CNR-INO
ISTITUTO NAZIONALE DI OTTICA
CONSIGLIO NAZIONALE DELLE RICERCHE

BEC

REALIZING ANALOG MODELS WITH TWO-COMPONENT BOSE-EINSTEIN CONDENSATES

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Collaborators: R. Cominotti, A. Farolfi, G. Lamporesi, A. Zenesini, G. Ferrari (from the BEC lab)
A. Recati, L.Giacomelli, L. Fernandes, S. G. Butera (theory and simulations)

Anna Berti

Pitaevskii BEC Center, University of Trento
Benasque, 1 June 2023

Q@TN

OUTLINE

- General introduction on atomic two-component BEC mixtures

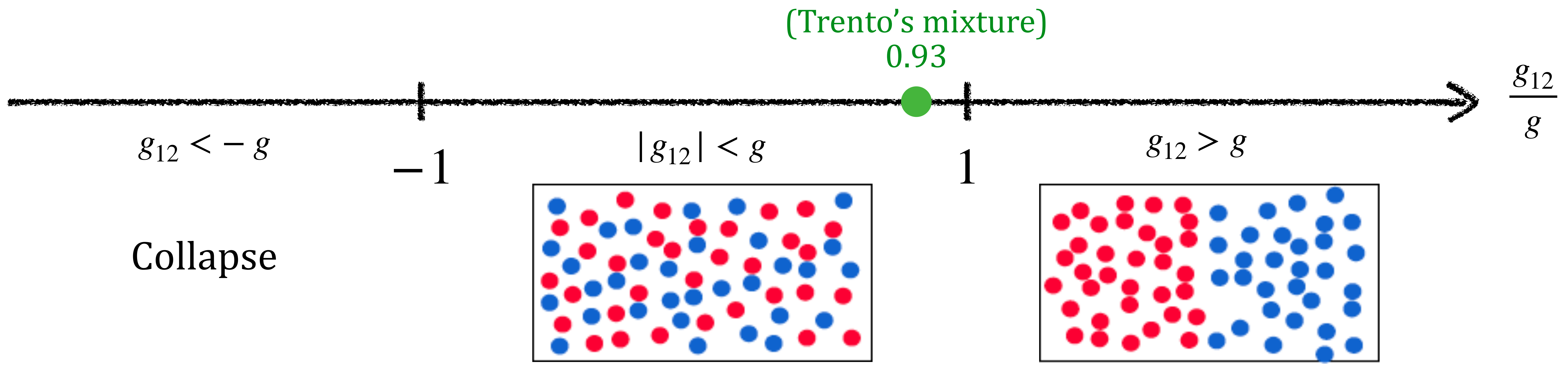
- **Pair-creation** in a time-evolving background
  R.Cominotti, **AB**, A. Farolfi, A. Zenesini, G. Lamporesi, I. Carusotto, A. Recati, and G. Ferrari, Phys. Rev. Lett. **128**, 210401 (2022)
- Analog **Hawking emission** in one-dimensional BH geometry
  AB, L. Fernandes, S.G.Butera, A.Recati and I. Carusotto (work in progress)
- Analog **ergoregion instabilities** with vortex configurations
  AB, L. Giacomelli and I. Carusotto, arXiv:2212.07337 (2022, accepted on Comptes Rendus Physique)

TWO-COMPONENT ATOMIC SUPERFLUIDS

Can be realized with two different atomic species or exploiting two internal states of the same atoms. In the latter case, a coherent coupling allows to transfer atoms from one component to the other.

$$i\hbar\frac{\partial\psi_1}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V + g|\psi_1|^2 + g_{12}|\psi_2|^2\right)\psi_1 - \frac{\Omega}{2}\psi_2$$

$$i\hbar\frac{\partial\psi_2}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V + g|\psi_2|^2 + g_{12}|\psi_1|^2\right)\psi_2 - \frac{\Omega}{2}\psi_1$$

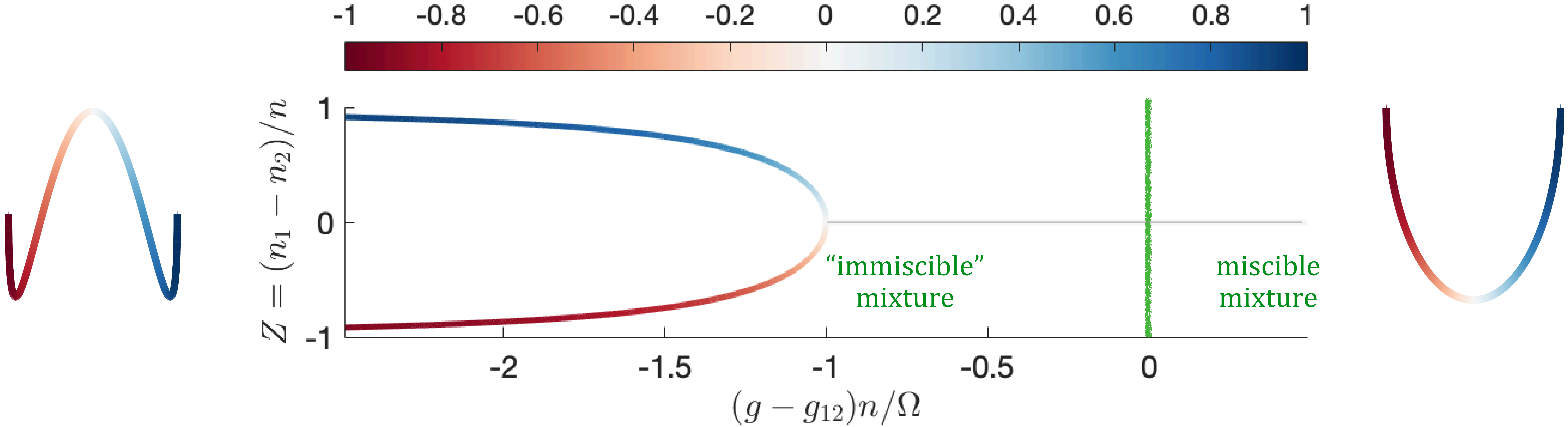


Abad and Recati, *Eur. Phys. J. D* **67,** 148 (2013)
Bose-Einstein Condensation and Superfluidity, Stringari and Pitaevskii (2016)

COHERENTLY-COUPLED MIXTURES

para-to-ferromagnetic quantum phase transition (driven by interactions) takes place at

$$(g - g_{12})n + \Omega = 0$$



Cominotti, **AB**, Dulin, Rogora, Lamporesi, Carusotto, Recati, Zenesini and Ferrari, arXiv:2209.13235 (2022, accepted on PRX)
Zenesini, **AB**, Cominotti, Rogora, Moss, Billam, Carusotto, Lamporesi, Recati and Ferrari, arXiv:2305.05225 (2023)

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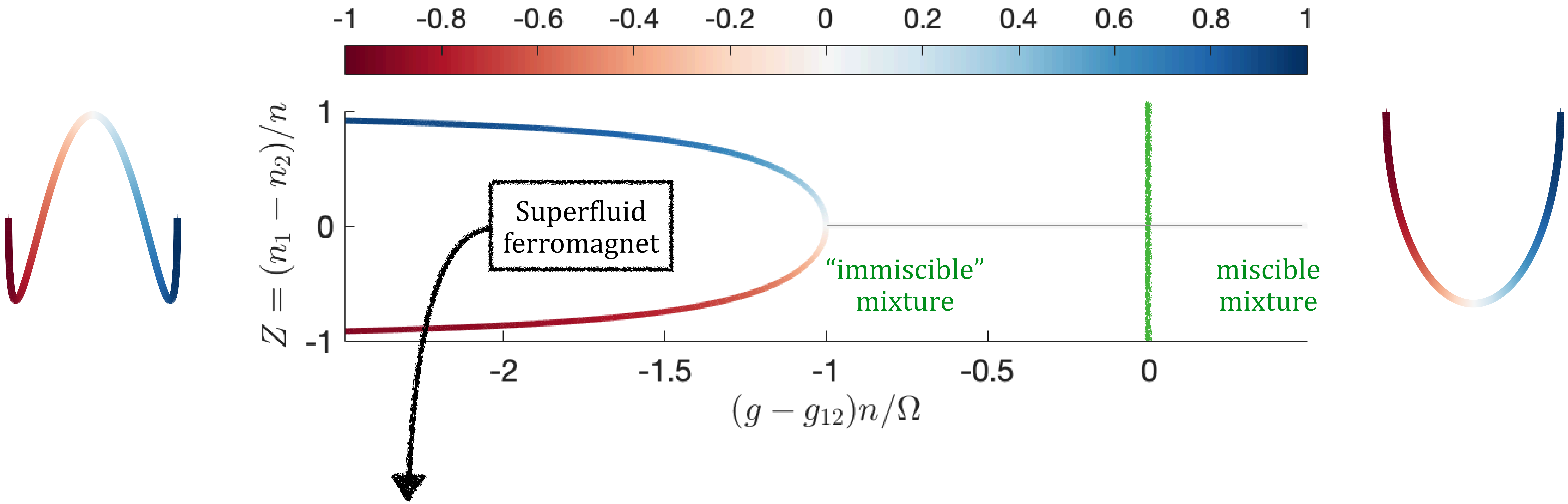

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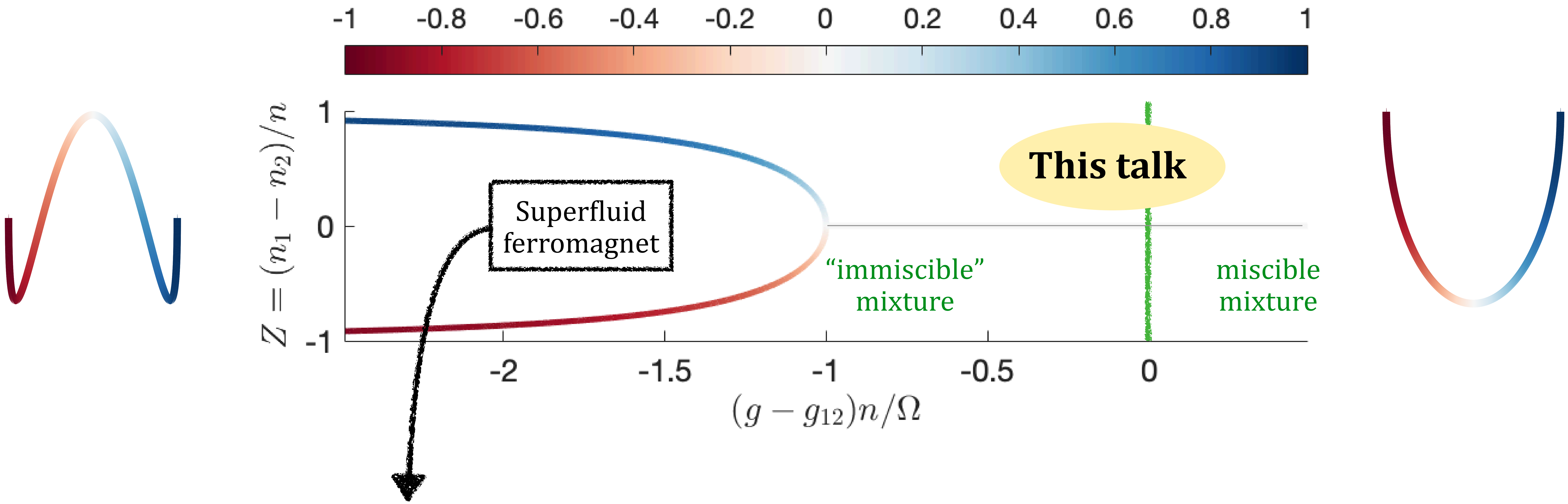

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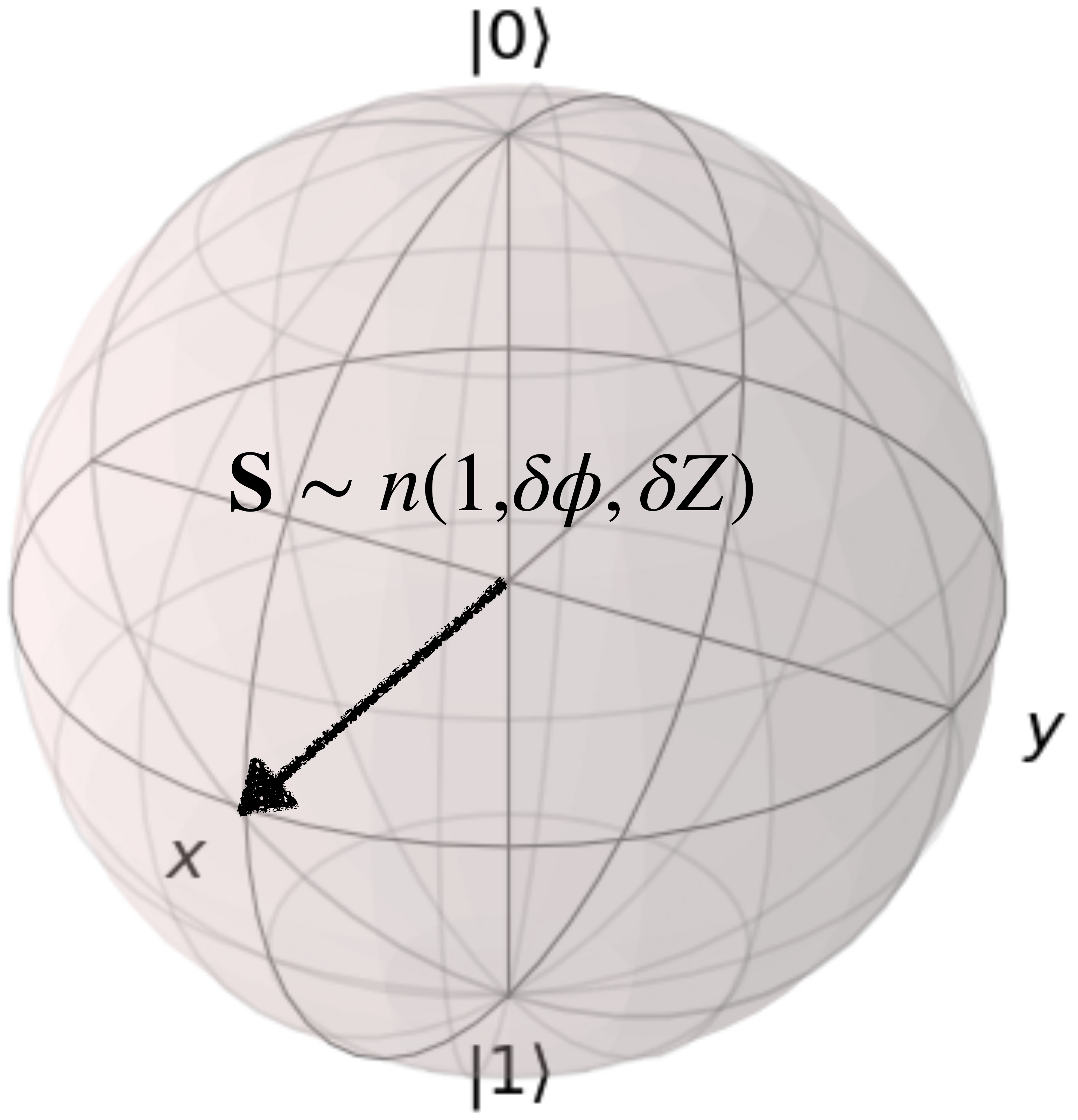
BLOCH SPHERE FORMALISM

State and dynamics of a mixture can be visualised on the Bloch sphere:

$$\mathbf{S} = \left(2\mathrm{Re}(\psi_1^*\psi_2), 2\mathrm{Im}(\psi_1^*\psi_2), |\psi_1|^2 - |\psi_2|^2\right)$$
$$= n\left(\sqrt{1-Z^2}\cos\phi, \sqrt{1-Z^2}\sin\phi, Z\right)$$

- relative phase can be measured by applying a $\pi/2$-pulse before measuring the relative density

The spin vector of a symmetric mixture lies on the x axis of the Bloch sphere, so small perturbations can be interpreted as fluctuations along z (relative density fluctuations) and along y (relative phase fluctuations)



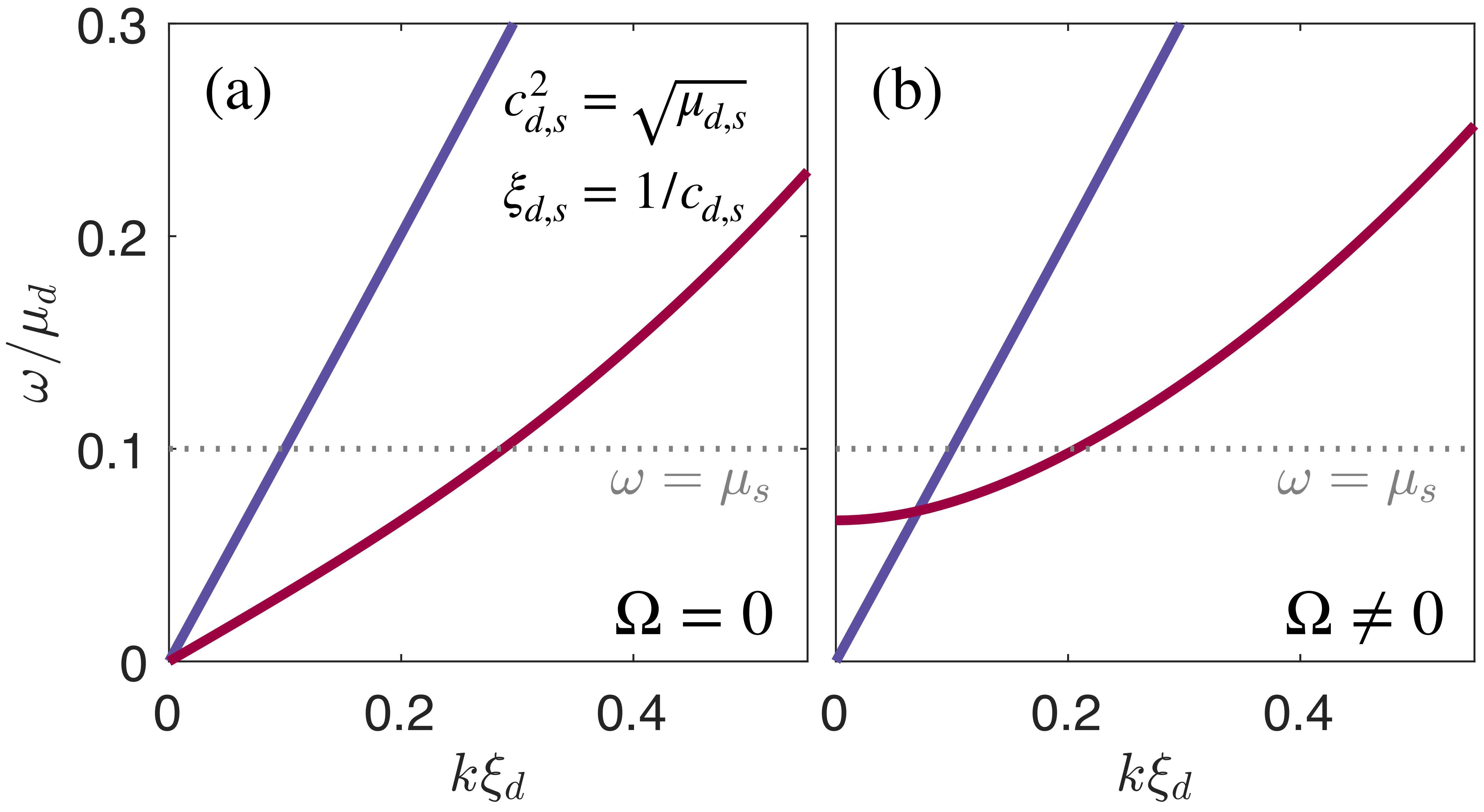
If the mixture is symmetric, excitations of total density and global phase (density modes) are independent from excitations of relative density and relative phase (spin modes).

- Density modes are unaffected by the presence of the coupling, which opens a gap in the spin dispersion
- Sound speeds can be much smaller for spin modes if $g_{12} \sim g$ (Trento's mixture has $\mu_d/\mu_s \sim 27$)

$$2\mu_d = (g + g_{12})n$$
$$2\mu_s = (g - g_{12})n$$

Doppler shift

$$(\omega_d - \mathbf{k}\cdot\mathbf{v})^2 = \frac{k^2}{2}\left(\frac{k^2}{2} + 2\mu_d\right)$$
$$(\omega_s - \mathbf{k}\cdot\mathbf{v})^2 = \left(\frac{k^2}{2} + \Omega\right)\left(\frac{k^2}{2} + \Omega + 2\mu_s\right)$$



TIME-DEPENDENT BACKGROUND

Time-dependent Hamiltonians allow to simulate evolving spacetimes:

- Cosmic inflation and accelerated expansion, preheating, cosmological particle creation, dynamical Casimir effect, back-reaction effects

PRA **70**, 063615 (2004), PRA **69**, 033602 (2004), PRA **76**, 033616 (2007), PRL 109, 220401 (2012), EPJ D 56, 391 (2010)
PRD 104, 083503 (2021), arXiv:2207.00311 (2022)

PAIR CREATION

Perturbation of "chemical potentials" $\mu_{d,s} \to \mu_{d,s}[1+f(t)]$ is realizable through a density modulation or by exploiting Feshback resonances to modify the interaction constants.

Quantized Bogoliubov Hamiltonian for spin modes (1D infinite system):

$$\widehat{\delta\mathcal{H}}_s = \sum_{k\neq 0}\left[\omega_s(k) + \mu_s S(k) f(t)\right]\hat{b}_k^\dagger b_k + \sum_{k>0} \mu_s S(k) f(t)\left(\hat{b}_k^\dagger \hat{b}_{-k}^\dagger + h.c.\right)$$

Pair creation

Matrix element depends on the structure factor of the mixture:

$$S(k) = \frac{\sqrt{\omega_s^2(k) + \mu_s^2} - \mu_s}{\omega_s(k)}$$

FARADAY PATTERNS

Dynamics for the amplitude β of the excited modes

$$\frac{\partial\beta(t)}{\partial t^2} + \omega_s^2(k)\left[1 + \frac{2\mu_s S(k)}{\omega_s(k)} f(t)\right]\beta(t) = 0$$

R.Cominotti, **AB**, A. Farolfi, A. Zenesini, G. Lamporesi, I. Carusotto,
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If $f(t) \propto \sin(\omega_M t)$, this coincides with the Mathieu equation, which describes the generation of Faraday patterns

R.Cominotti, **AB**, A. Farolfi, A. Zenesini, G. Lamporesi, I. Carusotto, A. Recati, and G. Ferrari, Phys. Rev. Lett. **128**, 210401 (2022)

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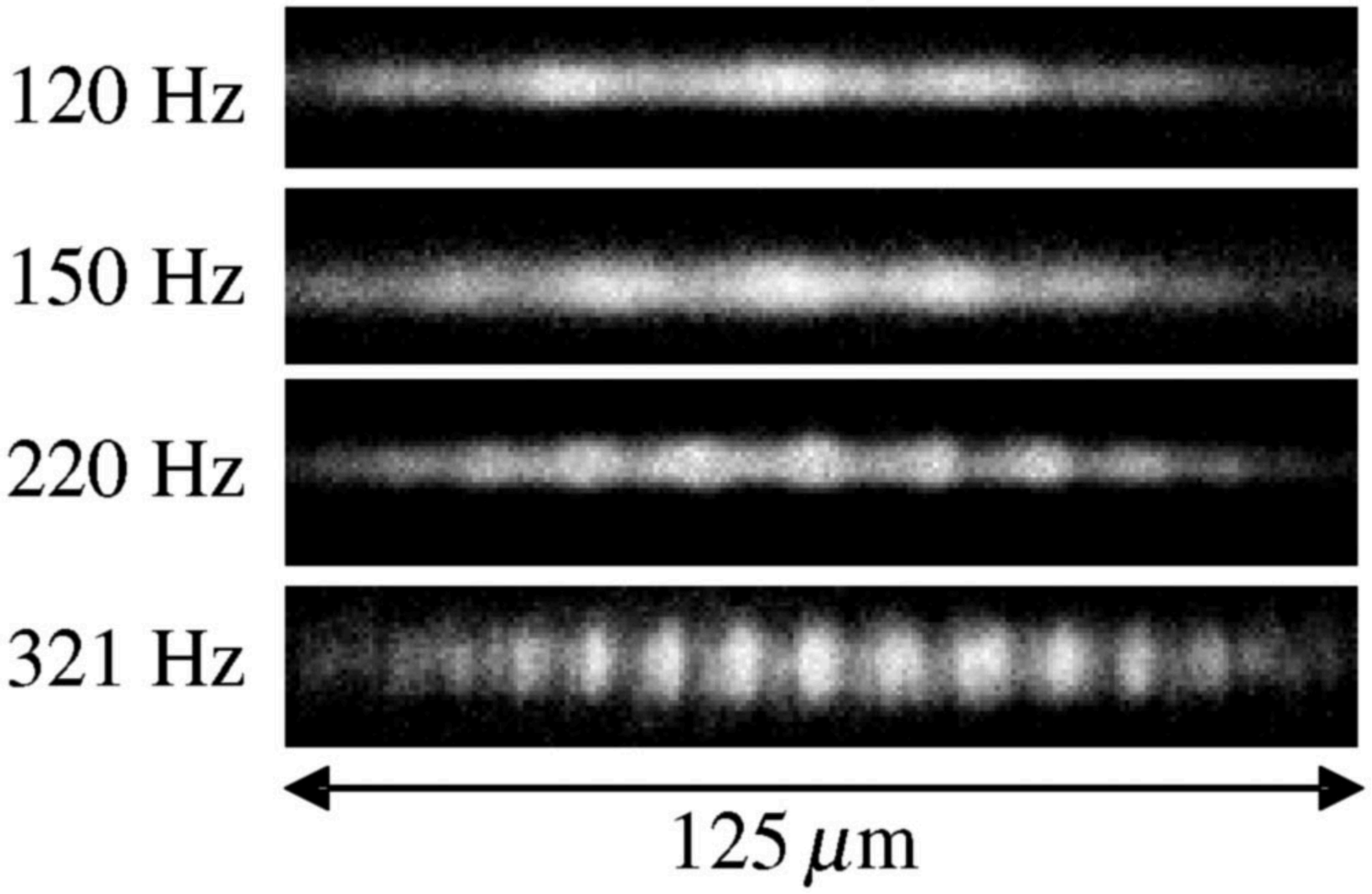
PRL **98**, 095301 (2007)



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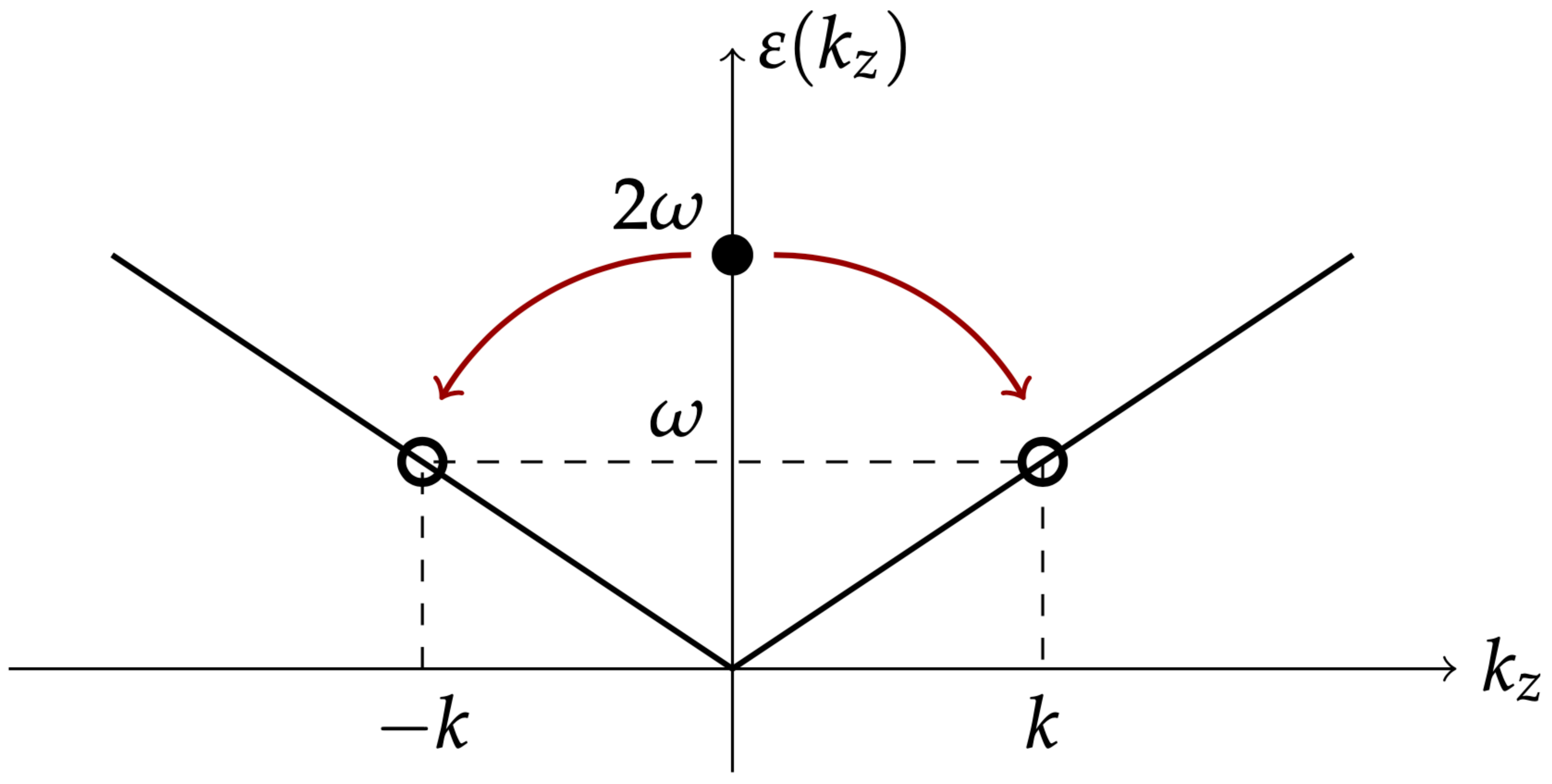

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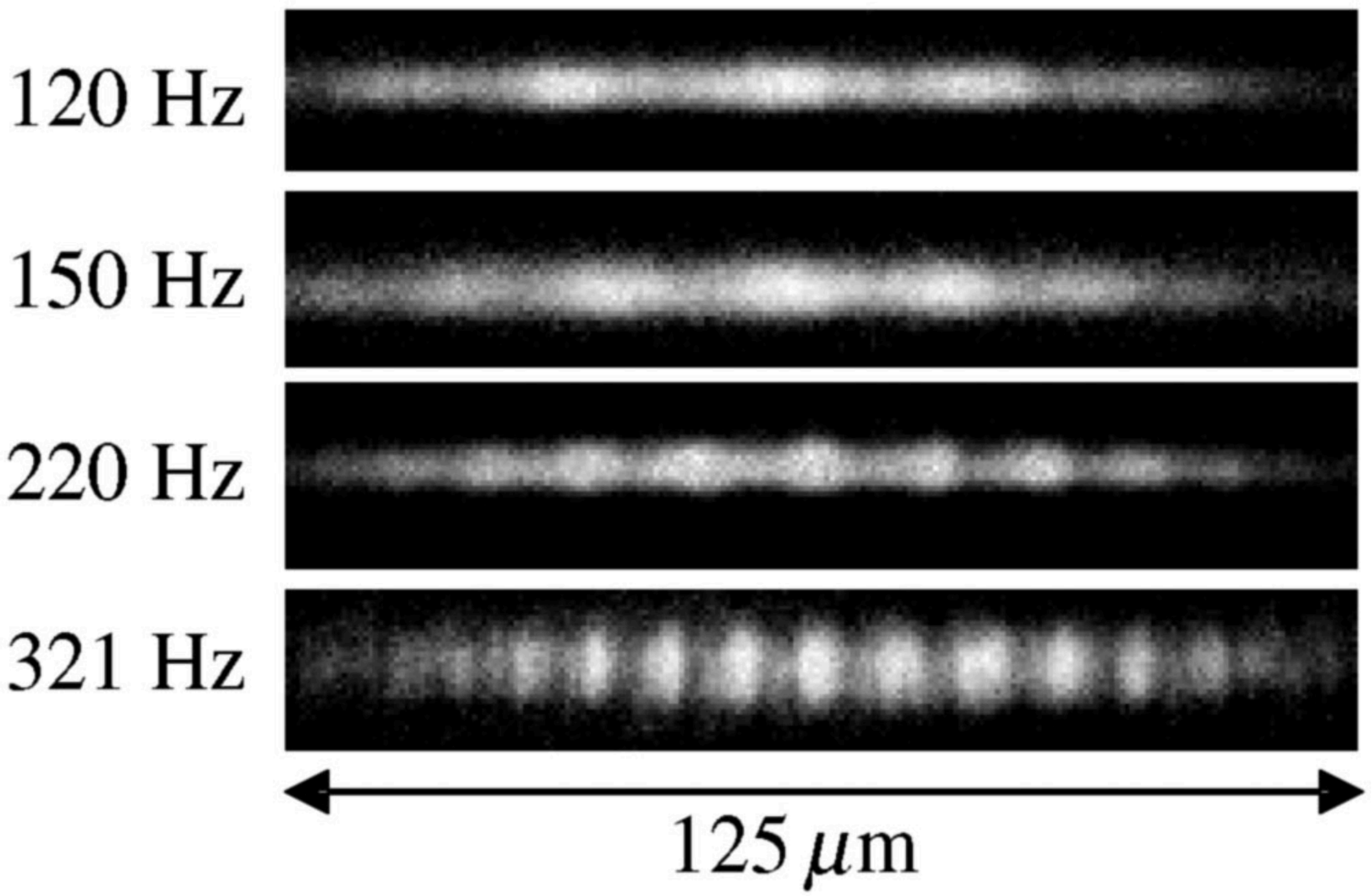

FARADAY PATTERNS

Cigar-shaped mixture of Sodium atoms in $(F, m_F) = (1, \pm 1)$

Modulation of both chemical potentials is realized through a periodic driving of the transverse trapping frequency

Focus on the longitudinal dynamics by integrating along the transverse direction

R.Cominotti, **AB**, A. Farolfi, A. Zenesini, G. Lamporesi, I. Carusotto, A. Recati, and G. Ferrari, Phys. Rev. Lett. **128**, 210401 (2022)

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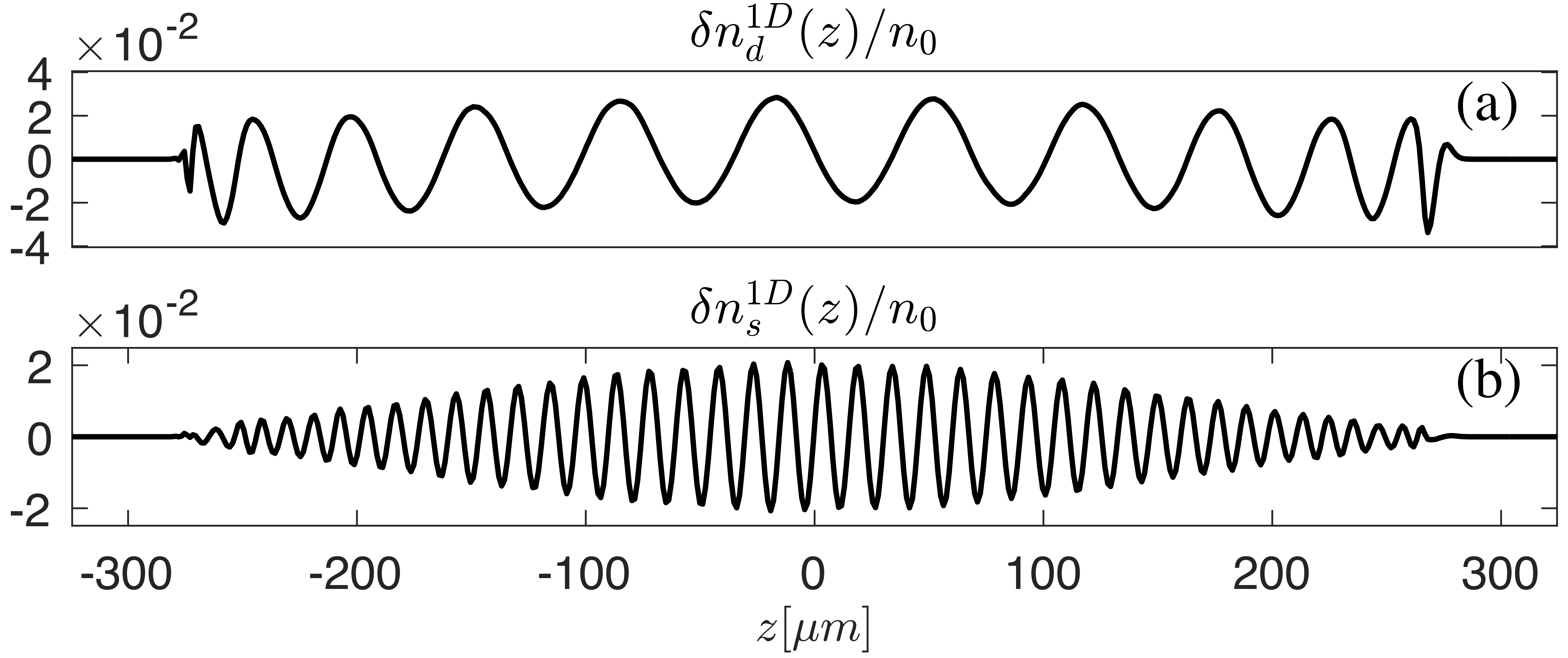
Cigar-shaped mixture of Sodium atoms in $(F, m_F) = (1, \pm 1)$
Modulation of both chemical potentials is realized through a periodic driving of the transverse trapping frequency
Focus on the longitudinal dynamics by integrating along the transverse direction

$$\mu_d \simeq 2\pi \times 3kHz, \quad \mu_s \simeq 2\pi \times 130Hz, \quad \omega_M \simeq 2\pi \times 200Hz$$



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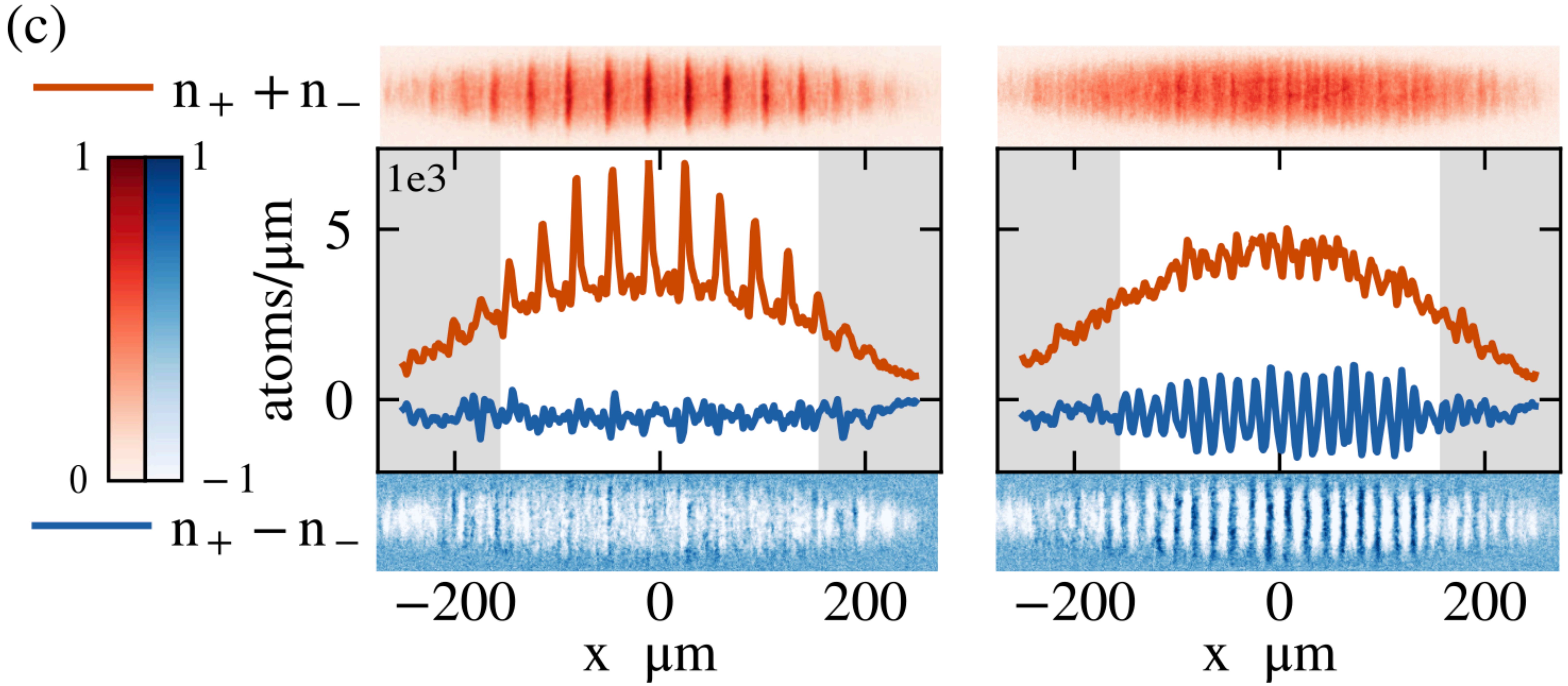

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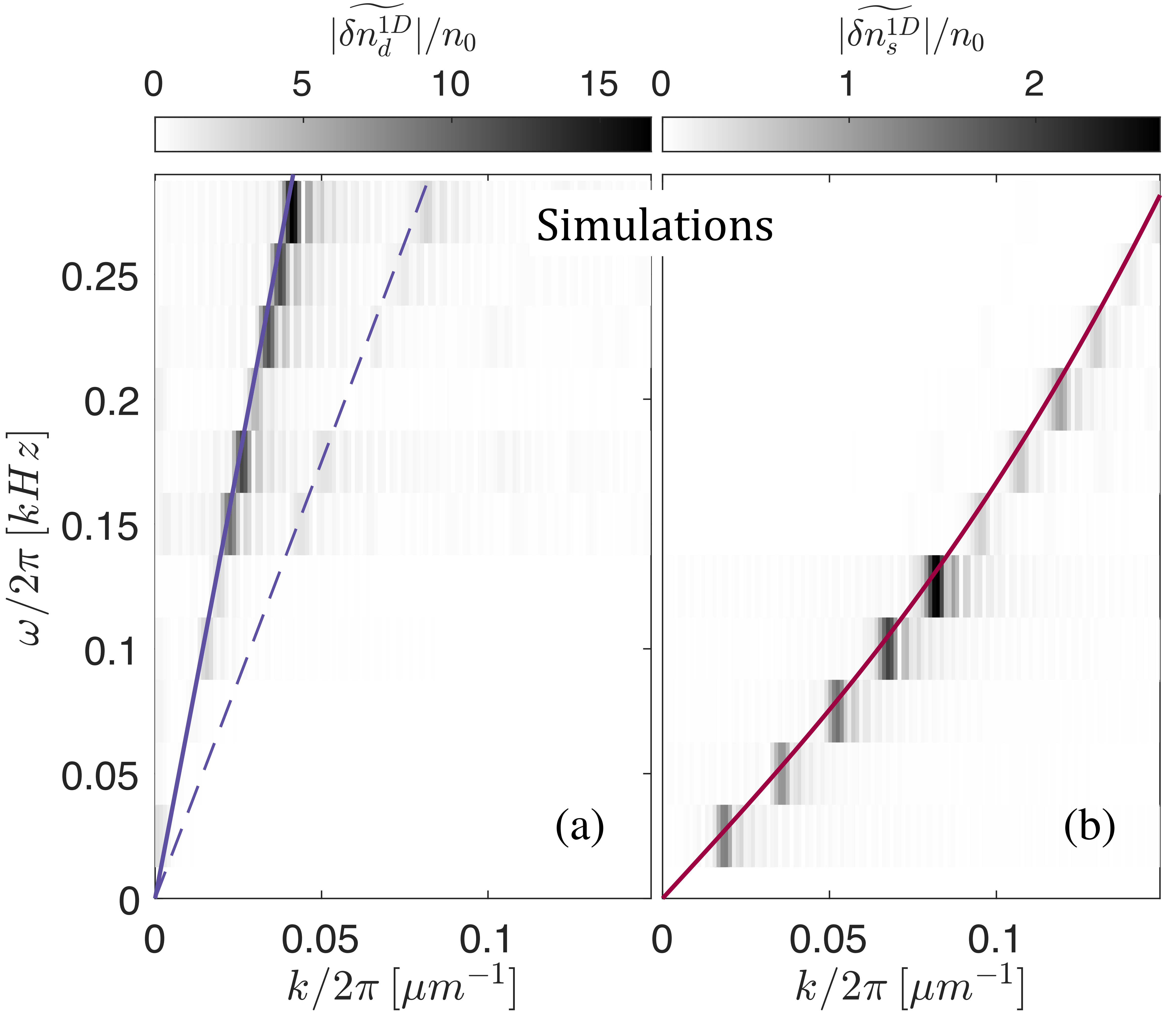

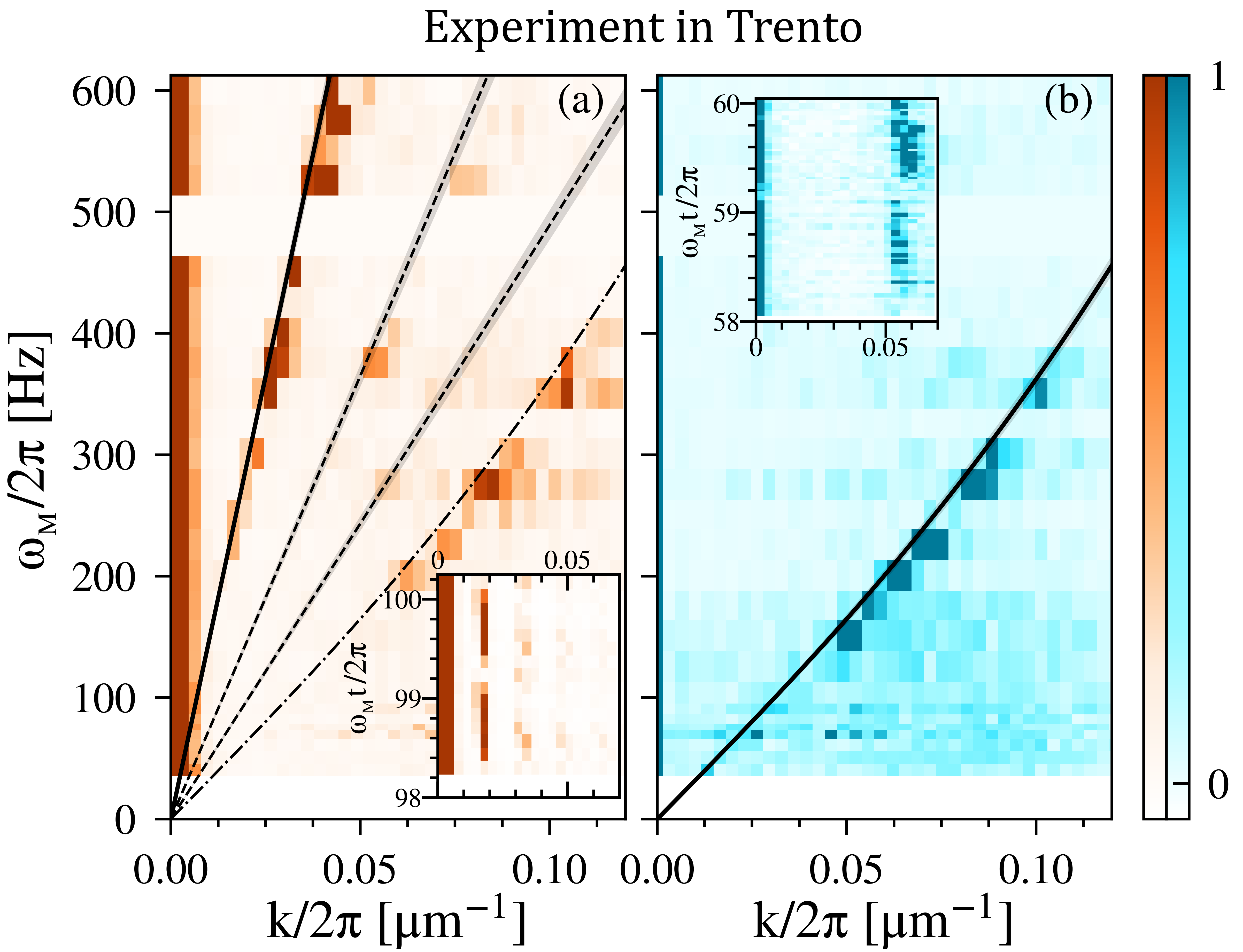

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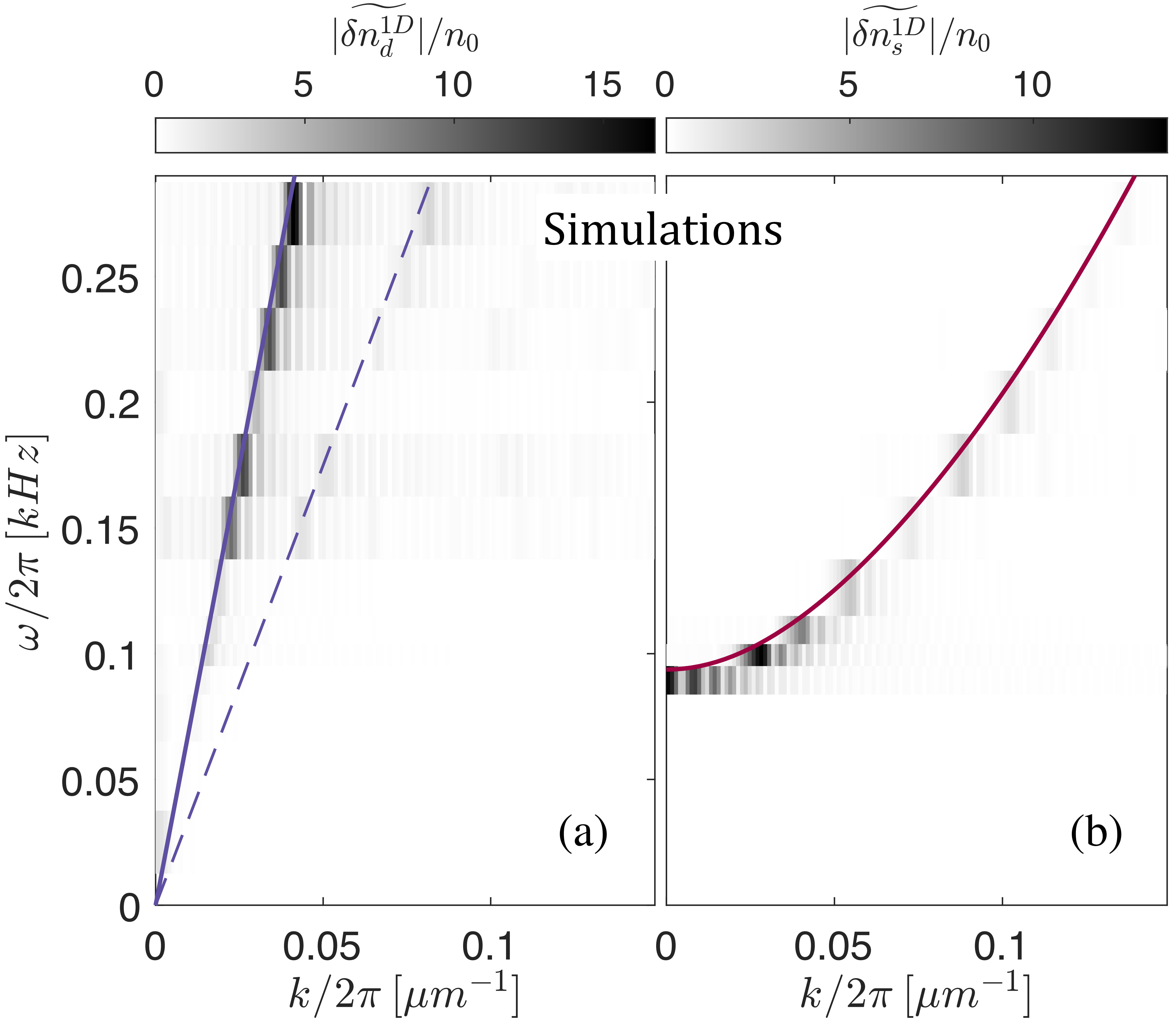
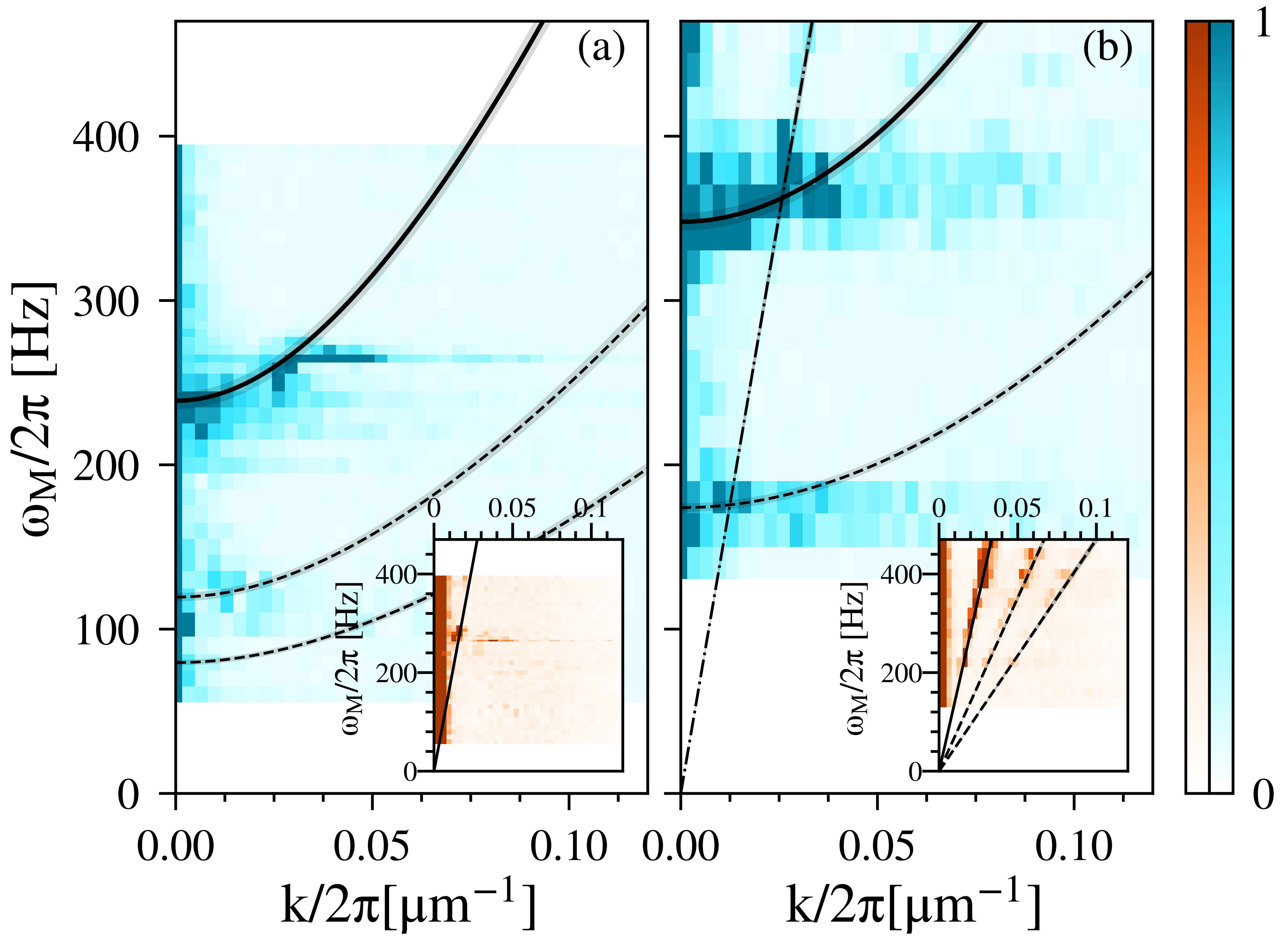
The protocol works for massive excitations as well!



Experiment in Trento



R.Cominotti, **AB**, A. Farolfi, A. Zenesini, G. Lamporesi, I. Carusotto, A. Recati, and G. Ferrari, Phys. Rev. Lett. **128**, 210401 (2022)

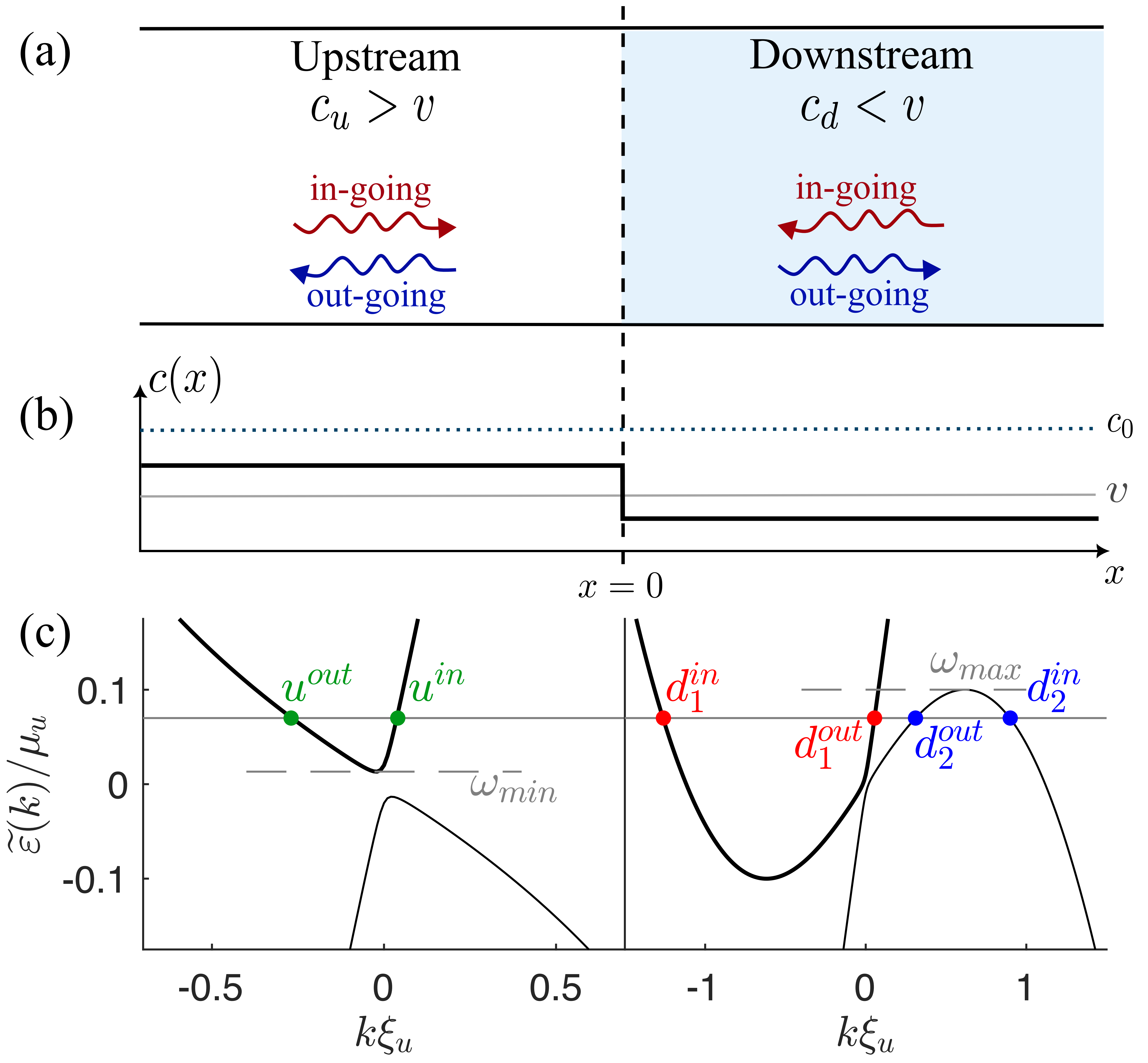
ANALOG HAWKING EMISSION

Stimulated or spontaneous emission of pairs of particles with opposite energy at an analog event horizon.

PRA 80, 043603 (2009), EPL 103 60001 (2013), Nat. Phys 12, 959-965 (2016)

One-dimensional configuration with an sharp interface separating a subsonic from a supersonic region.

The dispersion relation is tilted by the Doppler shift: propagating negative energy modes appear in the supersonic region.

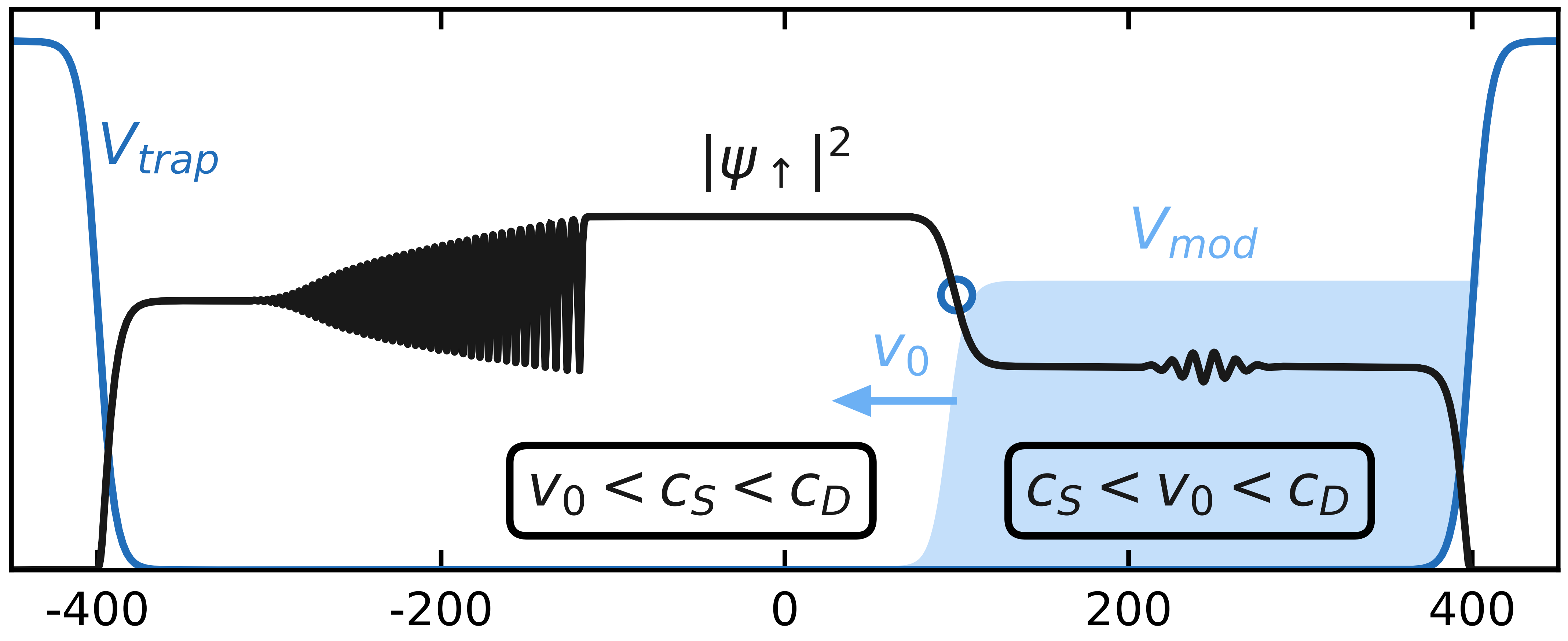


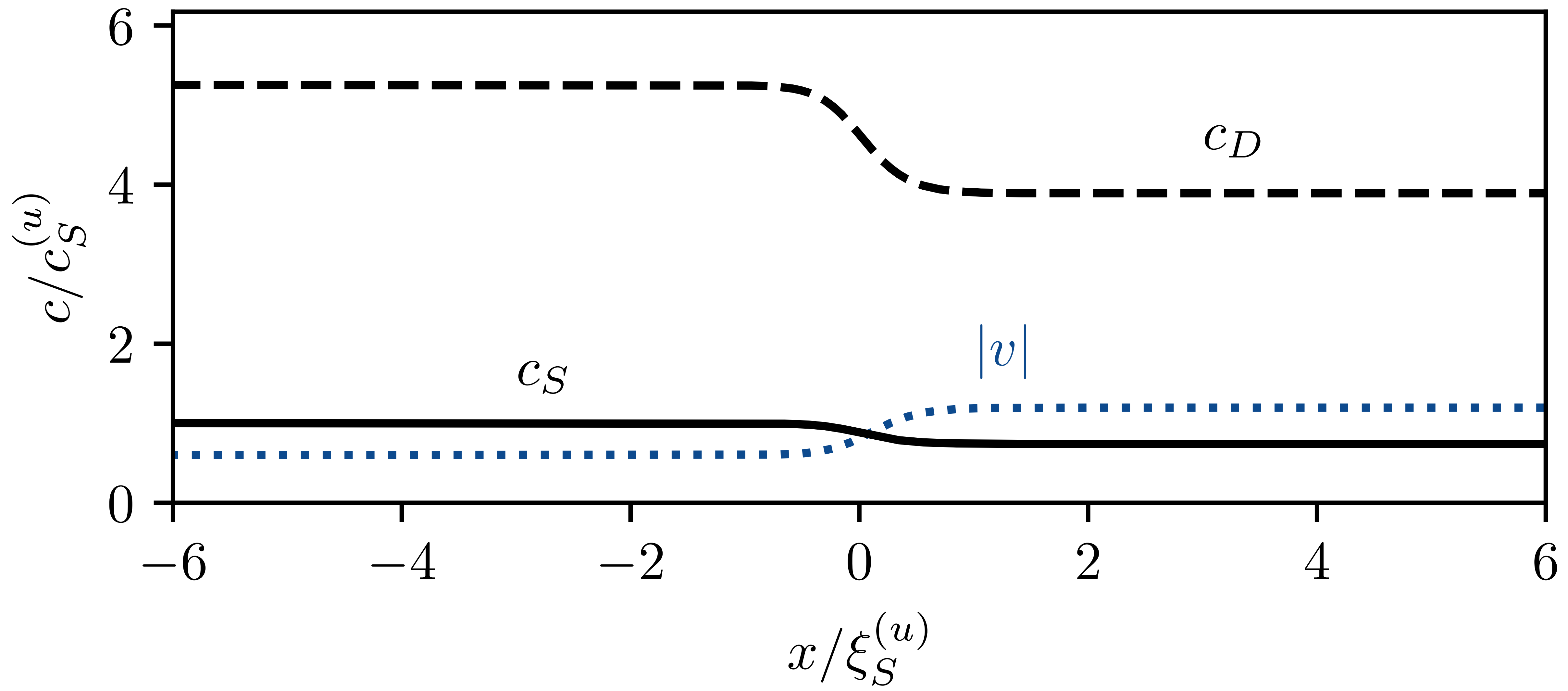
Realistic implementation in a box trap with a moving potential

Can be realized in the spin channel without affecting the stability of the total density.

Requires smaller flow velocity

The relevant length scale is the spin healing length: smoother horizon profile + easier to realize effectively one-dimensional configurations without going in the quasi-1D regime





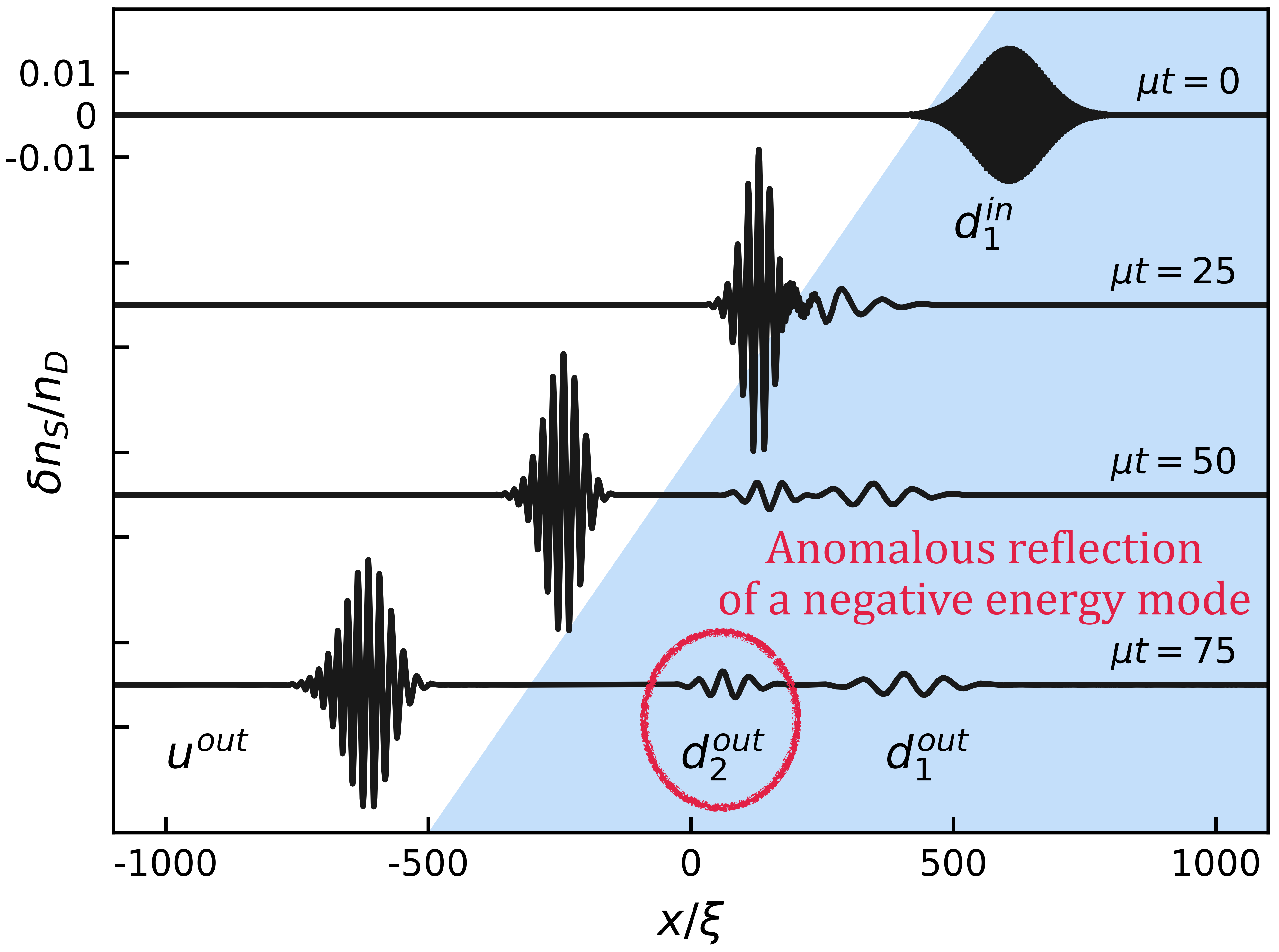
AB, **L. Fernandes**, S.G.Butera, A.Recati and I. Carusotto (manuscript in preparation)

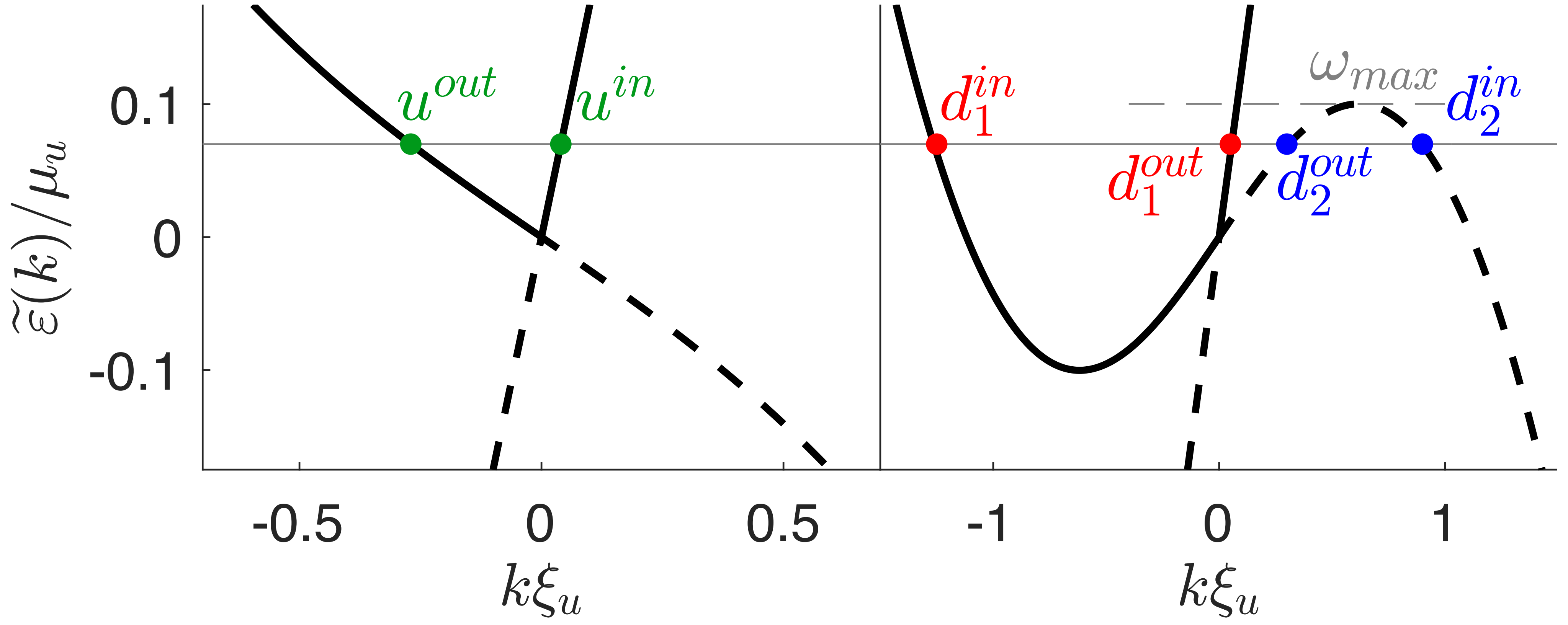
STIMULATED HAWKING RADIATION

Scattering properties of the sonic horizon can be summarized in the form of a scattering matrix.

$$\begin{pmatrix} \beta_u^{\text{out}} \\ \beta_{d_1}^{\text{out}} \\ \beta_{d_2}^{\text{out}} \end{pmatrix} = \mathcal{S}(\omega) \begin{pmatrix} \beta_u^{\text{in}} \\ \beta_{d_1}^{\text{in}} \\ \beta_{d_2}^{\text{in}} \end{pmatrix}$$

- anomalous reflection/transmission due to the presence of negative energy modes
- “boomerang” effect
- “undulation” instability inside the analog BH
- BH superradiance (total internal reflection+amplification)





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CORRELATIONS

Spontaneous ($T = 0$) analog Hawking emission in BECs can be detected through correlation patterns [Pavloff, Recati and Carusotto, PRA 80, 043603 (2009)].

$$\mathbf{S} \sim n(1, \delta\phi, \delta Z)$$

- third component of the spin vector $\longrightarrow$ relative density fluctuations
- second component of the spin vector $\longrightarrow$ relative phase fluctuations

$\langle : S_3(x) S_3(x') : \rangle$ $\qquad \propto \sqrt{\text{structure factor}}$

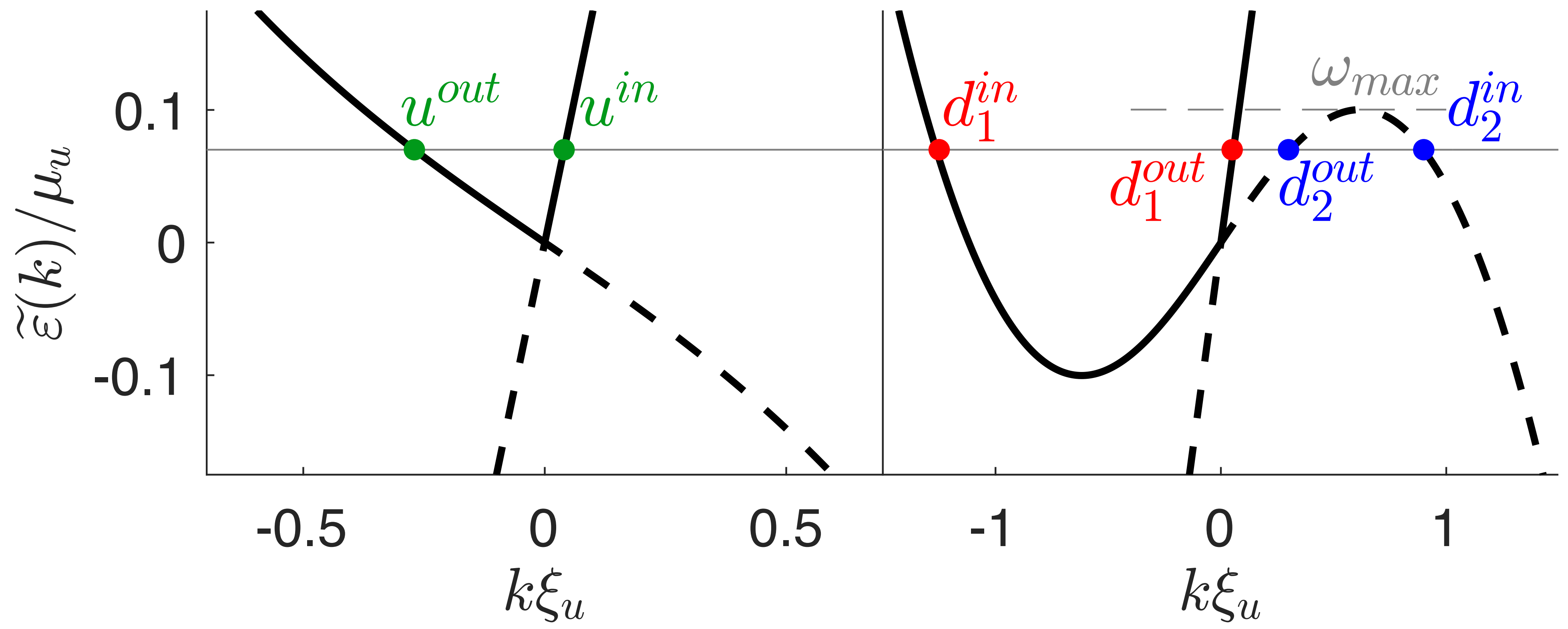
$$nG_{33}(x,x') = \frac{1}{4\pi}\int_0^{\omega_{\max}} \mathrm{d}\omega \sum_{IJ}\left[\frac{(U_I+V_I)(U_J+V_J)}{\sqrt{|w_I w_J|}} \mathcal{S}^*_{Id2}\mathcal{S}_{Jd2} e^{i(k_I x - k_J x')} + \text{c.c.}\right]\theta_I(x)\theta_J(x')$$

$$nG_{22}(x,x') = \frac{1}{4\pi}\int_0^{\omega_{\max}} \mathrm{d}\omega \sum_{IJ}\left[\frac{(U_I-V_I)(U_J-V_J)}{\sqrt{|w_I w_J|}} \mathcal{S}^*_{Id2}\mathcal{S}_{Jd2} e^{i(k_I x - k_J x')} + \text{c.c.}\right]\theta_I(x)\theta_J(x')$$

$\langle : S_2(x) S_2(x') : \rangle$ $\qquad \propto \frac{1}{\sqrt{\text{structure factor}}}$

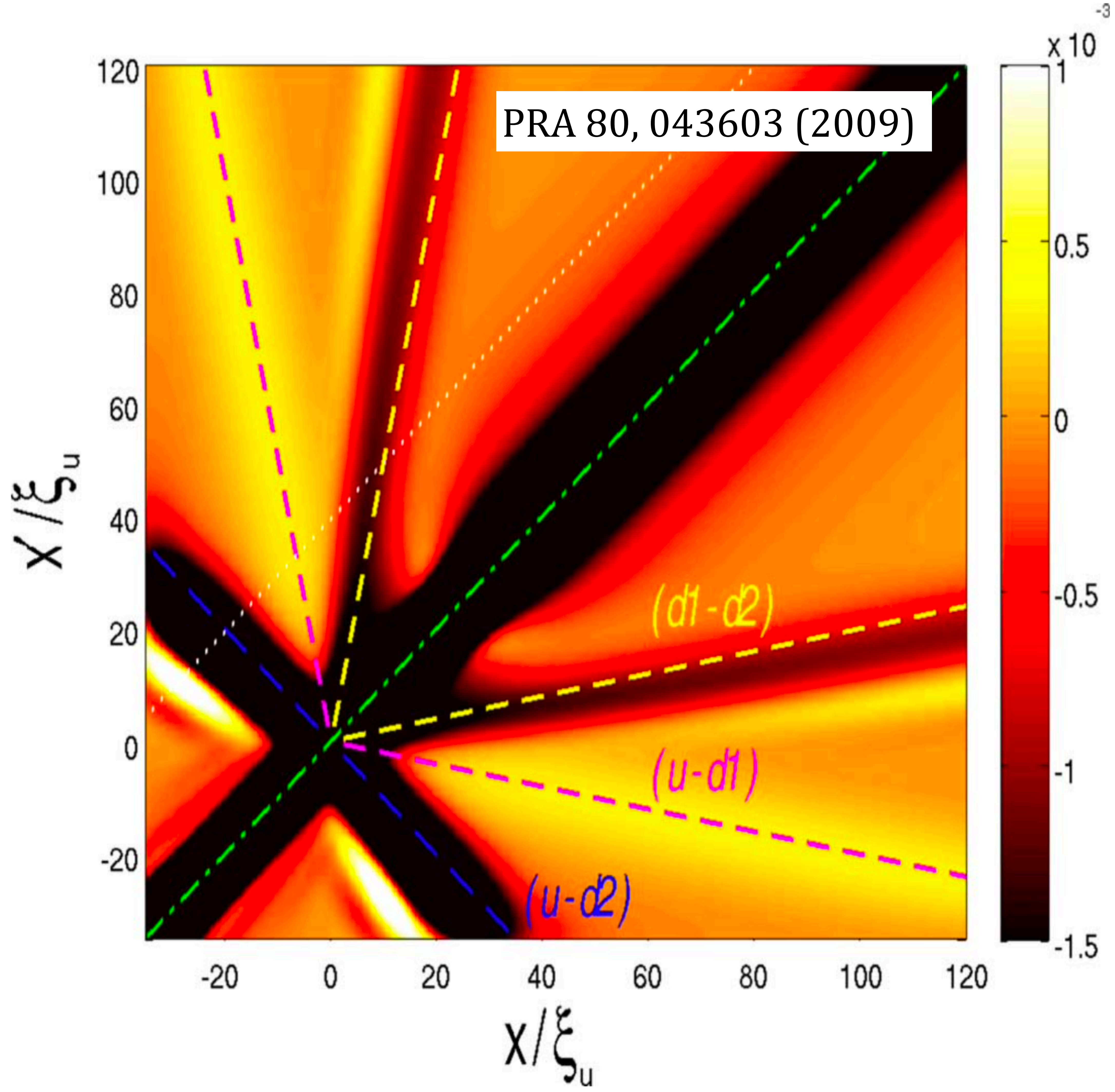
$I, J \in \{u, d1, d2\}$

A No coupling



$$U_I + V_I \propto \sqrt{\omega}$$
$$U_I - V_I \propto \sqrt{1/\omega}$$
$$\mathcal{S}^*_{Id2}\mathcal{S}_{Jd2} \propto 1/\omega$$
$$G_{22}(x, x') = \infty$$
$$G_{33}(x, x') \propto \text{sinc}\left[\omega_{\max}\left(\frac{x}{v_I} - \frac{x'}{v_J}\right)\right]$$

Peaked at $x = (v_I/v_J)x'$



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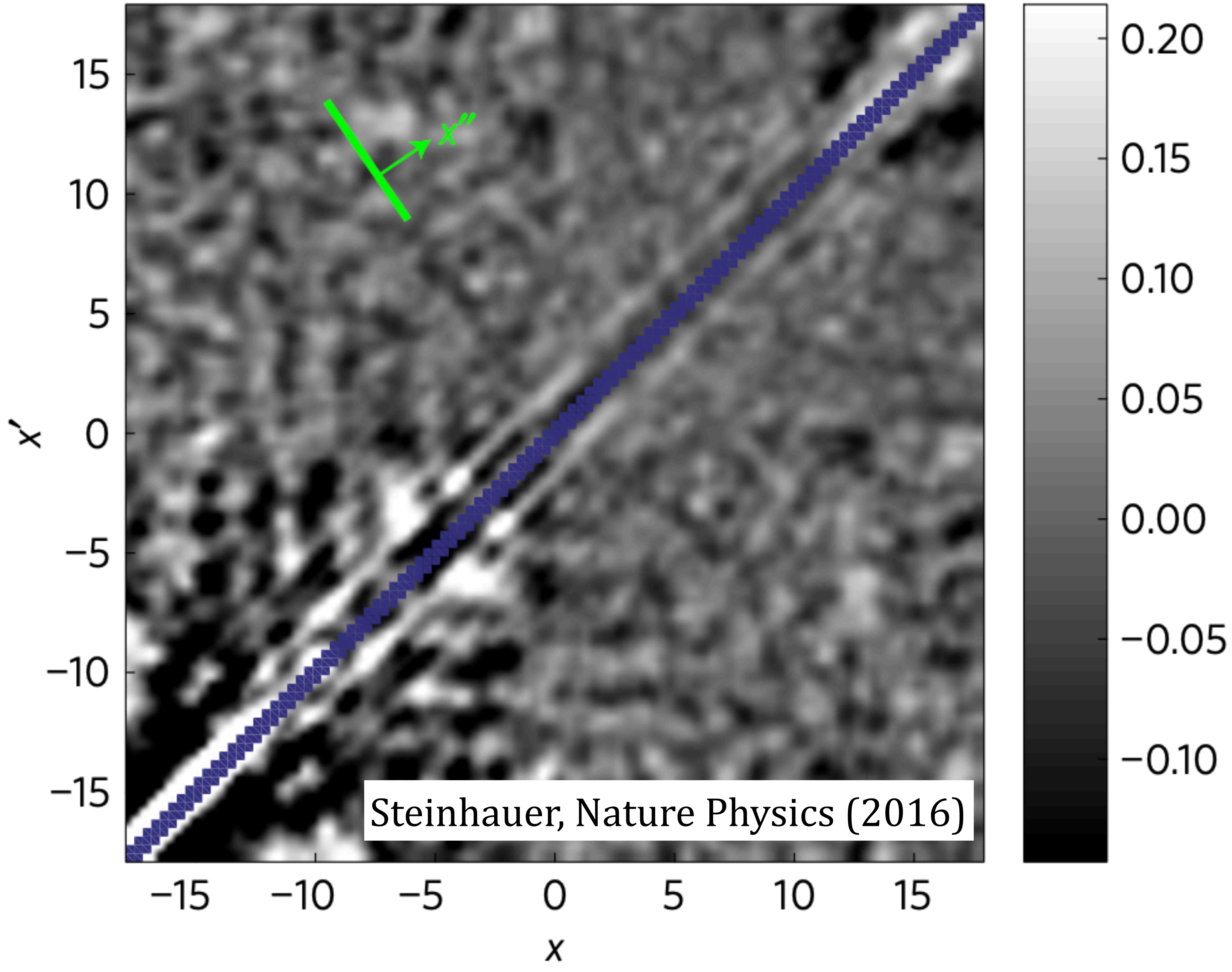
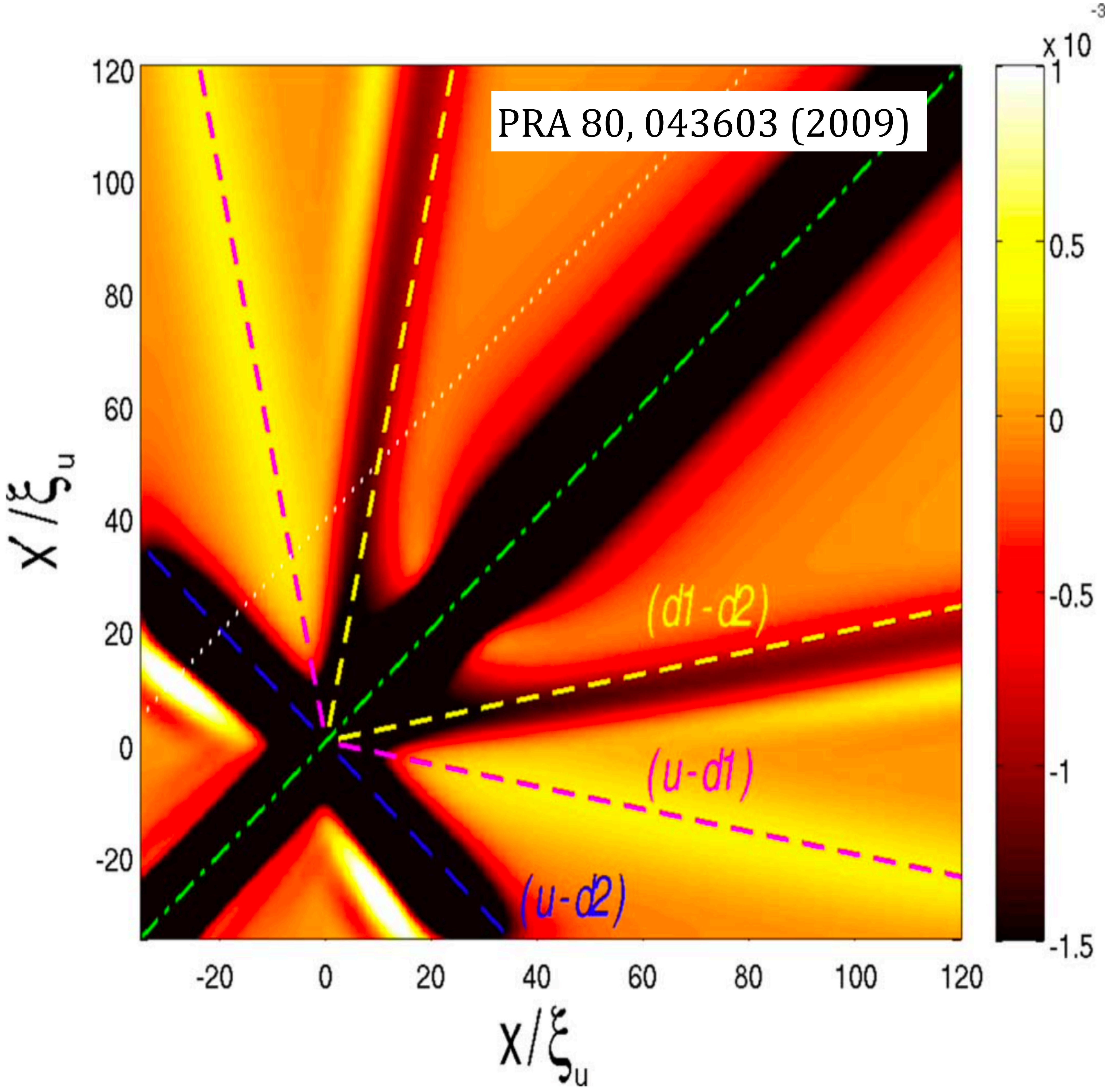
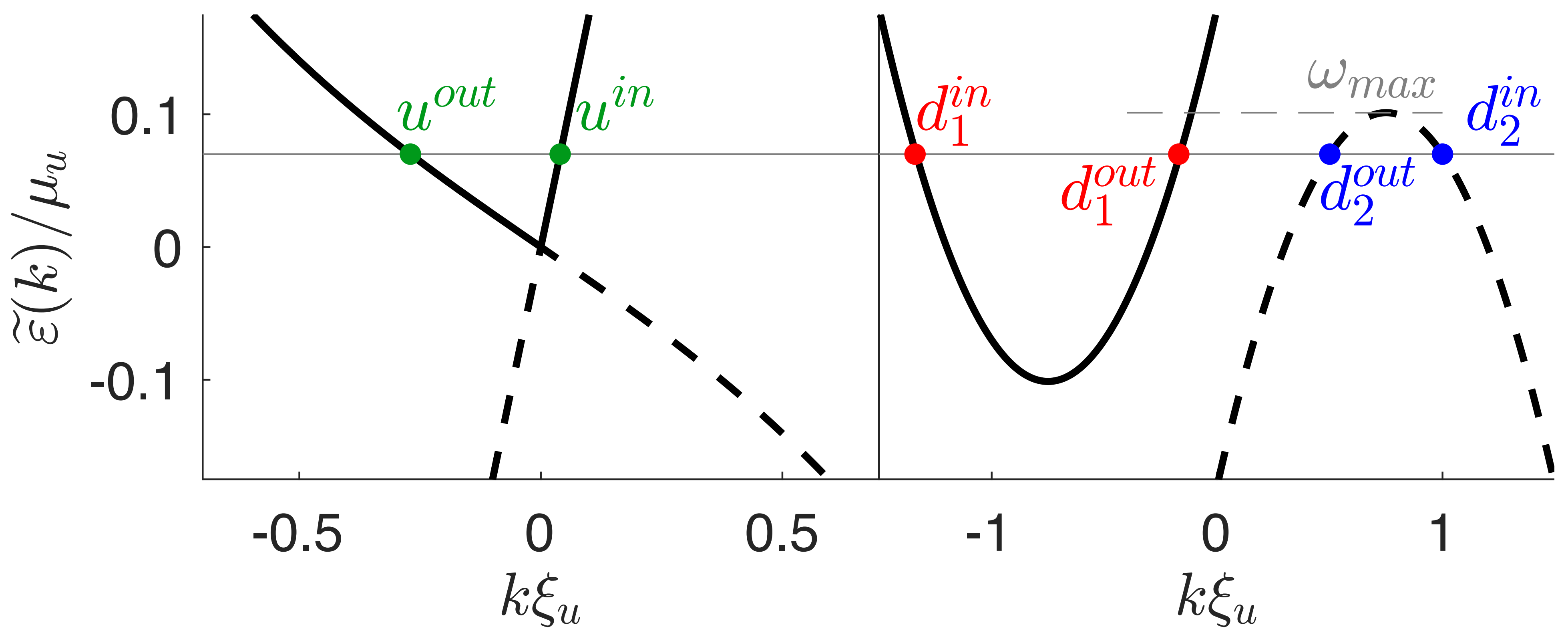


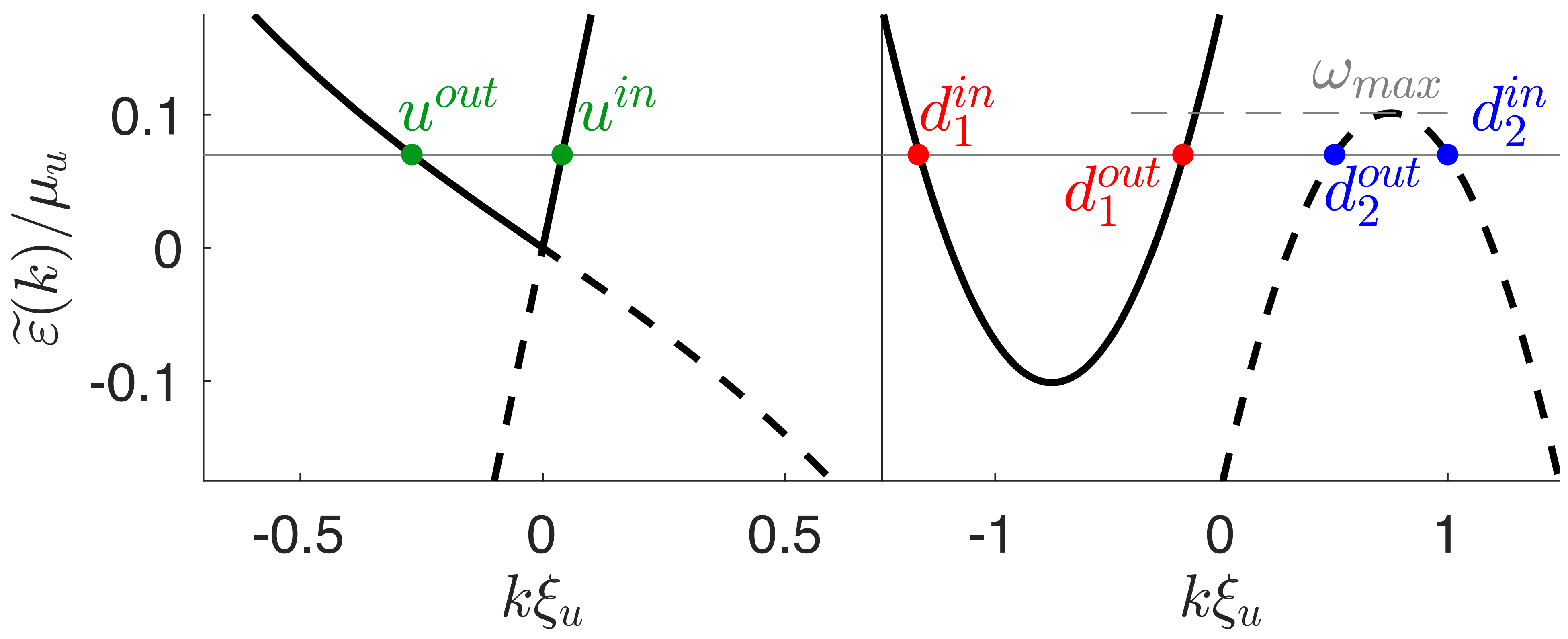


(B) Massive case, $g = g_{12}$



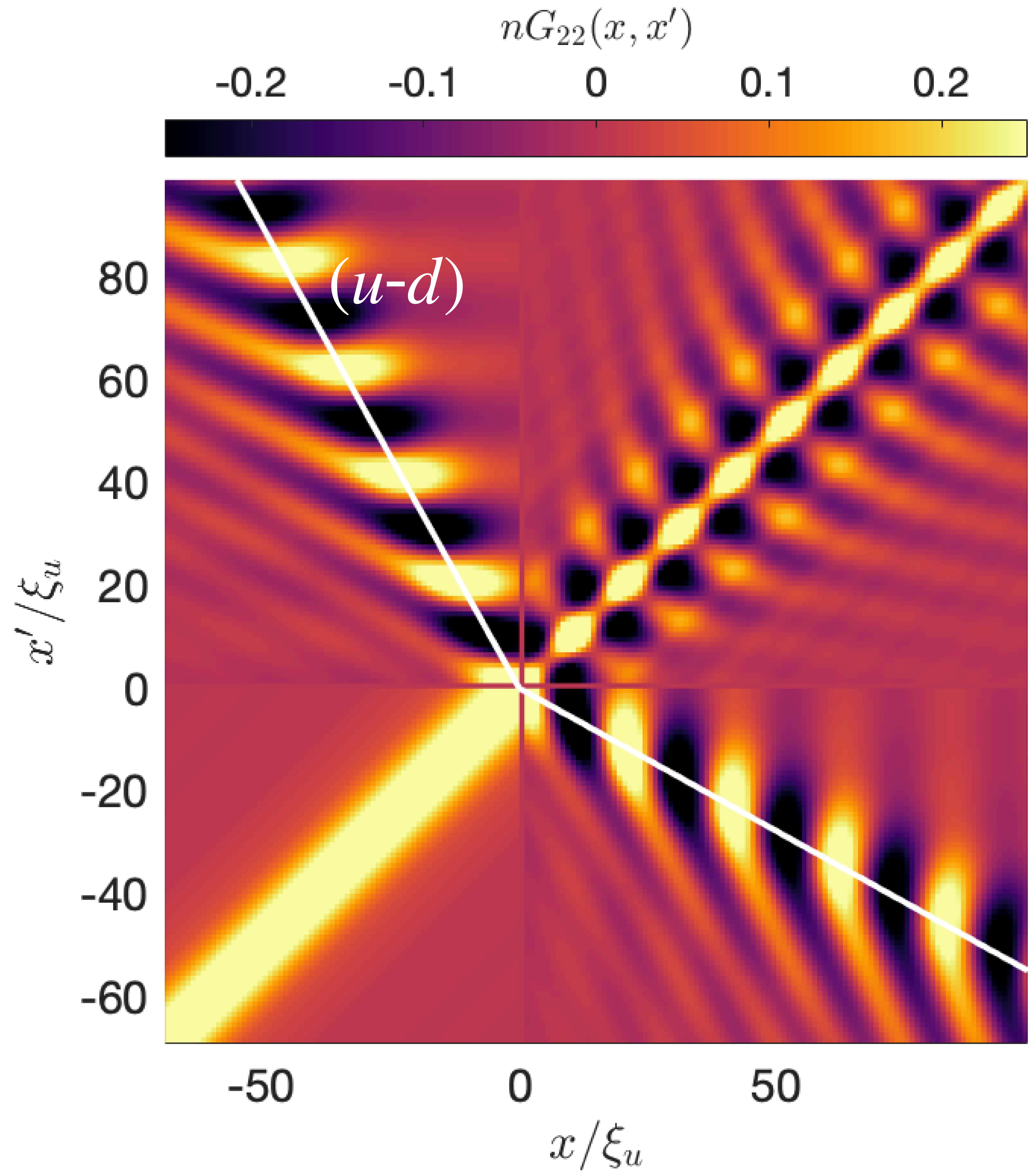
AB, L. Fernandes, S.G.Butera, A.Recati and I. Carusotto (manuscript in preparation)

B Massive case, $g = g_{12}$



Oscillations with wave-vector k_0 corresponding to the one of zero-frequency modes.

Hawking signal peaked at $x' = (v_u/w_0)x$ where w_0 is the group velocity at $\omega = 0$.
$(u\text{-}d1)$ and $(u\text{-}d2)$ correlations are superimposed, while $(d1\text{-}d2)$ are along the main diagonal, as well as all self-correlations.



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ERGOREGION INSTABILITIES

Self-amplification of negative-energy modes within an analog ergoregion, due to superradiant scattering.

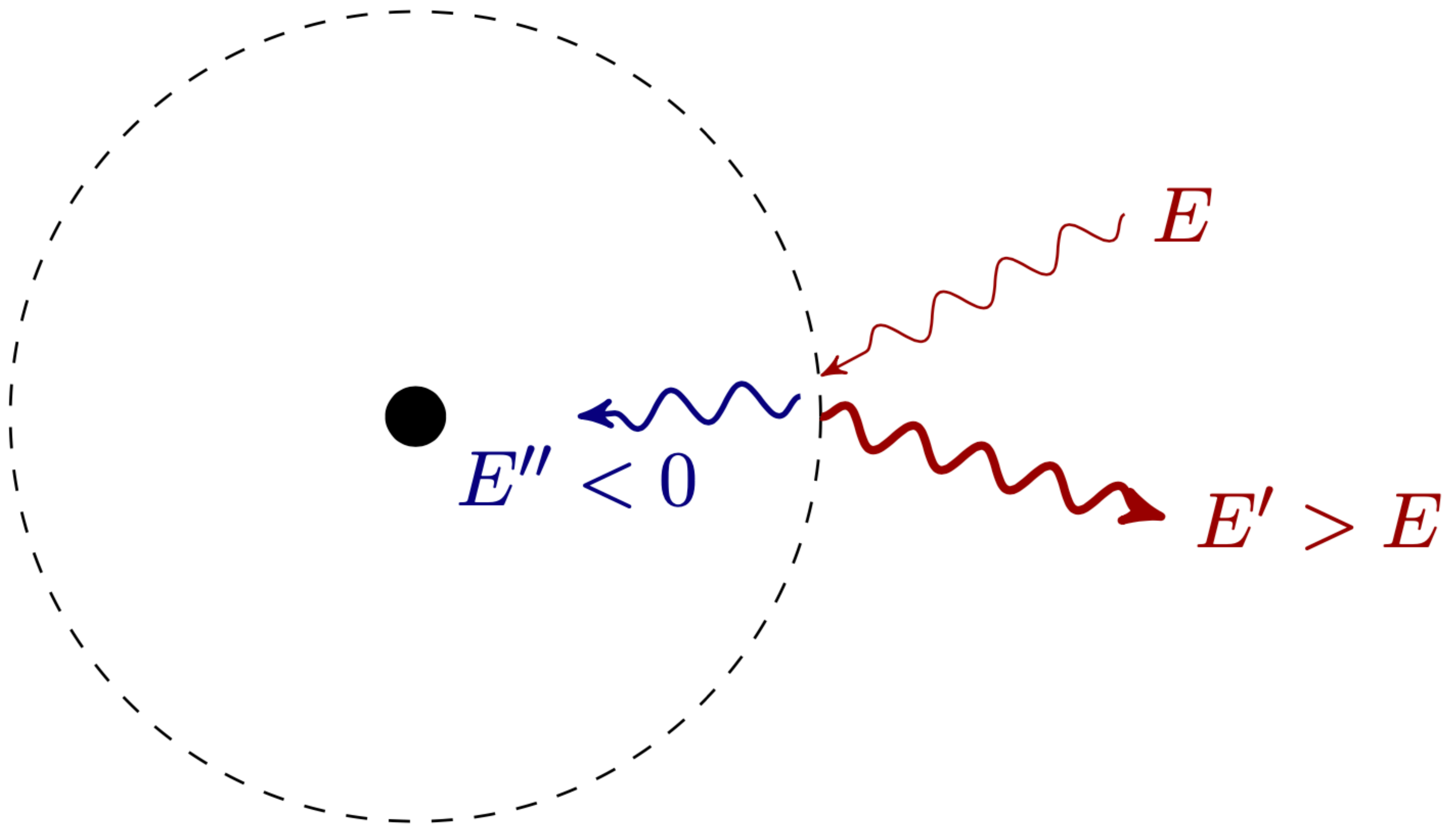
Superradiance, Richard Brito, Vitor Cardoso, Paolo Pani (2020),
Giacomelli and Carusotto, Phys. Rev. Research 2, 033139 (2020)

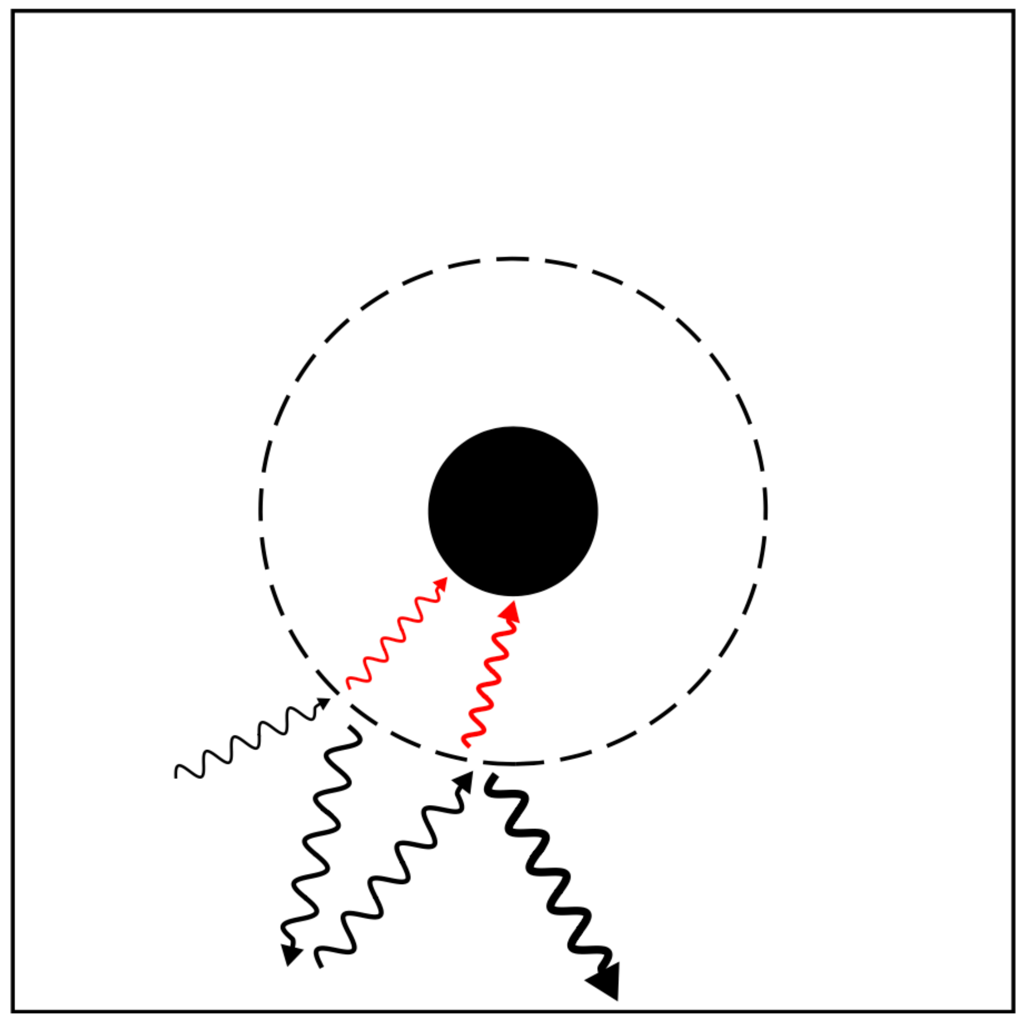
ROTATIONAL SUPERRADIANCE

Rotational superradiance is the amplified scattering of radiation from a rotating object, at the expense of the rotational energy of the object itself.

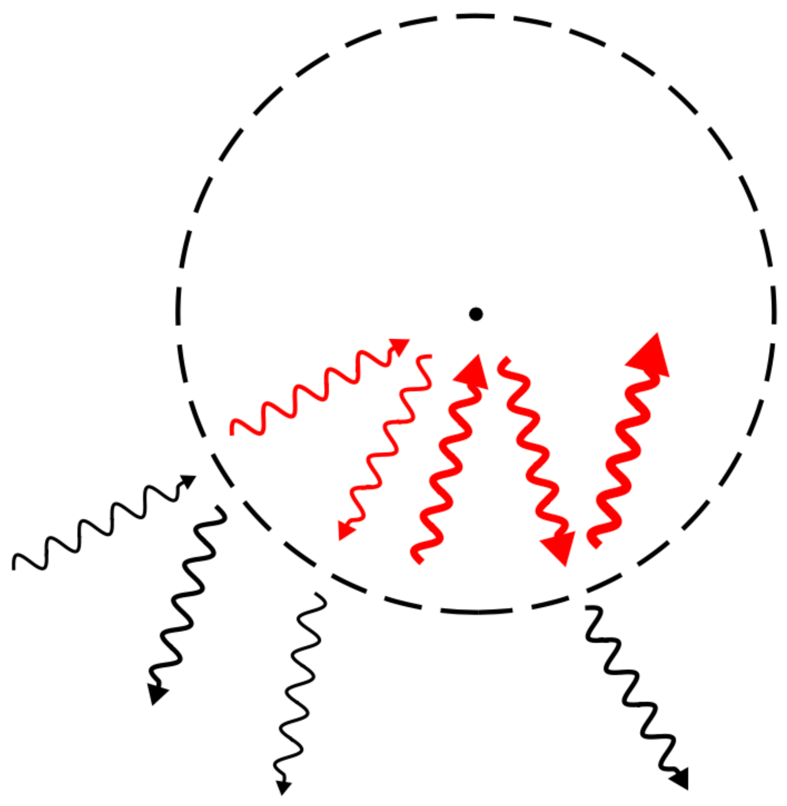
Predicted to occur in rotating spacetime geometries that provide an ergosphere (e.g. Kerr black hole).

Reflecting boundaries on either side of the ergoregion give rise to dynamical instabilities (exponentially growing modes)





BH bomb



Ergoregion instability

ANALOG METRIC OF A VORTEX IN A BEC

Vortices in BECs have tangential velocity flow, with speed diverging in the vortex core; the total density instead vanishes at the vortex core and saturates at infinite distance.

The analog metric therefore displays an ergoregion but not an analog horizon.

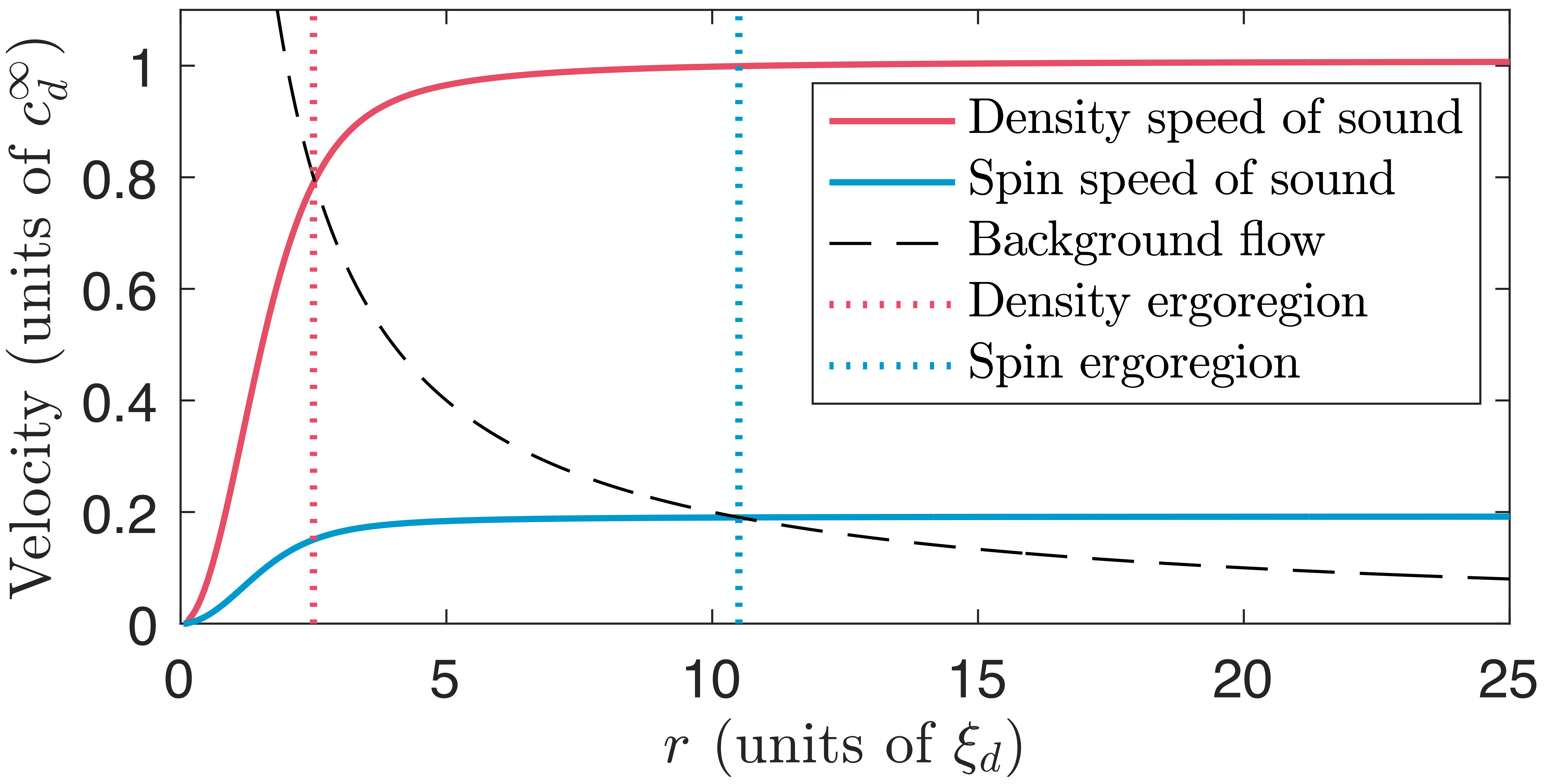
The density ergosurface is located close to the vortex core, while the spin ergoregion is much larger.

$$\psi(r,\theta,t) = \sqrt{n(r)}e^{iL\theta}e^{-i\mu t}$$

$$n(r) \sim n^{\infty} \begin{cases} (r/\xi_d)^{2L} & \text{if } r \ll \xi_d \\ 1 & \text{if } r \gg \xi_d \end{cases}$$

$$v(r) = L/r$$

$$r_E \sim L\xi_{d,s}$$

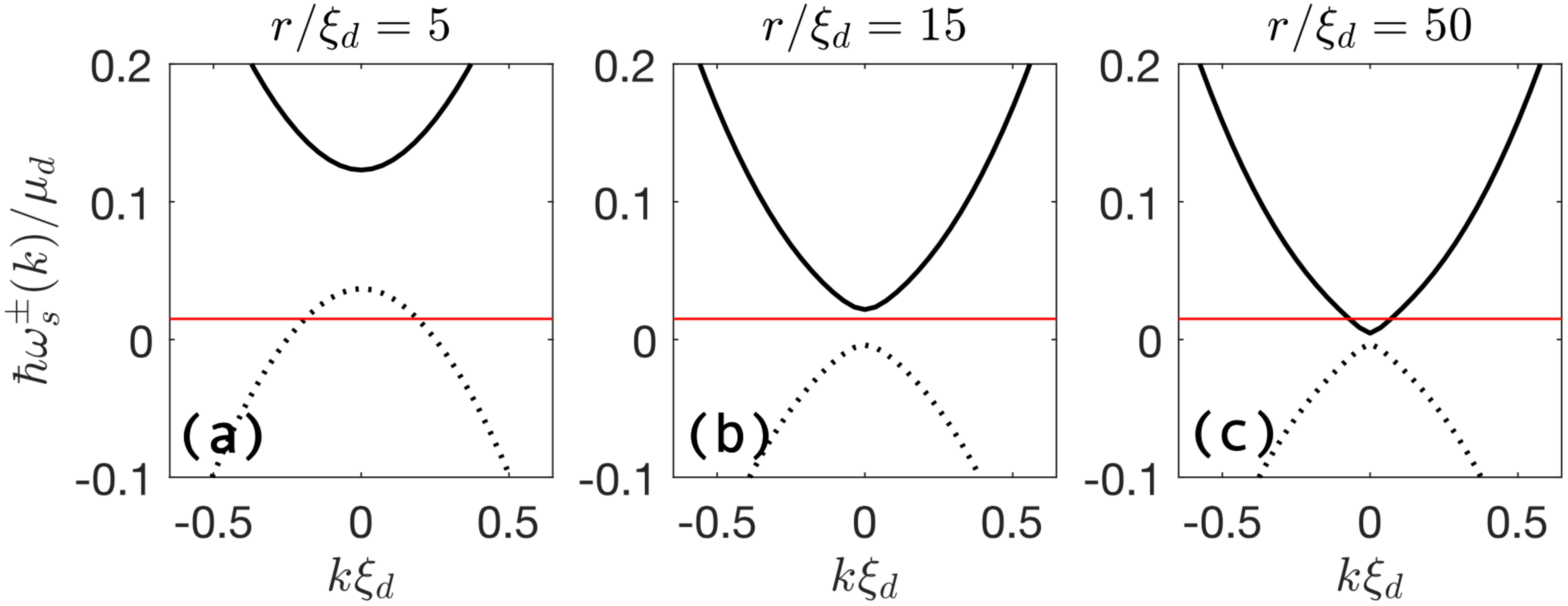


(LOCAL) COLLECTIVE MODES

$$\delta\psi_s(r,\theta,t) = \left[u(r,t)e^{iM\theta} + v^*(r,t)e^{-iM\theta}\right] e^{iL\theta}e^{-i\mu t}$$

The dispersion relation is locally shifted upwards by the Doppler shift; moreover a centrifugal gap opens due to the non-vanishing angular momentum M of the perturbation.

$$\omega_s^{\pm}(k) = \frac{LM}{r^2} \pm \sqrt{\left(\frac{k^2}{2} + \frac{M^2}{2r^2}\right)\left(\frac{k^2}{2} + \frac{M^2}{2r^2} + 2\mu_s\right)}$$



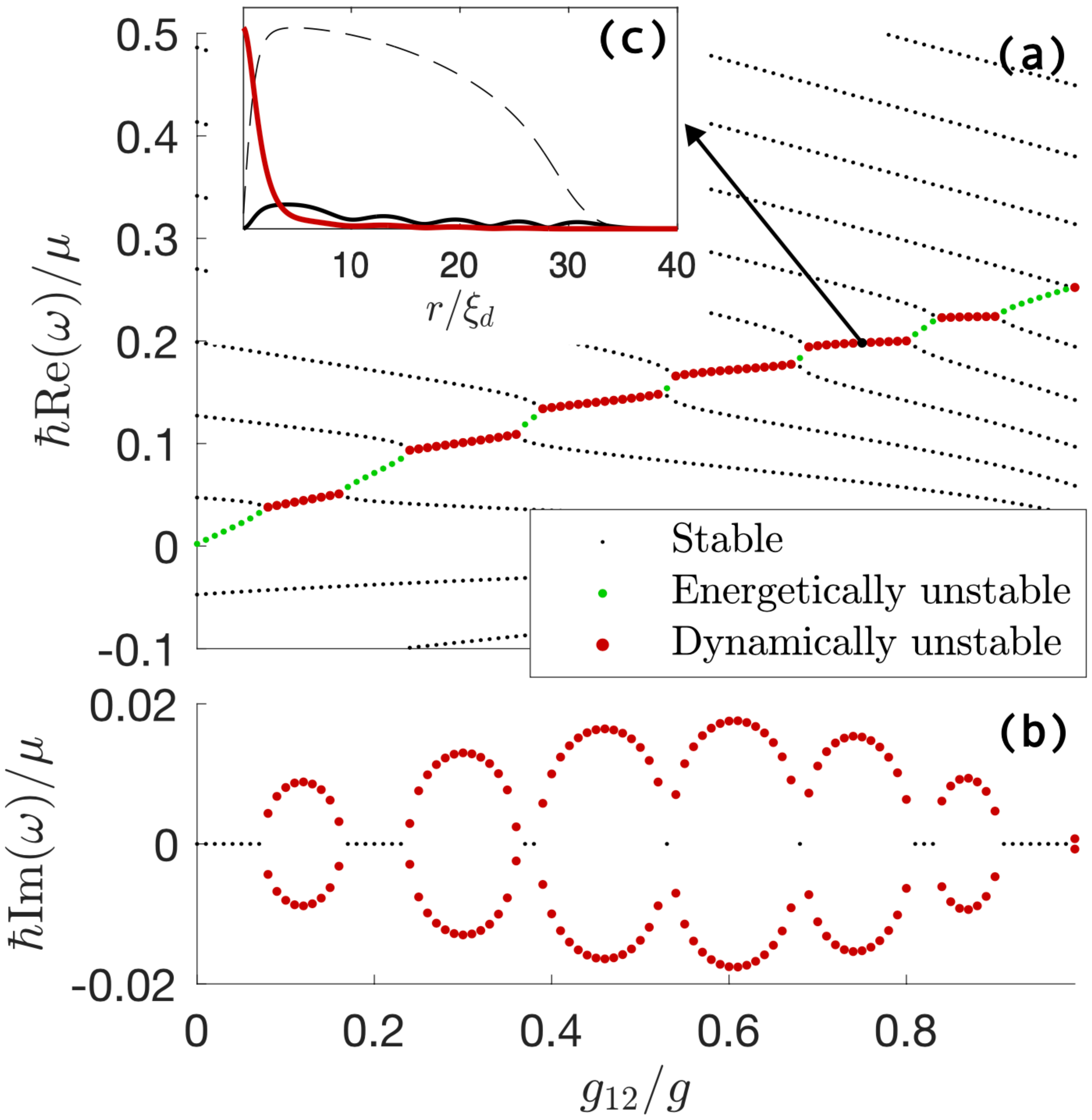
ERGOREGION INSTABILITIES

The stability of vortex configurations can be studied by exactly diagonalizing the Bogoliubov problem: the system is dynamically unstable if its spectrum shows complex frequency modes.

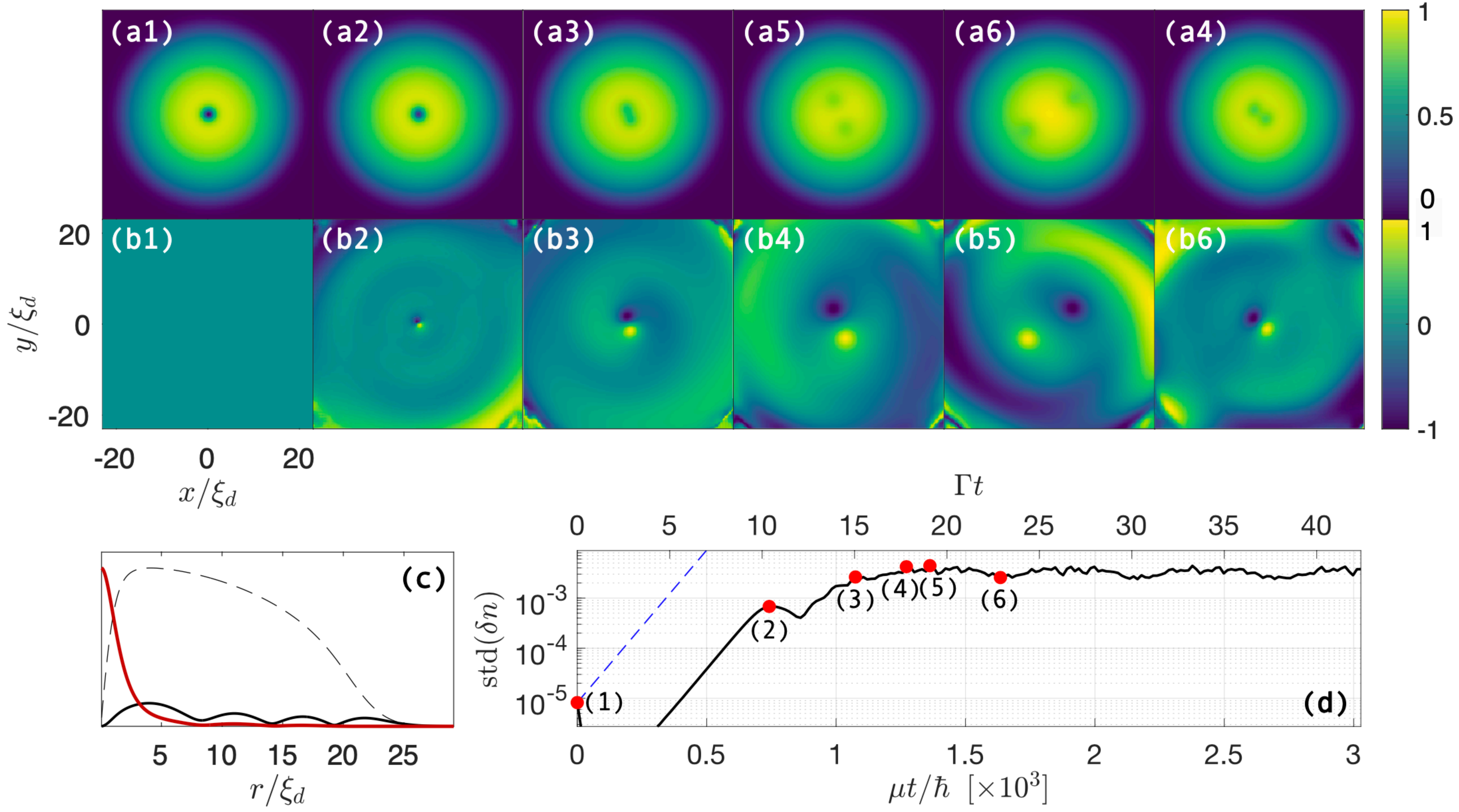
Unstable modes have a negative-energy component located inside the analog ergoregion

In most cases, a single negative norm mode crosses the band of positive energy ones.

$$L = M = 1 \text{ (spin)}$$



AB, L. Giacomelli and I. Carusotto, arXiv:2212.07337 (2022)



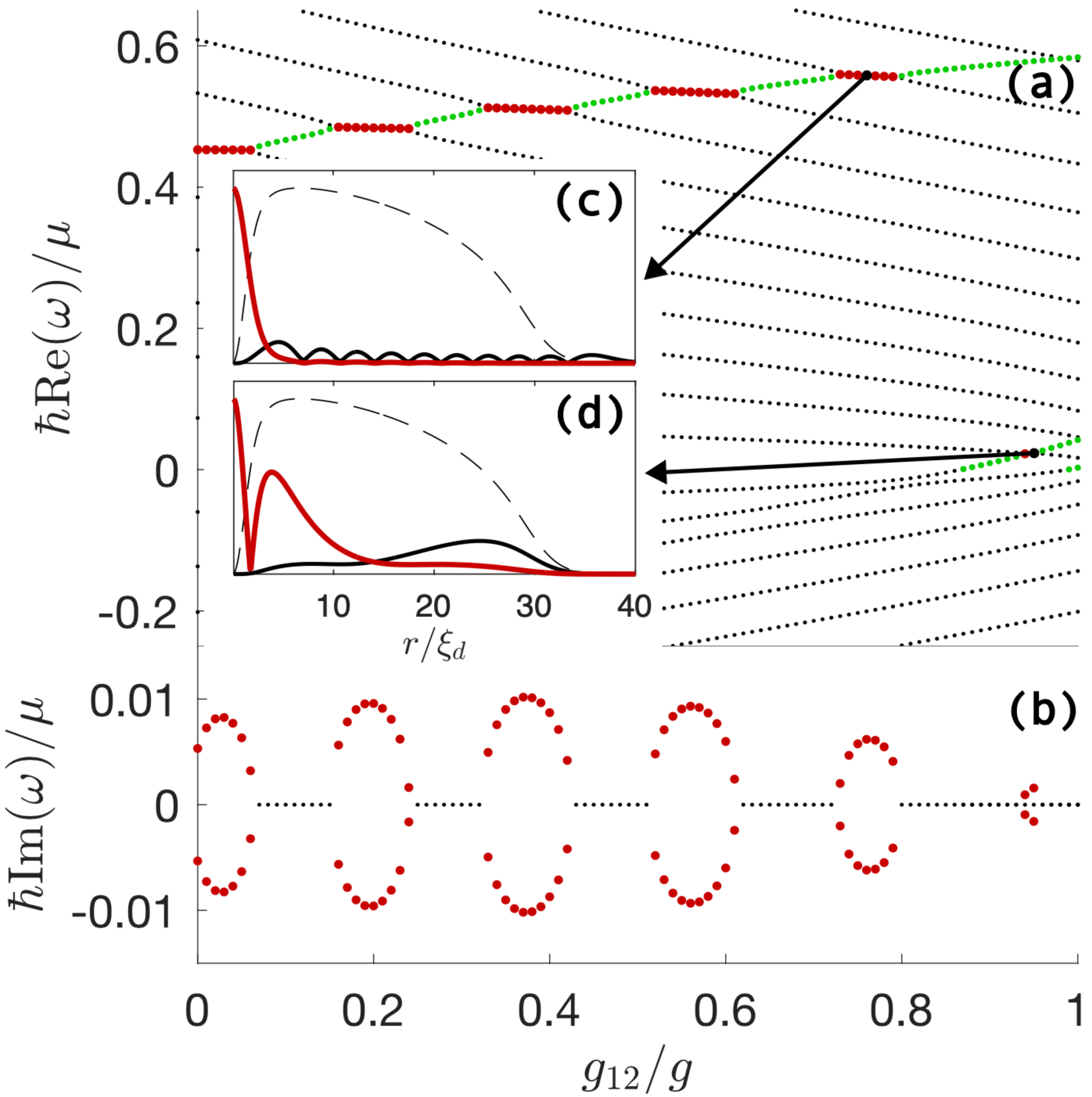
AB, L. Giacomelli and I. Carusotto, arXiv:2212.07337 (2022)

The stability of vortex configurations can be studied by exactly diagonalizing the Bogoliubov problem: the system is dynamically unstable if its spectrum shows complex frequency modes.

Unstable modes have a negative-energy component located inside the analog ergoregion

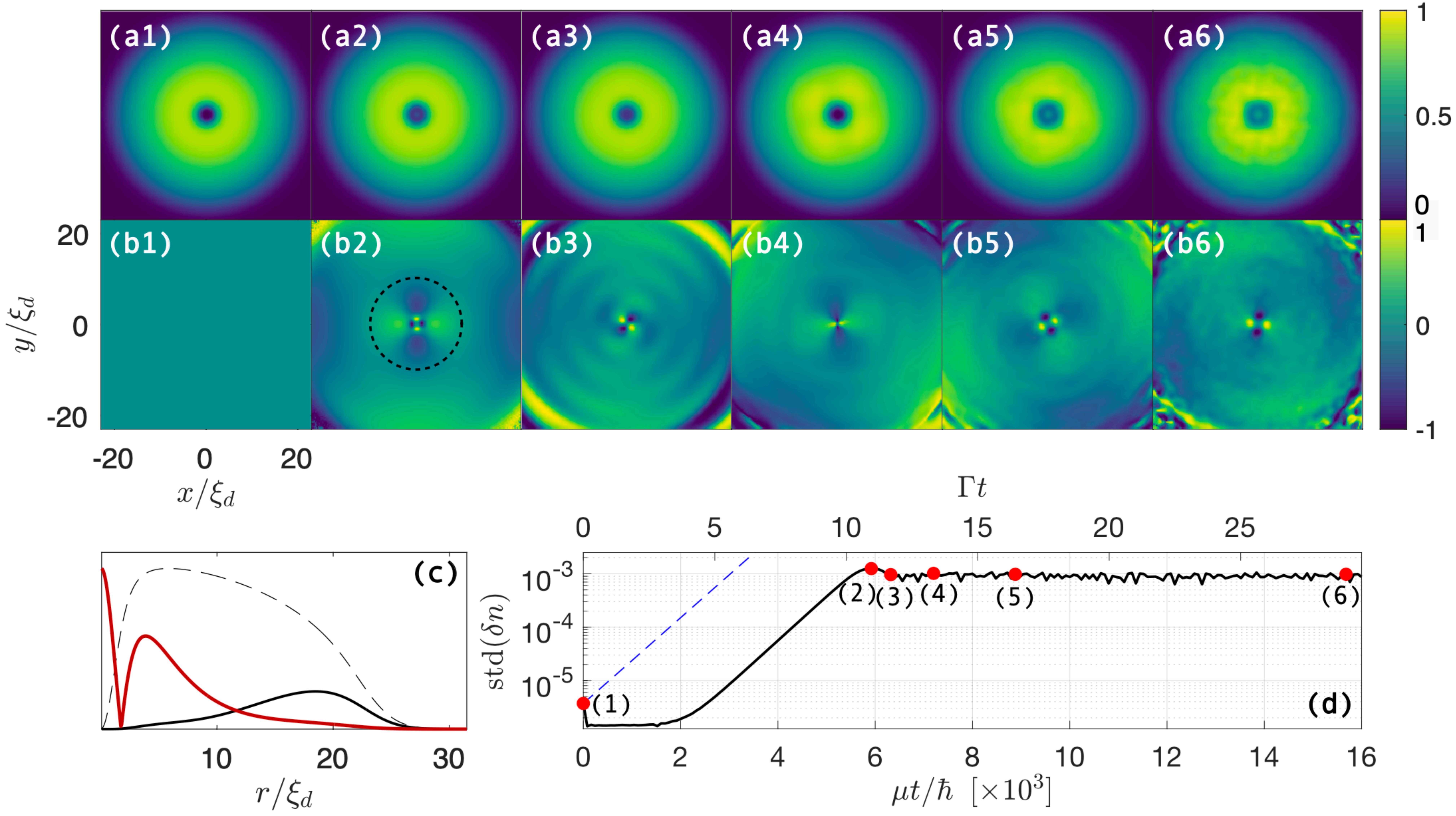
For $L \geq 2$, the whole bands of positive and negative norm modes cross each other.

$$L = M = 2 \text{ (spin)}$$



AB, L. Giacomelli and I. Carusotto, arXiv:2212.07337 (2022)

ERGOREGION INSTABILITIES



AB, L. Giacomelli and I. Carusotto, arXiv:2212.07337 (2022)

CONCLUSIONS

- Symmetric two-component mixtures can be used as quantum simulators of gravitational phenomena, thanks to the decoupling between the spin and density degrees of freedom.
- Working in the spin channel of a mixture, rather than a single component system, has practical advantages, mainly due to the smaller sound speed (or larger healing length).
- Conceptually, it allows to investigate a broader spectrum of physical phenomena, related to the presence of massive gaps.
- Possibility of strongly enhance the response of the system by approaching the ferromagnetic critical point.

THANK YOU
FOR YOUR ATTENTION