Measurement of Penrose superradiance in a photon superfluid using off-axis digital holography

Radivoje Prizia (he/him)

University of Nottingham (UK)

radivoje.prizia@nottingham.ac.uk

INTRODUCTION

Penrose Superradiance [1] (SR): extraction of energy from a rotating black hole via a scattering process in which an object splits in two at the ergoregion - one part falls into the ergoregion (i.e. negative energy), one part is reflected off the ergoregion and amplified (i.e. positive energy gain).

In a different setting: Zel'dovich process [2] – amplification of EM waves incident radially on a rotating metallic cylinder, allowing negative Doppler-shifted wave frequencies.

In SR, energy modes scattering from the ergoregion are amplified.

Rotational SR has been observed in ***analogue systems,*** such as: nonlinear optics [3], acoustics [4], hydrodynamics. [5]

Here, we look at **superfluids of light,** where Bogoliubov excitations act as particles in the Penrose picture.[6]

[1] R. Penrose, Riv. Nuovo Cimento **1**, 252 (1969)
[2] Y.B. Zel'dovich, Pis'ma Zh. Eksp. Teor. Fiz. **14**, 270 (1971) [JETP Lett. **14**, 180 (1971)]
[3] M.C. Braidotti *et al.,* Phys. Rev. Lett. **128**, 013901 (2022)
[4] M. Cromb *et al.,* Nat. Phys. **16**, 1069 (2020)
[5] T. Torres *et al.,* Nat. Phys. **13**, 833–836 (2017)
[6] M.C. Braidotti *et al.*, Phys. Rev. Lett. **125**, 193902 (2020)

PHOTON FLUID

Let us consider the propagation of a monochromatic optical beam centered at a frequency ω_0, with wavenumber k, in a self-defocusing thermal nonlinear medium. [7][8][9]

In the paraxial approximation, the slowly-varying electric field envelope E satisfies the Nonlinear Schrodinger Equation (NLSE),

$$i\frac{\partial E}{\partial z} + \frac{1}{2k}\nabla_\perp^2 E - k\frac{|n_2|}{n_0}|E|^2 E = 0$$

n_0 : linear refractive index

$k = \frac{2\pi n_0}{\lambda} = k_0 n_0$: wavenumber

$-|n_2|$: nonlinear coefficient for the defocusing nonlinearity of the medium

[7] R. W. Boyd, Nonlinear Optics, 3rd ed. Academic Press (2008)
[8] G. Fibich, The Nonlinear Schrodinger Equation - Singular Solutions and Optical Collapse. Springer - Applied Mathematical Sciences, vol. 192 (2015)
[9] F. Marino *et al.*, Phys. Rev. A **80**, 065802 (2009)

PHOTON FLUID

It is possible to reformulate the NLSE as [10]

$$i\frac{\partial E}{\partial t} + \alpha\nabla_\perp^2 E - \beta|E|^2 E = 0, \qquad \alpha = \frac{c}{2n_0 k}, \qquad \beta = \frac{ck|n_2|}{n_0^2}, \qquad t = \frac{n_0 z}{c}$$

The photon fluid is the 2D transverse part of the electric field E, while z plays the role of a timelike coordinate.

It is possible to show the connection of the NLSE with hydrodynamics by using the Madelung transform $E = \sqrt{\rho}\, e^{i\phi}$, so that the NLSE can be re-expressed as a set of hydrodynamical equations

Continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla_\perp \cdot (\rho\, \boldsymbol{v}) = 0, \qquad \frac{\partial \Phi}{\partial t} + \frac{1}{2}\boldsymbol{v}^2 + {c_s}^2 - \alpha^2 \frac{\nabla_\perp^2 \sqrt{\rho}}{\sqrt{\rho}} = 0$$

Euler equation of an incompressible fluid

where $\boldsymbol{v} = 2\alpha\, \nabla\phi = \nabla\Phi, \qquad c_s = \sqrt{c^2 n_2 \rho / n_0^3}$

[10] A. Prain *et al.*, Phys. Rev. D 100, 024037 (2019)

PHOTON FLUID

When the quantum pressure term $\nabla_\perp^2\sqrt{\rho}/\sqrt{\rho}$ is kept in the equations, small perturbations ψ on top of a background beam with electric field E_0 can be inserted in the total field E_{tot} as [10]

$$E_{tot} = E_0(1+\psi), \qquad |\psi| \ll 1$$

These perturbations satisfy the Bogoliubov-de Gennes (BdG) equations [11][12]

$$\left(\partial_t + v\cdot\nabla - i\frac{\alpha}{\rho}\nabla\cdot\rho\nabla\right)\psi + \mathrm{i}\beta\rho(\psi+\psi^*) = 0$$

with solutions $\psi = \psi_s + \psi_i = A_s(\boldsymbol{x})e^{-i\omega t} + A_i^*(\boldsymbol{x})e^{i\omega t}$

The elementary excitations associated with the photon fluid follow the Bogoliubov dispersion for superfluidity [13][14] - hence we have a ***photon superfluid***.

[10] A. Prain *et al.*, Phys. Rev. D 100, 024037 (2019)
[11] P. Gennes, Superconductivity of Metals and Alloys. Benjamin, New York (1966)
[12] L. Pitaevskii and S. Stringari, Bose-Einstein Condensation. Clarendon, Oxford (2003)
[13] D. Vocke *et al.*, Optica **2**, 5, 484-490 (2015)
[14] Q. Fontaine *et al.*, Phys. Rev. Lett. **121**, 183604 (2018)

PHOTON FLUID

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Signal (positive) mode | Idler (negative) mode

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SIGNAL AND IDLER PROPAGATION EQUATIONS

SR in nonlinear optics involves the interaction between a strong pump and a weak signal fields [6].

Pump: a vortex beam with Orbital Angular Momentum (OAM) charge ℓ, stationary along propagation z.
In cylindrical coordinates,

$$E_0(r,\theta,z) = \mathcal{E}_0(r)e^{i(\beta_\ell z+\ell\theta)} = \sqrt{I_\ell}\, u_\ell(r)e^{i(\beta_\ell z+\ell\theta)}$$

with

$$\beta_\ell u_\ell = \frac{1}{2k}\nabla_\ell^2 u_\ell + k_0 n_2 I_\ell u_\ell^3$$

I_ℓ : background pump beam intensity
$u_\ell(r) = tanh^{|\ell|}(r/r_\ell)$: vortex profile amplitude with vortex core size r_ℓ
$\beta_\ell = k_0 n_2 I_\ell = k_0 \Delta n < 0$: pump wavevector

$$u_\ell \to 1 \text{ for } r \gg r_\ell, \qquad \nabla_\ell^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{\ell^2}{r^2}$$

[6] M.C. Braidotti *et al.*, Phys Rev. Lett. **125**, 193902 (2020)

SIGNAL AND IDLER PROPAGATION EQUATIONS

Let us consider the presence of a weak **signal** field E_s with OAM charge n, interacting with the pump.[6]

The total field E is

$$
\begin{aligned}
E(r,\theta,z) &= E_0 + E_s + E_i \\
&= \left(\mathcal{E}_0(r)e^{i\ell\theta} + \mathcal{E}_s(r,z)e^{in\theta} + \mathcal{E}_i(r,z)e^{iq\theta}\right)e^{i\beta_\ell z} \\
&= \left(\mathcal{E}_0(r) + \mathcal{E}_s(r,z)e^{i(n-\ell)\theta} + \mathcal{E}_i(r,z)e^{i(q-\ell)\theta}\right)e^{i(\beta_\ell z+\ell\theta)}
\end{aligned}
$$

E_i is the **idler** field generated by the nonlinear interaction of pump and signal fields with OAM q.

From the solution to the Bogoliubov-de Gennes equations, this yields

$$
\begin{aligned}
n - \ell &= \ell - q \\
q &= 2\ell - n
\end{aligned}
$$

[6] M.C. Braidotti *et al.*, Phys Rev. Lett. **125**, 193902 (2020)

SIGNAL AND IDLER PROPAGATION EQUATIONS

By substituting the expression for the total field E into the NLSE + linearizing in the signal and idler fields + inserting the expression of the vortex pump, we obtain the propagation equations for $\mathcal{E}_s$ and $\mathcal{E}_i$, [6]

$$\frac{\partial \mathcal{E}_s}{\partial z} = \frac{i}{2k}\nabla_n^2 \mathcal{E}_s + i\beta_\ell u_\ell^2(r)(2\mathcal{E}_s + \mathcal{E}_i^*) - i\beta_\ell \mathcal{E}_s \,,$$

$$\frac{\partial \mathcal{E}_i}{\partial z} = \frac{i}{2k}\nabla_q^2 \mathcal{E}_i + i\beta_\ell u_\ell^2(r)(2\mathcal{E}_i + \mathcal{E}_s^*) - i\beta_\ell \mathcal{E}_i$$

These equations describe the parametric interaction between signal and idler fields in the presence of a strong pump field, the parametric interaction arising from **Four Wave Mixing (FWM).**

[6] M.C. Braidotti *et al.*, Phys Rev. Lett. **125**, 193902 (2020)

NOETHER CURRENT FORMALISM

The propagation equations for $\mathcal{E}_s$ and $\mathcal{E}_i$ have a conserved quantity N – proportional to the total energy of the Bogoliubov modes - associated with the Noether current $J^0(r,z)$ [10]

$$J^0(r,z) = |E_s(r,z)|^2 - |E_i(r,z)|^2 \,, \quad \partial_z J^0 = 0$$

$$N = \int_0^\infty J^0(r,z) r dr = \int_0^\infty (|E_s(r,z)|^2 - |E_i(r,z)|^2) r dr = const.$$

The reflection and transmission coefficients $R_N(z)$ and $T_N(z)$ for scattering from the pump ergoregion (whose radius r_e is defined by equating the flow speed and the speed of sound in the medium) with $r_e = (|n-\ell|/k)\sqrt{n_0/|n_2 I_\ell|}$ are [3]

$$R_N(z) = \frac{1}{N}\int_{r_e}^\infty (|E_s(r,z)|^2 - |E_i(r,z)|^2) r dr$$

$$T_N(z) = \frac{1}{N}\int_0^{r_e} (|E_s(r,z)|^2 - |E_i(r,z)|^2) r dr$$

[10] A. Prain *et al.*, Phys. Rev. D 100, 024037 (2019)
[3] M.C. Braidotti *et al.,* Phys. Rev. Lett. **128**, 013901 (2022)

NOETHER CURRENT FORMALISM

Penrose superradiance can be observed by extracting the currents J^0. [10][6]

The **amplification** of the positive mode is identified by a reflection coefficient larger than 1 **– the signal beam reflected off the ergoregion has gained energy**.

At the same time, since $R_N(z) + T_N(z) = 1 \ \ \forall z$, the **trapping** of the negative mode (**the idler mode**) is identified by a negative transmission coefficient.

It is important to note that the current formalism also works in the transient regime, i.e. where the signal field is still in proximity of the ergoregion.

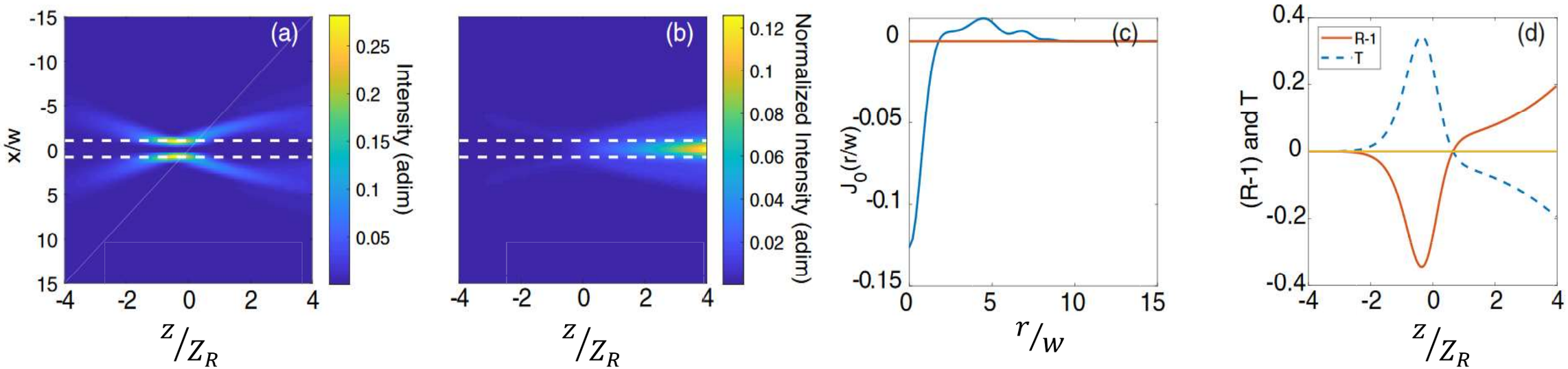
To summarise: the occurrence of Penrose superradiance is indicated by

- $J^0 > 0$ outside the ergoregion (reflected signal); and
- $J^0 < 0$ inside the ergoregion (trapped idler).

[10] A. Prain *et al.*, Phys. Rev. D 100, 024037 (2019)
[6] M.C. Braidotti *et al.*, Phys Rev. Lett. **125**, 193902 (2020)

NOETHER CURRENT FORMALISM

From numerical simulations,



(a,b) Section at y = 0 of the signal (a) and idler (b) field intensities evolution, versus x/w and z/Z_R (w is the signal spot size, Z_R is the Rayleigh range). Signal has OAM n = 2 and Idler has OAM q = 0. (c) Current J_N versus r/w for $z/Z_R = 4$. (d) Reflection R-1 and transmission T versus z/Z_R.[6]

[6] M.C. Braidotti *et al.*, Phys Rev. Lett. **125**, 193902 (2020)

TRAPPING OF THE IDLER FIELD

Let us consider a signal field as a loosely focused Laguerre-Gauss (LG) beam with radial index $p = 0$, OAM charge n , and focused spot size w_0 , [6]

$$\mathcal{E}_s(r,z) \approx c_s(z) V_n(r,z) e^{-i(1+|n|)\phi_G(z)}\, e^{i(2\beta_\ell \Gamma_n(z) - \beta_\ell)z}$$

$c_s(z)$: signal field amplitude
$V_n(r,z)$: normalized z-dependent amplitude of the LG mode profile
$\phi_G(z) = tan^{-1}(z/z_0)$: Gouy phase at the focus with Rayleigh range $z_0 = k\, w_0^2/2$
$\Gamma_n(z) = \int_0^\infty 2\pi r dr\, |V_n(r,z)|^2 u_\ell^2(r)$: signal phase variation caused by the overlap with pump.
Signal wavevector nonlinear shift : $\Delta K_s(z) = 2\beta_\ell \Gamma_n(z) - \beta_\ell$

Around $z = 0$, $\Delta K_s(z) \approx \Delta K_s(0) = 2\beta_\ell \Gamma_n(0) - \beta_\ell$, $0 \leq \Gamma_n(0) \leq 1$

Most of the nonlinear interaction occurs within a Rayleigh range around the signal beam focus at $z = 0$, and $\Gamma_n(0)$ can be evaluated numerically.

[6] M.C. Braidotti *et al.*, Phys Rev. Lett. **125**, 193902 (2020)

TRAPPING OF THE IDLER FIELD

Let us rearrange the propagation equation for $\mathcal{E}_i$, [6]

$$\frac{\partial \mathcal{E}_i}{\partial z} = \frac{i}{2k}\nabla_q^2 \mathcal{E}_i + 2i\beta_\ell(u_\ell^2(r) - 1)\mathcal{E}_i + i\beta_\ell \mathcal{E}_i + i\beta_\ell u_\ell^2 \mathcal{E}_s^*$$

where

$2\beta_\ell(u_\ell^2(r) - 1) = 2|\beta_\ell|(1 - u_\ell^2(r)) = 2k_0|\Delta n|(1 - u_\ell^2(r))$: **waveguiding term.** It defines a 2D refractive index profile.

The pump vortex induces a **cross-phase modulation** on the idler, trapping it inside the ergoregion.

$\beta_\ell u_\ell^2 \mathcal{E}_s^*$: **source term**. It describes how the idler field is driven by the signal field via parametric interaction.

[6] M.C. Braidotti *et al.*, Phys Rev. Lett. **125**, 193902 (2020)

TRAPPING OF THE IDLER FIELD

Let us consider an idler field as a Laguerre-Gauss (LG) beam with radial index $p = 0$, OAM charge q , and ignoring the source term in the propagation equation, [6]

$$\mathcal{E}_i(r,z) = c_i(z) U_{pq}(r)\, e^{i(\beta_\ell + \Lambda_{pq})z}$$

$c_s(z)$: idler field amplitude
$U_{pq}(r)$: guided idler mode arising in the presence of the pump waveguide
$\Delta K_i = \beta_\ell + \Lambda_{pq}$: idler wavevector shift

$$\left(\frac{1}{2k}\nabla_q^2 + 2\beta_\ell\left(u_\ell^2(r) - 1\right)\right) U_{pq}(r) = \Lambda_{pq} U_{pq}(r)$$

This eigenvalue problem must be solved to verify the existence of guided idler modes.
If $\Lambda_{pq} > 0$, the trapping of the idler modes takes place and $\Delta K_i > 0$.

[6] M.C. Braidotti *et al.*, Phys Rev. Lett. **125**, 193902 (2020)

PHASE-MATCHING CONDITION

By inserting the expressions for $\mathcal{E}_s$, $\mathcal{E}_i$ into the idler propagation equation (source term included), [6]

$$\frac{dc_i}{dz} = ic_s^* \beta_\ell F(z) e^{-i(2\Delta K z - (1+|n|)\phi_G(z))} = ic_s^* \beta_\ell F(z) e^{-i\Xi},$$

$\Delta K = \dfrac{\Delta K_s + \Delta K_i}{2}$: average wavevector shift of the whole perturbation (signal + idler)

$$F(z) = \int_0^\infty 2\pi r dr V_n^*(r,z) u_\ell^2(r) U_q^*(r)$$

Let us now look at the phase-matching condition.

Around $z = 0$, $\Xi \approx 2\Delta K z - (1+|n|)\, z/z_0$, so that $\Delta K = \frac{1}{2}\left[(2\beta_\ell \Gamma_n(0) - \beta_\ell) + \left(\beta_\ell + \Lambda_{pq}\right)\right]$

A general condition $\Delta K > 0$ can guarantee phase matching. In our case, this is used to determine the presence of trapped (guided) idler modes inside the ergoregion.

[6] M.C. Braidotti *et al.*, Phys Rev. Lett. **125**, 193902 (2020)

PHASE-MATCHING CONDITION

In terms of frequency shifts, $\Delta K > 0$ reads $\Delta\omega = \frac{\Delta\omega_s + \Delta\omega_i}{2} = -(c/n_0)\Delta K < 0$

$$\Delta\omega_{s,i} = \omega_{s,i} - \omega_p = -\frac{c\Delta \mathrm{K}_{s,i}}{n_0}\,, \qquad \omega_p = -\frac{c\beta_\ell}{n_0} = ck_0|n_2|\, I_\ell/n_0$$

The frequency shifts $\Delta\omega_{s,i}$ correspond to oscillation frequencies in the transverse plane – analogous to **phononic modes** in a 2D fluid. [6][11]

Since $\omega_s = \omega_i = \omega$ and $\omega_p = (n - \ell)\,\Omega$ (where Ω is the rotational frequency of the pump), it follows [6]

$$\Delta\omega = \omega - \omega_p = \omega - m\Omega\,, \quad m = n - \ell$$

To observe superradiance, $\Delta\omega = \omega - m\Omega < 0$ (Zel'dovich-Misner condition) [2][15]

As $\Omega\,, \omega > 0\,,$ it also follows that $m = n - \ell > 0$

[6] M.C. Braidotti *et al.*, Phys Rev. Lett. **125**, 193902 (2020)
[11] D. Vocke *et al.*, Optica **2**, 5, 484-490 (2015)
[2] Y.B. Zel'dovich, Pis'ma Zh. Eksp. Teor. Fiz. **14**, 270 (1971) [JETP Lett. **14**, 180 (1971)]
[15} C.W. Misner, Phys. Rev. Lett. **28**, 994 (1972)

SUPERRADIANT SCATTERING CONDITIONS

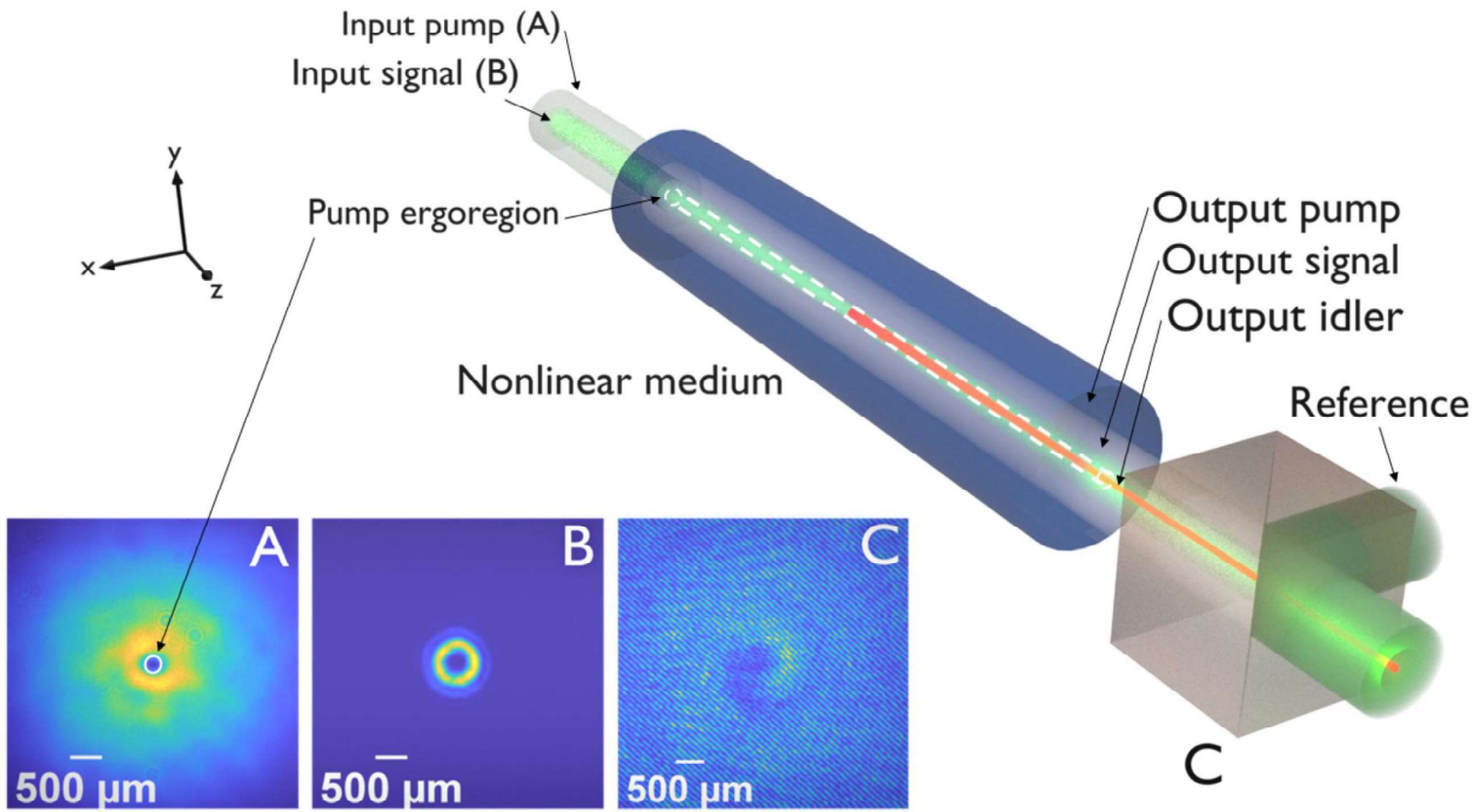
To summarise, the three conditions to satisfy in order to observe Penrose superradiance in the photon superfluid regime are [3]

- The phase-matching (Zel'dovich-Misner) condition $\Delta\omega \propto -\Delta K < 0$;
- The guided idler modes must have $\Delta K_i > 0$, or equivalently $\Delta\omega_i < 0$ in order to be trapped inside the ergoregion;
- The OAM charge of the signal beam must be larger than the OAM charge of the pump beam, $m = n - \ell > 0$.

For the experiment, the first two conditions are verified by numerically evaluating the signal modes and the idler modes that can be trapped inside the pump waveguiding potential. The third condition is verified simply by choosing the proper combination of pump and signal OAMs.

[3] M.C. Braidotti *et al.,* Phys. Rev. Lett. **128**, 013901 (2022)

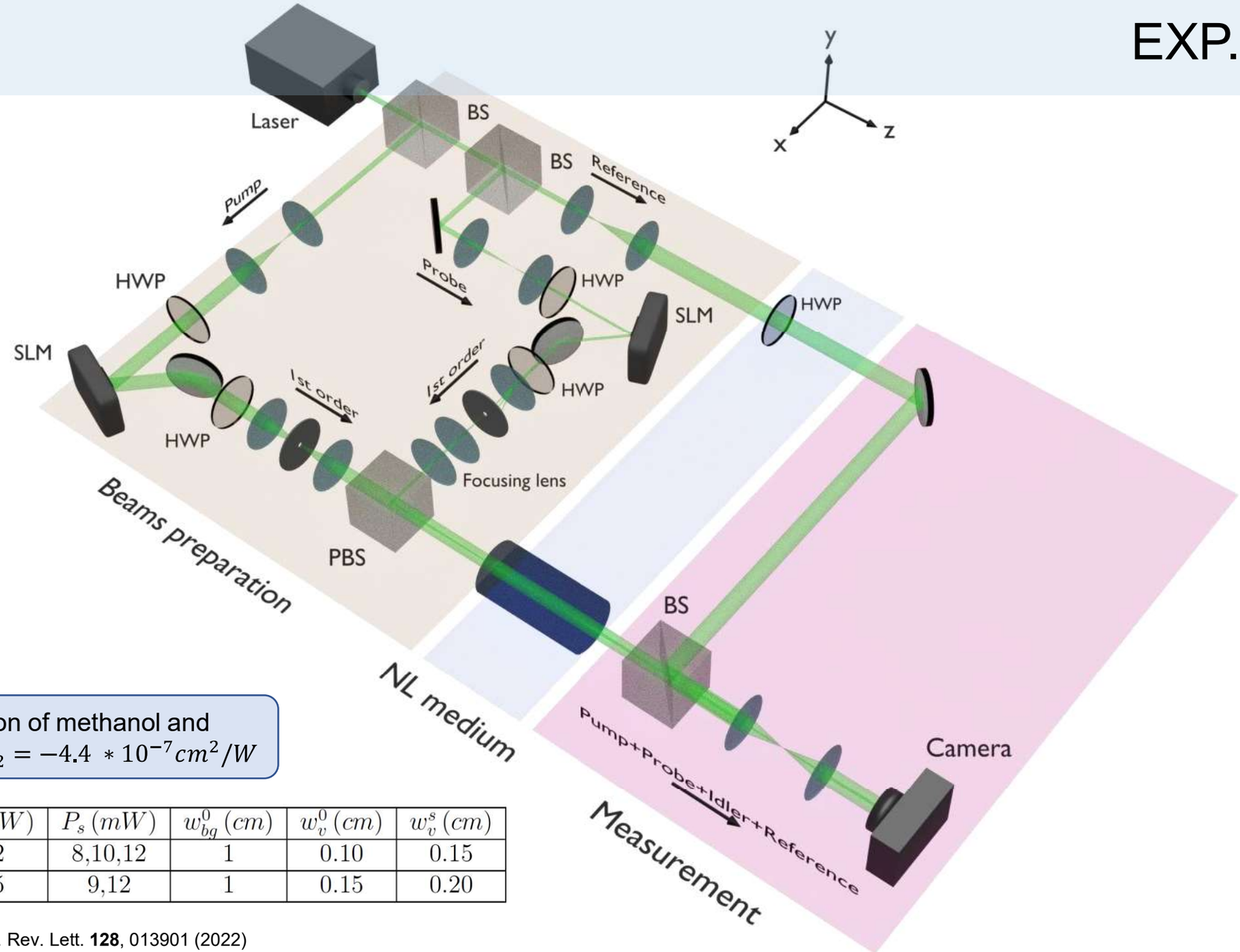
PUMP-PROBE INTERACTION GEOMETRY



Superradiant interaction geometry. The pump beam (grey) propagates inside the nonlinear sample along the z-axis, while co-propagating with the signal beam (green). The white dashed line is the ergoregion encircling the pump vortex core, and the experimental value of its radius is calculated for each combination of the experimental parameters. The signal beam is loosely focused onto the pump vortex core. If superradiance occurs, an idler beam (red) is generated and trapped inside the ergoregion. A reference beam interferes with the total field at the output of the sample. [3]

[3] M.C. Braidotti *et al.,* Phys. Rev. Lett. **128**, 013901 (2022)

EXP. SETUP [3]



Sample: solution of methanol and graphene flakes, $n_2 = -4.4 * 10^{-7} cm^2/W$

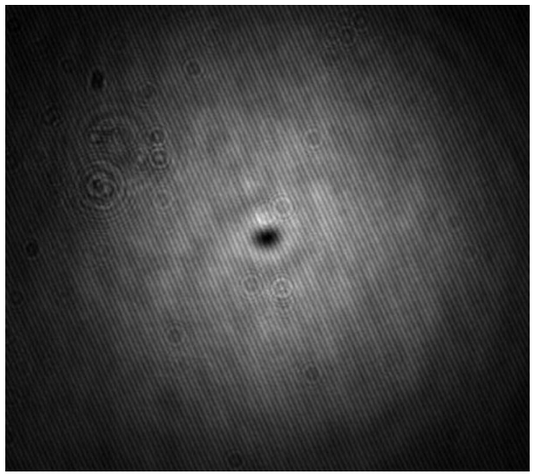
OAMs	$P_p\,(mW)$	$P_s\,(mW)$	$w_{bg}^0\,(cm)$	$w_v^0\,(cm)$	$w_v^s\,(cm)$
$\ell = \mathbf{1}, \mathbf{n} = \mathbf{2}$	252	8,10,12	1	0.10	0.15
$\ell = \mathbf{2}, \mathbf{n} = \mathbf{4}$	175	9,12	1	0.15	0.20

[3] M.C. Braidotti *et al.,* Phys. Rev. Lett. **128**, 013901 (2022)

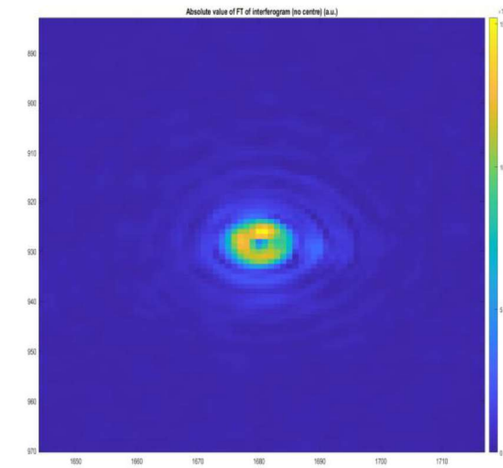
EXPERIMENTAL TECHNIQUE: OFF-AXIS DIGITAL HOLOGRAPHY [16]

Pump, OAM ℓ = 1

Interferogram

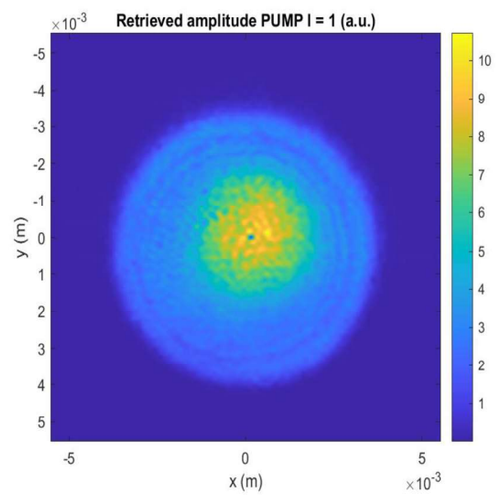


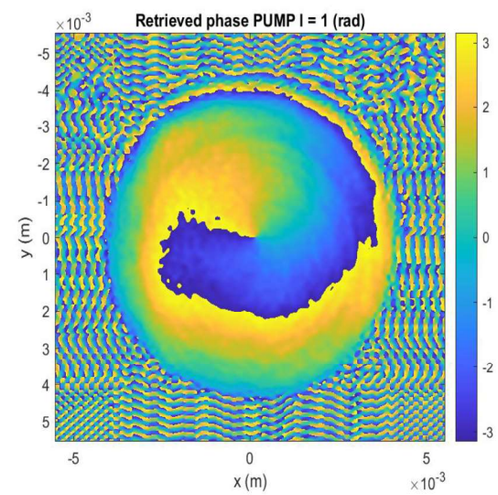
Fourier Transform

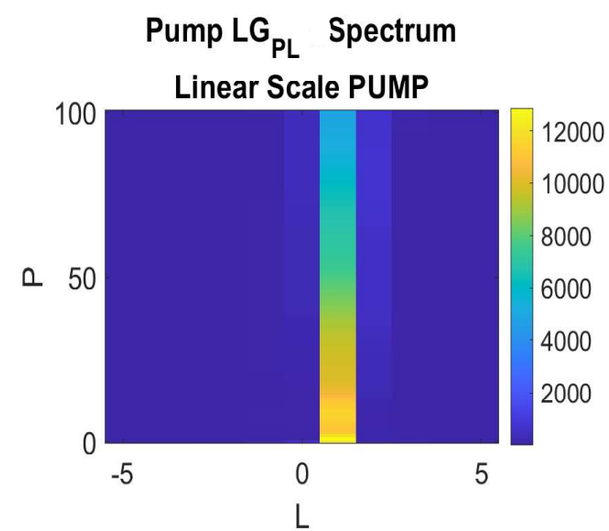


Inverse Fourier Transform

Laguerre-Gauss Decomposition







[16] E. Cuche *et al.*, Appl. Opt. **39**, 4070 (2000)

EXPERIMENTAL RESULTS

A **Laguerre-Gauss (LG) decomposition** of the complex field $\bar{E}(x,y) = \bar{\mathcal{E}}(x,y)\, e^{i\phi(x,y)}$ retrieved through Off-Axis Digital Holography allows to separate the components of the OAM spectrum. [3]

The weights $w_{j,p}$ quantify the contribution from a single LG mode (identified by radial index $p = 0, 1, 2$ ... and azimuthal index (i.e. the OAM charge) $j \in \mathbb{Z}$ in the mixed complex field $\bar{E}(x,y)$.

$$w_{j,p} = \int\int dxdy L_{j,p}^{LG}(x,y)\, \bar{E}^*(x,y)\ ,$$

where $L_{j,p}^{LG}(x,y)$ are the Laguerre-Gauss modes.

[3] M.C. Braidotti *et al.,* Phys. Rev. Lett. **128**, 013901 (2022)

EXPERIMENTAL RESULTS

In cylindrical coordinates, [17][18]

$$L_{j,p}^{LG}(\rho,\theta,z) = C_{j,p}^{LG}\frac{w_0}{w(z)}\left(\frac{\rho\sqrt{2}}{w(z)}\right)^{|j|} exp\left(\frac{-\rho^2}{w(z)^2}\right) L_p^{|j|}\left(\frac{2\rho^2}{w(z)^2}\right) exp\left(\frac{-ik\rho^2}{2R(z)}\right)\exp(ij\theta)\exp(-i\phi_{j,p}^{G}(z))$$

$C_{j,p}^{LG} = \sqrt{\frac{2p!}{\pi(p+|l|)!}}$: normalization constant

$L_p^{|j|}$: generalised Laguerre polynomial

$w_0, w(z)$: waist and radius of the mode

$R(z) = z + (z_0^2/z)$: radius of curvature with Rayleigh range $z_0 = \pi w_0^2/\lambda$

$\phi_{j,p}^{G}(z) = \tan^{-1}(z/z_0)$: Gouy phase

[17] L. Allen *et al.*, Phys. Rev. A **45**, 8185-8189 (1992)
[18] G. Grynberg, A. Aspect, and C. Fabre, Introduction to Quantum Optics - From the Semi-classical Approach to Quantized Light, 1st ed. Cambridge University Press (2010)

EXPERIMENTAL RESULTS

From the weights $w_{j,p}$ the jth components of the experimental fields are reconstructed through [3]

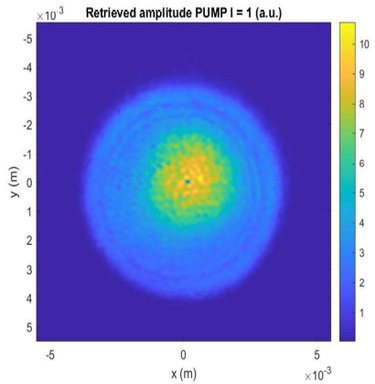
$$E^{(j)}(x,y) = \int dp\, w_{j,p}\, L_{j,p}^{LG}(x,y)$$

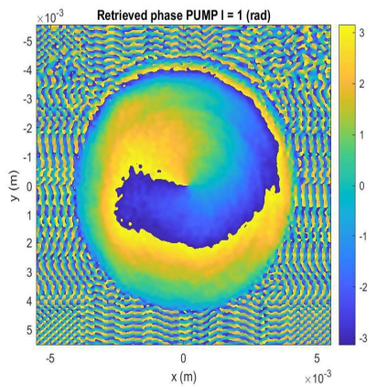
thereby finding the form of the field for each OAM charge j component.

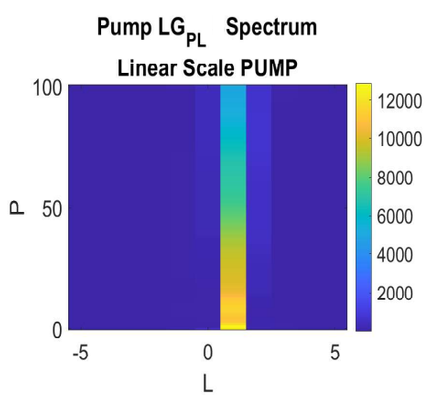
This allows to access the (x,y) amplitude and phase profiles of the experimental pump $E_0^{(\ell)}$, signal $E_s^{(n)}$, and idler $E_i^{(q)}$ fields.

[3] M.C. Braidotti *et al.,* Phys. Rev. Lett. **128**, 013901 (2022)

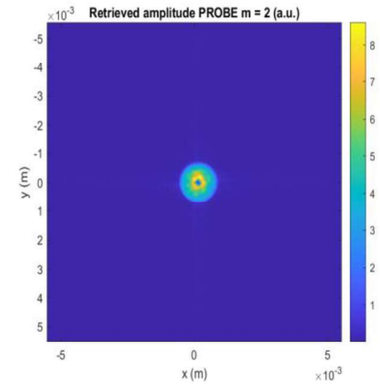
Pump, OAM ℓ = 1, P = 10 mW

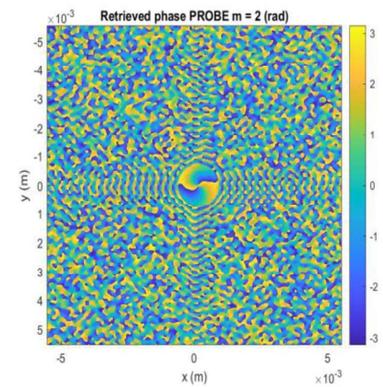


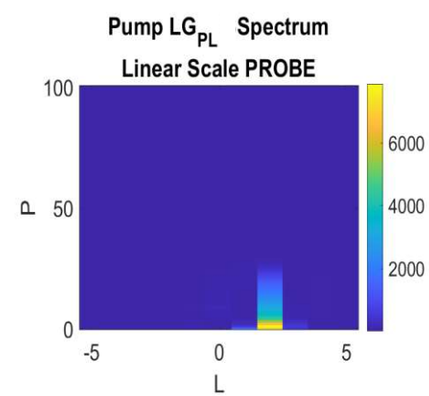




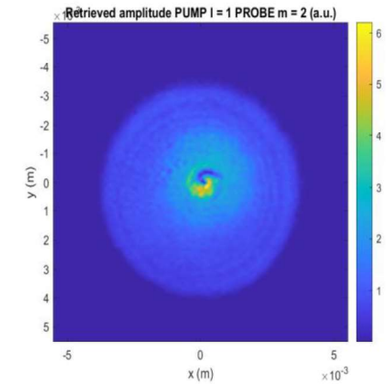
Probe, OAM n = 2, P = 1 mW

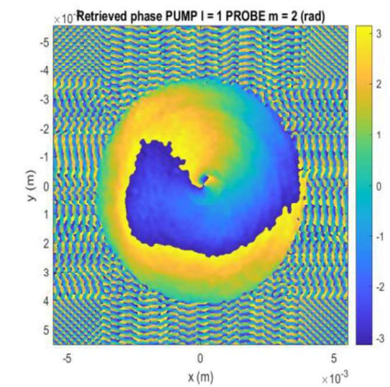


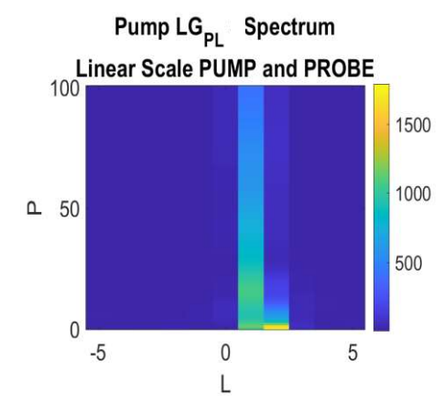




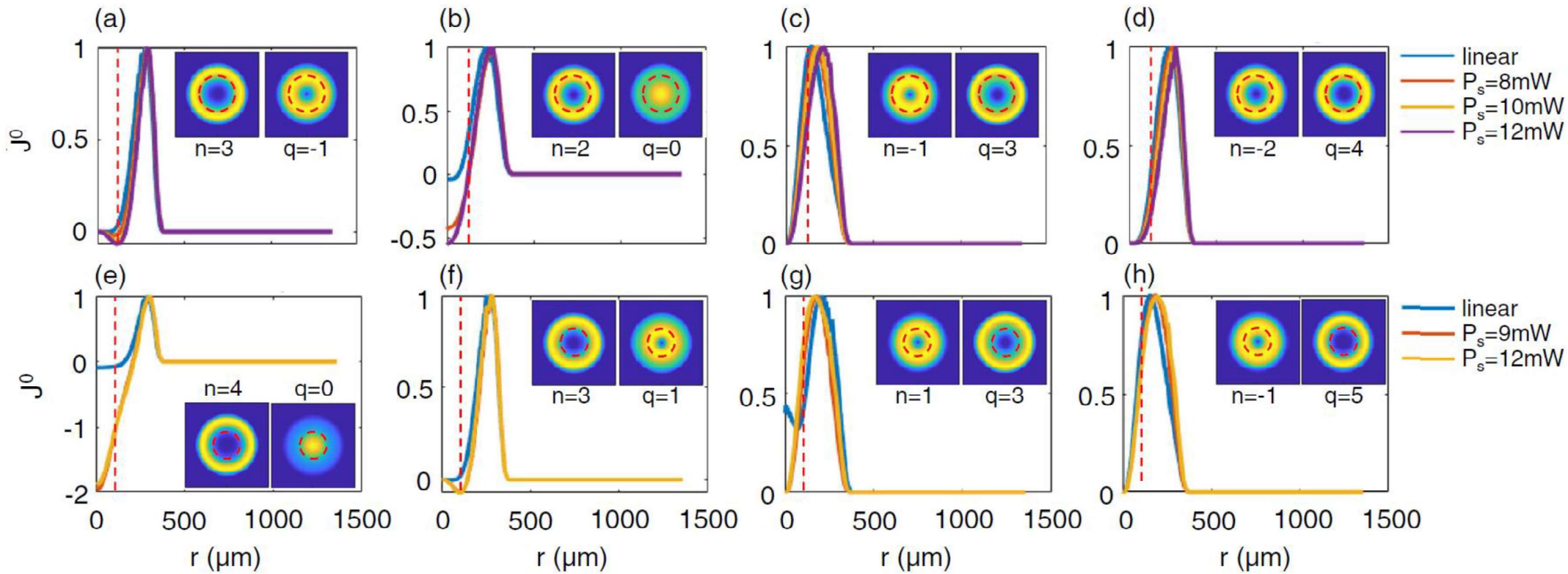
Pump+Probe, linear case







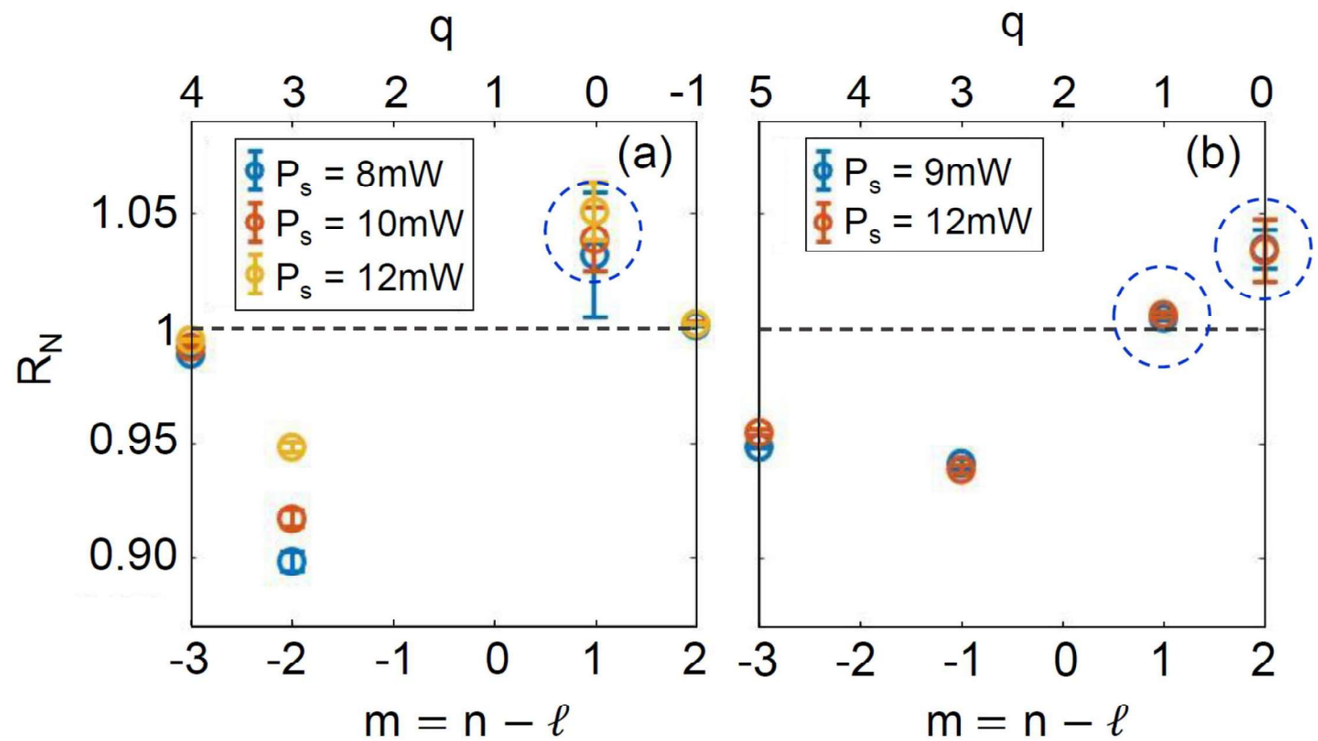
EXPERIMENTAL RESULTS



Experimental values of current J^0 as a function of radial coordinate r. (a-d) Pump $P_p = 250mW$ and OAM ℓ = 1; (e-h) Pump $P_p = 175mW$ and OAM ℓ = 2. SR occurs in (b), (e), and (f). Insets are the reconstructed signal and idler transverse field intensities. $J^0 < 0$ for $r > r_e$(red dotted line) in (e,f) indicates SR also in the transient regime – where signal is in proximity of the ergoregion. [3]

[3] M.C. Braidotti *et al.,* Phys. Rev. Lett. **128**, 013901 (2022)

EXPERIMENTAL RESULTS



Reflection coefficients R_N calculated at the sample output ($z = 13cm$), as a function of signal and pump OAM charges difference $m = n - \ell$ (lower axes) and idler OAM charge q (upper axes) for: (a) pump $P_p = 250mW$ and ℓ = 1, and (b) pump $P_p = 175mW$ and ℓ = 2. Amplification i.e. R_N > 1 is observed for $m = n - \ell$ > 0. R_N is calculated from the average over 20 different acquisitions and the standard deviation is used to determine the error bars. Blue dashed circles indicate the configurations where superradiance conditions (ΔK > 0, ΔK_i > 0, and $m = n - \ell$ > 0) are satisfied. [3]

[3] M.C. Braidotti *et al.,* Phys. Rev. Lett. **128**, 013901 (2022)

COMPARISON WITH NUMERICAL SIMULATIONS

In order to verify the trapping of the guided idler modes in the presence of the cross-phase modulation induced pump waveguide, the spectra of the idler modes (renamed $U_q \rightarrow U$ for simplicity) are computed by numerically solving the eigenvalue equation in cylindrical coordinates [3]

$$\frac{1}{2k}\left[\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-\frac{q^2}{r^2}\right]U(r)+\Delta n^*(r)U(r)=\Lambda_q U(r)$$

$$\Delta n^*(r)=2\beta_\ell(u_\ell^2(r)-1)$$

If $\Lambda_q > 0$, the idler mode can be trapped inside the waveguiding potential.

[3] M.C. Braidotti *et al.,* Phys. Rev. Lett. **128**, 013901 (2022)

COMPARISON WITH NUMERICAL SIMULATIONS

The experimental conditions are tested by numerically simulating the NLSE where an absorption term is included, [3]

$$i\frac{\partial E}{\partial z} + \frac{1}{2k}\nabla_{\perp}^{2}E + k\frac{\Delta n\,(|E|^{2})}{n_{0}}E = -\frac{i\alpha}{2}E$$

$$\Delta n = n_{2}\int R(\boldsymbol{r}-\boldsymbol{r}')|E(\boldsymbol{r}')|^{2}dr' \;,\; \alpha = 2m^{-1}$$

The absorption, present in the experiment, is included here to verify the persistence of the physics of Penrose SR. Numerical simulations show that this is verified, and the definition of reflection and transmission coefficients holds also in the case of small losses.
Moreover, simulations confirm SR also in the transient regime, i.e. showing an amplication $R > 1$ even if the signal beam is still in the vicinity of the ergoregion.

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COMPARISON WITH NUMERICAL SIMULATIONS

OAMs	$m = n - \ell > 0$	$\omega - m\Omega < 0$	$\Delta\omega_i < 0$	n_g	OAMs	R_{BPM}	R_{NLS}	$R_{exp}\,(\sigma_{R_{exp}})$ $(P_s = 8, 10, 12$ mW$)$
$\ell = 1,\ n = 3,\ q = -1$	yes	no	no	2	$\ell = 1,\ n = 3,\ q = -1$	0.996	0.998	1.001 (0.006), 1.001 (0.005), 1.001 (0.006)
$\boldsymbol{\ell = 1, n = 2, q = 0}$	**yes**	**yes**	**yes**	2	$\boldsymbol{\ell = 1, n = 2, q = 0}$	**1.193**	**1.848**	**1.032 (0.028), 1.044 (0.017), 1.054 (0.015)**
$\ell = 1,\ n = -1,\ q = 3$	no	no	no	0	$\ell = 1,\ n = -1,\ q = 3$	0.995	0.921	0.896 (0.005), 0.917 (0.004), 0.948 (0.002)
$\ell = 1,\ n = -2,\ q = 4$	no	no	no	0	$\ell = 1,\ n = -2,\ q = 4$	0.989	0.996	0.981 (0.001), 0.987 (0.001), 0.993 (0.002)
OAMs	$m = n - \ell > 0$	$\omega - m\Omega < 0$	$\Delta\omega_i < 0$	n_g	OAMs	R_{BPM}	R_{NLS}	$R_{exp}\,(\sigma_{R_{exp}})$ $(P_s = 9, 12$ mW$)$
$\boldsymbol{\ell = 2, n = 4, q = 0}$	**yes**	**yes**	**yes**	4	$\boldsymbol{\ell = 2, n = 4, q = 0}$	**1.696**	**1.091**	**1.037 (0.009), 1.039 (0.017)**
$\boldsymbol{\ell = 2, n = 3, q = 1}$	**yes**	**yes**	**yes**	3	$\boldsymbol{\ell = 2, n = 3, q = 1}$	**1.411**	**1.007**	**1.004 (0.002), 1.007 (0.002)**
$\ell = 2,\ n = 1,\ q = 3$	no	yes	no	2	$\ell = 2,\ n = 1,\ q = 3$	0.713	0.408	0.944 (0.003), 0.937 (0.003)
$\ell = 2,\ n = -1,\ q = 5$	no	no	no	1	$\ell = 2,\ n = -1,\ q = 5$	0.993	0.508	0.947 (0.001), 0.954 (0.002)

Summary of superradiance conditions in the various OAM configurations used in the experiment. When the three SR conditions ($\omega - m\Omega < 0, \Delta\omega_i < 0, m = n - \ell > 0$, or equivalently $\Delta K > 0, \Delta K_i > 0, m = n - \ell > 0$) are satisfied simultaneously, the reflection coefficient R is greater than 1, as shown in the rows with bold text font. Moreover, n_g is the number of idler guided modes, i.e. the number of modes with $\Lambda_q > 0$. The cases in which all three SR conditions are verified are indicated in bold font. The three reflection coefficients are obtained, respectively: by the local and stationary (along z) pump Beam Propagation Method for the signal and idler (R_{BPM}), by the full NLSE numerical simulation (R_{NLS}), and by experimental measurements (R_{exp}). The error bars σ (calculated as standard deviations) for the experimental reflection coefficients R_{exp} are also reported. [3]

[3] M.C. Braidotti *et al.,* Phys. Rev. Lett. **128**, 013901 (2022)

CONCLUSIONS

- These results show the arising of a novel process of wave mixing in nonlinear optics inspired by Penrose superradiance physics.

- The amplification of positive energy modes with OAM in the scattering with a rotating background is experimentally detected, together with the trapping of the negative modes supported by the Noether current formalism.

- Over-reflection (reflectivity greater than one) reveals the presence of superradiance even in the transient regime.

This experiment provides a novel and accessible platform for investigating Penrose superradiance, deepening the understanding of the physics at a fundamental level and providing a potential platform for future studies investigating energy extraction from superfluid vortices.

Editors' Suggestion

Measurement of Penrose Superradiance in a Photon Superfluid

Maria Chiara Braidotti,[1,*] Radivoje Prizia,[1,2] Calum Maitland,[2] Francesco Marino,[3,4] Angus Prain,[1] Ilya Starshynov,[1] Niclas Westerberg,[1] Ewan M. Wright,[5] and Daniele Faccio[1,5,†]

[1]*School of Physics and Astronomy, University of Glasgow, G12 8QQ Glasgow, United Kingdom*
[2]*Institute of Photonics and Quantum Sciences, Heriot-Watt University, EH14 4AS Edinburgh, United Kingdom*
[3]*CNR-Istituto Nazionale di Ottica, Largo Enrico Fermi 6, I-50125 Firenze, Italy*
[4]*INFN, Sezione di Firenze, Via Sansone 1, I-50019 Sesto Fiorentino (FI), Italy*
[5]*Wyant College of Optical Sciences, University of Arizona, Tucson, Arizona 85721, USA*

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The superradiant amplification in the scattering from a rotating medium was first elucidated by Sir Roger Penrose over 50 years ago as a means by which particles could gain energy from rotating black holes. Despite this fundamental process being ubiquitous also in wave physics, it has only been observed once experimentally, in a water tank. Here, we measure this amplification for a nonlinear optics experiment in the superfluid regime. In particular, by focusing a weak optical beam carrying orbital angular momentum onto the core of a strong pump vortex beam, negative norm modes are generated and trapped inside the vortex core, allowing for amplification of a reflected beam. Our experiment demonstrates amplified reflection due to a novel form of nonlinear optical four-wave mixing, whose phase-relation coincides with the Zel'dovich-Misner condition for Penrose superradiance in our photon superfluid, and unveil the role played by negative frequency modes in the process.

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