Probing the Vacuum with Continuous Unruh Detectors

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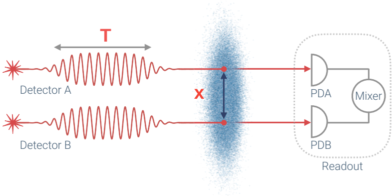
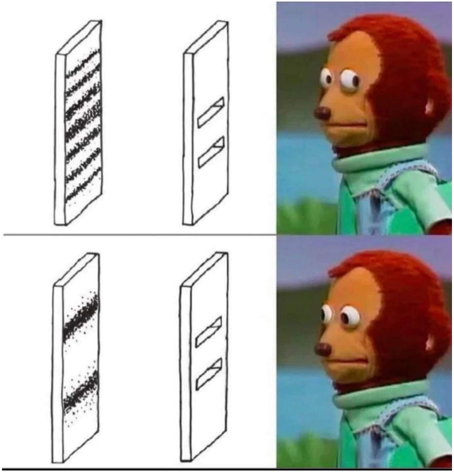


Overview



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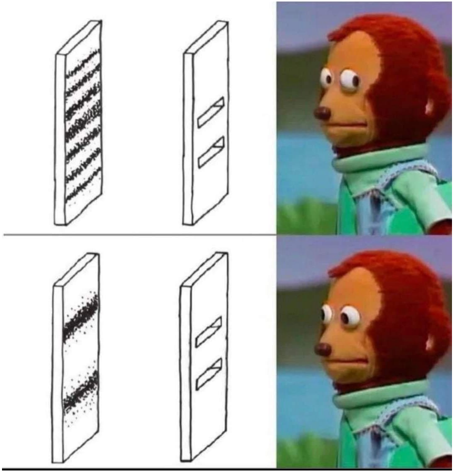

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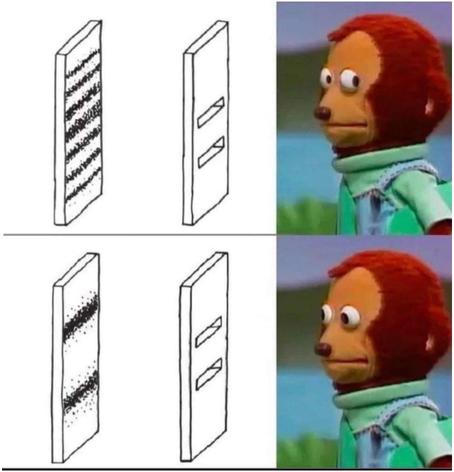

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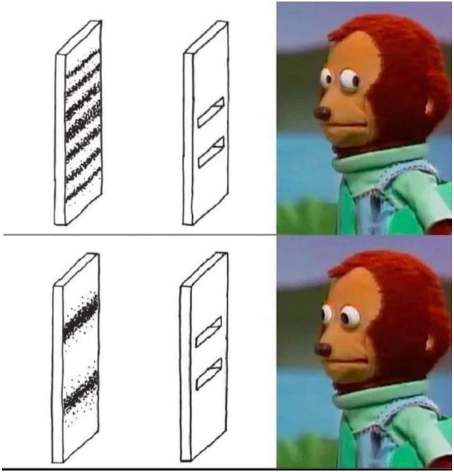

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- Outlook

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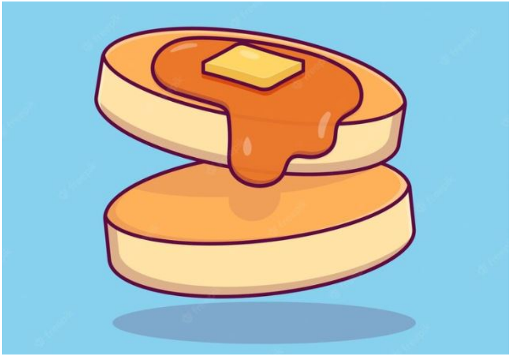

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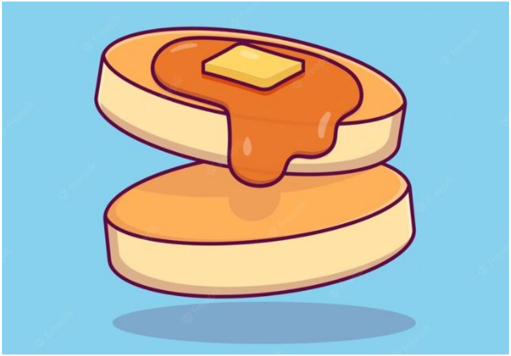

$(2+1)$ BEC Lagrangian, confined to the (x, y) plane:

$$\mathcal{L}_{BEC} = i\hbar\Phi\partial_t\Phi^* + \frac{\hbar^2}{2m}\left|\nabla\Phi\right|^2 + \frac{g_{2d}}{2}\left|\Phi\right|^4$$

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At the next order, BEC density fluctuations get transduced into the laser phase.

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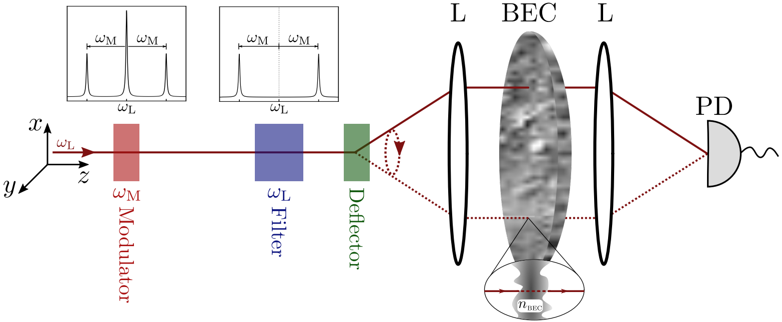
(Credit: arXiv:1911.06002)

Interferometric Unruh Detectors for BECs



Experimental proposal: use a circularly-moving interaction point between a laser and a 2*d* BEC to probe the "vacuum" along an accelerated trajectory [C. Gooding et al. **PhysRevLett.125.213603(2020)**].

Common Mode and Difference Signal

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$$Z_\nu^\varphi = \frac{1}{\sqrt{2}}\left(e^{-i\varphi}Z_\nu + e^{i\varphi}Z_{-\nu}^\dagger\right)$$

$$\Pi_\nu^\varphi = \frac{1}{i\sqrt{2}}\left(e^{-i\varphi}Z_\nu - e^{i\varphi}Z_{-\nu}^\dagger\right)$$

obeying the commutation relation $[Z_\nu^\varphi, \Pi_{\nu'}^{\varphi\dagger}] = i \cdot 2\pi\delta[\nu - \nu']$.

Fluctuations in the Photon Flux

The photon fluctuations can be expressed as

$$\frac{\delta\tilde{n}(t)}{2\alpha} = z^{\varphi}(t) + \frac{1}{2}\left[e^{-2i(\omega_M t+\psi_0)}\left(z^{\varphi}(t) + i\tilde{\Pi}^{\varphi}(t)\right) + h.c.\right]$$

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It is convenient to decompose fluctuations in the photon flux such that

$$\delta\tilde{n}(t) = \delta n(t) + \Delta n(t)$$

where $\delta n(t)$ is the noninteracting fluctuation and $\Delta n(t)$ is the perturbation caused by interaction with the BEC.

Power Spectra

The interaction between the BEC and the laser field leads to photon flux fluctuations $\delta\tilde{n}(t)$ with **power spectrum**

$$S_{nn}[\omega] = \int_{-\infty}^{\infty} dt\, e^{-i\omega t}\langle\delta\tilde{n}(t)\delta\tilde{n}(0)\rangle = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi}\langle\delta\tilde{n}[\omega]^{\dagger}\delta\tilde{n}[\omega']\rangle$$

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$S_{nn}[\omega]$ can be decomposed into four parts:

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$$S_{\Delta\Delta}[2\omega_M + \nu] = -4i\mu^2\alpha^2 e^{2i\psi_0}\sin 2\psi_0\, S_{\phi_r\phi_r}[\nu]$$

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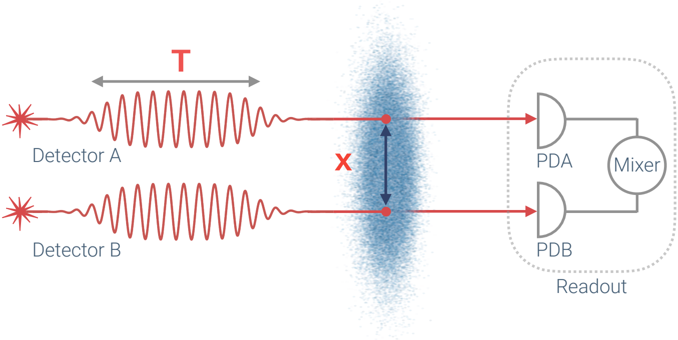
- Signal extraction ambiguities. More rigorous analysis? Extra optical processing?

Idea: steal entanglement from the vacuum!



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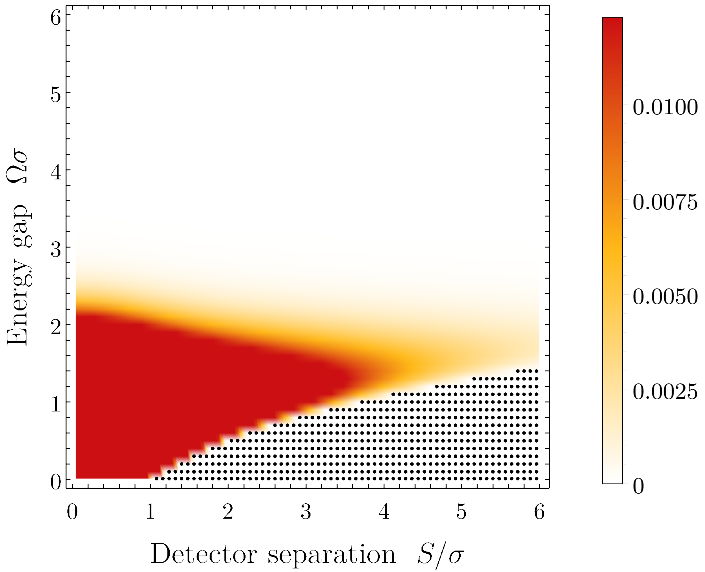
$$G_{\mathcal{M}}(\boldsymbol{k}) = -J_0(|\boldsymbol{k}|\,\Delta \boldsymbol{x}) e^{-\frac{T^2(\Omega^2+\omega_{\boldsymbol{k}}^2)}{2}} \cdot \left[1 + i\,\mathrm{erfi}\left(\frac{T\omega_{\boldsymbol{k}}}{\sqrt{2}}\right)\right]$$

Negativity and Parameter Optimization



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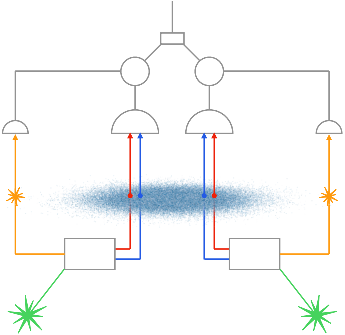
- UDW detectors couple only to $\hat{\phi}$, and not to its conjugate momentum

Analogue Implementation



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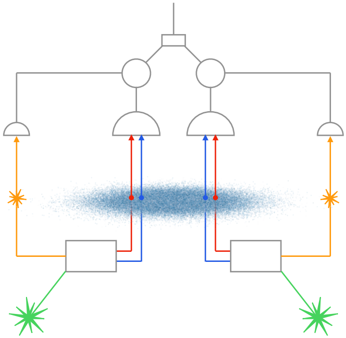

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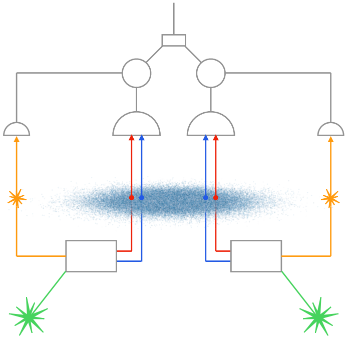

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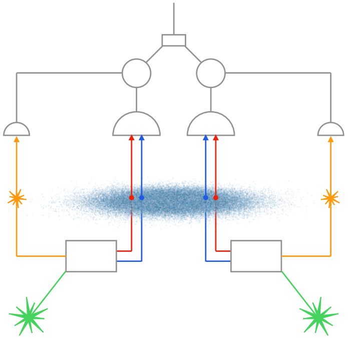

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$$\mathcal{I} = V(\hat{q}_{\pm}) + V(\hat{p}_{\mp}) < 1$$

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$$\hat{p}_{\pm}(\Omega, T) = \frac{1}{\sqrt{2}} \left(\hat{\Pi}_A^{\varphi_A}(\Omega, T) \pm \hat{\Pi}_B^{\varphi_B}(\Omega, T)\right)$$

- **Inseparability condition:**

$$\mathcal{I}(\Omega, T) = V(\hat{q}_+(\Omega, T)) + V(\hat{p}_-(\Omega, T)) < 1$$

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Hence, for $T \lesssim \Omega^{-1}$, there will be effective detector gaps throughout the measurement band: $\omega \in (-\Delta, \Delta)$. However, **the operator $\hat{O}(\Omega, T)$ still has largest contribution from $\omega = \Omega$.**

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with $\hat{H}(t) = \hat{H}_{\mathrm{A}}(t) + \hat{H}_{\mathrm{B}}(t)$ and $\hat{H}_i(t) \equiv \varepsilon(t)\hat{\phi}_i(t)\hat{\psi}'_i(t)$

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$$\begin{aligned}\hat{1} = |00\rangle\langle 00| &+ \int \frac{dK}{2\pi(2\Omega_K)} \left(|1_K 0\rangle\langle 1_K 0| + |01_K\rangle\langle 01_K|\right) \\ &+ \int \frac{dK}{2\pi(2\Omega_K)} \int \frac{dK'}{2\pi(2\Omega_{K'})} |1_K 1_{K'}\rangle\langle 1_K 1_{K'}|\end{aligned}$$

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The inseparability then parallels the usual UDW negativity.

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Acknowledgements

Thanks for listening!

(Silke's Gravity Lab team, 2020.)