

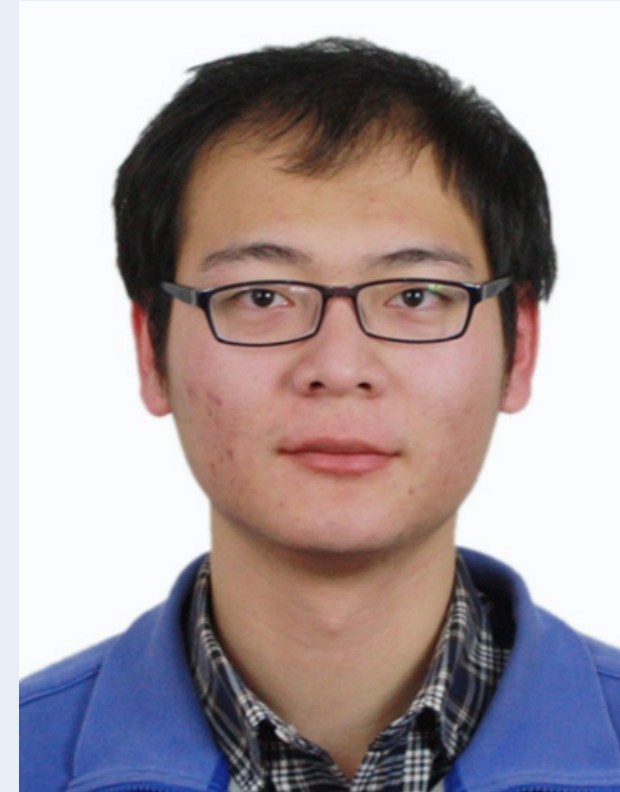
Analog simulation of effective field theories with 1D Bose gases

Amin - Schmiedmayer Lab - VCQ, TU Wien

KRb Experiment



Jörg Schmiedmayer



Si-Cong Ji



Federica Cataldini

Theory support



Igor Mazets



Nataliia Bezhan



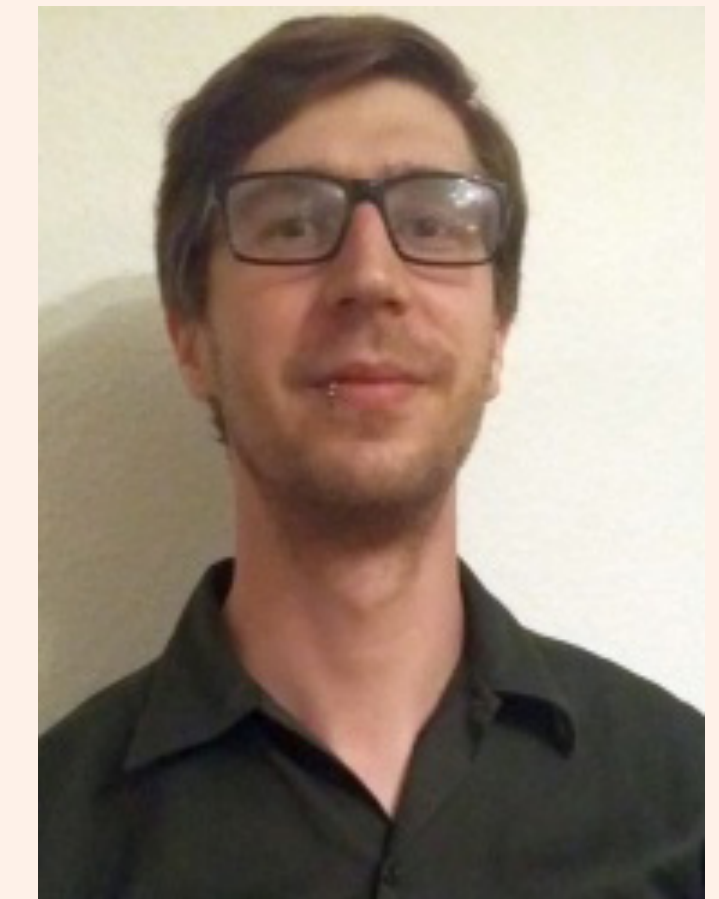
João Sabino



Frederik Møller

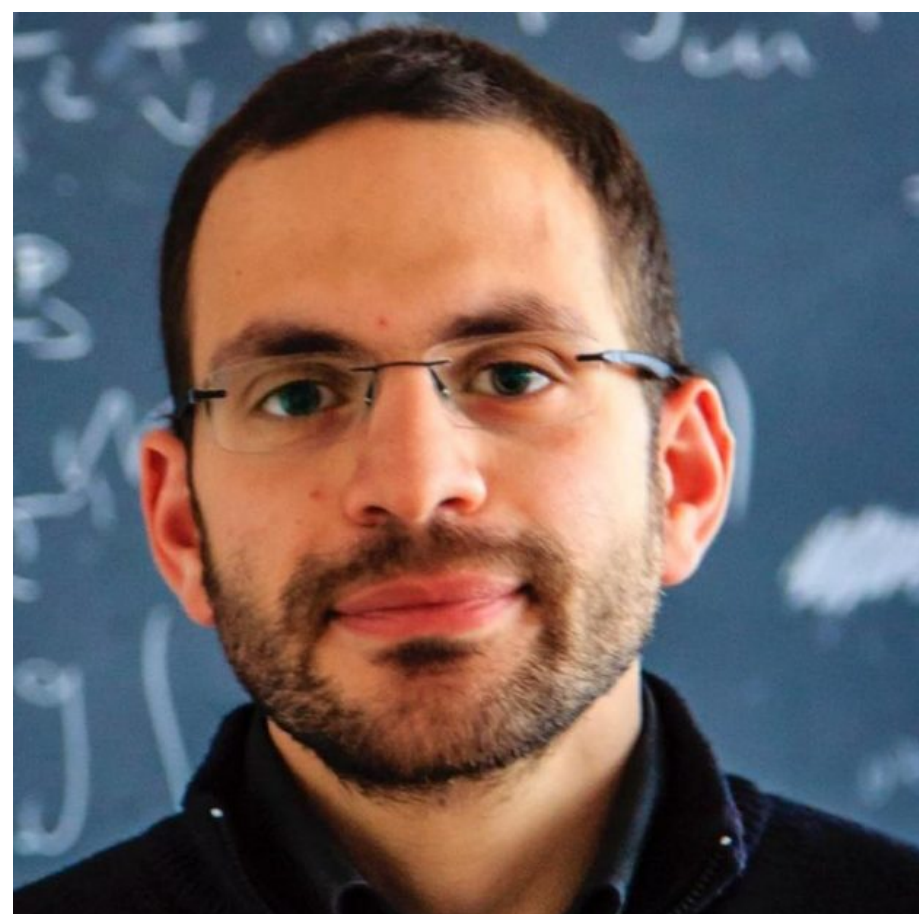


Philipp Schüttelkopf



Sebastian Erne

Theory Collaborators



Spyros Sotiriadis
University of Crete



Nicolas Sebe
EP Paris



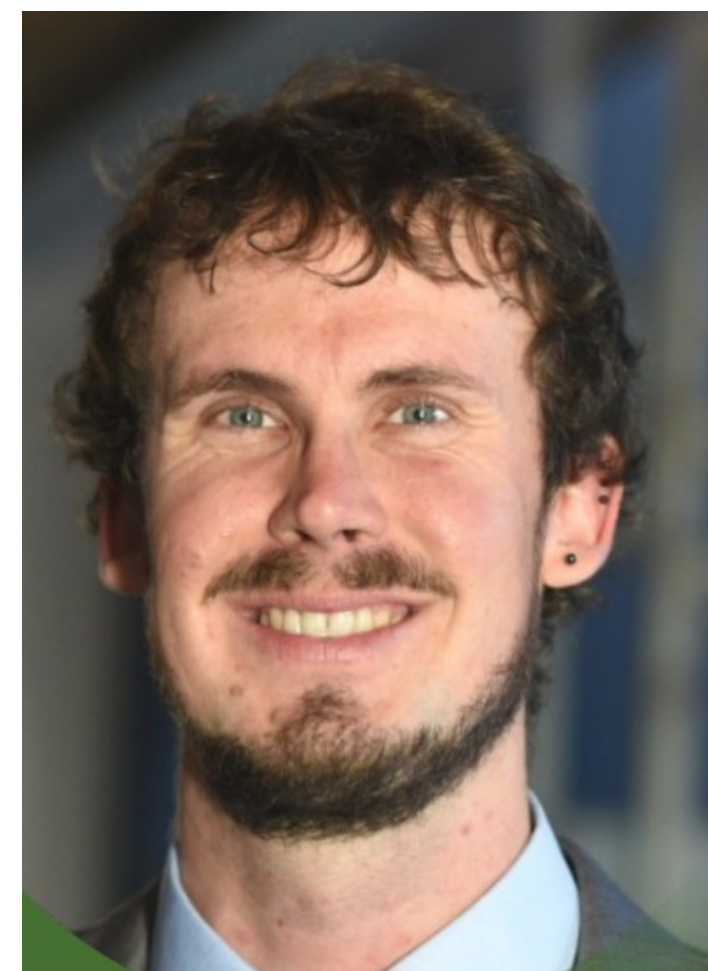
Marek Gluza
NTU Singapore



Giacomo Guarnieri
FU Berlin



Jens Eisert
FU Berlin



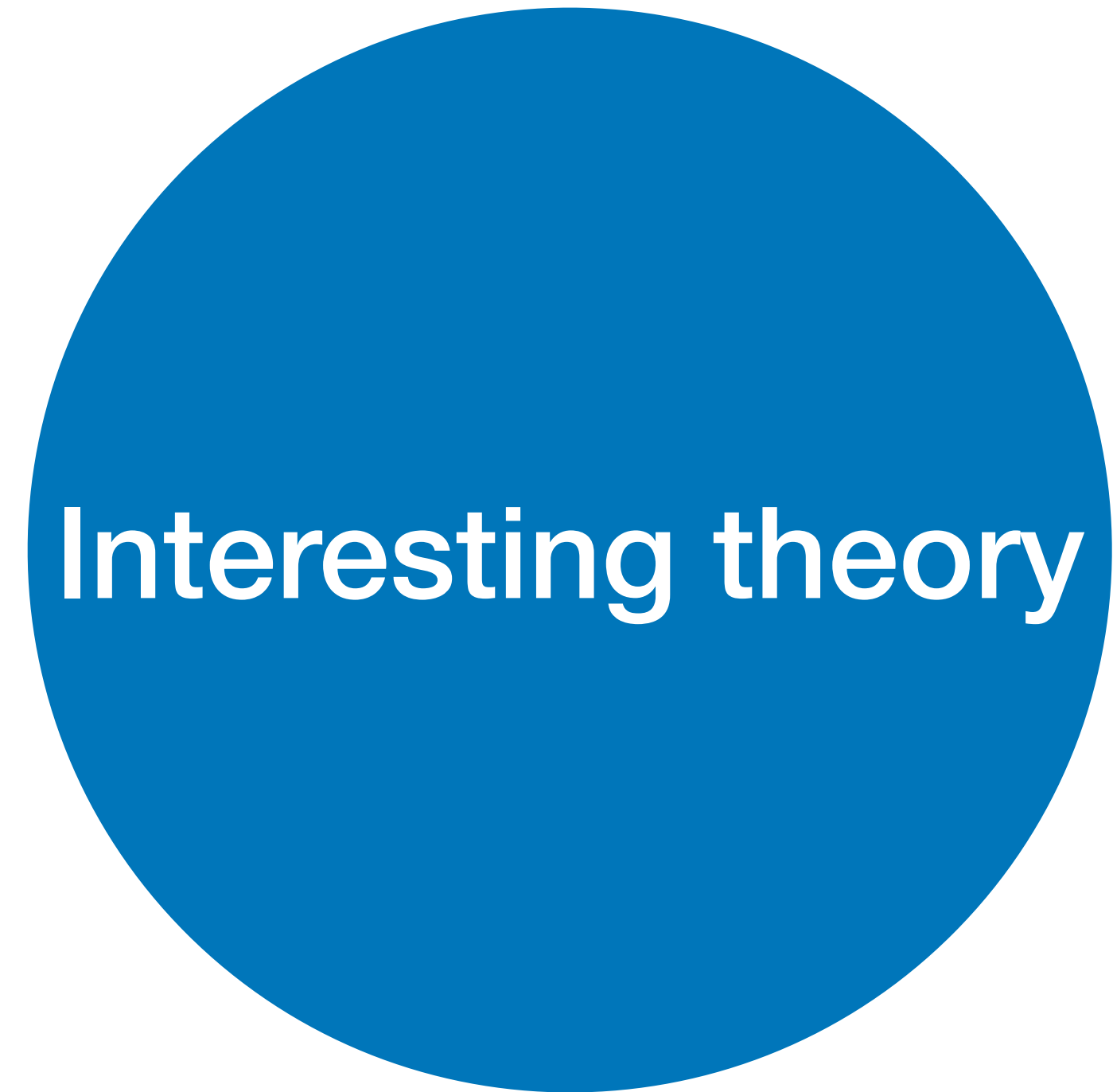
Ivan Kukuljan
MPI

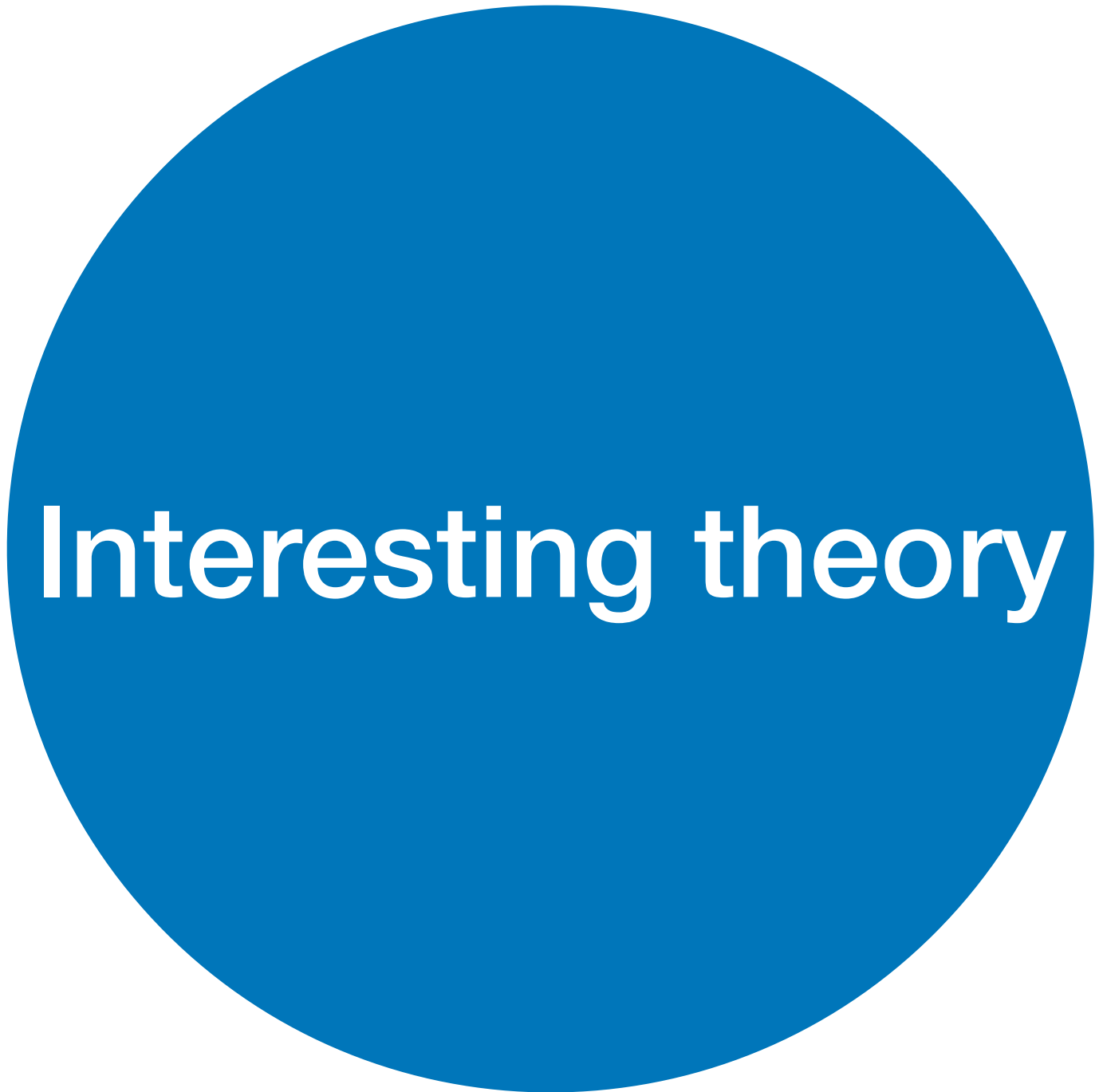


Dries Sels
NYU



Eugene Demler
ETH

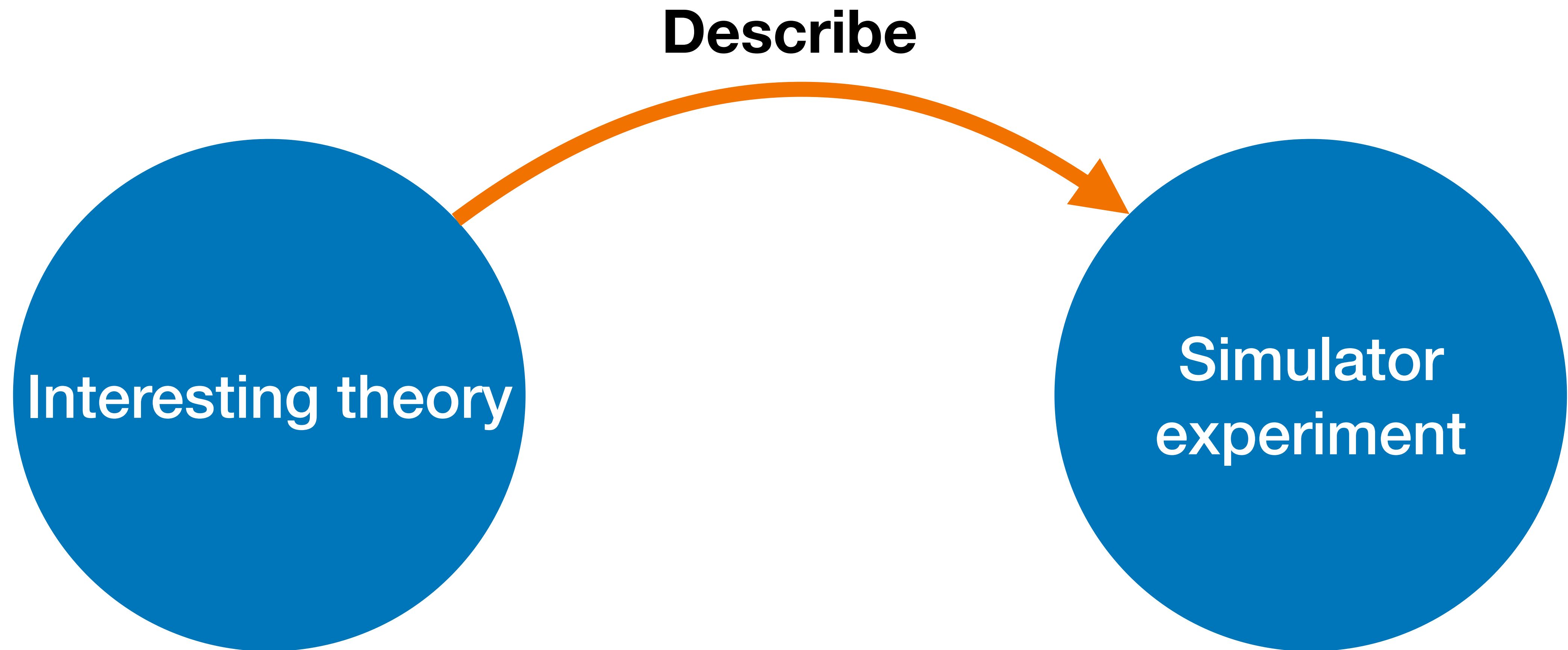


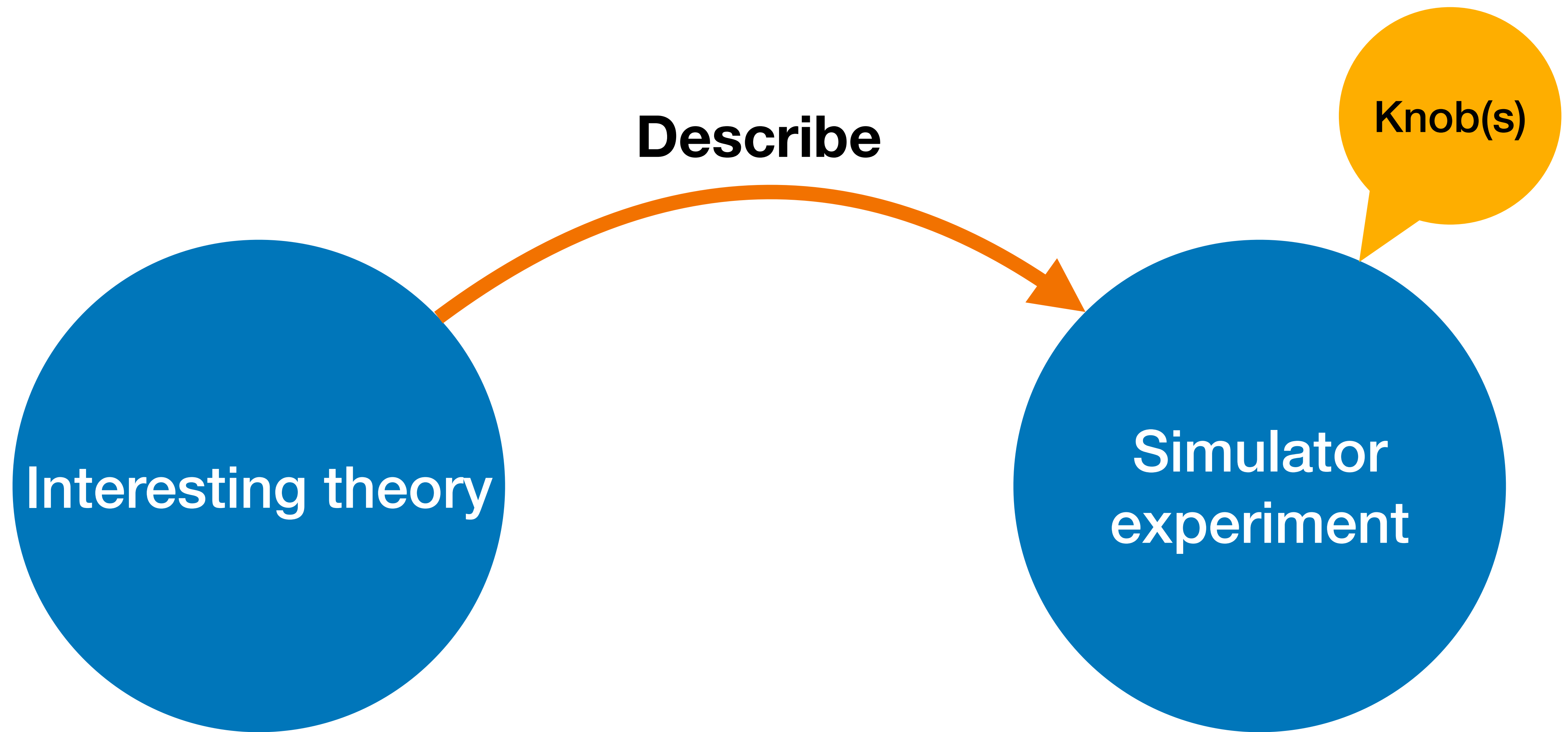


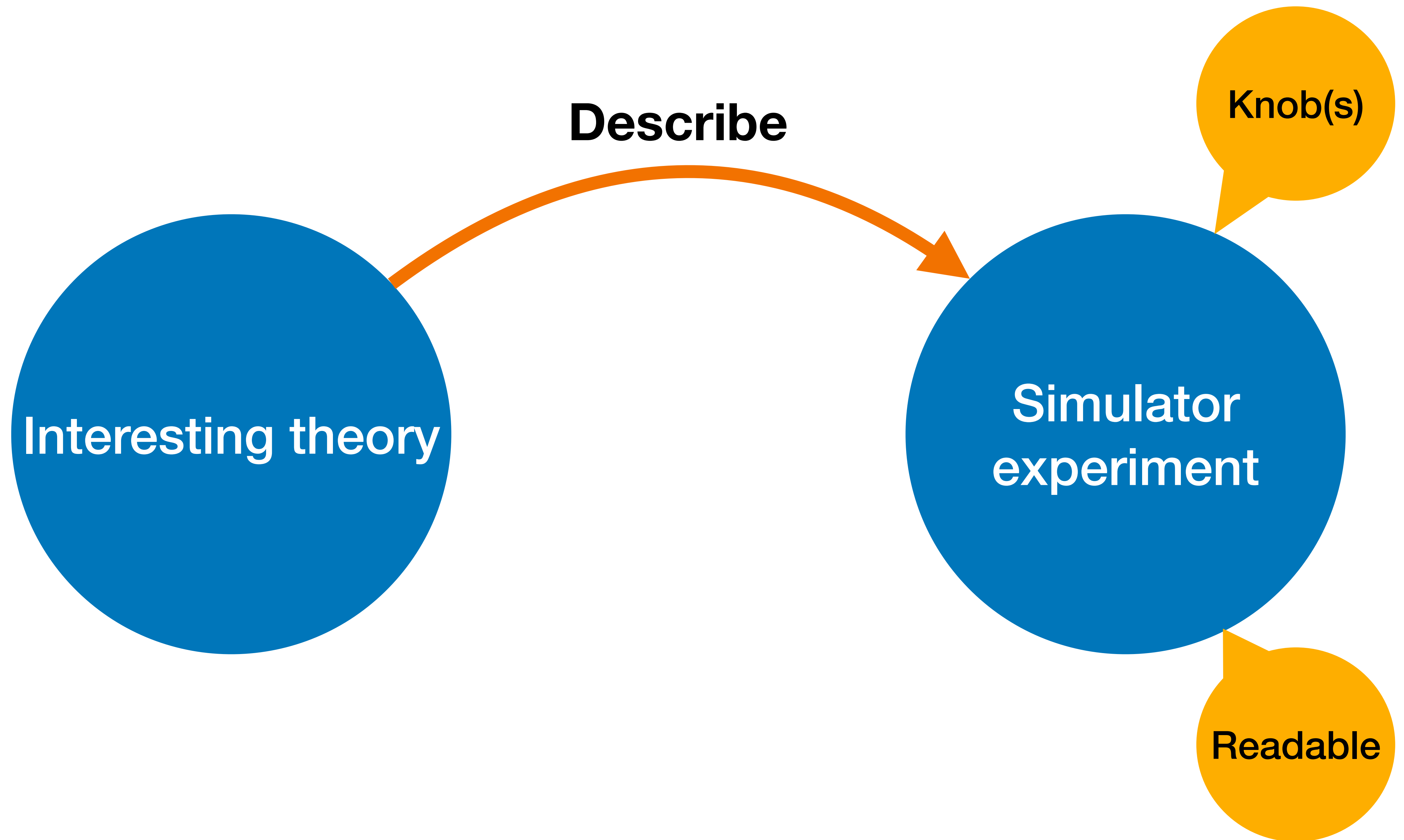
Interesting theory

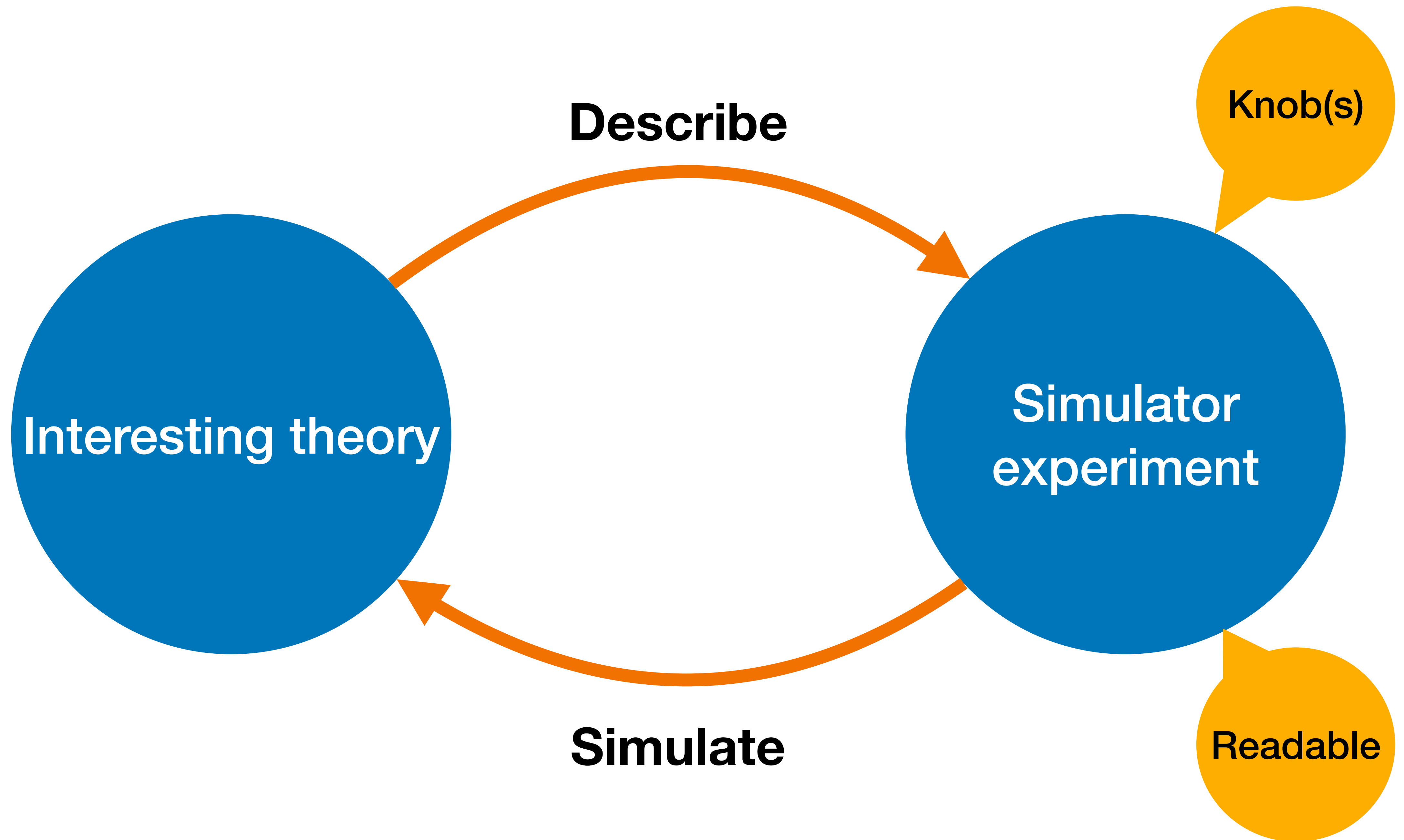


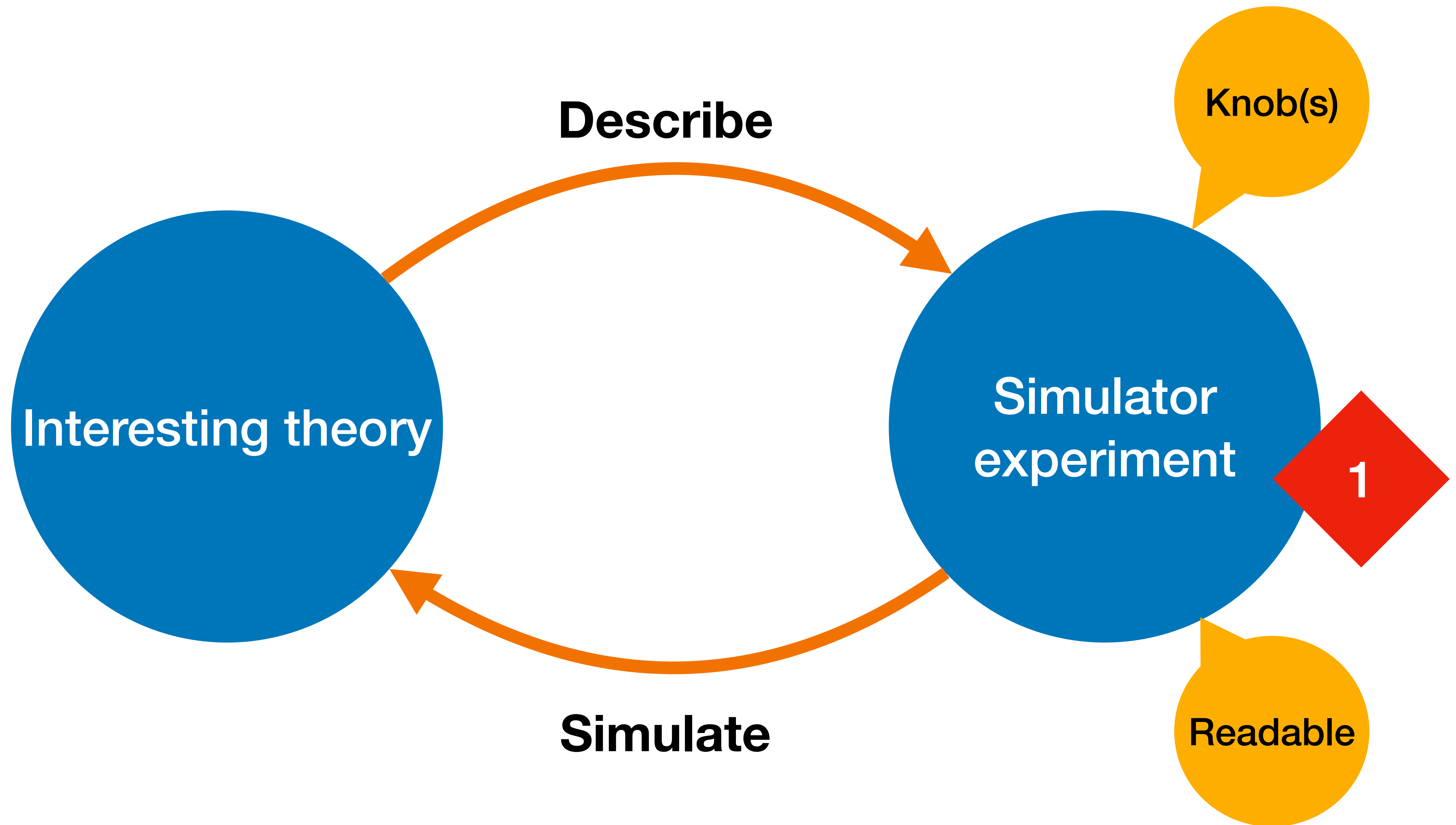
Simulator
experiment

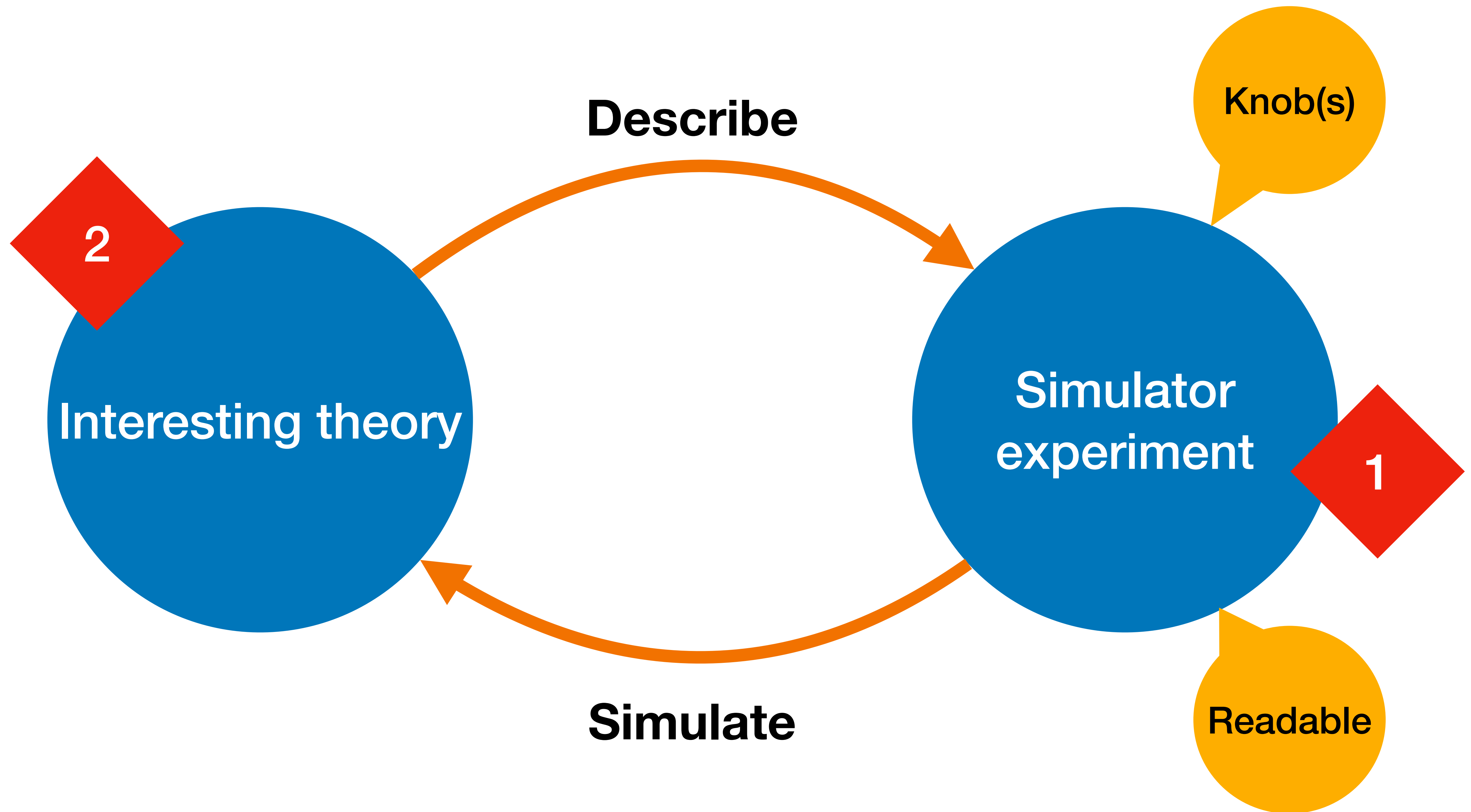


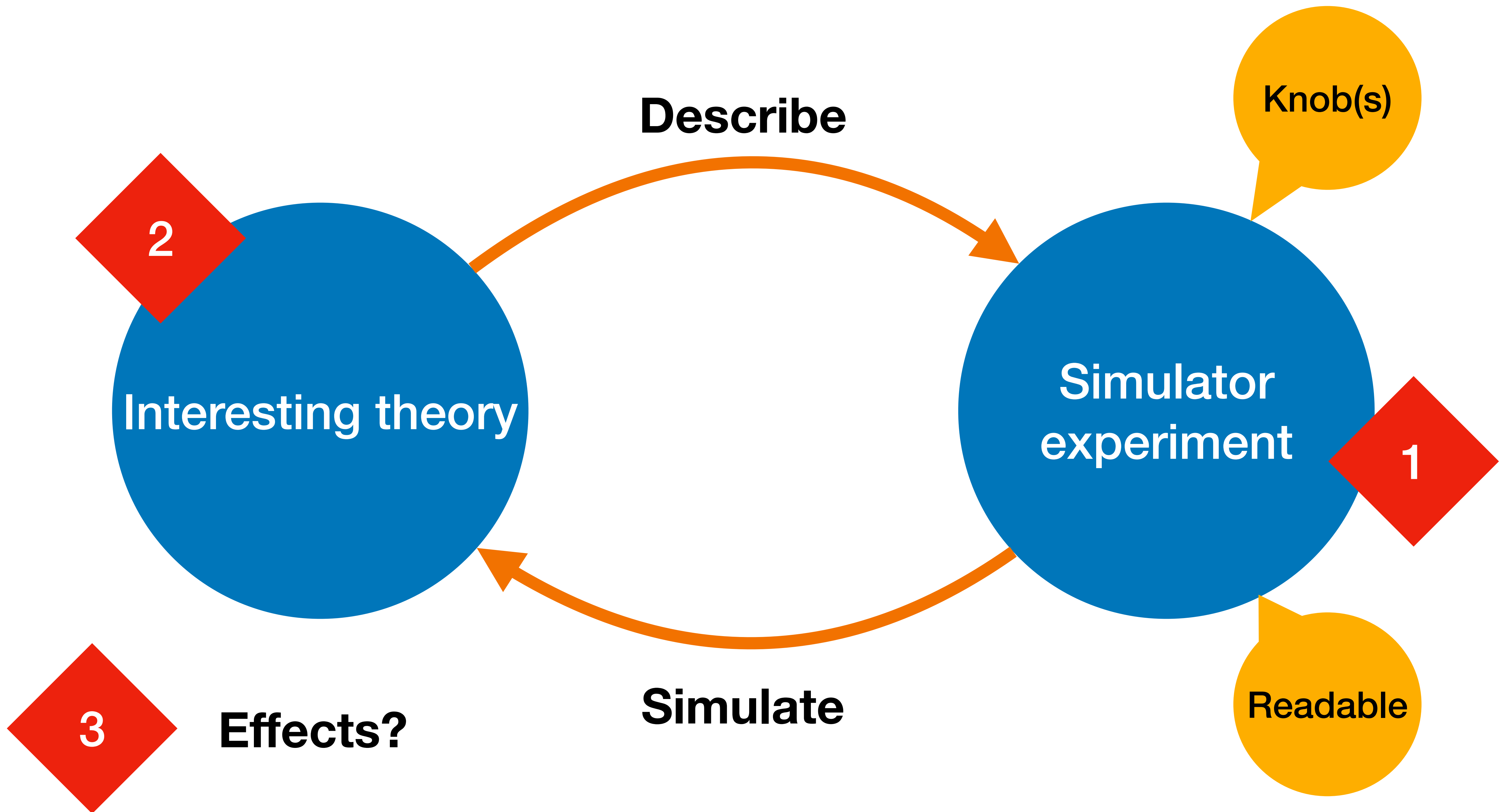




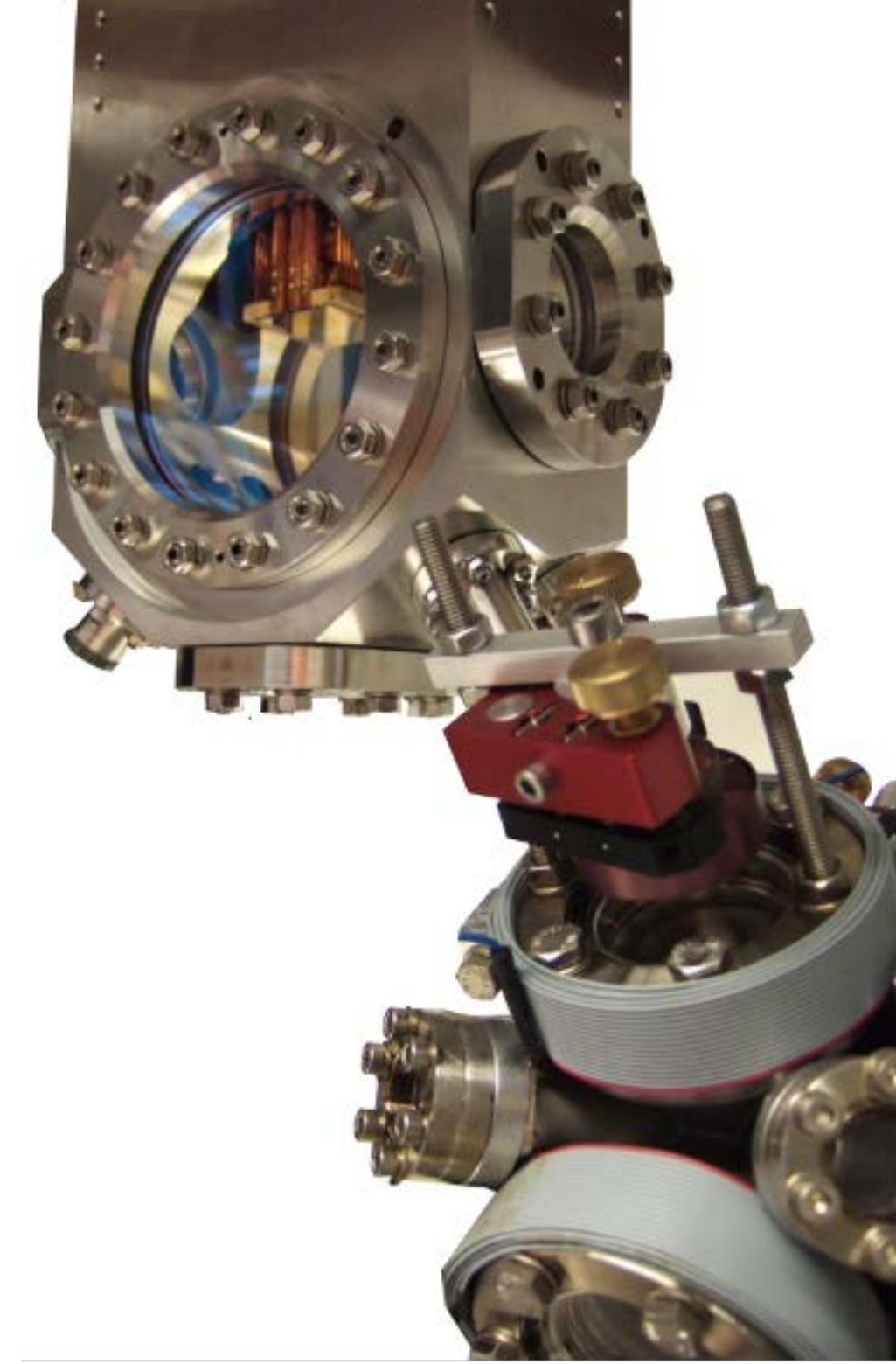






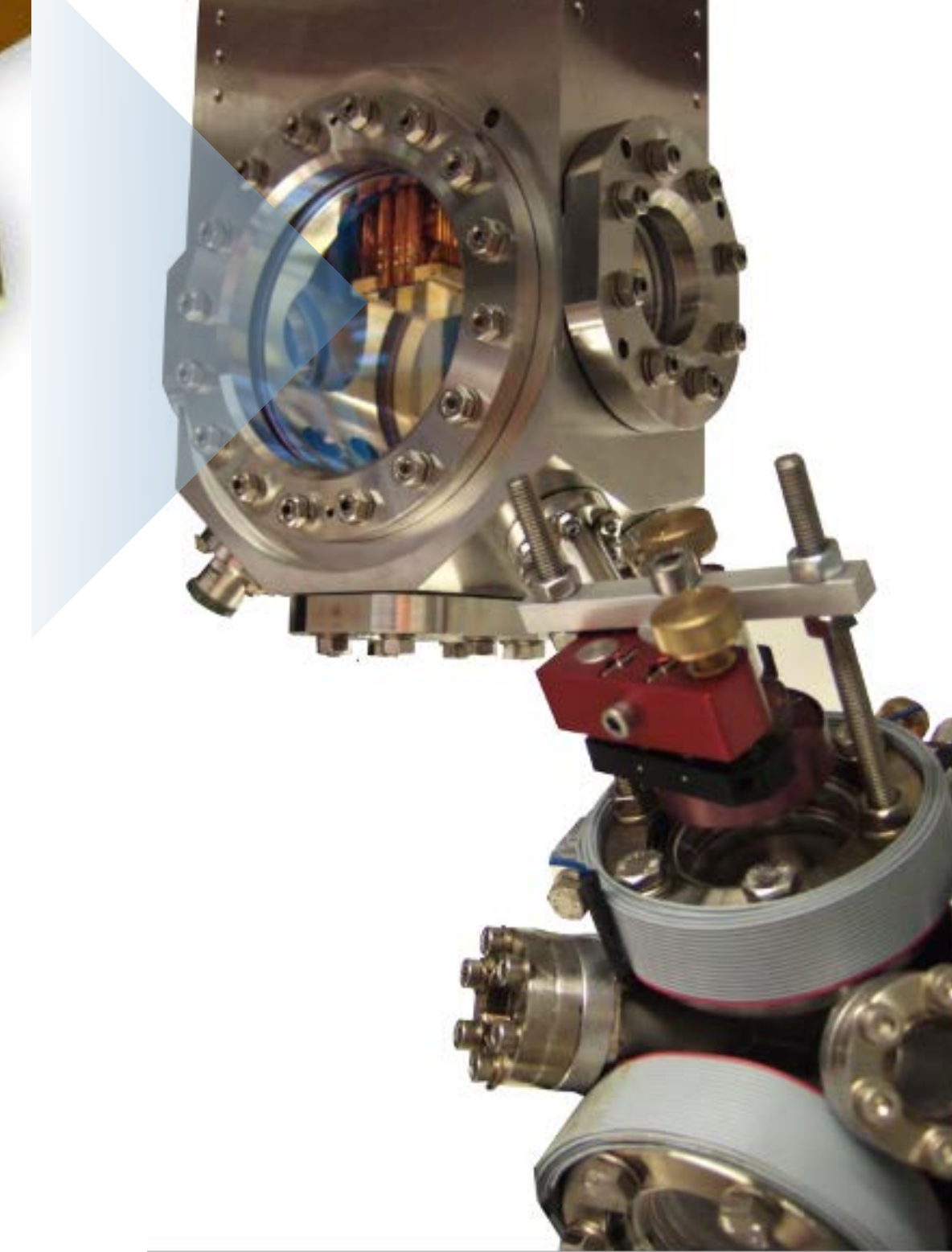
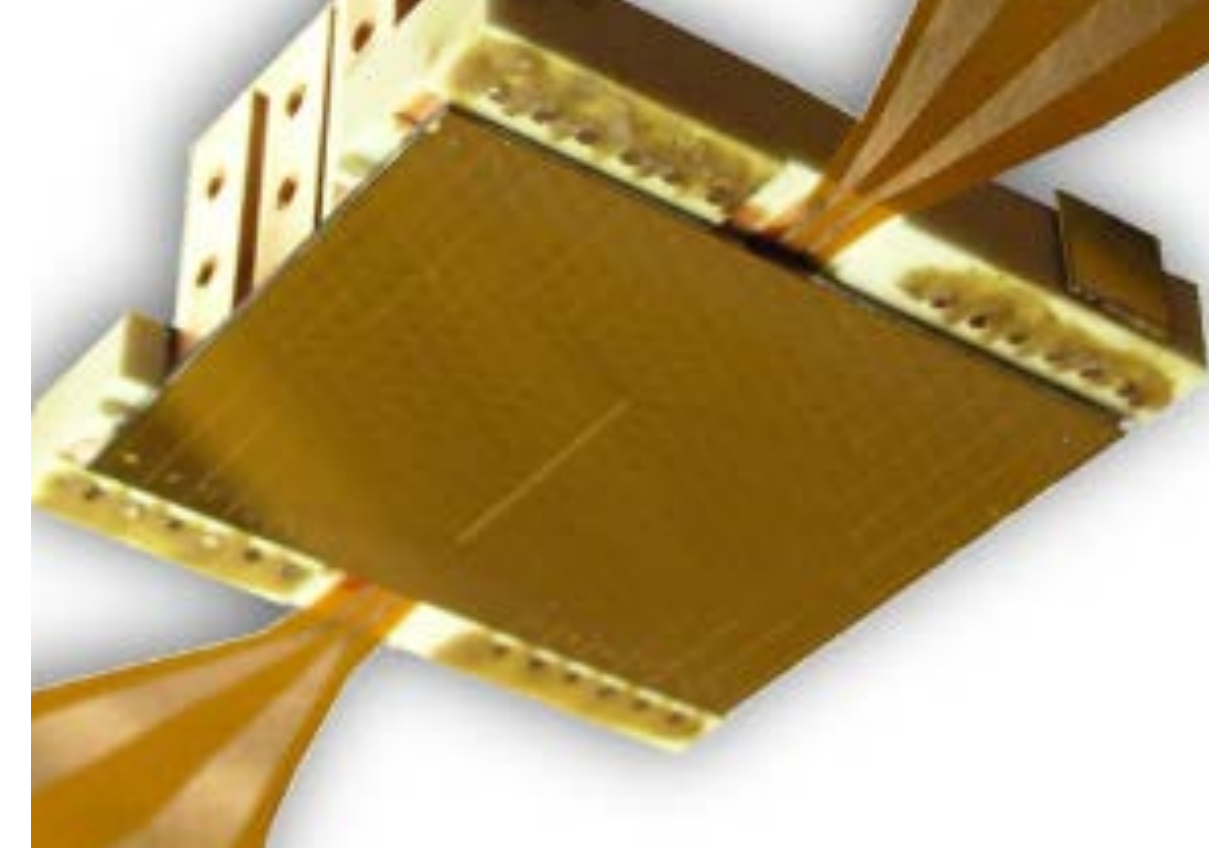


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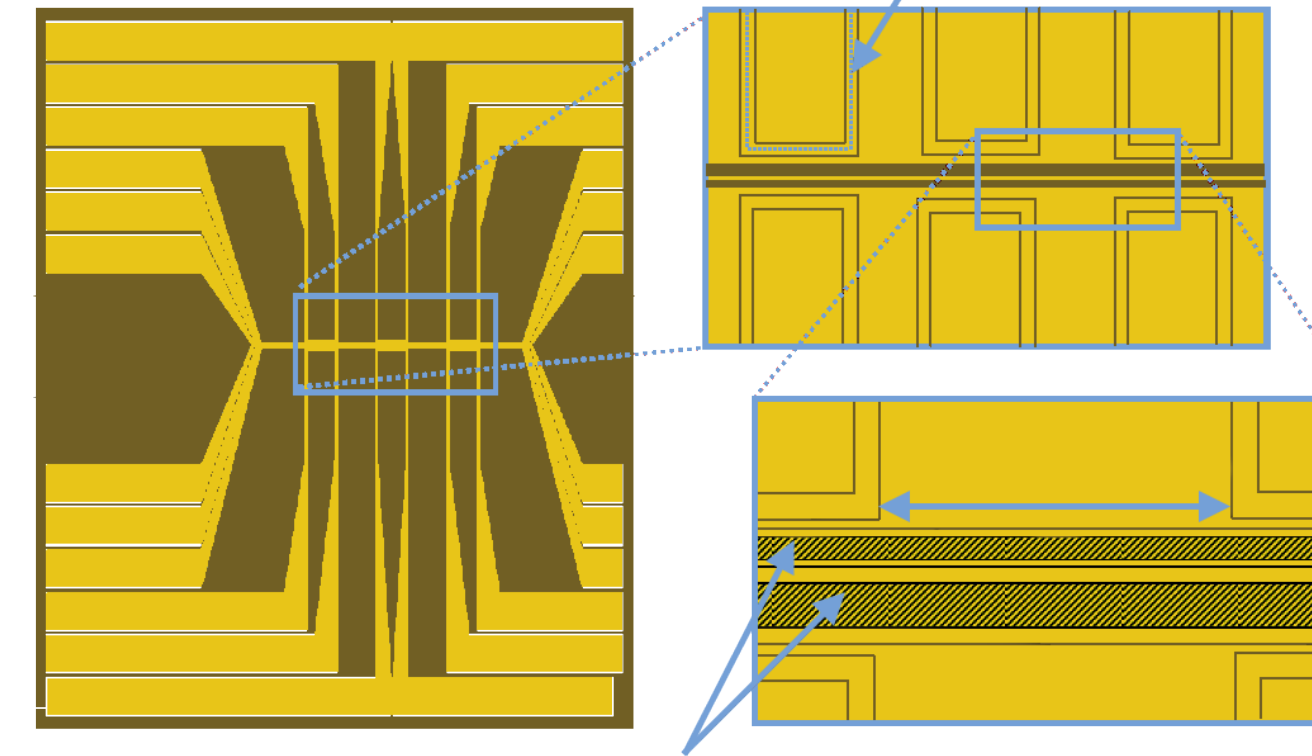
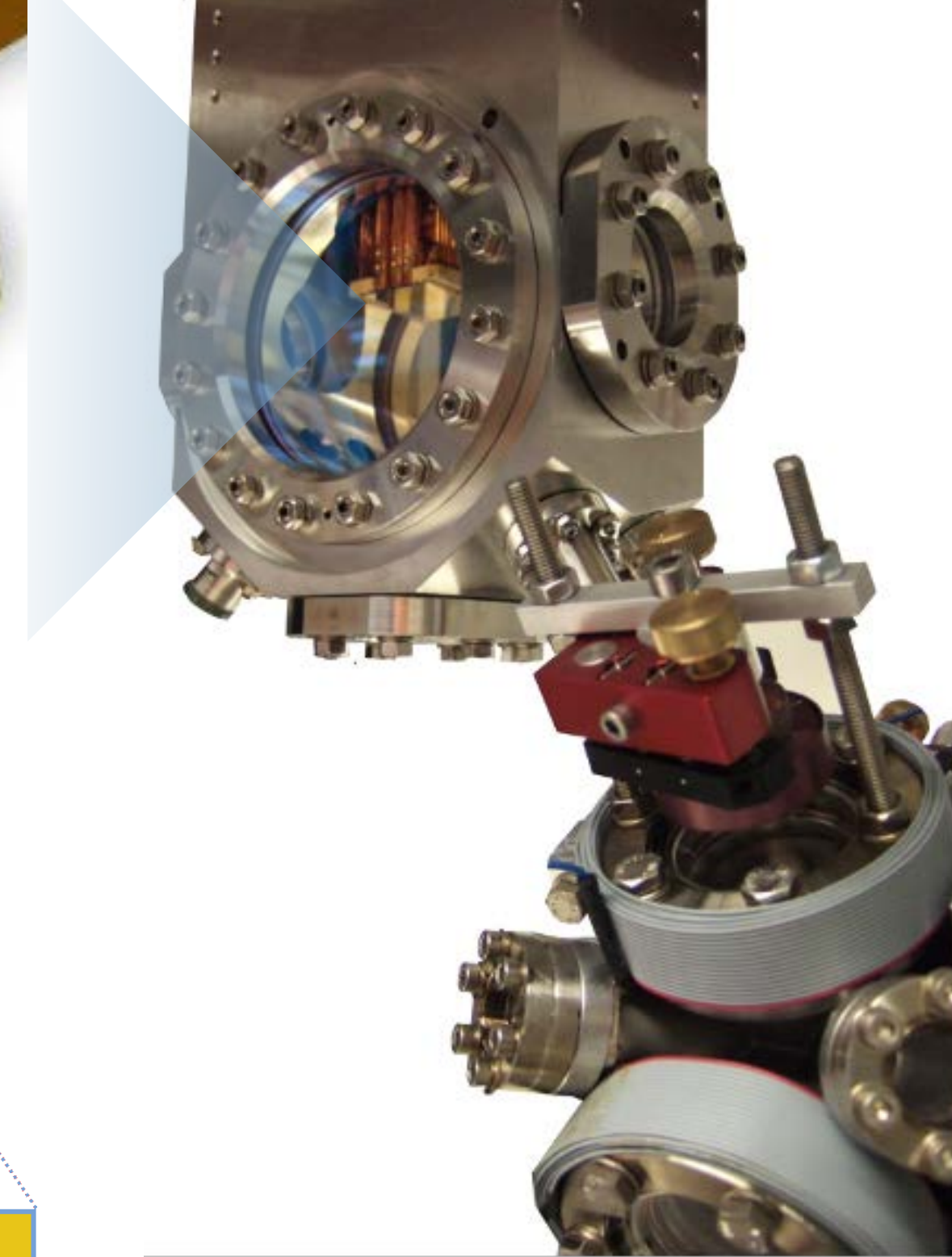
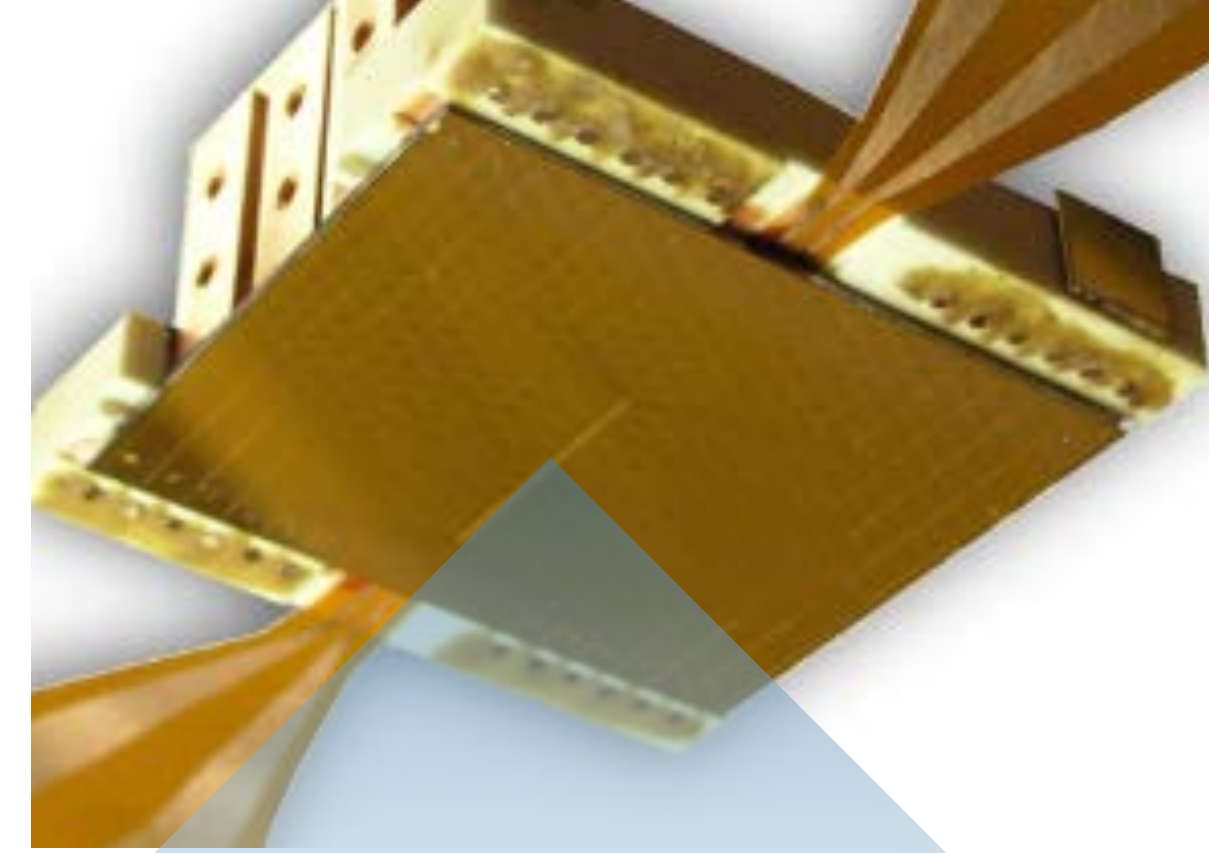
For more information on experiment, read T. Schweigler or B. Rauer thesis

1



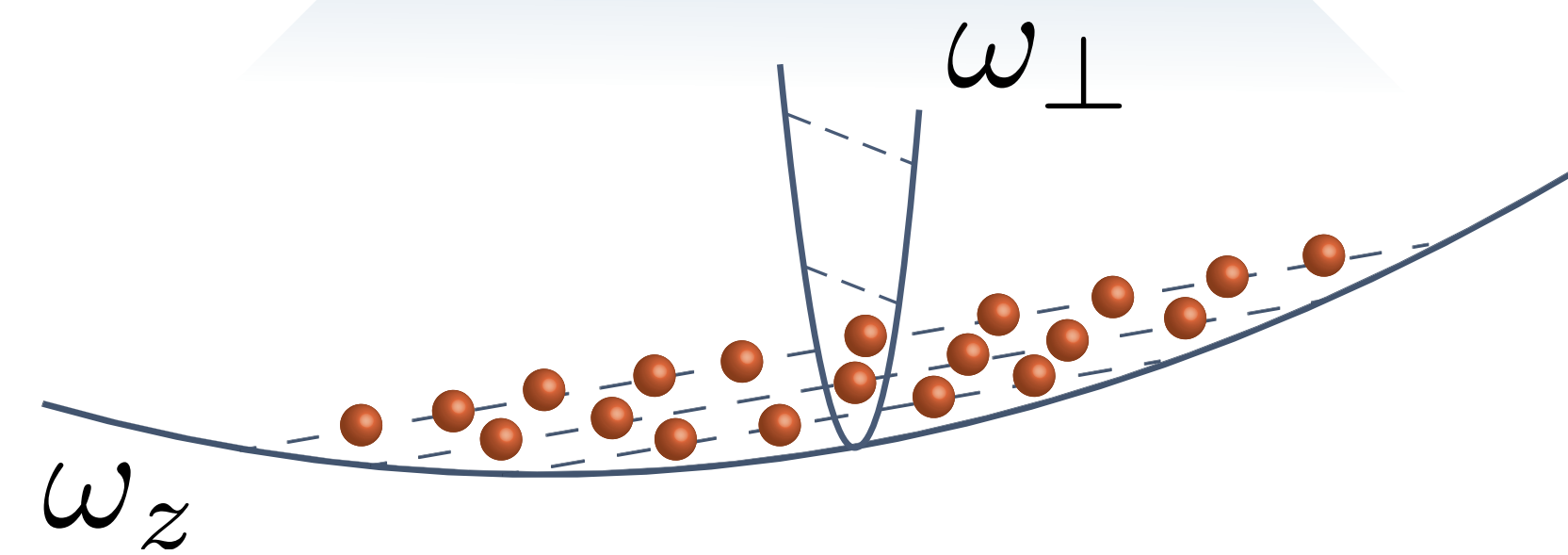
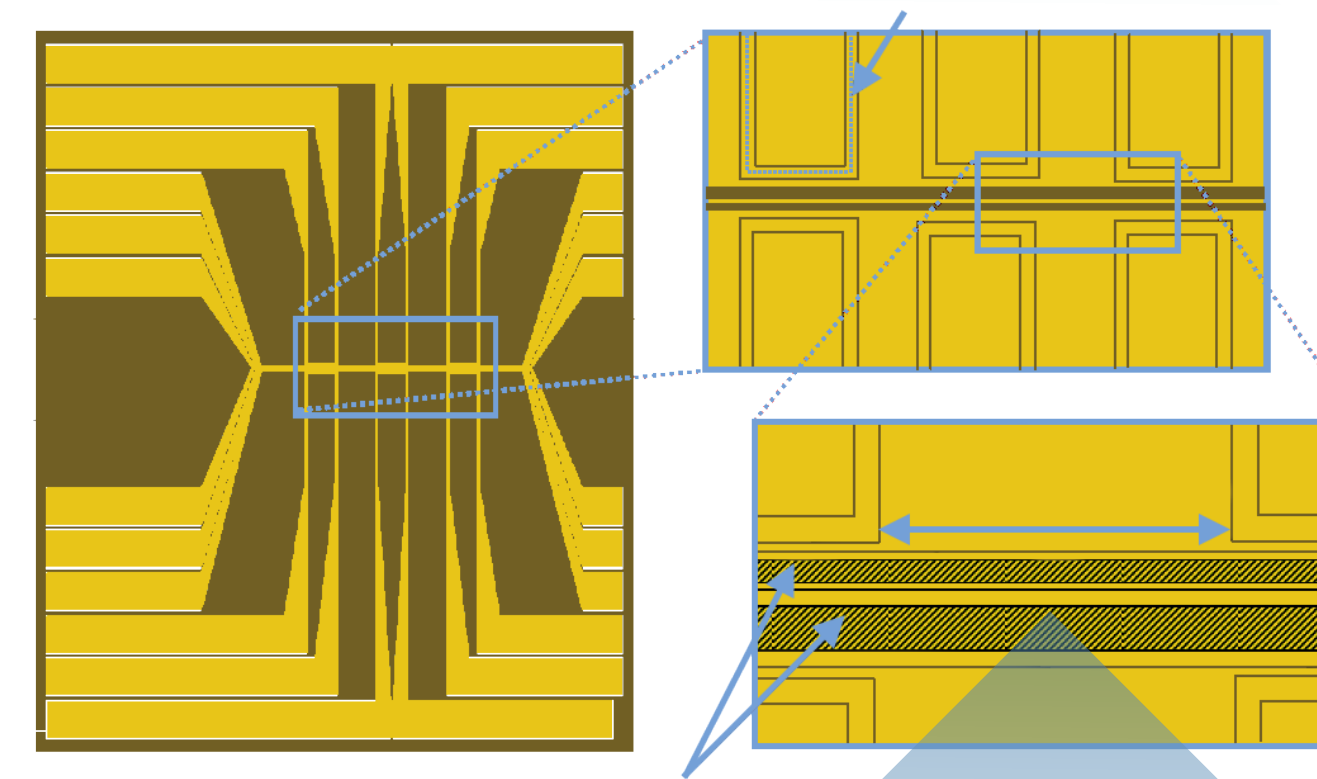
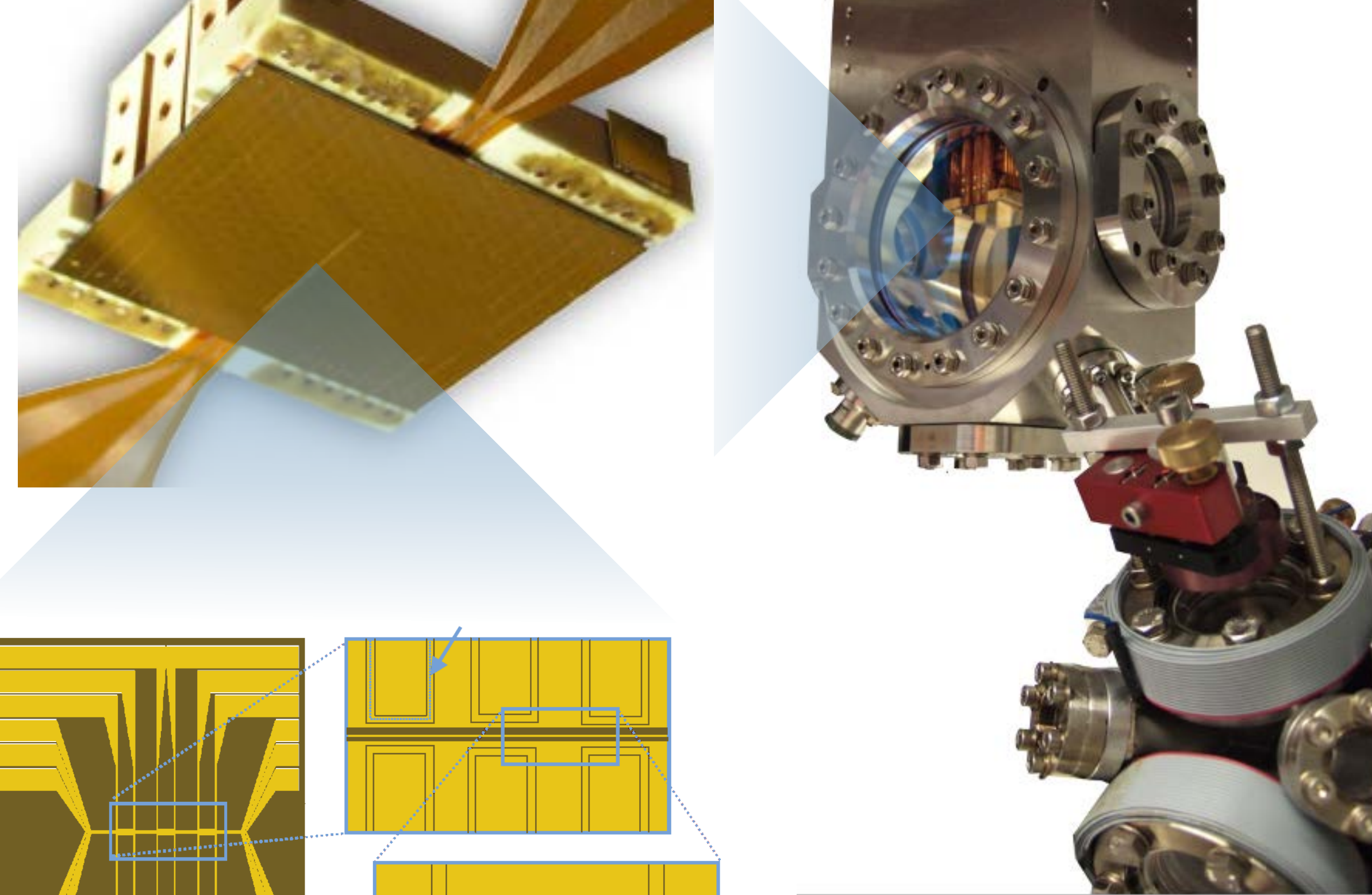
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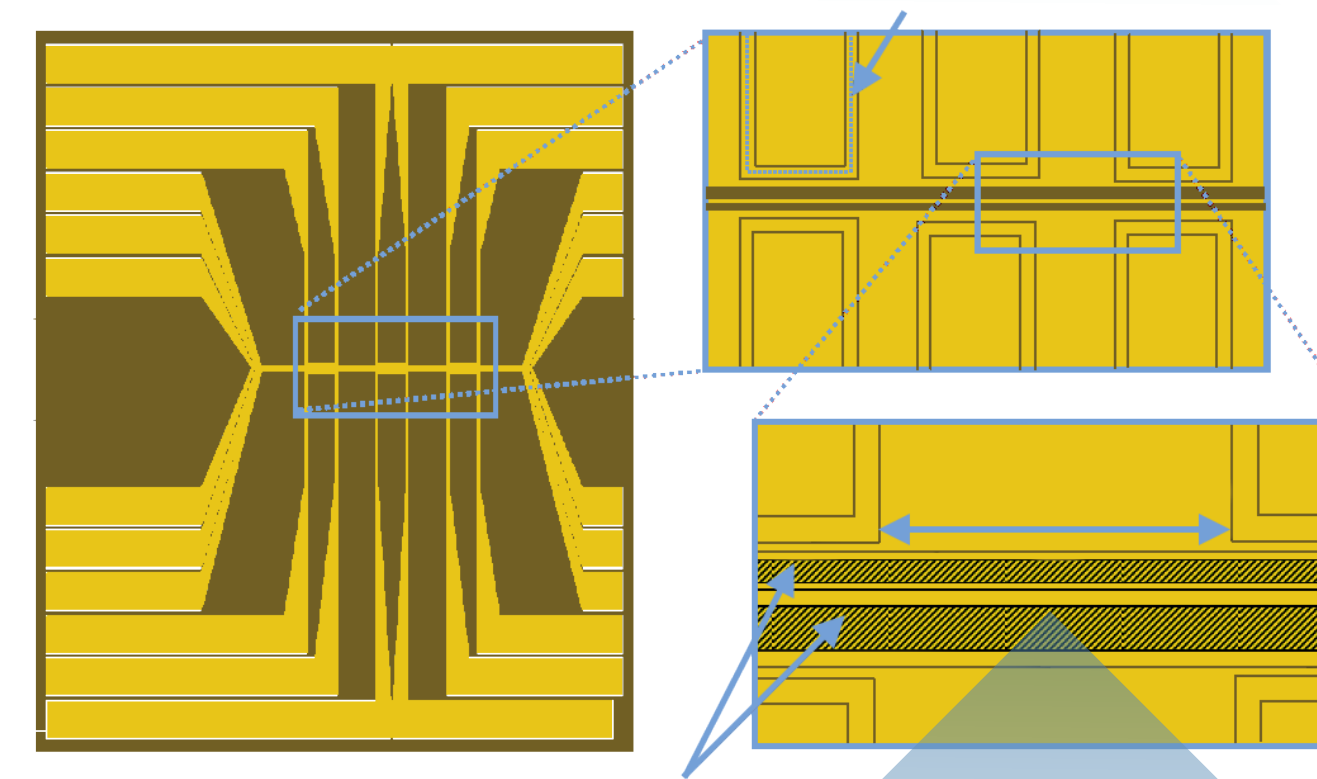
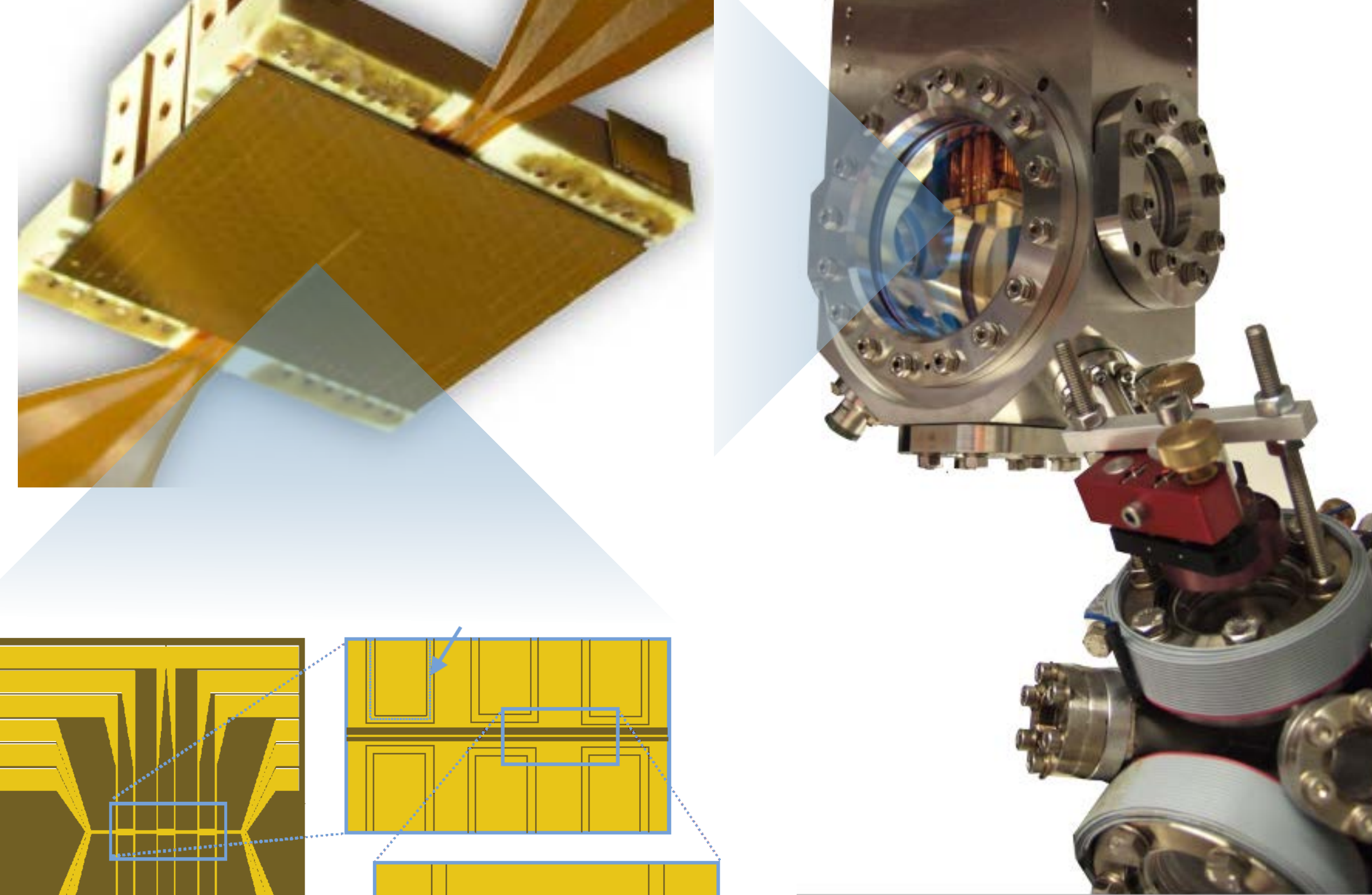
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1



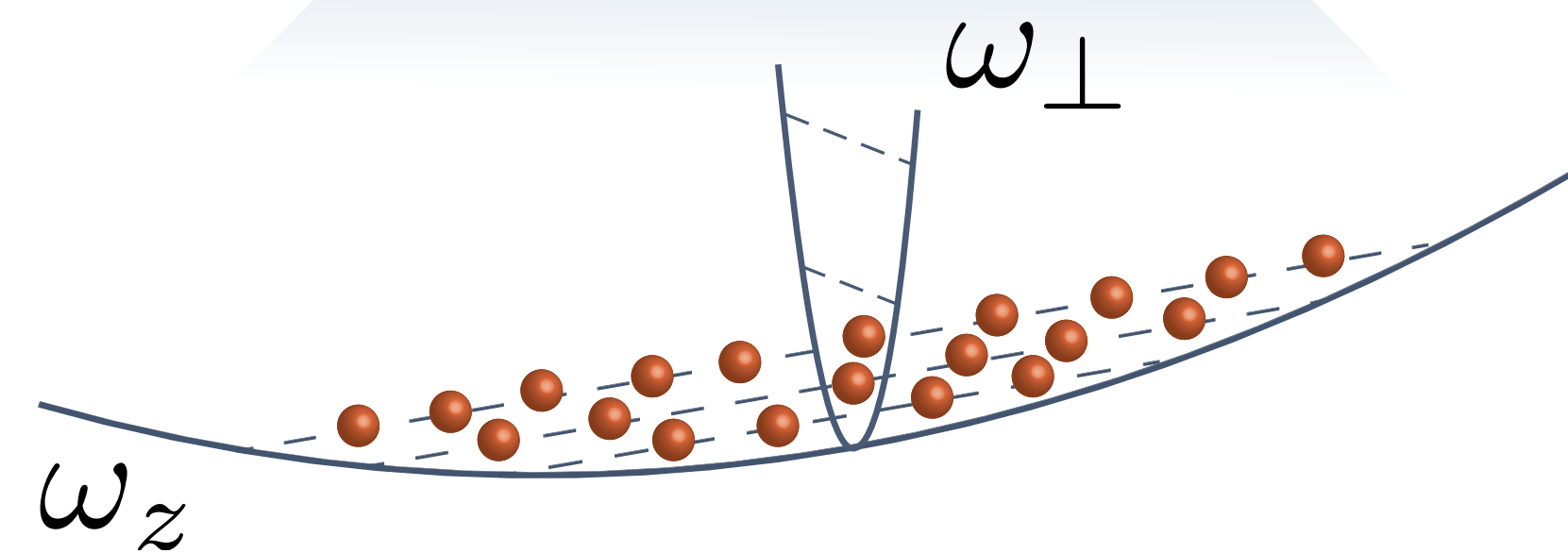
For more information on experiment, read T. Schweigler or B. Rauer thesis

1



1D condition:

$$\mu, k_B T < \hbar \omega_{\perp}$$



For more information on experiment, read T. Schweigler or B. Rauer thesis

1

Experimental parameters:

$$N_{\text{atoms}} = 2000 - 20000$$

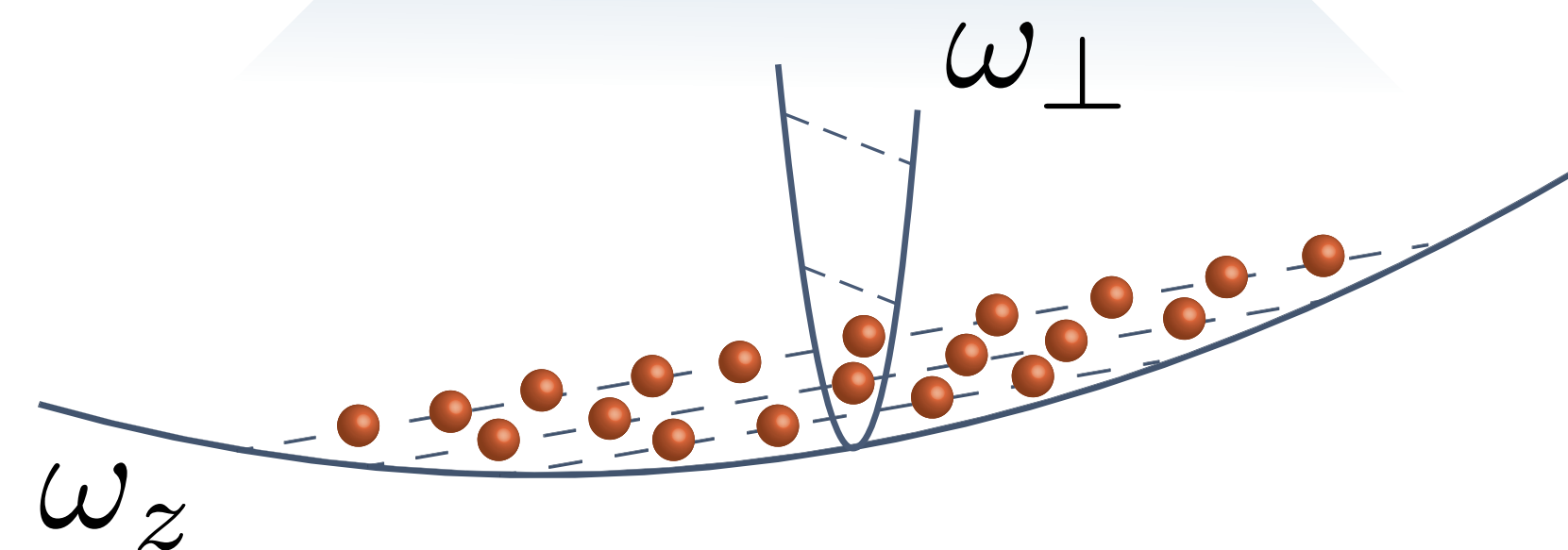
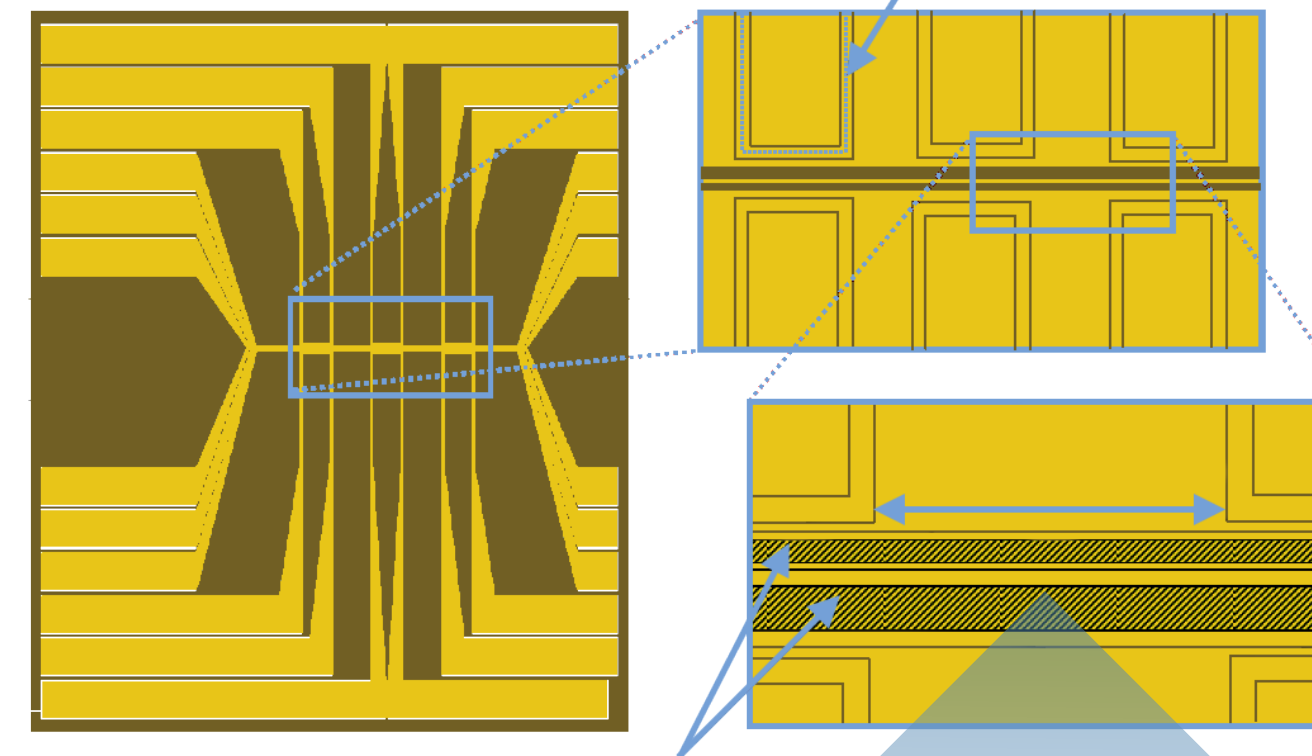
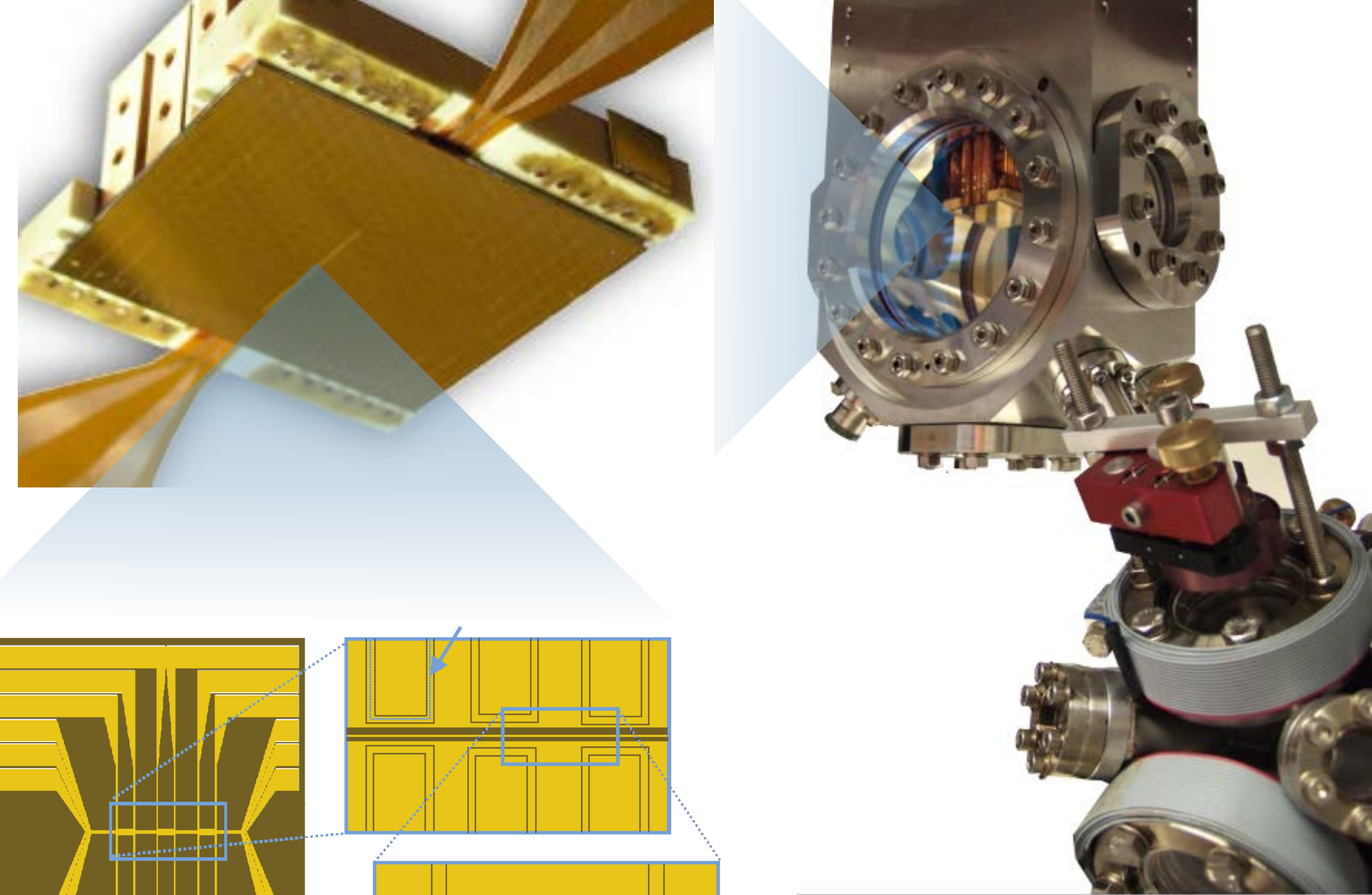
$$T = 20 - 150 \text{ nK}$$

$$\omega_{\perp} \approx 2\pi \times 2 \text{ kHz}$$

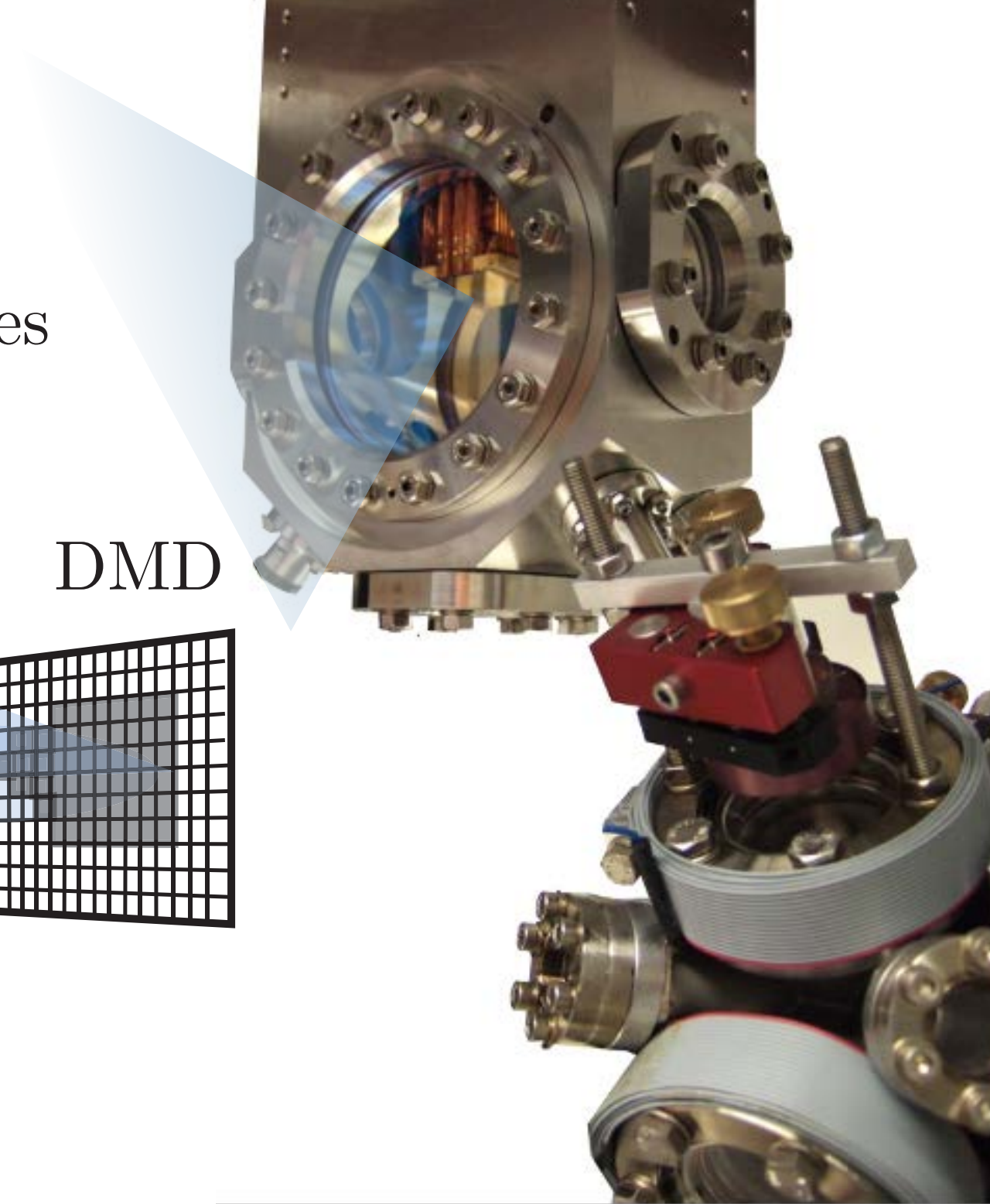
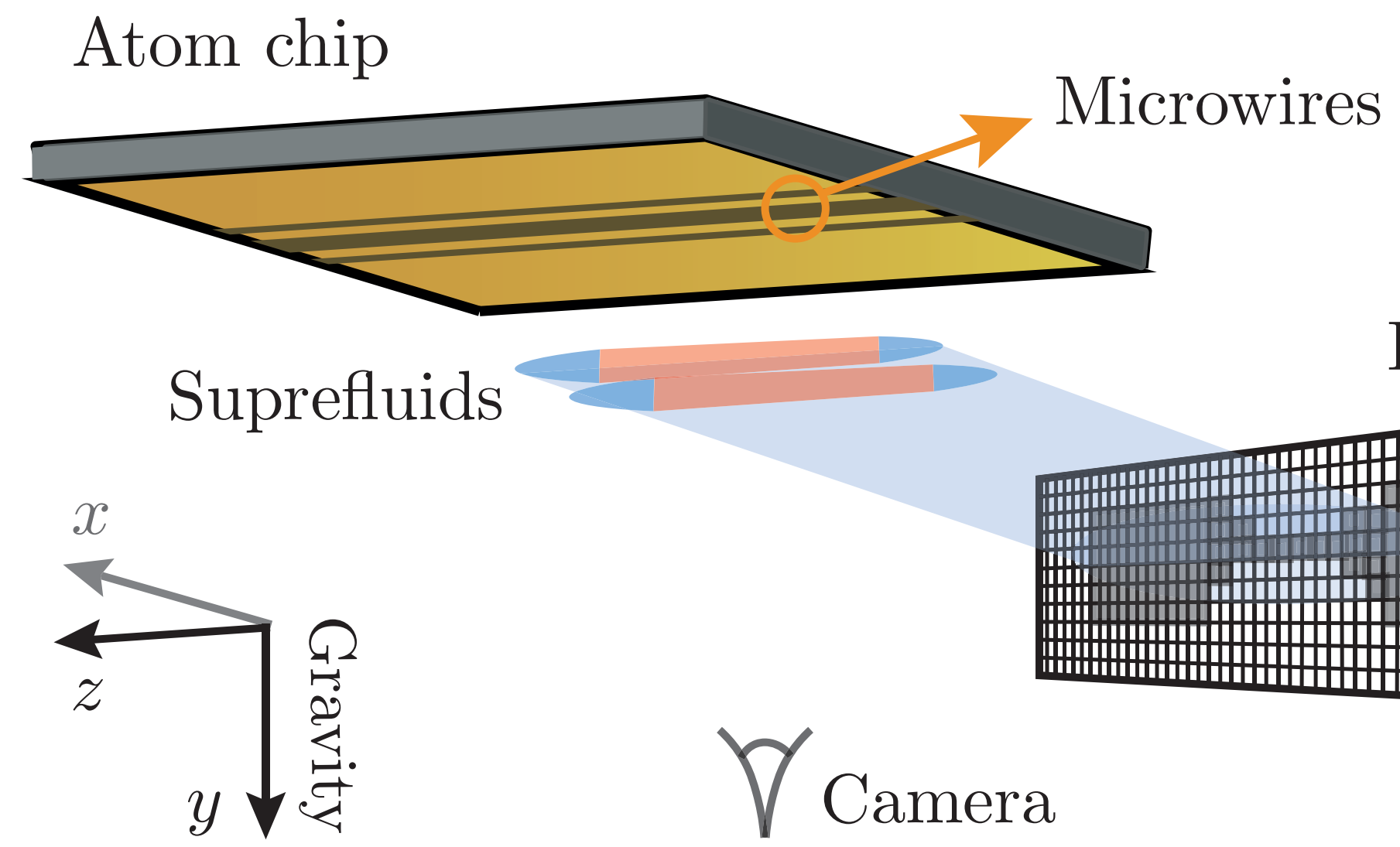
$$\omega_z \approx 2\pi \times 10 \text{ Hz}$$

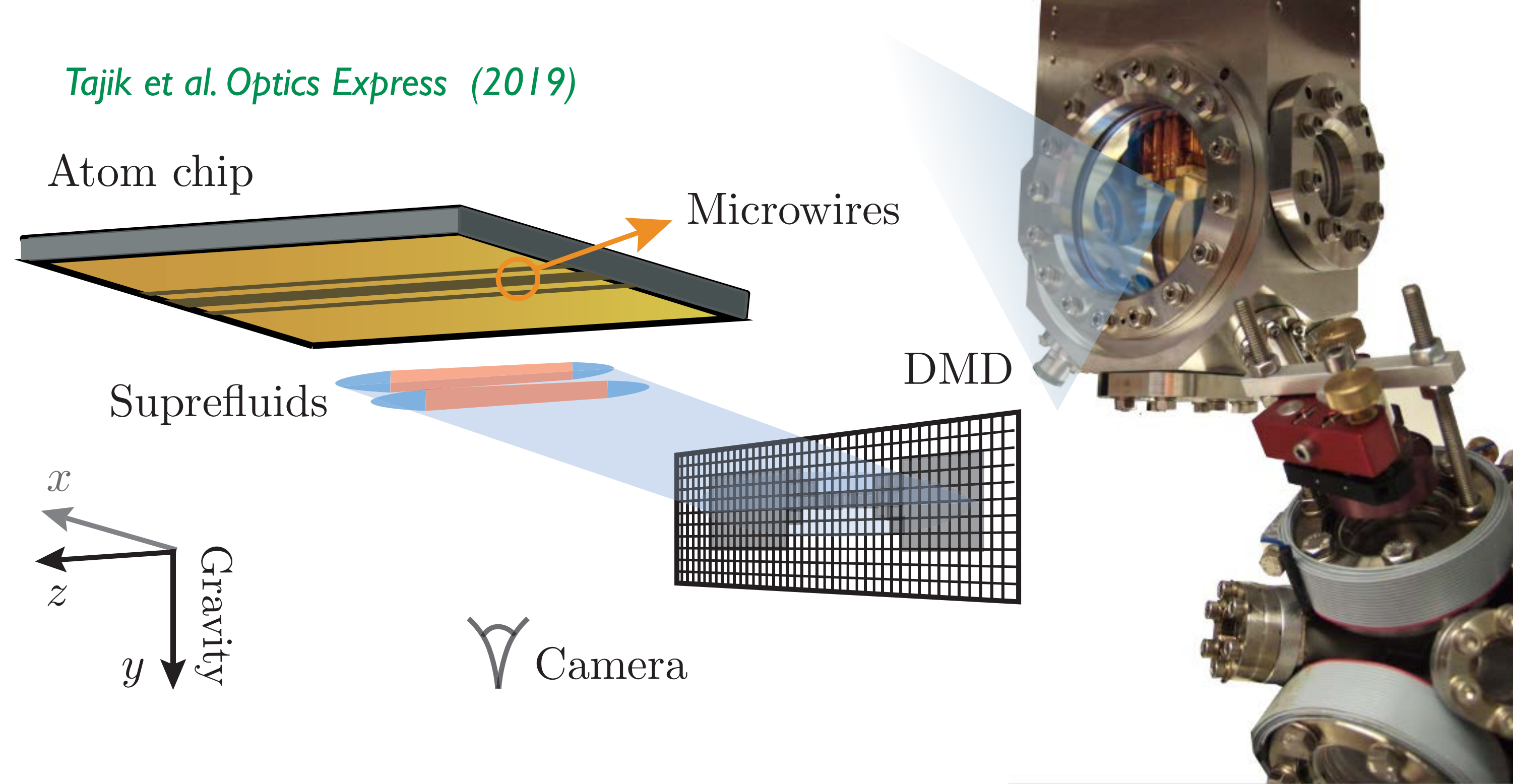
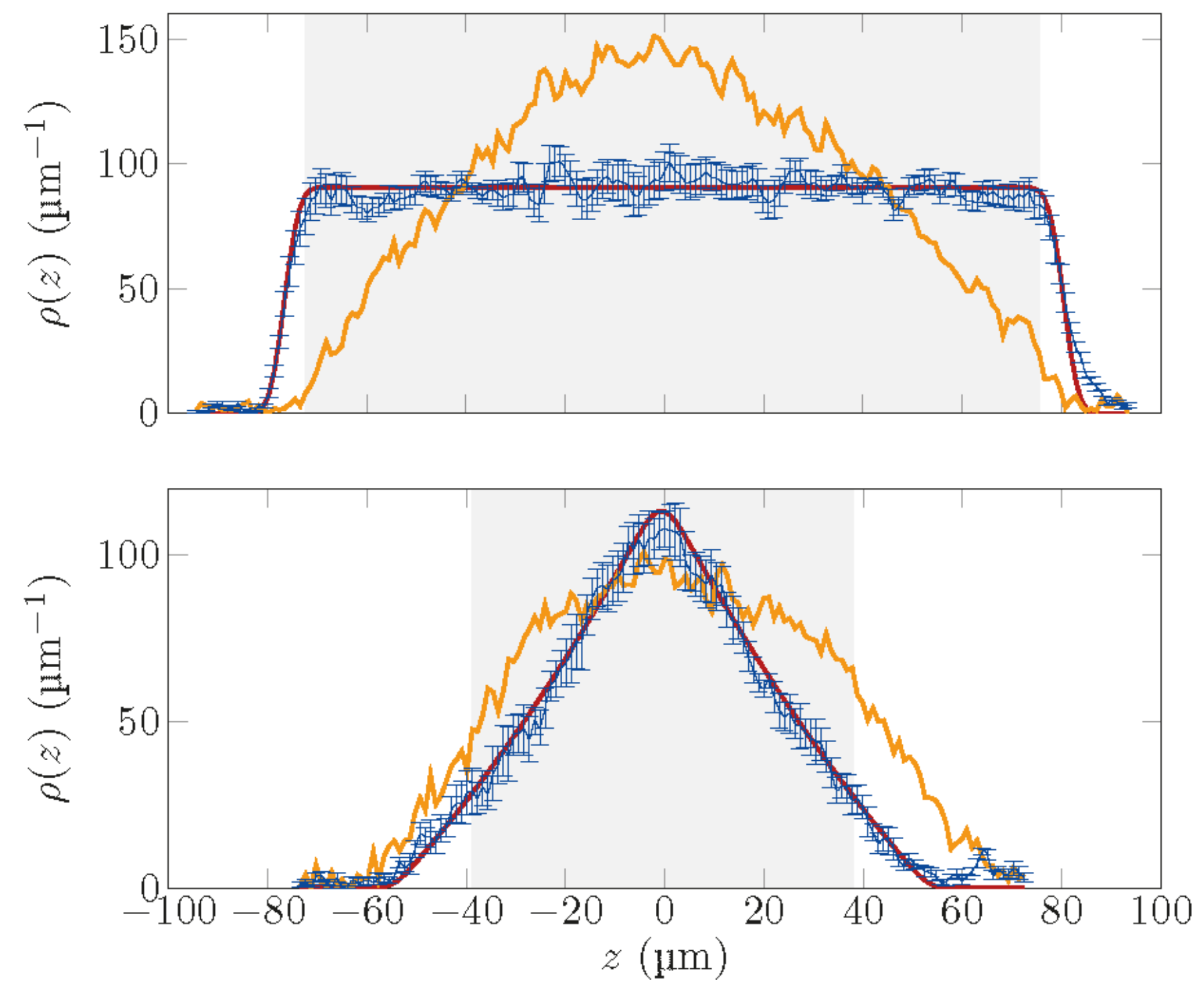
1D condition:

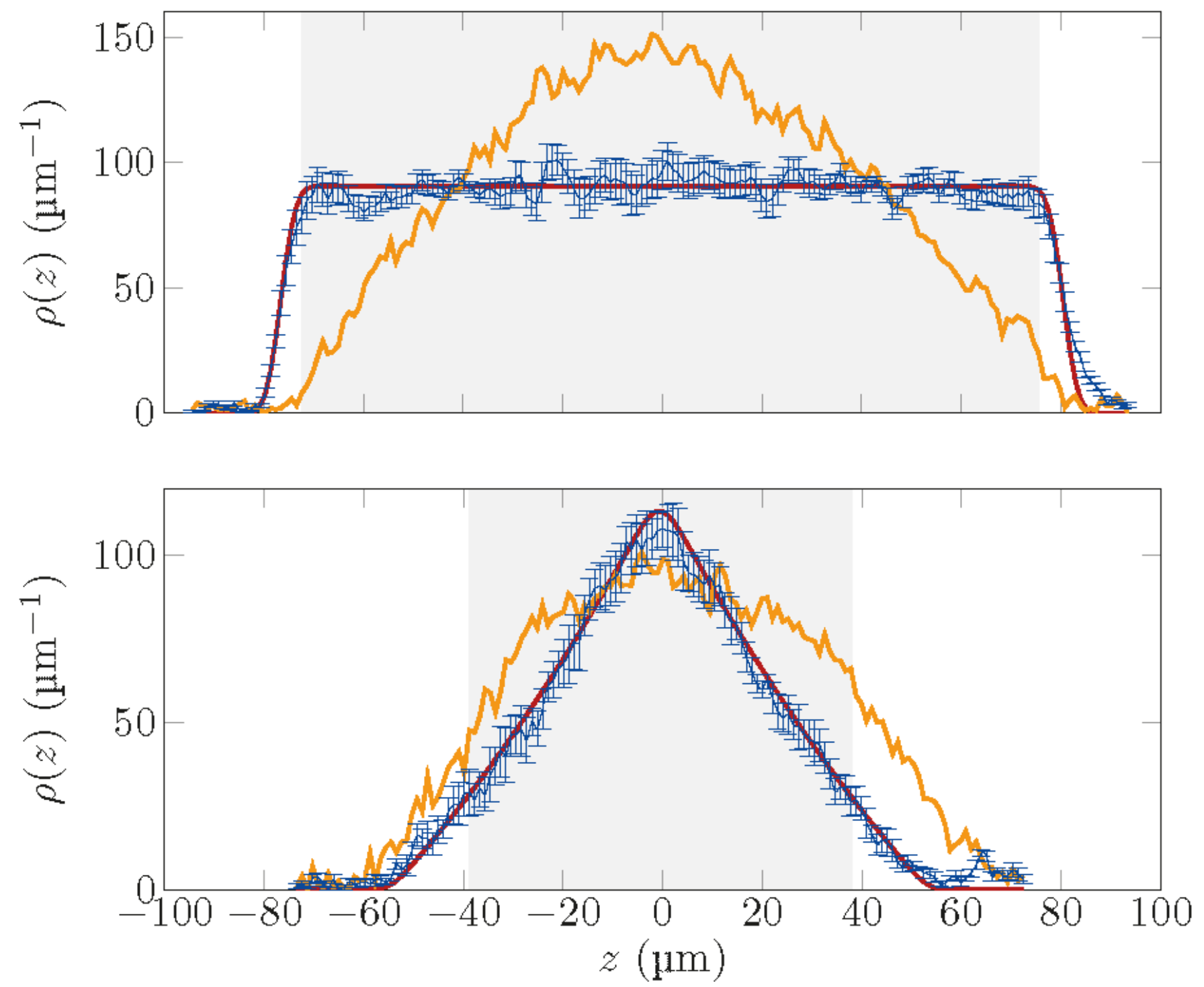
$$\mu, k_B T < \hbar \omega_{\perp}$$



Tajik et al. Optics Express (2019)







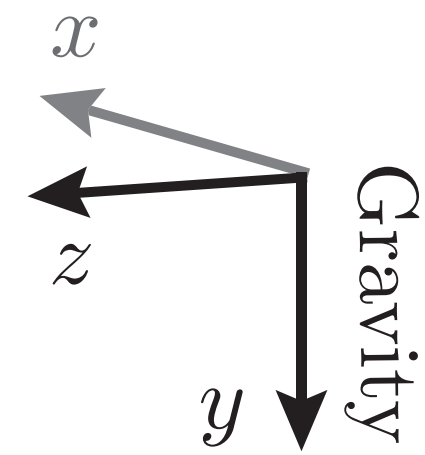
Tajik et al. Optics Express (2019)

Atom chip

Microwires

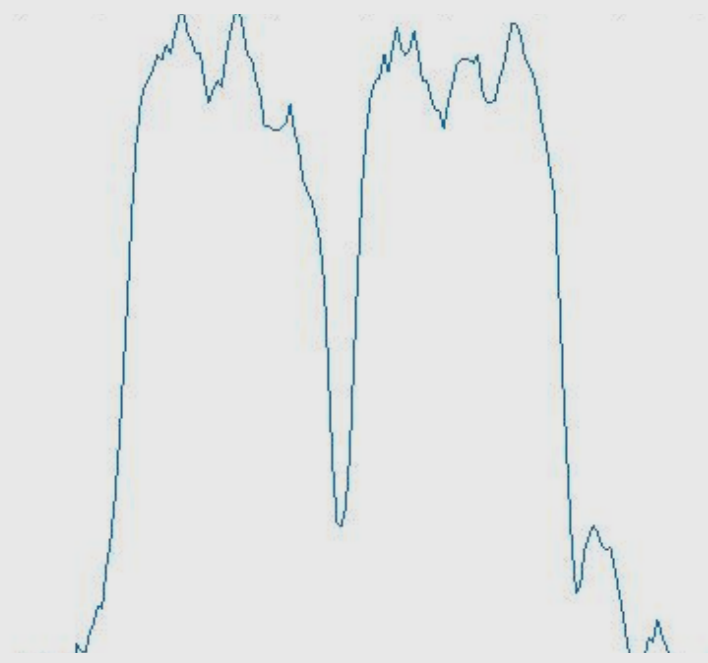
Suprefluids

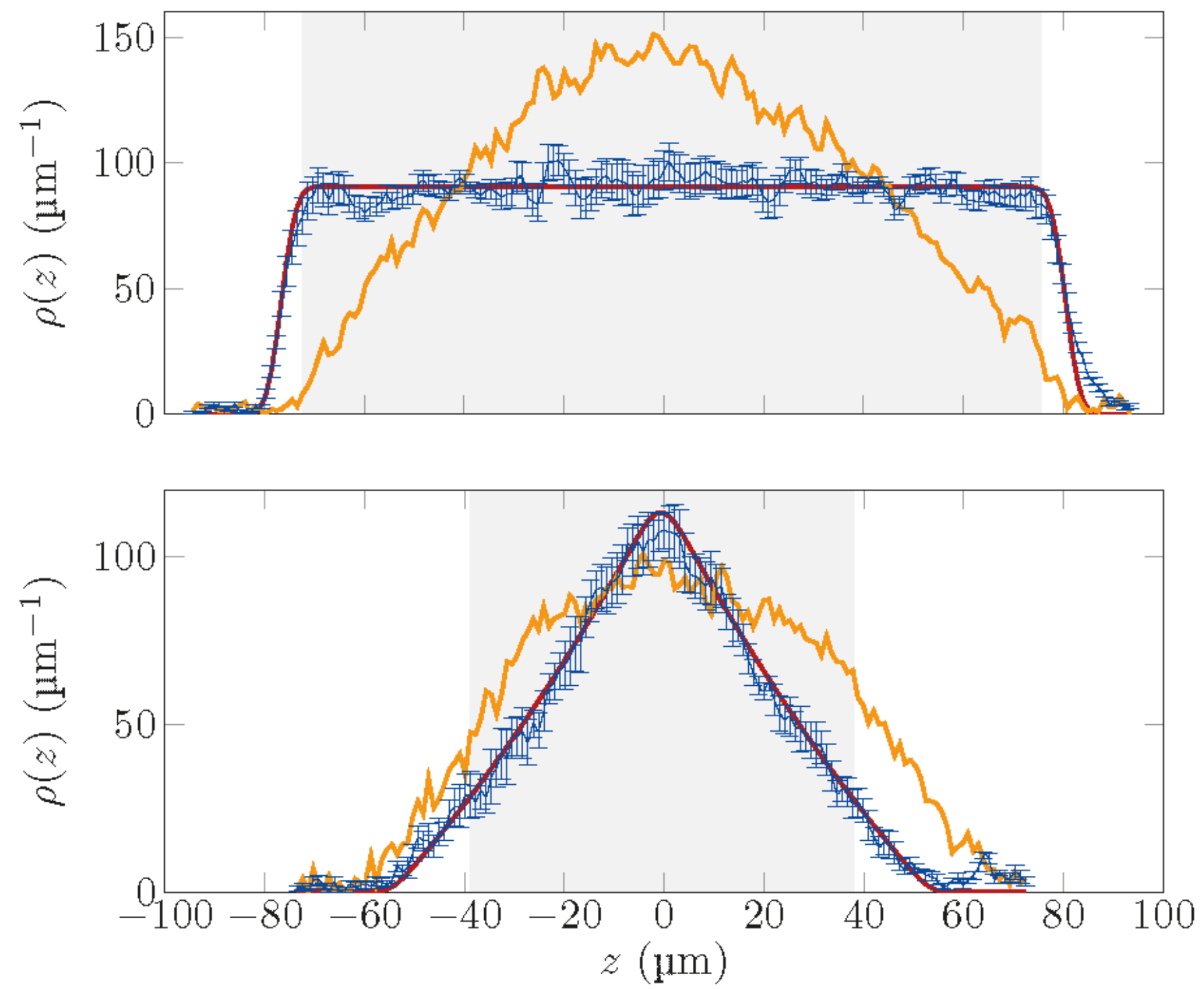
DMD



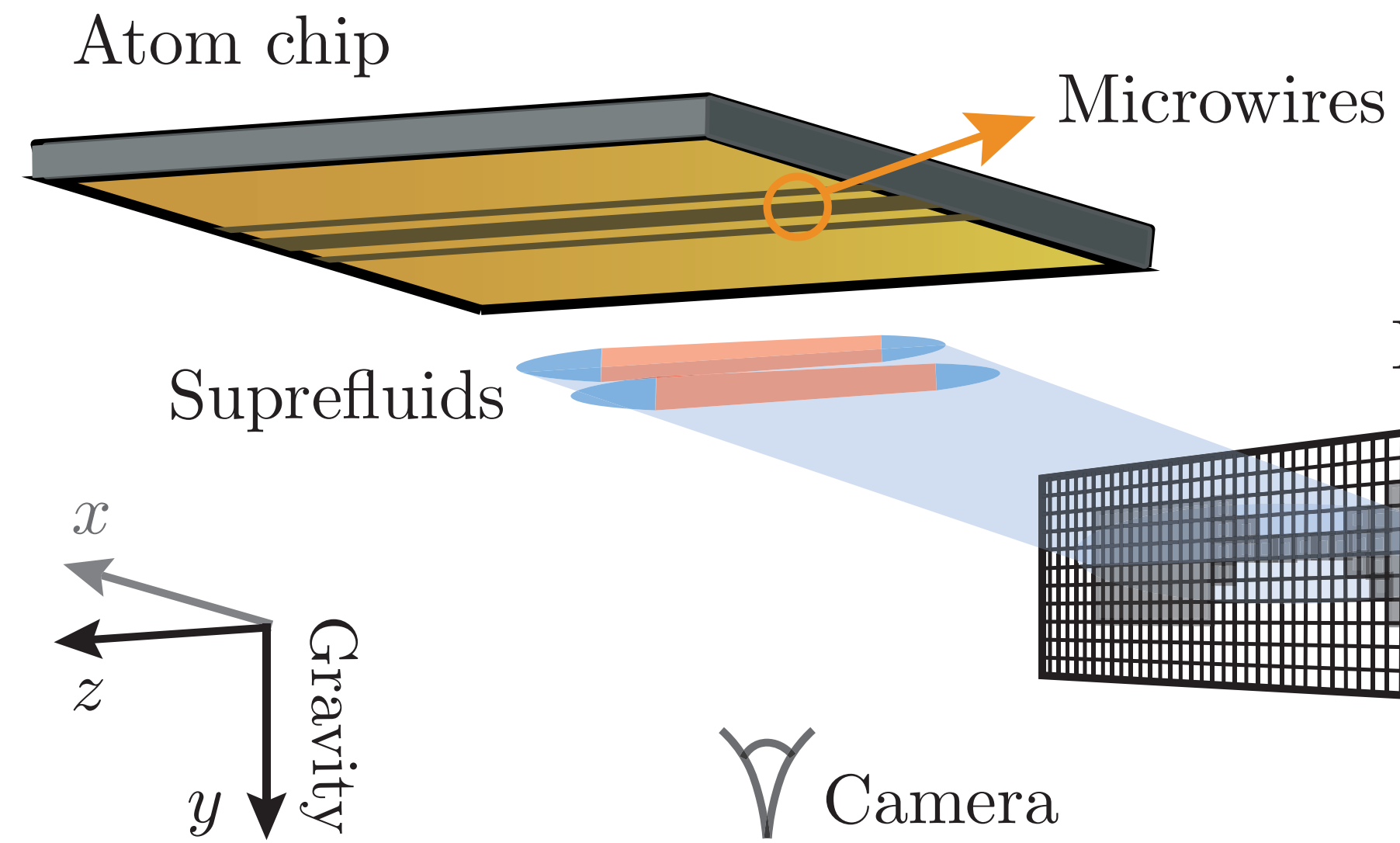
Camera

Quantum field machines
João S. & Philipp S.

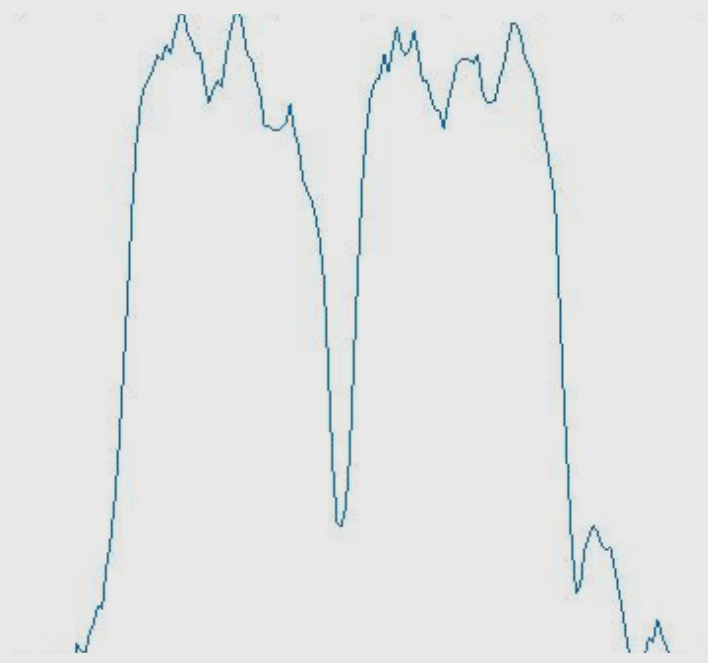




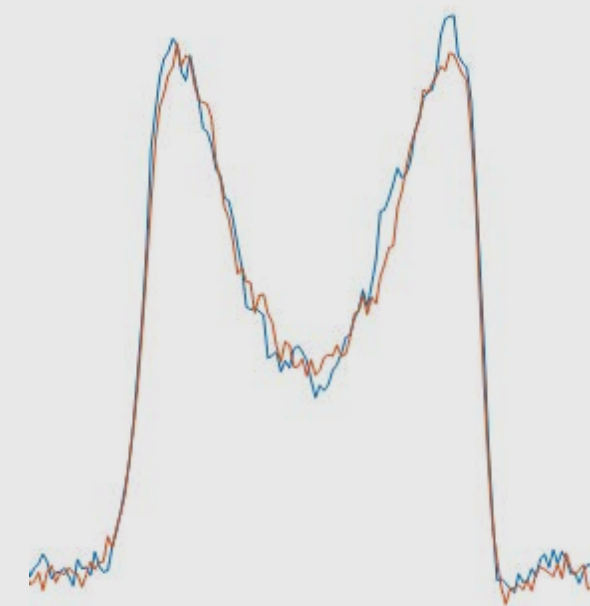
Tajik et al. Optics Express (2019)

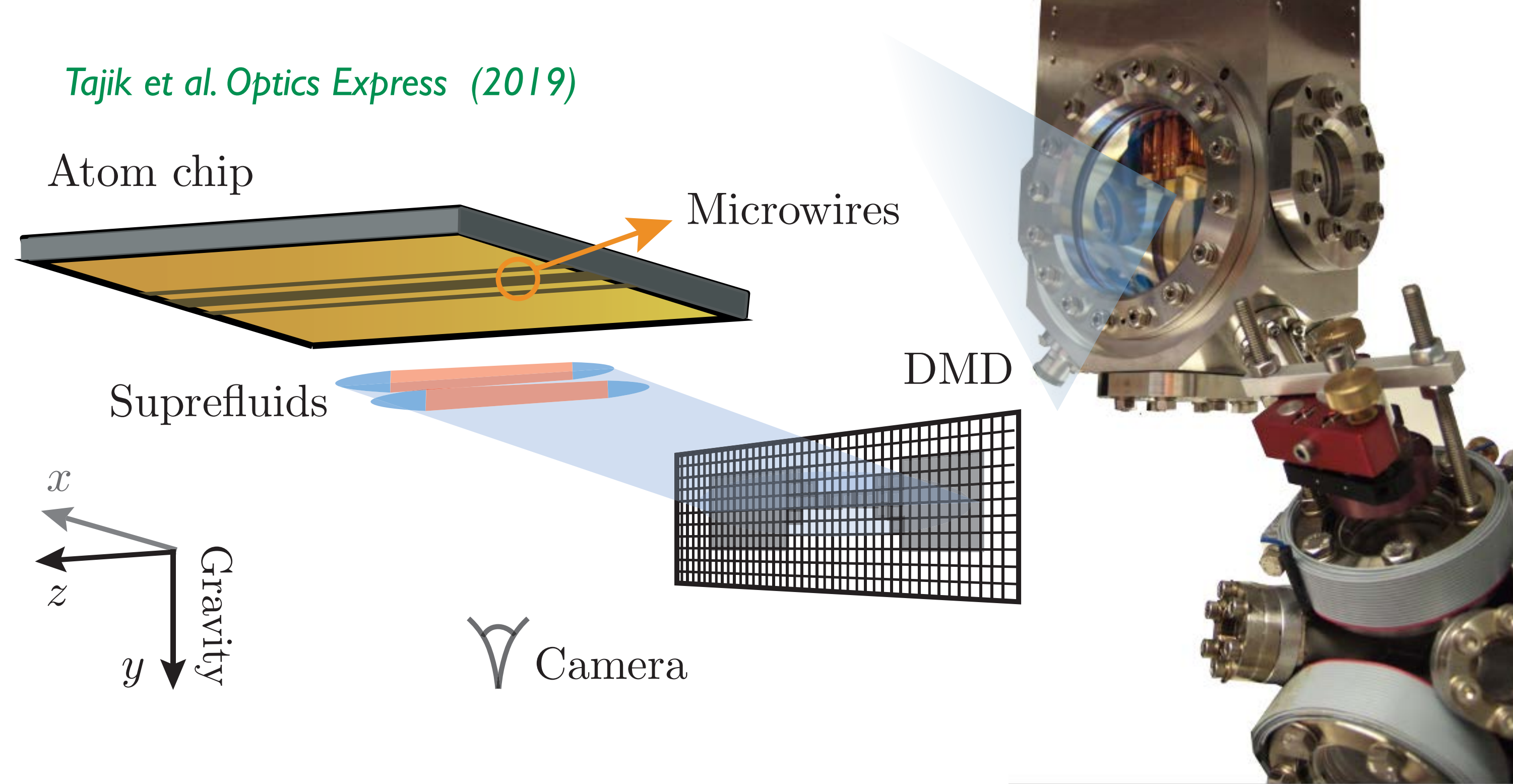
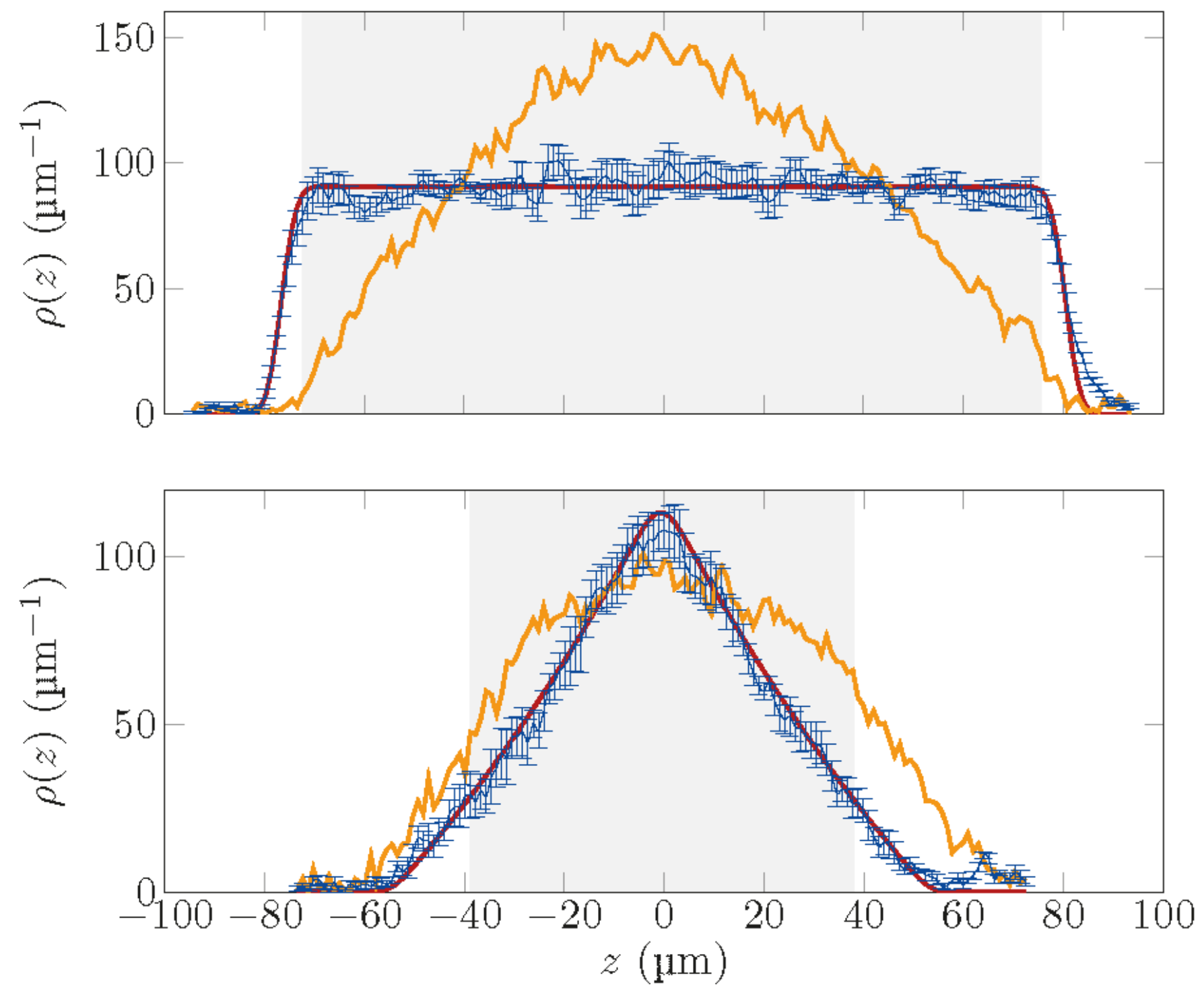


Quantum field machines
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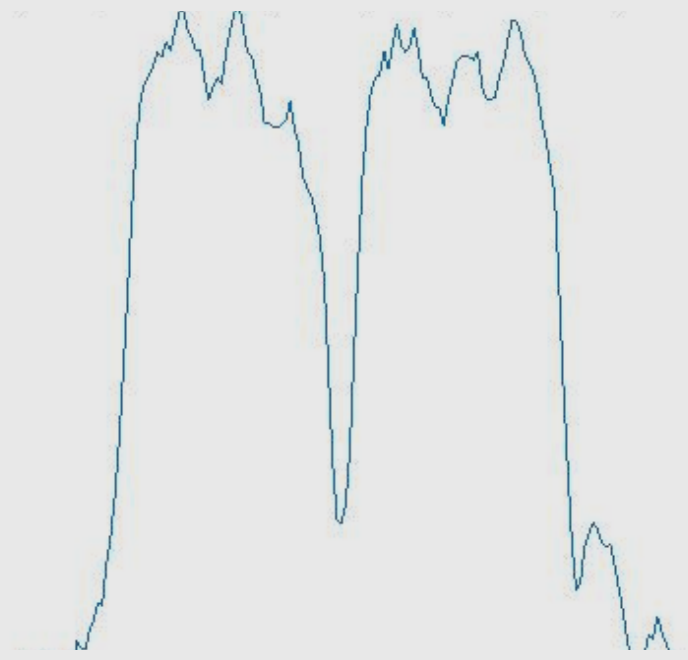


Excitation propagation & decay + GHD
Federica C. & Frederik M.

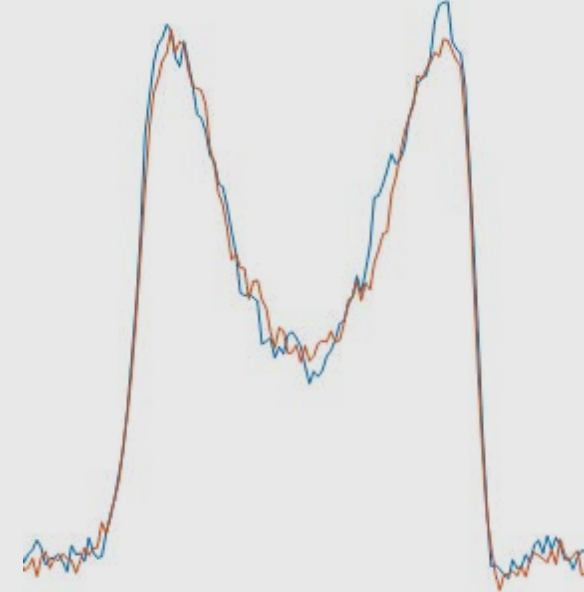




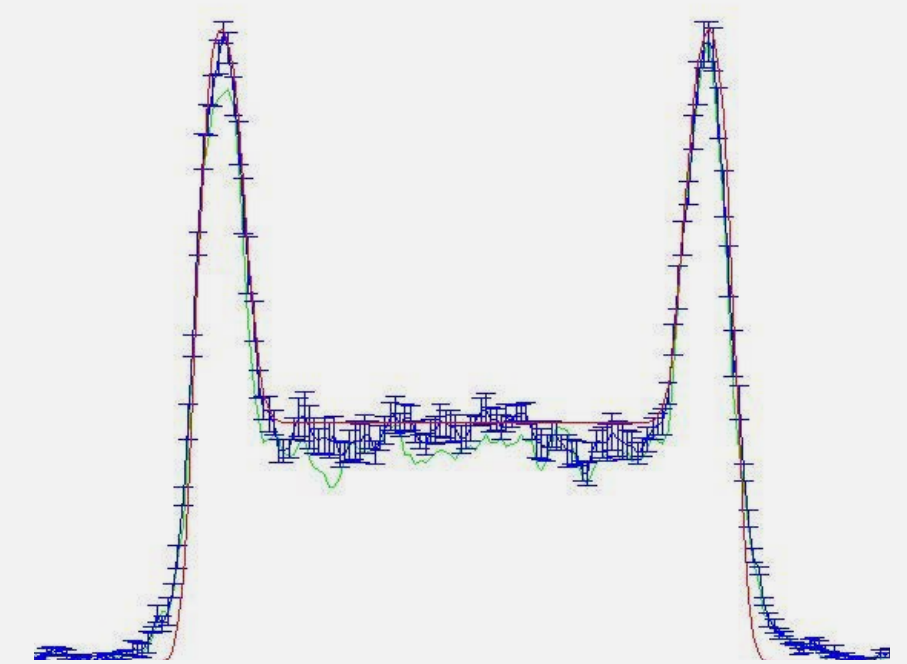
Quantum field machines João S. & Philipp S.



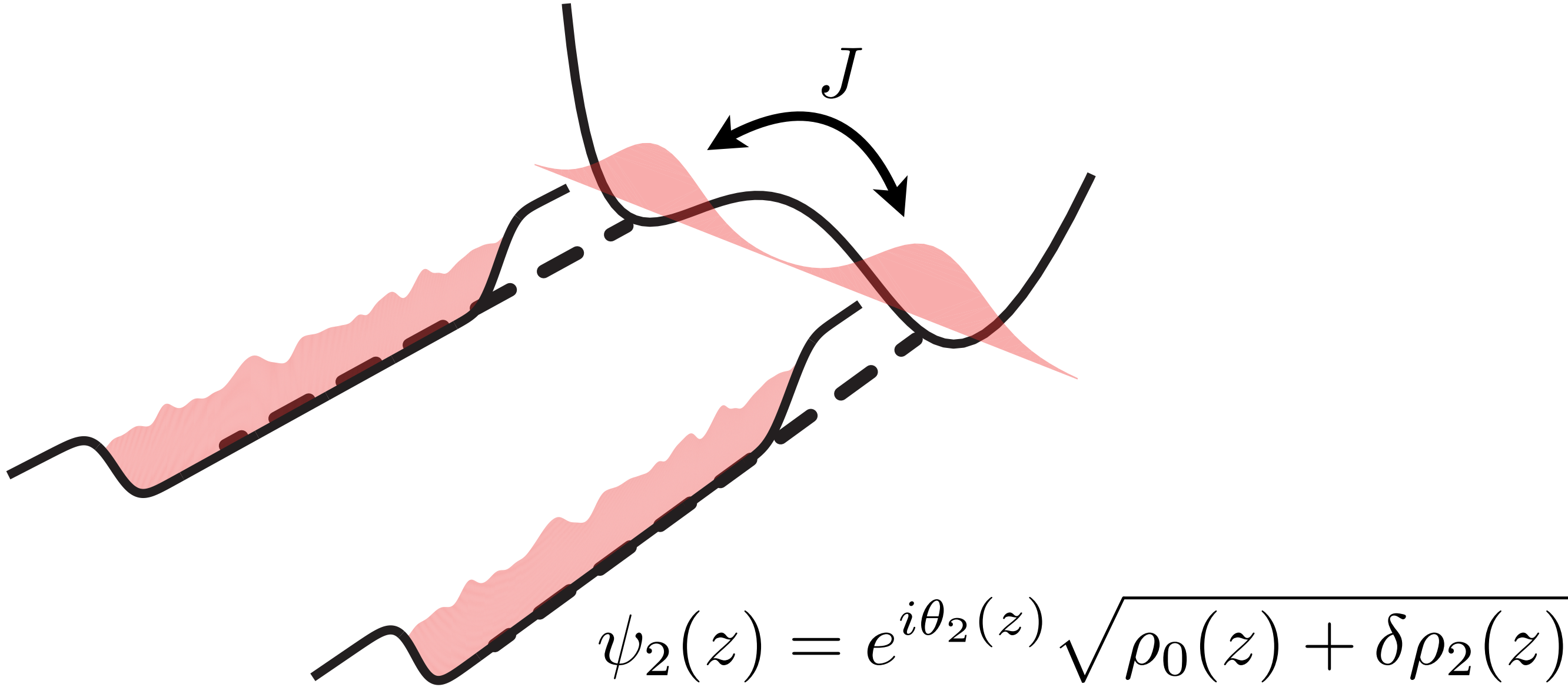
Excitation propagation & decay + GHD Federica C. & Frederik M.



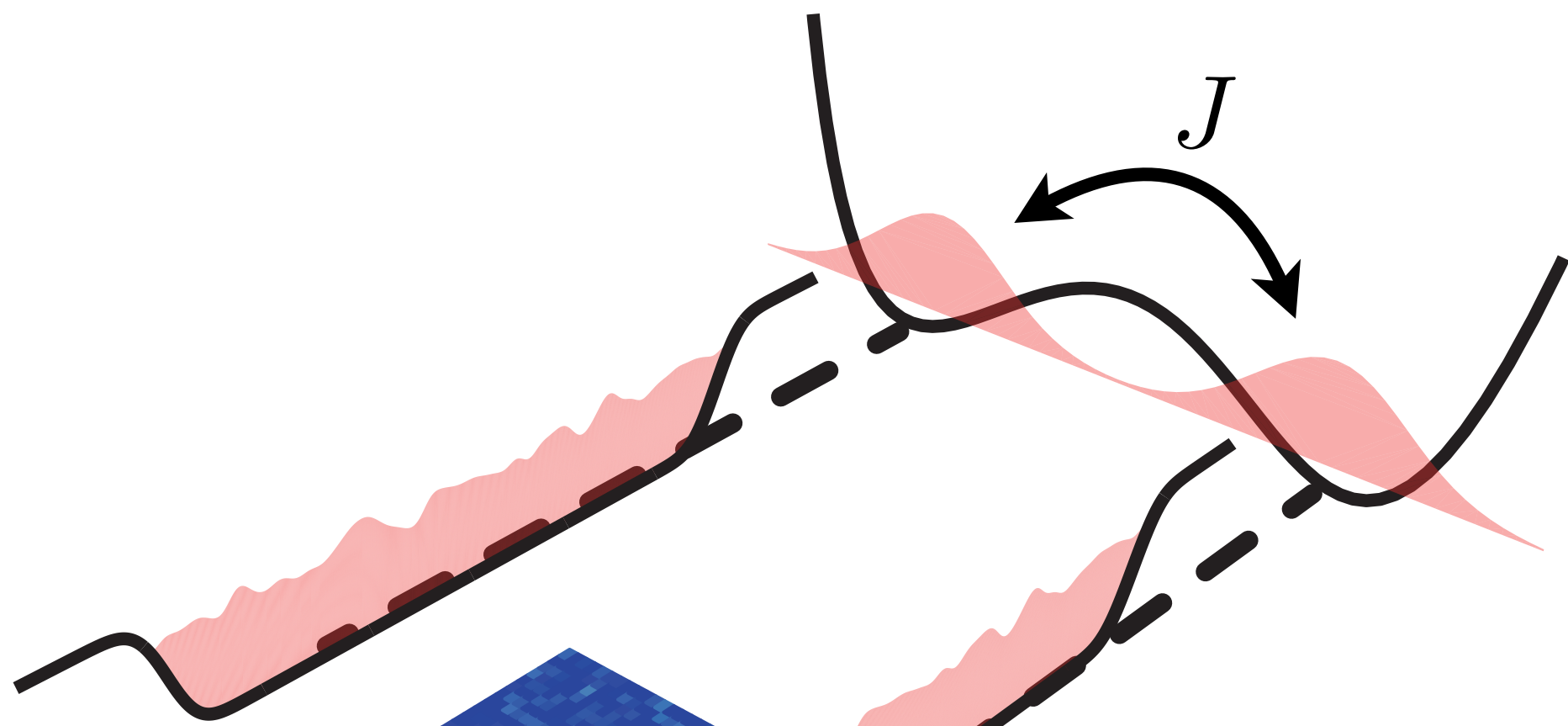
Boundary Sine-Gordon Amin T. , Sebastian E. & Natalia B.



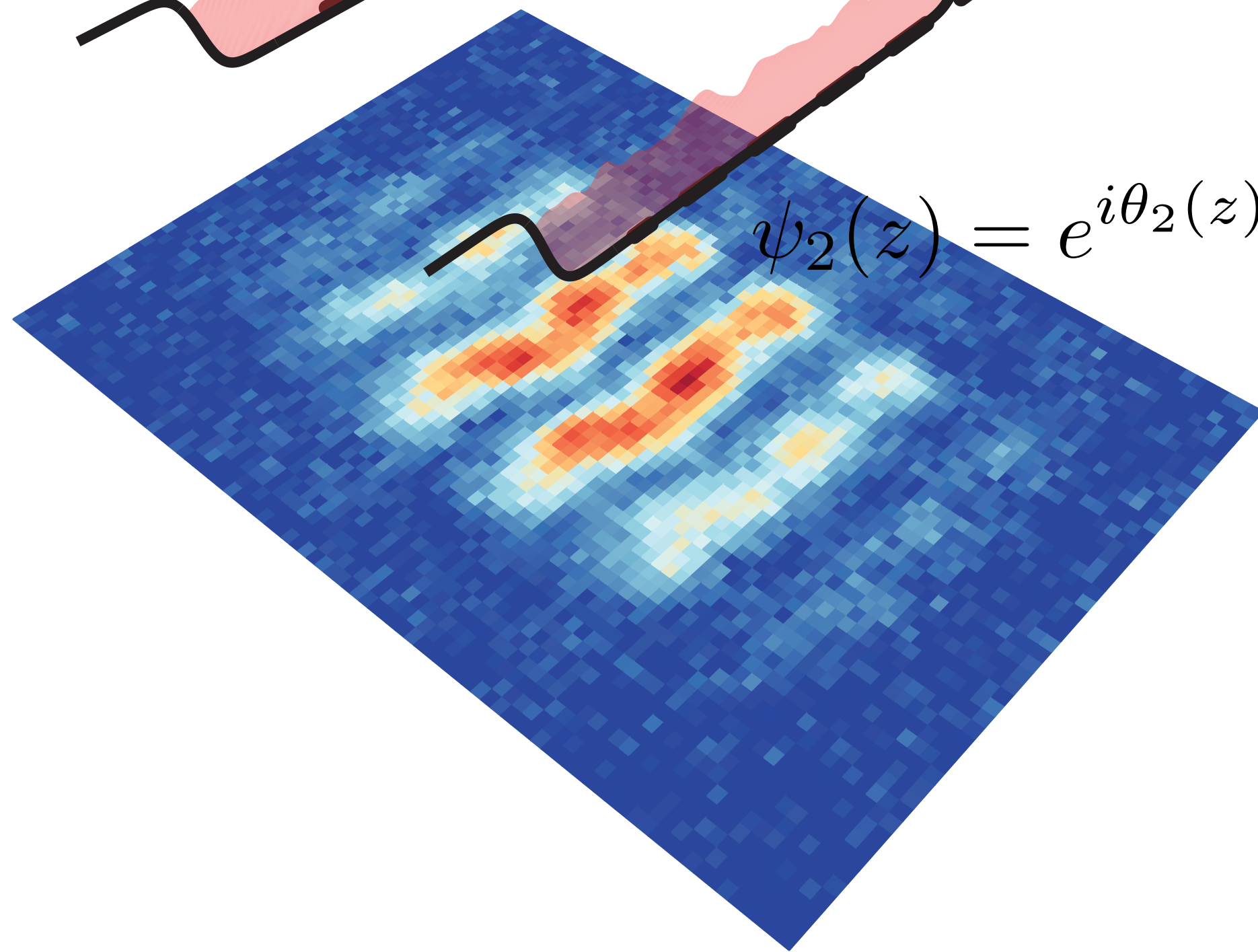
$$\psi_1(z) = e^{i\theta_1(z)} \sqrt{\rho_0(z) + \delta\rho_1(z)}$$



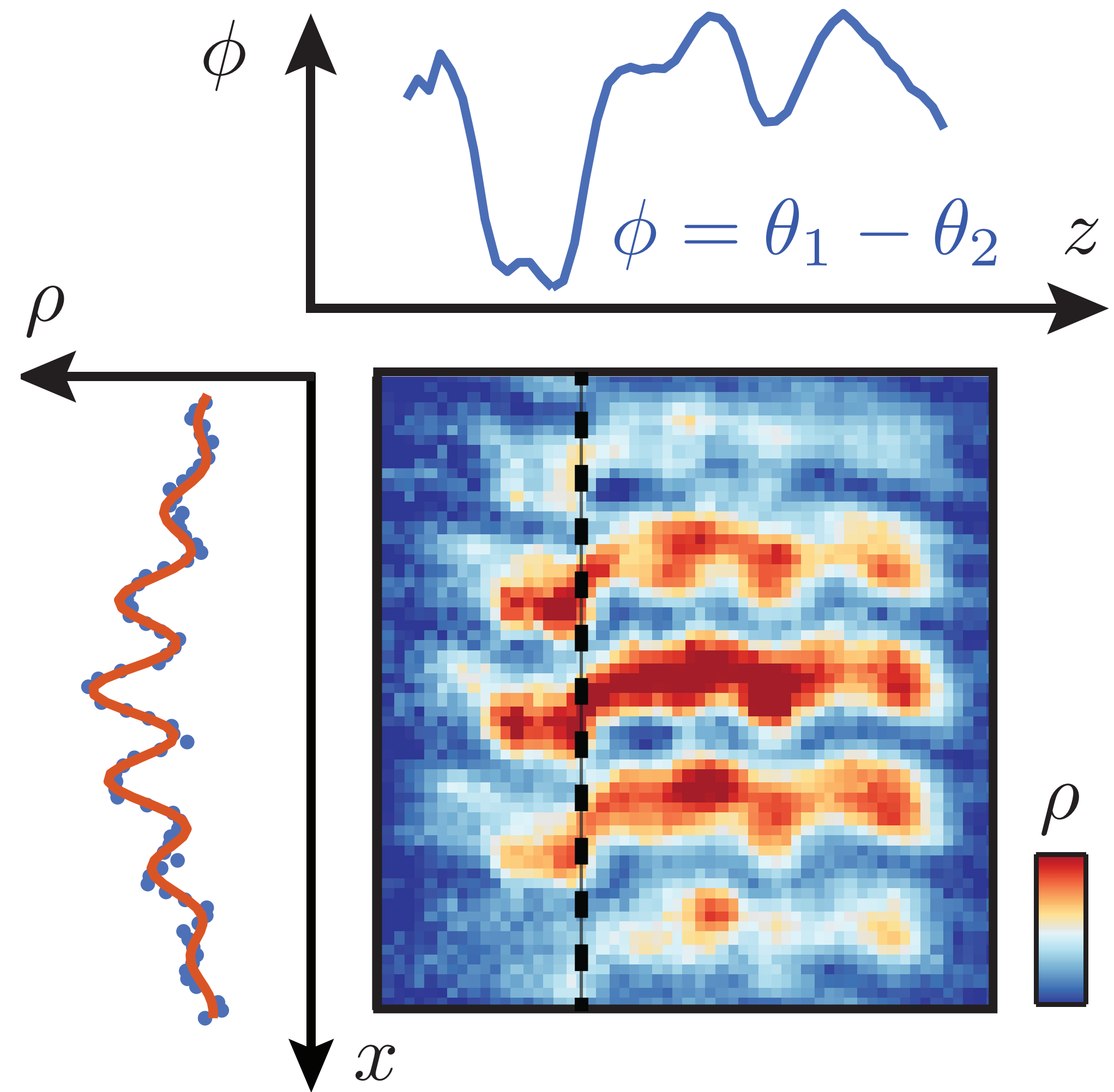
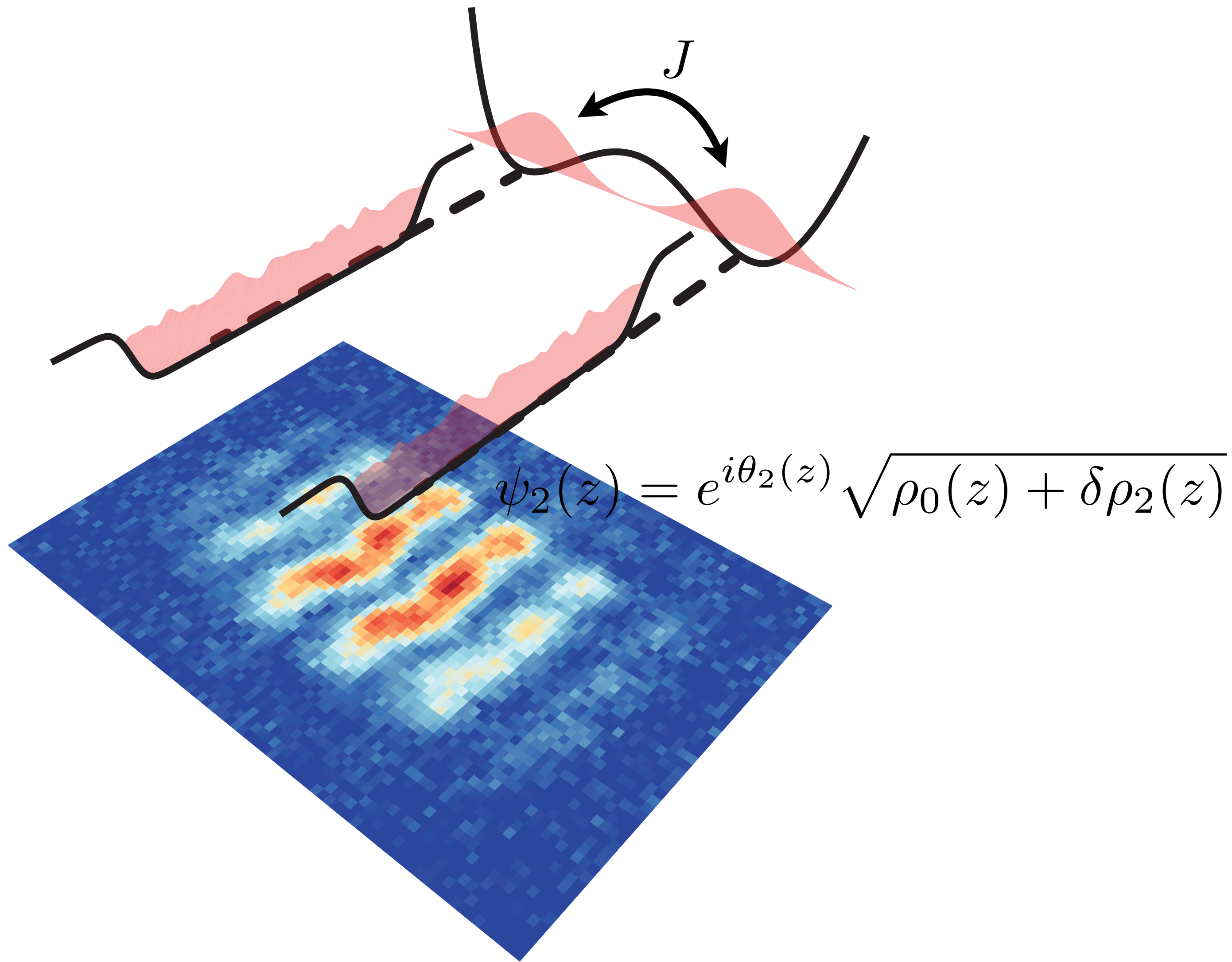
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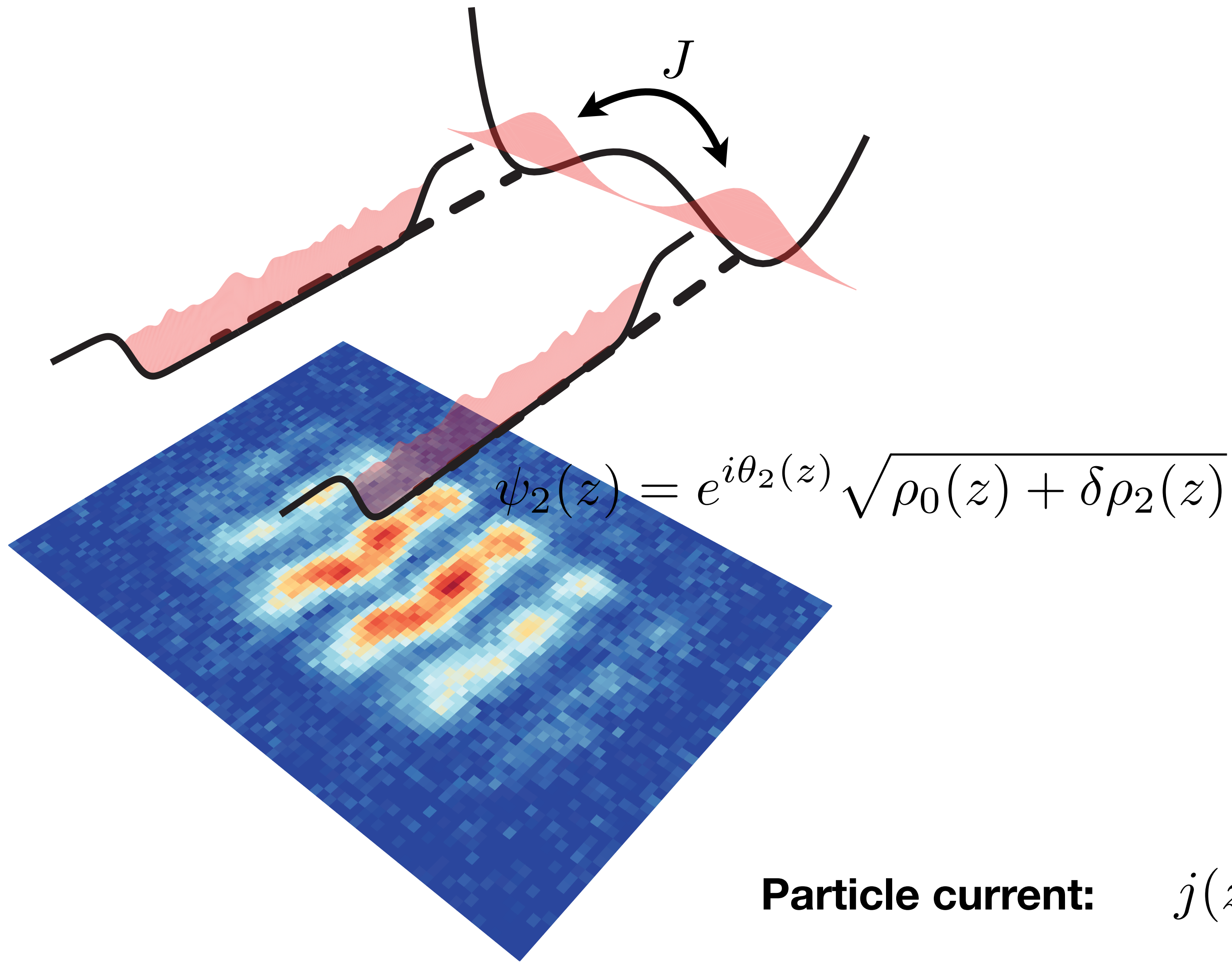
$$\psi_2(z) = e^{i\theta_2(z)} \sqrt{\rho_0(z) + \delta\rho_2(z)}$$



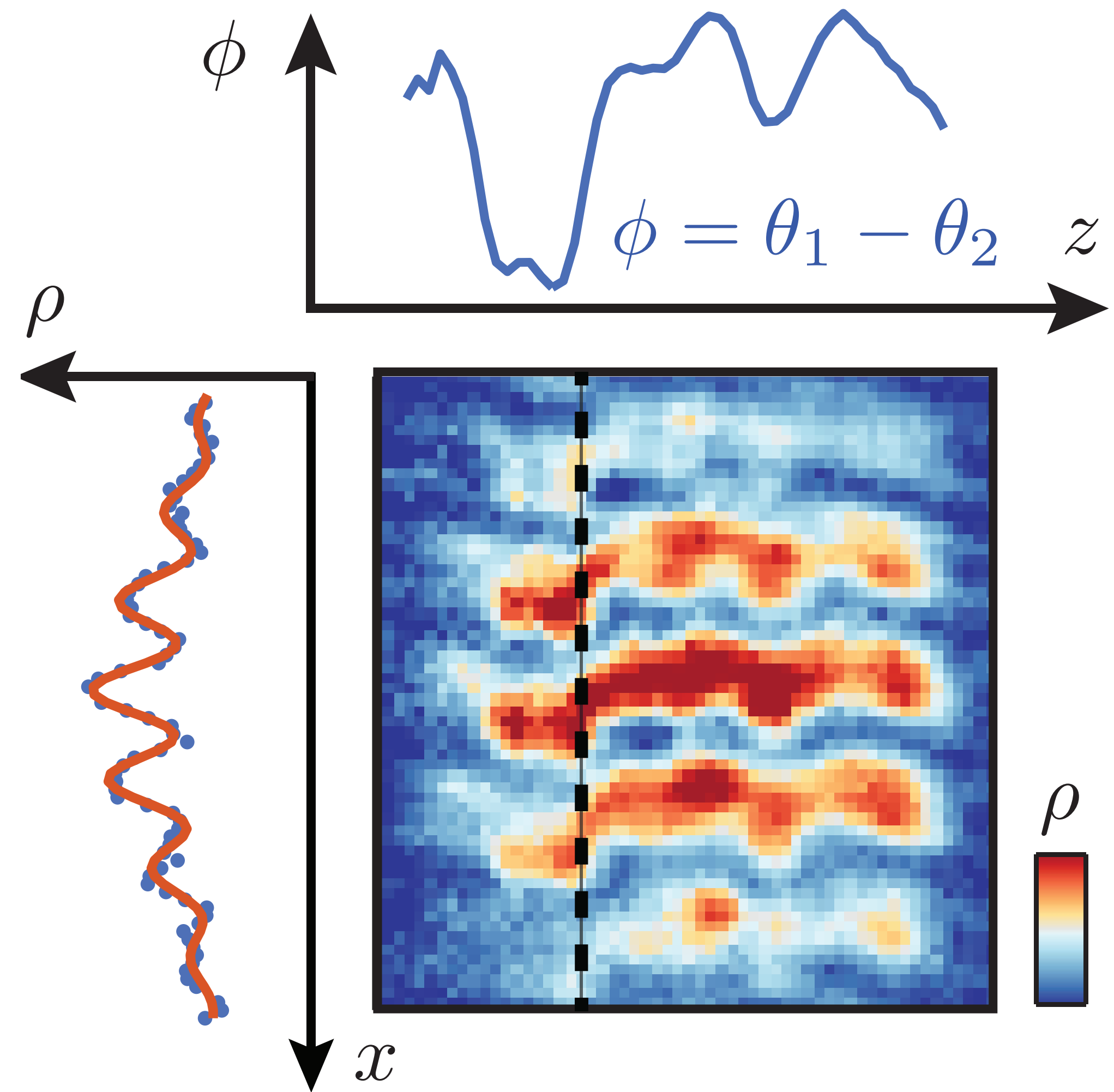
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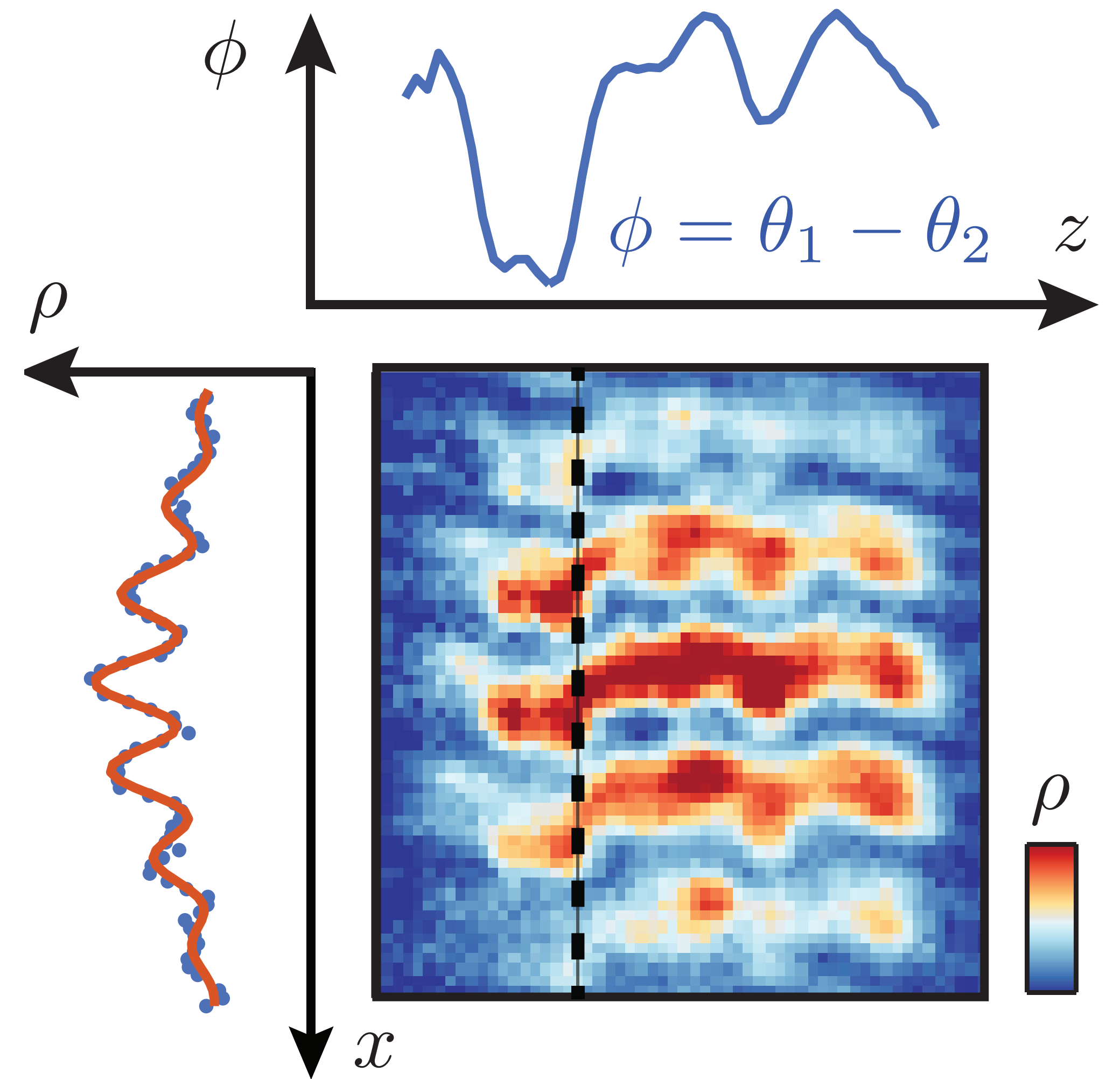
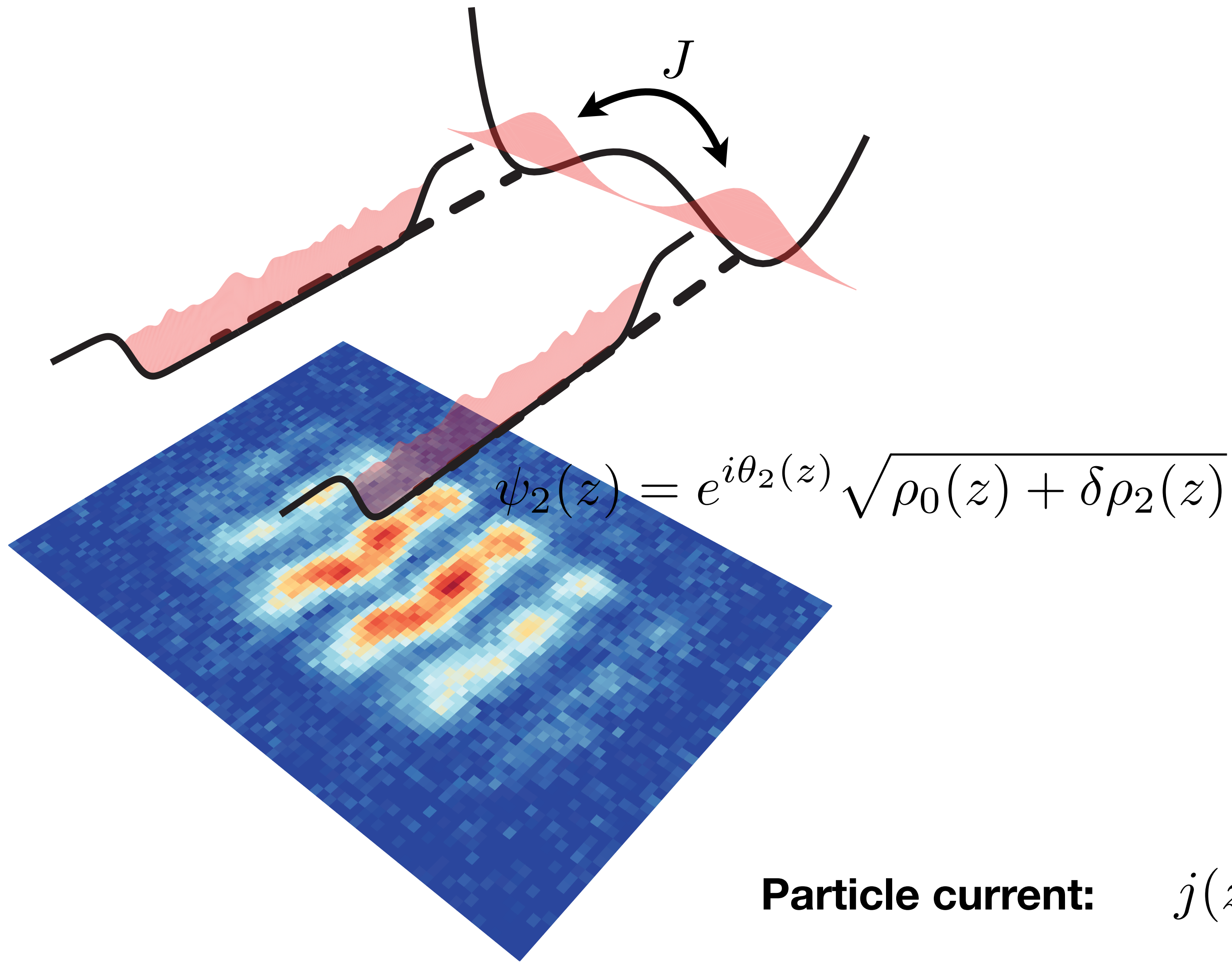
$$\psi_1(z) = e^{i\theta_1(z)} \sqrt{\rho_0(z) + \delta\rho_1(z)}$$



Particle current: $j(z) = \rho_0(z)u(z)$



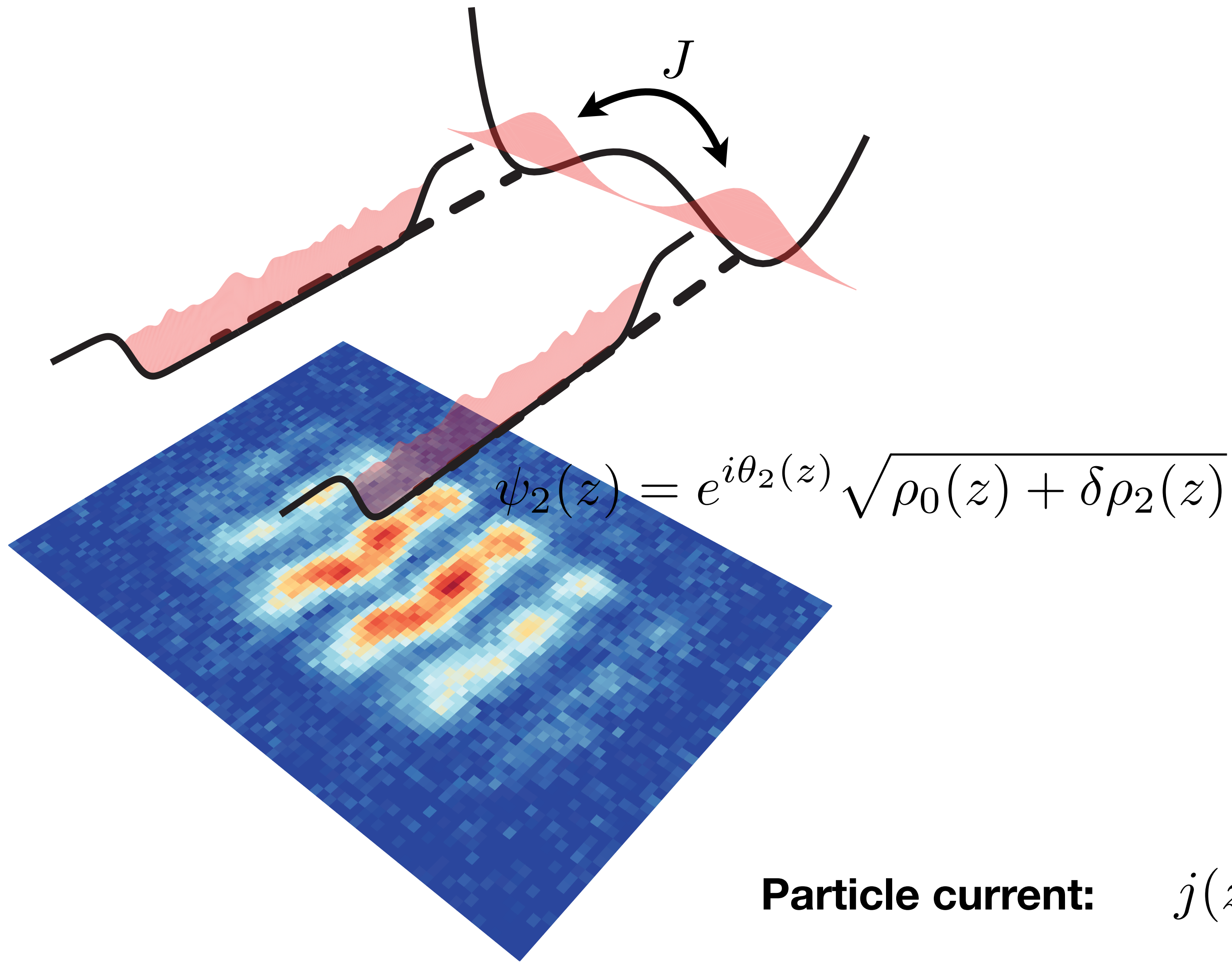
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Particle current: $j(z) = \rho_0(z)u(z)$

Velocity field: $u(z) = (\hbar/m)\partial_z\phi(z)$

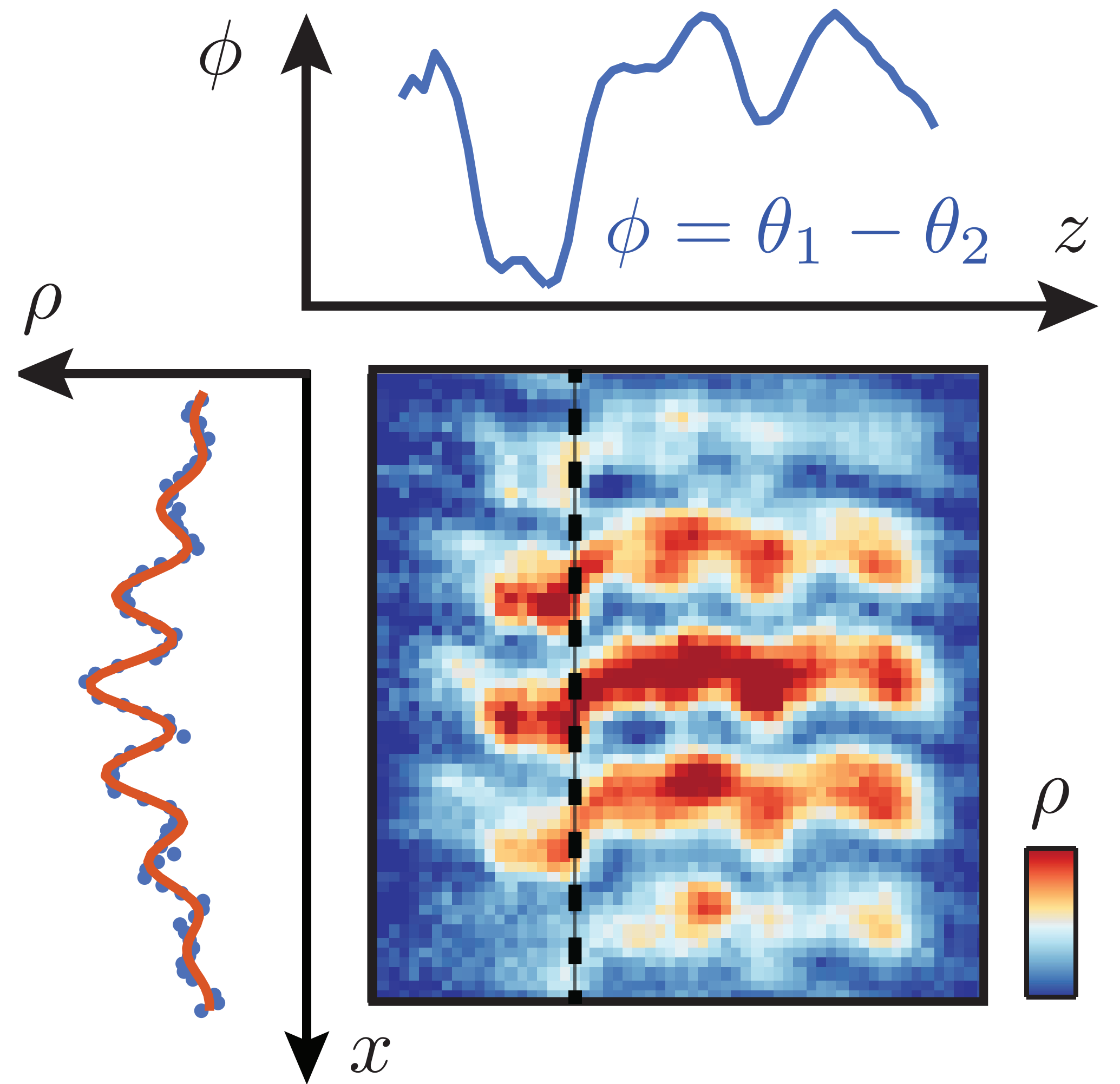
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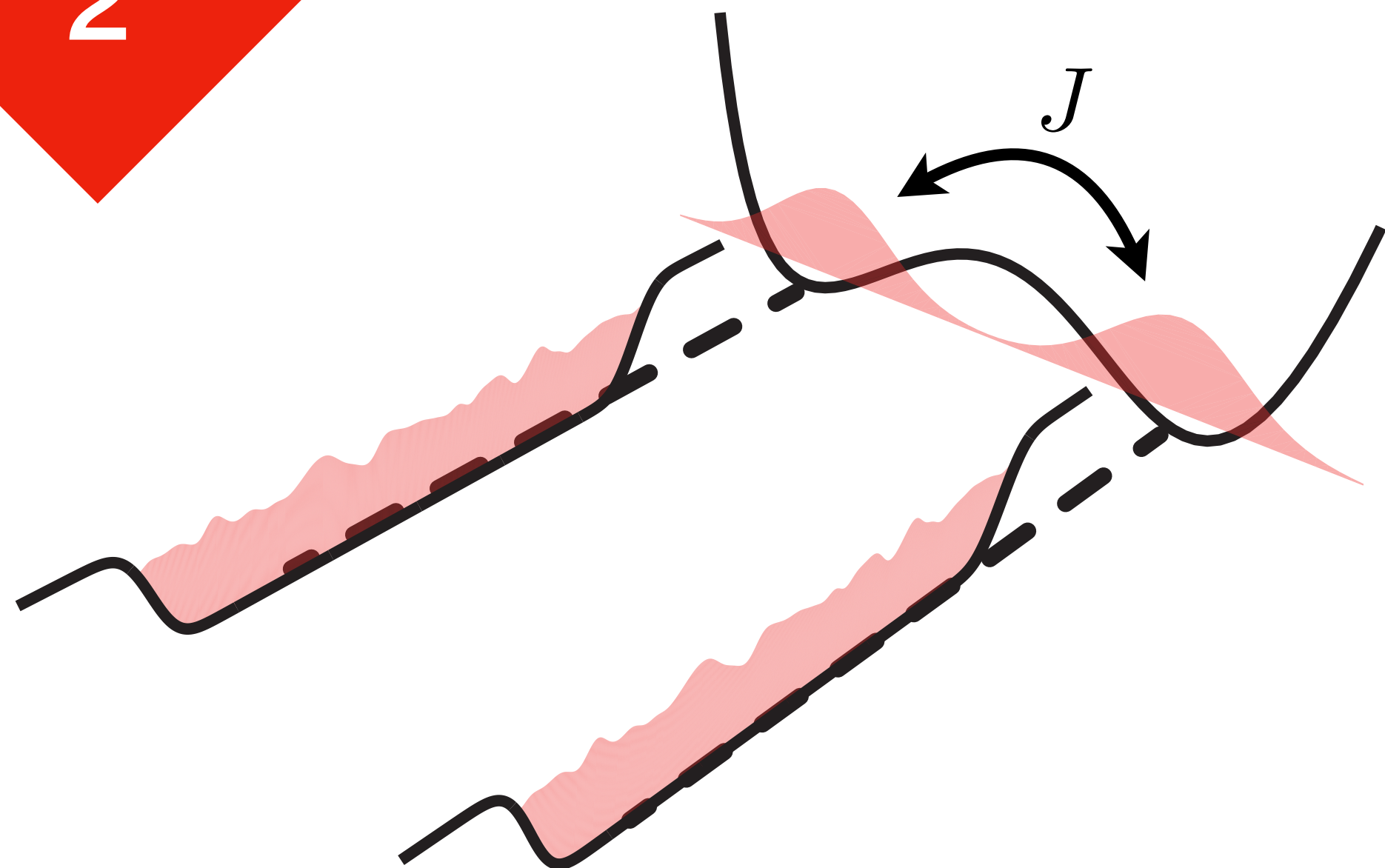
$$\psi_2(z) = e^{i\theta_2(z)} \sqrt{\rho_0(z) + \delta\rho_2(z)}$$

Particle current: $j(z) = \rho_0(z)u(z)$

Velocity field: $u(z) = (\hbar/m)\partial_z\phi(z)$ $C_u(z, z') = \langle u(z)u(z') \rangle$



2



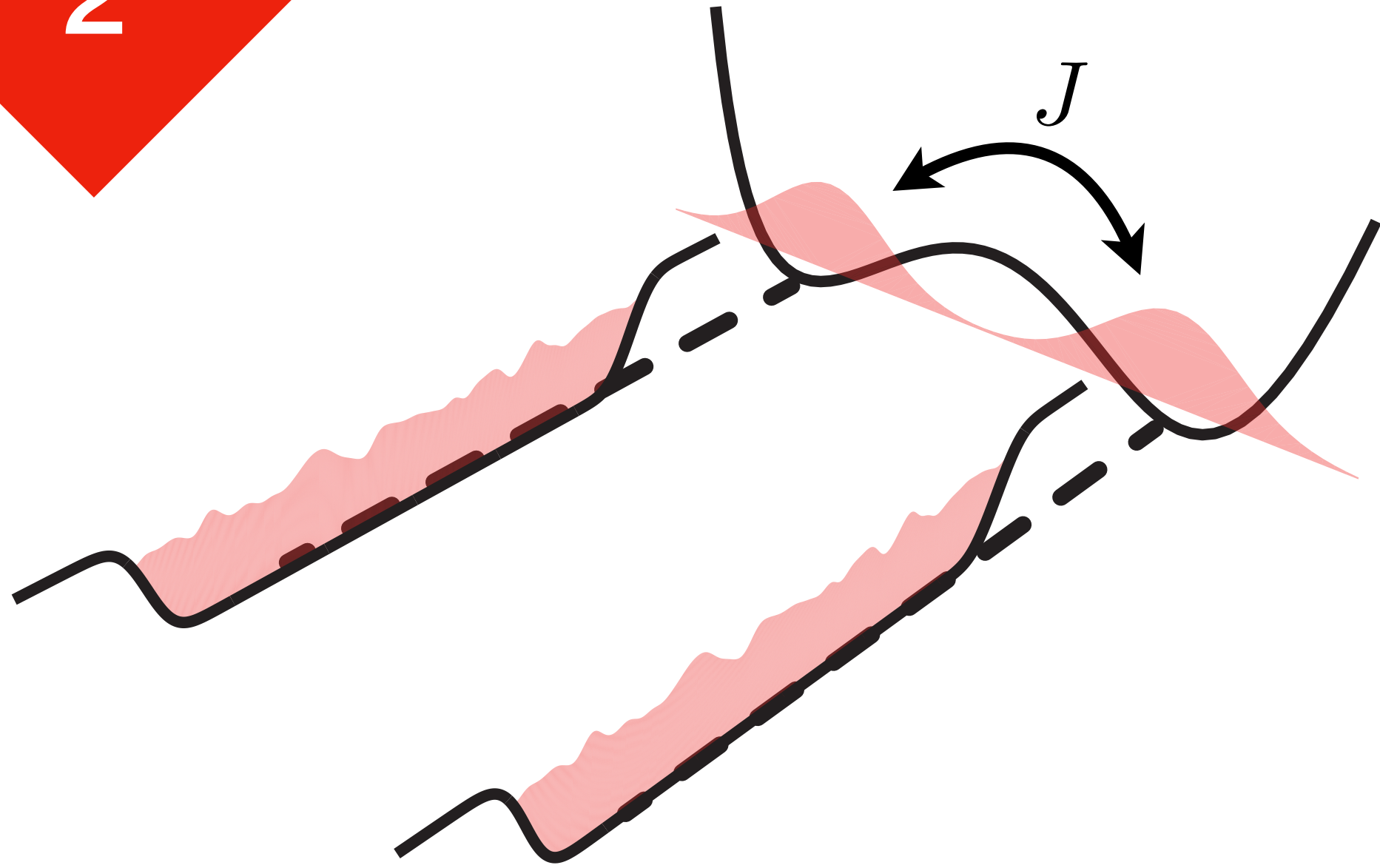
From sine-Gordon to Klein-Gordon - Dynamics of relative DOF

$$H_{\text{sG}} = \int dz \left[\frac{\hbar^2 \rho_0(z)}{4m} (\partial_z \phi)^2 + \gamma \delta \rho^2 - 2\hbar J \rho_0(z) \cos(\phi) \right]$$

Schweigler et al., Nature (2017)
Schweigler et al., Nat. Phys. (2021)

Rauer et al., Science (2018)

2



From sine-Gordon to Klein-Gordon - Dynamics of relative DOF

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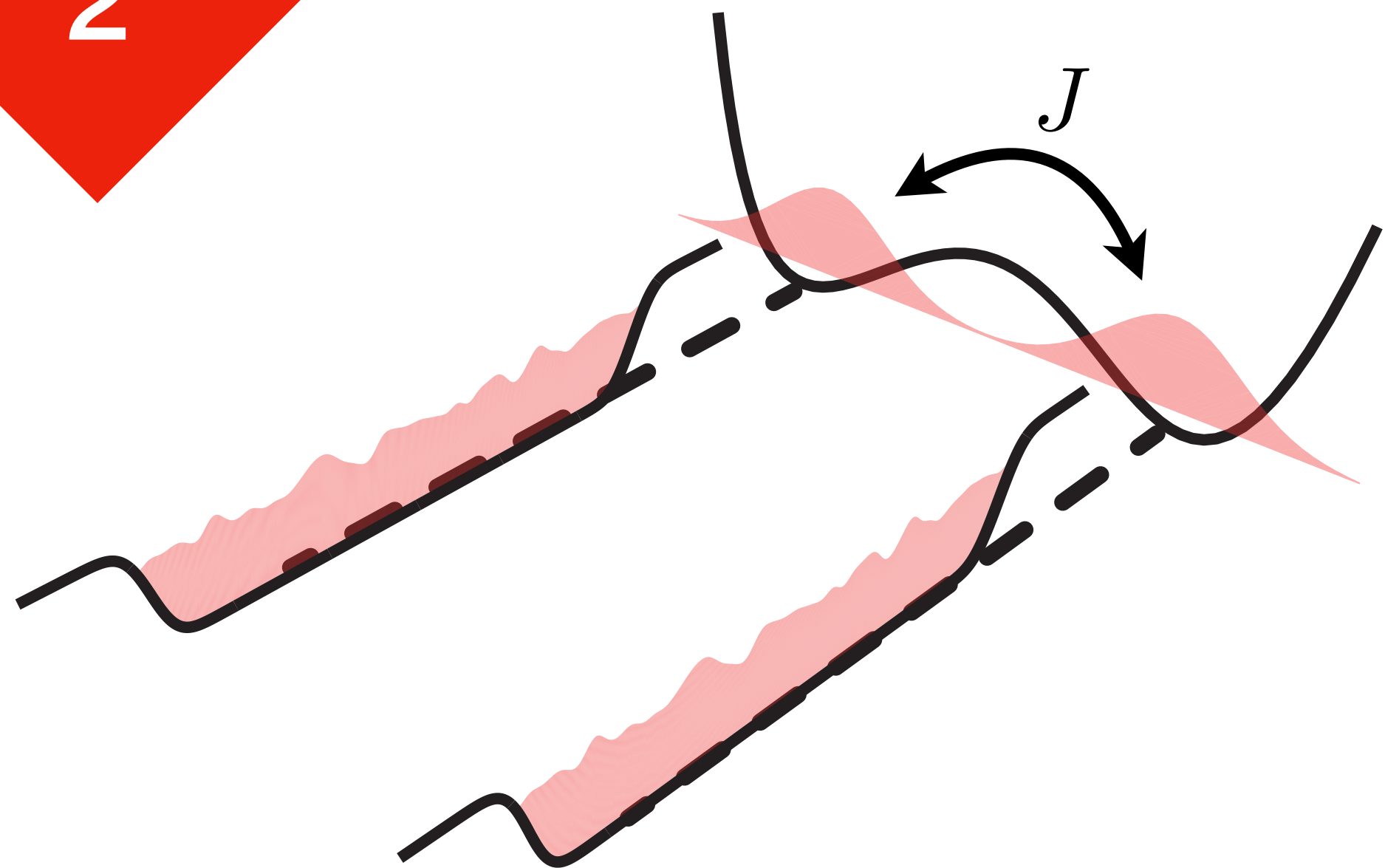
Schweigler et al., Nature (2017)
Schweigler et al., Nat. Phys. (2021)

For strong tunnelling

$$H_{\text{KG}} = \int dz \left[\frac{\hbar^2 \rho_0(z)}{4m} (\partial_z \phi)^2 + \gamma \delta \rho^2 + \hbar J \rho_0(z) \phi^2 \right]$$

Rauer et al., Science (2018)

2



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Schweigler et al., Nature (2017)
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1+1 D action

Rauer et al., Science (2018)

$$\mathcal{S}[\phi] \sim \int dz dt \sqrt{-g} K(z) \left[g^{\mu\nu} (\partial_\mu \phi) (\partial_\nu \phi) + \frac{1}{2} M^2 \phi^2 \right]$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -v(z)^2 dt^2 + dz^2$$

$$v(z) = \sqrt{\gamma \rho_0(z) / m}$$

$$K(z) = \frac{\hbar \pi}{2} \sqrt{\frac{\rho_0(z)}{m \gamma}} \quad M = \sqrt{2 \hbar m J / \gamma}$$

3

1+1 D action

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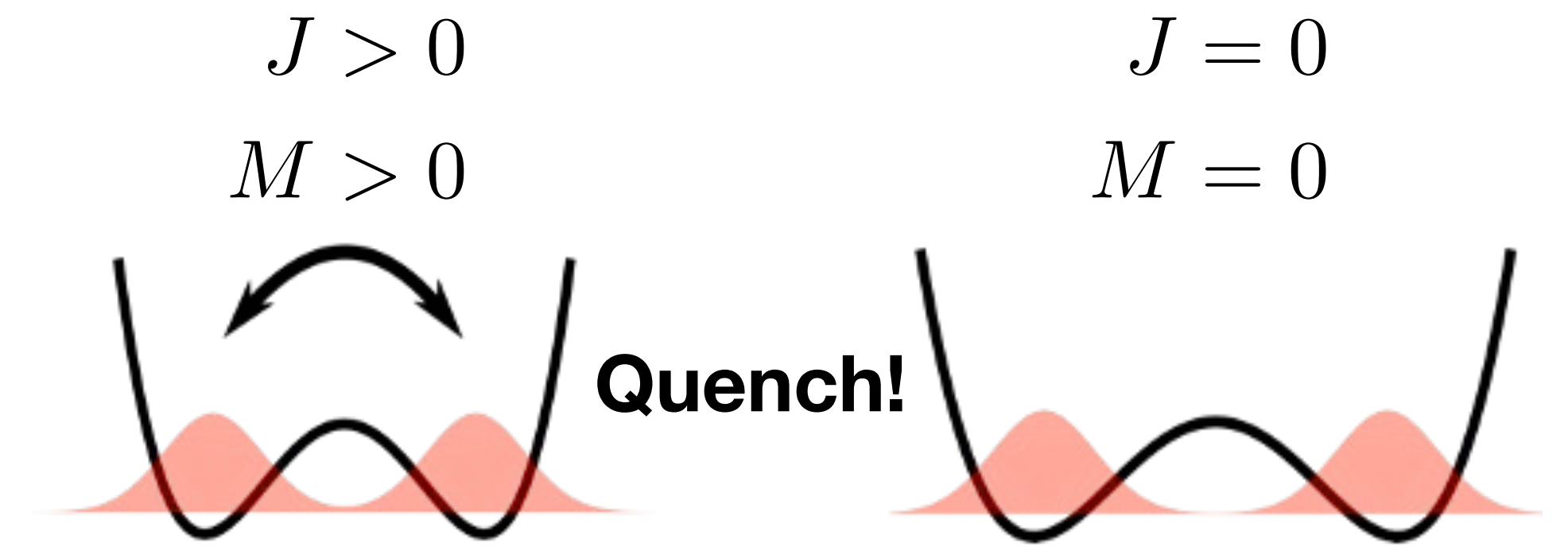
Tajik et al., PNAS (2023)

3

1+1 D action

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Tajik et al., PNAS (2023)





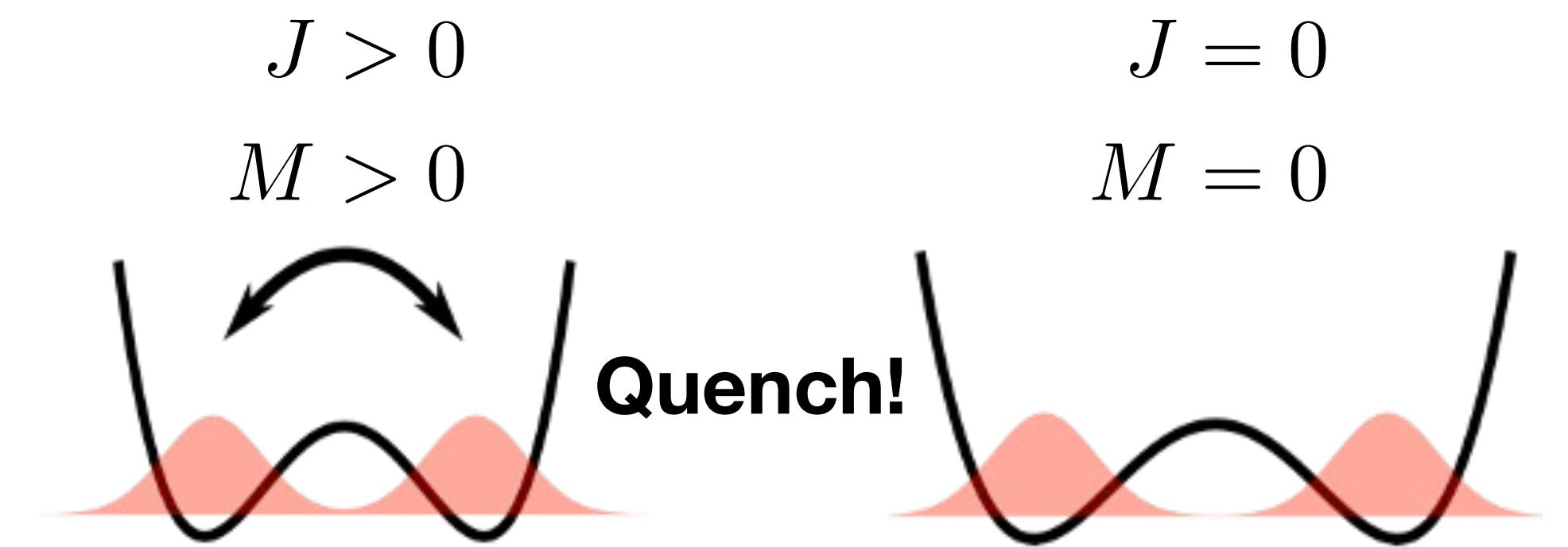
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Tajik et al., PNAS (2023)

Velocity field:

$$u(z) = (\hbar/m) \partial_z \phi(z)$$



3

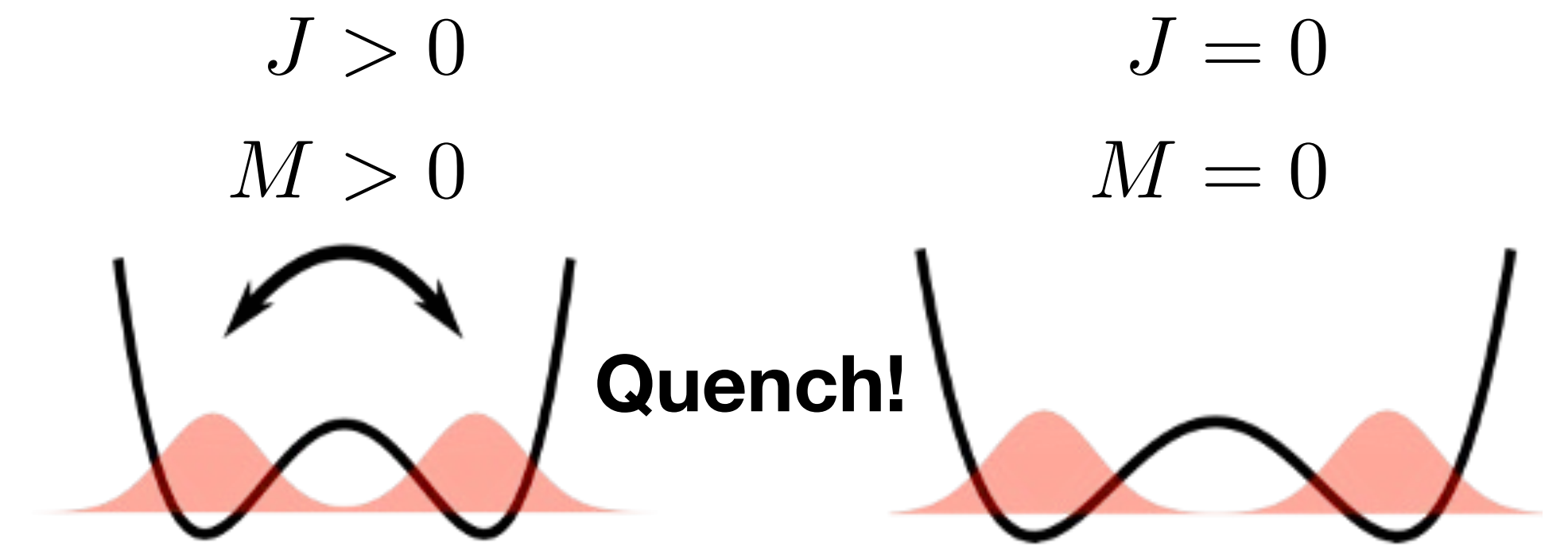
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Tajik et al., PNAS (2023)

Velocity field: $u(z) = (\hbar/m) \partial_z \phi(z)$

$$C_u(z, z') = \langle u(z) u(z') \rangle$$



3

1+1 D action

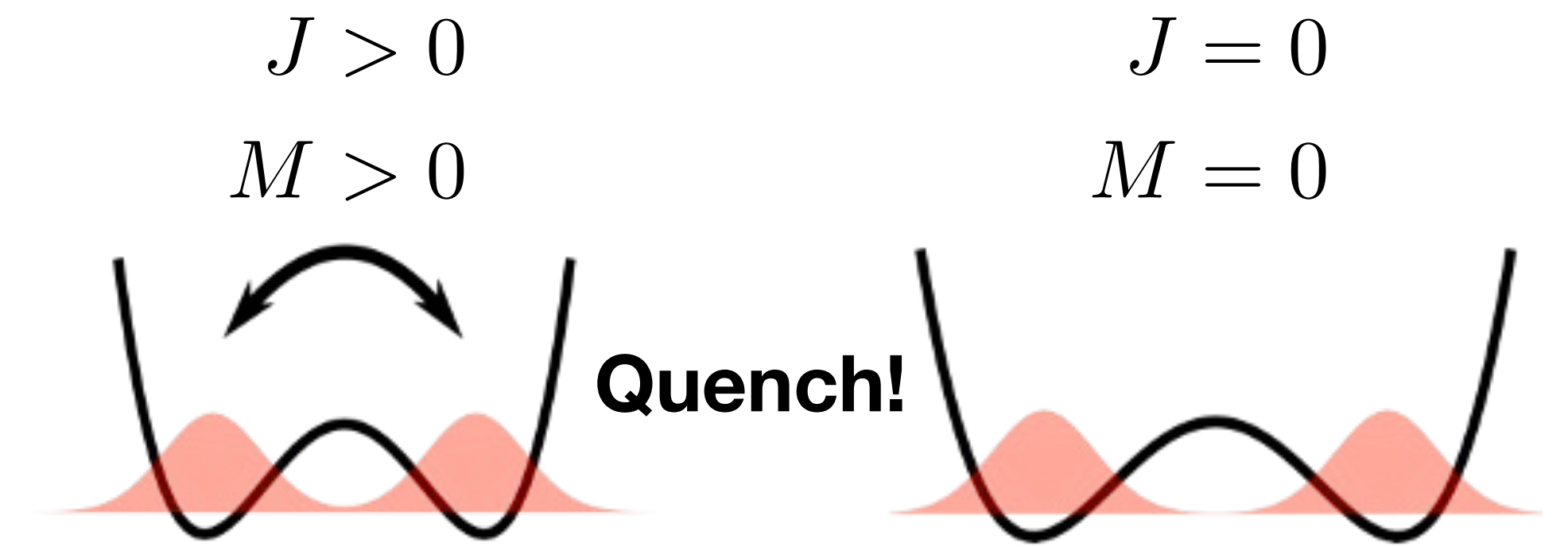
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Tajik et al., PNAS (2023)

Velocity field: $u(z) = (\hbar/m) \partial_z \phi(z)$

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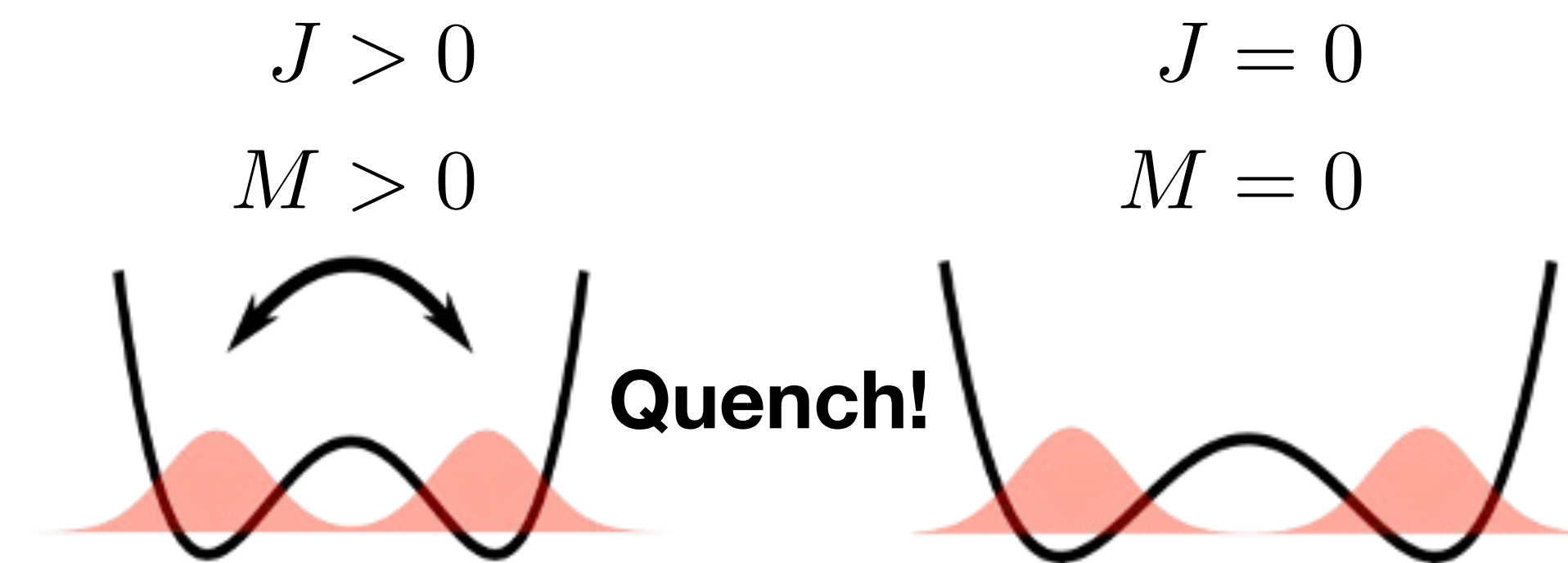
$$\langle u(z) \rangle = 0$$



3

1+1 D action

$$\mathcal{S}[\phi] \sim \int dz dt \sqrt{-g} K(z) \left[g^{\mu\nu} (\partial_\mu \phi) (\partial_\nu \phi) + \frac{1}{2} M^2 \phi^2 \right]$$



Tajik et al., PNAS (2023)

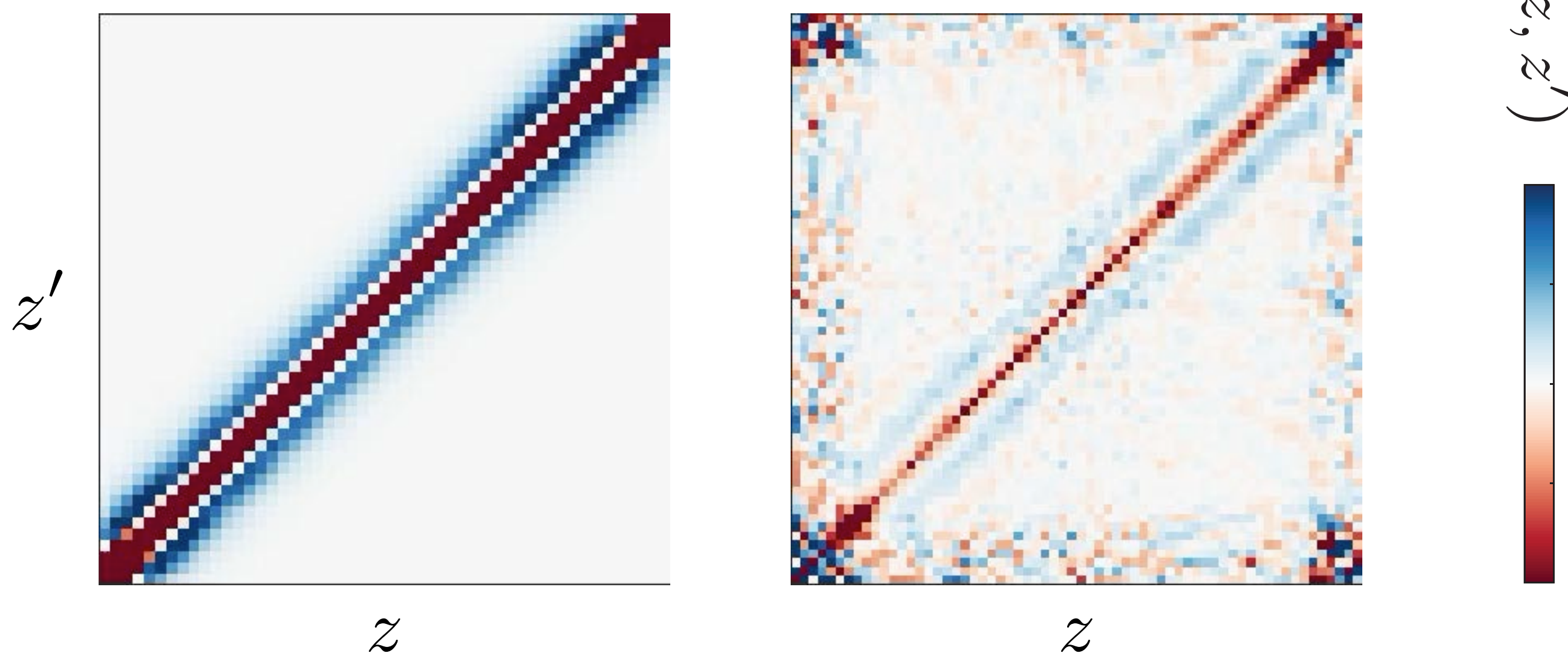
Correlation length $\ell_J = \sqrt{\frac{\hbar}{4mJ}}$

Velocity field:

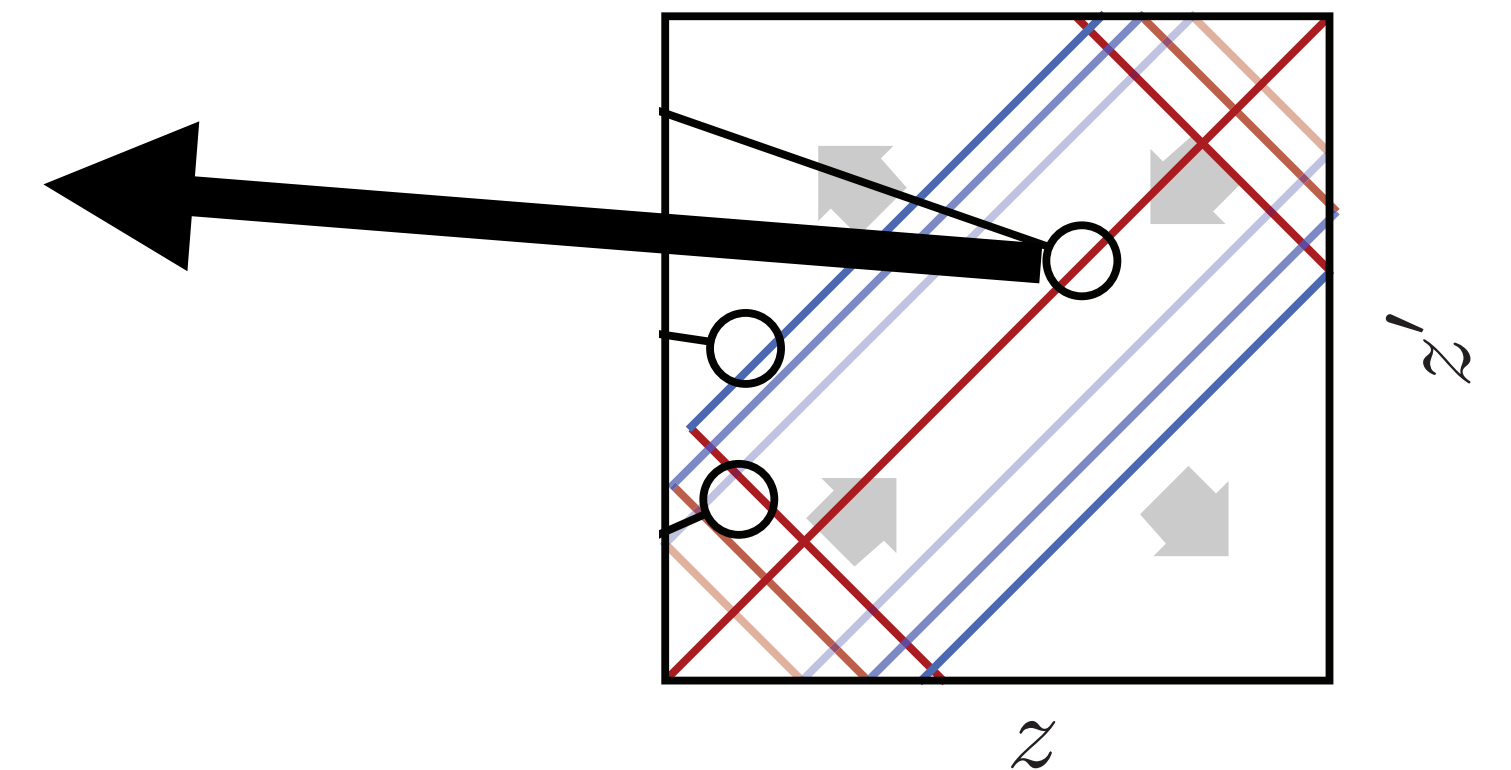
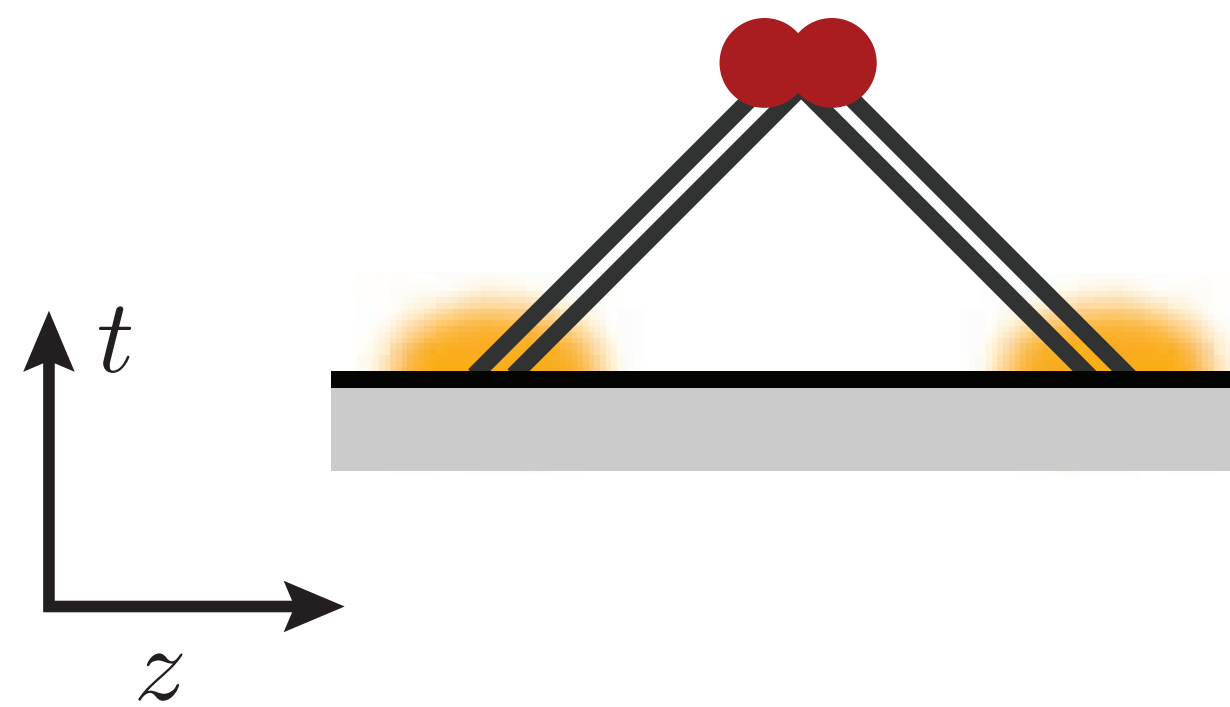
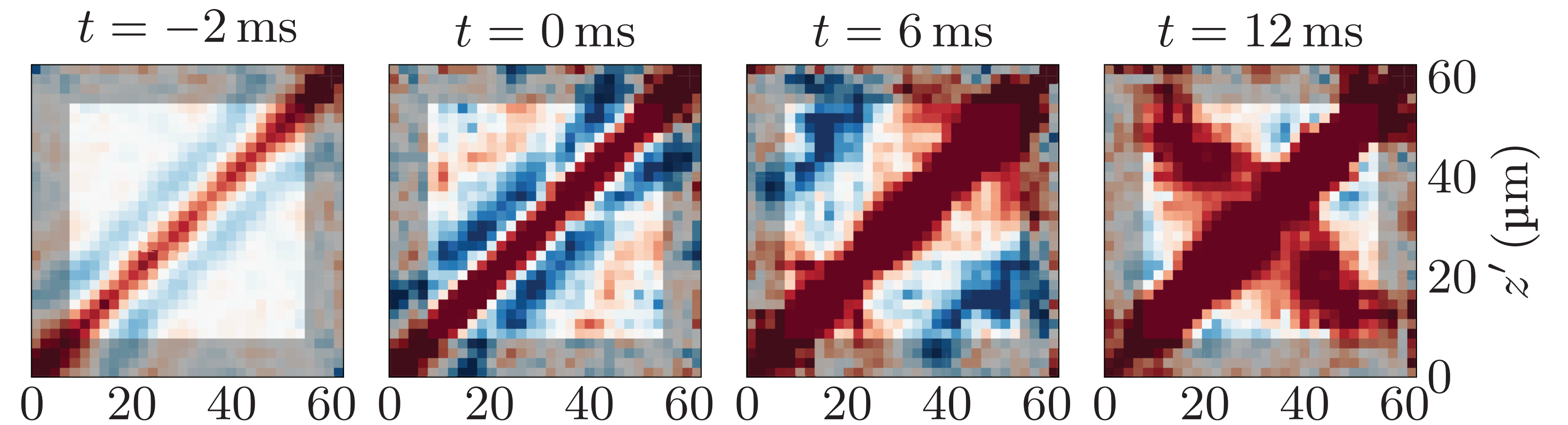
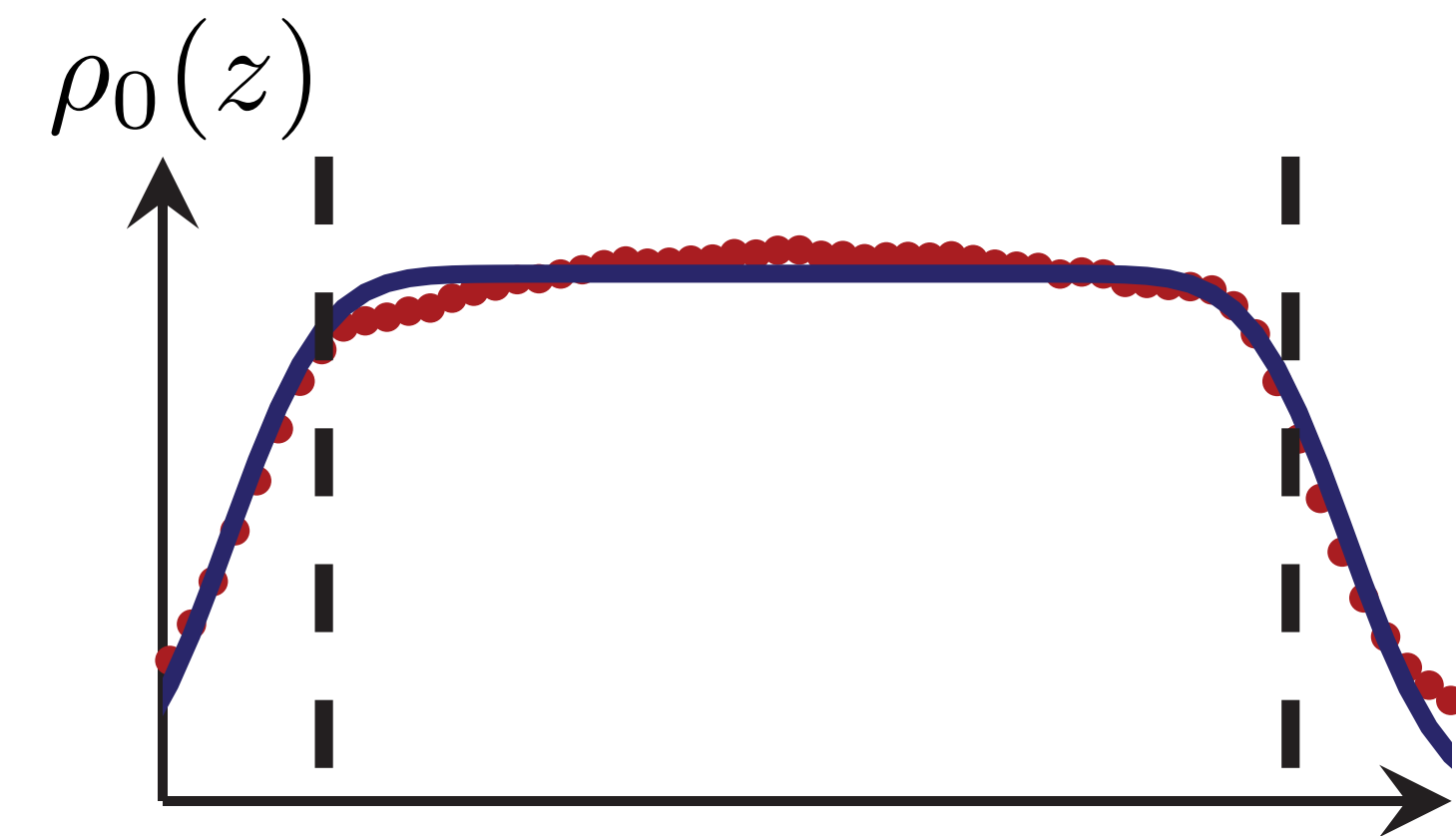
$$u(z) = (\hbar/m) \partial_z \phi(z)$$

$$C_u(z, z') = \langle u(z) u(z') \rangle$$

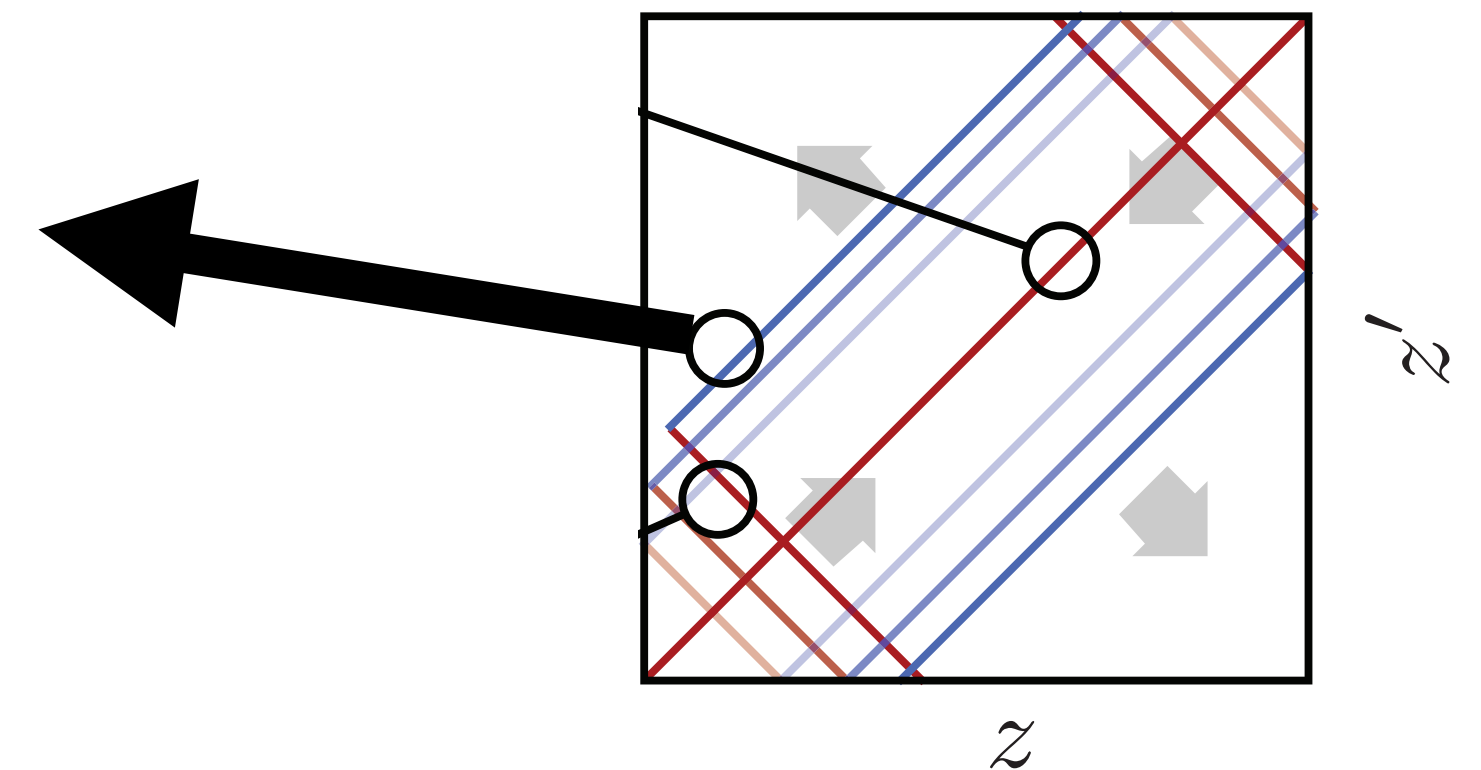
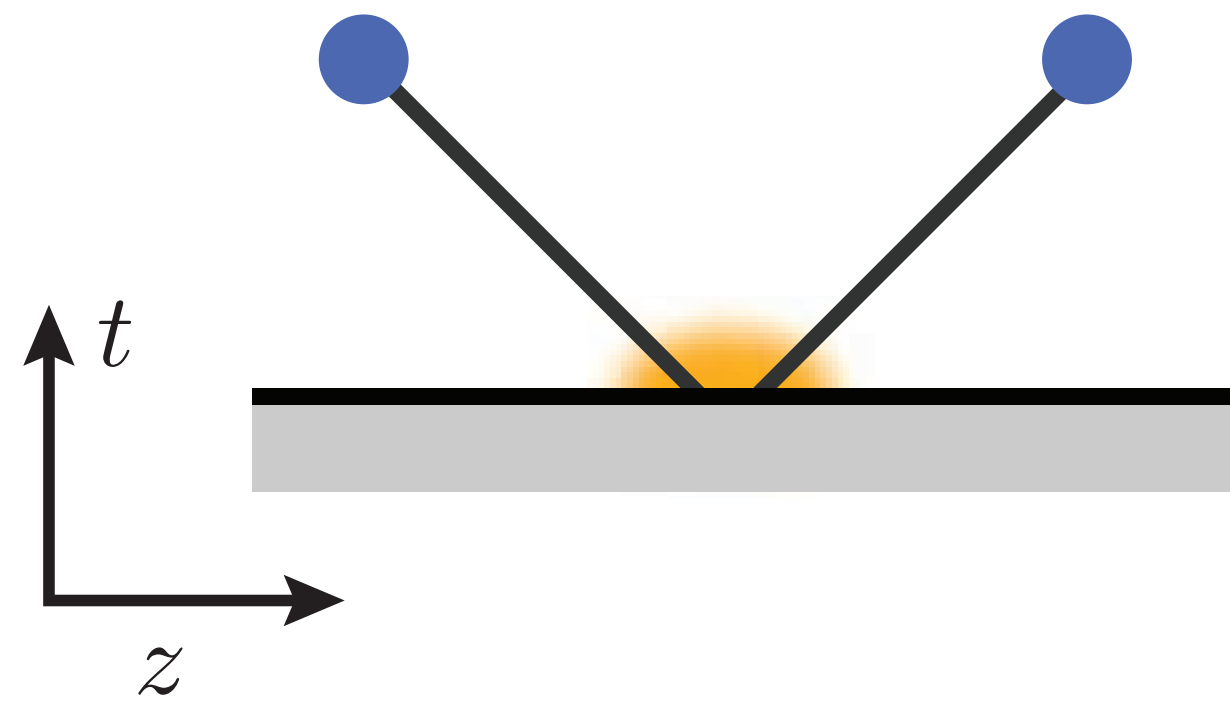
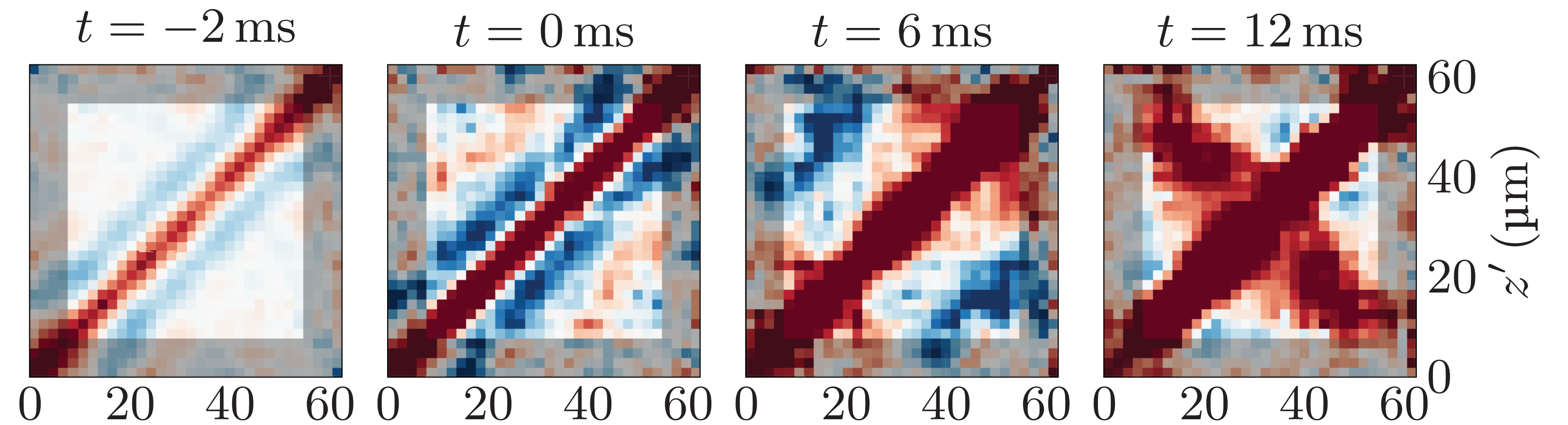
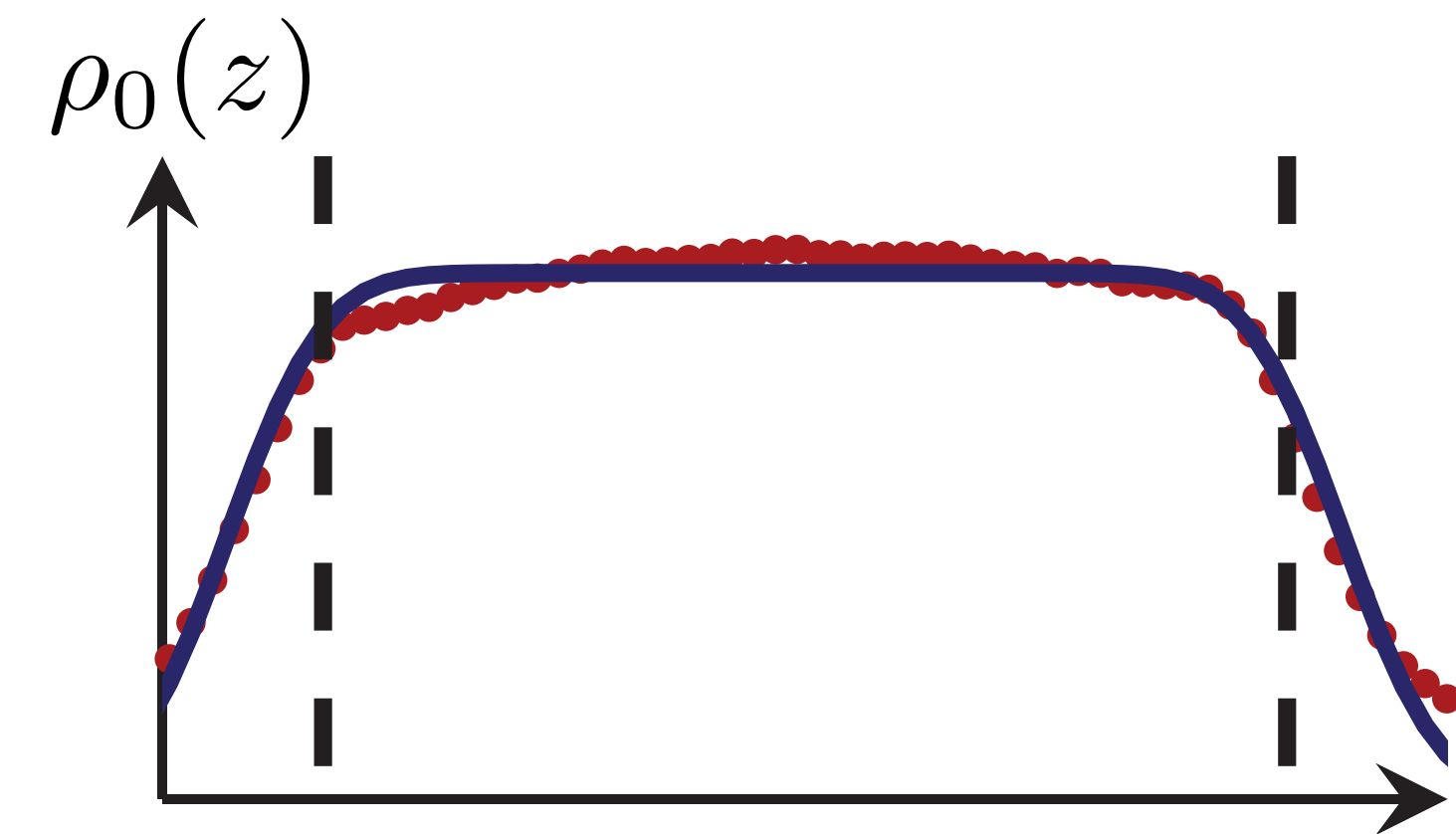
$$\langle u(z) \rangle = 0$$



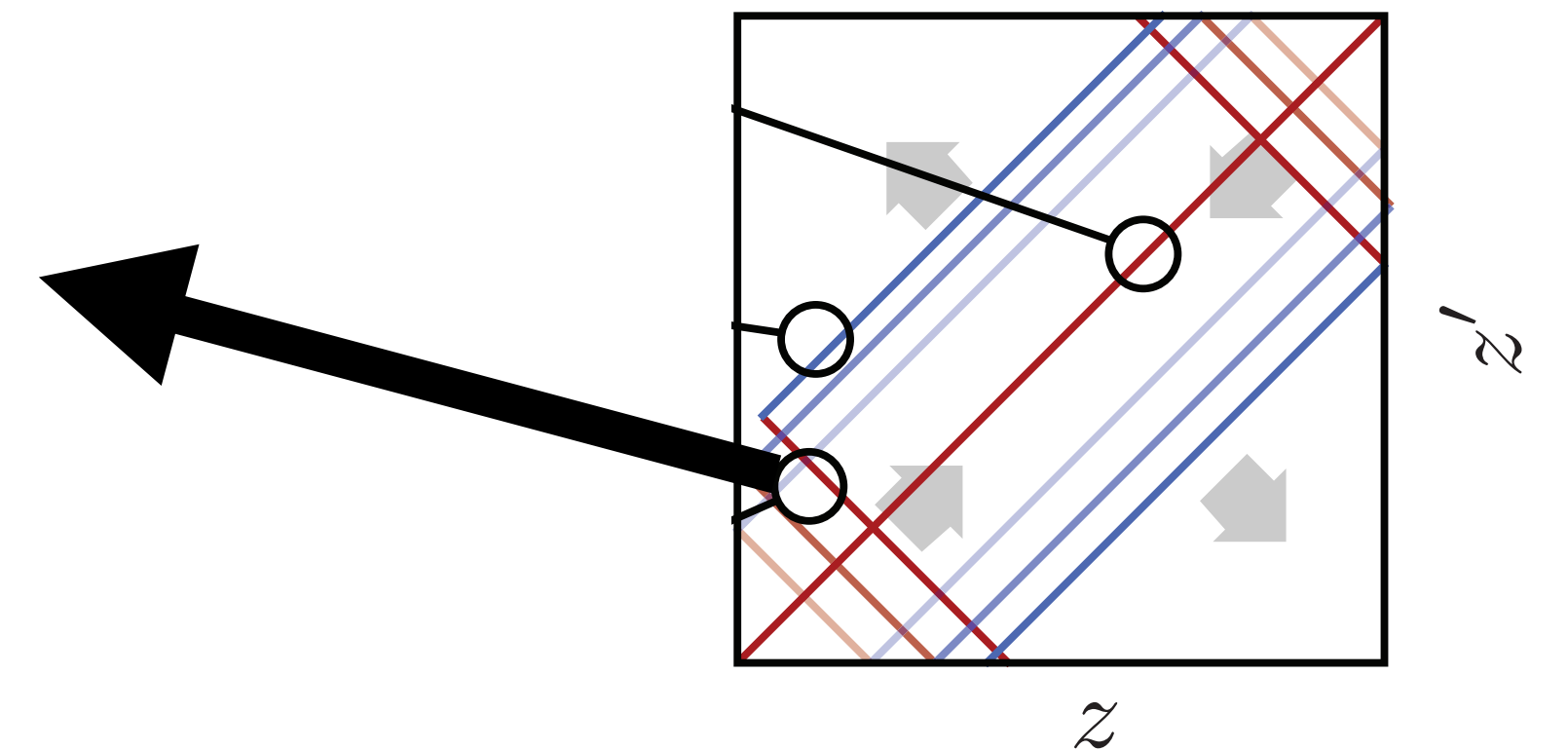
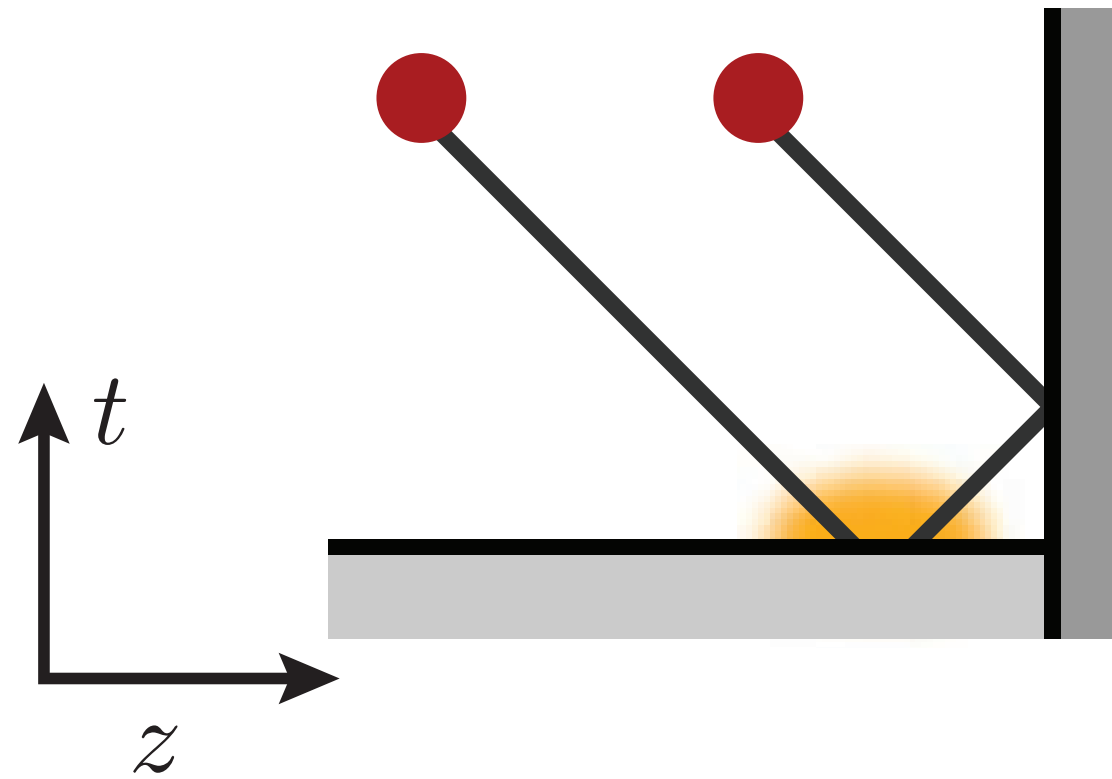
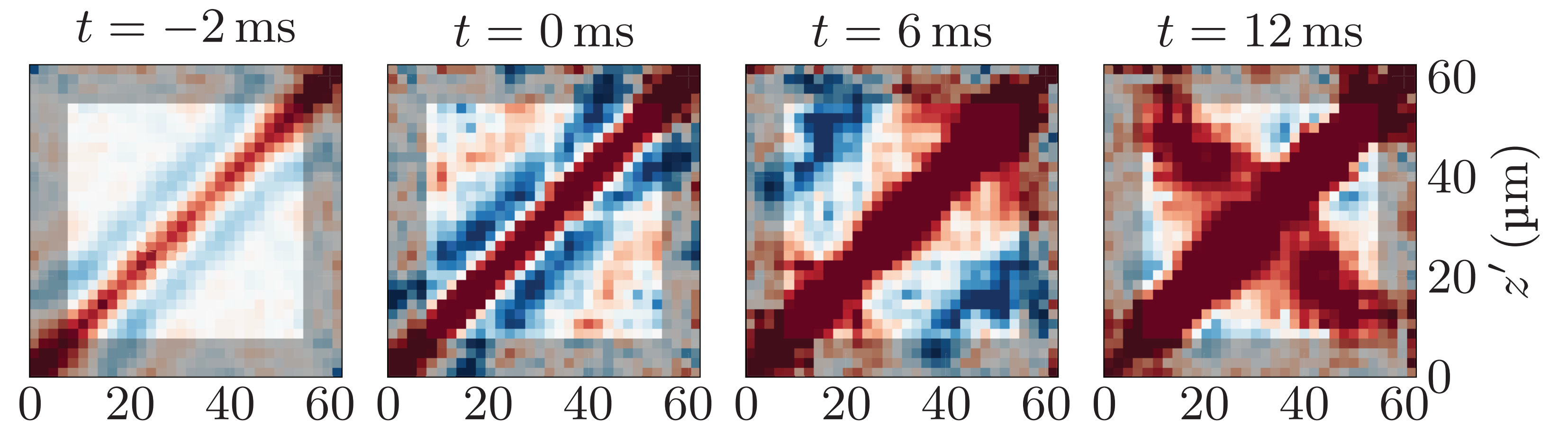
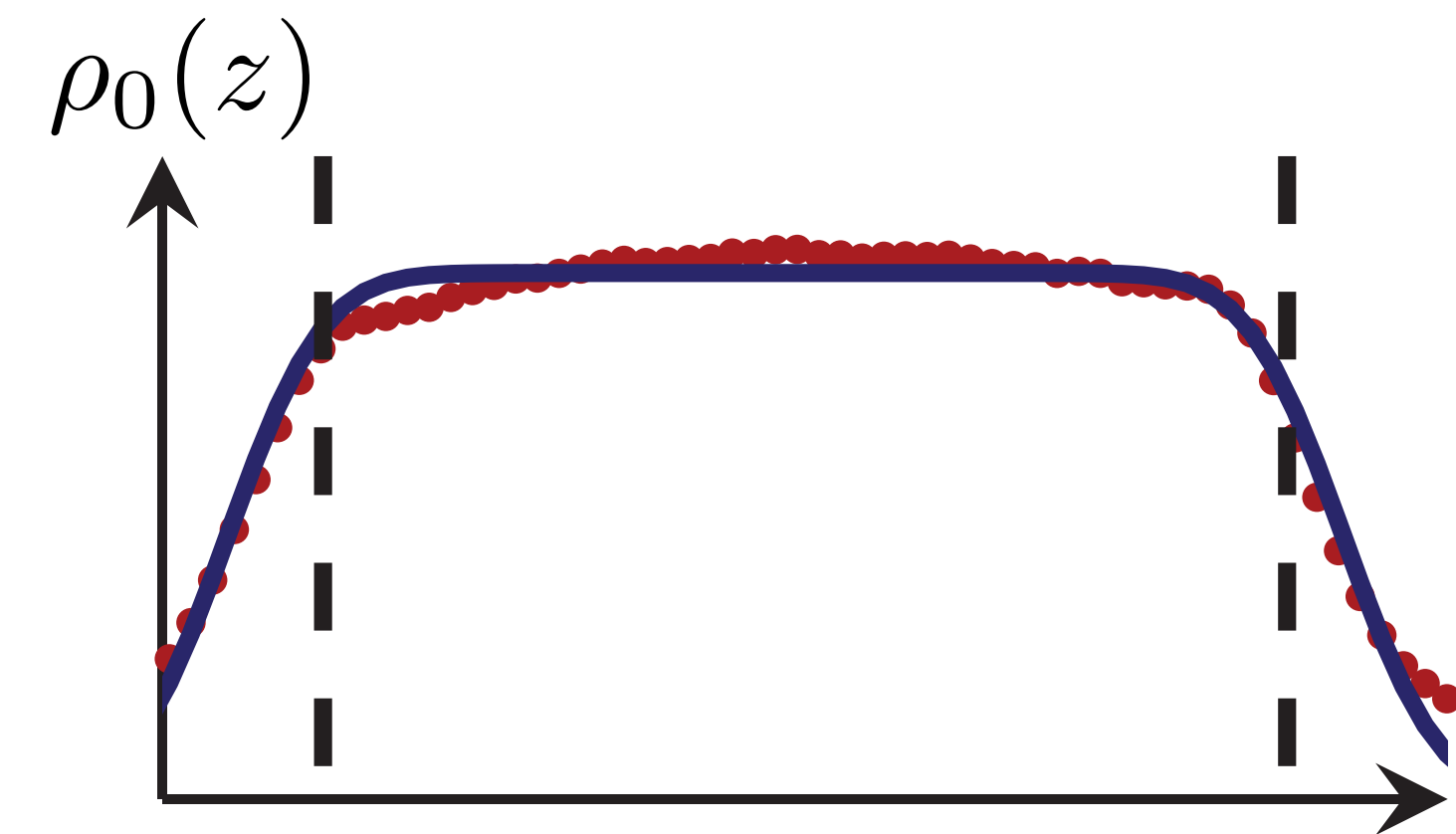
Homogeneous background density



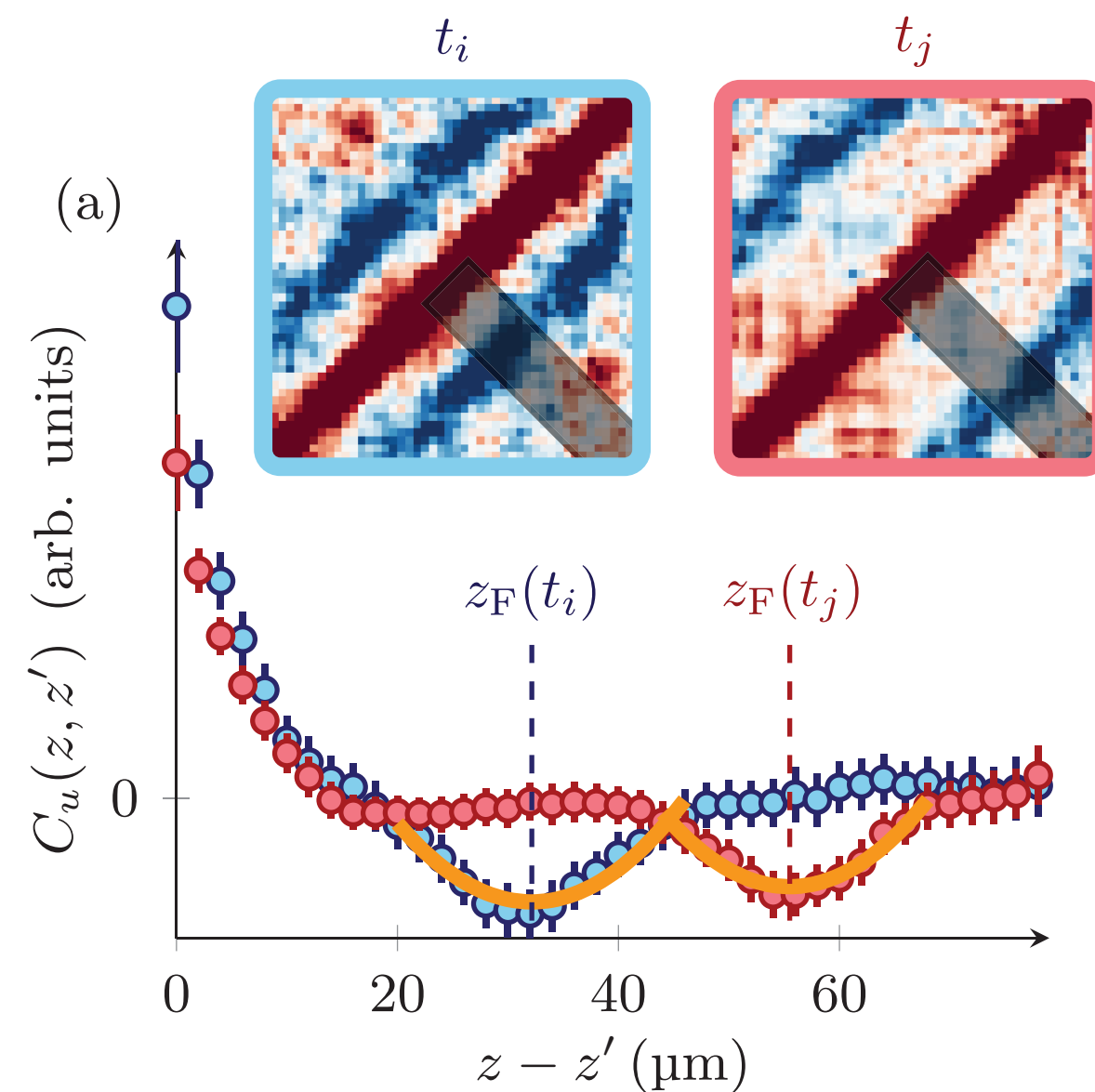
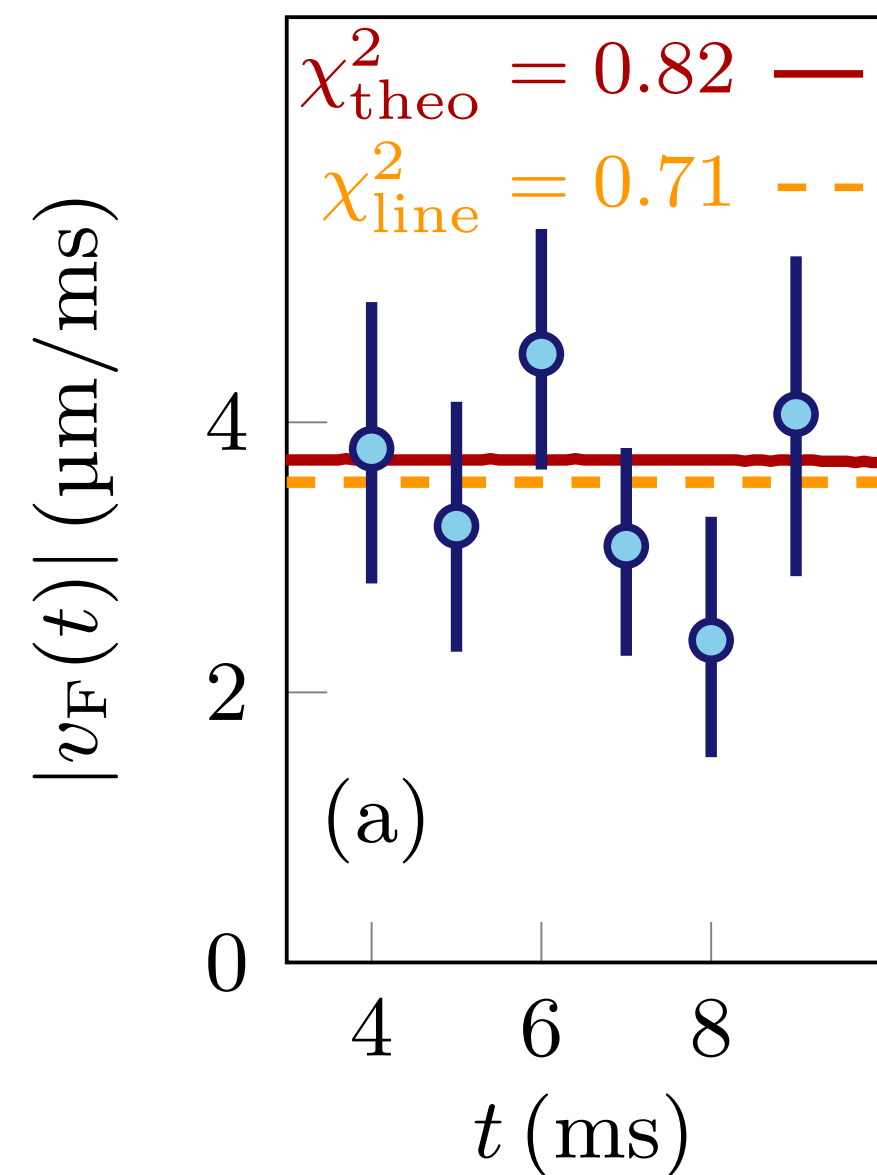
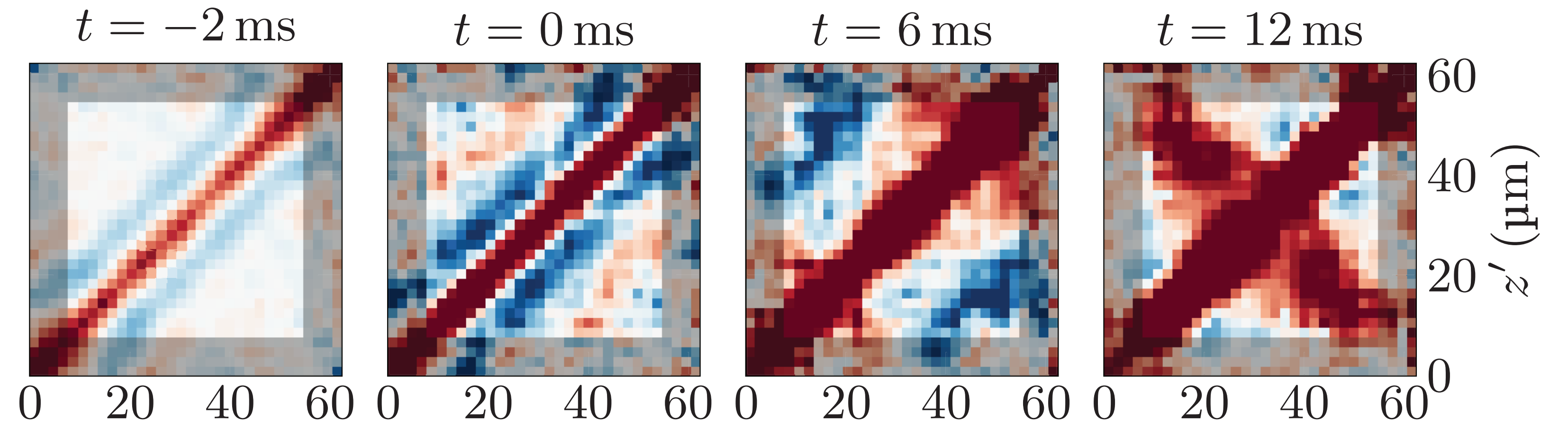
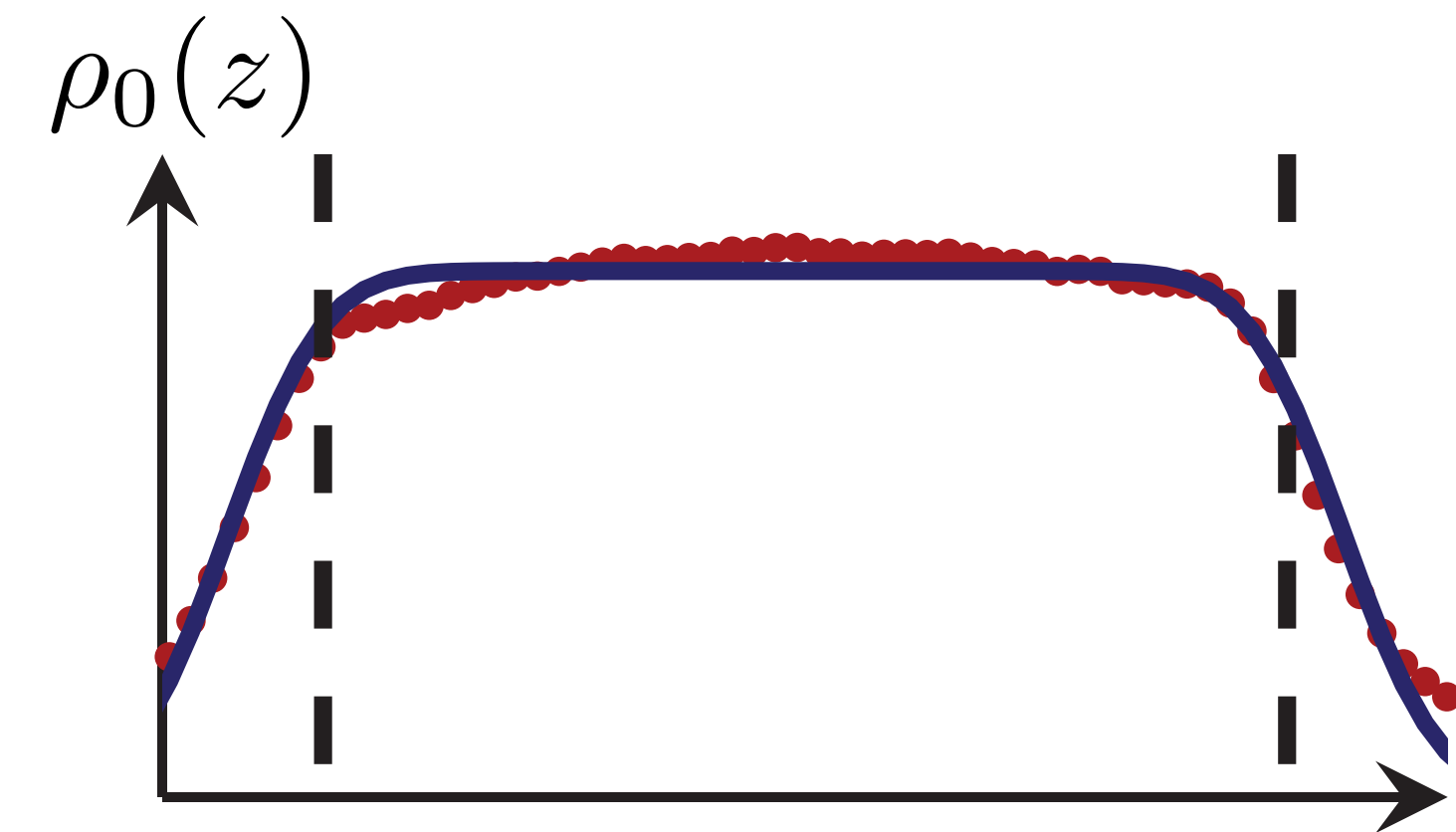
Homogeneous background density



Homogeneous background density



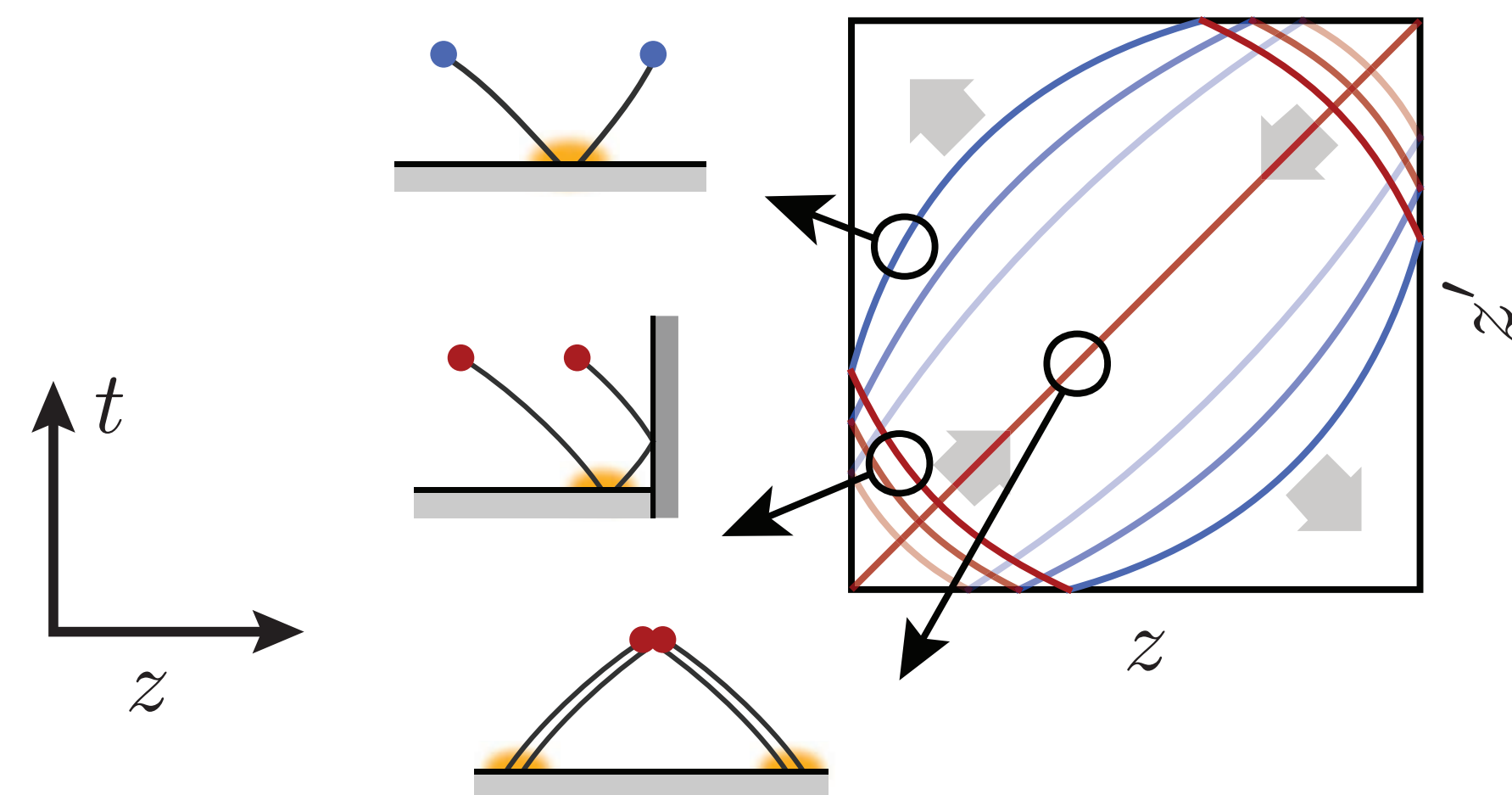
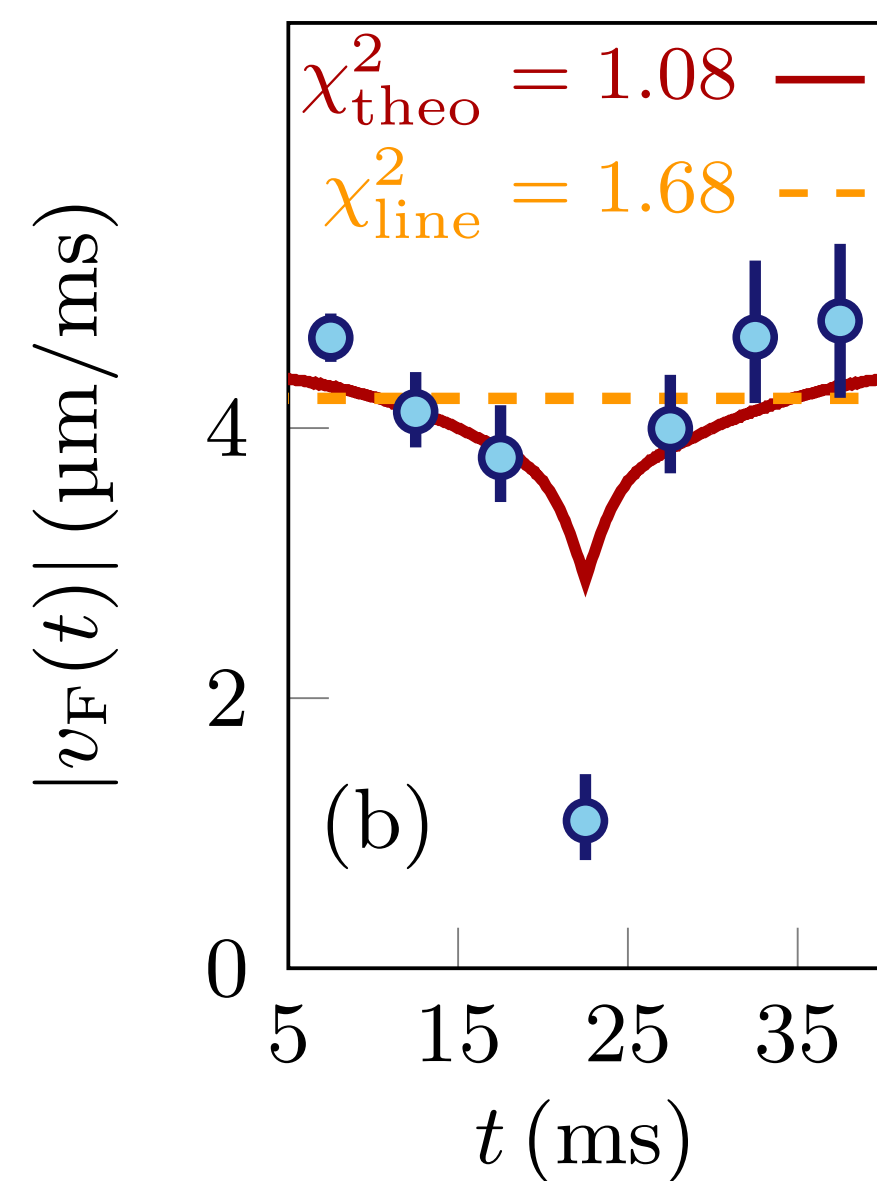
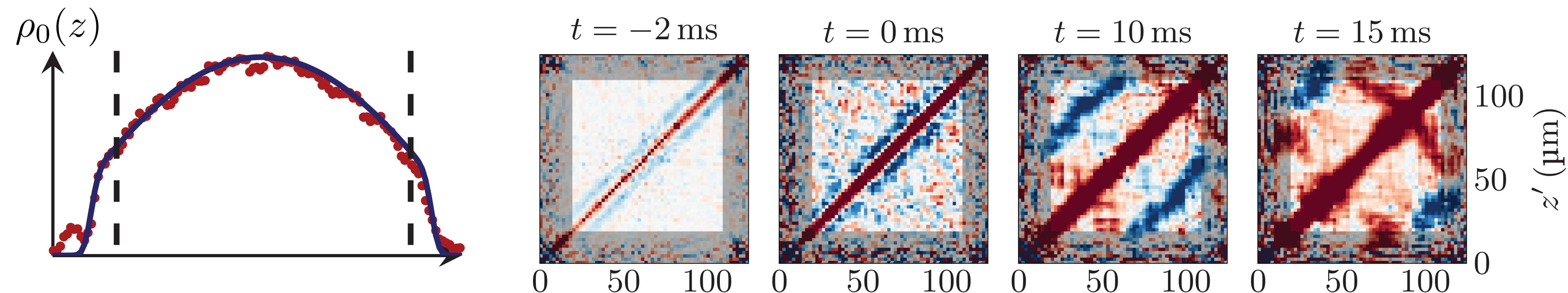
Homogeneous background density



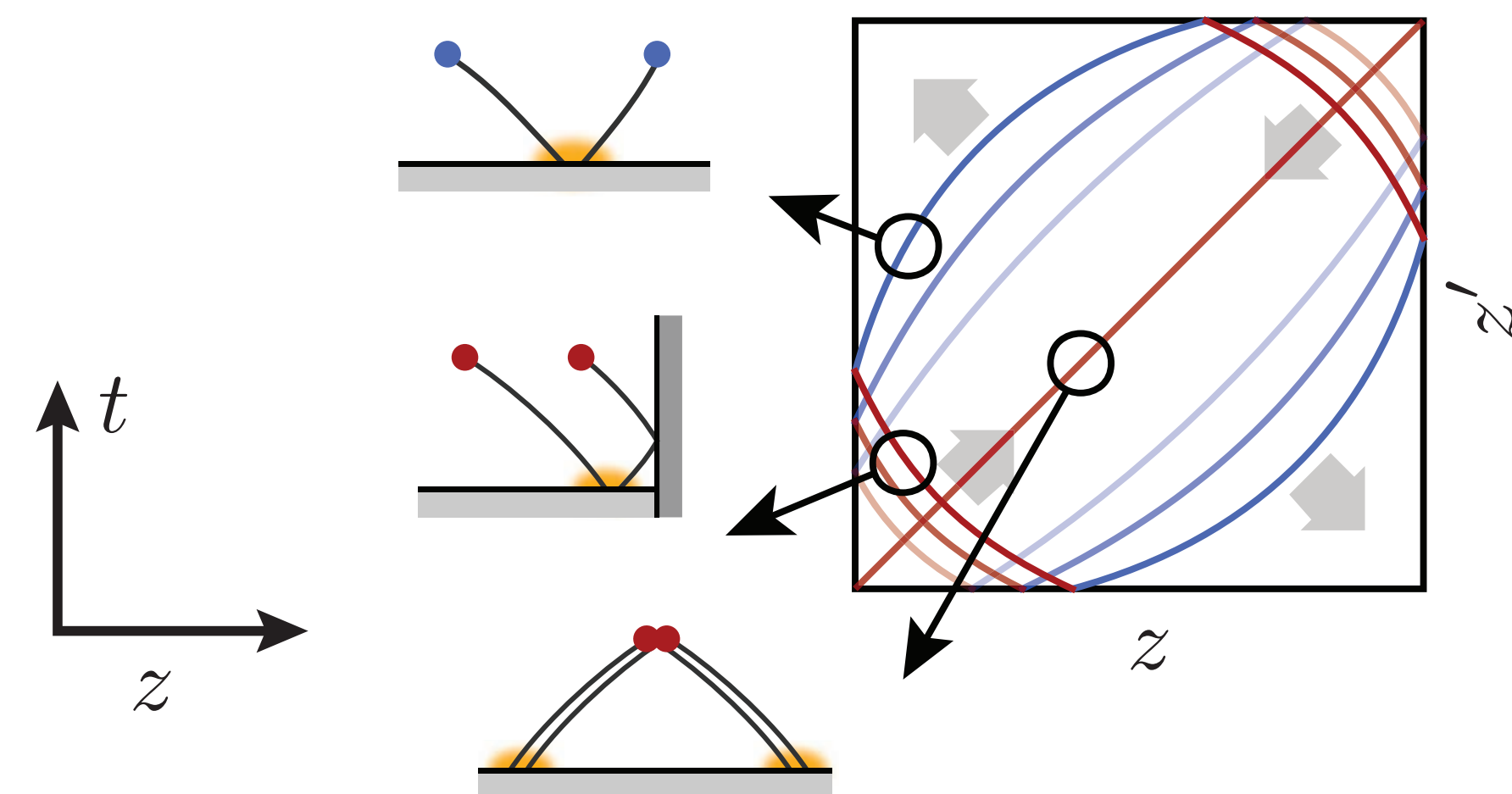
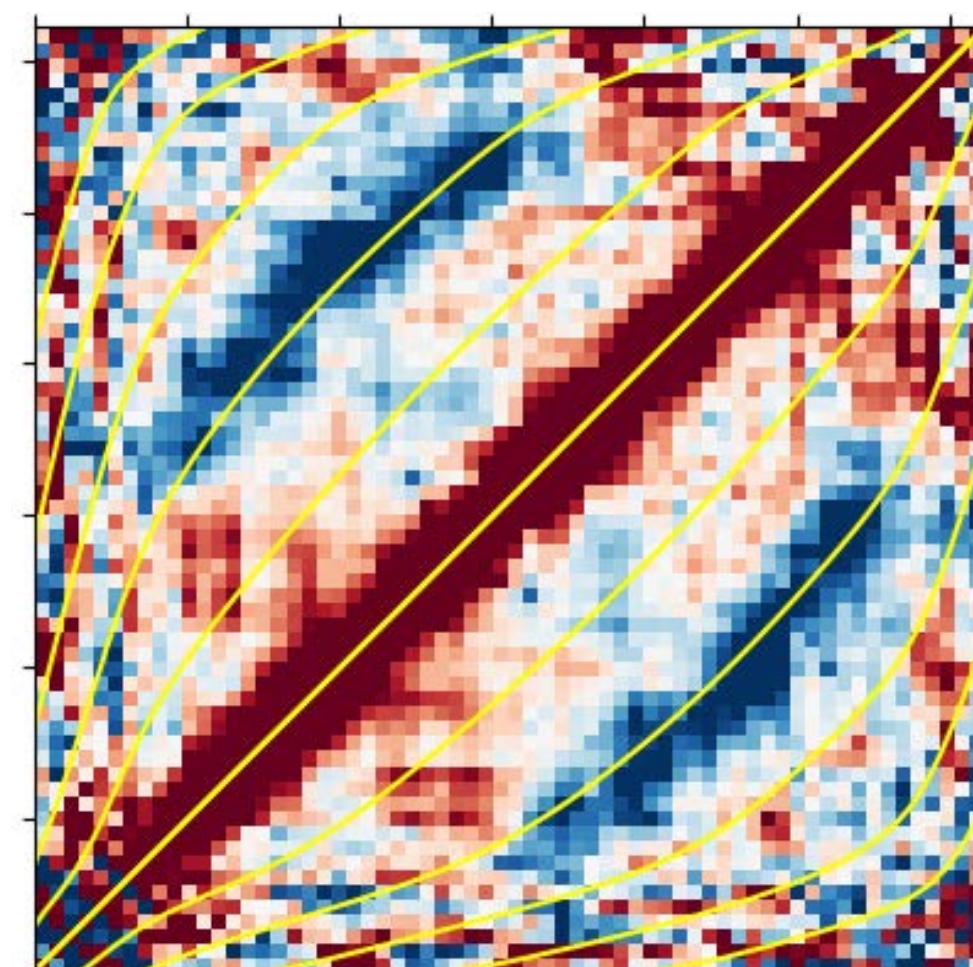
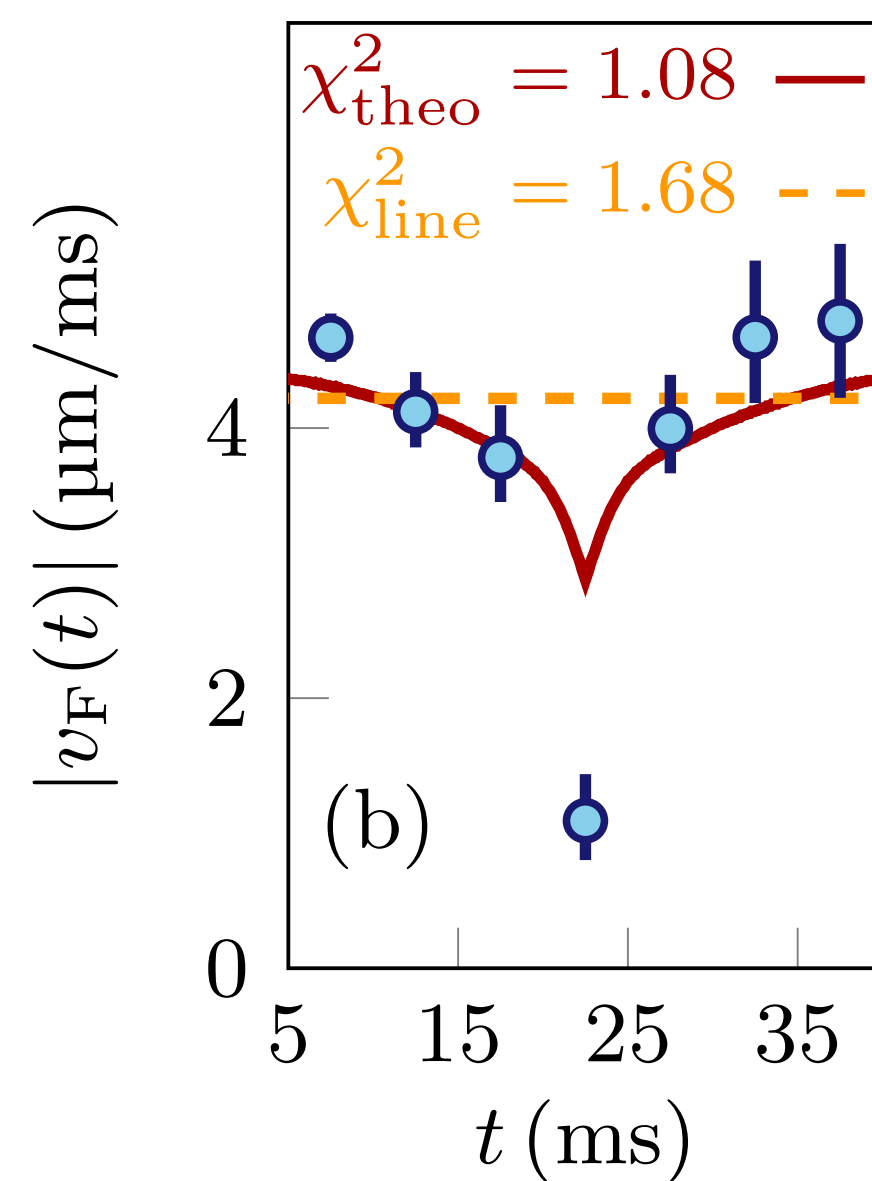
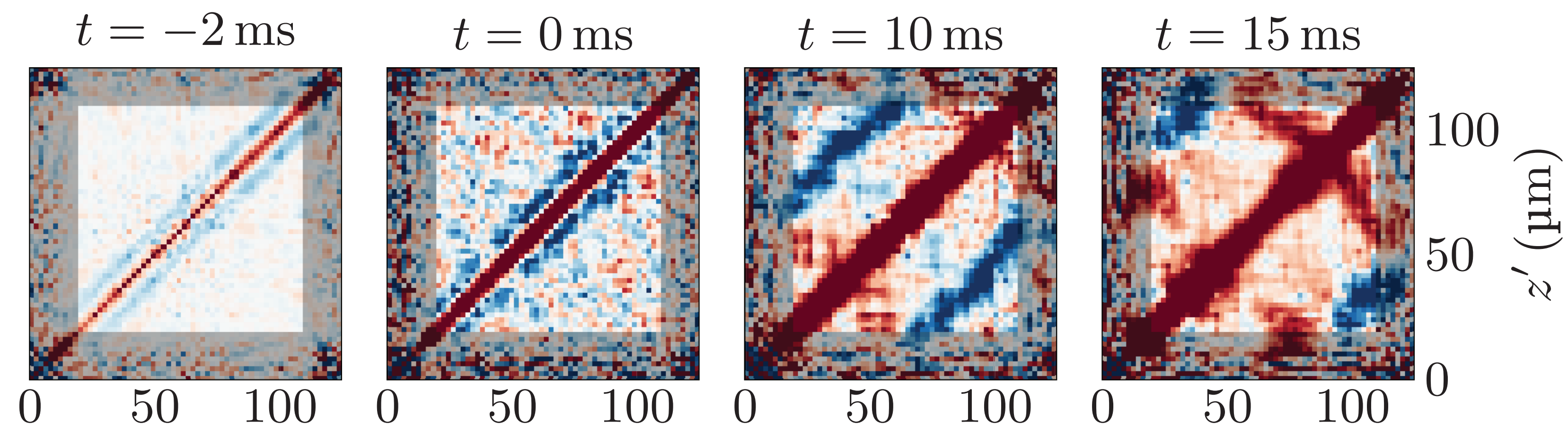
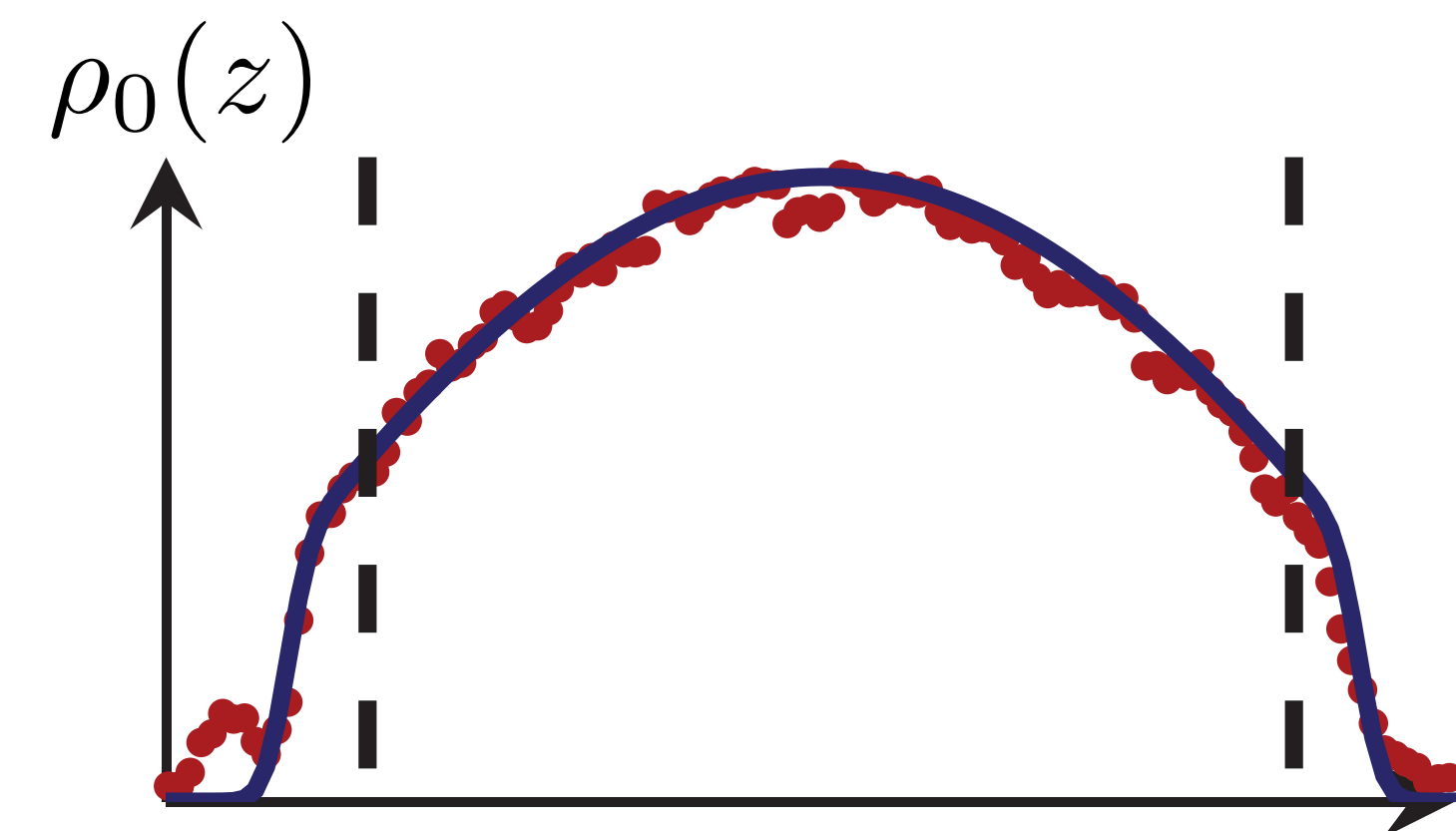
$$v_F(t) = \frac{z_F(t_i) - z_F(t_j)}{t_i - t_j}, \quad t = \frac{t_i + t_j}{2}$$

$$\chi^2_{\text{red}} = \frac{1}{N-1} \sum_{i=1}^N \frac{(v_F(t_i) - v_F^{\text{theo}}(t_i))^2}{\sigma_i^2}$$

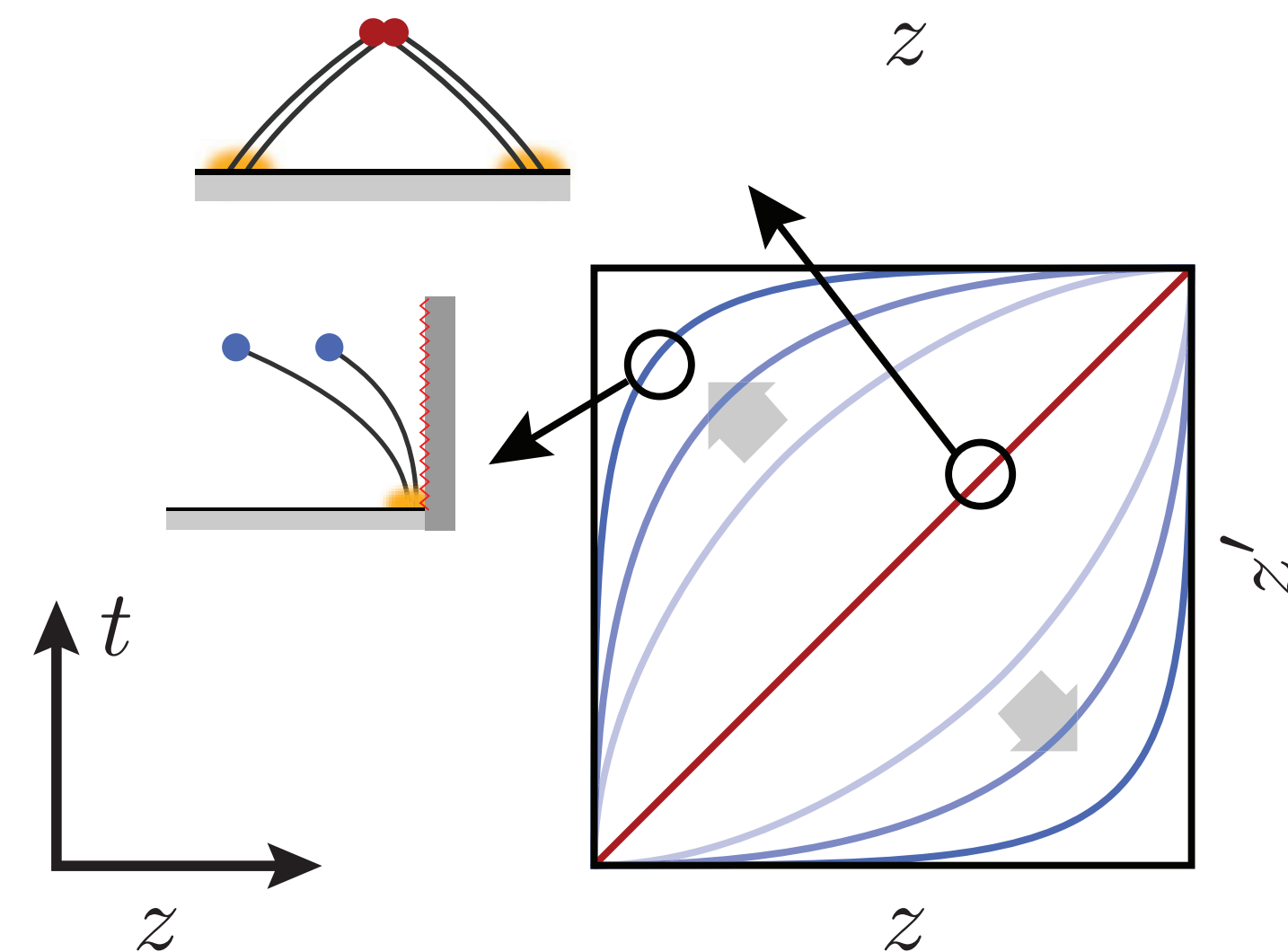
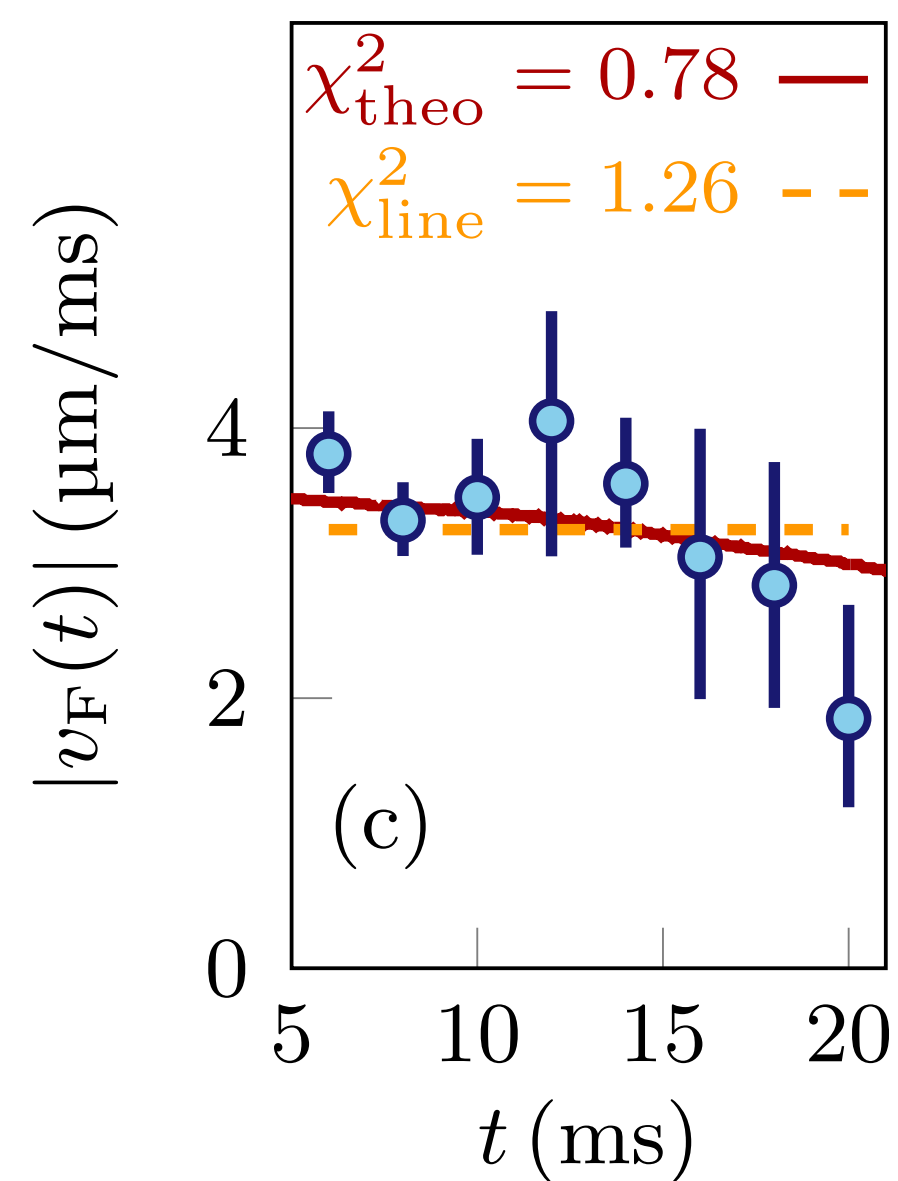
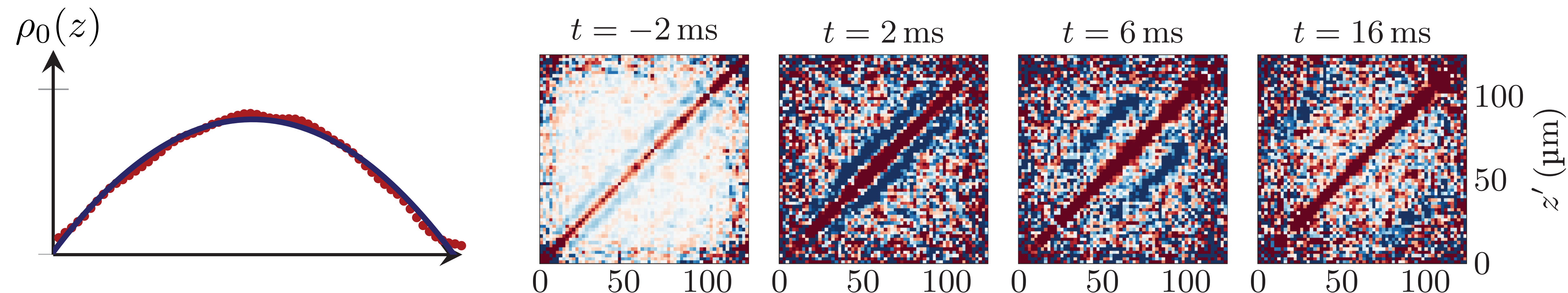
Inhomogeneous background density with walls



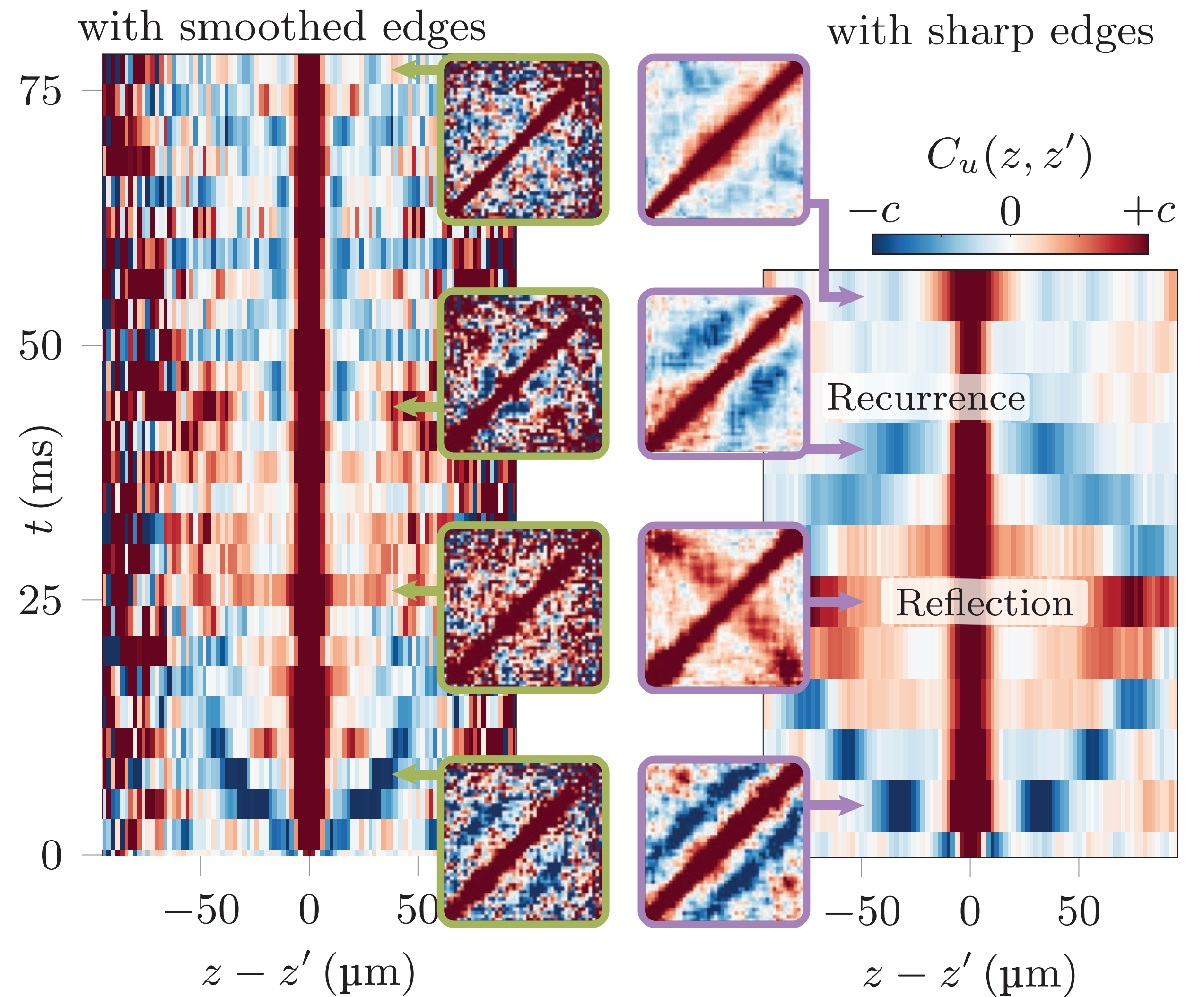
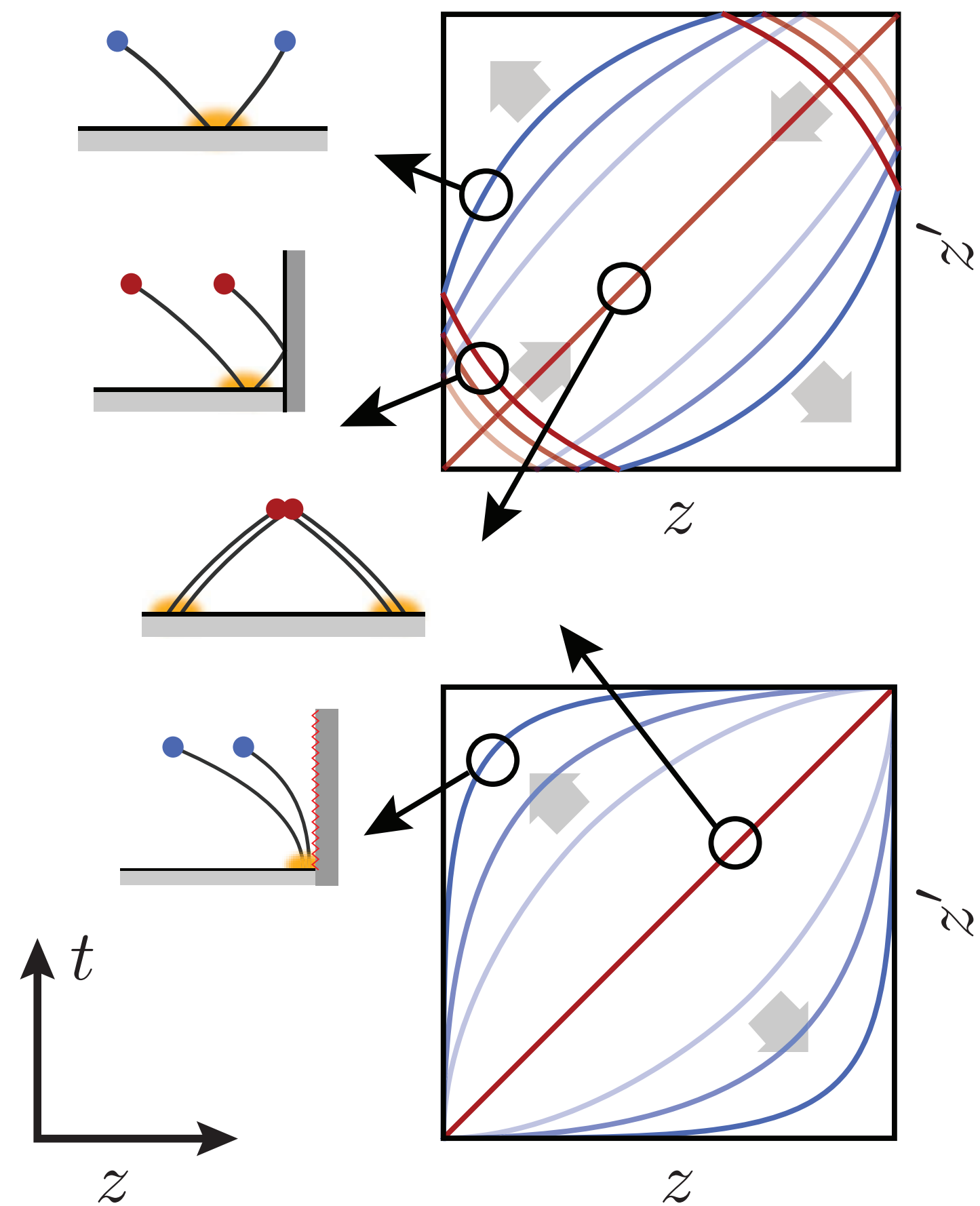
Inhomogeneous background density with walls



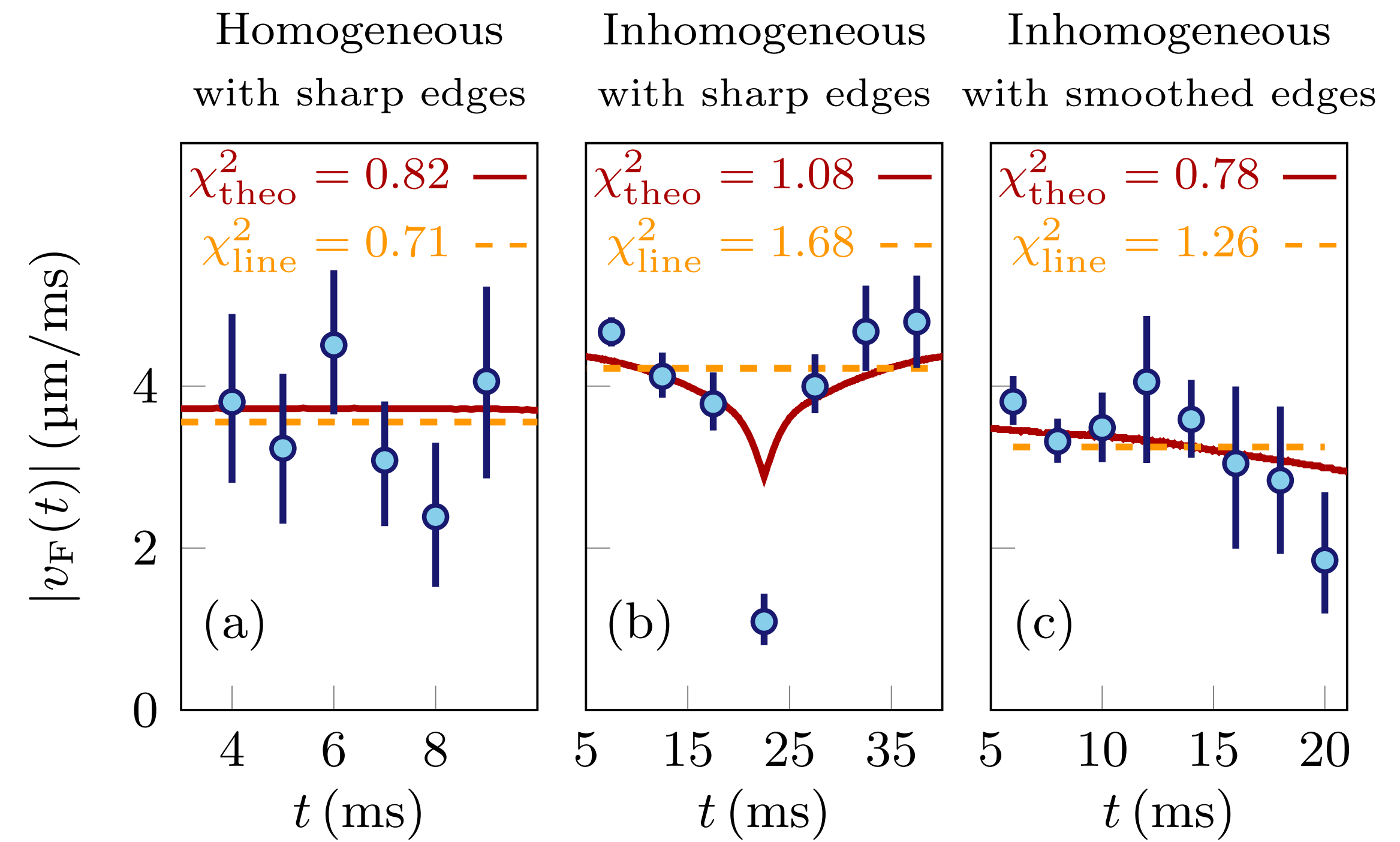
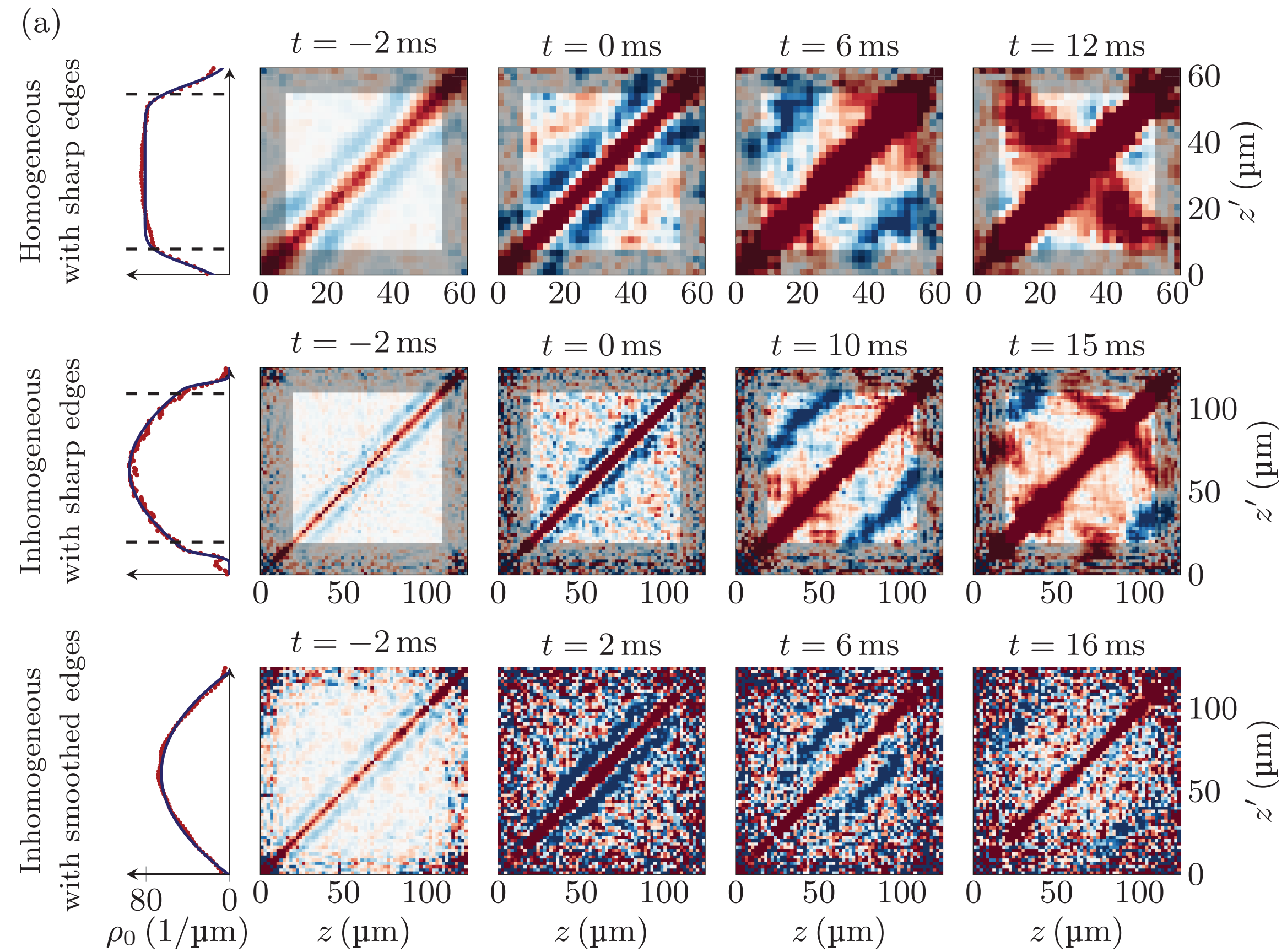
Inhomogeneous background density without walls



Reflection and recurrence



Summary



Measuring von Neumann entropy and mutual information

- Definition:

$$I(A : B) = S_A + S_B - S_{A \cup B}$$



von Neumann entropy for subsystem A

$$S_A = -\text{Tr}(\rho_A \ln \rho_A) \quad \text{with} \quad \rho_A = -\text{Tr}_B(\rho_{A \cup B})$$

Gluza et al. Nat Com Phys (2020)

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- Covariance matrix

Gluza et al. Nat Com Phys (2020)

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Gluza et al. Nat Com Phys (2020)

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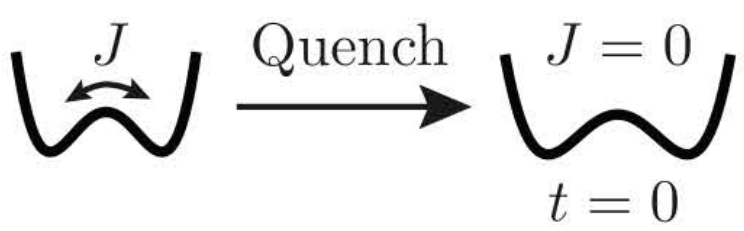
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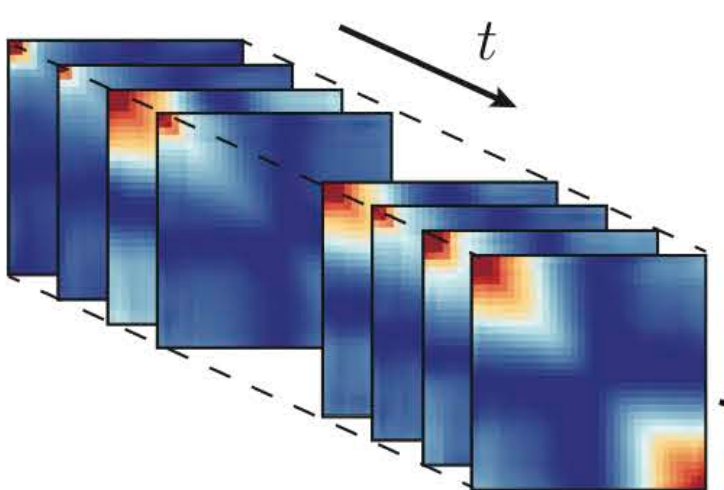
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Independent evolution



von Neumann entropy of Gaussian states



$$S_A = \sum_k \left(\gamma_k^A + \frac{1}{2} \right) \ln \left(\gamma_k^A + \frac{1}{2} \right) - \left(\gamma_k^A - \frac{1}{2} \right) \left(\gamma_k^A - \frac{1}{2} \right)$$

von Neumann entropy of Gaussian states



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Eigenvalues of $i \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix} \Gamma^A$

von Neumann entropy of Gaussian states



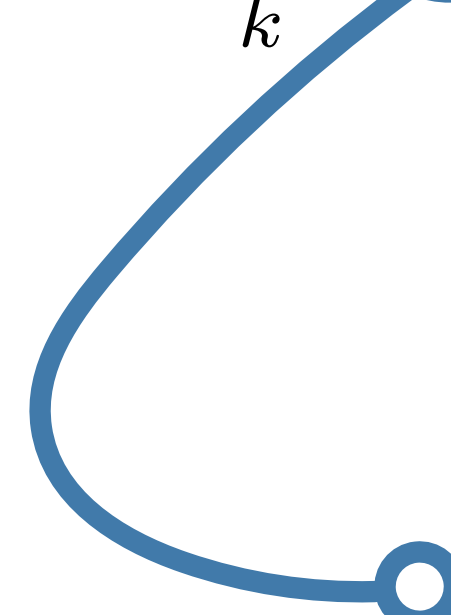
$$S_A = \sum_k \left(\gamma_k^A + \frac{1}{2} \right) \ln \left(\gamma_k^A + \frac{1}{2} \right) - \left(\gamma_k^A - \frac{1}{2} \right) \left(\gamma_k^A - \frac{1}{2} \right)$$

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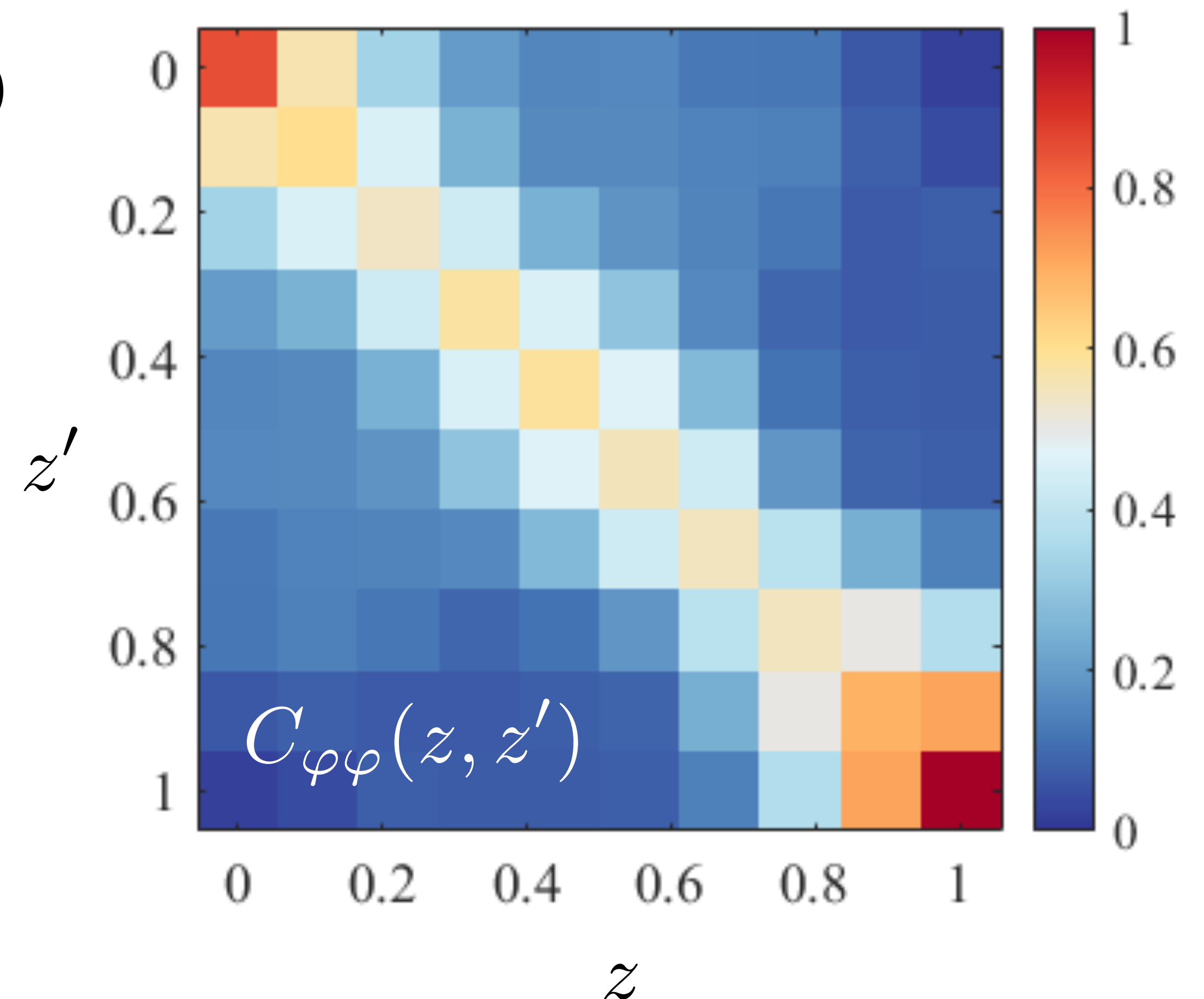
$$\Gamma^A = \begin{pmatrix} C_{\varphi\varphi}^A & C_{\varphi\delta\rho}^A \\ C_{\delta\rho\varphi}^A & C_{\delta\rho\delta\rho}^A \end{pmatrix}$$

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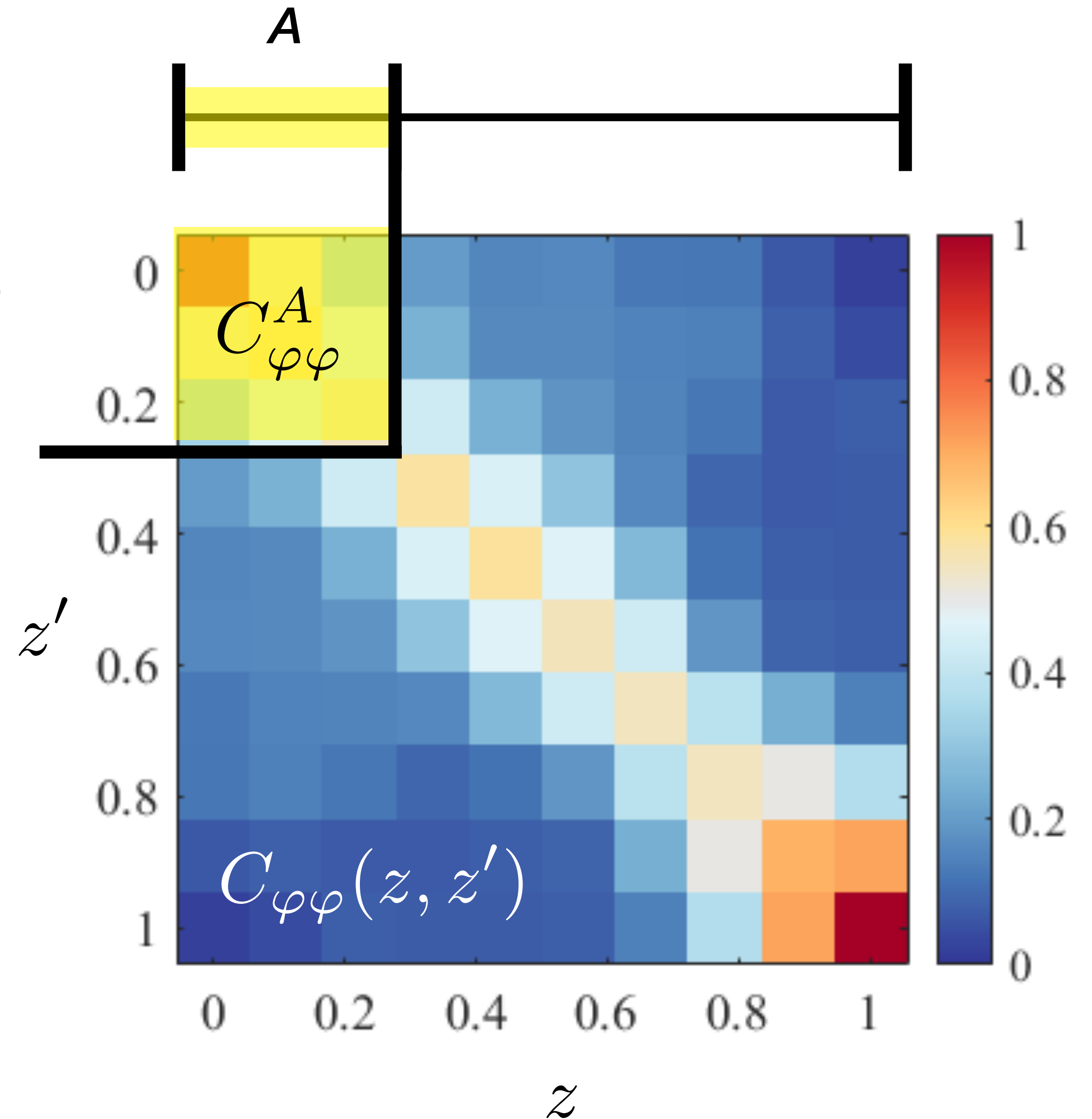


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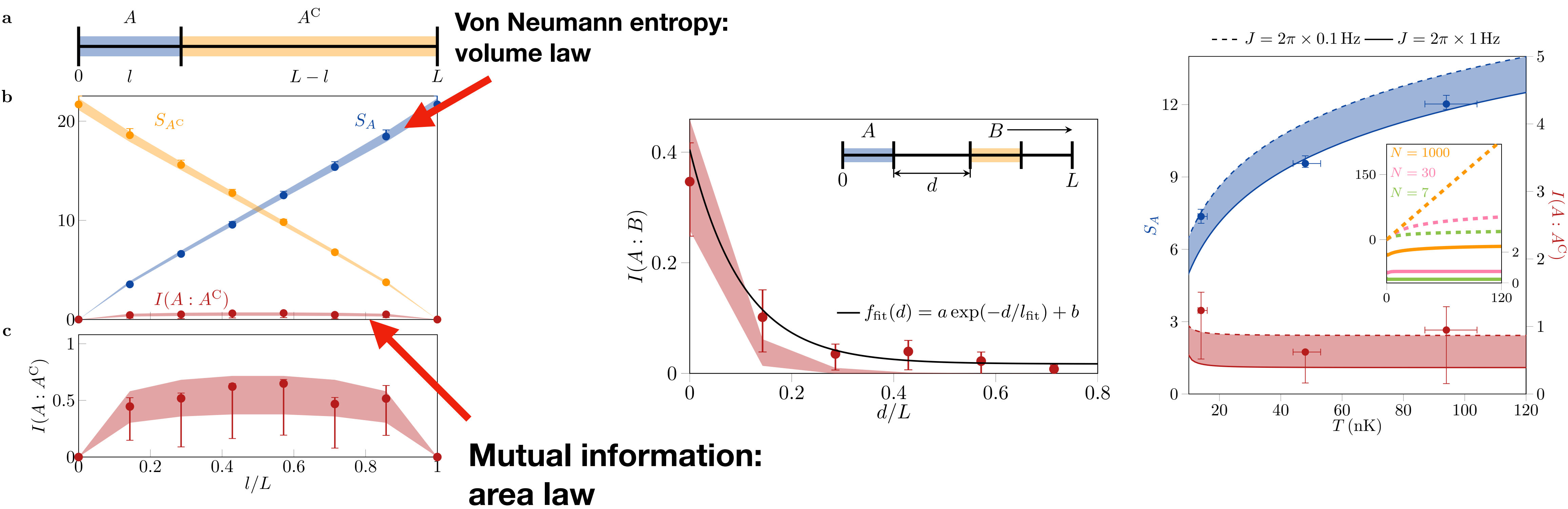
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Measuring von Neumann entropy and mutual information

Comparison with theory: Analytical correlations of massive Klein-Gordon model

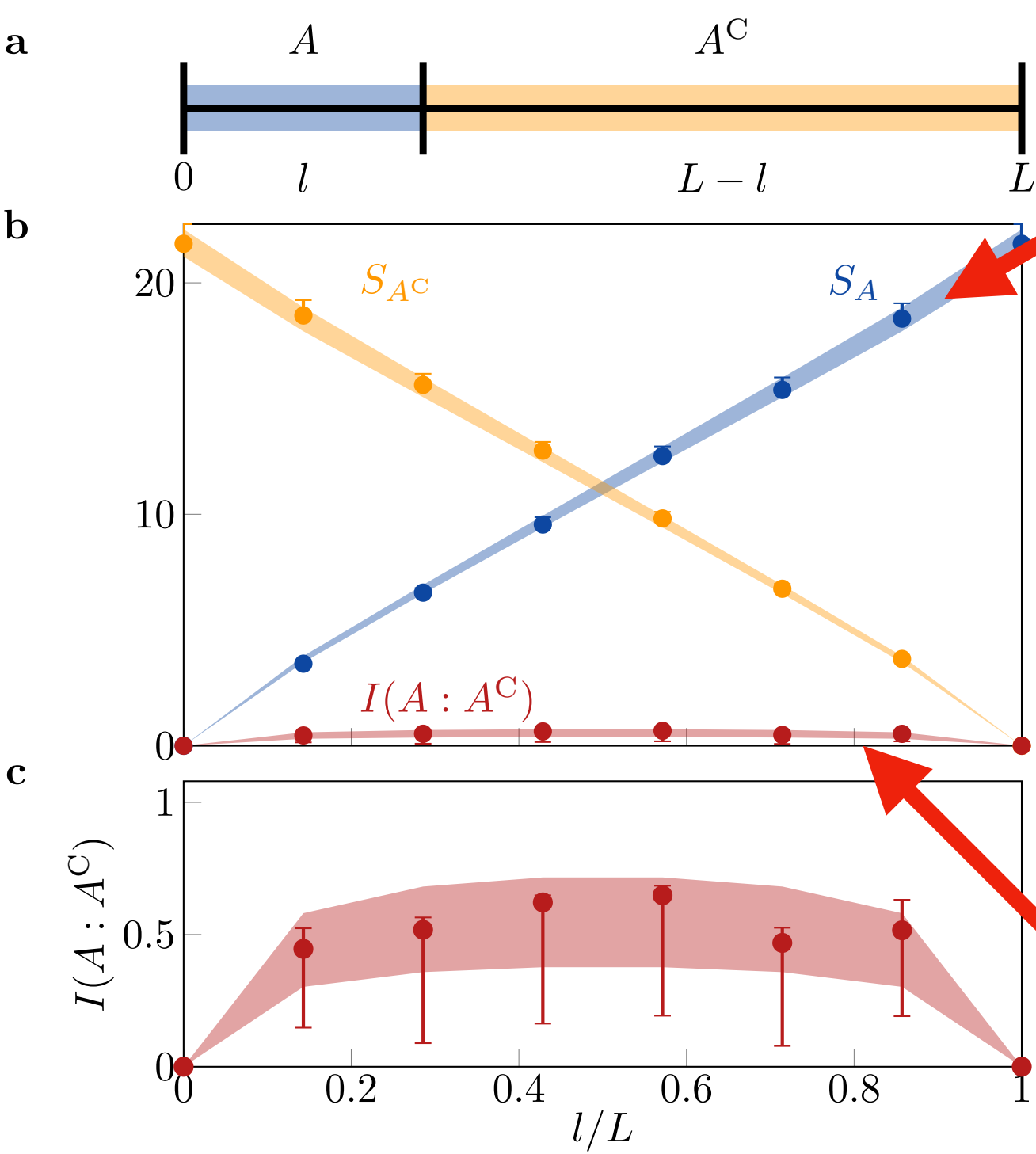
Tajik et al., Nat. Phys.(2023)



Measuring von Neumann entropy and mutual information

Comparison with theory: Analytical correlations of massive Klein-Gordon model

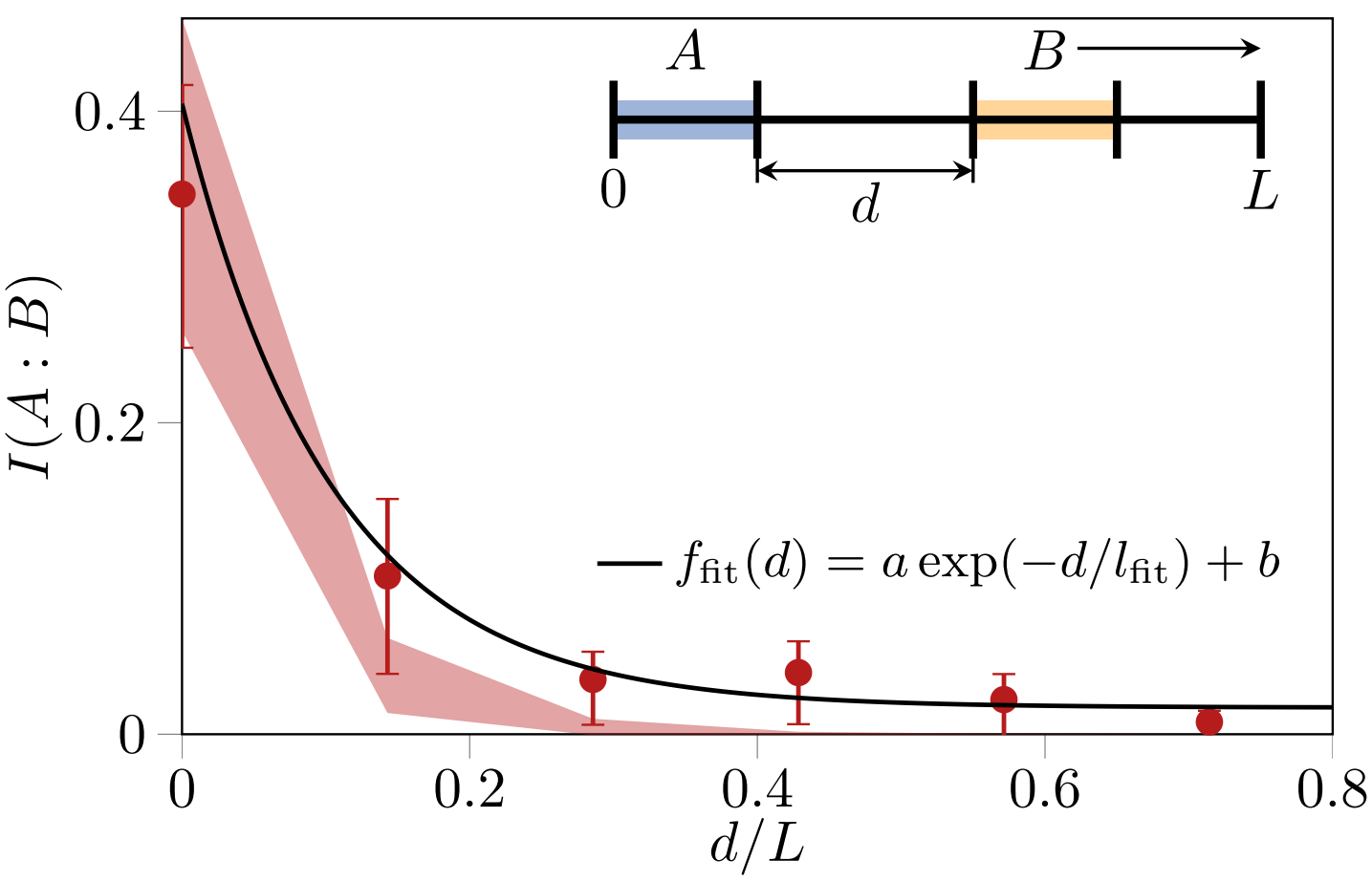
Tajik et al., Nat. Phys.(2023)



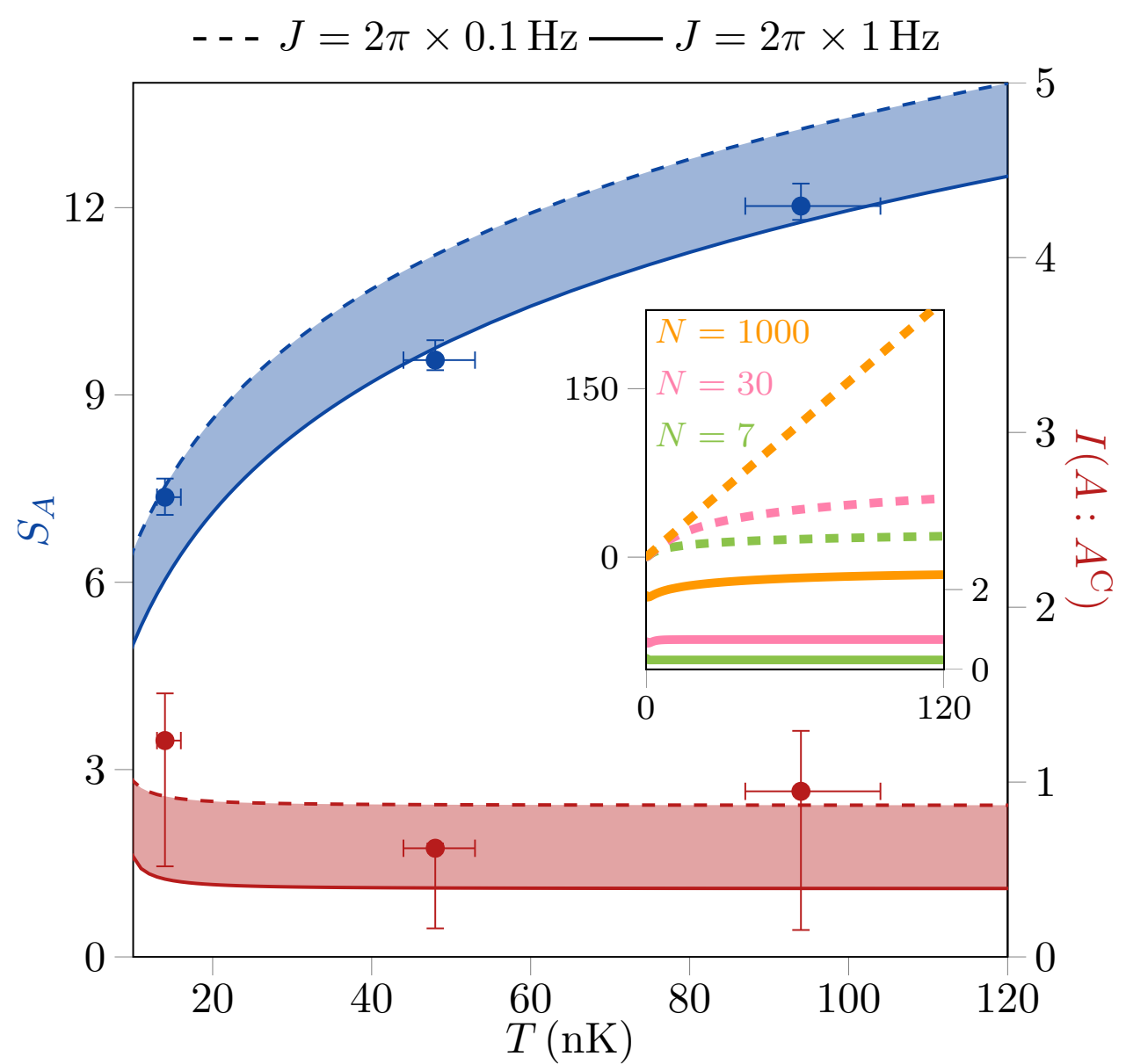
**Von Neumann entropy:
volume law**

**Mutual information:
area law**

Spatial dependence



Decay with separation



Temperature dependence

END