Entanglement in the stimulated Hawking emission of optical white-black holes

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Analogue gravity in 2023, Benasque, Spain

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In collaboration with I. Agullo, A. J. Brady, A. Delhom

Goals

- Advertise simple yet powerful tools from the quantum information theory of continuous variable systems and Gaussian states to quantify the amount of entanglement produced in the Hawking effect.

- Study the **quantum aspects** of the **stimulated** Hawking process.

Main references:

— I. Agullo, A. J. Brady, and D. Kranas, Phys. Rev. Lett. (2022)

— A. J. Brady, I. Agullo, and D. Kranas, Phys. Rev. D 106 (2022)

Elements of quantum information theory of Gaussian states

Reference: A. Serafini, *Quantum Continuous Variables: A Primer of Theoretical Methods* (2017)

- Consider a system of N quantum bosonic degrees of freedom (harmonic oscillators): $\hat{\boldsymbol{R}} = (\hat{x}_1, \hat{p}_1, ..., \hat{x}_N, \hat{p}_N)$.

  Commutation relations: $[\hat{x}, \hat{p}] = i\hbar \rightarrow [\hat{R}^i, \hat{R}^j] = i\hbar\Omega^{ij}$, $\quad \Omega^{ij} = \bigoplus \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

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- Gaussian state $\hat{\rho}$: Completely characterized by the **first** and **secnond** moments.
  $\rightarrow\ \mu^i \equiv \mathrm{Tr}\left[\hat{\rho}\hat{R}^i\right]$
  $\rightarrow\ \sigma^{ij} \equiv \mathrm{Tr}\left[\hat{\rho}\{(\hat{R}^i - \mu^i), (\hat{R}^j - \mu^j)\}\right]$

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- The properties of the system can be derived in an elegant manner from μ^i and σ^{ij}.
  $\to\ \sigma^{ij} + i\Omega^{ij} \geq 0$
  $\to\ \hat{\rho}$: pure iff $\text{eigen}\{\sigma^{ik}\Omega_{kj}\} = \pm i$
  $\to\ \langle\hat{n}\rangle = \frac{1}{4}\sigma^i{}_i + \frac{1}{2}\mu^i\mu_i - N/2$

Sometimes, it is more illuminating to write down expressions in terms of annihilation and creation operators. Let us, therefore define the vector $\hat{\boldsymbol{A}} = (\hat{a}_1, \hat{a}_1^\dagger, ...\hat{a}_N, \hat{a}_N^\dagger)$.

$$\hat{a}_l = \frac{1}{\sqrt{2}}\left(\hat{x}_l + i\hat{p}_l\right), \qquad \hat{a}_l^\dagger = \frac{1}{\sqrt{2}}\left(\hat{x}_l - i\hat{p}_l\right), \quad l = 1, .., N$$

We can jump between $\hat{\boldsymbol{A}}$ and $\hat{\boldsymbol{R}}$ via

$$\hat{\boldsymbol{A}} = \boldsymbol{U}\hat{\boldsymbol{R}}, \quad \boldsymbol{U} = \bigoplus_{k=1}^{N} \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}$$

$$\hat{\boldsymbol{R}} = \boldsymbol{V}\hat{\boldsymbol{A}}, \quad \boldsymbol{V} = \boldsymbol{U}^{-1} = \bigoplus_{k=1}^{N} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix}$$

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→ Thermal state: $\boldsymbol{\mu} = 0_{2N}$, $\boldsymbol{\sigma} = \oplus_i^N (2\bar{n}_i + 1)\boldsymbol{I}_2$

Evolution

For quadratic Hamiltonians, Gaussian states evolve to Gaussian states

$$(\boldsymbol{\mu}^{\text{in}}, \boldsymbol{\sigma}^{\text{in}}) \longrightarrow (\boldsymbol{\mu}^{\text{out}}, \boldsymbol{\sigma}^{\text{out}})$$

$$\boldsymbol{\mu}^{\text{out}} = \boldsymbol{S}\boldsymbol{\mu}^{\text{in}}, \quad \boldsymbol{\sigma}^{\text{out}} = \boldsymbol{S}\boldsymbol{\sigma}^{\text{in}}\boldsymbol{S}^{\mathsf{T}}, \quad \boldsymbol{S} \cdot \boldsymbol{\Omega} \cdot \boldsymbol{S}^{\mathsf{T}} = \boldsymbol{\Omega}$$

Forget about Schrödinger equation, infinite by infinite density matrices, etc. The evolution of Gaussian states is implemented by simple matrix multiplications of finitely dimensional matrices.

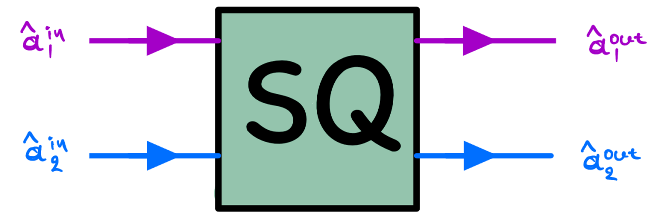
Entanglement

Logarithmic Negativity

To quantify entanglement of quantum states, including mixed ones, we will use *Logarithmic Negativity* LN, associated to the PPT criterion.

- Can be used to quantify the entanglement of mixed states.
- Based on the Positivity of Partial Transposition (PPT) criterion.
- For Gaussian states where either subsystem is made of a single degree of freedom, LN is a **faithful** entanglement quantifier.
- Can be computed from $\boldsymbol{\sigma}$.
- Measures entanglement in units of Bell states. For an operational interpretation look at [X. Wang, M. M. Wilde, Phys. Rev. Lett. 125, 040502 (2020)].

Two-mode squeezing



$$\hat{a}_1^{\text{out}} = \cosh r \, \hat{a}_1^{\text{in}} + e^{i\varphi} \sinh r \, \hat{a}_2^{\dagger\,\text{in}},$$
$$\hat{a}_2^{\text{out}} = e^{i\varphi} \sinh r \, \hat{a}_1^{\dagger\,\text{in}} + \cosh r \, \hat{a}_2^{\text{in}}$$

Two-mode squeezing for vacuum input

- **State before squeezing**:

$$\boldsymbol{\mu}^{\text{in}} = (0, 0, 0, 0), \quad \sigma^{\text{in}} = I_4$$

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$$\boldsymbol{S}_{\text{2sq}} = \begin{pmatrix} \cosh r & 0 & \cos\phi \sinh r & \sin\phi \sinh r \\ 0 & \cosh r & \sin\phi \sinh r & -\cos\phi \sinh r \\ \cos\phi \sinh r & \sin\phi \sinh r & \cosh r & 0 \\ \sin\phi \sinh r & -\cos\phi \sinh r & 0 & \cosh r \end{pmatrix}$$

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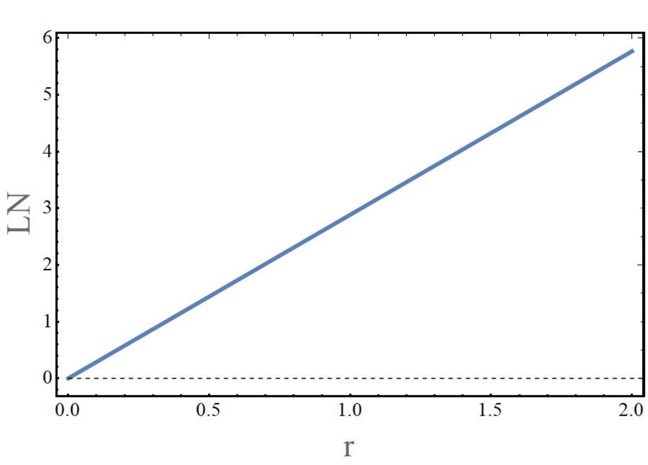
- **Entanglement:**

$$LN(r) = \max\{0, -\log_2 e^{-2r}\} = \frac{2}{\ln 2} r \simeq 2.89\, r$$

Two-mode squeezing for vacuum input

Entanglement vs squeezing amplitude



Cauchy-Schwarz inequality

Entanglement witness [X. Busch, I. Carusotto, and R. Parentani (2014)], [X. Busch and R. Parentani, (2014)].

$$\Delta = \langle \hat{a}_1^\dagger \hat{a}_1 \rangle + \langle \hat{a}_2^\dagger \hat{a}_2 \rangle - |\langle \hat{a}_1 \hat{a}_2 \rangle|^2$$

$\Delta < 0$ is a sufficient condition for entanglement. Only for some states Δ is a sufficient and necessary condition [X. Busch and R. Parentani, (2014)].

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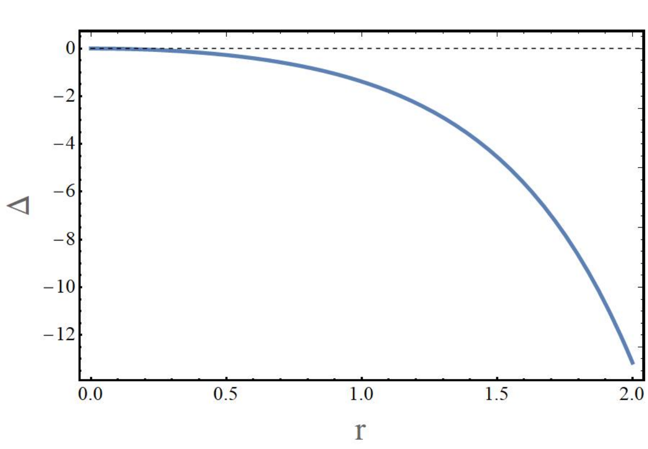
- **Advantages**: It can be computed by measuring a few observables.
- **Disadvantages**: It does not quantify entanglement.

Cauchy-Schwarz inequality and two-mode squeezing

Δ vs squeezing amplitude



Two-mode squeezing for displaced input

- **State before two-mode squeezing**:

$$\boldsymbol{\mu}^{\text{in}} = \sqrt{2}(\text{Re}[\alpha], \text{Im}[\alpha], \text{Re}[\alpha], \text{Im}[\alpha]), \quad \boldsymbol{\sigma} = \boldsymbol{I}_4$$

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The amount of entanglement produced is the same as in the case of *vacuum input*.

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The state is entangled only if $r > \frac{1}{2}\ln(2n+1)$

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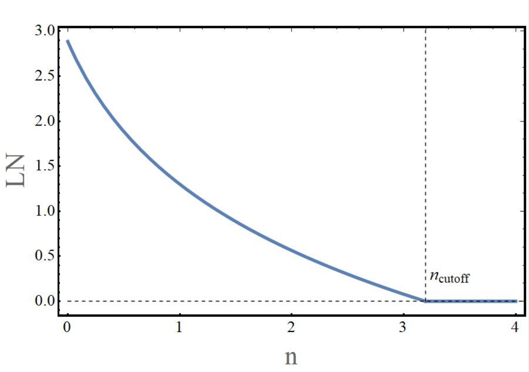

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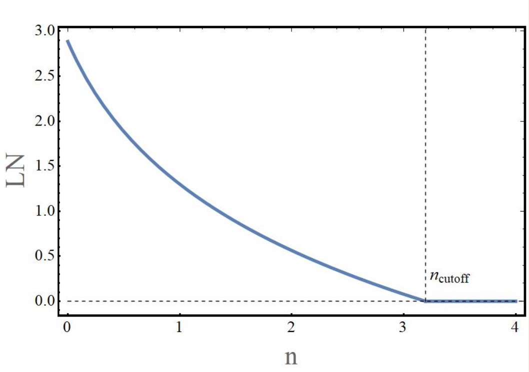

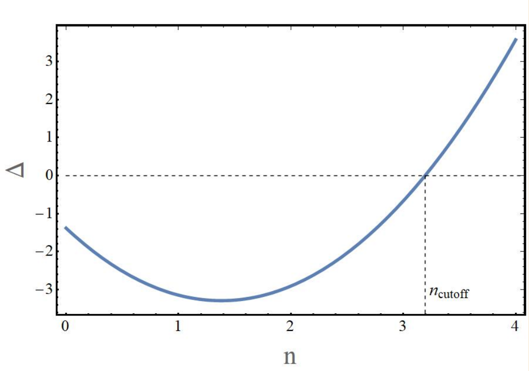

Messages: 1) Entanglement decreases with thermal noise. 2) *LN* measures correctly the *amount* of entanglement but Δ does not; it only tells us when the state is entangled,

Two-mode squeezing for thermal-single-mode squeezed ($\phi = 0$) input

- **State before two-mode squeezing**:

$$\boldsymbol{\mu}^{\text{in}} = (0, 0, 0, 0), \quad \boldsymbol{\sigma} = (2n+1)\begin{pmatrix} e^{-\xi} & 0 & 0 & 0 \\ 0 & -e^{\xi} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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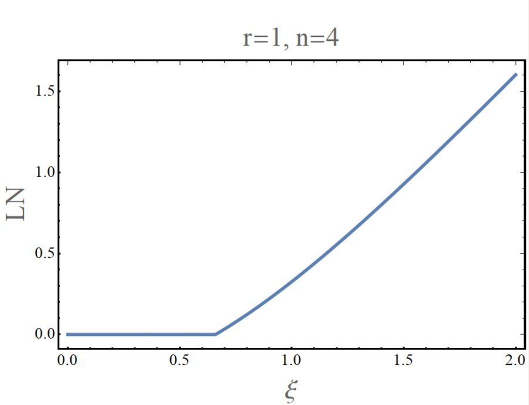
Whether the state is entangled or not, as well as the amount of entanglement it may contain depends on the combination of r, n, ξ.

Two-mode squeezing for thermal-single-mode squeezed ($\phi = 0$) input

Entanglement vs initial squeezing amplitude



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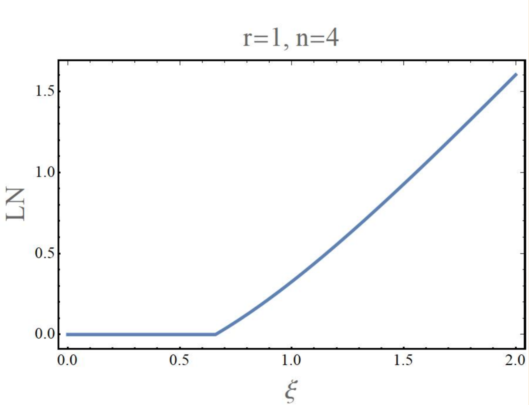
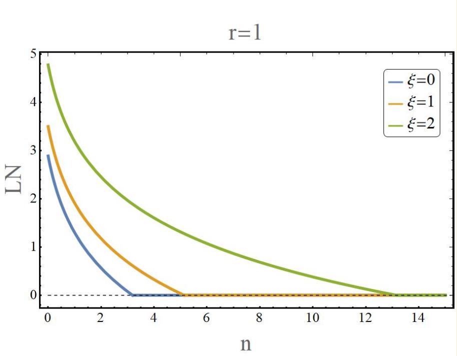


Message: One can amplify the amount of entanglement produced by the two-mode squeezer by tuning appropriately the input state.

Two-mode squeezing for thermal-single-mode squeezed ($\phi = 0$) input

Entanglement vs noise



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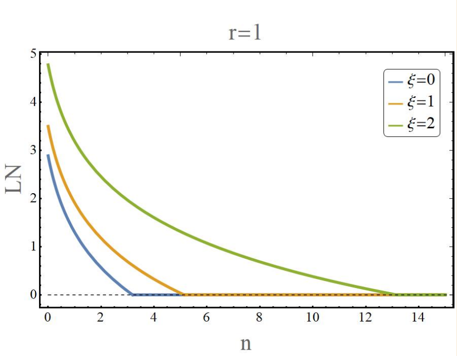

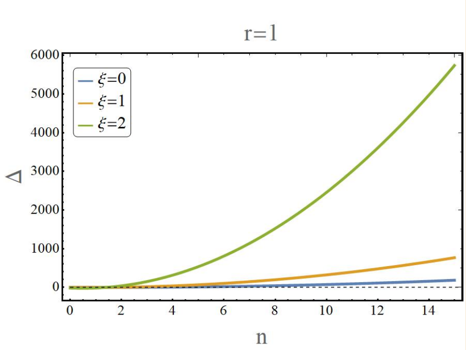

Messages: 1) Entanglement increases with the amount of initial squeezing. 2) Δ does not capture correctly the entanglement,

Hawking effect in optical analogue white-black holes

Modes and emergent causal structure

- **Kerr effect**: $n_{eff}(\omega, x, t) = n(\omega) + \delta n(x, t), \quad \delta n(x, t) = \alpha E_s^2(x, t).$
- In the comoving frame (χ, τ) w.r.t to the strong pulse, the system admits static solutions $\rightarrow$ Conservation of frequency.

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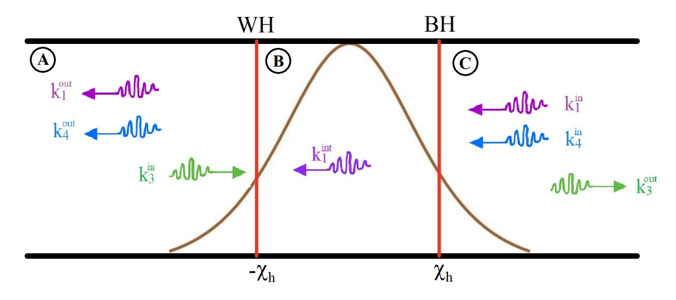
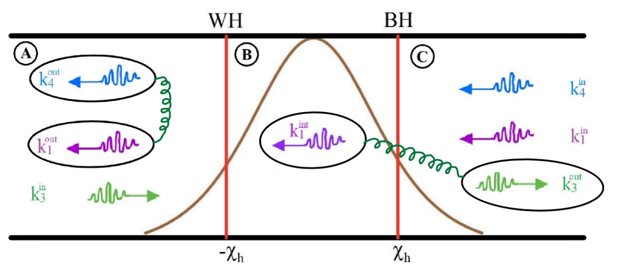


Modes and emergent causal structure



Each horizon behaves as a two-mode squeezer producing entangled Hawking pairs!!

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- Further confirmation needed!!

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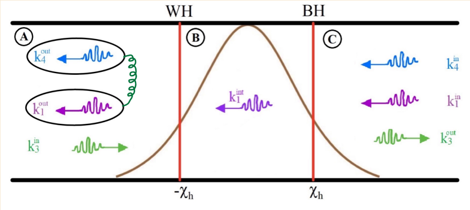

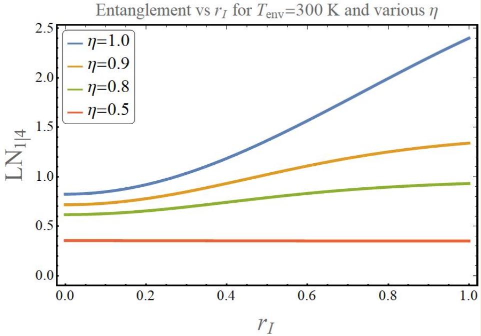

- **Single-mode squeezed inputs enhance the entanglement generated by the horizons.**

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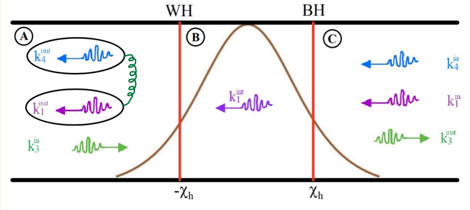
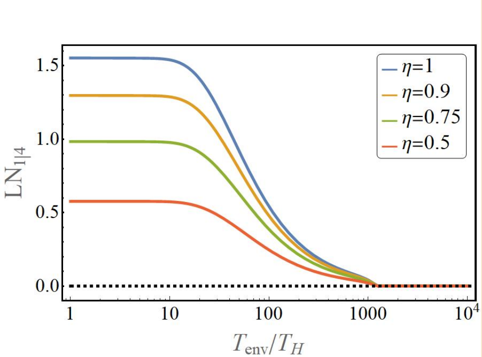
These results open a promising avenue for the detection of the Hawking effect and its quantum origin in the lab.

Additional Slides

Thermal noise and detector losses

To make contact with a realistic situation, we studied how noisy environments (i.e. thermal fluctuations) and detector losses affect the entanglement produced in the Hawking process.





Take-home message: Environment noise and detector inefficiencies reduce the amount of entanglement and can, even, make it completely vanish.

Bloch–Messiah decomposition: Any symplectic transformation can be decomposed to a set of squeezers, beam splitters, and phase shifters.

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The production of entangled quanta in the Hawking effect is a two-mode squeezing process.

Separability and entanglement

Let us consider a composite system that can be split into two subsystems A and B. Let $\hat{\rho}_A \in \mathcal{D}(\mathcal{H}_A)$ and $\hat{\rho}_B \in \mathcal{D}(\mathcal{H}_B)$ be the density operators describing A and B, respectively. The composite system is characterized by $\hat{\rho} \in \mathcal{D}(\mathcal{H}_A \bigotimes \mathcal{H}_B)$.

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where $\hat{\rho}_{A,j} \in \mathcal{D}(\mathcal{H}_A)$, $\hat{\rho}_{B,j} \in \mathcal{D}(\mathcal{B}_A)$, $0 \leq p_j \leq 1$ for $\forall\ j = 1, .., m$ and $\sum_{j=1}^{m} p_j = 1$.

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A quantum state $\hat{\rho}$ is said to be **entangled** if it is not separable.

Entanglement: von Neumann entropy

Standard entanglement quantifier: **entanglement entropy** $\rightarrow$ von Neumann entropy of one of the subsystems.

Let $\hat{\rho}_A = \mathrm{Tr}_B[\rho\hat{_{AB}}]$ be the state describing subsystem A. The entanglement entropy is given by

$$E[\hat{\rho}_A] = -\mathrm{Tr}[\hat{\rho}_A \log_2(\hat{\rho}_A)] = -\Sigma_j \, \lambda_{A,j} \log_2(\lambda_{A,j}), \quad \lambda_{A,j} \equiv \mathrm{eigen}\{\hat{\rho}_A\}$$

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For Gaussian states

$$E = \sum_{j=1}^{n} h(\nu_i^A), \quad h(\nu_i^A) = \frac{\nu_i^A + 1}{2} \log_2\left(\frac{\nu_i^A + 1}{2}\right) - \frac{\nu_i^A - 1}{2} \log_2\left(\frac{\nu_i^A - 1}{2}\right),$$

where $\{\nu_i^A\}$, for $i = 1, ..., N$, is the set of **symplectic eigenvalues** of $\boldsymbol{\sigma}_A$, i.e. $|\text{eigen}\{\boldsymbol{\Omega}\boldsymbol{\sigma}_A\}|$.

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- But if the total state $\hat{\rho}$ is mixed, $E[\hat{\rho}_A]$ could be positive even if $\hat{\rho}_{AB} = \hat{\rho}_A \otimes \hat{\rho}_B$.
- von Neumann entropy **cannot** be used to quantify entanglement in **mixed** states.

PPT criterion

To study the entanglement of quantum states, including mixed ones, we will use the well-known **positivity of the partial transposition (PPT)** criterion [A. Peres (1996), P. Horodecki (1997)].

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$$\hat{\rho}_{AB} = \sum_{i,j,k,\ell} p_{i,j,k,\ell} \left|i\right\rangle \left\langle j\right|_A \otimes \left|k\right\rangle \left\langle \ell\right|_B .$$

The partial transposition with respect to B is given by

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Let $\{\lambda_i^{\text{PT}}\}$ be the set of eigenvalues of $\hat{\rho}^{\text{PT}}$.

- If $\hat{\rho}_{AB}$ is **separable**, then $\lambda_i^{\text{PT}} > 0 \;\forall\; i$.
- If $\exists\; \lambda_i^{\text{PT}} < 0$, then $\hat{\rho}_{AB}$ is **entangled**.

PPT for Gaussian states

- For Gaussian states, all statements about correlations, separability, and entanglement can be extracted solely from the covariance matrix $\boldsymbol{\sigma}$.
- The operation of partial transposition of a system of $M + K = N$ d.o.f.s, partitioned as $(M - \text{d.o.f.s}|K - \text{d.o.f.s})$, is implemented by

$$\boldsymbol{\sigma}^{\mathrm{PT}} = \mathbf{T}\boldsymbol{\sigma}\mathbf{T}, \quad \mathbf{T} = \mathbf{I}_{2M} \bigoplus \boldsymbol{\Sigma}_{2K}, \quad \boldsymbol{\Sigma}_{2K} = \bigoplus_{i=1}^{K} \boldsymbol{\sigma}_z$$

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Let $\{\nu_i^{\mathrm{PT}}\}$ be the set of symplectic eigenvalues of σ^{PT}.

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Two-mode squeezing for vacuum input

- **Logarithmic Negativity**

$$LN(r) = \max\{0, -\log_2 \nu^{\mathrm{PT}}_{\min}\} = \frac{2}{\ln 2} r \simeq 2.89\, r$$

- **Entanglement entropy**

$$E = \frac{\nu^A + 1}{2} \log_2\left(\frac{\nu^A + 1}{2}\right) - \frac{\nu^A - 1}{2} \log_2\left(\frac{\nu^A - 1}{2}\right),$$

where $\nu^A = \cosh 2r$.

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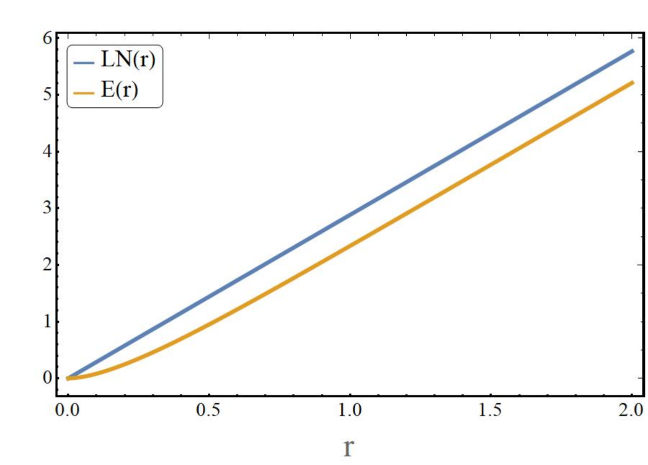

Both E and LN increase monotonically with r and capture the entanglement produced by the squeezing.

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— For the particular case where of Gaussian systems partitioned as $(1 - \text{d.o.f.s}|M - \text{d.o.f.s})$ (which are most of the situations we are interested in), LN is given by

$$LN = \max\{0, -\log_2 \nu_{\text{min}}^{\text{PT}}\},$$

where $\nu_{\text{min}}^{\text{PT}}$ is the lowest symplectic eigenvalue of σ^{PT}.

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The state is entangled only if $\nu_{\text{min}}^{\text{PT}} < 1 \Rightarrow r > \frac{1}{2}\ln(2n+1)$

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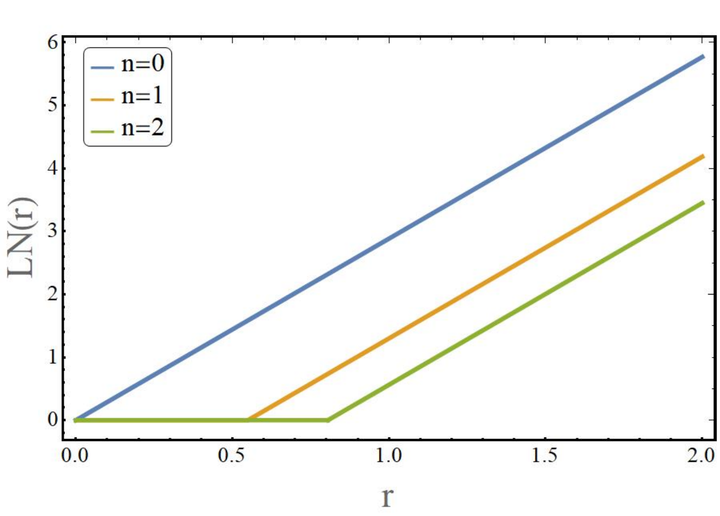

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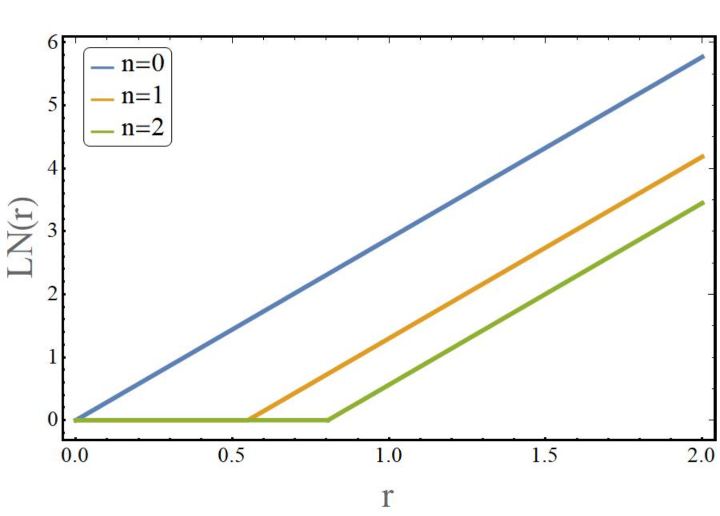

Message: Entanglement increases with r and decreases with n.

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Mutual information: $I = E_A + E_B - E_{AB}$

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- Mutual information encodes the **total** amount of correlation in the state, both *classical* and **quantum**.
- For **pure** states ($E_{AB} = 0$): $I = 2E_A = 2E_B$

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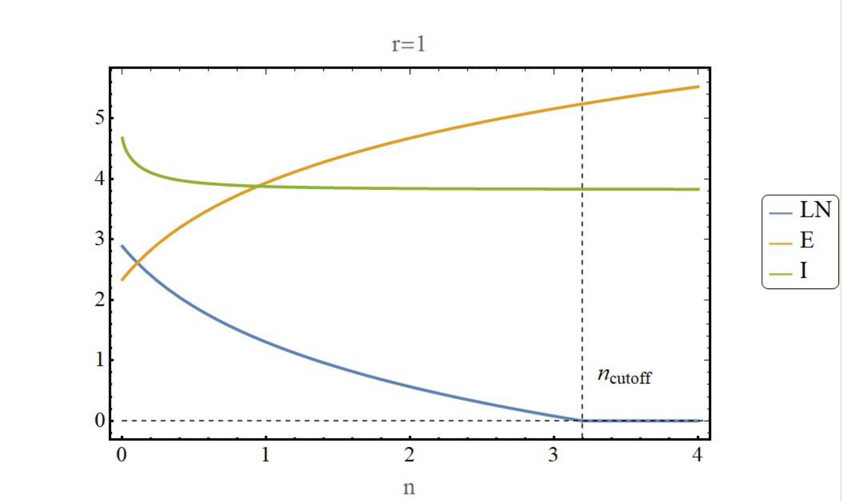

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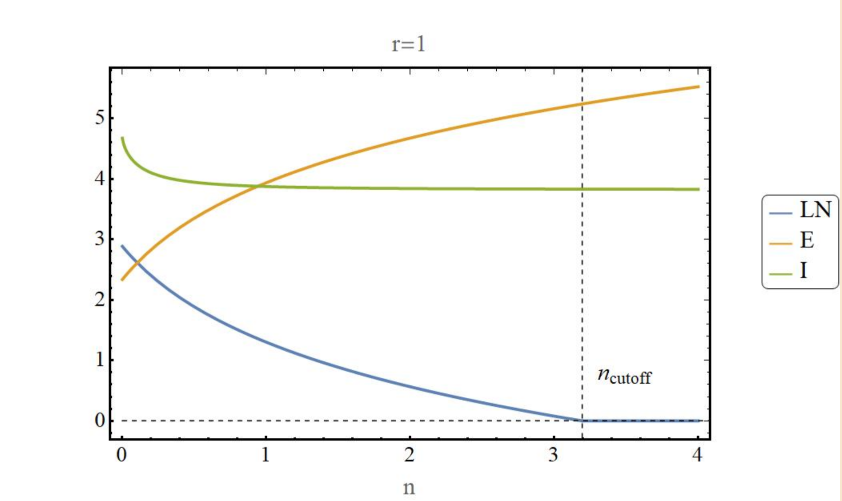

Message: The quantum state contains correlations even when entanglement disappears.

Two-mode squeezing for thermal-single-mode squeezed ($\phi = 0$) input

- **State before two-mode squeezing**:

$$\boldsymbol{\mu}^{\text{in}} = (0, 0, 0, 0), \quad \boldsymbol{\sigma} = (2n+1)\begin{pmatrix} e^{-\xi} & 0 & 0 & 0 \\ 0 & -e^{\xi} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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Whether the state is entangled or not, as well as the amount of entanglement it may contain depends on the combination of r, n, ξ.

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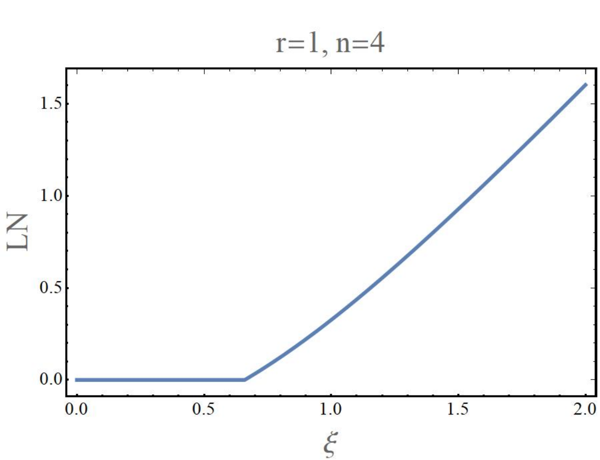

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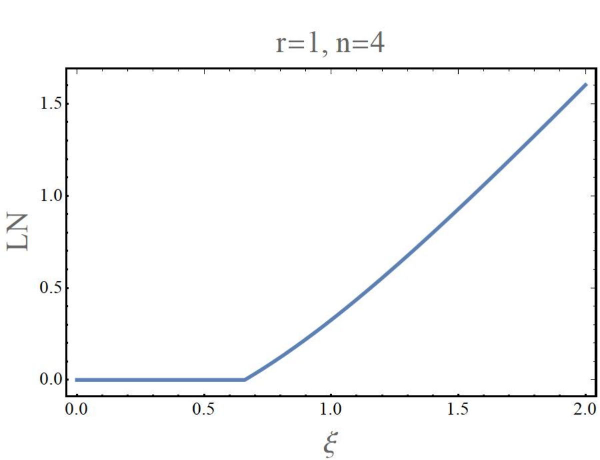


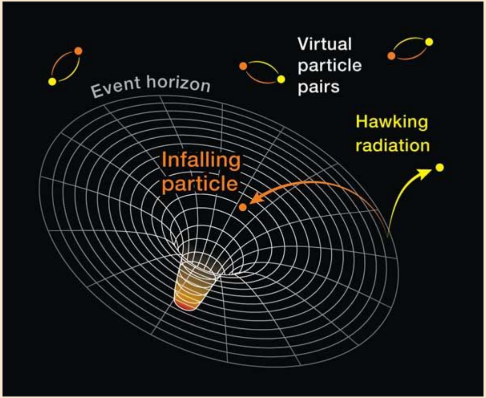
Message: One can amplify the amount of entanglement produced by the two-mode squeezer by tuning appropriately the input state.

Hawking effect

Hawking effect: Spontaneous creation of entangled particle pairs by black hole event horizons.

[S. W. Hawking (1974)]

Hawking process in a nutshell



- Ingredients: Black hole horizon + a quantum field.
- Thermal radiation emitted from the exterior of black holes.
- Hawking temperature:
  $$T_H = \frac{\hbar c^3}{8\pi G k_B M}$$
- Carries a quantum signature: Entanglement

One of the main contributions of this work is the incorporation of quantum information tools of Gaussian states into the physics of field theory to reformulate the Hawking process in a simple yet efficient manner.

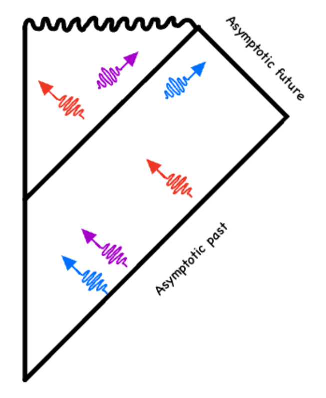
Goal: Quantify the amount of entanglement generated in the Hawking process.

From ∞ to 3

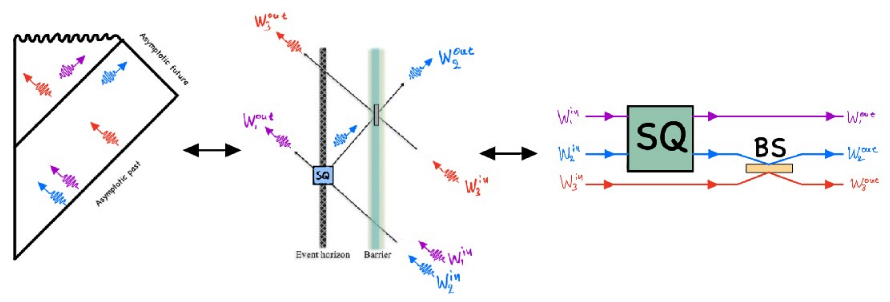
- QFT: Infinitely-many degrees of freedom.

From ∞ to 3

- QFT: Infinitely-many degrees of freedom.
- Wald (1975): Found the progenitors of the Hawking modes $\rightarrow$ evolution diagonalizes to interactions among sets of three modes.



Hawking process as symplectic transformations



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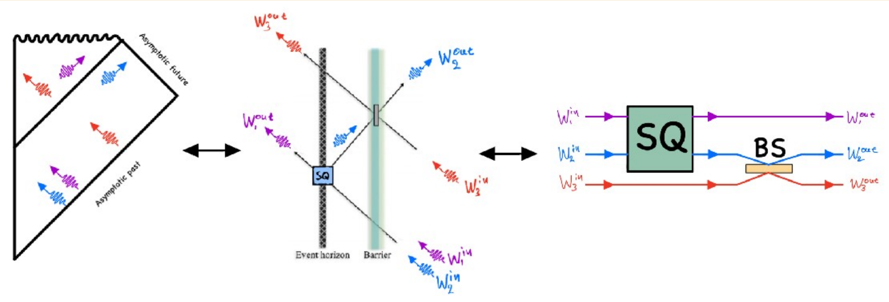


The scattering process at the black hole can be modeled via a **two-mode squeezer** followed by a **beam splitter**.

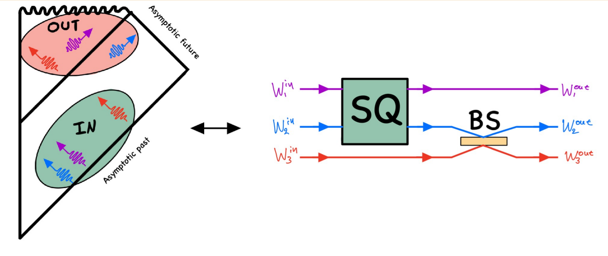
Squeezer

$$\hat{a}_1^{\text{out}} = \cosh r_\omega \ \hat{a}_1^{\text{in}} + e^{i\phi} \sinh r_\omega \ (\hat{a}_2^{\text{in}})^\dagger$$
$$\hat{a}_2^{\text{up}} = e^{i\phi} \sinh r_\omega \ (\hat{a}_1^{\text{in}})^\dagger + \cosh r_\omega \ \hat{a}_2^{\text{in}}$$

Beam splitter

$$\hat{a}_2^{\text{out}} = T_\omega \ \hat{a}_2^{\text{up}} - R_\omega \ \hat{a}_3^{\text{in}}$$
$$\hat{a}_3^{\text{out}} = R_\omega \ \hat{a}_2^{\text{up}} + T_\omega \ \hat{a}_3^{\text{in}}$$

Hawking process as symplectic transformations



$$\begin{pmatrix} \hat{a}_1^{\text{out}} \\ (\hat{a}_1^{\text{out}})^\dagger \\ \hat{a}_2^{\text{out}} \\ (\hat{a}_2^{\text{out}})^\dagger \\ \hat{a}_3^{\text{out}} \\ (\hat{a}_3^{\text{out}})^\dagger \end{pmatrix} = \begin{pmatrix} \cosh r_\omega & 0 & e^{i\phi}\sinh r_\omega & 0 & 0 & 0 \\ 0 & \cosh r_\omega & 0 & e^{-i\phi}\sinh r_\omega & 0 & 0 \\ 0 & e^{i\phi}T_\omega \sinh r & T_\omega \cosh r & 0 & -R_\omega & 0 \\ e^{-i\phi}T_\omega \sinh r_\omega & 0 & 0 & T_\omega \cosh r_\omega & 0 & -R_\omega \\ 0 & e^{i\phi}R_\omega \sinh r_\omega & R_\omega \cosh r_\omega & 0 & T_\omega & 0 \\ e^{-i\phi}R_\omega \sinh r_\omega & 0 & 0 & R_\omega \cosh r_\omega & 0 & T_\omega \end{pmatrix} \begin{pmatrix} \hat{a}_1^{\text{in}} \\ (\hat{a}_1^{\text{in}})^\dagger \\ \hat{a}_2^{\text{in}} \\ (\hat{a}_2^{\text{in}})^\dagger \\ \hat{a}_3^{\text{in}} \\ (\hat{a}_3^{\text{in}})^\dagger \end{pmatrix}$$

Hawking process as symplectic transformations

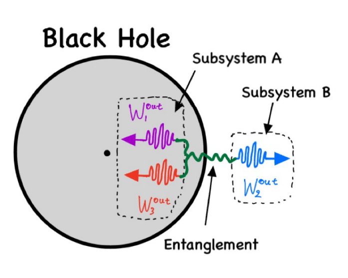
Number of emitted quanta:

$$\langle 0|\left(\hat{a}_2^{\text{out}}\right)^{\dagger}\hat{a}_2^{\text{out}}|0\rangle_{\text{in}} = T_\omega \sinh^2 r_\omega = T_\omega\left(e^{\hbar\omega/k_{\text{B}}T_{\text{H}}} - 1\right)^{-1}$$

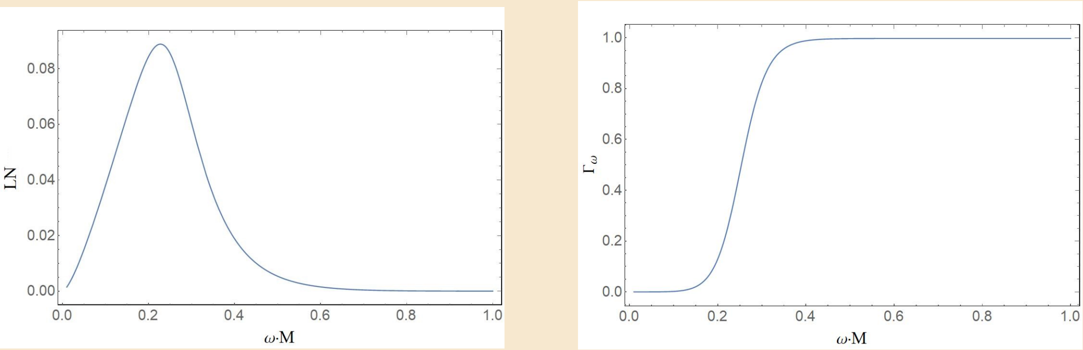
.

Entanglement produced by black holes

Let us compute the entanglement between Hawking radiation and the modes falling inside the black hole.



Entanglement in the Hawking effect



- At low ω, $\Gamma_\omega \to 0$: the gravitational barrier becomes fully reflective $\to$ No Hawking quanta escape.
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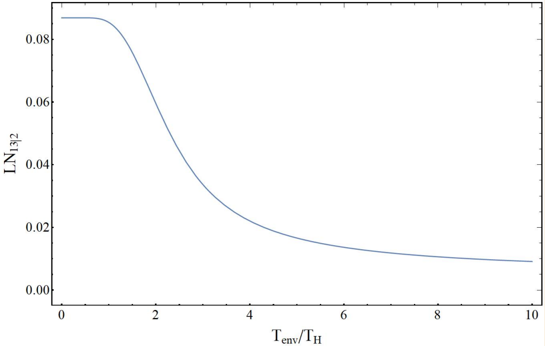
Entanglement for BHs in a thermal bath



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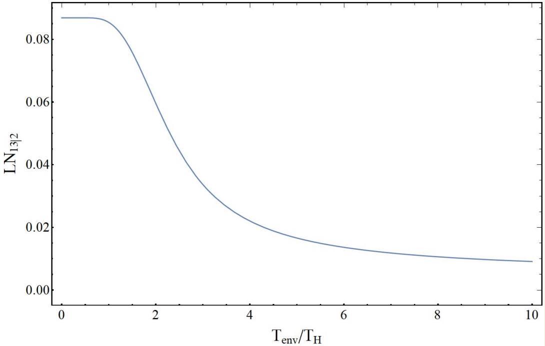

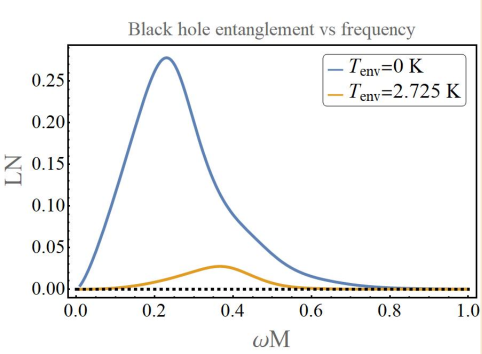

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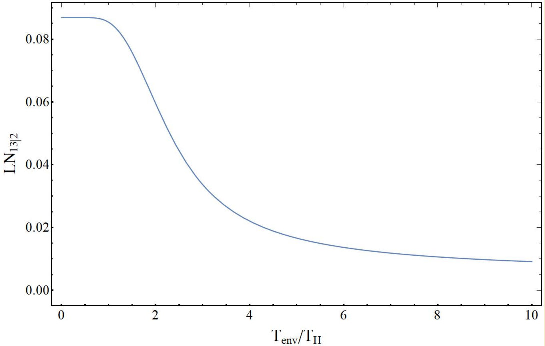
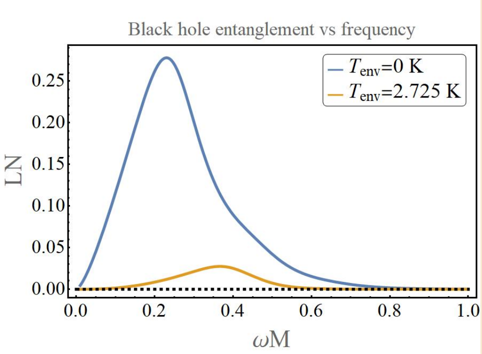




Main message: Thermal baths (mixed input quantum states) reduce the amount of entanglement produced in the Hawking process. In some cases, they can make entanglement completely vanish.

Observations?

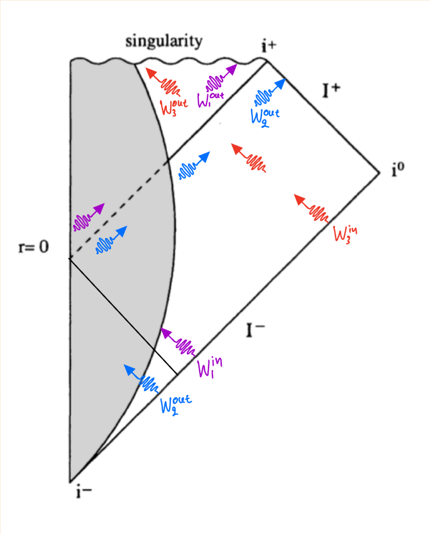
- **Entanglement**: Impossible as it would require extracting information from the interior of the black hole.
- **Hawking radiation**: Potentially...But, for "standard" black holes the resulting signal is extremely weak. Recall

$$T_{\mathrm{H}} = \frac{\hbar c^3}{8\pi G k_{\mathrm{B}} M}.$$

For $M = M_\odot \to T_{\mathrm{H}} = 61.7\,\mathrm{nK}$. On the other hand, $T_{\mathrm{CMB}} = 2.7\,\mathrm{K}$

Conclusion: Hawking radiation emitted by BHs of a typical mass is extremely weak and, thus, will be buried under other cosmic signals (e.g. CMB).

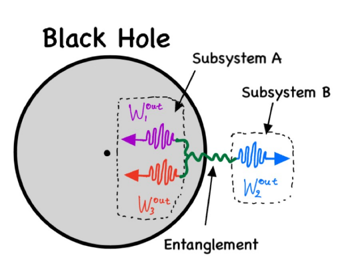
Hawking effect in a nutshell



- The Hawking process is a 3-mode interaction of a field (for concreteness we consider a masssless field).
- We associate $(\hat{a}_i, \hat{a}_i^\dagger)$, $i = 1, 2, 3$ to the three modes.
- At I^-, the field is in the vacuum state $|0\rangle_{\text{in}}$, i.e. $\hat{a}_i^{\text{in}} |0\rangle_{\text{in}} = 0$, $\forall\ i$.No quanta initially: $\langle 0| \hat{n}_i^{\text{in}} |0\rangle_{\text{in}} = \langle 0| (\hat{a}_i^{\text{in}})^\dagger \hat{a}_i^{\text{in}} |0\rangle_{\text{in}} = 0$.
- At I^+, a detector would measure $\langle 0| \hat{n}_2^{\text{out}} |0\rangle_{\text{in}} = \langle 0| (\hat{a}_2^{\text{out}})^\dagger \hat{a}_2^{\text{out}} |0\rangle_{\text{in}} = \Gamma_\omega \left(e^{\frac{\hbar\omega}{k_B T_H}} - 1 \right)^{-1}$.
- Black holes radiate as blackbodies of temperature $T_H = \frac{\hbar c^3}{8\pi G k_B M}$.
- The Hawking mode W_2^{out} is entangled with the interior modes W_1^{out} and W_3^{out}.

Entanglement in the Hawking effect

Let us compute the entanglement between Hawking radiation and the modes falling inside the black hole.



- Entanglement is directly produced in modes W_2^{out} and W_1^{out} by the two-mode squeezer.
- Due to the gravitational barrier (modeled by a beam splitter), some of the Hawking quanta are backscattered and follow into the black hole via the mode W_3^{out}. Hence, this mode will also be entangled with the W_1^{out}.

Elements of quantum information theory of Gaussian states

We work with the family Gaussian states, where all the information about the quantum state is encoded in the first moments $\boldsymbol{\mu} = \langle \hat{\boldsymbol{A}} \rangle$ and the covariance matrix $\sigma = \langle \{ \hat{\boldsymbol{A}} - \boldsymbol{\mu}, \hat{\boldsymbol{A}} - \boldsymbol{\mu} \} \rangle$, where $\hat{\boldsymbol{A}} = (\hat{a}_1, \hat{a}_1^\dagger, \hat{a}_2, \hat{a}_2^\dagger, ..., \hat{a}_N, \hat{a}_N^\dagger)^T$.

Examples of Gaussian States: Let us, for illustration purposes, consider a single degree of freedom $\hat{\boldsymbol{A}} = (\hat{a}, \hat{a}^\dagger)$.

→ Vacuum state

$|0\rangle : \hat{a}\,|0\rangle = 0$. Moments: $\boldsymbol{\mu} = (0,0)$, $\boldsymbol{\sigma} = \boldsymbol{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

→ Coherent state

$|coh\rangle = \exp\left(\alpha \hat{a}^\dagger - \alpha^* \hat{a}\right)|0\rangle$. Moments: $\boldsymbol{\mu} = (\alpha, \alpha^*)$, $\boldsymbol{\sigma} = \boldsymbol{\sigma}_x$.

→ Single-mode squeezed state

$|SMSV\rangle = \exp\left[\frac{1}{2}\xi\left(\hat{a}^{\dagger\,2} - \hat{a}^2\right)\right]|0\rangle$. Moments: $\boldsymbol{\mu} = (0,0)$, $\boldsymbol{\sigma} = \begin{pmatrix} \sinh 2\xi & \cosh 2\xi \\ \cosh 2\xi & \sinh 2\xi \end{pmatrix}$.

→ Thermal state

$\rho \propto \Sigma_n e^{-\frac{\omega}{T}(n+\frac{1}{2})}|n\rangle\langle n|$. Moments: $\boldsymbol{\mu} = (0,0)$, $\boldsymbol{\sigma} = (2\bar{n}+1)\boldsymbol{\sigma}_x$, $\bar{n} = (e^{\omega/T} - 1)^{-1}$.

Summary of our main results for the spontaneous Hawking effect (i.e. vacuum input)

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- We study the energy scale (frequency) where **effects of dispersion** become important and the Hawking particle creation loses its thermal character.

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- The evolution of the state is given by: $\hat{\boldsymbol{A}}_{out} = \boldsymbol{S} \cdot \hat{\boldsymbol{A}}_{in}$, $\boldsymbol{\mu}_{out} = \boldsymbol{S} \cdot \boldsymbol{\mu}_{in}$, $\boldsymbol{\sigma}_{out} = \boldsymbol{S} \cdot \boldsymbol{\sigma}_{\text{in}} \cdot \boldsymbol{S}^T$.

- We use Logarithmic negativity (LN), a well-known entanglement monotone in quantum information theory, to study and quantify the entanglement produced in the Hawking process. LN can be easily computed from the covariance matrix. [A. Serafini, *Quantum Continuous Variables: A Primer of Theoretical Methods* (2017)], [X. Wang, M. M. Wilde, Phys. Rev. Lett. 125, 040502 (2020).]

- LN will allow us to extend previous calculations in the literature based on entanglement entropy, e.g. [D. N. Page (2013)], to study more realistic scenarios, for instance, when the initial quantum state is mixed.

Black holes immersed in a thermal bath

→ The previous calculations were made for black holes in isolation. What about black holes immersed in a thermal bath of photons (such as the CMB)?

→ Does the thermal bath affect particle production and generation of entanglement?

- The initial quantum state of the field is not the vacuum anymore, but rather a mixed state.
- The covariance matrix of each mode is $(2n_{\mathrm{env},i}+1)\boldsymbol{I}_2$. But, modes W_1^{in} and W_2^{in} have an ultra-high frequency and therefore $n_{\mathrm{env},1}=n_{\mathrm{env},2}\approx 0$. For W_3^{in}, $n_{\mathrm{env},3}\equiv n_{\mathrm{env}}=\left(e^{-\omega/T_{\mathrm{env}}}-1\right)^{-1}$. The initial state is $\boldsymbol{\mu}^{\mathrm{in}}=(0,0)$, $\boldsymbol{\sigma}=\boldsymbol{I}_4\oplus(2n_{\mathrm{env}}+1)\boldsymbol{I}_2$.(I should probably remove this last bullet as it is technical and doesn't offer much in the global discussion.)

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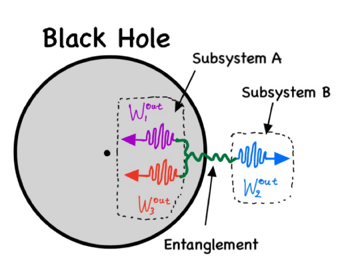

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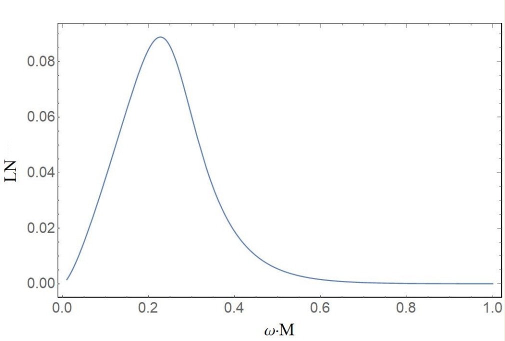

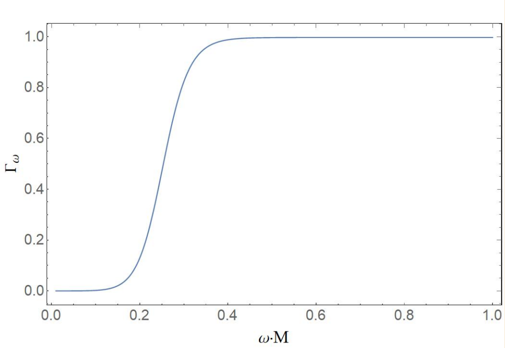

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Experimental status

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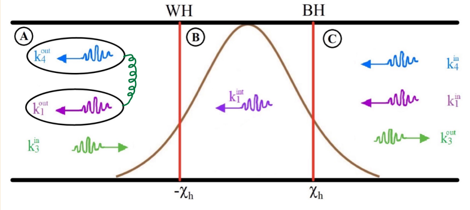
Open questions

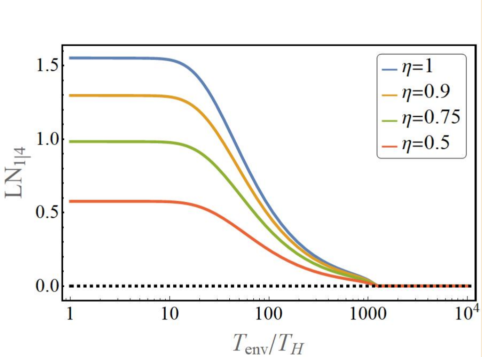
- Observation of a blackbody spectrum (dispersion the main obstacle and absence of perfect horizons).
- Generation of entanglement (fragile to background thermal noise).

Conclusion: Observing the Hawking effect, even in highly controllable analog gravity platforms, has proven to be an extremely challenging task.

Thermal noise and detector losses

To make contact with a realistic situation, we studied how noisy environments (i.e. thermal fluctuations) and detector losses affect the entanglement produced in the Hawking process.





Take-home message: Environment noise and detector inefficiencies reduce the amount of entanglement and can, even, make it completely vanish.

The protocol to extract T_H from observations

Intensities (classical signal)

- $\langle \hat{n}_3(\omega) \rangle = A(\omega)(e^{\omega/T_H} - 1)^{-1}$.

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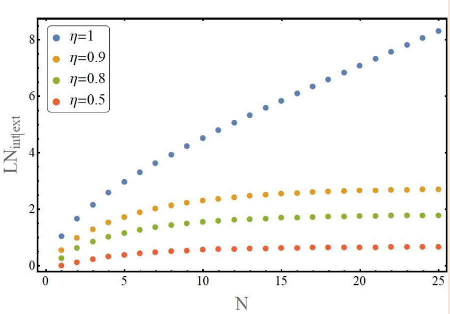
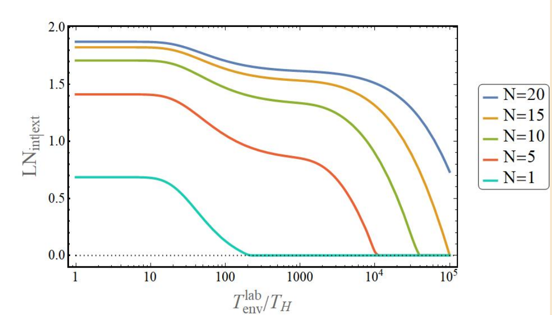
- $\langle \hat{n}_3(\omega) \rangle = A(\omega)(e^{\omega/T_H} - 1)^{-1}$.

Entanglement (quantum signal)

- Theoretically, we compute the function $LN_{1|4}(\omega) = f(\omega; T_H, T_{env}, r_I)$.
- Measure all mode correlations $\left\langle \{\hat{a}_i^{out}(\omega), \hat{a}_j^{out}(\omega)\} \right\rangle$, $\left\langle \{\hat{a}_i^{out}(\omega), \hat{a}_j^{out\,\dagger}(\omega)\} \right\rangle$, $\left\langle \{\hat{a}_i^{out\,\dagger}(\omega), \hat{a}_j^{out\,\dagger}(\omega)\} \right\rangle$.
- Construct $LN_{1|4}(\omega)$.
- Obtain T_H from $LN_{1|4}(\omega)$.

Entanglement in the laser effect subject to thermal noise and losses





- Entanglement maintains its monotonic growth as a function of the total loops N, even in the case of inefficient detectors. However, the amount of entanglement is reduced and it saturates quickly after the first few loops.
- Entanglement decreases with T_{env} and increases with η and N. The more loop the laser operates the more entanglement is produced and the T_{env} threshold where entanglement vanishes is pushed to higher values.
- The lasing setup provides us with a mechanism to overcome spurious effects induced by noise and detector inefficiencies.

Take-home messages

- The black hole causal structure can be reproduced in dispersive, inhomogeneous media.
- Stimulating the process with a single-mode squeeze state is a promising strategy for the detection of the Hawking effect as it enhances both the Hawking intensity and the entanglement generated by the Hawking process.
- Connecting observations (intensities and entanglement) with the causal horizon is essential for the experimental confirmation of the Hawking effect.

Context

- **Hawking effect**: Creation of entangled pairs of particles by black hole event horizons. S. W. Hawking, Black hole explosions, Nature 248, 30 .
- **Direct observation?** For typical black hole masses $T_H/T_{CMB} \ll 1 \rightarrow$ signal burried under CMB.
- **Analog black holes**: Dispersive media where propagating perturbations experience causal horizons mimicking the structure of black hole/white hole spacetimes. Popular analog models include: 1) Hydrodynamic systems, Bose Einstein Condensates, Optical media, etc.

River-analog of black holes

River-analog of black holes

River metric in 1+1D:

$$ds^2 = -u^2 dt^2 + (dx - V(x)dt)^2$$

Acoustic horizon condition $|V(x)| = u$.

River-analog of black holes

River metric in 1+1D:

$$ds^2 = -u^2 dt^2 + (dx - V(x)dt)^2$$

Acoustic horizon condition $|V(x)| = u$.

$$ds^2 = -\left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2$$

$$ds^2 = -c^2 d\tilde{t}^2 + \left(dr + c\sqrt{\frac{r_s}{r}}\, d\tilde{t}\right)^2$$

Flow velocity: $V(r) = -c\sqrt{\frac{r_s}{r}}$.

Optical black holes: The (microscopic) model

$$\mathcal{L} = \underbrace{\frac{1}{2}\left[|\partial_t A|^2 - |\partial_x A|^2\right]}_{\text{EM field}} + \underbrace{\frac{1}{2}\left[|\partial_t \psi|^2 - \Omega^2 \psi\right]}_{\text{medium}} + \underbrace{gRe\left[\psi \partial_t A^*\right]}_{\text{linear interaction}}$$

$$\Omega(x,t) = \Omega_o + \underbrace{\alpha_{ST}|\mathcal{E}_s(x,t)|^2}_{\text{nonlinear interaction}}$$

$$n_{eff}(\omega_{lab}, x, t) = \sqrt{1 + \frac{g^2}{\Omega^2(x,t) - \omega_{lab}^2}}$$

Wave equation

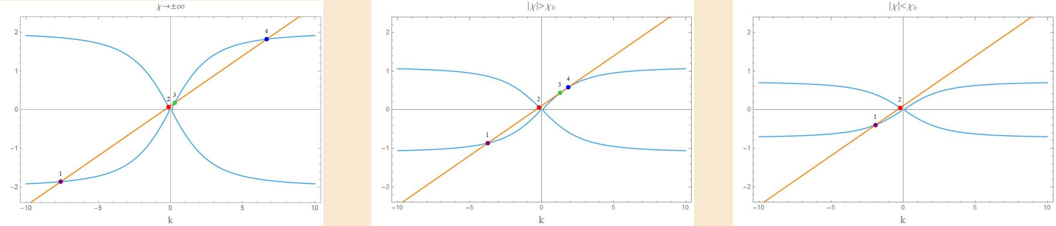
$$\partial_t^2 A(x,t) - \partial_x^2 A(x,t) = -g\partial_t\psi(x,t)$$
$$\partial_t^2\psi(x,t) + \Omega^2(x,t)\psi(x,t) = g\partial_t A(x,t)$$

↓

$$\left\{\left(\partial_\chi^2 + \omega^2\right)\left[\gamma^2\left(u\partial_\chi + i\omega\right)^2 + \Omega^2(\chi)\right] - \gamma^2 g^2\left(u\partial_\chi^2 + i\omega\right)^2\right\}\psi_\omega(\chi) = 0$$

Dispersion relation in the comoving frame

$$\gamma(\omega + uk) = \pm\Omega(\chi)\sqrt{1 + \frac{g^2}{\omega^2 - k^2 - g^2}}$$



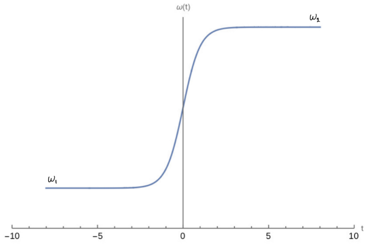
figures/wavepackets_in_and_out.jpg

Particle creation: a simple example

Consider a time-dependent quantum harmonic oscillator.

$$\hat{H} = \frac{1}{2}\hat{p} + \frac{1}{2}\omega^2(t)\hat{x}^2 \rightarrow \frac{d^2\hat{x}}{dt^2} + \omega(t)\hat{x} = 0$$

$$[\hat{x}, \hat{p}] = i, \quad [\hat{a}, \hat{a^\dagger}] = 1$$



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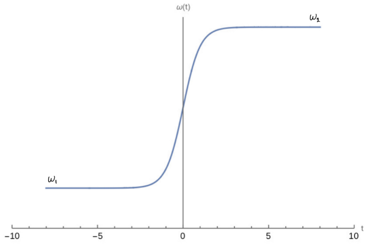

For $t \ll 0$:

- $\omega(t) = \omega_1 \rightarrow \hat{x}(t) = \frac{1}{\sqrt{2\omega_1}}\left(\hat{a}_1 e^{-i\,\omega_1\,t} + \hat{a^\dagger}_1 e^{+i\,\omega_1\,t}\right)$
- $\hat{a}_1 \left|0\right\rangle_1 = 0,\ \langle 0|\, \hat{a}_1^\dagger \hat{a}_1 \left|0\right\rangle_1 = 0$

For $t \gg 0$:

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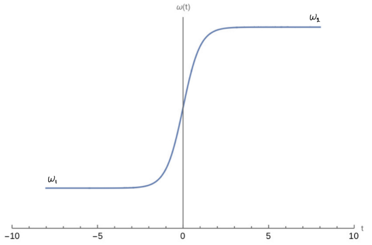

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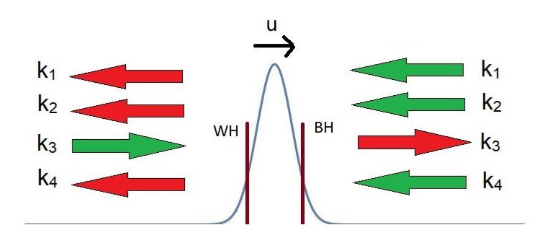
Time evolution:

$$e^{-i\,\omega_1\,t} \rightarrow \alpha e^{-i\,\omega_2\,t} + \beta e^{+i\,\omega_2\,t}, \quad \hat{a}_1 \rightarrow \alpha^* \hat{a}_2 - \beta^* \hat{a}_2^\dagger \quad \rightarrow \quad \left\langle 0\right| \hat{a}_2 \hat{a}_2^\dagger \left|0\right\rangle_2 = |\beta|^2$$

White hole-Black hole scattering



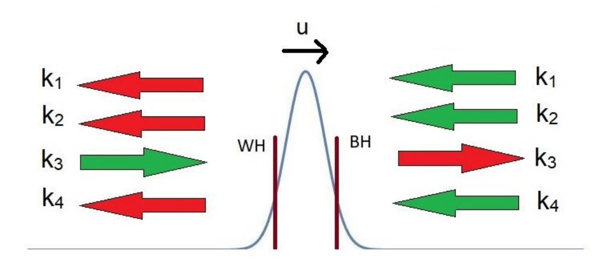
In the asymptotic regions ($\chi \to \pm\infty$):

$$\hat{A}(\chi,\tau) = \int d\omega \sum_{j=1}^{4} \left(\hat{a}_{j,\omega} e^{ik_j(\omega)\chi} e^{-i\omega\tau} + \hat{a}^{\dagger}_{j,\omega} e^{-ik_j(\omega)\chi} e^{+i\omega\tau} \right)$$

$$\hat{a}_i^{out} = \sum_{j}^{4} \left(\alpha_{ij} \hat{a}_i{}^{in} + \beta_{ij} \hat{a}_j{}^{in\,\dagger} \right)$$

Hawking radiation

Hawking radiation is emitted via the particle creation process at the analog BH horizon.



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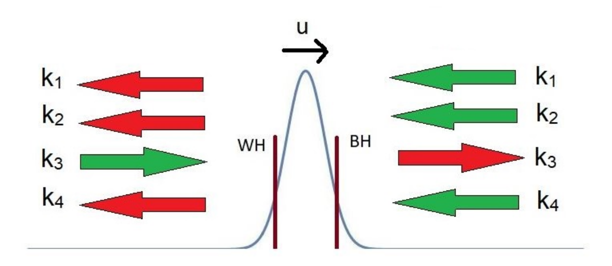

$$\hat{a}_3^{out} = \beta_{31}\left(\hat{a}_1^{in}\right)^\dagger + \alpha_{32}\,\hat{a}_2^{in} + \alpha_{33}\,\hat{a}_3^{in} + \alpha_{34}\,\hat{a}_4^{in}$$

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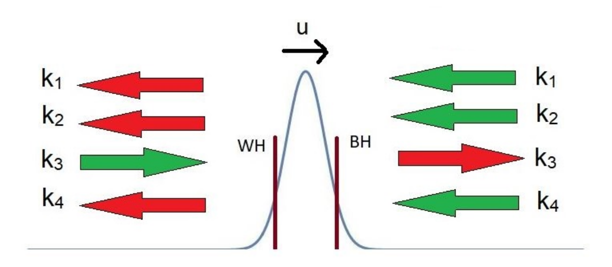

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$$\left\langle \hat{N}_3^{out} \right\rangle = \langle 0|\left(\hat{a}_3^{out}\right)^\dagger \hat{a}_3^{out}|0\rangle_{in} = |\beta_{31}|^2$$

Particle creation from quantum nothing!

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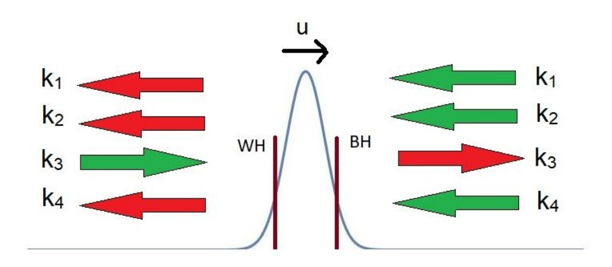

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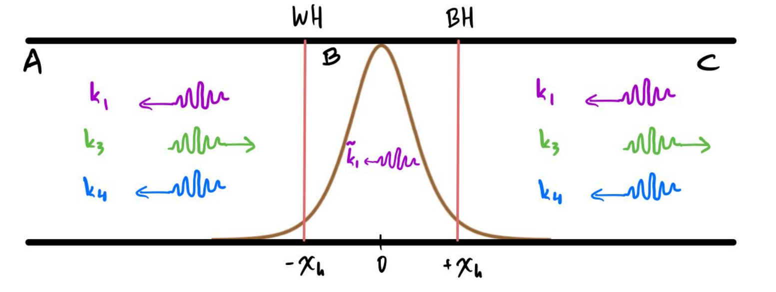
Particle creation from quantum nothing!

$$|\beta_{31}|^2 = \frac{1 - f(\omega)}{e^{\omega/T_H} - 1}, \qquad T_H = \frac{u\xi}{2\pi}$$

Linder, Schützhold, Unruh, 2018

Modes and emergent causal structure

- In the comoving frame (χ, τ) w.r.t to the strong pulse, the system has time symmetry $\rightarrow$ Conservation of frequency.
- For $|\chi| > \chi_\text{h}$: 4 modes (1 right-moving and 3 left-moving).
- For $|\chi| < \chi_\text{h}$: 2 modes (both left-moving).



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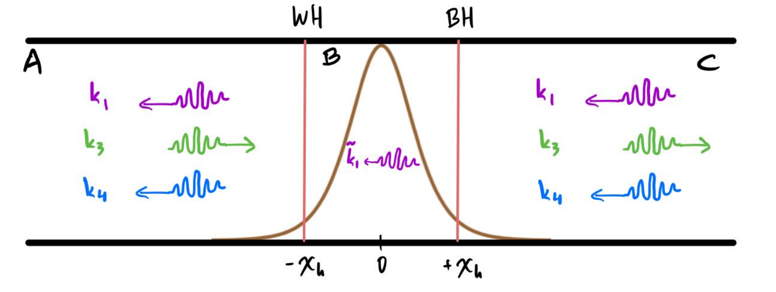
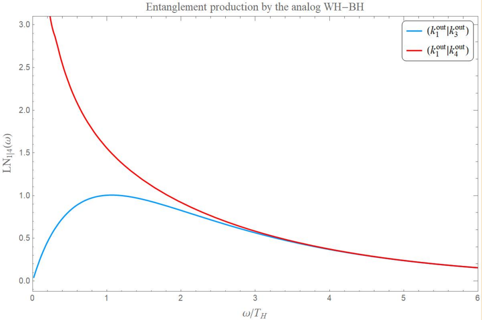


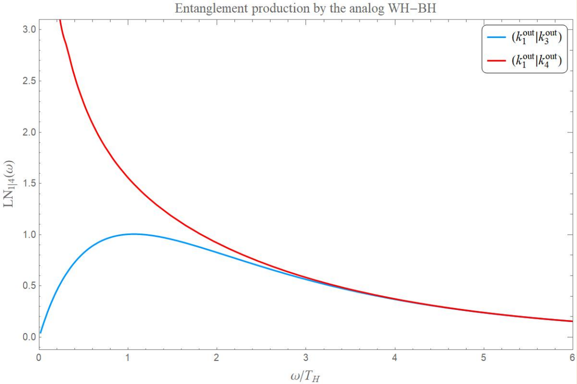
- Hawking particle creation at the WH and BH horizons out of vacuum fluctuations!

Gaussian states and Logarithmic Negativity

- We work with the family Gaussian states, where all the information about the quantum state is encoded in the first moments $\boldsymbol{\mu} = \langle \boldsymbol{A} \rangle$ and the covariance matrix $\sigma = \langle \{\boldsymbol{A} - \mu, \boldsymbol{A} - \mu\} \rangle$, where $\boldsymbol{A} = (\hat{a}_1, \hat{a}_1^\dagger, \hat{a}_2, \hat{a}_2^\dagger, \hat{a}_3, \hat{a}_3^\dagger, \hat{a}_4, \hat{a}_4^\dagger)^T$.
- The evolution of the state is given by: $\boldsymbol{A}_{out} = S \cdot \boldsymbol{A}_{in}$, $\boldsymbol{\mu}_{out} = S \cdot \boldsymbol{\mu}_{in}$, $\sigma_{out} = S \cdot \sigma_{in} \cdot S^T$.
- We use Logarithmic negativity (LN), a well-known entanglement monotone in quantum information theory, to study and quantify the entanglement produced in the Hawking process. LN can be easily computed from the first and second moments.

Entanglement structure of the analog WH-BH





- The BH and WH horizons behave as frequency-dependent two-mode squeezers generating entanglement in the modes $(k_1^{out}|k_3^{out})$ and $(k_1^{out}|k_4^{out})$, respectively.
- At low frequencies: $LN_{1|4} > LN_{1|3}$.
- At larger frequencies: $LN_{1|4} \approx LN_{1|3}$.

Generation of entanglement from vacuum fluctuations

- To study the evolution, we use Gaussian states and we quantify entanglement by means of Logarithmic Negativity (LN).
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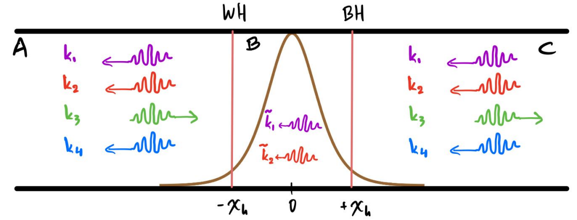
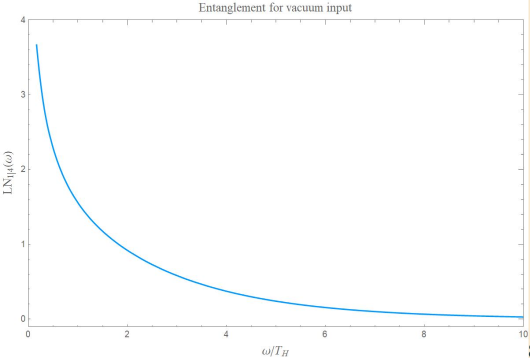




Observations?

- T. G. Philbin et al, (2008), Science 319, 1367.
- S. Weinfurtner et al, (2011), Phys. Rev. Lett. 106, 021302
- J. Steinhauer, (2016), Nature Phys. 12, 959-965.
- J. Drori et al, (2019), Phys. Rev. Lett. 122, 010404.

Successes

- Generation of horizons.
- Particle production via the stimulated process.

Open questions

- Observation of a blackbody spectrum (dispersion the main obstacle and absence of perfect horizons).
- Generation of entanglement (fragile to background thermal noise).

The protocol to extract T_H from observations

Intensities (classical signal)

- $\langle \hat{n}_3(\omega) \rangle = A(\omega)(e^{\omega/T_H} - 1)^{-1}$

Entanglement (quantum signal)

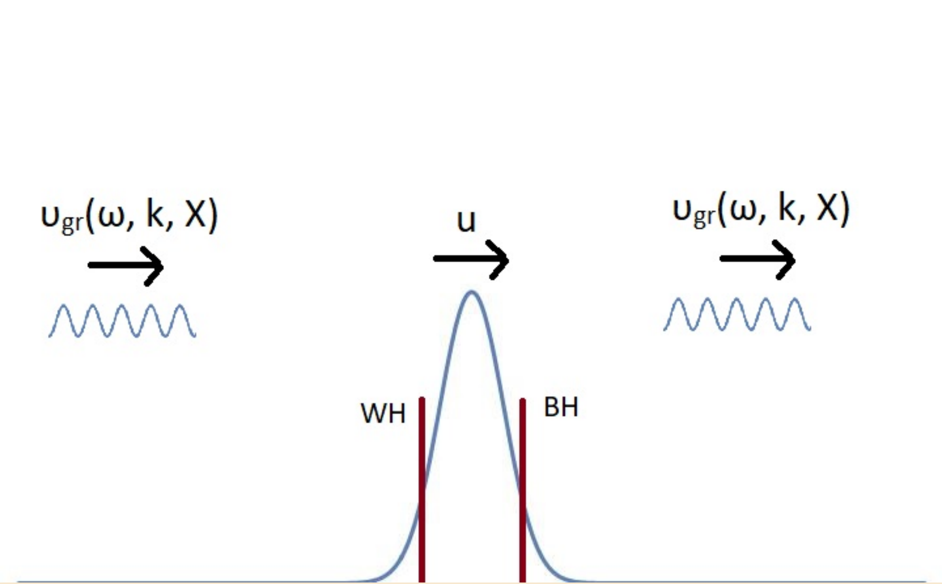
$$LN_{1|4}(\omega) = -\log_2 \left\{ \frac{1}{4} \Big[7 - 4\cosh 2r(\omega) + 8\cosh 4r(\omega) + 4\cosh 6r(\omega) + \cosh 8r(\omega) - 16\cosh^2 r(\omega) \cosh 2r(\omega)^{3/2} \left(9 + 6\cosh 2r(\omega) + \cosh 4r(\omega)\right)^{1/2} \sinh r(\omega) \Big]^{1/2} \right\}$$

- Measure all mode correlations $\langle \{\hat{a}_i(\omega)\hat{a}_j(\omega)\} \rangle$
- Construct $LN_{1|4}(\omega)$
- Obtain T_H from $LN_{1|4}(\omega)$

Take home messages

- The black hole causal structure can be reproduced in dispersive, inhomogeneous media.
- Frequency conservation and dispersion single out a finite number of degrees of freedom interacting with each other, allowing us to import theoretically rigorous and experimentally accessible tools from quantum information theory to study the entanglement in the Hawking process.
- Stimulating the process with a single-mode squeeze state is a promising strategy for the detection of the Hawking effect as it enhances both the Hawking intensity and the entanglement generated by the Hawking process.

Optical black holes: Schematic representation



The effect of background noise

- As input states, we restrict ourselves to the family of Gaussian quantum states and we quantify entanglement by means of Logarithmic Negativity (LN).
- We assume that the input modes are in thermal equilibrium with a blackbody bath of "environment" photons.

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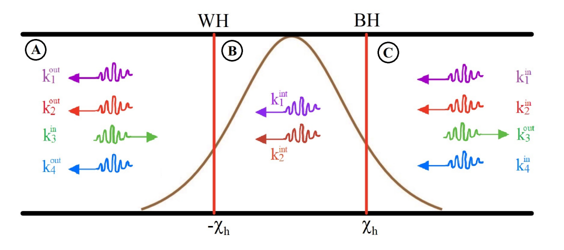
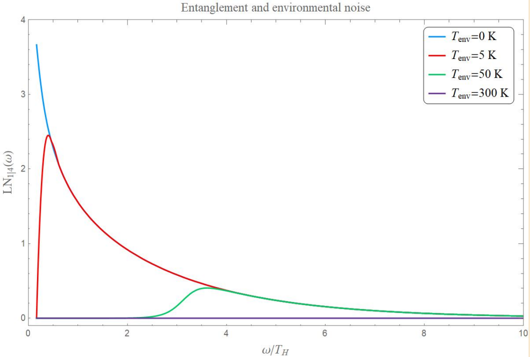
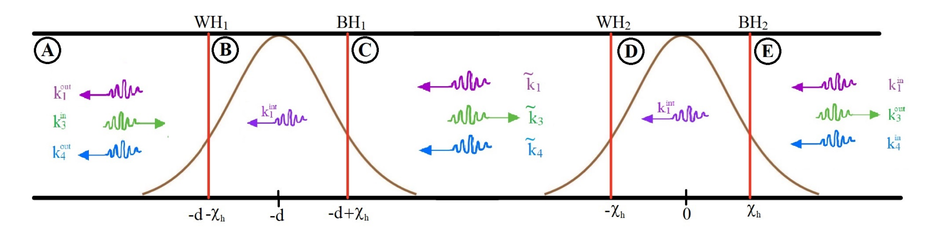




- We find that the presence of the thermal background degrades the entanglement generated in the Hawking process.

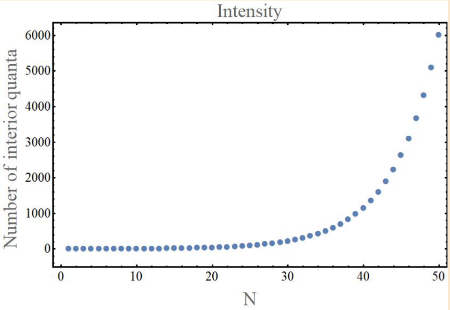
Hawking laser effect

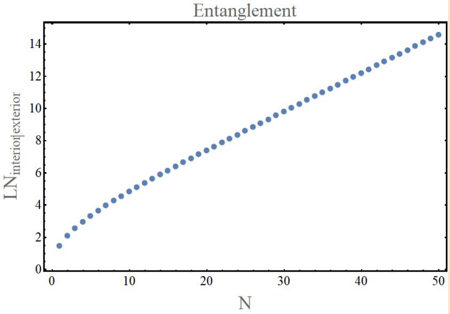
The setup



- Consider the configuration of two strong electric pulses each reproducing an analog white-black hole.
- The BH_1 horizon and the BH_2 horizons exchange Hawking quanta stimulating each other.
- We numerically solve the scattering problem and compute intensities and entanglement.

Intensity and entanglement in the optical laser setup





- The intensity of the trapped mode increases exponentially in time (manifestaton of the lasing effect). [U. Leonhardt and T. G. Philbin (2008), S. Finazzi and R. Parentani (2010), A. Coutant and R. Parentani (2010), D. Bermudez and U. Leonhardt (2018), H. Katayama (2021)] .
- In addition, we find that the entanglement shared between the interior and the exterior modes increases linearly in time.
- Laser configuration behaves as an entanglement factory!!!

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