Astroparticle Physics Exercises

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Astro-quiz

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1. Gamma-ray horizon

Exercise Compute the energy of background photons at threshold for pair-production for incoming photons of 1 TeV or 1 PeV energy. What type of "light" are these bands corresponding to? In the case of CMB, described by a blackbody spectrum at 2.7 K, compute the mean-free path of a photon of typical PeV energies.

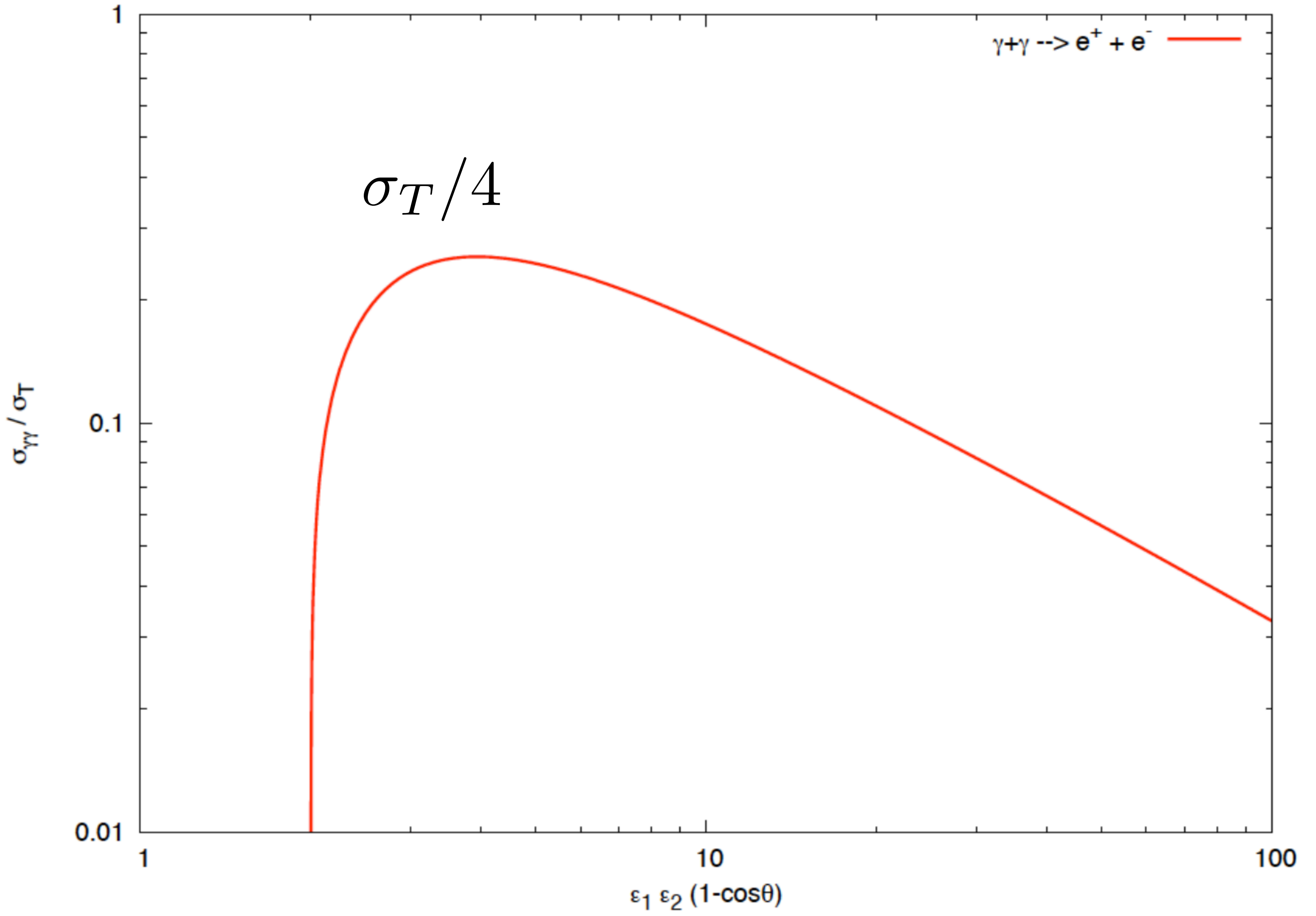
The exercise above is meant to illustrate the fact that the gamma-ray sky may not always be optically thin, with the $e^{\pm}$ pair production mechanism constituting a serious limitation for the remote extragalactic sky already at hundred of GeV, for the near extragalactic sky at about 10 TeV, and even for Galactic objects in the PeV range (see the *photon horizon* in Fig. 15). Put otherwise, we expect that beyond the TeV range, we can only perform astronomy in the "local neighborhood".

$$n_\gamma = 411\,\mathrm{cm}^{-3}$$

$$\sigma_{\gamma\gamma}(\beta) = \frac{3\pi\sigma_T}{16}\left(1-\beta^2\right)\left[2\beta\left(\beta^2-2\right)+\left(3-\beta^4\right)\ln\left(\frac{1+\beta}{1-\beta}\right)\right]$$

β is either lepton velocity in the COM frame.

$$\sigma_T = 6.65\times10^{-25}\,\mathrm{cm}^2$$



2. SNR and Galactic CRs

Exercise : Knowing that SN are estimated to happen 2-3 times per century in the Galaxy, each releasing a few times 10^{51} erg in kinetic energy, what it their "kinetic" luminosity? Compare that with the power needed to sustain a steady-state population of CRs, with integrated energy density of about $0.5\mathrm{eV/cm}^3$ filling a confinement volume of the Milky Way assumed to be a cylinder with radius 15 kpc and height 4 kpc, and typical "lifetime" of $\tau_{CR} \simeq$10 Myr. Is it enough for the SNRs to power CRs? What is the ratio of the two, or if you wish the efficiency of macrocopic kinetic energy conversion into CR acceleration?

3. The Crab spectrum

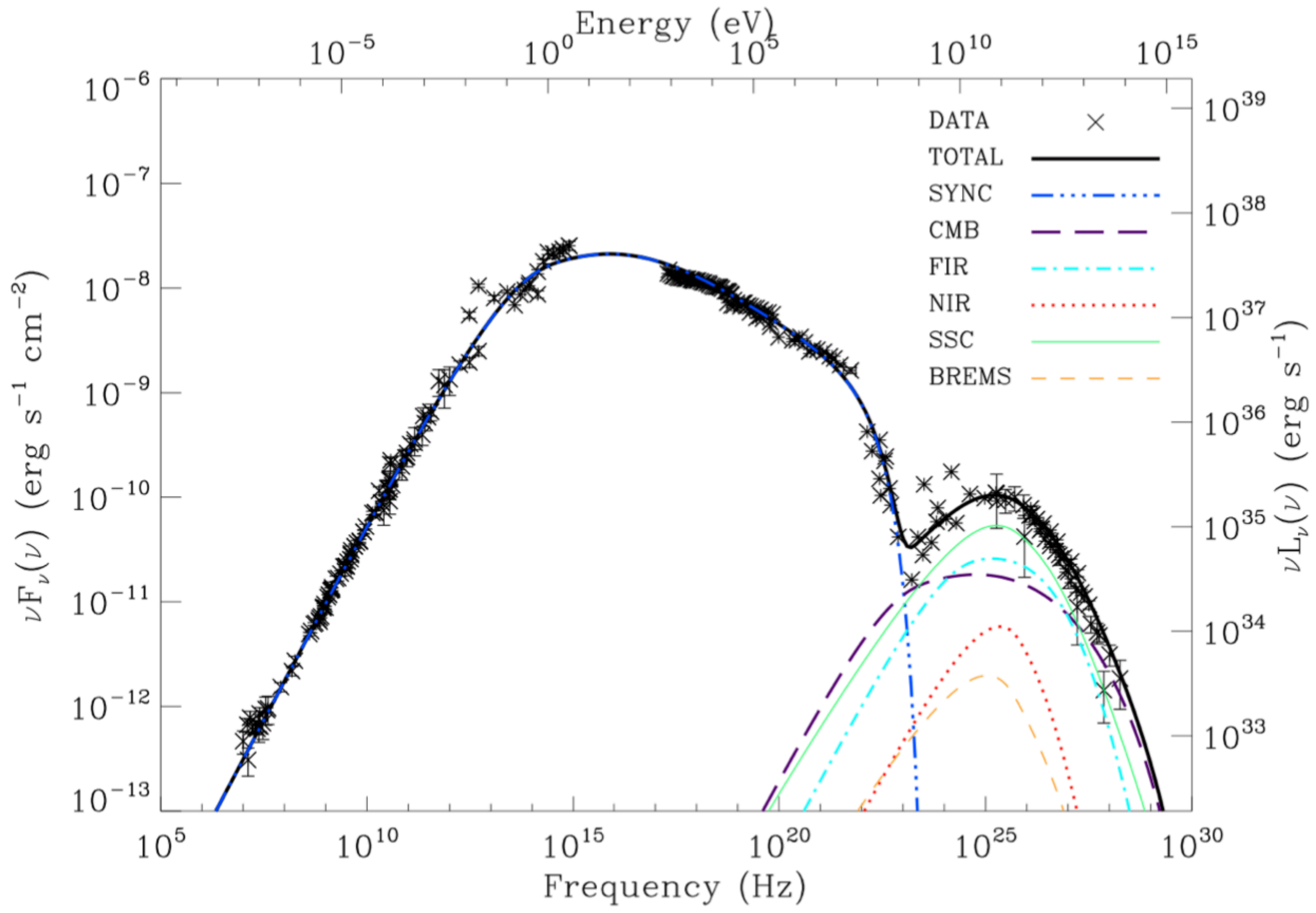


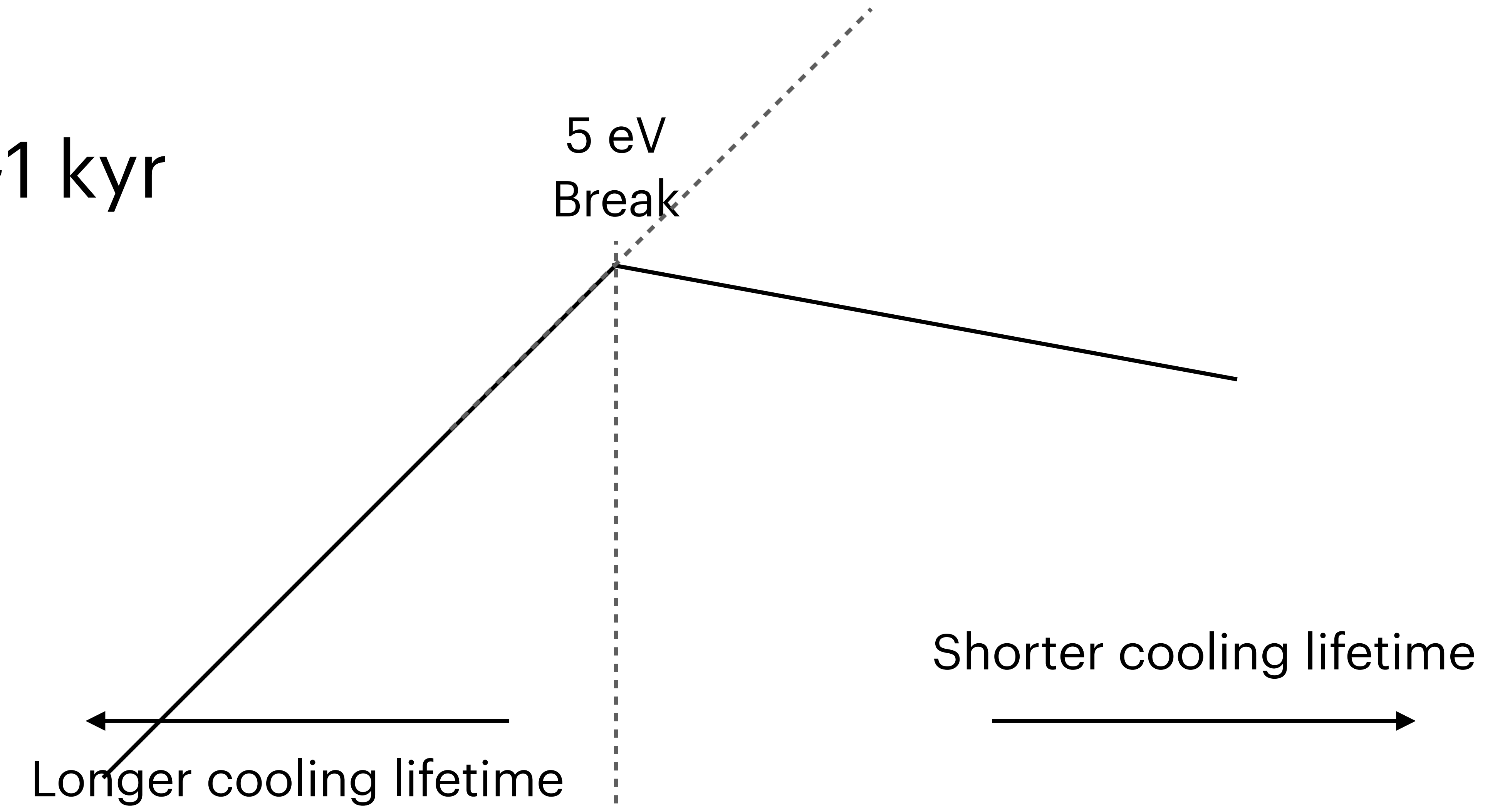
FIG. 20: The multiwavelength photon spectrum of the Crab nebula, also compared to models, taken from [43].

3. The Crab spectrum

Exercise The Crab nebula, associated to the explosion of a SN in AD 1054, presents a roughly broken power-law spectrum, with a steepening around 5 eV, interpreted as synchrotron radiation, see Fig. 20. If we attribute this phenomenon to a "cooling break" (i.e. the electrons producing the photons below the break have a cooling lifetime longer than its age, and the ones producing photons above it have a shorter cooling timescale), determine the magnetic field in the source, and the energy of the electrons associated to this "break point". What is the energy of IC photons produced when those electrons hit the very synchrotron photons at the break point?

Synchrotron self-Compton emission (SSC): IC on synchrotron photons from electron population

Age of the Crab ~1 kyr



4. CR secondaries and primaries

Exercise

Consider two CR species: primaries with number density n_p and secondaries (initially not produced!) with number density n_s. If the two are coupled by the spallation process $p \to s + \ldots$, then

$$\frac{dn_p}{dX} = -\frac{n_p}{\lambda_p} \tag{7}$$

$$\frac{dn_s}{dX} = -\frac{n_s}{\lambda_s} + \frac{p_{\mathrm{p}\to\mathrm{s}} n_p}{\lambda_p} \tag{8}$$

where $X \equiv \int \mathrm{d}\ell \rho(\ell)$ is the *grammage*, the density integrated along the actual path followed by the particle, measured in g/cm^2; $\lambda_i = \rho_{ISM}/(n_{ISM}\sigma_i) = m_{ISM}/\sigma_i$ is the *interaction length* of the species i in the ISM medium, in terms of the effective ISM mass $m_{ISM} \simeq m_p$, and total inelastic cross-section σ_i (appropriately weighted by the medium composition); it is measured in g/cm^2, as X; $p_{\mathrm{p}\to\mathrm{s}}$ is the probability that, in an inelastic interaction of species p, the species s is produced (the relevant b.r.), and is dimensionless. Note that λ_i are dependent from laboratory measurements and the ISM composition, not really from the CR path. For instance, one has $\sigma_{\mathrm{CNO}} \simeq 6.7$ g/cm^2, $\sigma_{\mathrm{LiBeB}} \simeq 10$ g/cm^2, and $p \simeq 0.35$ between these two groups. From the measured value $n_s/n_p \simeq 0.25$, deduce X, and compare with the predictions for X for straight lines crossing the Milky Way disk with an angle between $30°$ and $60°$, assuming a density of 1 hydrogen atom/cm^3 and a Galactic half-thickness $h \simeq$100 pc. Based on this, estimate the typical timescales the CRs spent in the gaseous disk.

5. Leaky box from slab model

Exercise : "Leaky box" from slab model

Let us assume that sources are confined to an infinitesimal disk. The steady state transport equation simplifies into

$$-\frac{\partial}{\partial z}\left(K\frac{\partial \phi}{\partial z}\right) = 2\,q_0(p)\,h\delta(z)\ . \tag{9}$$

Find the CR flux $\phi(z,p)$. Compare it with the solution of a "leaky box" type of model, where injection at rate Q happens in a homogeneous and isotropic medium from which cosmic rays take a (momentum dependent) time $\tau_{\rm esc}(p)$ to escape, leading to the steady state solution $\phi(p) = Q(p)\tau_{\rm esc}(p)$.