Entanglement generated in the Hawking process:

Astrophysical BHs and rotating analogs

Adrià Delhom
Louisiana State University

College of Science

In collaboration with: I. Agullo, A. J. Brady, D. Kranas, P. Calizaya Cabrera, K. Guerrero, K. Falque, and M. Jacquet

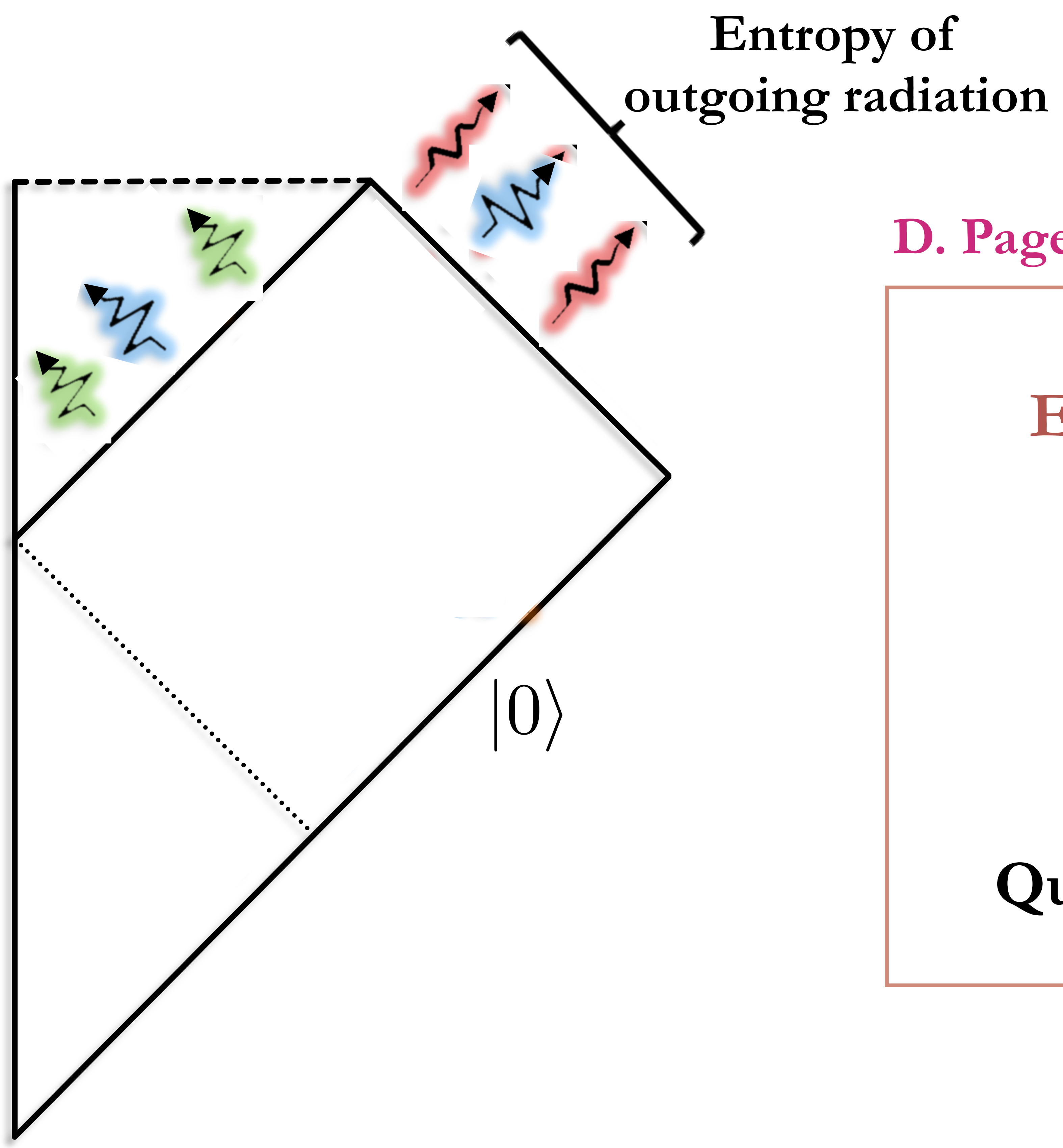
Goals

- Understand the role of superradiance in the generation of entanglement.
- Quantify entanglement in evaporation of realistic BHs (CMB input, rotating).
- Apply similar techniques to rotating analogues to yield testable predictions.

Part I: Astrophysical BHs

In collaboration with: I. Agullo, A. J. Brady, and D. Kranas

Previous work



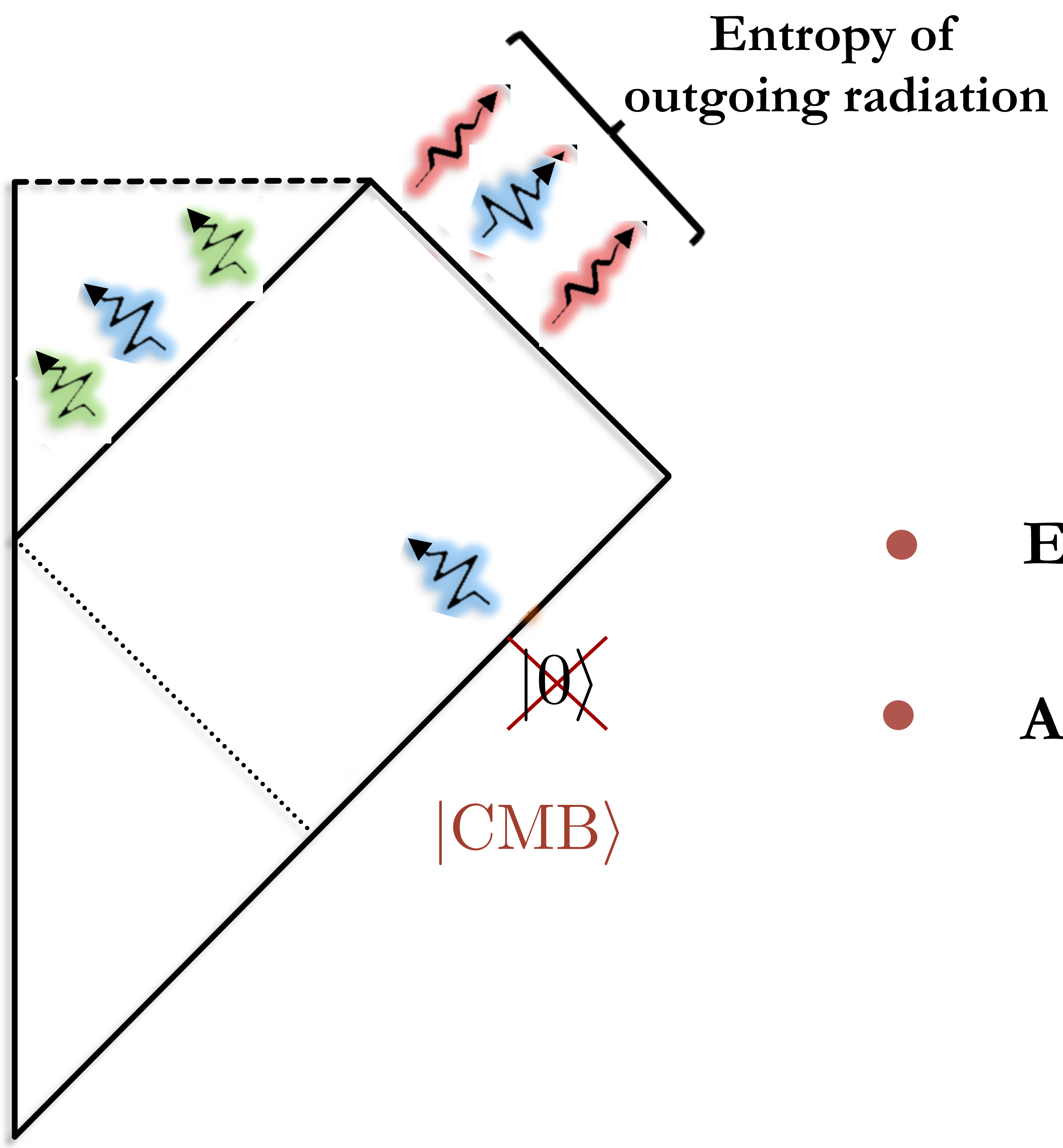
D. Page 2013

Entanglement entropy is a quantifier of Hawking-generated entanglement

Entropy of the radiation reaching infinity = Entanglement entropy

↓

Quantify generated entanglement as entropy of Hawking radiation at infinity



Problem:

- Entanglement entropy quantifies entanglement only if state is pure.
- Astrophysical black holes are immersed in a thermal bath: the CMB

  Known cases where thermal inputs destroy all entanglement.

  Agullo, Brady, Kranas '22

Entanglement entropy is not a quantifier for entanglement generated by realistic BHs.

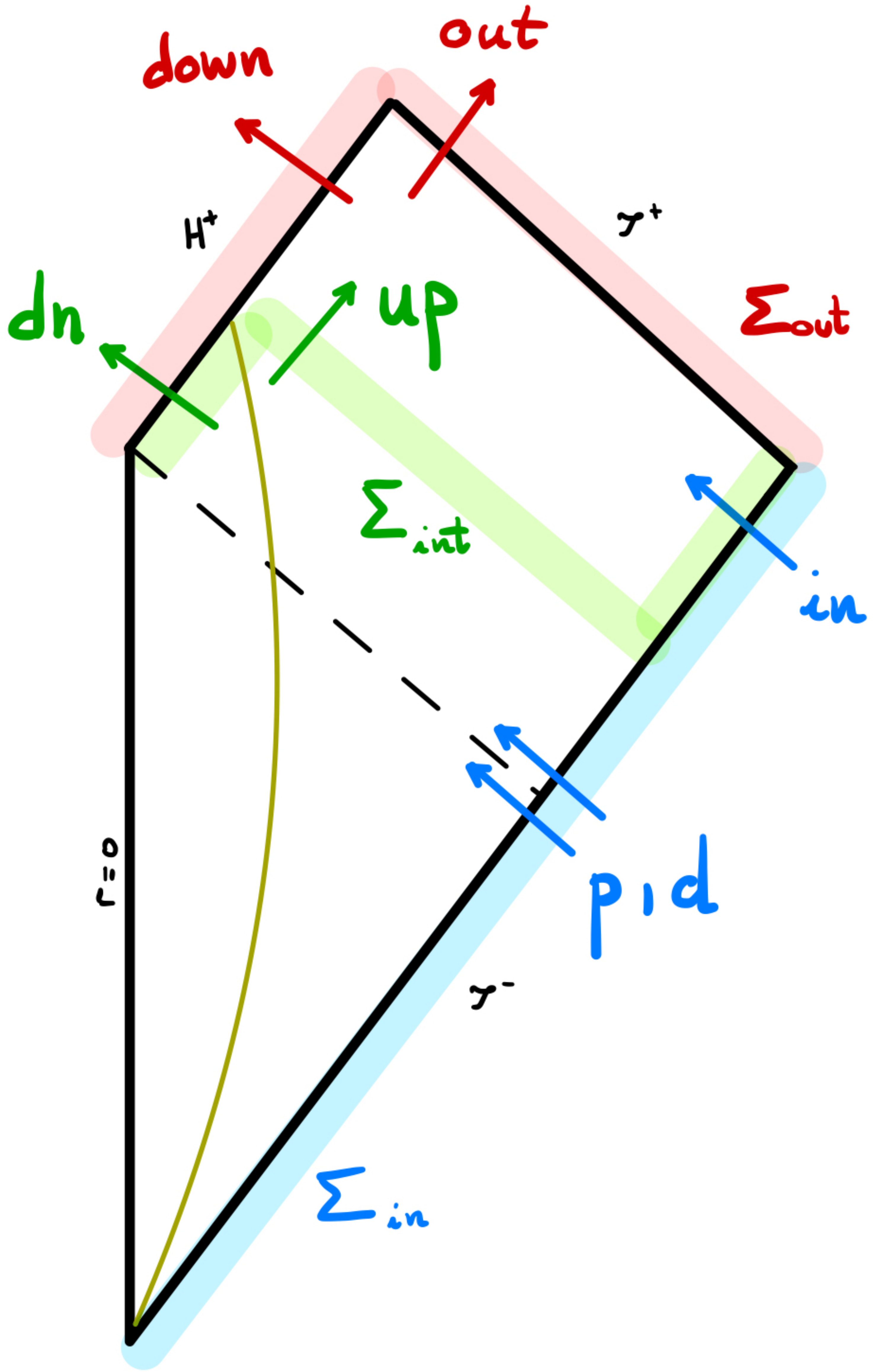
Use Logarithmic Negativity

The role of ergoregions in the Hawking process

Hawking process in two steps

1- Particle creation near horizon, early times: p + d ⟶ up + dn

2-Scattering at potential barrier, late times: up + in ⟶ out + down



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1- Particle creation near horizon, early times: p + d $\longrightarrow$ up + dn

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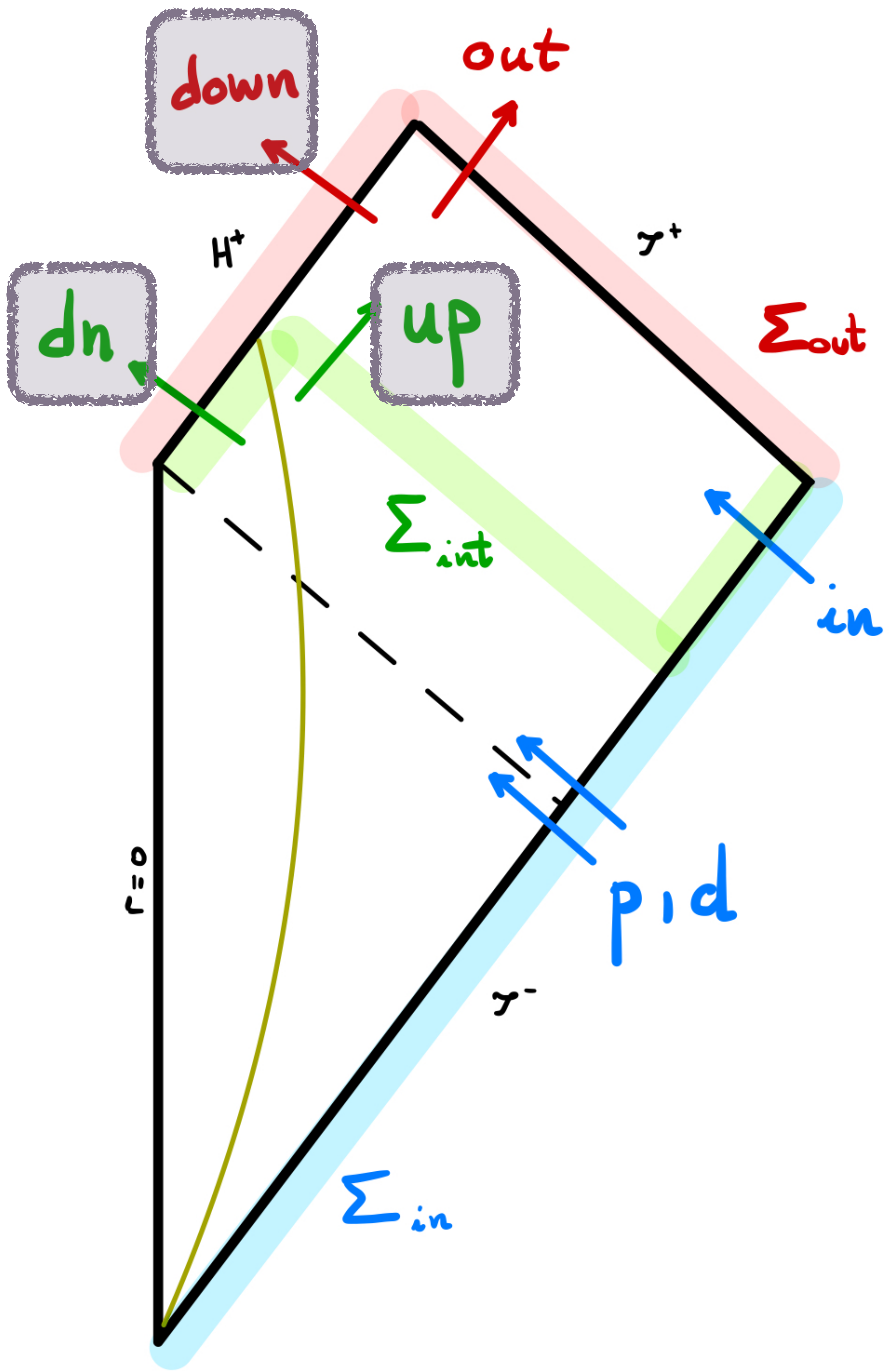
NORM (near the horizon)

Schwarzschild: $sign(\omega)$

Kerr: $sign\boxed{(\omega - m\Omega_h)}$ Superradiant conditon



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Particle creation at the horizon (Scwarzschild and Kerr)

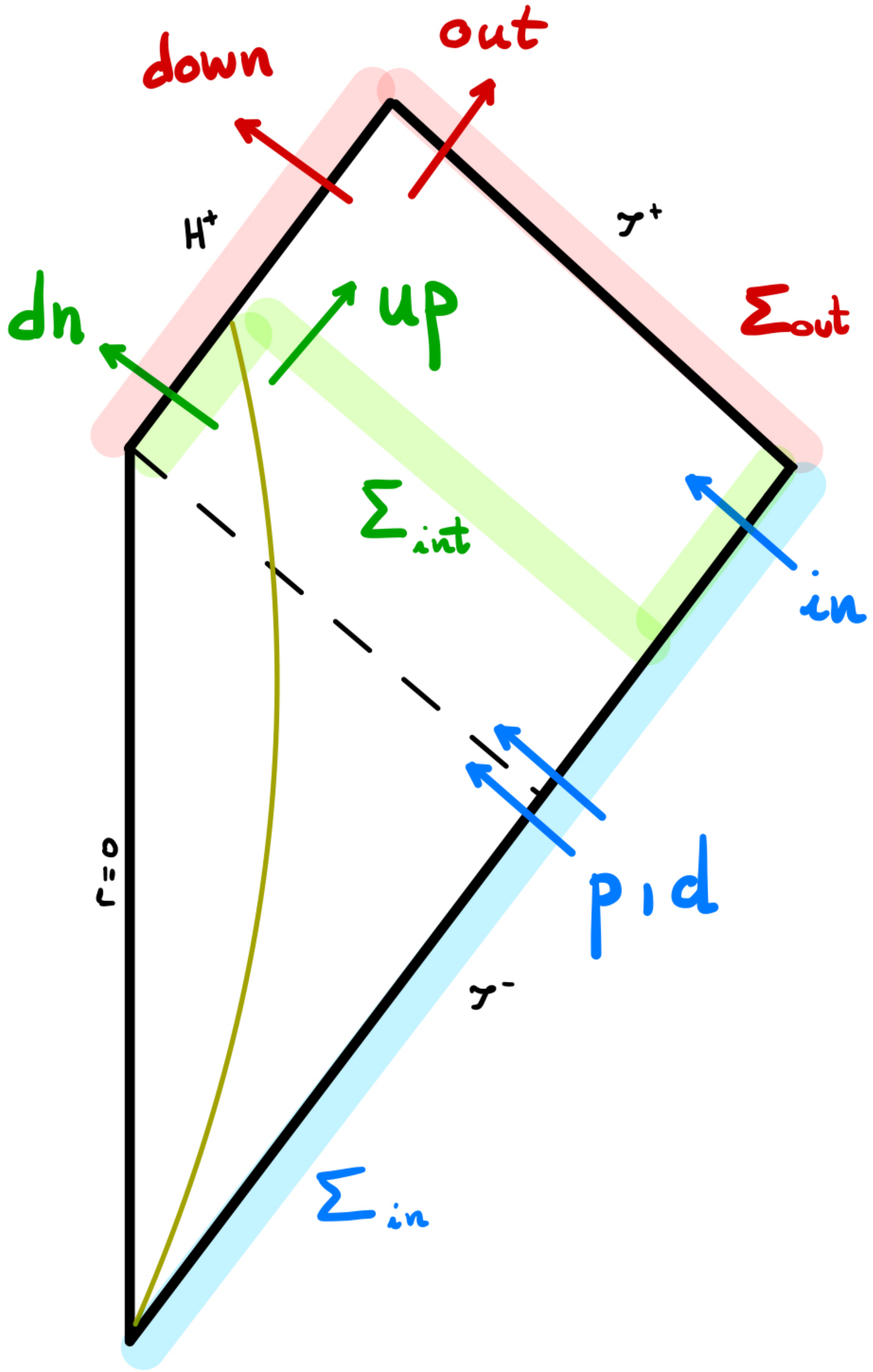
$$\boxed{\begin{aligned} \hat{a}^p_\omega \longrightarrow \hat{a}^{up}_\omega &= \cosh r_H\, \hat{a}^p_\omega - \sinh r_H\, \hat{a}^{d\,\dagger}_\omega \\ \hat{a}^d_\omega \longrightarrow \hat{a}^{dn}_\omega &= -\sinh r_H\, \hat{a}^{p\,\dagger}_\omega + \cosh r_H\, \hat{a}^d_\omega \end{aligned}}$$

TWO-MODE SQUEEZER!

where

$$\boxed{r_H(\omega, m) = \tanh^{-1} e^{-\frac{\omega - m\Omega_h}{3T_H}}}$$

Hawking squeezing intensity



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SUPERRADIANT

$$\hat{a}^p_\omega \longrightarrow \hat{a}^{dn}_\omega = \cosh r_H \, \hat{a}^p_\omega - \sinh r_H \, \hat{a}^{d\,\dagger}_\omega$$

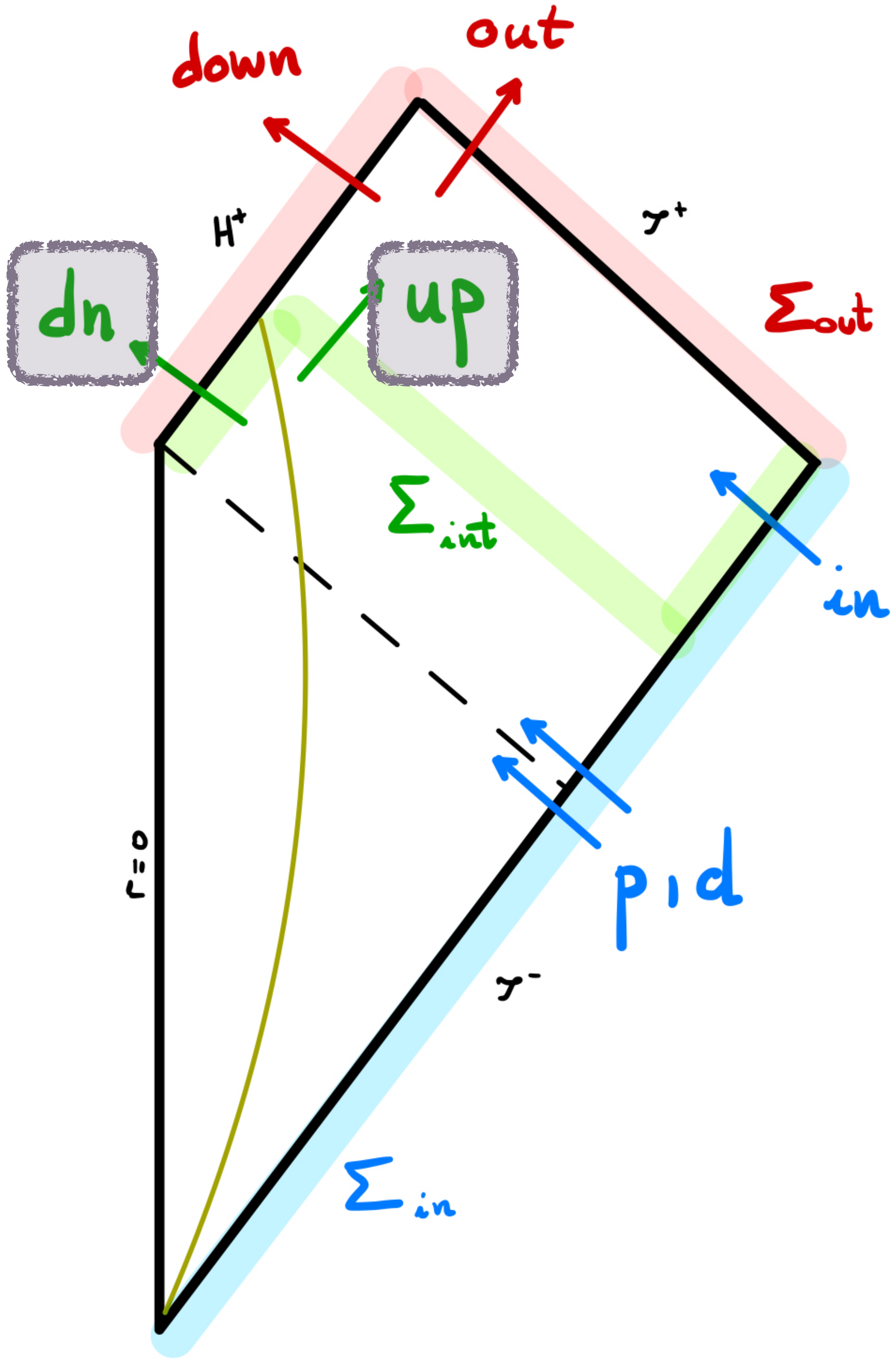
$$\hat{a}^d_\omega \longrightarrow \hat{a}^{up}_\omega = -\sinh r_H \, \hat{a}^{p\,\dagger}_\omega + \cosh r_H \, \hat{a}^d_\omega$$

TWO-MODE SQUEEZER!

where

$$r_H(\omega, m) = \tanh^{-1} e^{-\frac{\omega - m\Omega_h}{2T_H}}$$

Hawking squeezing intensity



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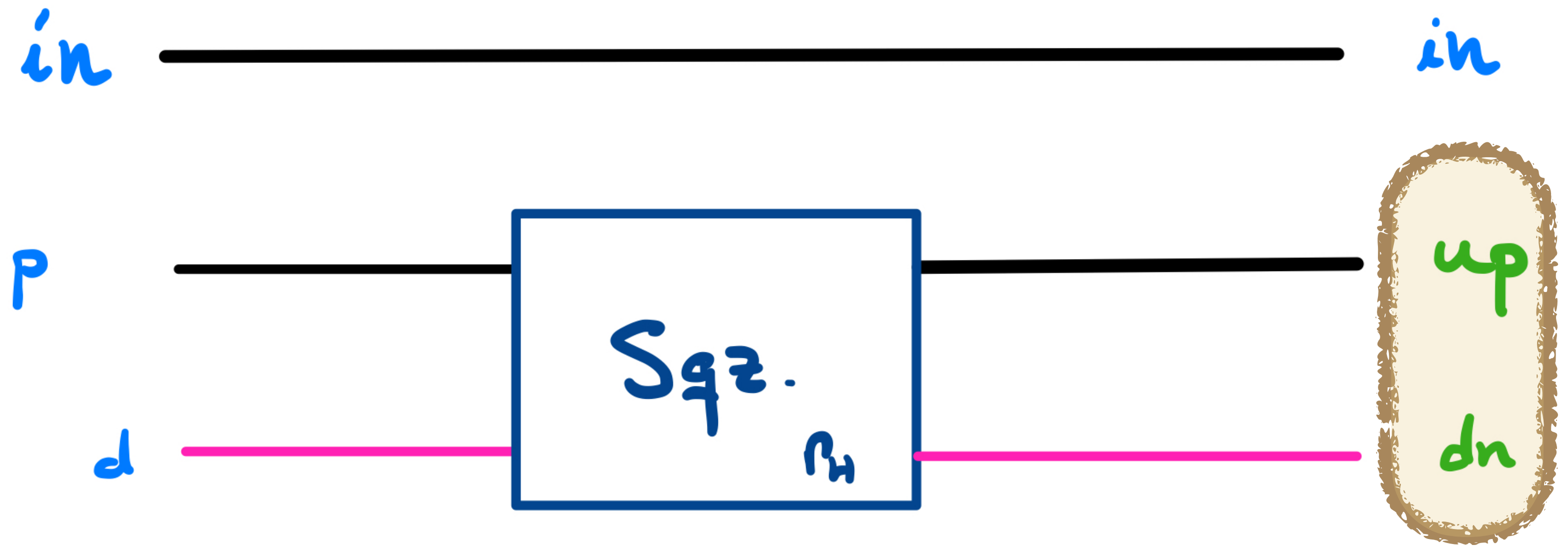
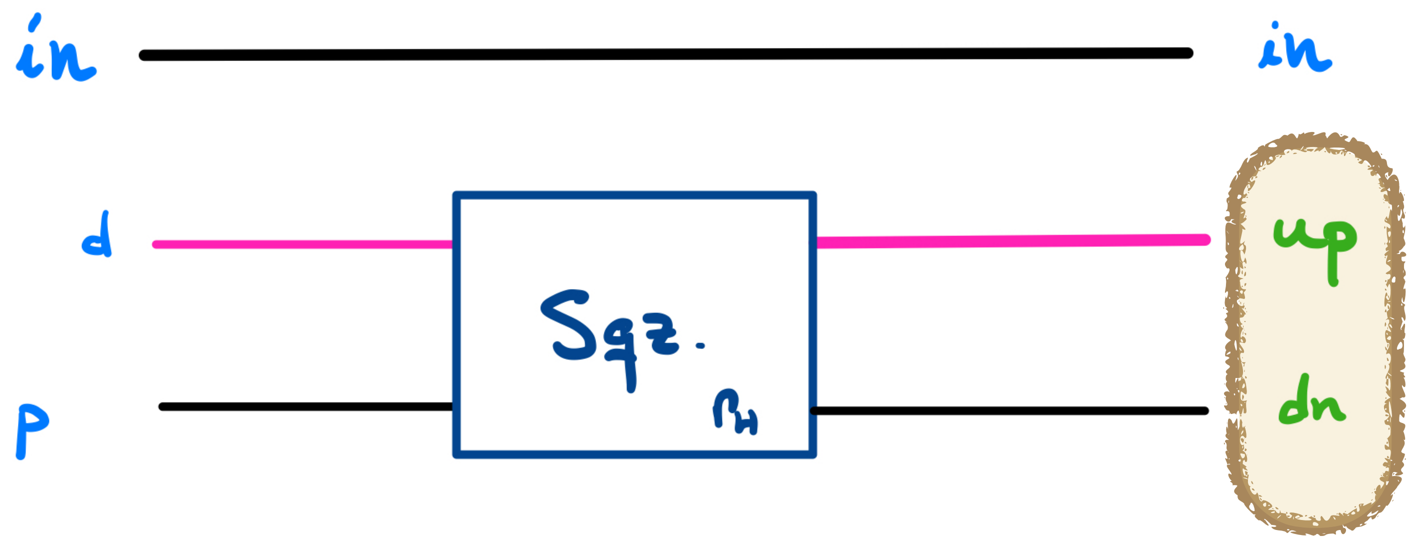
Non-superradiant



Entangled

Superradiant



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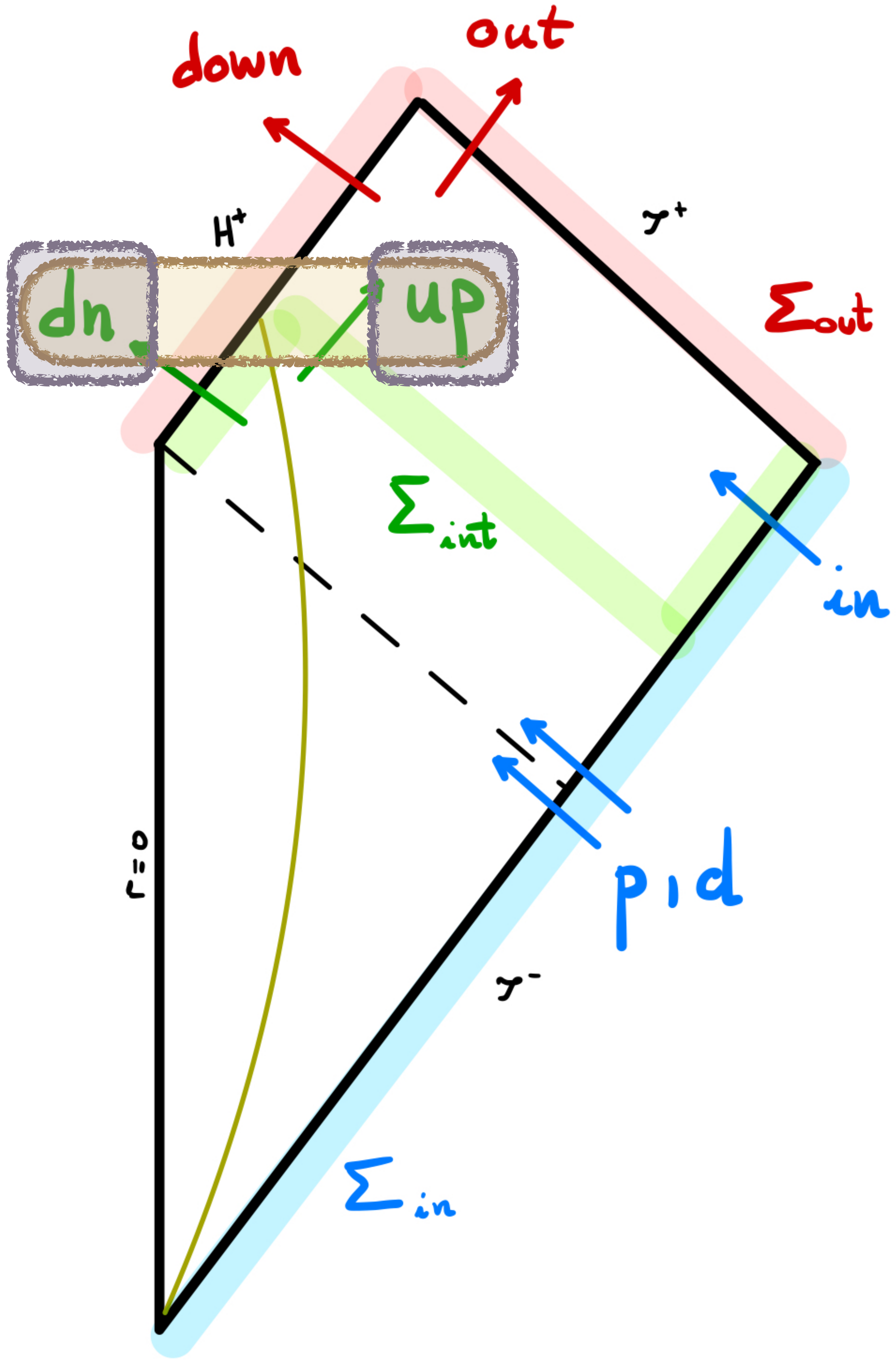

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Scattering with gravitational potential

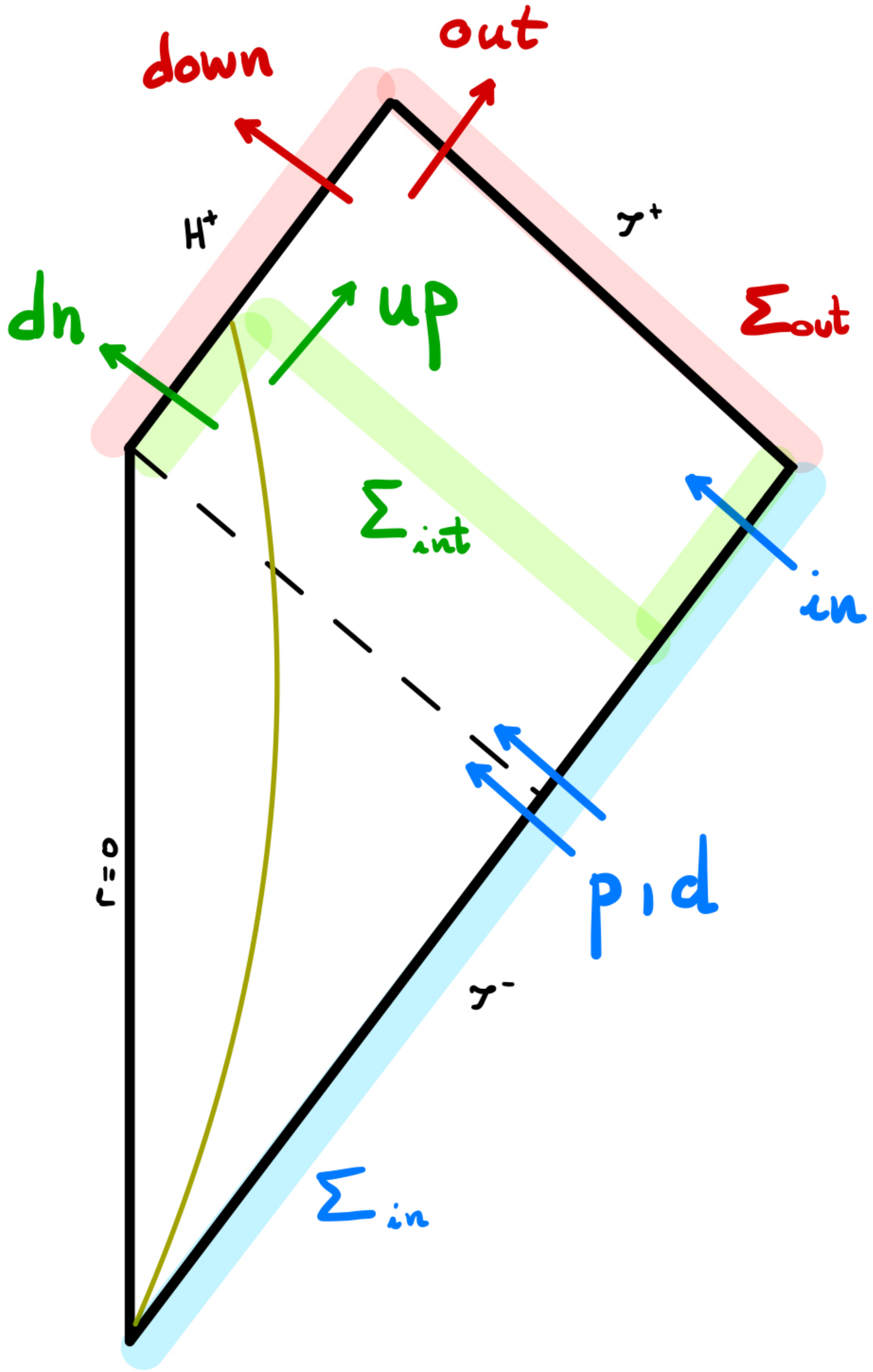
$$\hat{a}^{up}_{\omega} \longrightarrow \hat{a}^{out}_{\omega} = \cos\theta_{\Gamma}\,\hat{a}^{up}_{\omega} + \sin\theta_{\Gamma}\,\hat{a}^{in}_{\omega}$$

$$\hat{a}^{in}_{\omega} \longrightarrow \hat{a}^{down}_{\omega} = -\sin\theta_{\Gamma}\,\hat{a}^{up}_{\omega} + \cos\theta_{\Gamma}\,\hat{a}^{in}_{\omega}$$

BEAM SPLITTER

Where: $\Gamma_{\omega\ell} = \sin^2\theta_{\Gamma}$

(greybody factors from Teukolsy equation)



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TWO-MODE SQUEEZER

Where: $\Gamma_{\omega\ell} = \sinh^2 r_\Gamma$

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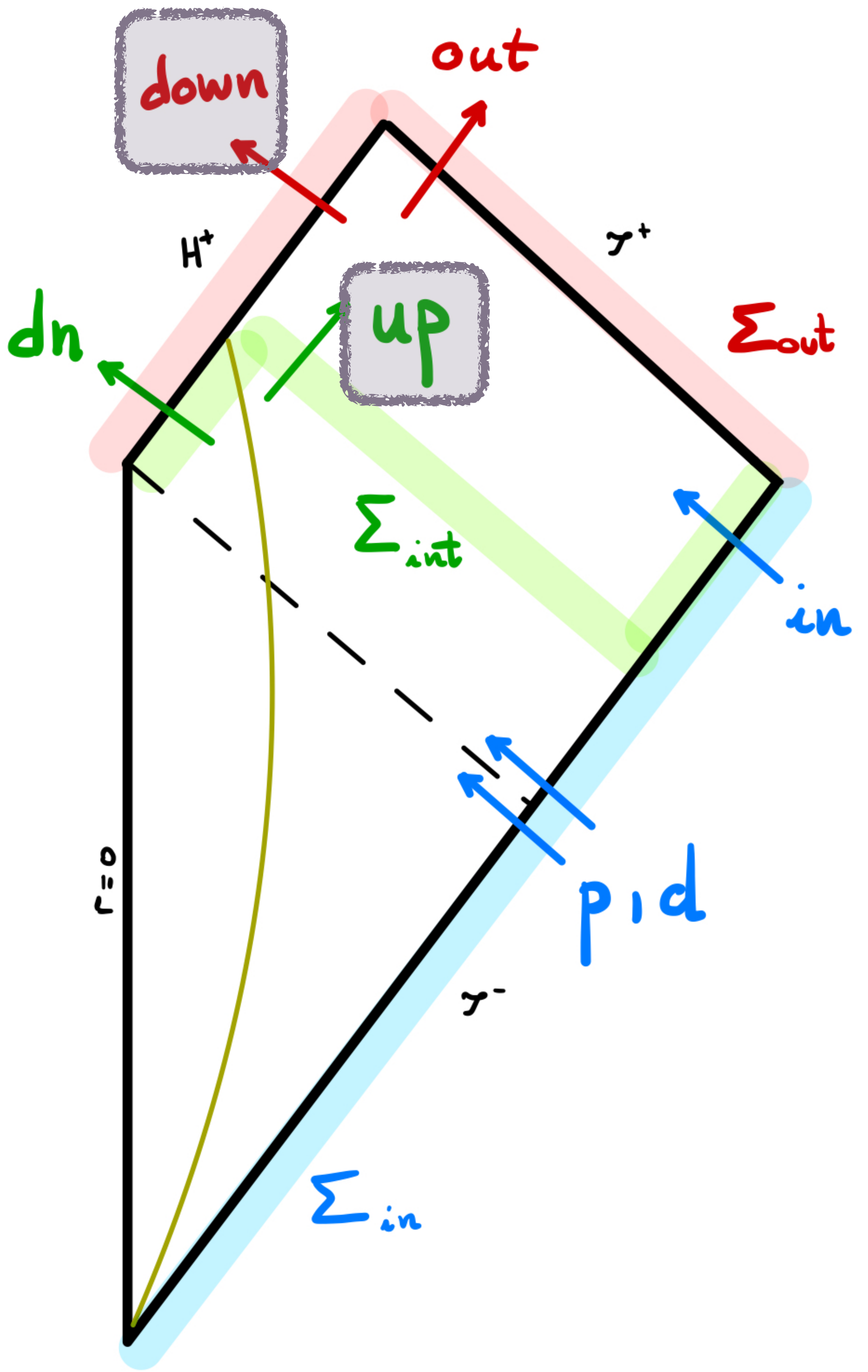

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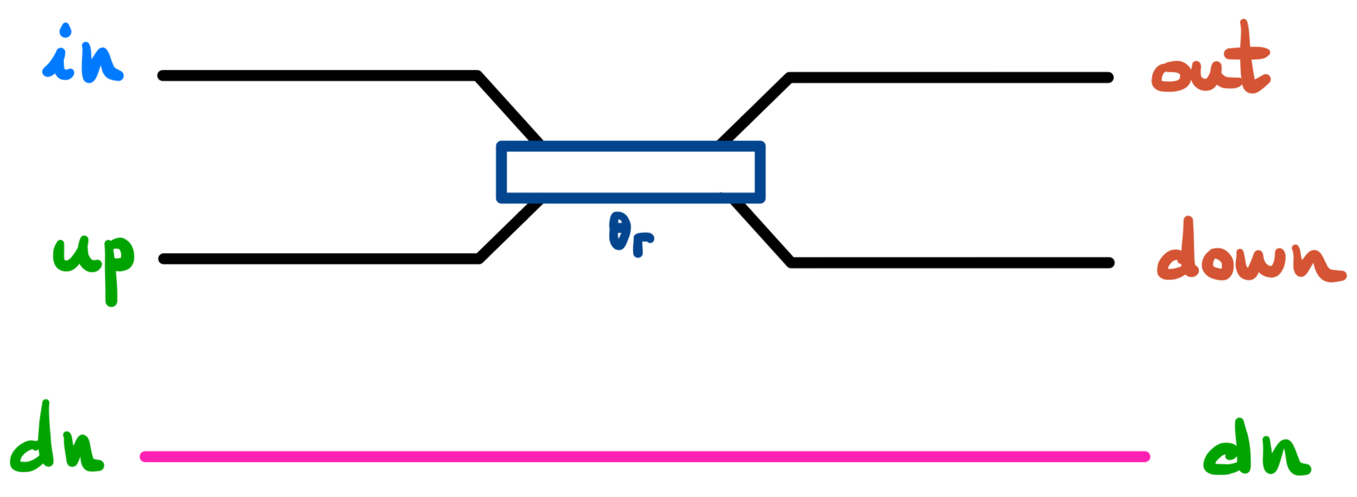
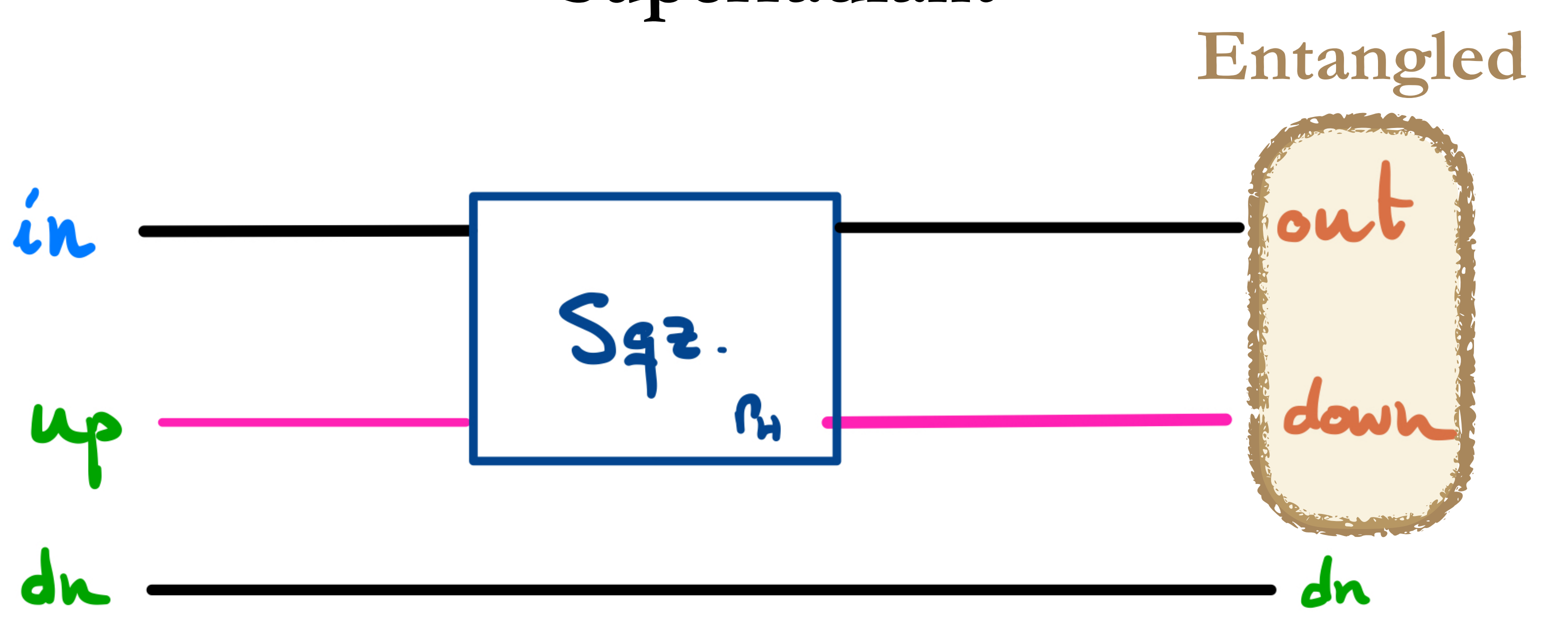
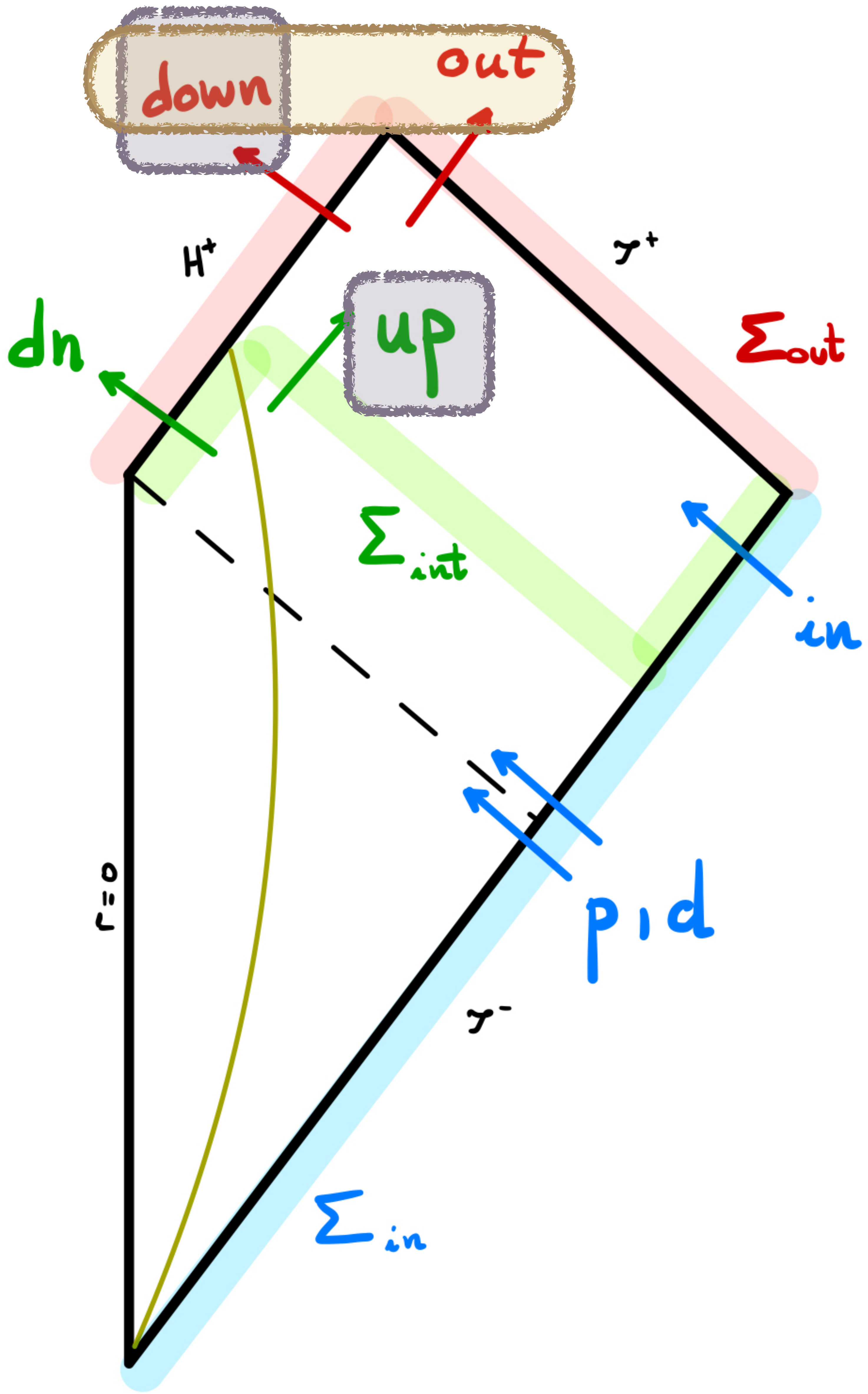
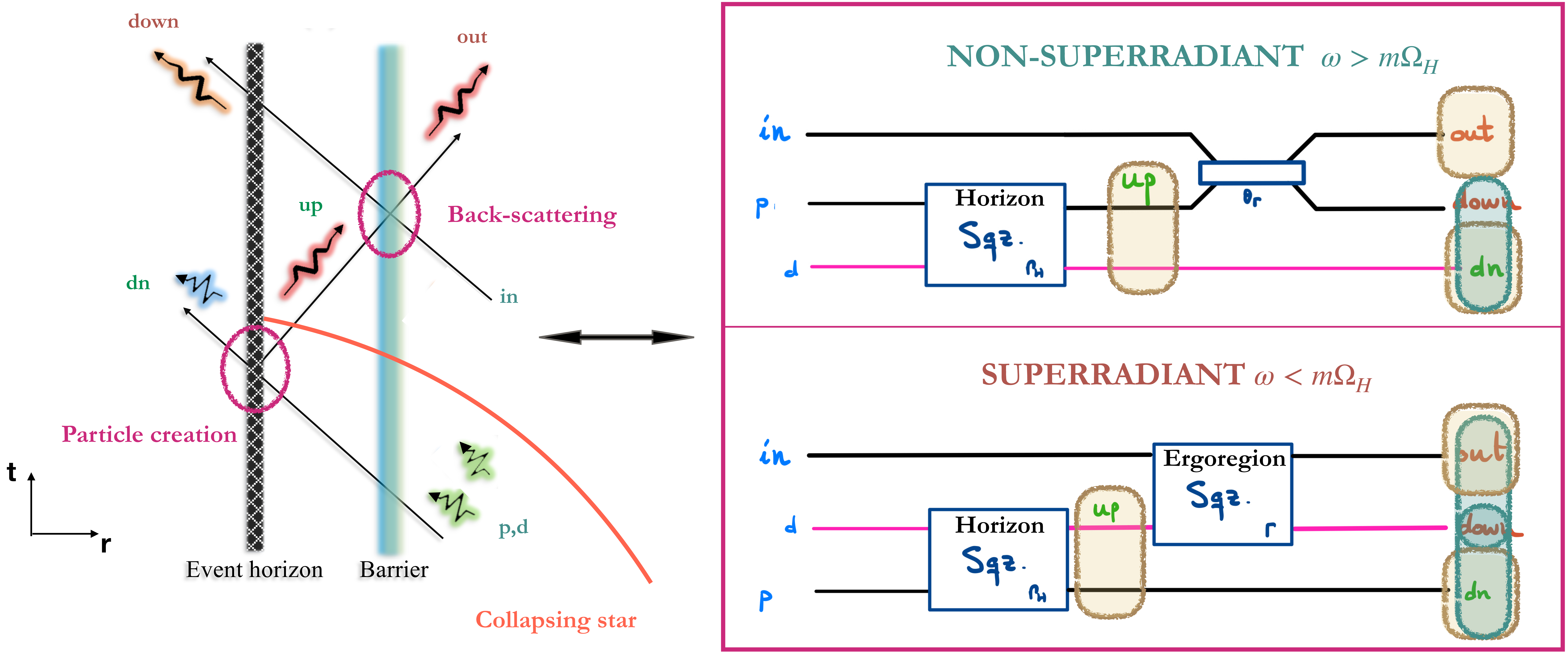


Superradiant







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Back-scattering
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Particle creation
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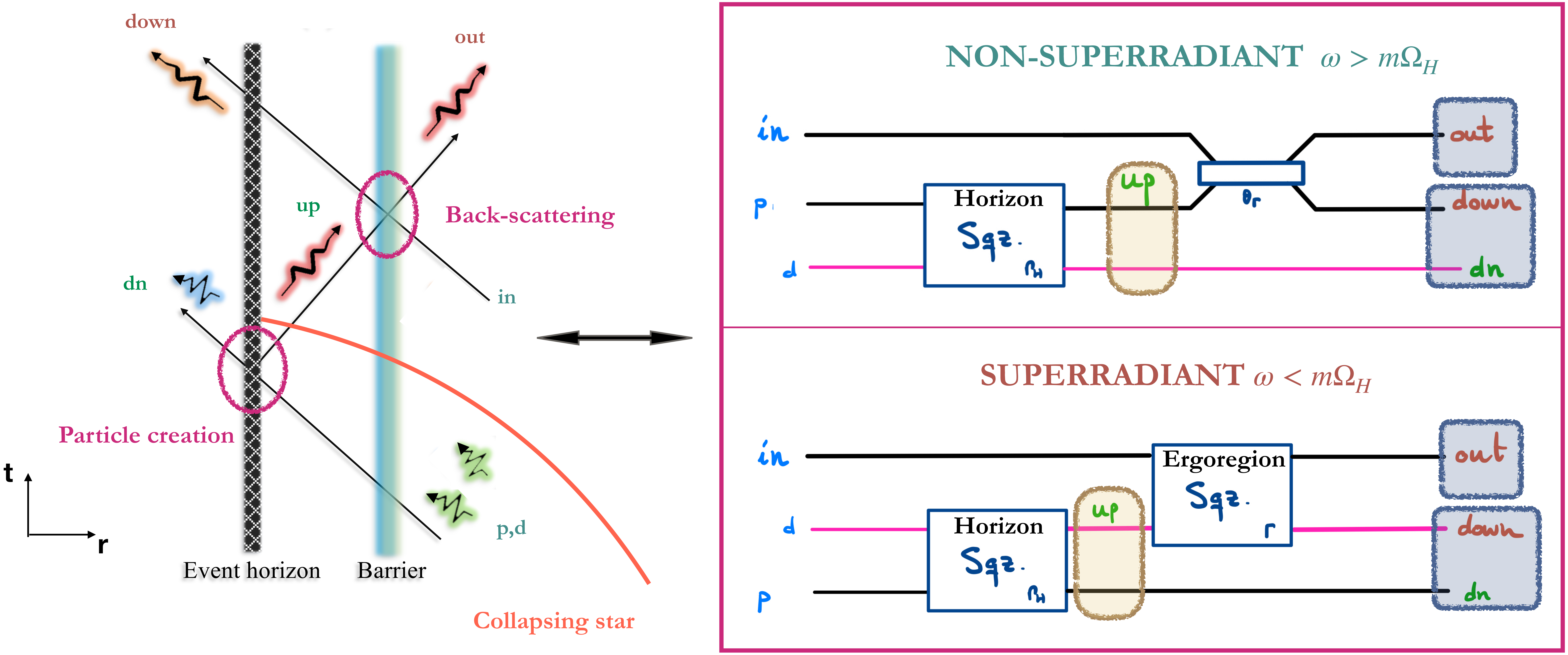

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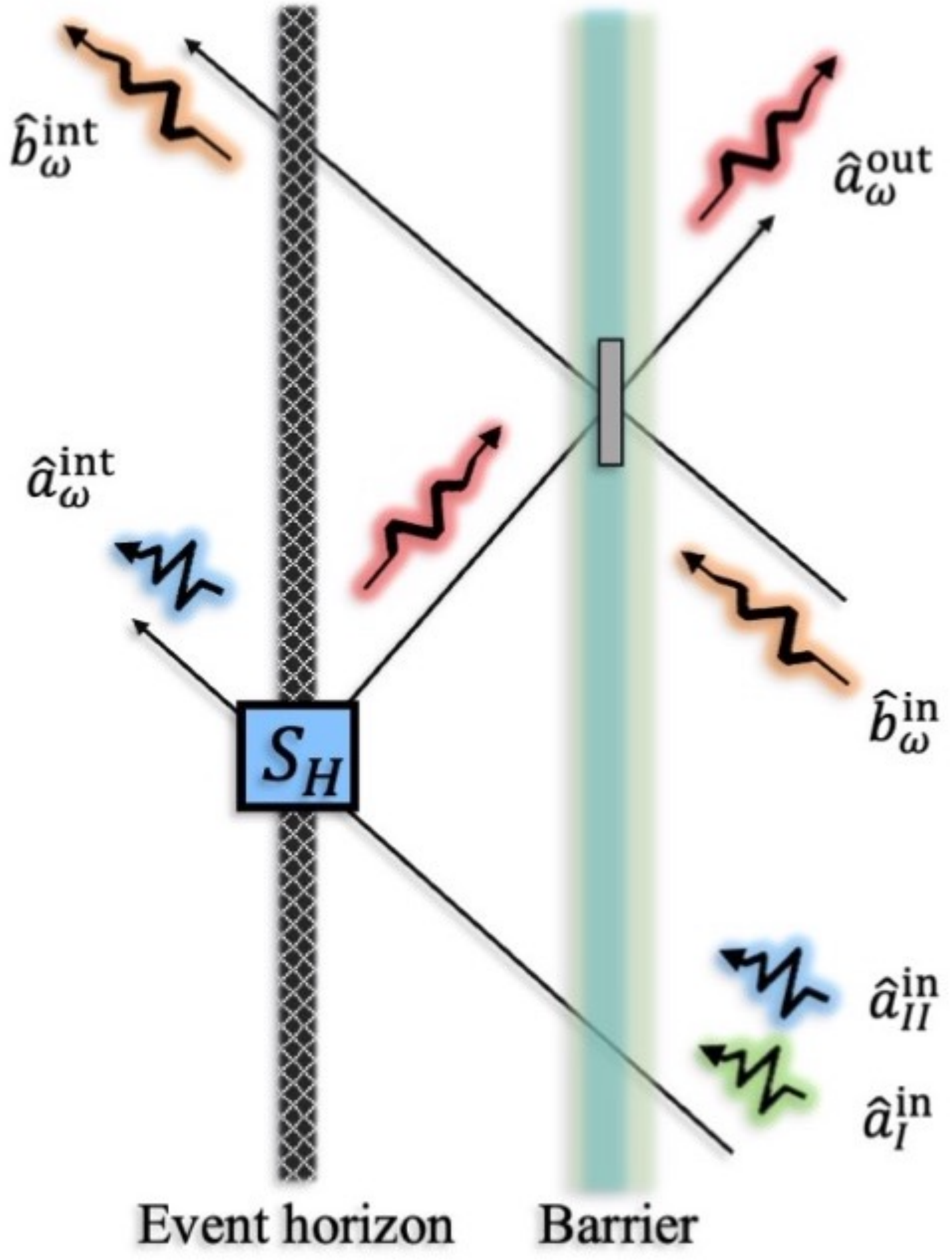
Generation of entanglement during the evaporation

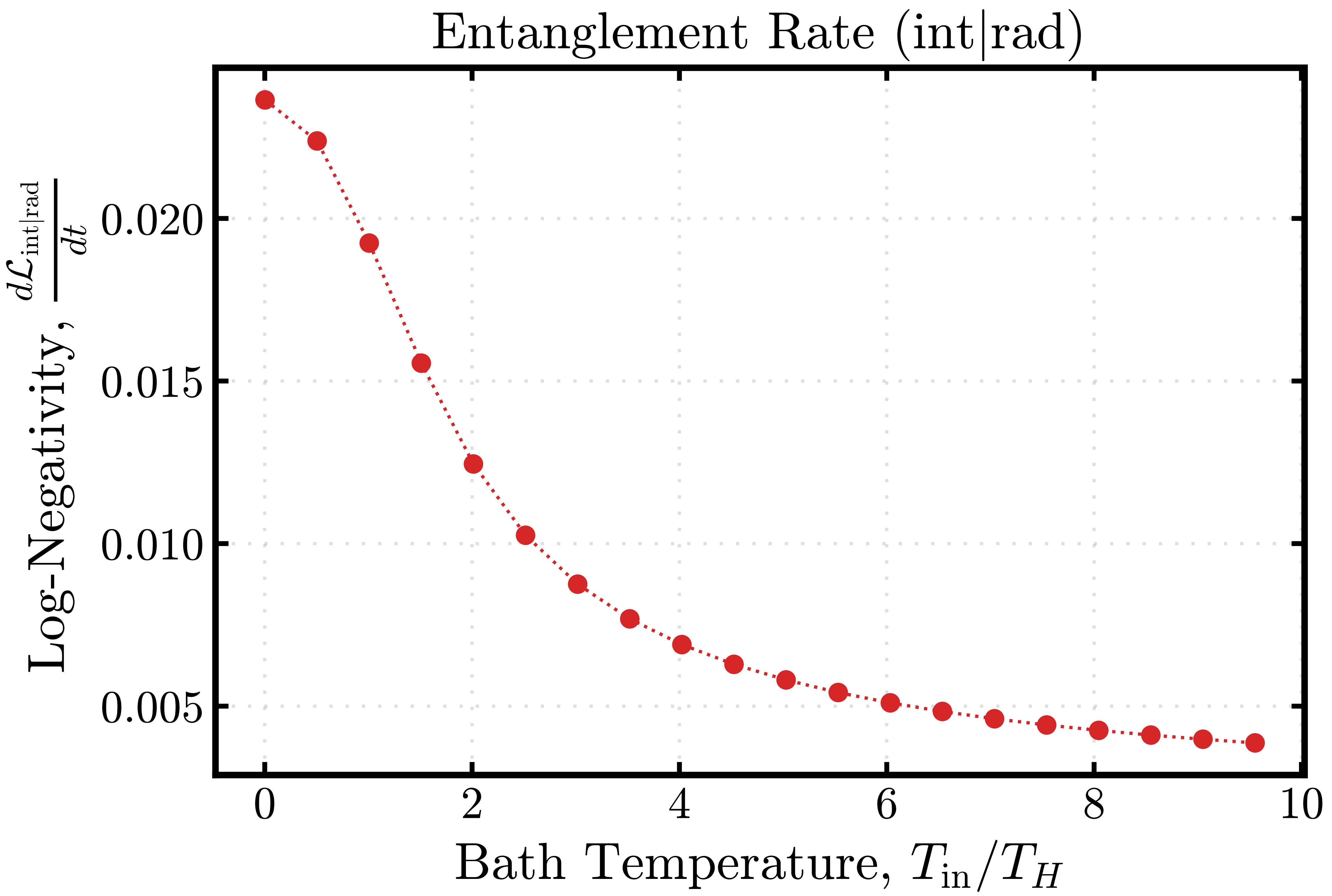
Thermal input

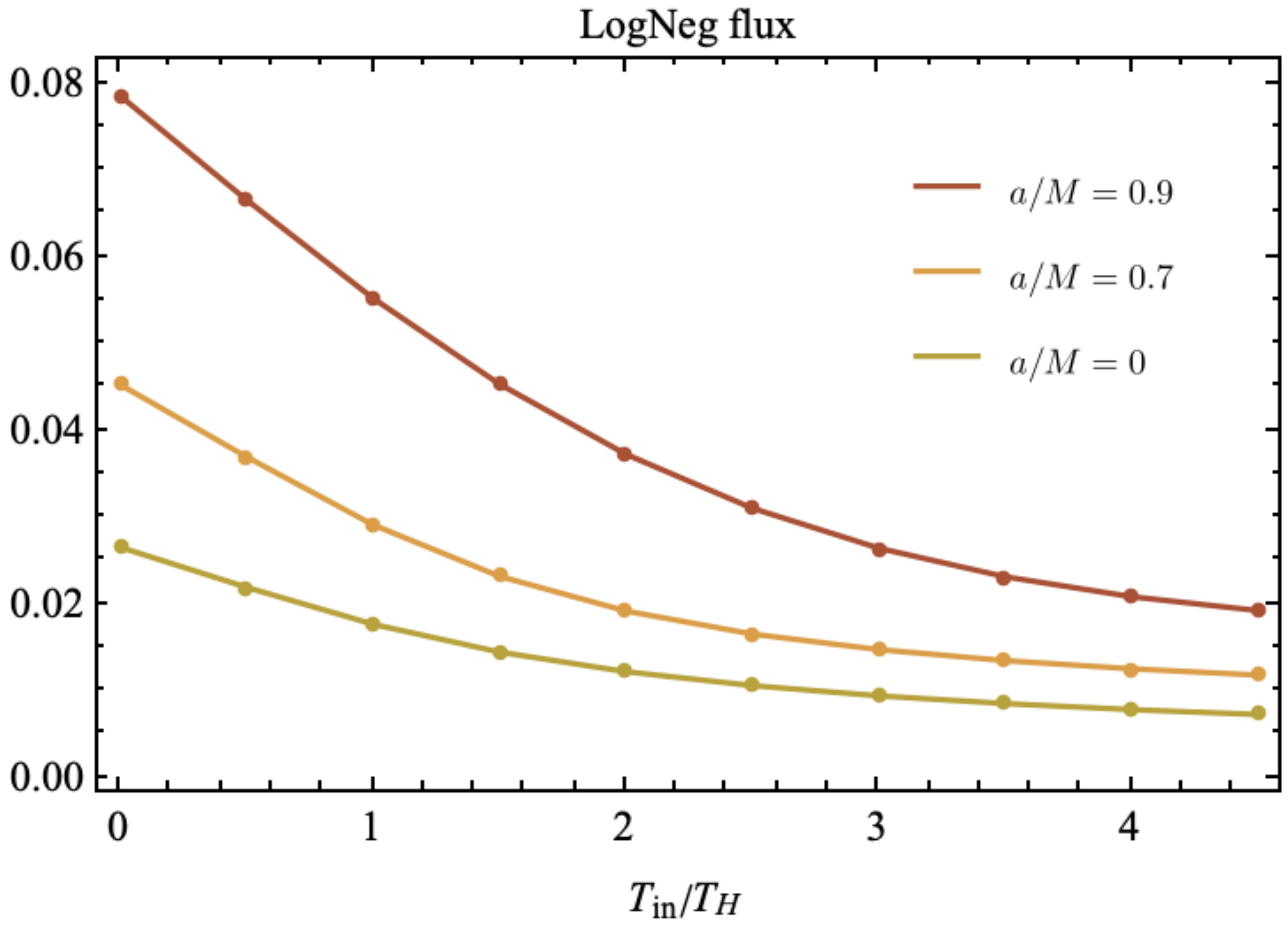
$$\mu^i = 0 \qquad \sigma^{ij} = \oplus_i^N (2\, n_i + 1)\, \mathbb{I}_2$$



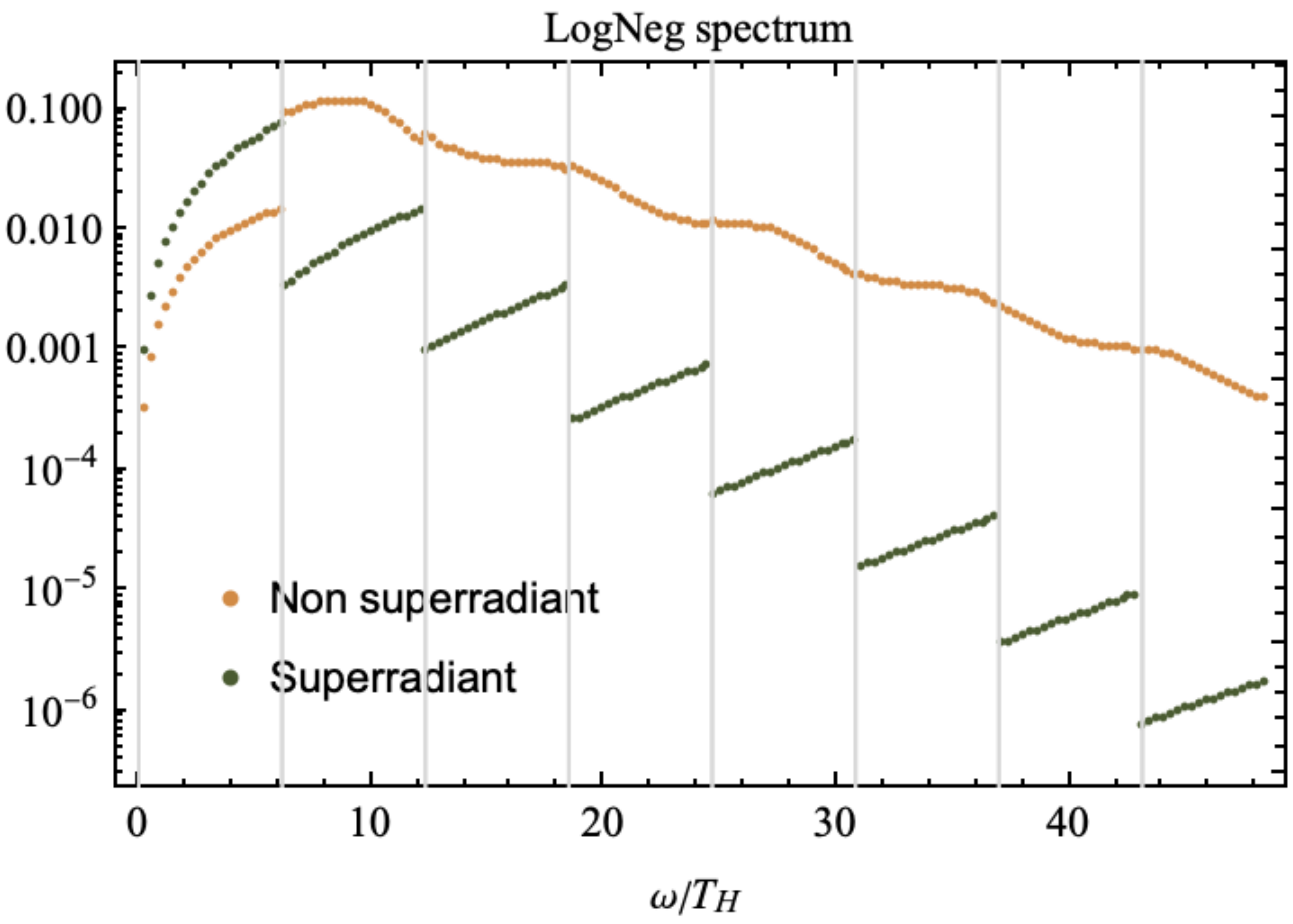
Thermal input **(CMB)**

Vacuum input **(very UV, not seeded by CMB)**

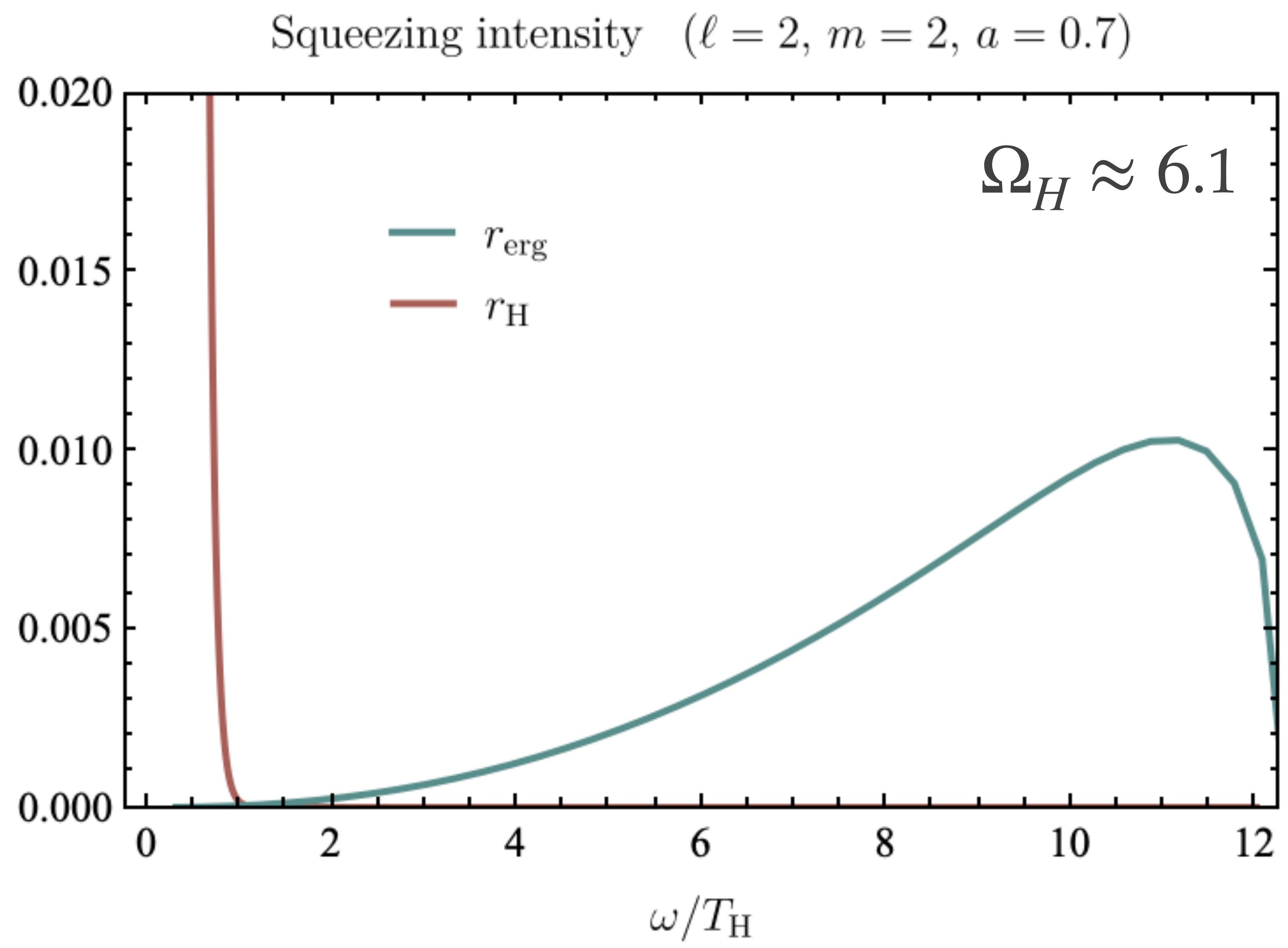




Sum over ℓ, m and w



Sum over ℓ, m



Part II: Rotating analogs with polaritons

In collaboration with: I. Agullo, P. Calizaya Cabrera, K. Falque, K. Guerrero, and M. Jacquet

Analogue gravity with polariton fluids

Driven-dissipative Gross-Pitaevskii equation: $i\partial_t \Psi = \left[\frac{\hbar}{2m}\nabla^2 + V_{ext} + \omega_0 + g\,|\Psi|^2 - i\frac{\gamma}{2} \right] \Psi + E_p$

Polaritons allow a great control on velocity profile through pump (see Killian's talk)

DBT profile: $\vec{v}_p = -\frac{D}{r}\hat{r} + \frac{C}{r}\hat{\theta}$

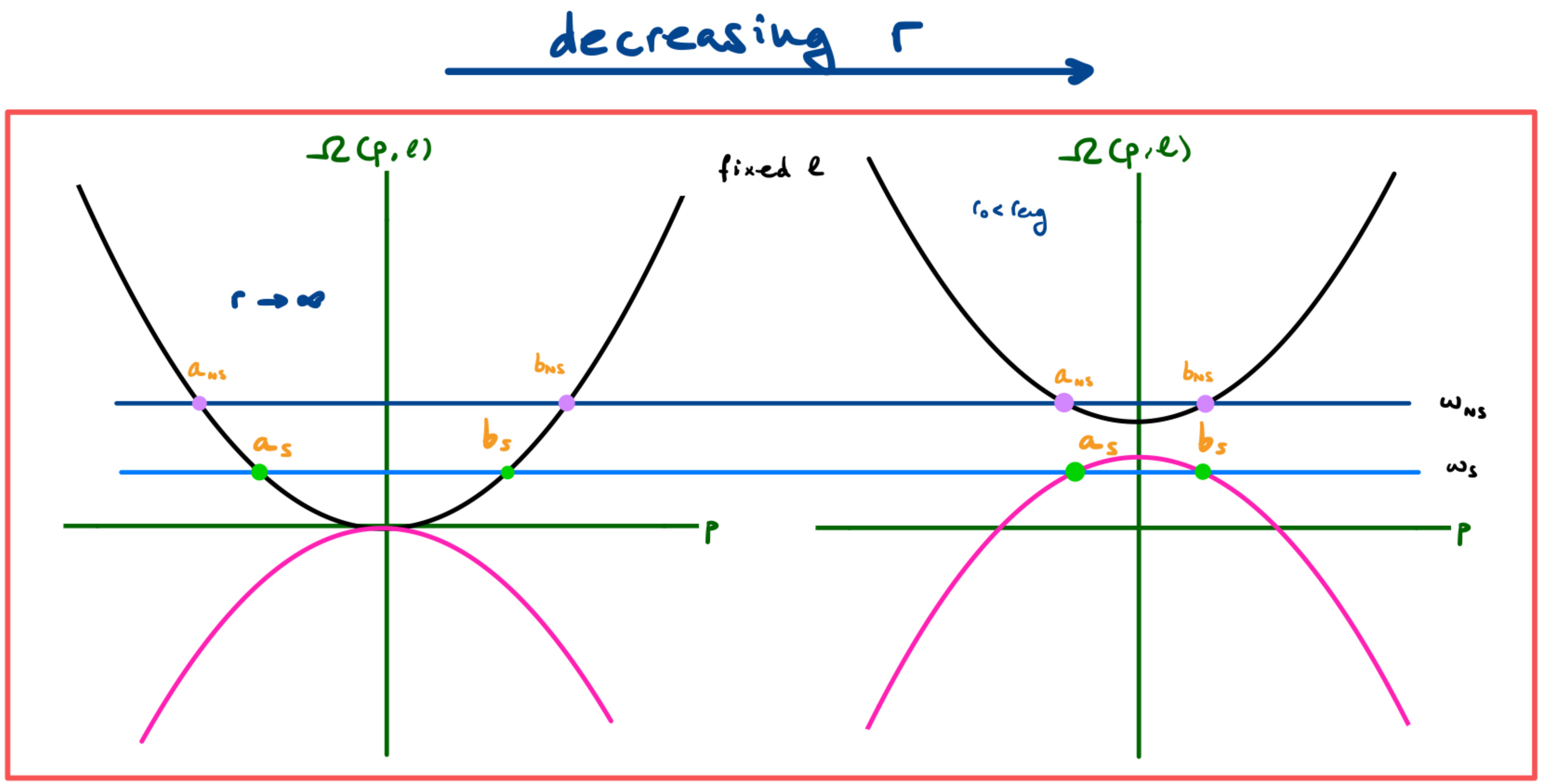
Causal structure: $r_{erg} = \frac{\sqrt{D^2 + C^2}}{c_s^2}$ **and** $r_H = \frac{D}{c_s}$

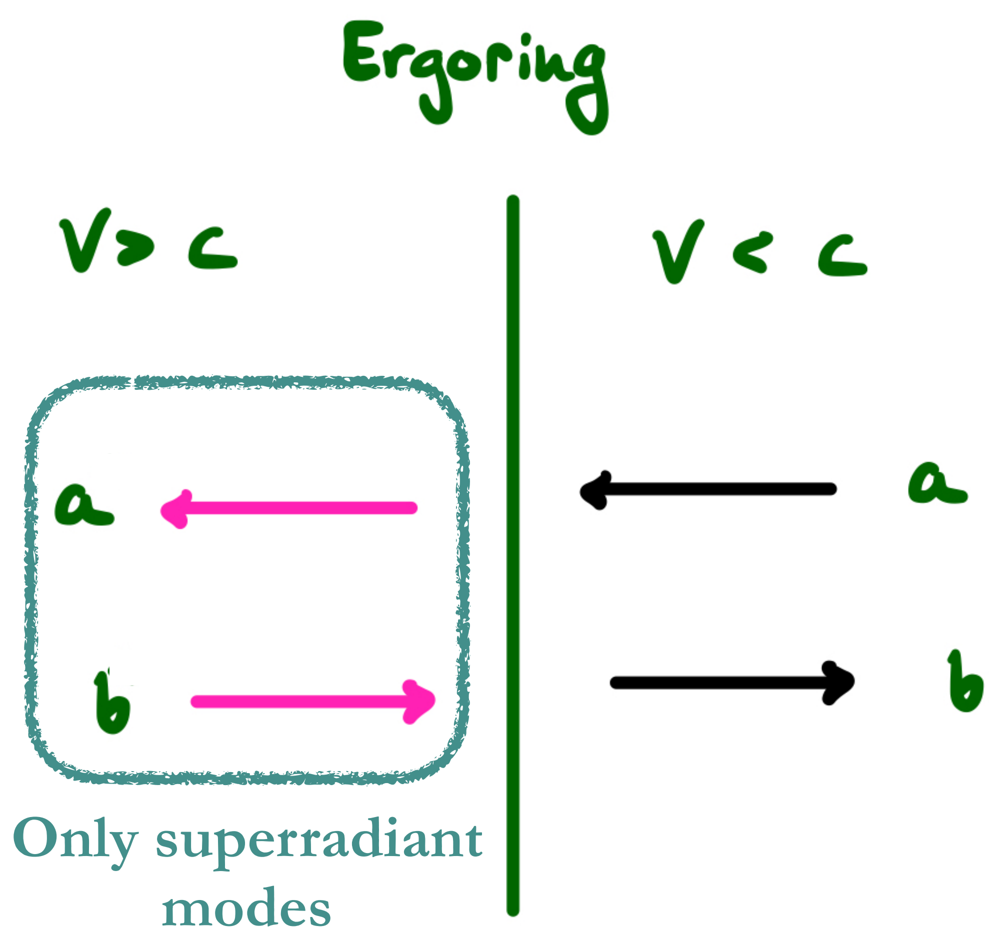
Different mode structures depending on D, C and $c_s(r)$ $\longrightarrow$ **Different analogues**

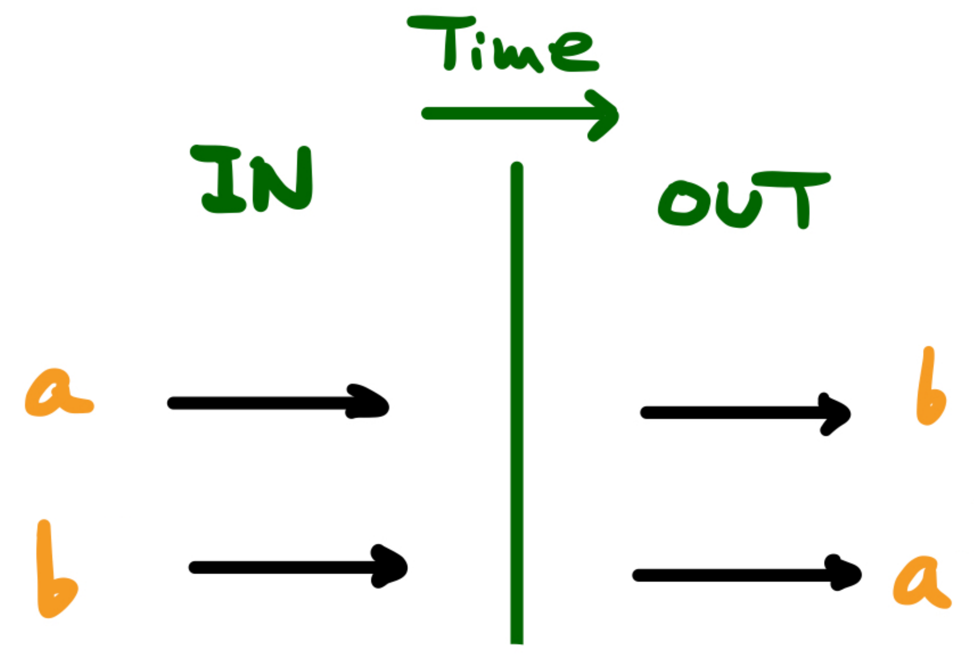
Analogue gravity with polariton fluids

$$\vec{v}_p = \frac{C}{r}\hat{\theta} \longrightarrow \omega = \boxed{\frac{C\ell}{r^2}} \pm F_{WKB}(p, \ell, r)$$

Lifts neg. branch







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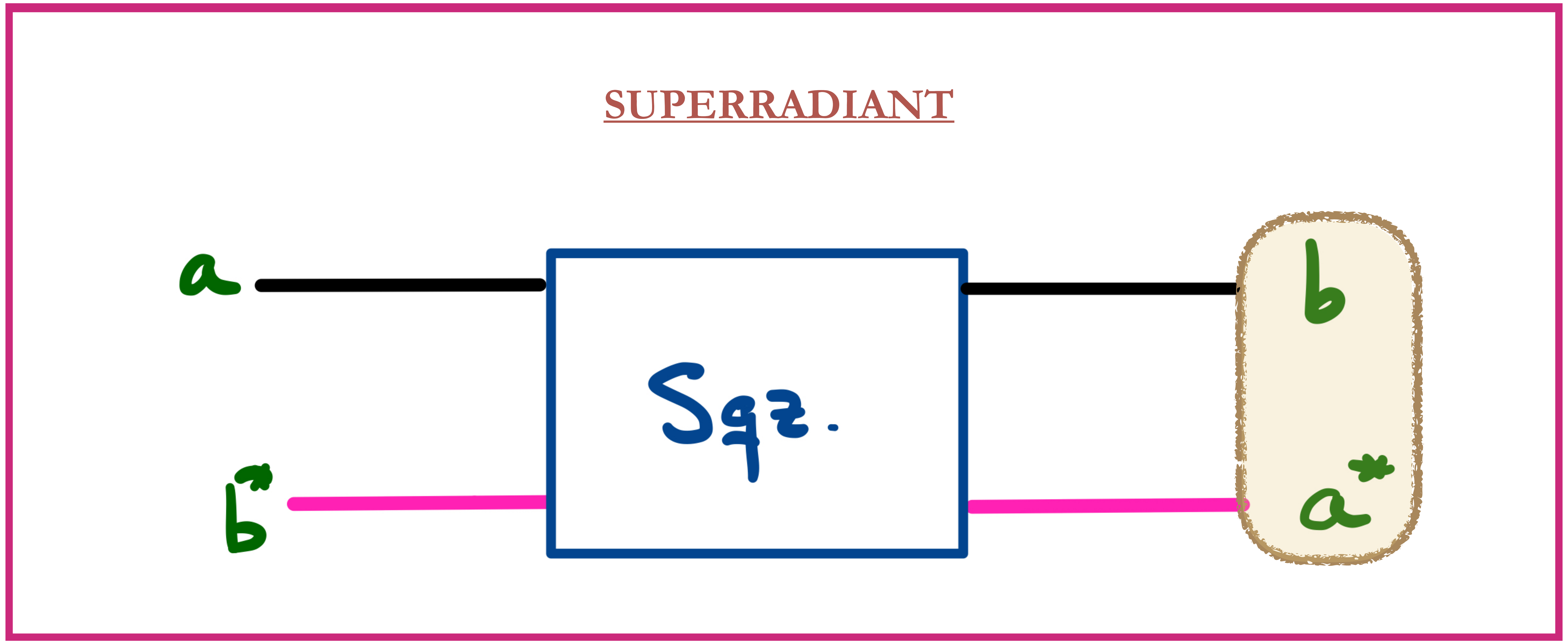
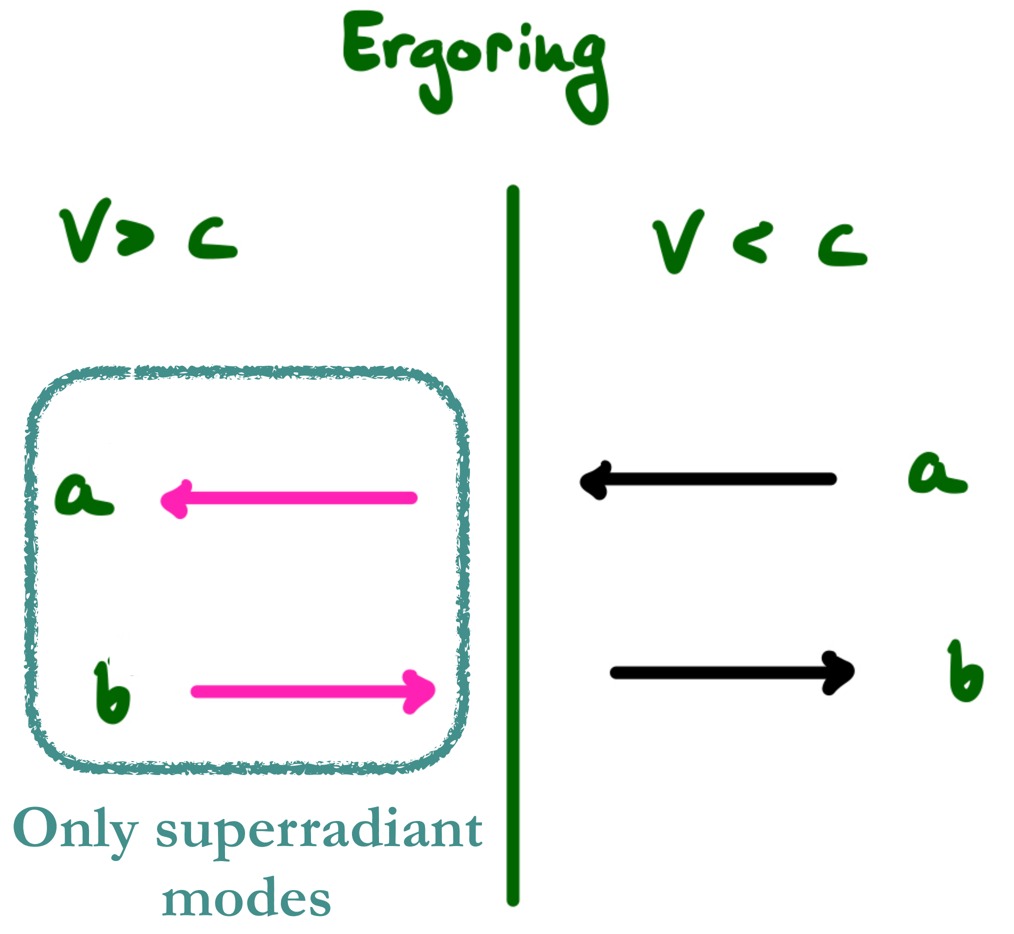
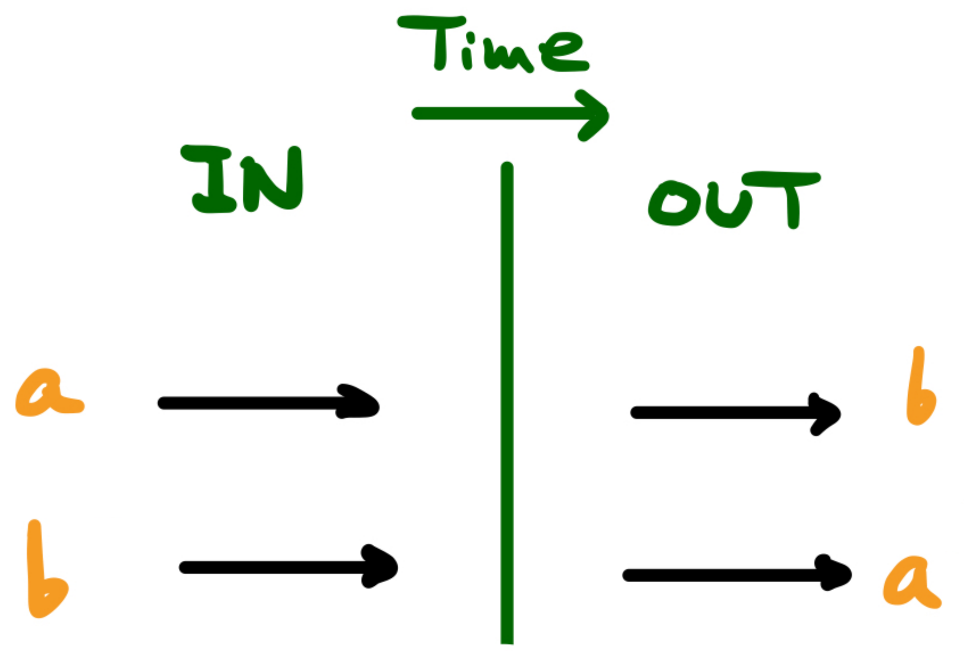


ANALOGUE OF
ISOLATED ERGOREGION

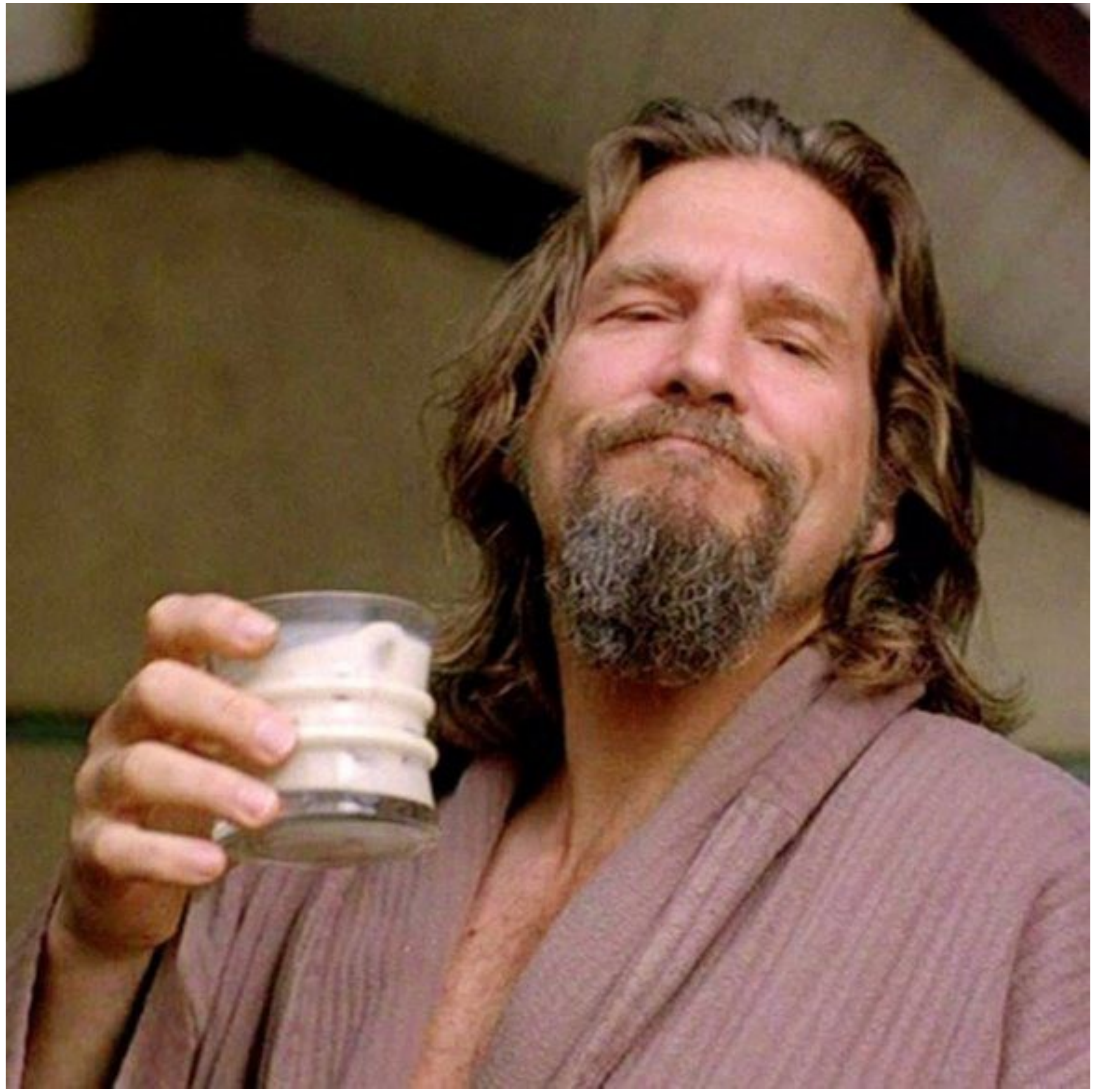




CONCLUDING REMARKS

- Hawking process is two-mode squeezer + beam splitter for non-superradiant modes, and two-mode squeezer + two-mode squeezer for superradiant modes.
- Entanglement is what makes the Hawking effect and superradiant emission quantum.
- CMB radiation degrades the entanglement generated in the Hawking process. Tough never vanishing.
- Entanglement radiated grows with spin. Due to super radiant squeezer.
- Subtle interplay between Horizon and ergoregion in pairwise entanglement.
- Polariton fluids offer great opportunity to study this interplay in the lab!
- Within experimental reach in the next years. (See Killian's amazing data!)

THANKS FOR ABIDING!



BACKUP

- **N-dimensional quantum (bosonic) system:** $\hat{x}_1, \hat{p}_1; \hat{x}_2, \hat{p}_2; \cdots \hat{x}_N, \hat{p}_N \equiv \hat{r}^i$

  C.C.R's: $[\hat{r}^i, \hat{r}^j] = i\,\hbar\,\Omega^{ij}$ $\qquad \Omega^{ij} = \oplus_N \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

- **Generic state** $\hat{\rho}$ **: Need all (infinitely many) moments** $\mathrm{Tr}[\hat{\rho}\, r^{i_1} \ldots r^{i_N}]$ **to fully characterise state.**

- **Gaussian state** $\hat{\rho}$ **: Completely and uniquely determined by its first and second moments**

  $\mu^i \equiv \mathrm{Tr}[\hat{\rho}\,\hat{r}^i]$ **mean**

  $\mathrm{Tr}[\rho\,\hat{r}^i\,\hat{r}^j] \longrightarrow \sigma^{ij} = \mathrm{Tr}[\hat{\rho}\,\{(\hat{r}^i - \mu^i), (\hat{r}^j - \mu^j)\}]$ **covariance matrix**

Examples:

Vacuum: $\mu^i = 0 \quad \sigma^{ij} = \mathbb{I}_{2N}$

Coherent state: $\mu^i \neq 0 \quad \sigma^{ij} = \mathbb{I}_{2N}$

$\Big\}$ **Pure**

Thermal: $\mu^i = 0 \quad \sigma^{ij} = \oplus_i^N (2\,n_i + 1)\,\mathbb{I}_2$ $\Big\}$ **Mixed**

- **Linear time evolution and restriction to a subsystem produce another Gaussian state:**

$$(\mu^i_{\text{in}}, \sigma^{ij}_{\text{in}}) \xrightarrow[S^i_j \text{ = evolution matrix } \in \mathrm{Sp}(2N)]{} (\mu^i_{\text{out}}, \sigma^{ij}_{\text{out}})$$

$$\boxed{\begin{aligned} \vec{\mu}_{\text{out}} &= S \cdot \vec{\mu}_{\text{in}} \\ \sigma_{\text{out}} &= S \cdot \sigma_{\text{in}} \cdot S^\top \end{aligned}}$$

$$\vec{\mu} = (\vec{\mu}_A^{\text{red}}, \vec{\mu}_B^{\text{red}}) \qquad \sigma = \begin{pmatrix} \sigma_A^{\text{red}} & \sigma_{AB} \\ \sigma_{AB}^\top & \sigma_B^{\text{red}} \end{pmatrix}$$

HOW TO QUANTIFY?

- **Entanglement entropy quantifies mixedness. Only equivalent to entanglement if state is pure.**
- **Logarithmic Negavity (based on the PPT criterion) is a convenient quantifier for use:**
  - **Gaussian state and if one two subsystem is a single mode, LogNeg is a faithful quantifier.**
  - **Need the full covariance matrix (full state tomography)**

Example: Two-mode squeezing

Evolution:

$$a_I = \frac{1}{\sqrt{2}}(x_I - i\,p_I)$$

$$\hat{a}_1^{\text{in}} \to \hat{a}_1^{\text{out}} = \hat{a}_1^{\text{in}} \cosh r + \hat{a}_2^{\text{in}\,\dagger} \sinh r$$

$$\hat{a}_2^{\text{in}} \to \hat{a}_2^{\text{out}} = \hat{a}_1^{\text{in}\,\dagger} \sinh r + \hat{a}_2^{\text{in}} \cosh r$$

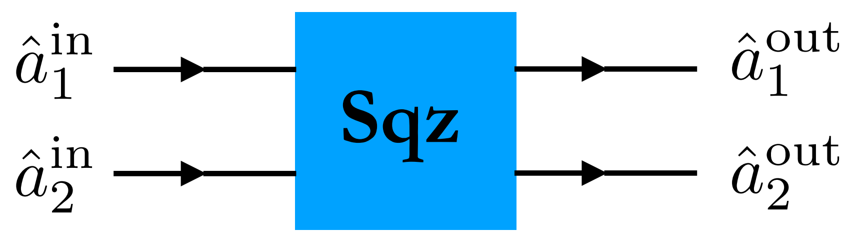
$$\longrightarrow \quad \hat{r}^i_{\text{out}} = S^i{}_j\, \hat{r}^j_{\text{in}}$$

$$\sigma_{\text{out}} = S \cdot \sigma_{\text{in}} \cdot S^\top$$

Entanglement quantifier after acting on vacuum:

$$\text{LogNeg} = \ln_2 e^{2\,r} \quad \text{(e-bits)}$$

where $$S^i{}_j = \begin{pmatrix} \cosh r & 0 & \sinh r & 0 \\ 0 & \cosh r & 0 & -\cosh r \\ \sinh r & 0 & \cosh r & 0 \\ 0 & -\sinh r & 0 & \cosh r \end{pmatrix}$$



Active transformation: Mixing of positive and negative norm modes. Create quanta and entanglement.

Use Wald's Basis to simplify evolution to a $2 \longrightarrow 2$ process.

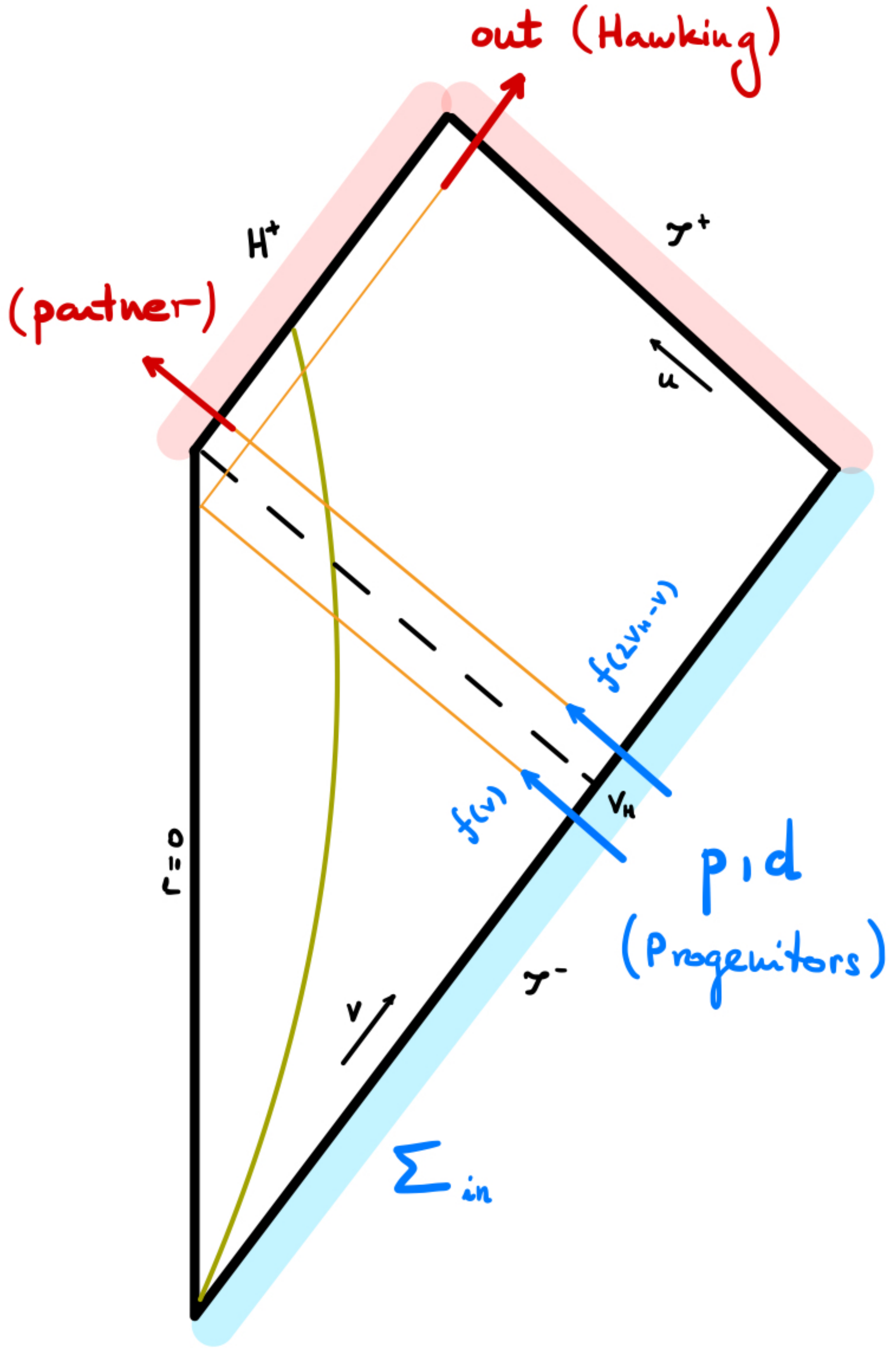
Wald '75

Wald's Basis:

Progenitors of the out modes: $F_p(\omega)$, $F_d(\omega)$

$$F_p(\omega) = N_{\omega\kappa}\left[f(v) + e^{-\frac{\pi\omega}{\kappa}} f(2v_H - v)\right]$$

$$F_d(\omega) = N_{\omega\kappa}\left[f^*(v) + e^{-\frac{\pi\omega}{\kappa}} f^*(2v_H - v)\right]$$



Linear combination of positive-frequency in modes hence define the same in vacuum.

Hawking process in two steps

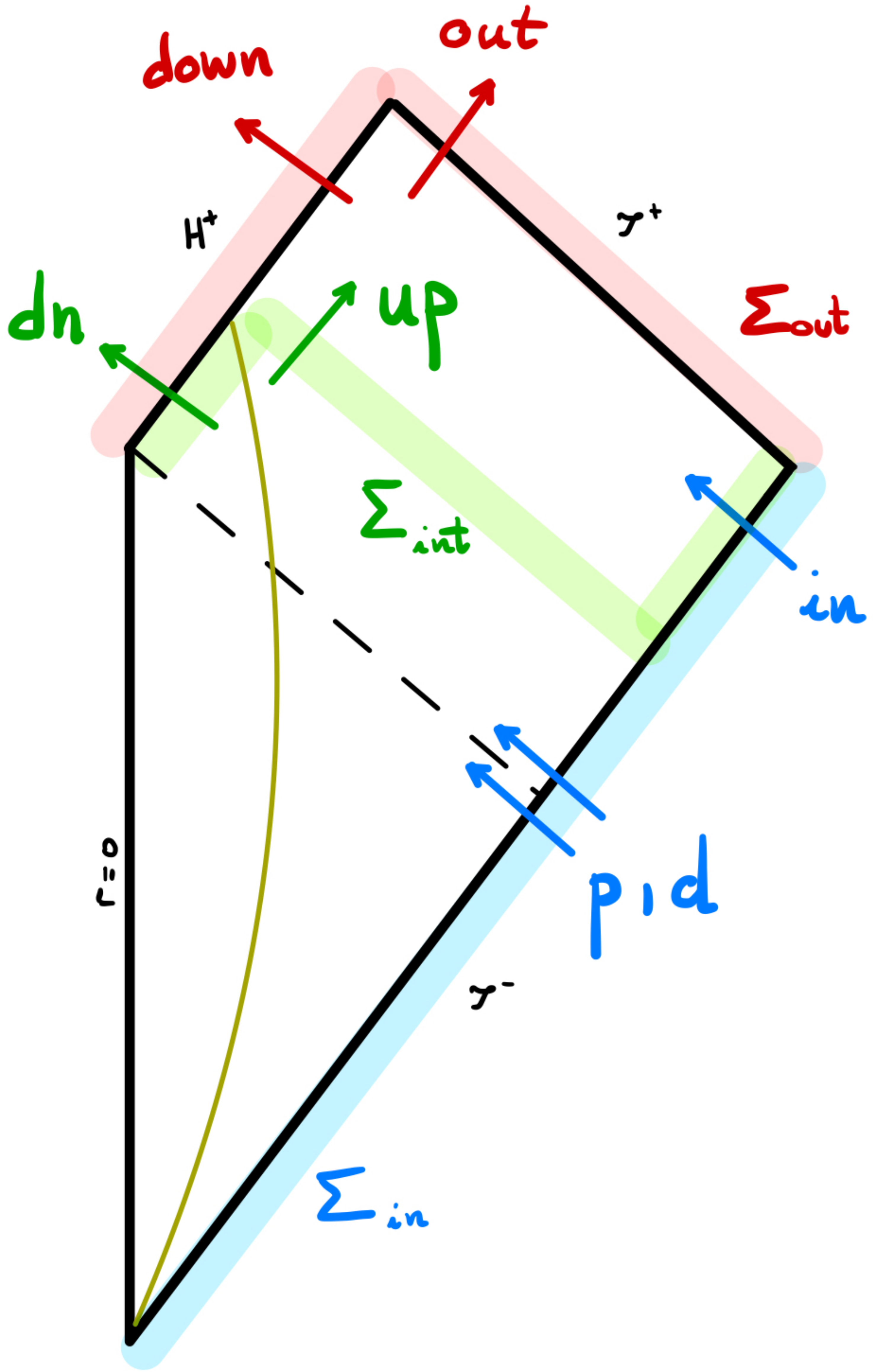
1- Particle creation near horizon, early times: p + d ⟶ up + dn entangled

2- Scattering at potential barrier, late times: up + in ⟶ out + down

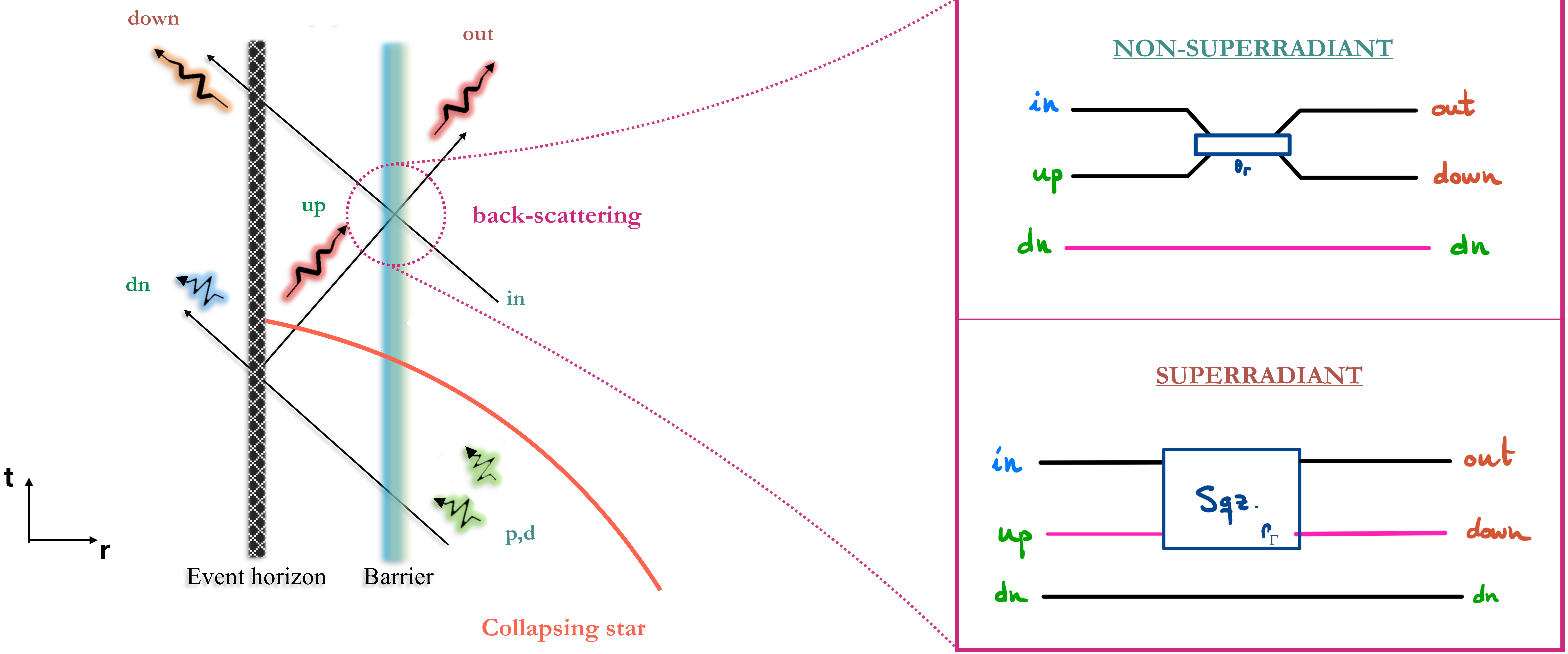
SIGN OF NORM ~ FREQUENCY conjugate to relevant KILLING vector on Σ

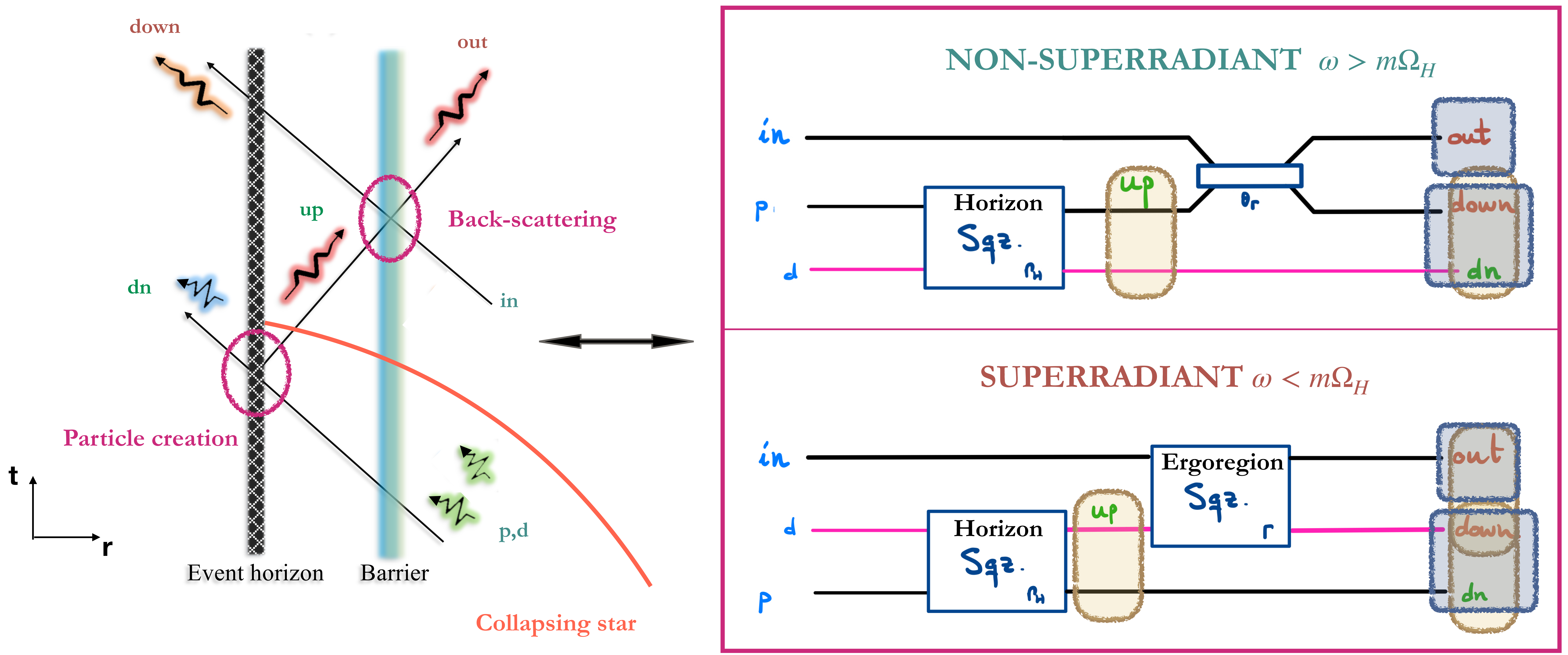
Schwarzschild: $\chi_H = \partial_t \quad \rightarrow \quad \tilde{\omega} = \omega$

Kerr: $\chi_H = \partial_t + \Omega_H \partial_\phi \quad \rightarrow \quad \tilde{\omega} = \boxed{\omega - m\Omega_H}$ Superradiant conditon



Scattering circuits





Non-Sup. evolution matrix: $S_{tot} = S_{BS_H} \cdot S_{SQ_H}$

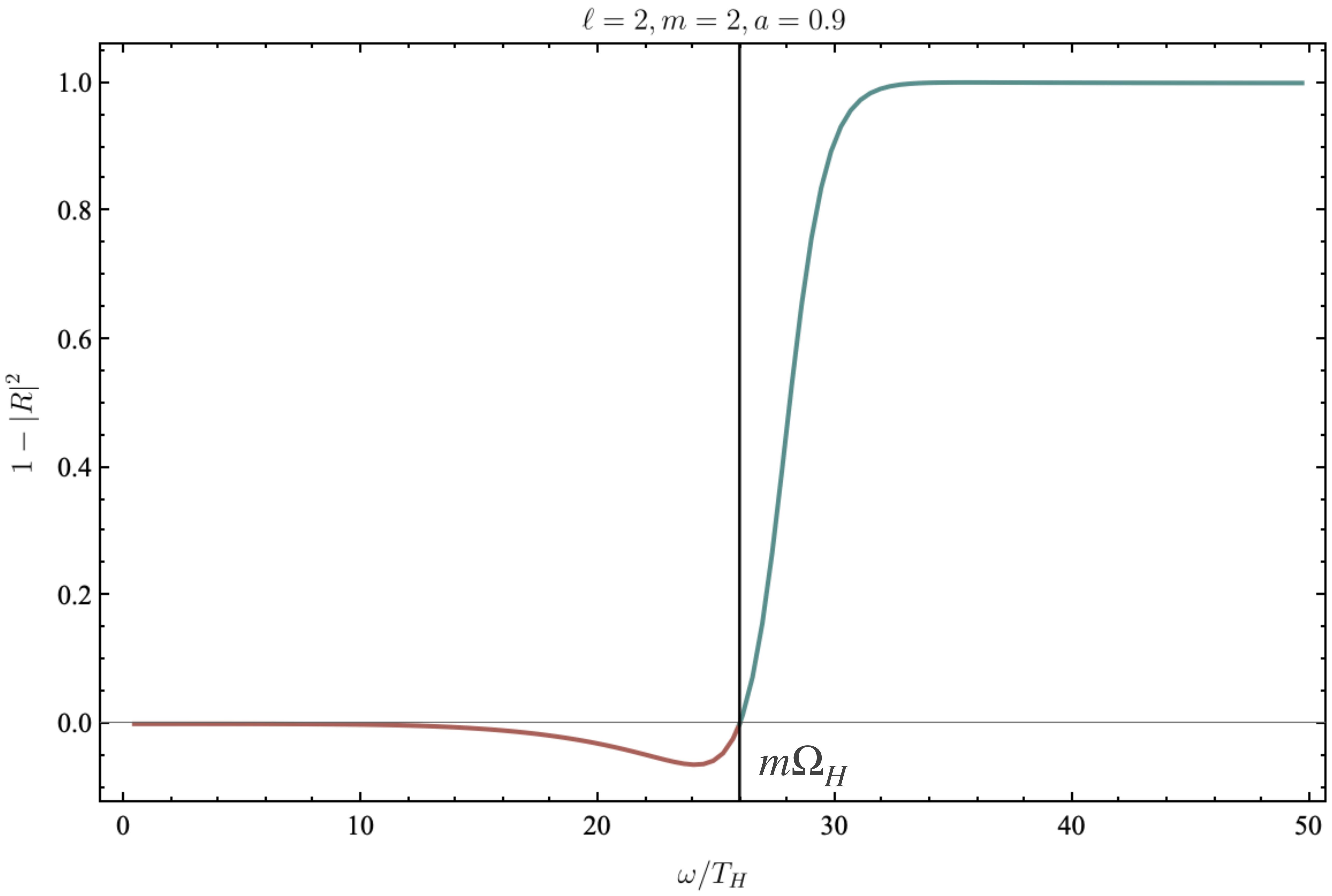
Sup. evolution matrix: $S_{tot} = S_{SQ_\Gamma} \cdot S_{SQ_H}$

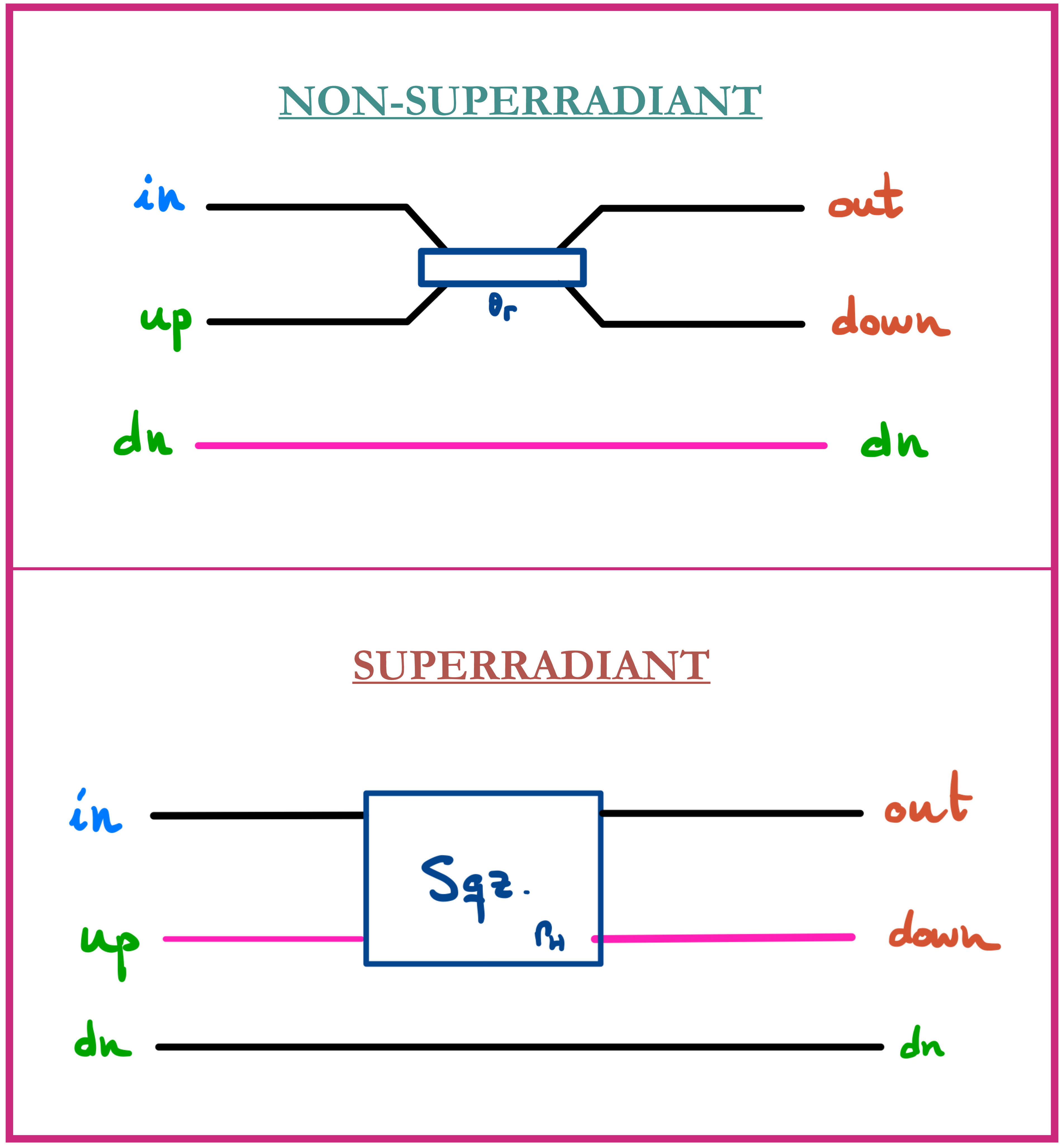
Evolution of "in" state to "out" state: $(\vec{\mu}_{\rm in}, \sigma_{\rm in}) \longrightarrow (\vec{\mu}_{\rm out} = S_{\rm tot} \cdot \vec{\mu}_{\rm in}, \sigma_{\rm out} = S_{\rm tot} \cdot \sigma_{\rm in} \cdot S_{\rm tot}^{\top})$

Greybody factors

- Spin 0, 1/2, 1 and 2 perturbations are separable in Kerr spacetime: Teukolsky equation. Teukolsky '73
- Angular part determined by spin weighted spheroidal harmonics.
  Eigenvalues can be computed in an expansion for small $a\omega$. Seidel '89
- Choose physical boundary conditions: no outgoing mode at horizon.
- Numerical errors for radial equation can be unstable due to exciting unwanted modes.
  Solution: Use a smart choice of radial functions to solve for.
- Extract greybody factors from transmission/reflection coefficients.
  (Relations between mode amplitudes at asymptotic and horizon).
- Superradiance is automatically accounted for from the computed value of the

Scattering circuits





Vacuum Input

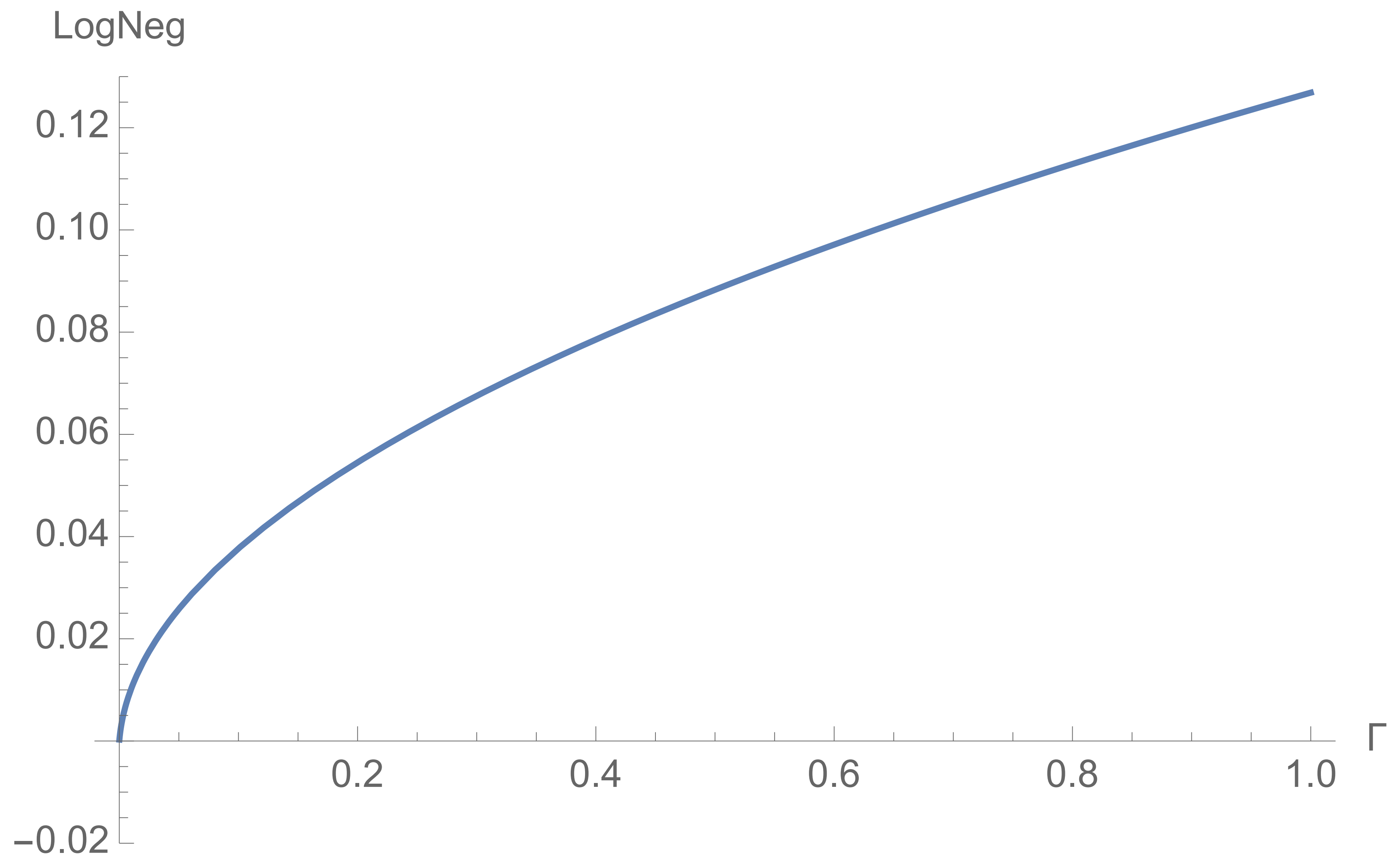
$$\vec{\mu}_{\mathrm{in}} = \vec{0} \qquad \sigma_{\mathrm{in}} = \mathbb{I}_6$$

Result of evolution:

$$\langle \hat{n}_{\text{out}}(w) \rangle = \Gamma_\ell(w)\, \sinh^2 r_H(w) = \frac{\Gamma_\ell(w)}{e^{w/T_H} - 1}$$

We recover the correct emission rate!

$\text{LogNeg}[\hat{a}_w^{\text{out}} | (\hat{a}_I^{\text{int}}, \hat{b}_w^{\text{int}})]$



Plot corresponding to Schwarzschild BH, $\ell = 1,\ w = 0.25\, M$

The potential barrier degrades the entanglement carried out to infinity

Analogue gravity with polariton fluids (and BECs)

Polaritons are bound states of photon-exciton pairs that occur in a semiconductor cavity when stimulated with light

Driven-dissipative Gross-Pitaevskii equation: $i\partial_t \Psi = \left[\frac{\hbar}{2m}\nabla^2 + V_{ext} + \omega_0 + g|\Psi|^2 - i\frac{\gamma}{2}\right]\Psi + E_p$

Stationary backgrounds for monochromatic pump: $E_p = E_{\omega_p} e^{-i\omega_p t}$ **and** $\Psi(\vec{x}, t) = \Psi_p(\vec{x}) e^{-i\omega_p t}$

$$\left[-\frac{\hbar}{2m}\nabla^2 - \delta_p + g|\Psi_p|^2 - i\frac{\gamma}{2}\right]\Psi_p + E_{\omega_p} = 0$$

where $\delta_p = \omega_p - \omega_0 - V_{ext}$

Perturbations around stationary background: $\Psi(\vec{x}, t) = \sqrt{n_p}(1+\varphi)e^{i(S_p - \omega_p t)}$ **with** $\vec{v}_p = \frac{\hbar}{m}\vec{\nabla} S_p$

$$i\left(\partial_t + \vec{v}_p \cdot \vec{\nabla}\right)\varphi = -\left[\frac{\hbar}{2m}\nabla^2 + \frac{E_{\omega_p} e^{-iS_p}}{\sqrt{n_p}}\right]\varphi + gn_p\left[\varphi + \varphi^\dagger\right]$$

WKB dispersion relation

$$\omega - \vec{v}_p \cdot \vec{k} = -i\frac{\gamma}{2} \pm \boxed{c_p}\sqrt{\frac{\hbar^2}{4m^2c_p^2}k^4 + k^2 + \frac{m^2c_p^2}{\hbar^2}\left(1 - \frac{\hbar^2 g^2 n_p^2}{m^2 c_p^4}\right)}$$

Pump-dependent SOUND SPEED

Low k approximation: Relativistic massive field

$$\omega - \vec{v}_p \cdot \vec{k} = \pm c\sqrt{k^2 + \left(\frac{m^2c^2}{\hbar^2} - \frac{g^2n_p^2}{c^2}\right)} + \mathcal{O}(k^2)$$

$\vec{v}_p$-dependent acoustic metric

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Pump-dependent SOUND SPEED

Pump frequency can be turned to make dispersion gapless

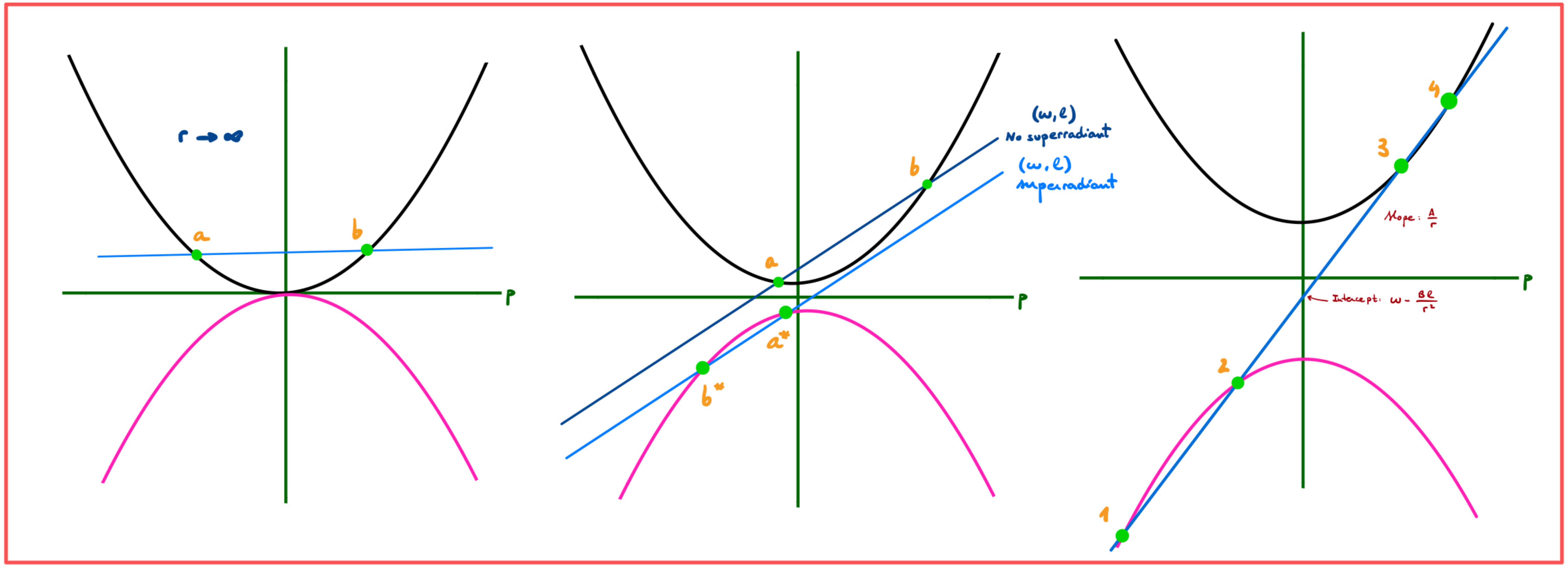
$$\Delta_p = \delta_p - \frac{m}{2\hbar} v_p^2 = g n_p \longrightarrow m c_p^2 = \hbar g n_p$$

Analogue gravity with polariton fluids (and BECs)

$$\omega + \underbrace{\boxed{\frac{A}{r}}}_{\text{Slope}} p - \underbrace{\boxed{\frac{B}{r^2}}}_{\text{Ordinate intercept}} \ell = -i\frac{\gamma}{2} \pm c_p \sqrt{\frac{\hbar^2}{4m^2c_p^2}p^4 + c_p^2\left(1 + \frac{\hbar^2}{2m^2c_p^2}\frac{\ell^2}{r^2}\right)p^2 + \underbrace{\boxed{\frac{\hbar^2}{4m^2c_p^2}\frac{\ell^4}{r^4} + \frac{\ell^2}{r^2}}}_{\text{r-dependent mass gap}} + \frac{m^2c_p^2}{\hbar^2}\left(1 - \frac{\hbar^2 g^2 n_p^2}{m^2 c_p^4}\right)}$$

Different mode structures depending on A, B and $c_p(r)$ ⟶ Different analogues

decreasing r ⟶

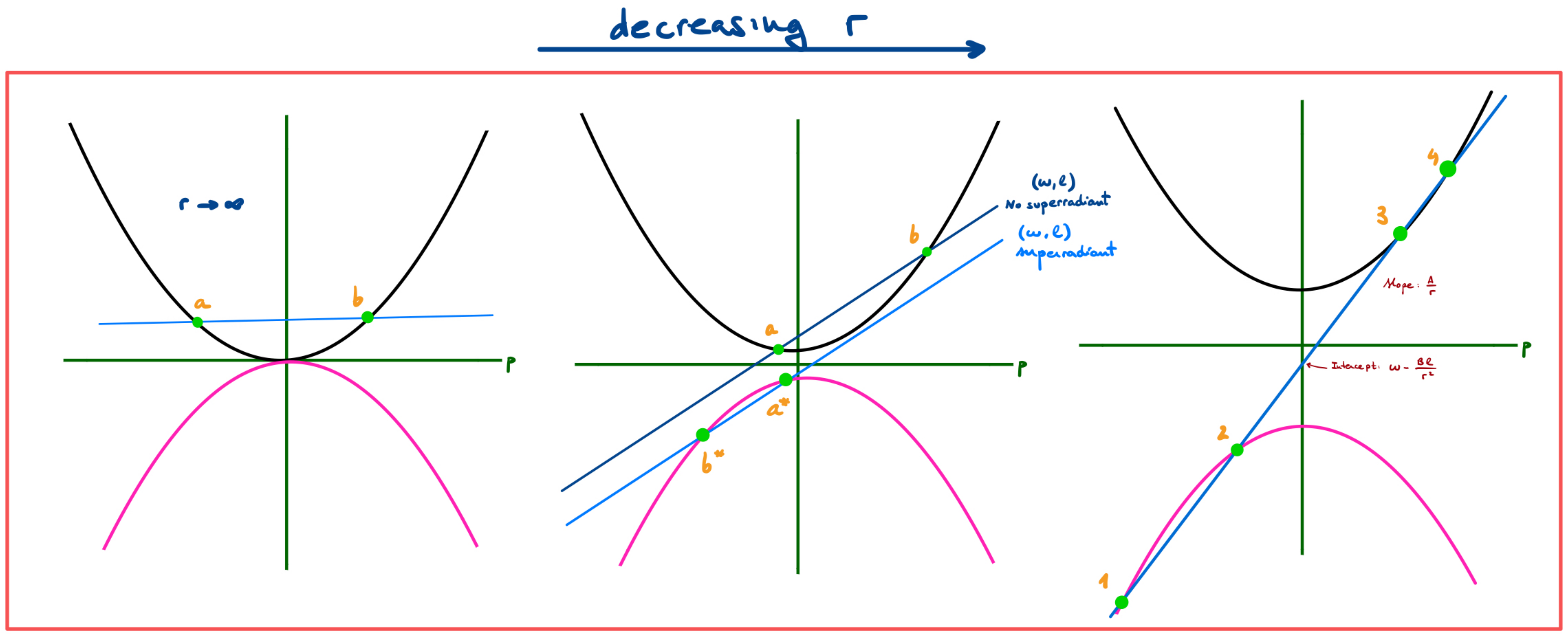


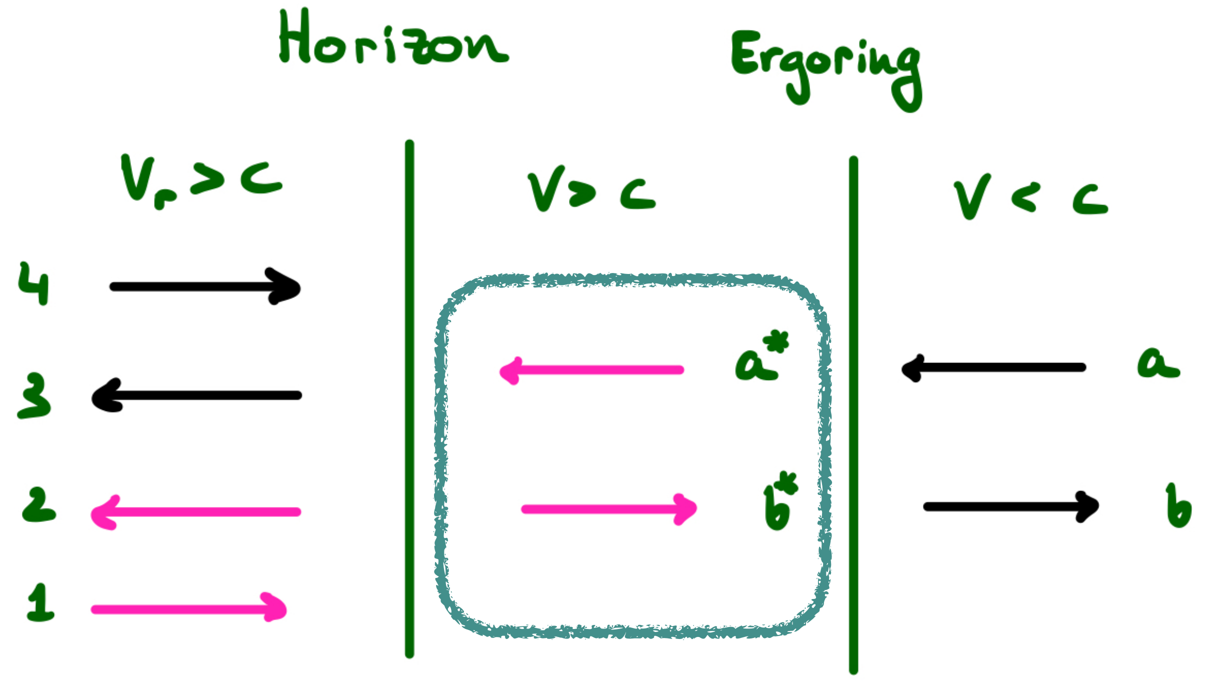
Analogue gravity with polariton fluids (and BECs)

$$\omega + \boxed{\frac{A}{r}}p - \boxed{\frac{B}{r^2}}\ell = -i\frac{\gamma}{2} \pm c_p\sqrt{\frac{\hbar^2}{4m^2c_p^2}p^4 + c_p^2\left(1+\frac{\hbar^2}{2m^2c_p^2}\frac{\ell^2}{r^2}\right)p^2 + \boxed{\frac{\hbar^2}{4m^2c_p^2}\frac{\ell^4}{r^4}+\frac{\ell^2}{r^2}} + \frac{m^2c_p^2}{\hbar^2}\left(1-\frac{\hbar^2 g^2 n_p^2}{m^2c_p^4}\right)}$$

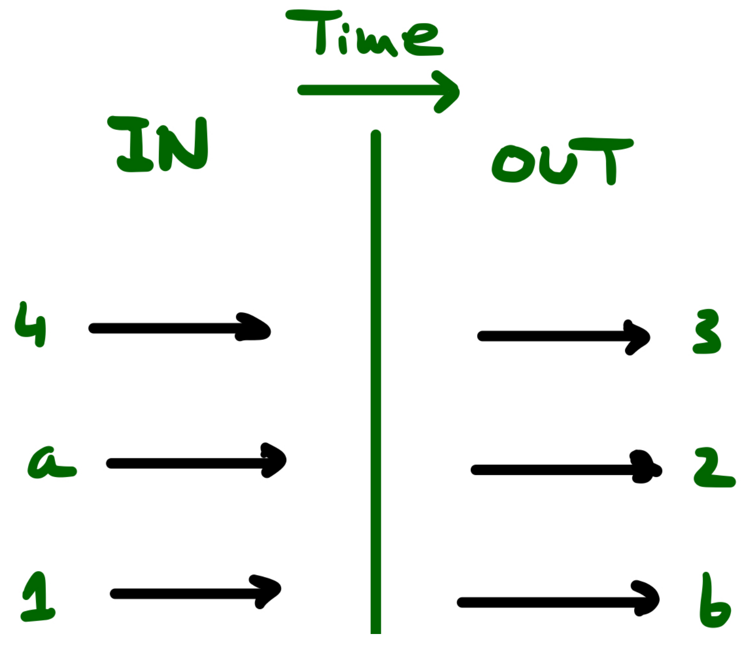
Slope　Ordinate intercept　r-dependent mass gap

Different mode structures depending on A, B and $c_p(r)$ ⟶ Different analogues





Only superradiant modes



Analogue gravity with polariton fluids (and BECs)

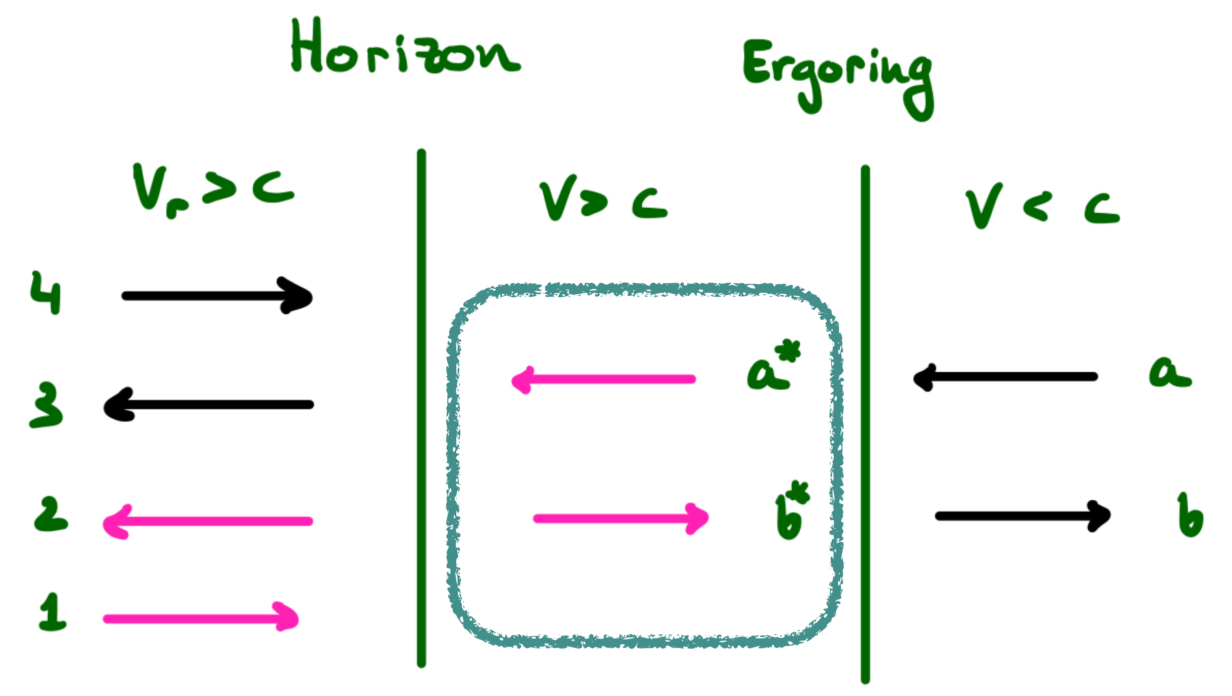
$$\omega + \boxed{\frac{A}{r}}p - \boxed{\frac{B}{r^2}}\ell = -i\frac{\gamma}{2} \pm c_p\sqrt{\frac{\hbar^2}{4m^2c_p^2}p^4 + c_p^2\left(1+\frac{\hbar^2}{2m^2c_p^2}\frac{\ell^2}{r^2}\right)p^2 + \boxed{\frac{\hbar^2}{4m^2c_p^2}\frac{\ell^4}{r^4}+\frac{\ell^2}{r^2}} + \frac{m^2c_p^2}{\hbar^2}\left(1-\frac{\hbar^2g^2n_p^2}{m^2c_p^4}\right)}$$

Slope

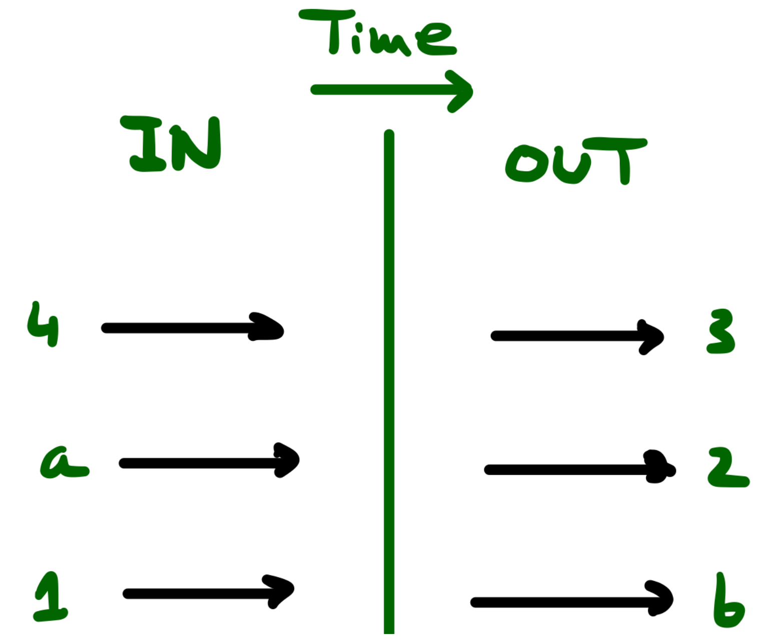
Ordinate intercept

r-dependent mass gap

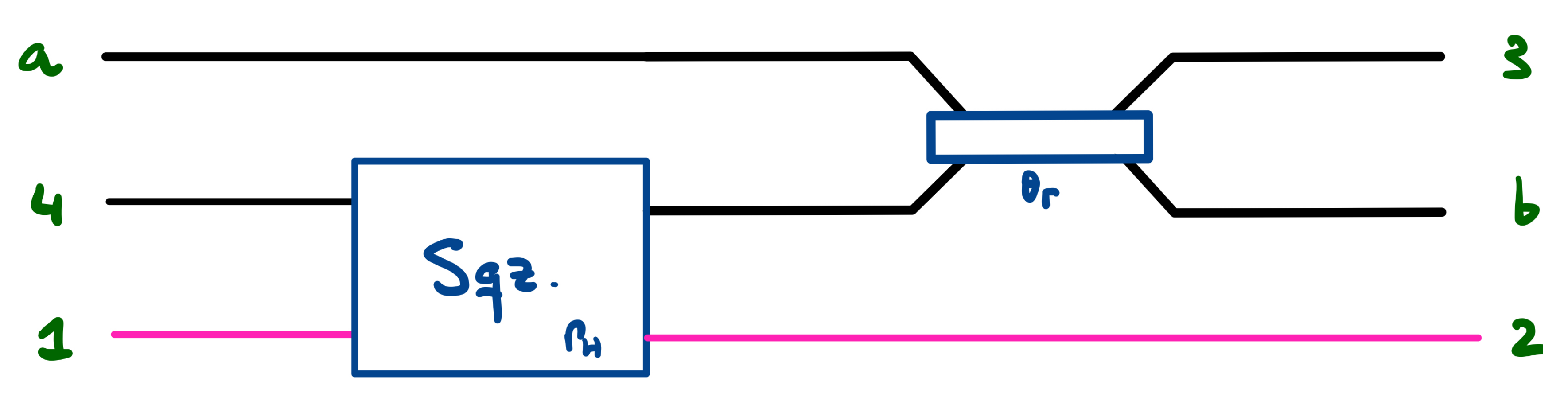
Different mode structures depending on A, B and $c_p(r)$ ⟶ Different analogues



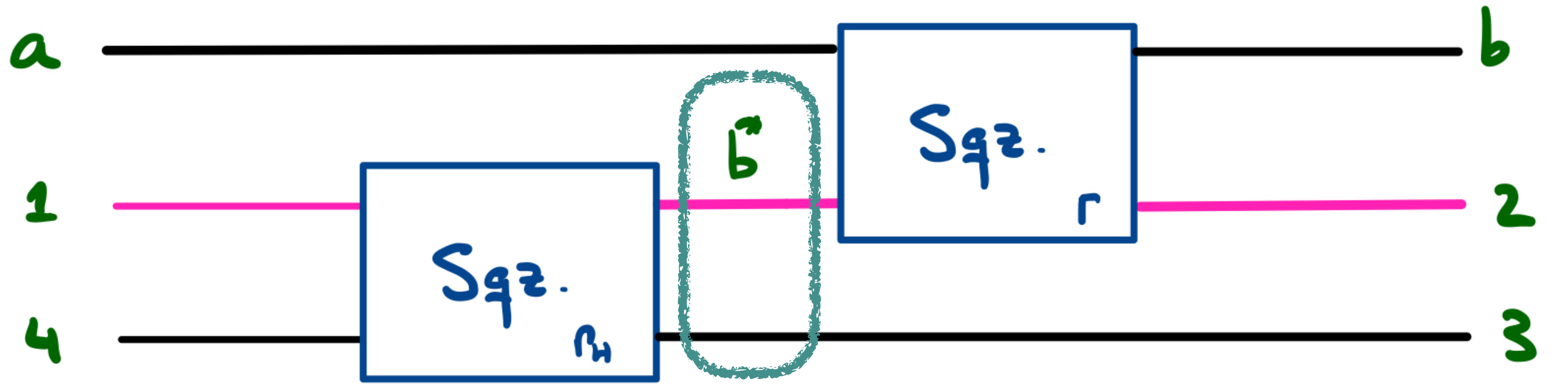
Only superradiant modes



NON-SUPERRADIANT

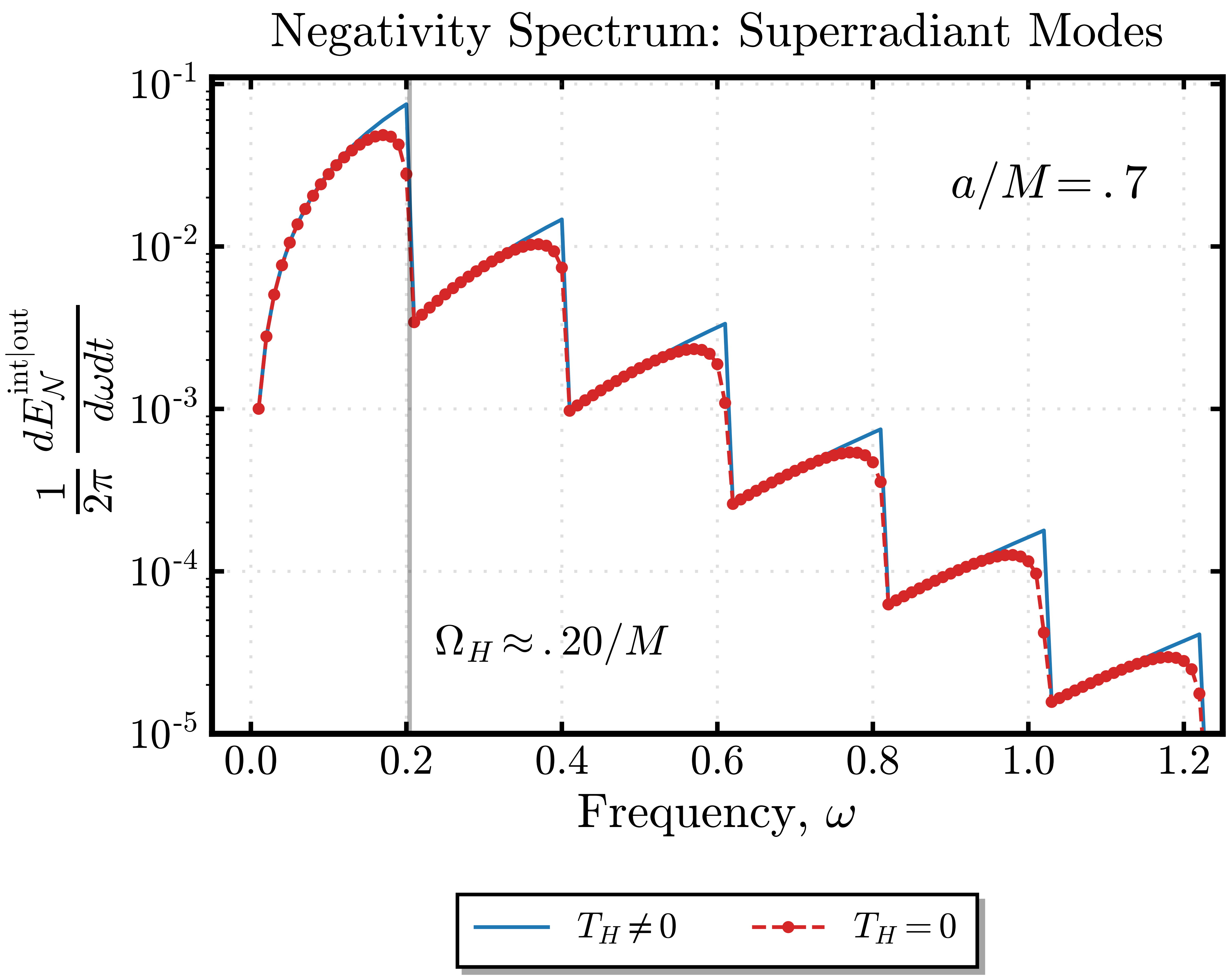


SUPERRADIANT



THIS IS A KERR ANALOGUE!

Practical problem: need gapless dispersion


Negativity Spectrum: Superradiant Modes
$a/M = .7$
$\Omega_H \approx .20/M$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0.0
0.2
0.4
0.6
0.8
1.0
1.2
Frequency, ω
$\frac{1}{2\pi}\frac{dE_{\mathcal{N}}^{\text{int}|\text{out}}}{d\omega dt}$
$T_H \neq 0$
$T_H = 0$