Evolution of entanglement in an analogue preheating experiment

Analogue Gravity in Benasque - 2023

A. Micheli and S. Robertson, Phys. Rev. B **106**, 214528 (2022).

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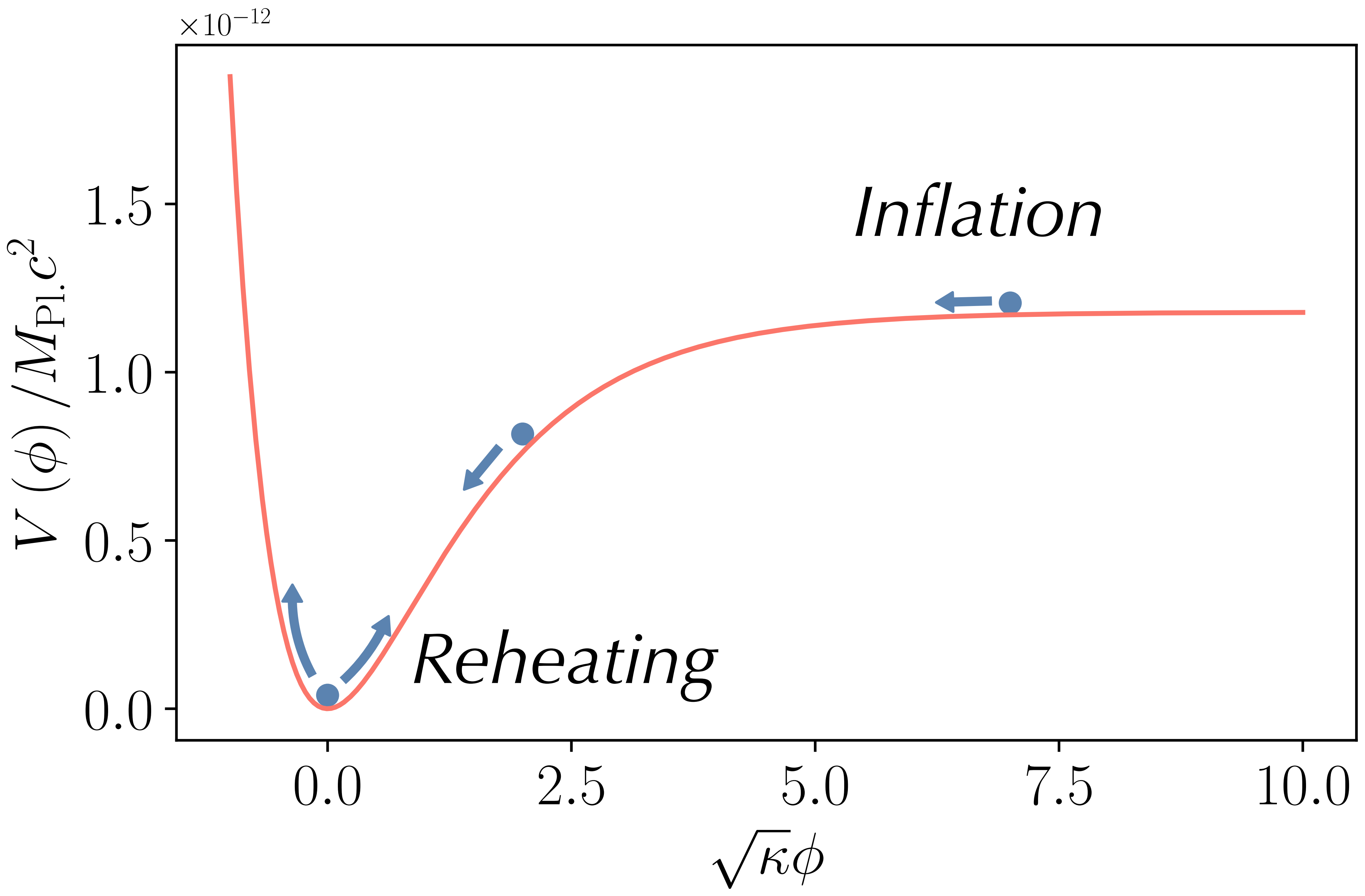
Introduction: Preheating



Inflation: a single field ϕ
and its perturbations $\delta\hat{\phi}$

Particles $\hat{\chi}$ produced during reheating via:

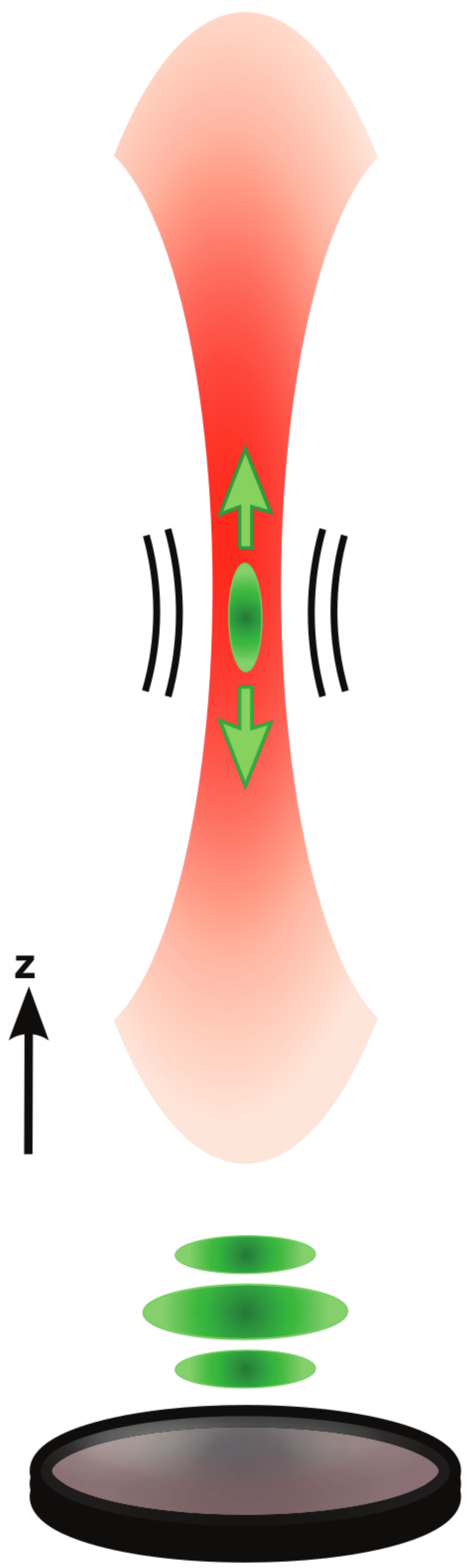
- Decay of inflaton $\phi \rightarrow \chi$
- Preheating[1]: parametric creation out of the vacuum.

Several stages: initial growth, saturation, redistribution, thermalisation.

Analogue of preheating?

1. L. Kofman, A. Linde, and A. Starobinsky, *Towards the Theory of Reheating After Inflation*, PRD (1997).

Analogue preheating with 1D Bose gas - I



Gas of cold He atoms whose trap is modulated[2] e.g.

$$\omega_\perp^2(t) = \omega_{\perp,0}^2\left[1 + A\cos(\omega_m t)\right]$$

effectively modulating speed of sound $c(t)$

→ Parametric creation of phonons $n_{\pm k}$ at $2\omega_k = \omega_m$ out of the vacuum (similar to DCE)

Signature of vacuum origin: pairs are entangled!

This talk: focus on initial pair creation and (early-times) dissipative processes[3]

2. Jaskula et al. *Acoustic Analog to the Dynamical Casimir Effect in a Bose-Einstein Condensate*, PRL (2012).
3. S. Robertson, F. Michel, and R. Parentani, *Nonlinearities Induced by Parametric Resonance in Effectively 1D Atomic Bose Condensates*, PRD (2018).

Analogue preheating with 1D Bose gas - II

How to experimentally access entanglement?

$$g^{(2)}_{(k,-k)} = \frac{\langle \hat{n}_k \hat{n}_{-k} \rangle}{\langle \hat{n}_k \rangle \langle \hat{n}_{-k} \rangle} > 2$$

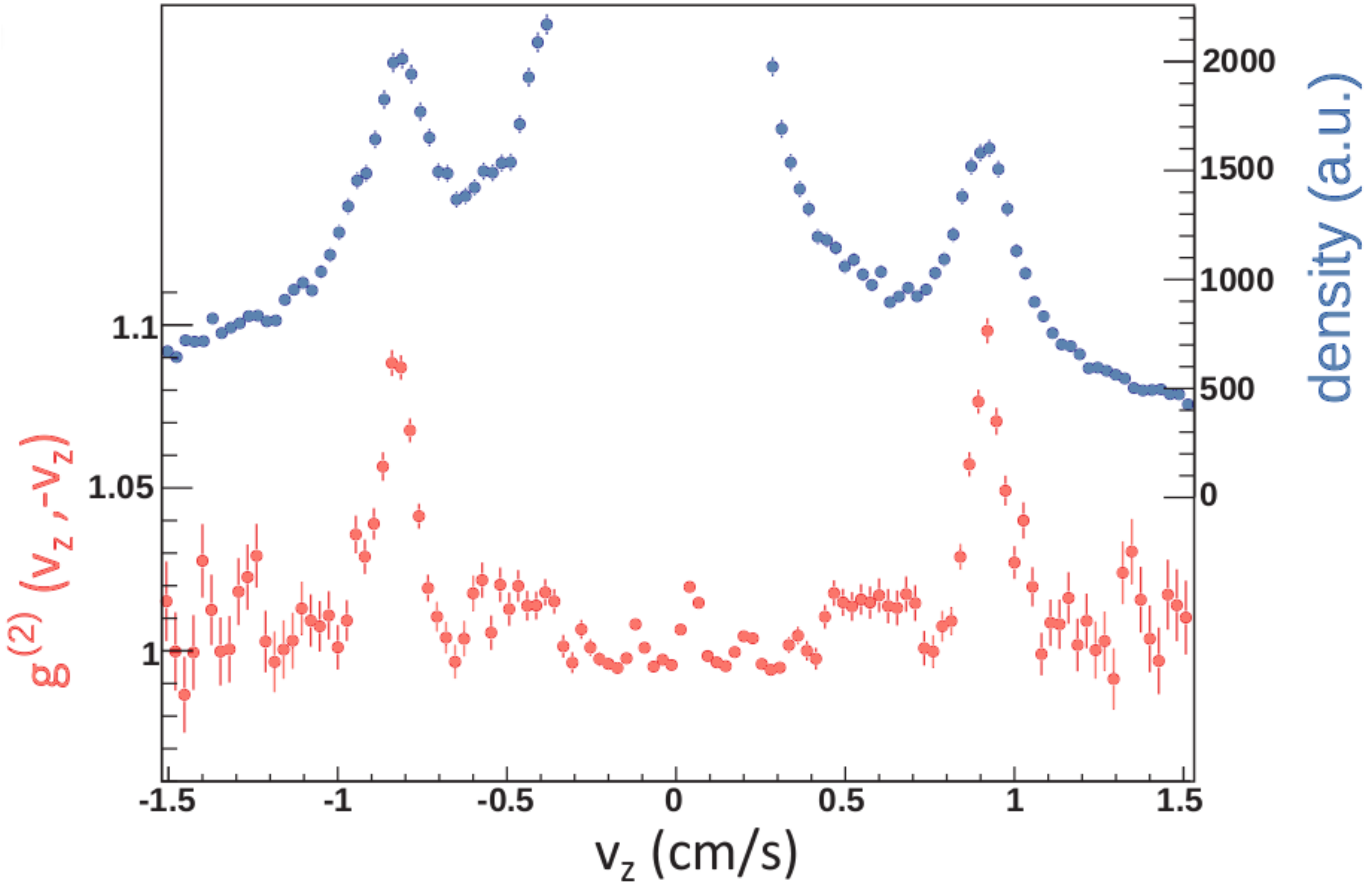
Original experiment failed to witness it[2].

Why?

A sufficient degree of interaction could explain the absence of entanglement[3].

↓

Modeling quasi-particle interactions in 1D Bose gas



2. Jaskula et al. *Acoustic Analog to the Dynamical Casimir Effect in a Bose-Einstein Condensate*, PRL (2012).
3. X. Busch, R. Parentani, and S. Robertson, *Quantum Entanglement Due to Modulated Dynamical Casimir Effect*, PRA (2014).

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I - Modeling 1D Bose Gas

Modeling evolution of quasi-particles

1D Bose gas time-dependent interaction $g(t)$, related to $\omega_\perp(t)$

$$\hat{H} = \int_0^L \left[\frac{\hbar^2}{2m} \partial_x \hat{\Psi}^\dagger \, \partial_x \hat{\Psi} + \frac{g(t)}{2} \hat{\Psi}^{\dagger 2} \hat{\Psi}^2 \right] \mathrm{d}x$$

Write in Madelung form

$$\hat{\Psi} = e^{i\hat{\theta}} \sqrt{\hat{\rho}}$$

$\longrightarrow$ (up to commutators…)

$$\hat{H} = \int_0^L \left[\frac{\hbar^2}{2m} \frac{\partial \hat{\theta}}{\partial x} \hat{\rho} \frac{\partial \hat{\theta}}{\partial x} + \frac{\hbar^2}{8m\hat{\rho}} \left(\frac{\partial \hat{\rho}}{\partial x} \right)^2 + \frac{g}{2} \hat{\rho}^2 \right] \mathrm{d}x$$

Homogeneous stationary background: $\hat{\rho} = \rho_0 \left(1 + \frac{\delta \hat{\rho}}{\rho_0} \right)$ and $\hat{\theta} = -g\rho_0 t/\hbar + \delta\hat{\theta}$

Expand Hamiltonian $\hat{H}(t) = E_0(t) + \hat{H}_2(t) + \hat{H}_3 + \ldots$

Defines quasi-particles

Interaction of quasi-particles

II - Free quasi-particles

Quadratic evolution of quasi-particles - I

$$\hat{H}_2 = \int_0^L \left[\frac{\hbar^2 \rho_0}{2m} \left(\frac{\partial \delta\hat{\theta}}{\partial x} \right)^2 + \frac{\hbar^2}{8m\rho_0} \left(\frac{\partial \delta\hat{\rho}}{\partial x} \right)^2 + \frac{g(t)}{2} \delta\hat{\rho}^2 \right] \mathrm{d}x$$ Quadratic Hamiltonian

For $g = \mathrm{cst.}$ it is diagonalised by defining quasi-particles

$$\hat{b}_k = \frac{1}{\sqrt{2}} \left(C_k^{-1}\, \delta\hat{\rho}_k + i\, C_k\, \delta\hat{\theta}_k \right)$$

$$\hat{H}_2 = \sum_{k \neq 0} \hbar\omega_k \hat{b}_k^\dagger \hat{b}_k$$ with $\omega_k^2 = c^2 k^2 + \frac{\hbar^2 k^4}{4m^2}$ and $c = \sqrt{\frac{g\rho_0}{m}}$

$\longrightarrow$ $\hat{b}_k(t) = e^{-i\omega_k t} \hat{b}_k(0)$ Free evolution

Quadratic evolution of quasi-particles - II

Make $g(t)$ time-dependent[3]

$$\partial_t \begin{pmatrix} \hat{b}_k \\ \hat{b}^\dagger_{-k} \end{pmatrix} = \begin{pmatrix} -i\omega_k(t) & \frac{\dot{\omega}_k}{2\omega_k} \\ \frac{\dot{\omega}_k}{2\omega_k} & i\omega_k(t) \end{pmatrix} \begin{pmatrix} \hat{b}_k \\ \hat{b}^\dagger_{-k} \end{pmatrix}$$

Mixing of creation / annihilation $\longrightarrow$ Quasi-particle creation

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Assume initial state to be thermal & linear evolution: state remains Gaussian homogeneous and isotropic.

- Quasi-particles are completely described by their number $n_k = \langle \hat{b}^\dagger_{\pm k} \hat{b}_{\pm k} \rangle$ and correlation $c_k = \langle \hat{b}_k \hat{b}_{-k} \rangle$

$$g^{(2)}_{(k,-k)} = 1 + \frac{|c_k|^2}{n_k^2}$$

- Entanglement of the pairs is equivalent to $|c_k| > n_k$

3. X. Busch, R. Parentani, and S. Robertson, *Quantum Entanglement Due to Modulated Dynamical Casimir Effect*, PRA (2014).

Quadratic evolution of quasi-particles - III

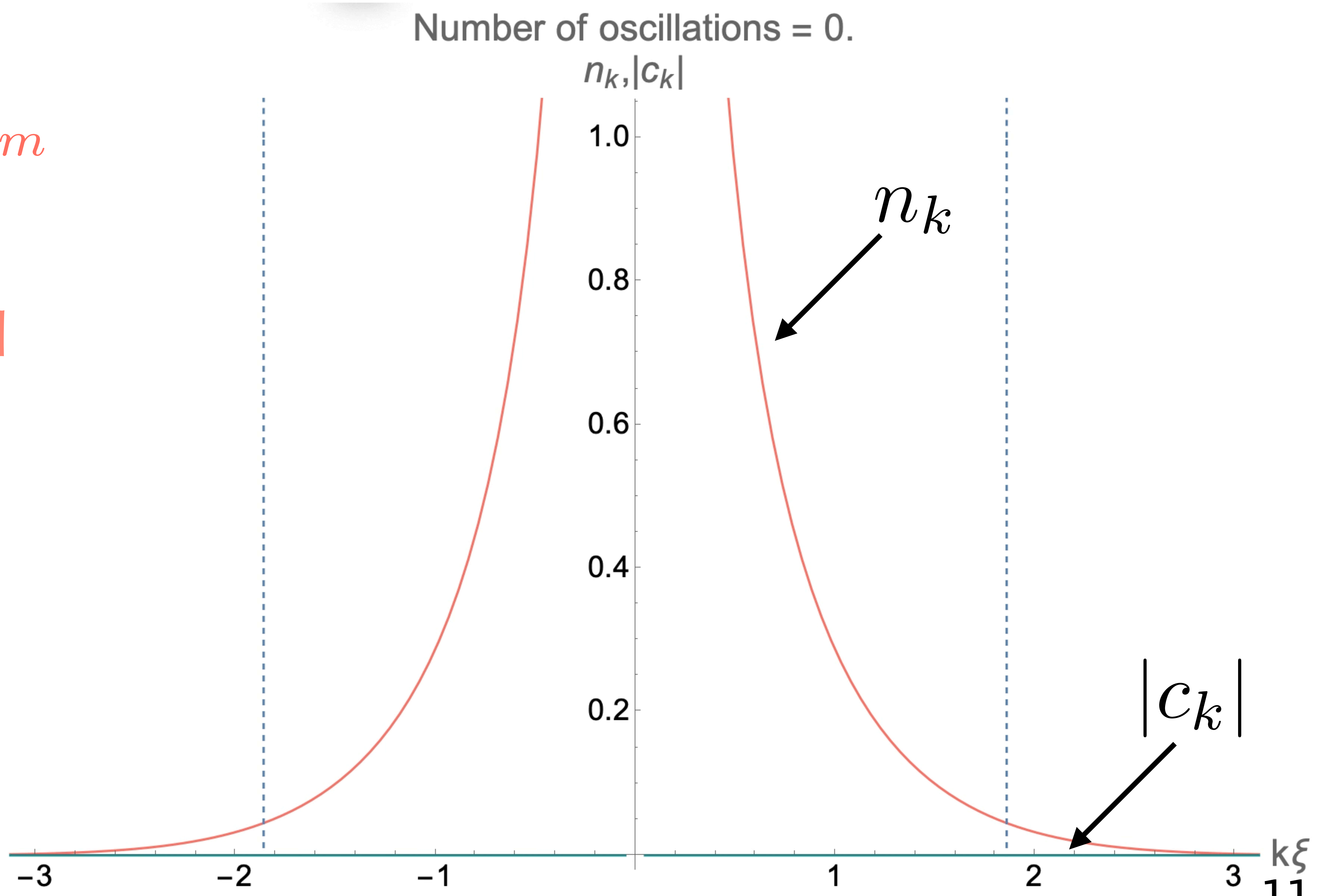
Modulating speed of sound $c^2(t) = c_0^2\left[1 + A\sin\left(\omega_m t\right)\right]$ $\longrightarrow$ Parametric resonance

Exponential growth[3] at $2\omega_k = \omega_m$

Create pairs, which are entangled

$$|c_k| > n_k$$

Can interactions change that?

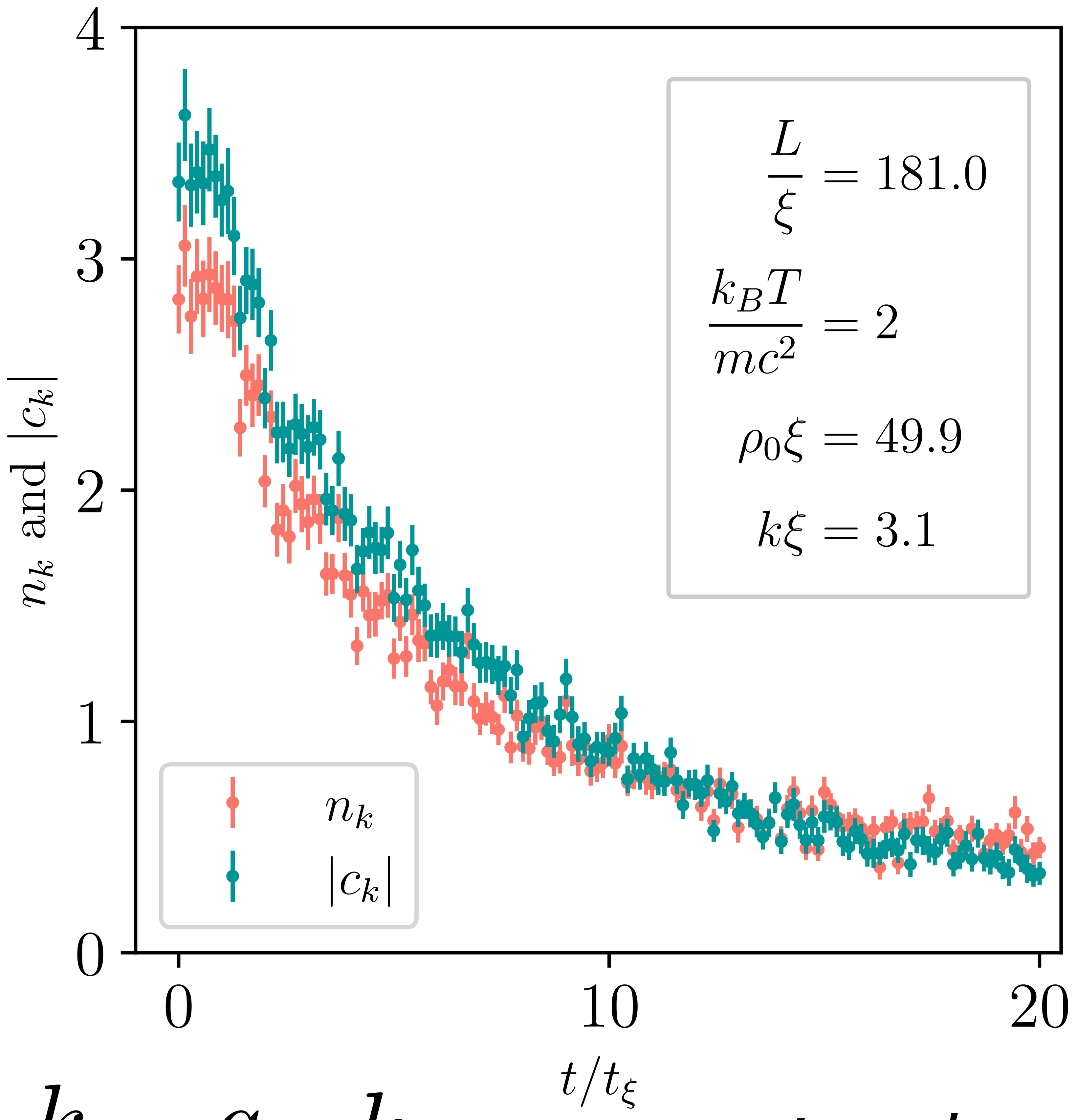


3. X. Busch, R. Parentani, and S. Robertson, *Quantum Entanglement Due to Modulated Dynamical Casimir Effect*, PRA (2014).

III - Quasi-particle interactions

Non-linear evolution of quasi-particles

TWA simulations of evolution from an initial entangled state in absence of modulation



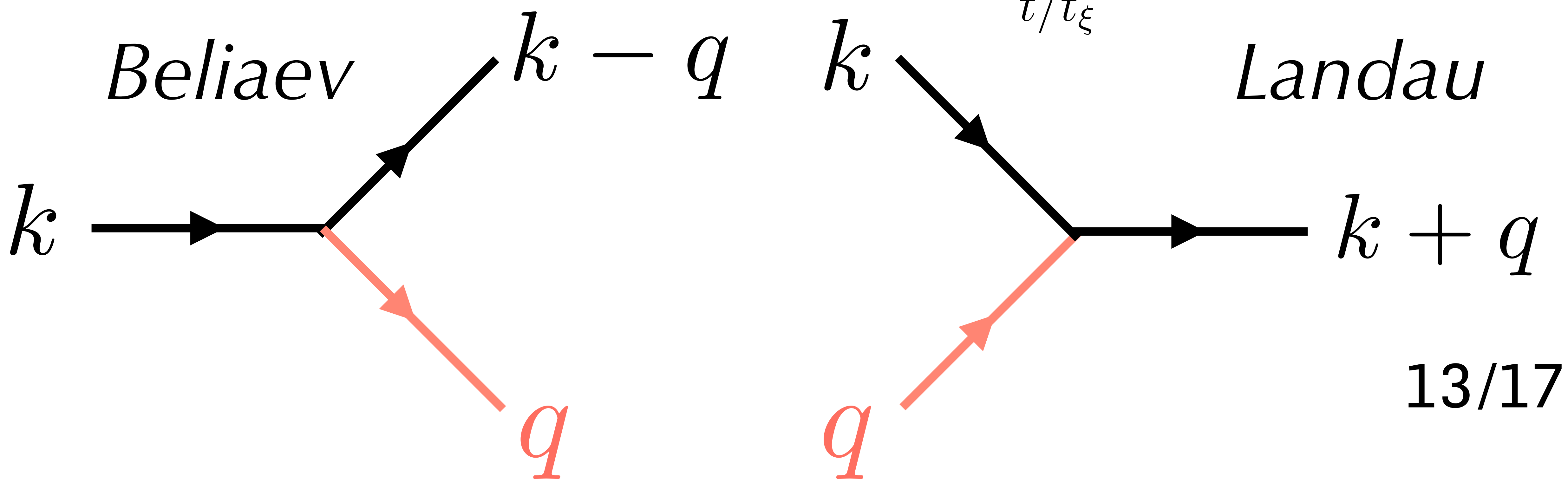
→ $|c_k|$ and n_k decrease exponentially.

→ During resonance expect a competition between decay and parametric growth

Can we explain it by considering $\hat{H}_3$?

$$\hat{H}_3 = \frac{1}{\sqrt{\rho_0 L}} \sum_{\substack{p,q\neq 0 \\ p+q\neq 0}} H_3(p,q) \left\{ \hat{b}_p^\dagger \hat{b}_q^\dagger \hat{b}_{p+q} + \hat{b}_{p+q}^\dagger \hat{b}_p \hat{b}_q \right\}$$

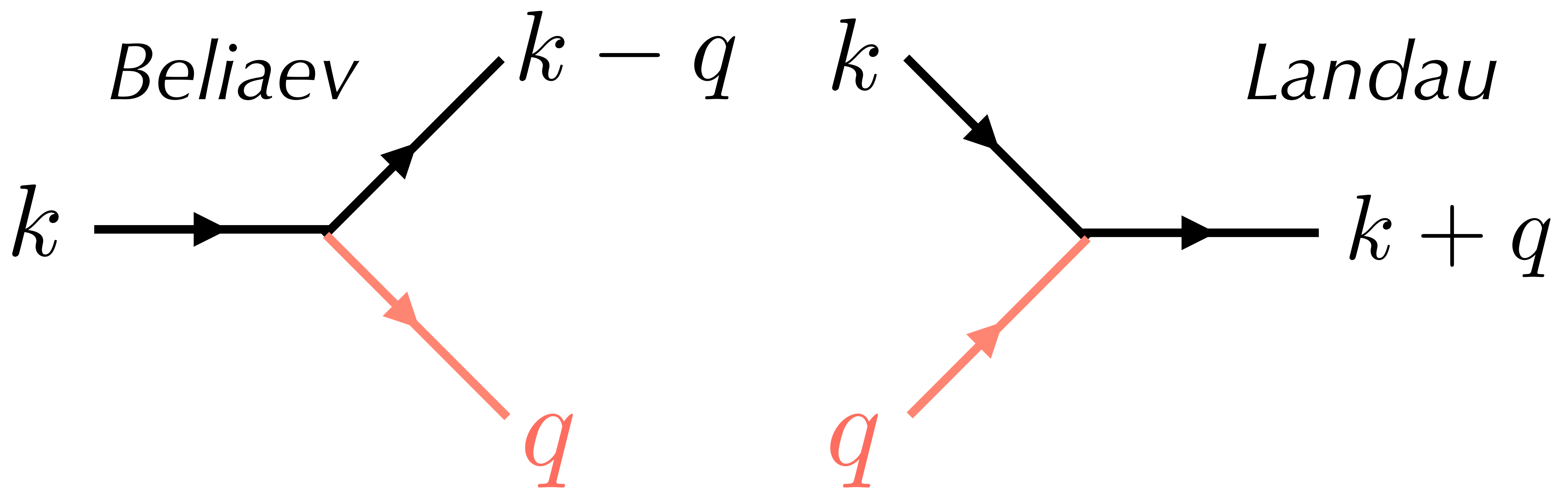
<u>3-body scattering of quasi-particles</u>

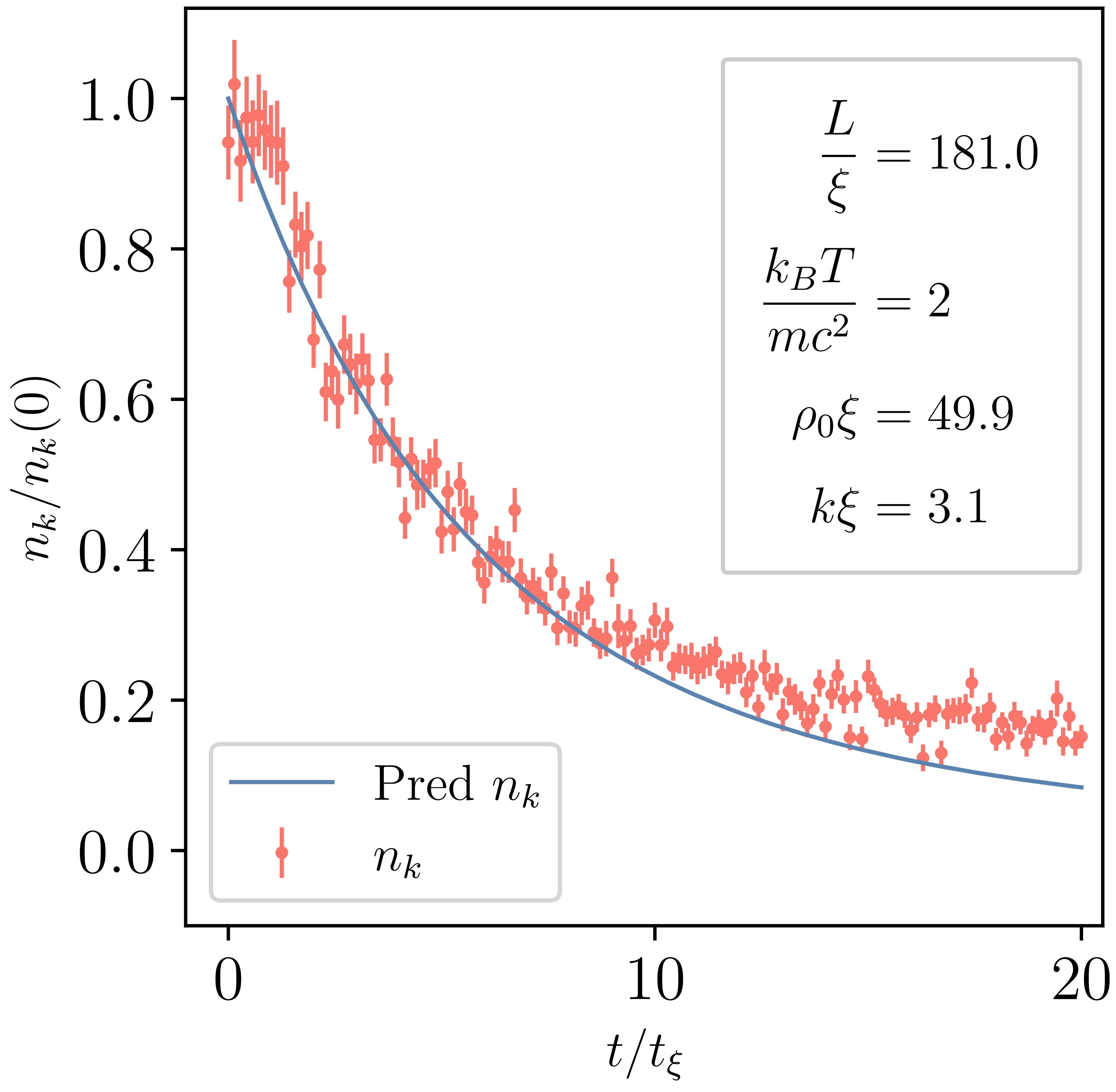


Non-linear evolution of n_k - I

$$\omega_k^2 = c^2k^2 + \frac{\hbar^2 k^4}{4m^2}$$

Energy-conserving for $q \sim 0$





Fermi Golden rule[4]

$$\Gamma_\pm(k) \propto \lim_{q\to 0^\pm} |H_3(k,q)|^2\, n_q$$

Finite! | Vanishes as q | Diverges as T/q

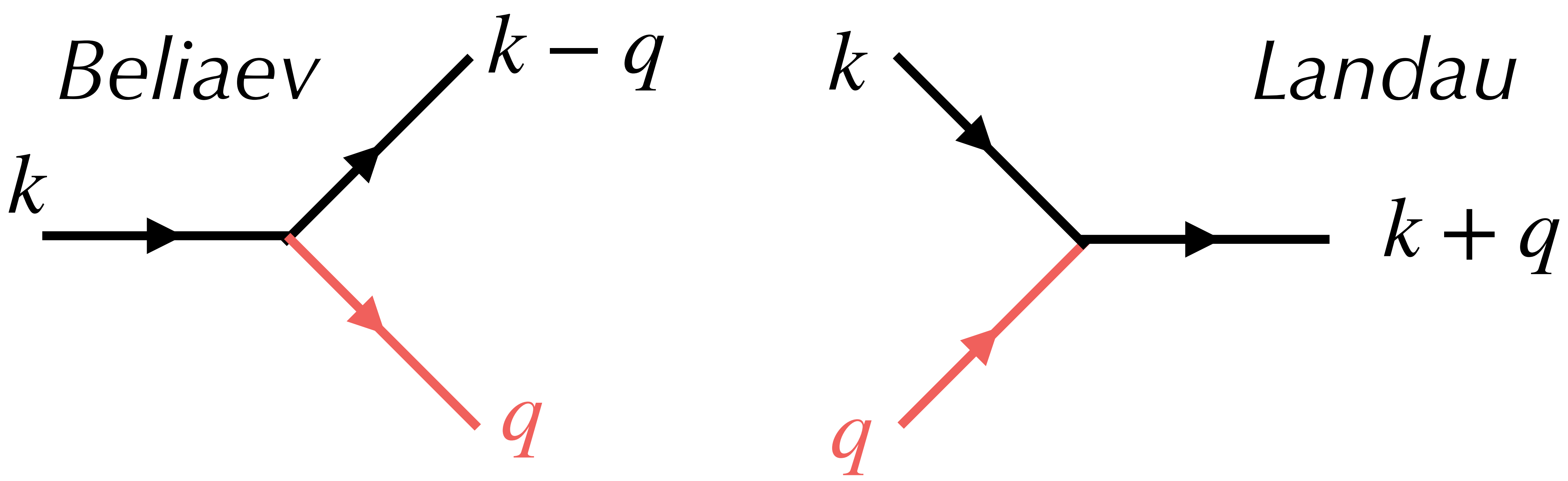
$$\Gamma_\pm(k) \propto \frac{k_BT}{mc^2}\frac{1}{\rho_0\xi}$$

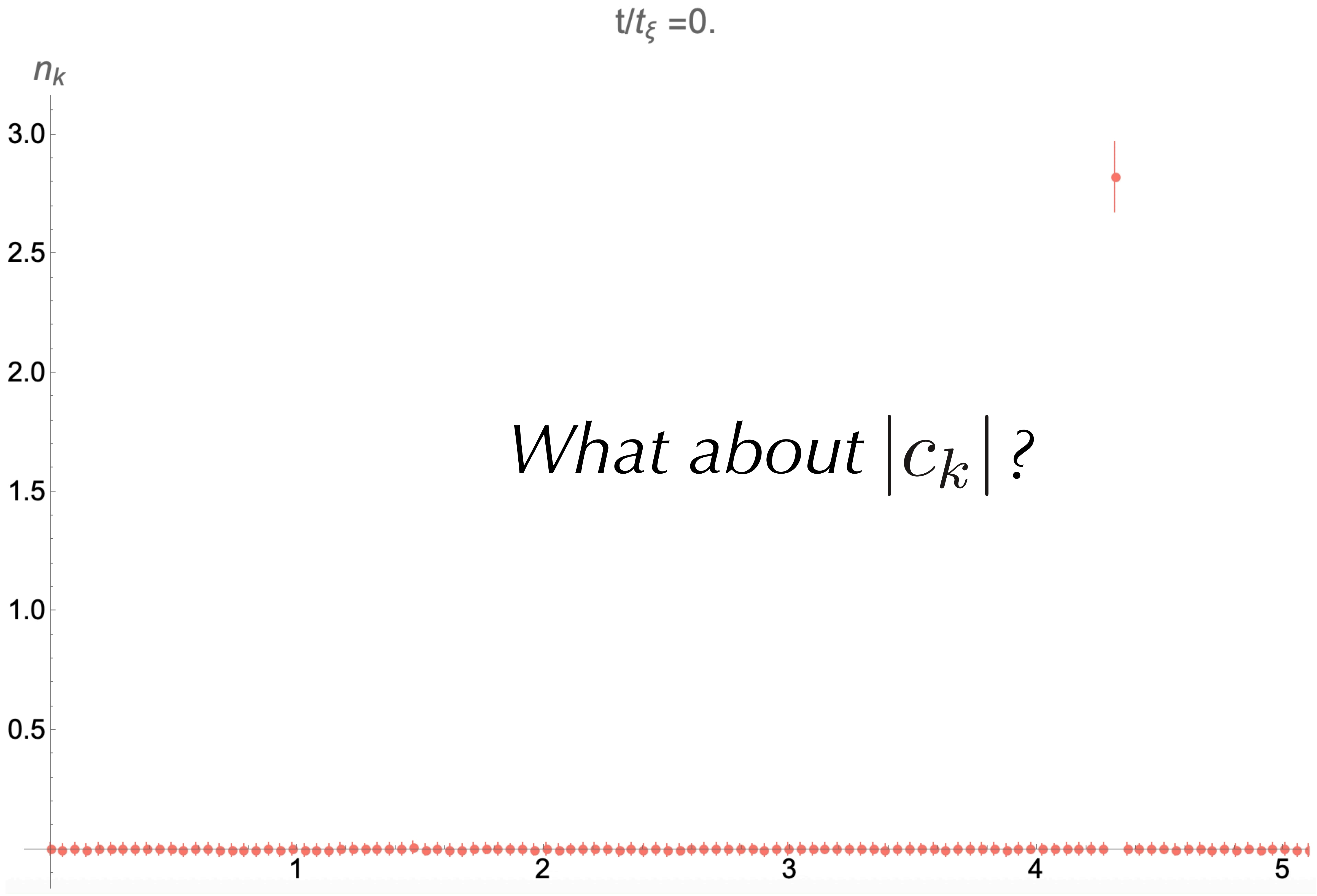
Matches well the simulations!

4. A. Micheli and S. Robertson, *Phonon Decay in 1D BEC via Beliaev-Landau Damping*, PRB (2022)

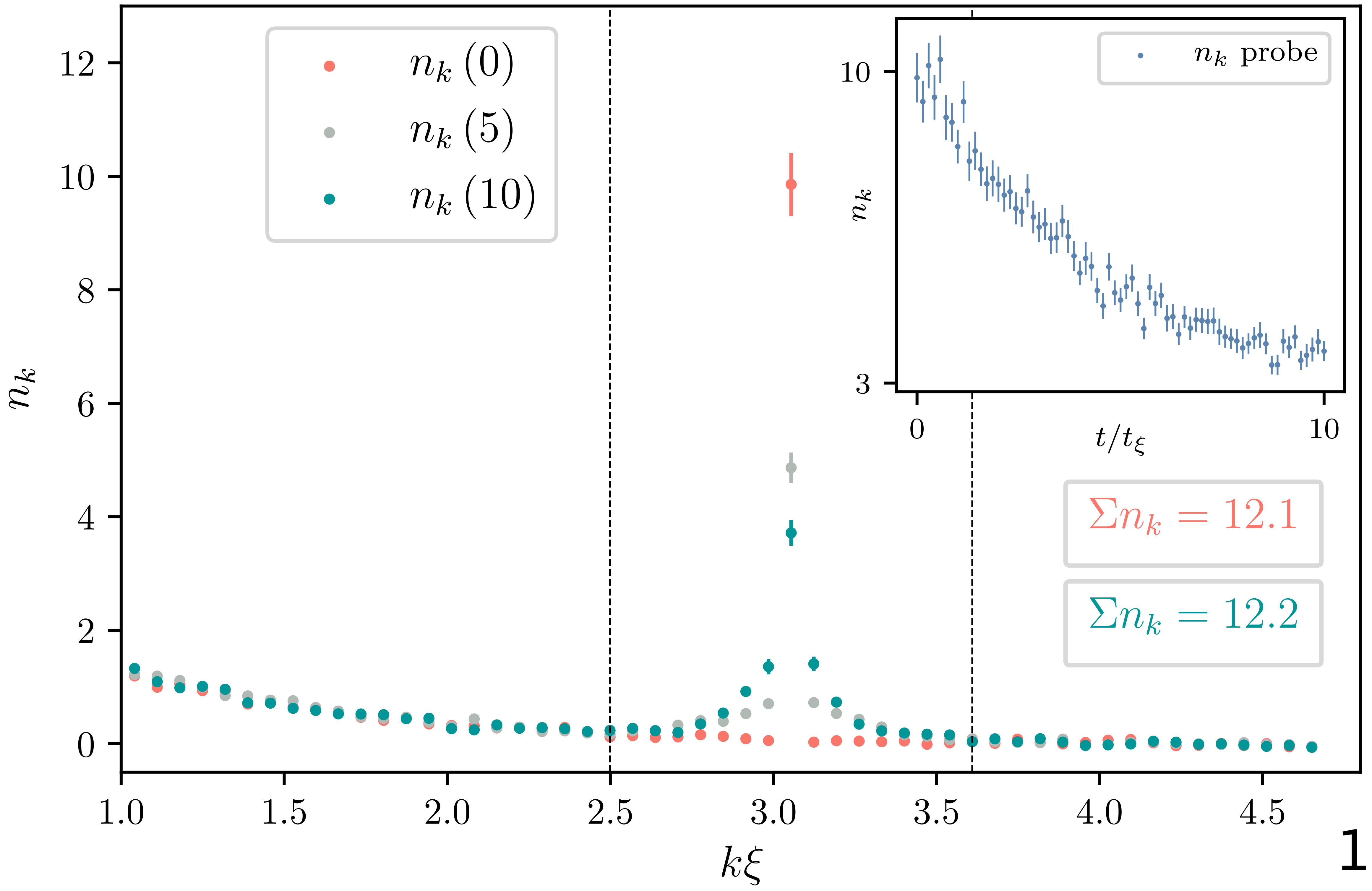
Non-linear evolution of n_k - II

Decay is a broadening; expect local conservation number of phonons[4].



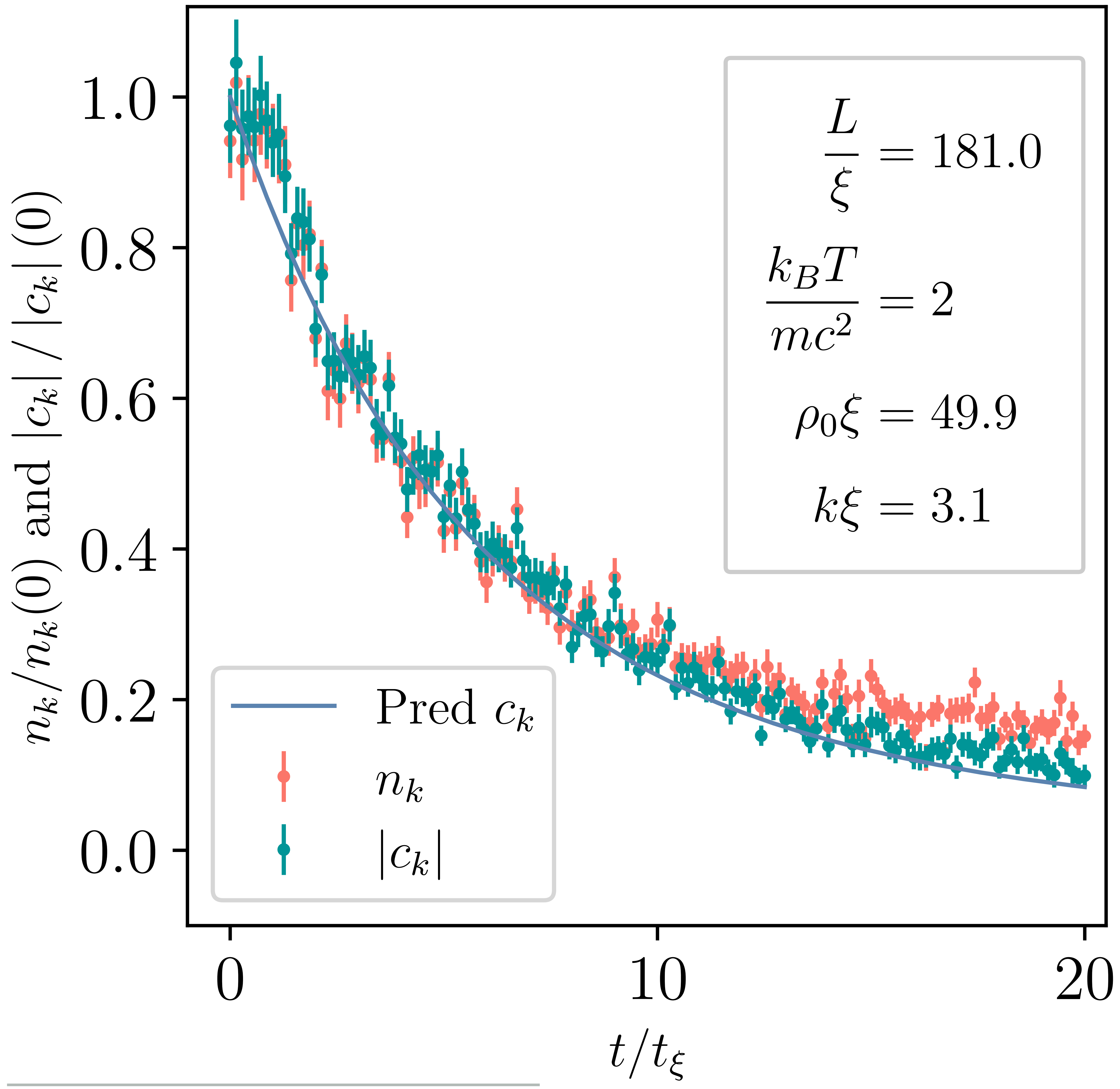


What about $|c_k|$ *?*



4. A. Micheli and S. Robertson, *Phonon Decay in 1D BEC via Beliaev-Landau Damping*, PRB (2022)

Non-linear evolution of $|c_k|$ (Preliminary results)



Seems you can repeat the computation for $|c_k|$ and get same result!

$$\Gamma_{\pm}(k) \propto \frac{k_B T}{mc^2}\frac{1}{\rho_0 \xi}$$

Using the same parameters as [3] we find $\Gamma_k/\omega_k \sim 5\%$ larger than the threshold of $\Gamma_k/\omega_k \geq 4.2\%$ to explain the absence of entanglement!

3. X. Busch, R. Parentani, and S. Robertson, *Quantum Entanglement Due to Modulated Dynamical Casimir Effect*, PRA (2014).

Conclusion & Future Directions

- Described general processes leading to particle n_k and correlation decay $|c_k|$ probably sufficient to explain absence of entanglement.

  $\longrightarrow$ Guide to tune experiment to observe entanglement!

- Application of these processes to the preheating scenario for most recent experimental data cf. Quentin's talk.

- Study of other stages of the analogue preheating.

Thank you for your attention!

Bibliography

1. L. Kofman, A. Linde, and A. Starobinsky, *Towards the Theory of Reheating After Inflation*, PRD (1997).
2. Jaskula et al. *Acoustic Analog to the Dynamical Casimir Effect in a Bose-Einstein Condensate*, PRL (2012).
3. S. Robertson, F. Michel, and R. Parentani, *Nonlinearities Induced by Parametric Resonance in Effectively 1D Atomic Bose Condensates*, PRD (2018).
4. A. Micheli and S. Robertson, Phonon Decay in 1D BEC via Beliaev-Landau Damping, PRB (2022)