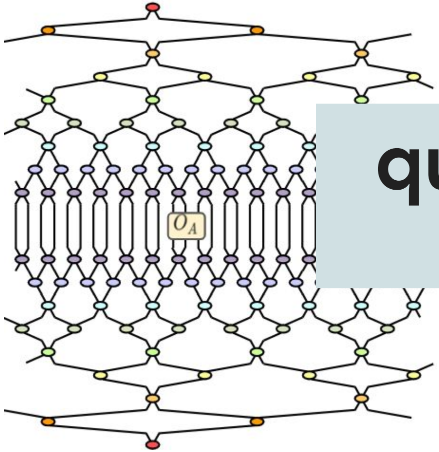
Entangle This: Randomness, Complexity and Quantum Circuits

Benasque's Centro de Ciencias Pedro Pascual

June 12th 2023



qubit MERA and quantum criticality

emergent structures inside the causal cone

Guifre Vidal

Google Quantum AI

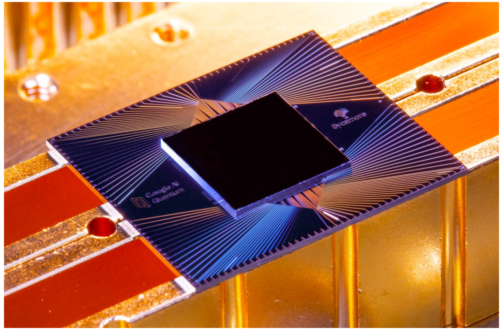
In collaboration with:

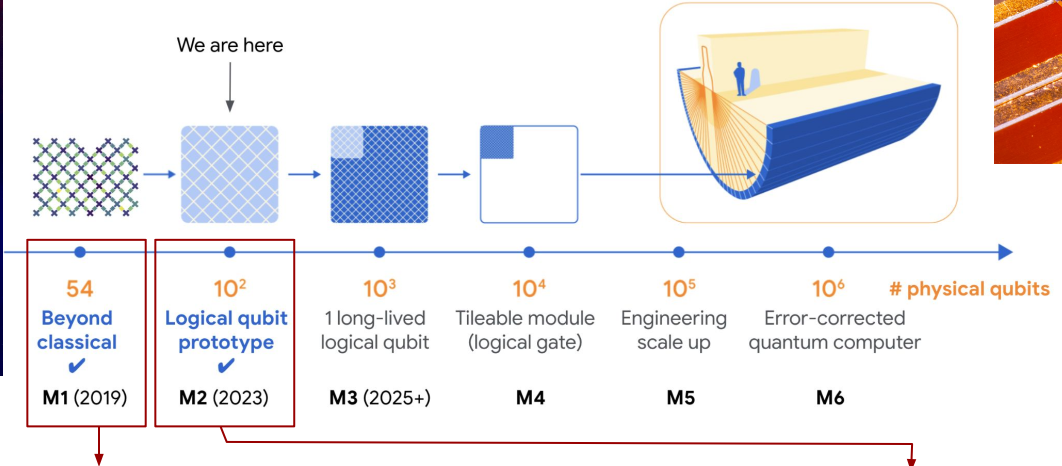
Riley Chien

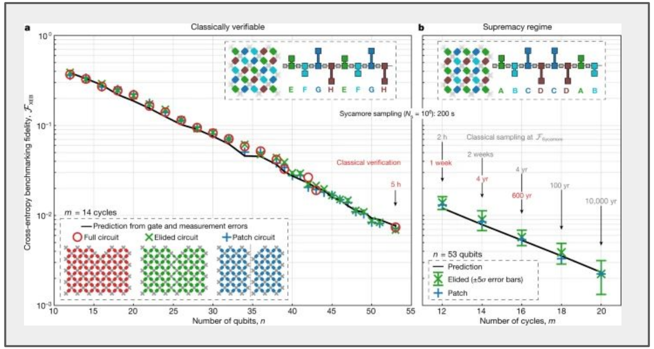
PhD at Dartmouth College (May 2023)

Student researcher at Google Quantum AI

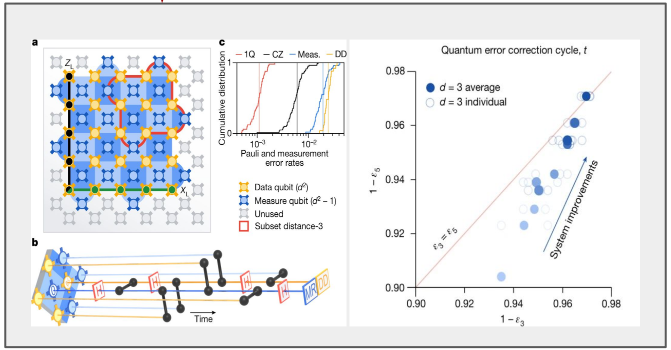
Quantum AI







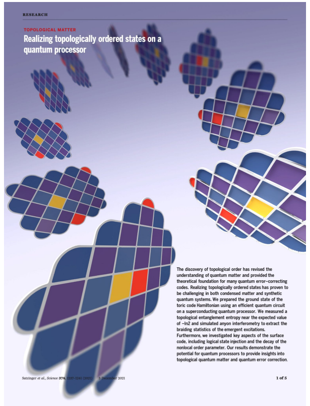
Beyond Classical demonstration (milestone 1)



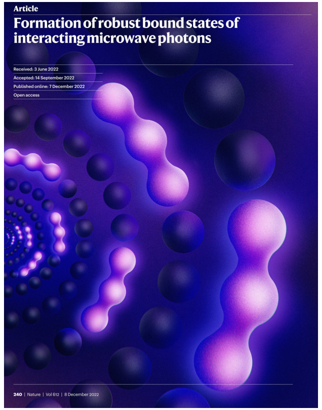
Quantum Error Correction break-even point (milestone 2)

Quantum AI

Beyond Classical
random circuit sampling



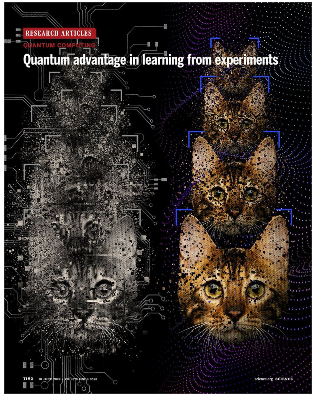
Topological Order
Abelian and non-Abelian



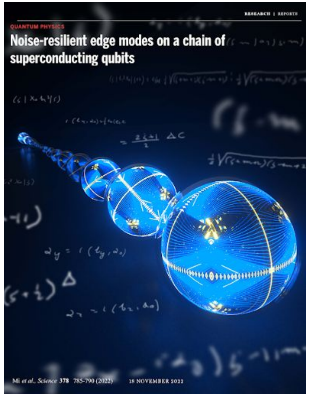
Bound States
in a quantum spin chain

Holographic Wormhole
simulation

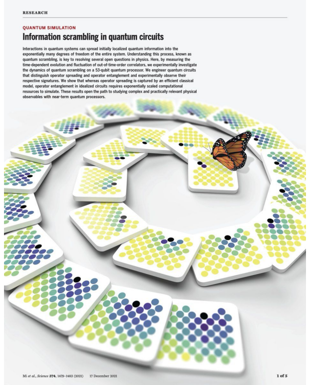
Molecular Isomerization
simulation



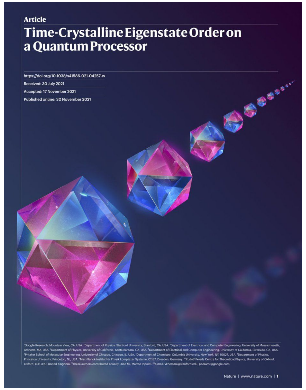
Quantum Advantage
in quantum machine learning



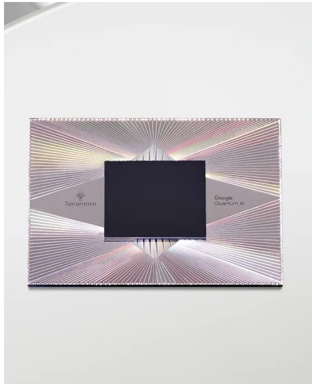
Majorana Edge modes
in a quantum spin chain



Quantum Scrambling
in 2d quantum evolution



Time Crystal
in a quantum spin chain



Quantum Error Correction
break-even milestone

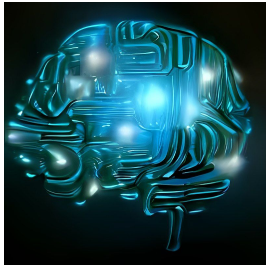
Quantum AI

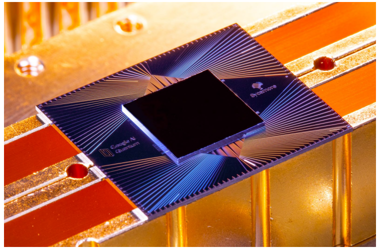
Commercial Applications of Quantum Computing ?

Drug Design

Computational Chemistry

Cybersecurity & Cryptography





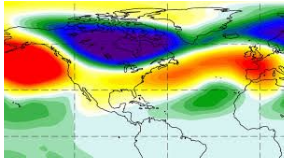
Financial Modelling

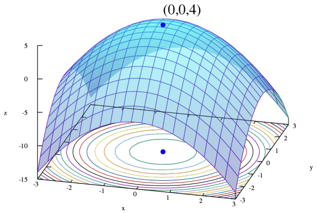
Artificial Intelligence

Complex Manufacturing

Logistics Optimisation

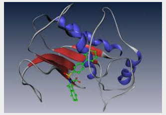
Weather Forecasting





Are Quantum Computers needed for Quantum Chemistry / Materials Science?

Quantum computers may be able to efficiently solve the **ground state electronic structure** of complex molecules and materials:



- Fertilizers
- Solar Energy

- Batteries
- Catalyzers

- Drug discovery
- High-Tc Superconductors

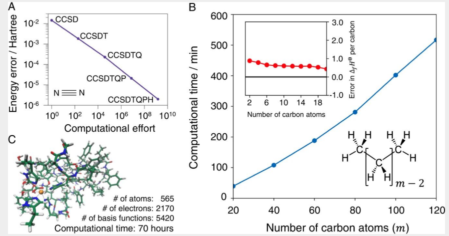
- New Materials

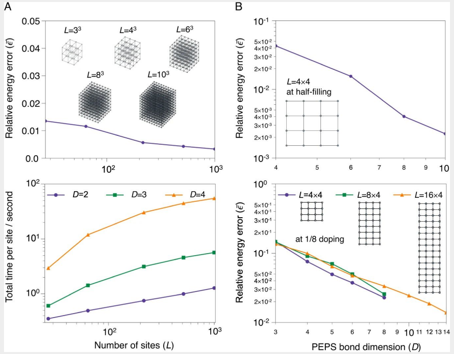
However… heuristic classical methods might be enough

Evaluating the evidence for exponential quantum advantage in ground-state quantum chemistry

Seunghoon Lee, Joonho Lee, Huanchen Zhai, Yu Tong, Alexander M. Dalzell, Ashutosh Kumar, Phillip Helms, Johnnie Gray, Zhi-Hao Cui, Wenyuan Liu, Michael Kastoryano, Ryan Babbush, John Preskill, David R. Reichman, Earl T. Campbell, Edward F. Valeev, Lin Lin & Garnet Kin-Lic Chan

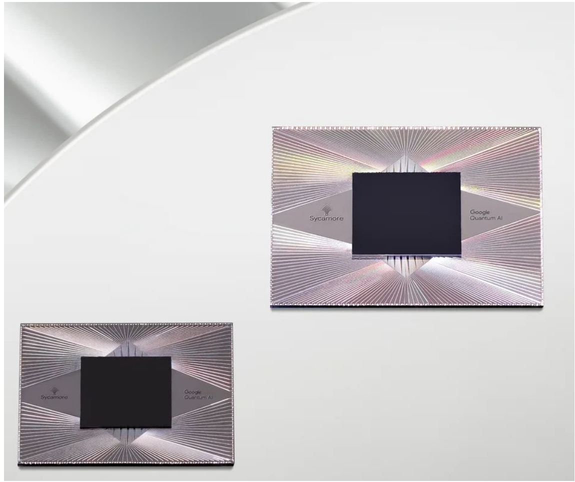
Nature Communications **14**, Article number: 1952 (2023) | Cite this article

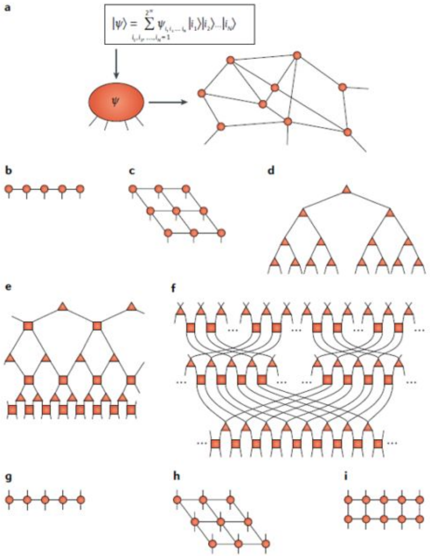




Quantum AI

Quantum Computers vs **Tensor Network** algorithms?



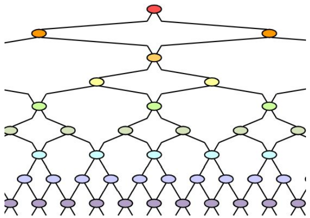


Actually, quantum computers can accelerate (exponentially!?) tensor network algorithms…

Outline

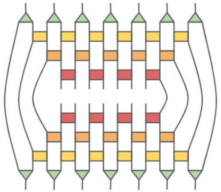
1 - Motivation:

- MERA on qubits (q-MERA)



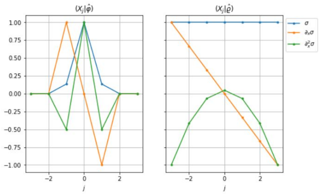
2 - MERA quantum channel

- Eigenvalue decomposition
- Symmetries
- Derivative descendants



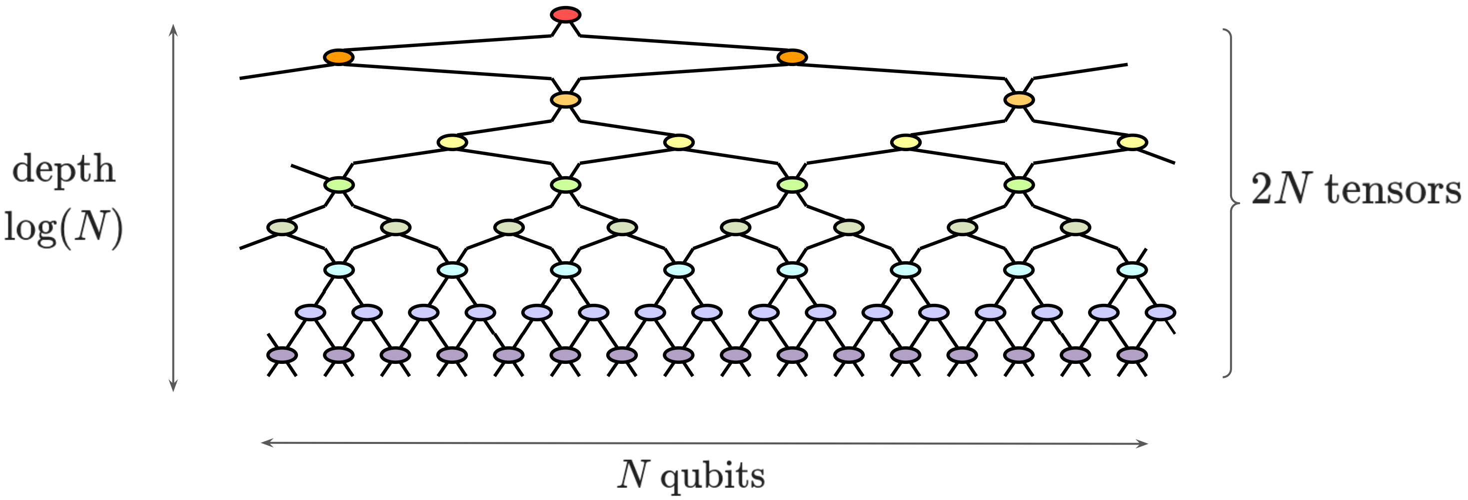
3 - Emergent structures in the causal cone

- Space resolved patterns
- MPO for channel eigen-operators



MERA is a 'holographic' tensor network:

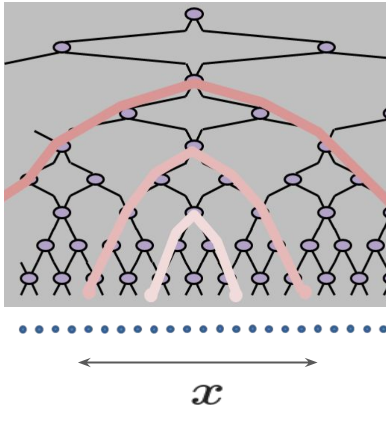
Vidal 2007, 2008 *(talk in Benasque 2005?)*
Evenbly, Vidal 2009



ground state of **1d system** (e.g. spin chain)
represented as a **2d tensor network** (space + scale)

accurate representation of ground states of critical systems

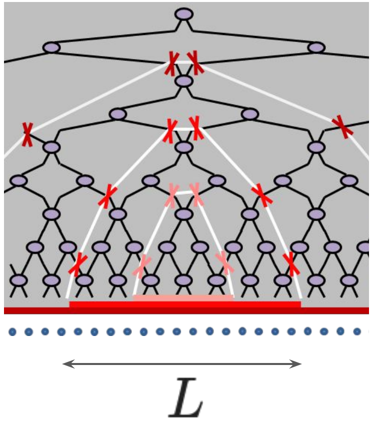
correlations



$$\langle O(0)O(x)\rangle \sim \frac{1}{x^p}$$

power law

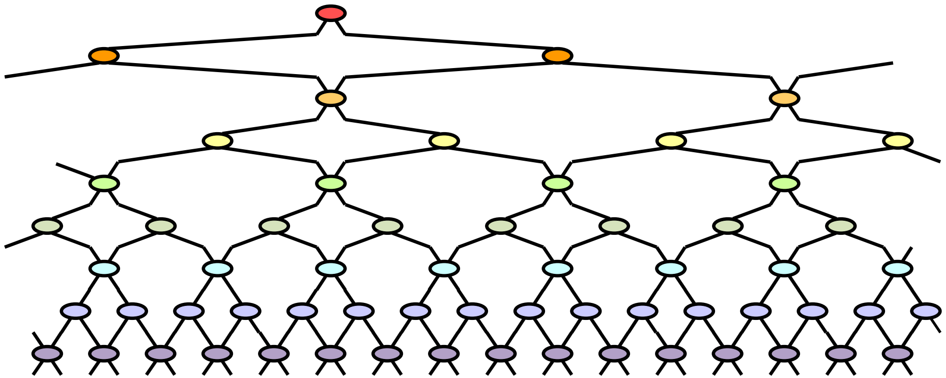
entanglement

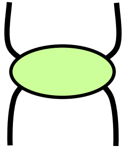


$$S(L) \sim \log(L)$$

logarithmic correction to area law

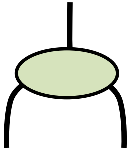
MERA is a quantum circuit:





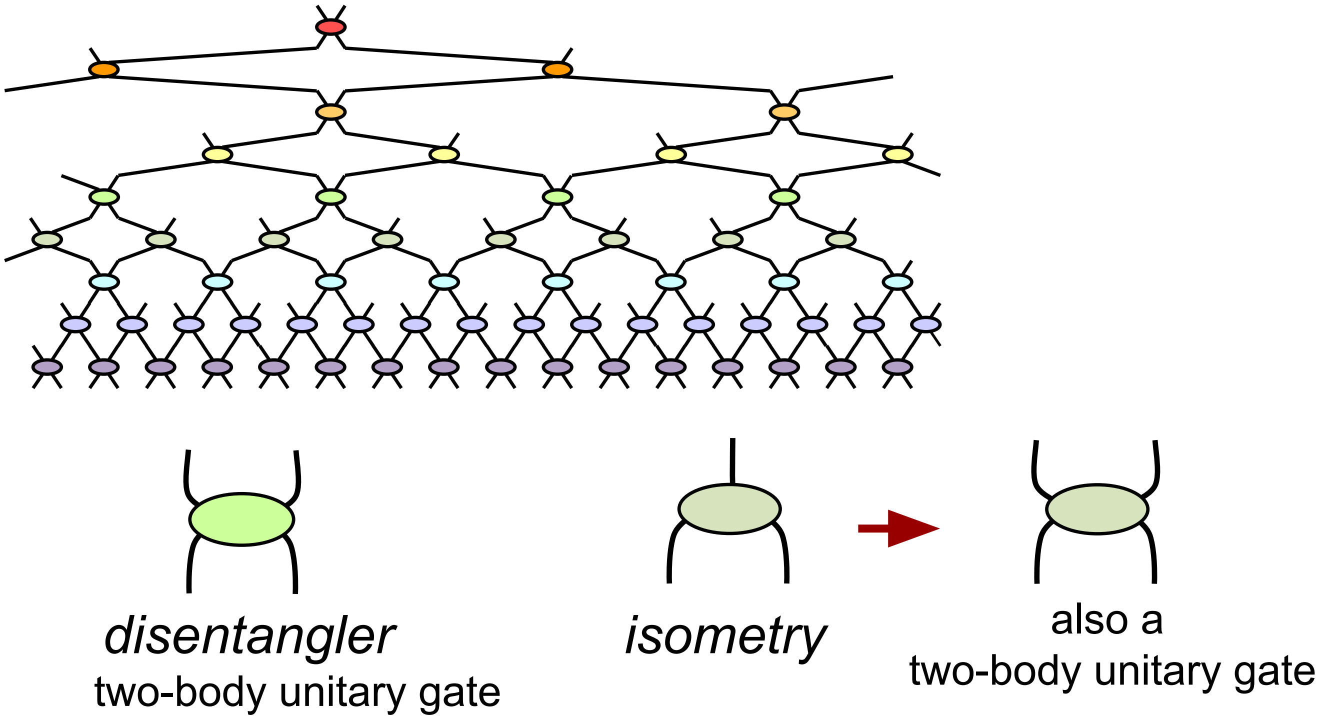
disentangler

two-body unitary gate



isometry

MERA is a quantum circuit:



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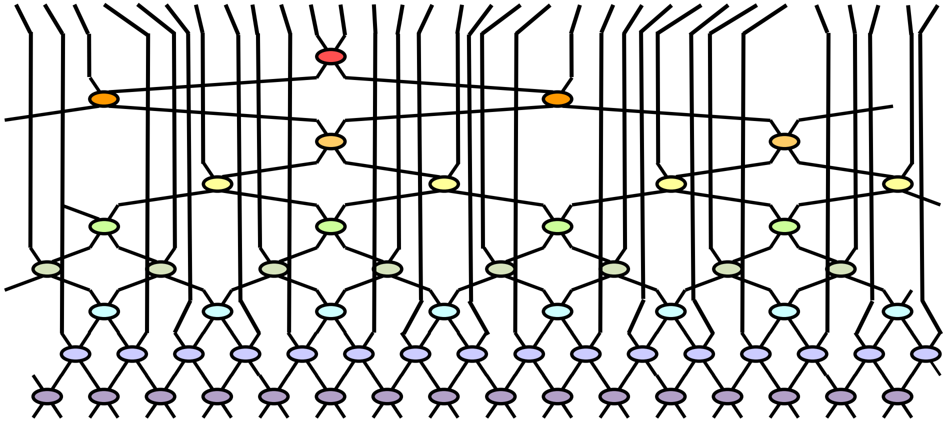
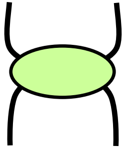
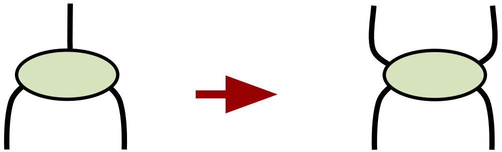




disentangler
two-body unitary gate



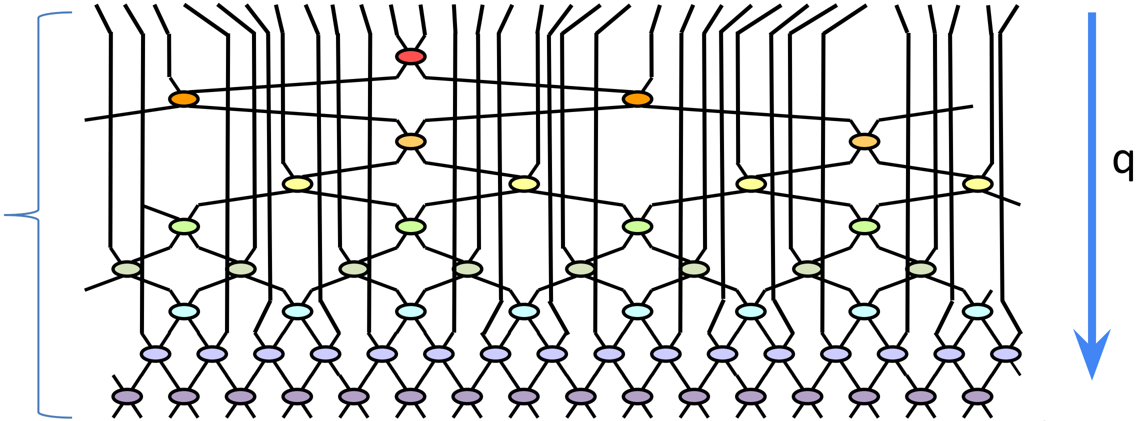
isometry

also a
two-body unitary gate

MERA is a quantum circuit:

$|0^{\otimes N}\rangle$

quantum circuit



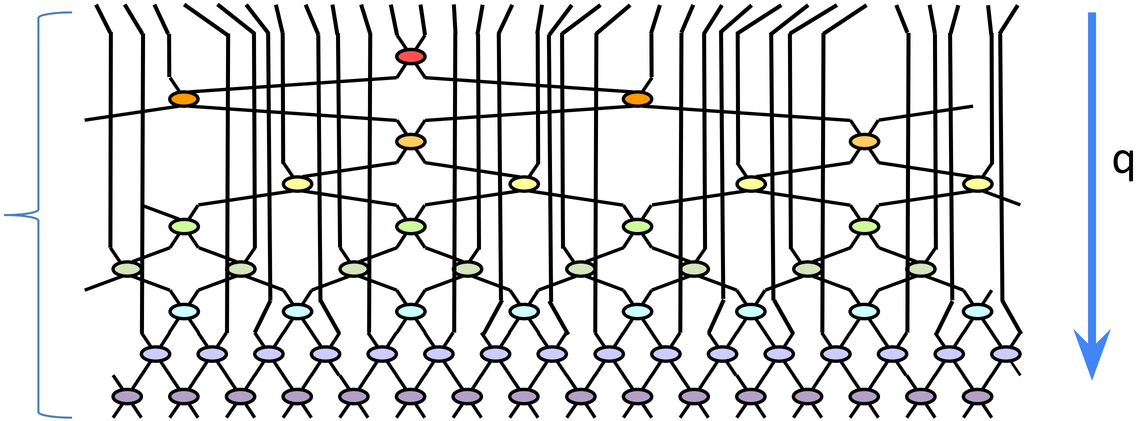
quantum circuit time
= **scale**

$|\Psi_{MERA}\rangle$

$$|\Psi_{MERA}\rangle = U|0^{\otimes N}\rangle$$

MERA is a quantum circuit:

quantum circuit

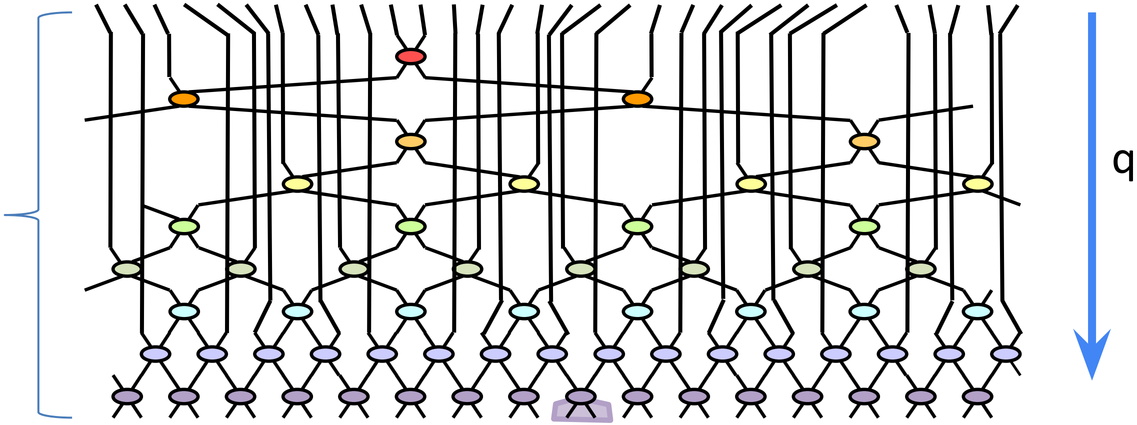


quantum circuit time
= **scale**

entanglement introduced sequentially at different length scales

MERA is a quantum circuit:

quantum circuit



quantum circuit time
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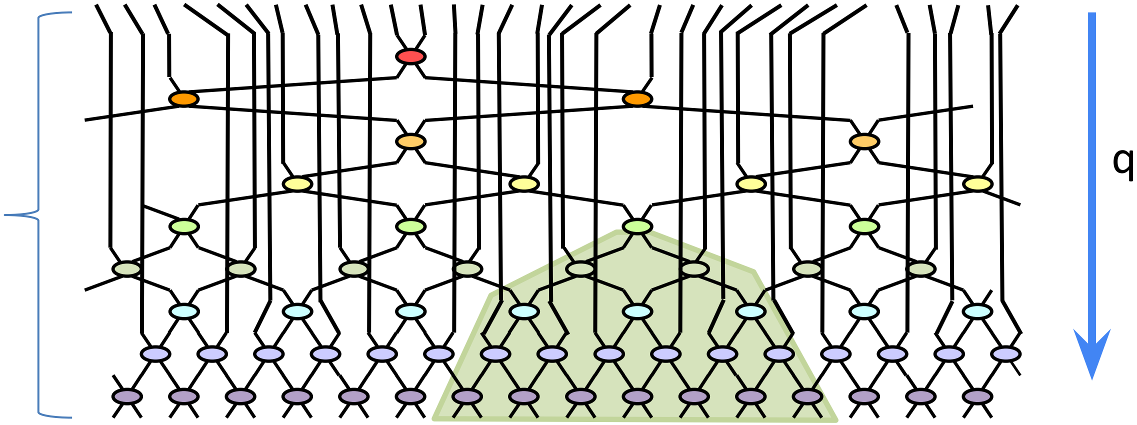
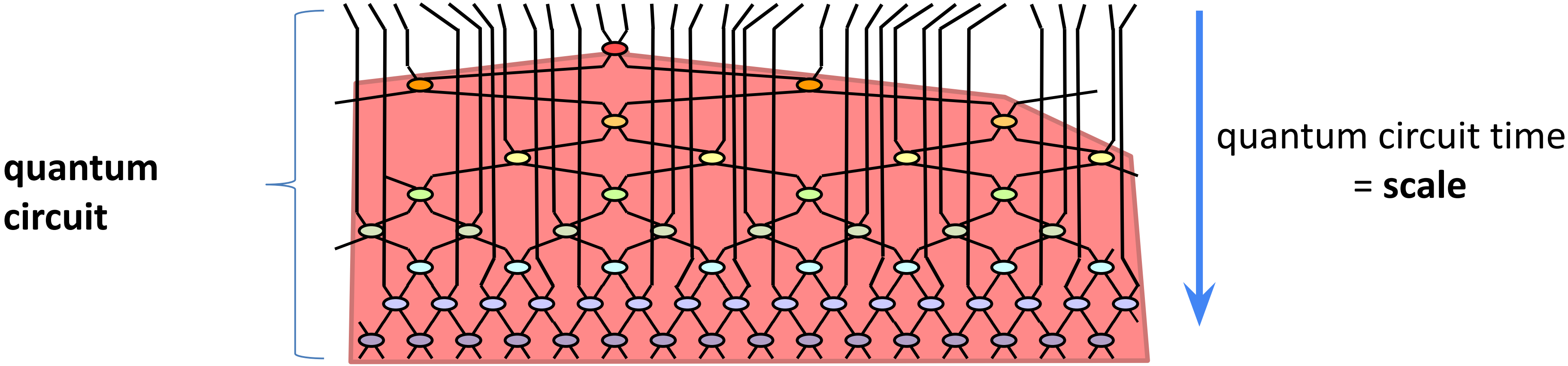
quantum circuit



quantum circuit time
= **scale**

entanglement introduced sequentially at different length scales

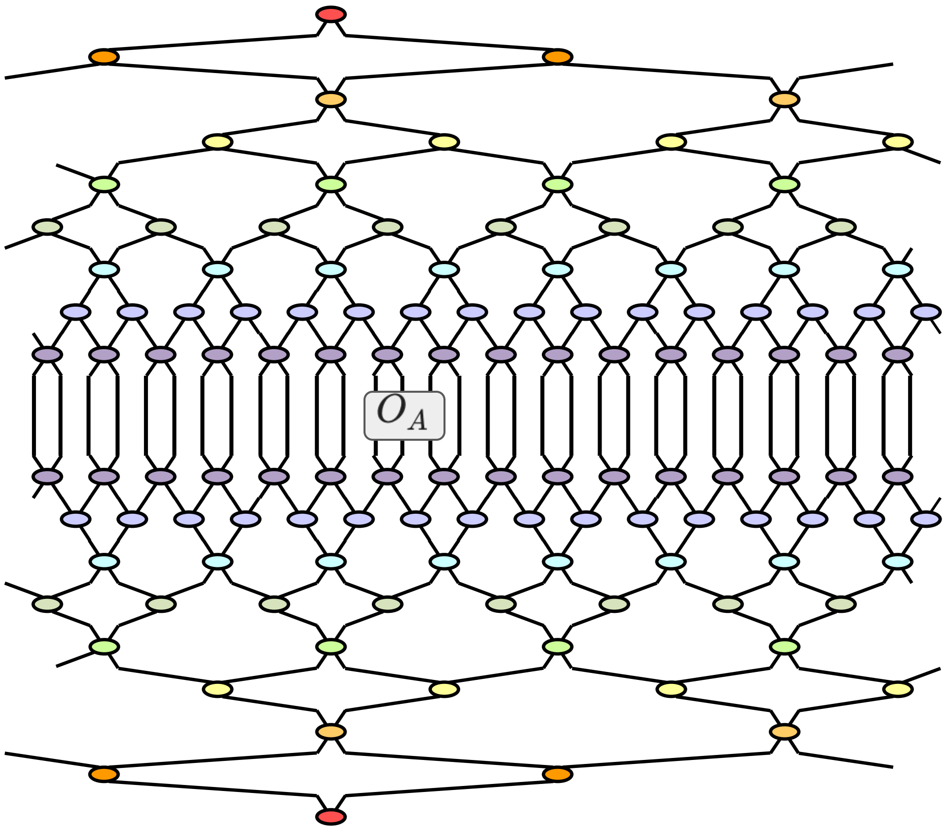
MERA is a quantum circuit:



entanglement introduced sequentially at different length scales

causal cone

$$\langle O_A \rangle = \langle \Psi_{MERA} | O_A | \Psi_{MERA} \rangle$$



$|\Psi_{MERA}\rangle$

$\langle\Psi_{MERA}|$

causal cone

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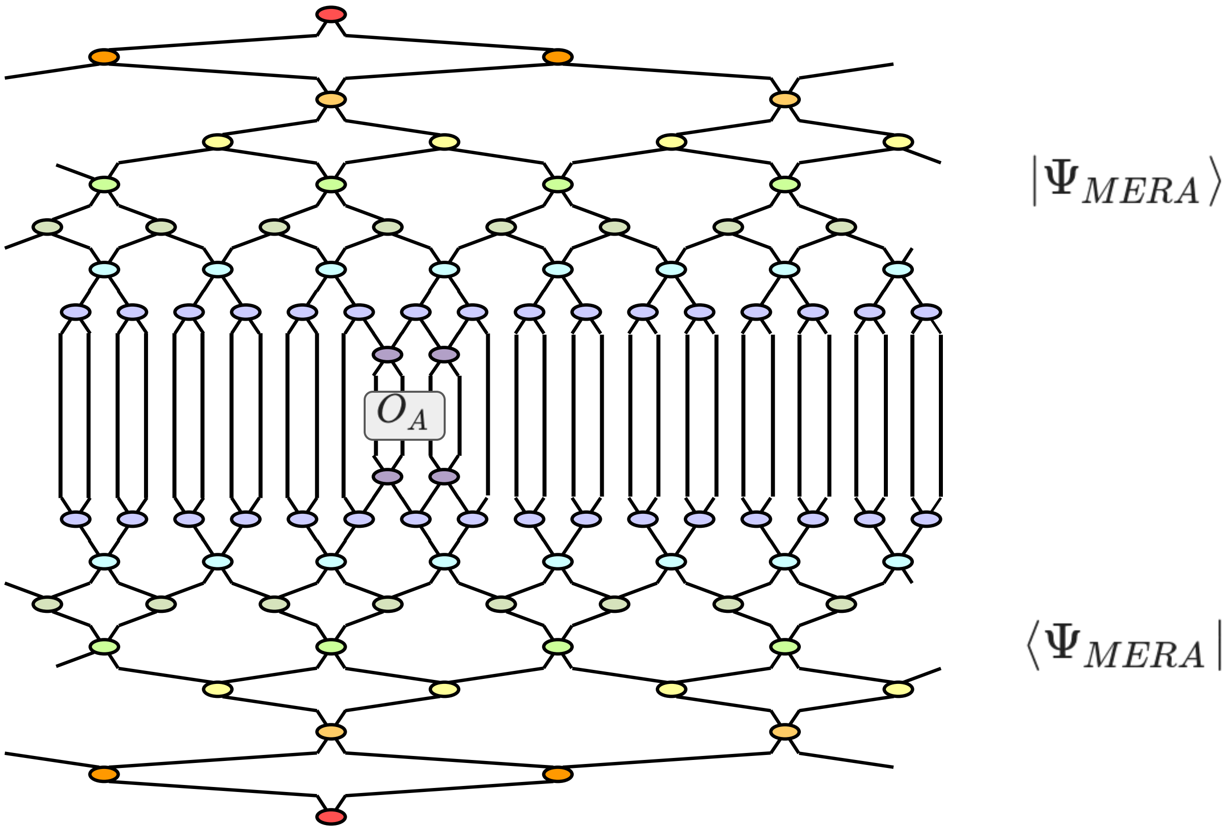

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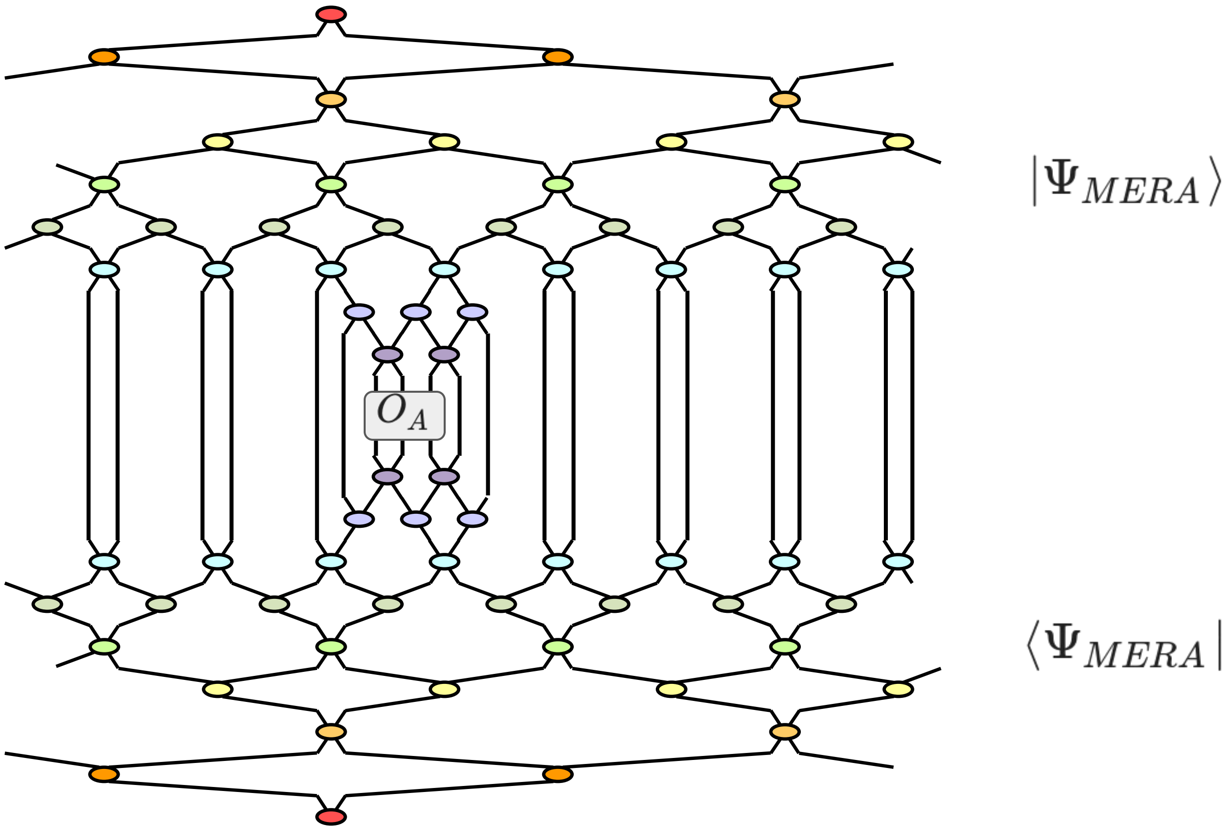

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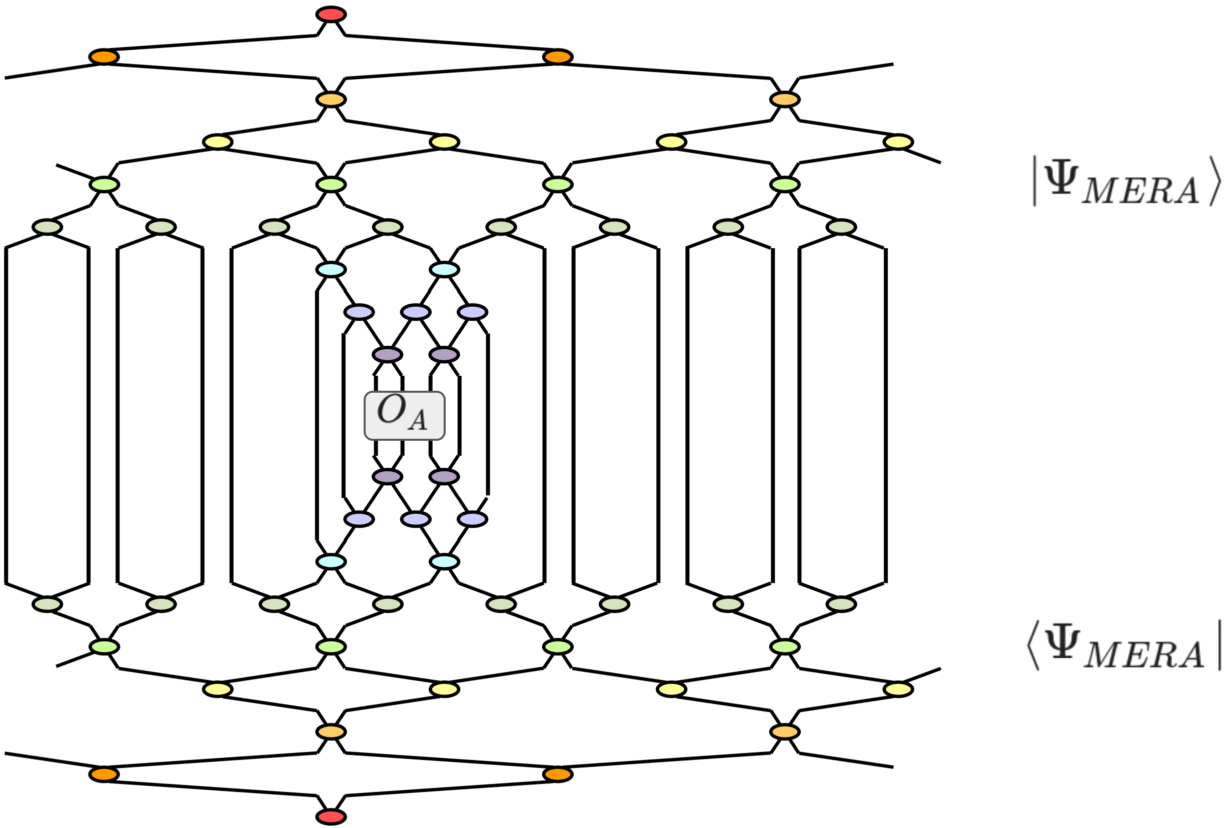

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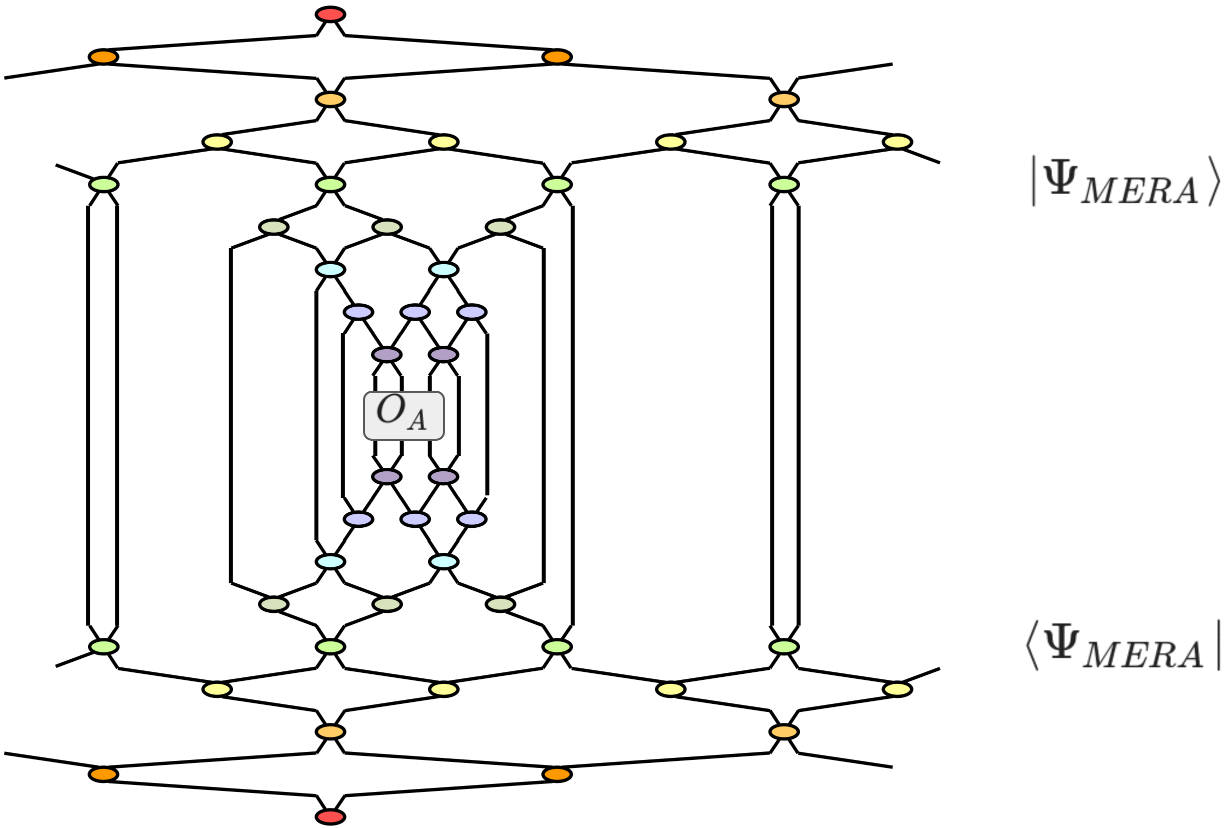

causal cone

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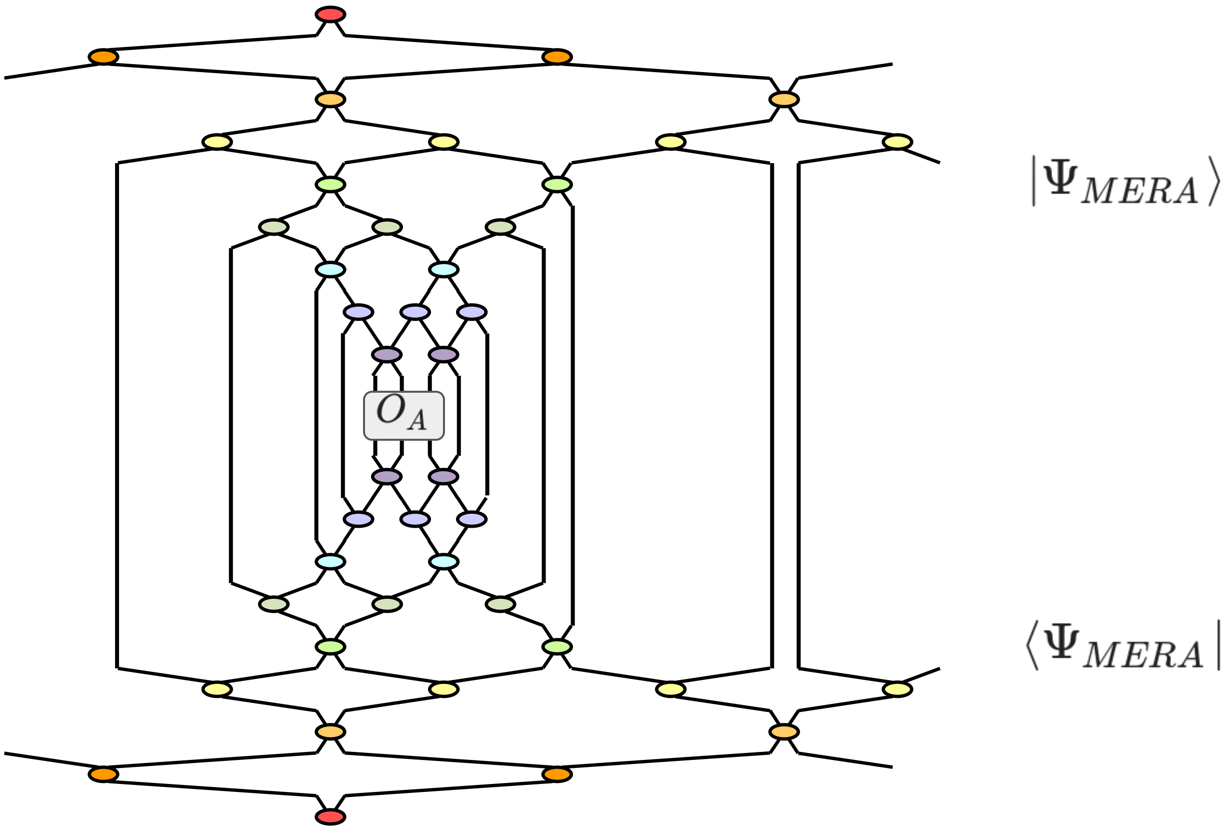
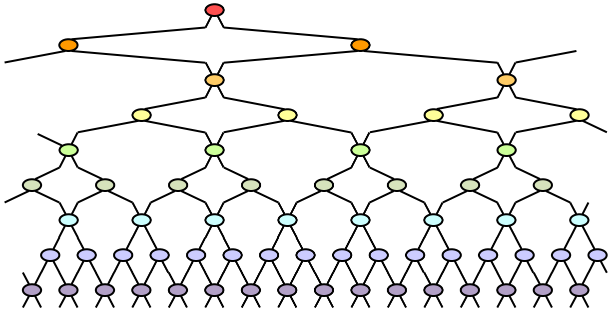
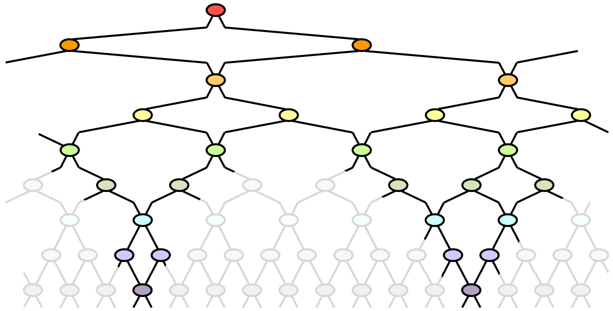


Simulating an N-qubit wavefunction with O(1) qubits

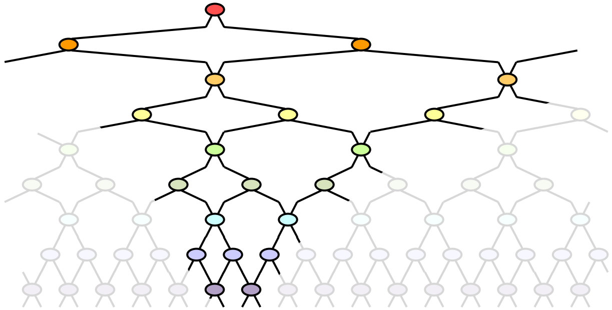


full N-qubit wavefunction: N-qubits



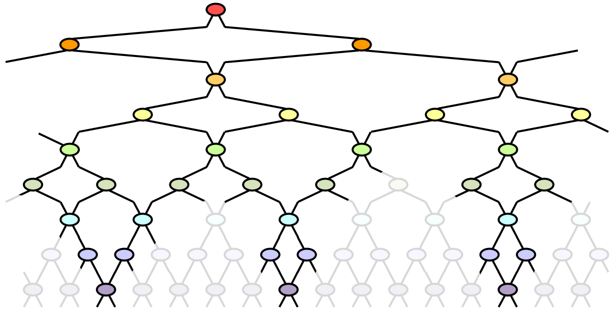
Two-point correlator: O(1) qubits (e.g. 6)

(Sufficient for optimization with 1D non-local Hamiltonian, e.g. $V = \sum_{i,j} \frac{n_i n_j}{|i-j|}$)



Local observable: O(1) qubits (e.g. 3)

(Sufficient for optimization with 1D local Hamiltonian)



k-point correlator: exp(k) qubits
(e.g. 9 for 3-point correlator)

Experimental implementation of MERA on a quantum processor

arXiv > quant-ph > arXiv:2109.09787

Quantum Physics

[Submitted on 20 Sep 2021]

Preparing Renormalization Group Fixed Points on NISQ Hardware

Troy J. Sewell, Stephen P. Jordan

Quantinuum/Honeywell HØ using **6 qubits**

arXiv > quant-ph > arXiv:2203.00886

Quantum Physics

[Submitted on 2 Mar 2022]

Holographic quantum simulation of entanglement renormalization circuits

Sajant Anand, Johannes Hauschild, Yuxuan Zhang, Andrew C. Potter, Michael P. Zaletel

Quantinuum/Honeywell H1 using **10 qubits**

arXiv > quant-ph > arXiv:2305.01650

Quantum Physics

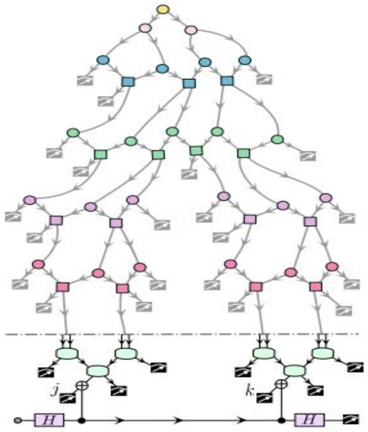
[Submitted on 2 May 2023 (v1), last revised 3 May 2023 (this version, v2)]

Probing critical states of matter on a digital quantum computer

Reza Haghshenas, Eli Chertkov, Matthew DeCross, Thomas M. Gatterman, Justin A. Gerber, Kevin Gilmore, Dan Gresh, Nathan Hewitt, Chandler V. Horst, Mitchell Matheny, Tanner Mengle, Brian Neyenhuis, David Hayes, Michael Foss-Feig

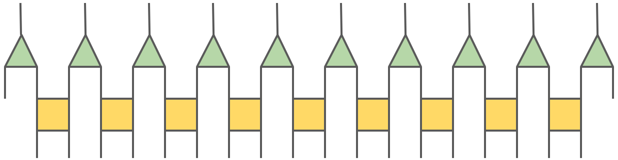
Quantinuum/Honeywell H1-1 using **20 qubits**

two-point correlator
for critical ising model
on 128 quantum spins



Qubit MERA (q-MERA)

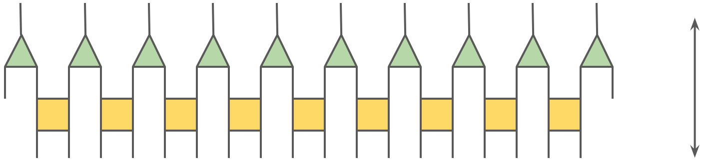
A layer of MERA is a coarse-graining transformation



How can we make MERA
more expressive / accurate?

Qubit MERA (q-MERA)

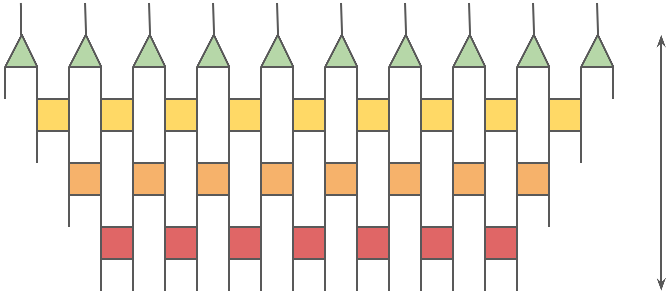
A layer of MERA is a coarse-graining transformation



depth $D = 2$

How can we make MERA more expressive / accurate?

q-MERA (MERA on qubits)



D

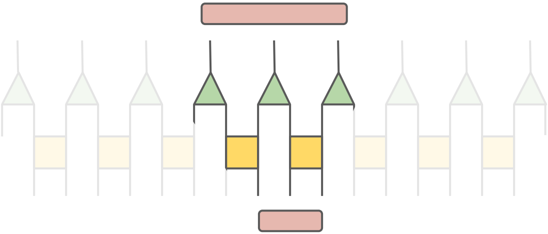
By increasing the depth D

Fishman, White 2015, Evenbly, White 2016

See also:
Arguello-Luengo 2017
Haegeman, Swingle, Walter, Cotler, Evenbly, Scholz, 2017
Kim, Swingle 2017
Haghschensas, Gray, Potter, Chan, 2021
Miao, Barthel, 2021
…

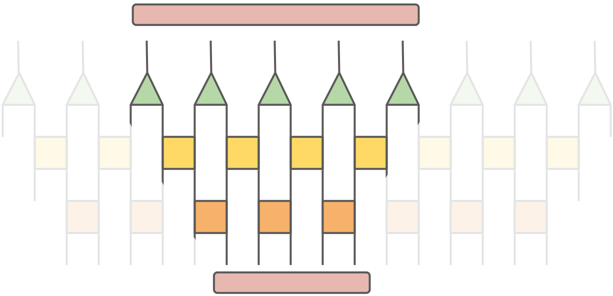
Qubit MERA (q-MERA)

Single layer of MERA and width of causal cone



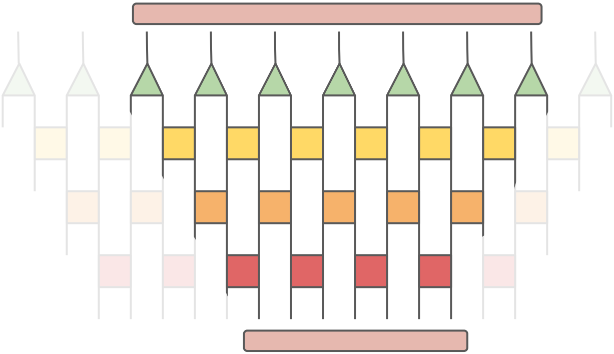
depth $D = 2$

$(2D-1=)$ 3 qubits



depth $D = 3$

$(2D-1=)$ 5 qubits

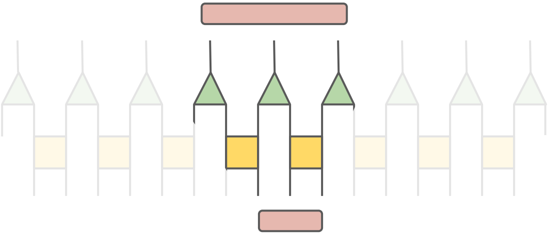


depth $D = 4$

$(2D-1=)$ 7 qubits

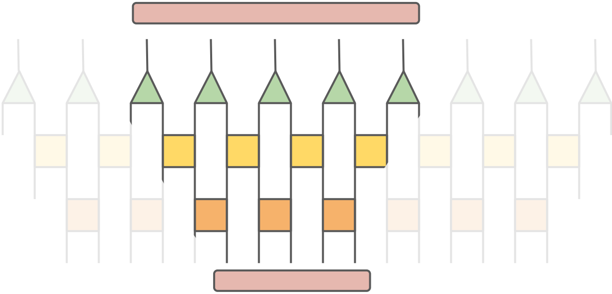
Qubit MERA (q-MERA)

Single layer of MERA and width of causal cone



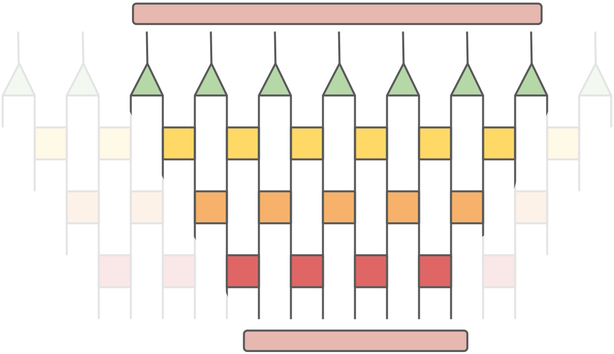
depth $D = 2$

$(2D-1=)$ 3 qubits



depth $D = 3$

$(2D-1=)$ 5 qubits



depth $D = 4$

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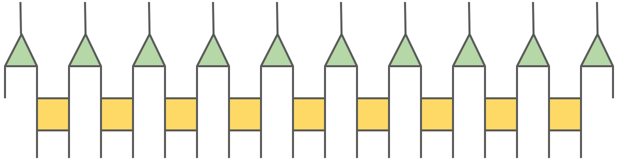
ρ is $2^{2D-1} \times 2^{2D-1}$

computational resources

	classical	quantum
memory	$O(\exp(D))$	$O(D)$ qubits on
time	$O(\exp(D))$	depth D circuit

Exponential quantum advantage?

* A comment for tensor network **experts**:

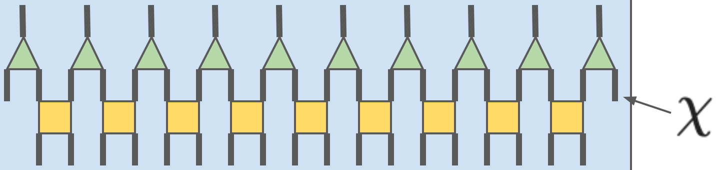


bond dimension $\chi = 2$

depth $D = 2$

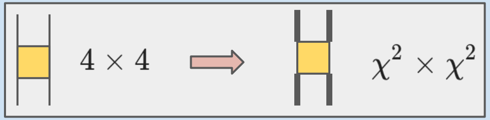
How can we make MERA more expressive / accurate?

traditionally: χ-MERA

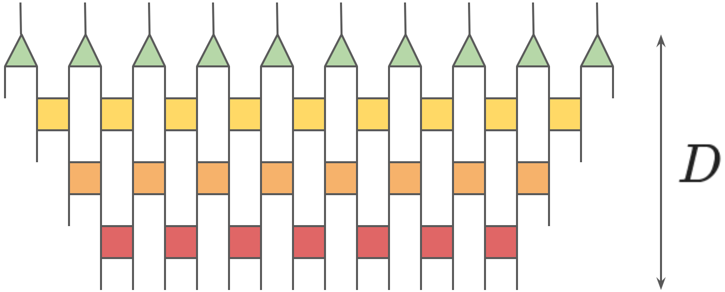


By increasing the bond dimension χ

Vidal 2007
Evenbly, Vidal 2009



q-MERA (MERA on qubits)



By increasing the depth D

Fishman, White 2015, Evenbly, White 2016

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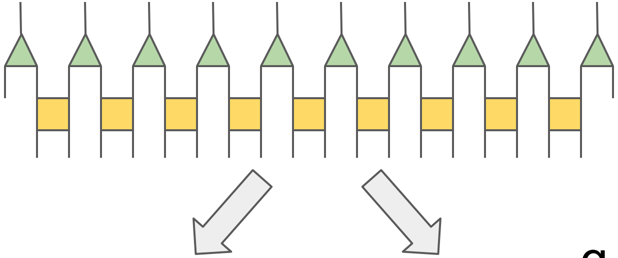

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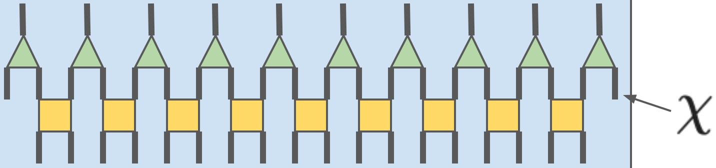

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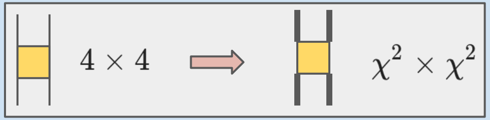

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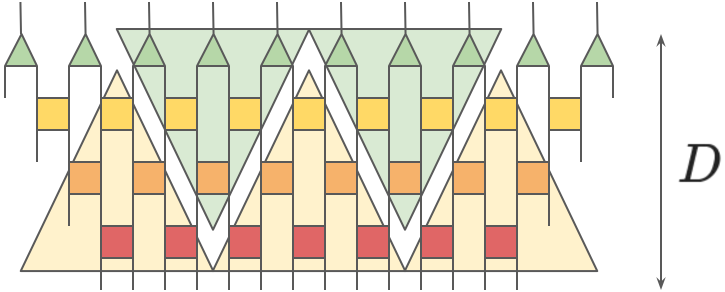
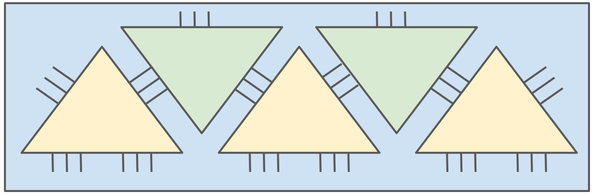
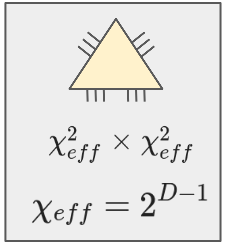


By increasing the depth D

Fishman, White 2015, Evenbly, White 2016





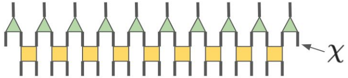
Quantum AI

Summary so far:

MERA is a variational ansatz for (quantum critical) many-body ground states

χ-MERA

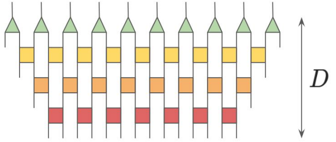
(increase bond dimension χ)



→ basis for classical algorithms

q-MERA

(increase layer depth D)



→ basis for quantum algorithms (e.g. VQE)

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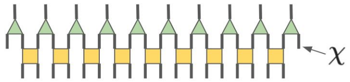
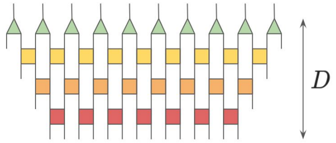


→ basis for classical algorithms

This talk:

q-MERA

(increase layer depth D)



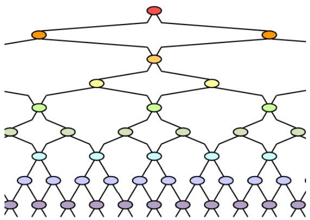
→ basis for classical algorithms

→ basis for quantum algorithms (e.g. VQE)

Outline

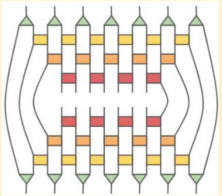
1 - Motivation:

- MERA on qubits (q-MERA)



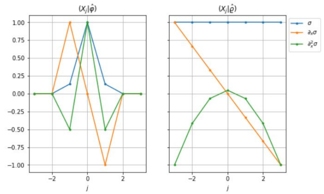
2 - MERA quantum channel

- Eigenvalue decomposition
- Symmetries
- Derivative descendants



3 - Emergent structures in the causal cone

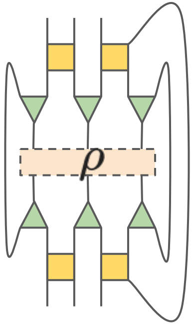
- Space resolved patterns
- MPO for channel eigen-operators



MERA quantum channel

Key step in MERA **algorithms**, both **classical** and **quantum**

$$\rho' = \mathcal{C}[\rho]$$



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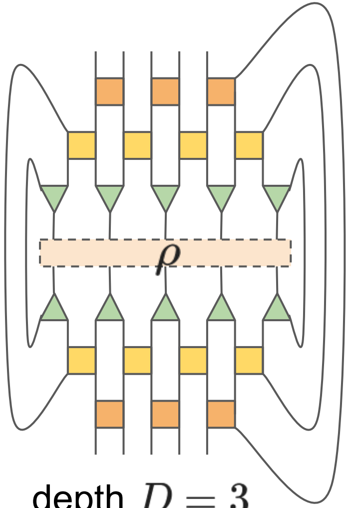
$(2D-1 =)$ 3 qubits



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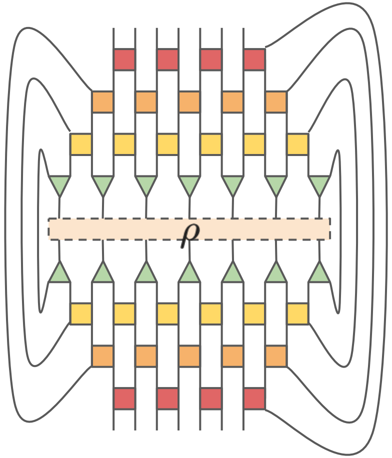

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computational resources

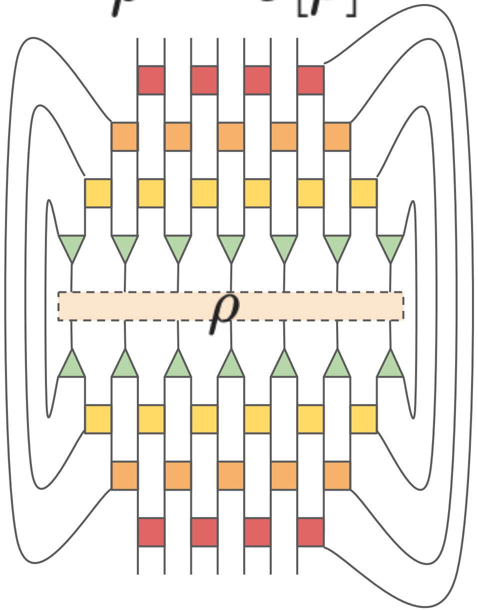
	classical	**quantum**
	memory $O(\exp(D))$ time $O(\exp(D))$	$O(D)$ qubits on depth D circuit

Quantum AI

MERA quantum channel

Our goal: diagonalize this channel

$$\rho' = \mathcal{C}[\rho]$$



depth $D = 4$

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Why? Extraction of universal (conformal) data, e.g. scaling dimensions

input: MERA tensors optimized for ground state of (modified) critical transverse field Ising chain

from Evenbly, White 2016

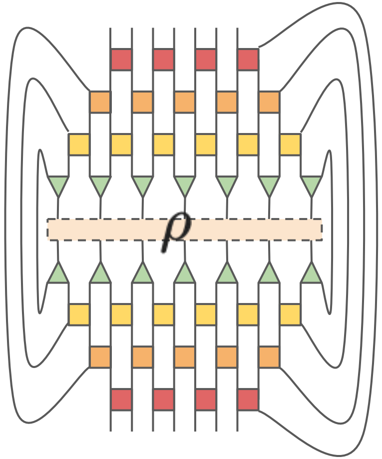
$$H = -\sum_{i=-\infty}^{\infty} (X_i X_{i+1} - X_{i-1} Z_i X_{i+1})$$

$$\left[H_{\text{Ising}} = -\sum_{i=-\infty}^{\infty} (X_i X_{i+1} + Z_i) \right]$$

output: Dominant eigenvalue decomposition of MERA quantum channel $\mathcal{C}$

$$\mathcal{C} = \sum_{\alpha=0}^{\chi-1} \lambda_\alpha |\hat{\rho}_\alpha)(\hat{\varphi}_\alpha|$$

MERA quantum channel



$$\rho' = \mathcal{C}[\rho]$$

depth $D = 4$

$(2D-1 =)$ 7 qubits

Eigenvalue decomposition of MERA quantum channel $\mathcal{C}$

lots of terms!!!

eigenvalues (real or complex pairs), with $|\lambda_\alpha| \leq 1$

$$1 = \lambda_0 \geq |\lambda_1| \geq \cdots \geq |\lambda_{2^{2n}-1}|$$

$$\mathcal{C} = \sum_{\alpha=0}^{2^{2n}-1} \lambda_\alpha |\hat{\rho}_\alpha)(\hat{\varphi}_\alpha|$$

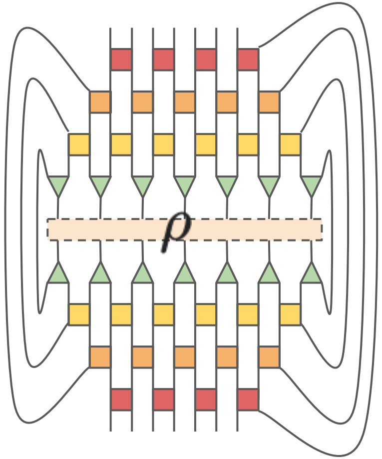
"density matrices"

primal eigen-operators

$$\mathcal{C}|\hat{\varrho}_\alpha) = \lambda_\alpha |\hat{\varrho}_\alpha)$$

or $\mathcal{C}[\hat{\varrho}_\alpha] = \lambda_\alpha \hat{\varrho}_\alpha$

MERA quantum channel



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"observables"

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$$(\hat{\varphi}_\alpha|\mathcal{C} = \lambda_\alpha(\hat{\varphi}_\alpha|$$

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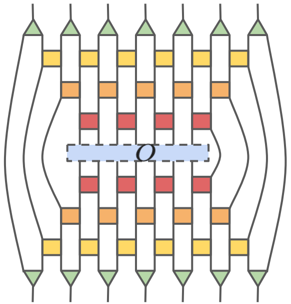
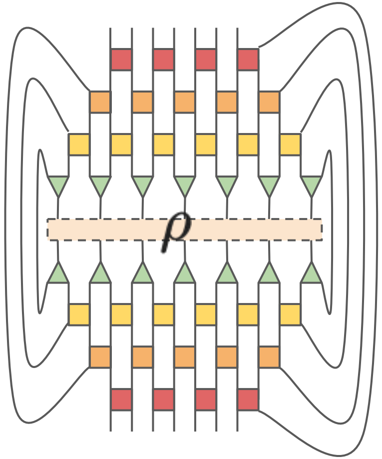


$$O' = \mathcal{C}^T[O]$$

MERA quantum channel



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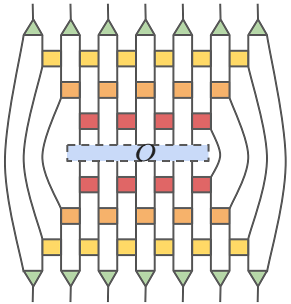


$O' = \mathcal{C}^T[O]$

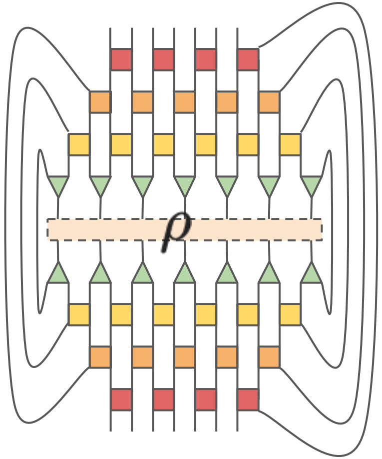
$|\hat{\varrho}_\alpha)$ and $(\hat{\varphi}_\alpha|$ are not Hermitian conjugates $(\hat{\varphi}_\alpha| \neq |\hat{\varrho}_\alpha)^\dagger$

example: $\lambda_0 = 1$ $\quad (\hat{\varphi}_0| = \mathbb{I}$

$|\hat{\varrho}_0)$ fixed-point density matrix

Quantum AI

MERA quantum channel



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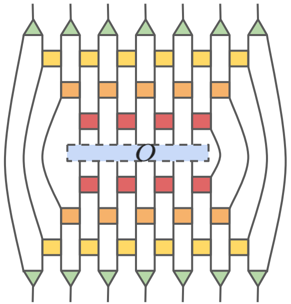

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$\{|\hat{\varrho}_\alpha)\}$ and $\{(\hat{\varphi}_\alpha|\}$

are not orthonormal bases

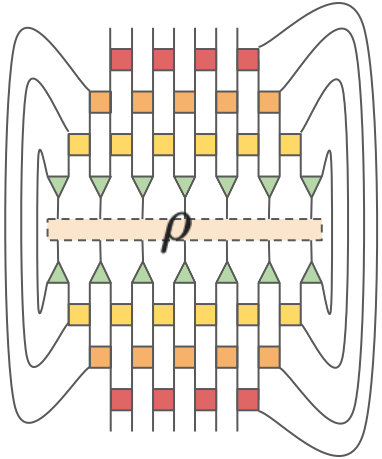
$(\hat{\varrho}_\alpha|\hat{\varrho}_\beta) \neq \delta_{\alpha\beta} \qquad (\hat{\varphi}_\alpha|\hat{\varphi}_\beta) \neq \delta_{\alpha\beta}$

they are instead *bi-orthonormal* bases

$$(\hat{\varphi}_\alpha|\hat{\varrho}_\beta) = \delta_{\alpha\beta}$$

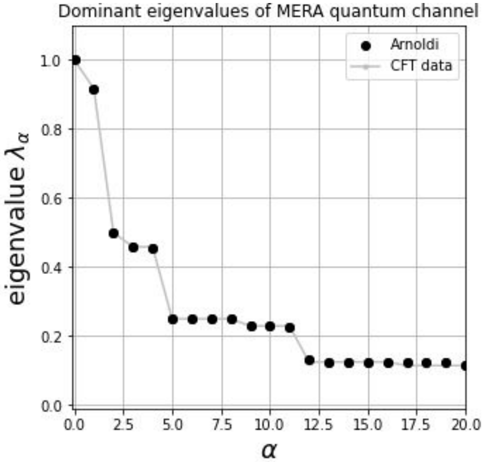
Quantum AI

MERA quantum channel



Arnoldi iteration

for dominant eigenvalues of non-normal matrix



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ρ is a 128x128 matrix

Quantum AI

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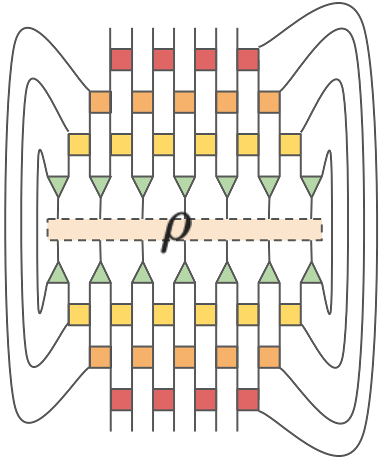

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eigenvalues

scaling dimensions

$$\Delta_\alpha = -\log_2(\lambda_\alpha)$$

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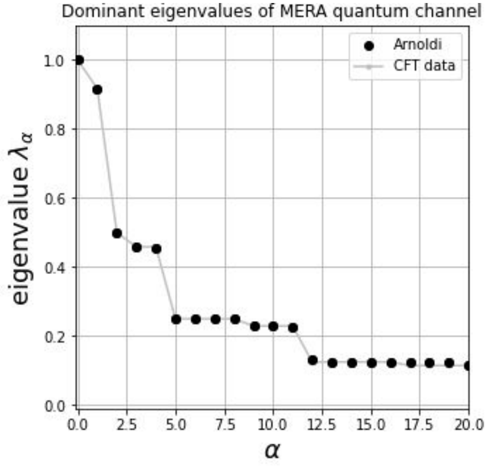

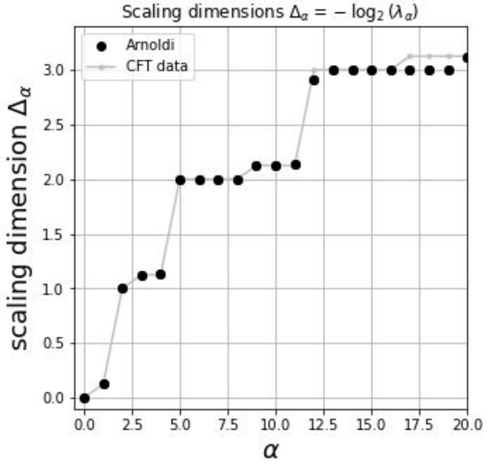

Quantum AI

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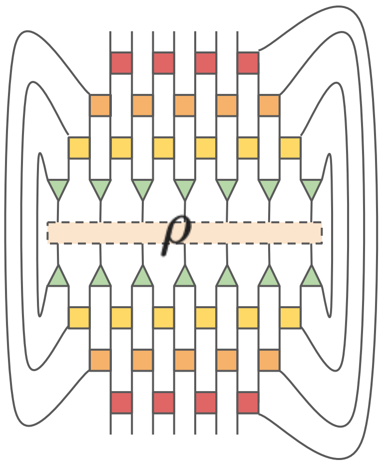

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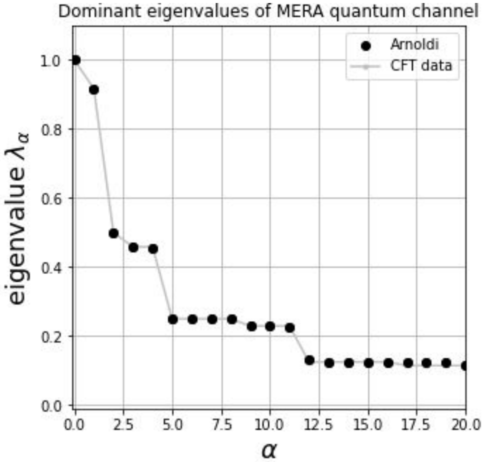

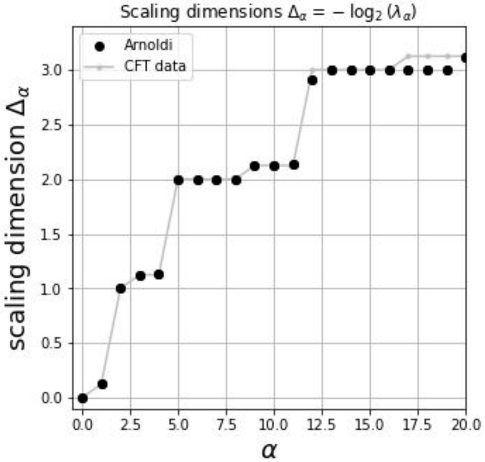

eigenvalue λ_α	symm. sect. (a_Z, a_T, a_R)	scaling operator	numer. Δ_α	exact $\Delta_\alpha^{\text{CFT}}$
1.00000	(0,0,0)	$\mathbb{I}$	0.0000	0
0.91807	(1,0,0)	σ	0.1233	0.125
0.50000	(0,0,0)	ϵ	1.0000	1
0.45918	(1,0,1)	$\partial_x \sigma$	1.1229	1.125
0.45537	(1,1,0)	$\partial_t \sigma$	1.1349	1.125
0.25000	(0,0,0)	h	2.0000	2
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0.25000	(0,0,1)	$\partial_x \epsilon$	2.0000	2
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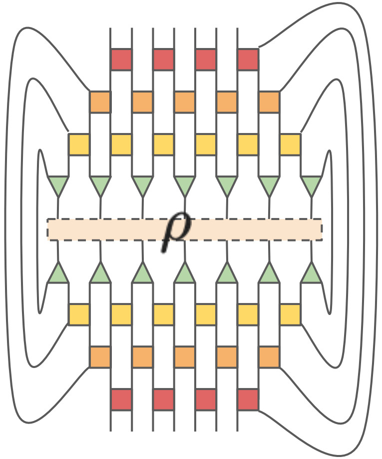
Quantum AI

MERA quantum channel



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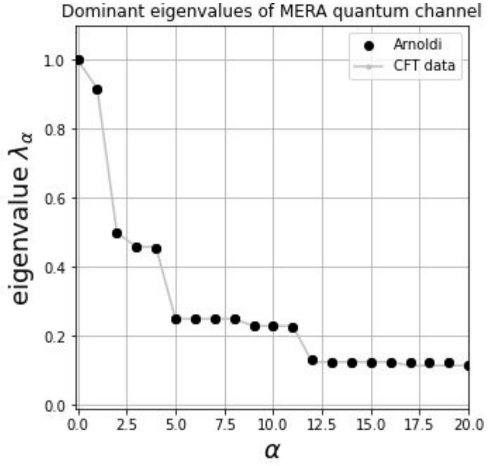

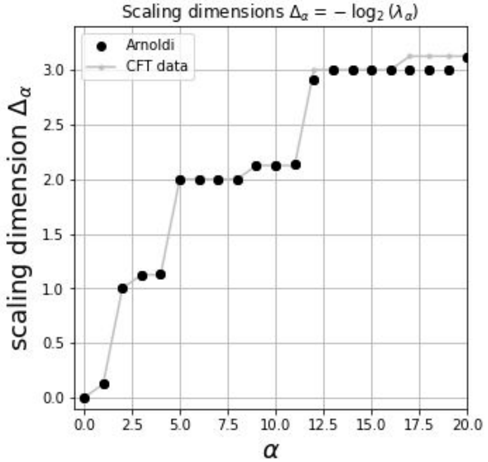

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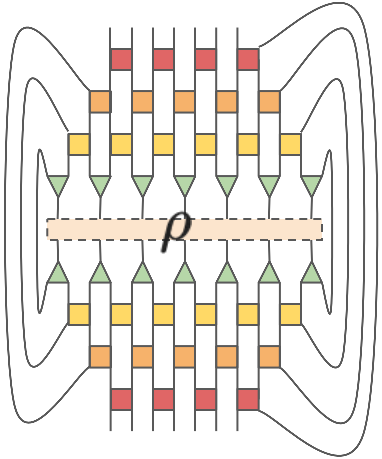
Amazing!!
[Evenbly, White 2016]

Quantum AI

MERA quantum channel



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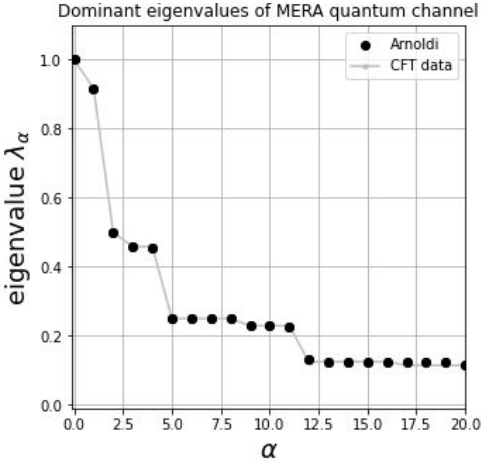
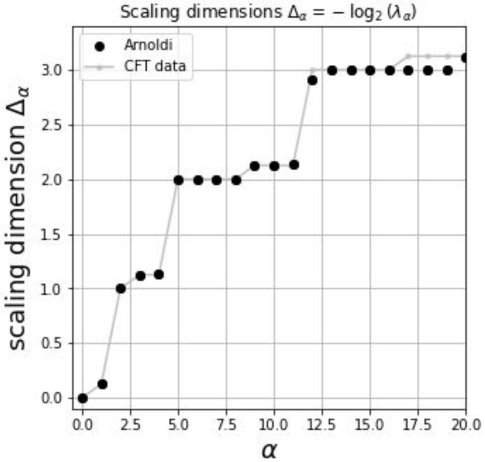
eigenvalues — scaling dimensions

$$\Delta_\alpha = -\log_2(\lambda_\alpha)$$





eigenvalue λ_α	symm. sect. (a_Z, a_T, a_R)	scaling operator	numer. Δ_α	exact $\Delta_\alpha^{\mathrm{CFT}}$
1.00000	(0,0,0)	$\mathbb{I}$	0.0000	0
0.91807	(1,0,0)	σ	0.1233	0.125
0.50000	(0,0,0)	ϵ	1.0000	1
0.45918	(1,0,1)	$\partial_x \sigma$	1.1229	1.125
0.45537	(1,1,0)	$\partial_t \sigma$	1.1349	1.125
0.25000	(0,0,0)	h	2.0000	2
0.25000	(0,1,1)	p	2.0000	2
0.25000	(0,0,1)	$\partial_x \epsilon$	2.0000	2
0.25000	(0,1,0)	$\partial_t \epsilon$	2.0000	2
0.22952	(1,0,0)	$\partial_t^2 \sigma$	2.1233	2.125
0.22921	(1,0,0)	$\partial_x^2 \sigma$	2.1253	2.125
0.22757	(1,1,1)	$\partial_x \partial_t \sigma$	2.1356	2.125
0.12500	(0,0,0)	$\partial_t^2 \epsilon$	3.0000	3
0.12500	(0,0,0)	$\partial_x^2 \epsilon$	3.0000	3
0.12500	(0,1,1)	$\partial_x \partial_t \epsilon$	3.0000	3
0.12500	(0,1,0)	$\partial_x h$	3.0000	3
0.12500	(0,0,1)	$\partial_x p$	3.0000	3

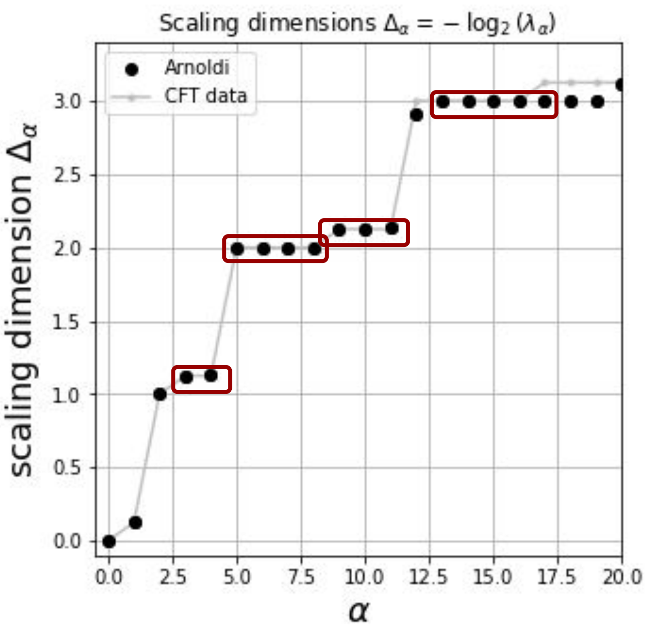
degeneracies!

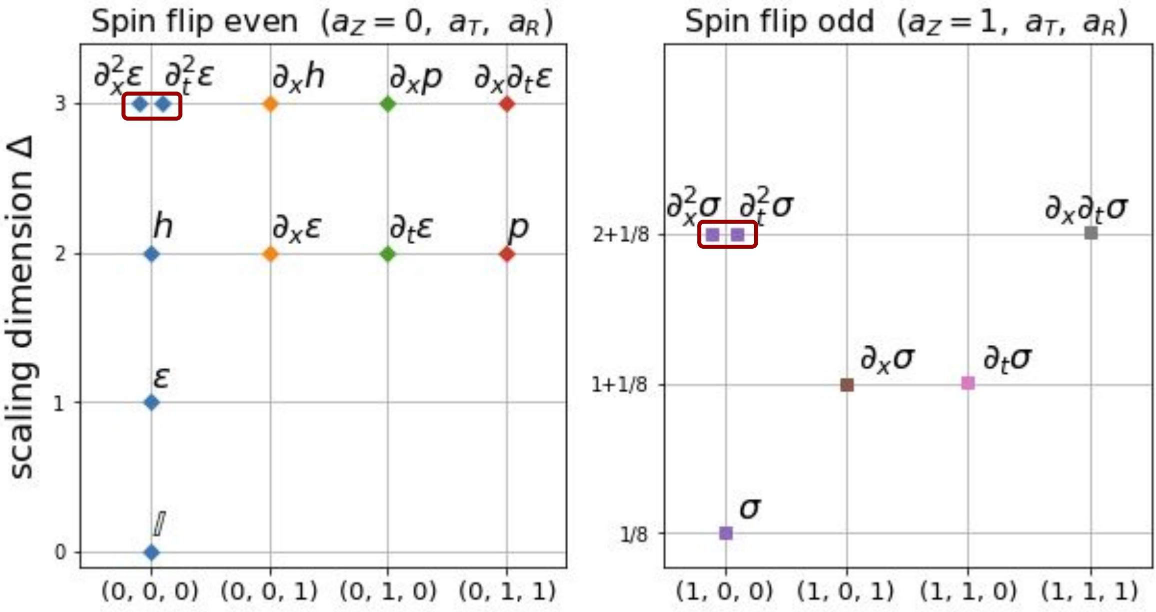
Quantum AI

without symmetries vs with symmetries

$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$

(spin-flip, matrix transposition, space reflection)





symmetries

Hamiltonian $H = -\sum_{i=-\infty}^{\infty} (X_i X_{i+1} - X_{i-1} Z_i X_{i+1})$

$\left[H_{\text{Ising}} = -\sum_{i=-\infty}^{\infty} (X_i X_{i+1} + Z_i) \right]$

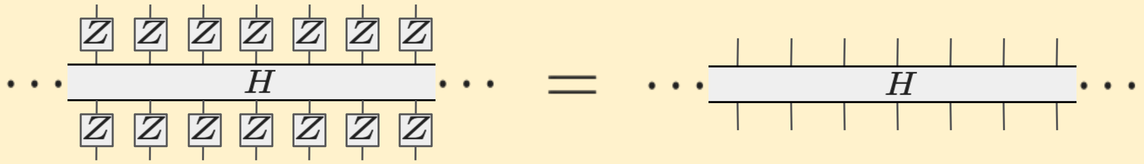
$\mathcal{Z}[Z] = Z$

$\mathcal{Z}[X] = -X$

spin flip

$\mathcal{Z}$

$$\mathcal{Z}[H] = H$$



symmetries

Hamiltonian $H = -\sum_{i=-\infty}^{\infty} \left(X_i X_{i+1} - X_{i-1} Z_i X_{i+1} \right)$

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spin flip
$\mathcal{Z}$

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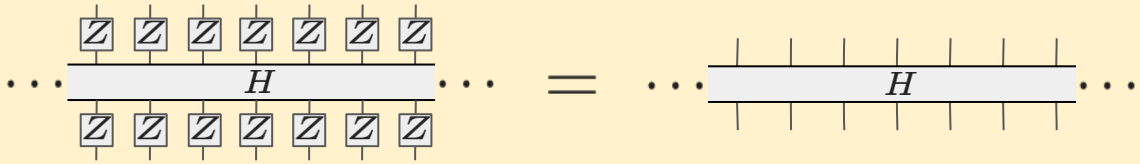
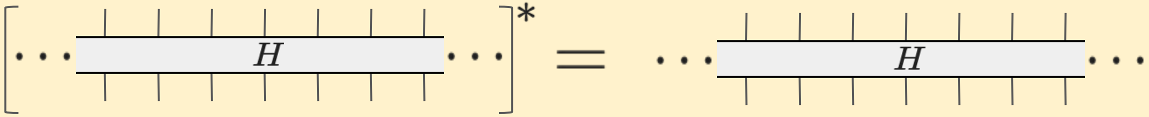


complex conjugation
$*$

$[H]^* = H$



symmetries

Hamiltonian $$H = -\sum_{i=-\infty}^{\infty} \left(X_i X_{i+1} - X_{i-1} Z_i X_{i+1}\right)$$

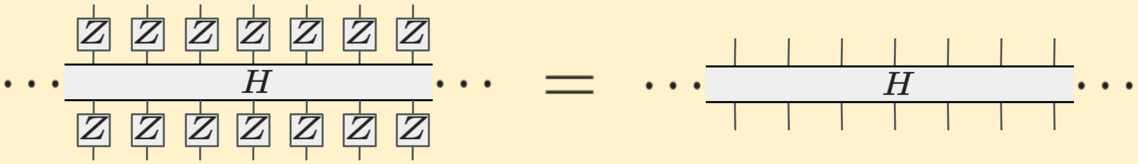
$$\left[H_{\text{Ising}} = -\sum_{i=-\infty}^{\infty} (X_i X_{i+1} + Z_i)\right]$$

$\mathcal{Z}[Z] = Z$

$\mathcal{Z}[X] = -X$

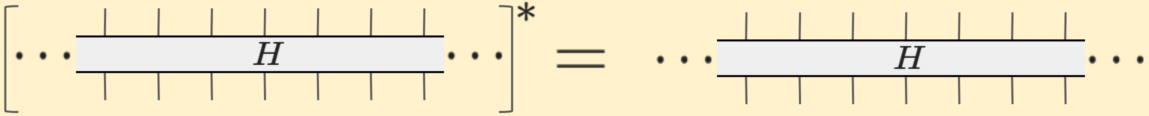
spin flip $\mathcal{Z}$

$$\mathcal{Z}[H] = H$$



complex conjugation $*$

$$[H]^* = H$$



matrix transposition $\mathcal{T}$

$$\mathcal{T}[H] = H$$

Quantum AI

symmetries

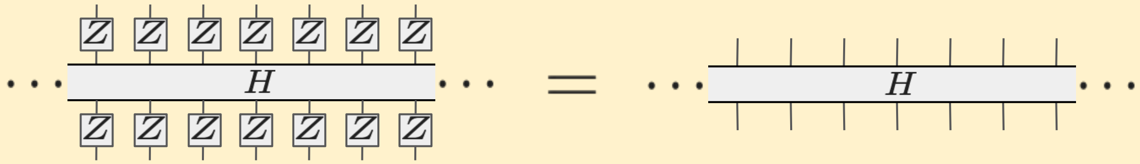
Hamiltonian $H = -\sum_{i=-\infty}^{\infty} (X_i X_{i+1} - X_{i-1} Z_i X_{i+1})$

$\left[H_{\text{Ising}} = -\sum_{i=-\infty}^{\infty} (X_i X_{i+1} + Z_i)\right]$

$\mathcal{Z}[Z] = Z$
$\mathcal{Z}[X] = -X$

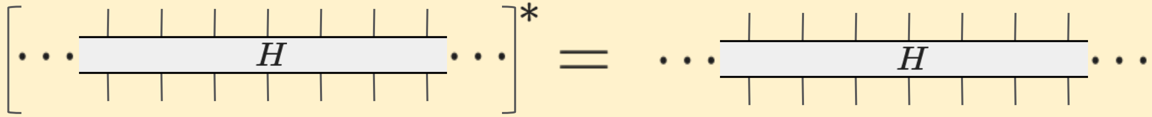
spin flip $\mathcal{Z}$

$\mathcal{Z}[H] = H$



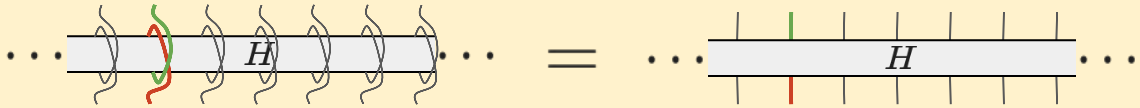
complex conjugation $*$

$[H]^* = H$



matrix transposition $\mathcal{T}$

$\mathcal{T}[H] = H$



space reflection $\mathcal{R}$

$\mathcal{R}[H] = H$

Quantum AI

symmetries

Hamiltonian $H = -\sum_{i=-\infty}^{\infty} (X_i X_{i+1} - X_{i-1} Z_i X_{i+1})$

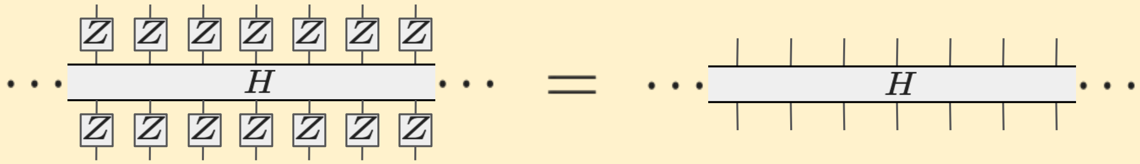
$\left[H_{\text{Ising}} = -\sum_{i=-\infty}^{\infty} (X_i X_{i+1} + Z_i) \right]$

$\mathcal{Z}[Z] = Z$

$\mathcal{Z}[X] = -X$

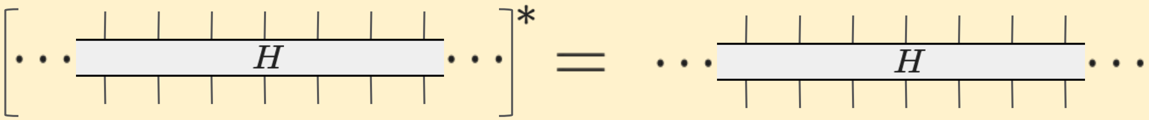
spin flip $\mathcal{Z}$

$\mathcal{Z}[H] = H$



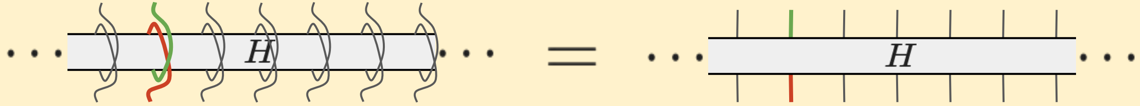
complex conjugation $*$

$[H]^* = H$



matrix transposition $\mathcal{T}$

$\mathcal{T}[H] = H$



space reflection $\mathcal{R}$

$\mathcal{R}[H] = H$

Quantum AI

(+ translation invariance)

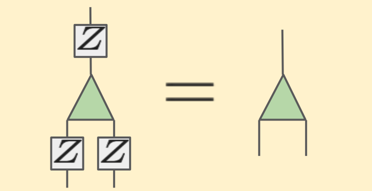
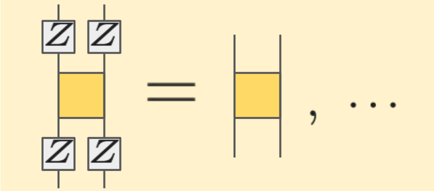
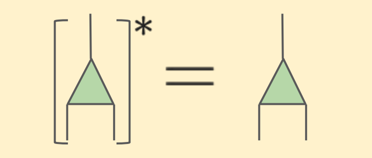
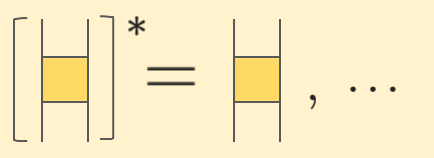
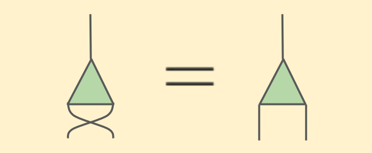
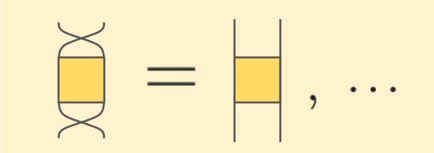
symmetries

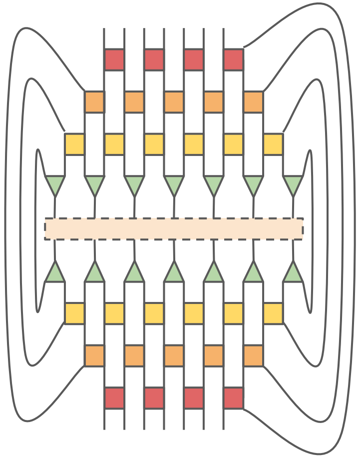
MERA tensors

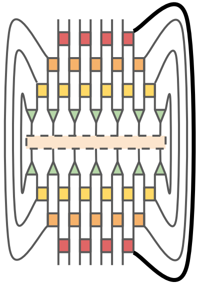
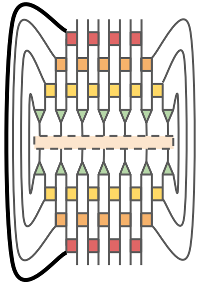
* These symmetries were already included in [Evenbly, White 2016]

$\mathcal{Z}[Z] = Z$

$\mathcal{Z}[X] = -X$

	isometry	disentanglers
spin flip $\mathcal{Z}$		
complex conjugation $*$		
matrix transposition $\mathcal{T}$	–	–
space reflection $\mathcal{R}$		



$\mathcal{C} = \frac{1}{2}$  $+ \frac{1}{2}$ 

Quantum AI

(+ translation invariance)

Symmetries of the MERA quantum channel

One can see that the super-operators

$$\mathcal{C}, \mathcal{Z}, [\,]^*, \mathcal{T}, \mathcal{R}$$

commute with each other.

Symmetries of the MERA quantum channel

$$H = -\sum_{i=-\infty}^{\infty} (X_i X_{i+1} - X_{i-1} Z_i X_{i+1})$$

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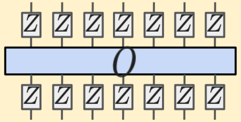
Therefore we can diagonalize the MERA quantum channel $\mathcal{C}$ using **real** 128x128 matrices O that are invariant under **spin-flip**, **matrix transposition** and **space reflection**.

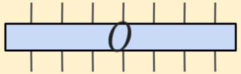
spin flip

$a_Z = 0, 1$ for spin flip even/odd

$$\mathcal{Z}[O] = \pm O$$



$$= (-1)^{a_Z}$$



examples:

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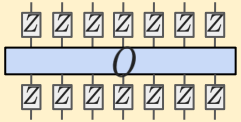
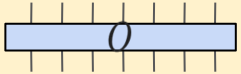
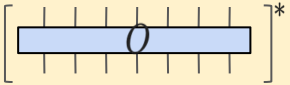


$= (-1)^{a_Z}$

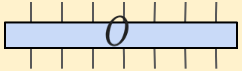


complex conjugation

$$[O]^* = O$$



$=$



examples:

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Symmetries of the MERA quantum channel

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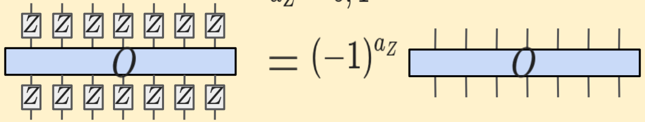
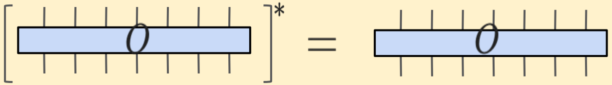


complex conjugation

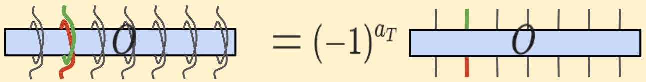
$[O]^* = O$



matrix transposition

$a_T = 0, 1$ for transposition even/odd

$\mathcal{T}[O] = \pm O$



examples:

$\mathcal{Z}[Z] = Z$

$\mathcal{Z}[X] = -X$

$[Z]^* = Z$

$[iY]^* = iY$

$\mathcal{T}[Z] = Z$

$\mathcal{T}[iY] = -iY$

Symmetries of the MERA quantum channel

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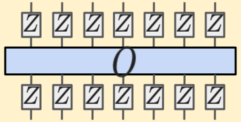
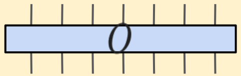
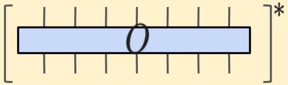
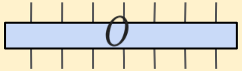
spin flip

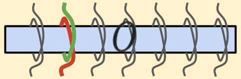
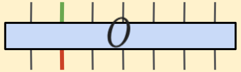
$\mathcal{Z}[O] = \pm O$  $= (-1)^{a_Z}$ 

$a_Z = 0, 1$ for spin flip even/odd

complex conjugation

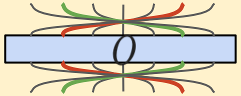
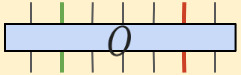
$[O]^* = O$  $=$ 

matrix transposition

$\mathcal{T}[O] = \pm O$  $= (-1)^{a_T}$ 

$a_T = 0, 1$ for transposition even/odd

space reflection

$\mathcal{R}[O] = \pm O$  $= (-1)^{a_R}$ 

$a_R = 0, 1$ for reflection even/odd

examples:

$\mathcal{Z}[Z] = Z$

$\mathcal{Z}[X] = -X$

$[Z]^* = Z$

$[iY]^* = iY$

$\mathcal{T}[Z] = Z$

$\mathcal{T}[iY] = -iY$

$\mathcal{R}[ZZ] = ZZ$

$\mathcal{R}[ZX - XZ] = -(ZX - XZ)$

$$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$$

$$|\hat{\varrho}_\alpha), (\hat{\varphi}_\alpha| \leftrightarrow (a_Z, a_T, a_R)$$

(spin-flip, matrix transposition, space reflection)

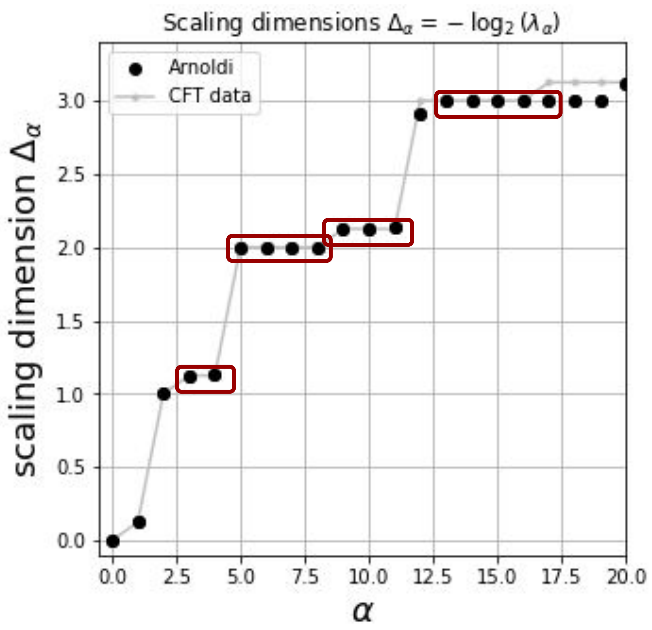
$$\mathcal{C} = \sum_{\alpha=0}^{\chi-1} \lambda_\alpha |\hat{\rho}_\alpha)(\hat{\varphi}_\alpha|$$

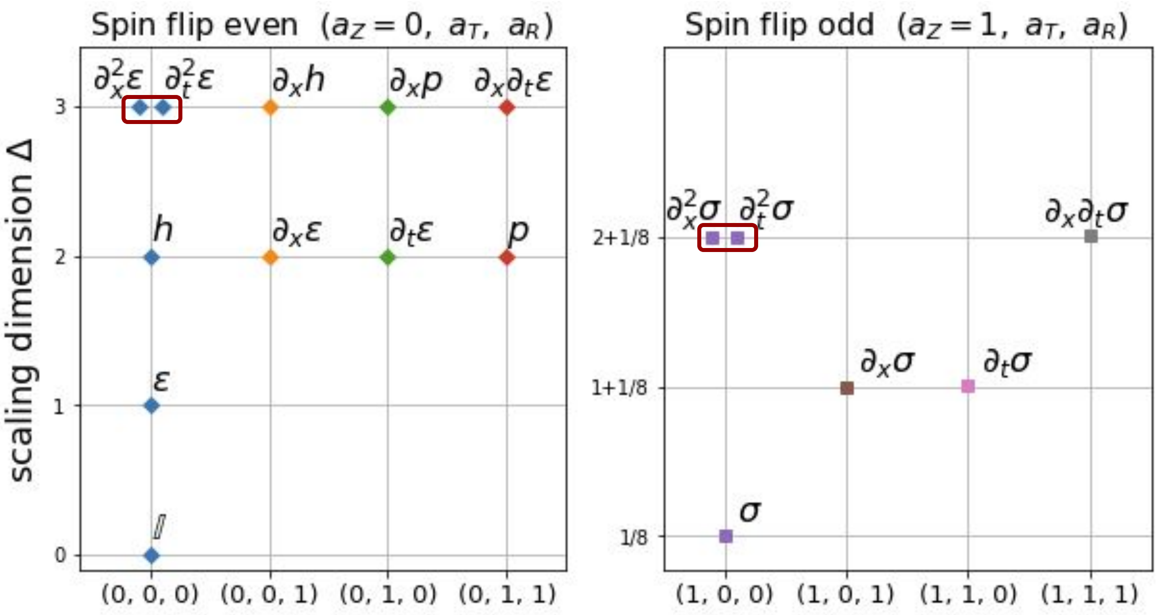
Quantum AI

without symmetries vs with symmetries

$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$

(spin-flip, matrix transposition, space reflection)

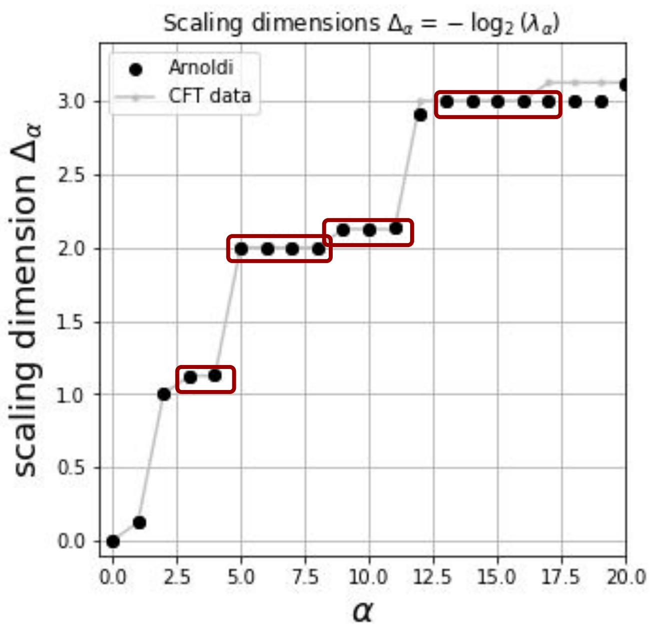


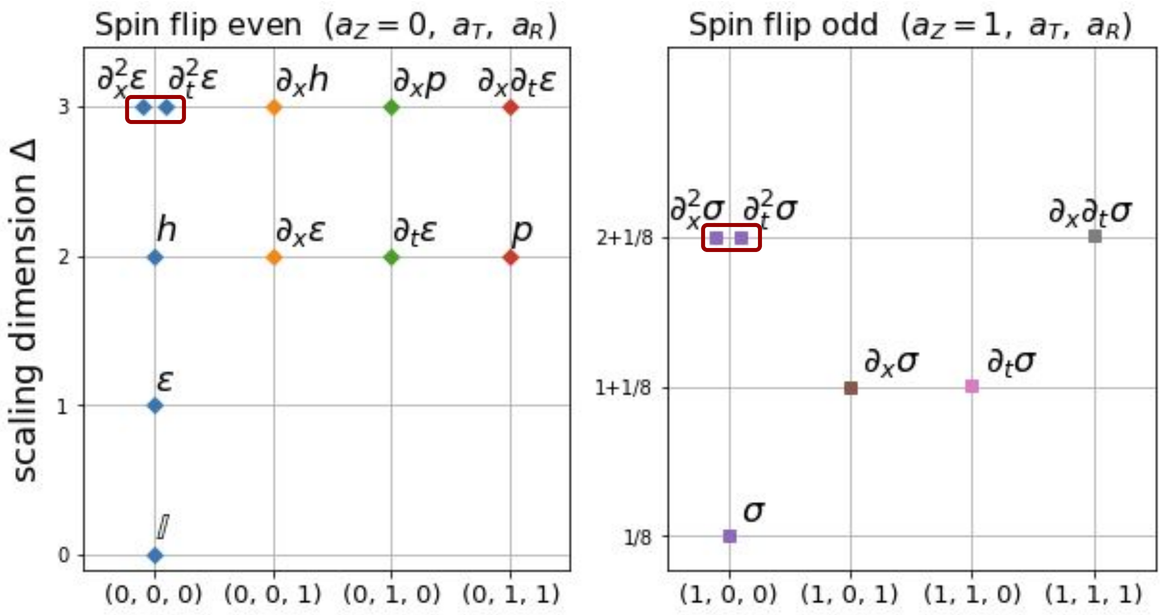


With spin flip $\mathcal{Z}$, matrix transposition $\mathcal{T}$ and space reflection $\mathcal{R}$ symmetries we can identify individual eigen-operators within degenerate multiplets

without symmetries vs with symmetries

$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$
(spin-flip, matrix transposition, space reflection)





With spin flip $\mathcal{Z}$, matrix transposition $\mathcal{T}$ and space reflection $\mathcal{R}$ symmetries we can identify individual eigen-operators within degenerate multiplets

We can still not distinguish second order derivative doublets $(\partial_x^2\sigma, \partial_t^2\sigma)$ and $(\partial_x^2\epsilon, \partial_t^2\epsilon)$

Quantum AI

Derivative descendants

How can we relate a scaling operator with its derivative descendants?

$$(\hat{\varphi}| \overset{?}{\longrightarrow} (\widehat{\partial_t \varphi}|$$

$$(\hat{\varphi}| \longrightarrow (\widehat{\partial_x \varphi}|$$

Derivative descendants

How can we relate a scaling operator with its derivative descendants?

$$(\hat{\varphi}| \; \overset{?}{\longrightarrow} \; (\widehat{\partial_t \varphi}| \qquad (\hat{\varphi}| \longrightarrow (\widehat{\partial_x \varphi}|$$

Given $O, H \longrightarrow \partial_t O = i[H, O]$

Given $O, P \longrightarrow \partial_x O = i[P, O]$

Hamiltonian operator

$$H = -\sum_{i=-\infty}^{\infty} (X_i X_{i+1} - X_{i-1} Z_i X_{i+1})$$

Derivative descendants

How can we relate a scaling operator with its derivative descendants?

$$(\hat{\varphi}| \quad \overset{?}{\longrightarrow} \quad (\widehat{\partial_t \varphi}| , \quad (\widehat{\partial_x \varphi}|$$

Given $O, H \longrightarrow \partial_t O = i[H, O]$

Given $O, P \longrightarrow \partial_x O = i[P, O]$

Hamiltonian operator

$$H = -\sum_{i=-\infty}^{\infty} (X_i X_{i+1} - X_{i-1} Z_i X_{i+1})$$

⇓

Hamiltonian density

$$h_i = -\frac{1}{2}(X_{i-1} X_i + X_i X_{i+1}) + X_{i-1} Z_i X_{i+1}$$

Derivative descendants

How can we relate a scaling operator with its derivative descendants?

$$(\hat{\varphi}| \quad \overset{?}{\longrightarrow} \quad (\widehat{\partial_t \varphi}| \,,\quad (\widehat{\partial_x \varphi}|$$

Given $O, H \longrightarrow \partial_t O = i[H, O]$

Given $O, P \longrightarrow \partial_x O = i[P, O]$

Hamiltonian operator

$$H = -\sum_{i=-\infty}^{\infty} (X_i X_{i+1} - X_{i-1} Z_i X_{i+1})$$

⇩

Hamiltonian density

$$h_i = -\frac{1}{2}(X_{i-1} X_i + X_i X_{i+1}) + X_{i-1} Z_i X_{i+1}$$

⇩

Energy conservation (continuum)

$$\partial_t h + \partial_x p = 0$$

⇨

Energy conservation (lattice)

$$\partial_t h_j = i[H, h_j] = -\left(p_{j+\frac{1}{2}} - p_{j-\frac{1}{2}}\right)$$

⇧

momentum density

$$p_{i+\frac{1}{2}} = -(X_i Y_{i+1} - Y_i X_{i+1})$$

⇧

Momentum operator

$$P = -\sum_{i=-\infty}^{\infty} (X_i Y_{i+1} - Y_i X_{i+1})$$

Derivative descendants

How can we relate a scaling operator with its derivative descendants?

$$(\hat{\varphi}| \;\overset{?}{\longrightarrow}\; (\widehat{\partial_t \varphi}| ,\quad (\widehat{\partial_x \varphi}|$$

Given $O, H \longrightarrow \partial_t O = i[H, O]$

Given $O, P \longrightarrow \partial_x O = i[P, O]$

(cheap alternative: use finite difference)

Hamiltonian operator

$$H = -\sum_{i=-\infty}^{\infty} (X_i X_{i+1} - X_{i-1} Z_i X_{i+1})$$

Momentum operator

$$P = -\sum_{i=-\infty}^{\infty} (X_i Y_{i+1} - Y_i X_{i+1})$$

$$\underset{(a_Z, a_T, a_R)}{(\hat{\varphi}|} \longrightarrow \underset{(a_Z, \boxed{\bar{a}_T}, a_R)}{(\widehat{\partial_t \varphi}|}$$

$$\underset{(a_Z, a_T, a_R)}{(\hat{\varphi}|} \longrightarrow \underset{(a_Z, a_T, \boxed{\bar{a}_R})}{(\widehat{\partial_x \varphi}|}$$

Example: $(\hat{\sigma}|$ $(1, 0, 0)$

$$(\partial_t \hat{\sigma}| = i[H, (\hat{\sigma}|] \sim (\widehat{\partial_t \sigma}|$$

$(1, 1, 0)$

$$(\partial_x \hat{\sigma}| = i[P, (\hat{\sigma}|] \sim (\widehat{\partial_x \sigma}|$$

$(1, 0, 1)$

Example: $(\hat{\sigma}|$ $(1,0,0)$

$$(\partial_t \hat{\sigma}| = i[H, (\hat{\sigma}|] \sim (\widehat{\partial_t \sigma}| \quad (1,1,0)$$

$$(\partial_x \hat{\sigma}| = i[P, (\hat{\sigma}|] \sim (\widehat{\partial_x \sigma}| \quad (1,0,1)$$

$$(\partial_t \widehat{\partial_t \sigma}| = i[H, (\widehat{\partial_t \sigma}|] \sim (\widehat{\partial_t^2 \sigma}| \quad (1,0,0)$$

$$\left.\begin{aligned} &(\partial_x \widehat{\partial_t \sigma}| = i[P, (\widehat{\partial_t \sigma}|] \\ &(\partial_t \widehat{\partial_x \sigma}| = i[H, (\widehat{\partial_x \sigma}|] \end{aligned}\right\} \sim (\widehat{\partial_t \partial_x \sigma}| = (\widehat{\partial_x \partial_t \sigma}| \quad (1,1,1)$$

$$(\partial_x \widehat{\partial_x \sigma}| = i[P, (\widehat{\partial_x \sigma}|] \sim (\widehat{\partial_x^2 \sigma}| \quad (1,0,0)$$

Example: $(\hat{\sigma}|$ (1, 0, 0)

$$(\partial_t \hat{\sigma}| = i[H, (\hat{\sigma}|] \sim (\widehat{\partial_t \sigma}| \quad (1,1,0)$$

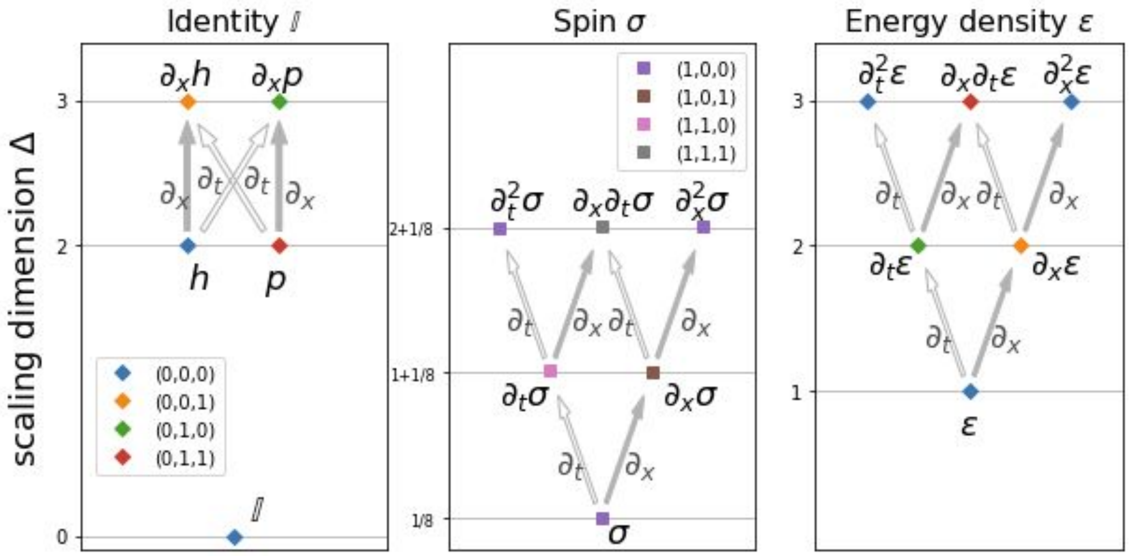
$$(\partial_x \hat{\sigma}| = i[P, (\hat{\sigma}|] \sim (\widehat{\partial_x \sigma}| \quad (1,0,1)$$

$$(\partial_t \widehat{\partial_t \sigma}| = i[H, (\widehat{\partial_t \sigma}|] \sim (\widehat{\partial_t^2 \sigma}| \quad (1,0,0)$$

$$\left.\begin{array}{l}(\partial_x \widehat{\partial_t \sigma}| = i[P, (\widehat{\partial_t \sigma}|] \\ (\partial_t \widehat{\partial_x \sigma}| = i[H, (\widehat{\partial_x \sigma}|]\end{array}\right\} \sim (\widehat{\partial_t \partial_x \sigma}| = (\widehat{\partial_x \partial_t \sigma}| \quad (1,1,1)$$

$$(\partial_x \widehat{\partial_x \sigma}| = i[P, (\widehat{\partial_x \sigma}|] \sim (\widehat{\partial_x^2 \sigma}| \quad (1,0,0)$$

Final numerical identification of eigen-operators:

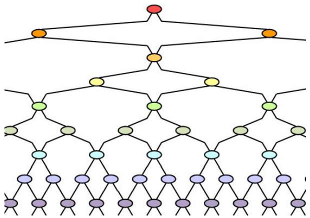


Quantum AI

Outline

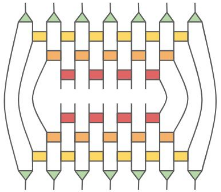
1 - Motivation:

- MERA on qubits (q-MERA)



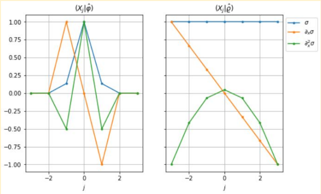
2 - MERA quantum channel

- Eigenvalue decomposition
- Symmetries
- Derivative descendants



3 - Emergent structures in the causal cone

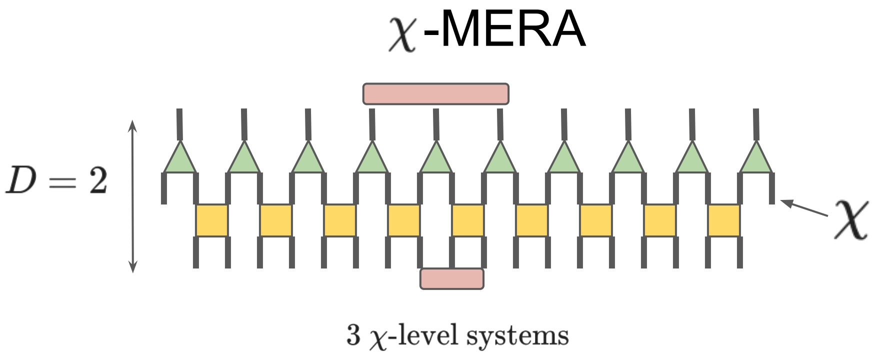
- Space resolved patterns
- MPO for channel eigen-operators

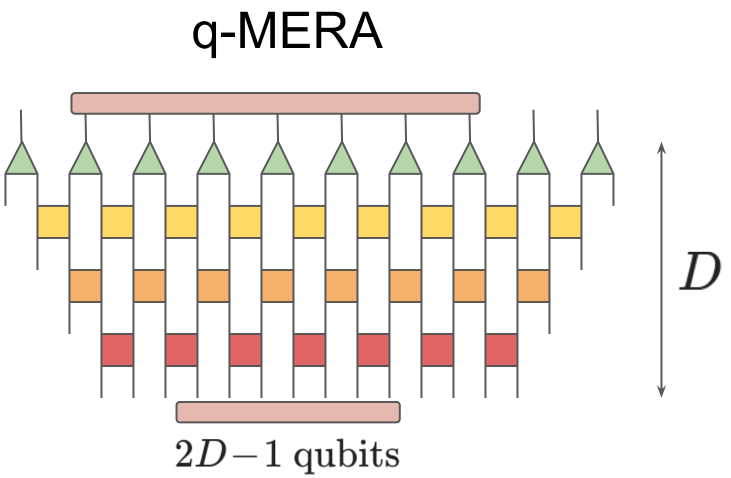


Emergent structure in *primal* and dual eigen-operators

traditional numerical simulations:

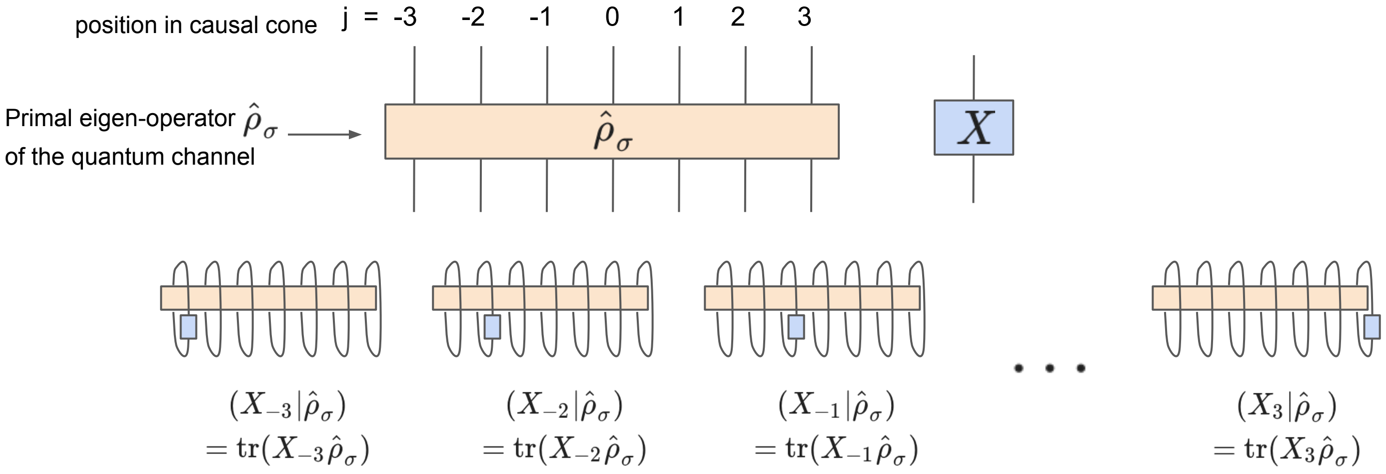
motivated by implementation with quantum processors:



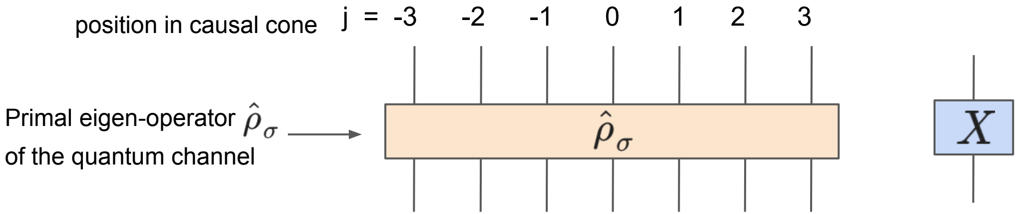


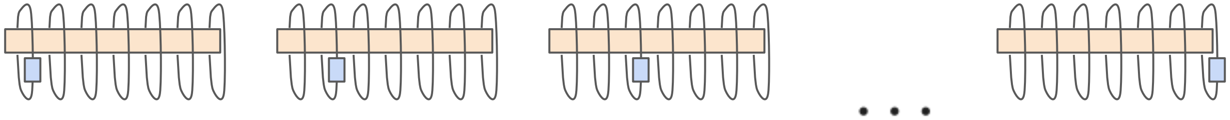
We can now investigate the space-resolved structure of the eigen-operators

Emergent structure in *primal* eigen-operators



Emergent structure in *primal* eigen-operators





$(X_{-3}|\hat{\rho}_\sigma) = \mathrm{tr}(X_{-3}\hat{\rho}_\sigma)$

$(X_{-2}|\hat{\rho}_\sigma) = \mathrm{tr}(X_{-2}\hat{\rho}_\sigma)$

$(X_{-1}|\hat{\rho}_\sigma) = \mathrm{tr}(X_{-1}\hat{\rho}_\sigma)$

. . .

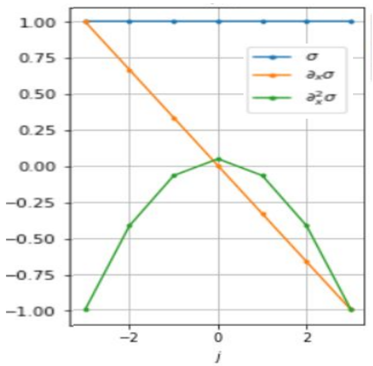
$(X_{3}|\hat{\rho}_\sigma) = \mathrm{tr}(X_{3}\hat{\rho}_\sigma)$

Empirical results:

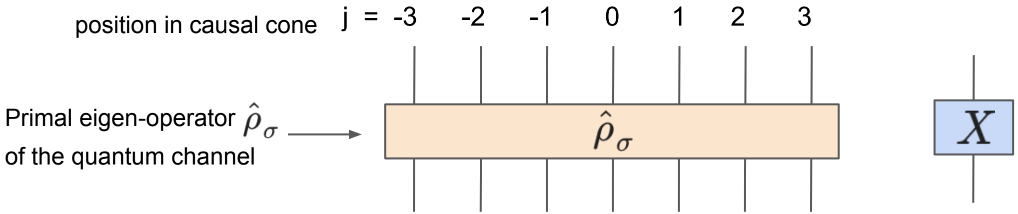
$$(X_j|\hat{\rho}_\sigma) = \mathrm{tr}(X_j\hat{\rho}_\sigma) \sim a_0$$

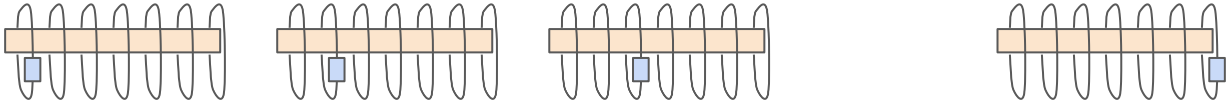
$$(X_j|\hat{\rho}_{\partial_x\sigma}) = \mathrm{tr}(X_j\ \hat{\rho}_{\partial_x\sigma}) \sim a_1\ j$$

$$(X_j|\hat{\rho}_{\partial_x^2\sigma}) = \mathrm{tr}(X_j\ \hat{\rho}_{\partial_x^2\sigma}) \sim a_2\ j^2$$



Emergent structure in *primal* eigen-operators





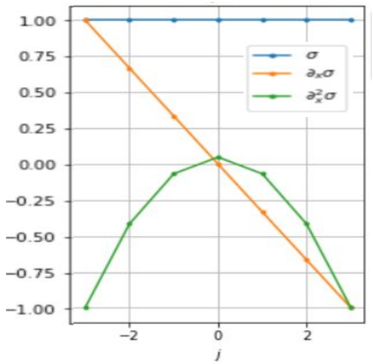
$(X_{-3}|\hat{\rho}_\sigma) = \mathrm{tr}(X_{-3}\hat{\rho}_\sigma)$ $\quad$ $(X_{-2}|\hat{\rho}_\sigma) = \mathrm{tr}(X_{-2}\hat{\rho}_\sigma)$ $\quad$ $(X_{-1}|\hat{\rho}_\sigma) = \mathrm{tr}(X_{-1}\hat{\rho}_\sigma)$ $\quad$ . . . $\quad$ $(X_{3}|\hat{\rho}_\sigma) = \mathrm{tr}(X_{3}\hat{\rho}_\sigma)$

Empirical results:

$$(X_j|\hat{\rho}_\sigma) = \mathrm{tr}(X_j\hat{\rho}_\sigma) \sim a_0$$

$$(X_j|\hat{\rho}_{\partial_x\sigma}) = \mathrm{tr}(X_j\ \hat{\rho}_{\partial_x\sigma}) \sim a_1\ j$$

$$(X_j|\hat{\rho}_{\partial_x^2\sigma}) = \mathrm{tr}(X_j\ \hat{\rho}_{\partial_x^2\sigma}) \sim a_2\ j^2$$

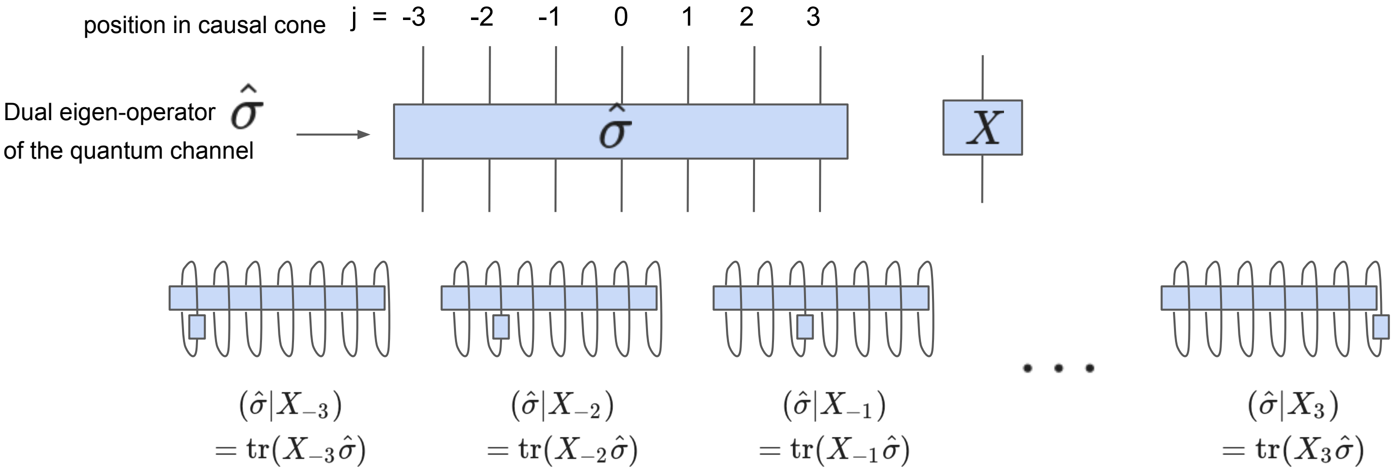


General expression (Taylor expansion of scaling fields):

$$(X_j|\hat{\rho}_{\partial_x^n\sigma}) = \mathrm{tr}(X_j\ \hat{\rho}_{\partial_x^k\sigma}) \sim a_k\ j^k$$

Application: we can finally distinguish between $\partial_x^2\sigma$ and $\partial_t^2\sigma$!!

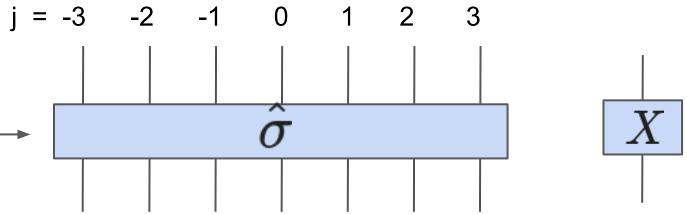
Emergent structure in *dual* eigen-operators

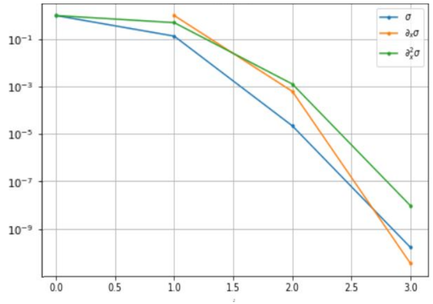


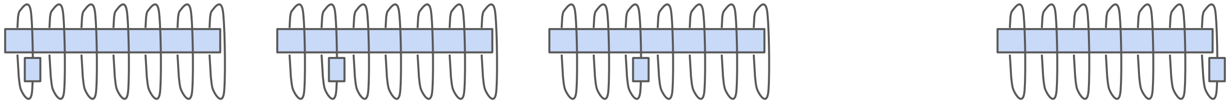
Emergent structure in *dual* eigen-operators

position in causal cone

Dual eigen-operator $\hat{\sigma}$
of the quantum channel







$(\hat{\sigma}|X_{-3})$
$= \mathrm{tr}(X_{-3}\hat{\sigma})$

$(\hat{\sigma}|X_{-2})$
$= \mathrm{tr}(X_{-2}\hat{\sigma})$

$(\hat{\sigma}|X_{-1})$
$= \mathrm{tr}(X_{-1}\hat{\sigma})$

. . .

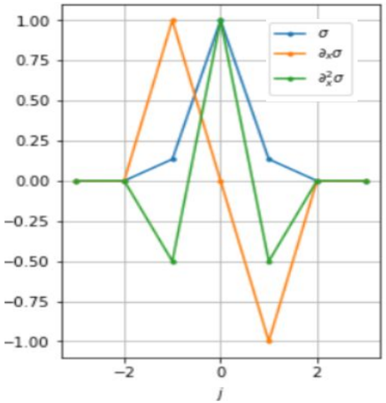
$(\hat{\sigma}|X_{3})$
$= \mathrm{tr}(X_{3}\hat{\sigma})$

Empirical results:

$$(\hat{\sigma}|X_j) \sim \exp\left(-\frac{1}{2}(j/a_0)^2\right)$$

$$(\widehat{\partial_x \sigma}|X_j) \sim j\exp\left(-\frac{1}{2}(j/a_1)^2\right)$$

$$(\widehat{\partial_x^2 \sigma}|X_j) \sim (j^2 - b)\exp\left(-\frac{1}{2}(j/a_2)^2\right)$$

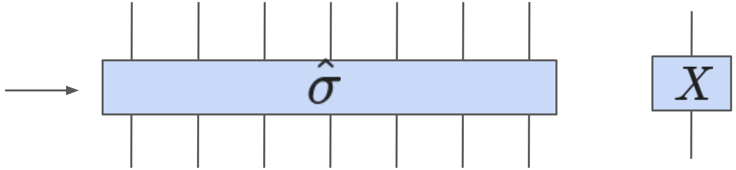


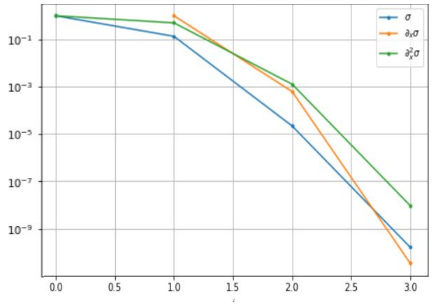
Quantum AI

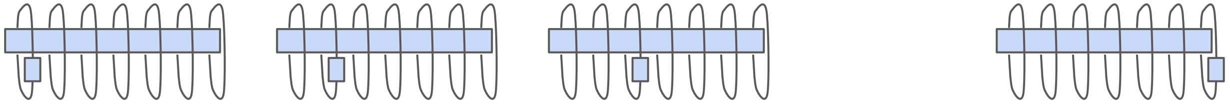
Emergent structure in *dual* eigen-operators

position in causal cone j = -3 -2 -1 0 1 2 3

Dual eigen-operator $\hat{\sigma}$ of the quantum channel →







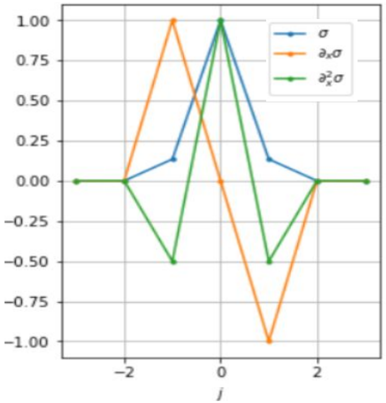
$(\hat{\sigma}|X_{-3}) = \mathrm{tr}(X_{-3}\hat{\sigma})$ $(\hat{\sigma}|X_{-2}) = \mathrm{tr}(X_{-2}\hat{\sigma})$ $(\hat{\sigma}|X_{-1}) = \mathrm{tr}(X_{-1}\hat{\sigma})$ . . . $(\hat{\sigma}|X_{3}) = \mathrm{tr}(X_{3}\hat{\sigma})$

Empirical results:

$$(\hat{\sigma}|X_j) \sim \exp\left(-\frac{1}{2}(j/a_0)^2\right)$$

$$(\widehat{\partial_x\sigma}|X_j) \sim j\exp\left(-\frac{1}{2}(j/a_1)^2\right)$$

$$(\widehat{\partial_x^2\sigma}|X_j) \sim (j^2-b)\exp\left(-\frac{1}{2}(j/a_2)^2\right)$$

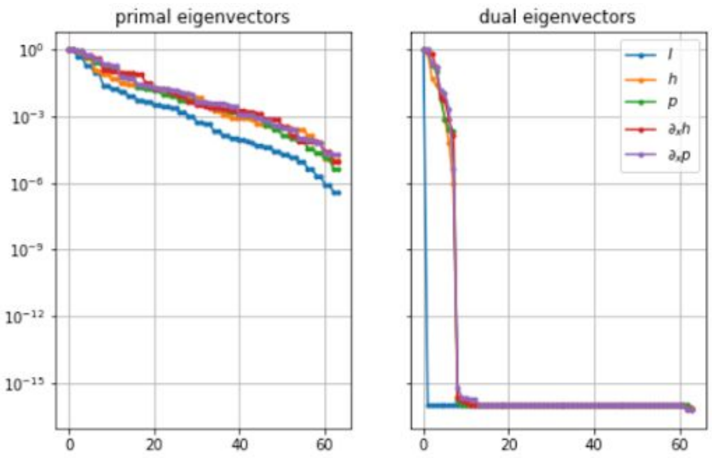


General expression (polynomial from finite differences):

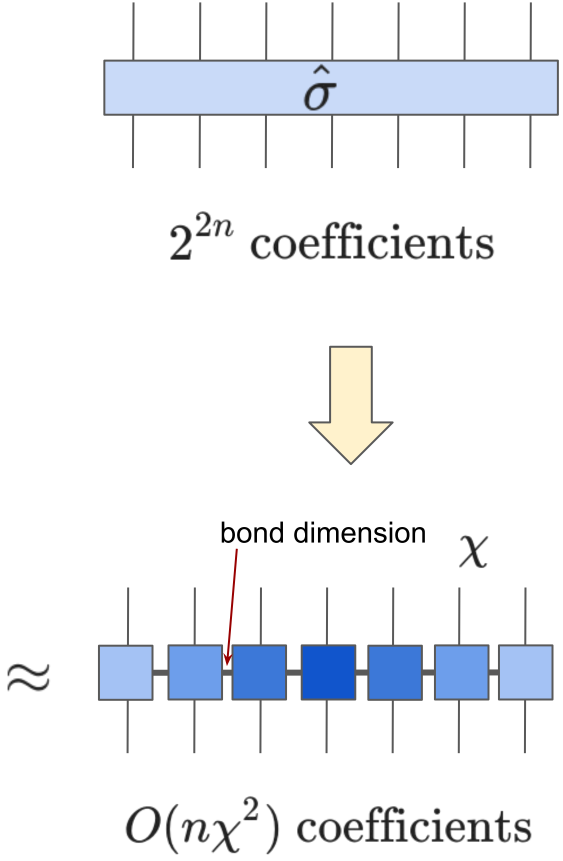
$$(\widehat{\partial_x^k\sigma}|X_j) \sim P_k(j)\exp\left(-\frac{1}{2}(j/a_k)^2\right)$$

$$\hat{\sigma} \approx X_0 \qquad \widehat{\partial_x\sigma} \approx \frac{X_1 - X_{-1}}{2} \qquad \widehat{\partial_x^2\sigma} \approx \frac{X_1 - 2X_0 + X_{-1}}{4}$$

Emergent structure in *dual* eigen-operators



Practical application: given operator entanglement structure, we can approximate with a matrix product operator (MPO)!

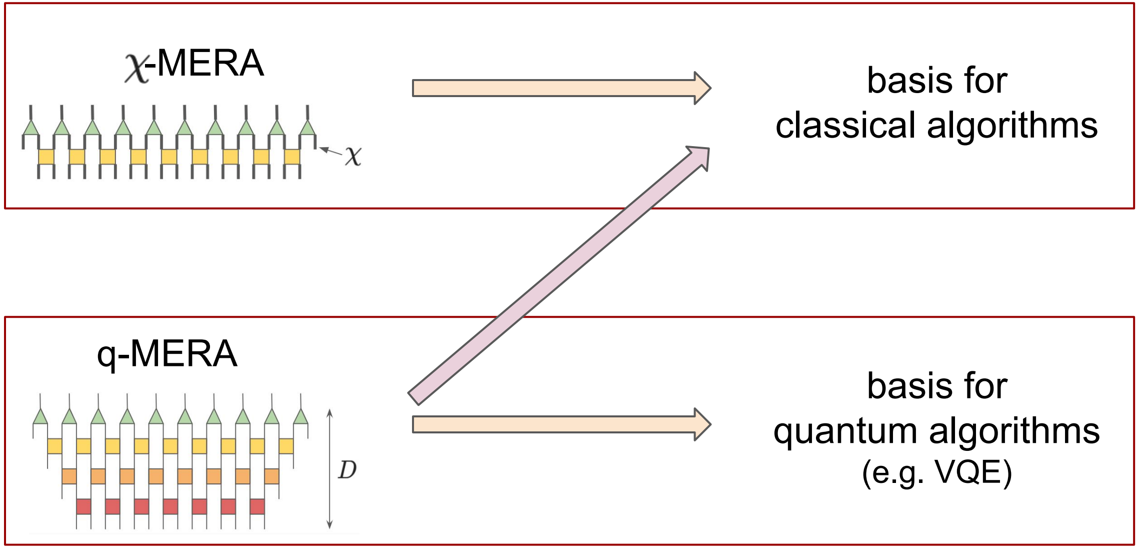


Summary

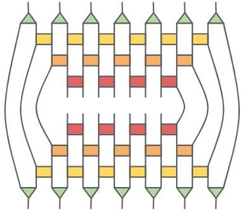
- Quantum Computer vs Tensor Networks?
- Quantum Computers can accelerate Tensor Networks
- MERA is already a quantum circuit (but χ-MERA $\rightarrow$ q-MERA)

In collaboration with:

Riley Chien
PhD at Dartmouth College (May 2023)
Student researcher at Google Quantum AI



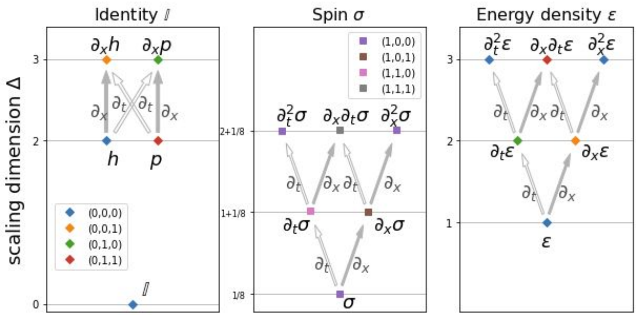
Summary



We diagonalized ***n-qubit*** MERA quantum channel
(for **n=7** instead of n=3 ⟶ we can now resolve in space)

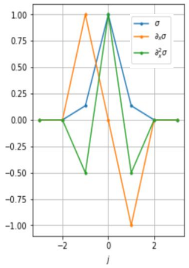
In collaboration with:

Riley Chien
PhD at Dartmouth College (May 2023)
Student researcher at Google Quantum AI



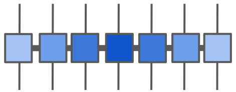
Symmetries help identify eigen-operators with CFT scaling operators

Space-time *derivatives* connect descendant eigen-operators



We discovered space-resolved *emergent properties* which allow us to

- *distinguish between derivative descendants* where symmetries are not enough
- suggest more *efficient MERA algorithm* (based on MPO)



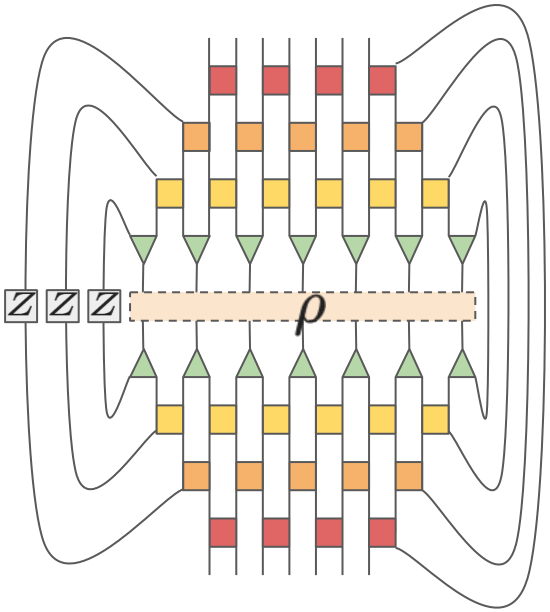
Quantum AI

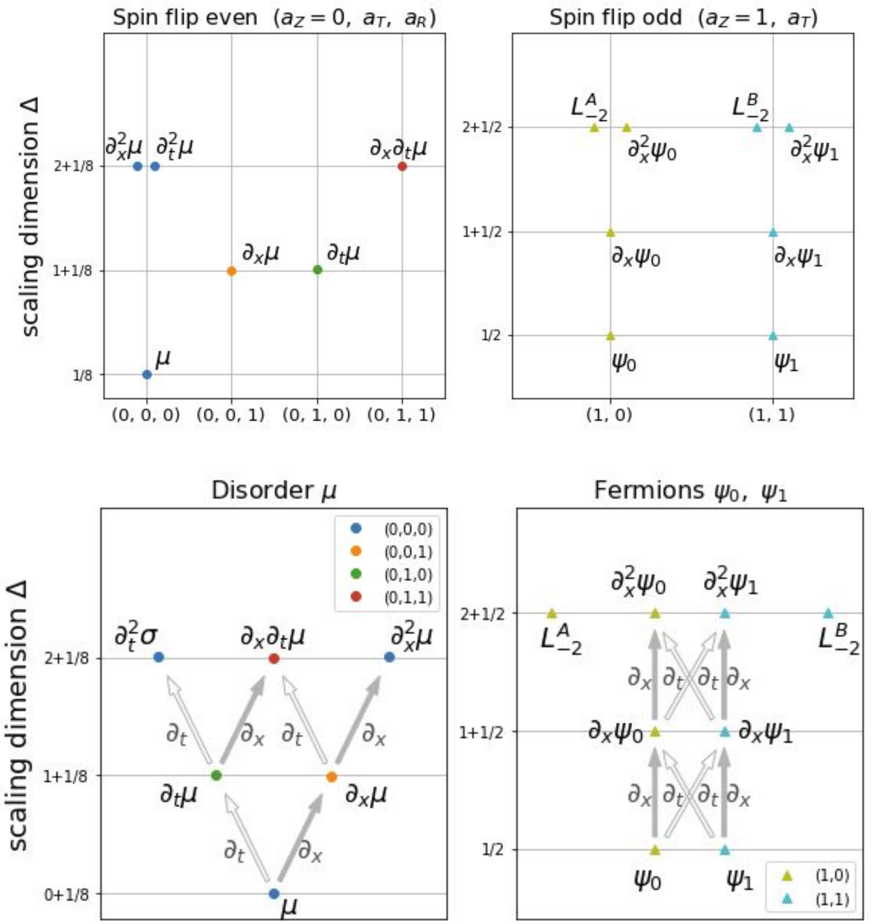
THANKS!

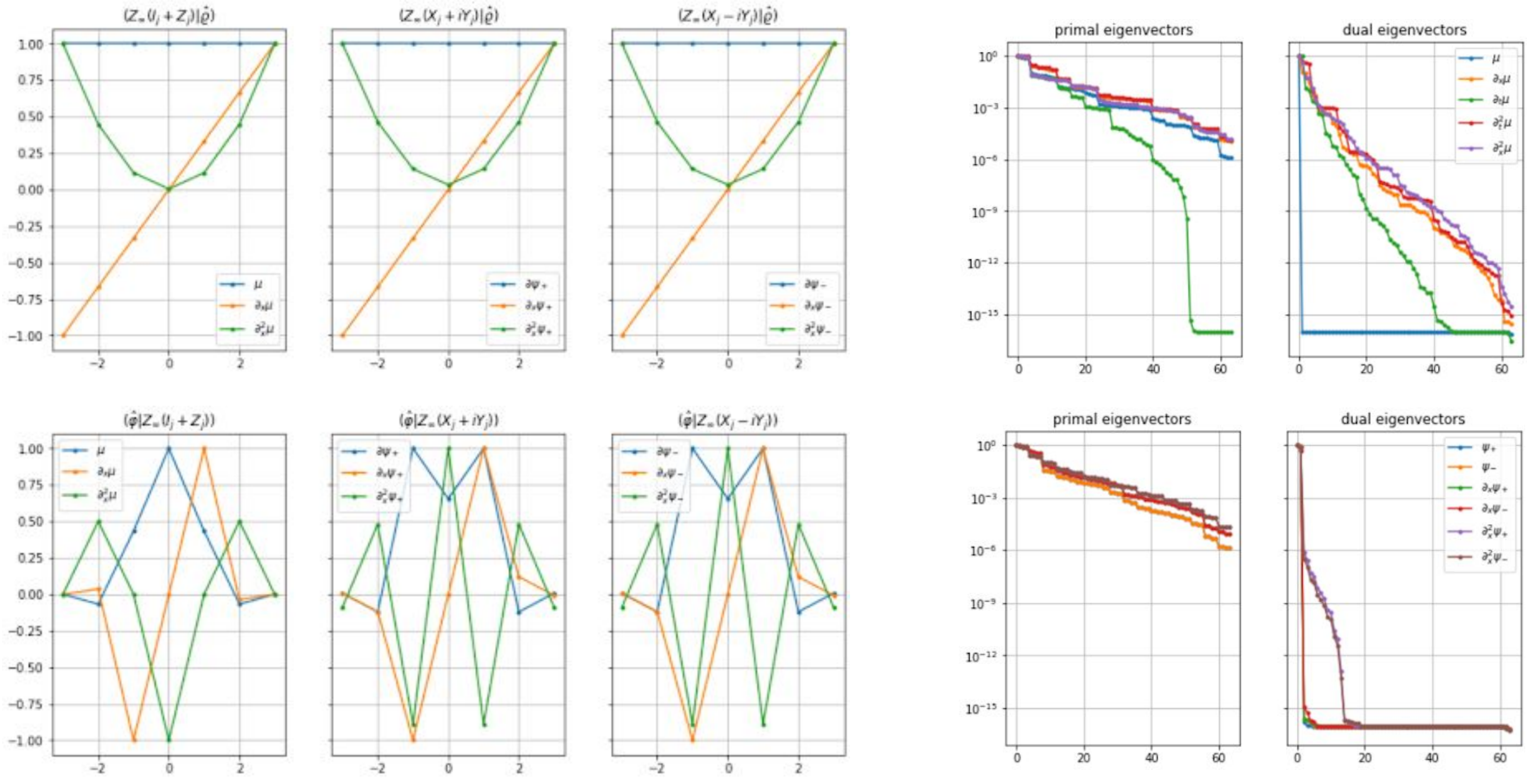
Fermionic super-operator

for non-local operators $\cdots ZZZZZ\, O$

$$\rho' = \mathcal{F}[\rho]$$






$\langle Z_\infty(I_j+Z_j)|\hat{\varrho}\rangle$
$\langle Z_\infty(X_j+iY_j)|\hat{\varrho}\rangle$
$\langle Z_\infty(X_j-iY_j)|\hat{\varrho}\rangle$
μ
$\partial_x\mu$
$\partial_x^2\mu$
$\partial\psi_+$
$\partial_x\psi_+$
$\partial_x^2\psi_+$
$\partial\psi_-$
$\partial_x\psi_-$
$\partial_x^2\psi_-$
primal eigenvectors
dual eigenvectors
μ
$\partial_x\mu$
$\partial_t\mu$
$\partial_t^2\mu$
$\partial_x^2\mu$
$\langle\hat{\varphi}|Z_\infty(I_j+Z_j)\rangle$
$\langle\hat{\varphi}|Z_\infty(X_j+iY_j)\rangle$
$\langle\hat{\varphi}|Z_\infty(X_j-iY_j)\rangle$
primal eigenvectors
dual eigenvectors
ψ_+
ψ_-
$\partial_x\psi_+$
$\partial_x\psi_-$
$\partial_x^2\psi_+$
$\partial_x^2\psi_-$

Quantum AI