LOOKING FOR THE BLACK HOLE LASER EFFECT IN INTERFACIAL HYDRODYNAMICS

Alexis BOSSARD (Pprime Institute,Poitiers, France)

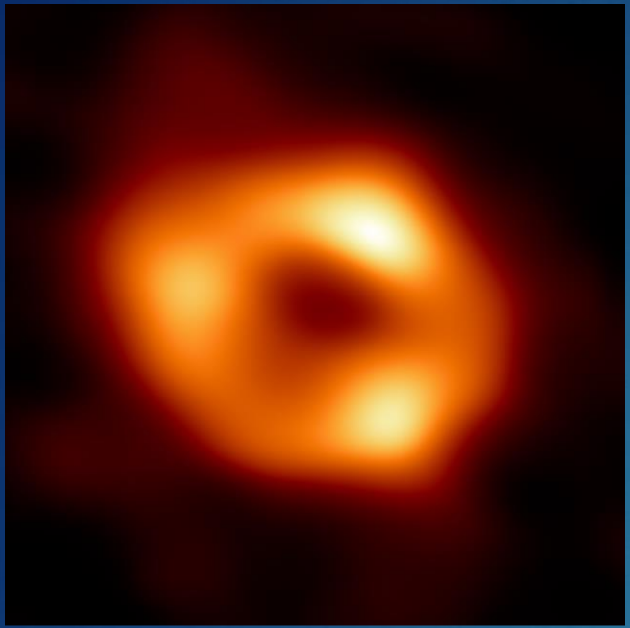
1st year of PhD thesis under the supervision of Germain Rousseaux (CNRS)

21 May 2023 – 3 June 2023

CENTRO DE CIENCIAS DE BENASQUE PEDRO PASCUAL

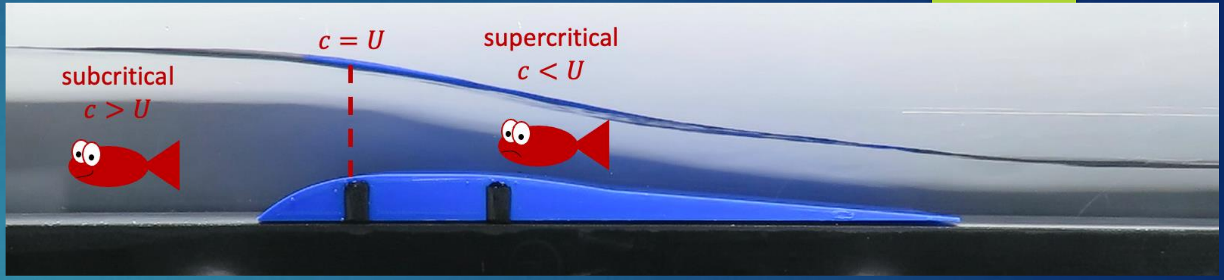
Analogue gravity in interfacial hydrodynamics

William Unruh (1981)[1]



Source: https://Beta.NSF.GOV/EHT

analogous to...
(kinematically)



$\mathbf{Fr_{local} = 1} \Rightarrow$ Existence of an analogous horizon

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\, g^{\mu\nu}\partial_\nu\phi\right) = 0 \text{ with } ds^2 = g_{\mu\nu}dx^\mu dx^\nu$$

- $\beta = \frac{v_{\text{escape}}(r)}{c} = \sqrt{\frac{r_s}{r}}$ with $r_s = \frac{2GM}{c^2}$
- $ds^2 = -c^2\left(1 - \frac{v(r)^2}{c^2}\right)dt^2 + 2v(r)dtdr + dr^2 + r^2 d\Omega^2$
- $(\omega - v_{\text{escape}}k)^2 = c^2k^2$ with $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \approx 3\times 10^8\ m.s^{-1}$
- $T_{\text{Hawking}} = \frac{\hbar}{4\pi k_B c}\left|\frac{\partial v^2_{\text{escape}}}{\partial r}\right|\Bigg)_{r=\beta^{-1}(1)} = \frac{\hbar c^3}{8\pi k_B GM}$

Stephen Hawking (1974)[2]

$$g^{\mu\nu}k_\mu k_\nu = 0$$

- $Fr = \frac{U(x)}{c} = \frac{U(x)}{\sqrt{gh}}$ with $g = 9.81\ m.s^{-2}$ and h the water depth
- $ds^2 = -c^2\left(1 - \frac{U^2}{c^2}\right)dt^2 + 2U dtdx + dx^2$
- $(\omega - Uk)^2 = c^2k^2$ with no dispersive terms
- $T_{\text{Visser}} = \frac{\hbar}{4\pi k_B}\left|\frac{1}{c}\frac{\partial(c^2 - U^2)}{\partial x}\right|\Bigg)_{x=Fr^{-1}(1)}$

Matt Visser (1998)[3]

White hole: time reversal of the black hole($t \to -t$) ~ White fountain

Euvé et al (2016) [4]

Black hole lasers

Steven Corley*
Theoretical Physics Institute, Department of Physics, University of Alberta, Edmonton, Alberta, Canada T6G 2J1

Ted Jacobson†
Department of Physics, University of Maryland, College Park, MD 20742-4111, USA

Vilenkin (1978) [5] : one active semi-transparent mirror + one mirror

Corley and Jacobson (1999) [6]: two active semi-transparent mirrors

Superluminal correction in supercritical region

Examples: BEC, circular jump

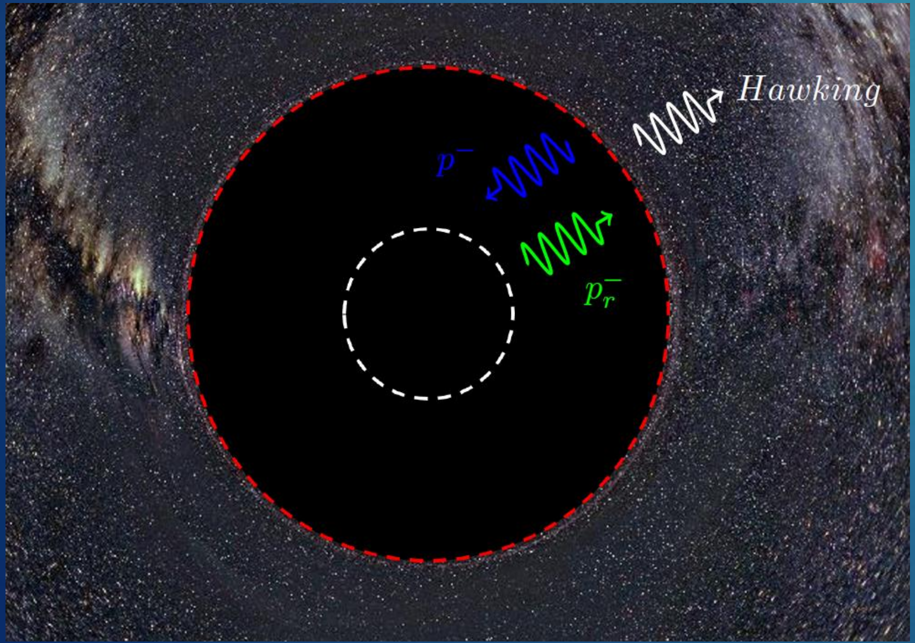
Definition of the black hole laser effect: it is the amplification of Hawking radiation due to successive bouncing of trapped modes on two horizons which would act as active mirrors as in an optical laser

$$(\omega - vk)^2 = c^2 F(k)^2$$

$$F(k)^2 = k^2 \pm \frac{1}{k_0^2} k^4$$

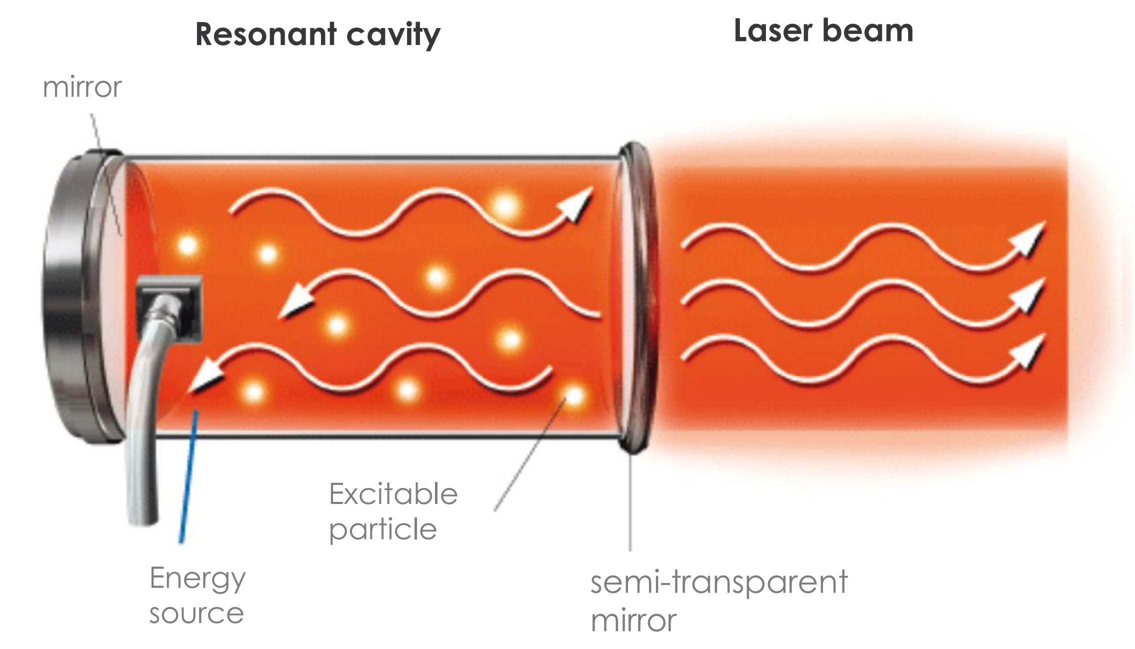
Subluminal correction in subcritical region

Example: Flow in a free surface channel

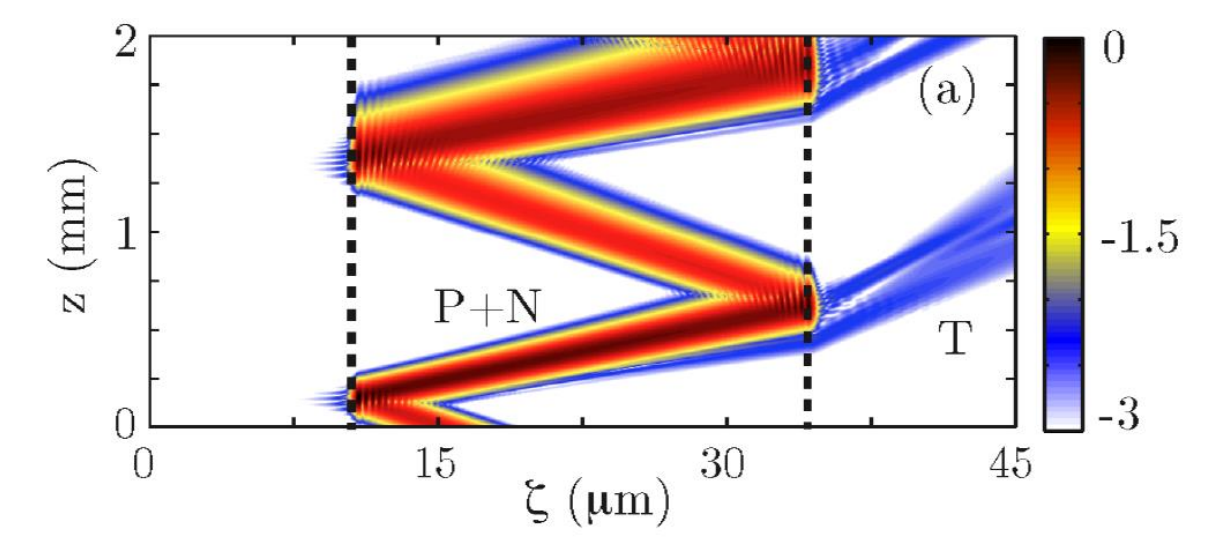


schematic representation of a laser effect between an inner and outer horizon

~ analogous to… (kinematically)



Numerical existence of the black hole laser effect



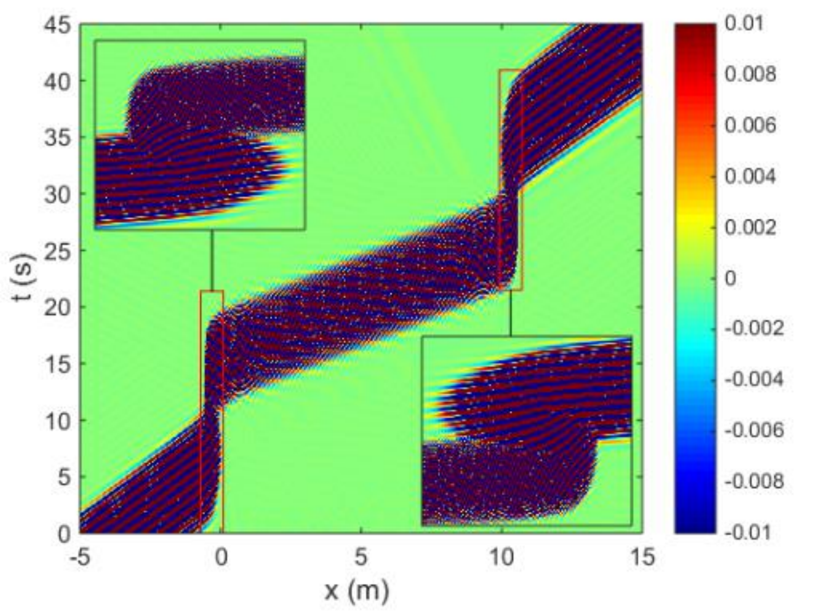
Faccio et al (2012) [7]: Optical black hole laser

Without dissipation

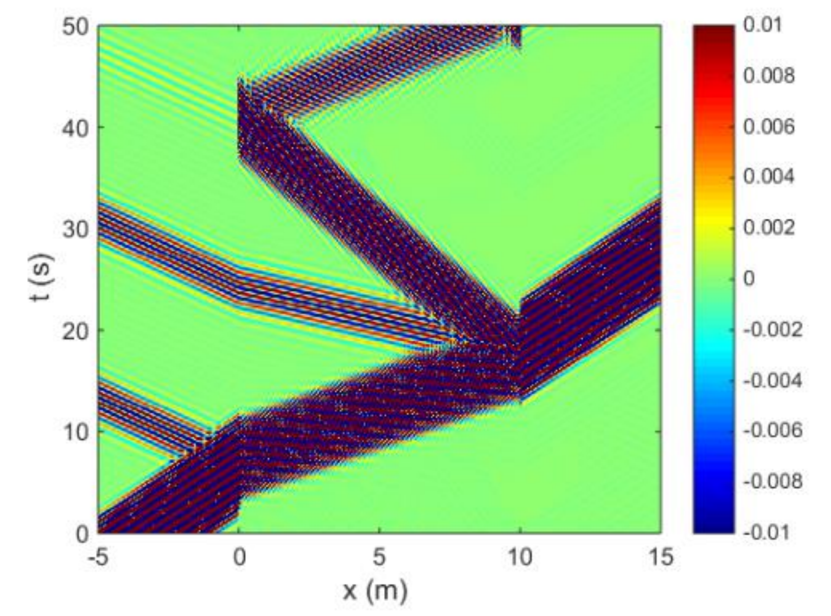
Robertson and Rousseaux [9]

Peloquin et al (2016) [8]:
Hydrodynamic black hole laser

- The velocity field is imposed
- Stable horizons



Low speeds gradients

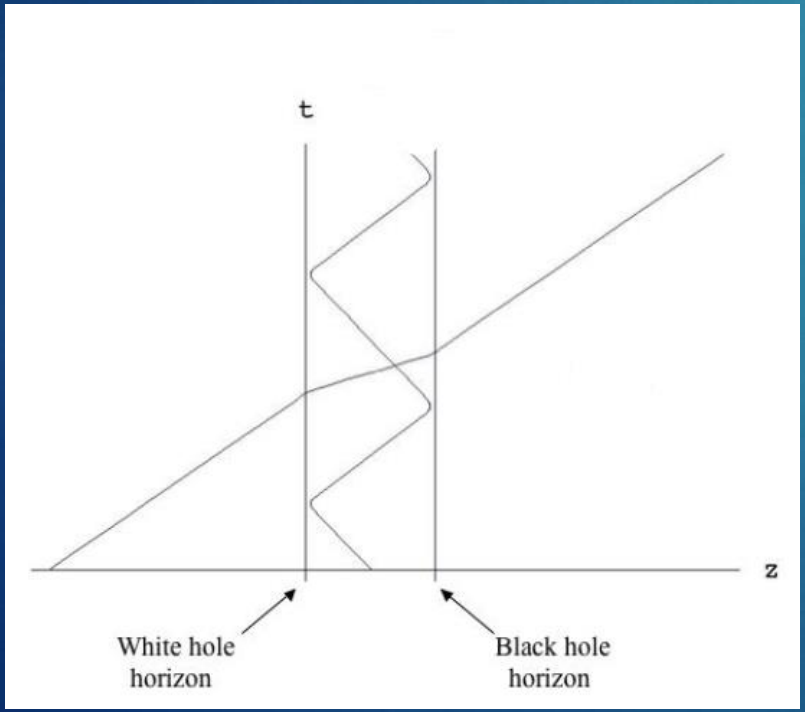


High speeds gradients

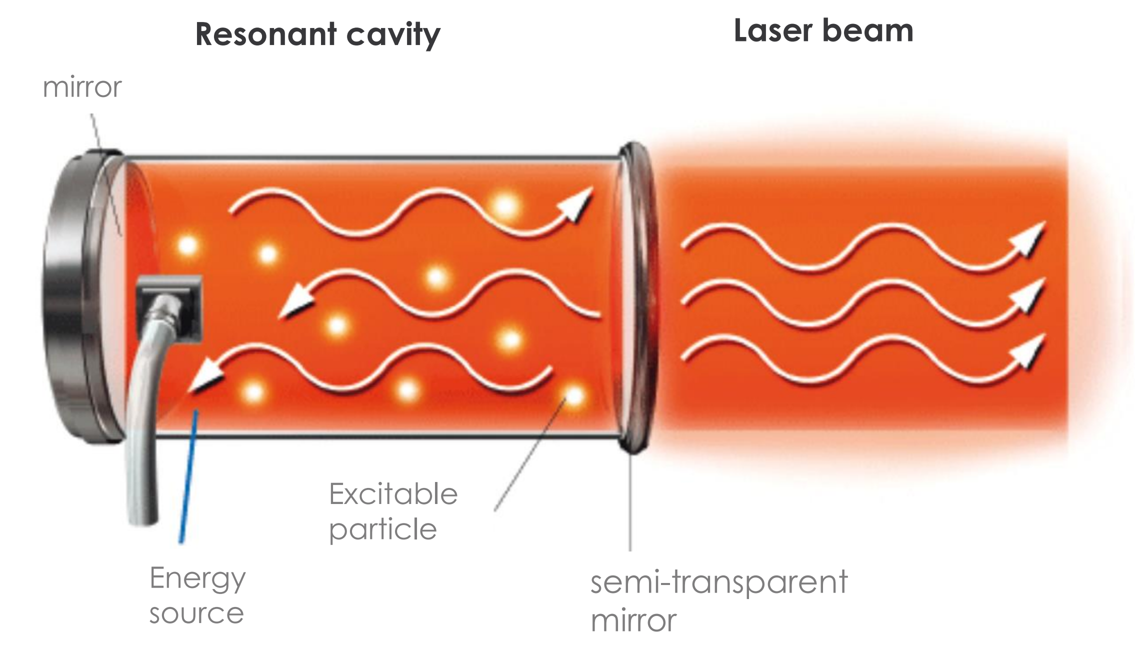
Ingredients for the usual black laser effect (as discuss in literature):

1. You need two stable horizons
2. A superluminal or subluminal dispersive correction
3. A trapping cavity with a flow regime compatible with the dispersive regime
4. Mixing of positive and negative modes
5. To avoid modes dissipation

No experimental measurement Steinhauer [10]



~ analogous to… (kinematically)



I) Flow over 2 obstacles :

Pratt (1984)[11]

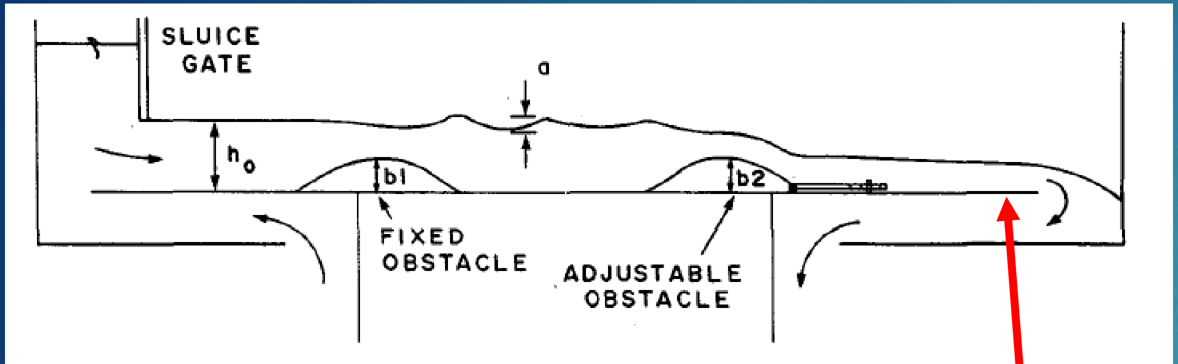


FIG. 3. Cross-section of laboratory flume.

Channel too short to observe the influence of downstream dissipation

Distance between obstacles:
4*length of the first obstacle

Definitions (from local Fr) :

- Transcritical flow:
Transition from Fr<1 to Fr>1 (or vice versa)
- Supercritical flow: Fr>1
- Subcritical flow: Fr<1

- Froude number

$$Fr = \frac{U}{c} = \frac{U}{\sqrt{gh}}$$

- Pratt number

$$\mathcal{P} = \frac{b_2 - b_1}{b_1}$$

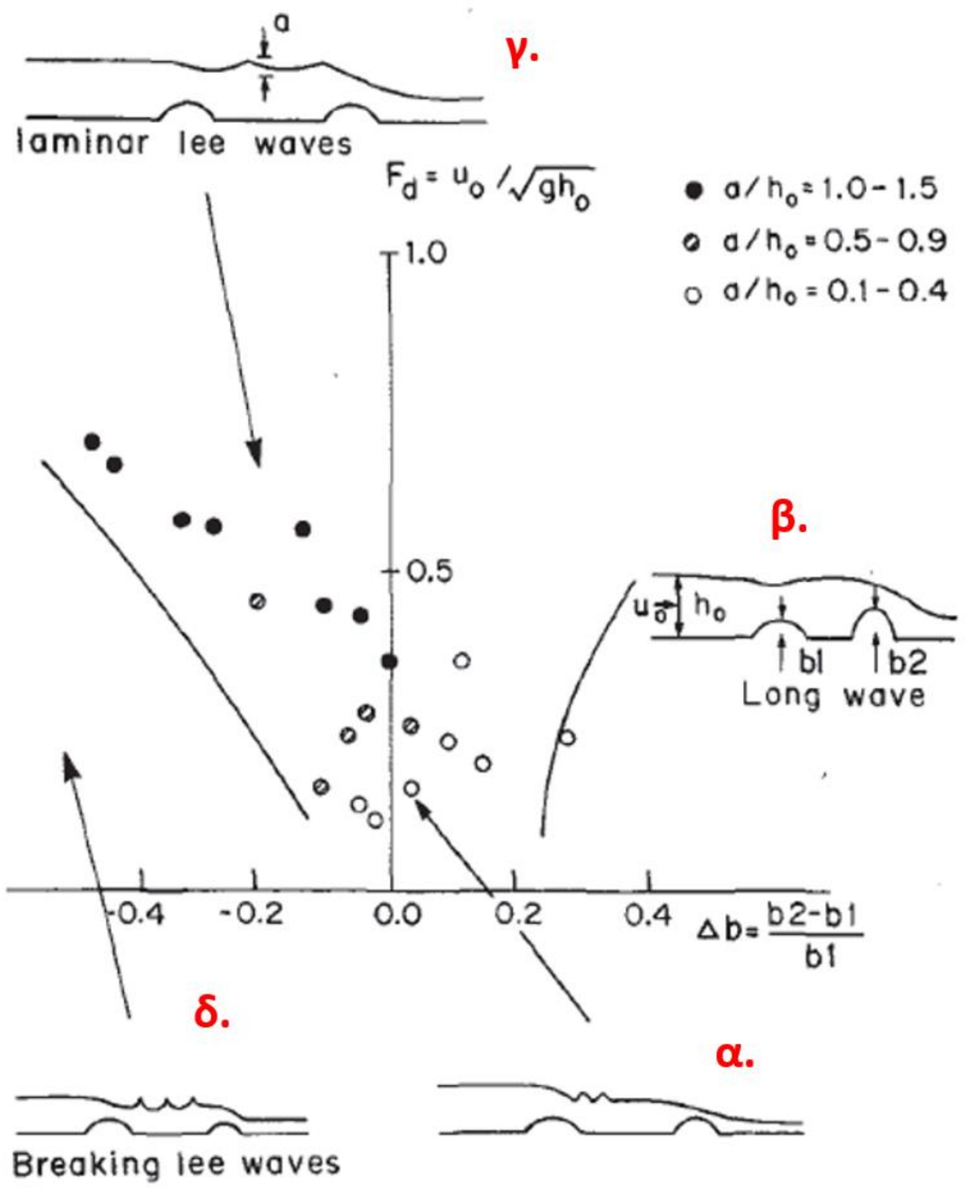


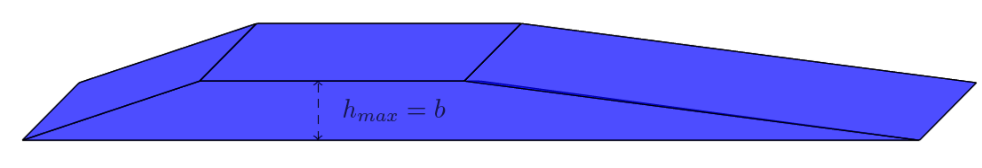
FIG. 5. Experimental results showing different flow regimes as function of $\Delta b = (b2 - b1)/b1$, and Froude number Fd = $u_0/(gh_0)^{1/2}$ measured upstream of first obstacle. The relative amplitude a/h_0 of the laminar wave nearest the upstream obstacle is indicated by the type of dot, as defined in the figure.

II) 1D free surface channel

Downstream gate

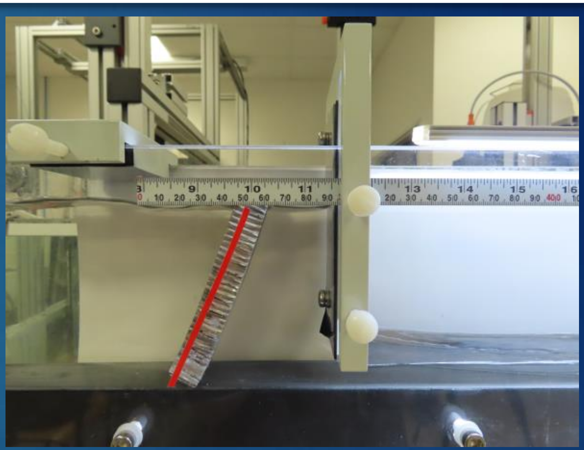
Flow inlet

pump



Type of asymmetric geometry used:
Length: 32.2 cm
Maximum heigth: 2.1 cm

Sluice gate

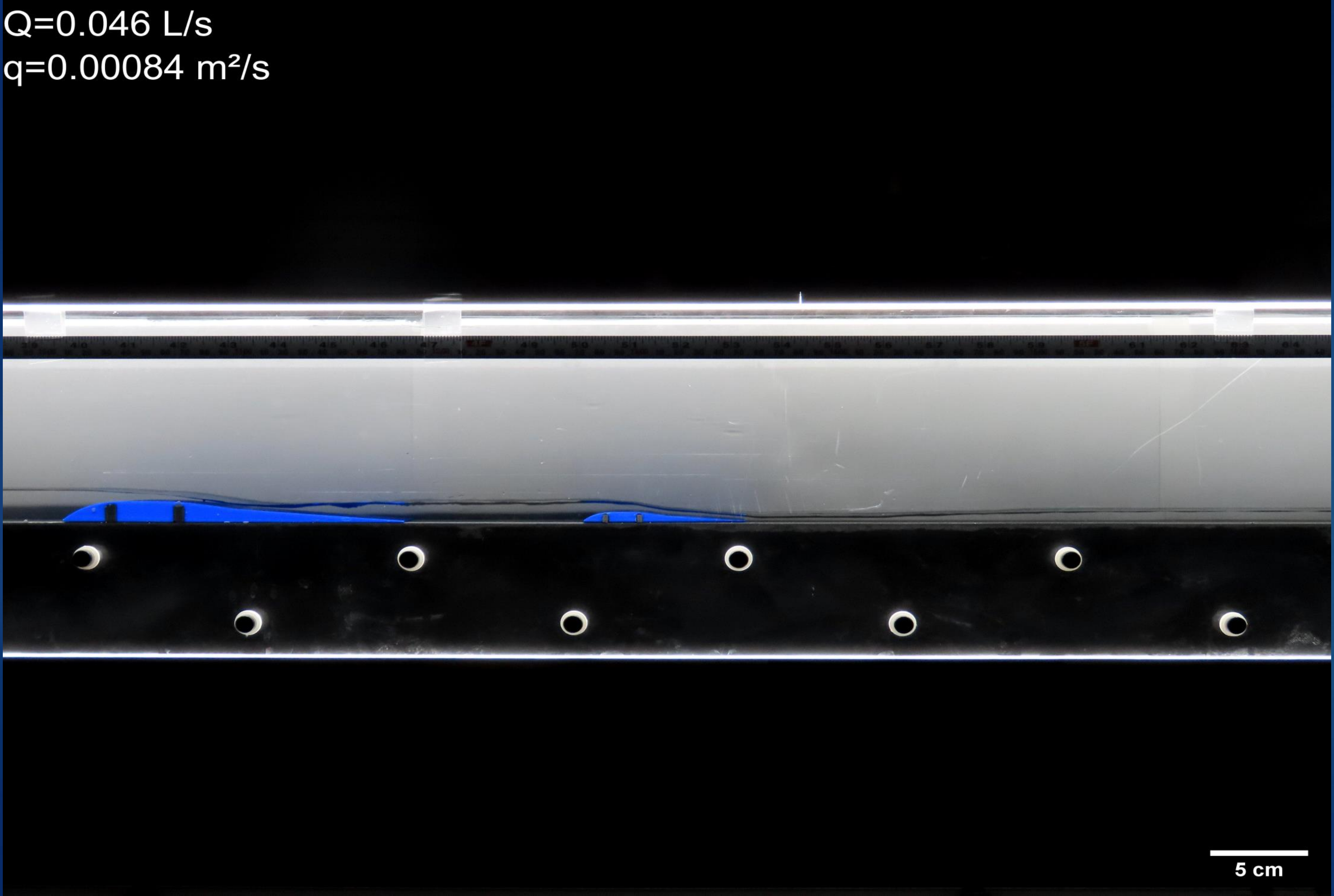


Hypotheses and experimental conditions:

- flow conservation
- $U_{upstream}=Q/(Wh)$
- No downstream condition (door open)
- No initial water level imposed
- Inter-obstacle distance set at 9.2 cm (arbitrary)
- Neglected boundary layer

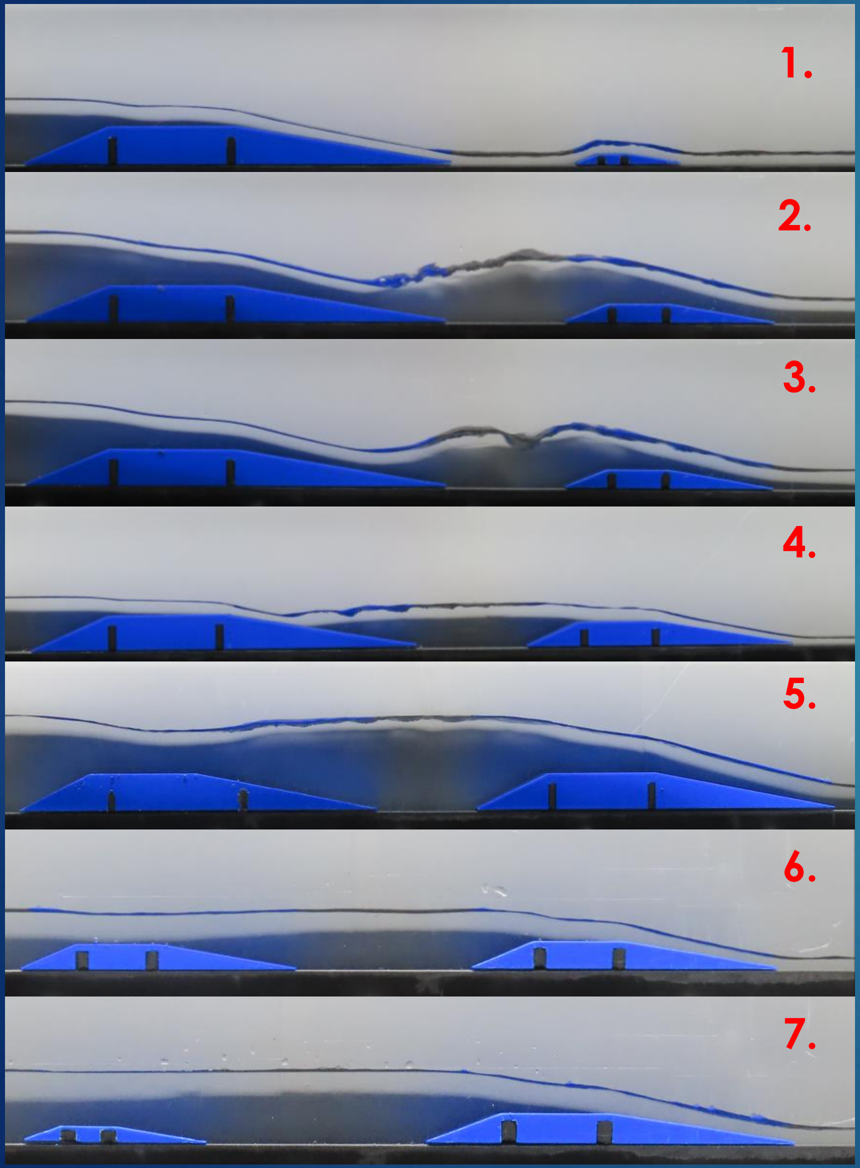
Channel characteristics:

- Length: L=2.5 m
- Wide: W=5.5 cm
- Range of the flow rate: 2 to 38 L/min
- Range of the flow rate: 0.0006 to 0.0115 m²/s



The flow rate is increased from one image to the other.

III) Flow regimes



1. T_a-S-S (Transcritical accelerating-Supercritical-Supercritical)

2. T_a-B-T_a (Transcritical accelerating-Breaking- Transcritical accelerating)

3. T_a-UB-T_a (Transcritical accelerating-Undular Breaking- Transcritical accelerating)

4. T_a-U*-T_a (Transcritical accelerating-Undular- Transcritical accelerating)

5. D-U-T_a (Depression-Undular-Transcritical accelerating)

6. D-E-T_a (Depression-Emitting-Transcritical accelerating)

7. F-F-T_a (Flat-Flat-Transcritical accelerating)

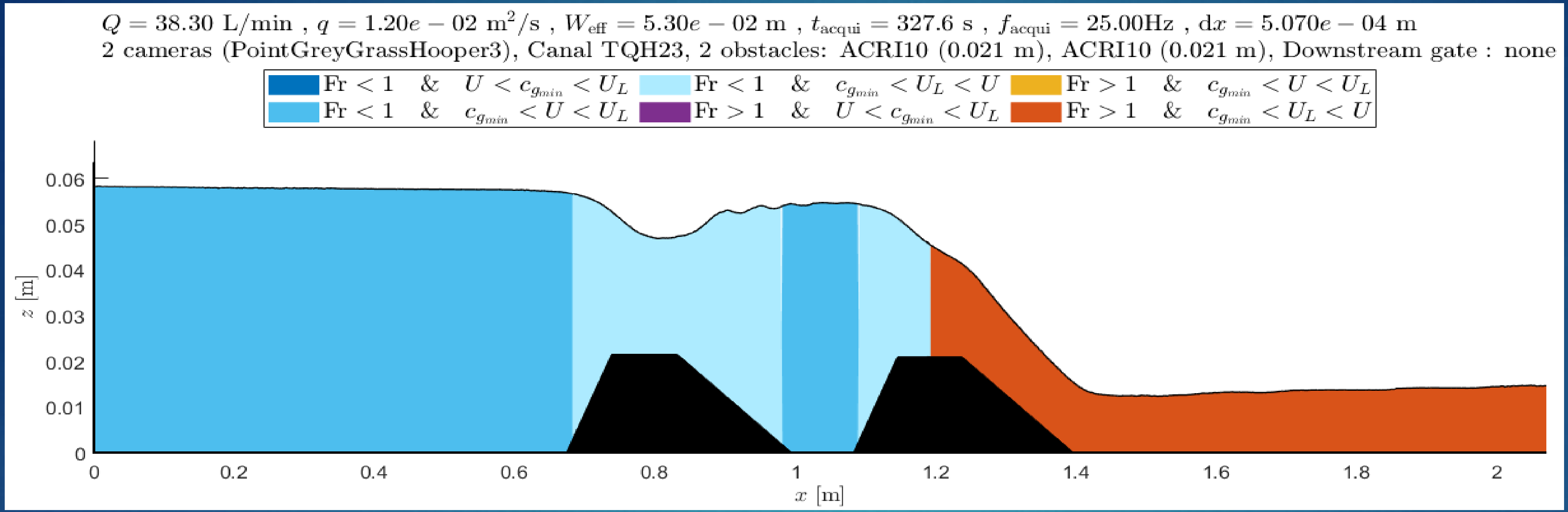
Identification in the Pratt diagram

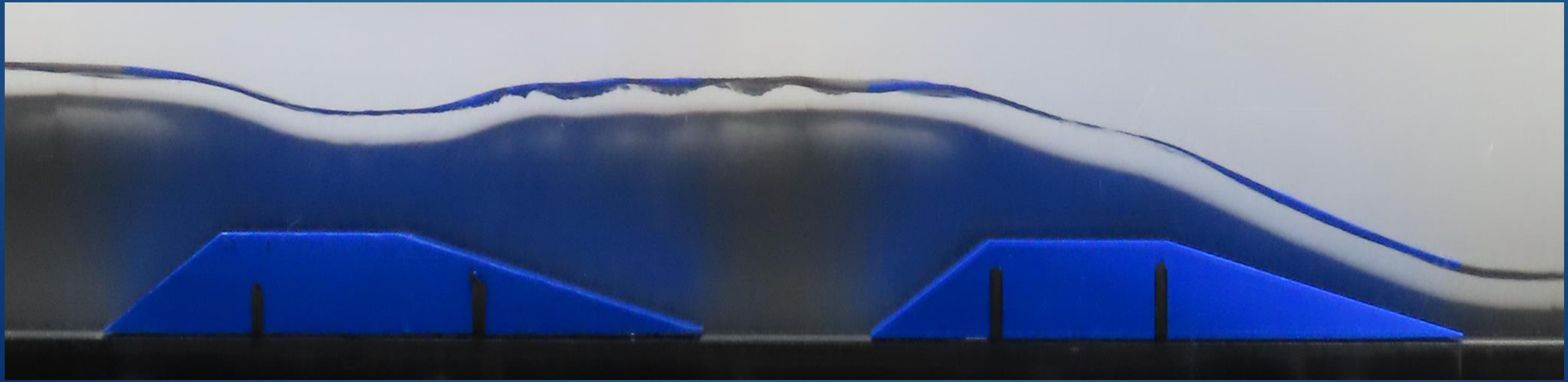
	Identification in the Pratt diagram	
1.	New regime	✗
2.	New regime	✓
3.	δ	✓
4.	α	✓
5.	γ	✓
6.	β	?
7.	New regime	✗

According to Coutant and Parentani (2014)[12] and Euvé (2016)[4]

Germain Rousseaux et Hamid Kellay (2020), Introduction of the nomenclature with one obstacle : F, D, U, B, etc[13]

Example of interface extraction on the regime 5.b) (D-U-T_a) :





$$U(x) = \frac{Q}{W h(x)}$$

$$c_{gmin} = \min_{k \in \mathbb{R}_+} (v_g)$$

$$c_{gmin} \underset{kh>>1}{=} \frac{\sqrt{3}}{\sqrt[4]{2\sqrt{3}+3}} \sqrt[4]{\frac{\gamma g}{\rho}}$$

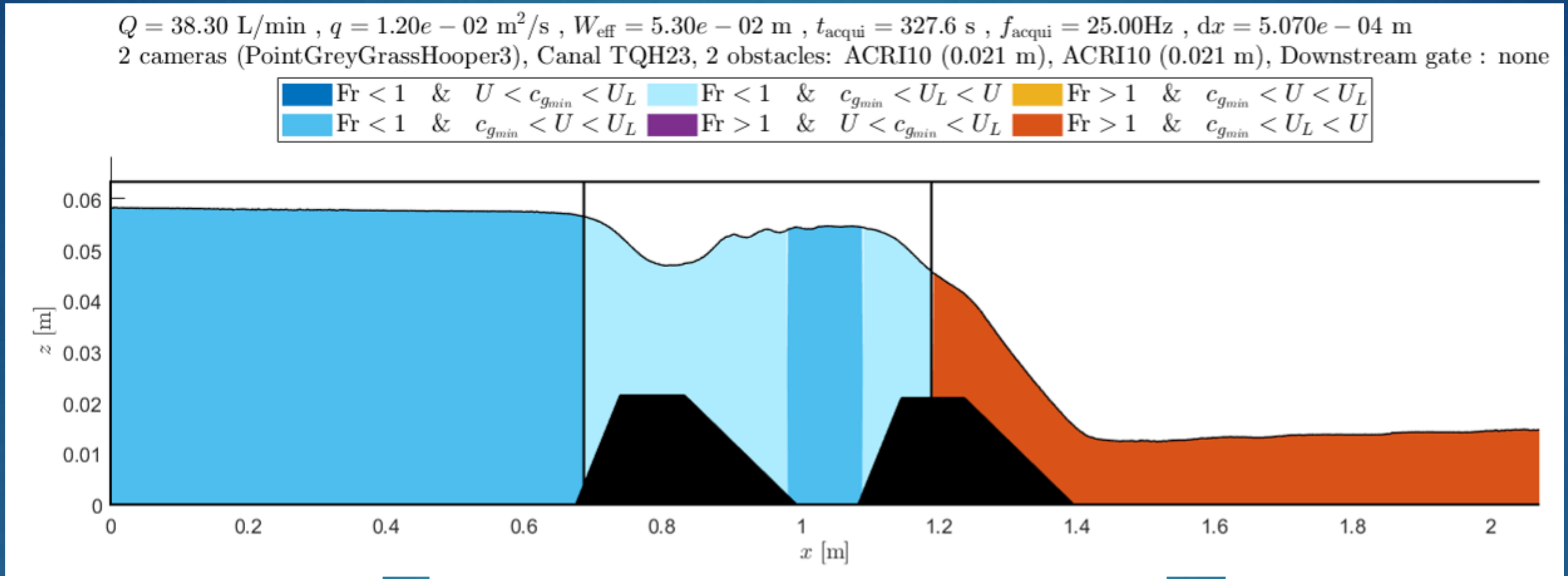
$$U_L = \min_{k \in \mathbb{R}_+} (v_\varphi)$$

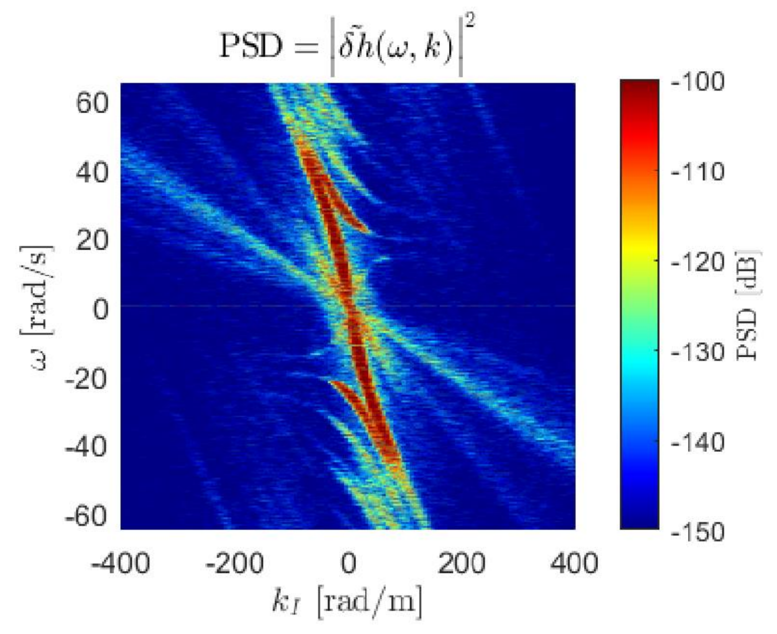
$$U_L \underset{kh>>1}{=} \sqrt{2} \sqrt[4]{\frac{\gamma g}{\rho}}$$

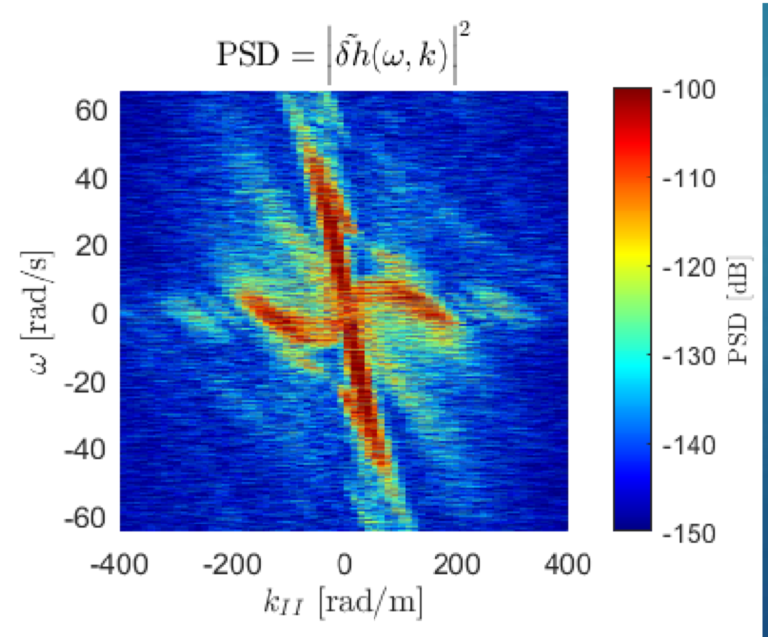
= possible existence of negative modes

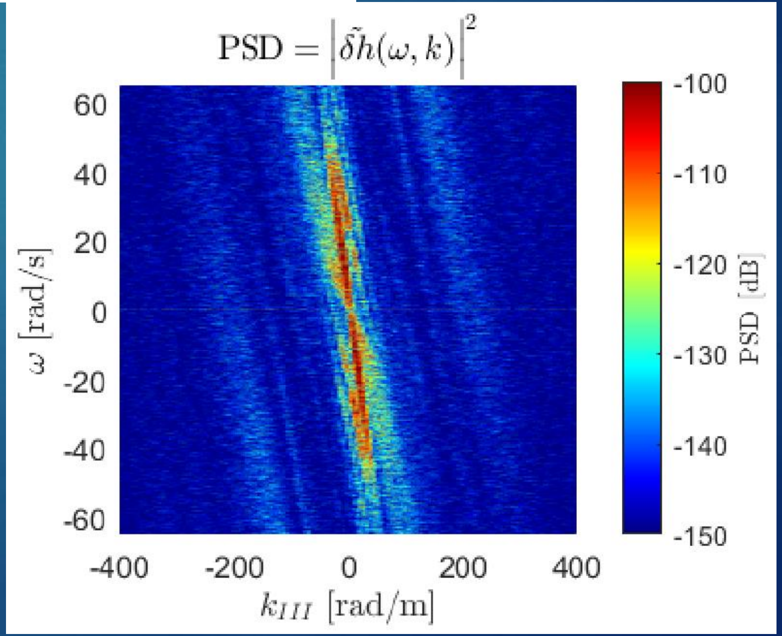
$$\left(\omega - \overrightarrow{U}\cdot\overrightarrow{k}\right)^2 = gk\left(1+\frac{\gamma}{\rho g}k^2\right)\mathrm{th}\,(kh)$$

Can modes be measured?

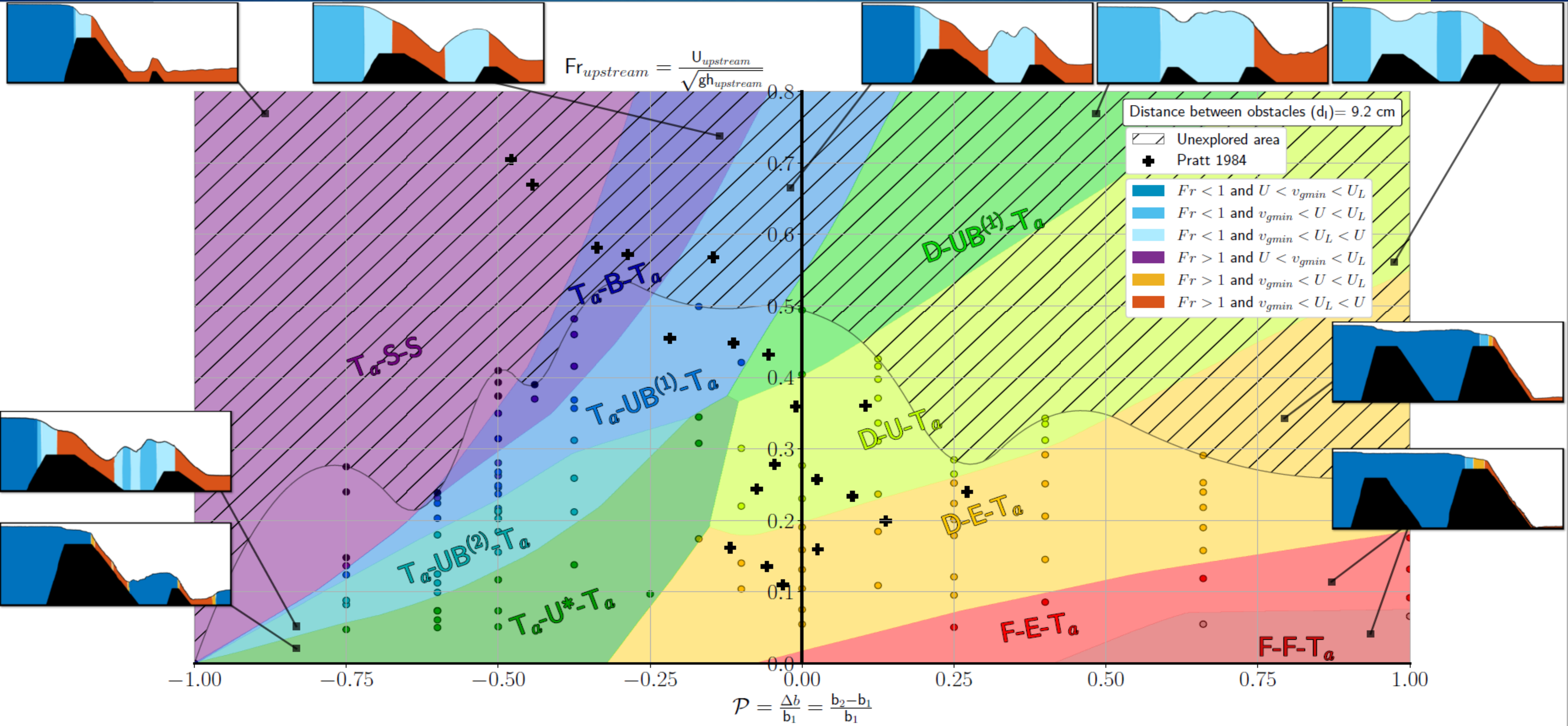








Improved Pratt diagram ($\mathrm{Fr}_{\mathrm{upstream}}(\mathcal{P})$)



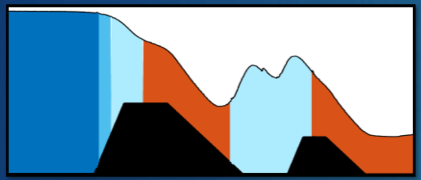
Inter-obstacles distance: 9.2 cm
without downstream condition

Number of experimental points to train the neural network: 119

Summary

1. You need two stable horizons
2. A superluminal or subluminal dispersive correction
3. A trapping cavity with a flow regime compatible with the dispersive regime
4. Mixing of positive and negative modes

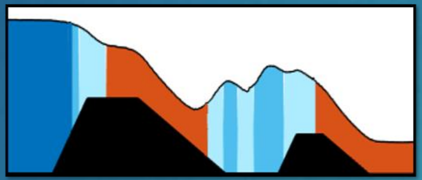
T_a-$UB^{(1)}$-T_a



1: ✓ 2: ✓ 3: ✓ 4: ?

Closest scenario to general relativity Because it is possible to have negative energy waves in the cyan subliminal cavity.

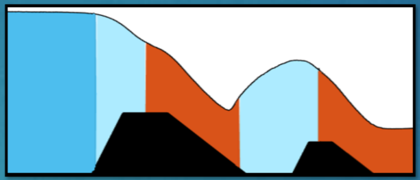
T_a-$UB^{(2)}$-T_a



1: ✓ 2: ✓ 3: ✓ 4: ?

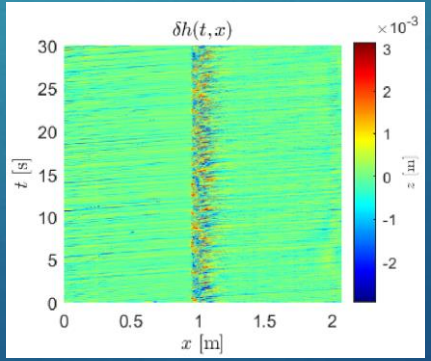
In this scenario, the hawking radiation can be blocked because of capillary dispersion at small scales

T_a-B-T_a

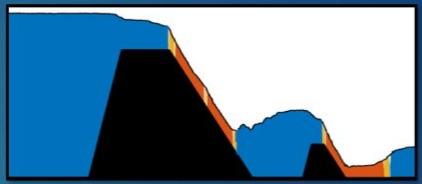


1: ✗ 2: ✓ 3: ✓ 4: ?

Presence of two horizons but the white horizon is unsteady



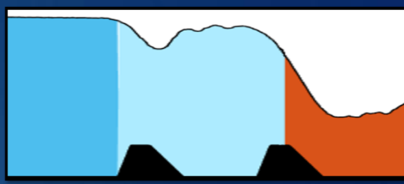
T_a-U^*-T_a



1: ✓ 2: ✓ 3: ✓ 4: ?

two superluminal red cavities on both obstacles are formed but damping will kill the superluminal capillary waves.

D-$UB^{(1)}$-T_a



1: ✓ 2: ✓ 3: ✓ 4: ?

The undulation is trapped between a non-dispersive downstream horizon and a dispersive Upstream horizon.

References:

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[2] Stephen W. Hawking. Black hole explosions? Nature,248(5443):30-31,1974.

[3] Matt Visser(1998). Acoustic black holes: horizons, ergospheres and Hawking radiation. *Classical and Quantum Gravity*, *15*(6), 1767.

[4]Léo-Paul Euvé, Florent Michel, Renaud Parentani, Thomas G. Philbin, and Germain Rousseaux. Observationof noise correlated by the hawking effect in a water tank. Physical review letters, 117(12) :121301, 2016.

[5]Alexander Vilenkin. Exponential amplification of waves in the gravitational field of ultrarelativistic rotatingbody. Physics Letters B, 78(2-3) :301–303, 1978.

[6] Steven Corley and Ted Jacobson. Black hole lasers. Physical Review D, 59(12) :124011, 1999.

[7] Daniele Faccio, Tal Arane, Marco Lamperti and Ulf Leonhardt. Optical black hole lasers. Classical and Quantum Gravity 29.22 : 224009,2012.

[8] Cédric Peloquin, Léo-Paul Euvé, Thomas Philbin and Germain Rousseaux. Analog wormholes and black hole laser effects in hydrodynamics. Physical Review D 93.8: 084032, 2016.

[9] Scott Robertson and Germain Rousseaux. "Viscous dissipation of surface waves and its relevance to analogue gravity experiments." *arXiv preprint arXiv:1706.05255* (2017).

[10] Jeff Steinhauer. "Confirmation of stimulated Hawking radiation, but not of black hole lasing." *Physical Review D* 106.10: 102007, 2022.

[11] Lawrence J.Pratt. *On Nonlinear Flow with Multiple Obstructions*, Journal of the Atmospheric sciences, 41(7):1214-1225,1984.

[12] Antonin Coutant and Renaud Parentani. Undulations from amplified low frequency surface waves. *Physics of Fluids*, *26*(4), 044106, 2014.

[13] Germain Rousseaux & Hamid Kellay, Classical hydrodynamics for analogue space–times: open channel flows and thin films. Philosophical Transactions of the Royal Society A, Volume 378, Issue 2177, 20190233, July 2020.

How to construct the improved Pratt diagram?

```python
from sklearn.neural_network import MLPClassifier
```

1. I define labels for the dataset
2. I train the network with the data
3. After training, the network predicts a label for the whole plan

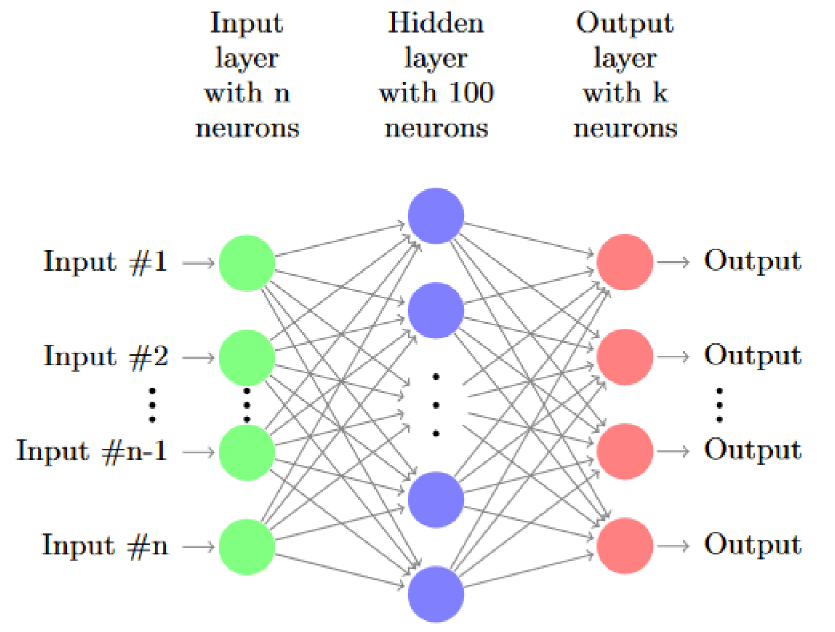


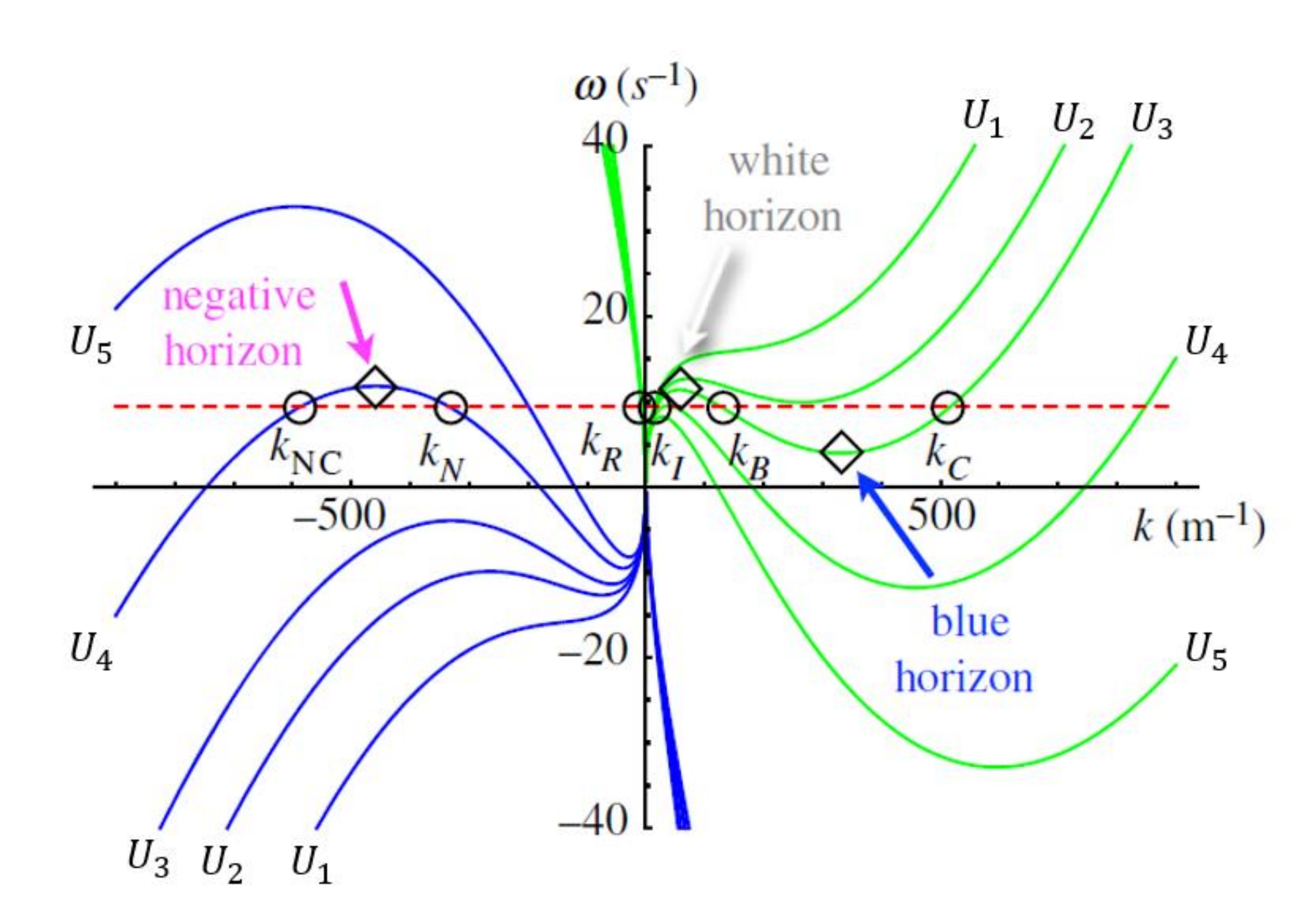
Figure 11. Architecture of the neural network with n the number of points in the training set and k the number of groups (here the number of hydrodynamic regimes).

<u>Parameters used :</u>

- 3 layers, 1 hidden layer
- 100 neurons in the hidden layer
- Maximum iteration : 3000

- Default setting:
  1. Cost function: entropy function
  2. Solver: adam
  3. Activation function: the rectified linear unit function

$$\left(\omega - \overrightarrow{U} \cdot \overrightarrow{k}\right)^2 = gk\left(1 + \frac{\gamma}{\rho g}k^2\right)\text{th}\,(kh)$$



Analogue Wormholes and Black Hole LASER Effect in Hydrodynamics

Cédric Peloquin,[1] Léo-Paul Euvé,[2] Thomas Philbin,[3] and Germain Rousseaux[2]

[1]*Université François Rabelais de Tours, 60 Rue du Plat d'Etain, 37000 Tours, France*
[2]*Pprime Institute, UPR 3346, CNRS - Université de Poitiers - ISAE ENSMA,*
11 Boulevard Marie et Pierre Curie, Téléport 2, BP 30179, 86962 Futuroscope Cedex, France
[3]*Physics and Astronomy Department, University of Exeter, Stocker Road, Exeter EX4 4QL, United Kingdom*

Viscous dissipation of surface waves and its relevance to analogue gravity experiments

S. Robertson[1*], G. Rousseaux[2]

1 Laboratoire de Physique Théorique, UMR 8627, CNRS,
Univ. Paris-Sud, Université Paris-Saclay, 91405 Orsay, France
2 Institut Pprime, UPR 3346, CNRS-Université de Poitiers-ISAE ENSMA,
11 Boulevard Marie et Pierre Curie-Téléport 2, BP 30179, 86962 Futuroscope, France
* scott.robertson@th.u-psud.fr

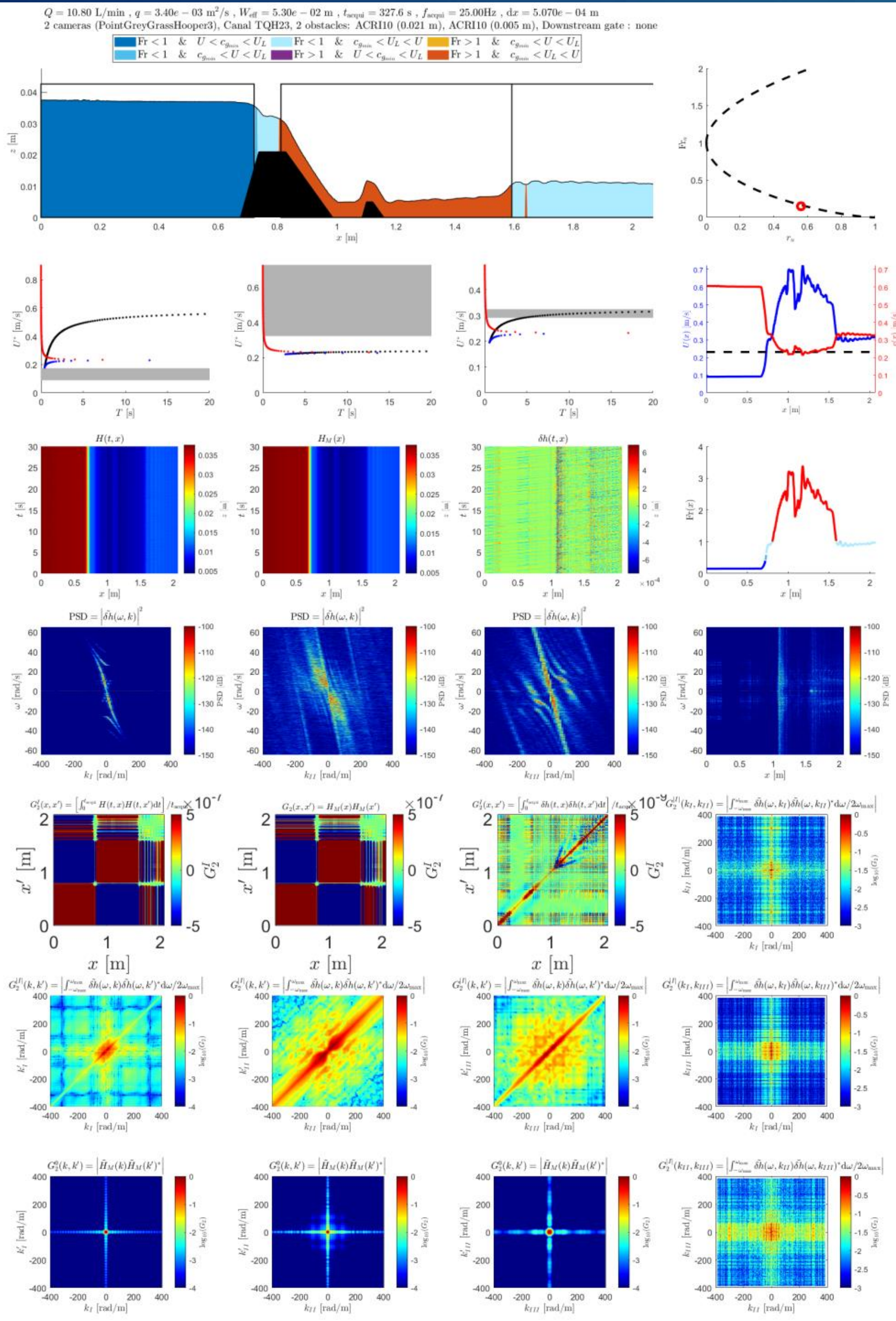
More difficult to detect the black hole laser effect!

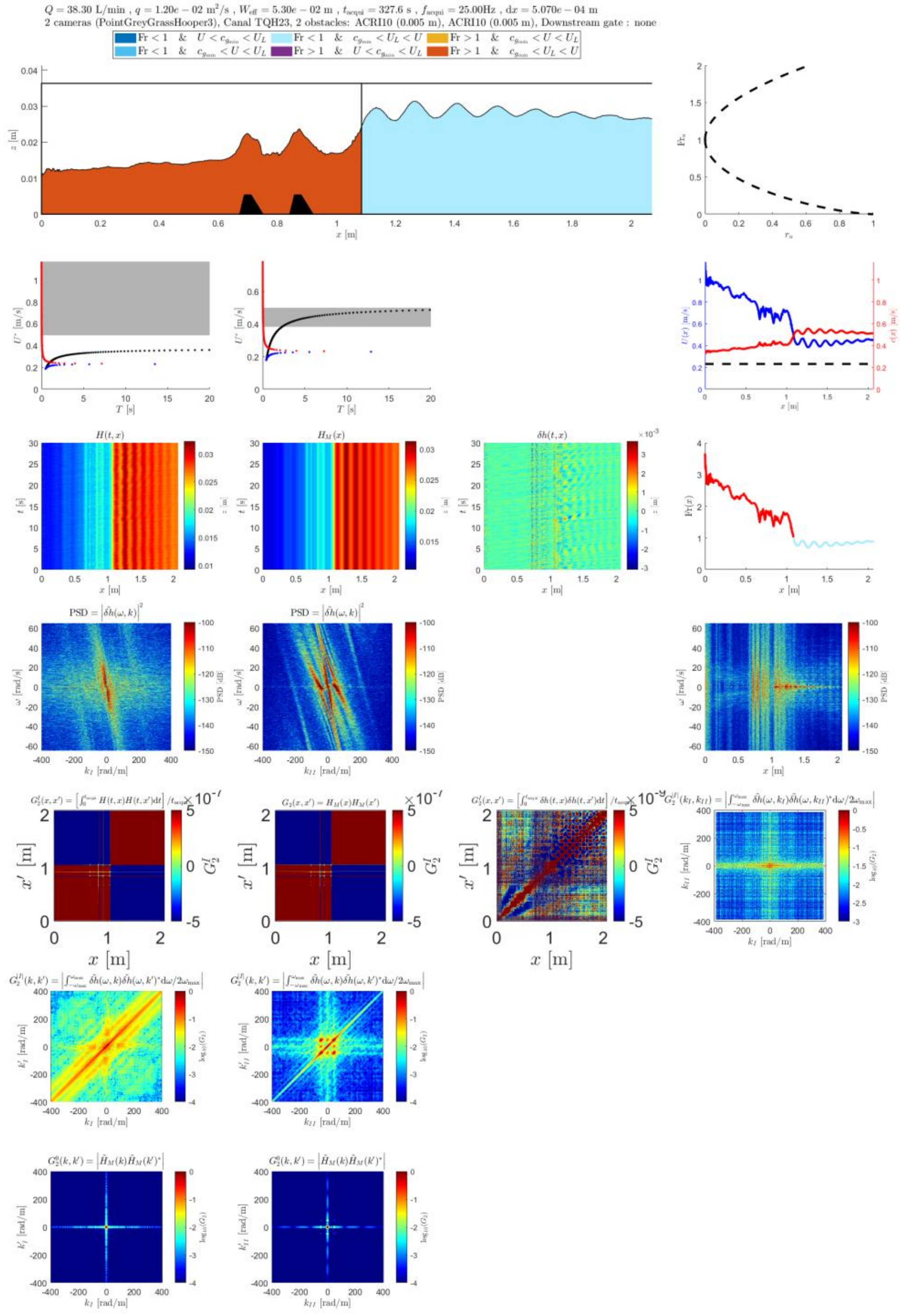
Confirmation of stimulated Hawking radiation, but not of black hole lasing

Jeff Steinhauer[1]

[1] *Department of Physics, Technion—Israel Institute of Technology, Technion City, Haifa 32000, Israel*

(Dated: October 14, 2021)


$Q = 10.80$ L/min , $q = 3.40e-03$ m^2/s , $W_{eff} = 5.30e-02$ m , $t_{acqui} = 327.6$ s , $f_{acqui} = 25.00$Hz , $dx = 5.070e-04$ m
2 cameras (PointGreyGrassHooper3), Canal TQH23, 2 obstacles: ACRI10 (0.021 m), ACRI10 (0.005 m), Downstream gate : none


$Q = 38.30$ L/min , $q = 1.20e-02$ m^2/s , $W_{eff} = 5.30e-02$ m , $t_{acqui} = 327.6$ s , $f_{acqui} = 25.00$Hz , $dx = 5.070e-04$ m
2 cameras (PointGreyGrassHooper3), Canal TQH23, 2 obstacles: ACRI10 (0.005 m), ACRI10 (0.005 m), Downstream gate : none

A particular regime : D-E-T_a