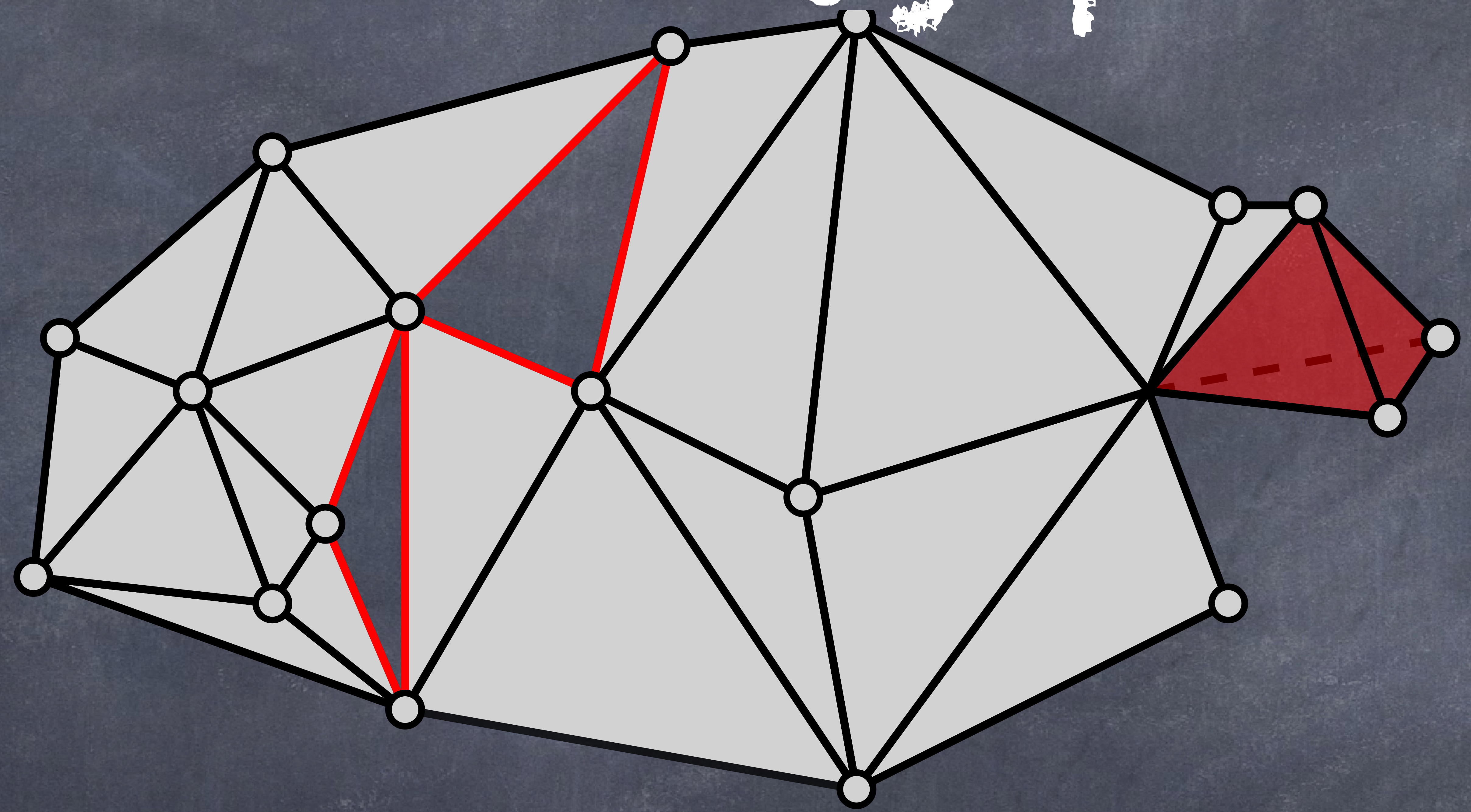
Clique Homology is QMA_1-hard

Joint work with Marcos Crichigno (Imperial College London & QC-ware)

arxiv:2209.11793

The homology problem

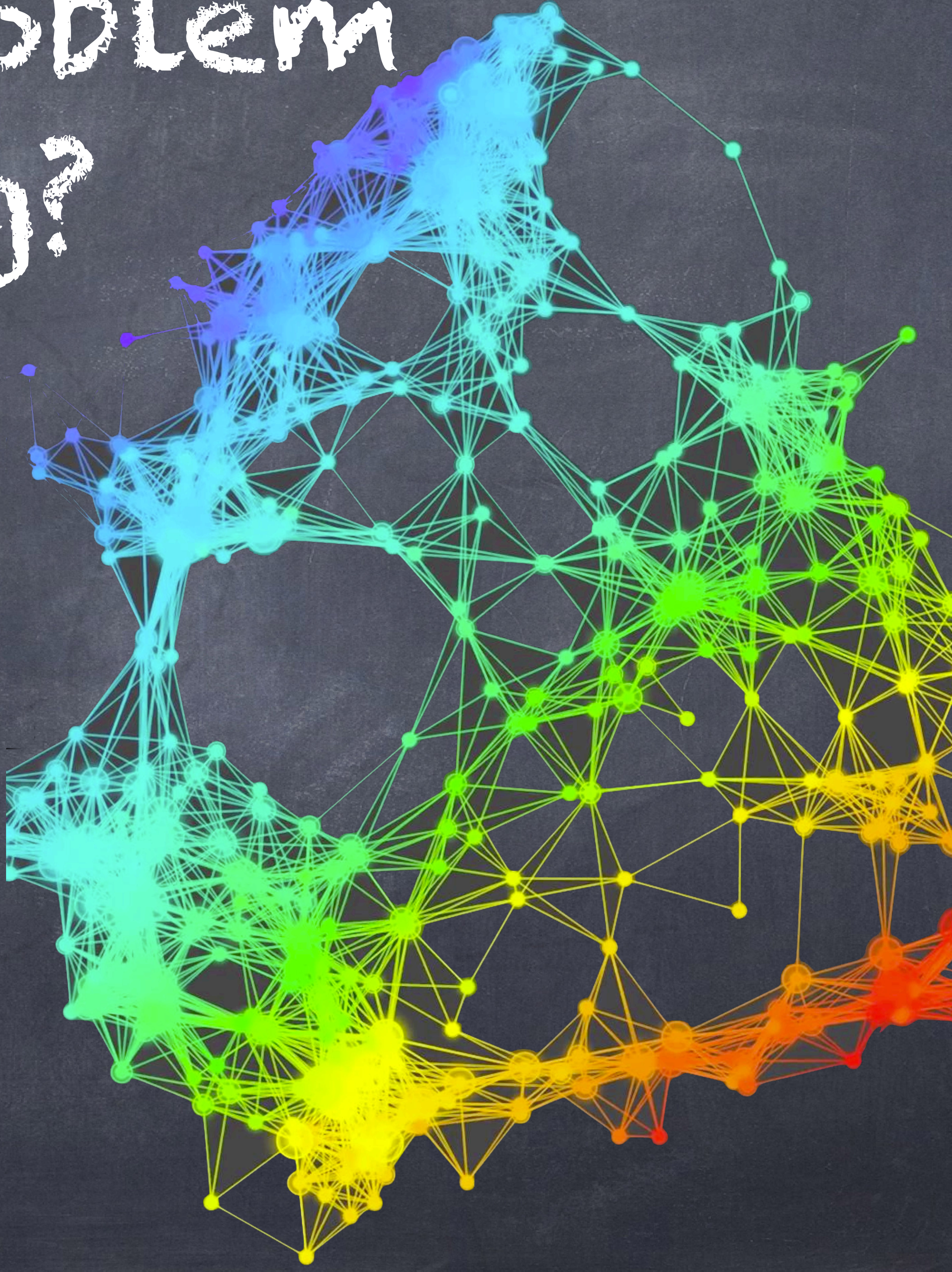


Input: a (efficient description of a) simplicial complex K and an integer, l

Output: yes if K has an l-dimensional hole, no otherwise

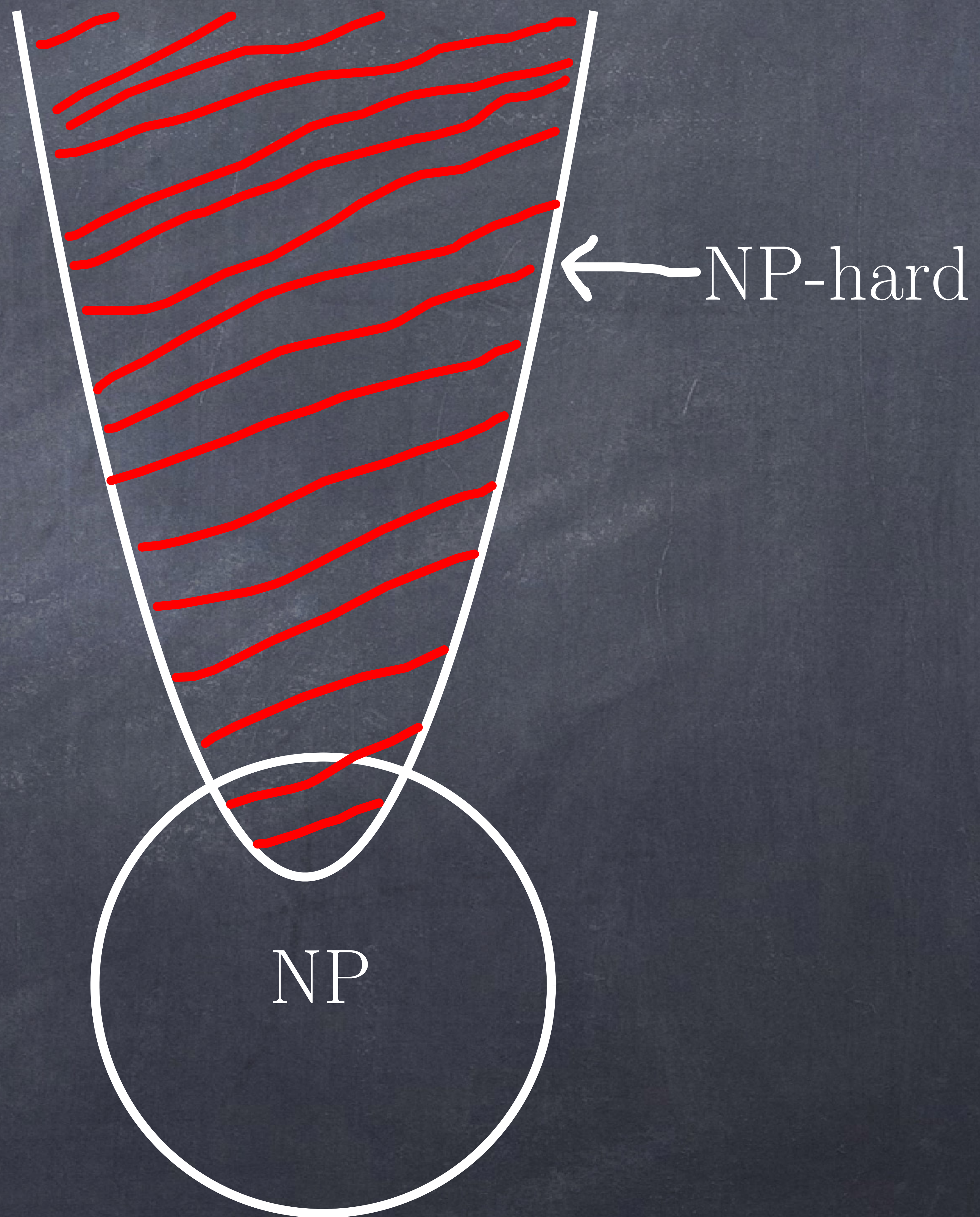
Why is this problem interesting?

- It has applications for topological data analysis - a practically useful problem! (Li et al, 2015)
- There is a quantum algorithm for a closely related problem - can that algorithm be dequantised? (Lloyd et al, 2014)



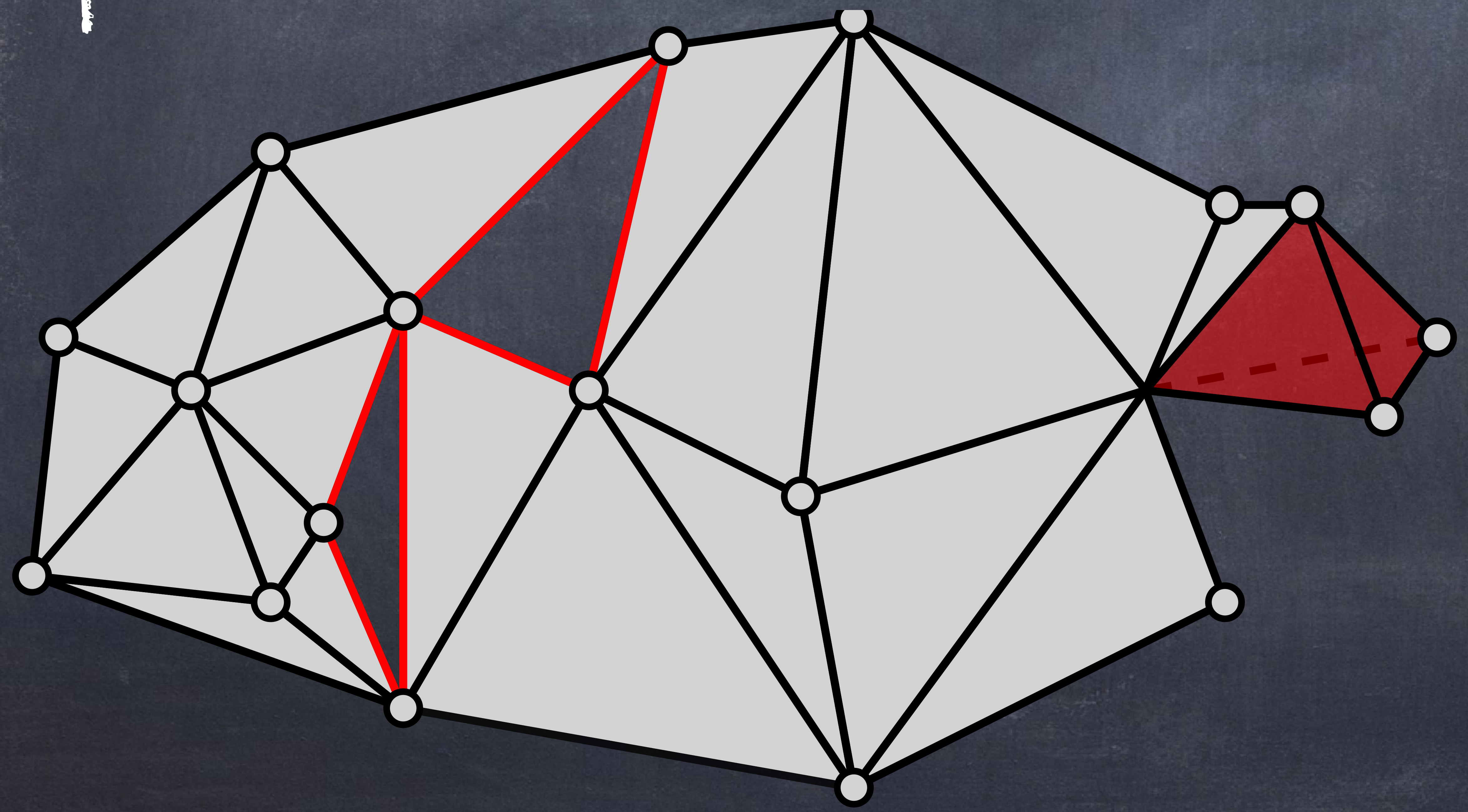
What's known about its complexity?

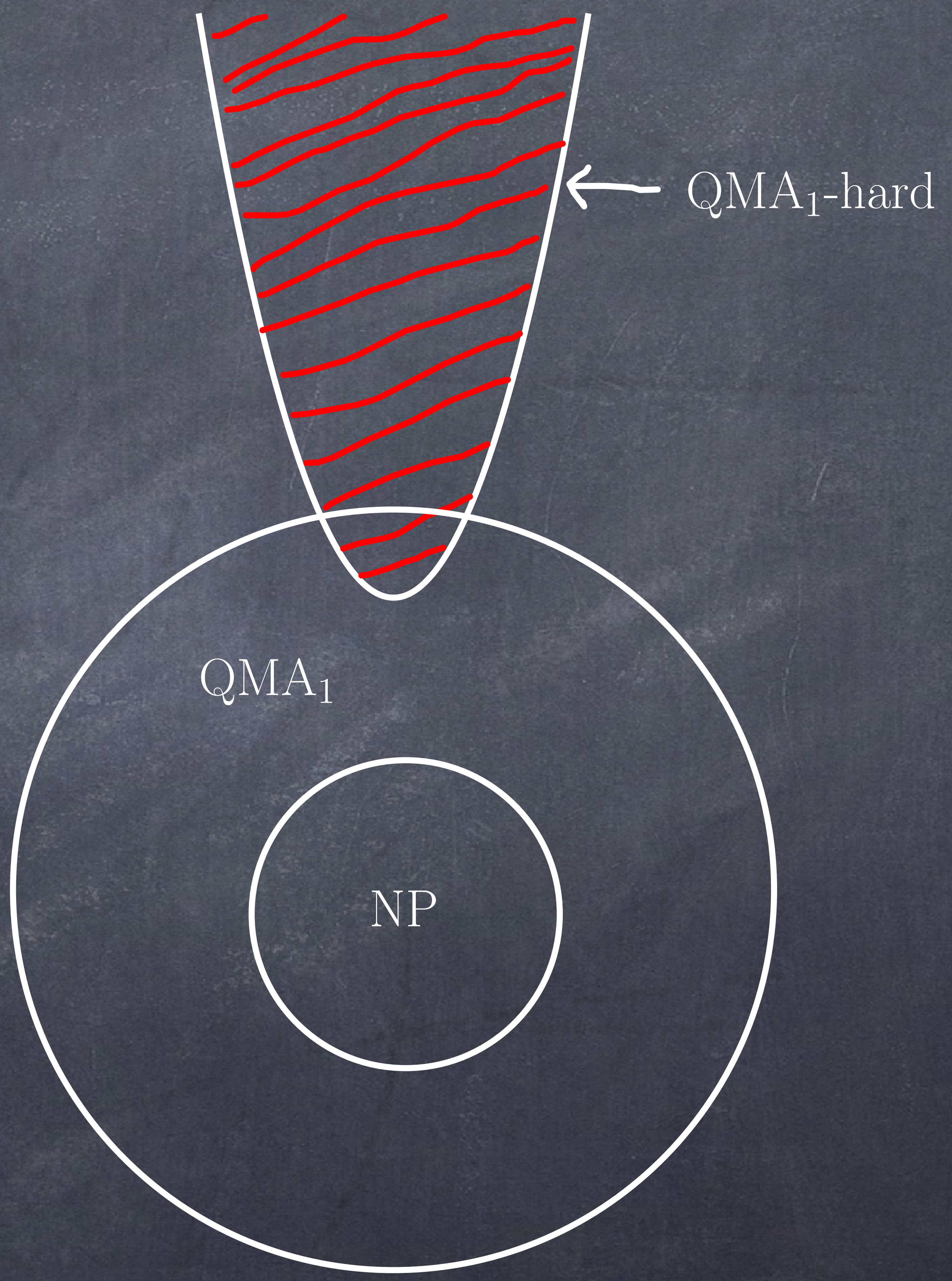
- The problem was first defined formally in 2002 (Kaibel-Pfetsch, 2002)
- It is known to be NP-hard and retains its hardness when restricted to clique complexes (Adamaszek-Stacho, 2016) and when restricted to clique dense complexes (Lloyd-Schmidhuber 2022)
- A similar problem for general chain complexes was shown to be QMA_1-hard last year (Crichigno-Cade, 2021)



Our main results

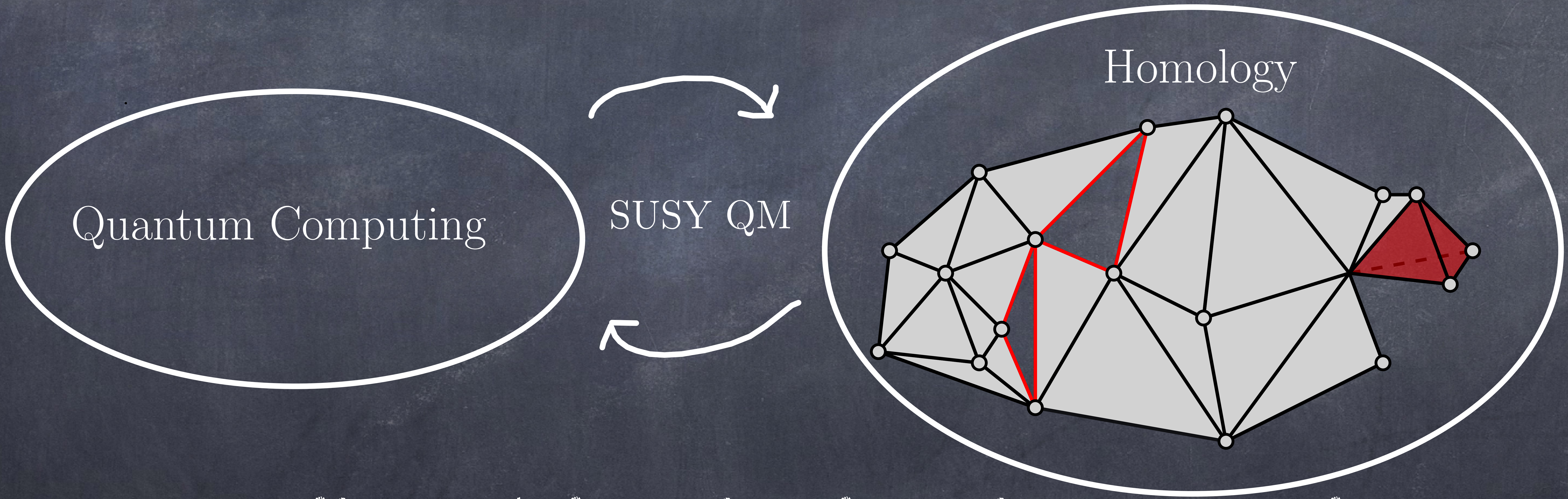
Homology is QMA_1-hard, and retains its hardness when restricted to clique complexes and to clique dense complexes.





Our main results

Why should this (seemingly classical problem) be related to quantum complexity classes?



Can we use this relationship to achieve quantum advantage for problems related to homology?

Outline of talk

- Overview of simplicial homology
- Quantum k-SAT
- Our reduction from quantum k-SAT to homology
- Quantum advantage for topological data analysis?

Simplicial homology

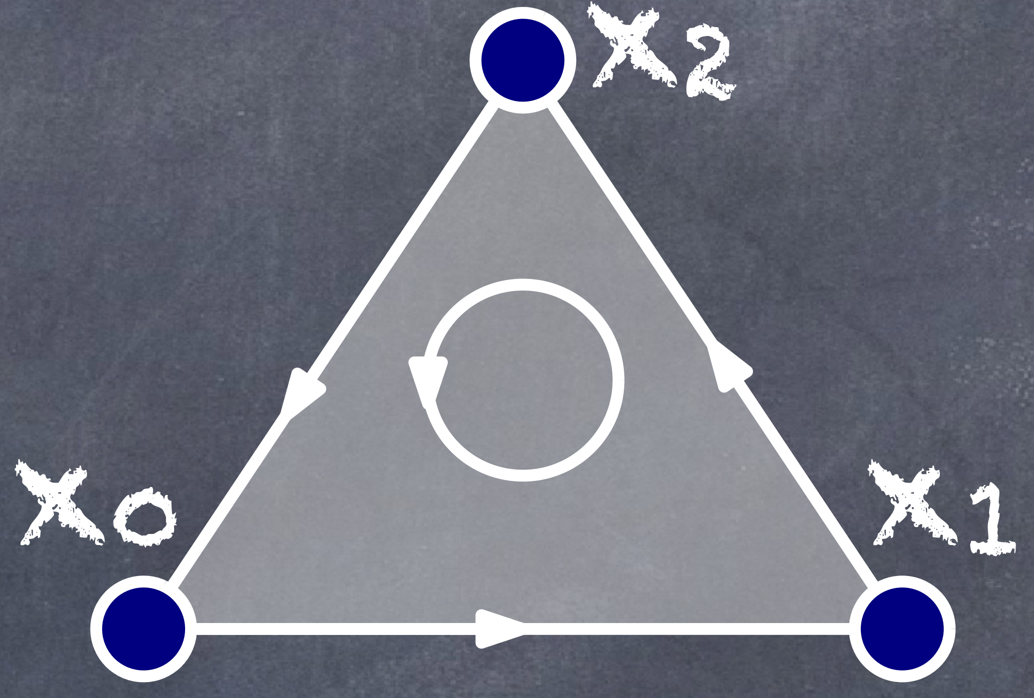
Simplicial homology

0-simplex

$[x_0]$

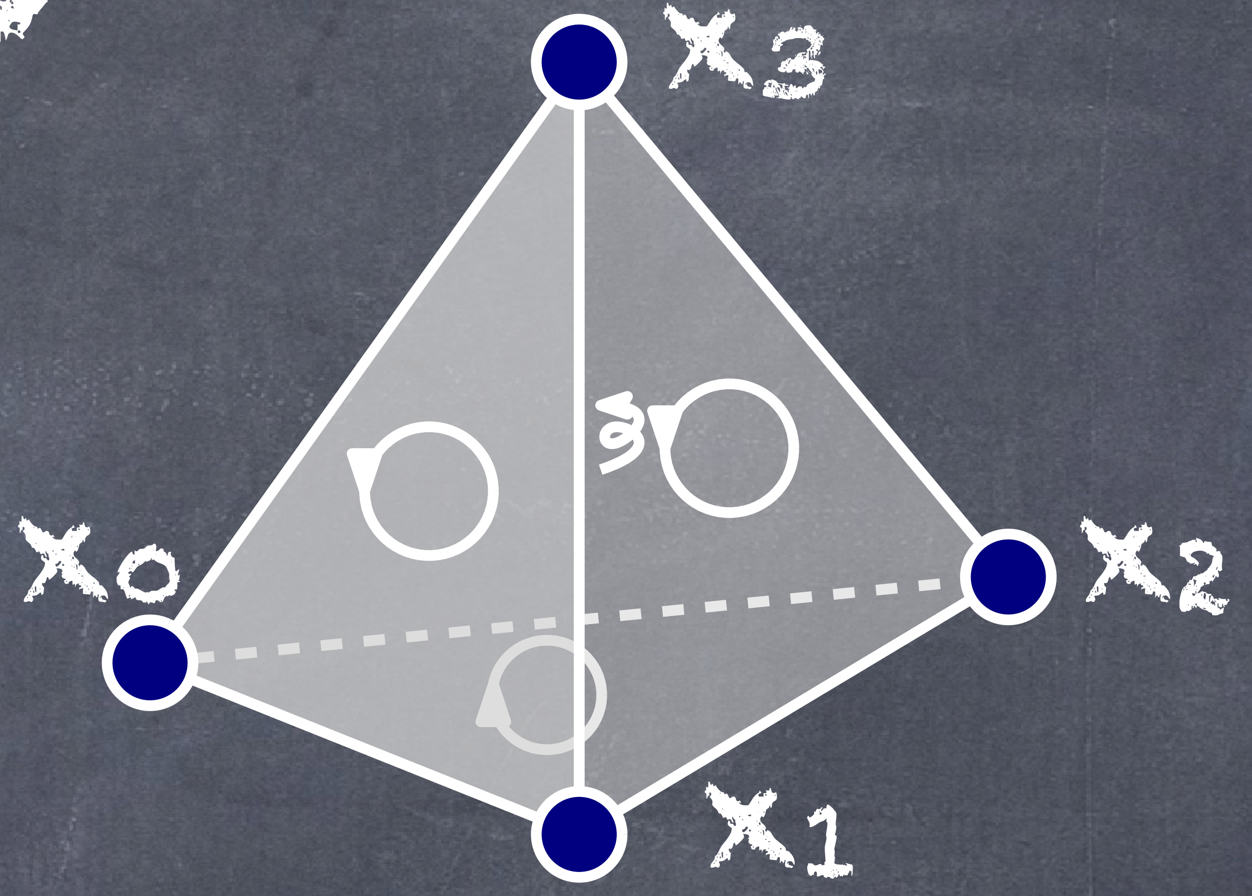
1-simplex

$[x_0x_1]$



2-simplex

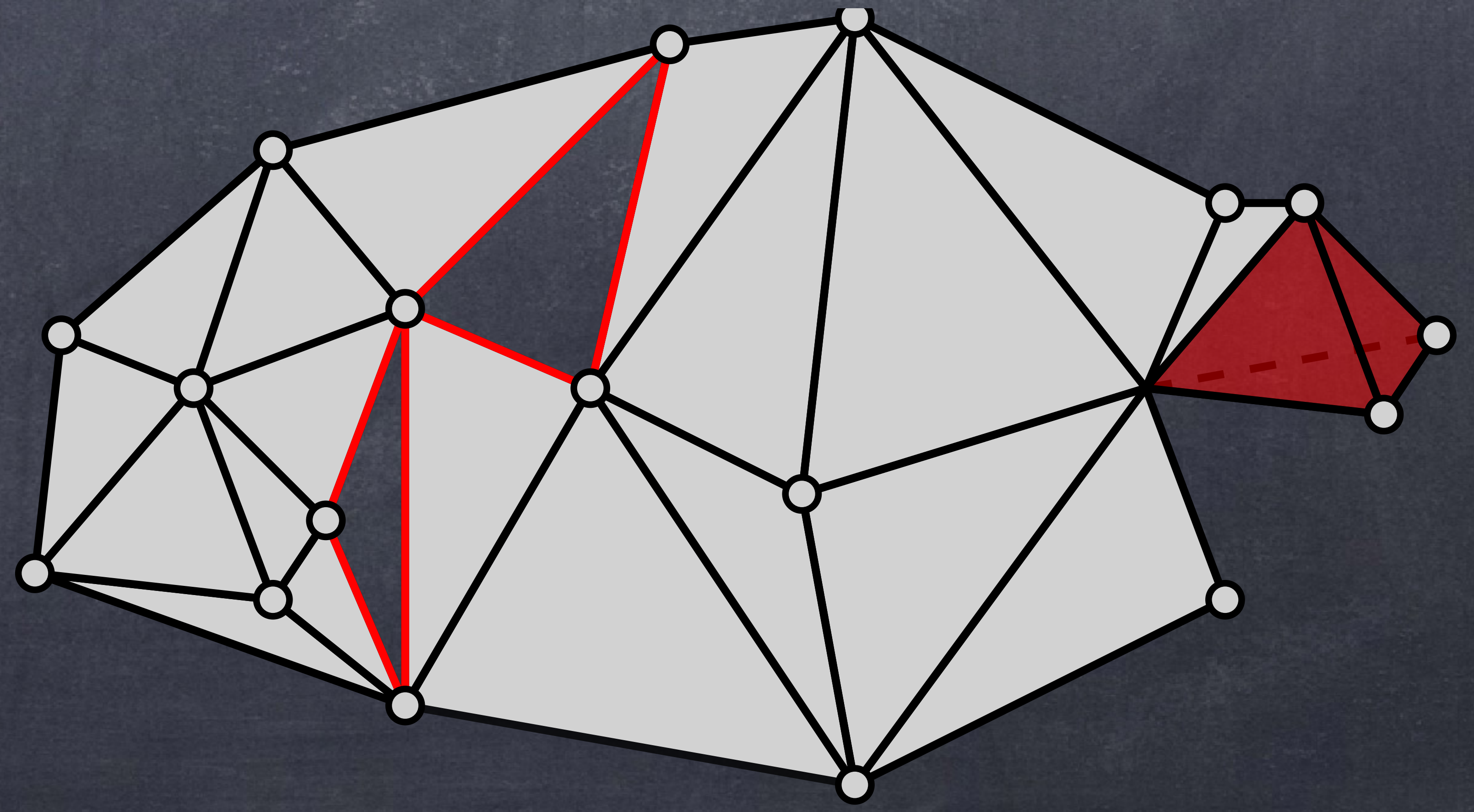
$[x_0x_1x_2]$



3-simplex

$[x_0x_1x_2x_3]$

Simplicial complexes are formed by gluing along faces



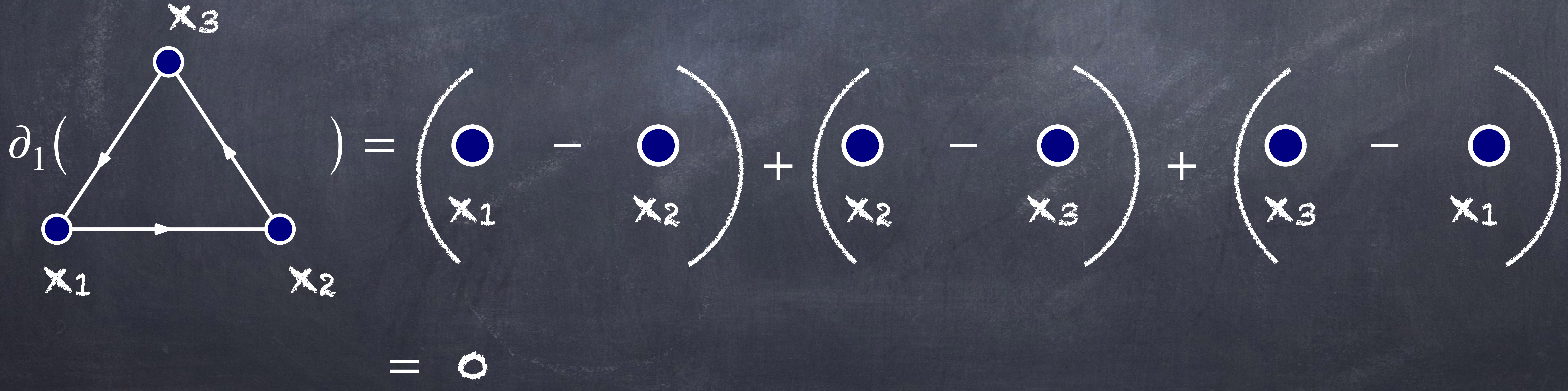
Boundary operator

$$\partial_n[x_0 x_1 \cdots x_n] = \sum_{i=0}^{n} (-1)^i [x_0 \cdots \hat{x}_i \cdots x_n]$$

delete i^{th} vertex

Properties of the boundary operator

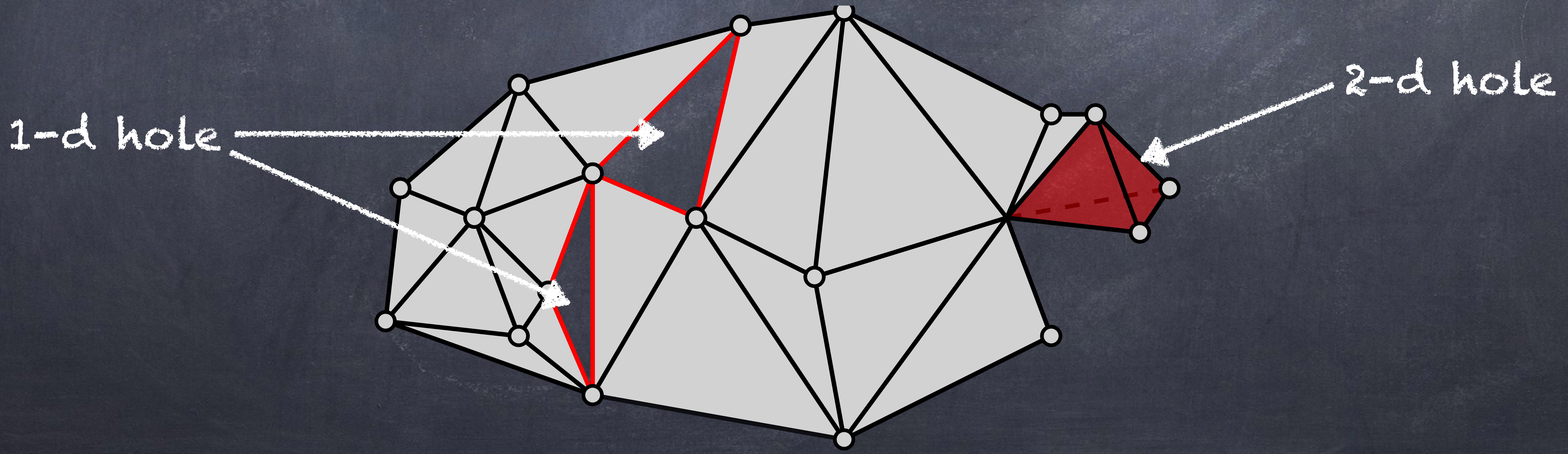
- Cycles dont have boundaries: $\partial_p c = 0$
- The boundary of a boundary vanishes: $\partial_{p-1}\partial_p = 0$



Holes in simplicial complexes

A hole, c:

- is a cycle, $\partial_p c = 0$ ⟶ $c \in \ker(\partial_p)$
- isnt a boundary $c \neq \partial_{p+1} v$ ⟶ $c \notin Im(\partial_{p+1})$



Homology groups

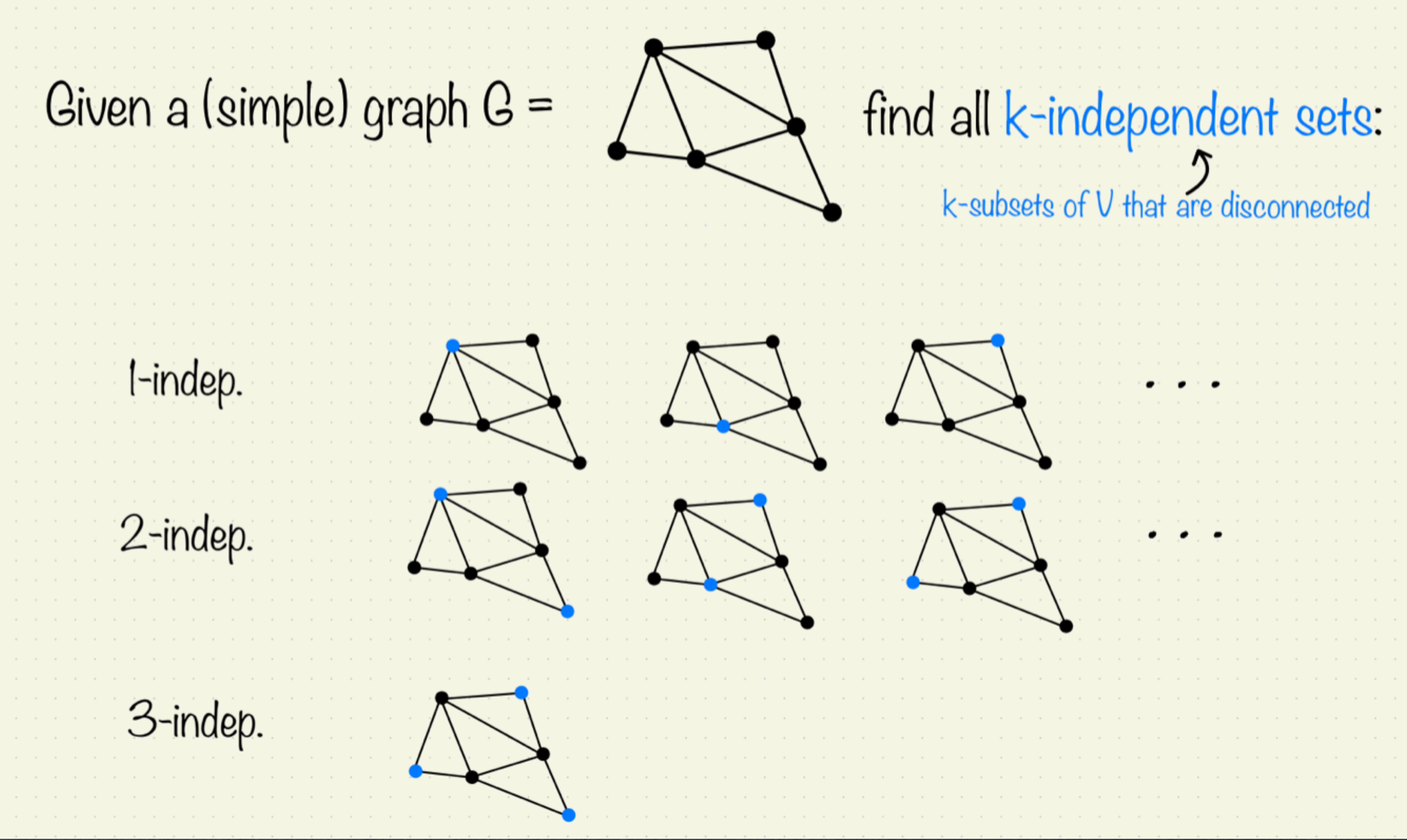
Given a simplicial complex, K, with boundary operator ∂ define:

$$H_p(K) := \frac{\ker(\partial_p)}{Im(\partial_{p+1})}$$

Given a simplicial complex, K, and an integer, p, decide if $H_p(K) \neq 0$ or $H_p(K) = 0$

Independence & clique complexes

- The independence (clique) complex $I(G)$ ($Cl(G)$) of a graph is the simplicial complex defined by its independent sets (cliques)
- We are interested in clique complexes, but $Cl(G) = I(\overline{G})$ and in the reduction we focus on independence complexes



Quantum k-SAT and QMA_1

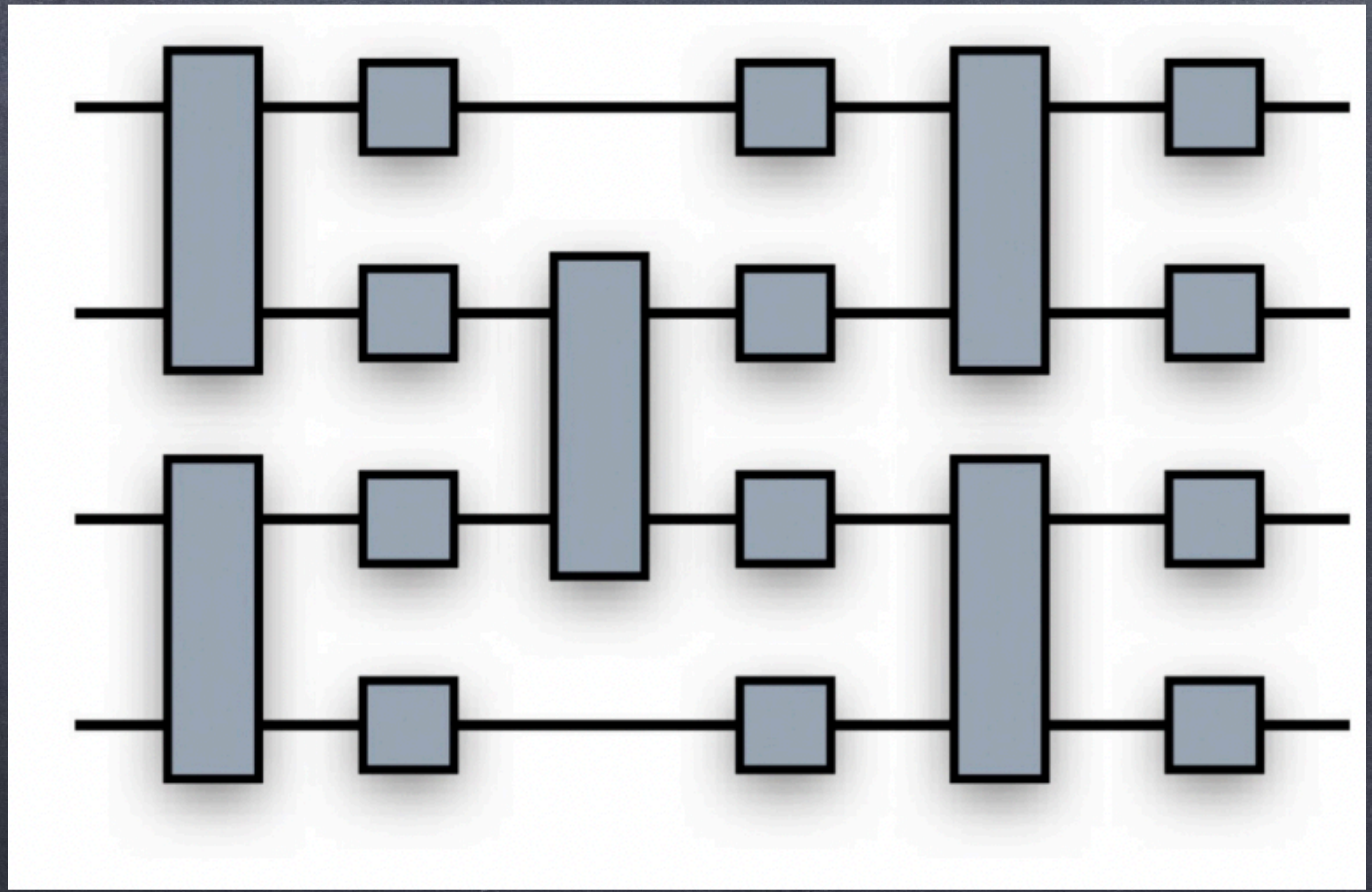
Complexity class QMA_1

Definition 2 ($\mathbf{QMA_1}$). A promise problem $L_{\text{yes}} \cup L_{\text{no}} \subset \{0,1\}^*$ is contained in QMA_1 if and only if there exists a uniform polynomial-size quantum circuit family U_X over the gate set $\mathcal{G}$ such that

If $X \in L_{\text{yes}}$ there exists a state $|W\rangle$ such that $\text{AP}(U_X, |W\rangle) = 1$ (perfect completeness).

If $X \in L_{\text{no}}$ then $\text{AP}(U_X, |W\rangle) \le \frac{1}{3}$ for any state $|W\rangle$ (soundness).

$|W\rangle \rightarrow$



U_X

$P(1) \begin{cases} = 1 \text{ if } X \text{ in } L_{YES} \\ \le 1/3 \text{ if } X \text{ in } L_{NO} \end{cases}$

Complexity class QMA_1

A common choice of universal gate set is:

$$\mathcal{G} = \{\widehat{H}, T, \mathrm{CNOT}\},$$

$$\widehat{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix}, \quad \mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

However any set {CNOT, U} is universal if U is basis changing.

We choose {CNOT, U, Toffoli} where:

$$U = \frac{1}{5}\begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix}$$ "Pythagorean gate"

(Rational coefficients - important later)

Quantum k-SAT [Bravyi, 2006]

Given set of n-qubit, k-local projectors

$$H = \sum_a \Pi_a \qquad a = (1, \ldots, m = \text{poly}(n))$$

decide if

$$H|\psi\rangle = 0 \quad \text{or} \quad \langle\psi|H|\psi\rangle \geq \varepsilon$$

for some $|\psi\rangle$ — YES

for all $|\psi\rangle$ — NO

ε ← 1/poly(n)

promised one is the case.

Theorem: [Bravyi, 2006] [Gosset-Nagaj, 2013]

Quantum 4-SAT is QMA_1-complete

Theorem: [Bravyi, 2006] [Gosset-Nagaj, 2013]

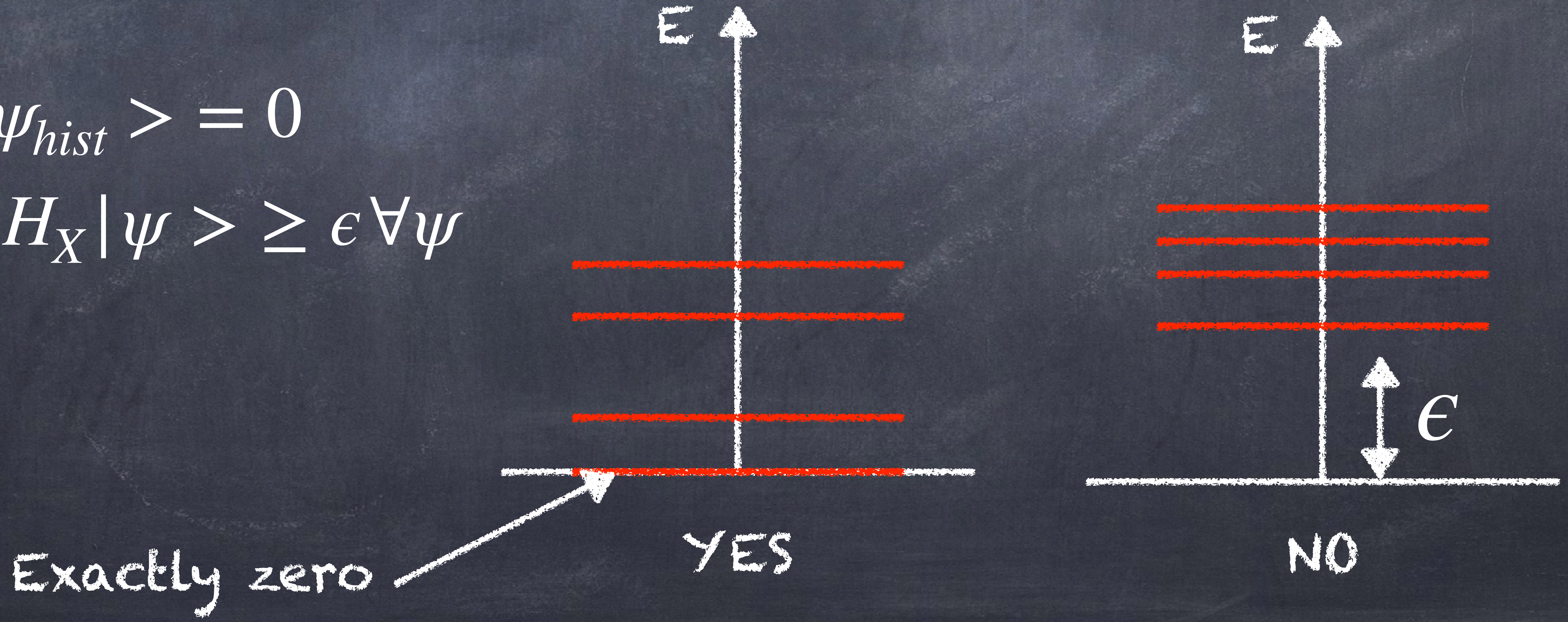
Quantum 4-SAT is QMA_1-complete

Given a QMA_1 verification circuit U_X construct a Hamiltonian:

$$H_X = \sum_a \Pi_a(U_X)$$

Such that:

- if $X \in L_{Yes}$, $H_X|\psi_{hist}> = 0$
- if $X \in L_{No}$, $<\psi|H_X|\psi> \geq \epsilon \forall \psi$



Theorem: [Bravyi, 2006] [Gosset-Nagaj, 2013]

Quantum 4-SAT is QMA_1-complete

The Hamiltonian $H_X = H_{in} + H_{clock} + H_{prop} + H_{out}$

$$\sum_{in} |011><011|$$

$$|011><011|$$

$$
\begin{aligned}
H^{(1)}_{\text{clock}} &= |u\rangle\langle u|_1, \\
H^{(2)}_{\text{clock}} &= |d\rangle\langle d|_L, \\
H^{(3)}_{\text{clock}} &= \sum_{1\le j<k\le L} (|a1\rangle\langle a1| + |a2\rangle\langle a2|)_j \otimes (|a1\rangle\langle a1| + |a2\rangle\langle a2|)_k, \\
H^{(4)}_{\text{clock}} &= \sum_{1\le j<k\le L} (|a1\rangle\langle a1| + |a2\rangle\langle a2| + |u\rangle\langle u|)_j \otimes |d\rangle\langle d|_k, \\
H^{(5)}_{\text{clock}} &= \sum_{1\le j<k\le L} |u\rangle\langle u|_j \otimes (|a1\rangle\langle a1| + |a2\rangle\langle a2| + |d\rangle\langle d|)_k, \\
H^{(6)}_{\text{clock}} &= \sum_{1\le j\le L-1} |d\rangle\langle d|_j \otimes |u\rangle\langle u|_{j+1}.
\end{aligned}
$$

$$H_{\text{prop},t} = \frac{1}{2}\left[(|a1\rangle\langle a1| + |a2\rangle\langle a2|)_t \otimes I_{\text{comp}} - |a2\rangle\langle a1|_t \otimes U_t - |a1\rangle\langle a2|_t \otimes U_t^\dagger\right],$$

$$H'_{\text{prop},t} = \frac{1}{2}(|a2,u\rangle\langle a2,u| + |d,a1\rangle\langle d,a1| - |d,a1\rangle\langle a2,u| - |a2,u\rangle\langle d,a1|)_{t,t+1} \otimes I_{\text{comp}}$$

In order to reduce from quantum k-SAT to homology we need to encode the following projectors into an independence complex:

Term in H_{Bravyi}	*Penalizes state* $\lvert\psi_S\rangle$
$H'_{\text{prop,t}}$	$\frac{1}{\sqrt{2}}\lvert 10\rangle(\lvert 11\rangle - \lvert 00\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\lvert 01\rangle(\lvert 10\rangle - \lvert 01\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\lvert 00\rangle(\lvert 10\rangle - \lvert 01\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\lvert 000\rangle(\lvert 10\rangle - \lvert 01\rangle)$
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$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\lvert 010\rangle(\lvert 10\rangle - \lvert 01\rangle)$
$H_{\text{prop,t}}(U_{Pyth.})$	$\frac{1}{5\sqrt{2}}(-5\lvert 011\rangle + 4\lvert 100\rangle + 3\lvert 101\rangle)$
$H_{\text{prop,t}}(U_{Pyth.})$	$\frac{1}{5\sqrt{2}}(-5\lvert 010\rangle + 3\lvert 100\rangle - 4\lvert 101\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\lvert 1\rangle(\lvert 101\rangle - \lvert 010\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\lvert 11\rangle(\lvert 101\rangle - \lvert 010\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\lvert 1\rangle(\lvert 011\rangle - \lvert 100\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\lvert 11\rangle(\lvert 011\rangle - \lvert 100\rangle)$
$H^{(1)}_{\text{clock}}$	$\lvert 00\rangle$
$H^{(2)}_{\text{clock}}$	$\lvert 11\rangle$
H_{in}, H_{out}	$\lvert 011\rangle$
$H^{(6)}_{\text{clock}}$, $H^{(4)}_{\text{clock}}$, $H^{(5)}_{\text{clock}}$, $H^{(3)}_{\text{clock}}$	$\lvert 1100\rangle$
$H^{(4)}_{\text{clock}}$	$\lvert 0111\rangle$
$H^{(5)}_{\text{clock}}$	$\lvert 0001\rangle$

Reduction from quantum k-SAT to homology

Proof idea

- Construct a graph where the independence complex has 2^n $(n-1)$-dimensional holes

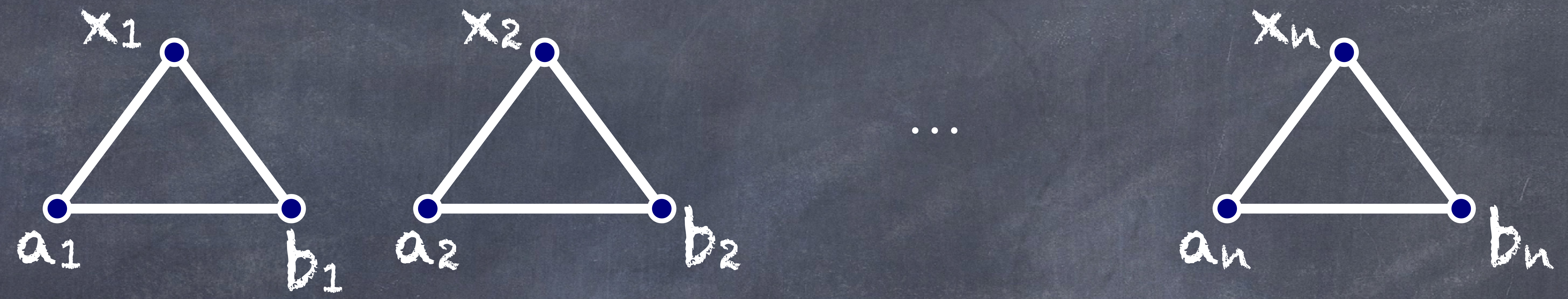
← 2^n holes!

- Construct gadgets which 'fill in' holes corresponding to the projectors in quantum k-SAT
- Build up the graph corresponding to H_X - any remaining holes are satisfying solutions to quantum k-SAT

Constructing the holes

Consider n disconnected triangles:

$G_n =$

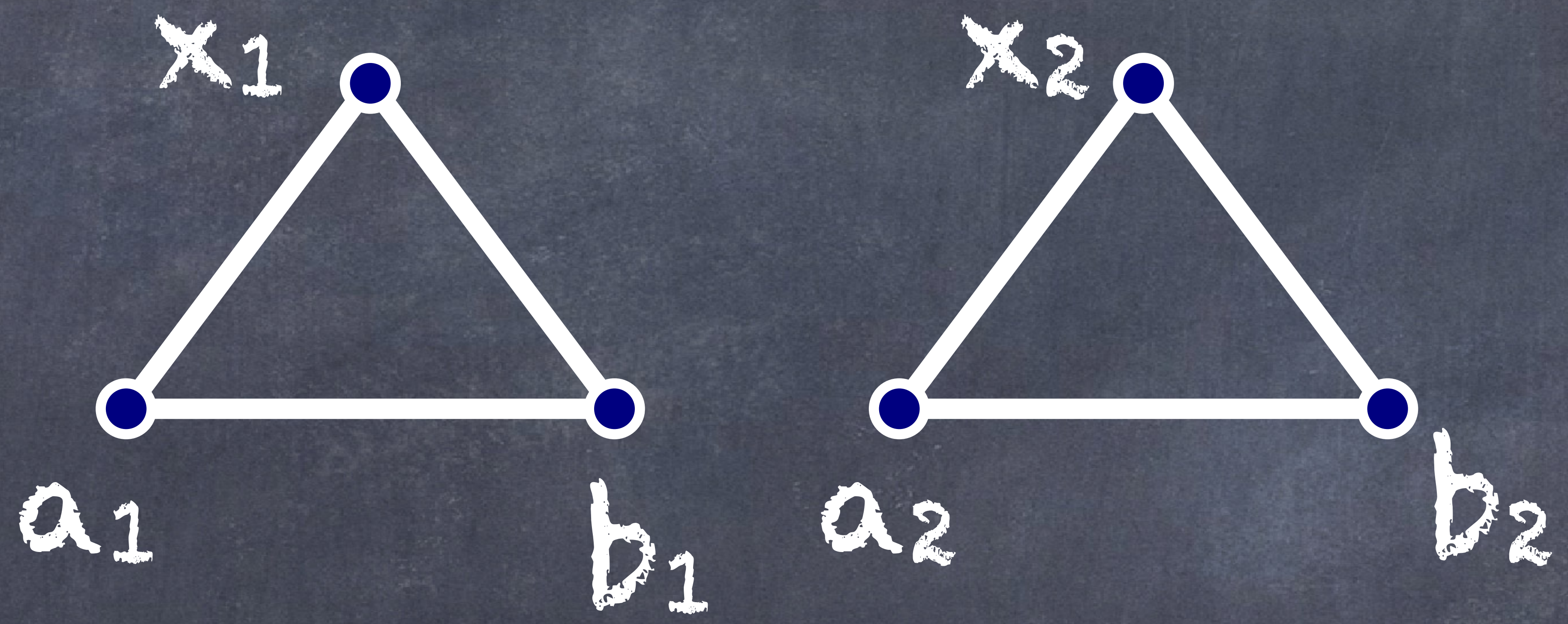


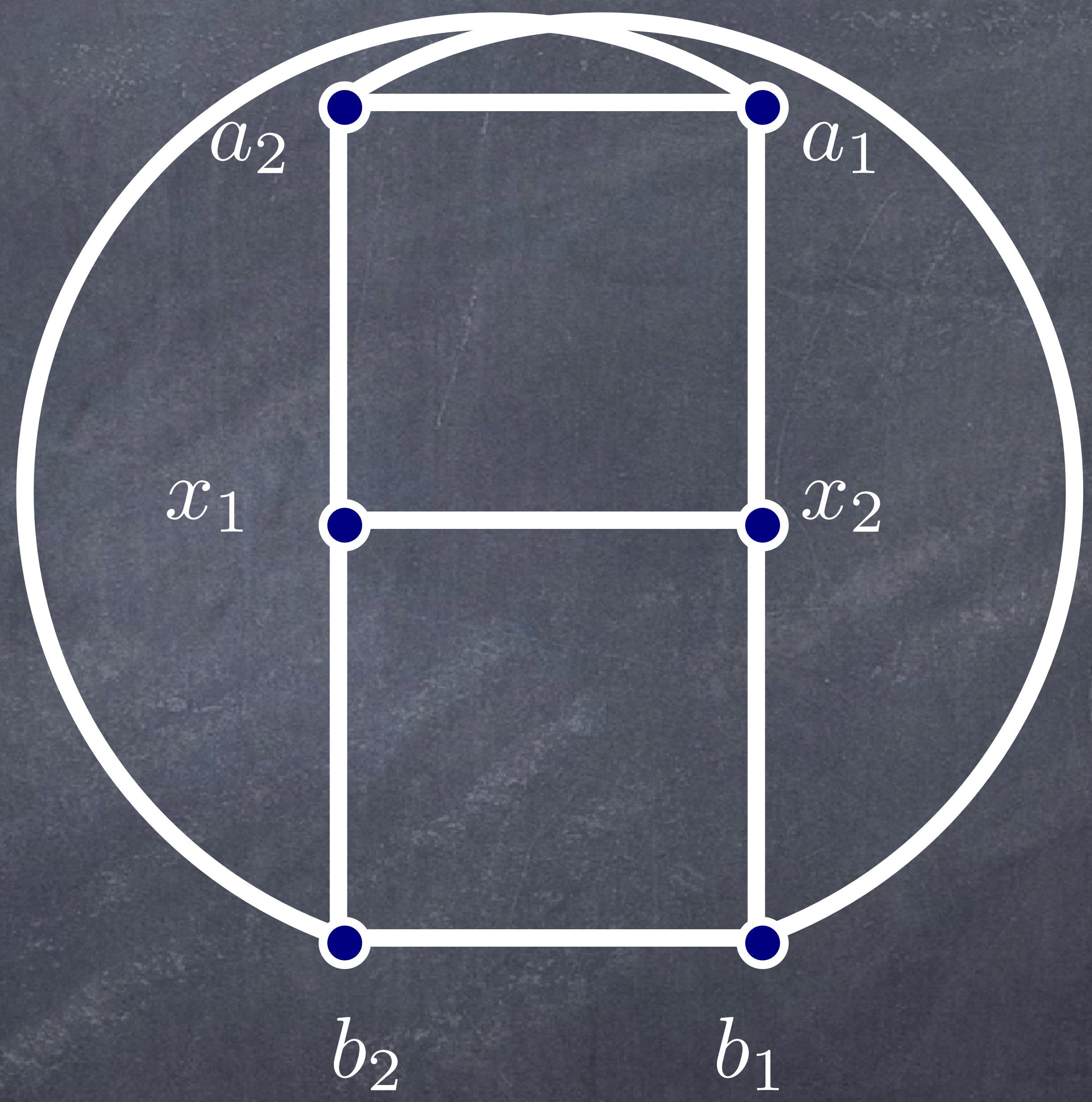
The independence complex, $\Sigma_n = I(G_n)$, has dimension n and:

$H_0(\Sigma_n) = \mathbb{C}$ $\quad H_{n-1}(\Sigma_n) = (\mathbb{C}^2)^{\otimes n}$ $\quad H_i(\Sigma_n) = 0 \, \forall i \neq 0, n-1$

Hilbert space of n qubits!

Constructing 2 qubit projectors

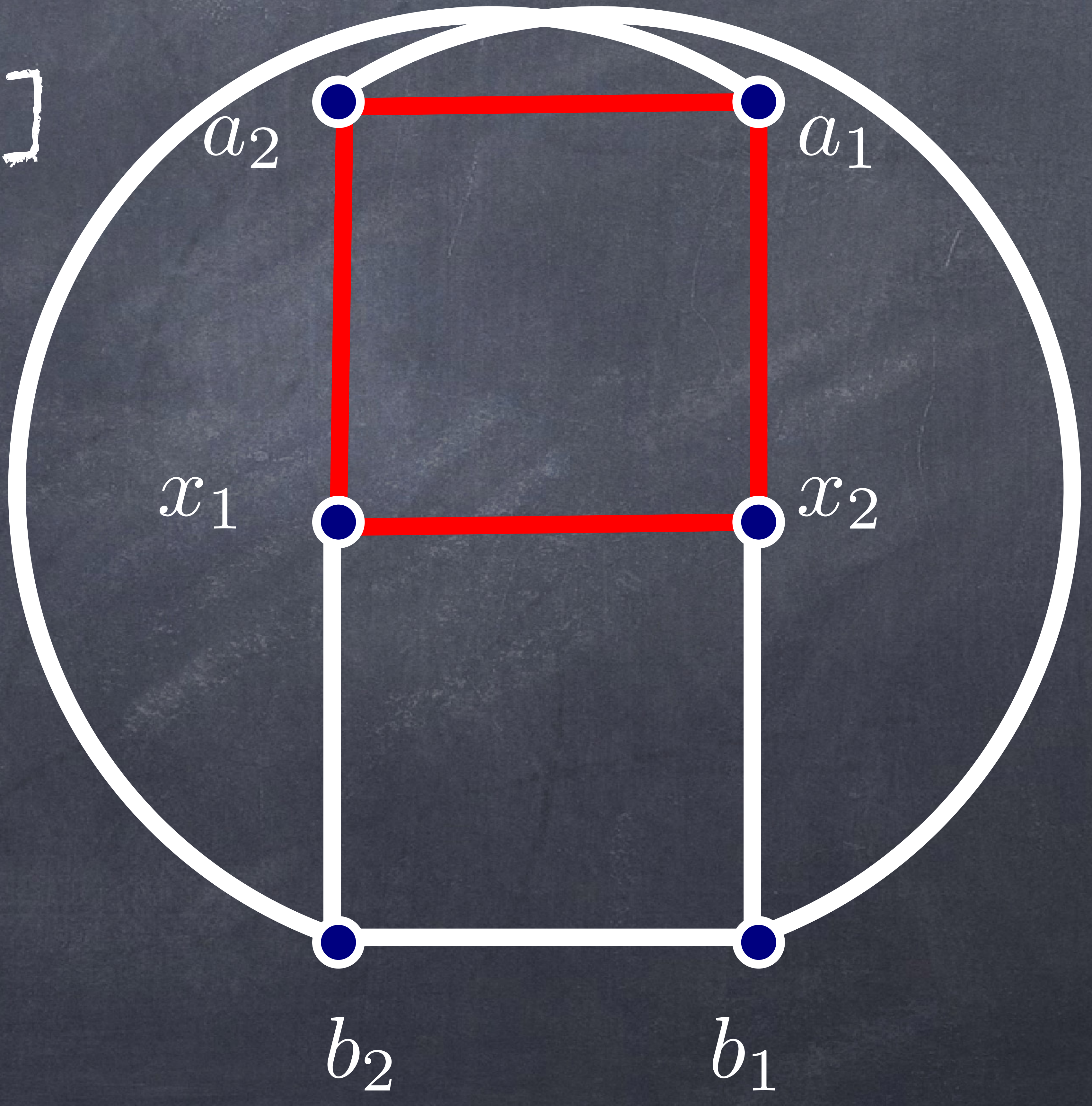
$G_2=$ 

$\Sigma_2=$ 

Constructing 2 qubit projectors

We have $H_1(\Sigma_n) = \mathbb{C}^4$ and can choose a basis:

$|00\rangle = [x_1 x_2] + [x_2 a_1] + [a_1 a_2] + [a_2 x_1]$



Constructing 2 qubit projectors

We have $H_1(\Sigma_n) = \mathbb{C}^4$ and can choose a basis:

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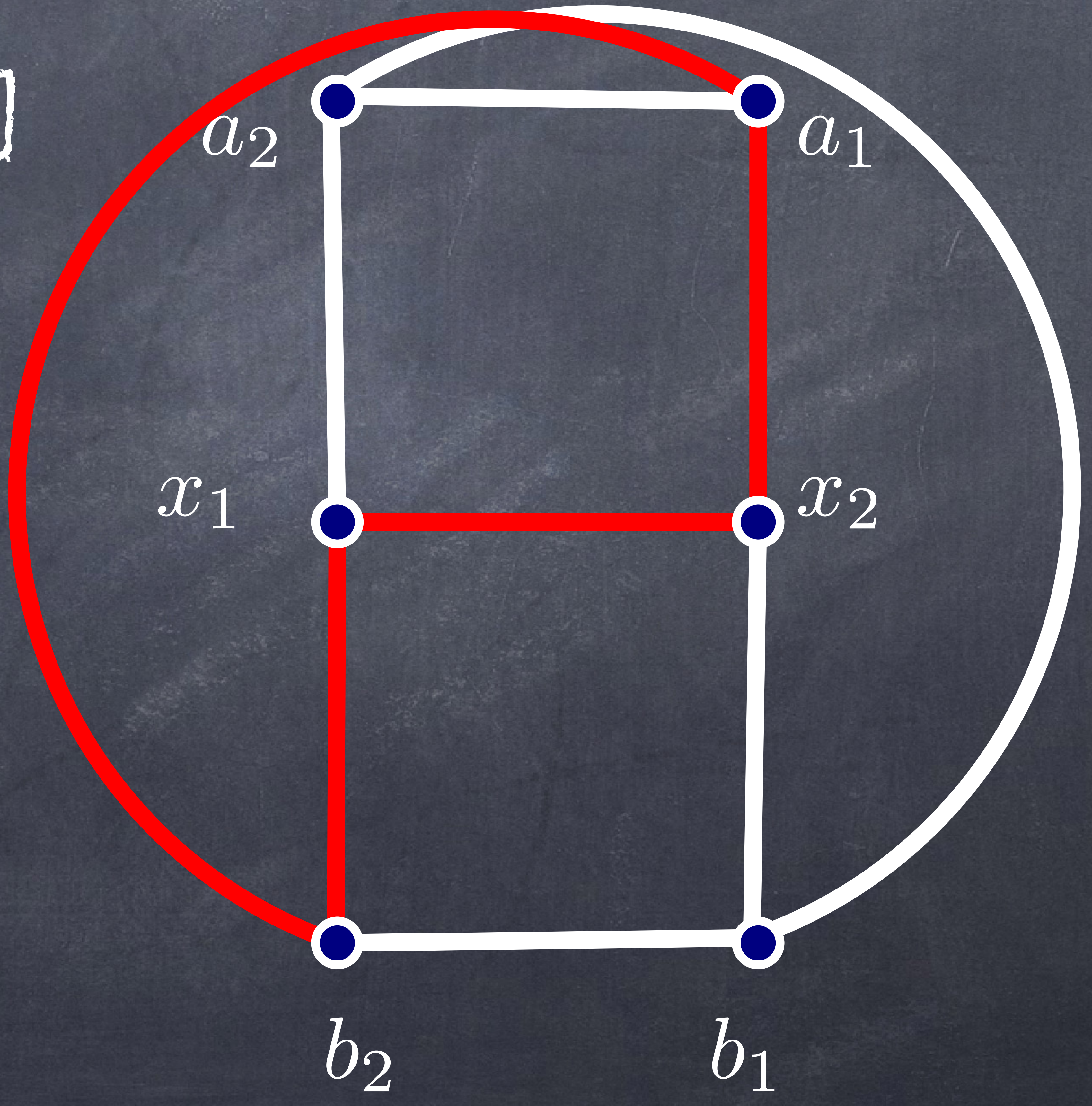
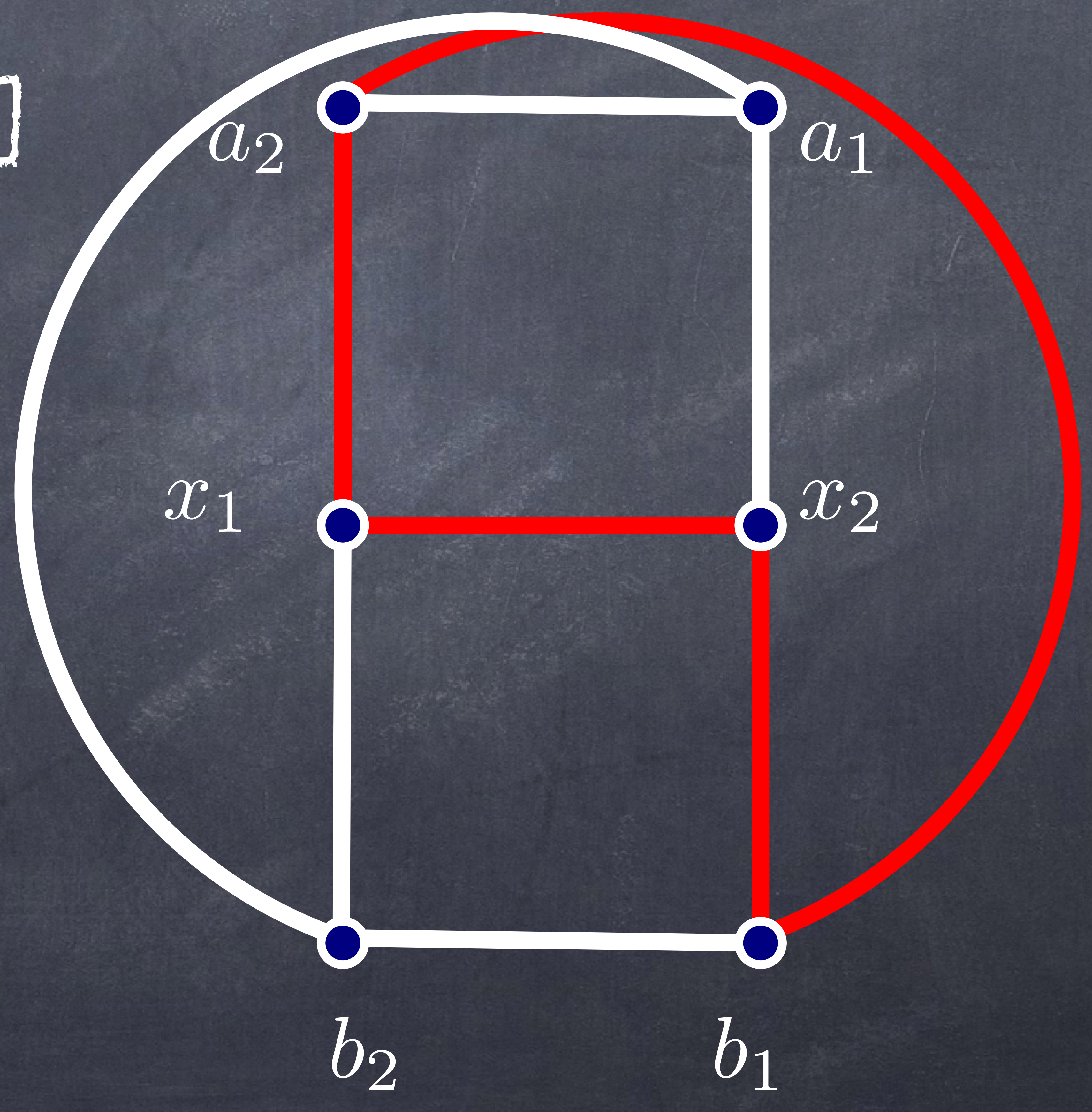


Constructing 2 qubit projectors

We have $H_1(\Sigma_n) = \mathbb{C}^4$ and can choose a basis:

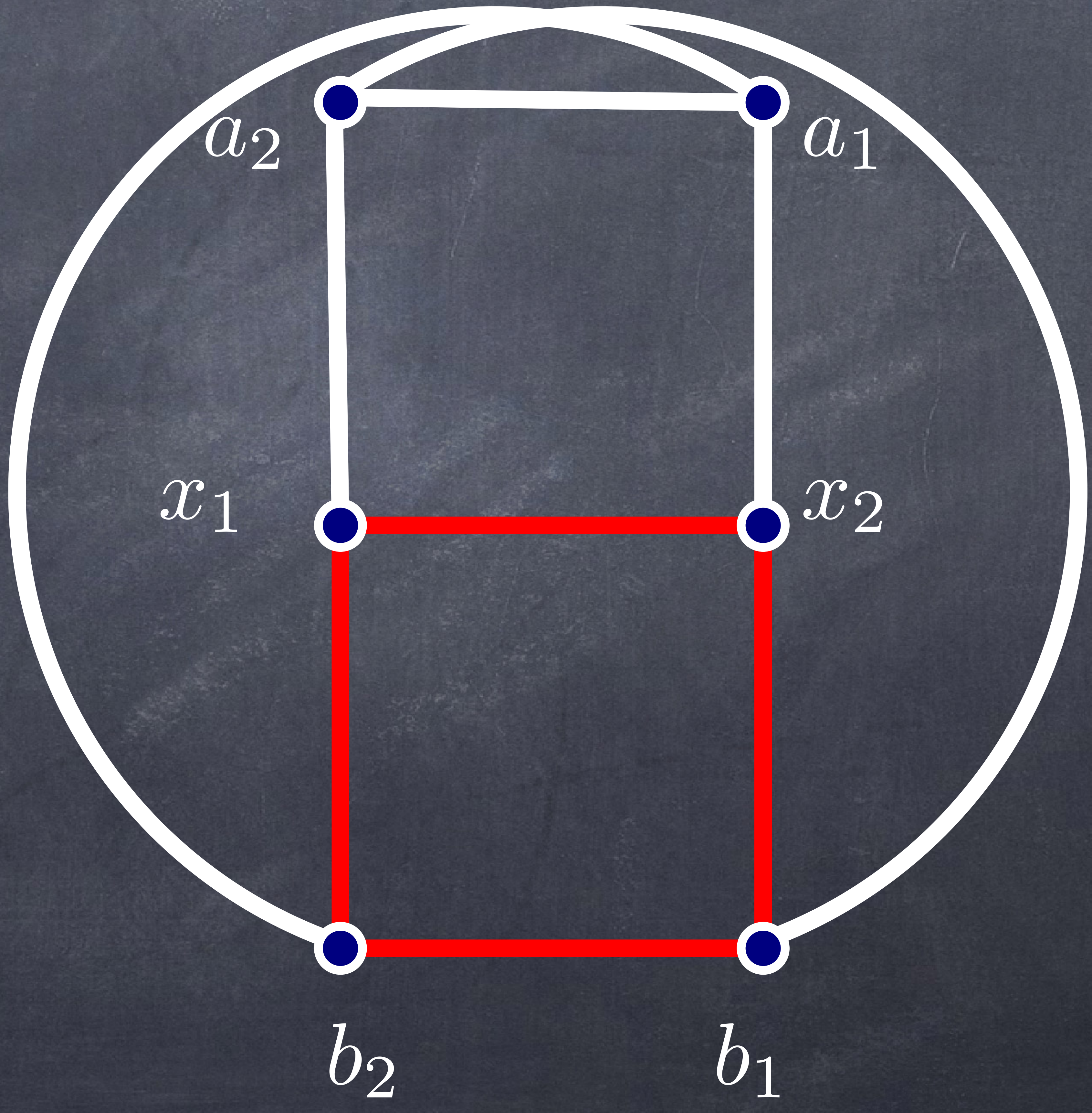
$|10\rangle = [x_1x_2] + [x_2b_1] + [b_1a_2] + [a_2x_1]$



Constructing 2 qubit projectors

We have $H_1(\Sigma_n) = \mathbb{C}^4$ and can choose a basis:

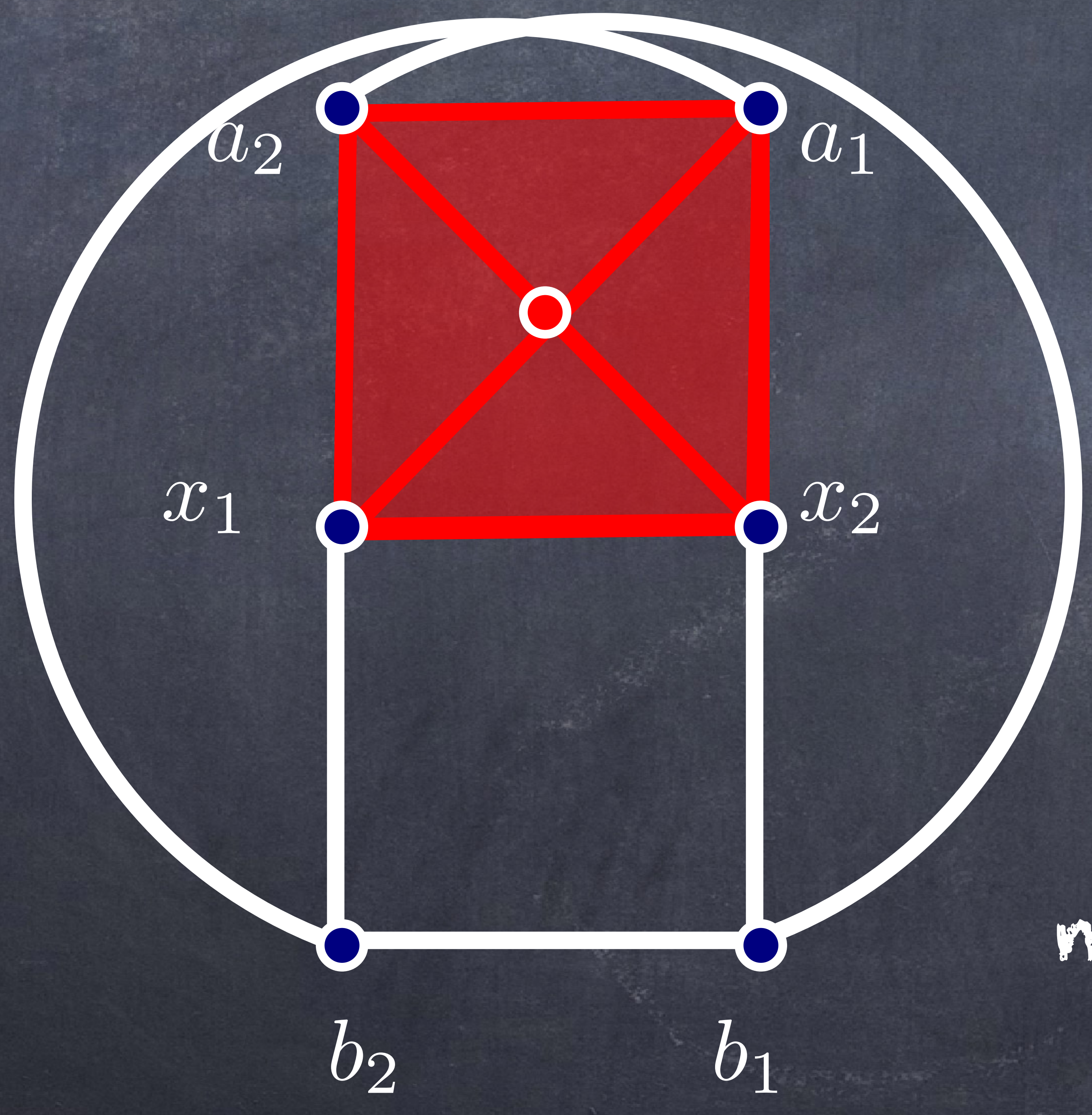
$|11\rangle = [x_1 x_2] + [x_2 b_1] + [b_1 b_2] + [b_2 x_1]$



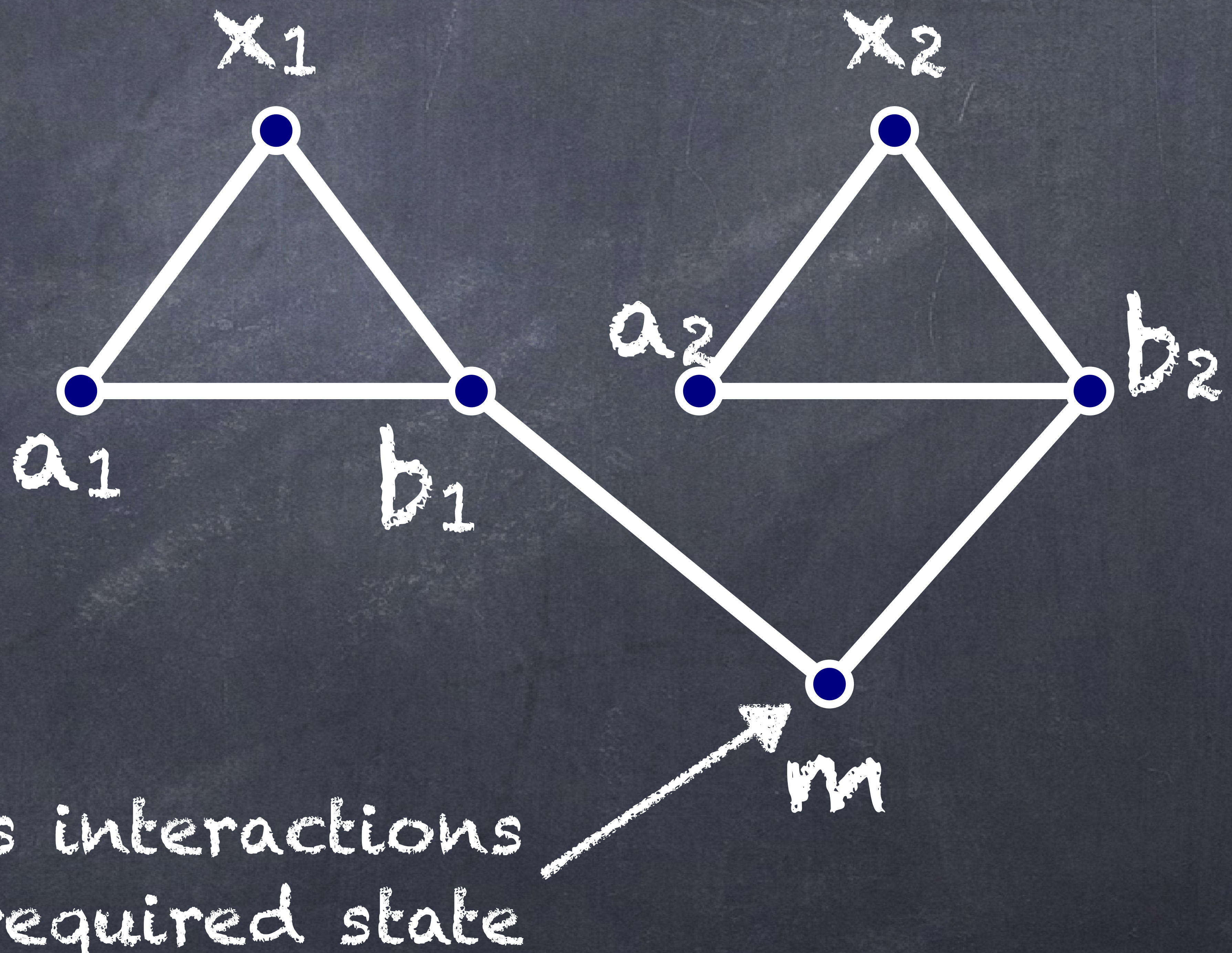
Constructing 2 qubit projectors

Consider the classical projector $\Pi = |00><00|$, we need to fill in the cycle corresponding to |00>:

$\Sigma_2' =$



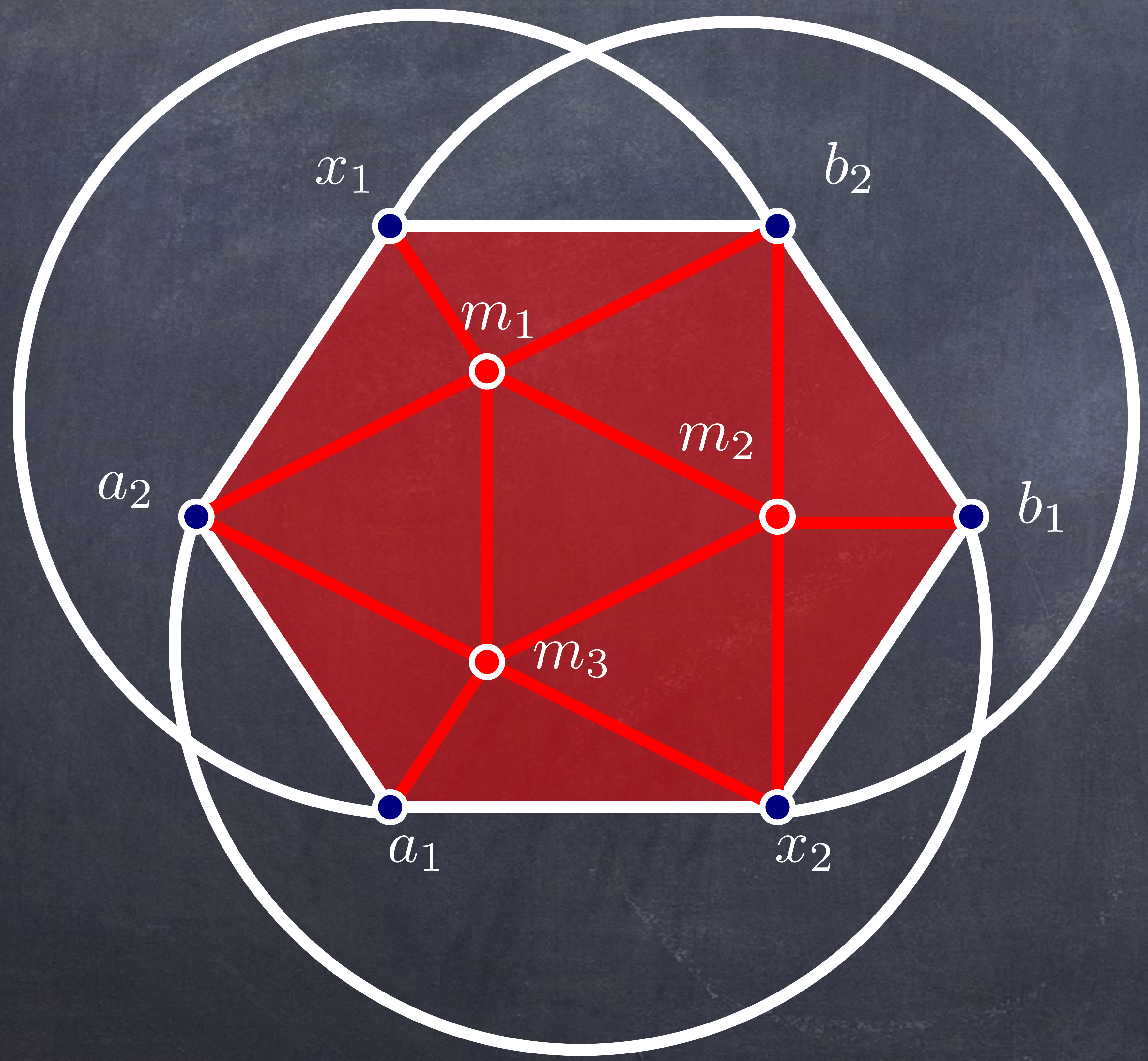
$G_2' =$



The same process works for the other computational basis states. For the entangled states things get a little more complicated...

$$\Pi = (|00> - |11>)(<00| - <11|)$$

$\Sigma_2' =$



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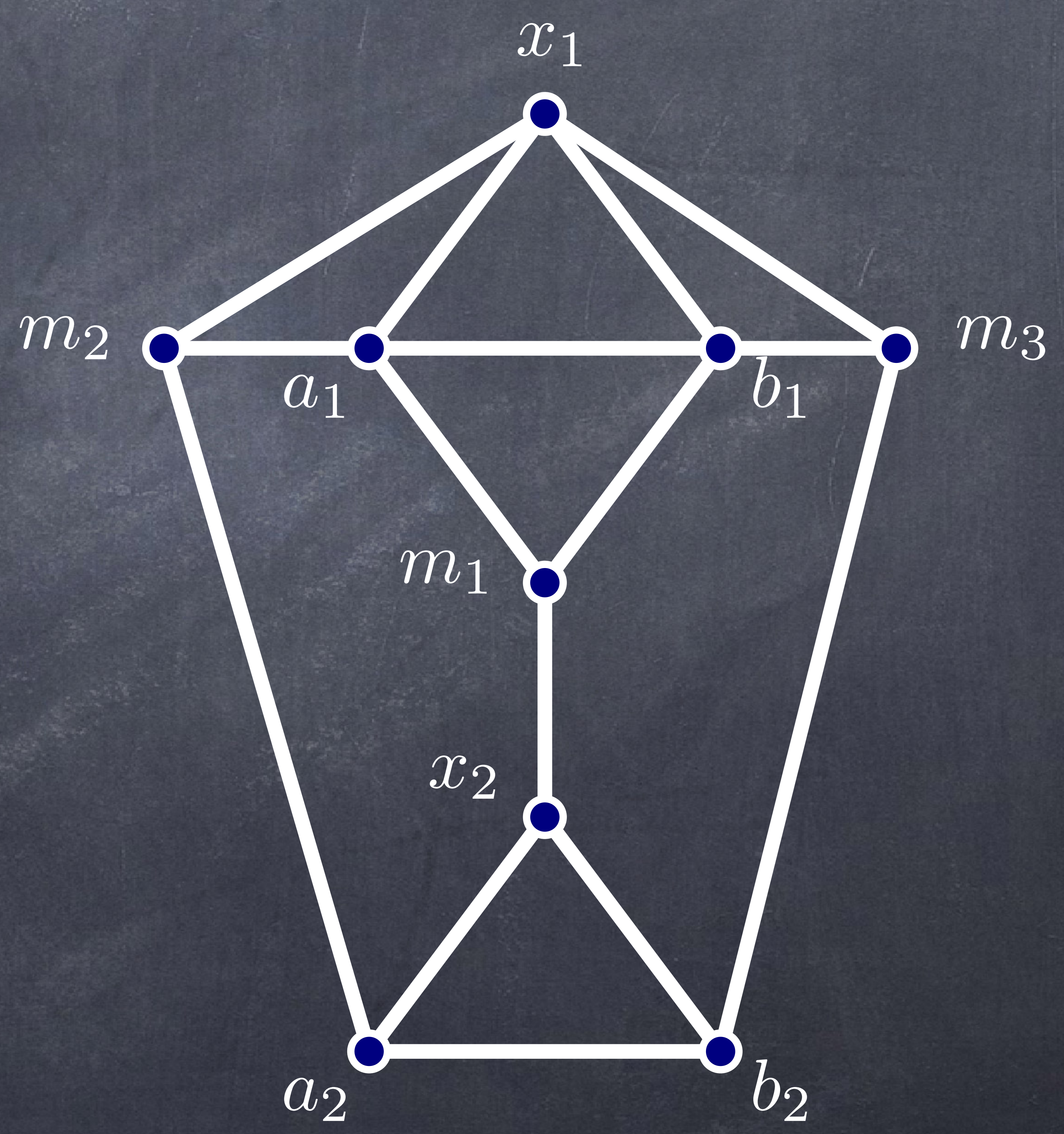
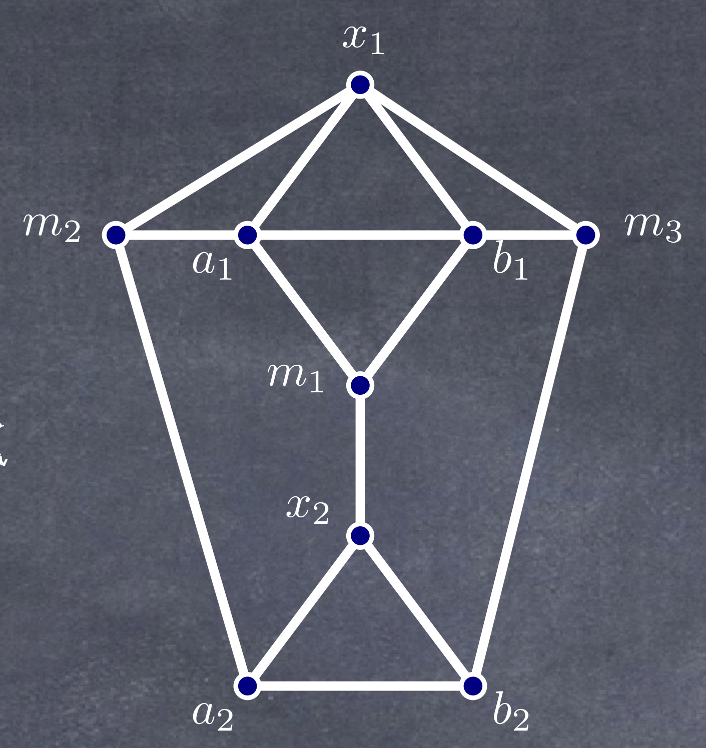


So, we created $G_2' =$  such that:

$$H_1(I(G_2'))$$

$$\|$$

$$\text{span}\{|01\rangle, |10\rangle, |00\rangle + |11\rangle\}$$

$=$

Space of satisfying solutions to

$$\Pi|\psi\rangle = 0$$

$$\Pi = (|00\rangle - |11\rangle)(\langle 00| - \langle 11|)$$

Taking stock...

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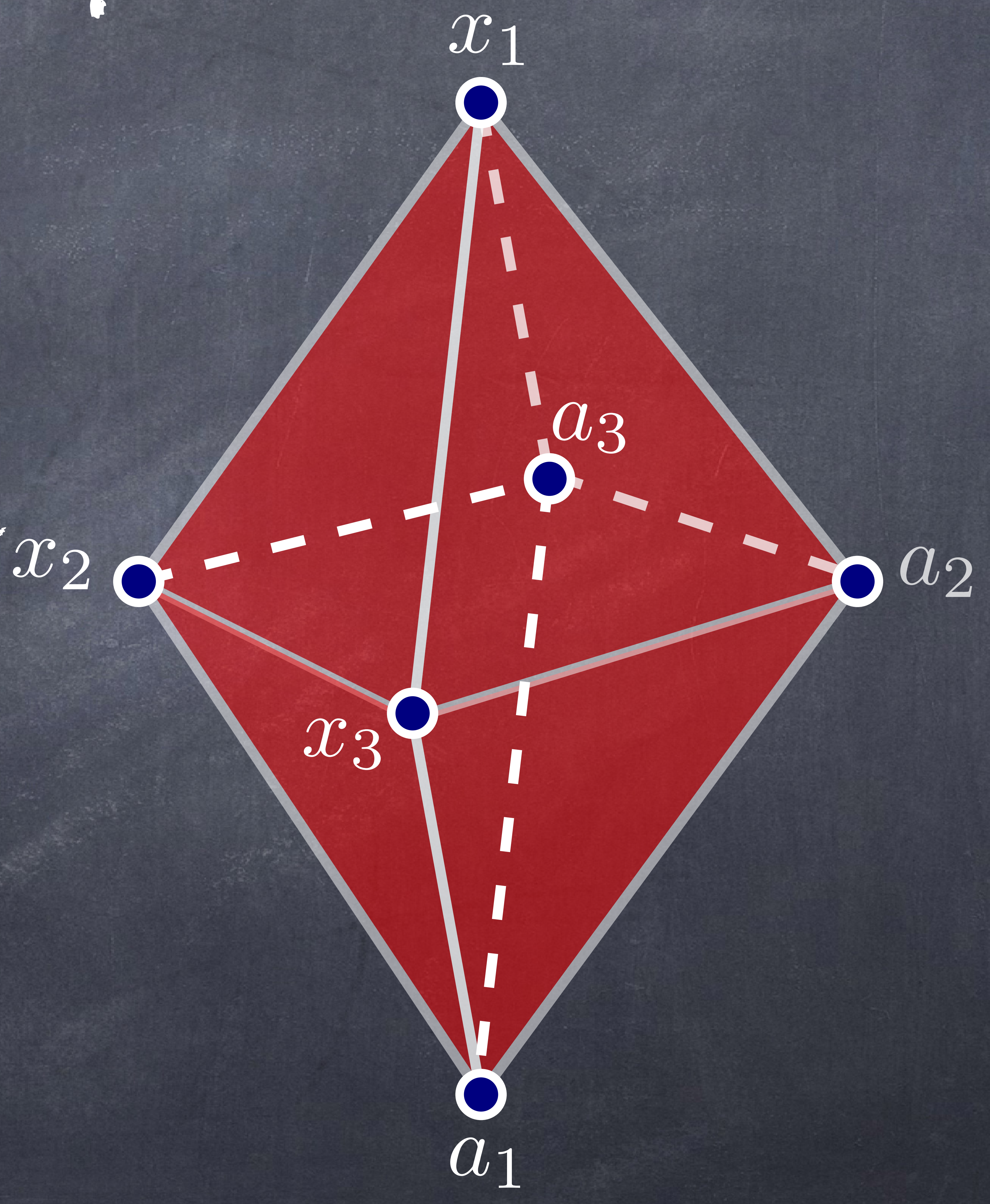
Constructing three qubit projectors

To construct three qubit projectors we need to fill in three dimensional voids.

E.g. to lift the state |000> we need to fill the void:



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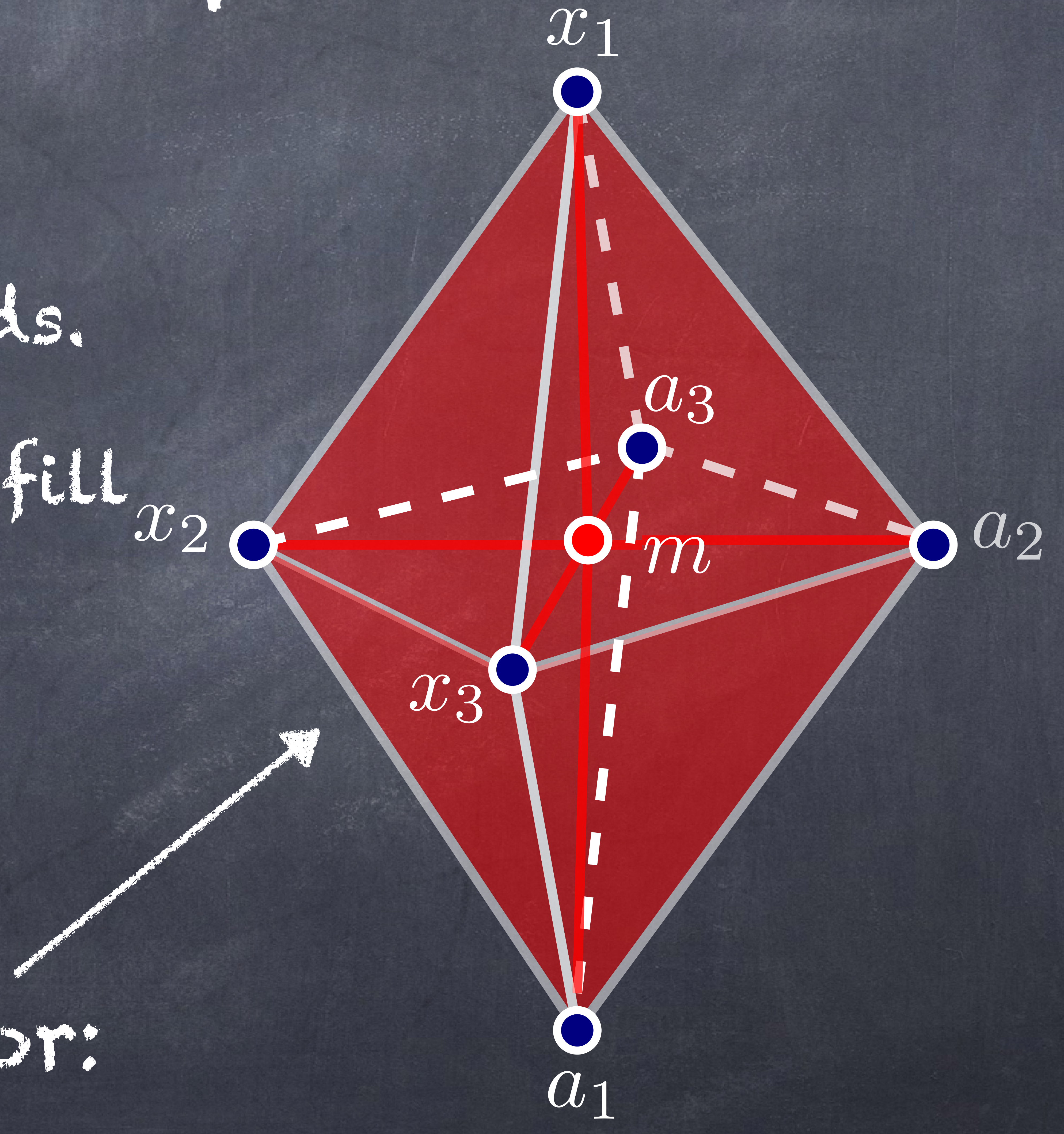

Which we do by adding a mediator:

Constructing three qubit projectors

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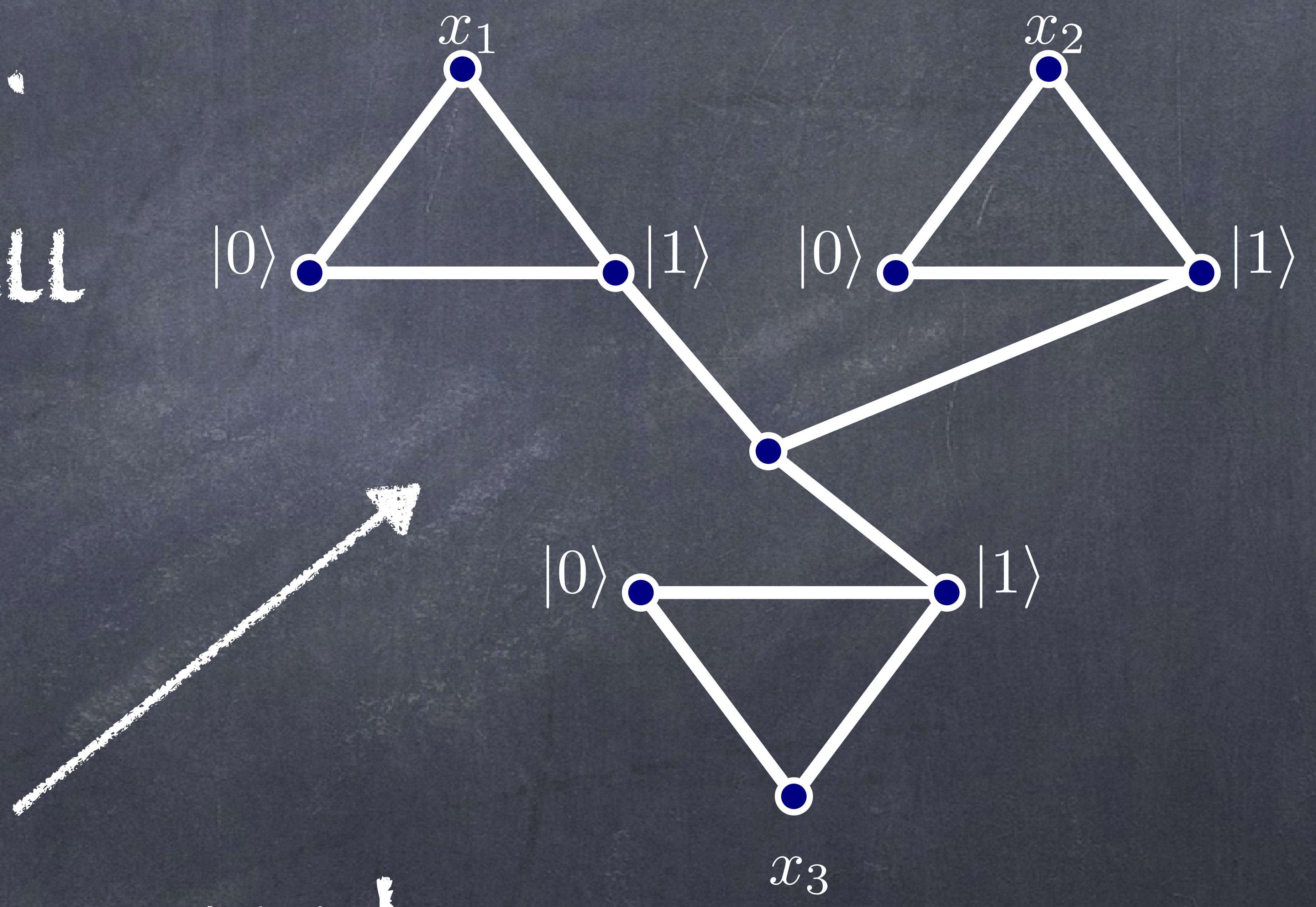
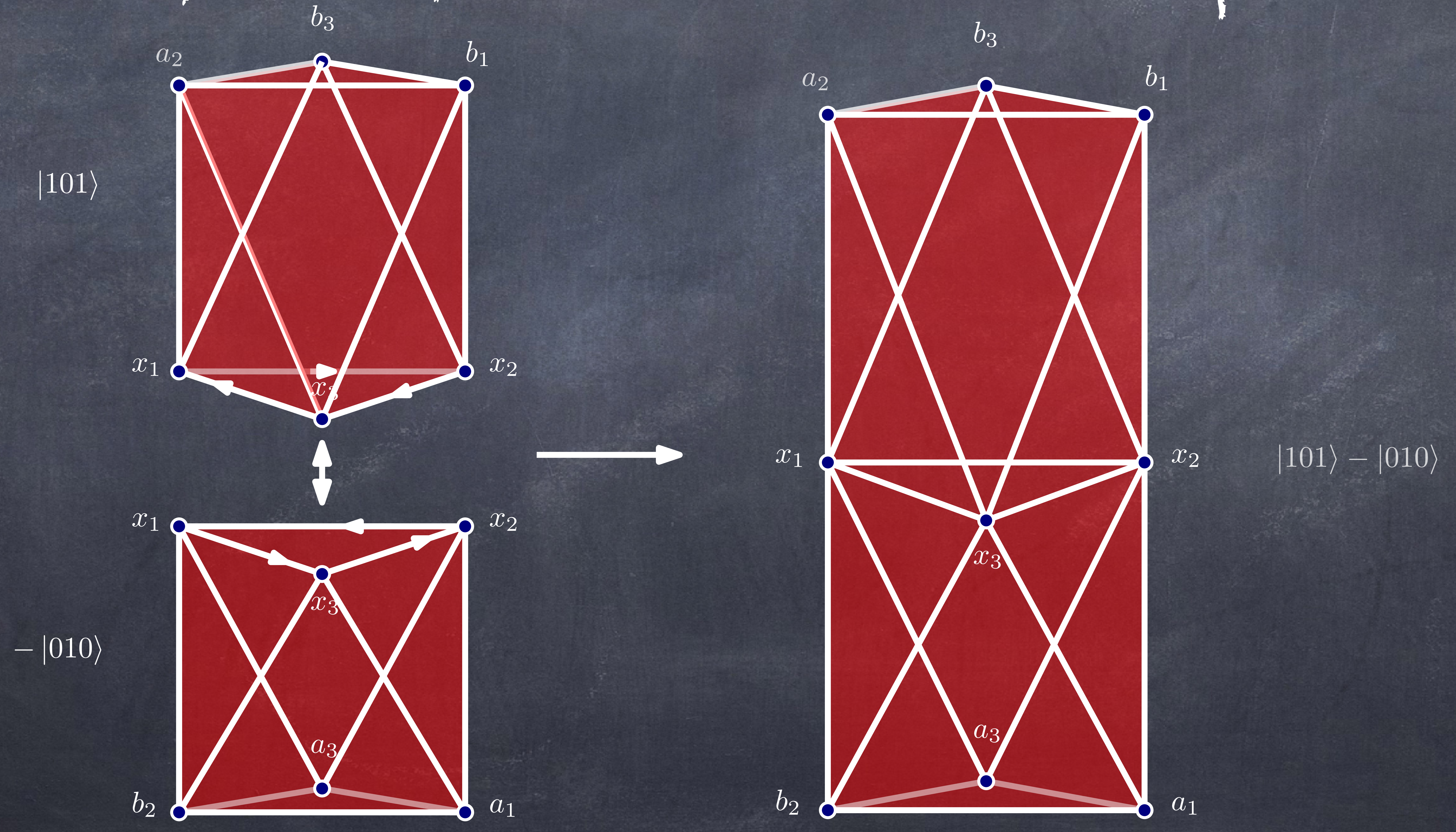


This corresponds to this new graph

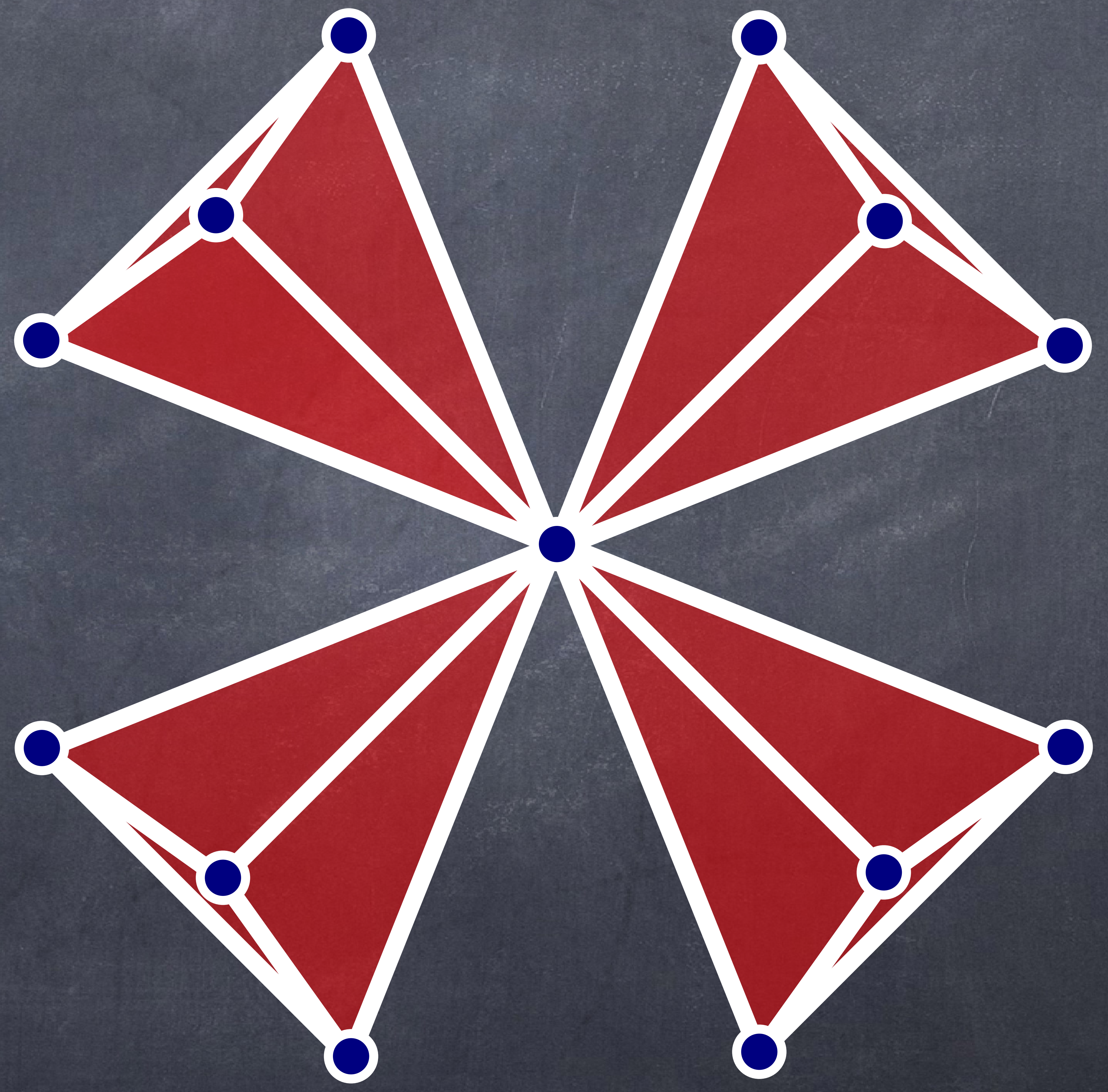
Constructing three qubit projectors

To fill in entangled projectors we have to fill take linear combinations of voids, as we did in the 2-qubit case:



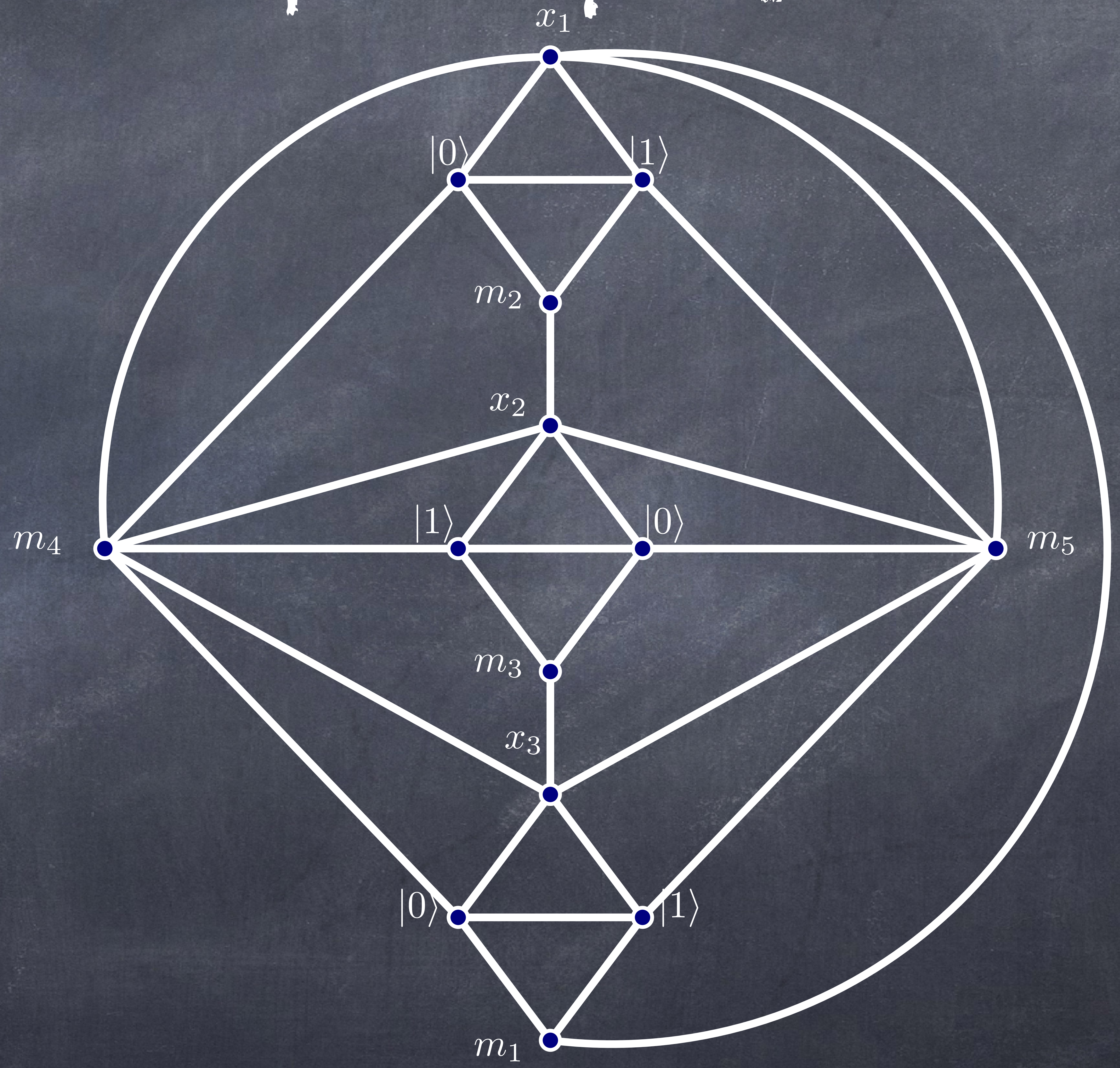
Constructing three qubit projectors

Again we do this by adding mediators, and connecting them to the faces surrounding the void to fill in.



Constructing three qubit projectors

This gives a graph:



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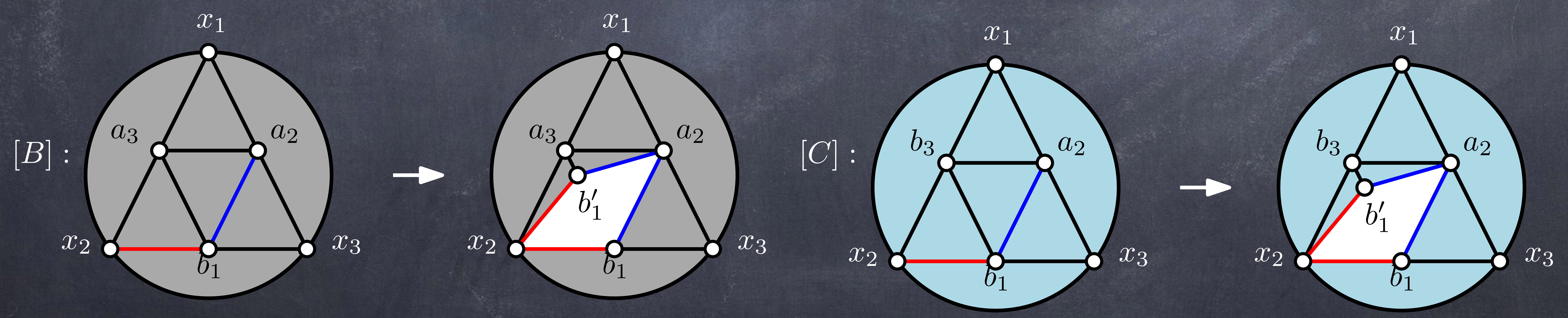
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}} \|00\rangle (\|10\rangle - \|01\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}} \|000\rangle (\|10\rangle - \|01\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}} \|101\rangle (\|10\rangle - \|01\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}} \|010\rangle (\|10\rangle - \|01\rangle)$
$H_{\text{prop,t}}(U_{Pyth.})$	$\frac{1}{5\sqrt{2}} (-5\|011\rangle + 4\|100\rangle + 3\|101\rangle)$
$H_{\text{prop,t}}(U_{Pyth.})$	$\frac{1}{5\sqrt{2}} (-5\|010\rangle + 3\|100\rangle - 4\|101\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}} \|1\rangle (\|101\rangle - \|010\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}} \|11\rangle (\|101\rangle - \|010\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}} \|1\rangle (\|011\rangle - \|100\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}} \|11\rangle (\|011\rangle - \|100\rangle)$
$H^{(1)}_{\text{clock}}$	$\|00\rangle$
$H^{(2)}_{\text{clock}}$	$\|11\rangle$
$H_{\text{in}}, H_{\text{out}}$	$\|011\rangle$
$H^{(6)}_{\text{clock}}, H^{(4)}_{\text{clock}}, H^{(5)}_{\text{clock}}, H^{(3)}_{\text{clock}}$	$\|1100\rangle$
$H^{(4)}_{\text{clock}}$	$\|0111\rangle$
$H^{(5)}_{\text{clock}}$	$\|0001\rangle$

Just the Pythagorean gate left!

The Pythagorean projectors are the most technical challenging gadgets to construct:

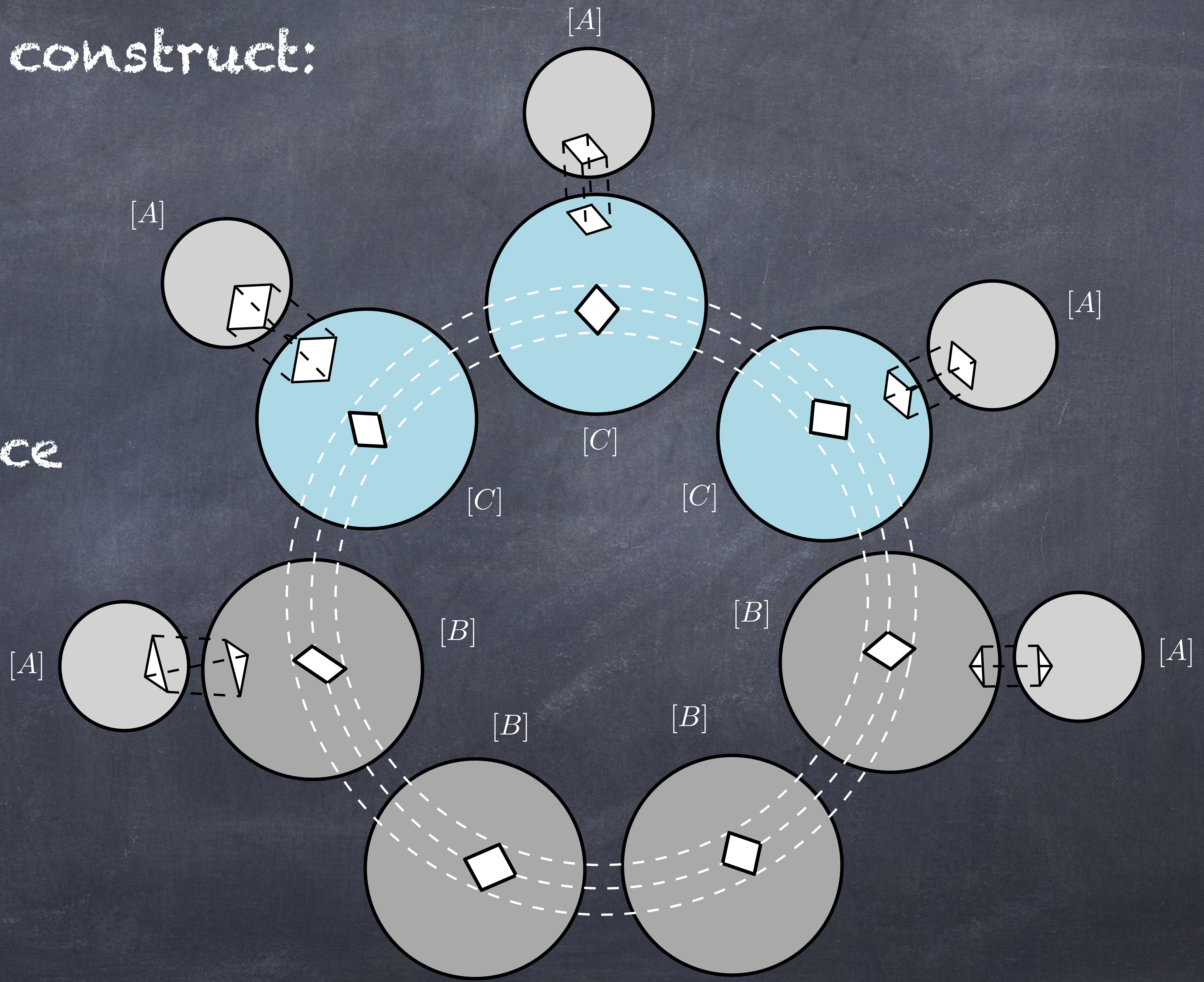
$$|\psi> = -5|011> + 4|100> + 3|101>$$

A-cycle B-cycle C-cycle

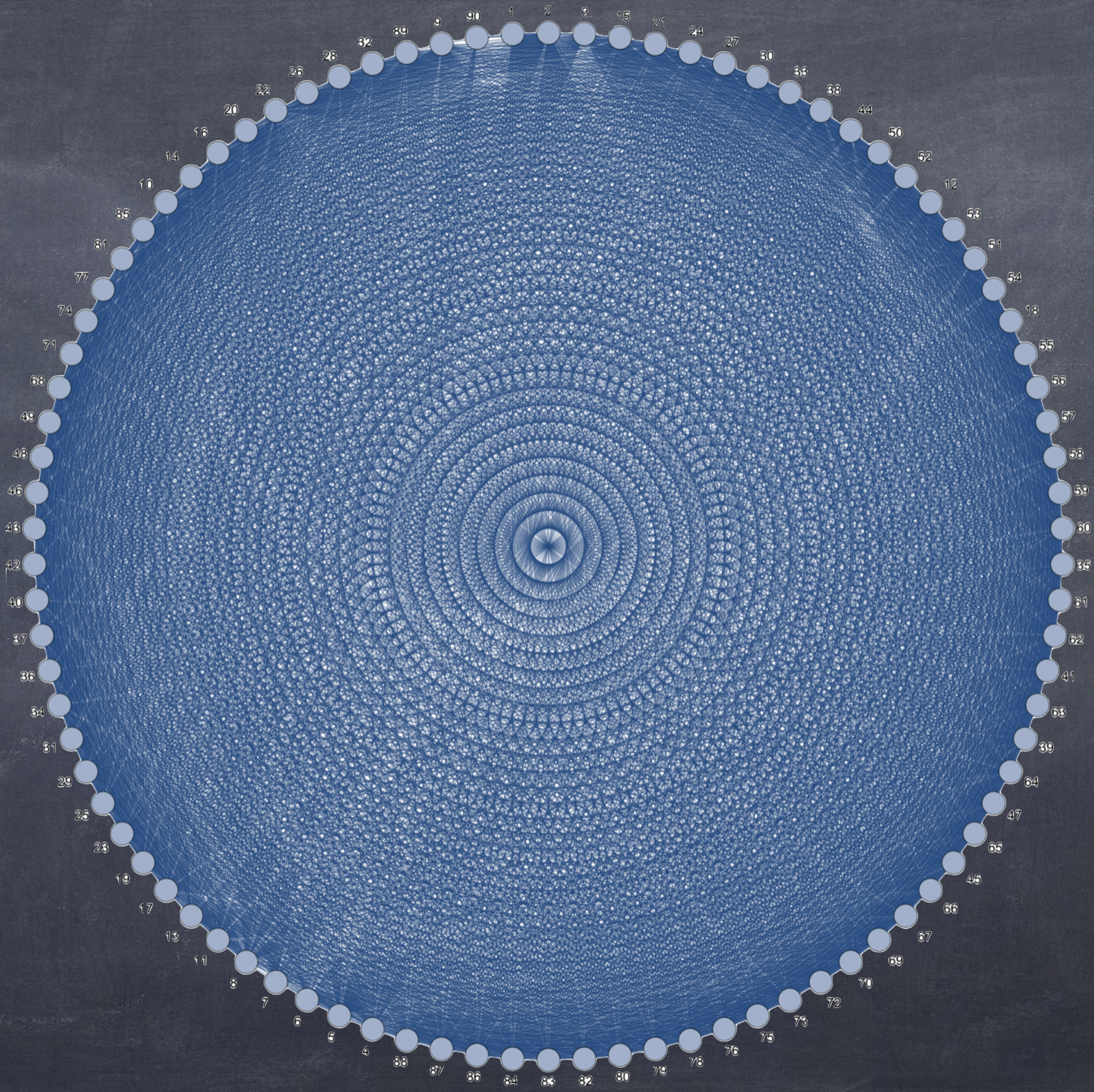


The Pythagorean projectors are the most technical challenging gadgets to construct:

We can then glue the A faces along the shared face with opposite orientation:

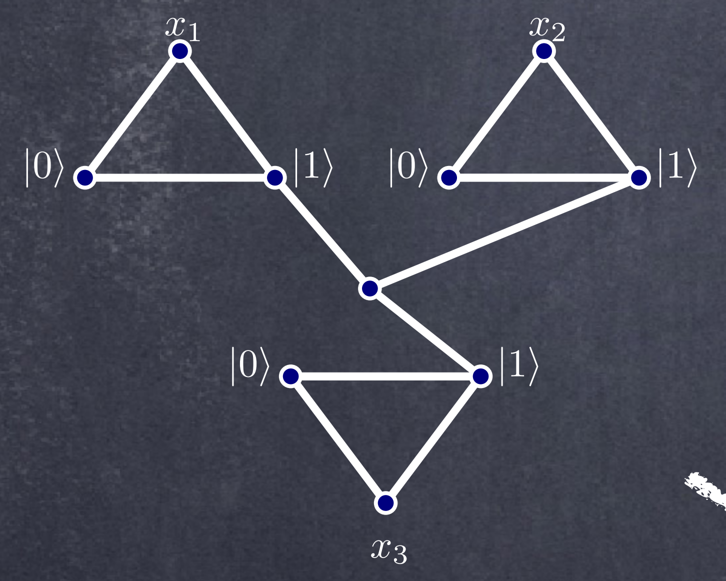


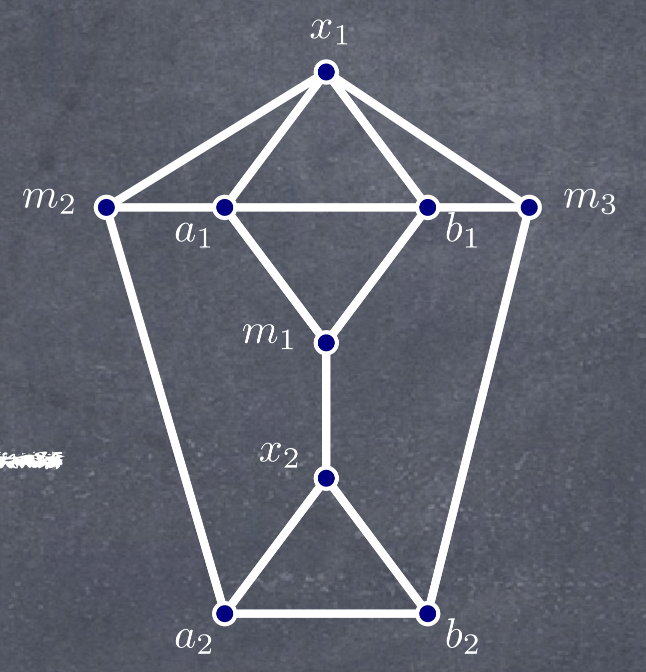
This is implemented by this graph!

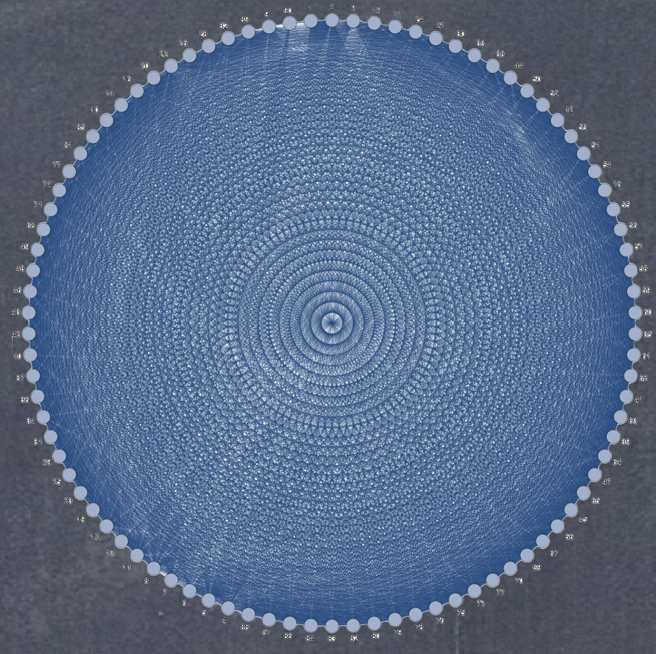


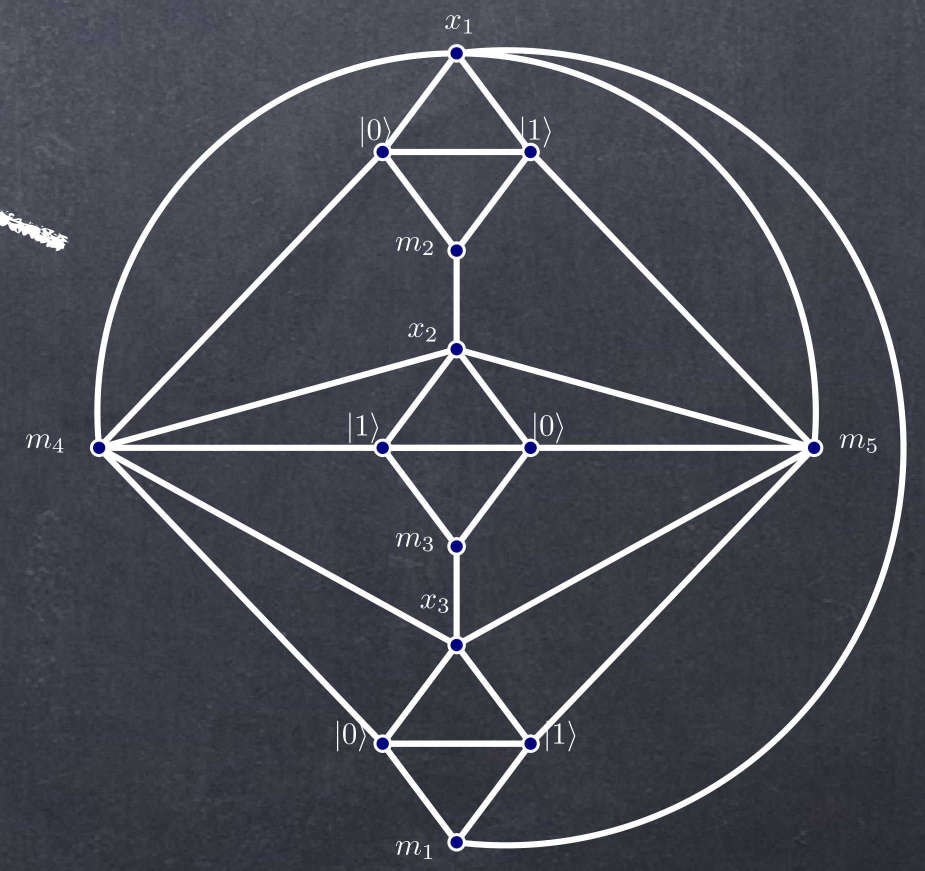
Summing up...

Term in H_{Bravyi}	*Penalizes state* $\lvert\psi_S\rangle$
$H'_{\text{prop,t}}$	$\frac{1}{\sqrt{2}}\lvert 10\rangle(\lvert 11\rangle - \lvert 00\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\lvert 01\rangle(\lvert 10\rangle - \lvert 01\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\lvert 00\rangle(\lvert 10\rangle - \lvert 01\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\lvert 000\rangle(\lvert 10\rangle - \lvert 01\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\lvert 101\rangle(\lvert 10\rangle - \lvert 01\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\lvert 010\rangle(\lvert 10\rangle - \lvert 01\rangle)$
$H_{\text{prop,t}}(U_{Pyth.})$	$\frac{1}{5\sqrt{2}}(-5\lvert 011\rangle + 4\lvert 100\rangle + 3\lvert 101\rangle)$
$H_{\text{prop,t}}(U_{Pyth.})$	$\frac{1}{5\sqrt{2}}(-5\lvert 010\rangle + 3\lvert 100\rangle - 4\lvert 101\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\lvert 1\rangle(\lvert 101\rangle - \lvert 010\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\lvert 11\rangle(\lvert 101\rangle - \lvert 010\rangle)$
$H_{\text{prop,t}}(\text{CNOT})$	$\frac{1}{\sqrt{2}}\lvert 1\rangle(\lvert 011\rangle - \lvert 100\rangle)$
$H_{\text{prop,t}}(\text{Toffoli})$	$\frac{1}{\sqrt{2}}\lvert 11\rangle(\lvert 011\rangle - \lvert 100\rangle)$
$H^{(1)}_{\text{clock}}$	$\lvert 00\rangle$
$H^{(2)}_{\text{clock}}$	$\lvert 11\rangle$
H_{in}, H_{out}	$\lvert 011\rangle$
$H^{(6)}_{\text{clock}}$, $H^{(4)}_{\text{clock}}$, $H^{(5)}_{\text{clock}}$, $H^{(3)}_{\text{clock}}$	$\lvert 1100\rangle$
$H^{(4)}_{\text{clock}}$	$\lvert 0111\rangle$
$H^{(5)}_{\text{clock}}$	$\lvert 0001\rangle$





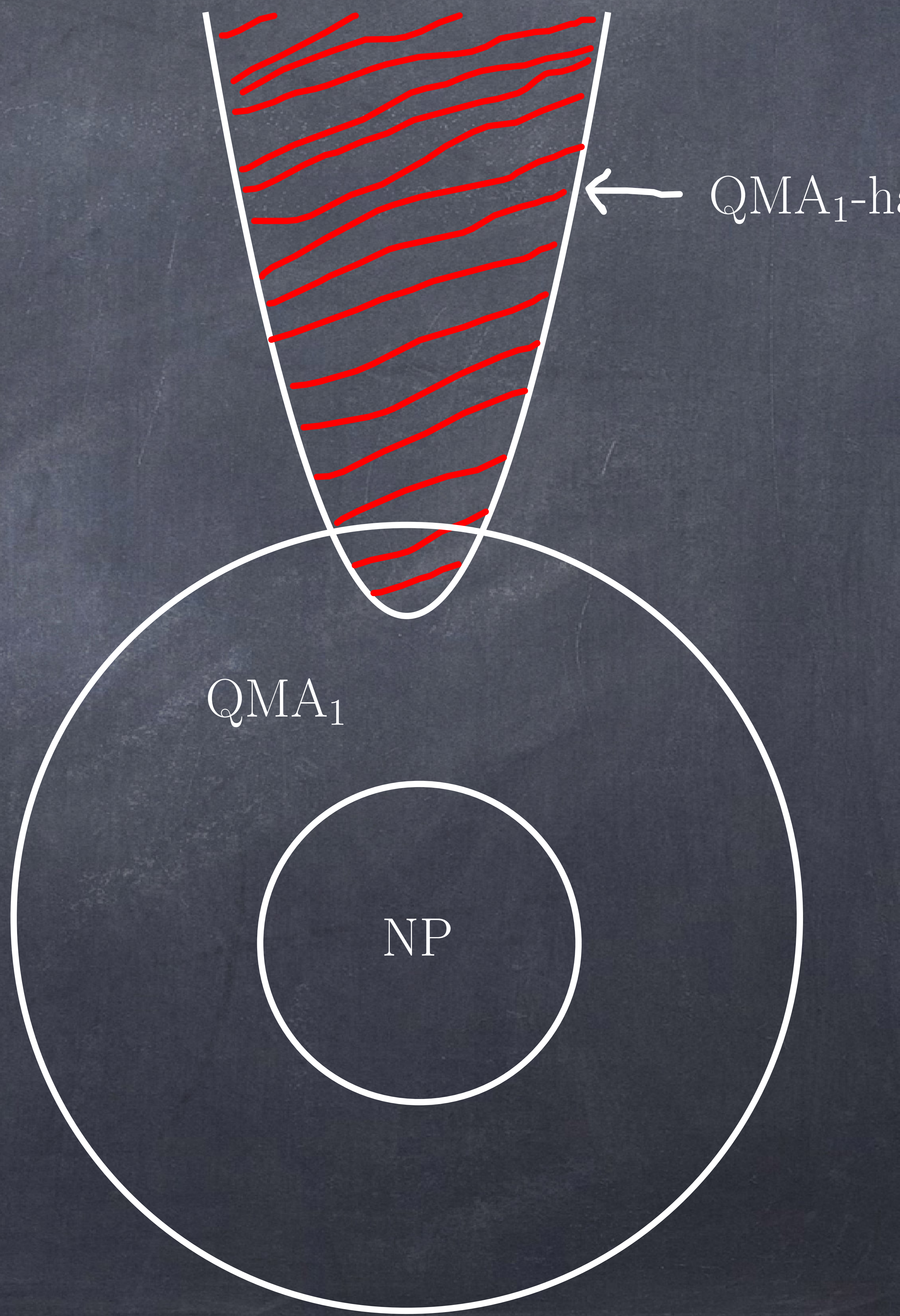




Putting everything together...

Clique / independence homology are QMA_1-hard with the graph, G, given as input.

We also show that the problem remains QMA_1-hard when restricted to clique-dense complexes.

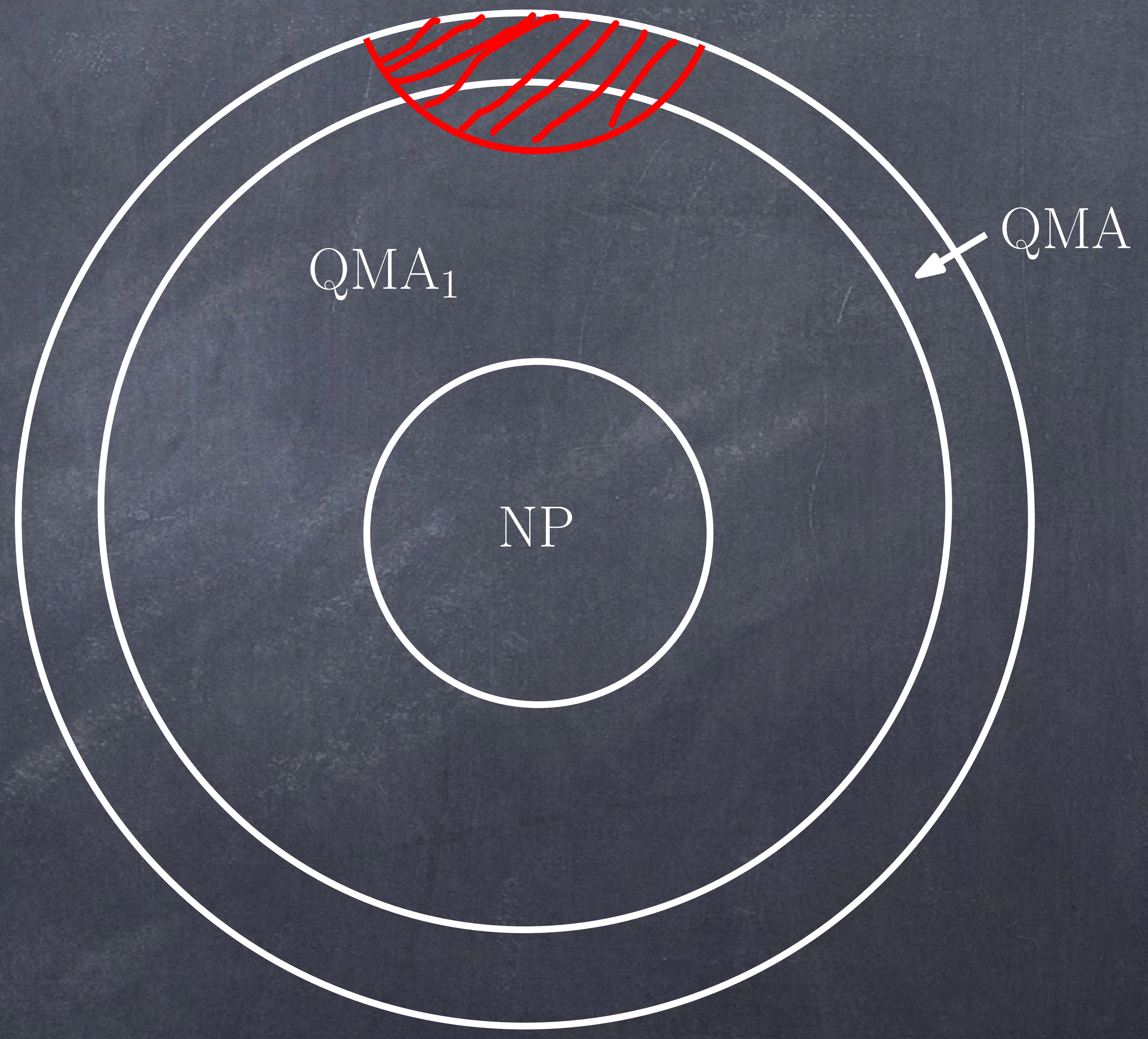


Future work

Can we pin down the complexity more?

A promise version of the problem is contained in QMA - but it's not clear that our initial construction satisfies the promise.

We are working on a new construction that does satisfy the promise - should appear shortly!



Future work

Quantum advantage for TDA?