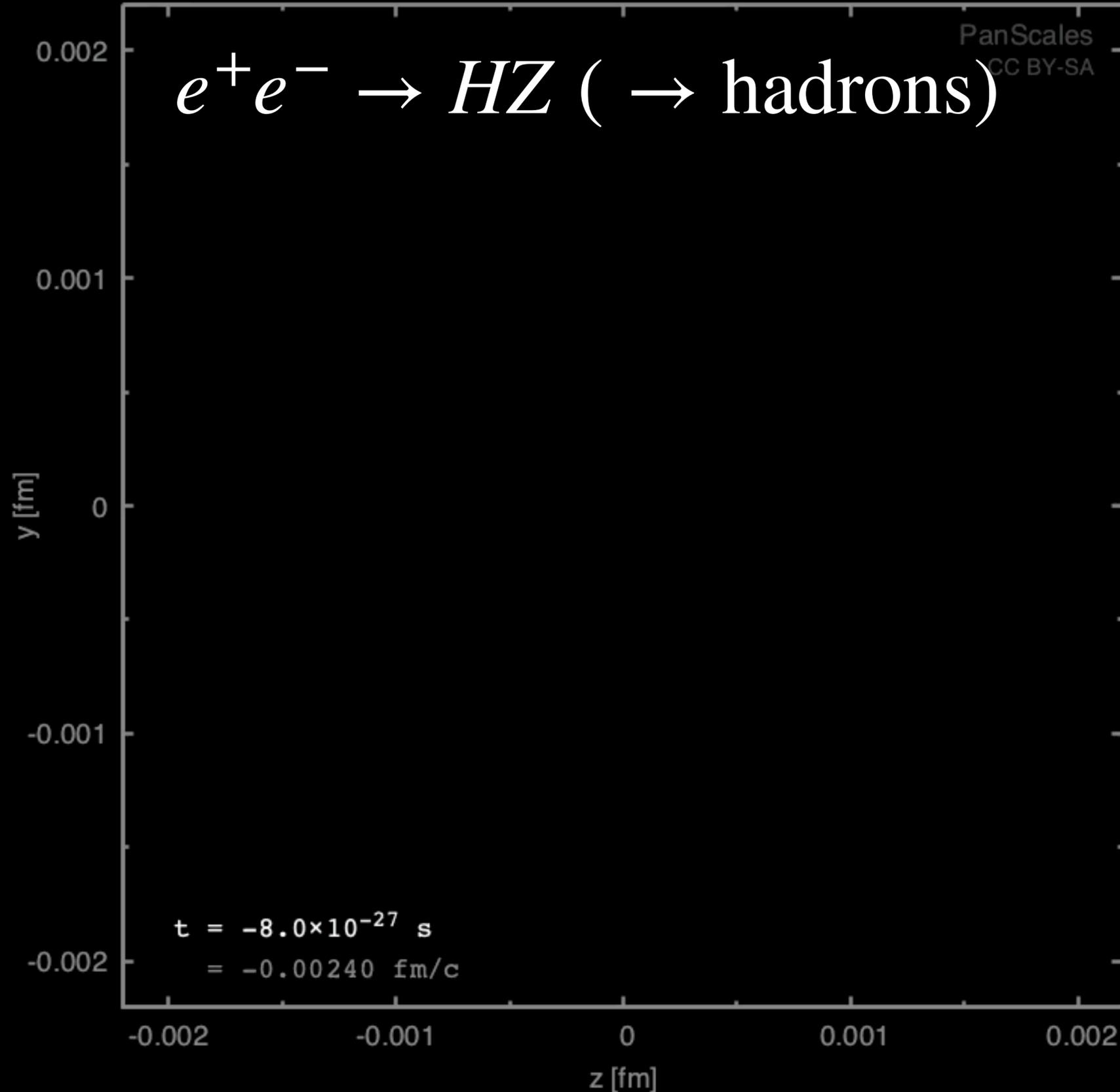


3TeV e^+e^- events

Initial particles in **yellow**
Intermediate particles in **blue**
Final particles in **red**

Boosted H and Z decay
into **collimated jets**

[Simulation of the events are
produced with Pythia 8
times estimated by clustering algorithm
- see details in the web page]

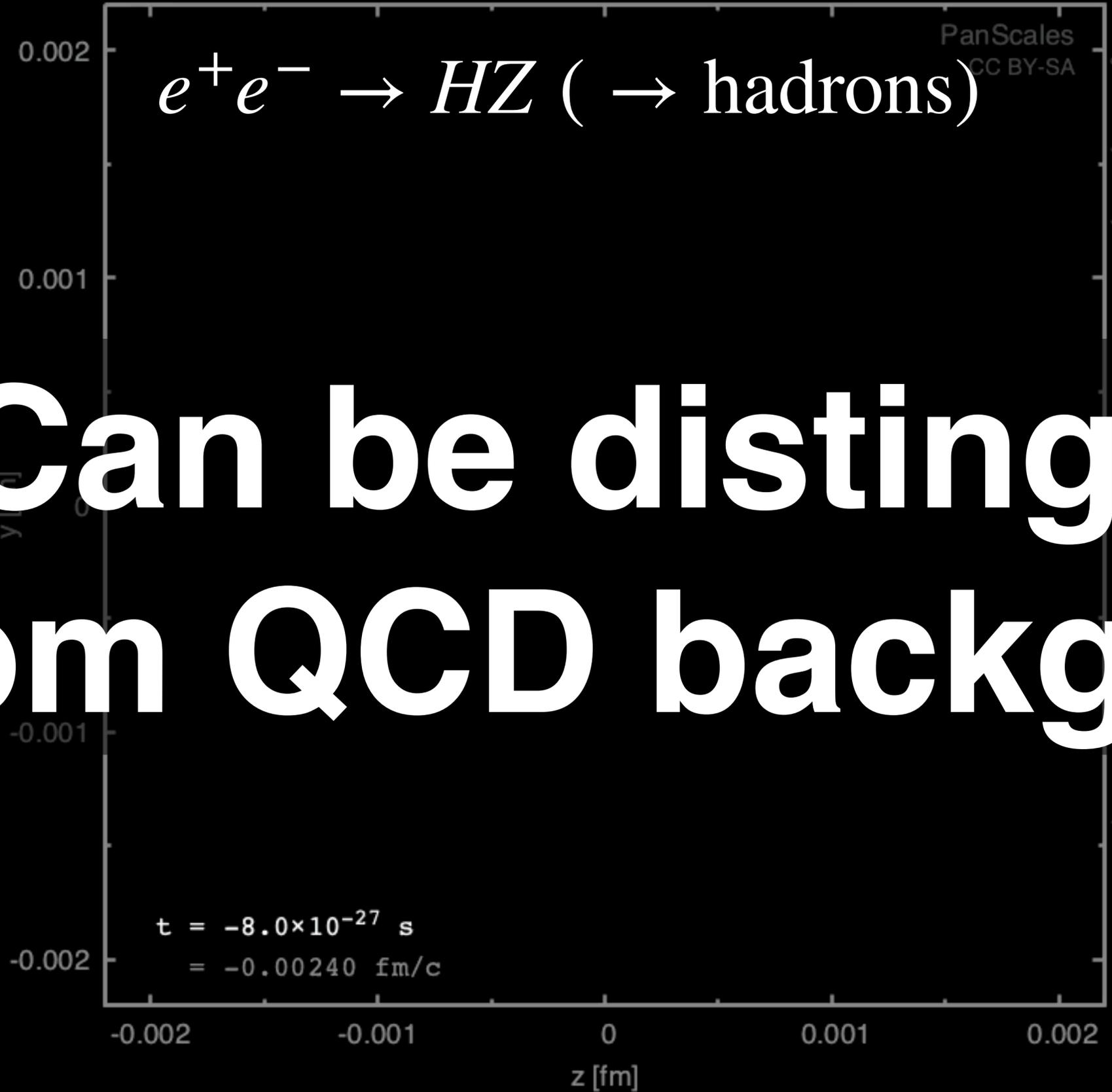


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Can be distinguished from QCD background???

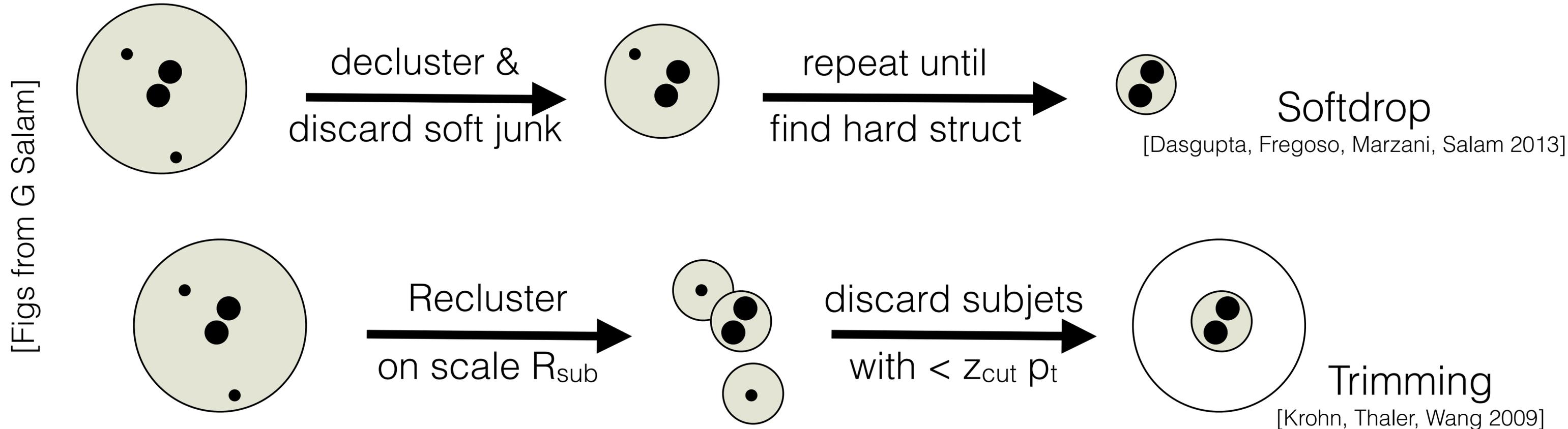
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Jet substructure

Find different substructures in identified jets

[very active area, lots of results in the last years]

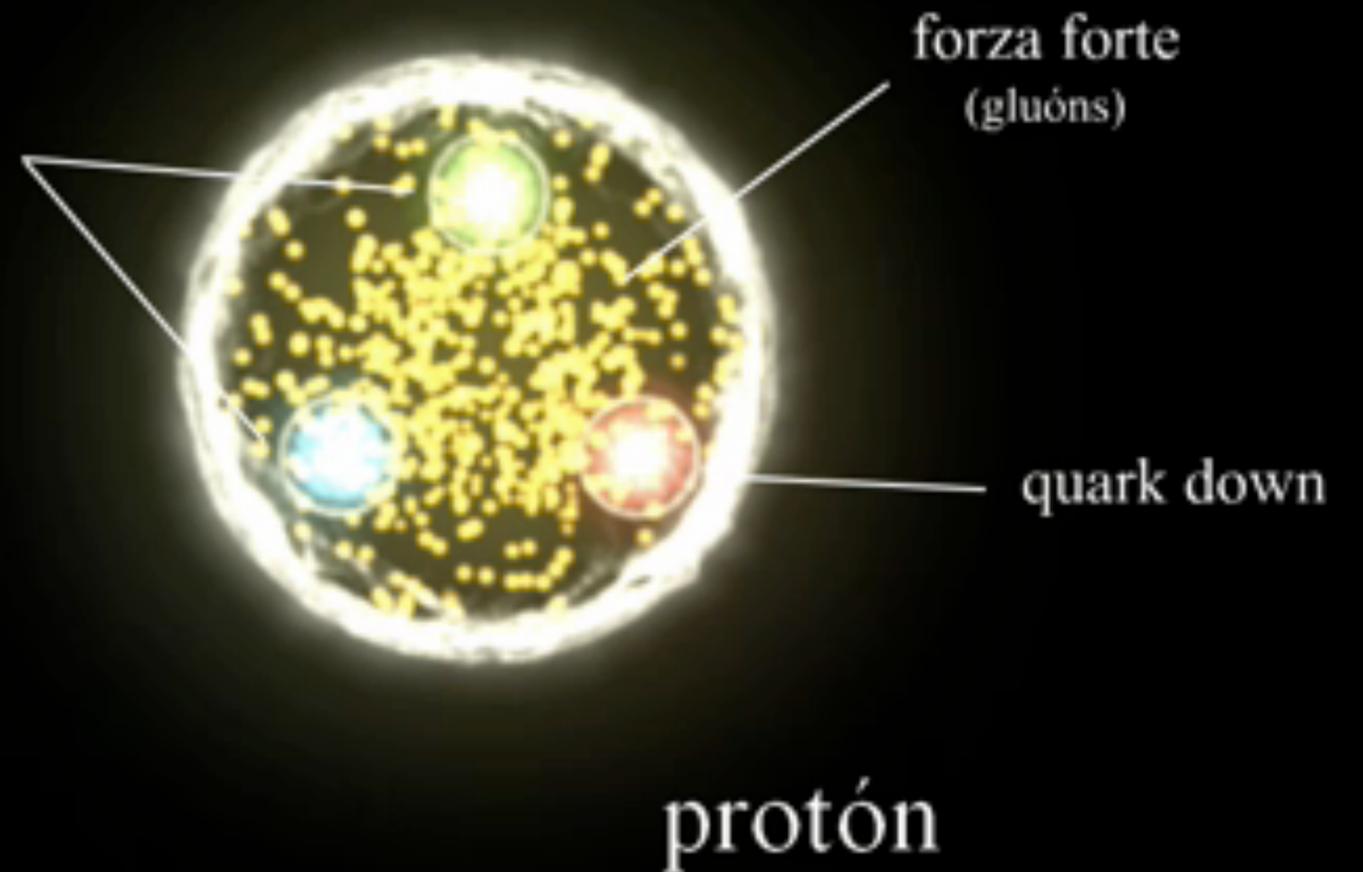


Also to identify two-pronged jet structures - boosted H/W/Z

QCD is a Quantum Field Theory

↳ Quantum fluctuations ↔ virtual particles

- * What is the structure of the proton?
- * How do we extract it from data?



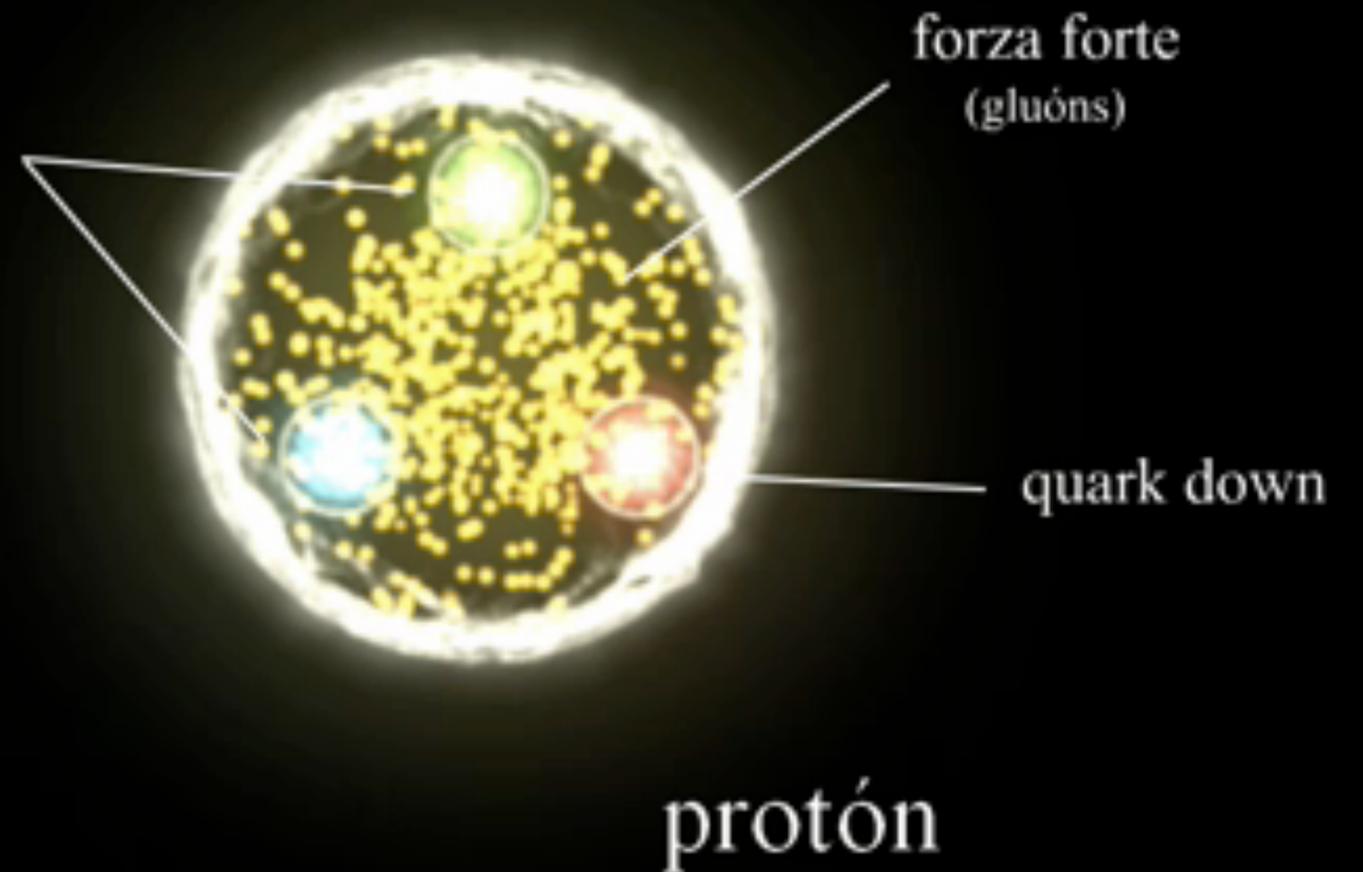
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GLUON MULTIPLICATION

See e.g. [arxiv:1709.04922](https://arxiv.org/abs/1709.04922) for a review

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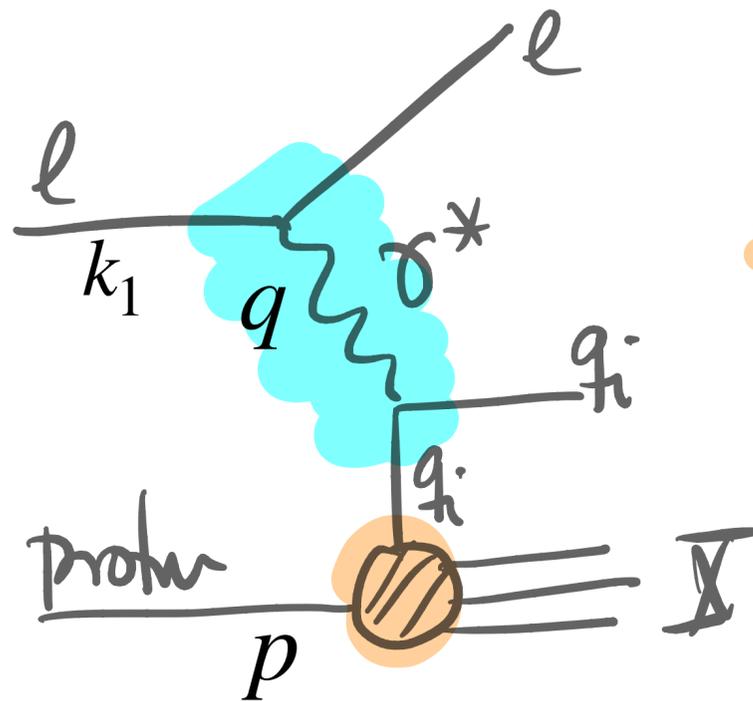
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DIS - parton model



~~≠~~ PARTON MODEL ~~⇒~~

$f_{q_i}(x)$ Probability to find q_i with the fraction of momentum x **PDF**

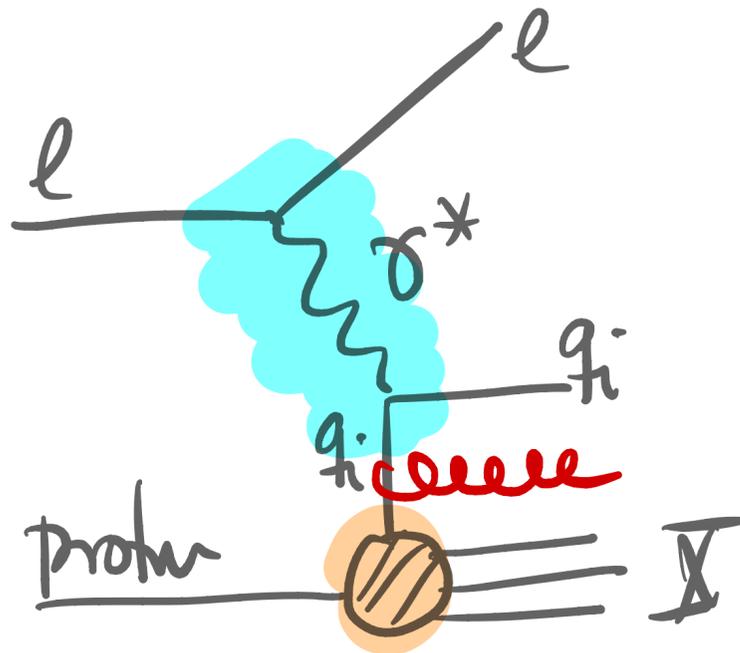
$$\frac{d\sigma}{dx dQ^2} = \sum_i \int dx f_{q_i}(x) \frac{d\hat{\sigma}_i}{dx dQ^2} = \underbrace{\frac{2\pi\alpha^2}{xQ^4} [1+(1-y)^2]}_{\text{Kinematics}} \underbrace{F_2(x)}_{\text{Structure of the proton}}$$

Structure function [Bjorken scaling]

$$F_2(x) = x \sum_{q_i} \frac{Q_{q_i}^2}{f_{q_i}} P_{q_i}(x) = x \left[\frac{4}{9} (u(x) + \bar{u}(x)) + \frac{1}{9} (d(x) + \bar{d}(x)) + s(x) + \bar{s}(x) + \dots \right]$$

Kinematical variables $Q^2 \equiv -q^2$, $x = \frac{Q^2}{2p \cdot q}$, $y = \frac{p \cdot q}{p \cdot k_1}$

Parton model + QCD



PARTON MODEL + SOFT GLUON

Now $\frac{d\hat{\sigma}}{dx dQ^2} \rightarrow \frac{d\hat{\sigma}}{dx dQ^2} \left[1 + \frac{2\alpha_s G_F}{\pi} \frac{1}{k_{\perp}^2} P(x) \right]$

We have to integrate in k_{\perp} & x $\rightarrow P(x) \rightarrow \frac{1}{x}$

$\int_{\mu_0}^Q \frac{dk_{\perp}^2}{k_{\perp}^2} = \log \frac{Q^2}{\mu_0^2} \xrightarrow{\mu_0 \rightarrow 0} \infty$

Renormalization

$$\frac{\partial F_2(x, \mu^2)}{\partial \log \mu^2} = 0$$



$$\frac{\partial f_q(x, \mu^2)}{\partial \log \mu^2} = \frac{\alpha_s}{2\pi} \int \frac{dy}{y} P(y) f_q\left(\frac{x}{y}\right)$$

[Dokshitzer, Gribov, Lipatov, Altarelli, Parisi - DGLAP - see next slide]

DIS - parton model

Full DGLAP are coupled integro-differential eqs.

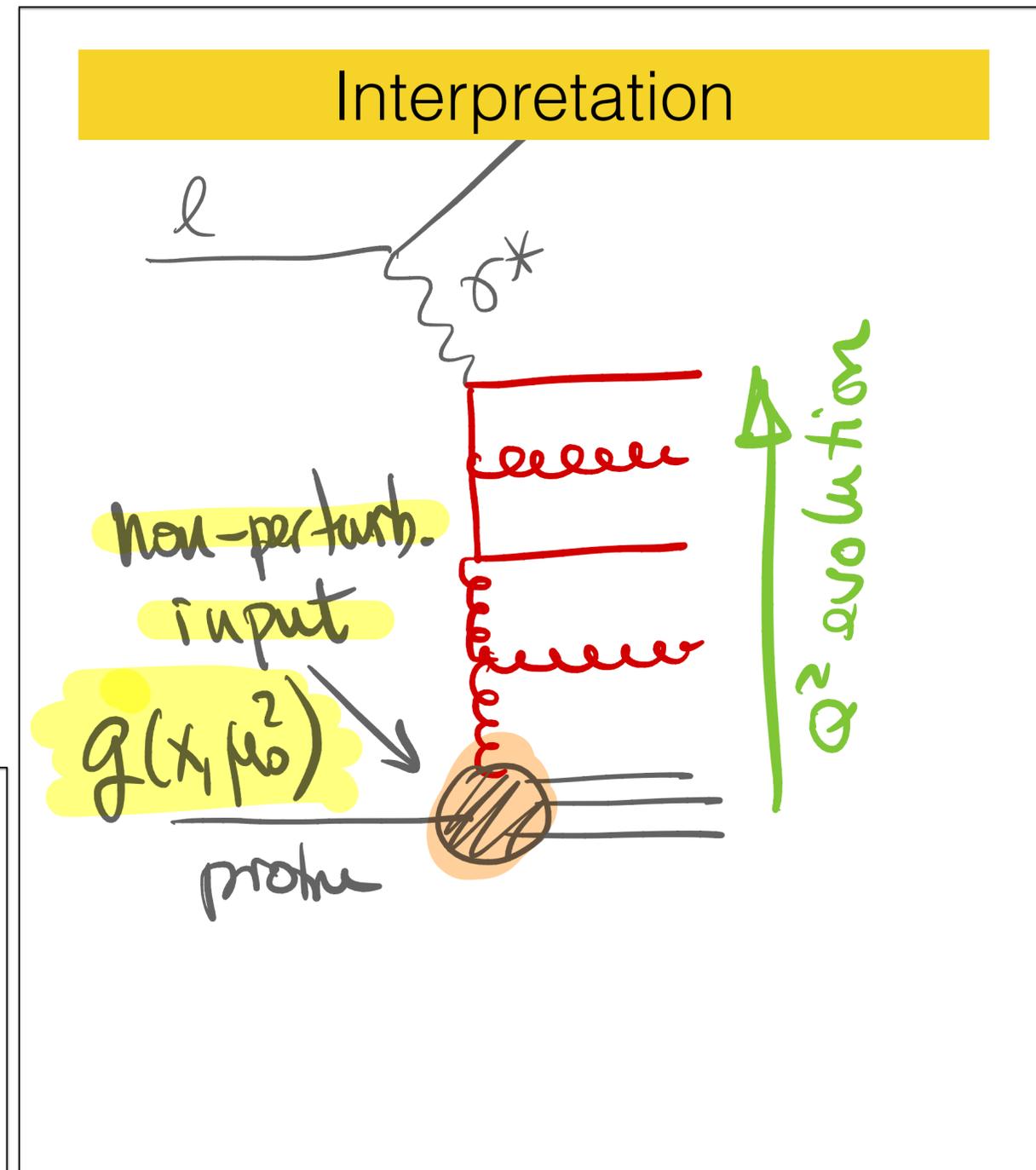
$$\frac{\partial q_i}{\partial \log Q^2} = \frac{\alpha_s}{2\pi} \left[\sum_j P_{q_i q_j} \otimes q_j + P_{q_i g} \otimes g \right]$$

$$\frac{\partial g}{\partial \log Q^2} = \frac{\alpha_s}{2\pi} \left[\sum_j P_{g q_j} \otimes q_j + P_{g g} \otimes g \right]$$

Initial conditions needed:

non-perturbative input at initial scale μ_0^2

$$u(x, \mu_0^2), \bar{u}(x, \mu_0^2), d(x, \mu_0^2), \bar{d}(x, \mu_0^2), s(x, \mu_0^2), \dots, g(x, \mu_0^2)$$



DIS - parton model

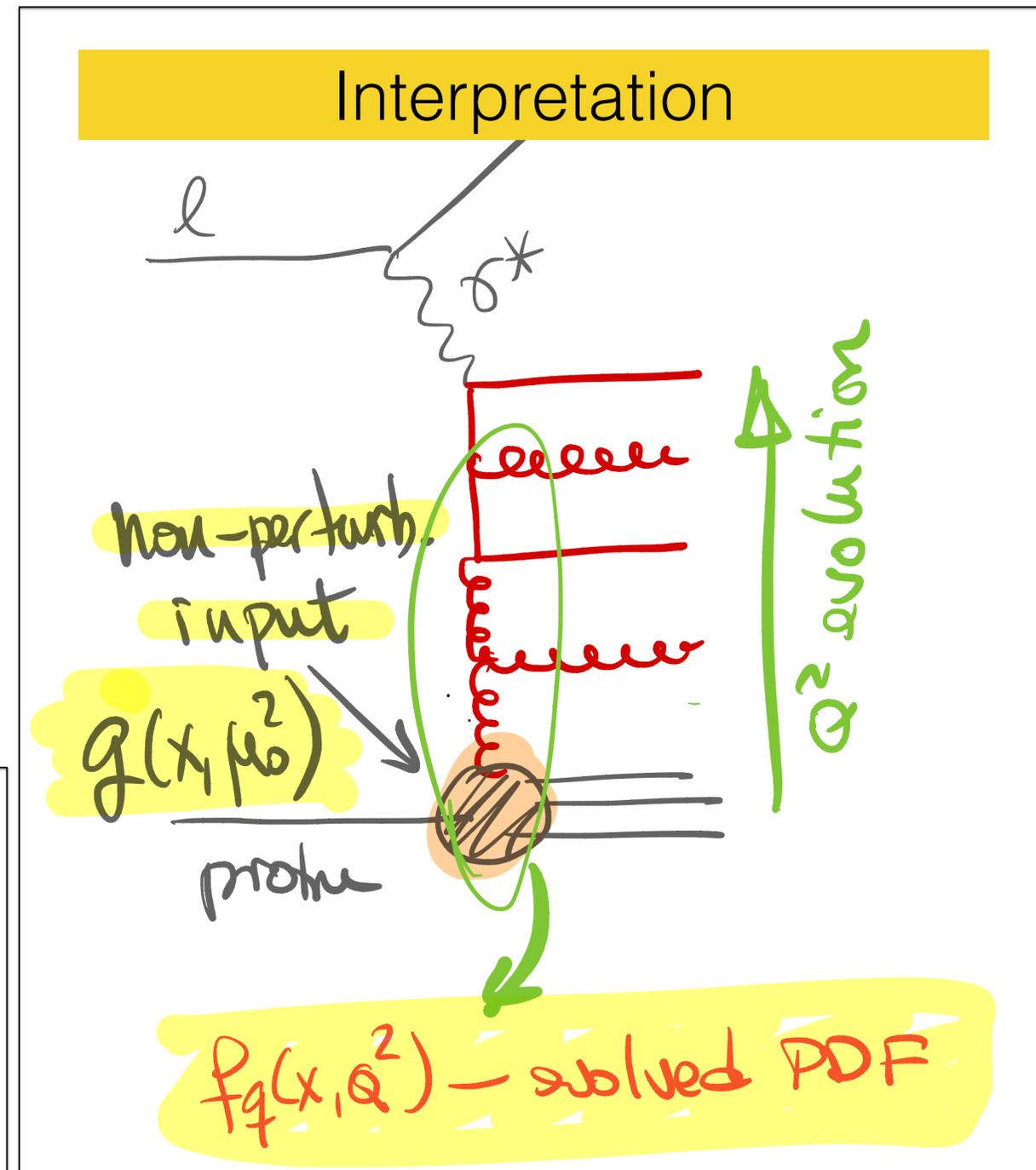
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Splitting functions

Building block of DGLAP equations: encode the probability of splitting [included here just for completeness]

$$P_{q \leftarrow g}(z) = T_F [z^2 + (1-z)^2],$$

$$P_{q \leftarrow q}(z) = C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2} \delta(1-z) \right],$$

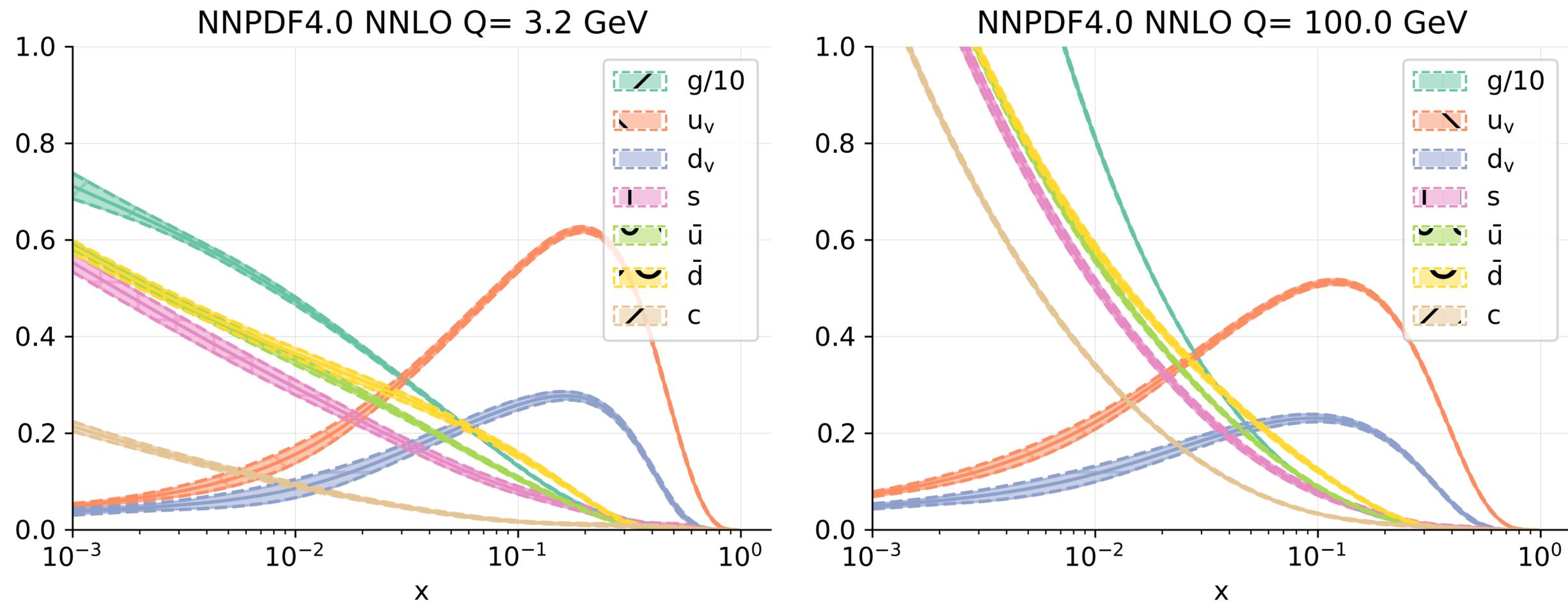
$$P_{g \leftarrow q}(z) = C_F \left[\frac{1+(1-z)^2}{z} \right],$$

$$P_{g \leftarrow g}(z) = 2C_A \left[\frac{1-z}{z} + \frac{z}{(1-z)_+} + z(1-z) \right] + \left(\frac{11}{6} C_A - \frac{2}{3} T_F n_f \right) \delta(1-z).$$

$$C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}, \quad C_A = N_c = 3 \quad \text{and} \quad T_F = \frac{1}{2}$$

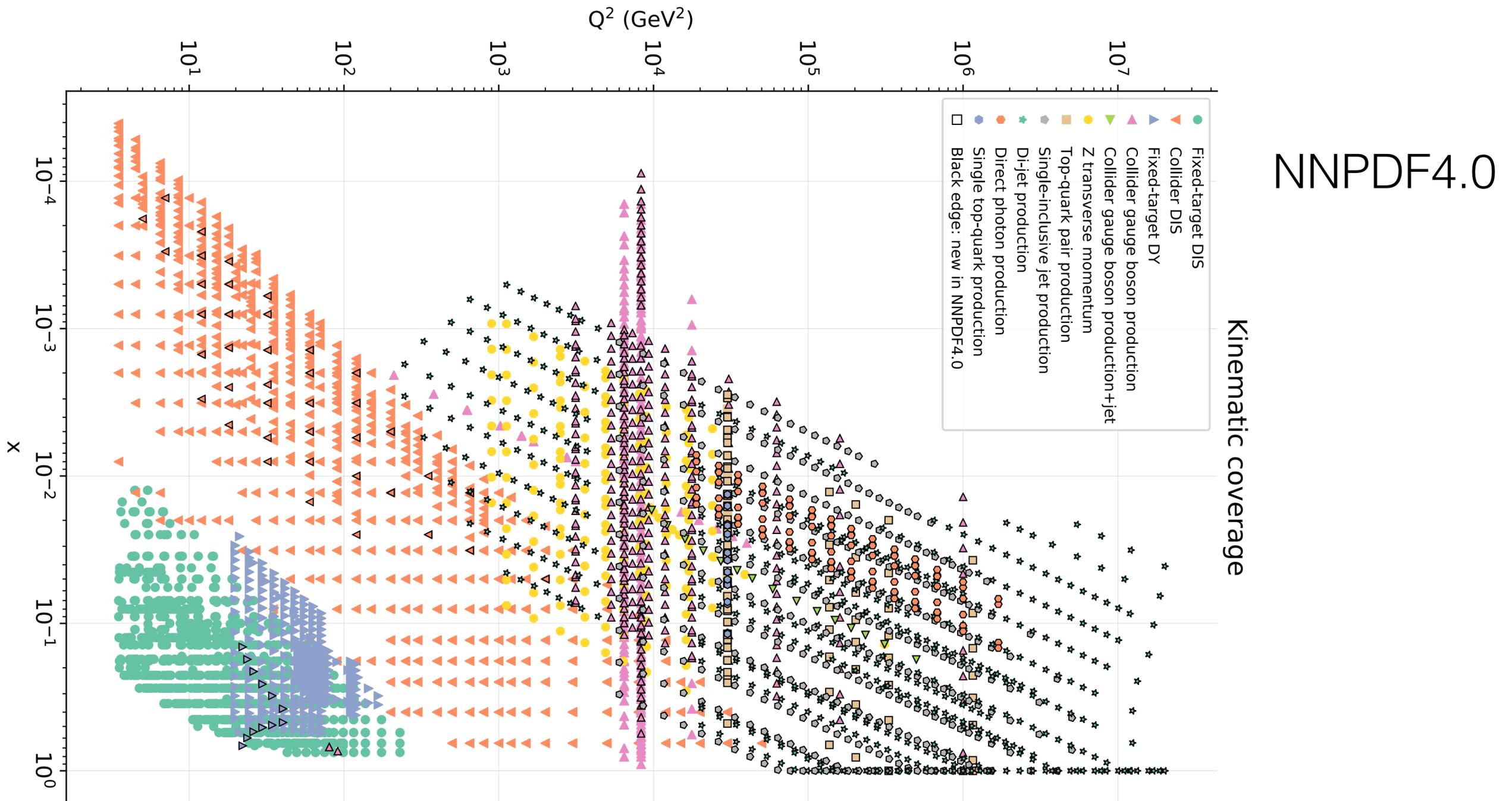
NNPDF4.0 set

Parton Distribution Functions for the proton from NNPDF global analysis



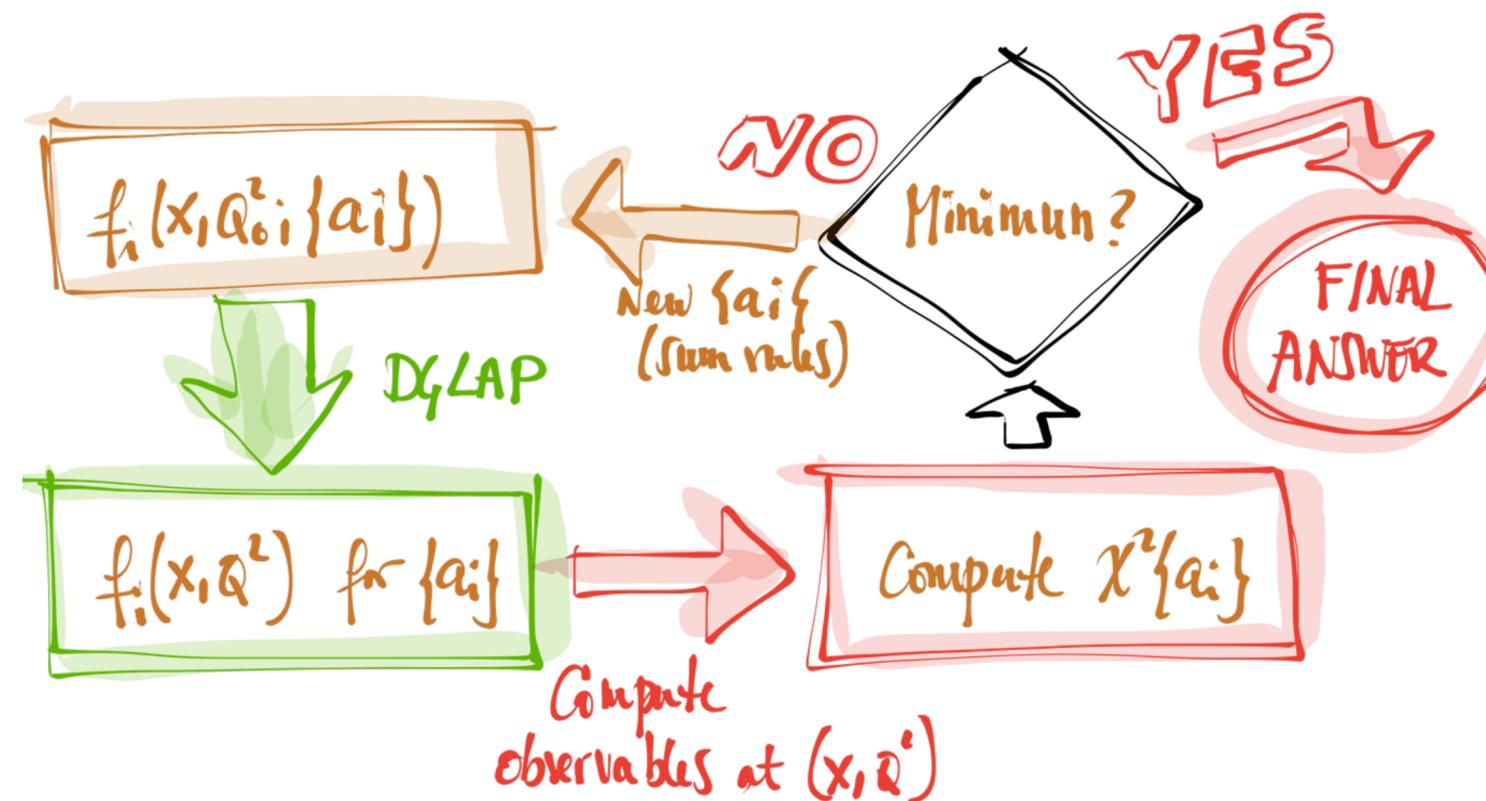
Huge number of data needed to achieve this degree of precision

Kinematic coverage by data



Global PDF fits

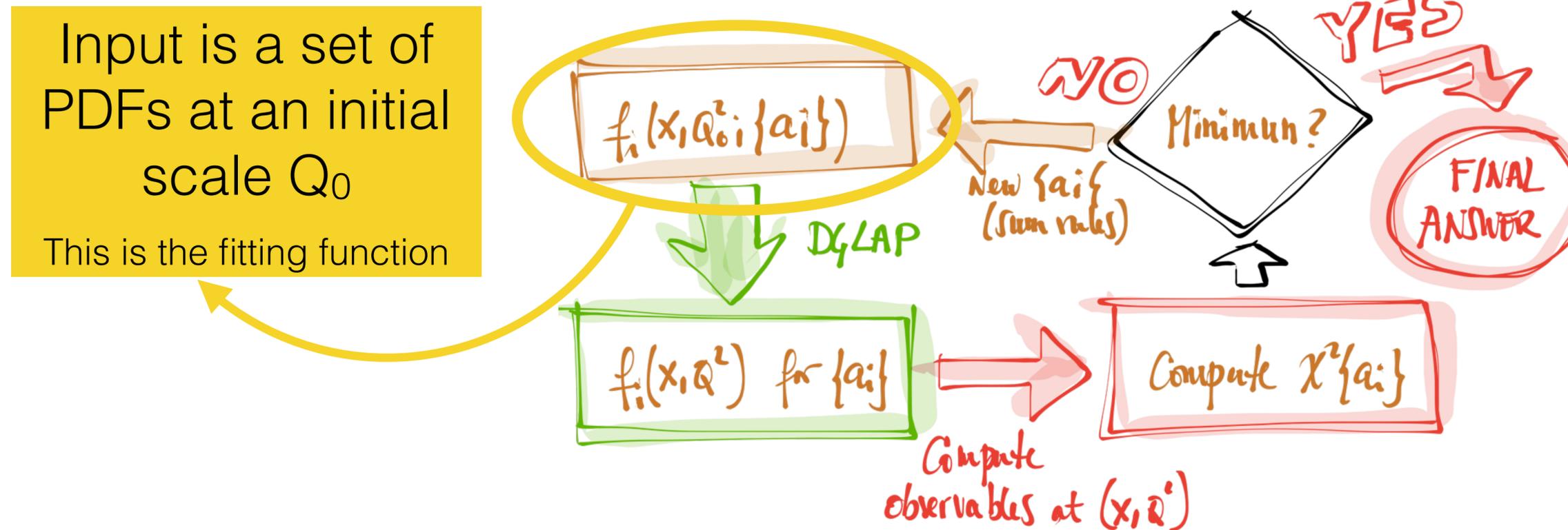
- ▶ One of the most standardized procedures in High-Energy Physics.
- ▶ Main goal: provide a set of Parton Distribution Functions (PDFs)



Different sets differ mostly on how I.C. are parametrized and how to treat the error analysis

Global PDF fits

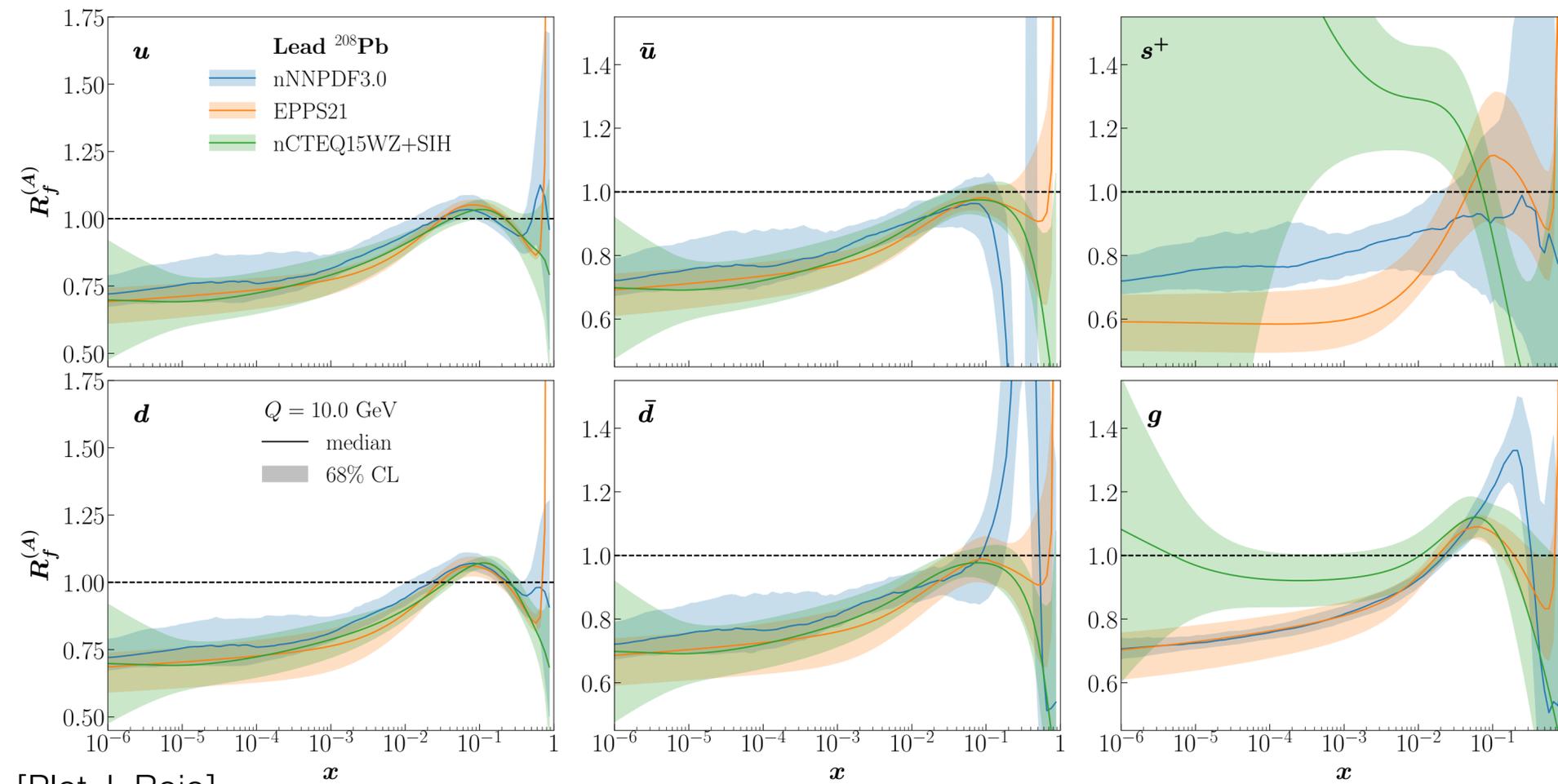
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Nuclear PDFs

Similar analyses performed for nuclei - especially Pb

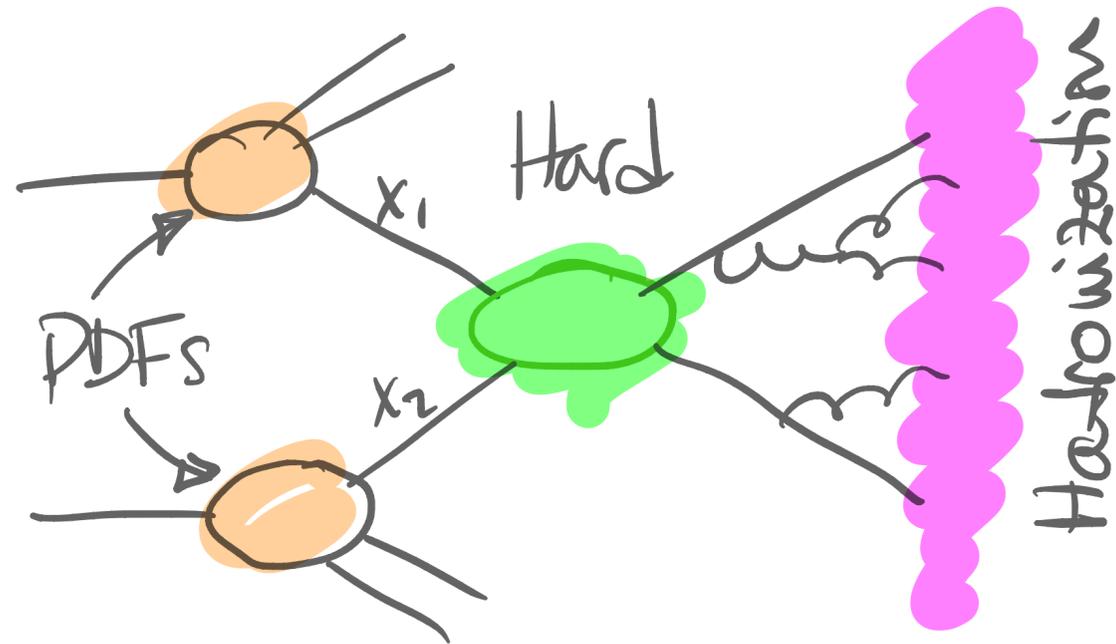


In terms of ratios to a known set of proton PDF

$$R_i(x, Q^2) = \frac{f_i^A(x, Q^2)}{f_i^p(x, Q^2)}$$

Two new analyses
EPPS21, nNNPDF3.0

Factorization



QCD factorization between small and long distance processes

- Long distance:** non-perturbative (PDFs and hadronization)
- Short distance:** perturbative elementary cross section

Example:
$$\frac{d\sigma^{pp \rightarrow h}}{dP_t} = f_i^p(x_1, Q^2) \otimes f_j^p(x_2, Q^2) \otimes \frac{d\hat{\sigma}^{ij \rightarrow k}}{dt} \otimes D(z, Q^2)$$

Region between small- and long-distance: parton resummation (e.g. DGLAP)

This formalism/procedure allows to compute any cross section at LHC involving large scales: jets, H/W/Z (+jets), etc... [any cross section need long-distance information: PDFs]

Summary

QCD is the theory of strong interactions

- **QCD has a rich dynamical content well within experimental reach**
- Needed for all phenomenology at hadron colliders (and most of other colliders as well)

Asymptotic freedom allows to perform perturbative computations at large scales

- However, non-perturbative contributions always present (due to confinement)
- Large regions of phase-space need to resum large logarithms - gluon multiplication

Jets and Parton Distribution Functions (PDFs) two main tools at the LHC

- Parton shower and DGLAP evolution

Huge effort in precision computations at NNLO and more [not mentioned here]