Black hole analogues of VdW phase transitions and other everyday critical phenomena

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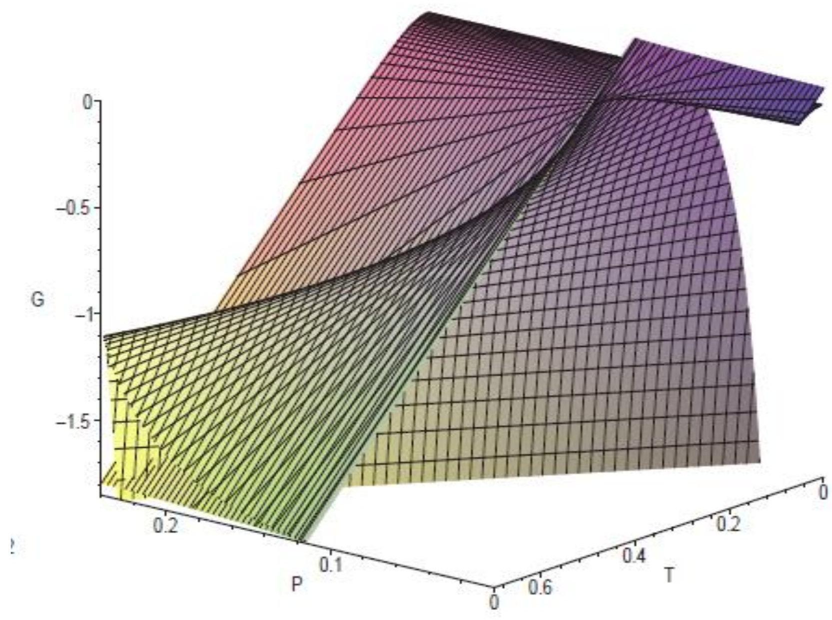


Analogue gravity
Benasque, Spain
May 29 – June 2, 2023

Plan of the talk

I. Black holes as thermodynamic objects

II. Black hole chemistry: black holes as "ordinary thermodynamic systems"

III. Black hole thermodynamics in analogue gravity systems?

IV. Summary

Based on:

- DK, R.B. Mann, *P-V criticality of charged AdS black holes*, JHEP 07 (2012) 033; ArXiv:1205:0559.
- DK, R.B. Mann, M. Teo, *Black hole chemistry: thermodynamics with Lambda*, CQG 34 (2017) 063001, Arxiv:1608.0614.

1) Black Holes as Thermodynamic Objects

Black holes as thermodynamic objects

If someone points out to you that your pet theory of the universe is in disagreement with Maxwell's equations-then so much the worse for Maxwell's equations. If it is found to be contradicted by observation-well these experimentalists do bungle things sometimes. But if your theory is found to be against the second law of thermodynamics I can give you no hope; there is nothing for it but to collapse in deepest humiliation.

Sir Arthur Stanley Eddington

Gifford Lectures (1927), *The Nature of the Physical World* (1928), 74.

Laws of black hole mechanics

- Bardeen, Carter, Hawking (1973)

- **Zeroth law:** The surface gravity κ is constant on the black hole horizon.
- **First law:**

$$\boxed{dM = \frac{\kappa}{2\pi}\frac{dA}{4} + \underbrace{\Omega dJ + \Phi dQ}_{\text{work terms}}\,.} \tag{5.8}$$

  Here, Ω is the angular velocity of the black hole horizon, and Φ is its 'electrostatic potential'.

- **Second law:** Classically, the area of the horizon never decreases (provided the null energy condition holds).

$$\boxed{dA \geq 0\,.} \tag{5.9}$$

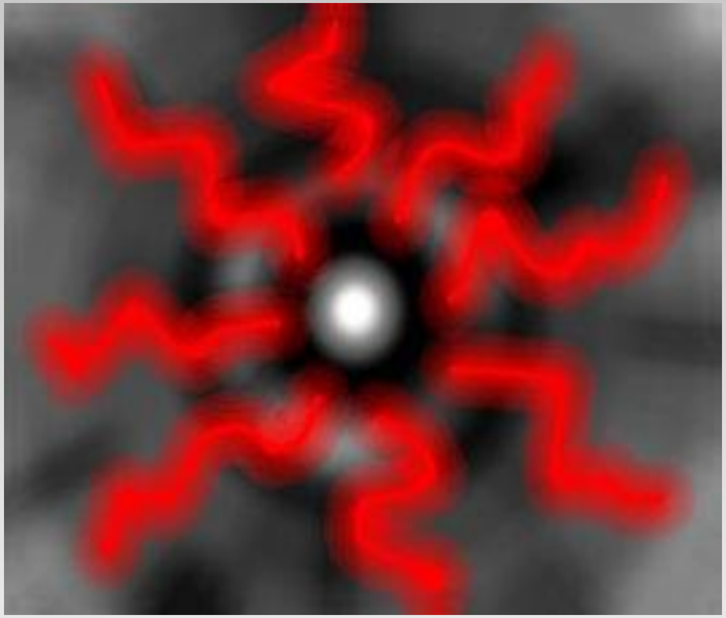
- **Third law:** It is impossible to reduce κ to zero in a finite number of steps.

- Essentially equivalent to **gravitational dynamics** (entropy is theory dependent,….)
- Despite the resemblance with laws of TDs, **classical** BHs are black

Black holes have temperature and entropy

Hawking (1974):

$$T = \frac{\kappa}{2\pi}\frac{\hbar c^3}{k_B} \quad \Rightarrow \quad S = \frac{A}{4}\frac{c^3 k_B}{\hbar G_N}$$



derived in framework of
QFT in curved spacetime

Other approaches: Euclidean path integral approach (Gibbons & Hawking-1977), tunnelling, LQG, string theory,…

Classical laws of black hole mechanics become laws of black hole thermodynamics

Black hole thermodynamics: AF BHs

- First law of black hole thermodynamics:

$$\delta M = T\delta S + \sum_i \Omega_i \delta J_i + \Phi \delta Q$$

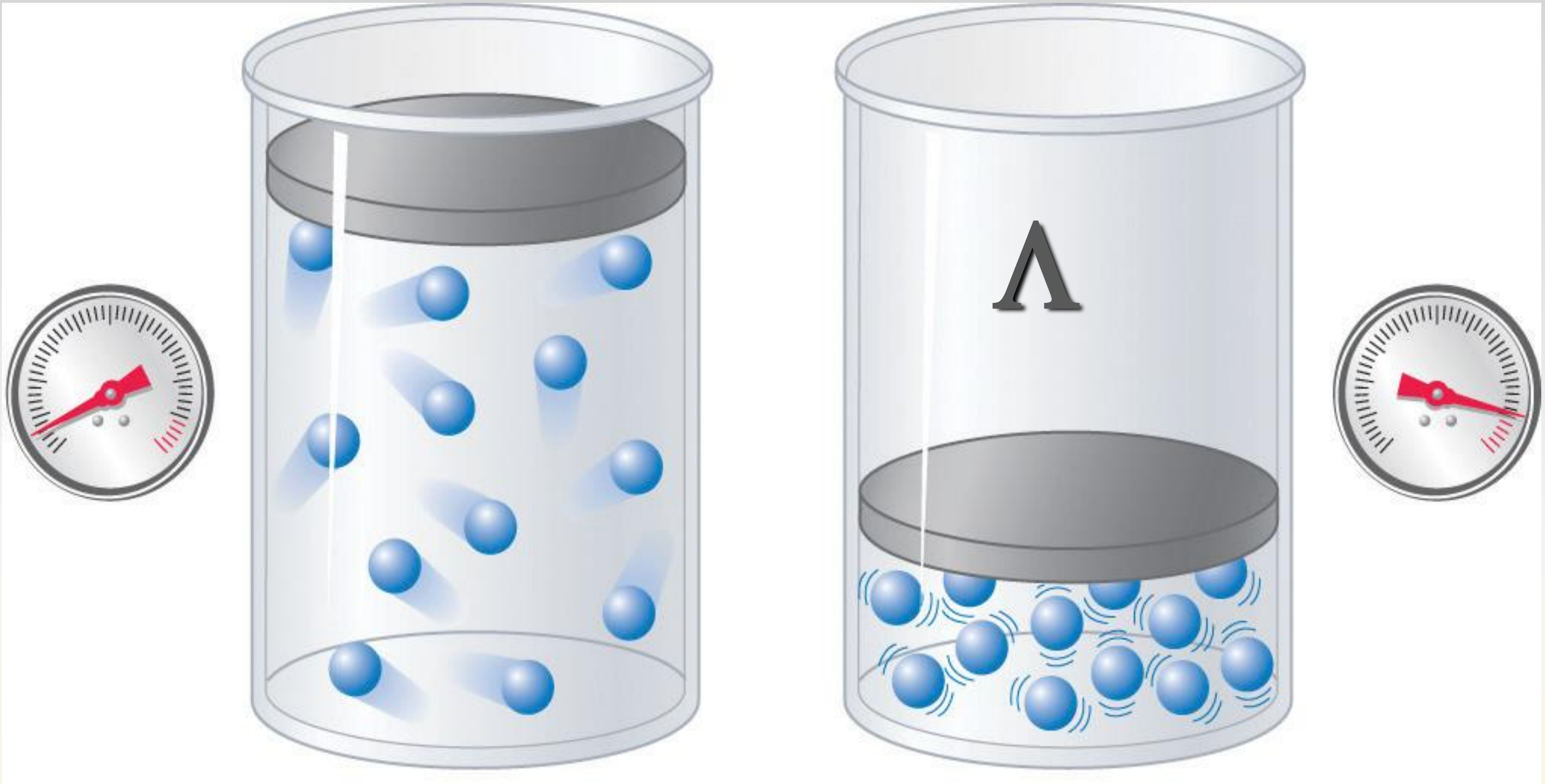
- Smarr-Gibbs-Duhem relation:

$$\frac{d-3}{d-2} M = TS + \sum_i \Omega_i J_i + \frac{d-3}{d-2} \Phi Q$$

Basic characteristics:

- Thermodynamic ensemble not well defined!
- Where is the standard PdV term?
- TD behaviour interesting, yet not exactly analogous to everyday thermodynamics!

2) Black hole chemistry: black holes as ordinary TD systems



Black hole chemistry

Simple idea:

- Consider an asymptotically **AdS black hole spacetime**

- Identify the cosmological constant with a thermodynamic pressure

$$P = -\frac{\Lambda}{8\pi G}\,, \quad \Lambda = -\frac{(D-1)(D-2)}{2l^2}$$

- Allow this to be a "dynamical" quantity

(Teitelboim and Brown – 1980's)

Immediate consequences

- **Extended black hole thermodynamics:**

D.Kastor, S.Ray, and J.Traschen, *Enthalpy and the Mechanics of AdS Black Holes*, Class. Quant. Grav. 26 (2009) 195011.

$$\delta M = T\delta S + V\delta P + \dots$$

• Introduces the standard -**PdV term** into black hole thermodynamics

• Black hole mass M no longer identified with energy but rather interpreted as **enthalpy**

$$U = M + \epsilon V = M - PV$$

Immediate consequences

- **Black hole volume:**

$$V=\left(\frac{\partial M}{\partial P}\right)_{S,\dots}$$

Schwarzschild(-AdS):

$$V=\frac{4}{3}\pi r_+^3$$

• More involved for more complicated black holes

• The fact this this provides a good definition of volume is supported by the **Reverse Isoperimetric Inequality** conjecture:

M. Cvetic, G.W Gibbons, DK, C.N. Pope, *Black hole enthalpy and an entropy inequality for the thermodynamic volume,* Phys. Rev. D84 (2011) 024037, [arXiv:1012.2888].

Immediate consequences

- **Consistent Smarr relation:**

$$\delta M = T\delta S + V\delta P + \phi\delta Q + \Omega\delta J\,,$$
$$M = \frac{D-2}{D-3}(TS + \Omega J) + \phi Q - \frac{2}{D-3}PV$$

- **Phase transitions:**

- AdS black holes can be in **thermal equilibrium**
- Exhibit interesting **phase transitions – exactly analogous to everyday TDs**
- Provide dual description of CFT at finite temperature via **AdS/CFT correspondence**

Canonical Example: VdW behavior of charged AdS black holes

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega_2^2 \,, \qquad A = -\frac{Q}{r} dt$$

$$f = 1 - \frac{2GM}{r} + \frac{GQ^2}{r^2} + \frac{r^2}{l^2} \,,$$

- Basic thermodynamic quantities:

$$M = \frac{r_+(l^2 + r_+^2)}{2l^2 G} + \frac{Q^2}{2r_+} \,, \qquad T = \frac{3r_+^4 + l^2 r_+^2 - GQ^2 l^2}{4\pi l^2 r_+^3}$$

$$S = \frac{\pi r_+^2}{G} \,, \qquad V = \frac{4\pi r_+^3}{3} \,, \qquad \phi = \frac{Q}{r_+} \,,$$

$$\boxed{F = M - TS = \frac{3GQ^2 l^2 + l^2 r_+^2 - r_+^4}{4G r_+ l^2}}$$

Example: VdW criticality

- DK, R.B. Mann, *P-V criticality of charged AdS black holes*, JHEP 1207 (2012) 033.

Van der Waals fluid

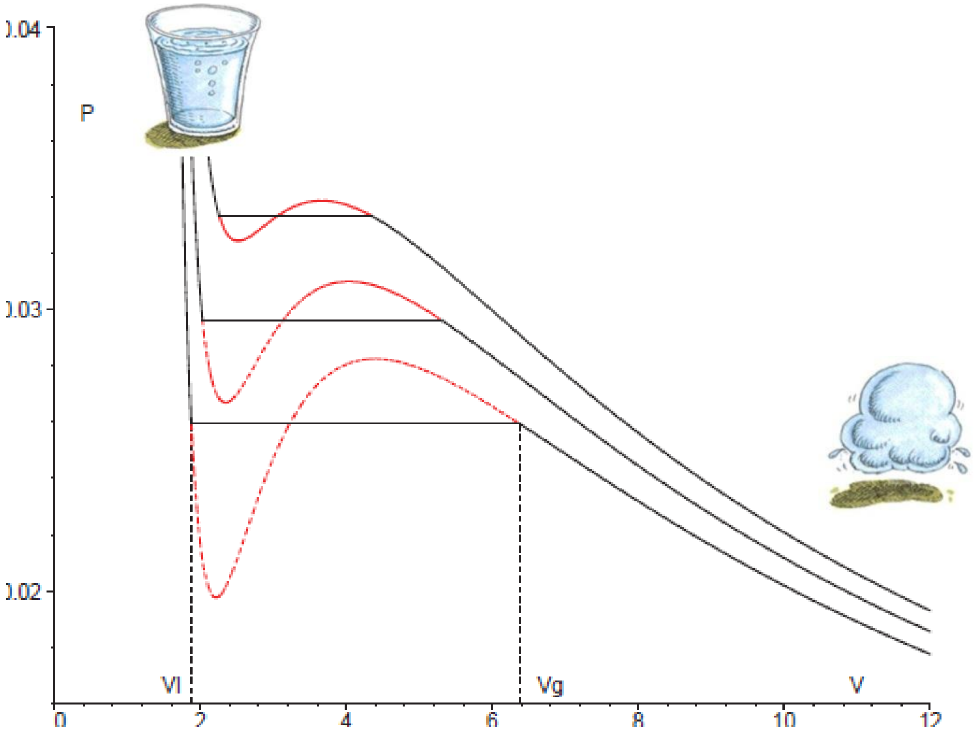


FIG. 2. Maxwell's equal area law. The 'oscillating' (dashed) part of the isotherm $T < T_c$ is replaced by an isobar, such that the areas above and below the isobar are equal one another.

$$\left(P + \frac{a}{v^2}\right)(v - b) = T$$

Parameter a measures the **attraction** between particles (a>0) and b corresponds to "**volume of fluid particles**".

Critical point:

$$\rho_c = \frac{P_c v_c}{T_c} = \frac{3}{8}$$

P-V criticality

- DK, R.B. Mann, *P-V criticality of charged AdS black holes*, JHEP 1207 (2012) 033.

Charged black hole

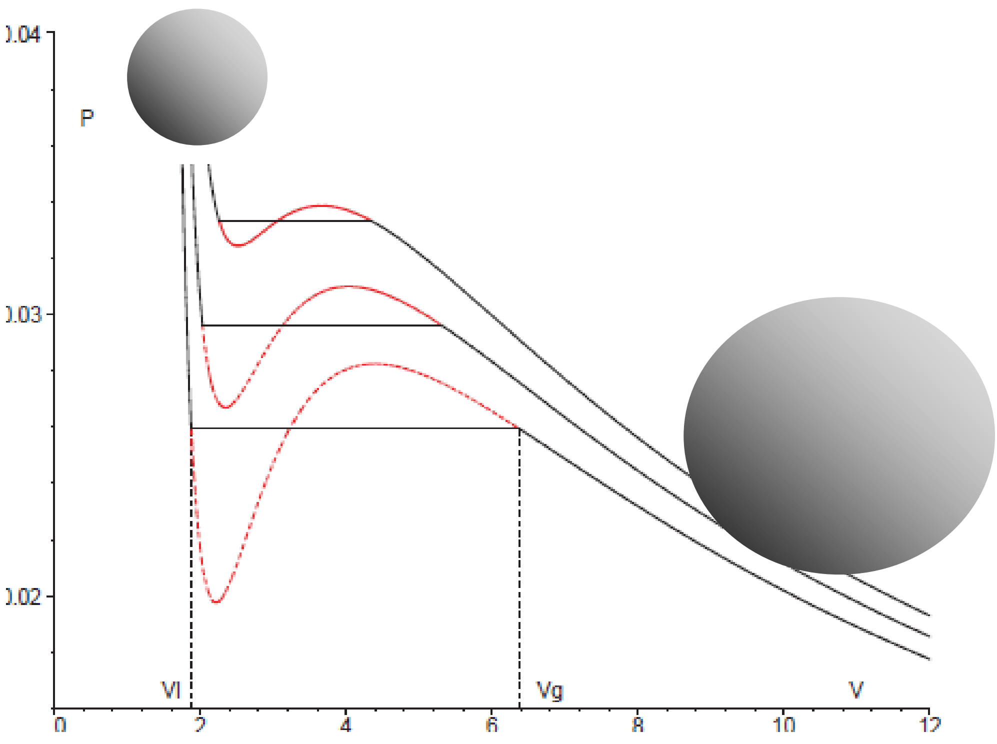


FIG. 2. Maxwell's equal area law. The 'oscillating' (dashed) part of the isotherm $T < T_c$ is replaced by an isobar, such that the areas above and below the isobar are equal one another.

$$\left(P + \frac{a}{v^2}\right)(v - b) = T$$

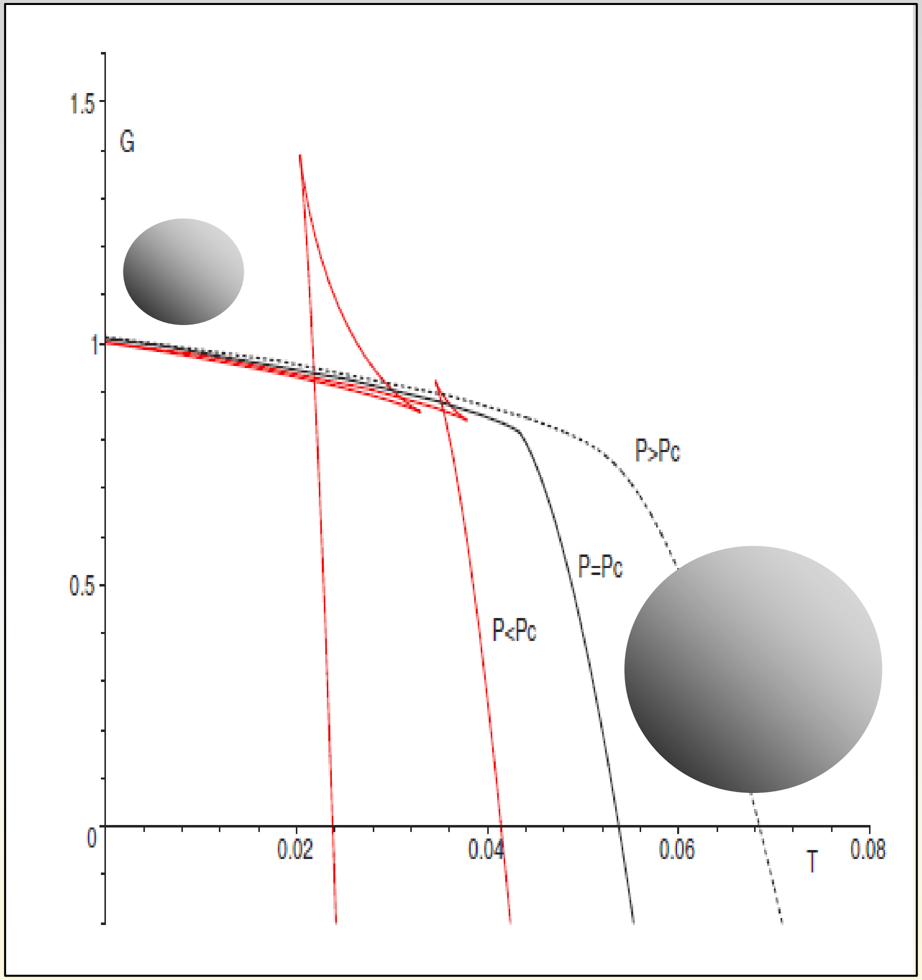
$$P = \frac{T}{v} - \frac{1}{2\pi v^2} + \frac{2Q^2}{\pi v^4}$$

Critical point:

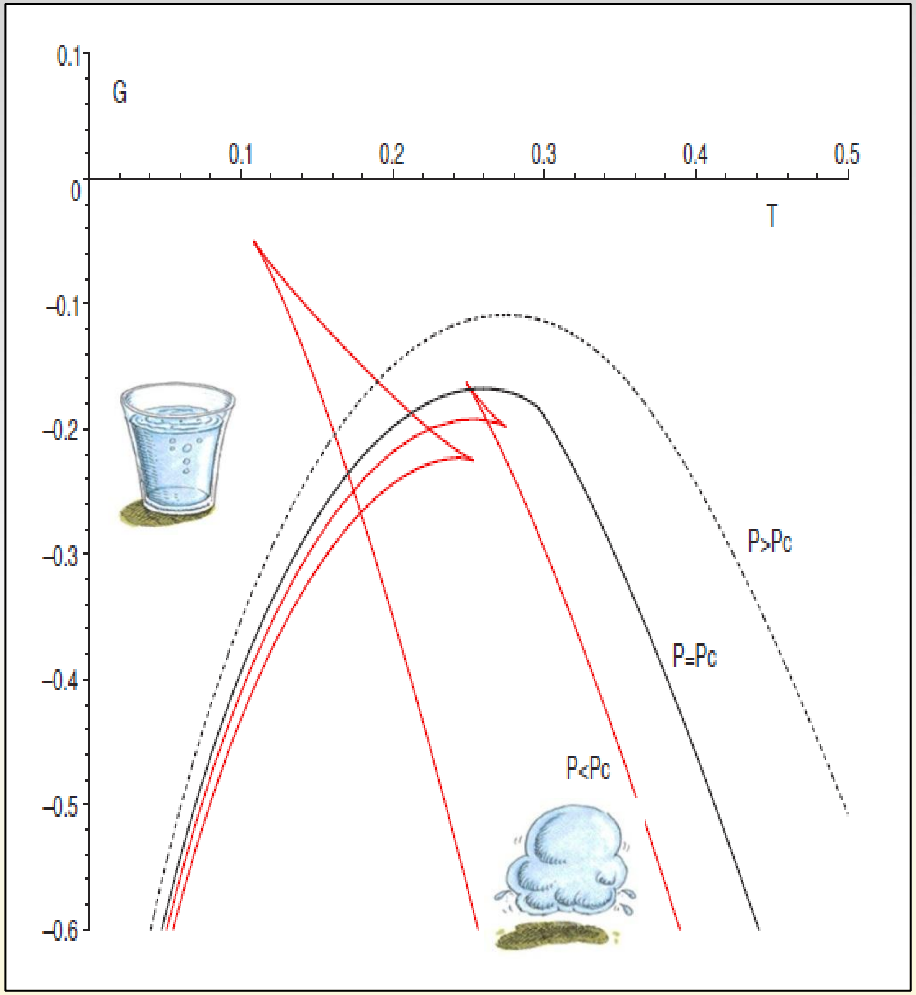
$$\rho_c = \frac{P_c v_c}{T_c} = \frac{3}{8}$$

Free energy: demonstrates standard swallow tail behavior

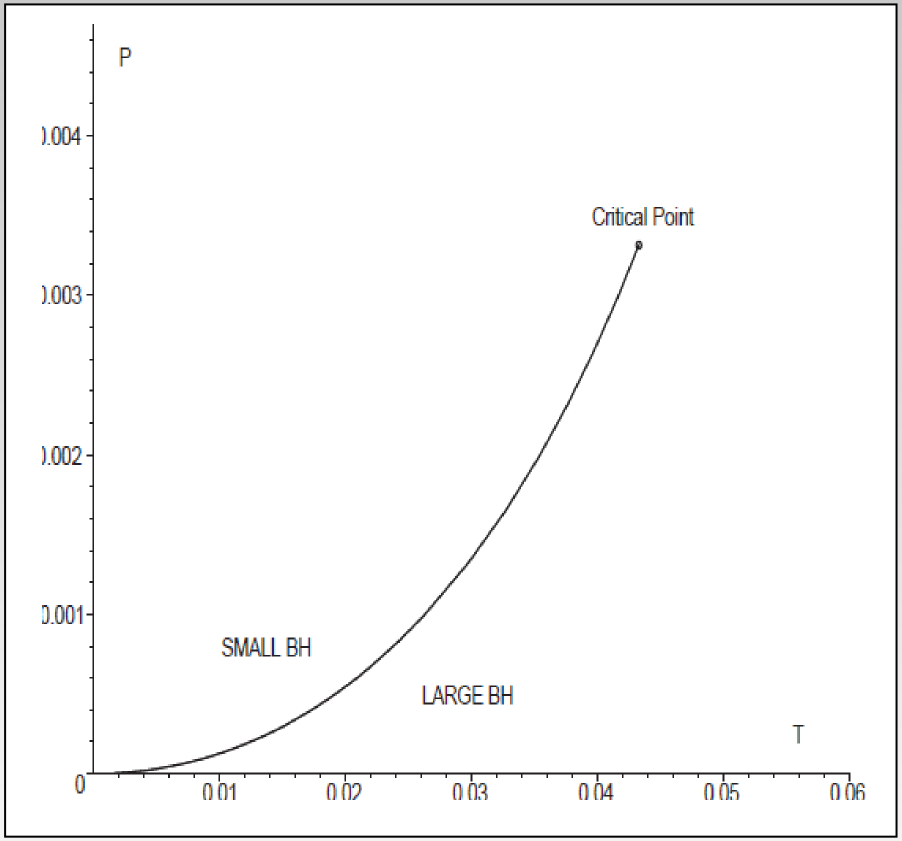
$$F = F(T,P,Q) = \frac{1}{4}\left(r_+ - \frac{8\pi}{3}Pr_+^3 + \frac{3Q^2}{r_+}\right)$$



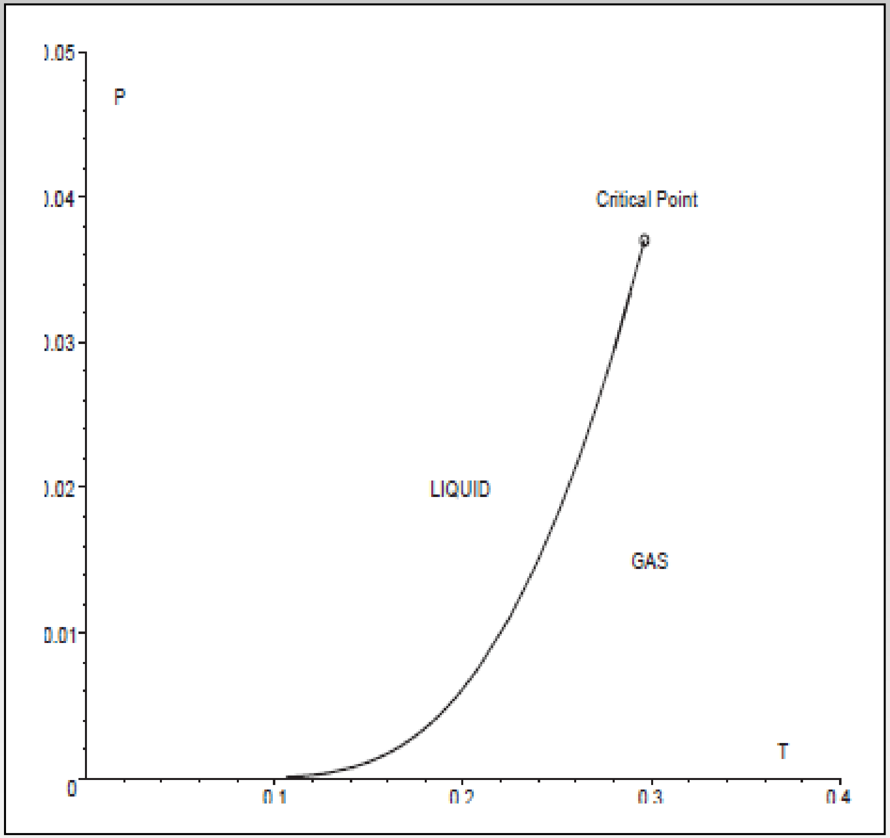
vs.



Phase diagrams: complete analogy



vs.

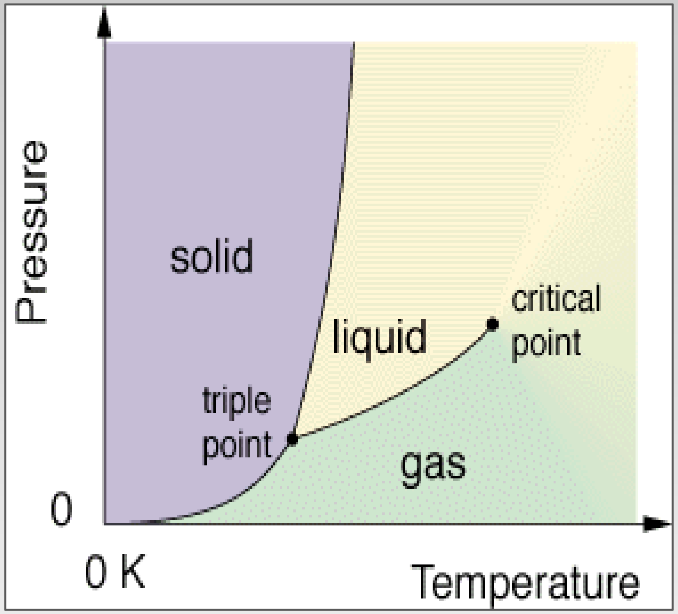


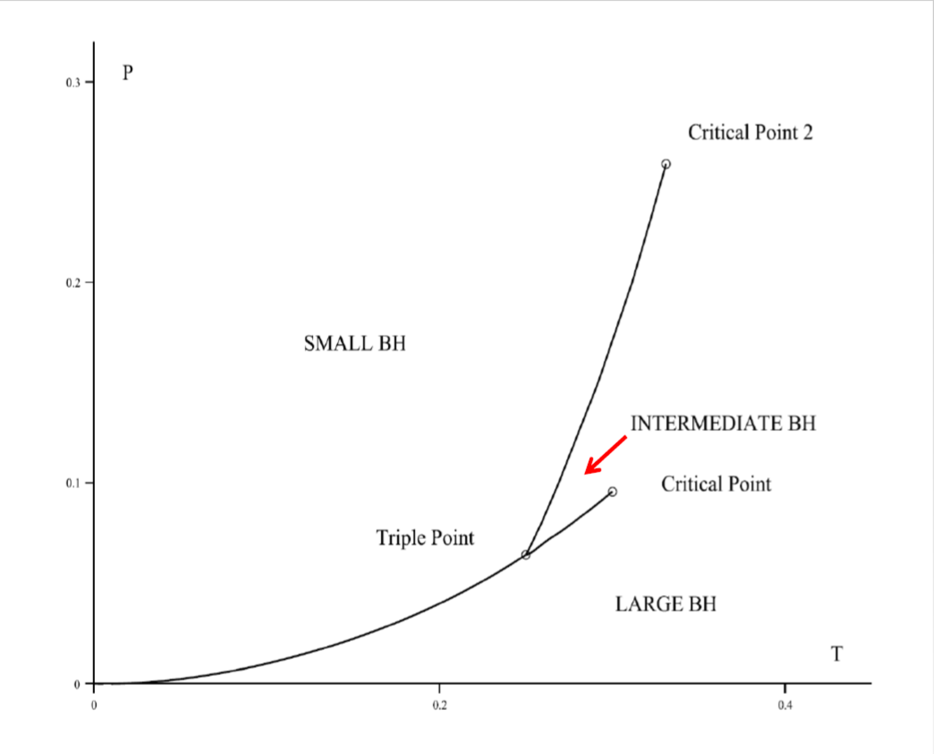
- Coexistence & critical point described by **Clausius-Clapeyron** and **Ehrenfest** equations
- **MFT critical exponents**

$$\alpha = 0\,, \quad \beta = \frac{1}{2}\,, \quad \gamma = 1\,, \quad \delta = 3$$

More generally: black hole chemistry

- **Triple point** and solid/liquid/gas analogue**:**





- **Many other**:
  - Isolated critical point
  - Reentrant PT
  - superfluid PT

(can have n-tuple points)

- DK, Mann, Teo, *Black hole chemistry: thermodynamics with Lambda*, CQG 34 (2017) 063001, Arxiv:1608.0614.

3) Black hole chemistry in analogue systems?

(million dollar questions for you)

- Construct **black hole analogues** with well defined TD ensemble (AdS-like black holes, BHs in cavity?)
- Can we go **beyond Hawking radiation** – assign entropy, energy, pressure,….?
- Can we go beyond kinematics -- consider backreaction of the system? **Capture 1st law** (dynamics of gravitating systems)? Can we probe other laws?
- Can we use **analogue systems** to study TD black hole phase transitions?

Summary

- Black hole thermodynamics goes beyond kinematic description of **Hawking radiation***

- Especially interesting is the framework of **black hole chemistry** (thermodynamics with variable Λ)

- It uncovers rich structure of **phase transitions** of BHs – **analogous** to **everyday TDs**

- Can some of these been studied using the **BH analogue systems** (how do we capture the dynamics of gravitating systems?)

*Temperature is non-local, and can also depend on the theory.