# OBSERVING AND MANIPULATING PARTICLES AND FIELDS IN SUPERFLUID $^3$He UNIVERSE

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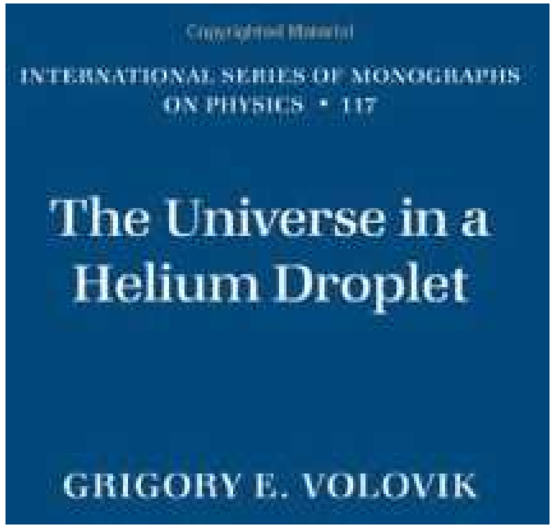
# SUPERFLUID $^3$He UNIVERSE

Fermi system with pairing in $L = 1$, $S = 1$ state. $3 \times 3$ order parameter, complex symmetry breaking and multiple superfluid phases.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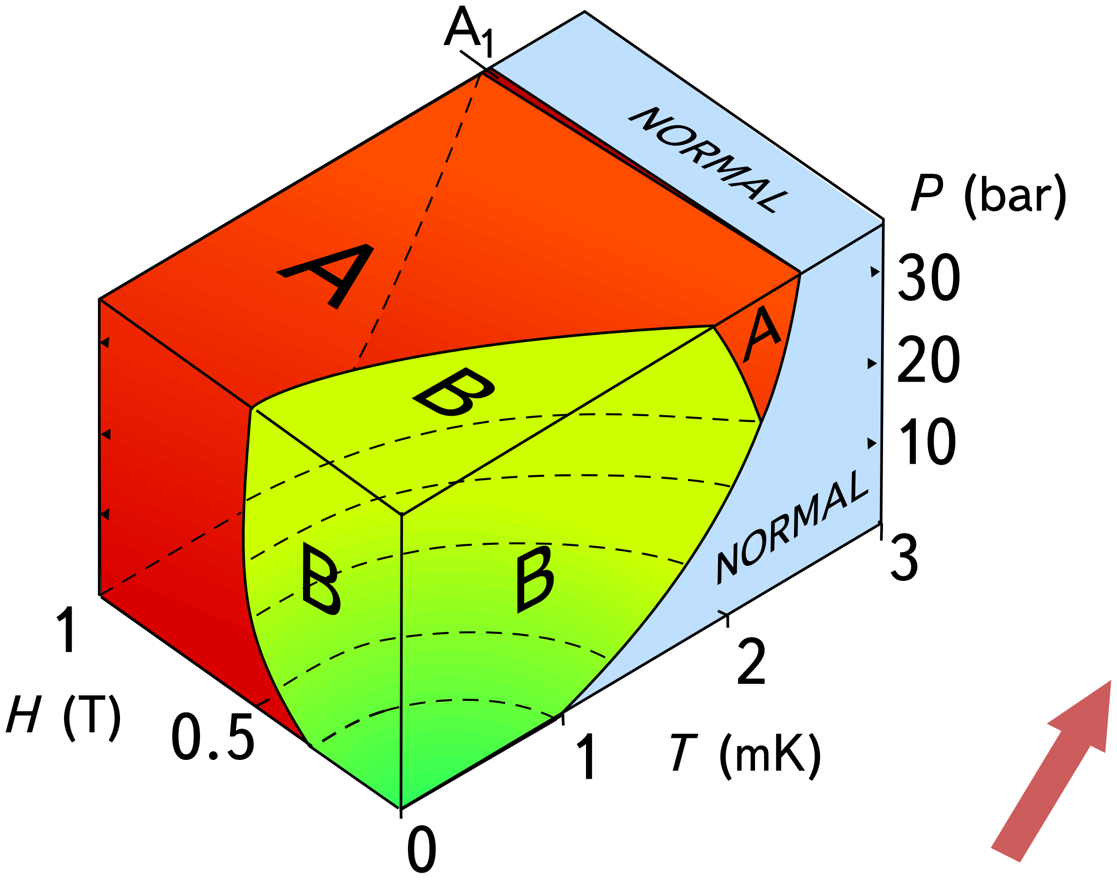
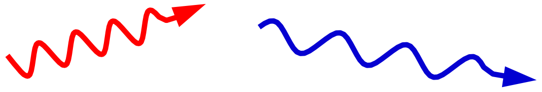
The Universe in a
Helium Droplet

GRIGORY E. VOLOVIK



- Bosonic excitations

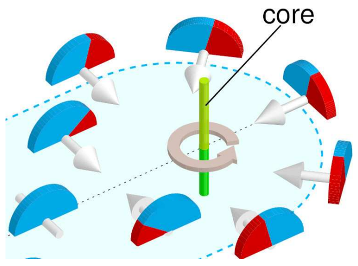


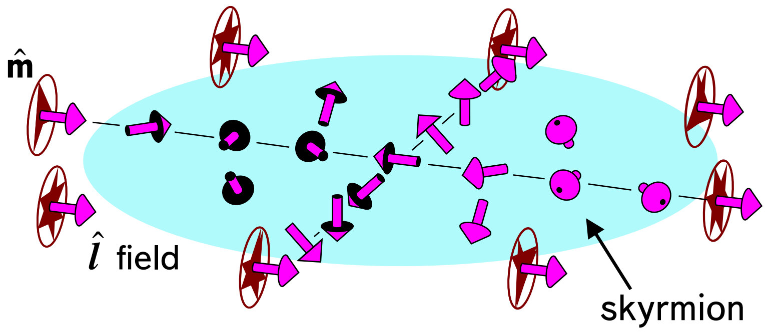
Collective modes: Heavy and light Higgs, sound, spin waves

- Fermionic quasiparticles

Massive and massless: Weyl, Majorana, Dirac ...

- Topological objects





Vortices, skyrmions, solitons, KLS walls ...

Effective gauge and geometry:

Synthetic electromagnetic and

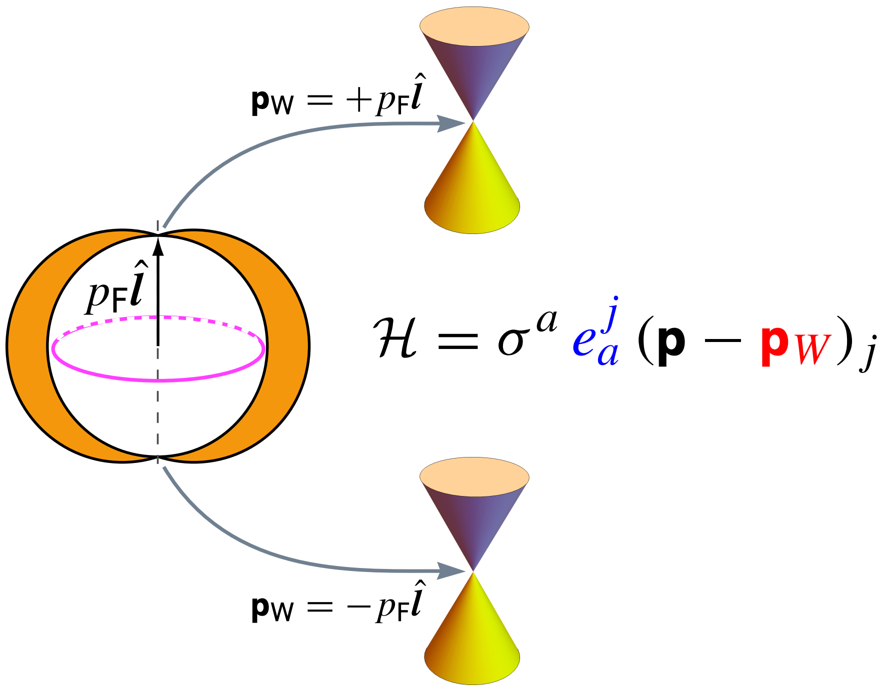
gravitational fields

create and observe feedback

# TALK OVERVIEW

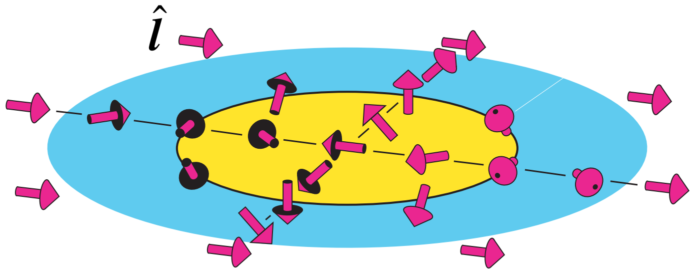
- Superfluid phases of $^3$He.
- Quasirelativistic Weyl fermions in the A phase:

  Effective vector potential and vierbein.



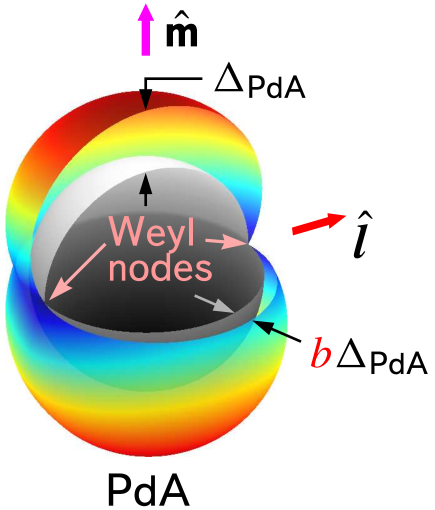
$$\mathcal{H} = \sigma^a \, e_a^j \, (\mathbf{p} - \mathbf{p}_W)_j$$

- Creating synthetic fields with flow and topological objects.
- Chiral anomaly and the zero-charge effect.
- Gravitational anomalies and challenges for observation.



- Engineering new phases of $^3$He with nanostructured confinement.
- Horizon analogue at the interface of type-I and type-II

  Weyl fermion spectra: PdA phase and flow.

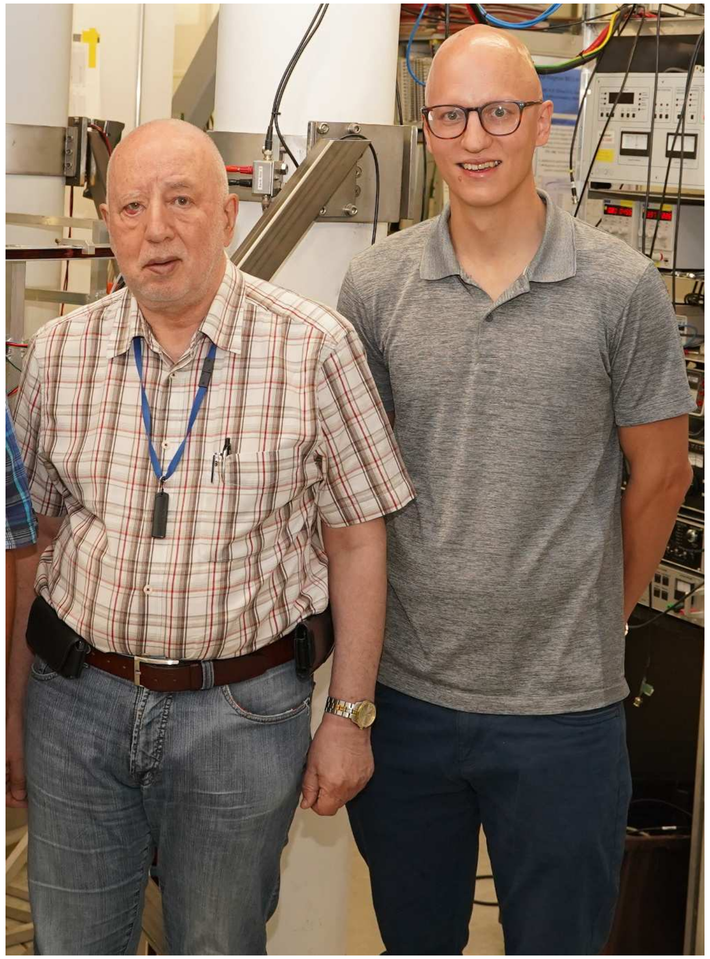


PdA

$0 < b < 1$

- Expereimental tools to observe quasiparticle emission (Hawking radiation).
- Route to antispacetime via PdA-polar-PdA phases.

Rota
cryostat



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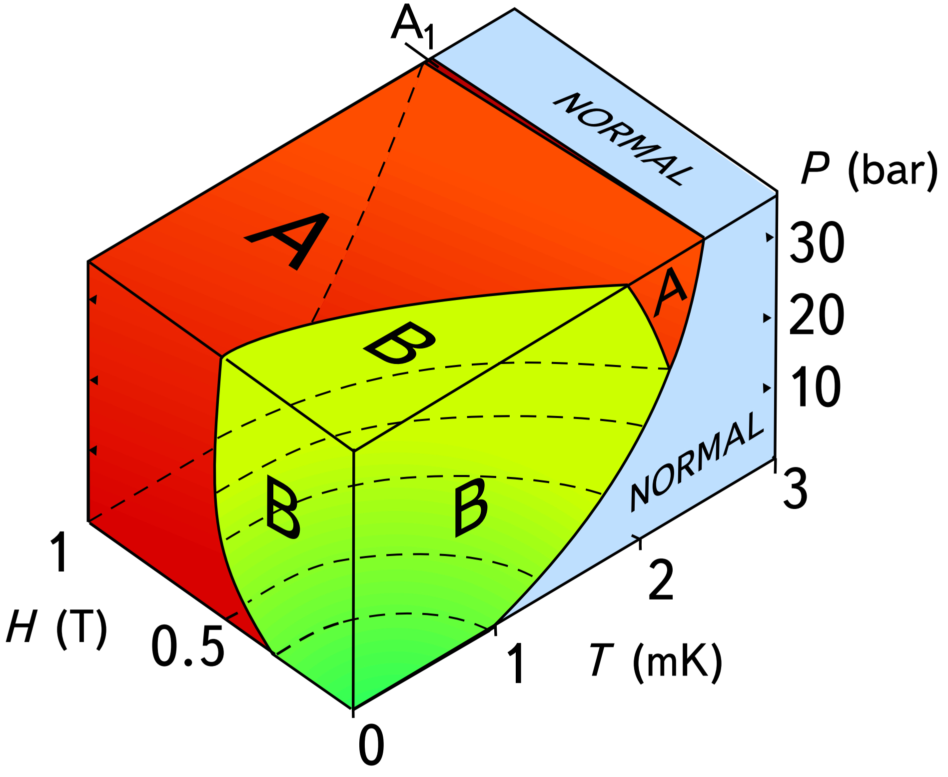
# SUPERFLUID PHASES OF $^3$He



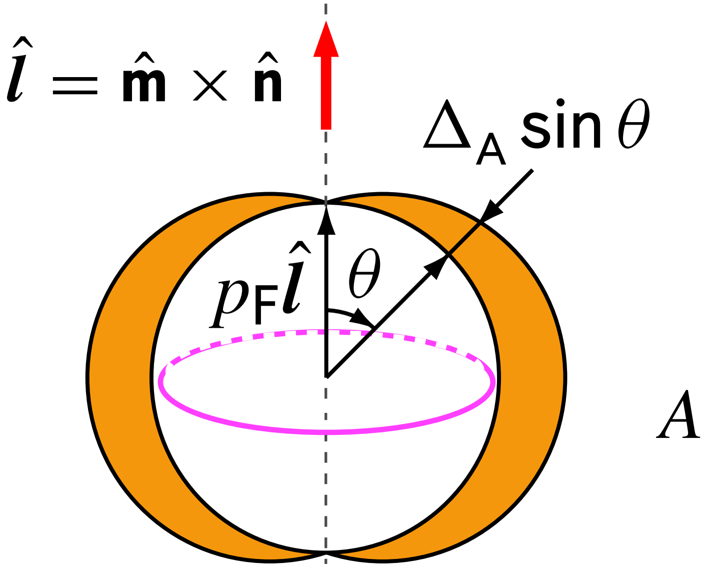
Triplet $p$-wave superfluid. Gap matrix $2 \times 2$

$$\boldsymbol{\Delta}_{\alpha\beta}(\mathbf{k}) = \underbrace{(i\sigma^2\sigma^\mu)_{\alpha\beta}}_{\text{Pauli matrices}}\, \mathbf{k}_j\, \underset{\overset{\uparrow}{\text{order parameter } 3\times 3}}{A_{\mu j}}$$

Mean-field BCS Hamiltonian

$$H(\mathbf{k}) = \begin{pmatrix} \epsilon(\mathbf{k})\sigma^0 & \boldsymbol{\Delta}(\mathbf{k}) \\ \boldsymbol{\Delta}^\dagger(\mathbf{k}) & -\epsilon(\mathbf{k})\sigma^0 \end{pmatrix} \qquad \epsilon(\mathbf{k}) = \hbar^2 \frac{k^2 - k_{\mathrm{F}}^2}{2m}$$

## A phase

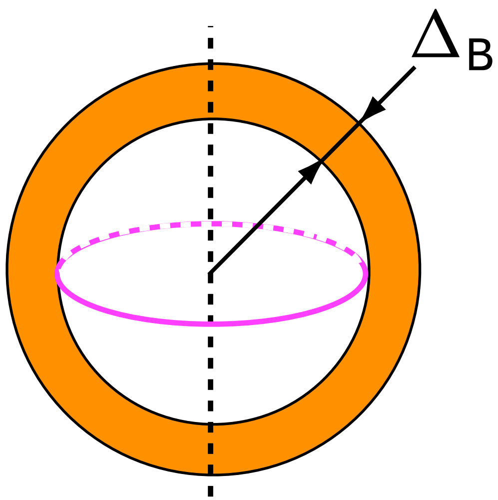


Two Weyl nodes.

Time-reversal symmetry broken.

$$A_{\mu j} = \Delta_{\mathrm{A}}\, \underset{\overset{\uparrow}{\text{spin}}}{\hat{d}_\mu}\, \underbrace{(\hat{m}_j + i\hat{n}_j)}_{\text{orbital+U(1)}}$$

## B phase



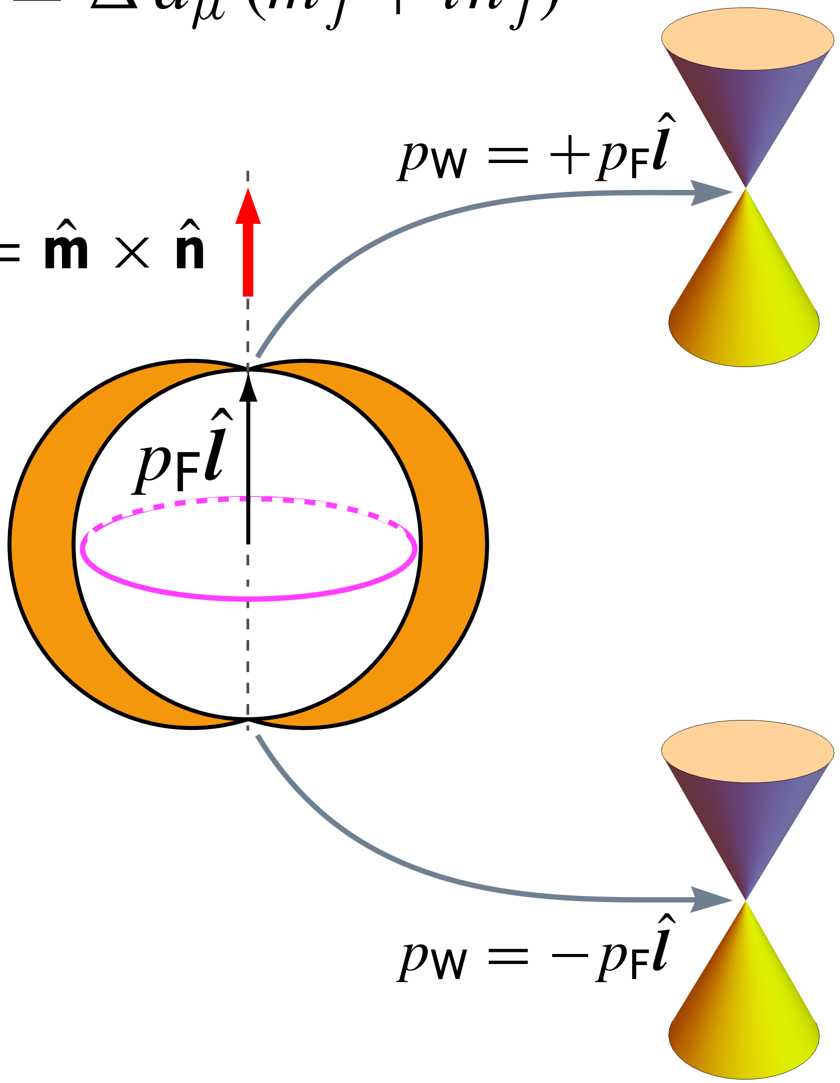
Fully gapped.

Time-reversal symmetric.

$$A_{\mu j} = \Delta_{\mathrm{B}}\, \underset{\overset{\uparrow}{\text{U(1)}}}{e^{i\phi}}\, \underset{\overset{\uparrow}{\text{rotation of spin vs orbital}}}{R_{\mu j}}$$

# QUASIRELATIVISTIC FERMIONS IN $^3$He-A

$$A_{\mu j} = \Delta\, \hat{d}_\mu\, (\hat{m}_j + i\hat{n}_j)$$



Expanding close to gap nodes, one gets Hamiltonian for left and right Weyl fermions living in effective gauge field and geometry:

$$\mathcal{H} = \pm\sigma^a \underset{\text{like vierbein}}{\underset{\uparrow}{e_a^j}} (\mathbf{p} \pm \overset{\text{like vector potential}}{\overset{\downarrow}{p_F \hat{\boldsymbol{l}}}})_j$$

Volovik & Vachaspati (1996)

$$c_\perp = \frac{\Delta}{p_\mathsf{F}} \ll c_\parallel = \frac{p_\mathsf{F}}{m} = v_\mathsf{F}$$

$$e_a^\mu = \begin{pmatrix} 1 & \mathbf{0} \\ 0 & c_\perp \hat{\mathbf{m}} \\ 0 & c_\perp \hat{\mathbf{n}} \\ 0 & c_\parallel \hat{\boldsymbol{l}} \end{pmatrix}$$

Effective metric $g_{\mu\nu} = e_\mu^a\, e_\nu^b\, \eta_{ab}$

$\eta_{ab} = \mathrm{diag}(1, -1, -1, -1)$

Nissinen & Volovik, PRRes **2**, 033269 (2020)

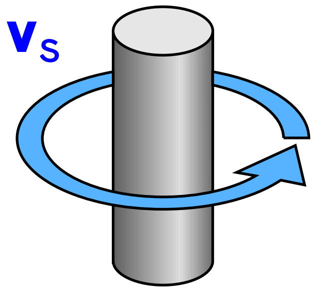
Control of effective fields: Change $c_\perp$, $c_\parallel$ with pressure, magnetic field and confinement.

Reorient $(\hat{\mathbf{m}}, \hat{\mathbf{n}}, \hat{\boldsymbol{l}})$ with boundaries, flow and topological objects.

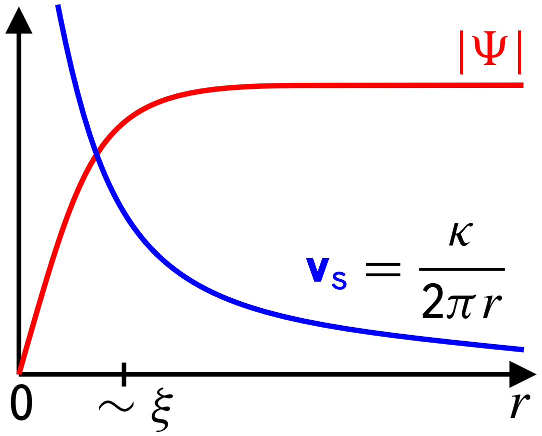
# VORTICITY IN $^3$He-A

Simple quantized vortex:

$$\Psi = |\Psi| e^{i\phi}, \quad \mathbf{v}_s = \frac{\hbar}{M} \nabla \phi.$$



$$\phi \to \phi + 2\pi$$



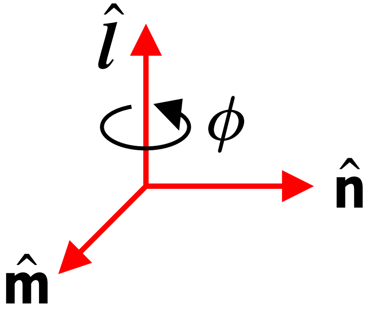
$$\oint \mathbf{v}_s \cdot d\mathbf{r} = \frac{2\pi\hbar}{M} = \kappa.$$

$M$ — mass of superfluid particle

$\kappa$ — circulation quantum

Order parameter:

$$A_{\mu j} = \Delta\, \hat{\mathbf{d}}_\mu\, (\hat{\mathbf{m}}_j + i\, \hat{\mathbf{n}}_j)\, e^{i\phi}$$



Local gauge-orbital symmetry

$$(\mathbf{v}_s)_i = \frac{\hbar}{M} \hat{\mathbf{m}} \cdot \nabla_i \hat{\mathbf{n}}$$

$$\boxed{(\nabla \times \mathbf{v}_s)_i = \frac{\hbar}{2M} \epsilon_{ijk}\, \hat{l} \cdot (\nabla_j \hat{l} \times \nabla_k \hat{l})}$$

*(Mermin and Ho, 1976)*

Continuous vorticity without suppressing superfluidity.

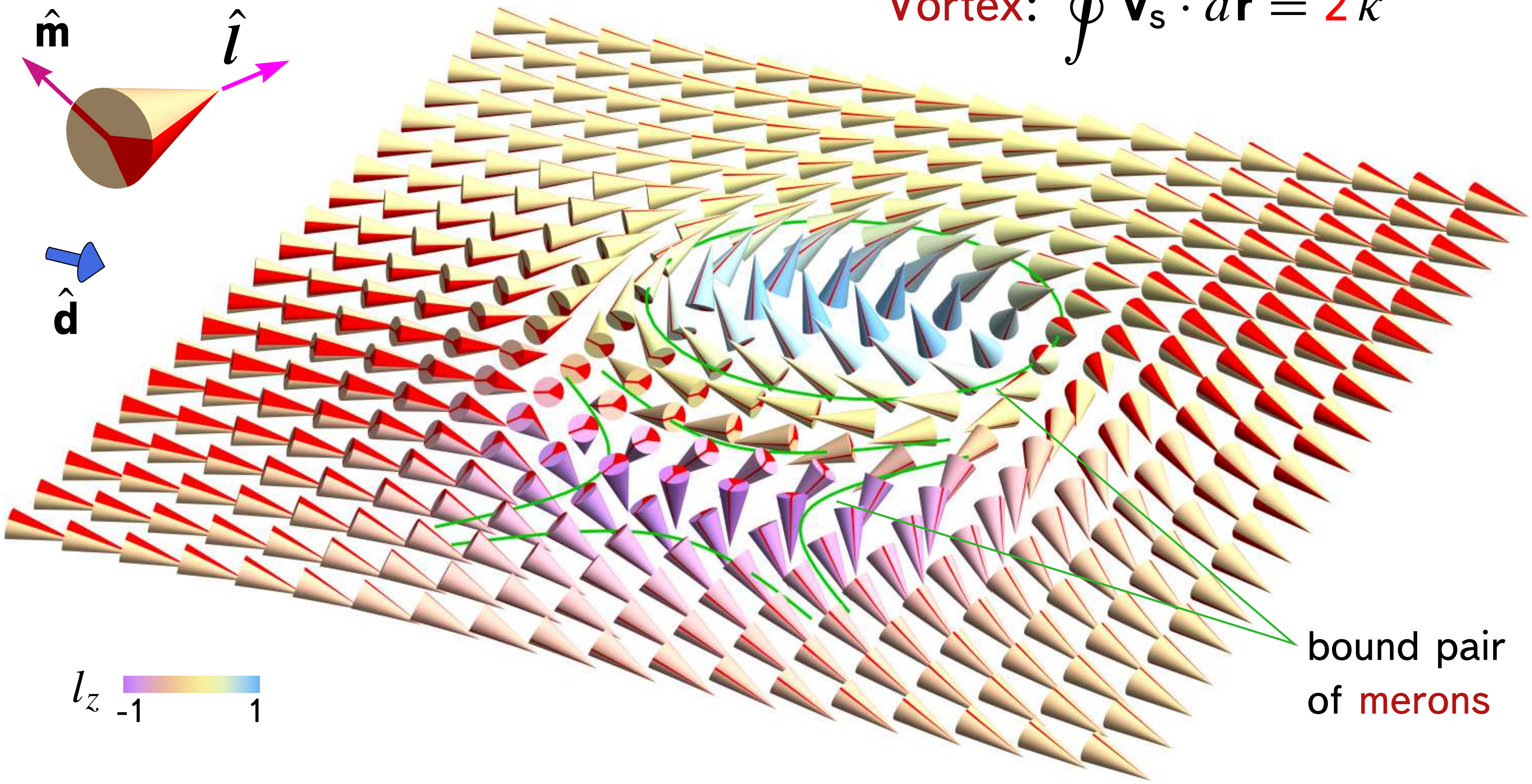
If $\hat{l}$ is in plane then $\nabla \times \mathbf{v}_s = 0$ and circulation is quantized.

Zeeman energy $F_H \propto (\hat{\mathbf{d}} \cdot \mathbf{H})^2$ Length scale $\xi_D \sim 10^3\, \xi \sim 10\, \mu\text{m}$

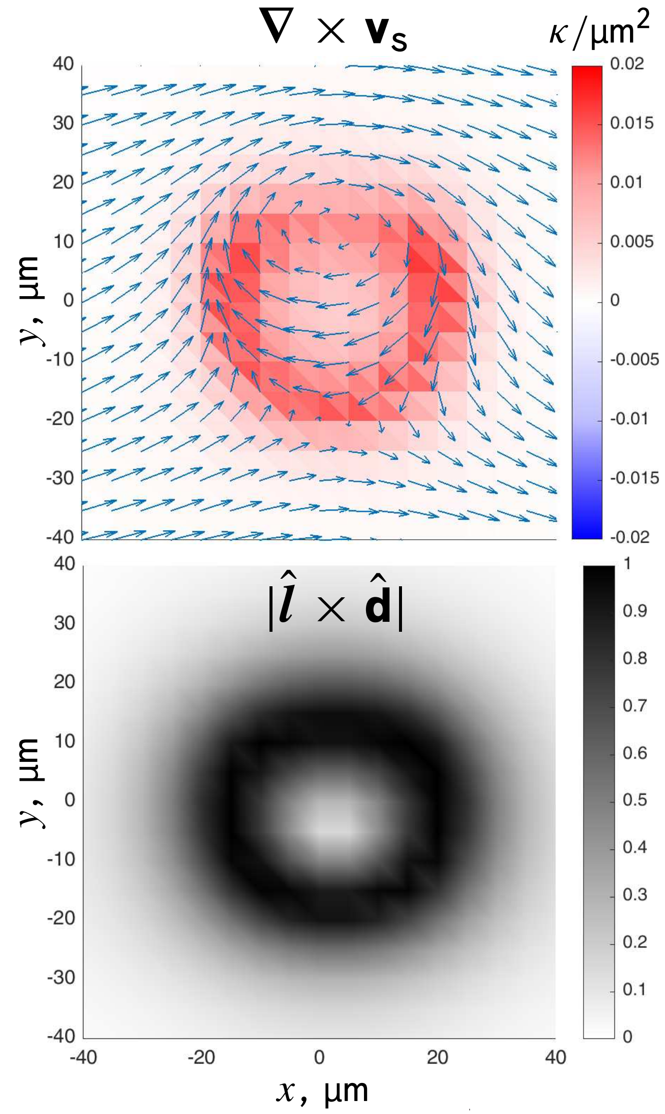
Spin-orbit interaction $F_D = -g_D\, (\hat{\mathbf{d}} \cdot \hat{l})^2 \sim 10^{-3}\, \Delta$

# DOUBLE QUANTUM VORTEX SKYRMION

Vortex: $\oint \mathbf{v}_s \cdot d\mathbf{r} = 2\,\kappa$

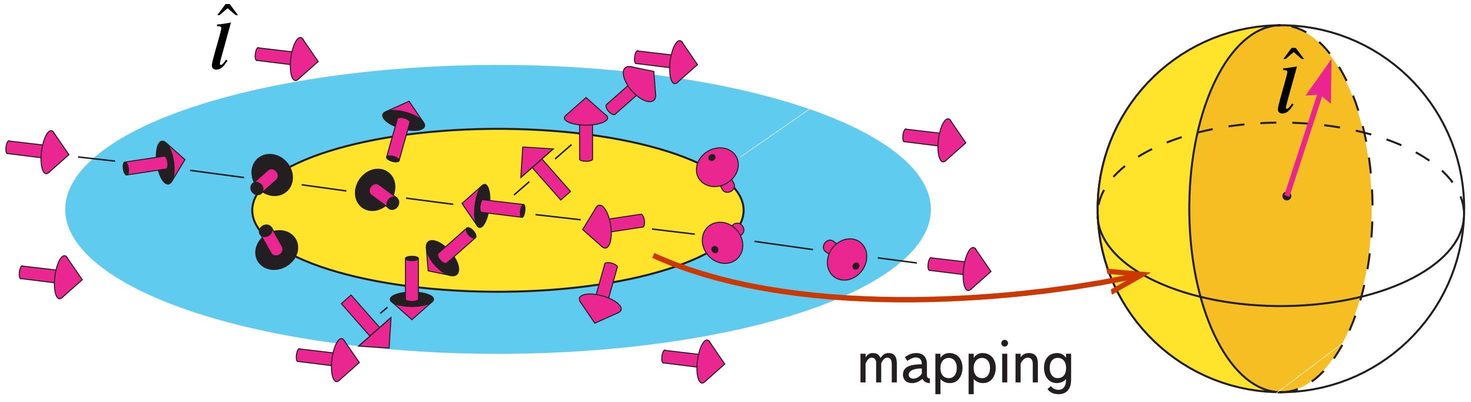


bound pair of merons



Skyrmion: Topological invariant

$$m_l = \frac{1}{8\pi}\epsilon_{ijk}\int d\mathbf{S}_i\,\hat{\boldsymbol{l}}\cdot(\nabla_j\hat{\boldsymbol{l}} \times \nabla_k\hat{\boldsymbol{l}}) = 1$$



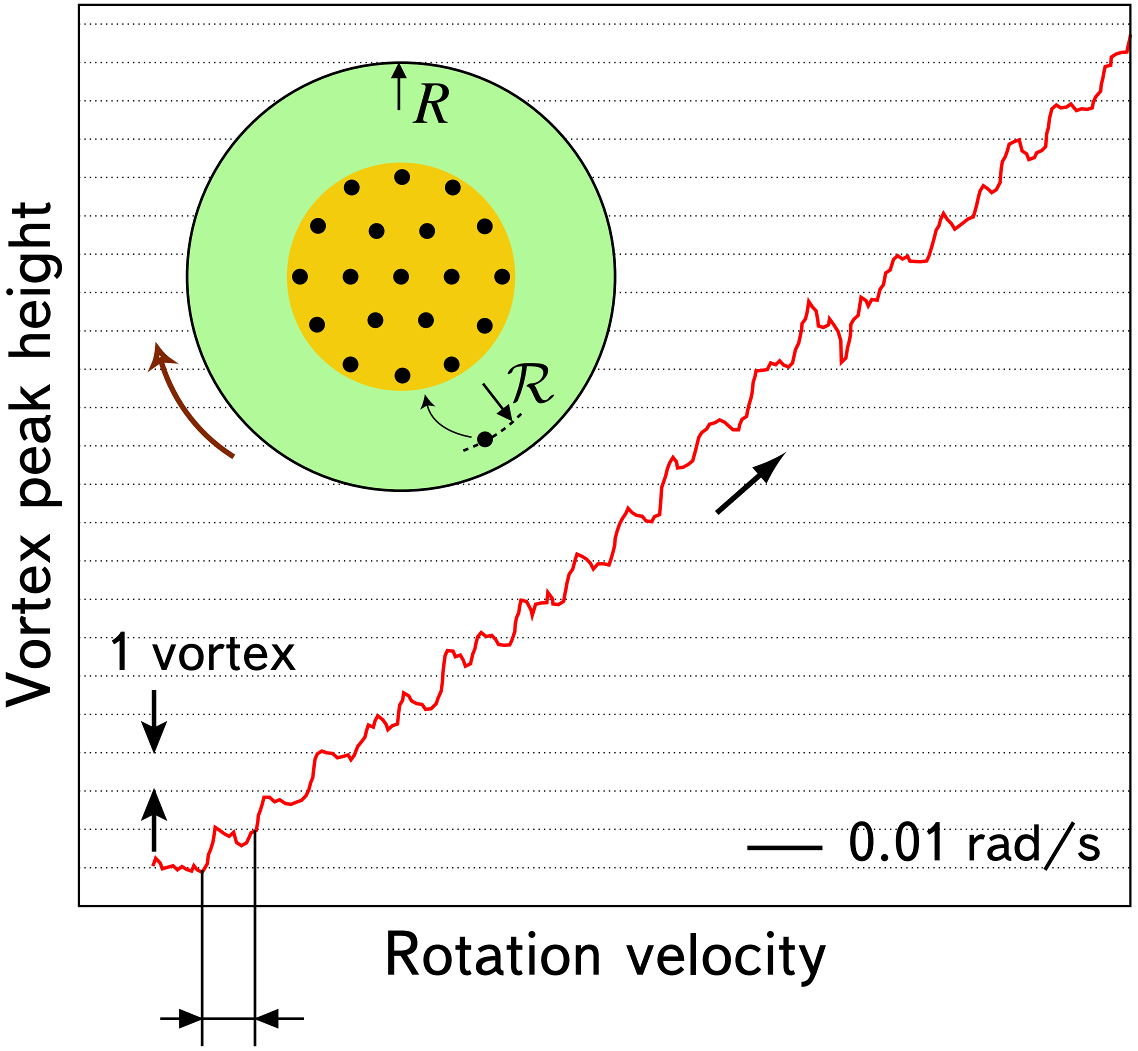
# DOUBLE-QUANTUM VORTEX IN EXPERIMENTS

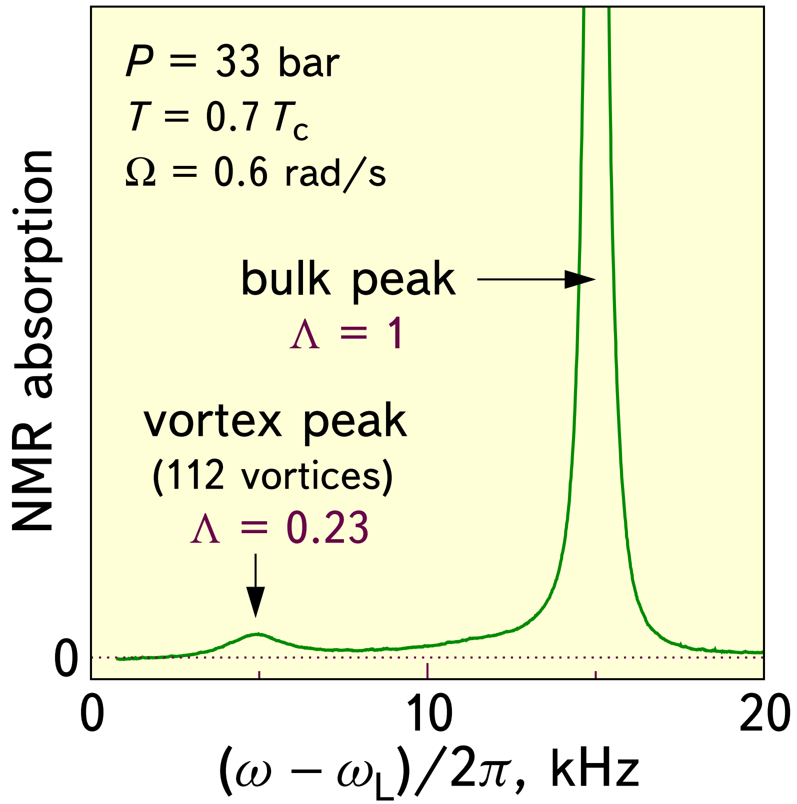
Satellite peak in the NMR spectrum with characteristic frequency shift.

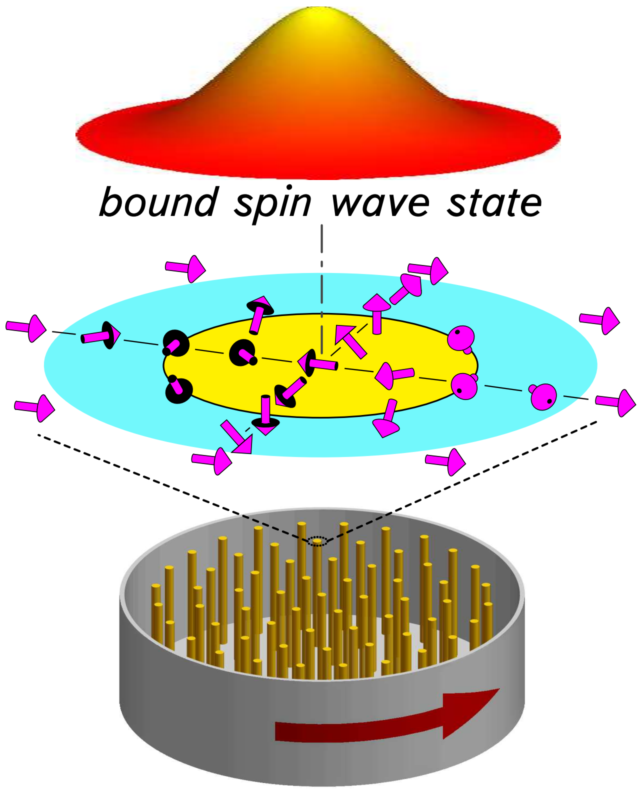
$$\omega^2 = \omega_\mathrm{L}^2 + \Lambda\,\Omega_\mathrm{A}^2$$

$\omega_\mathrm{L} = \gamma H$

$\Omega_\mathrm{A}$ – Leggett frequency



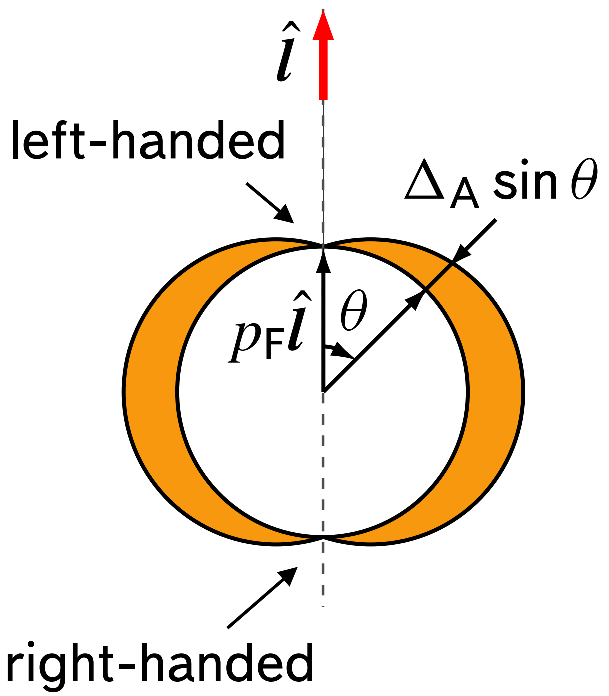




$$\Delta\Omega = \frac{n\kappa}{2\pi\mathcal{R}^2} = 6.24 \cdot 10^{-3}\,\mathrm{rad/s} \Rightarrow n = 2,\ R - \mathcal{R} \approx 0.1\,\mathrm{mm}$$

Blaauwgeers *et al*, Nature **404**, 471 (2000)

# WEYL FERMIONS AND CHIRAL ANOMALY



Chiral Weyl fermions in $^3$He-A in synthetic gauge field:

like vector potential

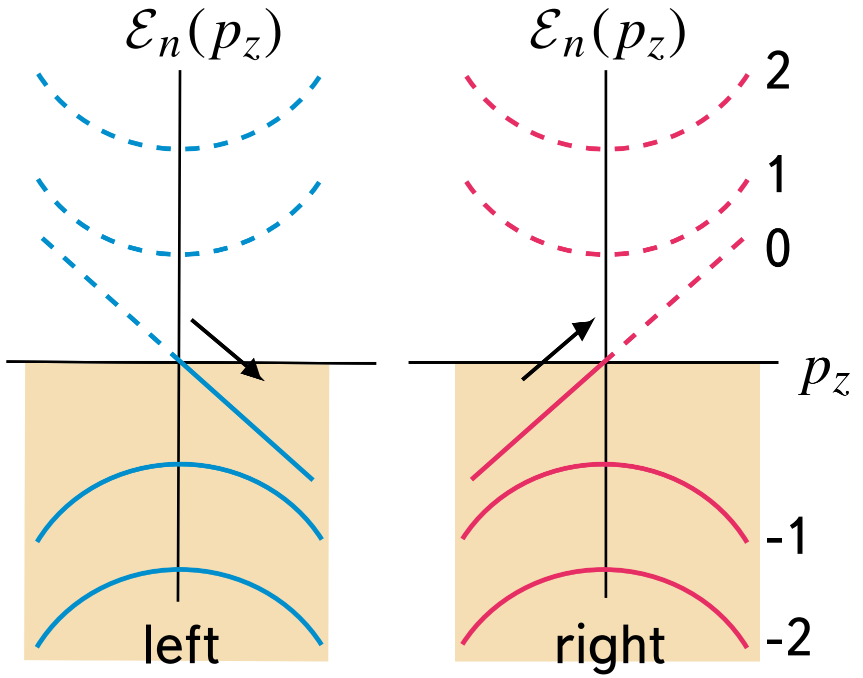
$$\mathcal{H} = \pm \sigma^a e_a^j (\mathbf{p} \pm p_F \hat{\boldsymbol{l}})_j$$

Synthetic fields: $\mathbf{B} = k_F \nabla \times \hat{\boldsymbol{l}}$ and $\mathbf{E} = k_F \partial_t \hat{\boldsymbol{l}}$.

## Chiral anomaly:

$\mathbf{B} \Rightarrow$ Landau levels $\propto n + \frac{1}{2} - \frac{1}{2} + \varepsilon(p_z) \Rightarrow$ level, crossing zero.

orbital spin



$\mathbf{E} \parallel \mathbf{B} \Rightarrow$ Spectral flow $\Rightarrow \frac{dn_{\text{chiral}}}{dt} = -\frac{1}{4\pi^2}\mathbf{B}\cdot\mathbf{E}$

Adler-Bell-Jackiw equation for anomalous creation of chiral charge from quantum vacuum.

$$n_{\text{chiral}} = n_{\text{right}} - n_{\text{left}}$$

carry $-p_F\hat{\boldsymbol{l}}$ carry $+p_F\hat{\boldsymbol{l}}$

In $^3$He-A: transfer of momentum from vacuum to excitations.

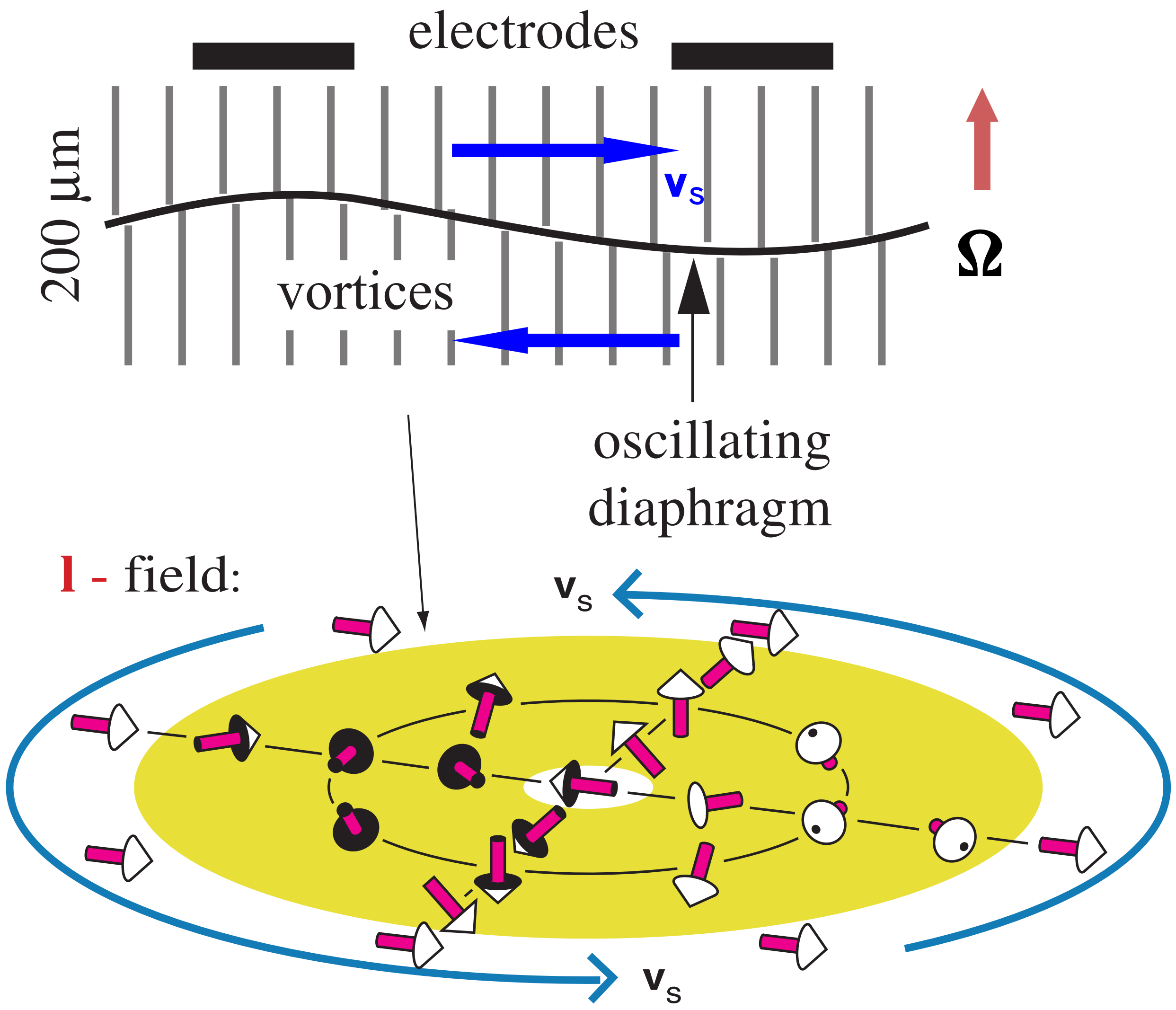
# OBSERVATION OF CHIRAL ANOMALY IN $^3$He-A

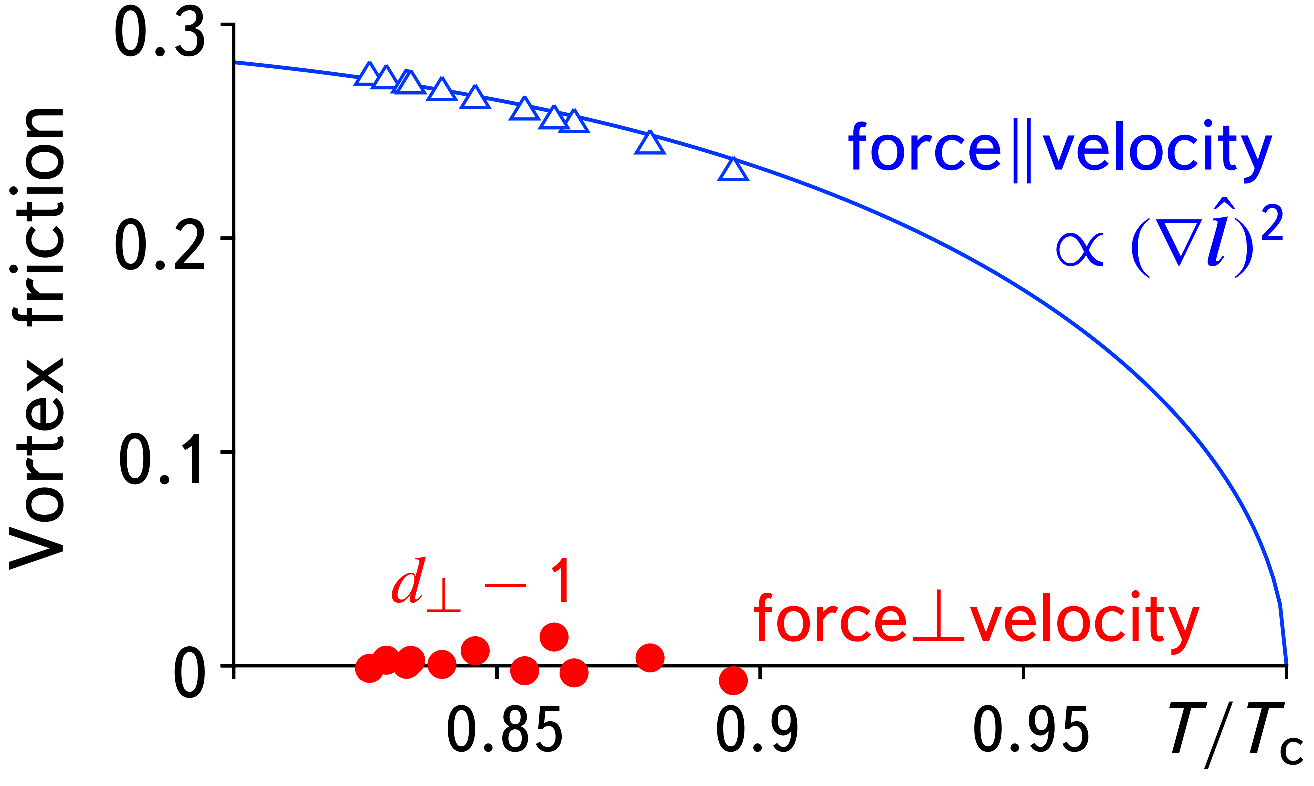
Spectral flow contribution to the transverse force acting on a moving vortex (mutual friction $\mathbf{F}_n$)

$$\frac{d\mathbf{P}}{dt} = -2p_F\hat{\boldsymbol{l}}\,\frac{dn_{\rm chiral}}{dt} = \frac{1}{2\pi^2}p_F\hat{\boldsymbol{l}}\,(\underset{\underset{k_F\partial_t\hat{\boldsymbol{l}}}{\nearrow}}{\mathbf{E}}\cdot\underset{\underset{k_F\nabla\times\hat{\boldsymbol{l}}}{\nwarrow}}{\mathbf{B}}) \propto \frac{p_F^3}{\hbar^2} \quad \Rightarrow \quad \text{full density } \rho$$

$$\mathbf{F}_{n\perp} = \kappa\rho_s\, d_\perp\, \hat{\mathbf{z}} \times (\mathbf{v}_n - \mathbf{v}_{\rm vortex})$$

$$d_\perp = \frac{\overset{\text{chiral anomaly}\ \searrow}{\rho} - \overset{\swarrow\ \text{Iordanski}}{\rho_n}}{\rho_s \longleftarrow \text{Magnus}} = 1$$





Bevan *et al*, Nature **386**, 689 (1997)

# ZERO-CHARGE EFFECT IN $^3$He-A

Synthetic electromagnetic field: $\mathbf{B} = k_{\mathrm{F}} \nabla \times \hat{\boldsymbol{l}}$ and $\mathbf{E} = k_{\mathrm{F}} \partial_t \hat{\boldsymbol{l}}$.

↑ in a vortex

↑ vortex moves

In $^3$He-A synthetic fields possess QED features.

Volovik, JETP Lett **47**, 55 (1988)

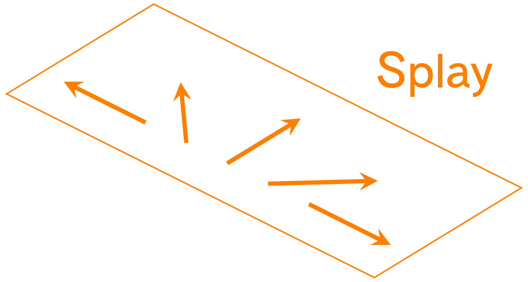
Running coupling constant: Action $S \propto \ln \left[ \dfrac{E^2_{\mathrm{Plank}}}{\max(T^2, \omega^2, \underbrace{(\hbar c/r)^2}, B, M^2)} \right]$

$$e^2 \sim \frac{e_0^2}{\ln(r p_{\mathrm{Plank}}/\hbar)} \to 0$$

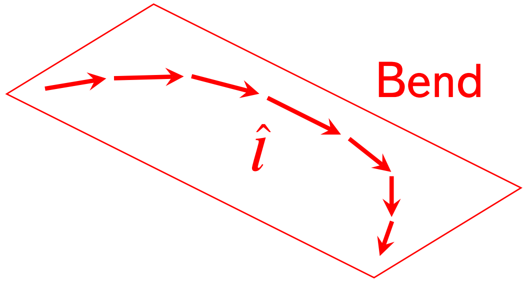
In $^3$He-A $\quad S \propto \ln(\Delta_{\mathrm{A}}^2 / T^2)$

Free energy = Magnetic + Spin-orbit + Kinetic + Gradient $\hat{\mathbf{d}}$

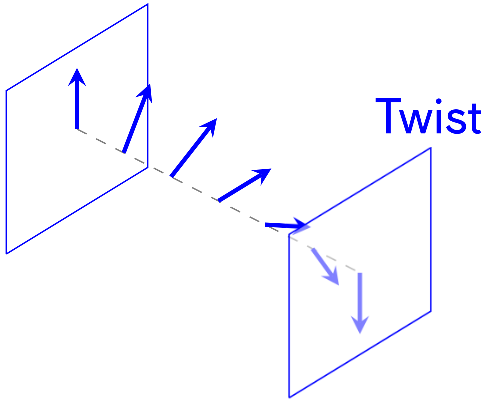
$+ K_{\mathrm{s}}$



$+ K_{\mathrm{b}}$



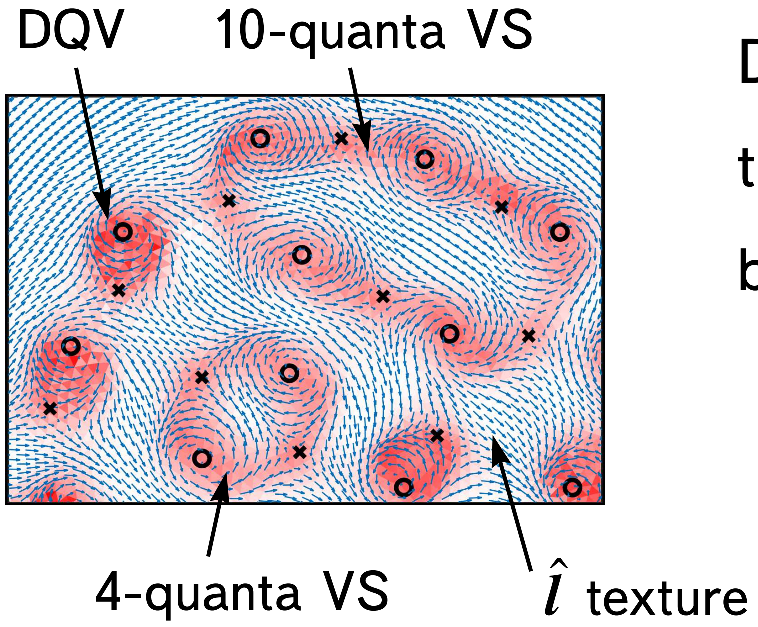
$+ K_{\mathrm{t}}$



$$K_{\mathrm{b}} = K_{\mathrm{b0}} + K_{\mathrm{b1}} \ln(\Delta_{\mathrm{A}}/T)$$

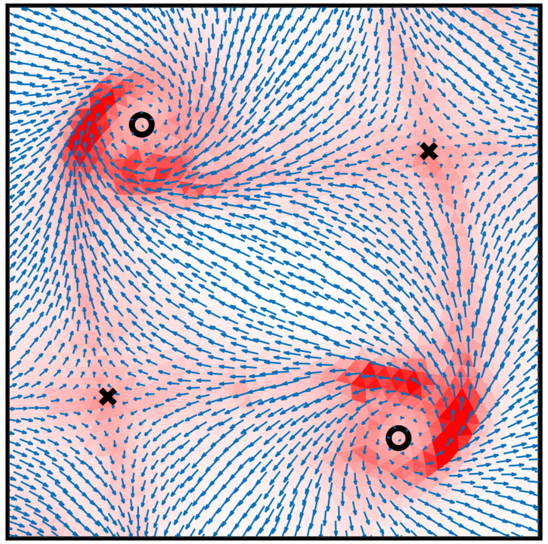
# "ZERO-CHARGE" TRANSITION IN THE VORTEX SHEET STRUCTURE

In $^3$He-A, double-quantum vortex (DQV) skyrmions can merge to vortex sheet (VS).

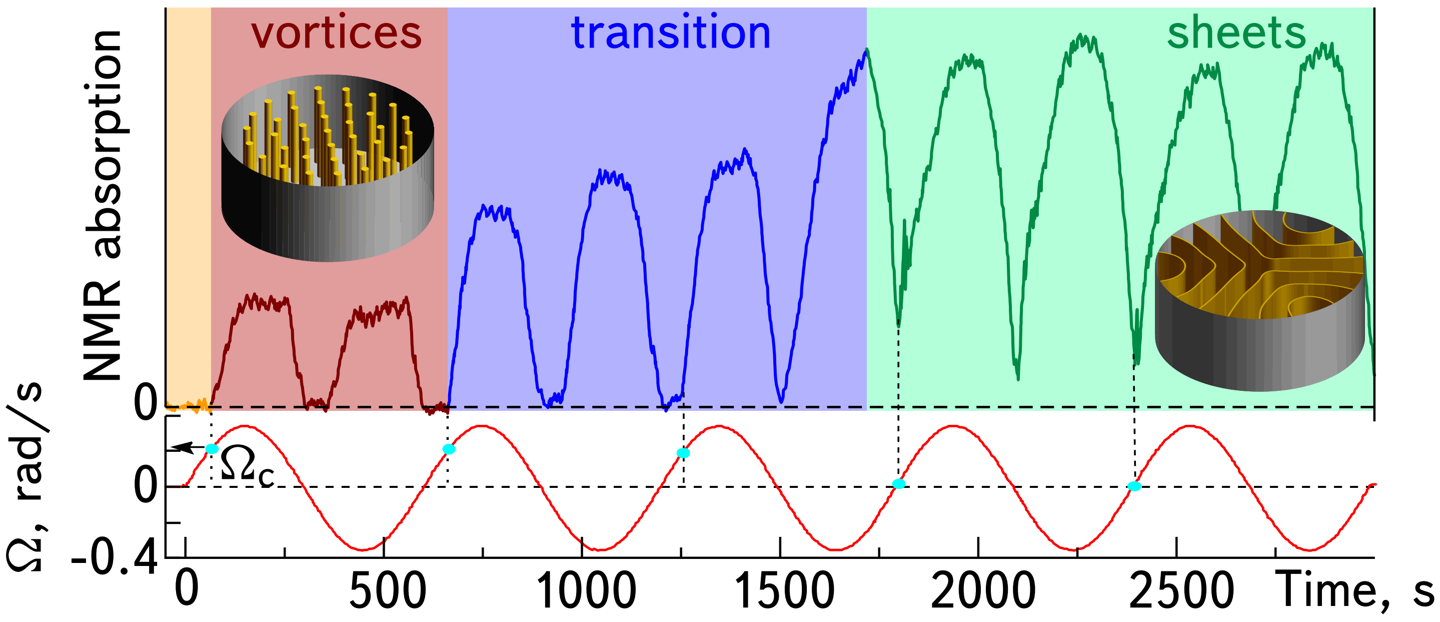


Diverging $K_b$ when $T \to 0$ causes transition to tube-like vorticity to avoid bending deformations.

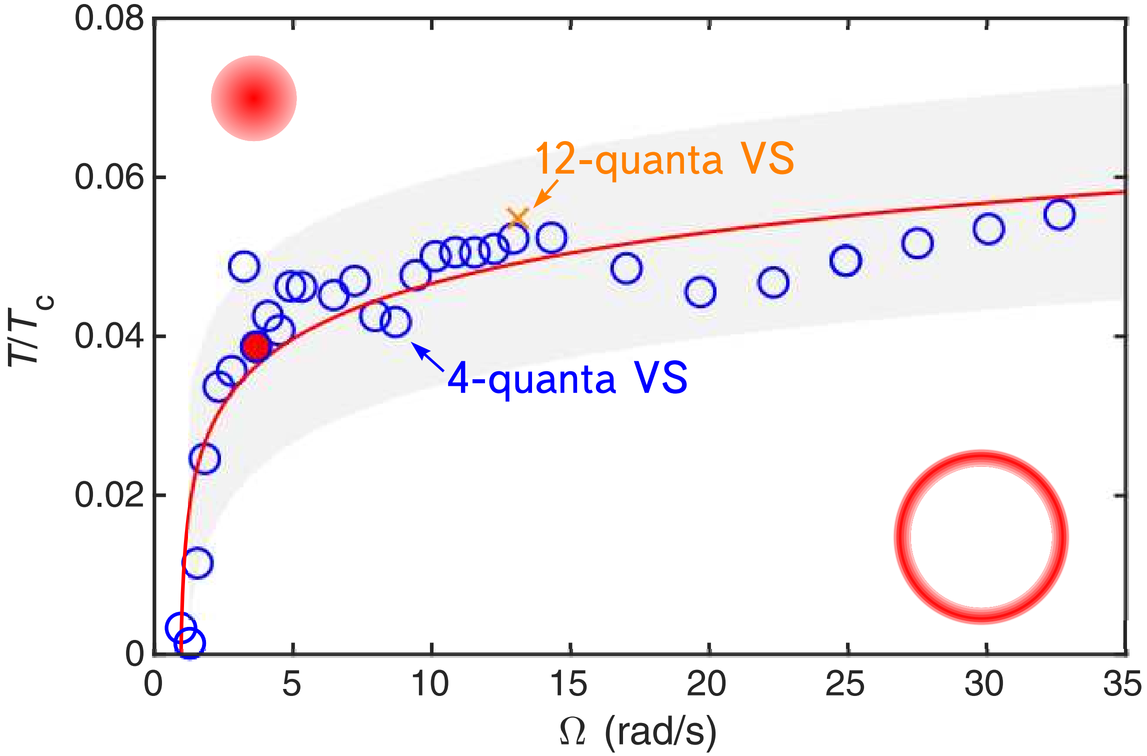
Rantanen & VE, PRB **107**, 104505 (2023)



4-quanta VS at ultra-low $T$

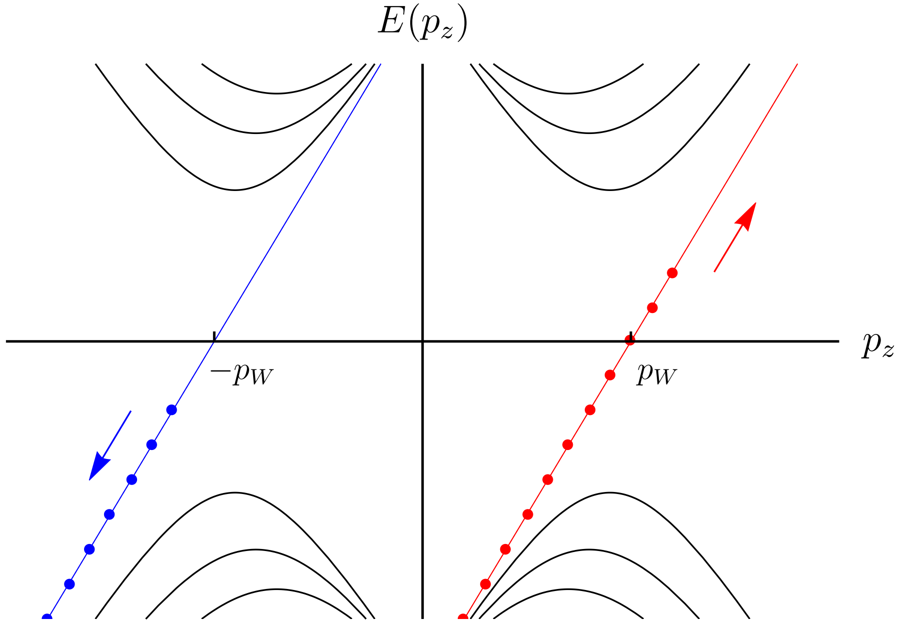


VE *et al*, PRL **88**, 065301 (2002)



# THERMAL NIEH-YAN ANOMALY IN WEYL SUPERFLUID

Weyl fermions in a gravitational field with torsion experience Nieh-Yan anomaly similar to chiral anomaly in a gauge field due to spectral flow over the anomalous Landau level.

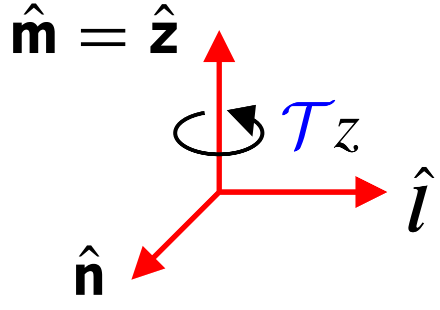


But the effective charge increases with momentum and overall result becomes dependent on the cutoff $\Lambda$:

$$\partial_\mu (e j_5^\mu) = \frac{\Lambda^2}{4\pi^2} \epsilon^{\mu\nu\lambda\rho} \left( \frac{1}{4} T^a_{\mu\nu} T_{a\lambda\rho} - \frac{1}{2} e^a_\mu e^b_\nu R_{ab\lambda\rho} \right)$$

$j_5^\mu = j^\mu_{\text{right}} - j^\mu_{\text{left}}$, $e = \det e^\mu_a$, $T^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu}$, $\Gamma^\lambda_{\mu\nu}$ is the coordinate connection

In $^3$He-A, torsion is produced by the twist texture of $\hat{l}$, while the cutoff is given by temperature $T$. As before, $j_5$ is the momentum trasferred.



$$\mathbf{P}_{\text{anom}}(T) = -\left[ \frac{p_F^3}{6\pi^2} - \boxed{\frac{p_F T^2}{6 c_\perp^2}} - \frac{p_F T^2}{12 c_\perp^2} \left( \frac{m^*}{m} - 1 \right) \right] \hat{l} \, (\hat{l} \cdot (\nabla \times \hat{l}))$$

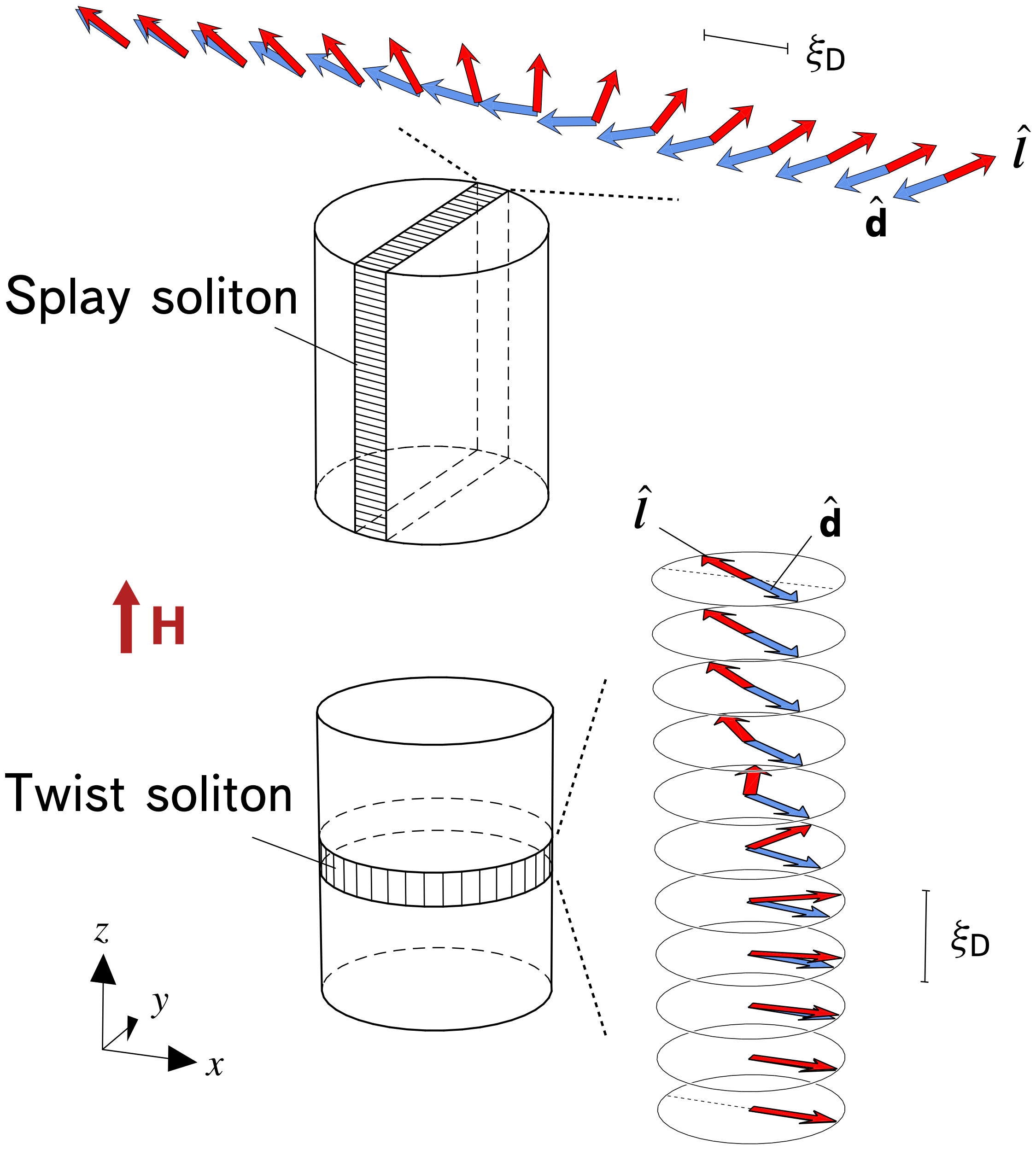
Nieh-Yan anomaly contribution

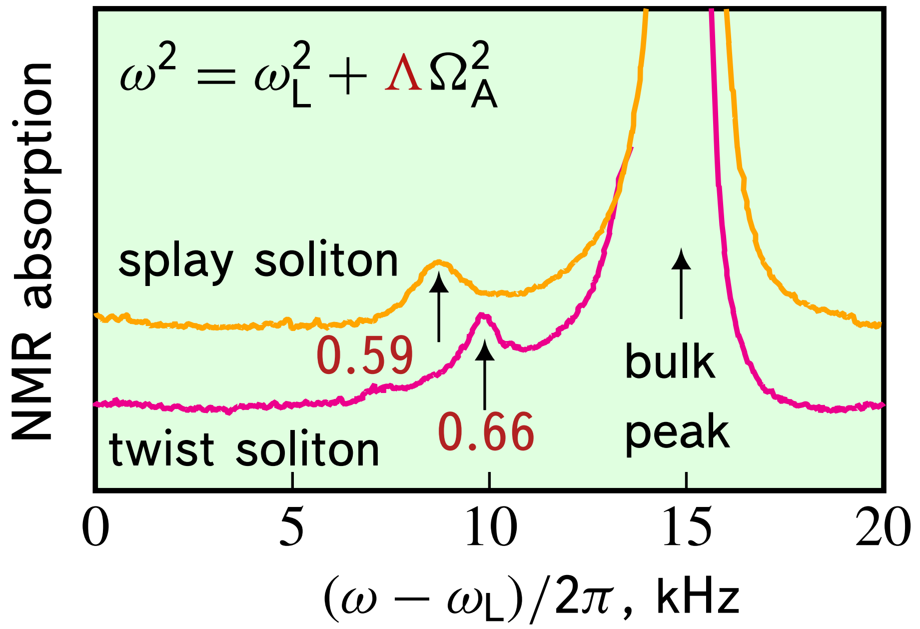
Nissinen, PRL **124**, 117002 (2020); Nissinen & Volovik, PRRes **2**, 033269 (2020); Laurila & Nissinen, PRB **102**, 235163 (2020)

# TOPOLOGICAL SOLITONS IN THE A PHASE

Protected by $Z_2$ symmetry:

$\hat{\mathbf{d}} \upuparrows \hat{l}$ and $\hat{\mathbf{d}} \uparrow\downarrow \hat{l}$





Ruutu *et al*,
JLTP **103**, 331 (1996)

Nieh-Yan anomaly changes the twist soliton structure and the NMR response contributing

$$F_{NY} = \frac{p_F m^*}{96 m^2} \frac{T^2}{c_\perp^2} (\hat{l} \cdot (\nabla \times \hat{l}))^2$$

to the free energy (at $\mathbf{v}_s = 0$).

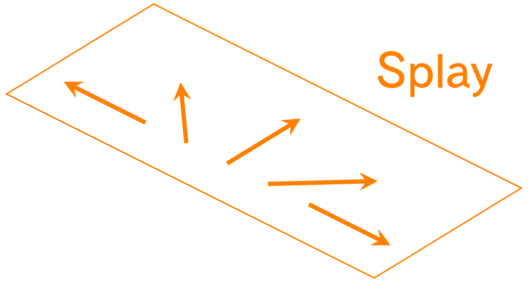
Nissinen & Volovik, PRRes **2**, 033269 (2020)

# TWIST CONTRIBUTION TO THE FREE ENERGY

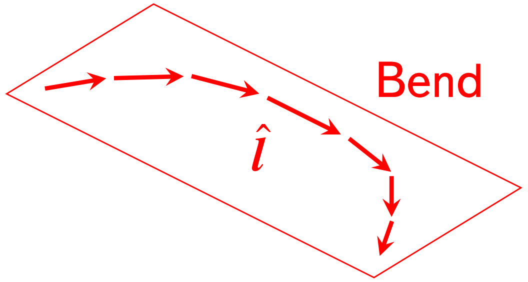
Free energy of $^3$He-A = Magnetic $(\Delta\chi)$ + Spin-orbit $(g_\mathrm{d})$ + Kinetic $(\rho_{\mathrm{s}\parallel}, \rho_{\mathrm{s}\perp}, C, C_0)$

+ Gradient $\hat{\mathbf{d}}$ $(K_5, K_6)$

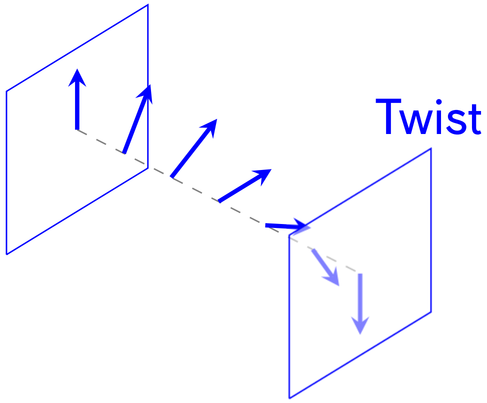
$+ K_\mathrm{s}$



$+ K_\mathrm{b}$

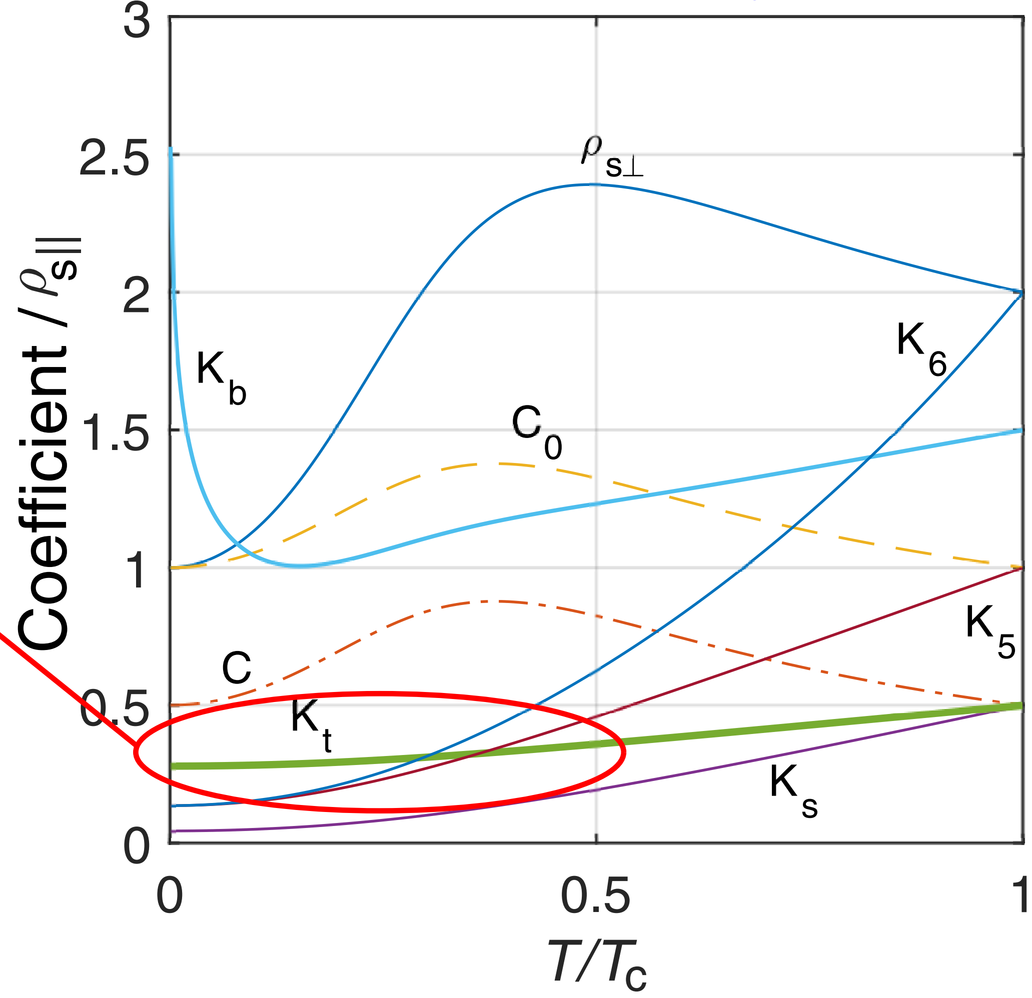


$+ K_\mathrm{t}$



This quadratic rise of $K_\mathrm{t}$ results from the Nieh-Yan gravitational anomaly.

How to measure it independently of other (not well-known) coefficients?



Cross, JLTP **21**, 525 (1975); Fetter, PRB **20**, 303 (1979)

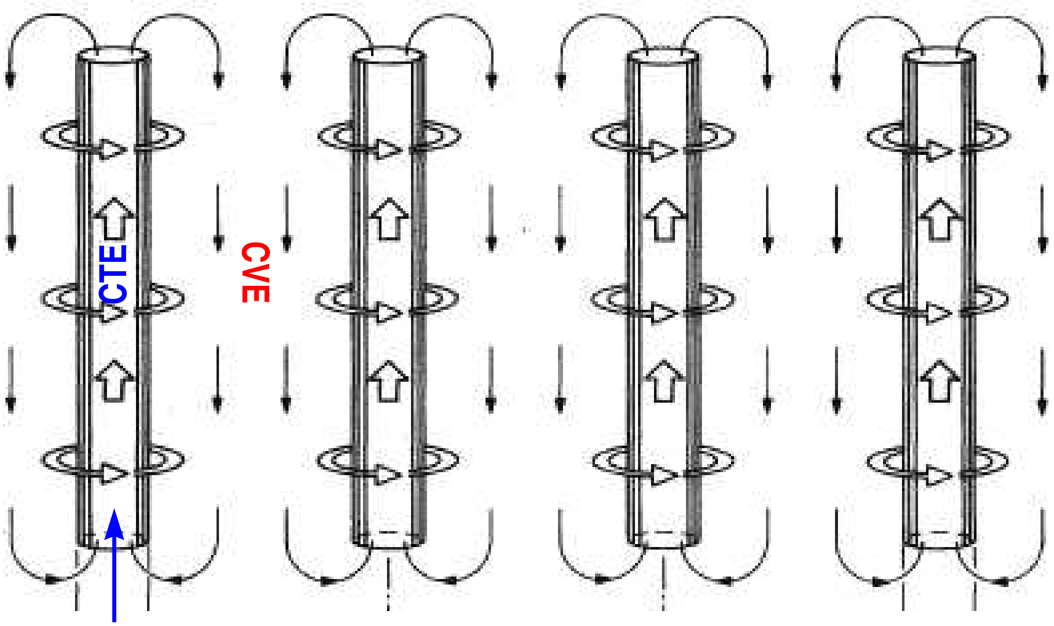
# CANCELLATION OF CHIRAL VORTICAL AND CHIRAL TORSIONAL EFFECTS

Chiral vortical effect — rotation $\mathbf{\Omega}$ polarizes spin of Weyl fermions and directs their momenta along/opposite to rotation for right/left particles.

$$\mathbf{J}_5^{\mathrm{CVE}} = -\frac{T^2}{12c_\perp^2} 2\mathbf{\Omega}$$

for two nodes

Volovik & Vilenkin, PRD **62**, 025014 (2000)



$\nabla \times \mathbf{v}_\mathrm{s}$ concentrated in vortex core

Chiral torsional effect — superflow $\mathbf{v}_\mathrm{s}$ generates tetrad gravity with torsion

$$e_a^\mu = \begin{pmatrix} 1 & -\mathbf{v}_\mathrm{s} \\ 0 & c_\perp \hat{\mathbf{m}} \\ 0 & c_\perp \hat{\mathbf{n}} \\ 0 & c_\parallel \hat{l} \end{pmatrix}$$

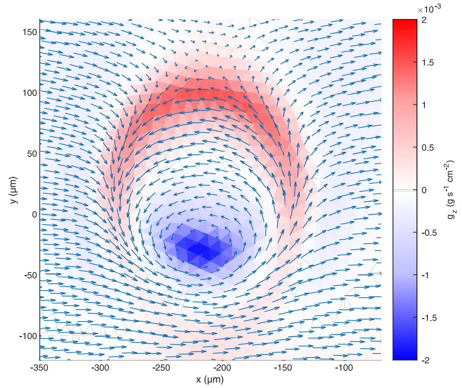
$$\mathbf{J}_5^{\mathrm{CTE}} = \frac{T^2}{12c_\perp^2} (\nabla \times \mathbf{v}_\mathrm{s})$$

$$\langle \mathbf{J}_5 \rangle = \frac{T^2}{12c_\perp^2} (\langle \nabla \times \mathbf{v}_\mathrm{s} \rangle - 2\mathbf{\Omega}) = 0$$

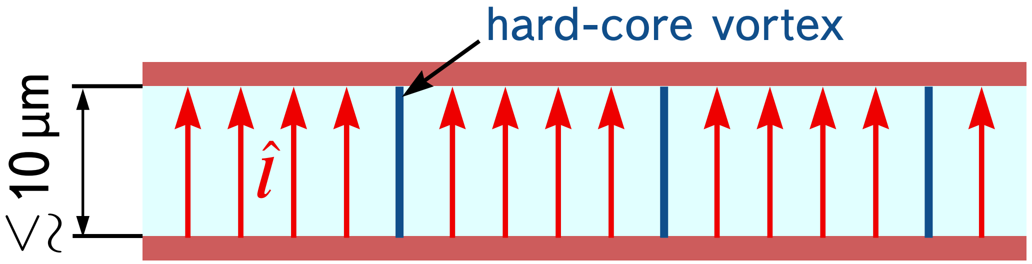
in rotating equilibrium

Nissinen & Volovik, PRD **106**, 045022 (2022)

When $^3$He-A is rotated: DQVs with large axial flow due to $\hat{l}$ texture.

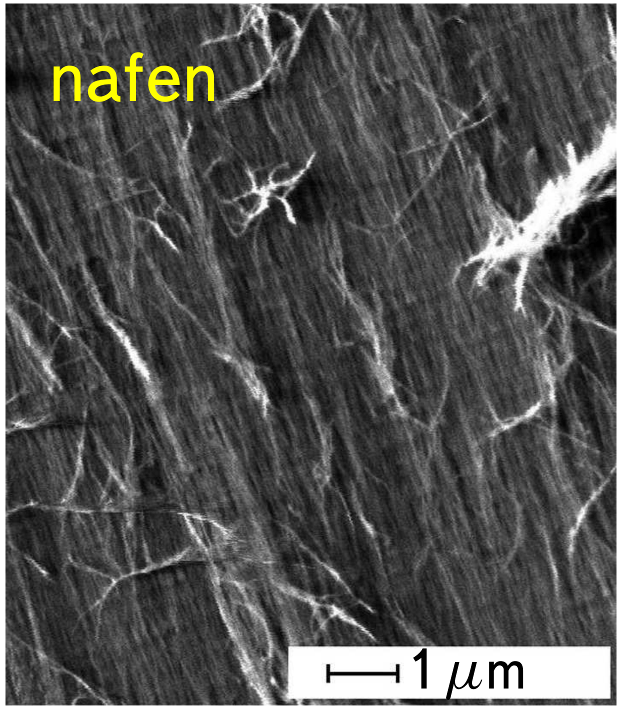


Slab is needed:



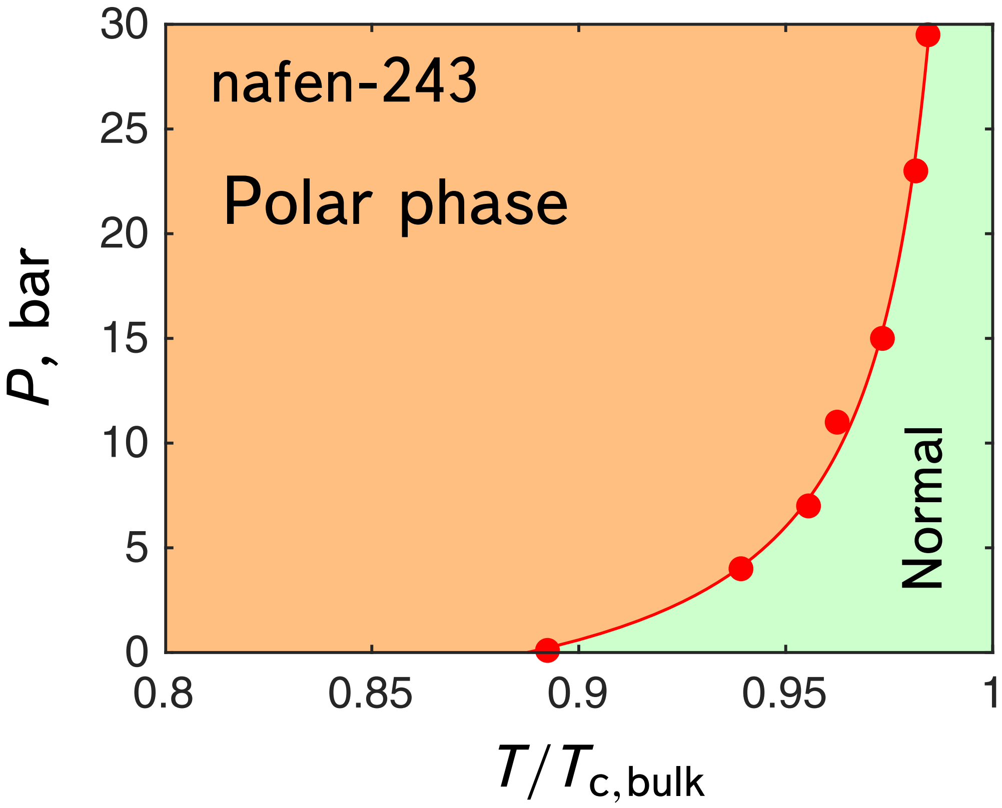
How to measure $\mathbf{J}_5$?

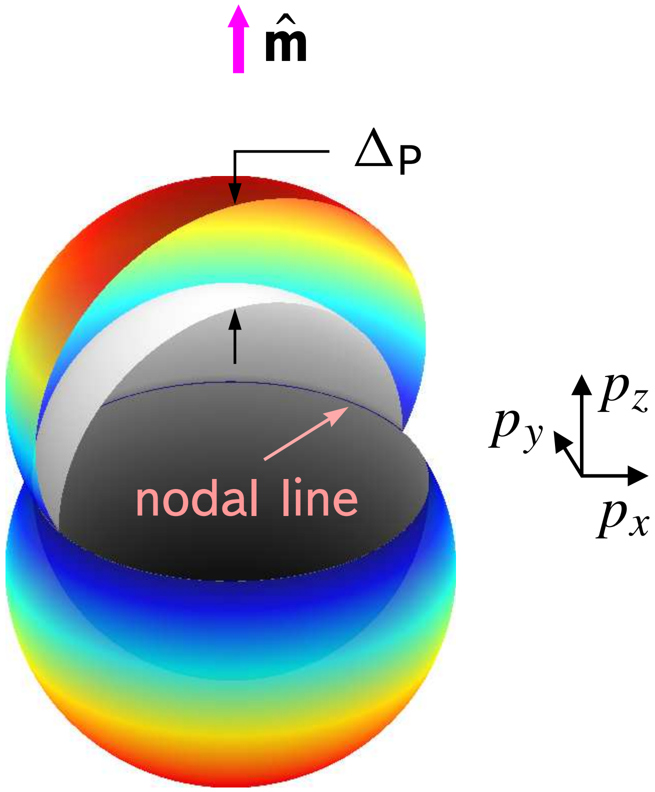
# ENGINEERING NEW PHASES OF SUPERFLUID $^3$He

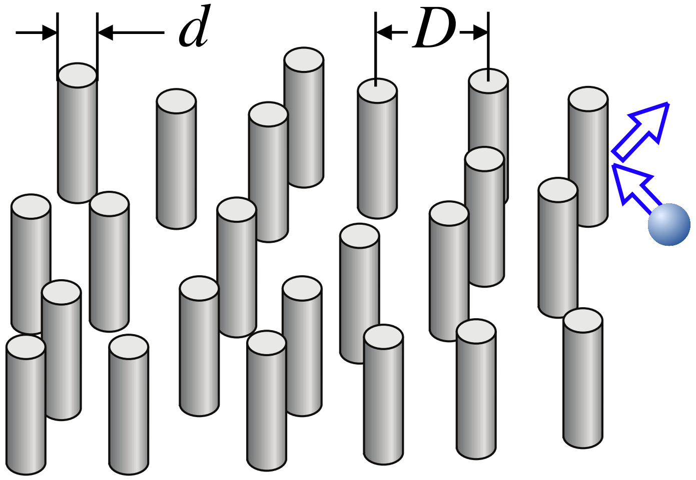


Polar phase stabilized with confinement between parallel nanostrands.

$$A_{\mu j} = \Delta_\mathrm{P}\, e^{i\phi}\, \hat{\mathbf{d}}_\mu\, \hat{\mathbf{m}}_j$$





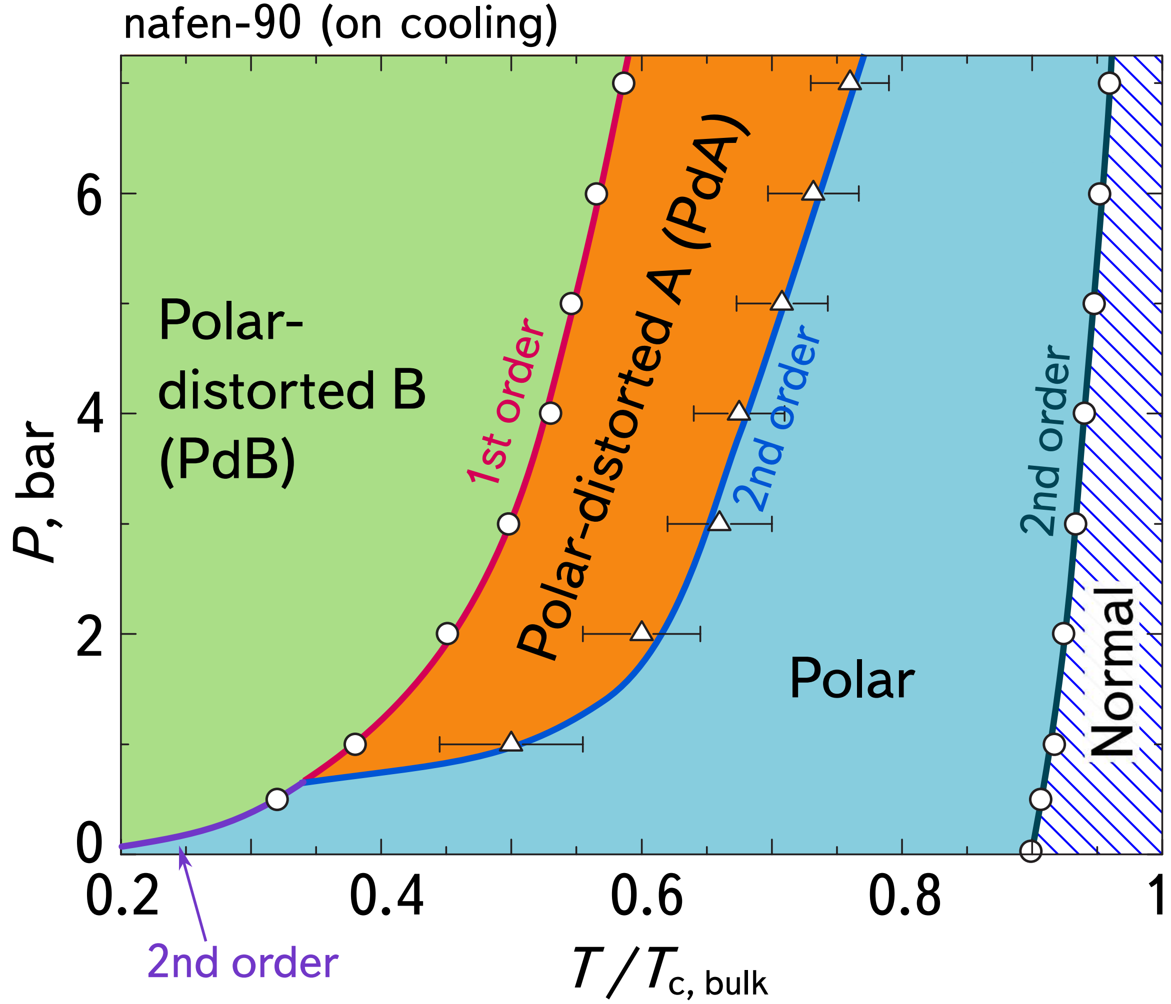


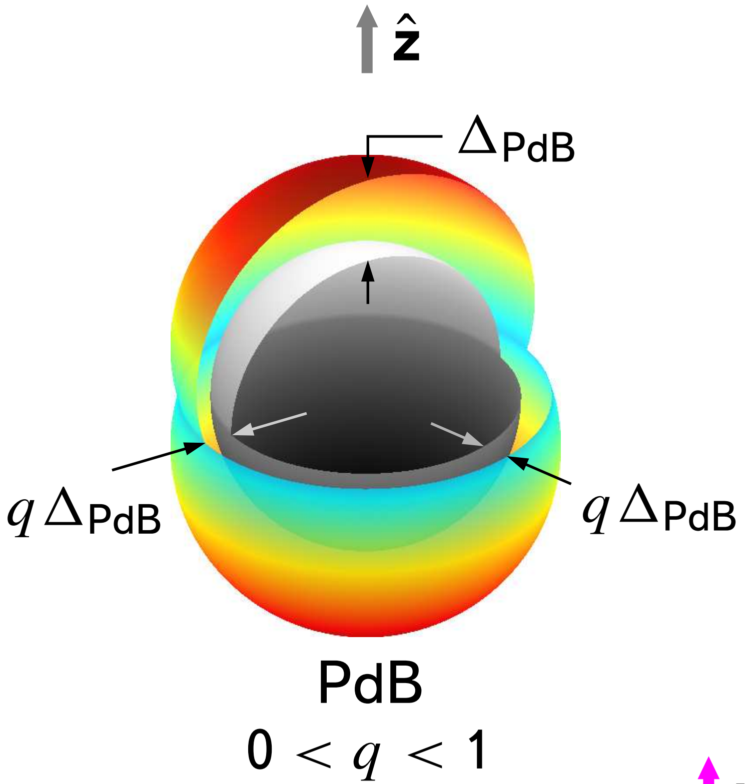
| | open | $d$, nm | $\langle D\rangle$, nm |
|---|---|---|---|
| nafen-90 | 98% | 8 | 47 |
| nafen-243 | 94% | 9 | 32 |

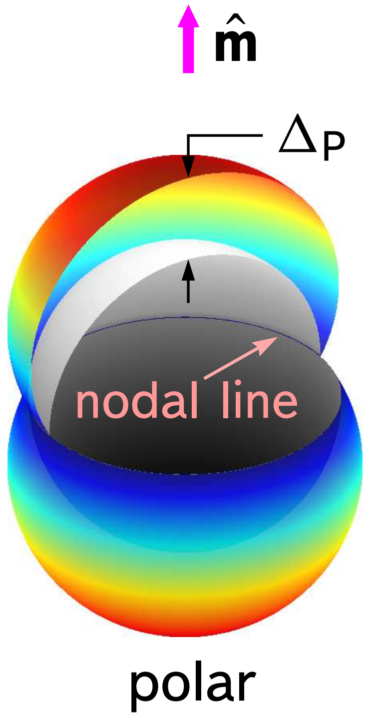
Scattering time $\tau < \hbar/\Delta$ but $T_{\mathrm{c,polar}} \approx T_{\mathrm{c,bulk}}$. Extension of the Anderson theorem applicable if $p_z$ is conserved.

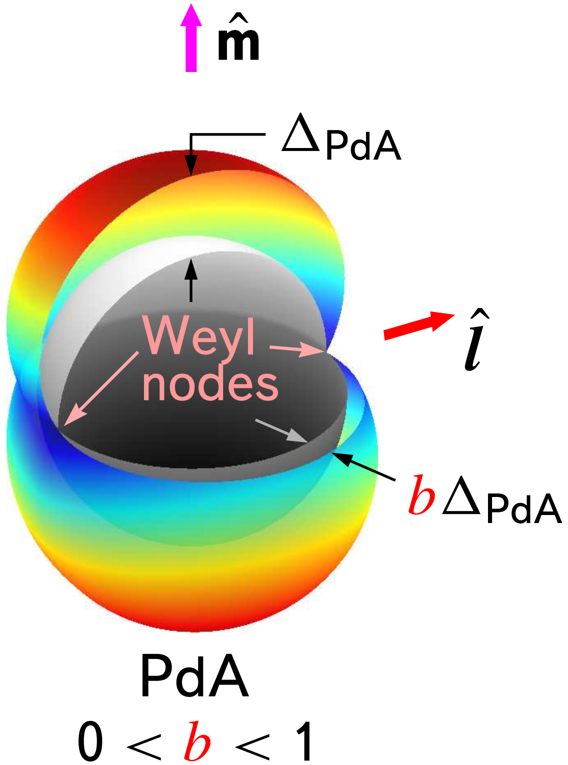
Aoyama & Ikeda, PRB **73**, 060504 (2006); Dmitriev *et al*, PRL **115**, 165304 (2015)
Fomin, JETP **127**, 933 (2018); Kamppinen *et al*, arXiv:1908.01645v4

# POLAR-DISTORTED A AND B PHASES









For Weyl fermions in the PdA phase, vierbein is

$$e^{\mu}_{a} = \begin{pmatrix} 1 & -\mathbf{v}_{\mathrm{s}} \\ 0 & c_{\perp 1}\hat{\mathbf{m}} \\ 0 & c_{\perp 2}\hat{\mathbf{n}} \\ 0 & c_{\parallel}\hat{l} \end{pmatrix}$$

$$c_{\perp 1} = \frac{\Delta_{\mathrm{PdA}}}{p_{\mathrm{F}}}, \quad c_{\parallel} = v_{\mathrm{F}}$$

$$c_{\perp 2} = b\, c_{\perp 1}$$

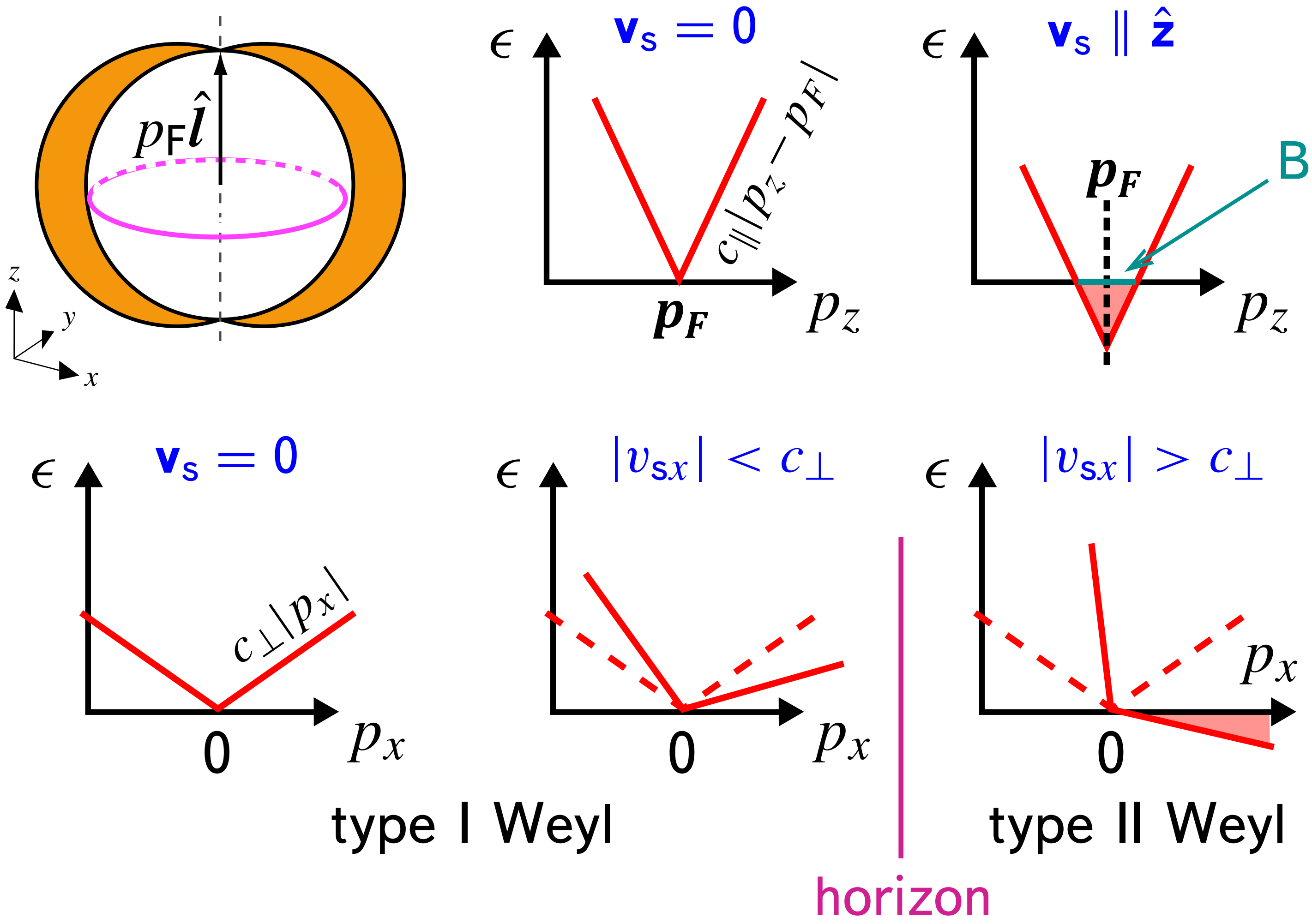
can be arbitrarily small

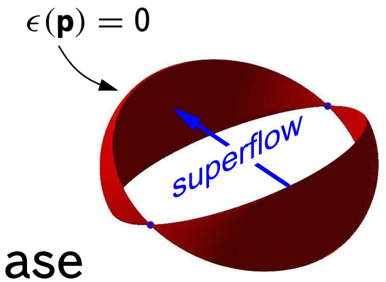
# SUPERFLOW IN NODAL SUPERFLUID

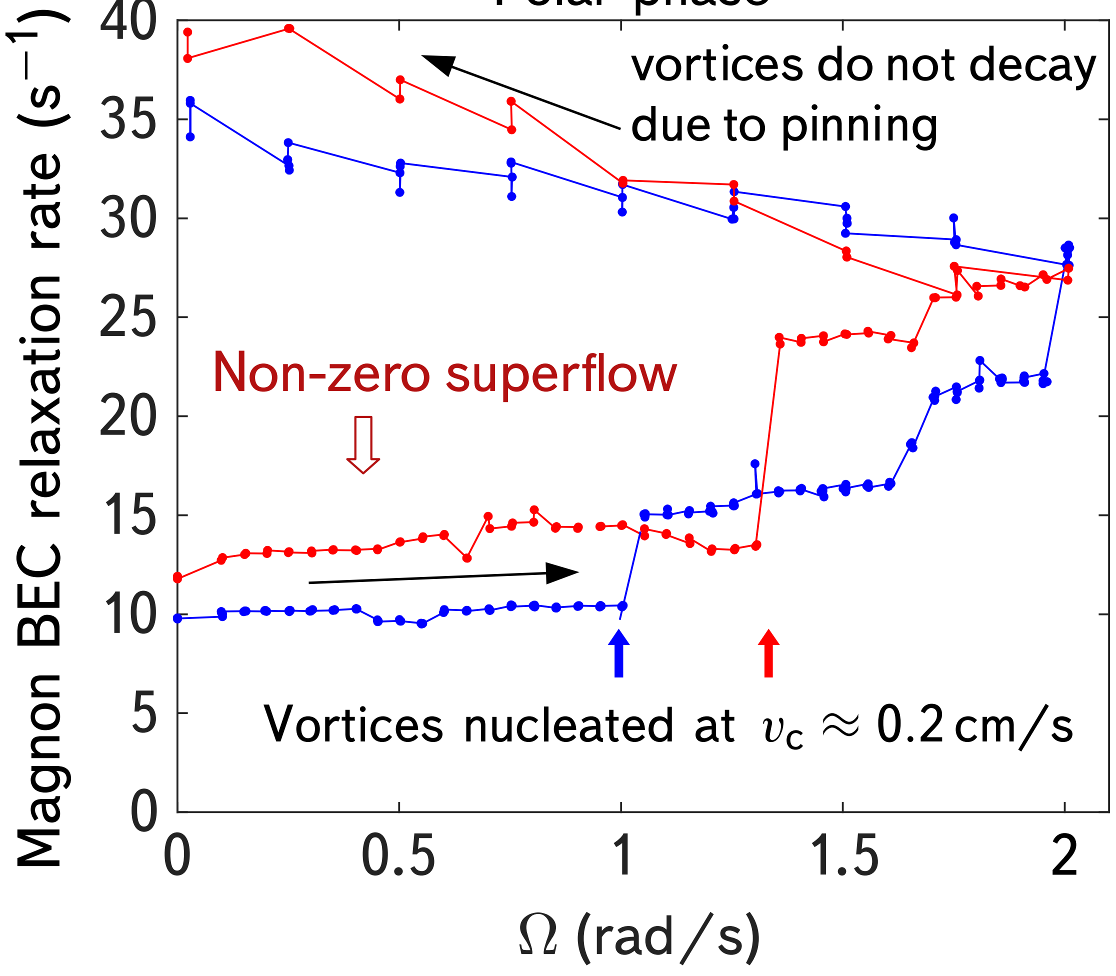
In a nodal superfluid Landau critical velocity $v_{\mathrm{cL}} = 0$ but superflow is stable.

$$\epsilon'_{\mathbf{p}} = \epsilon_{\mathbf{p}} + \mathbf{p}\mathbf{v}_{\mathrm{s}}$$

$$v_{\mathrm{cL}} = \min(\epsilon_{\mathbf{p}}/p)$$





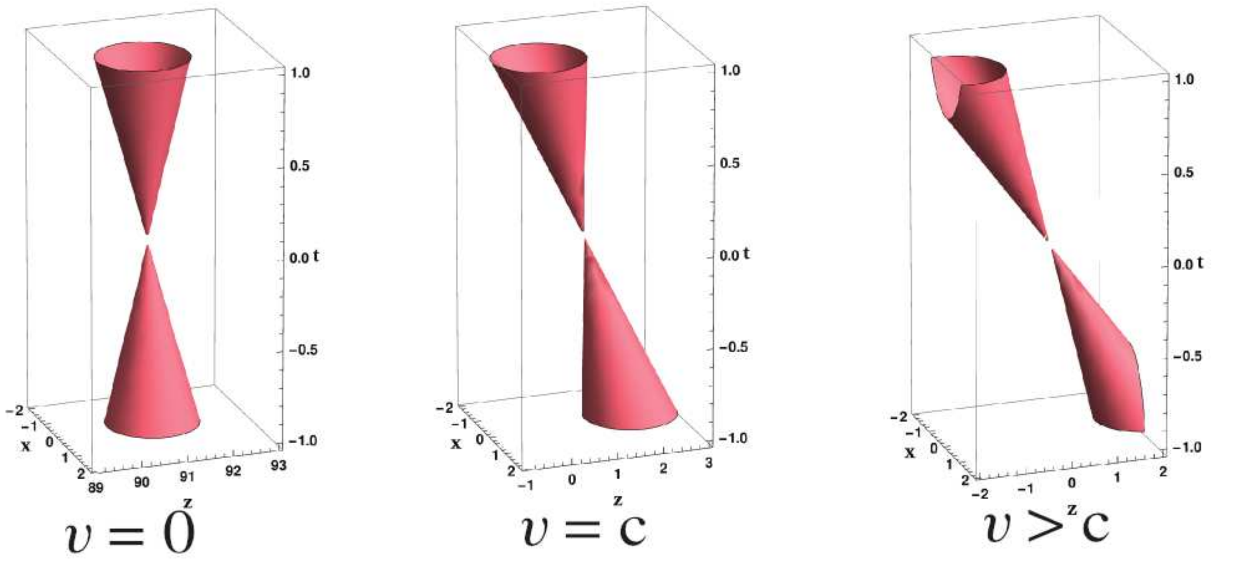


In nanoconfined phases with nodes (polar, PdA), sizeable superflow can be applied before vortex nucleation.

Autti, *et al*, Phys. Rev. Res. **2**, 033013 (2020)

# BLACK-HOLE HORIZON ANALOGUE WITH TYPE-I/II WEYL FERMIONS

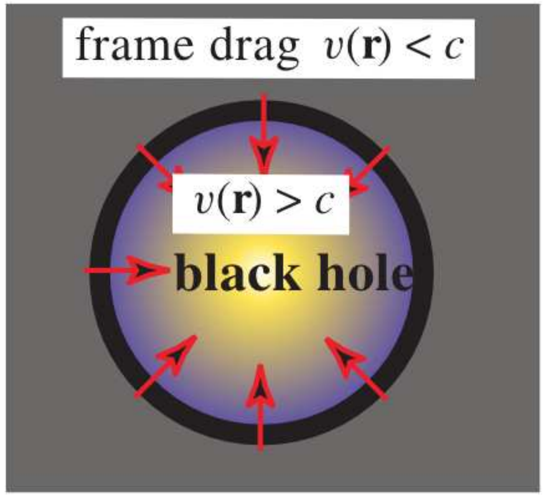
Light cone in a black hole with metric $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -c^2dt^2 + (d\mathbf{r} - \mathbf{v}dt)^2$



$v = 0$

$v = c$
event horizon

$v > c$
overtilted light cone



**Black hole:**

$\mathbf{v}(\mathbf{r}) = -c\,\hat{\mathbf{r}}\sqrt{\frac{r_h}{r}}$,

$c$ is speed of light.

**Horizon:** $v(\mathbf{r}) = c$.

**Superfluid:**

$\mathbf{v}(\mathbf{r})$ is applied flow,

$c \sim b\Delta/p_F$,

$b$ from confinement.

**Semimetal:**

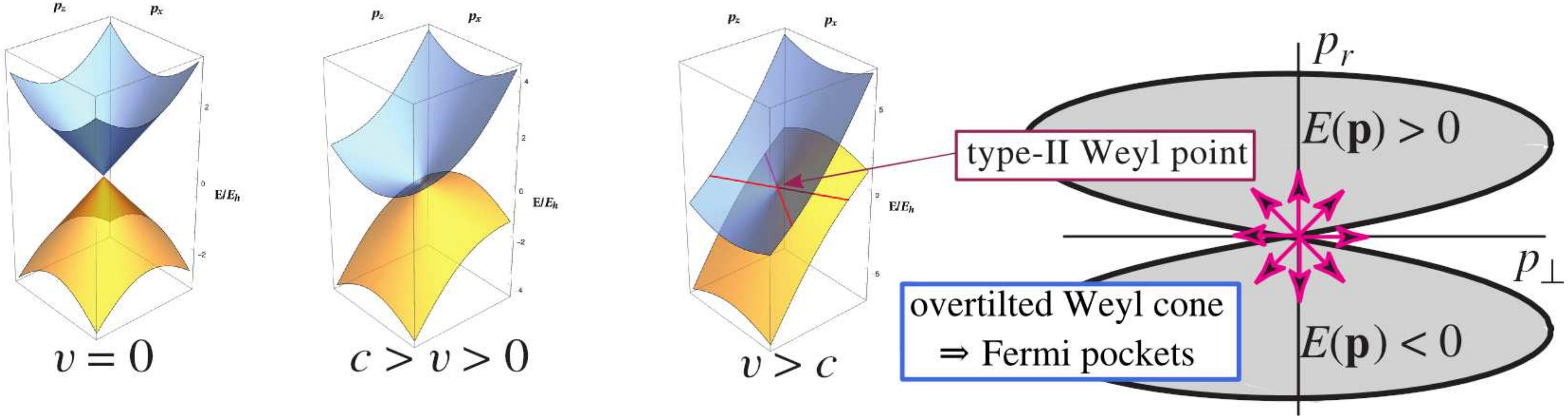
$\mathbf{v}(\mathbf{r})$ and $c$ are

material parameters.

**Hawking radiation:**

filling $E(\mathbf{p}) < 0$ pocket.

Energy spectrum in Weyl semimetal/superfluid: $g^{\mu\nu}p_\mu p_\nu = (E - \mathbf{p}\cdot\mathbf{v})^2 - c^2 p^2 = 0$

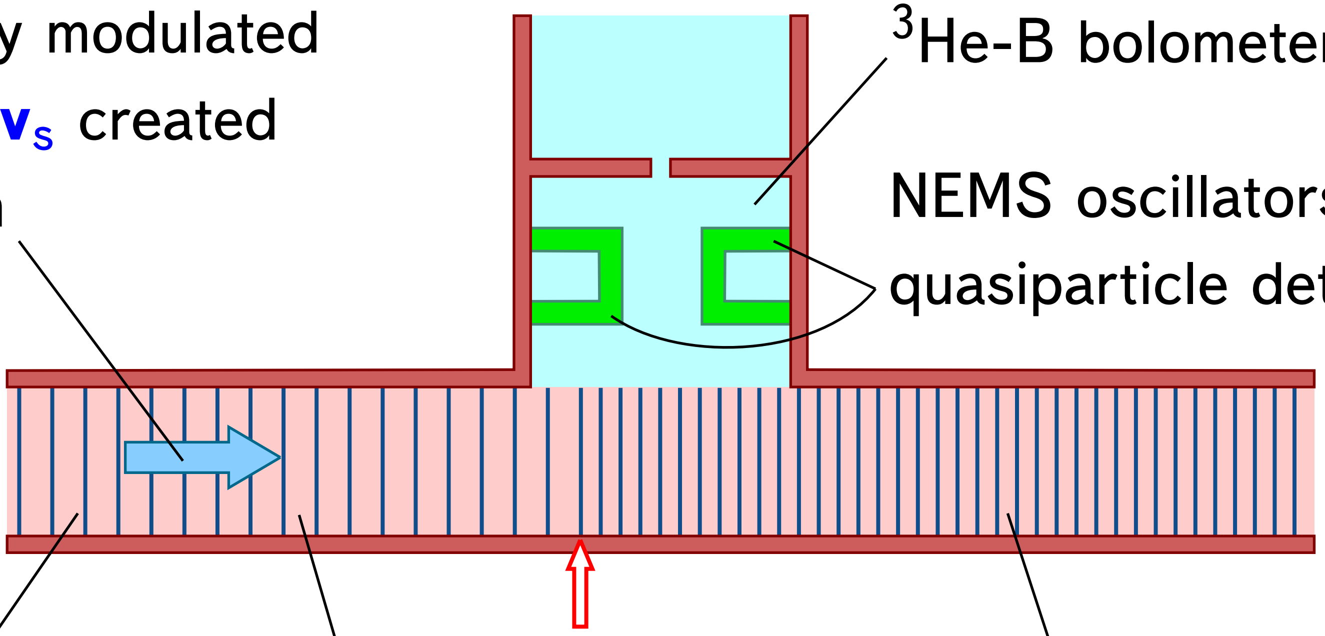


$v = 0$ $c > v > 0$ $v > c$

Volovik, JETP Lett. **104**, 645 (2016); Zhang & Volovik, JETP Lett. **105**, 519 (2017); Volovik & Zhang, JLTP **189**, 276 (2017)

# POTENTIAL HORIZON ANALOGUE IN THE PdA PHASE

Periodically modulated
superflow $\mathbf{v}_s$ created
by rotation



$^3$He-B bolometer

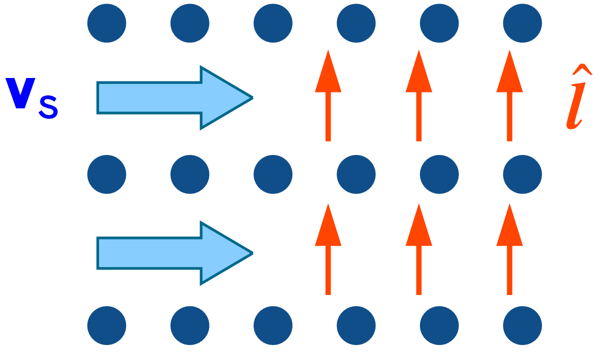
NEMS oscillators as
quasiparticle detectors

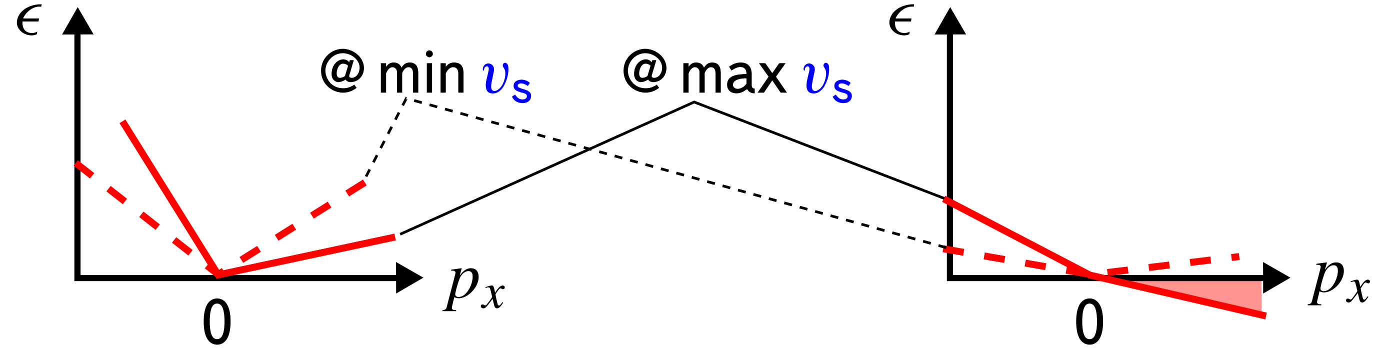
Black-hole horizon

Confinement with a
step in density and
in-plane anisotropy

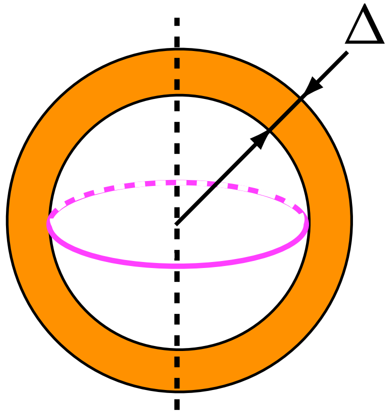
PdA phase with smaller
polar distortion,
always type I Weyl

PdA phase with larger
polar distortion,
type I/II Weyl depending on $|\mathbf{v}_s|$

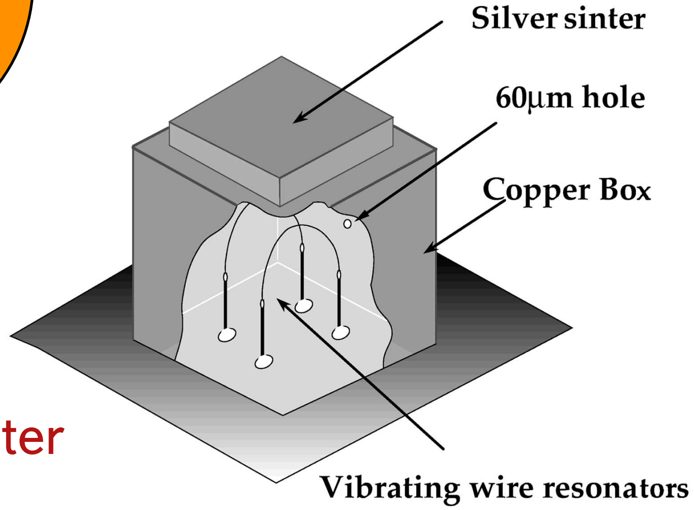




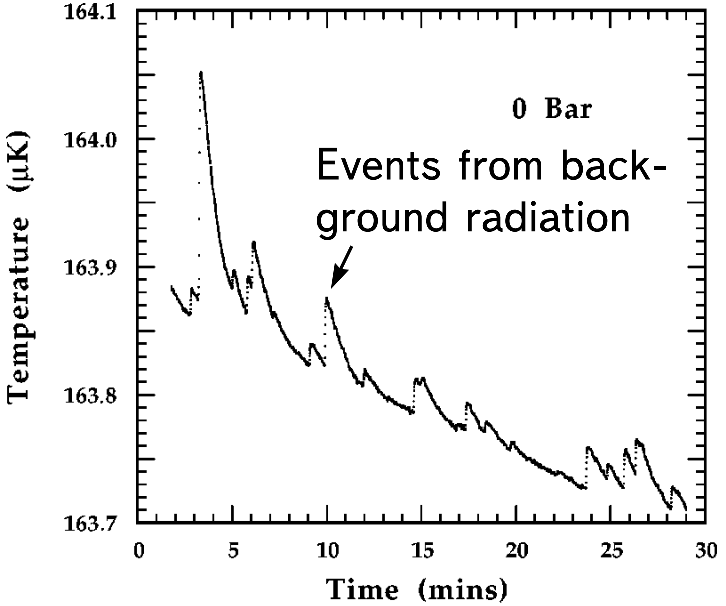
# $^3$He-B BOLOMETER FOR (QUASI)PARTICLE DETECTION



In the gapped $^3$He-B, heat capacity $C$, quasiparticle density $n$ and resonance width on an immersed mechanical oscillator $\Delta f \propto \exp(-\Delta / k_B T)$.



Bolometer



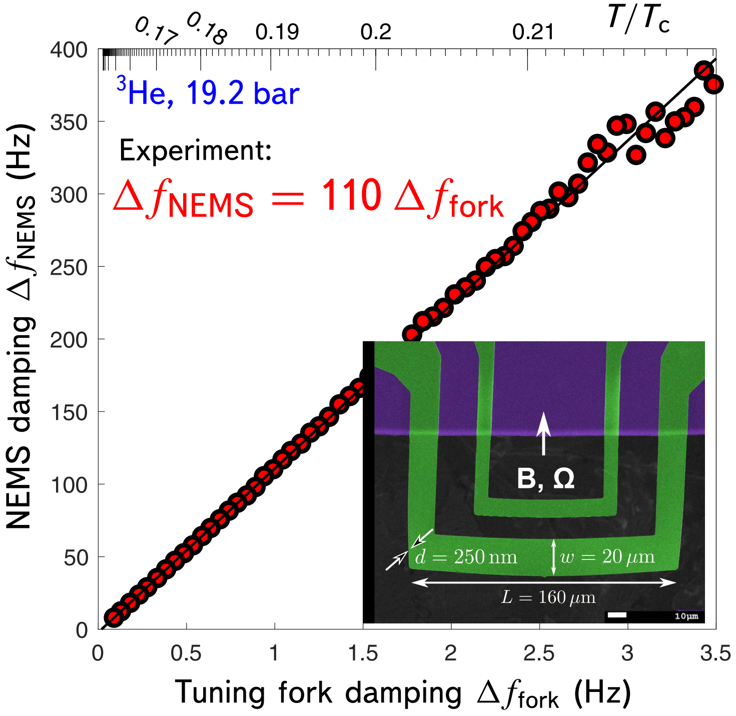
Events from background radiation

Bäurele *et al*, PRB **57**, 14381 (1997)

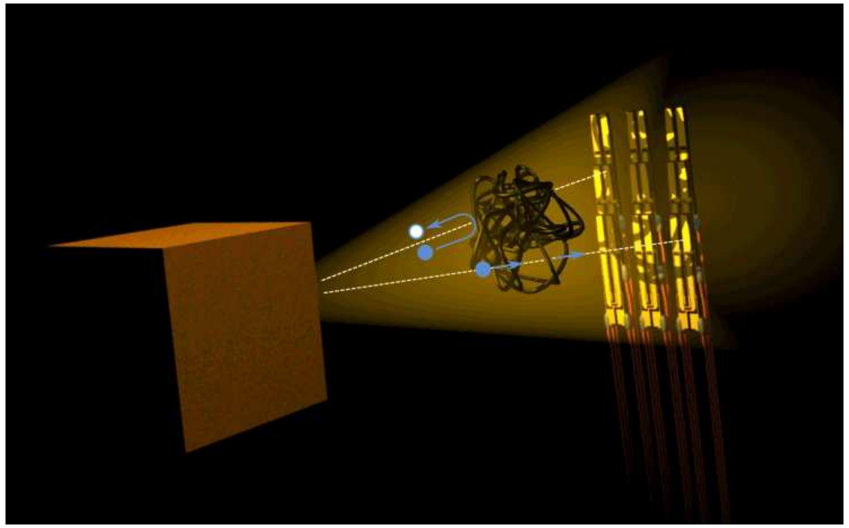
Planned to be used for dark matter search

QUEST DMC

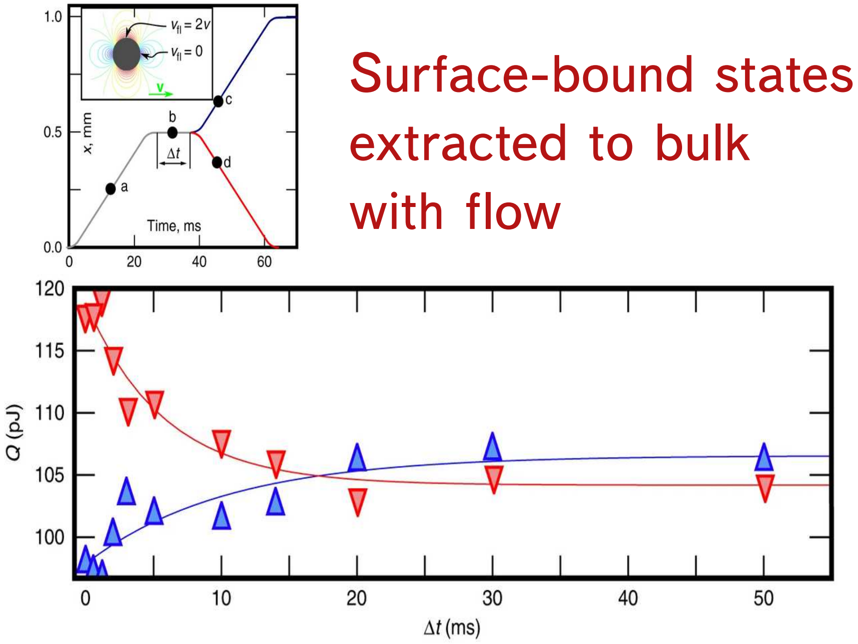
Enhanced sensitivity with NEMS oscillators



Quasiparticle camera



Surface-bound states extracted to bulk with flow



Noble *et al*, PRB **105**, 174515 (2022)

Autti *et al*, NatCom **11**, 4742 (2020)

Kamppinen *et al*, PRB **107**, 014502 (2023)

# ROUTE TO ANTISPACETIME VIA POLAR PHASE

Antispacetime: Reversal of (one of) the tetrad components so that

$$\det e_a^\mu > 0 \Longleftrightarrow \det e_a^\mu < 0$$

$$\text{right-handed fermions} \Longleftrightarrow \text{left-handed fermions}$$

What happens to action? Possible choices:

- $\det e_a^\mu < 0$ is prohibited
- action depends on $|\det e_a^\mu|$
- something else

$\mathbf{e}_1 = (\Delta/p_{\mathsf{F}})\,\hat{\mathbf{m}}$

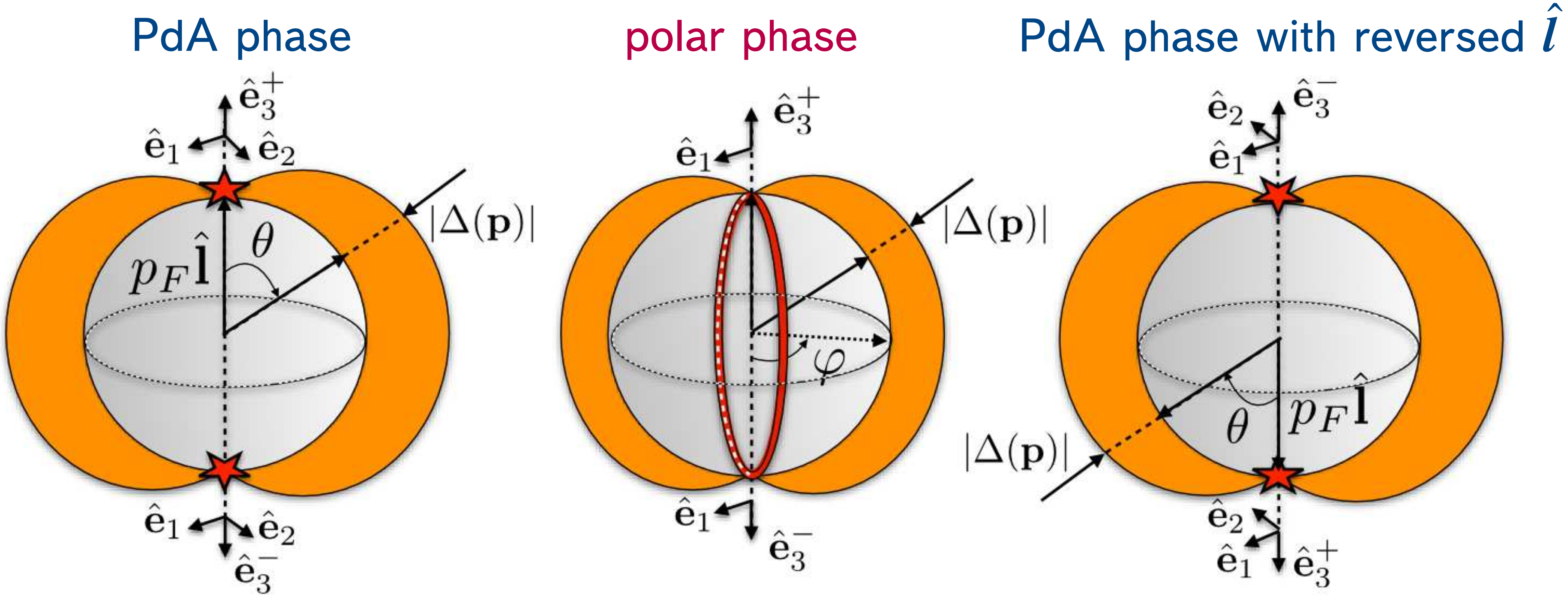
$\mathbf{e}_2 = b\,(\Delta/p_{\mathsf{F}})\,\hat{\mathbf{n}}$

$\uparrow$

controlled by confinement

$0 < b < 1$

$\mathbf{e}_3^\pm = \pm v_{\mathsf{F}}\,\hat{\boldsymbol{l}}$



Non-analitic behavior of effective fields at the transition (in the polar phase), action $\propto |\nabla\cdot\hat{\mathbf{m}}|^{3/2}$.

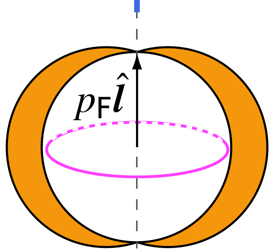
How to probe when $\hat{\mathbf{m}}$ is fixed by confinement?

Nissinen & Volovik, PRD **97**, 025018 (2018)

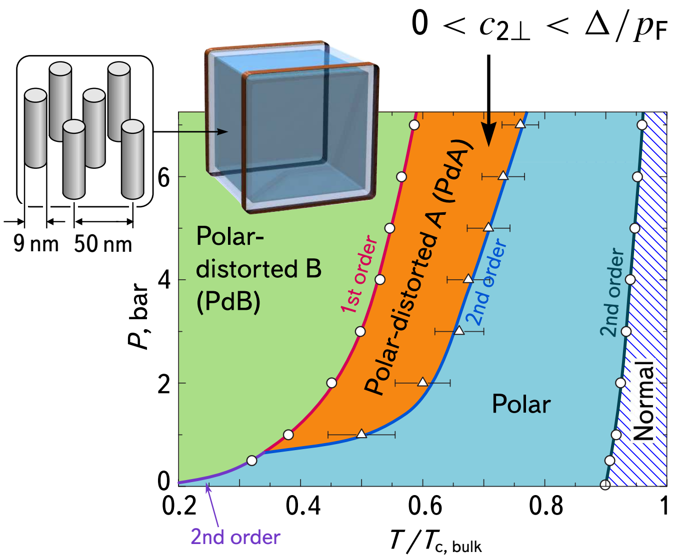
# CONCLUSIONS

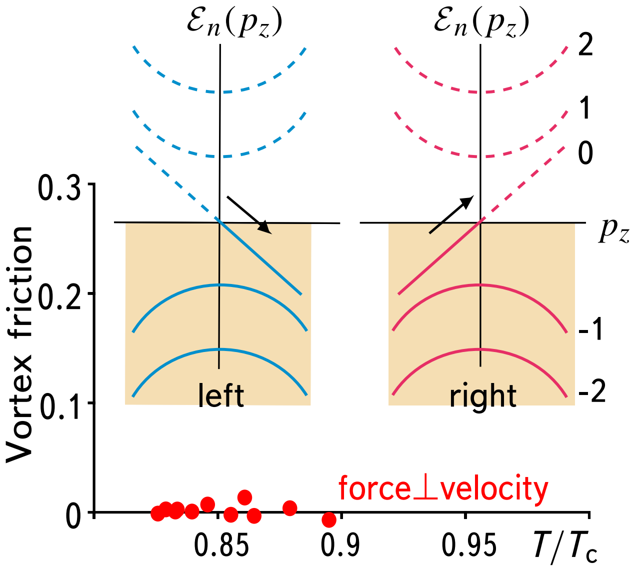
$$\hat{l} = \hat{\mathbf{m}} \times \hat{\mathbf{n}}$$

$$\mathcal{H} = \pm \sigma^a \, e_a^j \, (\mathbf{p} \mp p_F \hat{\boldsymbol{l}})_j$$



$$e_a^\mu = \begin{pmatrix} 1 & -\mathbf{v}_s \\ 0 & c_{\perp 1}\hat{\mathbf{m}} \\ 0 & c_{\perp 2}\hat{\mathbf{n}} \\ 0 & c_\parallel \hat{l} \end{pmatrix}$$





- In many cases, dynamics of quasiparticles in superfluid $^3$He can be described as interaction with synthetic electromagnetic and gravitational fields formed by the spatial variation of the order parameter and its time dependence.

- Such order-parameter distributions are created in the experiment with confinement, superflow and by topological objects like vortices and solitons. The quasiparticle response is observed using NMR and mechanical probes.

- While chiral anomaly in synthetic electromagnetic field is observed experimentally, analogue gravity effects are more subtle and so far remain theoretical suggestions: Nieh-Yan anomaly, CVE & CTE, black/white hole horizon, antispacetime.