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CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

Luca Tagliacozzo

S. Carignano (BSC), C. Ramos (UB) arXiv:2306.xxxx
M. Piani (evolutionQ) J. Surace (ICFO) arXiv:1810.01231 PRB 2019
M. Frias-Perez (MPQ) MC. Bañuls (MPQ) arXiv:2306.xxxx

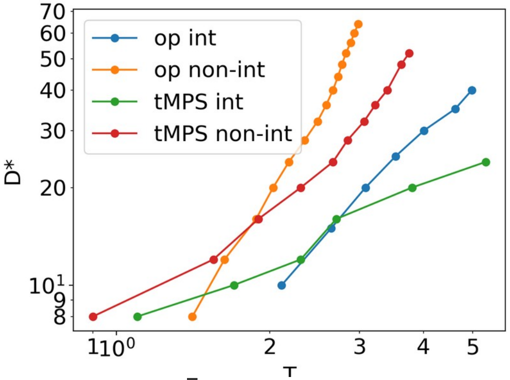
Tensor networks algorithms and their cost for out-of-equilibrium dynamics

Plan de Recuperación,
Transformación y Resiliencia

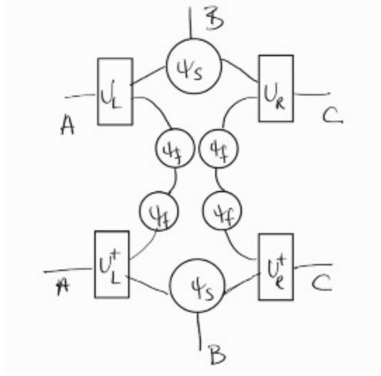
PGC2018-095862-B-C22
PID2021-127968NB-I00
TED2021-130552B-C22

GOBIERNO DE ESPAÑA
MINISTERIO DE CIENCIA E INNOVACIÓN

- Upper bound the scaling of the temporal entanglement with the one of the operator entanglement



- Algorithm that provides local access to the the evolved state with finite bond dimension



- Out of equilibrium
- Entanglement barrier
- On temporal entanglement
  - The transition matrix T
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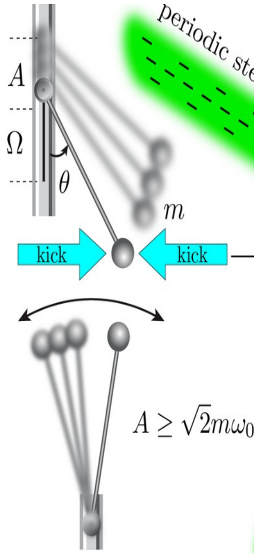
  - The upper bound to the rank of T
- Quasi-particles
- Trading entanglement for mixture
  - Unveiling local equilibration
  - Identifying fast degrees of freedom
  - Relation with decoherence

Classical:

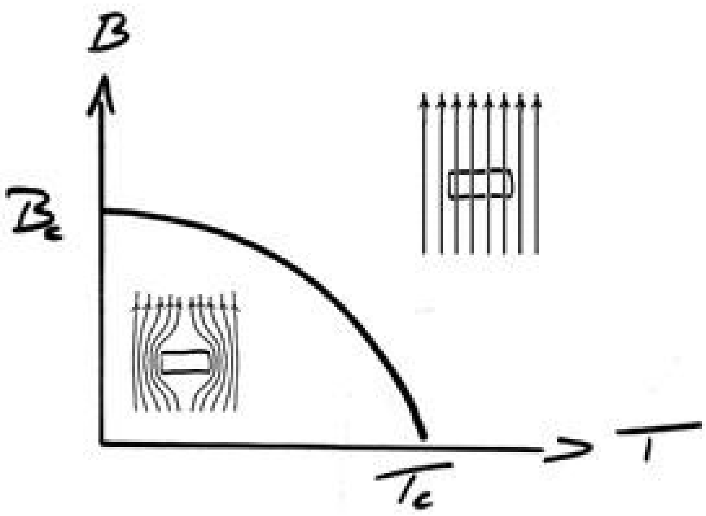
Kapitsa pendulum

Bukov

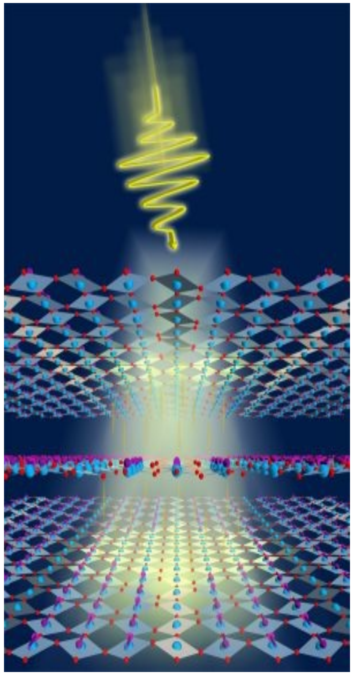


Quantum:

Light induced superconductor



Cavallieri



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$$H = \sum_i h_i$$

$$U(T) = \exp\left(-iHT\right)$$

$$O(T) = U^\dagger(T) O_i U(T)$$

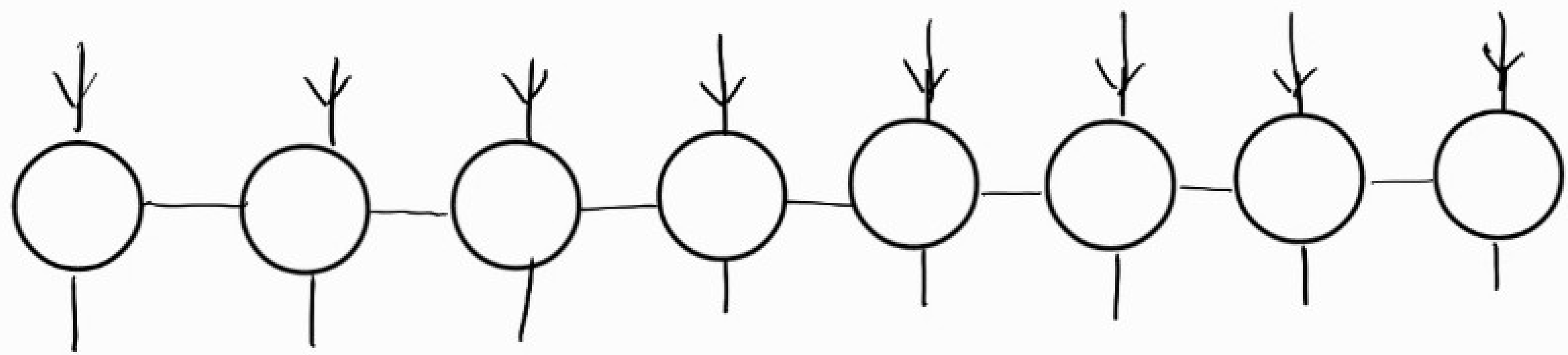
$$\langle \psi_0 | O(T) | \psi_0 \rangle$$

$$U(T) = \exp(-iHT)$$

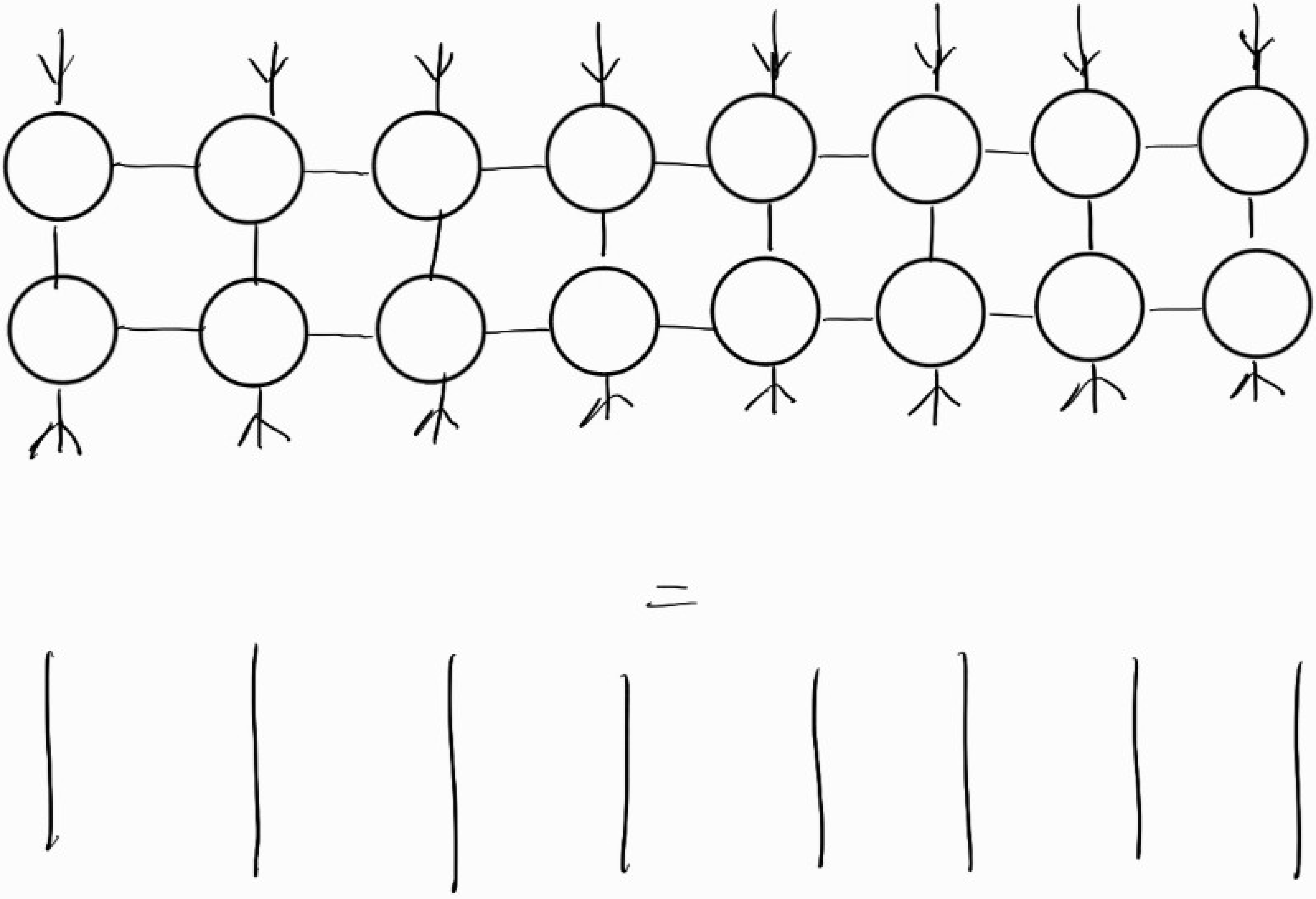
$$\delta t = t/N \ll 1$$

$$U(T) \simeq U(\delta t)^N$$

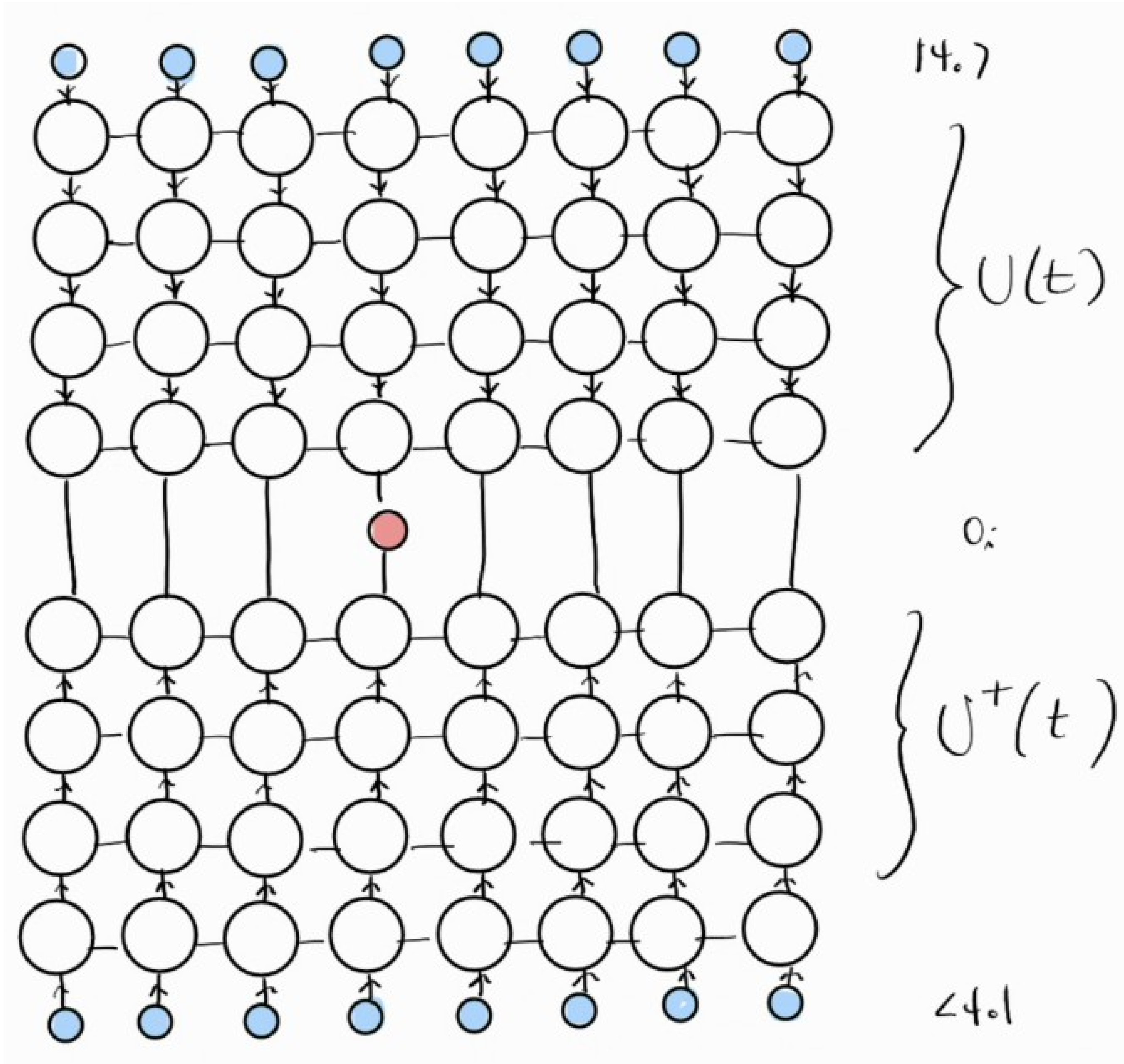
$$U(\delta t) =$$

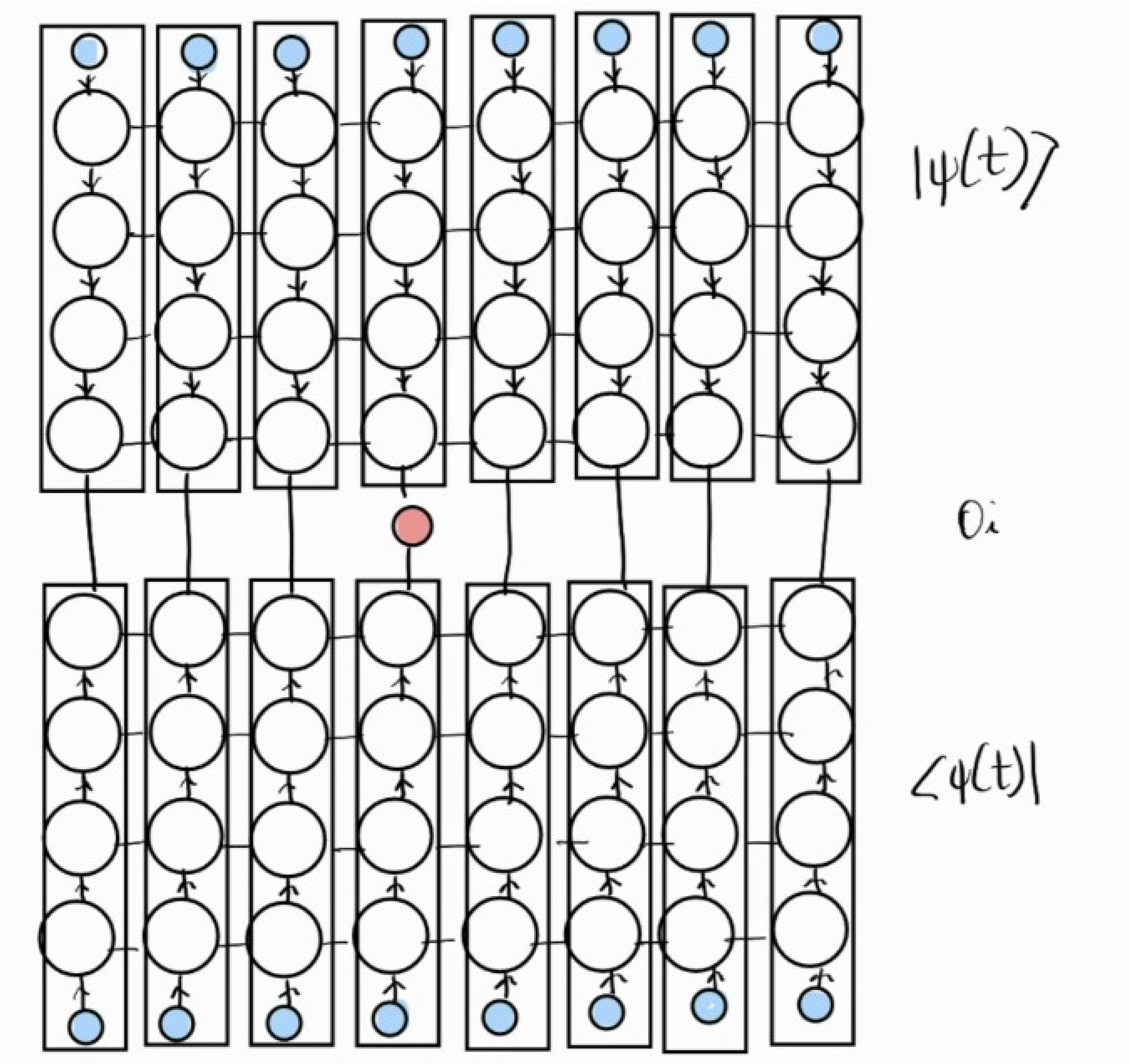


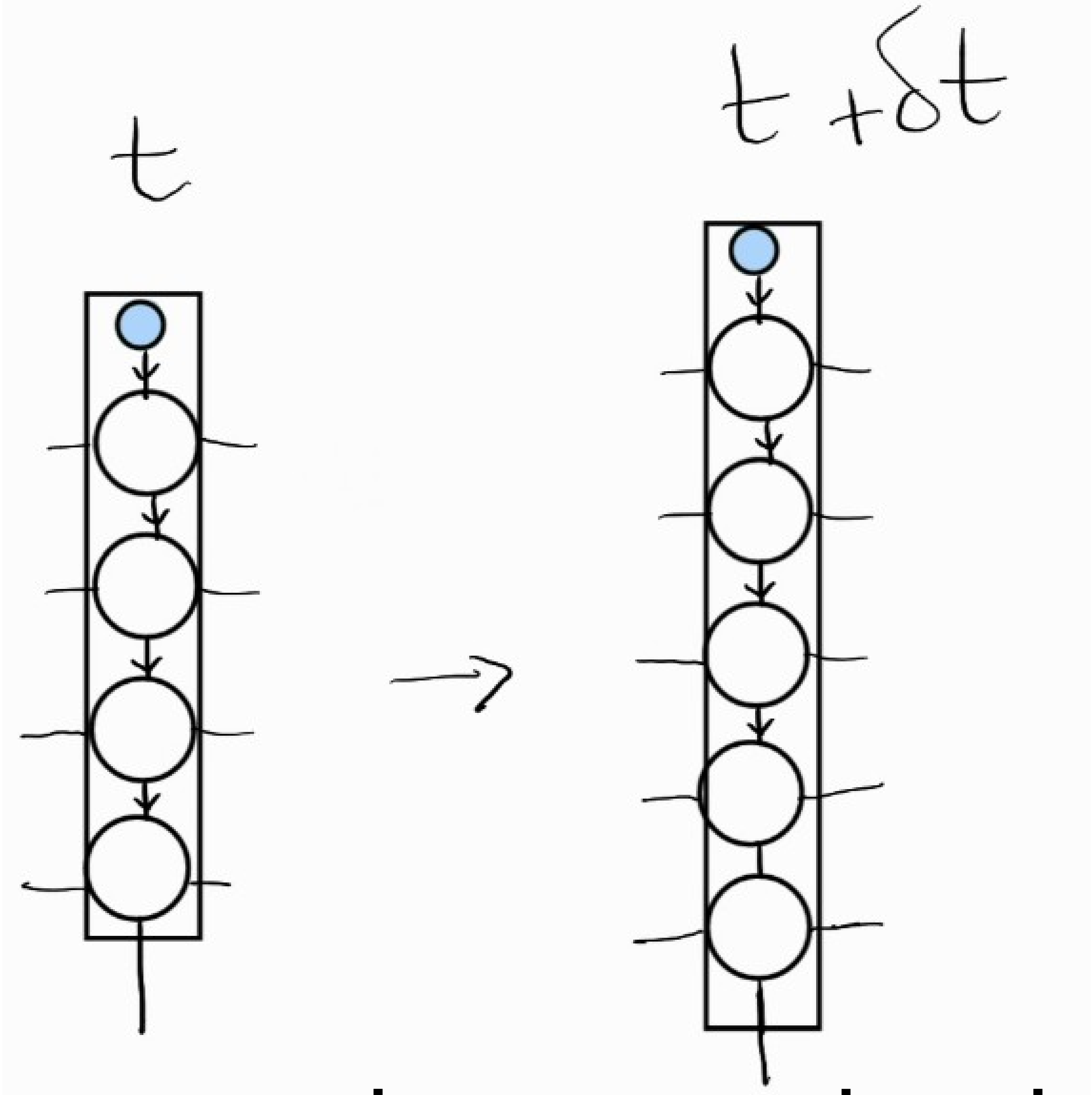
$$U(\delta t)U^\dagger(\delta t) = \mathbb{I}$$



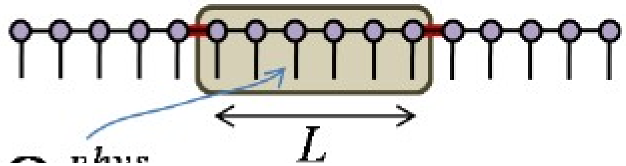
$$\langle \psi_0 | O(T) | \psi_0 \rangle$$

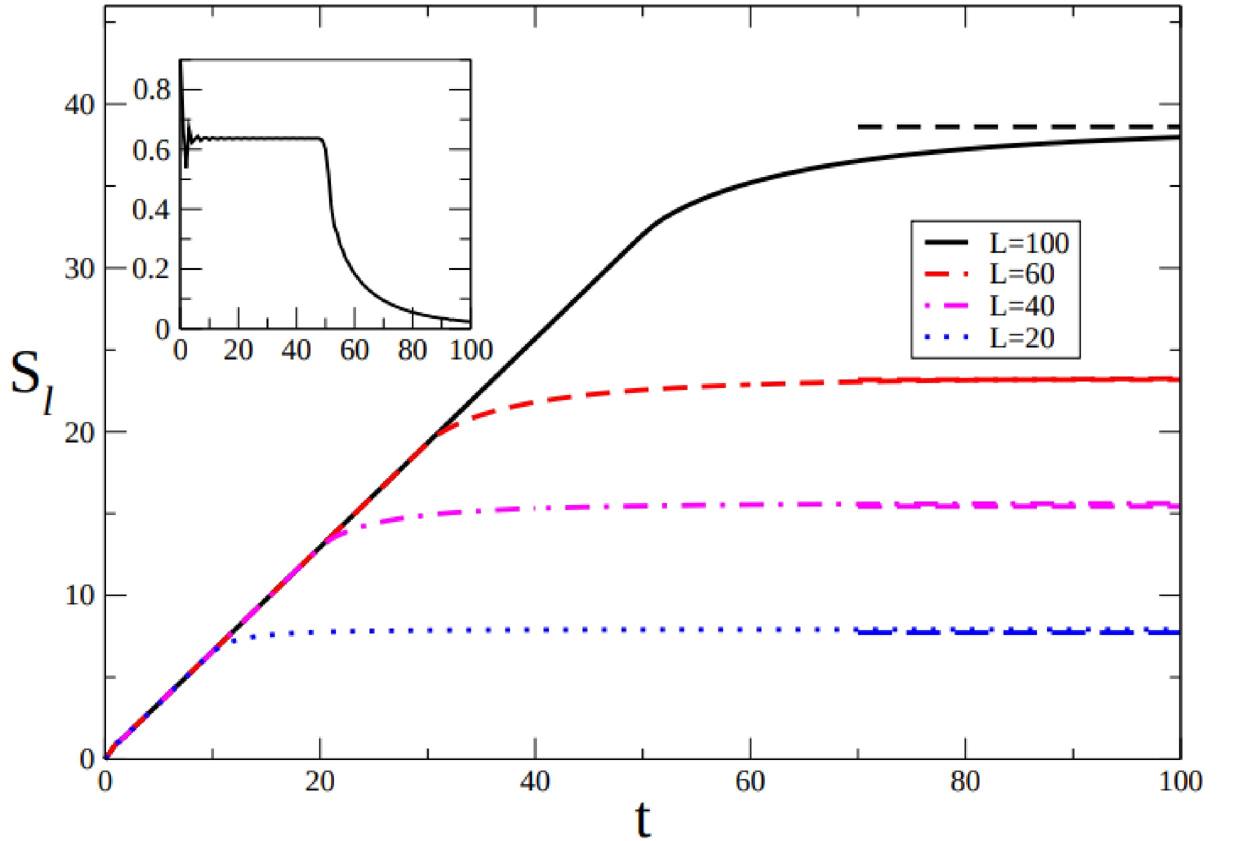



|ψ(t)⟩
O_i
⟨ψ(t)|



What is the actual computational cost?





from Calabrese Cardy 2005

$$S \leq n_{AB} \log(\chi)$$

$$\chi^{n_{AB}} \geq \exp S \propto \exp(t)$$

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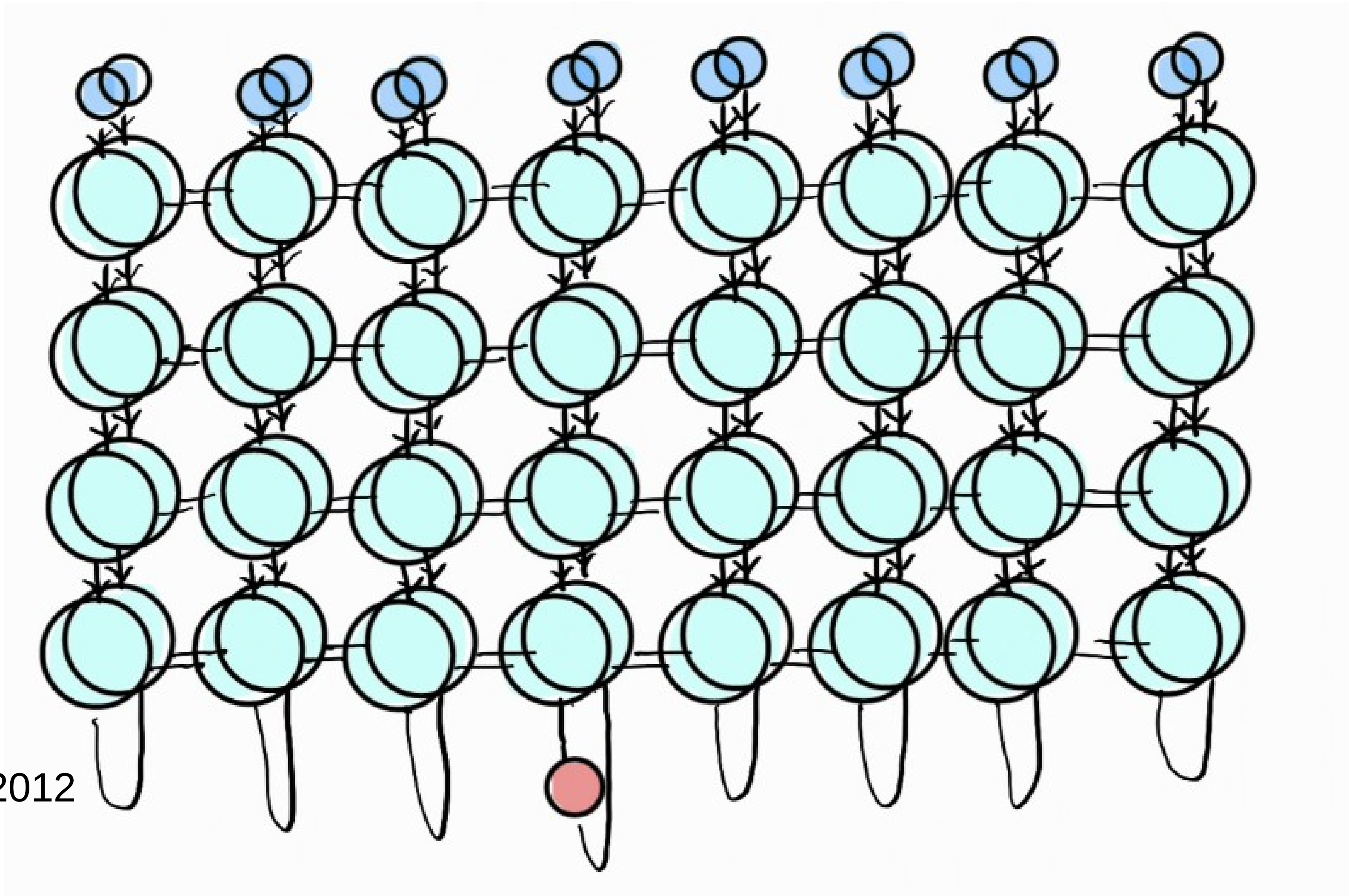
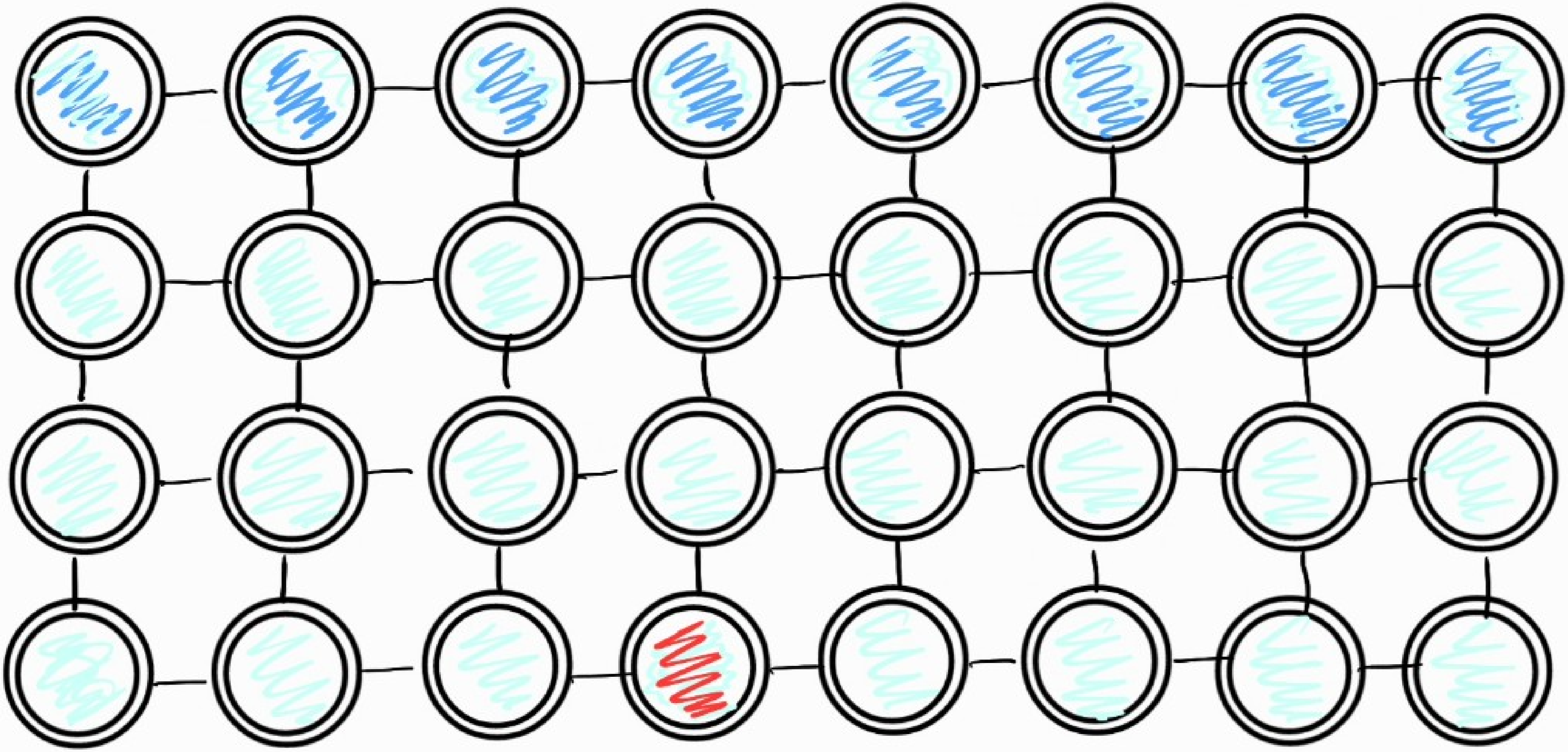
Upper bounding temporal entanglement

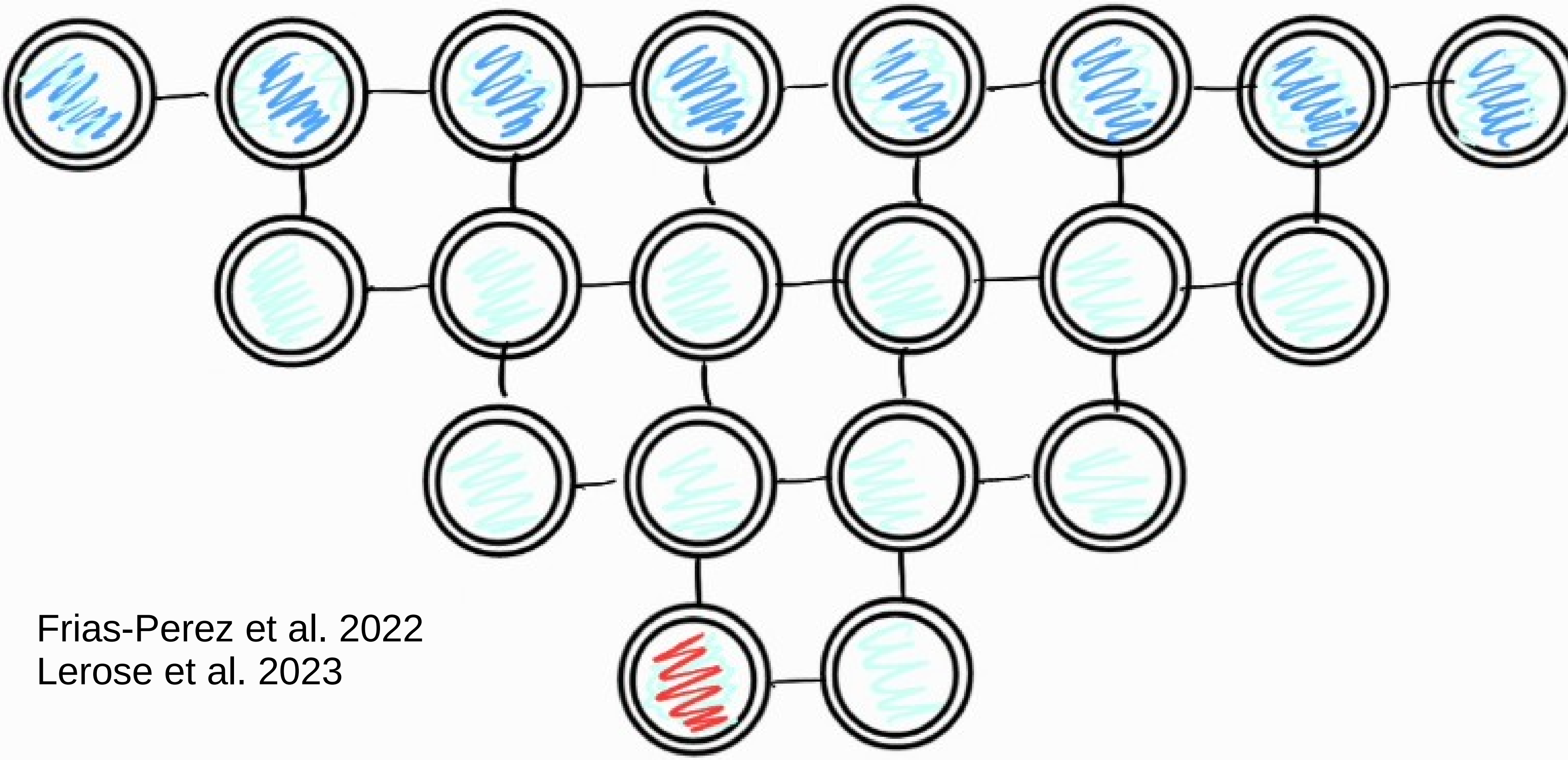
with S. Carignano C. Ramos arXiv:2306.xxxx



Bañuls et al. 2009
Muller Hermes et al 2012
Hastings et al. 2015
Tirrito et al (LT) 2018
Lerose et al 2021
Giudice et al 2021….

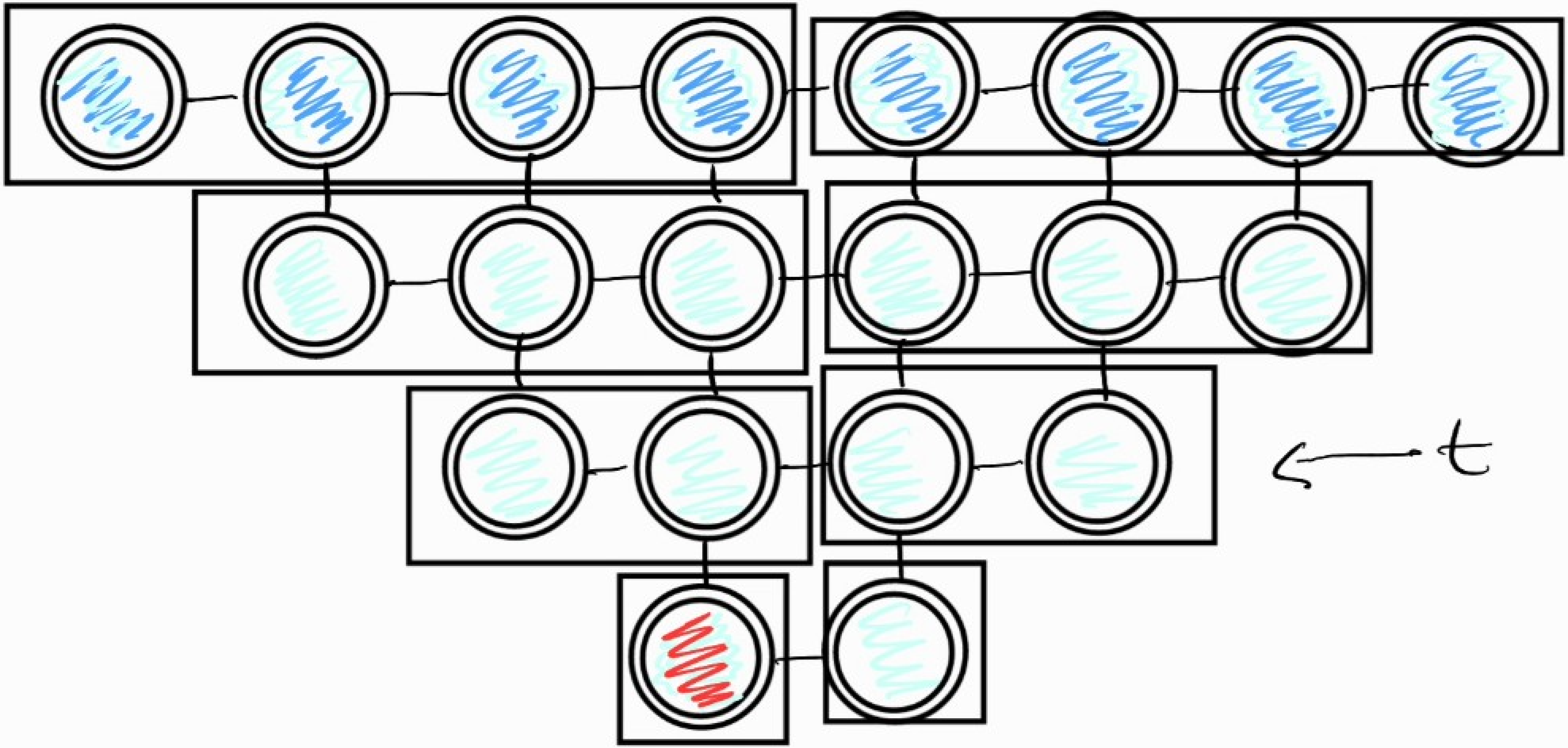


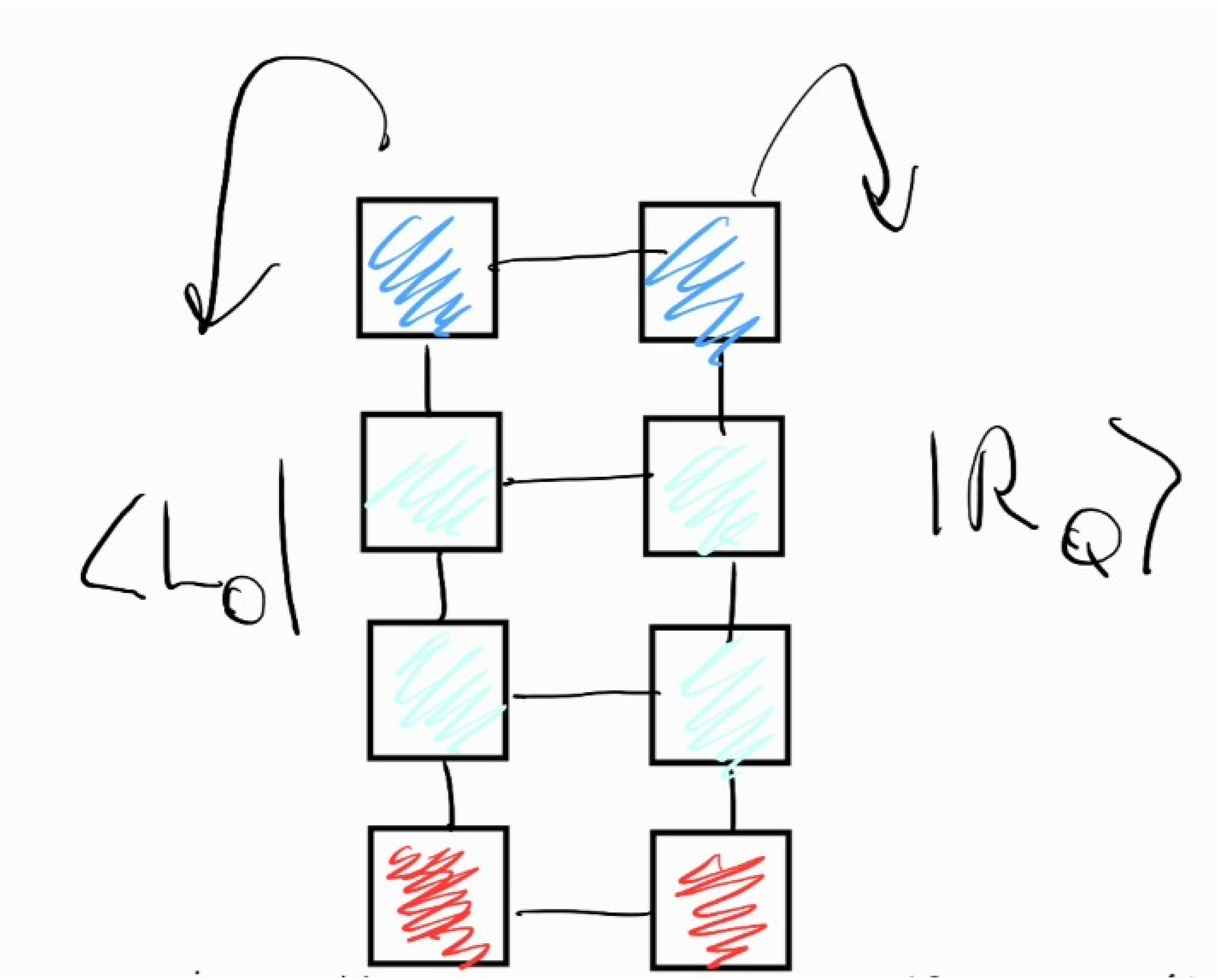
$$\langle \psi_0 | U^{\dagger}(t)\, O_i\, U(t) | \psi_0 \rangle$$



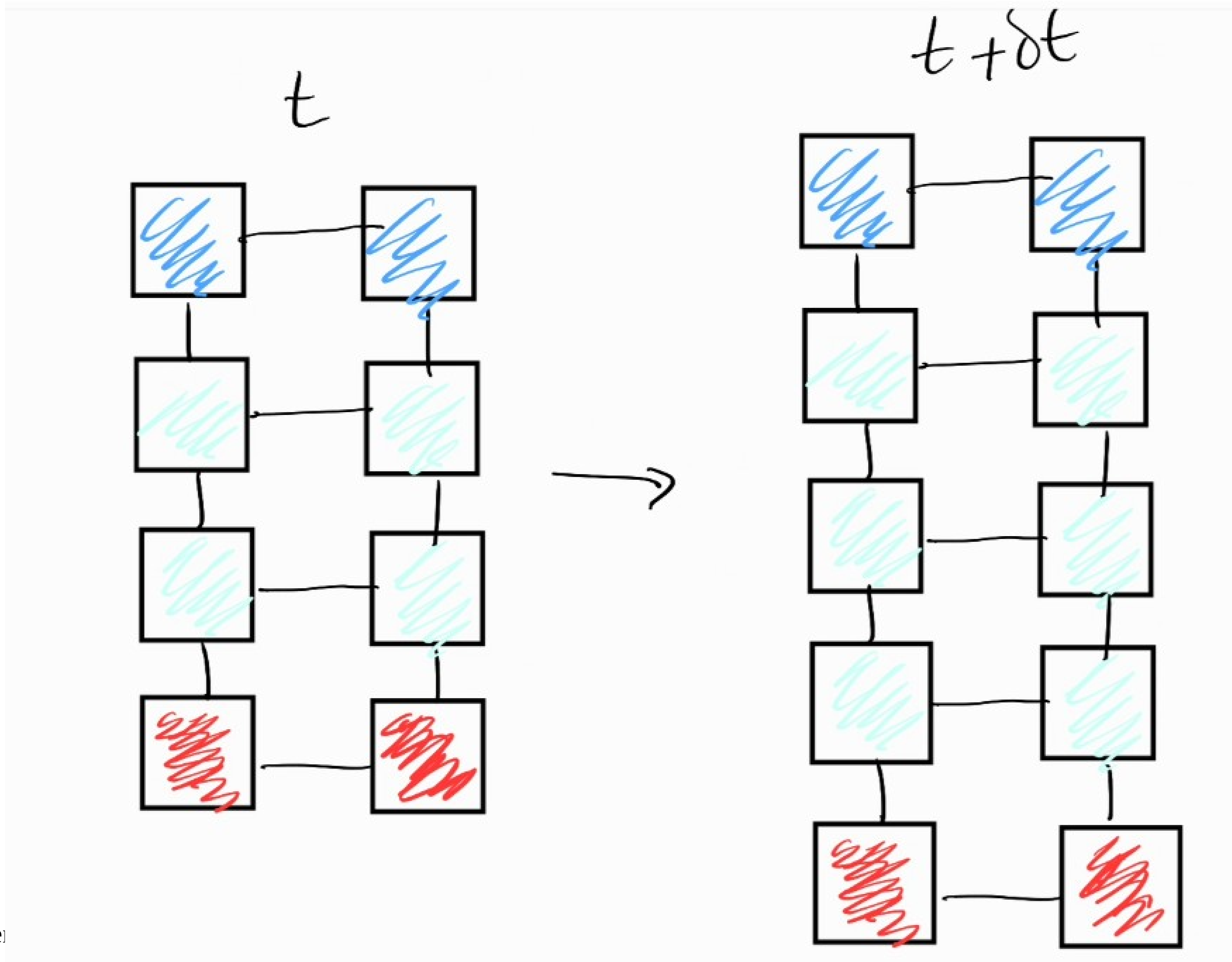
Frias-Perez et al. 2022
Lerose et al. 2023

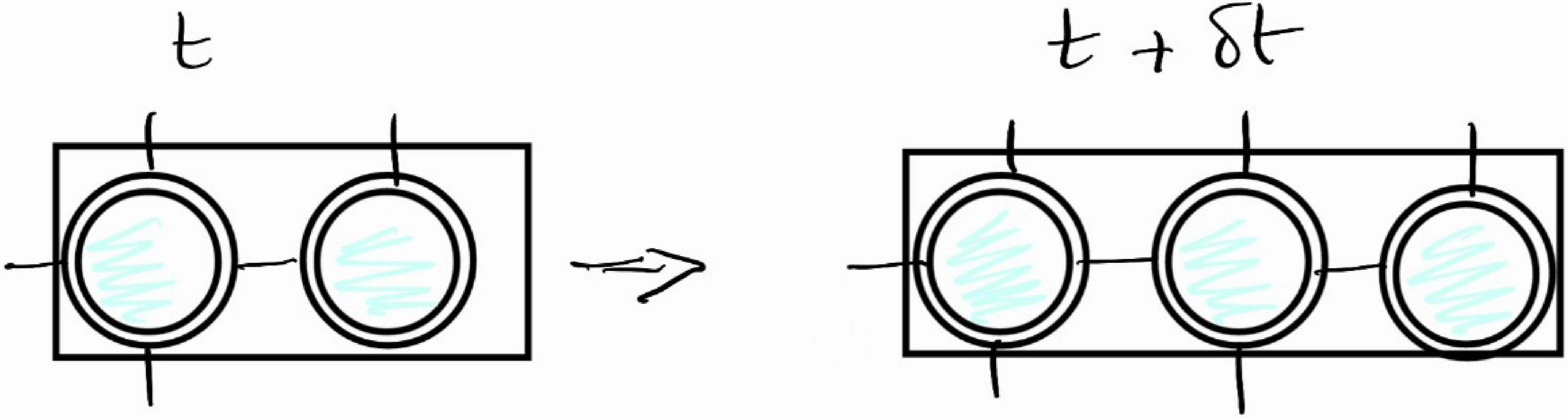
$$\langle \psi_0 | U^{\dagger}(t)\, O_i\, U(t) | \psi_0 \rangle$$


t

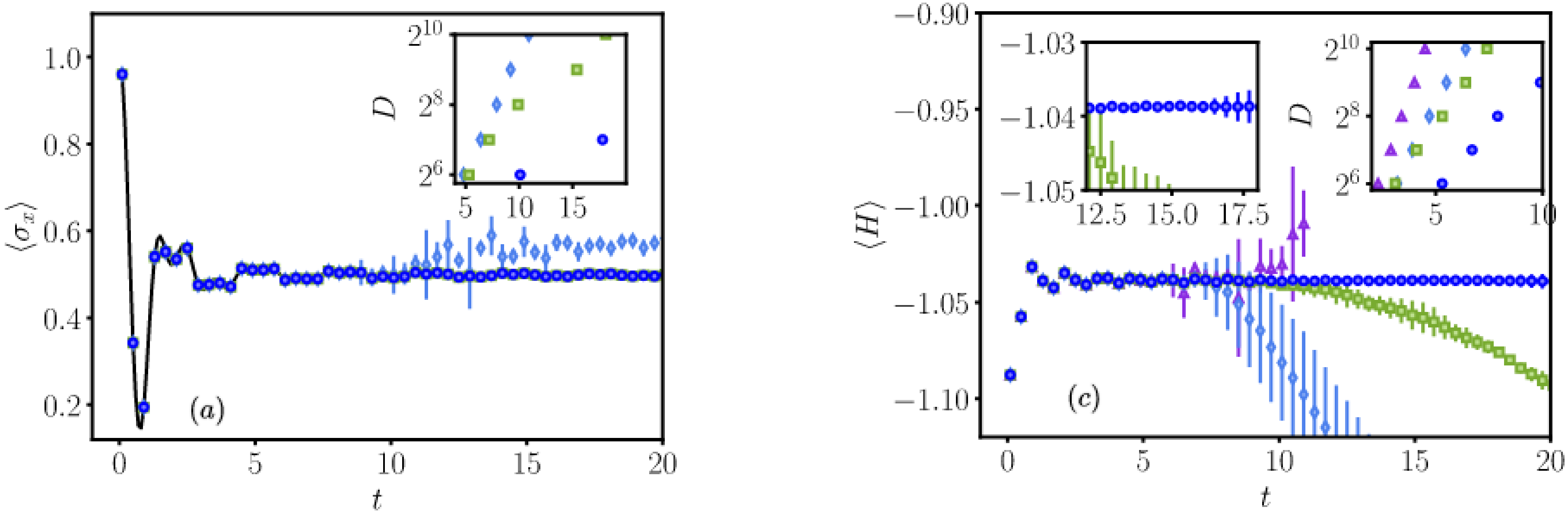


temporal MPS, tMPS


t
t + δt



What is the actual computational cost?



from Frias-Perez Bañuls 2022 similar results presented in talk by Dima

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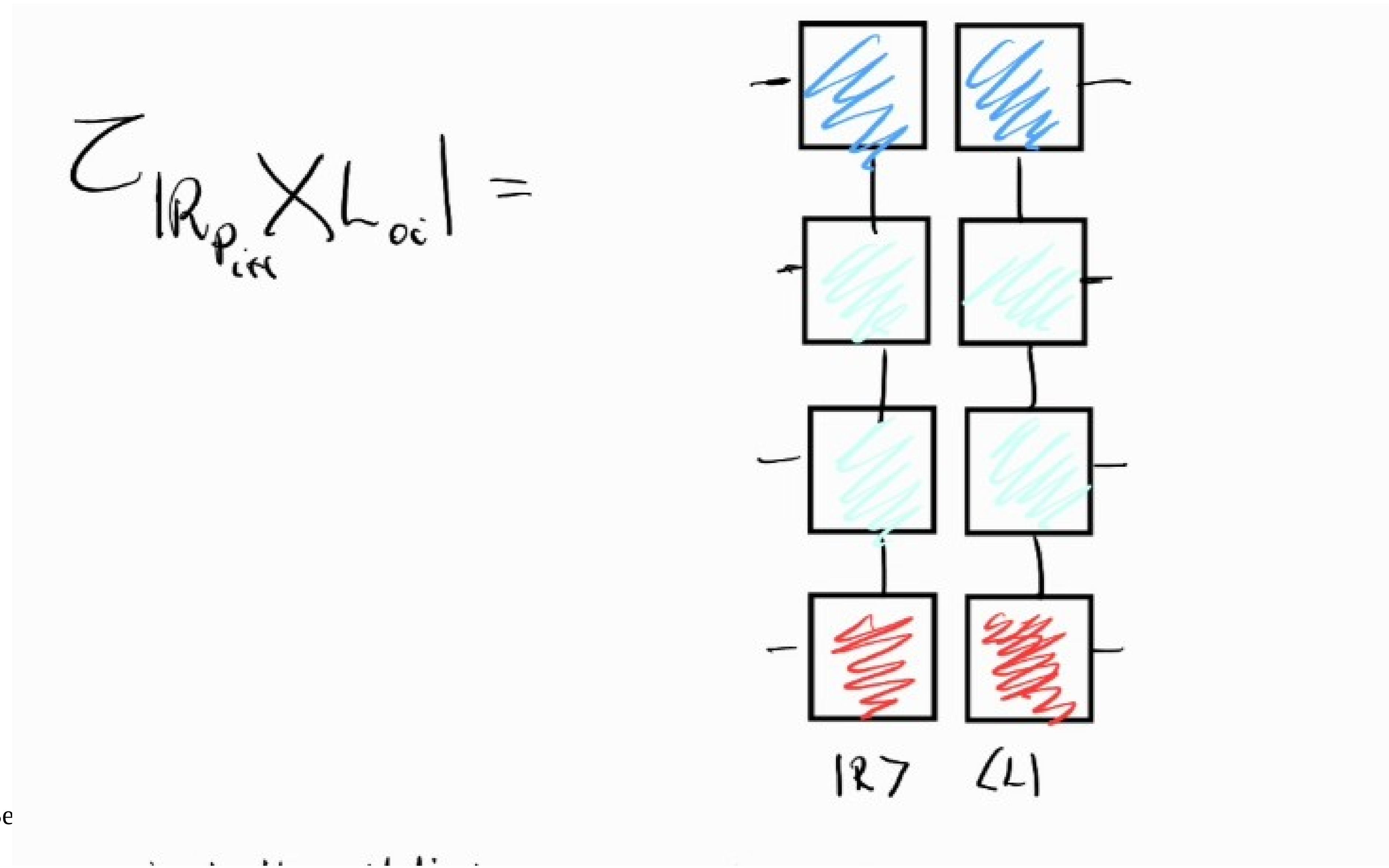
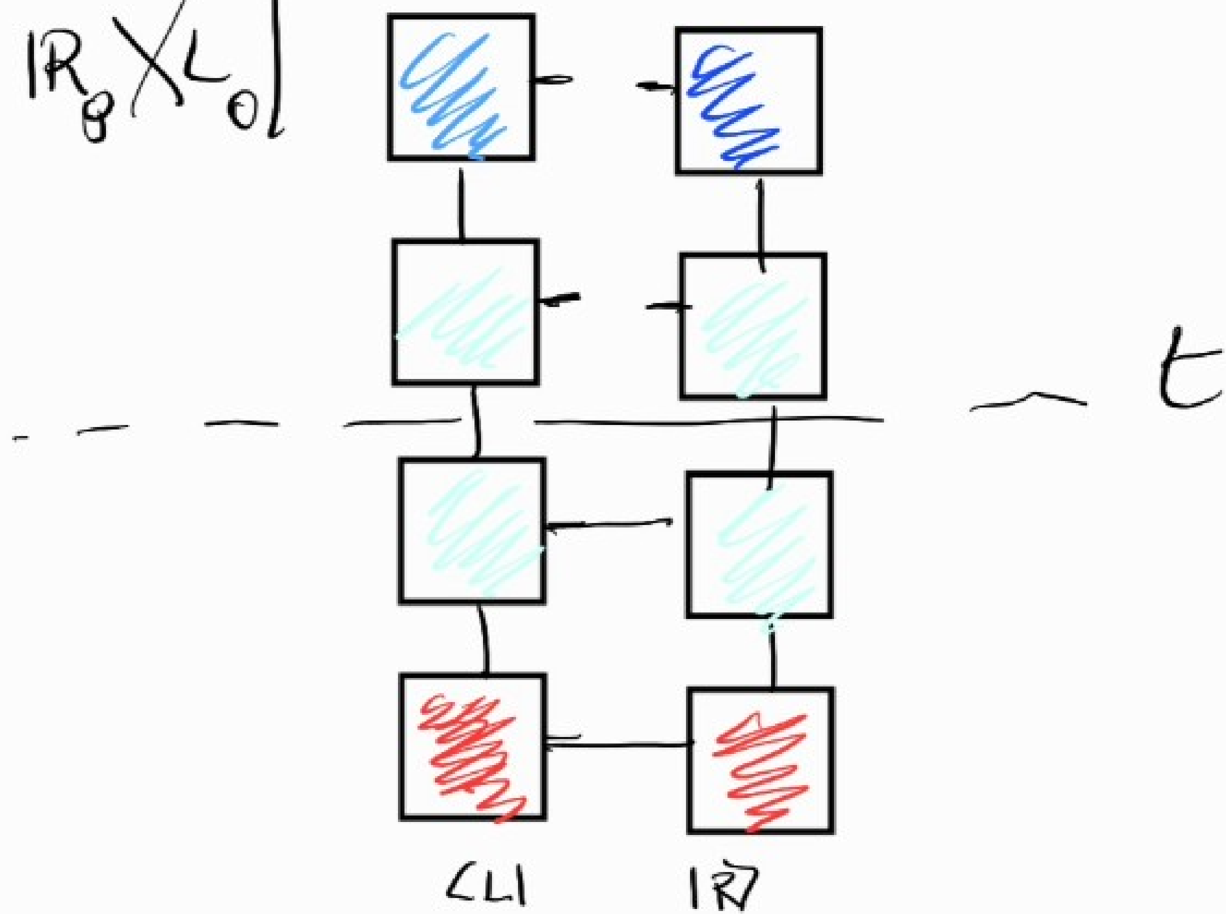


Be

$$\tau^t = \mathrm{tr}_{(T-t)} \tau |R_0\rangle\langle L_0|$$



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Simulation cost is dictated by
Cost of low rank approximation
of

$$\mathcal{T}^t \qquad \forall t \in \{0 ... T\}$$

Since operators are included,
we need the consider the worst
case scenario

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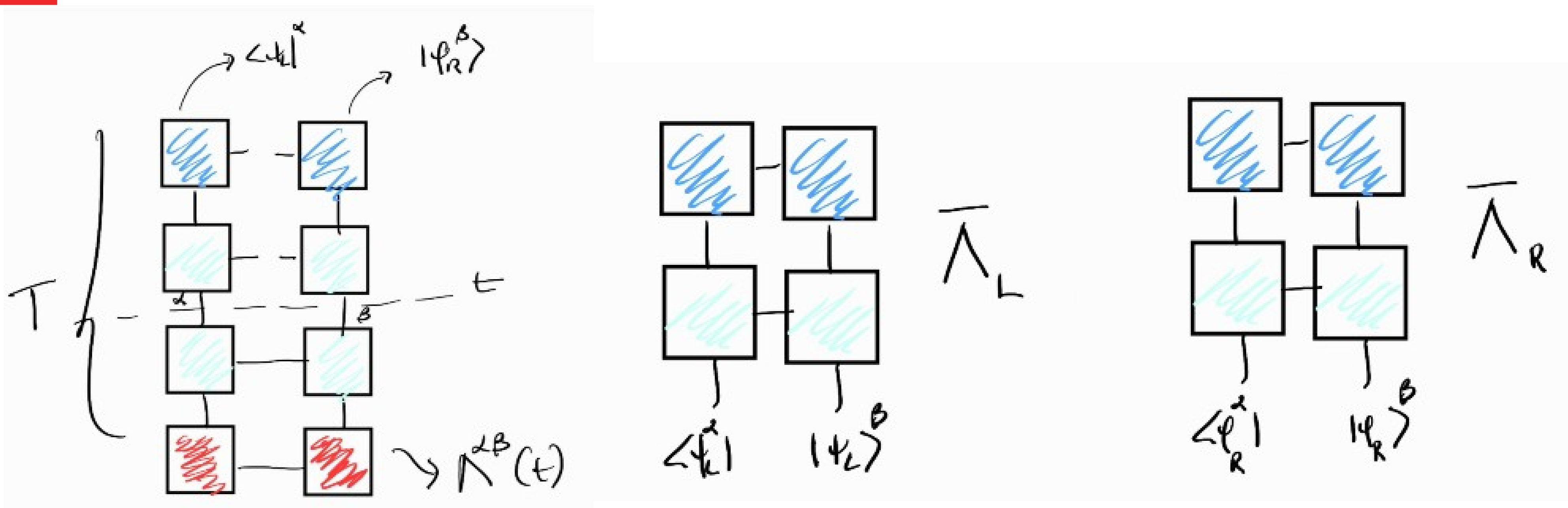
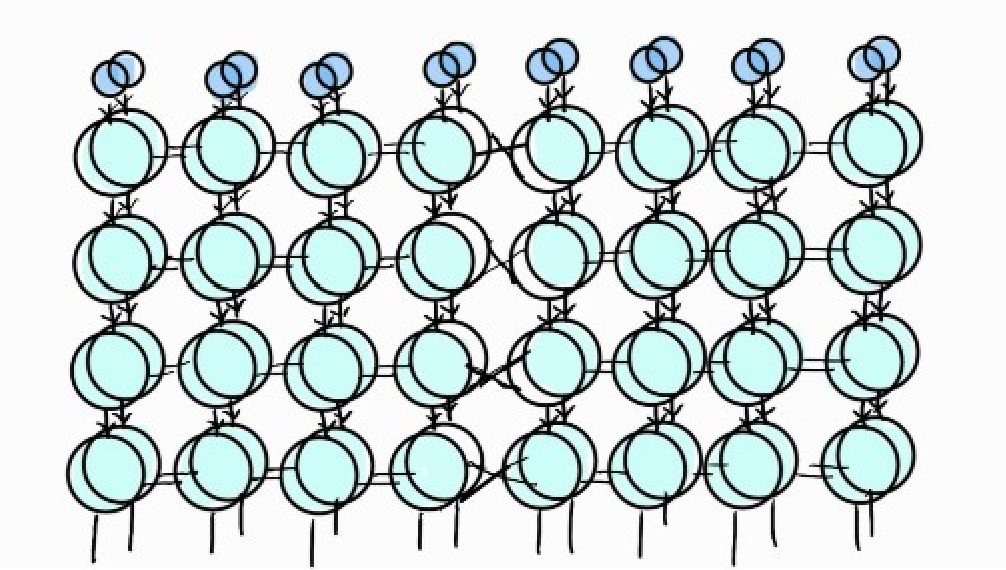


$$\mathcal{T}^t \simeq \sqrt{\bar{\Lambda}_L}\,\Lambda_t\,\sqrt{\bar{\Lambda}_R}$$

$$\bar{\Lambda}_L = \bar{\Lambda}_R$$

$$\mathrm{rank}\left(\mathcal{T}^t\right) \leq \min\left\{\mathrm{rank}\left(\Lambda_t\right), \mathrm{rank}\left(\bar{\Lambda}_L\right)\right\}$$

$$\bar{\Lambda}_L$$



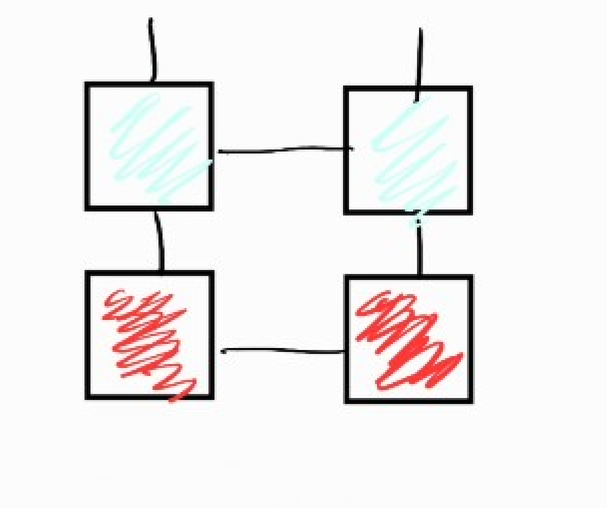
$$\rho(t)_{T_R}$$

$$\rho_R = tr_L \rho(t)$$

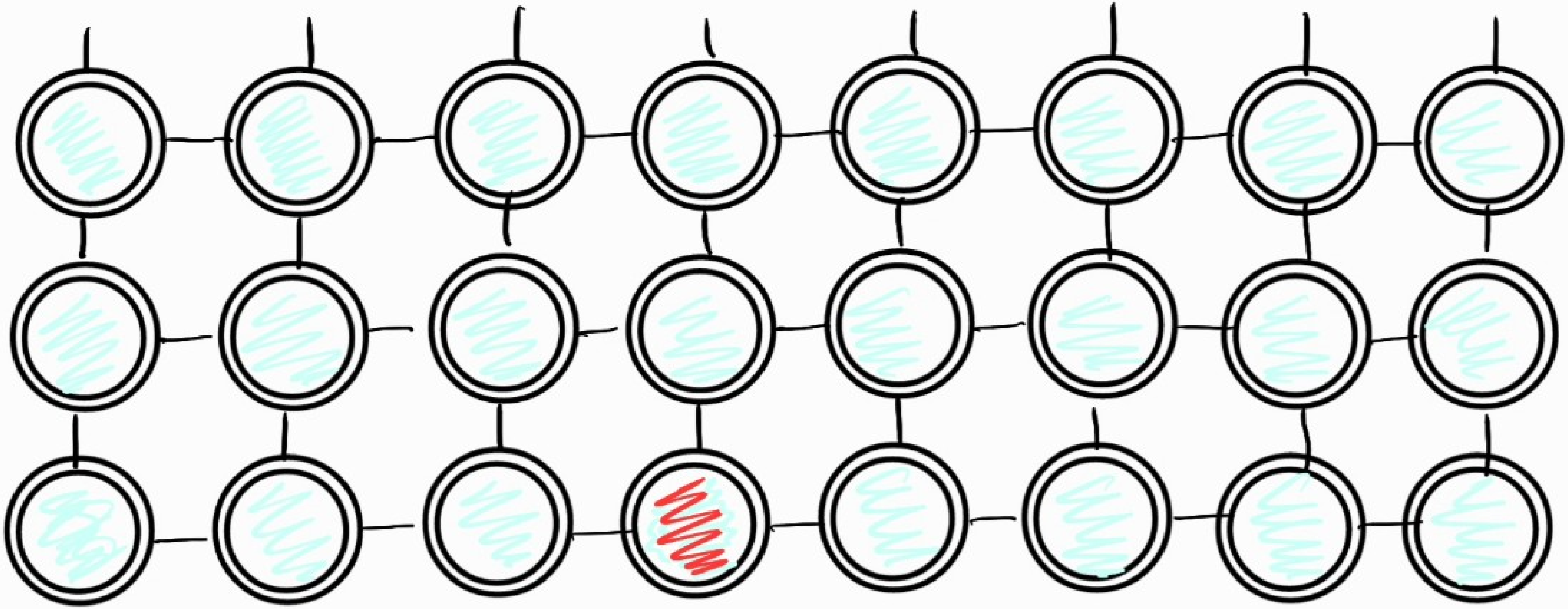
$$tr(\rho_R)^{n_{\rm odd}} = tr(\rho_{T_R})^{n_{\rm odd}}$$

$$\mathrm{rank}\left(\bar{\Lambda}_L\right) \propto \exp(t)$$

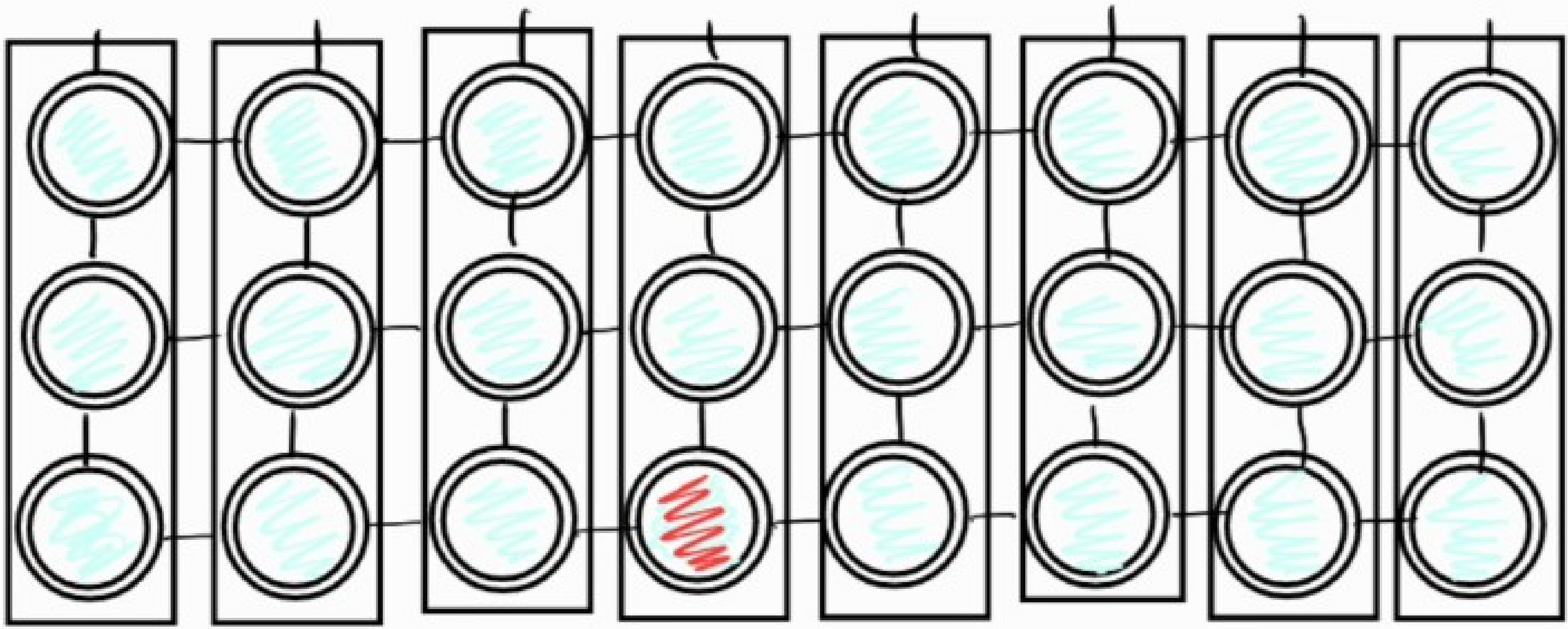
$$\Lambda_t$$

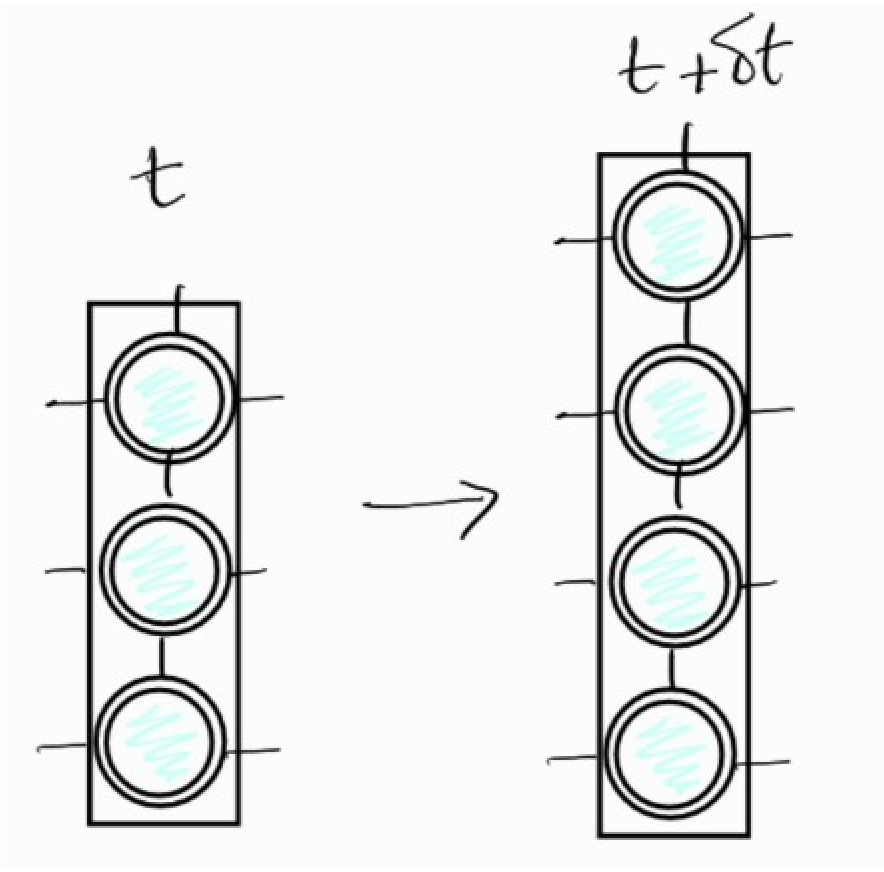


$$O_i(t) = U^{\dagger}(t)\, O_i\, U(t)$$



$$O_i(t) = U^{\dagger}(t)\, O_i\, U(t)$$





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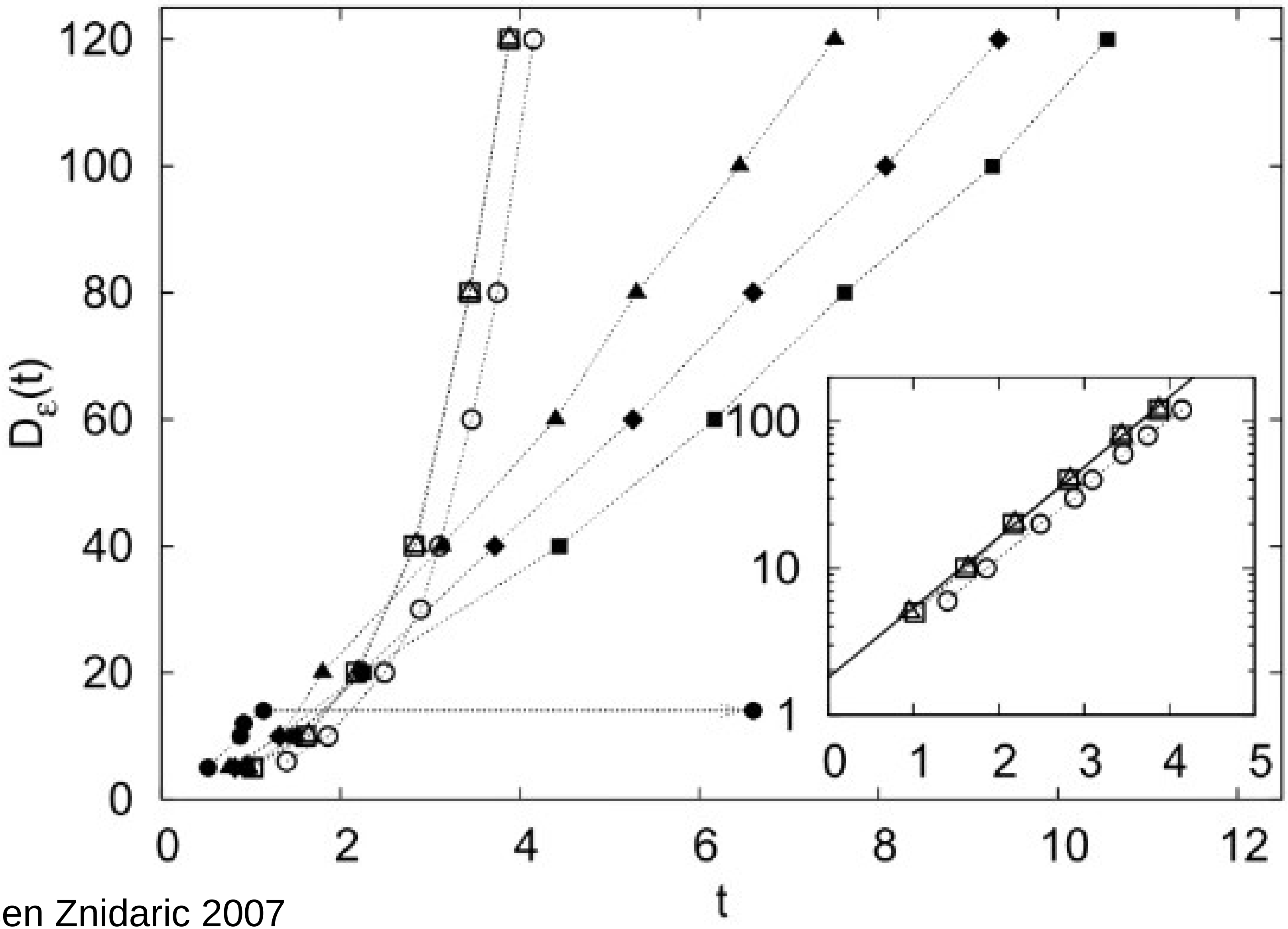


Prosen Znidaric 2007

$$\mathrm{rank}\left(\mathcal{T}^t\right) \leq \min\left\{\mathrm{rank}\left(\Lambda_t\right), \mathrm{rank}\left(\bar{\Lambda}_L\right)\right\}$$

$$\mathrm{rank}\left(\Lambda^t\right) \propto t^\alpha$$

$$\mathrm{rank}\left(\bar{\Lambda}_L\right) \propto \exp(t)$$

Prosen Znidaric 2007
Pizorn Prosen 2009
Dubail 2017……

$$\mathrm{rank}\left(\mathcal{T}^t\right) \leq t^\alpha$$

$$\mathrm{rank}\left(\mathcal{T}^t\right) \leq \min\left\{\mathrm{rank}\left(\Lambda_t\right), \mathrm{rank}\left(\bar{\Lambda}_L\right)\right\}$$

$$\mathrm{rank}\left(\Lambda^t\right) \propto \exp\left(t\right) \qquad \mathrm{rank}\left(\bar{\Lambda}_L\right) \propto \exp(t)$$

$$\mathrm{rank}\left(\mathcal{T}^t\right) \leq \exp(t)$$

$$H = \sum_i (-X_i X_{i+1} - gZ_i - hX_i)$$

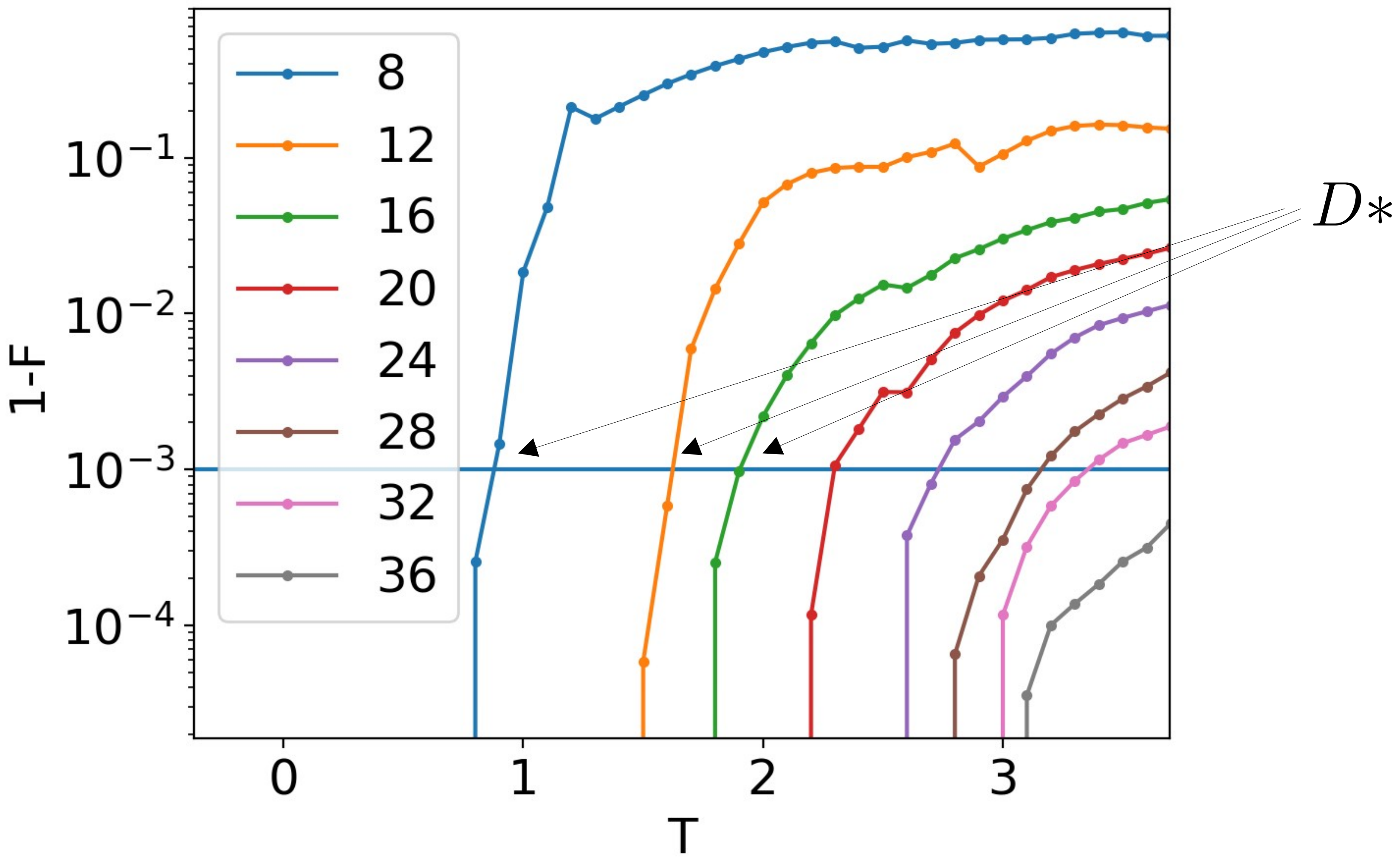
Integrable quench:

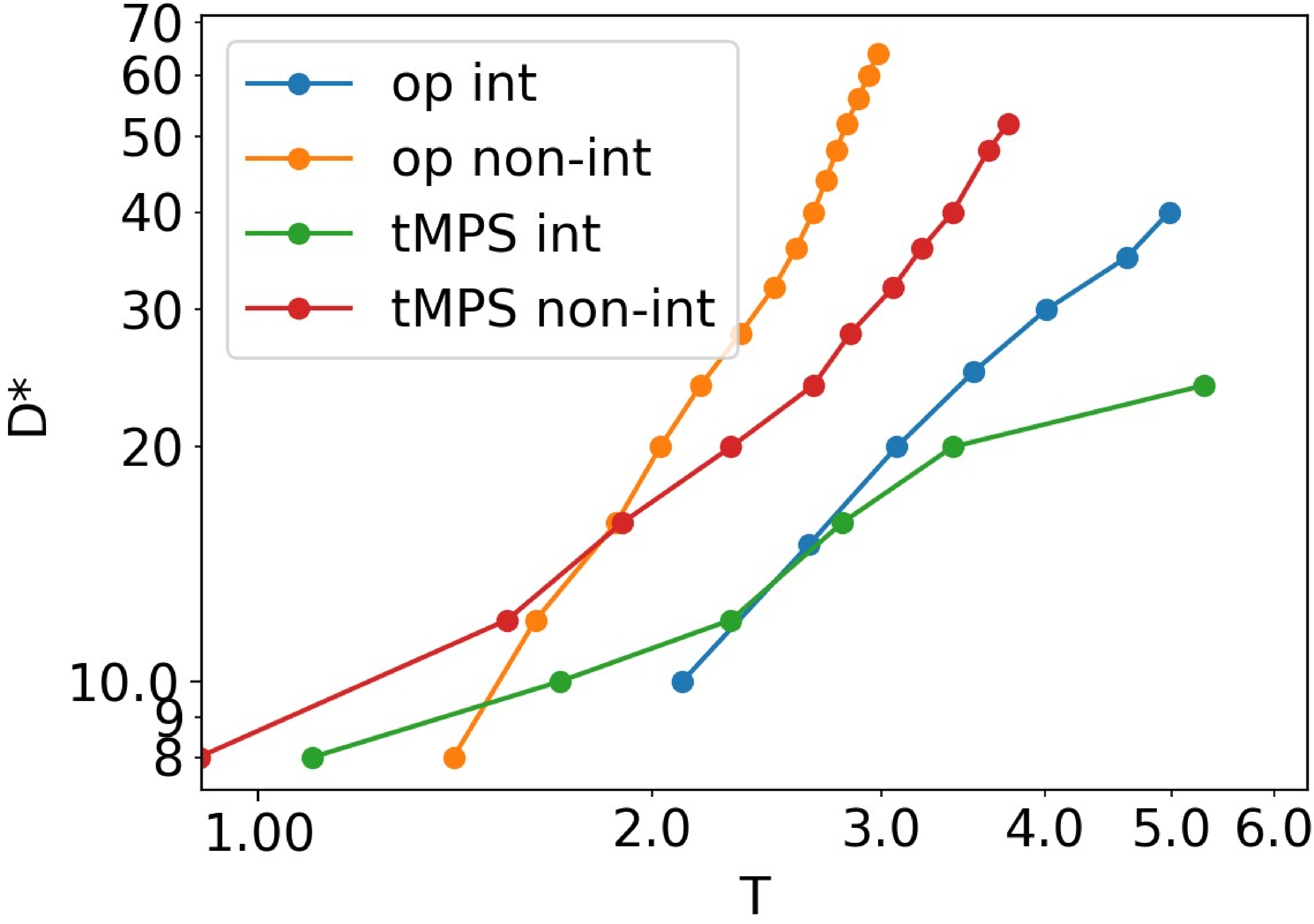
$$\{g = \infty, h = 0\} \rightarrow \{g = 0.7, h = 0\}$$

Non-integrable quench:

$$\{g = \infty, h = 0\} \rightarrow \{g = -1.05, h = 0.5\}$$

$$F = \frac{\langle L_X^D | L_X \rangle^2}{\langle L_X^D | L_X^D \rangle \langle L_X | L_X \rangle}$$





op int
op non-int
tMPS int
tMPS non-int
D*
T
70
60
50
40
30
20
10.0
9
8
1.00
2.0
3.0
4.0
5.0
6.0

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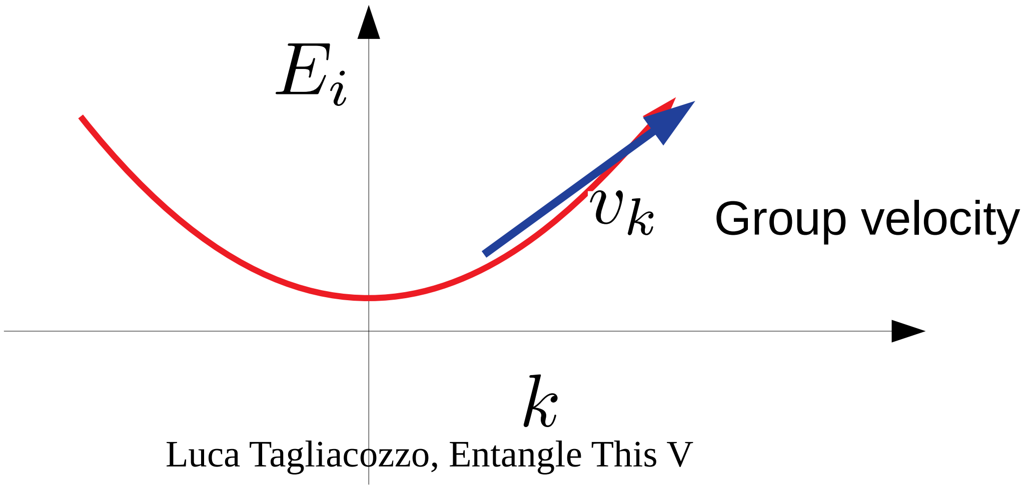
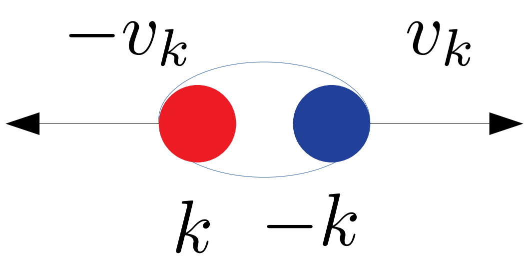
- The Hamiltonian is translational invariant,
- Momentum is a good quantum number

$$H = \oplus_k H_k$$

- We have bands, **quasi-particles** $E_i(k)$

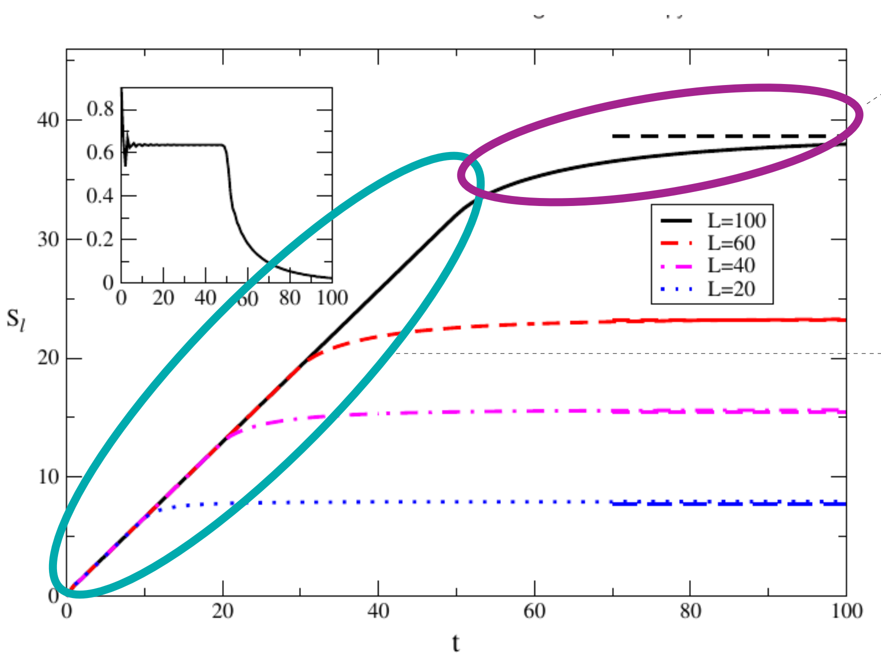


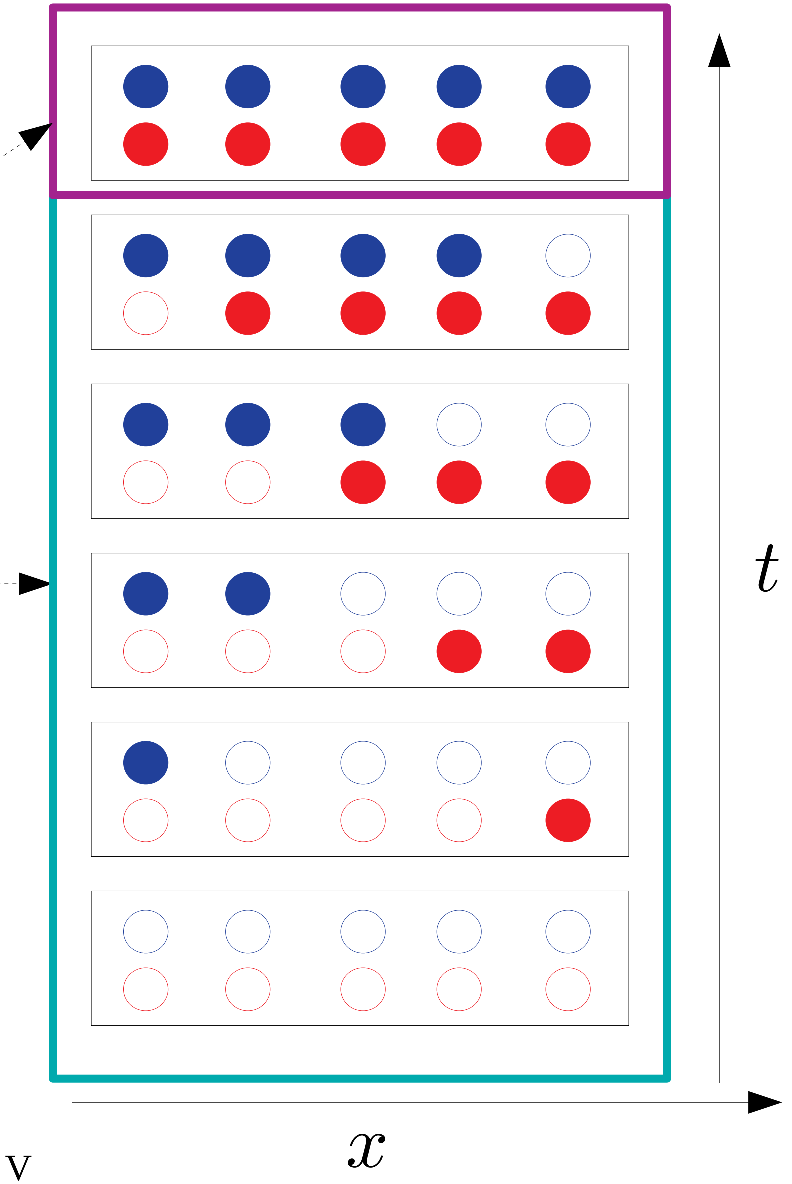
The **extra energy** in the quench is radiated by entangled **quasi-particle** with opposite momenta k and velocities v_k



Entanglement after a quench

Calabrese Cardy 05





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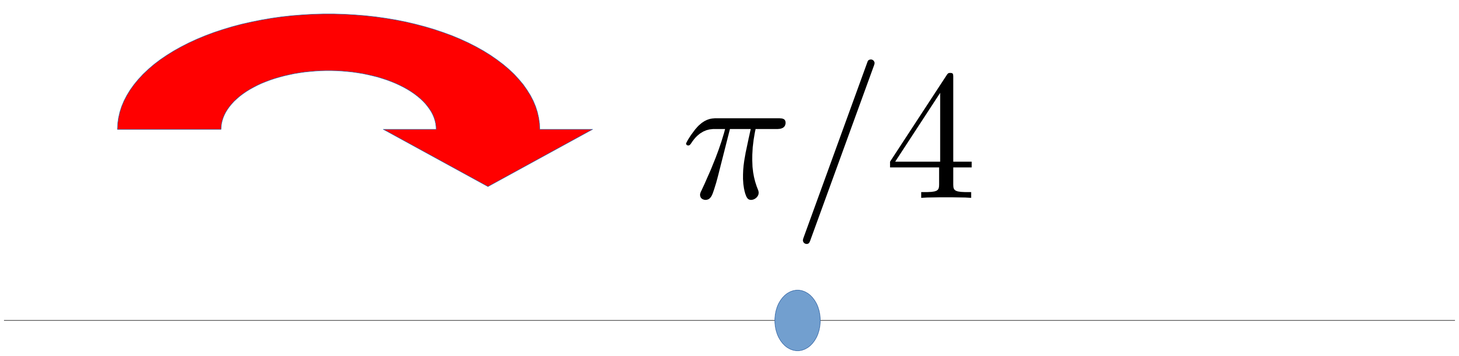
Trading entanglement for mixture

- Surace Piani Tagliacozzo PRB2019

The quench protocol

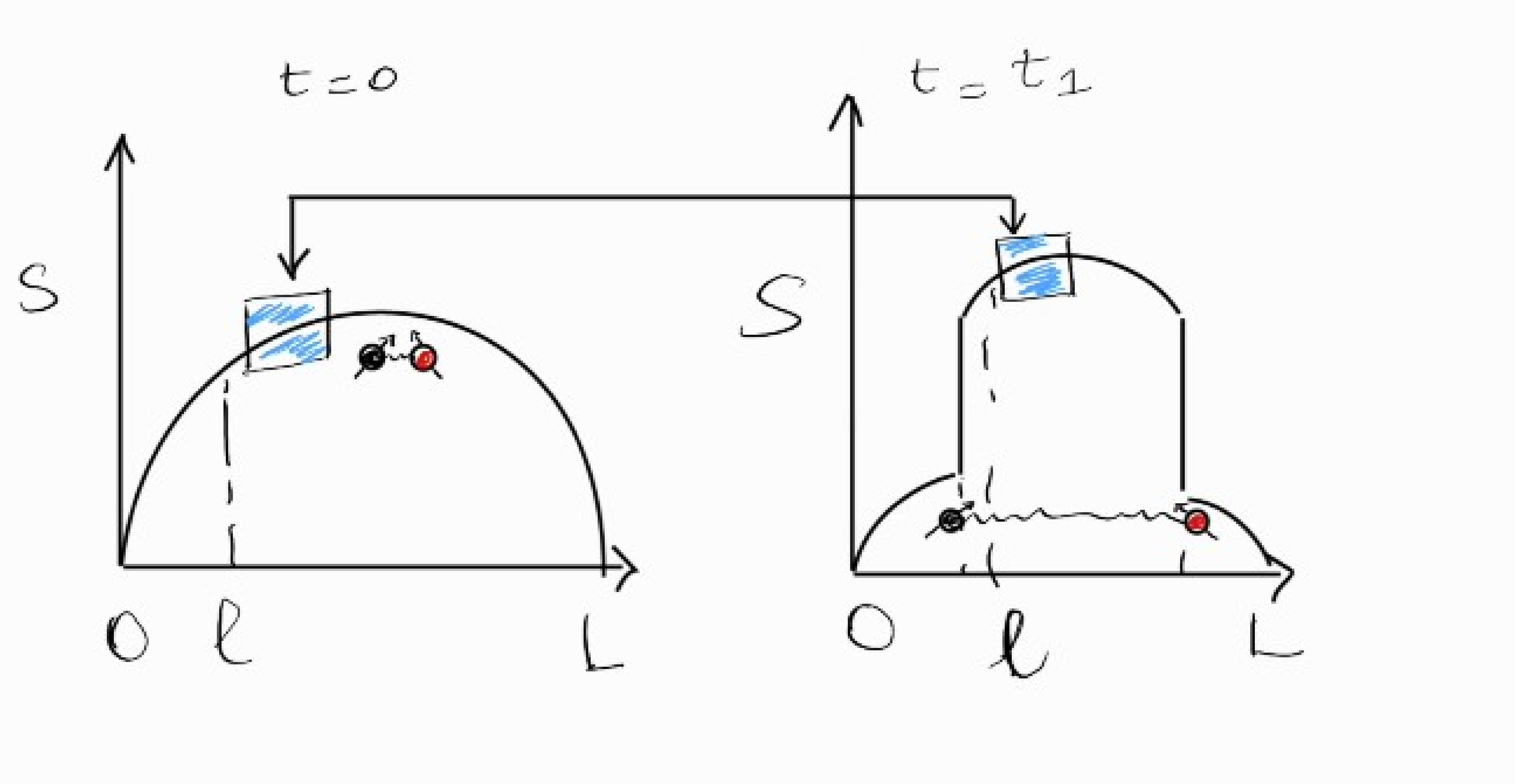
$$H(\theta) = -\sin(\theta)\sum_{i=0}^{N-1}\sigma_i^x\sigma_{i+1}^x - \cos(\theta)\sum_{i=0}^{N-1}\sigma_i^z$$



$$\lambda_{dB} = \frac{h}{mv}$$

$$mv^2 \sim k_B T$$

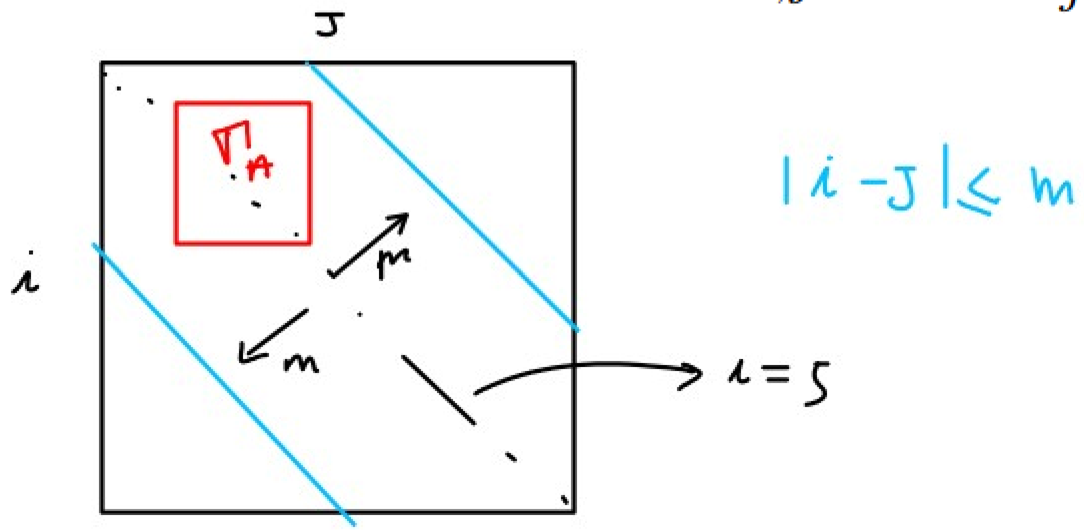
$$\lambda_{dB} \sim \frac{h}{\sqrt{m k_B T}} \, .$$


t = 0
S
O
ℓ
L
t = t1
S
O
ℓ
L

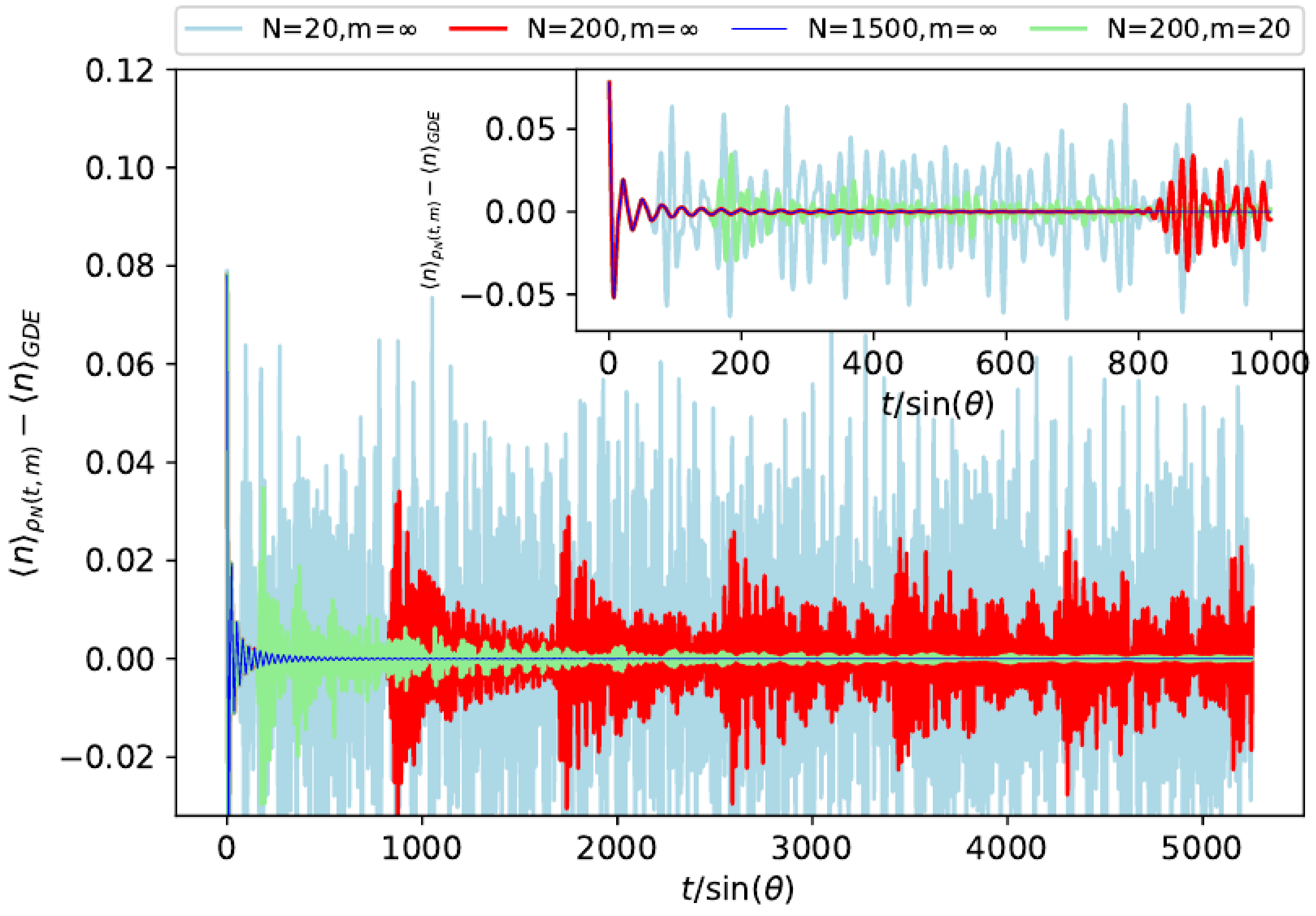
Gaussian states

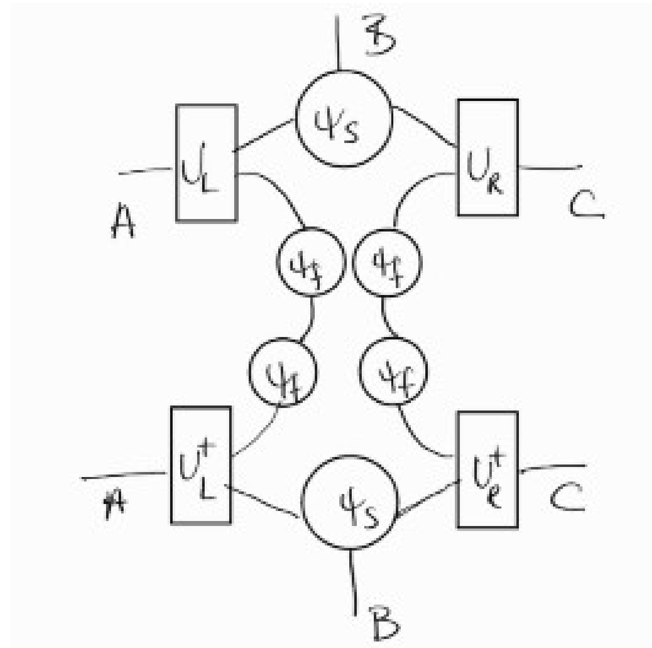
- The relevant correlations are inside a band of size m in the correlation matrix

$$\Gamma_{i,j} = \langle \vec{\alpha}_i \vec{\alpha}_j^\dagger \rangle,$$



- At the truncation stage all correlation that are outside the band are zeroed


N=20,m=∞
N=200,m=∞
N=1500,m=∞
N=200,m=20
0.12
0.10
0.08
0.06
0.04
0.02
0.00
−0.02
0
1000
2000
3000
4000
5000
t/sin(θ)
⟨n⟩ρN(t,m) − ⟨n⟩GDE
0.05
0.00
−0.05
0
200
400
600
800
1000
t/sin(θ)
⟨n⟩ρN(t,m) − ⟨n⟩GDE



Unveiling local equilibration in TN

Work in progress with M Frias-Perez and MC Bañuls (MPQ), arXiv2306xxxx

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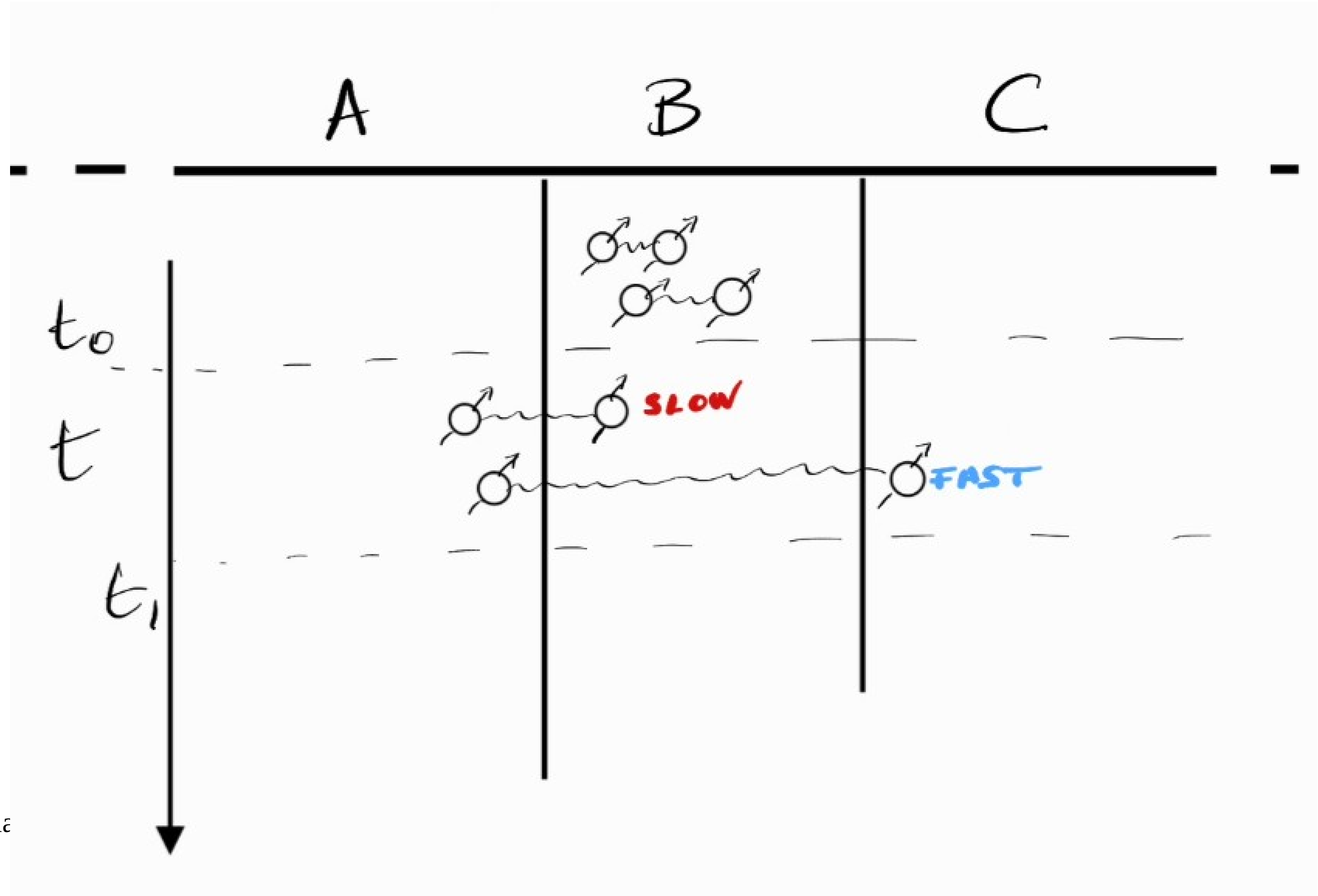
$$|\psi(t)\rangle \simeq |\psi_{A_s BC_s}\rangle \otimes |\psi_{A_f BC_f}\rangle$$



Bena

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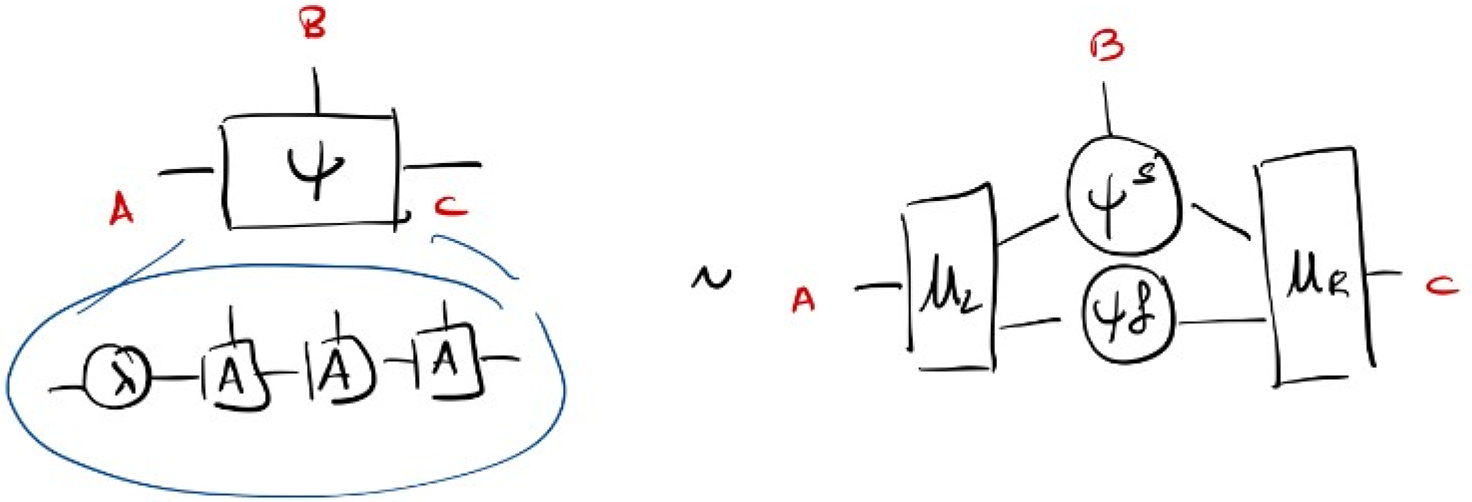
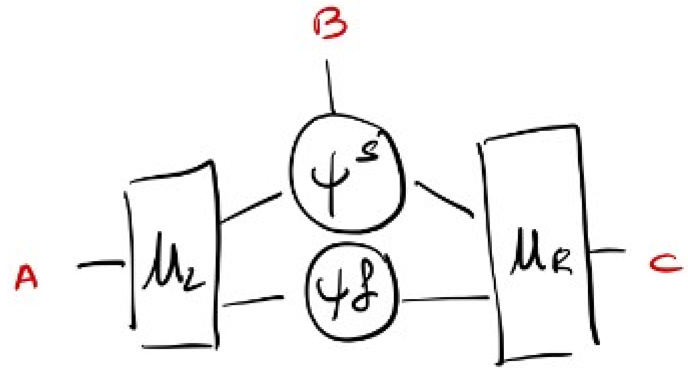
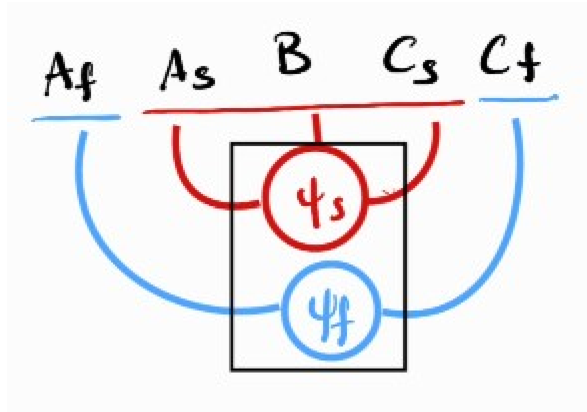
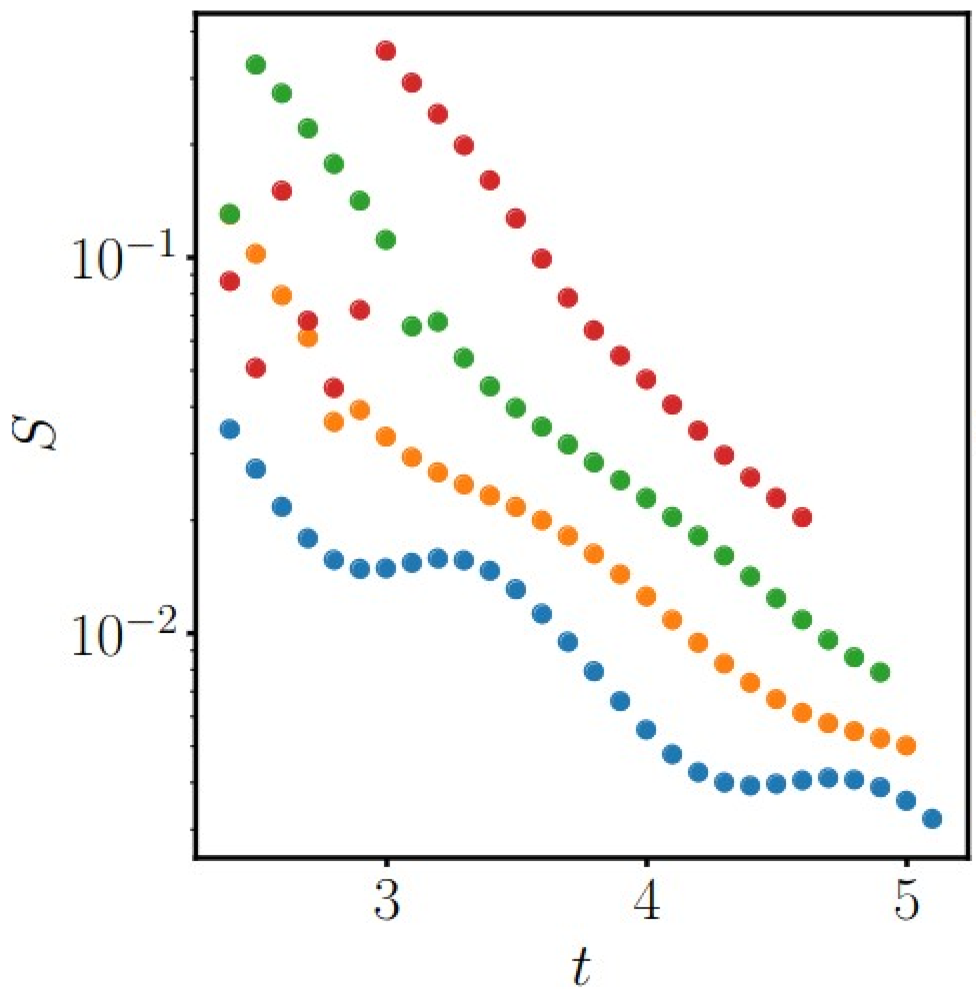
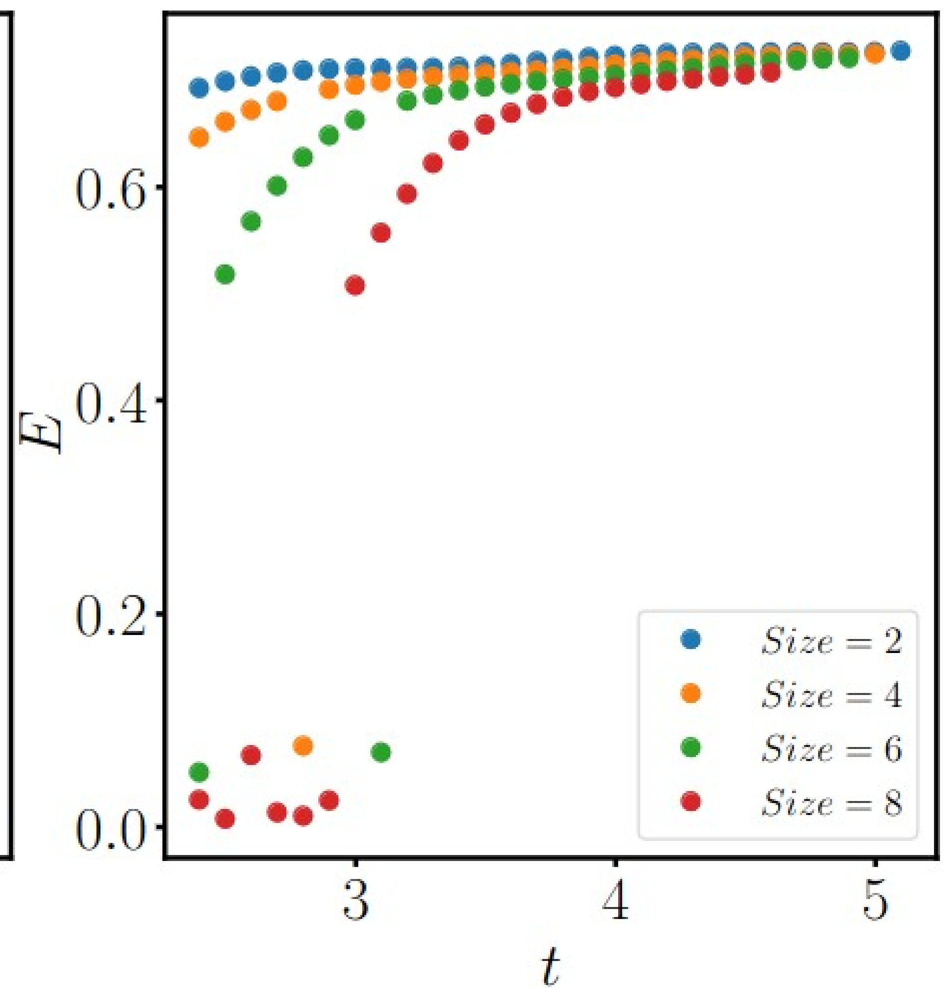

B
A
C
B
A
C


B
A
C


Af
As
B
Cs
Cf


S
10^{-1}
10^{-2}
3
4
5
t


E
0.0
0.2
0.4
0.6
3
4
5
t
$Size = 2$
$Size = 4$
$Size = 6$
$Size = 8$

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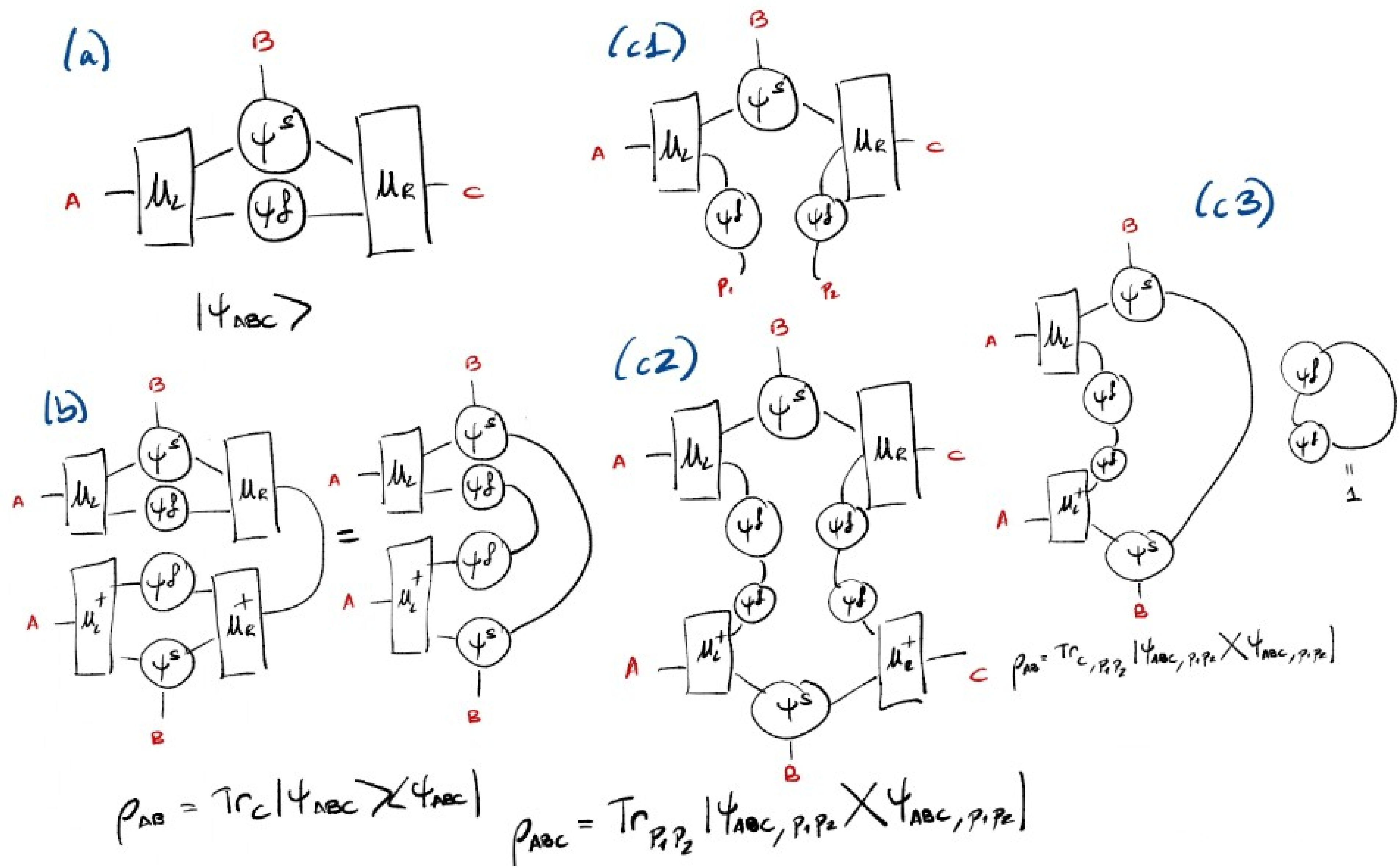
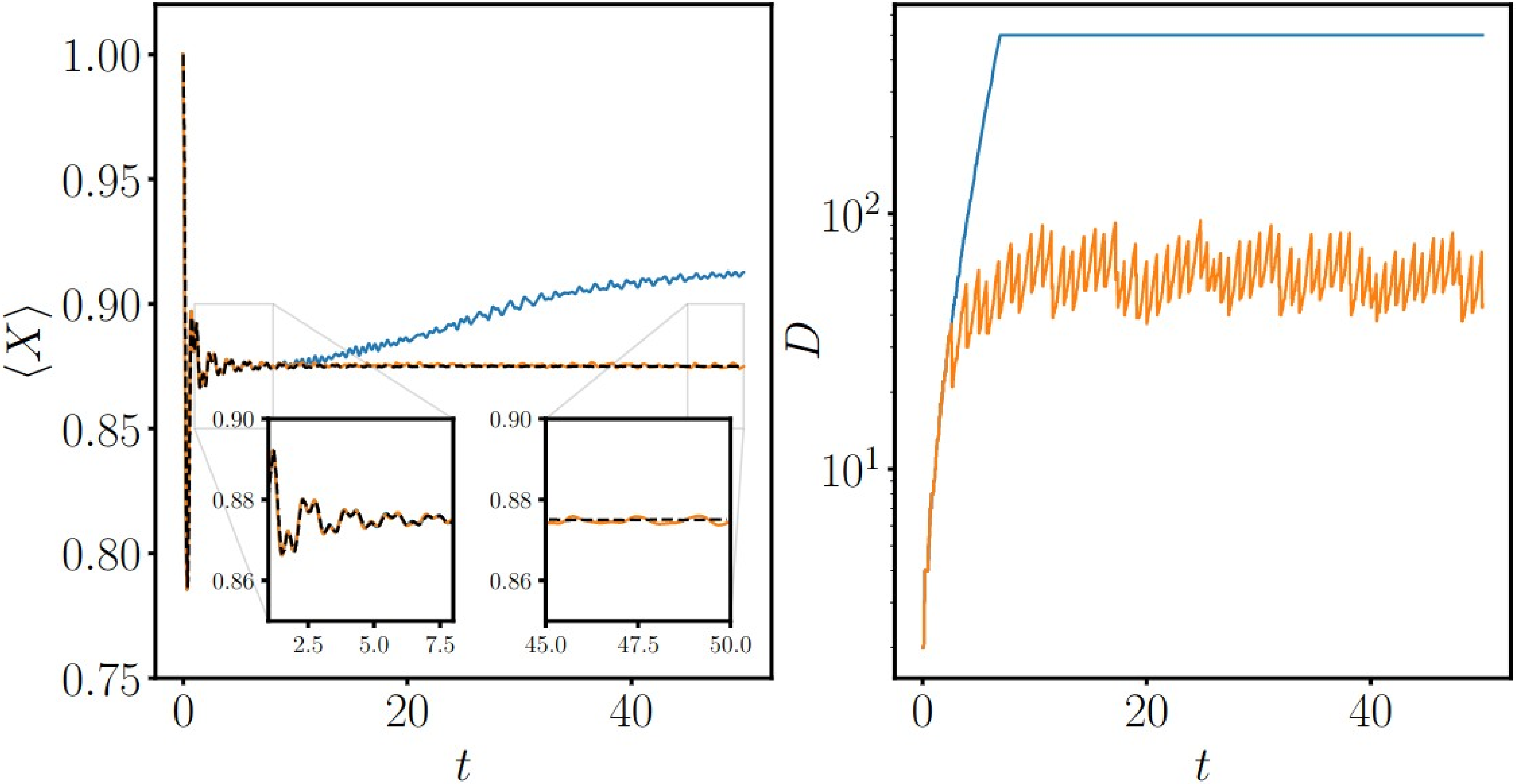
- Quasi-particles
- Trading entanglement for mixture
  - Unveiling local equilibration
  - Identifying fast degrees of freedom
  - Relation with decoherence

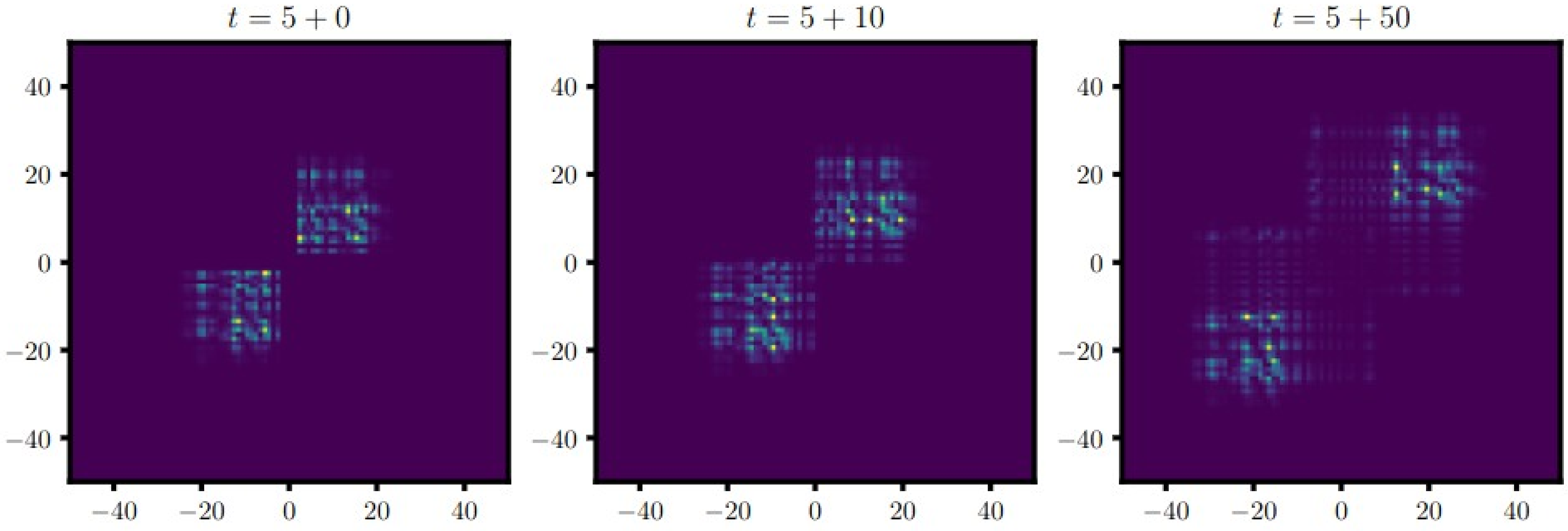


(a)
$|\psi_{ABC}\rangle$
(b)
(c1)
(c2)
(c3)
$\rho_{AB} = Tr_C |\psi_{ABC}\rangle\langle\psi_{ABC}|$
$\rho_{ABC} = Tr_{P_1 P_2} |\psi_{ABC, P_1 P_2}\rangle\langle\psi_{ABC, P_1 P_2}|$
$\rho_{AB} = Tr_{C, P_1 P_2} |\psi_{ABC, P_1 P_2}\rangle\langle\psi_{ABC, P_1 P_2}|$

$$g = \infty \rightarrow g = 2$$



de-cohere only fast degrees of freedom



- The entanglement barrier in 1D **can be circumvented**
- Folding and tMPS alone might not be enough, as the cost depends on the scaling of the operator entanglement
- Adding **Decoherence** in Ising allows to reach long times at finite computational resources

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