UNIVERSALITY OF QUANTUM TIME DILATION

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Universality of time dilation

- classical effect

Universality of time dilation

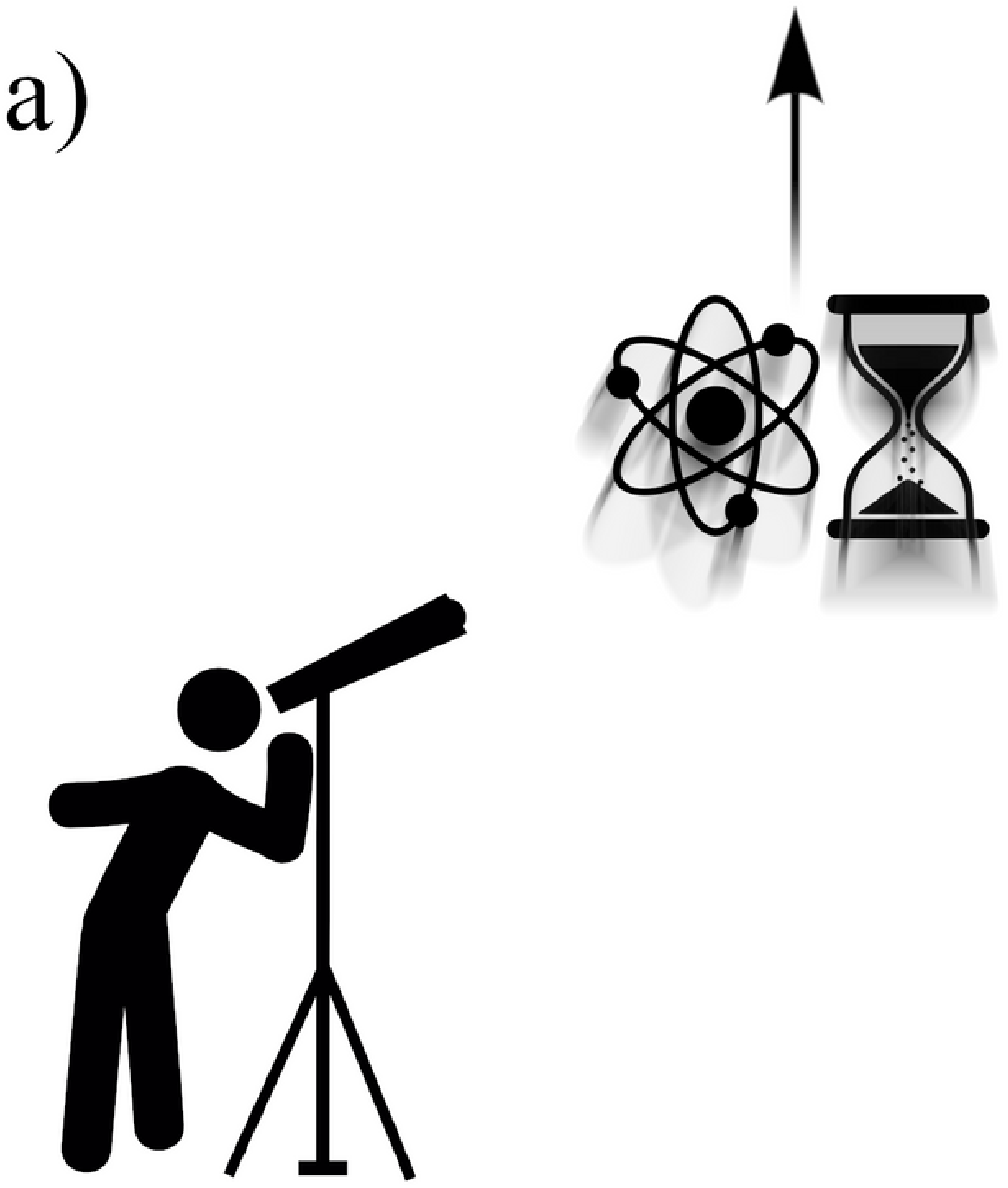
- classical effect

Universal = **independent** of the mechanism of the clock

Universality of time dilation

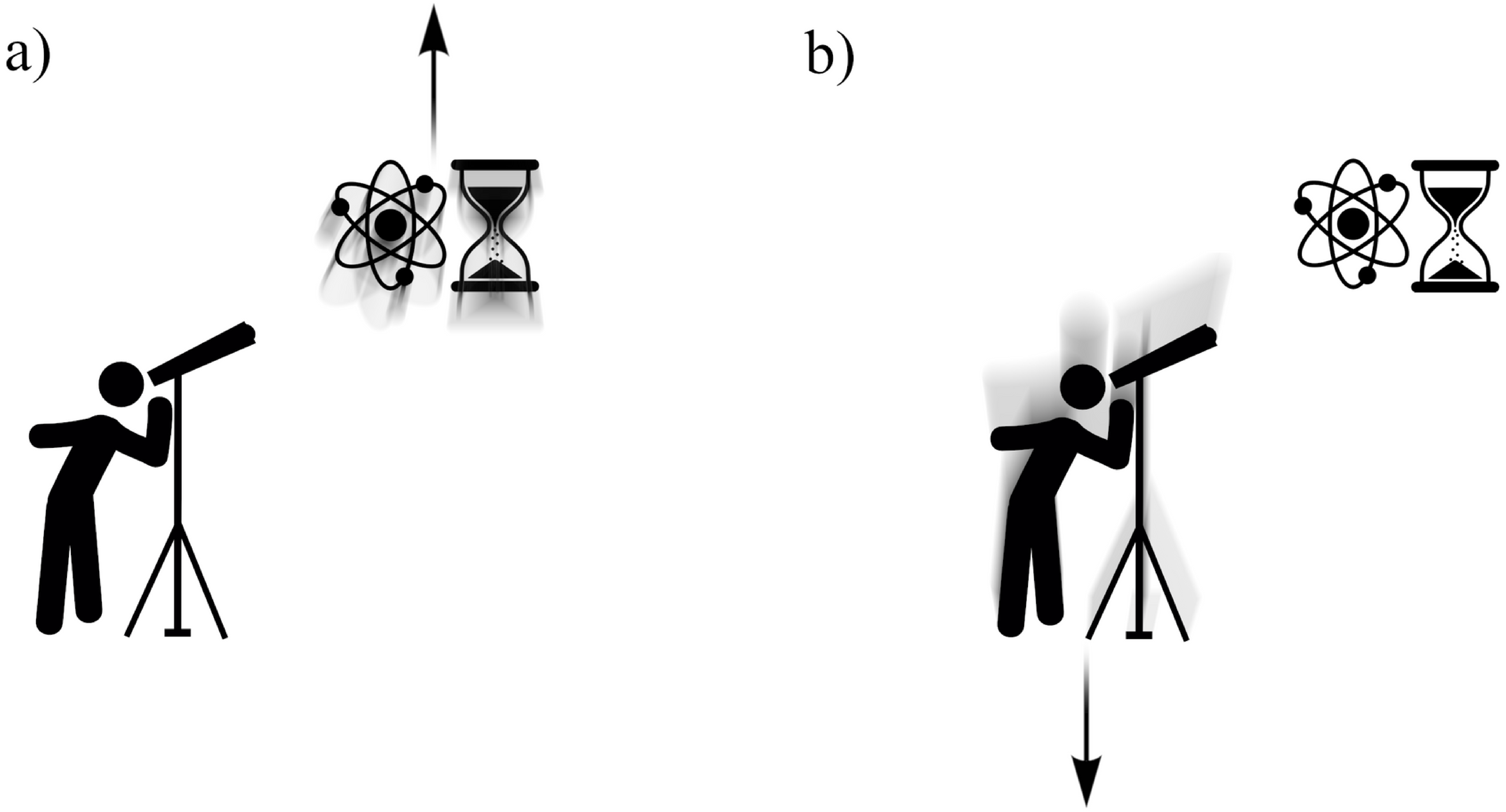
- classical effect

a)



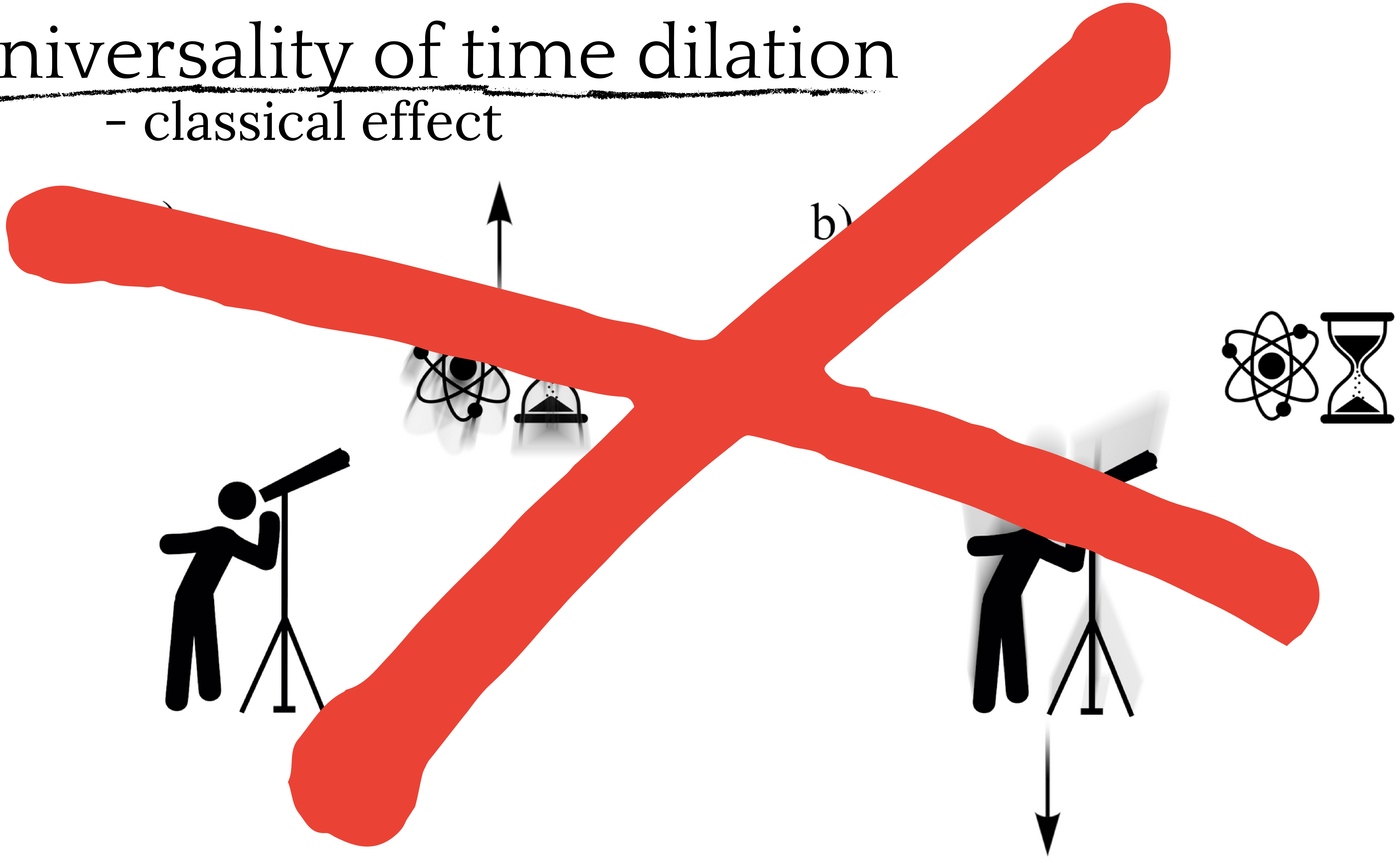
Universality of time dilation

- classical effect

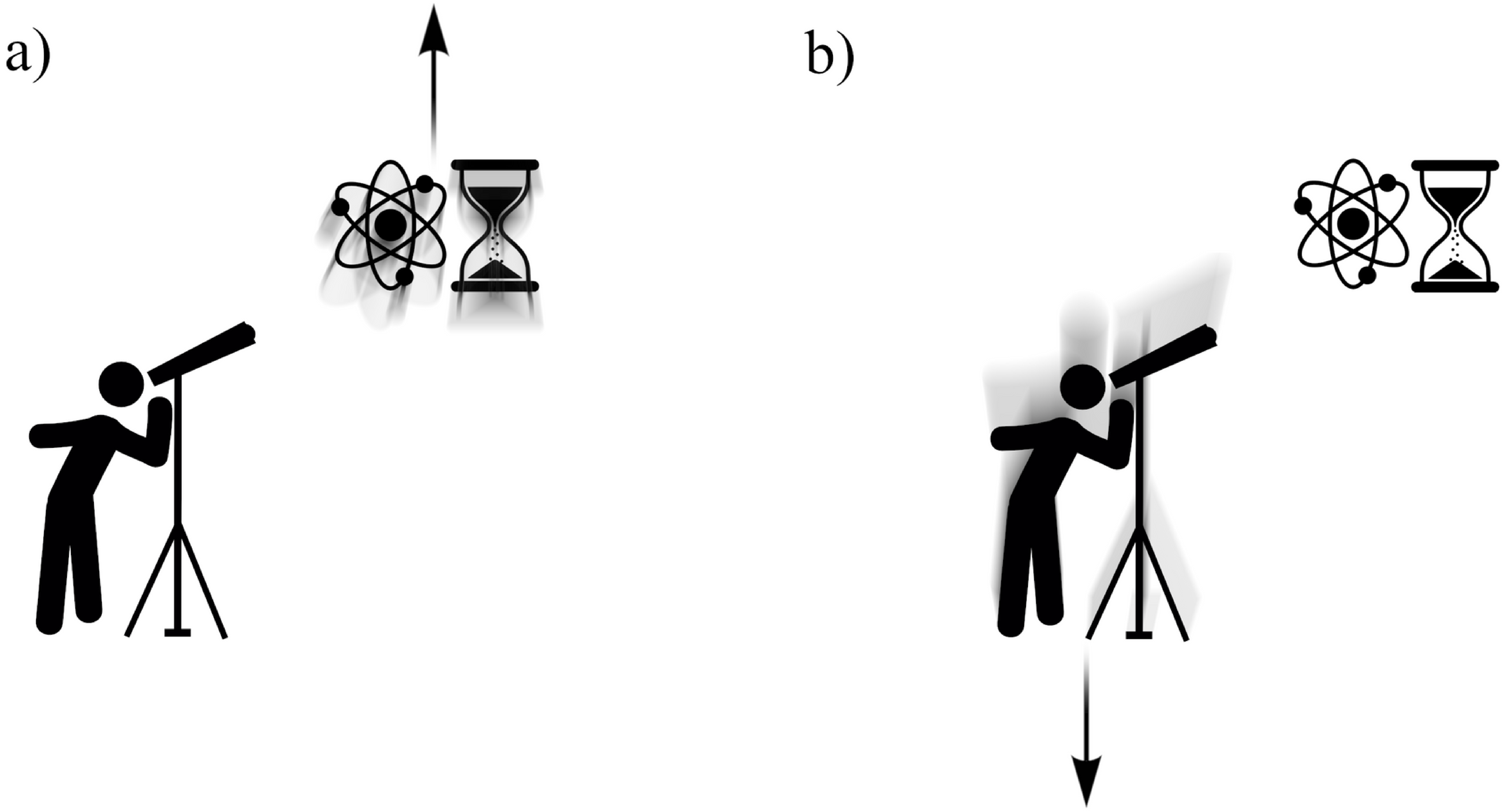


Universality of time dilation

- classical effect



Universality of ~~time dilation~~ kinematic time dilation
- classical effect



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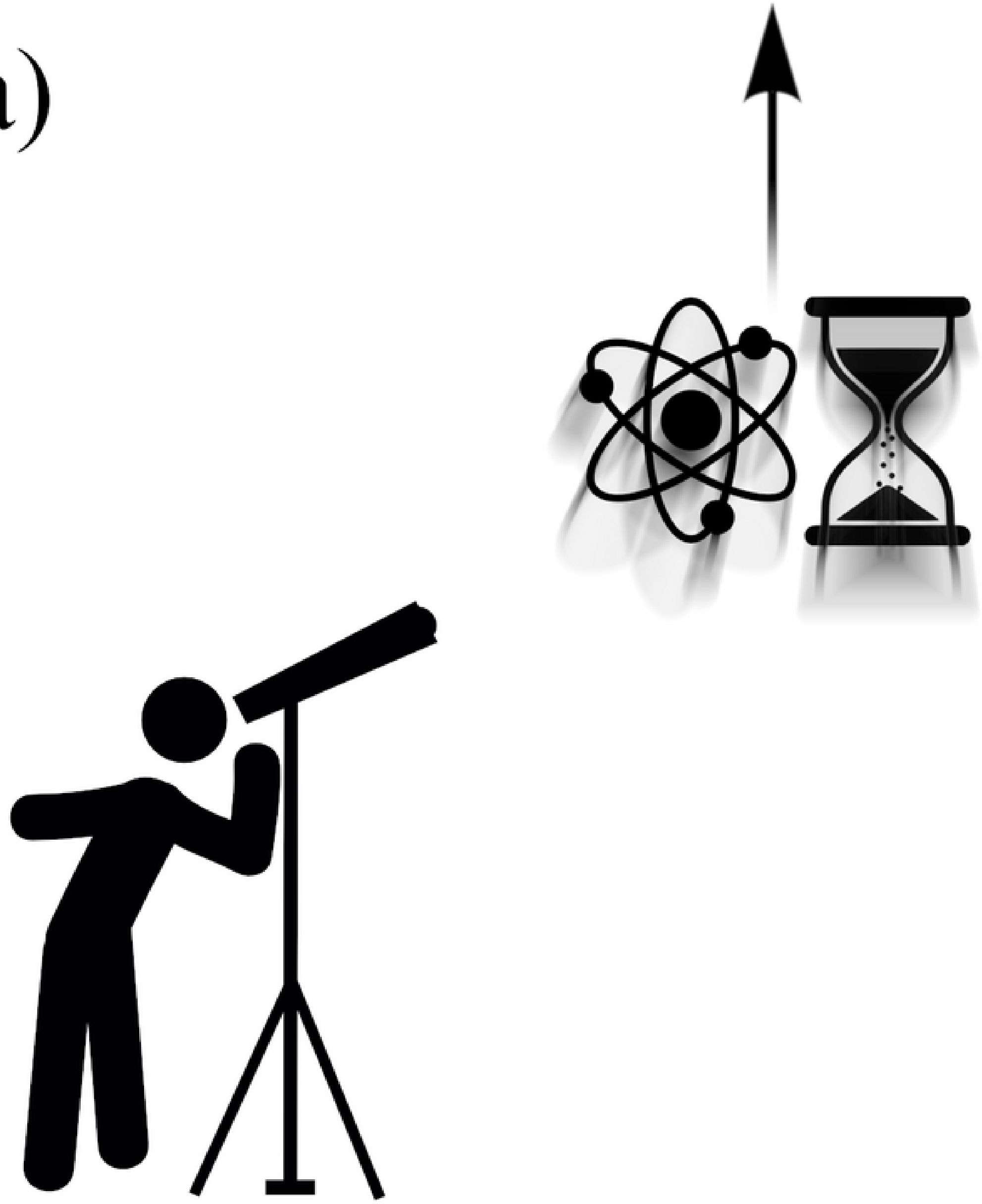

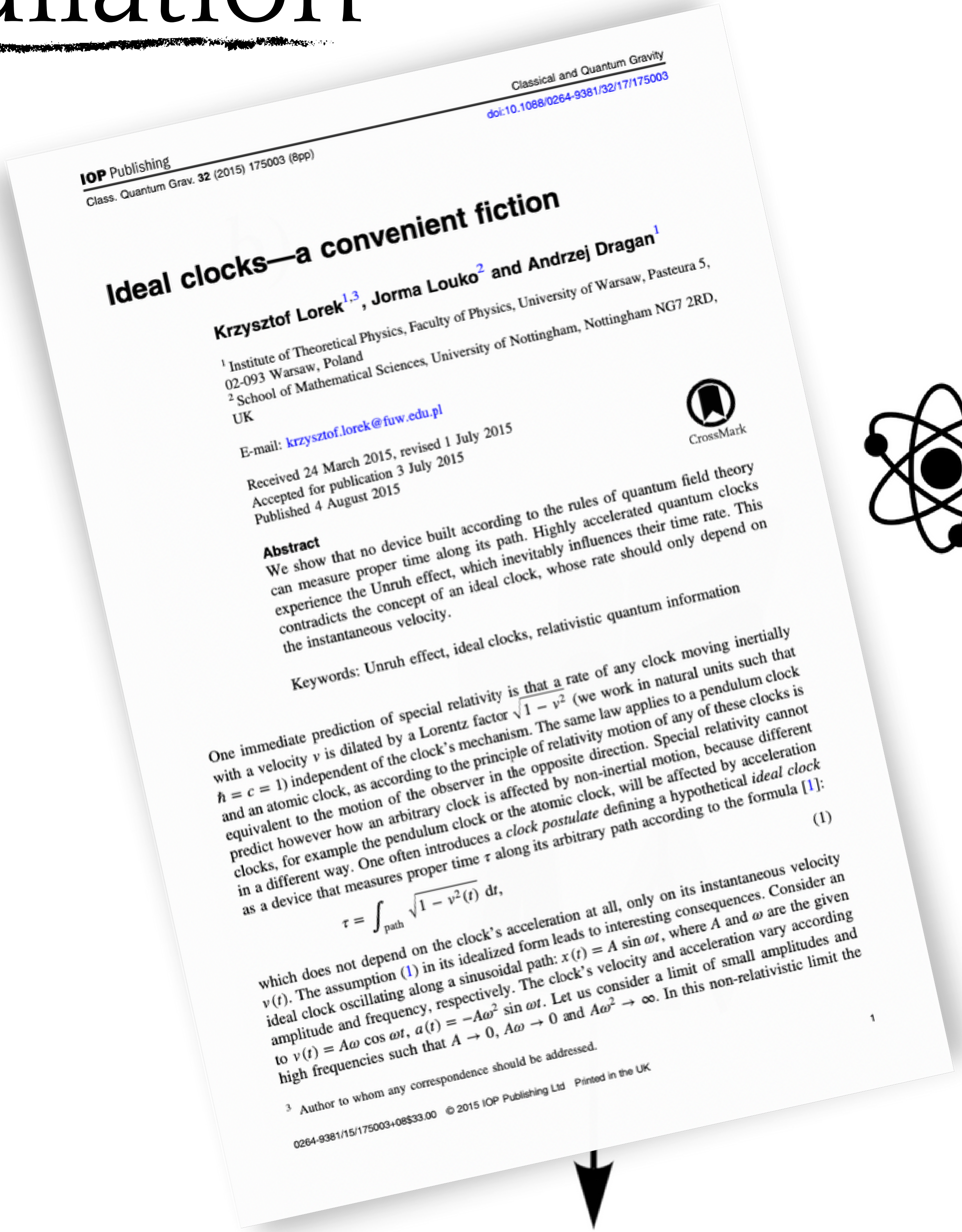

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Ideal clocks—a convenient fiction

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Abstract
We show that no device built according to the rules of quantum field theory can measure proper time along its path. Highly accelerated quantum clocks experience the Unruh effect, which inevitably influences their time rate. This contradicts the concept of an ideal clock, whose rate should only depend on the instantaneous velocity.

Keywords: Unruh effect, ideal clocks, relativistic quantum information

One immediate prediction of special relativity is that a rate of any clock moving inertially with a velocity v is dilated by a Lorentz factor $\sqrt{1-v^2}$ (we work in natural units such that $\hbar = c = 1$) independent of the clock's mechanism. The same law applies to a pendulum clock and an atomic clock, as according to the principle of relativity motion of any of these clocks is equivalent to the motion of the observer in the opposite direction. Special relativity cannot predict however how an arbitrary clock is affected by non-inertial motion, because different clocks, for example the pendulum clock or the atomic clock, will be affected by acceleration in a different way. One often introduces a *clock postulate* defining a hypothetical *ideal clock* as a device that measures proper time τ along its arbitrary path according to the formula [1]:

$$\tau = \int_{\text{path}} \sqrt{1-v^2(t)}\, dt, \quad (1)$$

which does not depend on the clock's acceleration at all, only on its instantaneous velocity $v(t)$. The assumption (1) in its idealized form leads to interesting consequences. Consider an ideal clock oscillating along a sinusoidal path: $x(t) = A \sin \omega t$, where A and ω are the given amplitude and frequency, respectively. The clock's velocity and acceleration vary according to $v(t) = A\omega \cos \omega t$, $a(t) = -A\omega^2 \sin \omega t$. Let us consider a limit of small amplitudes and high frequencies such that $A \to 0$, $A\omega \to 0$ and $A\omega^2 \to \infty$. In this non-relativistic limit the

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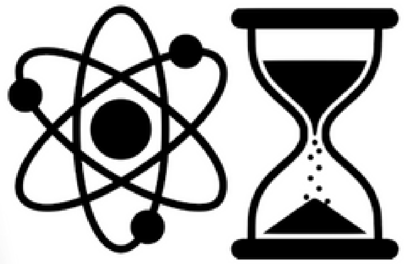


Quantum time dilation

Quantum time dilation

is

Quantum time dilation

is

the difference in

Quantum time dilation

is

the difference in

time measured by a clock prepared as:

Quantum time dilation

is

the difference in

time measured by a clock prepared as:

quantum superposition

Quantum time dilation

is

the difference in

time measured by a clock prepared as:

quantum superposition vs.

Quantum time dilation

is

the difference in

time measured by a clock prepared as:

quantum superposition vs. **classical mixture**

Quantum time dilation

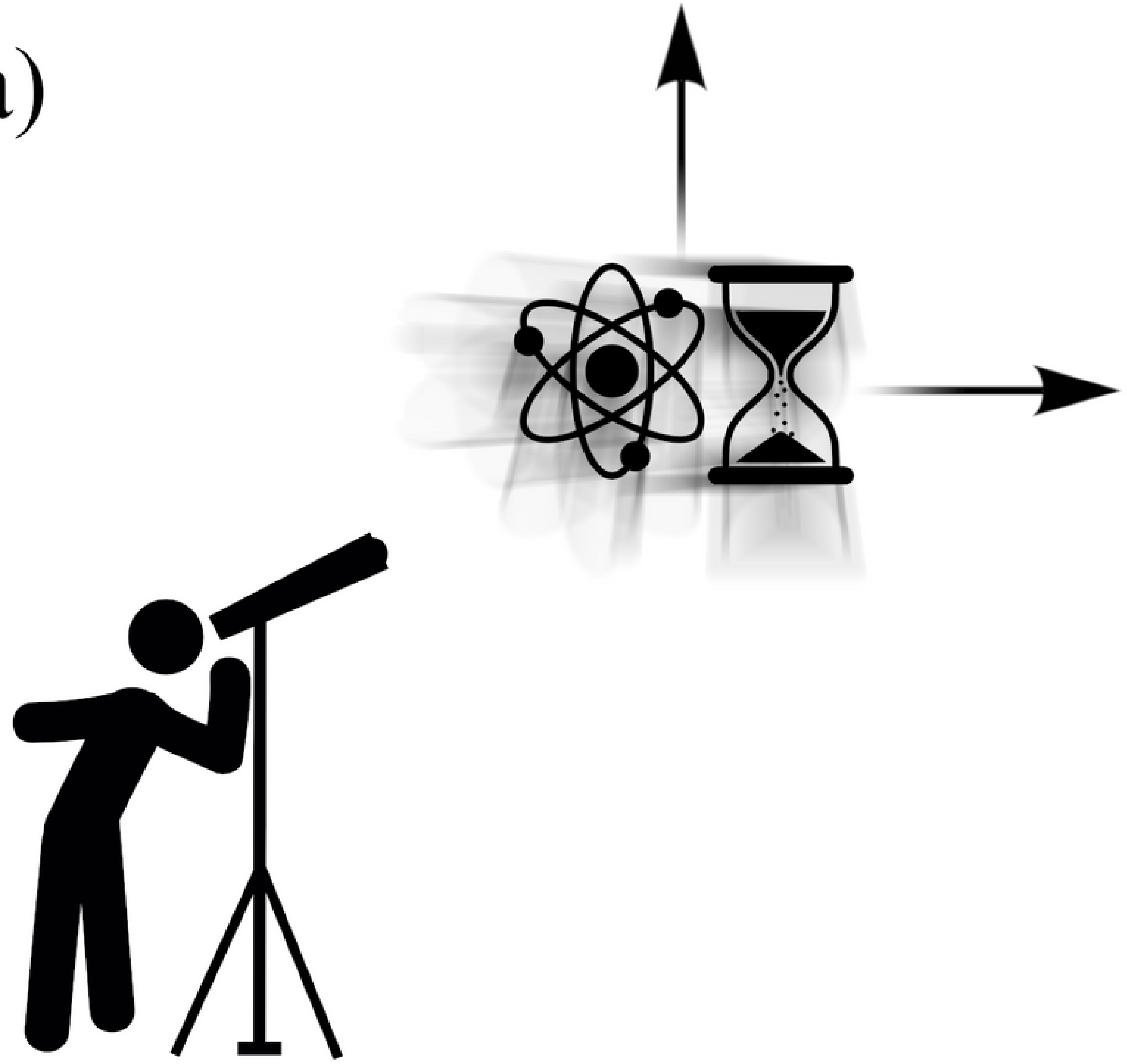
Quantum time dilation

- Is it universal?

Quantum time dilation

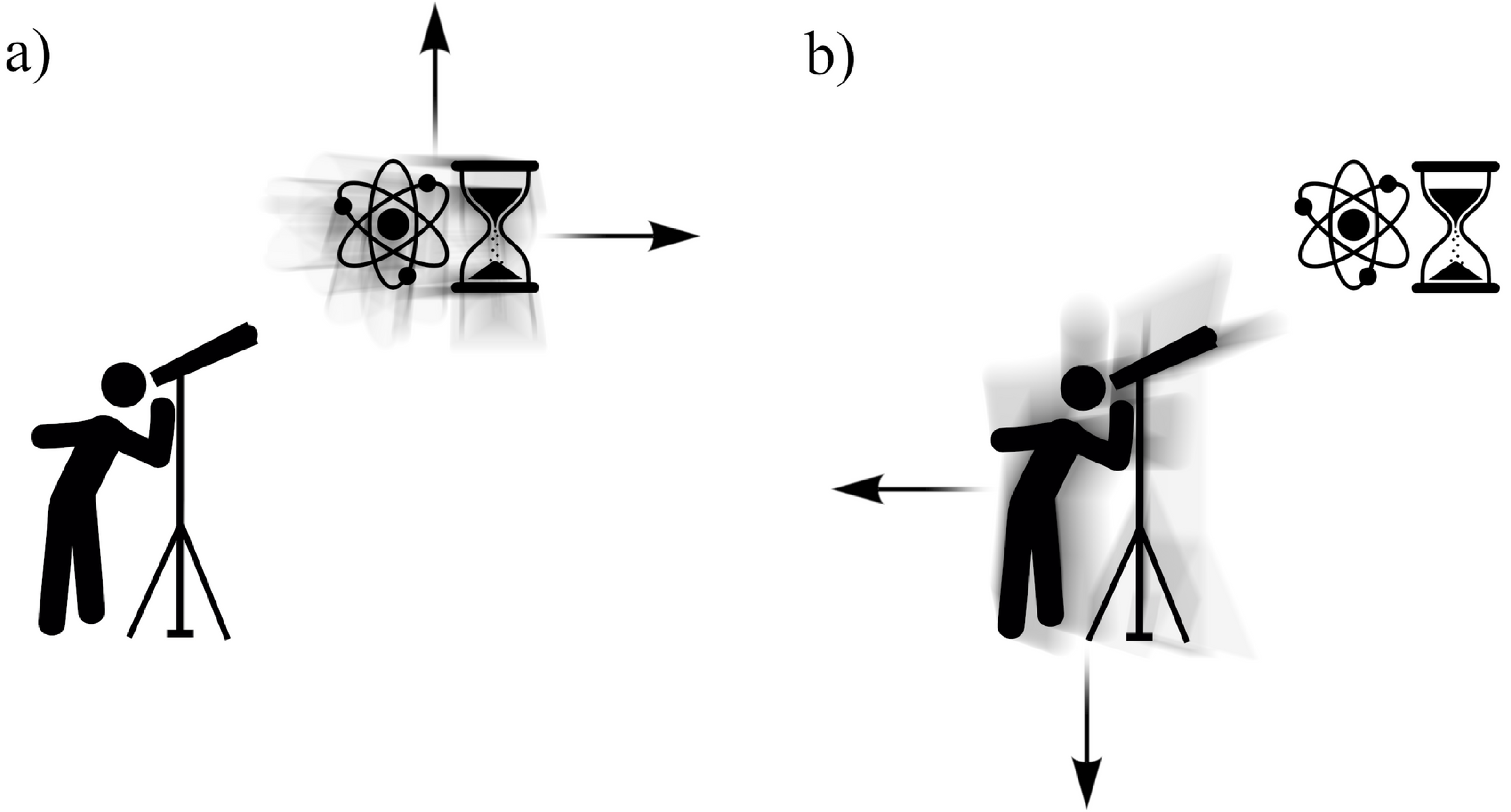
- Is it universal?

a)



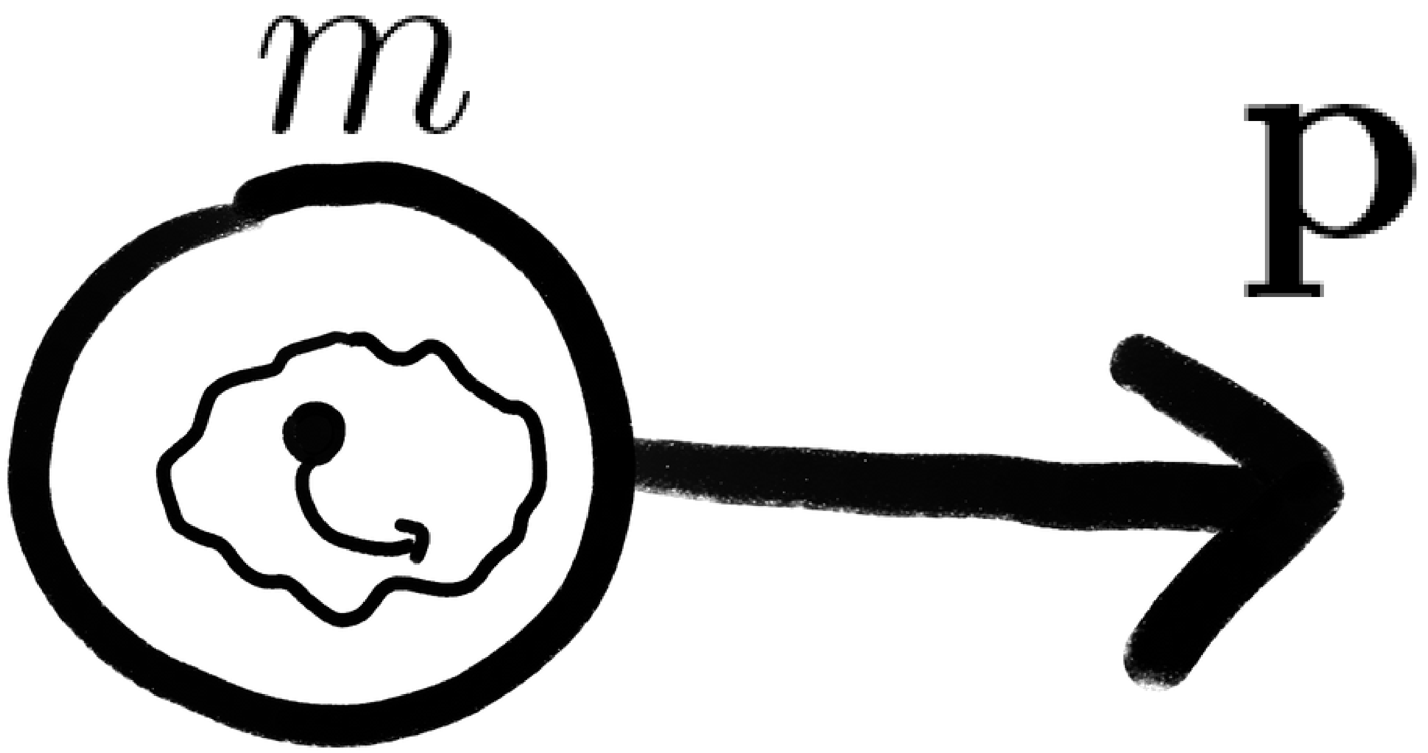
Quantum time dilation

- Is it universal?



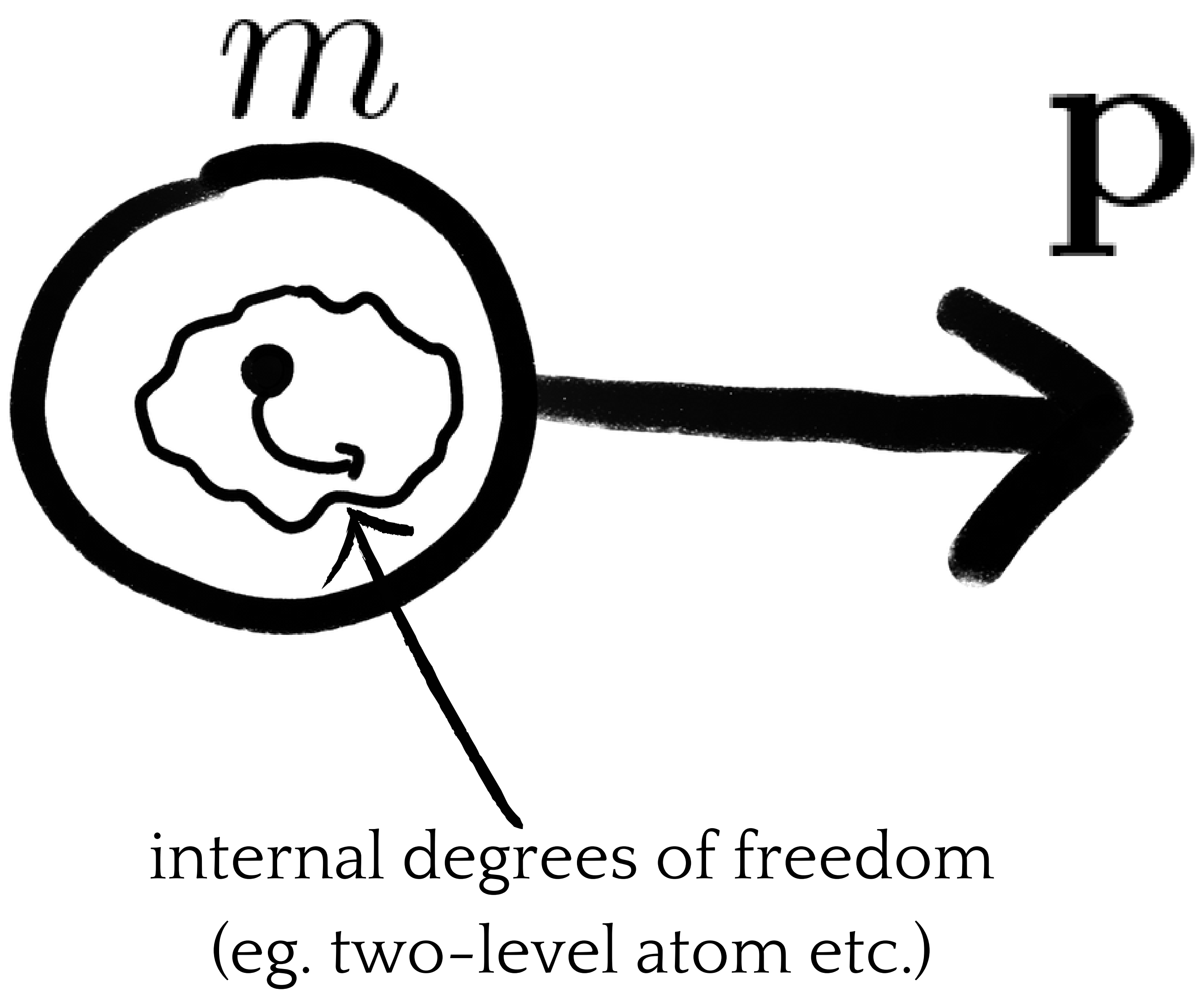
Quantum time dilation

- model



Quantum time dilation

- model

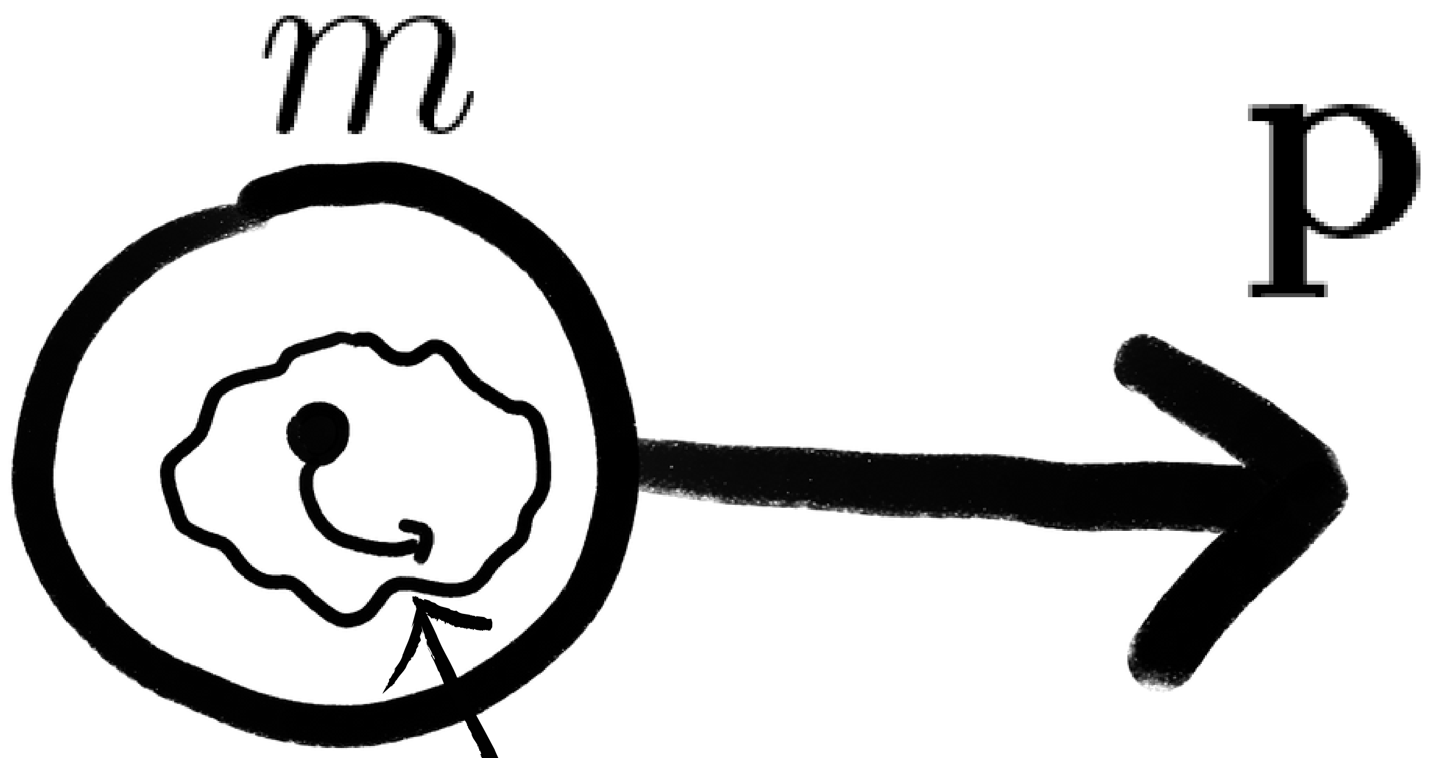


Quantum time dilation

- model

$$E = \sqrt{g_{00}(\boldsymbol{r})}\sqrt{(mc^2)^2 - g^{ij}(\boldsymbol{r})p_i p_j c^2}$$

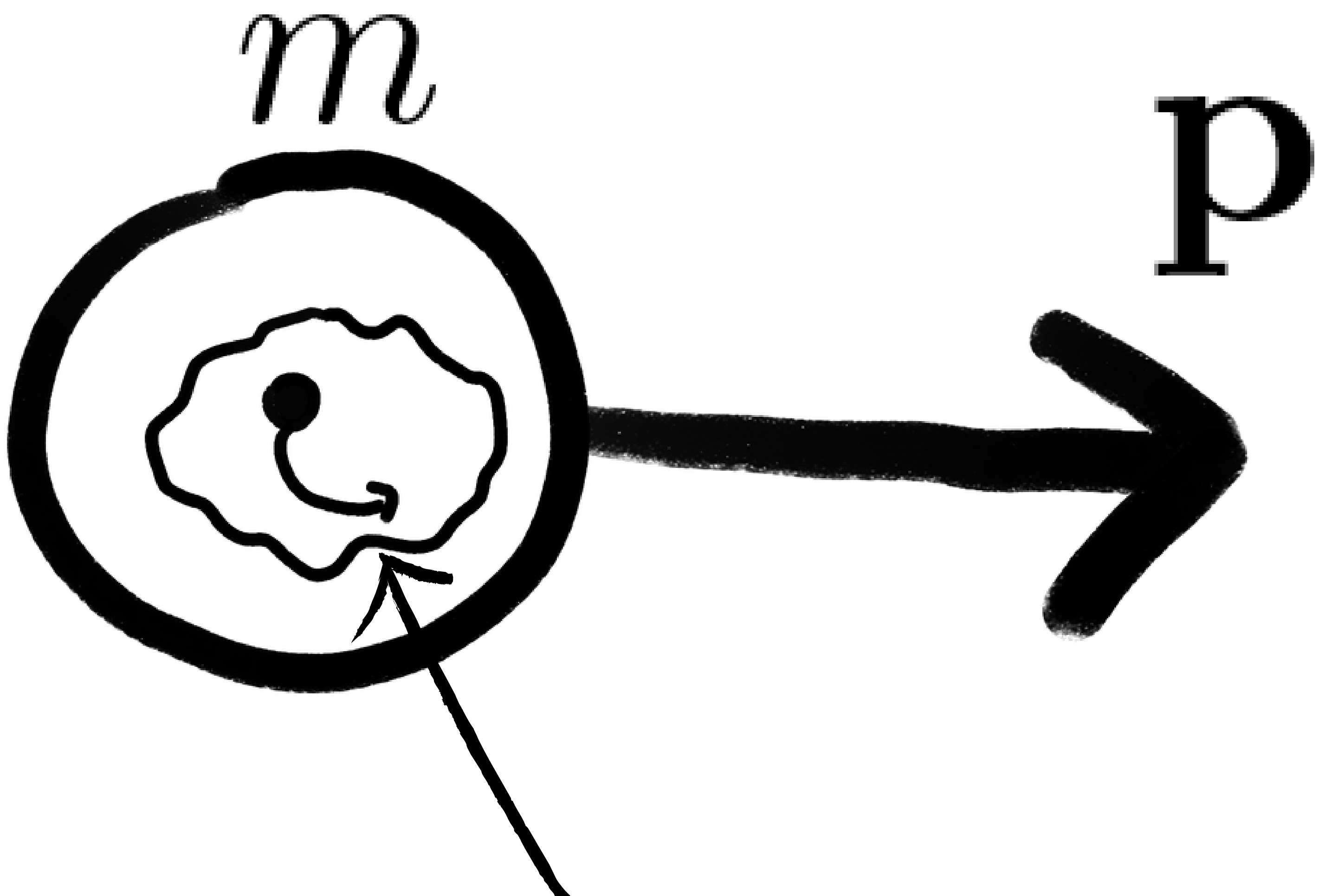
The Schwarzschild metric



internal degrees of freedom
(eg. two-level atom etc.)

Quantum time dilation
- model

$$\hat{H} = \sqrt{g_{00}(\hat{\boldsymbol{r}})}\sqrt{\left(mc^2 + \hat{H}_{\text{clock}}\right)^2 - g^{ij}(\hat{\boldsymbol{r}})\hat{p}_i\hat{p}_j c^2}$$



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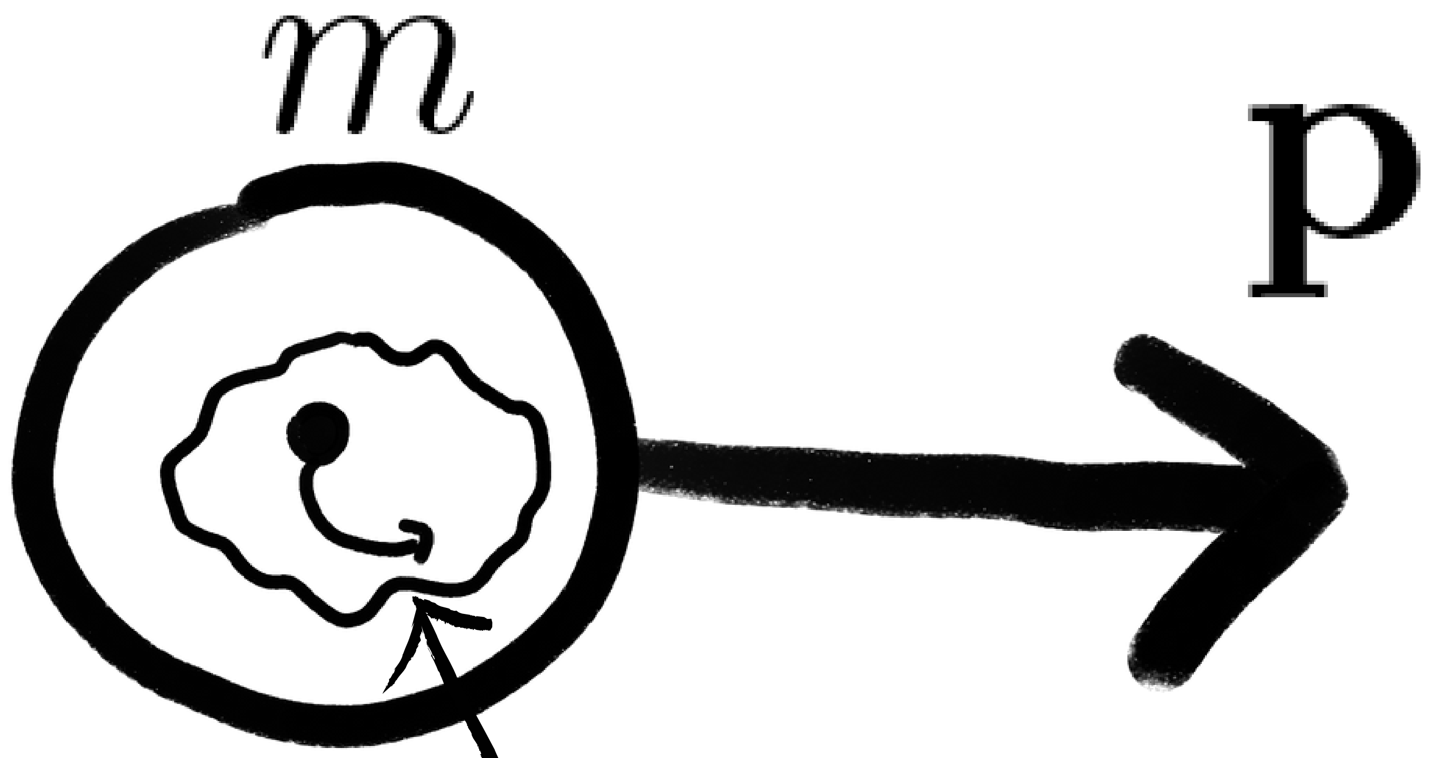
mass-energy equivalence



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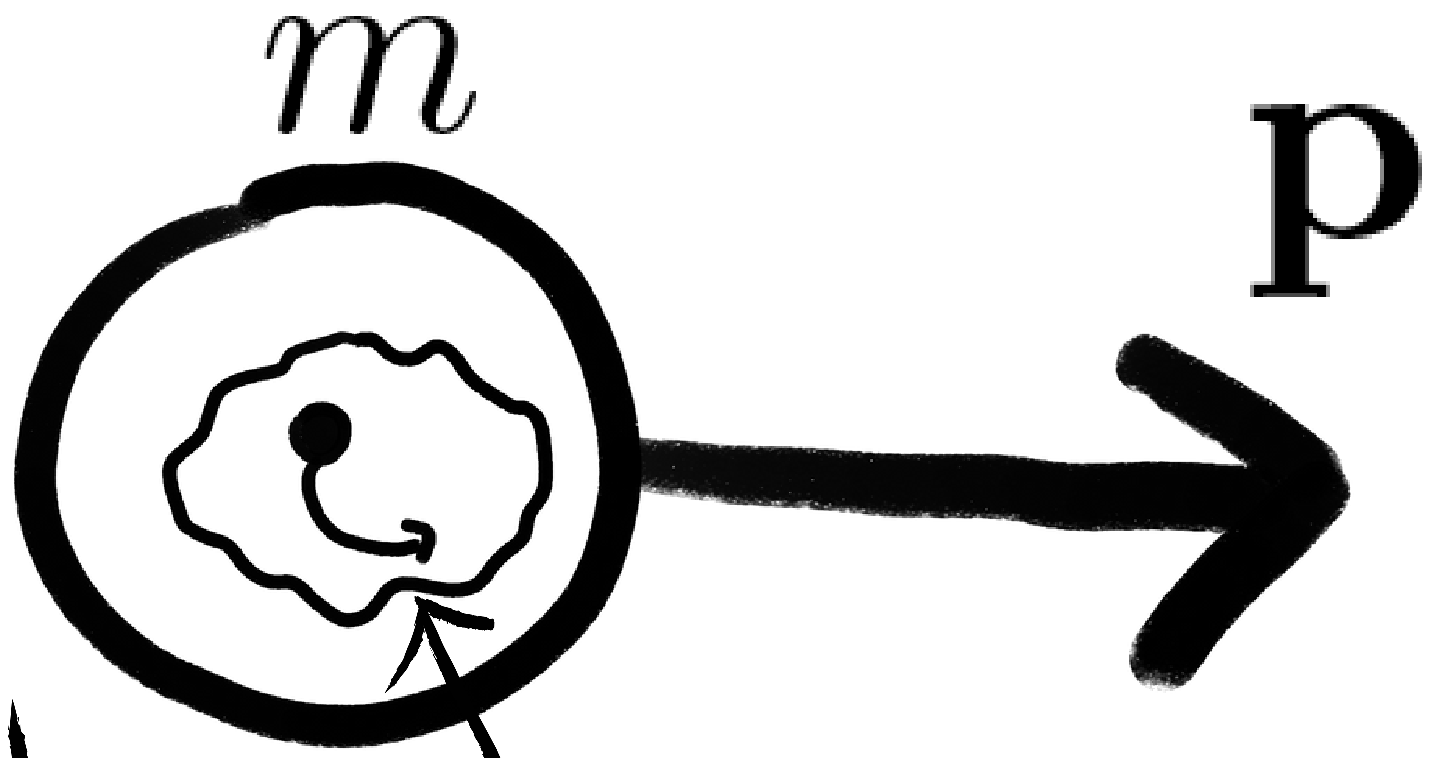

mass-energy equivalence

internal degrees of freedom
(eg. two-level atom etc.)

Quantum time dilation

- kinematic

Quantum time dilation

- kinematic

Quantum time dilation

- kinematic

$$\hat{H} = \sqrt{g_{00}(\hat{\boldsymbol{r}})}\sqrt{\left(mc^2 + \hat{H}_{\text{clock}}\right)^2 - g^{ij}(\hat{\boldsymbol{r}})\hat{p}_i\hat{p}_j c^2}$$

$$\hat{r}(t) = \hat{r}_0$$

Quantum time dilation

- kinematic

$$|\Psi(0)\rangle = \int \mathrm{d}^3\boldsymbol{p}\,\psi(\boldsymbol{p})|\boldsymbol{p}\rangle \otimes |0\rangle$$

Quantum time dilation
- kinematic

$$|\Psi(0)\rangle = \int \mathrm{d}^3\boldsymbol{p}\,\psi(\boldsymbol{p})|\boldsymbol{p}\rangle \otimes |0\rangle$$

initial state of the system

Quantum time dilation
- kinematic

eigenstate of momentum

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Quantum time dilation

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initial state of the system

initial state of the clock

Quantum time dilation

- kinematic

eigenstate of momentum

$$|\Psi(0)\rangle = \int \mathrm{d}^3\boldsymbol{p}\,\psi(\boldsymbol{p})|\boldsymbol{p}\rangle \otimes |0\rangle$$

initial state of the system

initial state of the clock

some distribution

eg. the sum of two Gaussians

Quantum time dilation

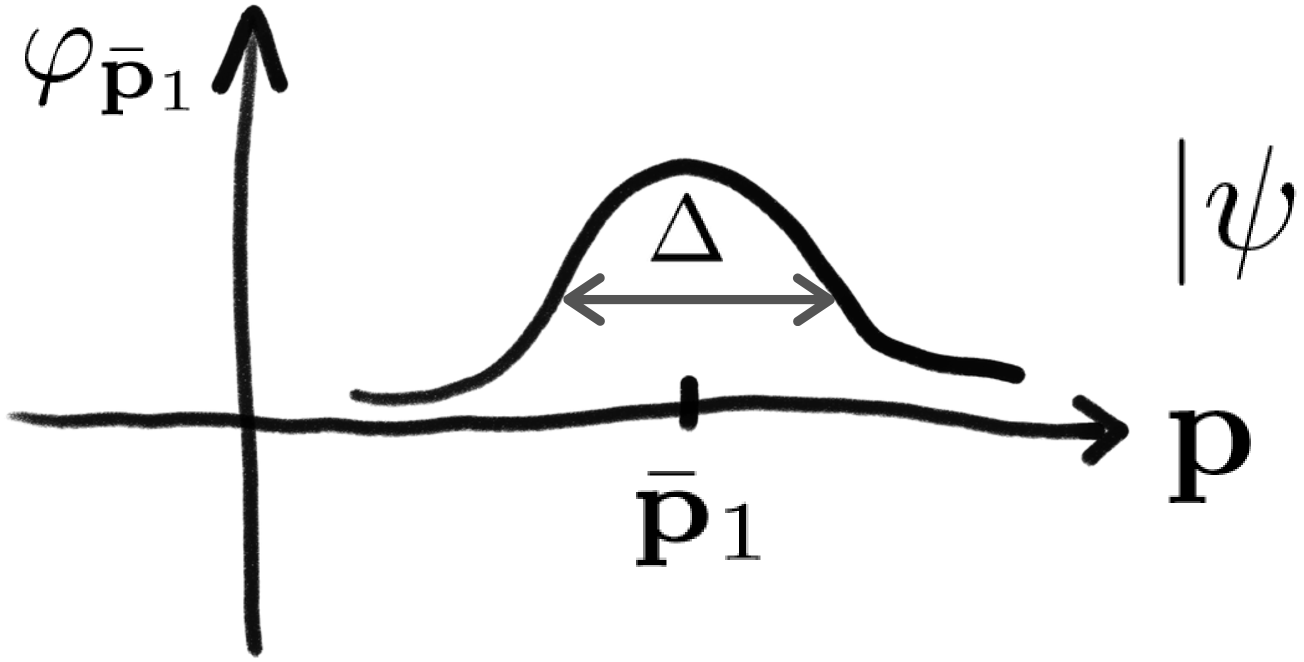
- kinematic

eigenstate of momentum

$$|\Psi(0)\rangle = \int \mathrm{d}^3\boldsymbol{p}\,\psi(\boldsymbol{p})|\boldsymbol{p}\rangle \otimes |0\rangle$$

initial state of the system

initial state of the clock



$$|\psi\rangle \sim \cos\theta\,|\varphi_{\bar{\mathbf{p}}_1}\rangle + e^{i\phi}\sin\theta\,|\varphi_{\bar{\mathbf{p}}_2}\rangle$$

$$\langle \boldsymbol{p}|\varphi_{\bar{\boldsymbol{p}}_i}\rangle \;=\; e^{-(\mathbf{p}-\bar{\mathbf{p}}_i)^2/2\Delta^2}/\pi^{1/4}\sqrt{\Delta}$$

Quantum time dilation

- kinematic

$$|\Psi(0)\rangle = \int \mathrm{d}^3\boldsymbol{p}\,\psi(\boldsymbol{p})|\boldsymbol{p}\rangle \otimes |0\rangle$$

$$|\Psi(t)\rangle = \int \mathrm{d}^3\boldsymbol{p}\,\psi(\boldsymbol{p})|\boldsymbol{p}\rangle \otimes |\phi(\boldsymbol{p}, t)\rangle$$

Quantum time dilation

- kinematic

$$|\Psi(0)\rangle = \int \mathrm{d}^3\boldsymbol{p}\,\psi(\boldsymbol{p})|\boldsymbol{p}\rangle \otimes |0\rangle$$

$$|\Psi(t)\rangle = \int \mathrm{d}^3\boldsymbol{p}\,\psi(\boldsymbol{p})|\boldsymbol{p}\rangle \otimes |\phi(\boldsymbol{p},t)\rangle$$

the evolution only affects a clock's state

Quantum time dilation

- kinematic

$$\hat{\rho}_{\text{clock}}(t) = \int \mathrm{d}^3\boldsymbol{p}\, |\psi(\boldsymbol{p})|^2 |\phi(\boldsymbol{p}, t)\rangle\langle\phi(\boldsymbol{p}, t)|$$

Quantum time dilation

- kinematic

$$\hat{\rho}_{\text{clock}}(t) = \int \mathrm{d}^3\boldsymbol{p}\, |\psi(\boldsymbol{p})|^2 |\phi(\boldsymbol{p},t)\rangle\langle\phi(\boldsymbol{p},t)|$$

we traced out the center-of-mass
degrees of freedom

Quantum time dilation
- kinematic

$$\hat{\rho}_{\mathrm{clock}}(t) = \int \mathrm{d}^3\boldsymbol{p}\,|\psi(\boldsymbol{p})|^2 |\phi(\boldsymbol{p},t)\rangle\langle\phi(\boldsymbol{p},t)|$$

time measurement

$$\mathcal{P}(\tau) = \mathrm{Tr}\left(\hat{E}(\tau)\hat{\rho}_{\mathrm{clock}}(t)\right)$$

Quantum time dilation
- kinematic

$$\hat{\rho}_{\text{clock}}(t) = \int \mathrm{d}^3\boldsymbol{p}\,|\psi(\boldsymbol{p})|^2|\phi(\boldsymbol{p},t)\rangle\langle\phi(\boldsymbol{p},t)|$$

the probability to obtain the measurement result

time measurement

$$\mathcal{P}(\tau) = \mathrm{Tr}\left(\hat{E}(\tau)\hat{\rho}_{\text{clock}}(t)\right)$$

Quantum time dilation

- kinematic

$$\mathcal{P}(\tau) = \int \mathrm{d}^3\boldsymbol{p}\, |\psi(\boldsymbol{p})|^2 \langle \phi(\boldsymbol{p}, t) | \hat{E}(\tau) | \phi(\boldsymbol{p}, t) \rangle$$

Quantum time dilation

- kinematic

$$\mathcal{P}(\tau) = \int \mathrm{d}^3\boldsymbol{p}\,|\psi(\boldsymbol{p})|^2 \langle \phi(\boldsymbol{p},t)|\hat{E}(\tau)|\phi(\boldsymbol{p},t)\rangle$$

a weighted average

Quantum time dilation
- kinematic

$$\mathcal{P}(\tau) = \int \mathrm{d}^3\boldsymbol{p}\, |\psi(\boldsymbol{p})|^2 \langle \phi(\boldsymbol{p}, t) | \hat{E}(\tau) | \phi(\boldsymbol{p}, t) \rangle$$

a weighted average

contribution from the eigenstate of momentum

Quantum time dilation

- kinematic

$$\mathcal{P}(\tau) = \int \mathrm{d}^3\boldsymbol{p}\, |\psi(\boldsymbol{p})|^2 \langle \phi(\boldsymbol{p}, t)|\hat{E}(\tau)|\phi(\boldsymbol{p}, t)\rangle$$

a weighted average

universal

contribution from the eigenstate of momentum

Gravitational quantum time dilation

Gravitational quantum time dilation

$$\hat{H} = \sqrt{g_{00}(\hat{\boldsymbol{r}})}\sqrt{\left(mc^2 + \hat{H}_{\text{clock}}\right)^2 - g^{ij}(\hat{\boldsymbol{r}})\hat{p}_i\hat{p}_j c^2}$$

$$\hat{p}_i = 0$$

Gravitational quantum time dilation

$$\widetilde{\mathcal{P}}(\tau) = \int \mathrm{d}^3\boldsymbol{r}\, |\widetilde{\psi}(\boldsymbol{r})|^2 \langle \widetilde{\phi}(\boldsymbol{r}, t)|\hat{E}(\tau)|\widetilde{\phi}(\boldsymbol{r}, t)\rangle$$

contribution from the
eigenstate of position

Gravitational quantum time dilation

$$\widetilde{\mathcal{P}}(\tau) = \int \mathrm{d}^3\boldsymbol{r}\, |\widetilde{\psi}(\boldsymbol{r})|^2 \langle \widetilde{\phi}(\boldsymbol{r},t)|\hat{E}(\tau)|\widetilde{\phi}(\boldsymbol{r},t)\rangle$$

NONuniversal contribution from the eigenstate of position

Kinematic/gravitational **quantum time dilation depends** only **on** the **probability distribution**.

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It can be **simulated** by **classical states**.

Kinematic/gravitational **quantum time dilation depends** only **on** the **probability distribution**.

It can be **simulated** by **classical states**.

Is it possible to **find** an
effect that depends on
combined distributions
of position and
momentum?

Combined kinematic and gravitational quantum time dilation

Combined kinematic and gravitational quantum time dilation

$$\hat{H} = \sqrt{g_{00}(\hat{\boldsymbol{r}})}\sqrt{\left(mc^2 + \hat{H}_{\text{clock}}\right)^2 - g^{ij}(\hat{\boldsymbol{r}})\hat{p}_i\hat{p}_j c^2}$$

Combined kinematic and gravitational quantum time dilation

$$\hat{H} = \sqrt{g_{00}(\hat{\boldsymbol{r}})}\sqrt{\left(mc^2 + \hat{H}_{\text{clock}}\right)^2 - g^{ij}(\hat{\boldsymbol{r}})\hat{p}_i\hat{p}_j c^2}$$

$$\downarrow$$

$$\hat{H} \approx \hat{H}_{\text{clock}} + \hat{H}_{\text{cm}}(\hat{\boldsymbol{r}}, \hat{\boldsymbol{p}}) + \hat{H}_{\text{int}}$$

Combined kinematic and gravitational quantum time dilation

$$\hat{H} \approx \hat{H}_{\text{clock}} + \hat{H}_{\text{cm}}(\hat{\boldsymbol{r}}, \hat{\boldsymbol{p}}) + \hat{H}_{\text{int}}$$

$$\hat{H}_{\text{cm}} \equiv mc^2 + \frac{\hat{p}^2}{2m} + mg\hat{x} + \frac{3g}{2mc^2} \overset{\text{w}}{:} \hat{p}^2\hat{x} \overset{\text{w}}{:},$$

$$\hat{H}_{\text{int}} \equiv \hat{H}_{\text{clock}} \underbrace{\left(-\frac{\hat{p}^2}{2m^2c^2} + \frac{g\hat{x}}{c^2} - \frac{3g}{2m^2c^4} \overset{\text{w}}{:} \hat{p}^2\hat{x} \overset{\text{w}}{:}\right)}_{\hat{V}_1}$$

$$+ \hat{H}^2_{\text{clock}} \underbrace{\left(\frac{\hat{p}^2}{2m^3c^4} + \frac{3g}{2m^3c^6} \overset{\text{w}}{:} \hat{p}^2\hat{x} \overset{\text{w}}{:}\right)}_{\hat{V}_2}$$

$$:\hat{p}\hat{x}: \equiv \lambda\hat{p}\hat{x} + (1-\lambda)\hat{x}\hat{p}$$

Combined kinematic and gravitational quantum time dilation

$$\hat{H} \approx \hat{H}_{\text{clock}} + \hat{H}_{\text{cm}}(\hat{\boldsymbol{r}}, \hat{\boldsymbol{p}}) + \hat{H}_{\text{int}}$$

$$\hat{H}_{\text{cm}} \equiv mc^2 + \frac{\hat{p}^2}{2m} + mg\hat{x} + \frac{3g}{2mc^2} \overset{\mathrm{W}}{\vdots}\, \hat{p}^2\hat{x} \,\overset{\mathrm{W}}{\vdots},$$

$$\hat{H}_{\text{int}} \equiv \hat{H}_{\text{clock}} \underbrace{\left(-\frac{\hat{p}^2}{2m^2c^2} + \frac{g\hat{x}}{c^2} - \frac{3g}{2m^2c^4} \overset{\mathrm{W}}{\vdots}\, \hat{p}^2\hat{x} \,\overset{\mathrm{W}}{\vdots}\right)}_{\hat{V}_1}$$

$$+ \hat{H}^2_{\text{clock}} \underbrace{\left(\frac{\hat{p}^2}{2m^3c^4} + \frac{3g}{2m^3c^6} \overset{\mathrm{W}}{\vdots}\, \hat{p}^2\hat{x} \,\overset{\mathrm{W}}{\vdots}\right)}_{\hat{V}_2}$$

elements that mix up position and momentum operators

Combined kinematic and gravitational quantum time dilation

$$\hat{H} \approx \hat{H}_{\text{clock}} + \hat{H}_{\text{cm}}(\hat{x}, \hat{p}) + \hat{H}_{\text{int}}$$

THE FINAL RESULT DEPENDS ON 2-POINT CORRELATION FUNCTION

elements that mix up position and momentum operators

Combined kinematic and gravitational quantum time dilation

$$\hat{\rho}_{\text{clock}}(t) \propto \text{Tr}\left[\overset{\text{W}}{:} \hat{p}^2 \hat{x} \overset{\text{W}}{:} \hat{\rho}_{\text{cm}}(0) \right] = \frac{1}{3} \text{Tr}\left[\left(\hat{p}^2 \hat{x} + \hat{p}\hat{x}\hat{p} + \hat{x}\hat{p}^2 \right) \hat{\rho}_{\text{cm}}(0) \right]$$

$$= \int \mathrm{d}p \mathrm{d}x \mathrm{d}x' \, \frac{3xp^2 - i\hbar p}{6\pi\hbar} e^{-\frac{i}{\hbar} p(x-x')} \langle x | \hat{\rho}_{\text{cm}}(0) | x' \rangle$$

2-point correlation function

Conclusions:

- QTD depends on a weighted average of probabilities (1-point correlation function)
- Kinematic QTD is UNIVERSAL
- Gravitational QTD is NOT
- There is a higher-order effect that can't be simulated by classical states and that depends on a 2-point correlation function