Optical analogue of the Schwarszchild-Planck metric

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Analogue Gravity in 2023

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Introduction

$$
\begin{tikzcd}
\text{Black mirror trajectroy} \arrow[d, "\text{New trajectory}"'] & \text{Schwarszchild metric} \arrow[l, "\text{Collapse condition}"'] \arrow[r, "\text{Surface gravity or geodesics}"] & \text{Optical Schwarszchild metric} \\
\text{Quantum pure black mirror trajectory} \arrow[r, "\text{Collapse condition}"'] & \text{Schwarszchild-Planck metric} \arrow[r, "\text{Surface gravity or geodesics}"'] & \text{Optical Schwarszchild-Planck metric} \\
\text{Fulling-Davis effect} & \text{Hawking effect} & \text{Analogue Hawking effect}
\end{tikzcd}
$$

Schwarzschild metric

The Schwarzschild metric is

$$\mathrm{d}s^2 = f(r)c^2\mathrm{d}t^2 - f^{-1}(r)\mathrm{d}r^2, \qquad f(r) = 1 - \frac{r_s}{r}, \tag{1}$$

where $r_s = 2GM/c^2$ is the Schwarzschild radius. This metric describes the spacetime around a non-rotating spherical black hole.
Due to the Hawking effect, a black hole radiates eternally as a black body with temperature

$$T = \frac{\hbar c^3}{8\pi GMk_B}. \tag{2}$$

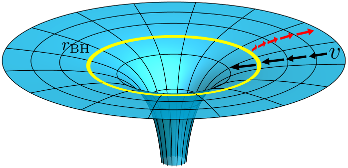

Figure 1: Schwarzschild spacetime.

Fulling-Davis effect

Fulling and Davis considered a massless scalar field in the spacetime created by an accelerating mirror and a detector in the late infinity that measures the resulting emission at the temperature

$$T = \frac{\hbar a}{2\pi c k_B}. \tag{3}$$

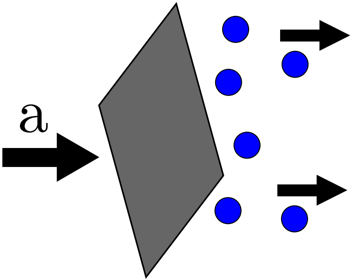

Figure 2: Accelerating mirror that reflects particles from the vacuum.

Black mirror trajectory

The black mirror is a trajectory that recreates the collapse condition for a spherical shell of matter in Schwarzschild spacetime. In this way, the emission due to F-D effect is the same as a Schwarzschild black hole with infinite energy and particles emitted.

The trajectory of the black mirror in null coordinates is

$$u_b(v) = v - \frac{c}{a} \ln\left(\frac{av}{c}\right), \qquad (4)$$

where $u_b = t - x/c$ is the retarded time as a function of the advanced time $v = t + x/c$ and a is the acceleration of the mirror.

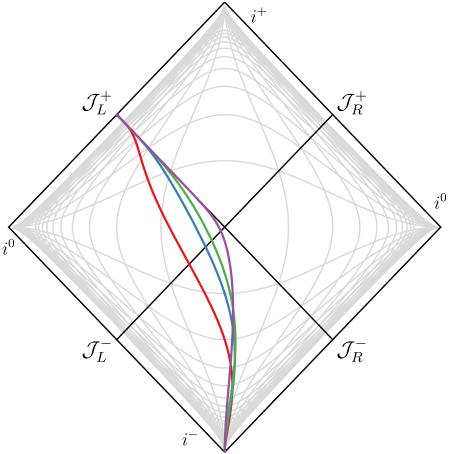

Figure 3: Black mirror trajectory in a Penrose diagram.

Quantum-pure black mirror trajectory

The quantum-pure black mirror trajectory is a trajectory that solves the infinite emission problem of the Schwarzschild metric.
The trajectory is

$$u_p(v) = v - \frac{c}{a} \sinh^{-1} \left| \frac{gv}{c} \right|, \quad (5)$$

where u_p is the new retarded time (subindex p refers to pure).
Now there is a new free parameter g written in terms of a new length scale $g = c^2/(2\ell)$.

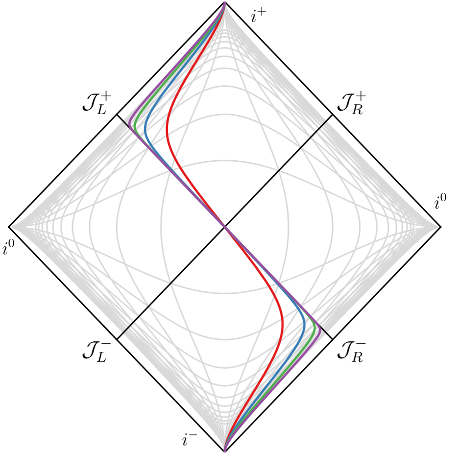

Figure 4: Quantum-pure black mirror trajectory in a Penrose diagram.

Schwarzschild-Planck metric

This metric is obtained from the new collapse condition as

$$ds^2 = \bar{f}(r)c^2 dt^2 - \bar{f}^{-1}(r)dr^2, \qquad \bar{f}(r) = 1 - \frac{r_s}{r_s + \sqrt{(r - r_s)^2 + \ell^2}}. \quad (6)$$

The original Schwarzschild metric is recovered in the limit $\ell \to 0$.

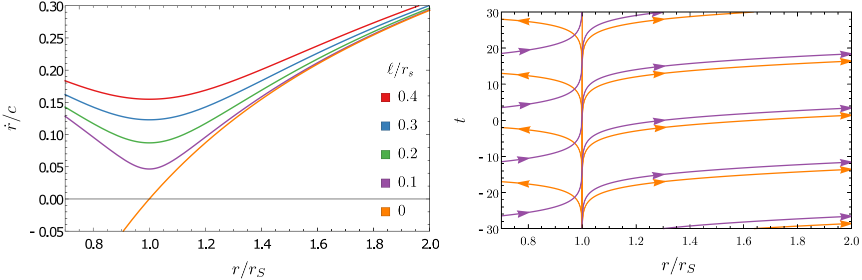

Figure 5: Coordinate velocity and light trajectories around the Schwarzschild radius.

Gravity in (1+1)D

Any stationary system in (1+1)D can be written in orthogonal coordinates through the Gram–Schmidt process using an independent field $\alpha(x)$, as

$$\mathrm{d}s^2 = \alpha(x) c^2 \mathrm{d}t^2 - \alpha(x)^{-1} \mathrm{d}x^2, \tag{7}$$

where t is time and x is position.
All geometric quantities are expressed as function of $\alpha(x)$ or its derivatives. We are interested in the surface gravity

$$\kappa(x) = \left| \frac{c^2}{2} \alpha'(x) \right|. \tag{8}$$

One of the most remarkable results in (1+1)D gravity is the fact that the Einstein tensor G is always zero

$$G_{\mu\nu} = R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = 0. \tag{9}$$

Optical analogue metric

The metric for a probe field travelling in a dielectric medium in the comoving frame is

$$\mathrm{d}s^2 = \left(\frac{u^2 n(\tau)^2}{c^2} - 1\right) c^2 \mathrm{d}\zeta^2 - \left(\frac{u^2 n(\tau)^2}{c^2} - 1\right)^{-1} c^2 \mathrm{d}\tau^2, \tag{10}$$

where ζ and τ are the propagation and delay time, respectively.
From where we can directly calculate the surface gravity

$$\kappa(\tau) = \left| \frac{c}{2} \frac{\mathrm{d}}{\mathrm{d}\tau} \left[\frac{u^2 n(\tau)^2}{c^2} - 1 \right] \right| . \tag{11}$$

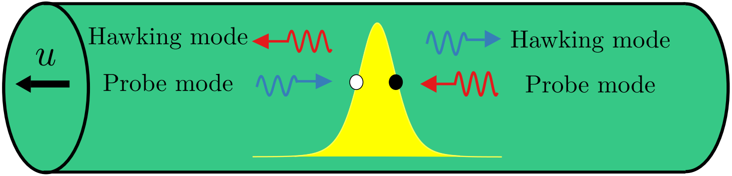

Figure 6: Pump field travelling through an optical fiber in the comoving frame.

Matching the surface gravity in both systems given by (8) and (11), we obtain the following relation for the refraction index

$$n(\tau) = \frac{c}{u}\sqrt{\frac{\tau_s}{\tau}}, \qquad n(\tau) = \frac{c}{u}\sqrt{\frac{\tau_s}{\tau_s + \sqrt{(\tau-\tau_s)^2 + \tau_\ell^2}}}. \tag{12}$$

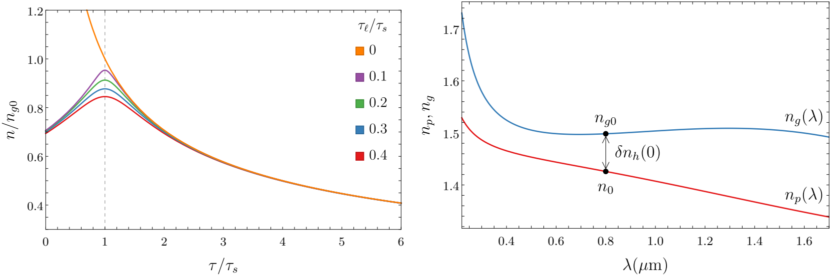

Figure 7: (a) Refractive index that with different values of τ_ℓ/τ_s, delay is in units of horizon τ_s. (b) Refractive (phase) index (red) and group index (blue) as function of wavelength (λ) for a fiber used in recent optical analogue experiments.

Quasi-thermal spectrum

In the limit $\tau_\ell \ll \tau_s$, we obtain the spectrum

$$N_\omega = \frac{1}{e^{\omega/T} - 1} \times \left[\frac{1}{\pi} \ln\left(\frac{1}{\omega\tau_\ell}\right) + \frac{1}{\pi}\mathrm{Re}(H_{i\omega}) - \frac{\gamma+1}{\pi} + \mathrm{Re}\left(\frac{(\omega\tau_\ell)^{2i\omega}\mathrm{csch}(\pi\omega)}{4(2\omega+i)\Gamma(1+i\omega)^2} \right) \right], \tag{13}$$

where $T = 1/2\pi$, H_n is the harmonic number, $\gamma \simeq 0.577$ is Euler's constant, and Γ is the gamma function.

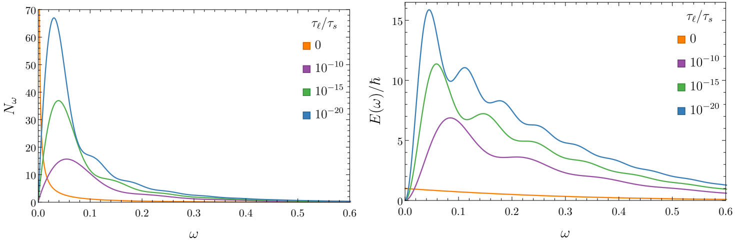

Figure 8: Particle spectrum N_ω and energy distribution $E(\omega)$.

Flux energy

The quantum stress-energy tensor is obtained by considering a massless scalar field in the curved spacetime.

$$\langle T_{uu}(U)\rangle = \frac{\hbar\tau_s}{12\pi}\frac{\tau_s + 2U}{(2\tau_s - U)^4} + O(\tau_\ell^2). \tag{14}$$

And total energy is

$$E = \frac{\hbar}{48\pi\tau_s}\ln\frac{2\tau_s}{\tau_\ell} + O(\tau_\ell^2). \tag{15}$$

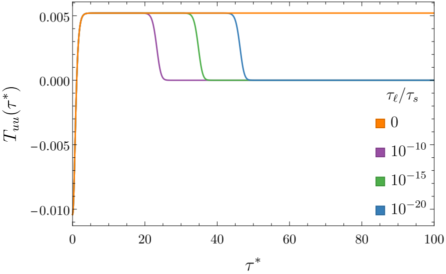

Figure 9: The energy flux is illustrated in tortoise coordinates.

Conclusions

- We related spacetime metrics with the pump field shape in the optical analogue. We applied the result for Schwarzschild and Schwarszchild-Planck metrics.
- The experiments of optical analogues have been performed using ultra-short pulses with a duration of $\sim$10-100 fs. If our hypothesis is correct and the microscopic time-scale of the optical metric is $\sim$1 ps, it means that these experiments would be in the regime of microscopic black-hole analogues.

Future work

- We mainly ignored the dispersion of the fiber, since we fixed the frequency of the pump wave, but this deserves further study as the dispersion also modifies the thermality of the analogue Hawking spectrum.
- A future perspective is to use the theory we developed here to invert the process and to find trajectories of accelerating mirrors that recreate commonly used shapes in optical and acoustic experiments.

Thank you for your time and attention

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