The Interacting Ising-Majorana chain model

randomness — topology — many body localization

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- Introduction to many body localization
- The non-interacting Majorana chain with disorder
- Effects of interactions
- Conclusions

Short introduction to MBL physics

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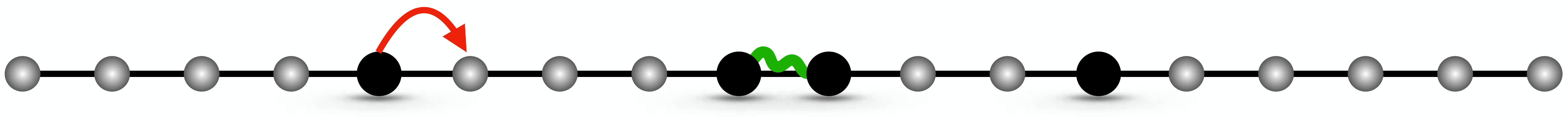
Interacting electrons in one dimension

$$\mathscr{H} = \sum_i \left[t\left(c_i^\dagger c_{i+1} + \text{hc}\right) + V n_i n_{i+1} + \epsilon_i n_i \right]$$

quantum tuneling

short-range interaction

local potential



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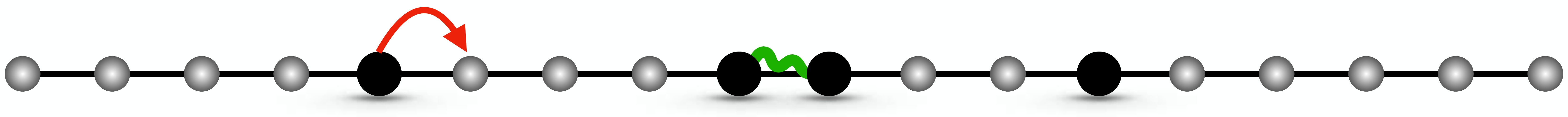


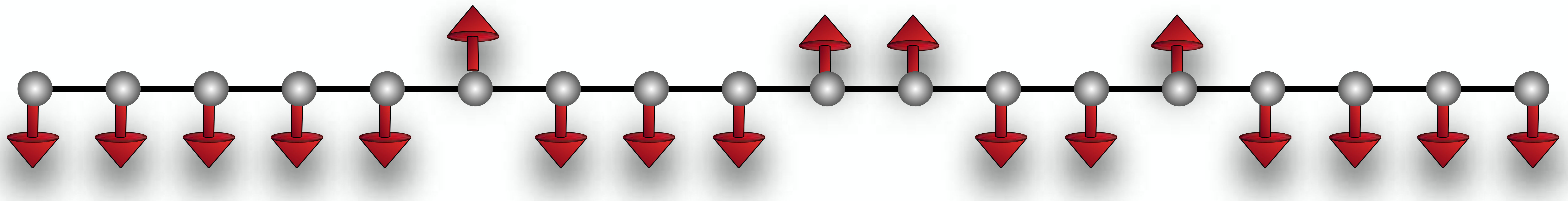
Jordan-Wigner transformation
spin-1/2 —> fermions

$$S_j^z = c_j^\dagger c_j - \frac{1}{2}$$
$$S_j^+ = c_j^\dagger \, \mathrm{e}^{i\pi \sum_{l<j} n_l}$$
$$S_j^- = c_j \, \mathrm{e}^{-i\pi \sum_{l<j} n_l}$$

Equivalent to a spin S=1/2 chain in a magnetic field

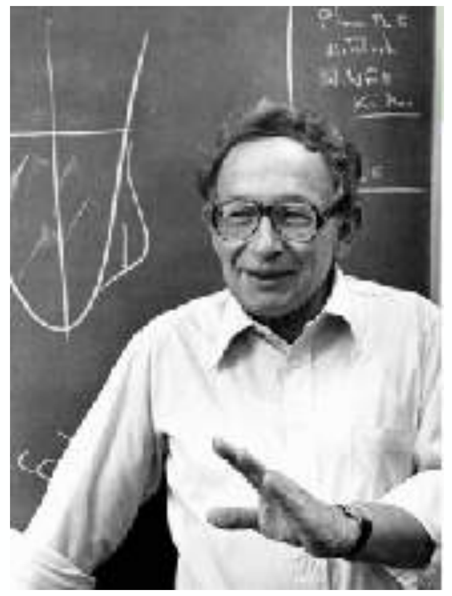
$$\Rightarrow \mathscr{H} = \sum_i \left[t \left(S_i^+ S_{i+1}^- + \mathrm{hc} \right) + V S_i^z S_{i+1}^z + \epsilon_i S_i^z \right] + C$$

spin-flip — Ising interaction — magnetic field



S^z_{total} is conserved!

From single to many: another “More is Different”



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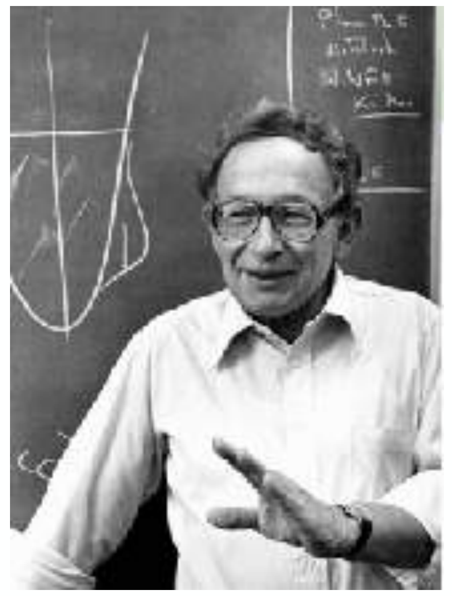

(i) Single particle

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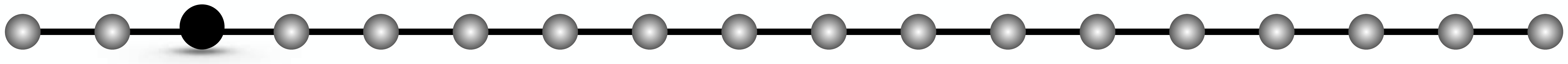

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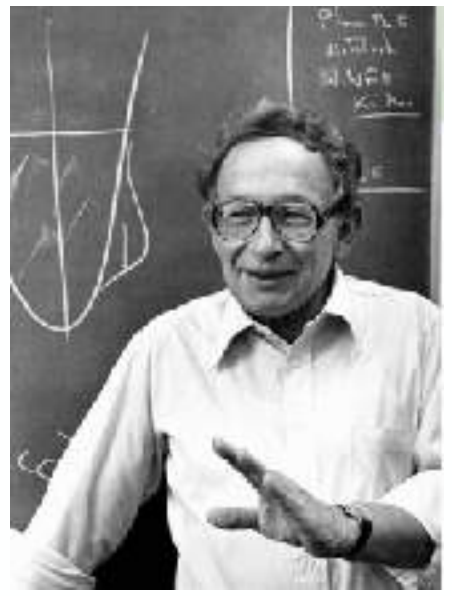

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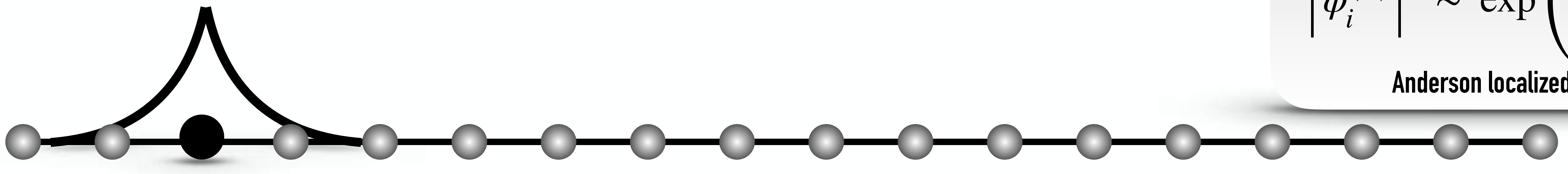
$$= \sum_m \tilde{\epsilon}_m b_m^\dagger b_m$$

$$b_m = \sum_i \phi_i^m c_i$$

$$\left| \phi_i^{(m)} \right|^2 \sim \exp\left(-\frac{|i - i_0^{(m)}|}{\xi_m} \right)$$

Anderson localized orbitals



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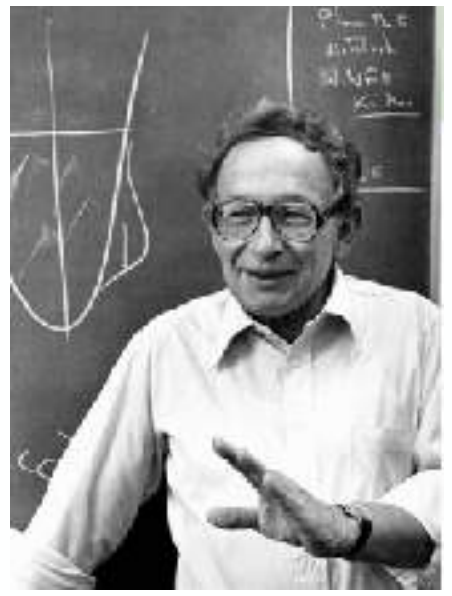

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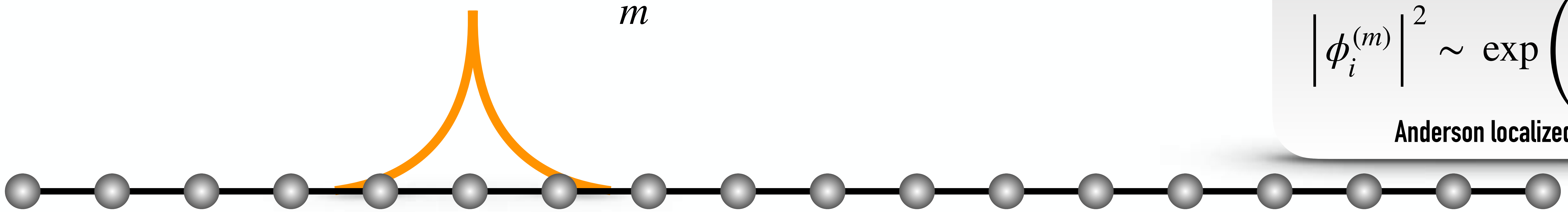

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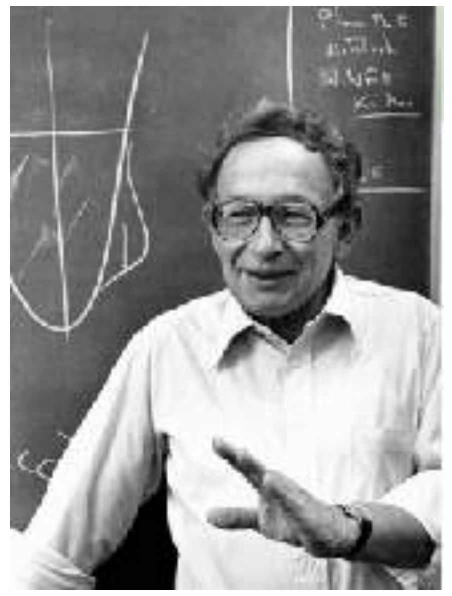

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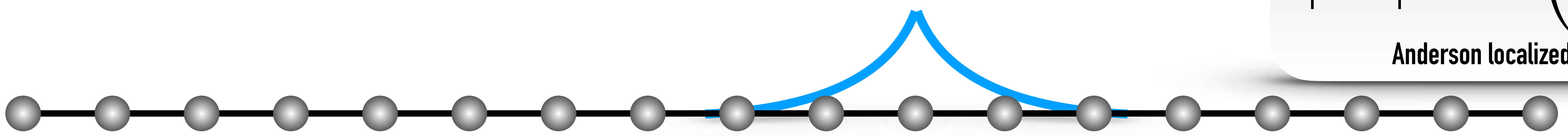
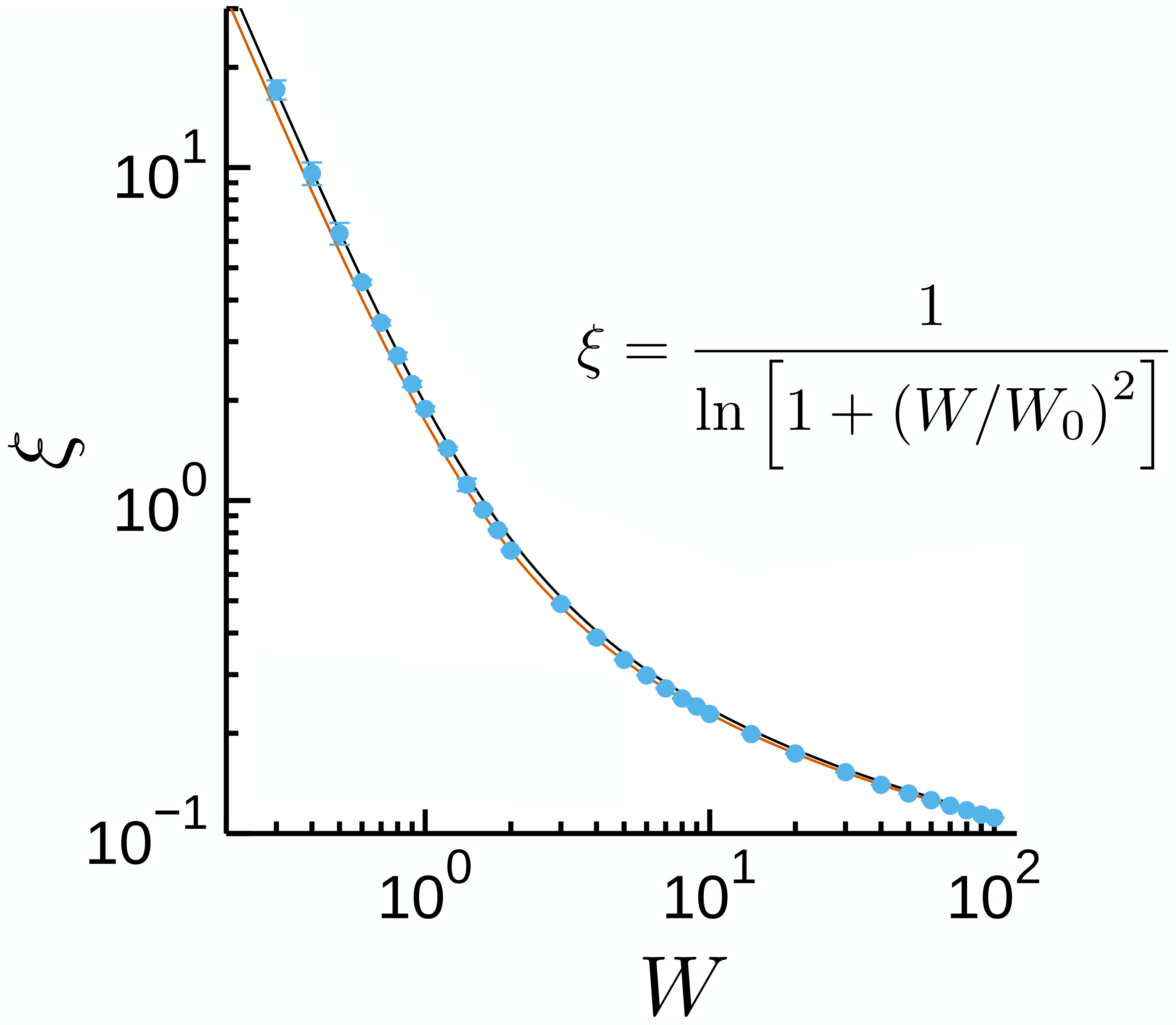


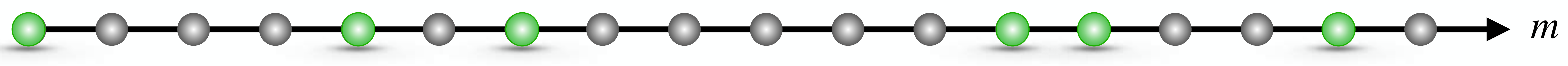


From single to many: another “More is Different”

(ii) Many particles

$$\mathscr{H} = \sum_i \left[t\left(c_i^\dagger c_{i+1} + \mathrm{hc}\right) + \epsilon_i n_i + V n_i n_{i+1} \right]$$

$$= \sum_m \tilde{\epsilon}_m b_m^\dagger b_m \quad + \quad \sum_{j,k,l,m} V_{jklm} b_j^\dagger b_k^\dagger b_l b_m$$



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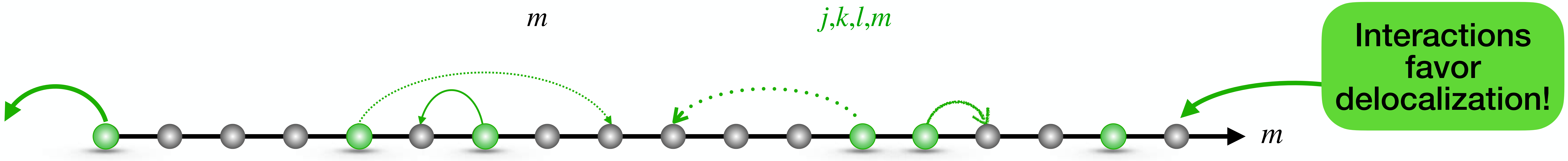

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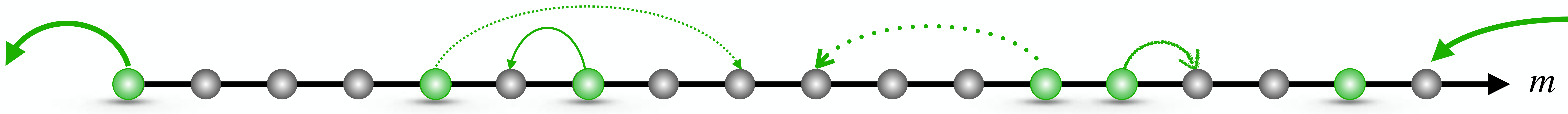


Interactions favor delocalization!

We expect a transition in the presence of interactions

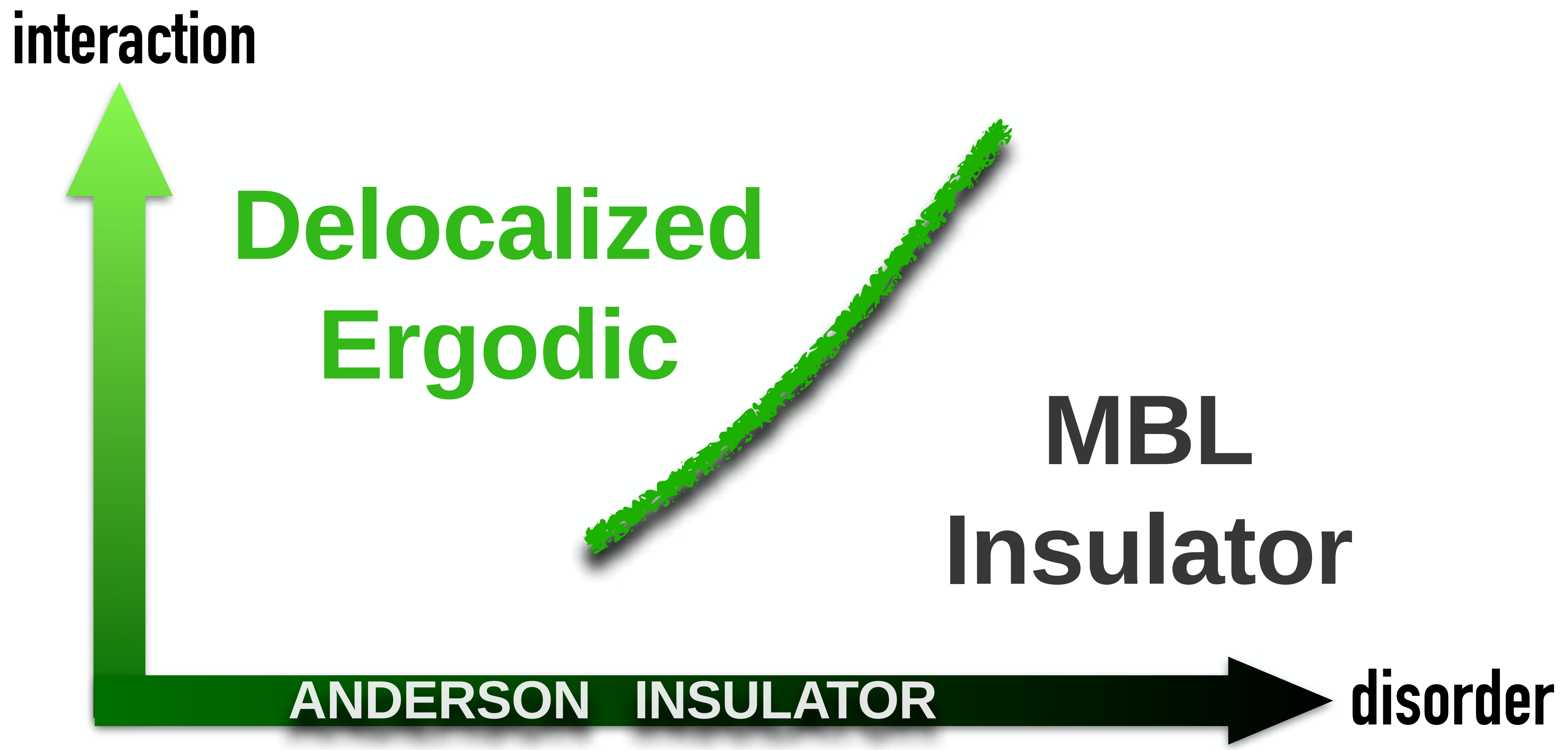
Quasiparticle Lifetime in a Finite System: A Nonperturbative Approach
Boris L. Altshuler,[1] Yuval Gefen,[2] Alex Kamenev,[2] and Leonid S. Levitov[3]
Phys. Rev. Lett. (1997)

Interactions and the Anderson transition
L. Fleishman* and P. W. Anderson†
Phys. Rev. B (1980)

Interacting Electrons in Disordered Wires: Anderson Localization and Low-*T* Transport
I. V. Gornyi,[1,*] A. D. Mirlin,[1,2,†] and D. G. Polyakov[1,*]
Phys. Rev. Lett. (2005)

Emergence of Quantum Chaos in Finite Interacting Fermi Systems
Ph. Jacquod[1] and D. L. Shepelyansky[2,*]
Phys. Rev. Lett. (1997)

Metal–insulator transition in a weakly interacting many-electron system with localized single-particle states
D.M. Basko [a,b,*], I.L. Aleiner [b], B.L. Altshuler [a,b,c]
Annals of Physics (2006)



Smoking guns for ergodicity breaking

Smoking guns for ergodicity breaking

Spectral statistics transition

PHYSICAL REVIEW B **75**, 155111 (2007)

Localization of interacting fermions at high temperature

Vadim Oganesyan*
Department of Physics, Yale University, New Haven, Connecticut 06520, USA

David A. Huse†
Department of Physics, Princeton University, Princeton, New Jersey 08544, USA

PHYSICAL REVIEW B **82**, 174411 (2010)

Many-body localization phase transition

Arijeet Pal and David A. Huse
Physics Department, Princeton University, Princeton, New Jersey 08544, USA

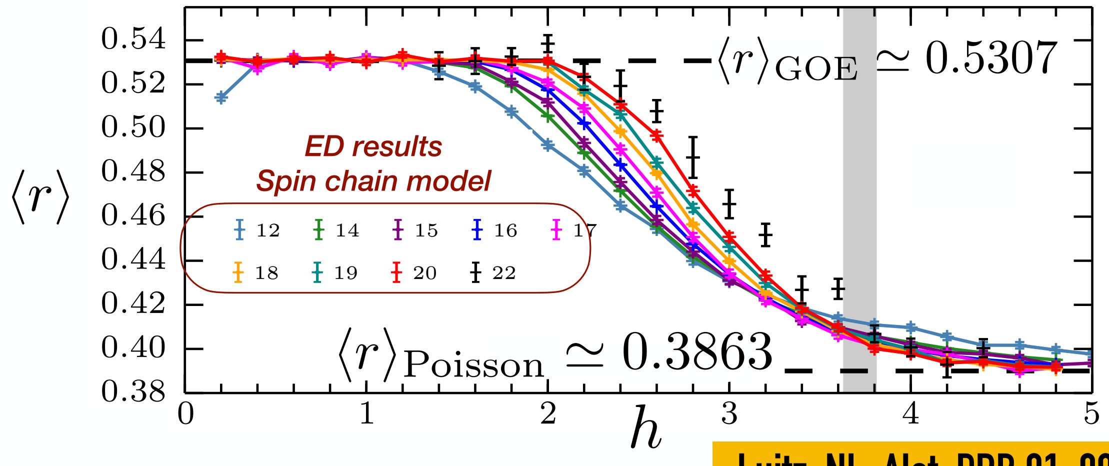
PHYSICAL REVIEW B **91**, 081103(R) (2015)

Many-body localization edge in the random-field Heisenberg chain

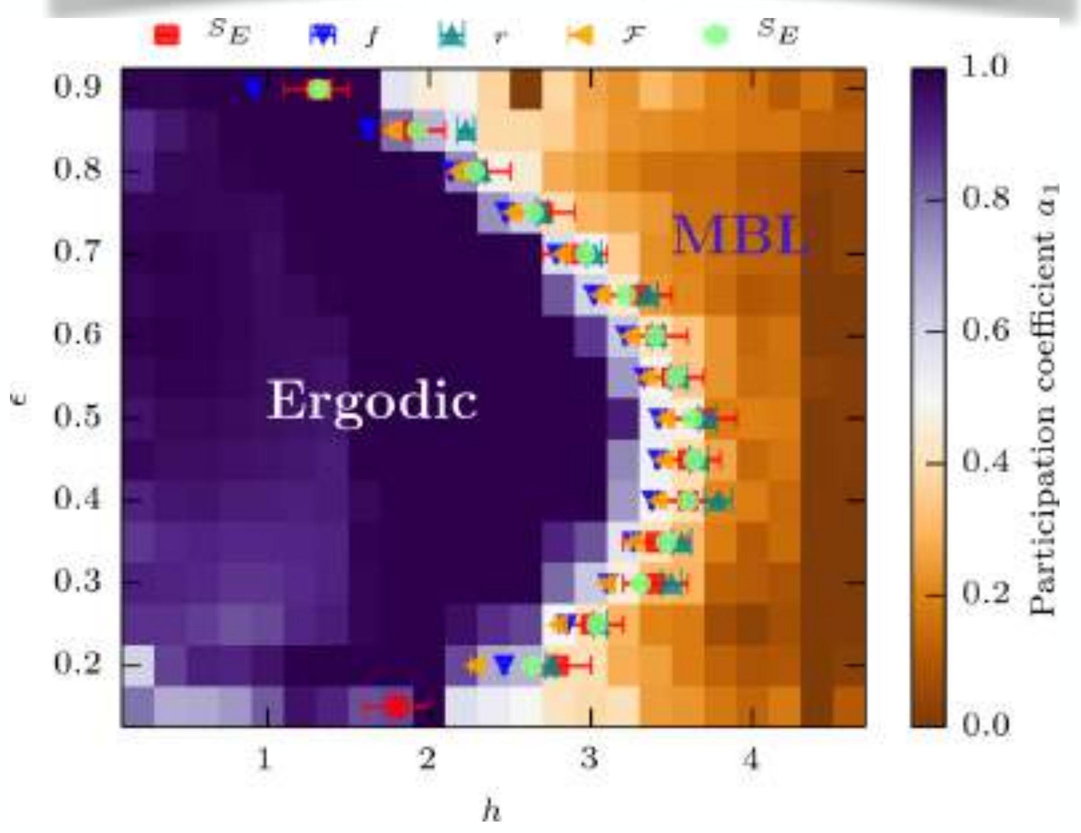
David J. Luitz,* Nicolas Laflorencie,† and Fabien Alet‡
Laboratoire de Physique Théorique, IRSAMC, Université de Toulouse, CNRS, 31062 Toulouse, France

$$g_n = |E_n - E_{n-1}|$$

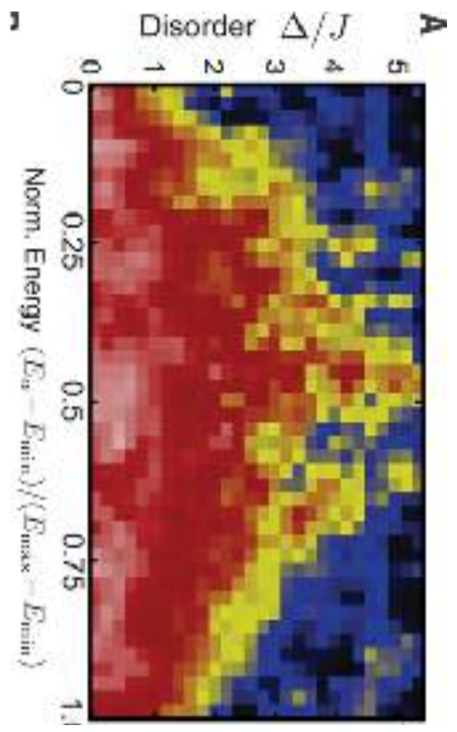
gap ratio $$r = \min(g_n, g_{n+1}) / \max(g_n, g_{n+1})$$



Luitz, NL, Alet, PRB 91, 081103(R) (2015)



➡ **Mobility edge also observed with superconducting qubits**



Roushan et al., Science 358, 1175 (2017)

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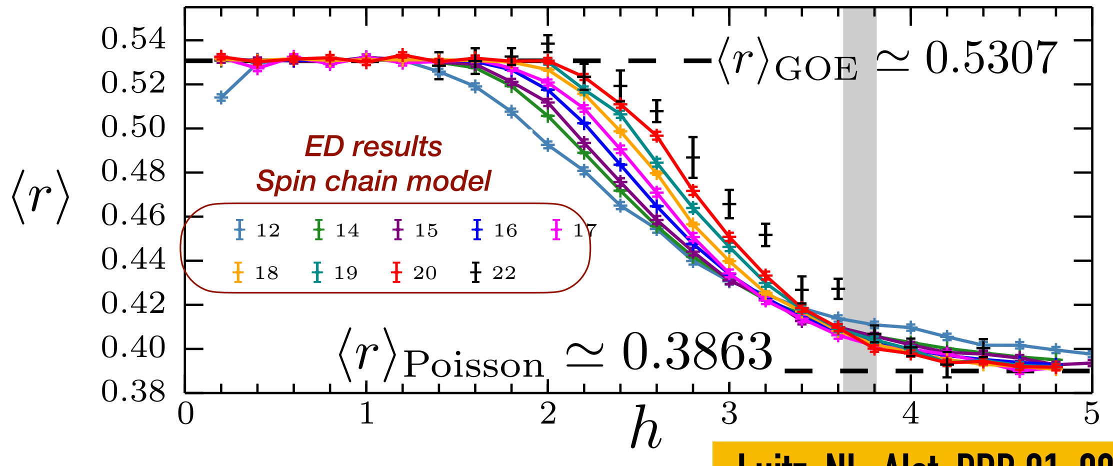

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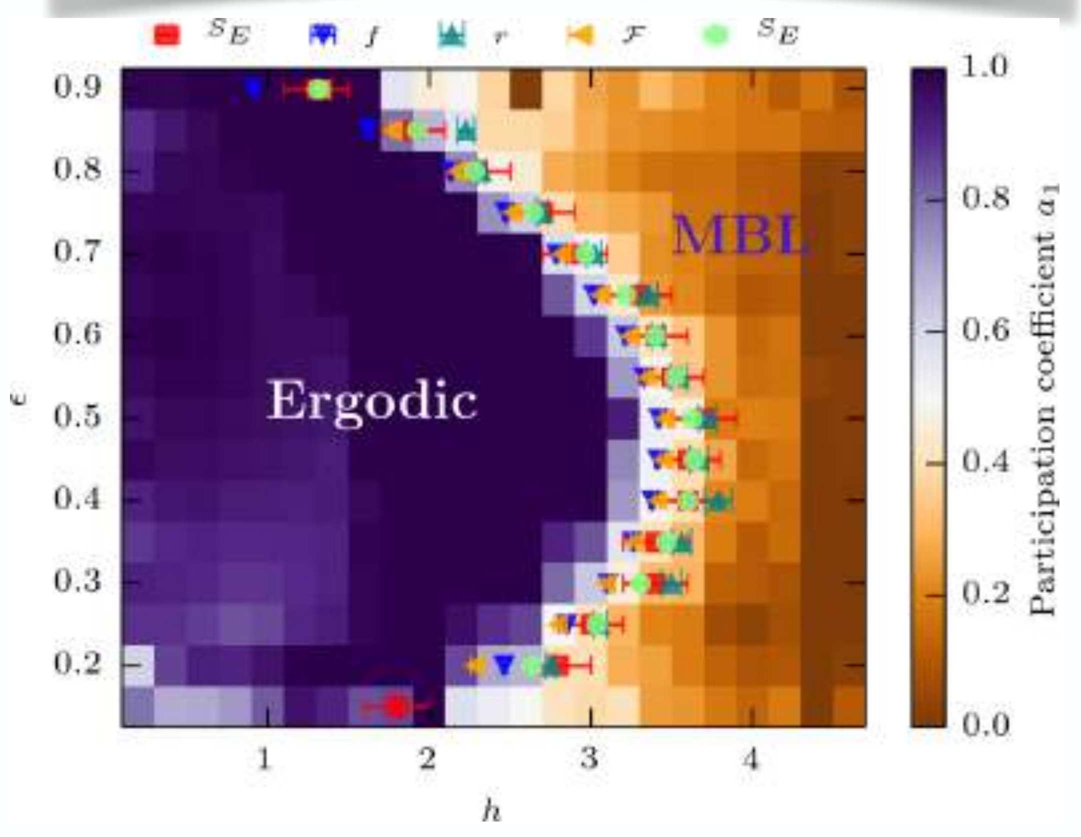

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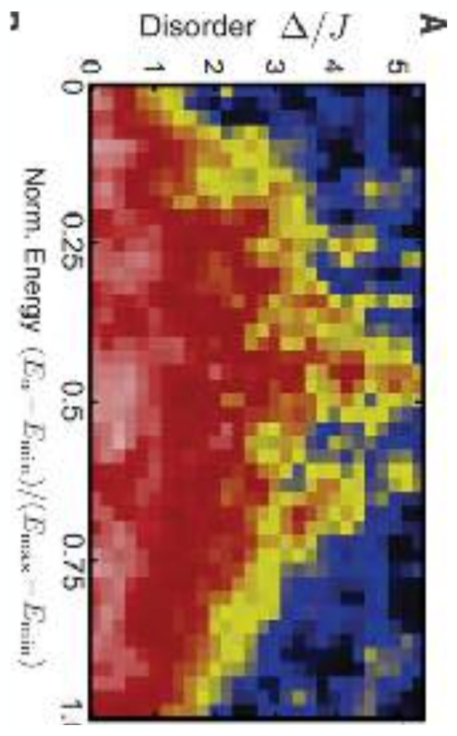
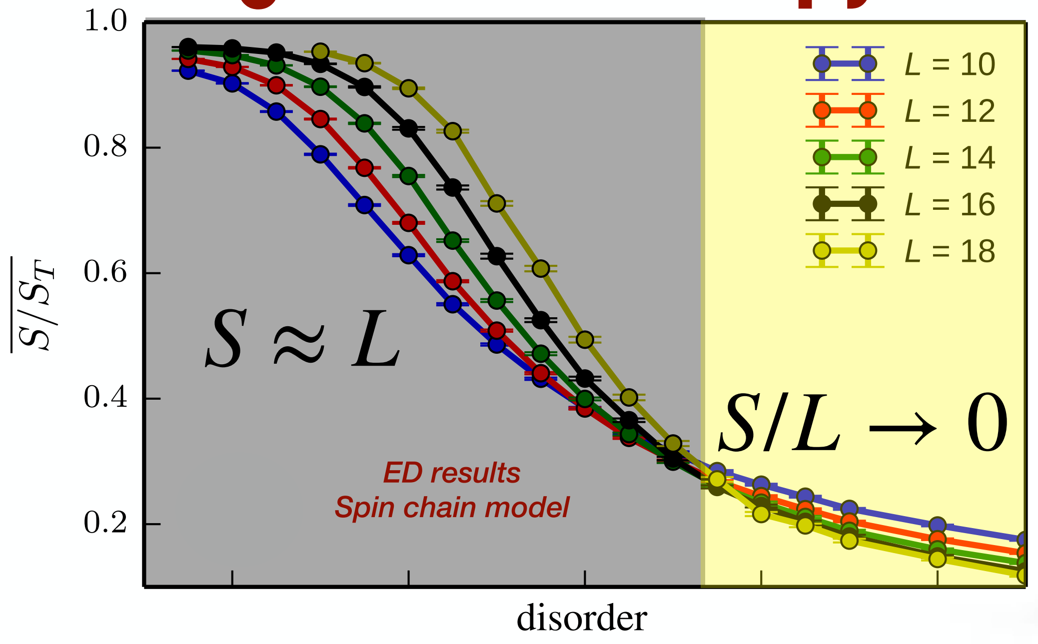


Roushan et al., Science 358, 1175 (2017)

Entanglement entropy: Volume vs. Area law



Khemani et al., PRX 7, 021013 (2017)

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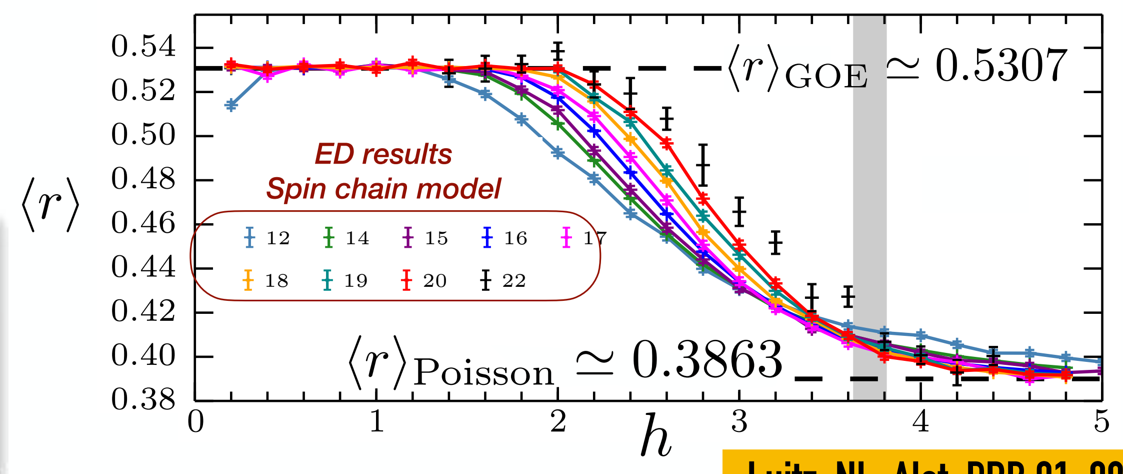

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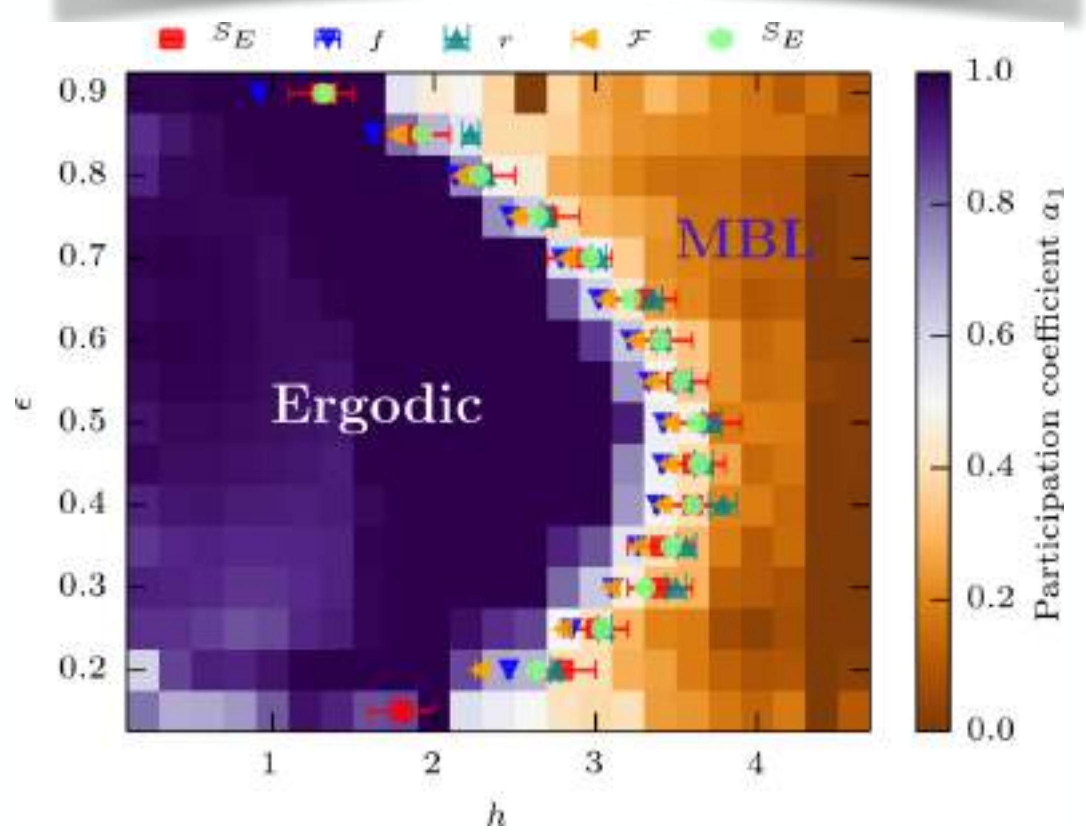

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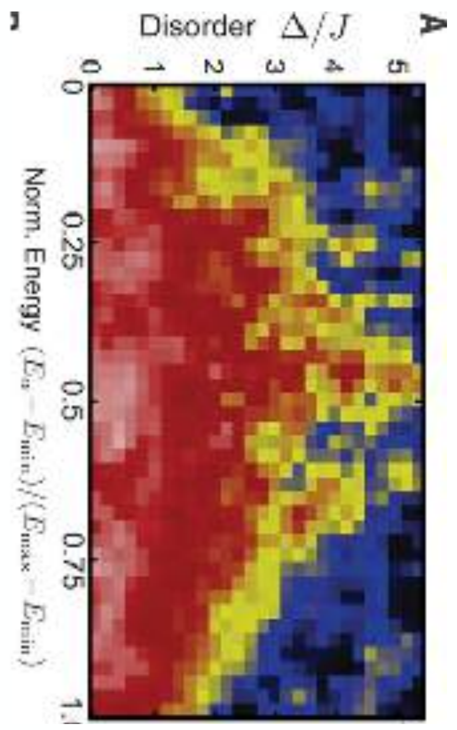

Roushan et al., Science 358, 1175 (2017)

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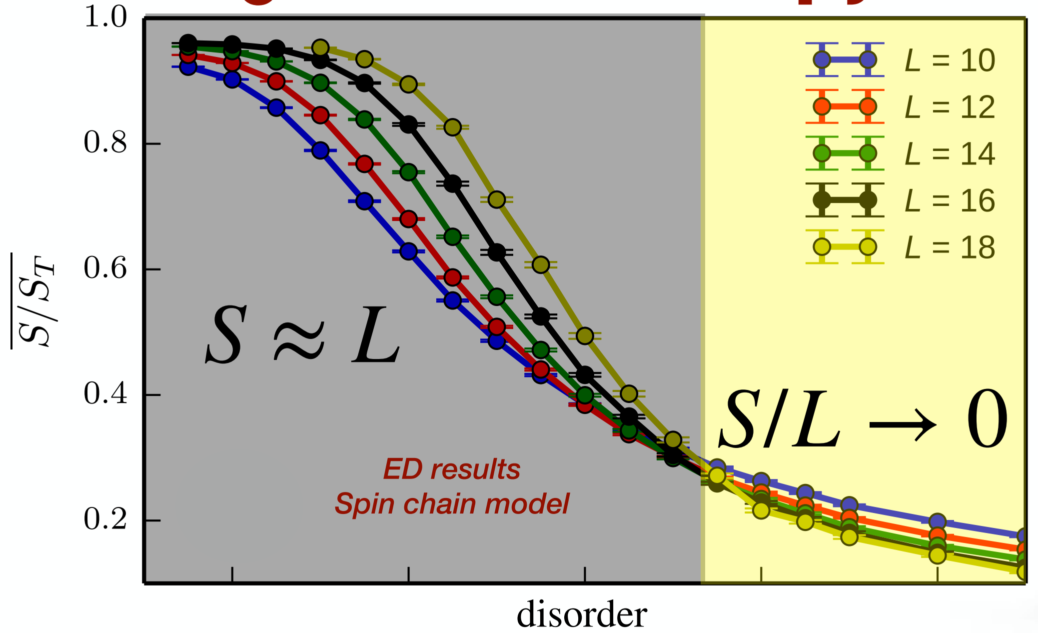
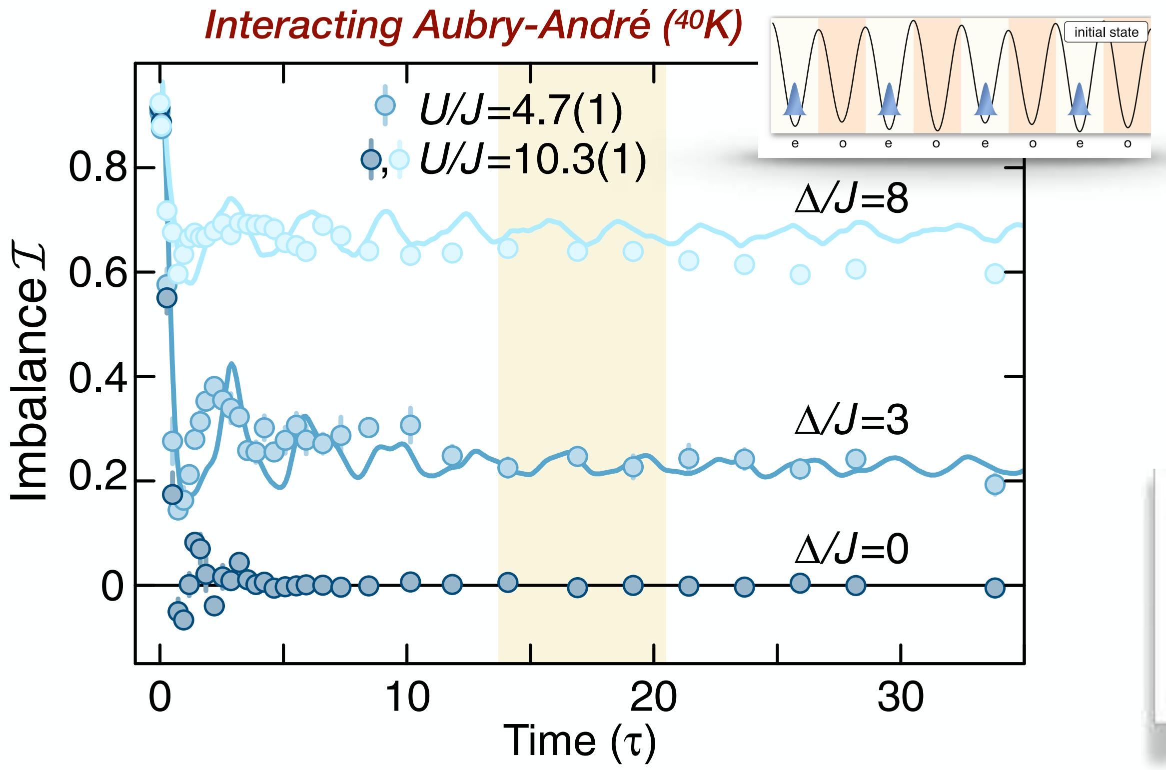


Khemani et al., PRX 7, 021013 (2017)

Out of equilibrium dynamics



Schreiber et al., Science 349, 842 (2015)

PHYSICAL REVIEW B 77, 064426 (2008)

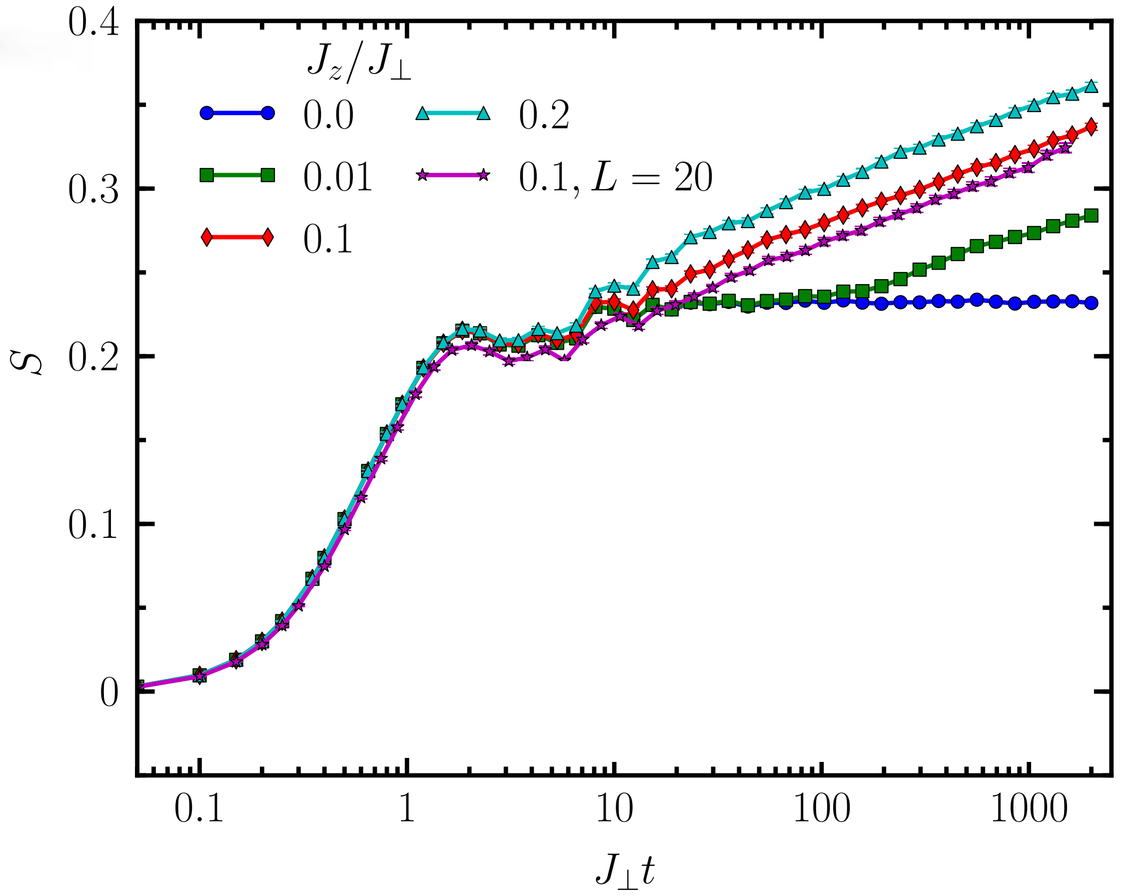
Many-body localization in the Heisenberg XXZ magnet in a random field

Marko Žnidarič,[1] Tomaž Prosen,[1] and Peter Prelovšek[1,2]

PRL 109, 017202 (2012) PHYSICAL REVIEW LETTERS week ending 6 JULY 2012

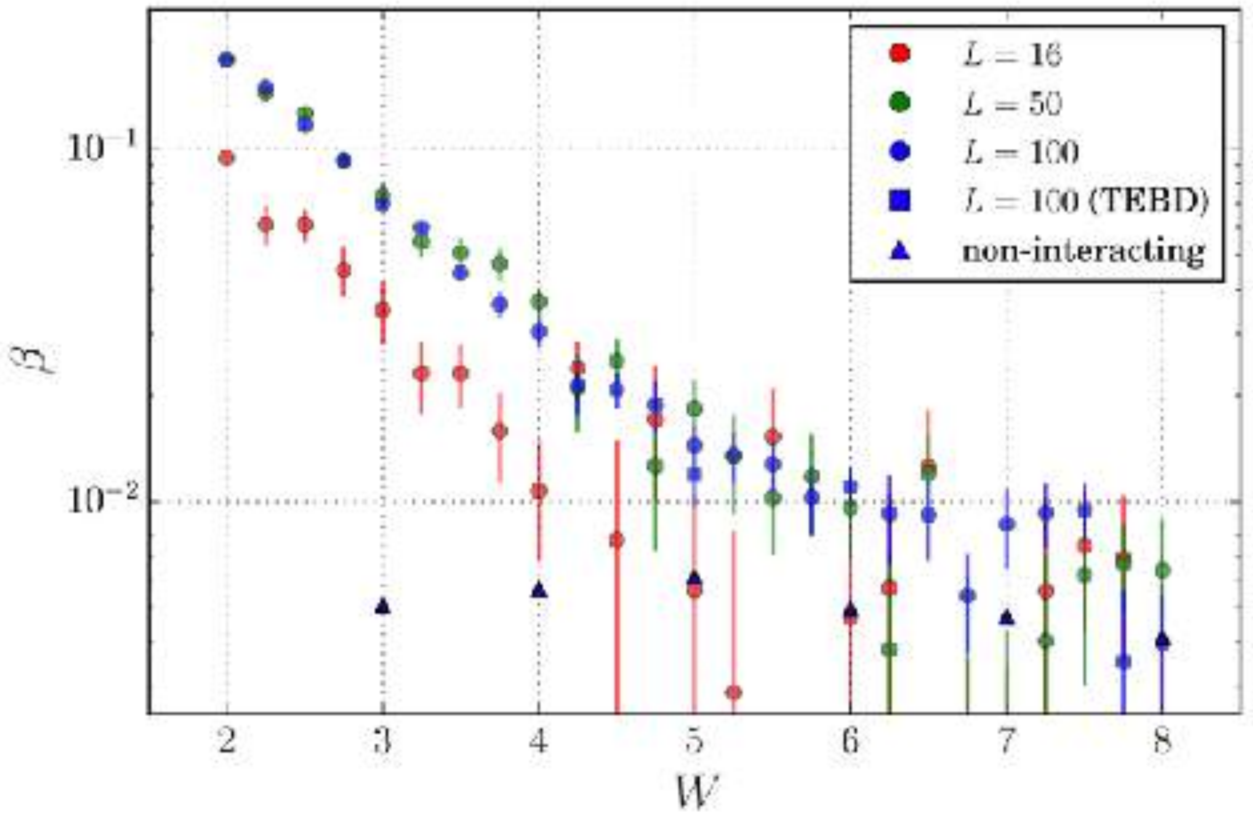
Unbounded Growth of Entanglement in Models of Many-Body Localization

Jens H. Bardarson,[1,2] Frank Pollmann,[3] and Joel E. Moore[1,2]



Many-body localization and delocalization in large quantum chains

Elmer V. H. Doggen,[1,*] Frank Schindler,[2] Konstantin S. Tikhonov,[1,3] Alexander D. Mirlin,[1,3,4,5] Titus Neupert,[2] Dmitry G. Polyakov,[1] and Igor V. Gornyi[1,3,4,6]



PRL 110, 260601 (2013) PHYSICAL REVIEW LETTERS week ending 28 JUNE 2013

Universal Slow Growth of Entanglement in Interacting Strongly Disordered Systems

Maksym Serbyn,[1] Z. Papić,[2] and Dmitry A. Abanin[3,4]

But also some debates…

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EPL, **128** (2019) 67003
doi: 10.1209/0295-5075/128/67003

Can we study the many-body localisation transition?

R. K. Panda[1,2], A. Scardicchio[1,3], M. Schulz[1], S. R. Taylor[1(a)] and M. Žnidarič[4]

PHYSICAL REVIEW E **102**, 062144 (2020)

Quantum chaos challenges many-body localization

Jan Šuntajs,[1] Janez Bonča,[2,1] Tomaž Prosen,[2] and Lev Vidmar[1,2]

Annals of Physics
Volume 427, April 2021, 168415

Distinguishing localization from chaos: Challenges in finite-size systems

D.A. Abanin[a 1], J.H. Bardarson[b 1], G. De Tomasi[c 1], S. Gopalakrishnan[d e 1], V. Khemani[f 1], S.A. Parameswaran[g 1], F. Pollmann[h i 1], A.C. Potter[j 1], M. Serbyn[k 1], R. Vasseur[l 1]

PHYSICAL REVIEW B **105**, 224203 (2022)
Editors' Suggestion

Challenges to observation of many-body localization

Piotr Sierant[1,2] and Jakub Zakrzewski[2,3,*]

But also some debates…

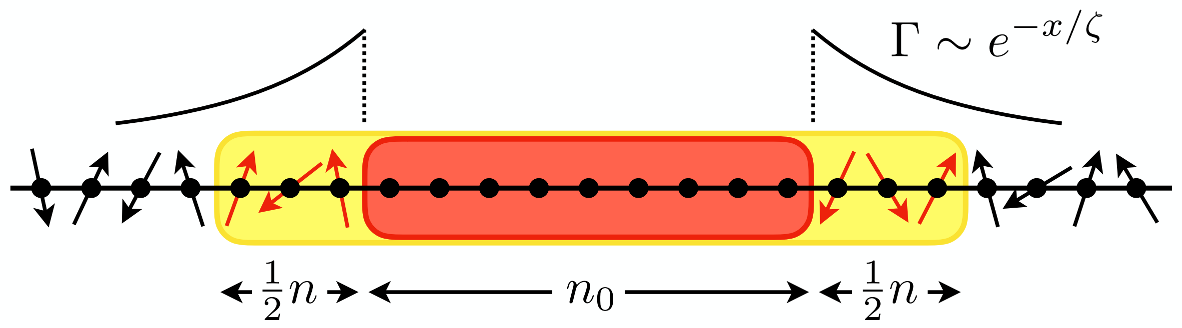
➡ (2018-2020) Analytical breakthrough
The Avalanche scenario

Delocalization transition
Avalanche of ergodic regions when $\xi_{\text{typ}} \geq (\ln 2)^{-1}$

PHYSICAL REVIEW LETTERS **121**, 140601 (2018)

Many-Body Delocalization as a Quantum Avalanche

Thimothée Thiery,[1,*] François Huveneers,[2,†] Markus Müller,[3,‡] and Wojciech De Roeck[1,§]
[1]*Instituut voor Theoretische Fysica, KU Leuven, 3001 Leuven, Belgium*
[2]*Université Paris-Dauphine, PSL Research University, CNRS, CEREMADE, 75016 Paris, France*
[3]*Paul Scherrer Institute, PSI, 5232 Villigen, Switzerland*



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PHYSICAL REVIEW B **105**, 174205 (2022)
Editors' Suggestion

Avalanches and many-body resonances in many-body localized systems

Alan Morningstar,[1] Luis Colmenarez,[2] Vedika Khemani,[3] David J. Luitz,[4,2] and David A. Huse[1,5]

PHYSICAL REVIEW E **102**, 062144 (2020)

Quantum chaos challenges many-body localization

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nature physics

Letter
https://doi.org/10.1038/s41567-022-01887-3

Probing the onset of quantum avalanches in a many-body localized system

Received: 23 December 2021
Accepted: 22 November 2022
Published online: 26 January 2023

Julian Léonard[1,5,6], Sooshin Kim[1,6], Matthew Rispoli[1], Alexander Lukin[1], Robert Schittko[1], Joyce Kwan[1], Eugene Demler[2], Dries Sels[3,4] & Markus Greiner[1]

Annals of Physics
Volume 427, April 2021, 168415

Distinguishing localization from chaos: Challenges in finite-size systems

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PHYSICAL REVIEW B **105**, 224203 (2022)
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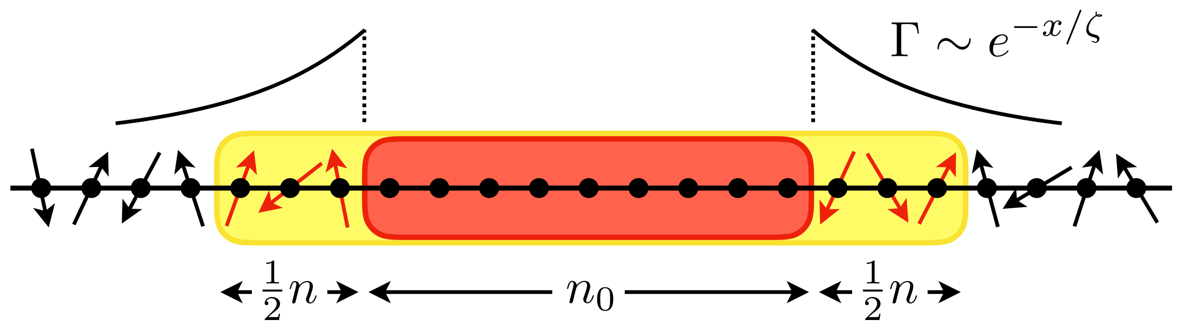
Thimothée Thiery,[1,*] François Huveneers,[2,†] Markus Müller,[3,‡] and Wojciech De Roeck[1,§]
[1]*Instituut voor Theoretische Fysica, KU Leuven, 3001 Leuven, Belgium*
[2]*Université Paris-Dauphine, PSL Research University, CNRS, CEREMADE, 75016 Paris, France*
[3]*Paul Scherrer Institute, PSI, 5232 Villigen, Switzerland*



PHYSICAL REVIEW B **105**, 174205 (2022)
Editors' Suggestion

Avalanches and many-body resonances in many-body localized systems

Alan Morningstar,[1] Luis Colmenarez,[2] Vedika Khemani,[3] David J. Luitz,[4,2] and David A. Huse[1,5]

nature physics
Letter
https://doi.org/10.1038/s41567-022-01887-3

Probing the onset of quantum avalanches in a many-body localized system

Received: 23 December 2021
Accepted: 22 November 2022
Published online: 26 January 2023

Julian Léonard[1,5,6], Sooshin Kim[1,6], Matthew Rispoli[1], Alexander Lukin[1], Robert Schittko[1], Joyce Kwan[1], Eugene Demler[2], Dries Sels[3,4] & Markus Greiner[1]

Avalanches are hard to probe but MBL seems to be still a plausible option, perhaps at disorder strengths larger than expected

EPL, **128** (2019) 67003
doi: 10.1209/0295-5075/128/67003

Can we study the many-body localisation transition?

R. K. Panda[1,2], A. Scardicchio[1,3], M. Schulz[1], S. R. Taylor[1(a)] and M. Žnidarič[4]

PHYSICAL REVIEW E **102**, 062144 (2020)

Quantum chaos challenges many-body localization

Jan Šuntajs,[1] Janez Bonča,[2,1] Tomaž Prosen,[2] and Lev Vidmar[1,2]

Annals of Physics
Volume 427, April 2021, 168415

Distinguishing localization from chaos: Challenges in finite-size systems

D.A. Abanin[a,1], J.H. Bardarson[b,1], G. De Tomasi[c,1], S. Gopalakrishnan[d,e,1], V. Khemani[f,1], S.A. Parameswaran[g,1], F. Pollmann[h,i,1], A.C. Potter[j,1], M. Serbyn[k,1], R. Vasseur[l,1]

PHYSICAL REVIEW B **105**, 224203 (2022)
Editors' Suggestion

Challenges to observation of many-body localization

Piotr Sierant[1,2] and Jakub Zakrzewski[2,3,*]

these debates have surprisingly focused on a restricted class of U(1) symmetric models

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(XXZ chains, interacting fermions, bosons…)
≈
99% of the studies

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$$\mathscr{H} = \sum_i \left[S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + S_i^z S_{i+1}^z + h_i S_i^z \right]$$

Random-field Heisenberg chain
≈
98% of the studies

MBL in disordered Ising chains ?

$$\mathcal{H}_{\mathrm{TFIM}} = \sum_i \left(J_i \sigma_i^x \sigma_{i+1}^x + h_i \sigma_i^z\right)$$

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with $K_j = \prod_{k=1}^{j-1} \sigma_k^z,$

$$\mathscr{H}_{\text{free-fermions}} = \sum_i \left(J_i\left[c_i^\dagger c_{i+1}^\dagger + c_i^\dagger c_{i+1} - c_i c_{i+1}^\dagger - c_i c_{i+1}\right] + h_i\left[1 - 2c_i^\dagger c_i\right]\right)$$

Particle number is NOT conserved

Duality transformation

$$\tau_i^z = \sigma_i^x \sigma_{i+1}^x \,,\ \tau_i^x = \prod_{k=1}^{i} \sigma_k^z \quad \Rightarrow \quad \mathscr{H}_{\text{TFIM}} = \sum_i \left(J_i \tau_i^z + h_i \tau_{i-1}^x \tau_i^x\right)$$

$\mathbb{Z}_2$ symmetry (parity)

$$\mathbb{P} = \prod_j \sigma_j^z = (-1)^{\sum_i n_i} = \pm 1$$

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PRL **117,** 027201 (2016) Selected for a Viewpoint in *Physics* PHYSICAL REVIEW LETTERS week ending 8 JULY 2016

Diagonalization and Many-Body Localization for a Disordered Quantum Spin Chain

John Z. Imbrie

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PHYSICAL REVIEW X 4, 011052 (2014)

Hilbert-Glass Transition: New Universality of Temperature-Tuned Many-Body Dynamical Quantum Criticality

David Pekker,[1,2] Gil Refael,[1] Ehud Altman,[3,4] Eugene Demler,[5] and Vadim Oganesyan[6,7]

III) $$\sum_i J_i^x\sigma_i^x\sigma_{i+2}^x = \sum_i J_i^x\left(c_i^\dagger + c_i\right)\left(1-2n_i\right)\left(1-2n_{i+1}\right)\left(c_{i+2}^\dagger + c_{i+2}\right)$$

$\mathbb{Z}_2$

PRL 113, 107204 (2014) PHYSICAL REVIEW LETTERS week ending 5 SEPTEMBER 2014

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PRL 113, 107204 (2014) PHYSICAL REVIEW LETTERS 5 SEPTEMBER 2014

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IV) $\sum_i g_i \left(\sigma_i^z \sigma_{i+1}^z + \sigma_i^x \sigma_{i+2}^x \right)$

$\mathbb{Z}_2$ and duality

PHYSICAL REVIEW LETTERS **126,** 100604 (2021)

Emergent Ergodicity at the Transition between Many-Body Localized Phases

Rahul Sahay,[1,*] Francisco Machado,[1,*] Bingtian Ye,[1,*] Chris R. Laumann,[2] and Norman Y. Yao[1,3]

Perturbative instability towards delocalization at phase transitions between MBL phases

Sanjay Moudgalya,[1] David A. Huse,[1] and Vedika Khemani[2]

arXiv:2008.09113v1

$$\mathscr{H} = -\sum_i \left(J_i\, \sigma_i^x \sigma_{i+1}^x + h_i\, \sigma_i^z\right) + \sum_i g_i \left(\sigma_i^x \sigma_{i+2}^x + \sigma_i^z \sigma_{i+1}^z\right)$$

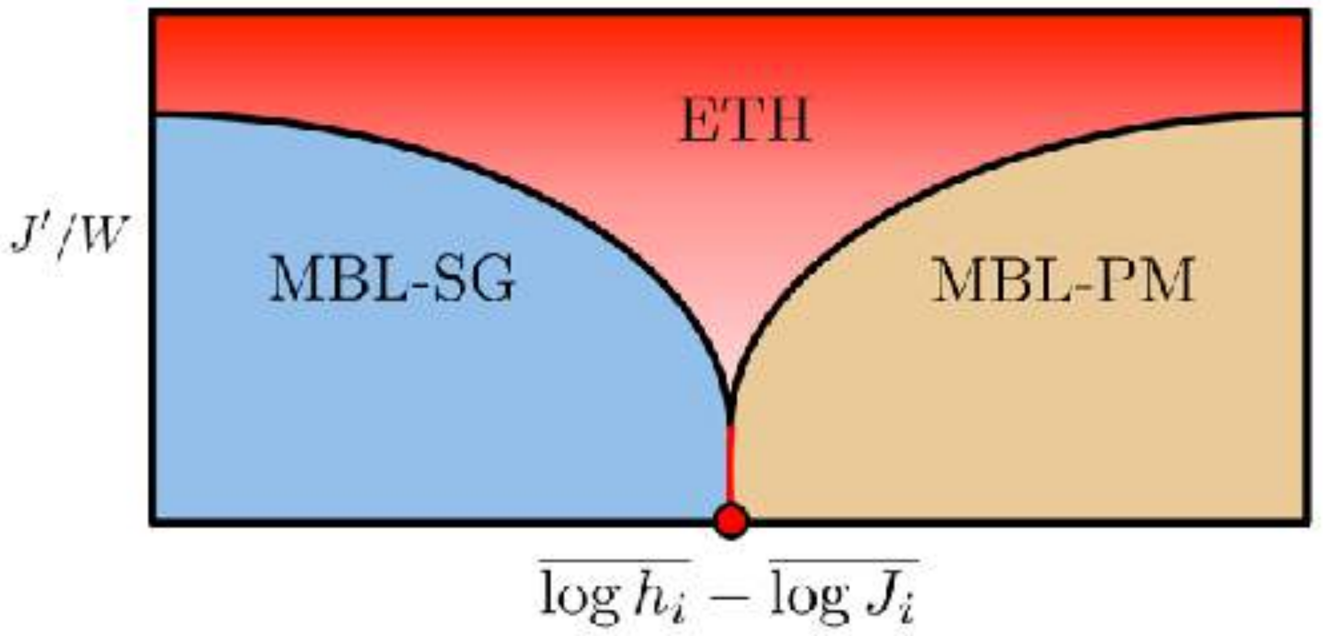
non – interacting

Interaction ($\mathbb{Z}_2$ sym. + duality)

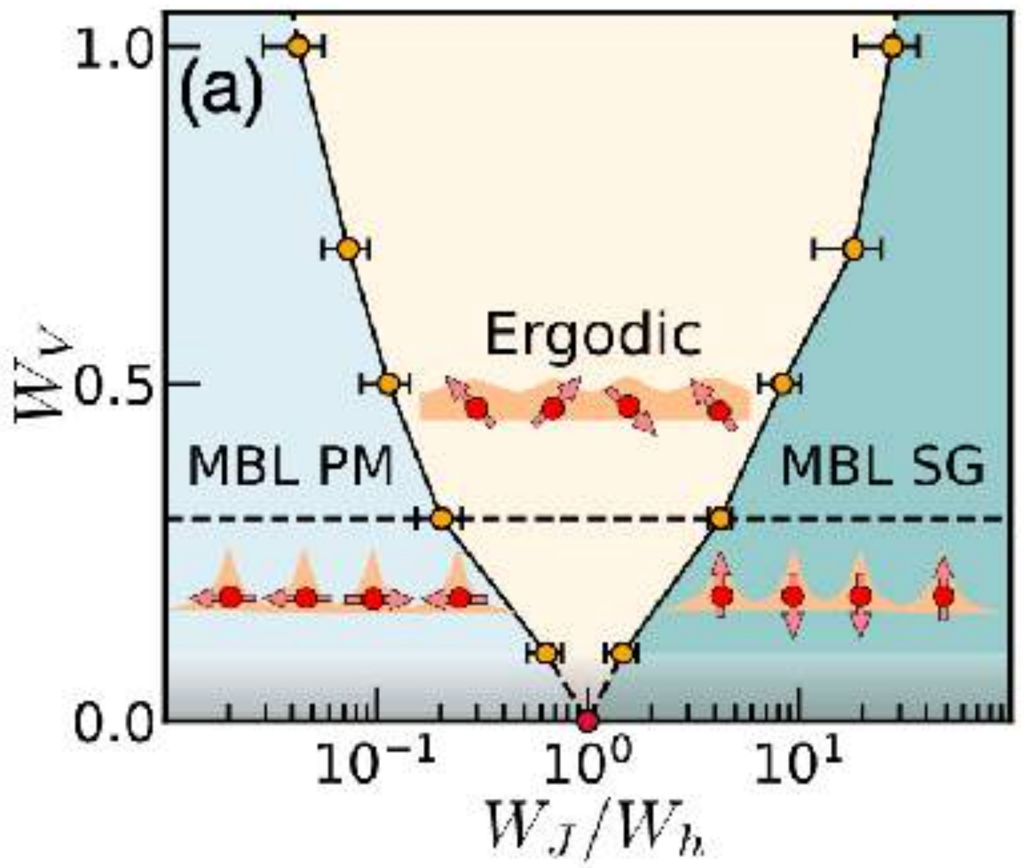
$$\mathscr{H} = -\underbrace{\sum_i \left(J_i \sigma_i^x \sigma_{i+1}^x + h_i \sigma_i^z\right)}_{\text{non-interacting}} + \underbrace{\sum_i g_i \left(\sigma_i^x \sigma_{i+2}^x + \sigma_i^z \sigma_{i+1}^z\right)}_{\text{Interaction } (\mathbb{Z}_2 \text{ sym. } + \text{ duality})}$$

$\mathbf{T = \infty}$ Phase diagram: Previous results/proposals

Parameswaran and Vasseur
Rep. Prog. Phys. 81 082501 (2018)



Sahay, Machado, Ye, Laumann, Yao
Phys. Rev. Lett. 126, 100604 (2021).



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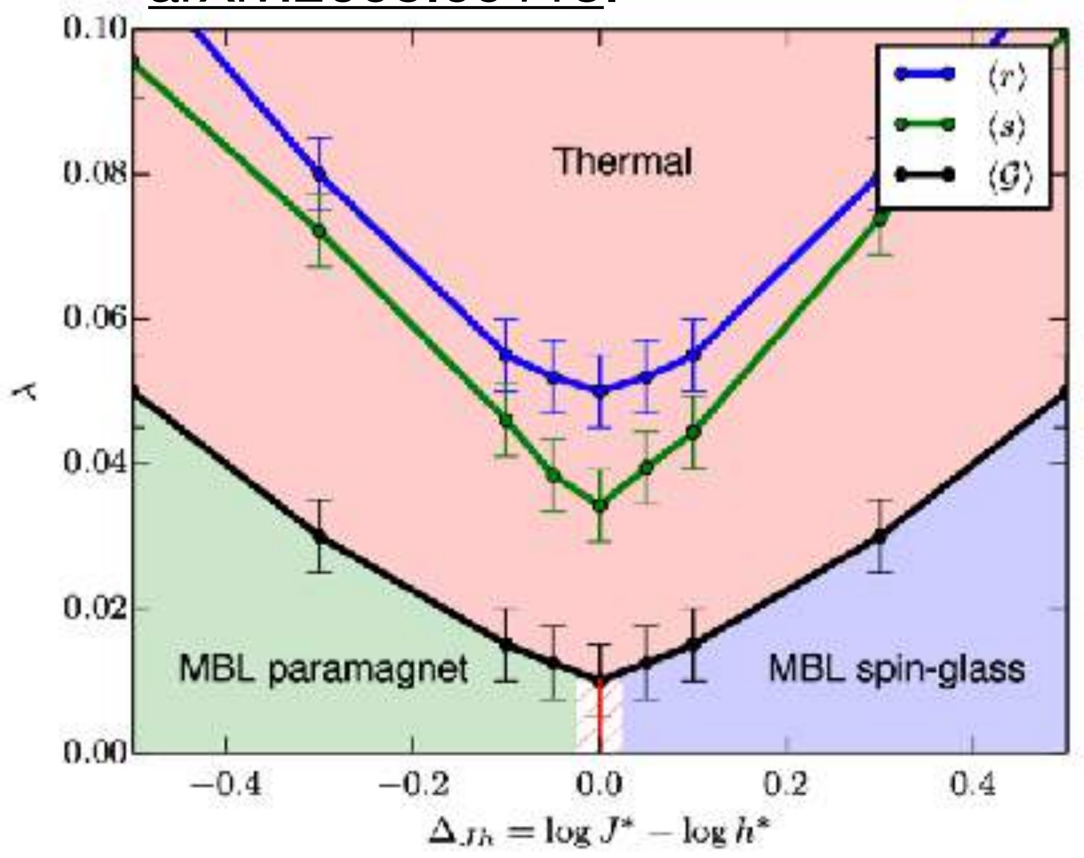

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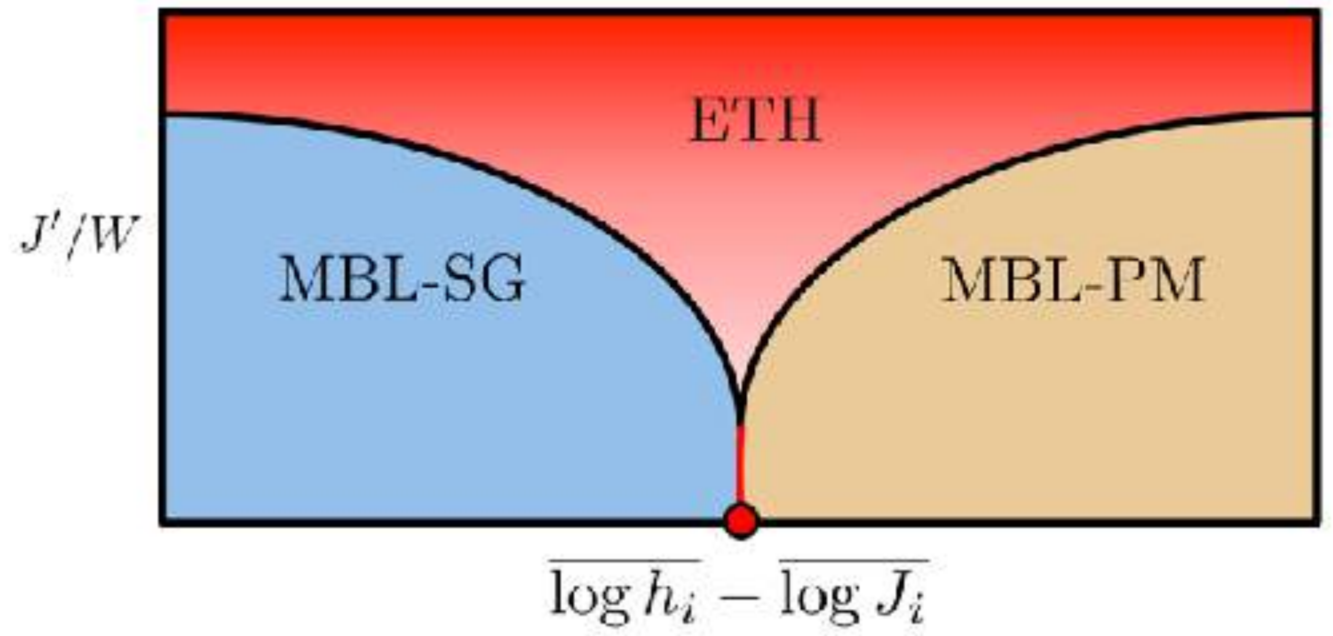

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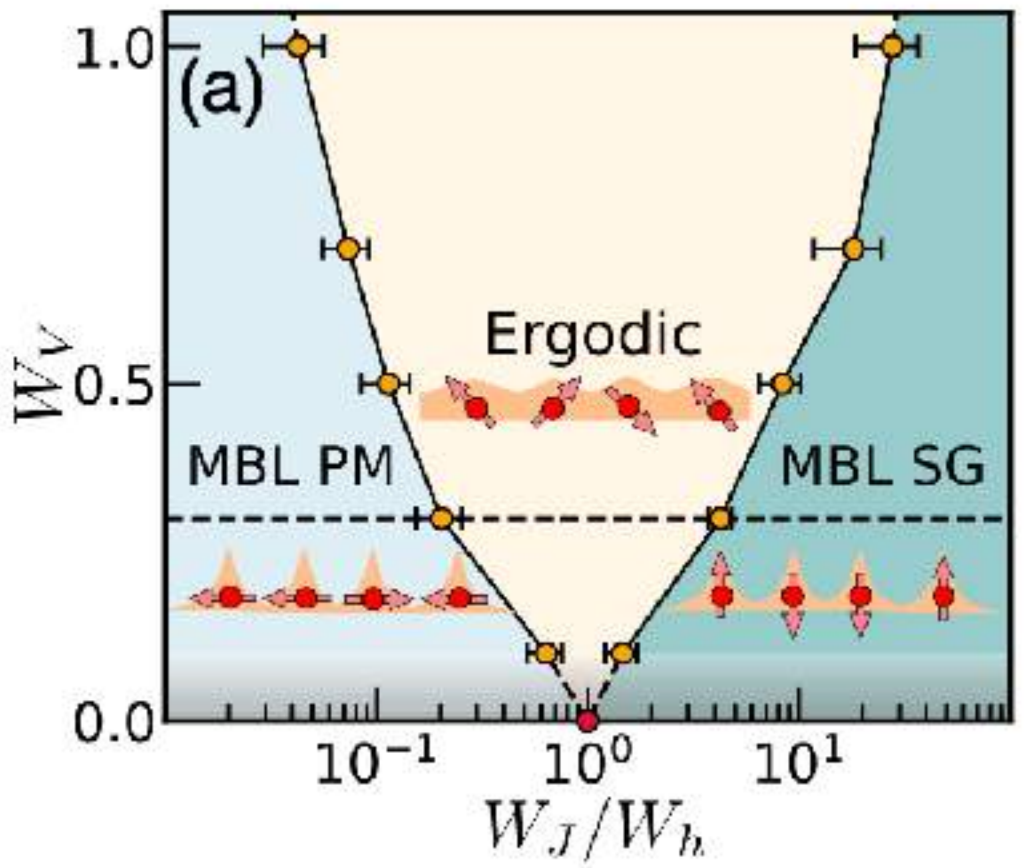

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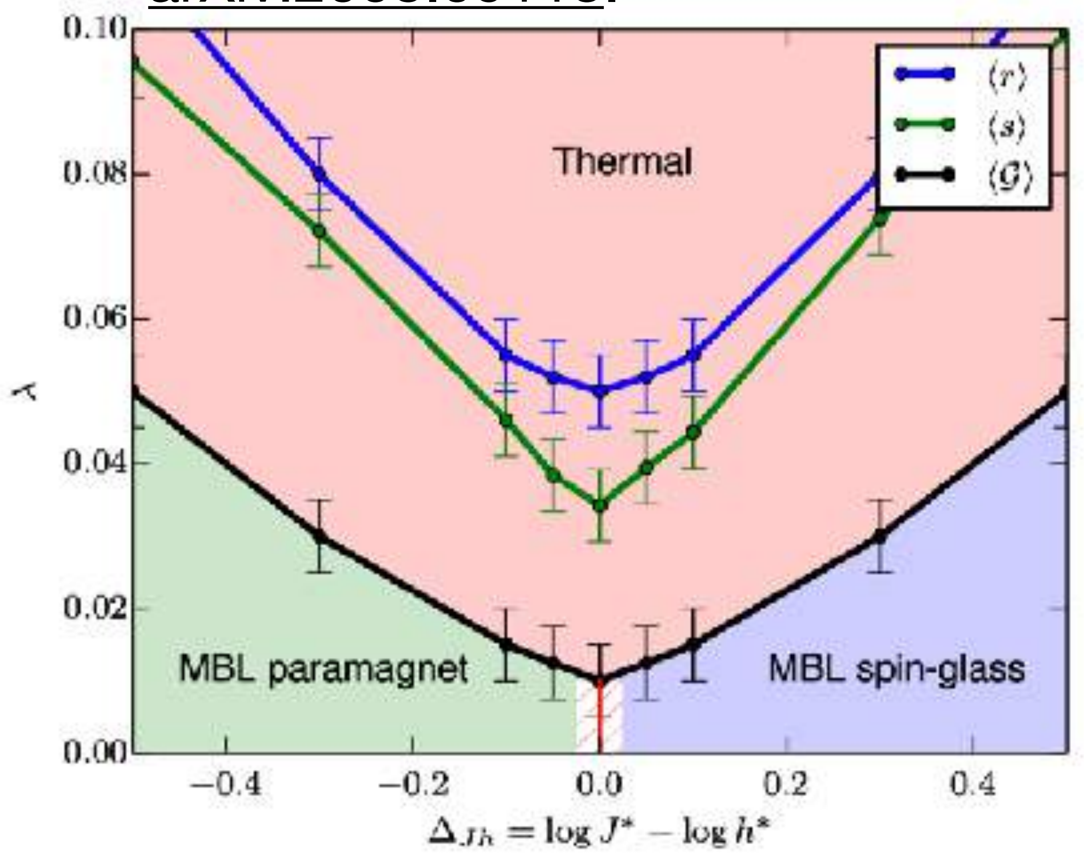

some key questions

- **Fate of the non-interacting "Infinite Randomness" critical point ?**
- **Possible direct transition between 2 different MBL phases ?**
- **Topological nature of the MBL Spin Glass phase ?**

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- *Dirac fermions*

Lieb-Schultz, Mattis (1961) ; Pfeuty (1970) ...

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Kitaev (2001) ; Fendley (2012) ...

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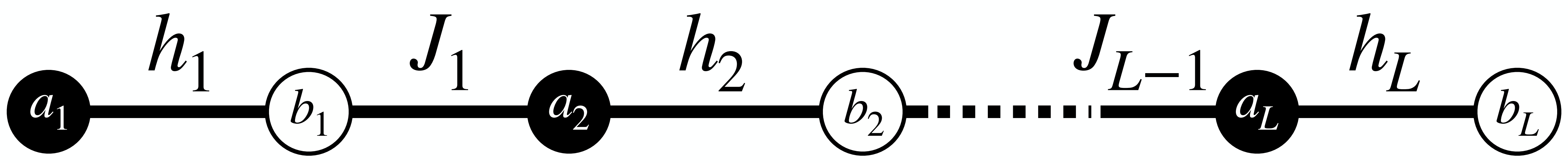

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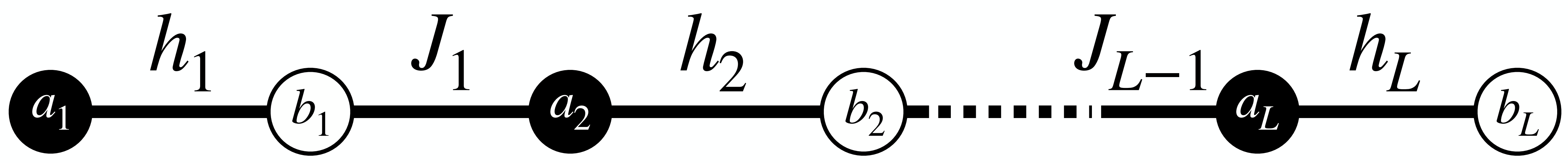


‣ Strong zero-modes

$$\Psi_{\text{Left}} = \frac{1}{\mathcal{N}^{1/2}} \left(a_1 + \frac{h_1}{J_1} a_2 + \cdots + \prod_{i=1}^{L-1} \frac{h_i}{J_i} a_L \right) \qquad \Psi_{\text{Right}} = \frac{1}{\mathcal{N}^{1/2}} \left(b_L + \frac{h_L}{J_{L-1}} b_{L-1} + \cdots + \prod_{i=1}^{L-1} \frac{h_i}{J_i} b_1 \right)$$

When $\prod_i \frac{h_i}{J_i} < 1$

(i) $[\mathcal{H}, \Psi_{\text{Left}}] = \frac{2\mathrm{i}}{\sqrt{\mathcal{N}}} \frac{h_1 h_2 \cdots h_L}{J_1 J_2 \cdots J_{L-1}} b_L \xrightarrow[(L \to \infty)]{} 0$

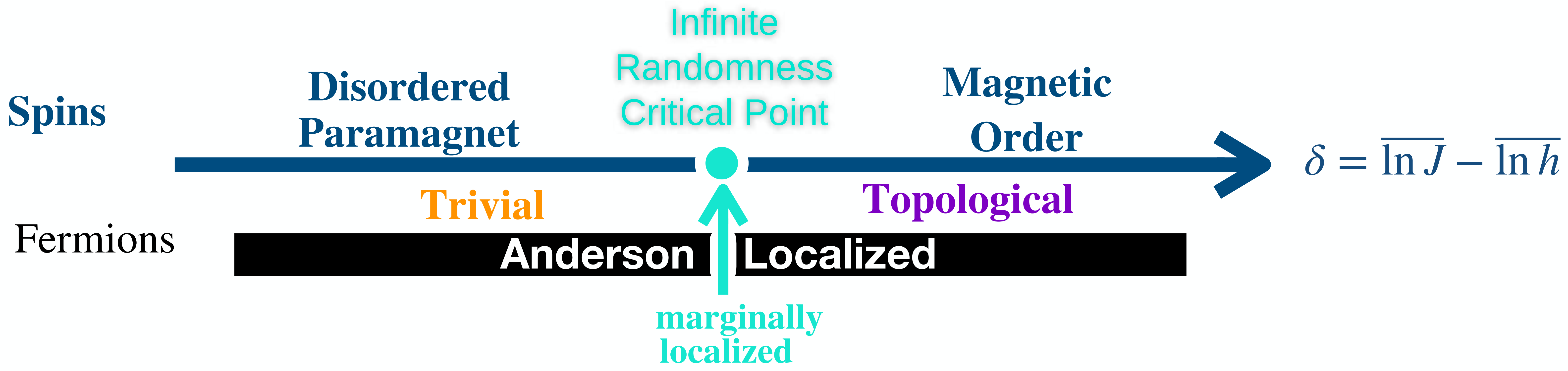
(ii) $\mathcal{O}^\dagger_{\text{zm}} = \frac{1}{2}\left(\Psi_{\text{Left}} - \mathrm{i}\Psi_{\text{Right}}\right)$

(iii) $\{\mathcal{O}^\dagger_{\text{zm}}, \hat{\mathbb{P}}\} = 0$

Create an exponentially small energy state localized at the edges ⇒ topological order

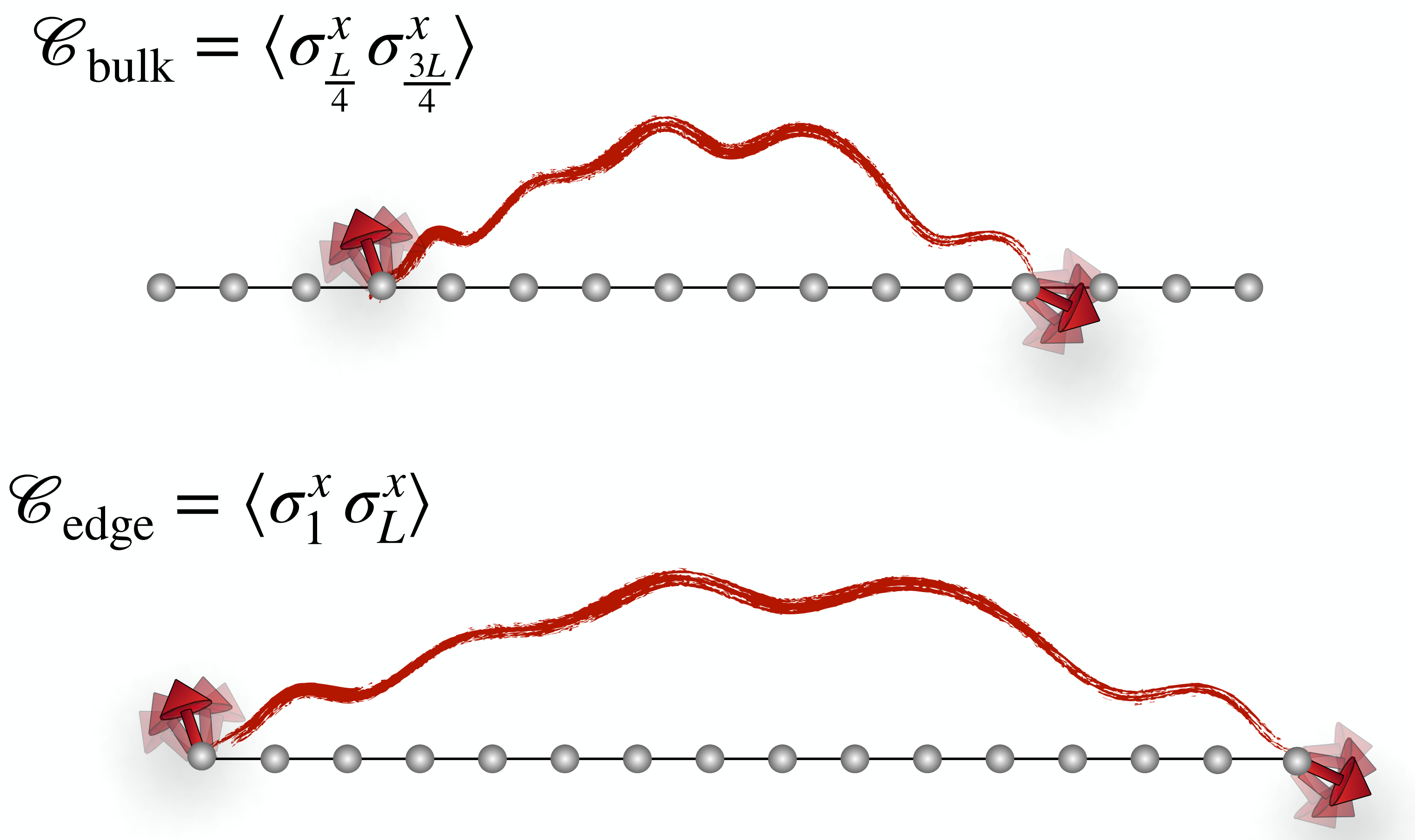
$\mathcal{O}^\dagger_{\text{zm}}$ maps between even and odd sectors

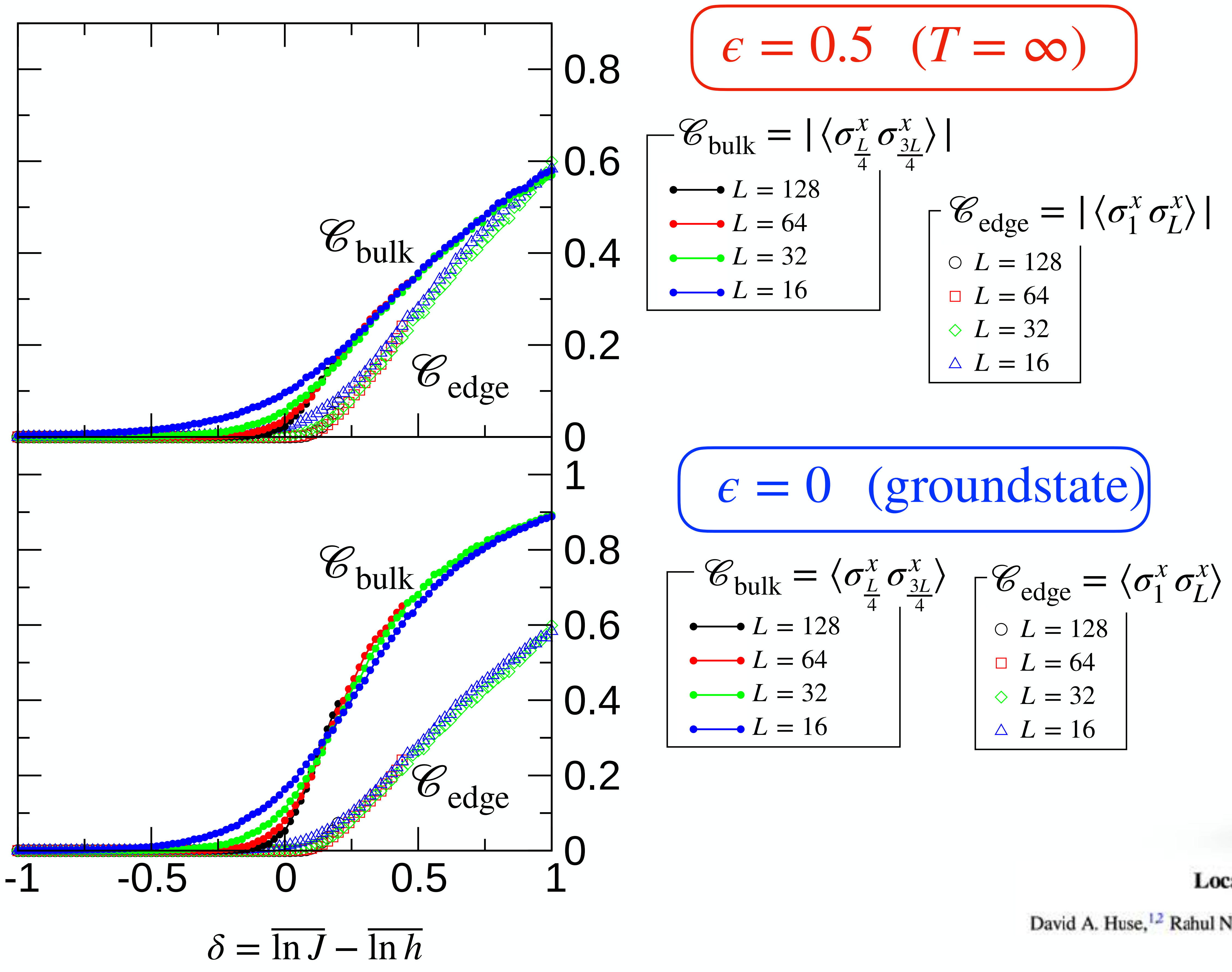
Non-interacting phase diagram



Non-interacting phase diagram

- *Bulk and End-End Spin Correlations*





Disorder "protects" magnetic order at ALL energies!

Localization-protected quantum order

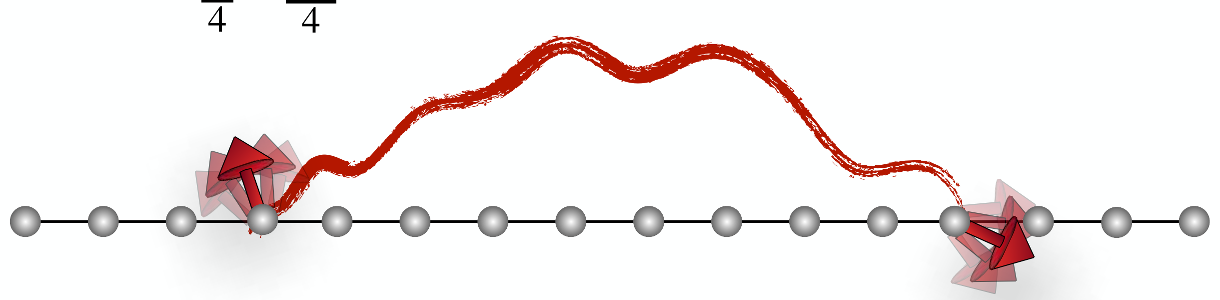
David A. Huse,[1,2] Rahul Nandkishore,[1] Vadim Oganesyan,[3,4] Arijeet Pal,[5] and S. L. Sondhi[2]

PRB 2013

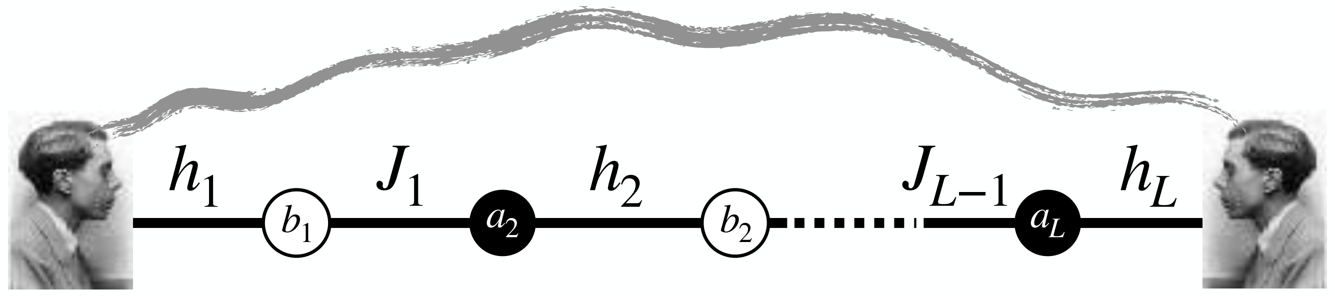
Non-interacting phase diagram

- *Bulk and End-End Spin Correlations*

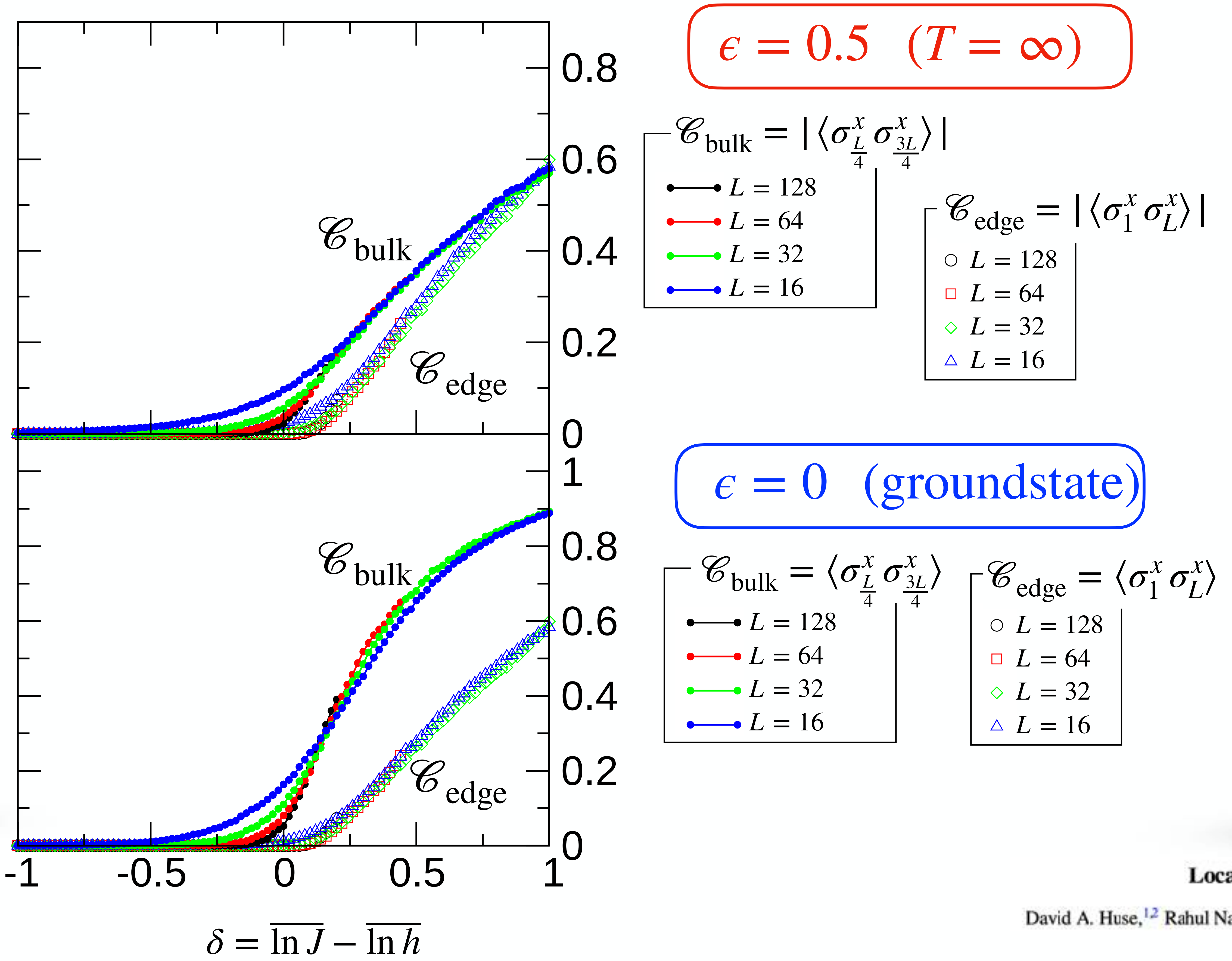
$\mathscr{C}_{\text{bulk}} = \langle \sigma^x_{\frac{L}{4}} \sigma^x_{\frac{3L}{4}} \rangle$



$\mathscr{C}_{\text{edge}} = \langle \sigma^x_1 \sigma^x_L \rangle = i\langle a_1 b_L \rangle$



$\mathscr{C}_{\text{edge}}$ **probes Majorana edge modes**



Disorder "protects" magnetic order at ALL energies!

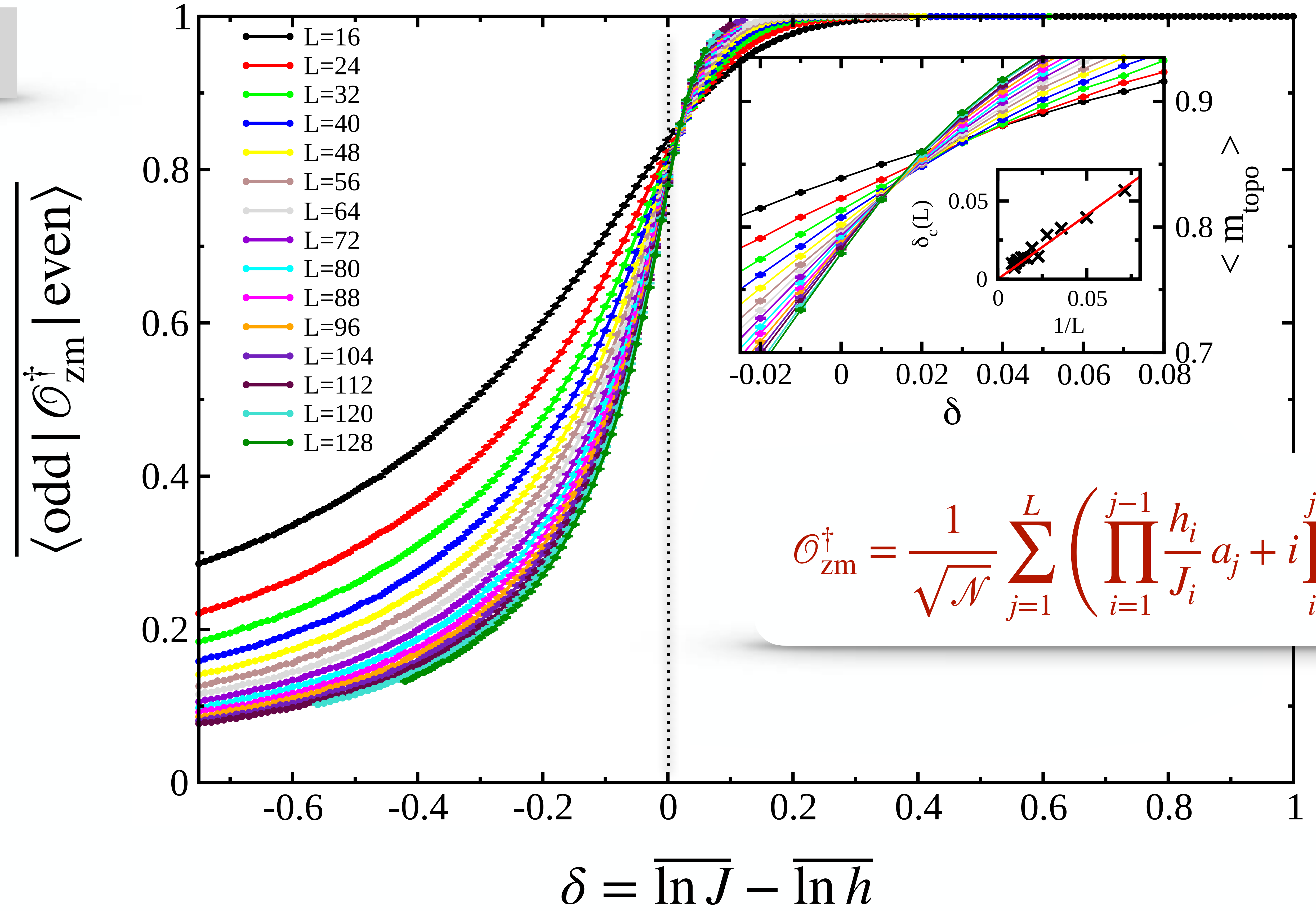
Localization-protected quantum order

David A. Huse,[1,2] Rahul Nandkishore,[1] Vadim Oganesyan,[3,4] Arijeet Pal,[5] and S. L. Sondhi[2]

PRB 2013

Non-interacting phase diagram

- *Strong Zero Mode*

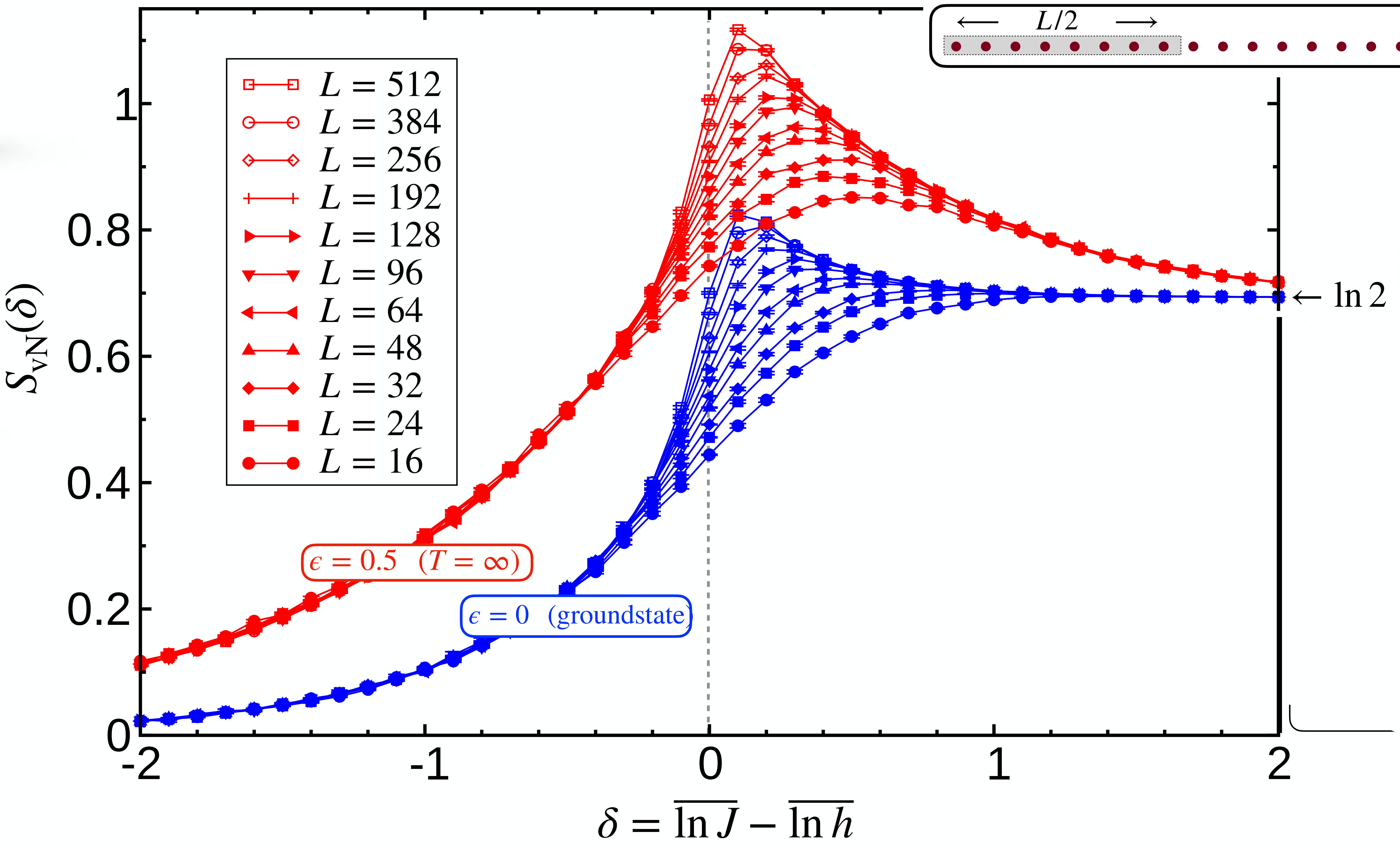


$$\mathcal{O}_{\mathrm{zm}}^{\dagger} = \frac{1}{\sqrt{\mathcal{N}}} \sum_{j=1}^{L} \left(\prod_{i=1}^{j-1} \frac{h_i}{J_i} a_j + i \prod_{i=1}^{j-1} \frac{h_{L+1-i}}{J_{L-i}} b_{L+1-j} \right)$$

Non-interacting phase diagram

- *Half-Chain Entanglement Entropy*

Area-law at all energies



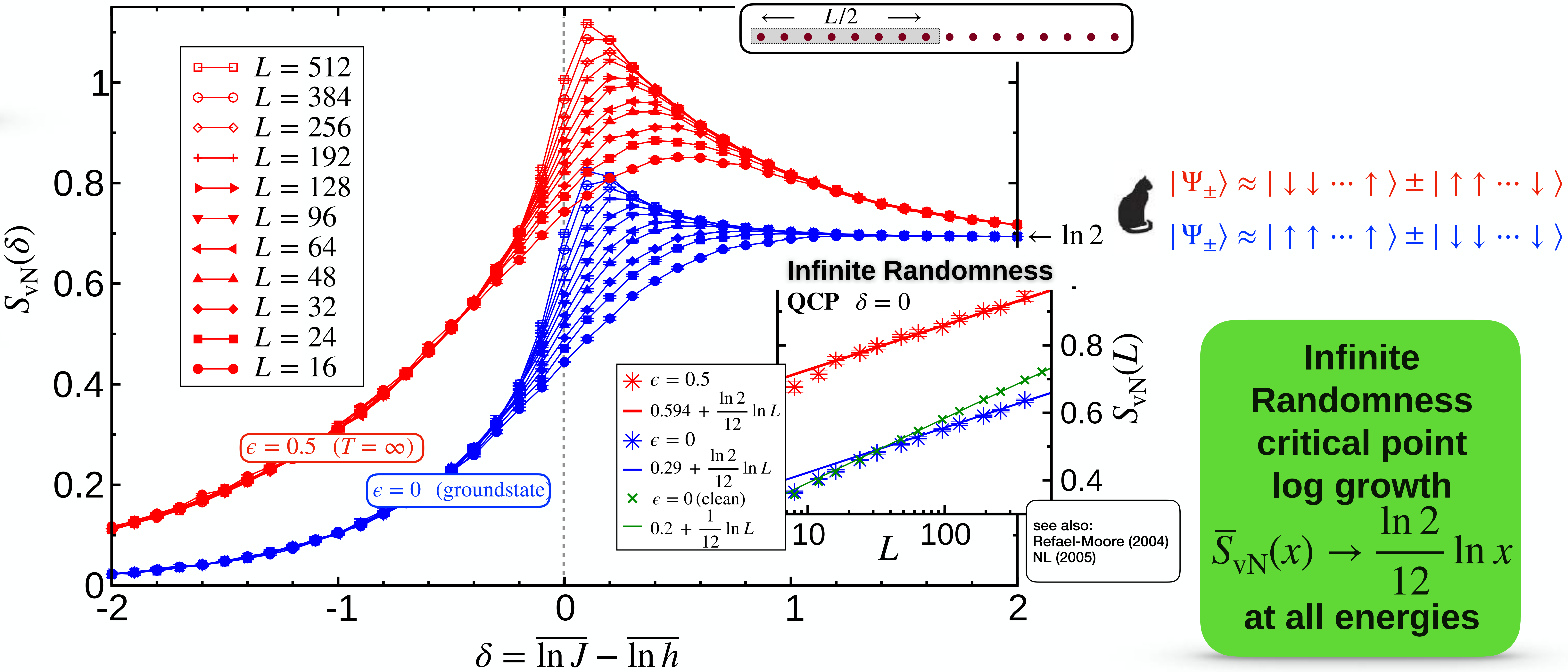
$|\Psi_\pm\rangle \approx |\downarrow\downarrow\cdots\uparrow\rangle \pm |\uparrow\uparrow\cdots\downarrow\rangle$

$|\Psi_\pm\rangle \approx |\uparrow\uparrow\cdots\uparrow\rangle \pm |\downarrow\downarrow\cdots\downarrow\rangle$

Non-interacting phase diagram

- *Half-Chain Entanglement Entropy*

Area-law at all energies



see also:
Refael-Moore (2004)
NL (2005)

Infinite Randomness critical point log growth

$$\overline{S}_{\rm vN}(x) \rightarrow \frac{\ln 2}{12} \ln x$$

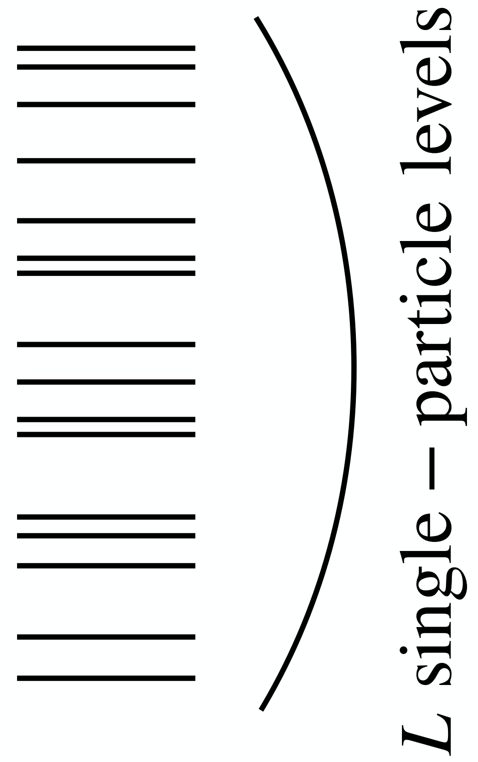
at all energies

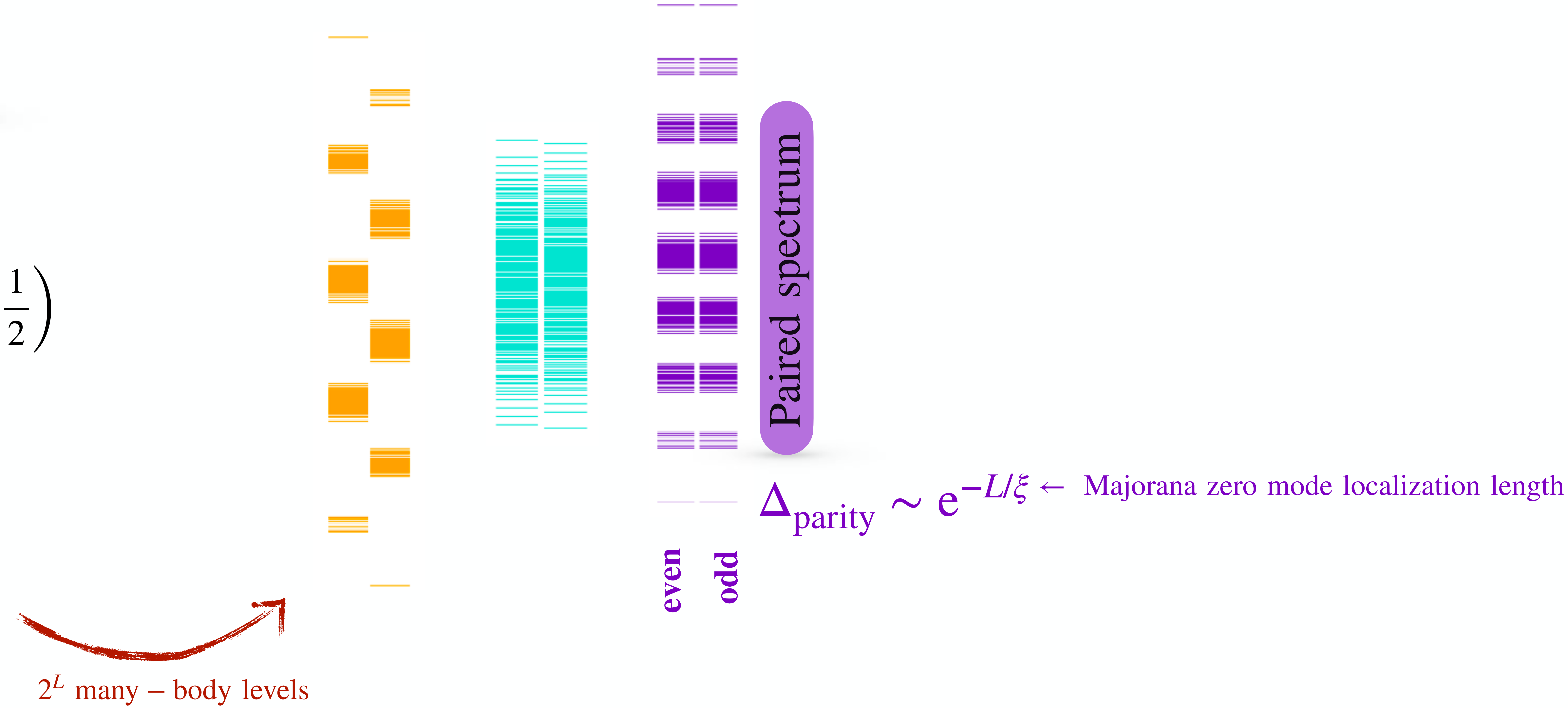
Non-interacting phase diagram

‣ *Many-Body Spectroscopy*

Reconstruct the exact many-body spectrum

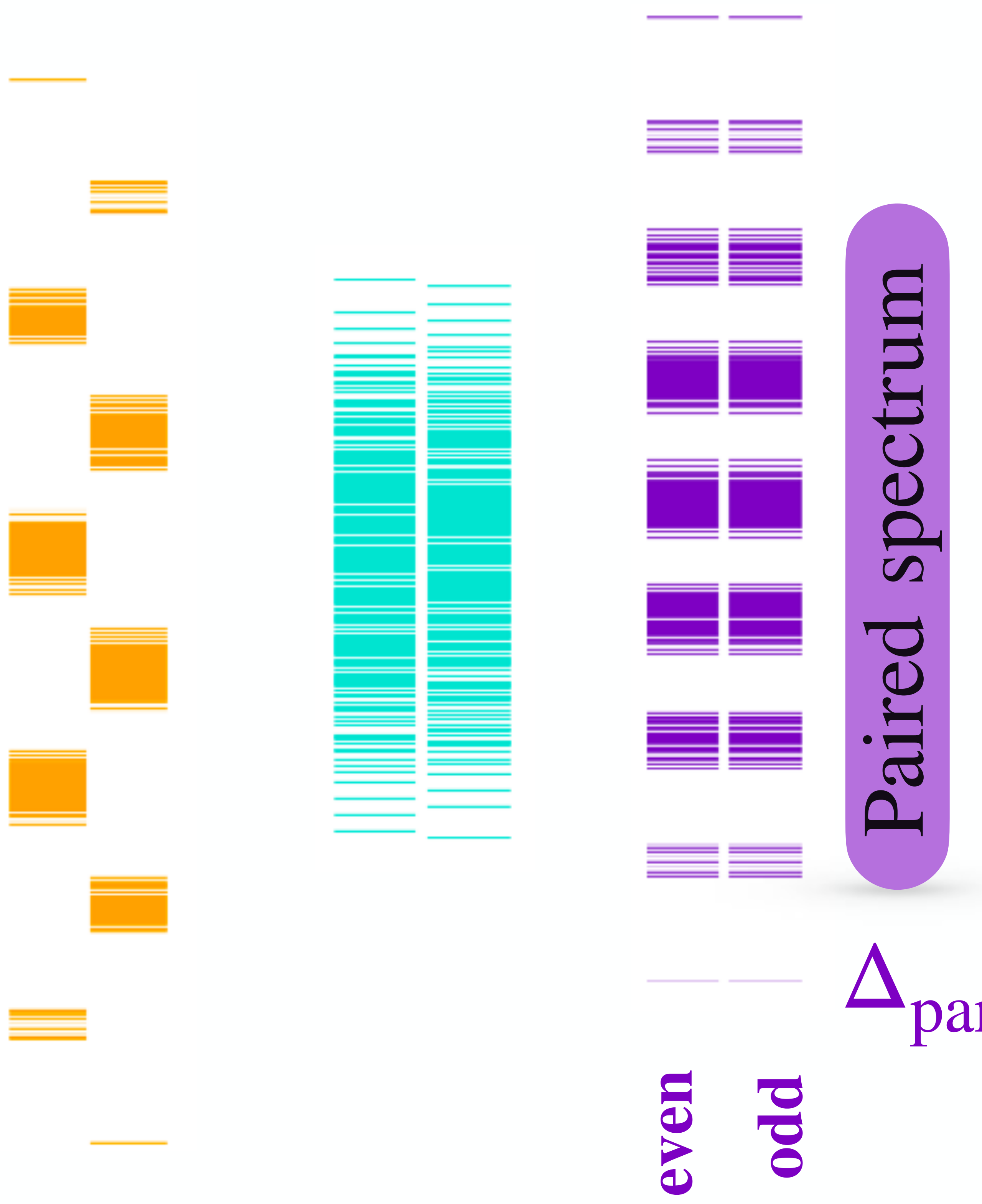
$$\mathscr{H}_{\mathrm{TFIM}} = \sum_{m=1}^{L} \epsilon_m \left(\phi_m^\dagger \phi_m - \frac{1}{2} \right)$$





Non-interacting phase diagram

- *Many-Body Spectroscopy*



$\Delta_{\text{parity}} \sim e^{-L/\xi} \leftarrow$ Majorana zero mode localization length

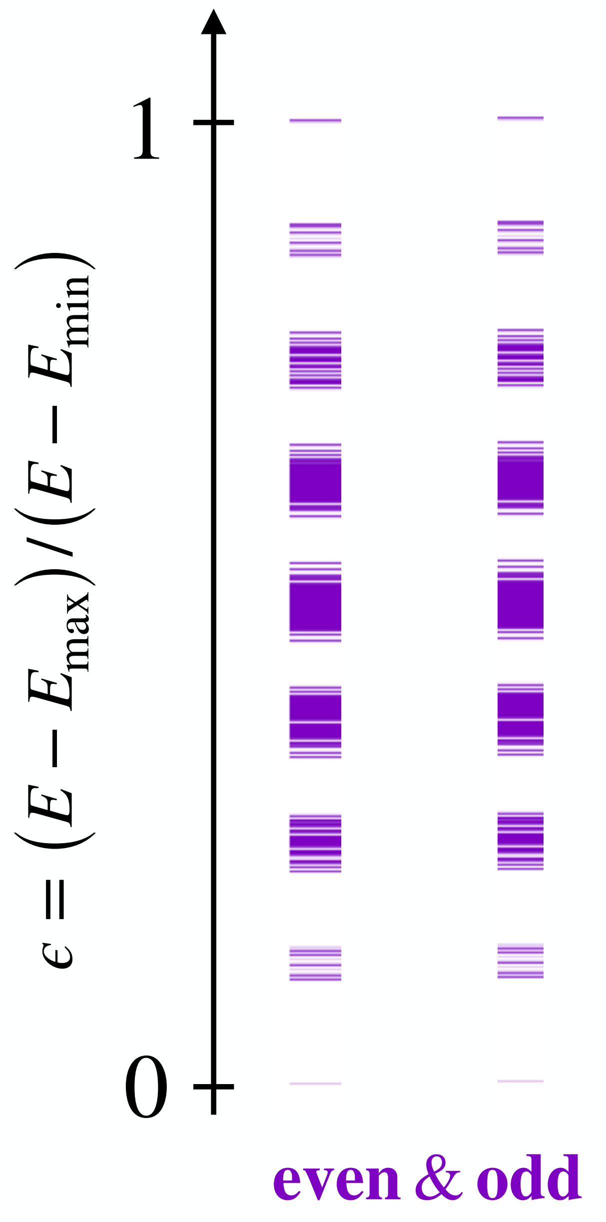
how to probe spectral pairing? (and MZM)

Non-interacting phase diagram

- *Many-Body Spectroscopy*

=> *Mixed level statistics* r'_{avg}

$$r'_{\text{avg}} = \frac{\min(\delta, \delta')}{\max(\delta, \delta')} = \begin{cases} \dfrac{\Delta_{\text{parity}}}{\Delta_{\text{many-body}}} \propto \dfrac{\exp(-L/\xi)}{\exp(-s_\epsilon L)} \longrightarrow 0 \quad \text{if } \Delta_{\text{parity}} \sim e^{-L/\xi} \ll \text{level spacing} \sim e^{-L s_\epsilon} \ (\text{if } \xi < 1/s_\epsilon) \\ r_{\text{Poisson}} = \ln 4 - 1 \approx 0.386 \quad (\text{otherwise}) \end{cases}$$



Non-interacting phase diagram

- *Many-Body Spectroscopy*

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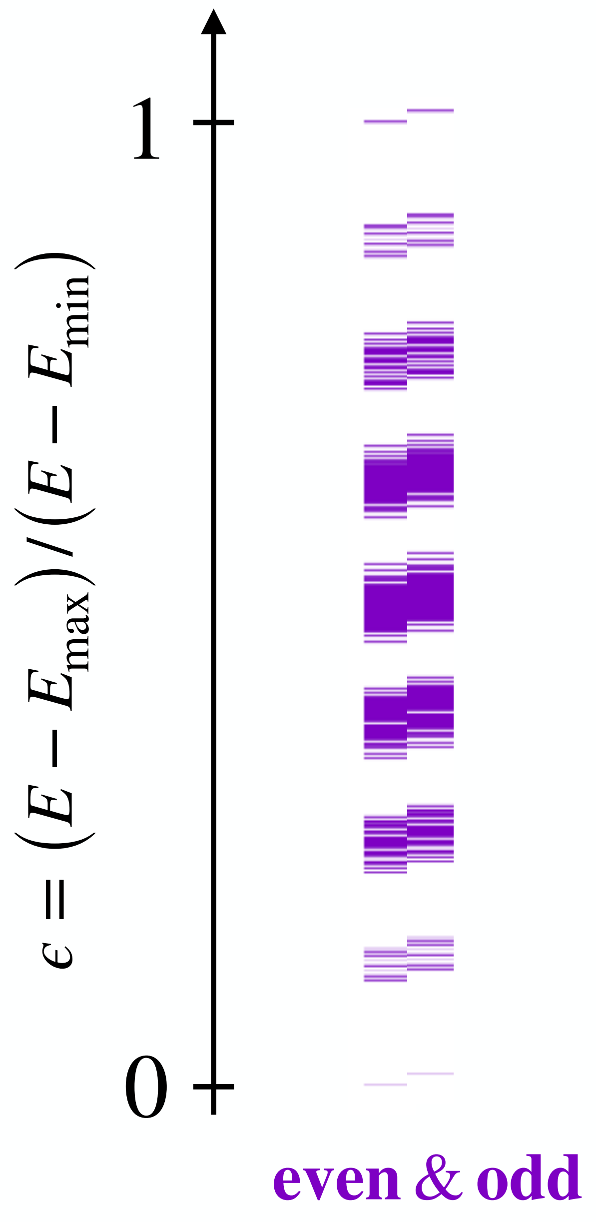
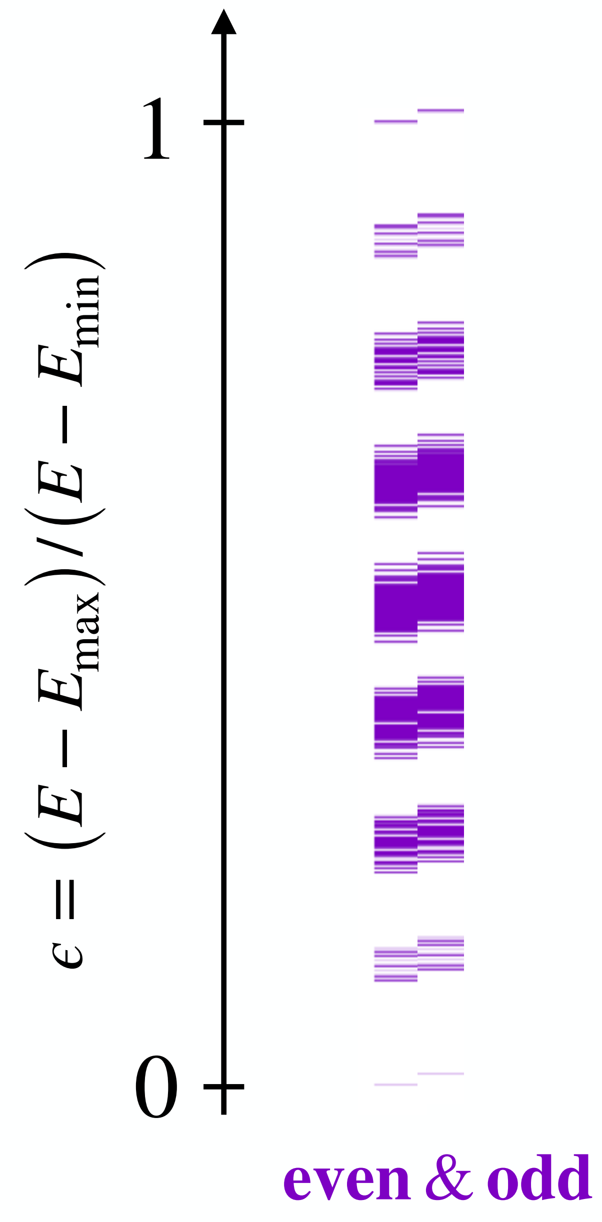


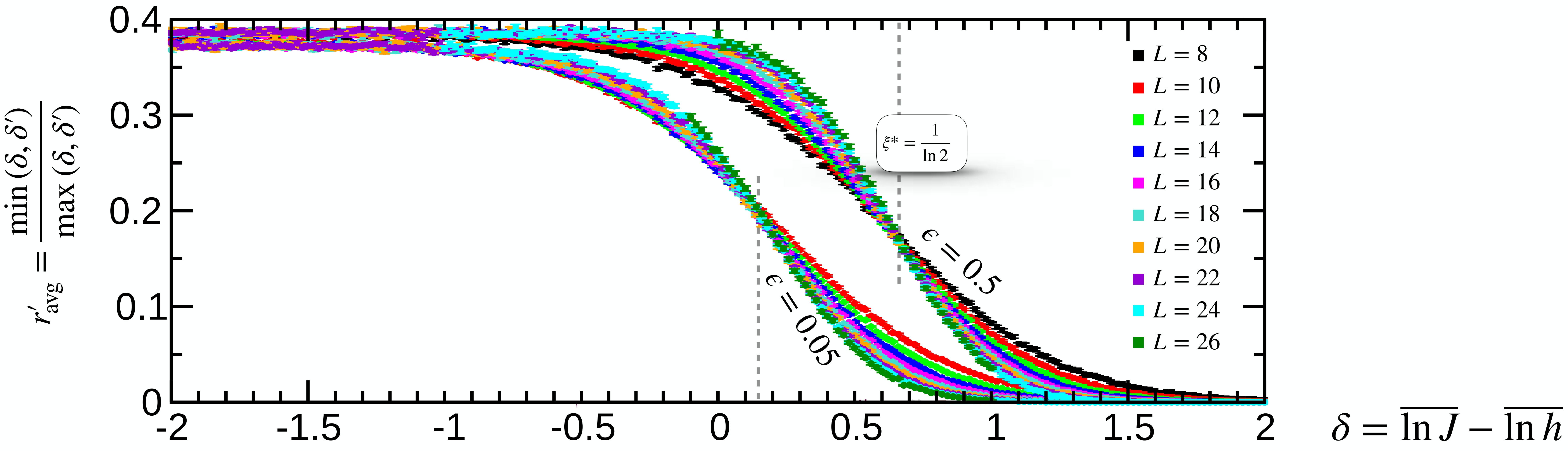
Non-interacting phase diagram

- *Many-Body Spectroscopy*

=> Mixed level statistics $r'_{\rm avg}$

$$r'_{\rm avg} = \frac{\min(\delta,\delta')}{\max(\delta,\delta')} = \begin{cases} \dfrac{\Delta_{\rm parity}}{\Delta_{\rm many-body}} \propto \dfrac{\exp(-L/\xi)}{\exp(-s_\epsilon L)} \longrightarrow 0 \quad \text{if } \Delta_{\rm parity} \sim e^{-L/\xi} \ll \text{level spacing} \sim e^{-Ls_\epsilon} \ (\text{if } \xi < 1/s_\epsilon) \\ r_{\rm Poisson} = \ln 4 - 1 \approx 0.386 \quad (\text{otherwise}) \end{cases}$$





Non-interacting phase diagram

- *Many-Body Spectroscopy*

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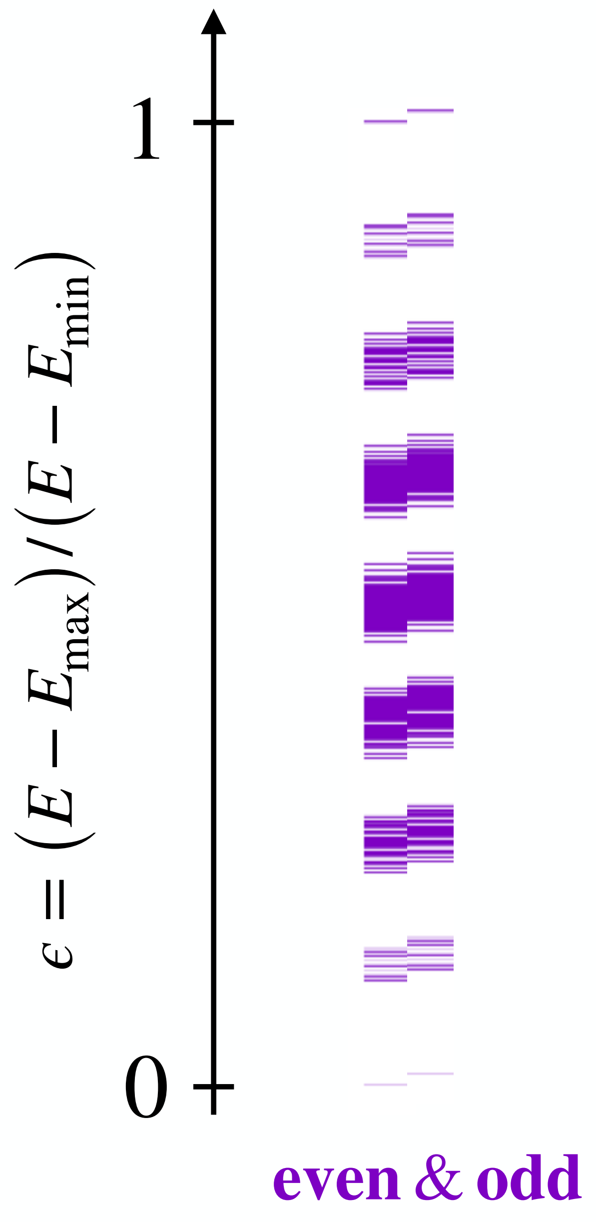
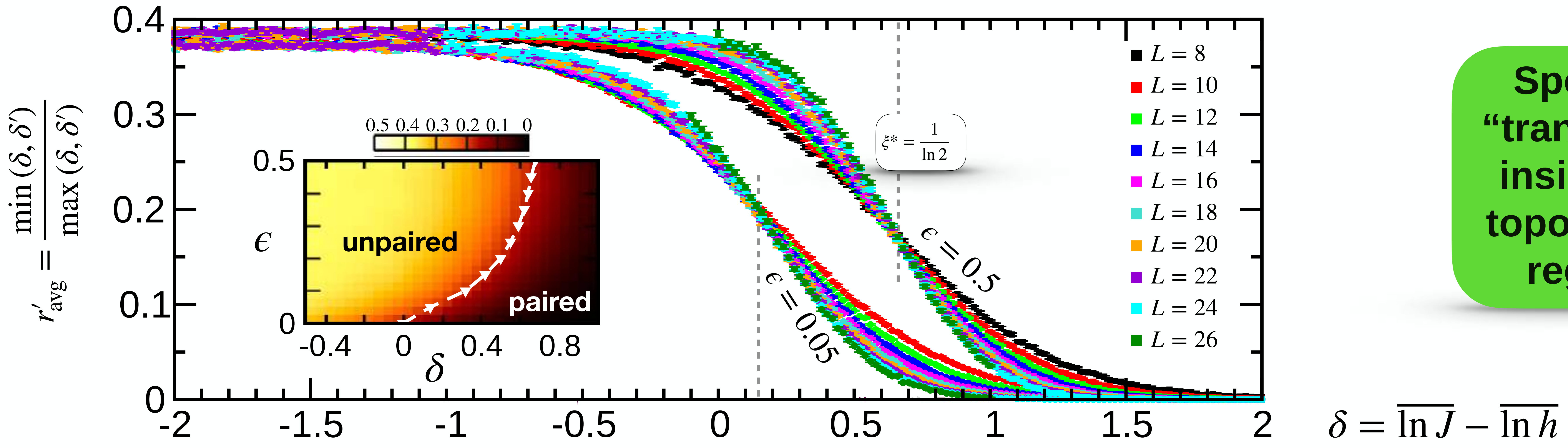
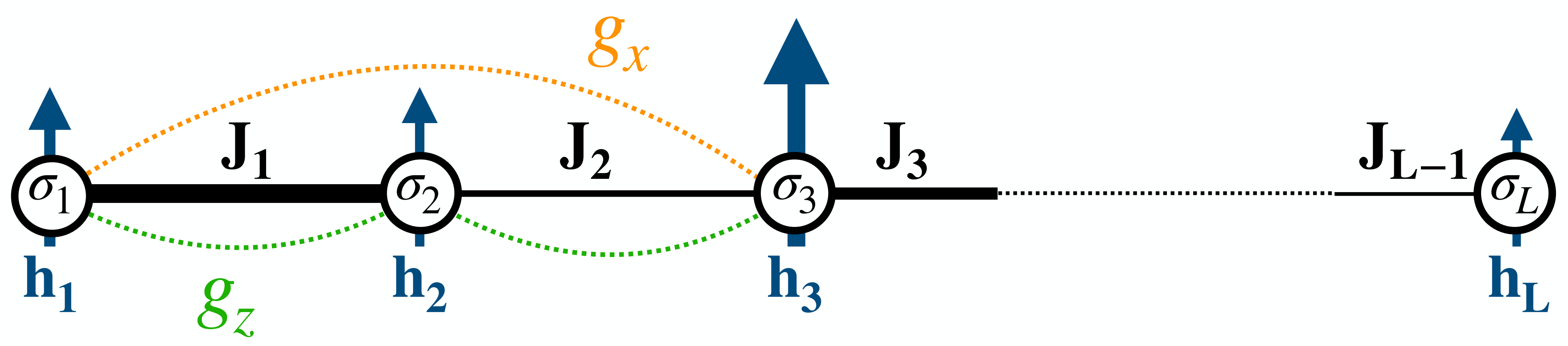




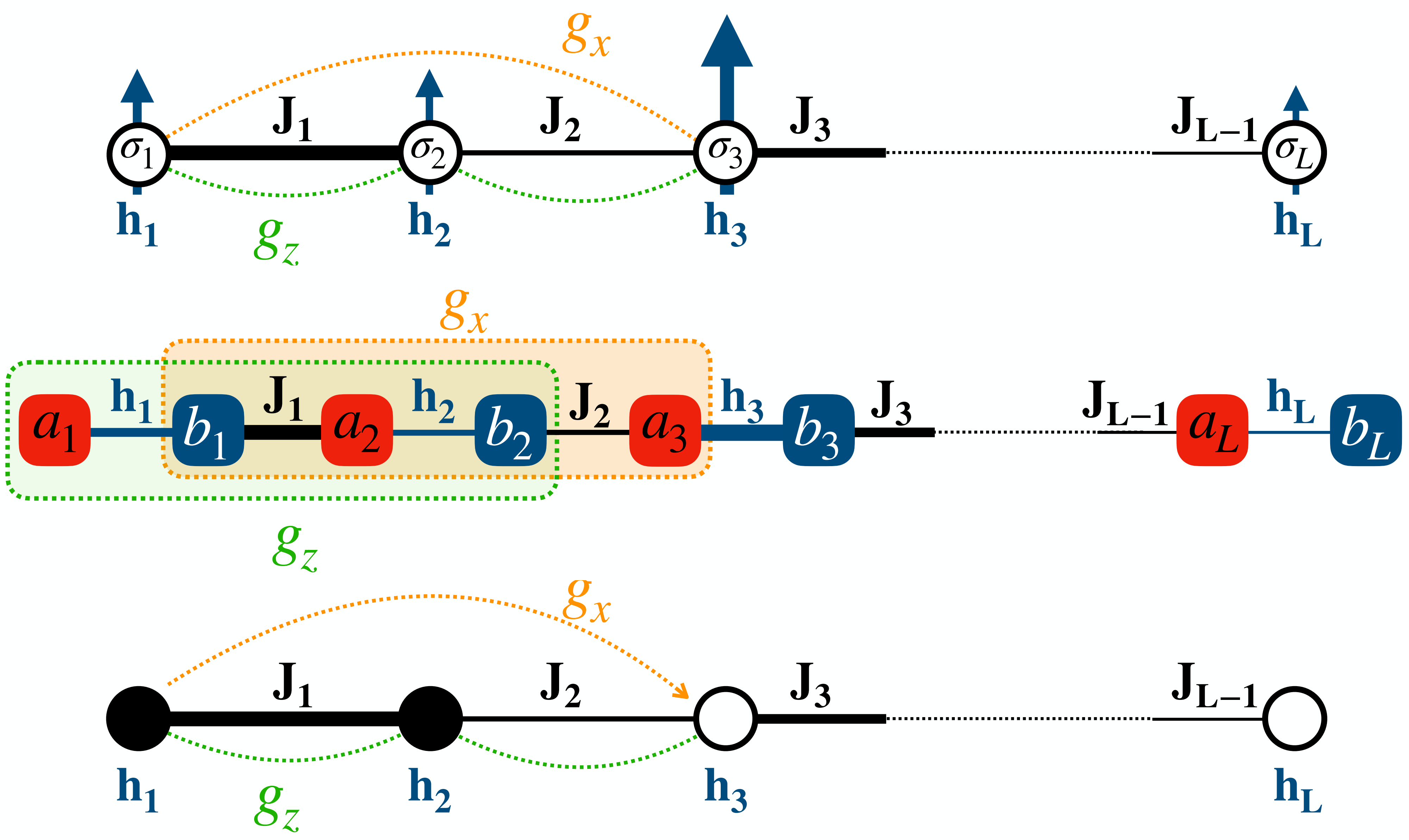
Spectral "transition" inside the topological regime

What are the effects of interactions?



$$\mathscr{H}_{\text{Pauli}} = \sum_j \left(J_j \sigma_j^x \sigma_{j+1}^x - h_j \sigma_j^z \right) + g_z \sum_j \sigma_j^z \sigma_{j+1}^z + g_x \sum_j \sigma_j^x \sigma_{j+2}^x$$

What are the effects of interactions?

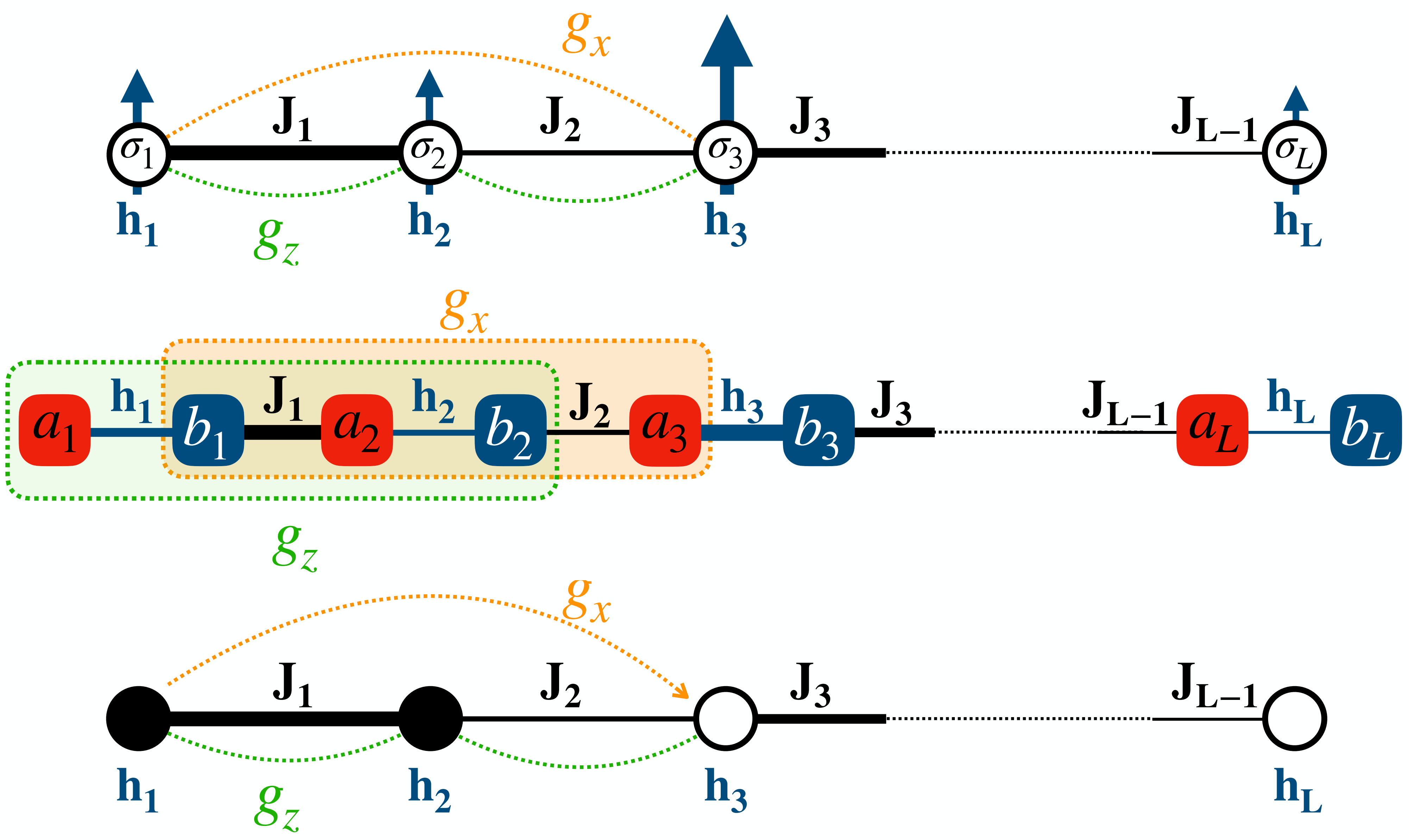


$$\mathscr{H}_{\text{Pauli}} = \sum_j \left(J_j \sigma_j^x \sigma_{j+1}^x - h_j \sigma_j^z \right) + g_z \sum_j \sigma_j^z \sigma_{j+1}^z + g_x \sum_j \sigma_j^x \sigma_{j+2}^x$$

$$\mathscr{H}_{\text{Majorana}} = -\mathrm{i} \sum_j \left(J_j b_j a_{j+1} - h_j a_j b_j \right) - g_z \sum_j a_j b_j a_{j+1} b_{j+1} - g_x \sum_j b_j a_{j+1} b_{j+1} a_{j+2}$$

$$\mathscr{H}_{\text{Dirac}} = \sum_j \left[J_j \left(c_j^\dagger c_{j+1} + c_j^\dagger c_{j+1}^\dagger + \mathrm{h.c.} \right) + 2 h_j n_j \right] + g_z \sum_j \left(1 - 2n_j\right)\left(1 - 2n_{j+1}\right)$$
$$+ g_x \sum_j \left(c_j^\dagger - c_j\right)\left(1 - 2n_{j+1}\right)\left(c_{j+2}^\dagger + c_{j+2}\right)$$

What are the effects of interactions?



$$\mathcal{H}_{\text{Pauli}} = \sum_j \left(J_j \sigma_j^x \sigma_{j+1}^x - h_j \sigma_j^z \right) + g_z \sum_j \sigma_j^z \sigma_{j+1}^z + g_x \sum_j \sigma_j^x \sigma_{j+2}^x$$

$$\mathcal{H}_{\text{Majorana}} = -\mathrm{i} \sum_j \left(J_j b_j a_{j+1} - h_j a_j b_j \right) - g_z \sum_j a_j b_j a_{j+1} b_{j+1} - g_x \sum_j b_j a_{j+1} b_{j+1} a_{j+2}$$

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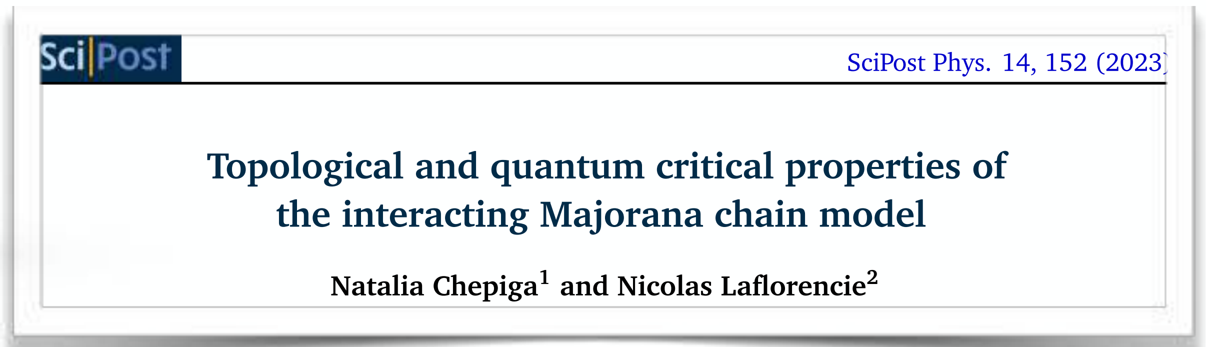
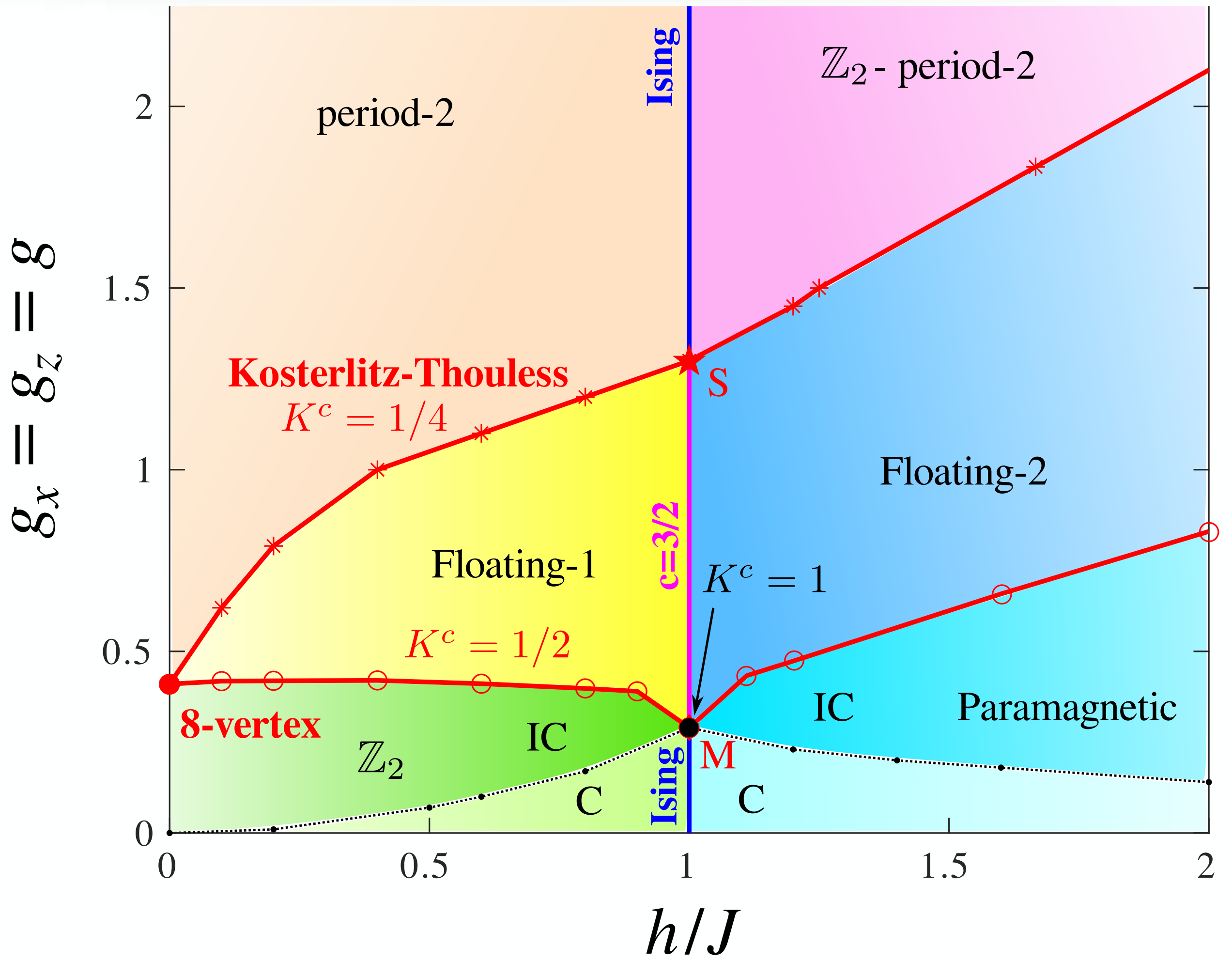

SciPost
SciPost Phys. 14, 152 (2023)

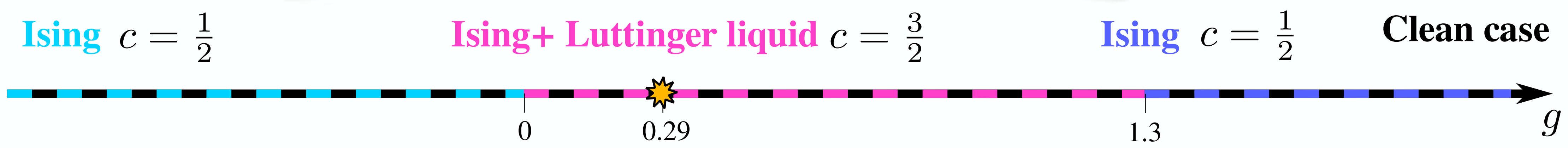
Topological and quantum critical properties of the interacting Majorana chain model

Natalia Chepiga[1] and Nicolas Laflorencie[2]

- *Ground-state of the clean interacting chain*

DMRG results





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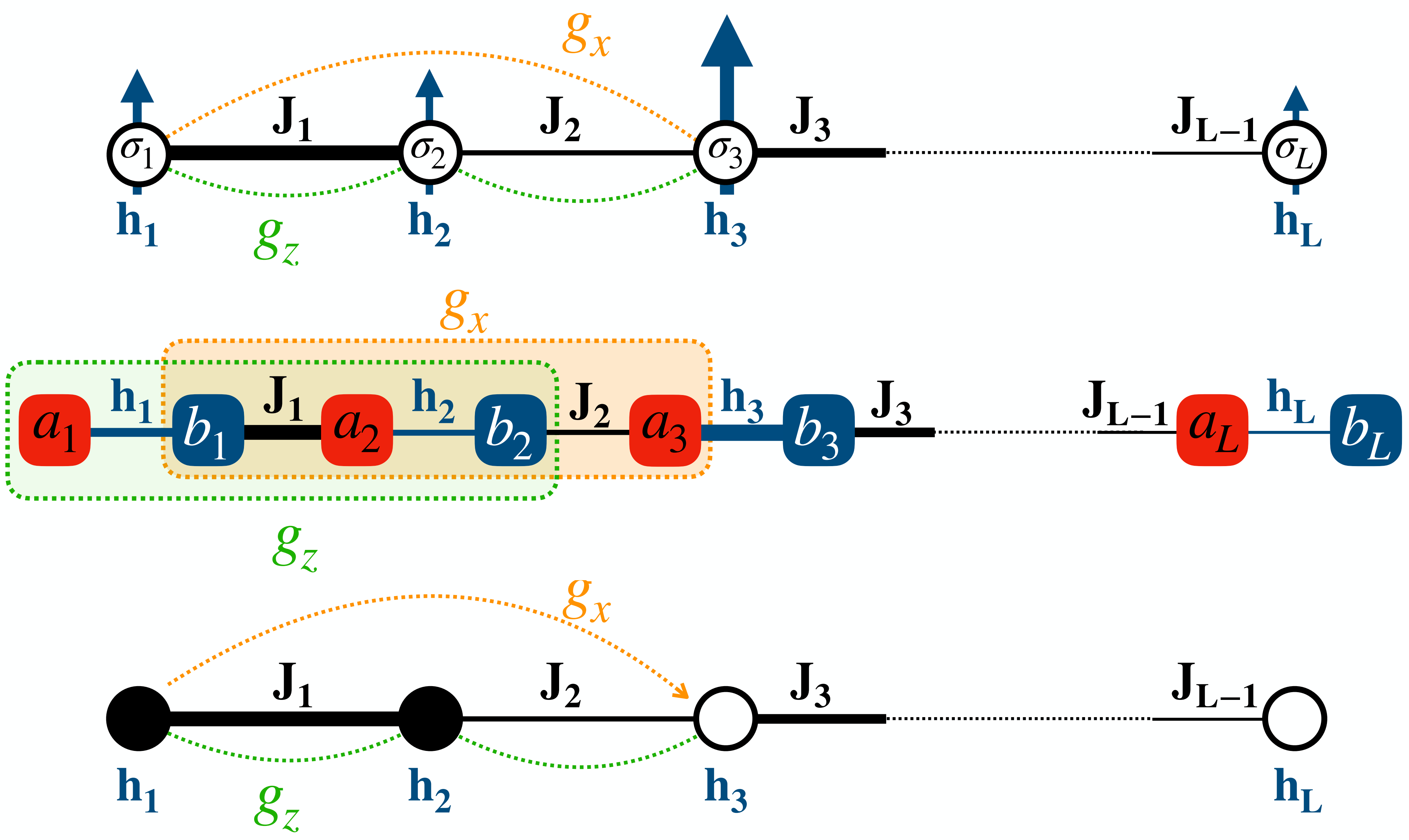

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$$\mathcal{H}_{\text{Majorana}} = -\mathrm{i}\sum_j \left(J_jb_ja_{j+1} - h_ja_jb_j\right) - g_z\sum_j a_jb_ja_{j+1}b_{j+1} - g_x\sum_j b_ja_{j+1}b_{j+1}a_{j+2}$$

$$\mathcal{H}_{\text{Dirac}} = \sum_j \left[J_j\left(c_j^\dagger c_{j+1} + c_j^\dagger c_{j+1}^\dagger + \mathrm{h.c.}\right) + 2h_jn_j\right] + g_z\sum_j\left(1-2n_j\right)\left(1-2n_{j+1}\right)$$
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‣ Ground-state of the clean interacting chain

DMRG results

SciPost Phys. 14, 152 (2023)

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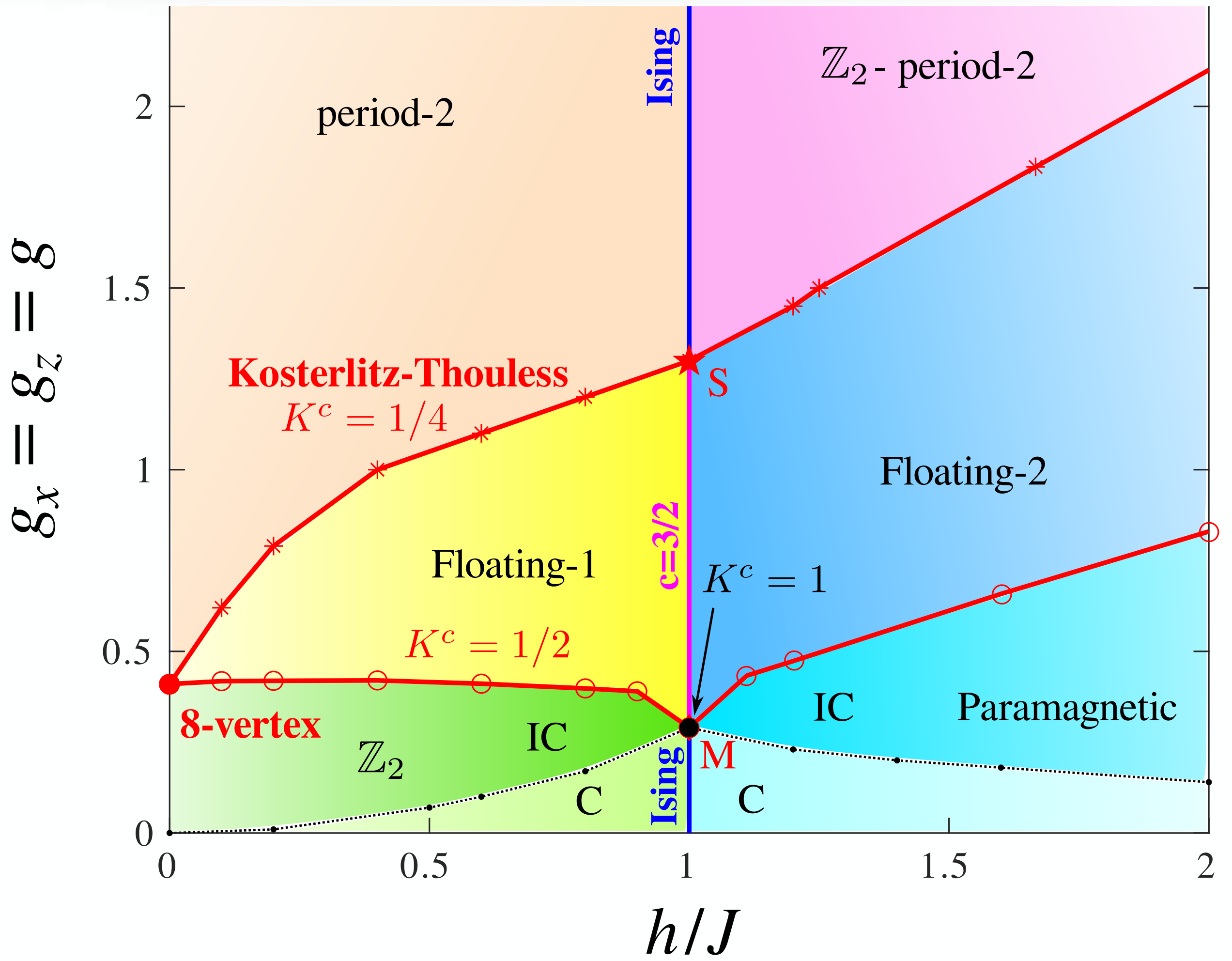
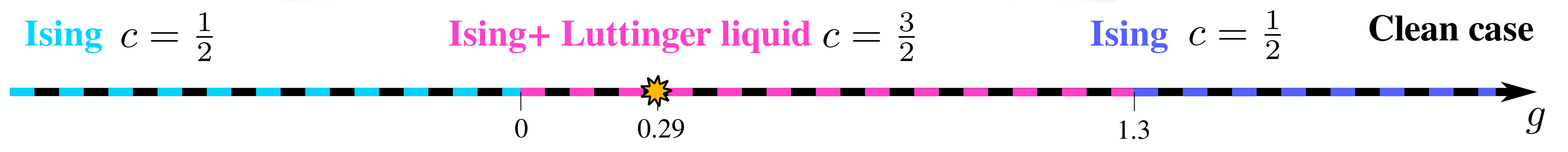


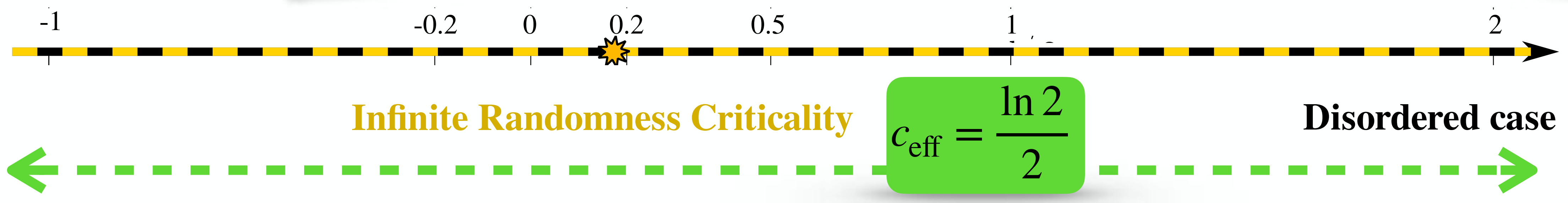


‣ Ground-state of the self-dual random model

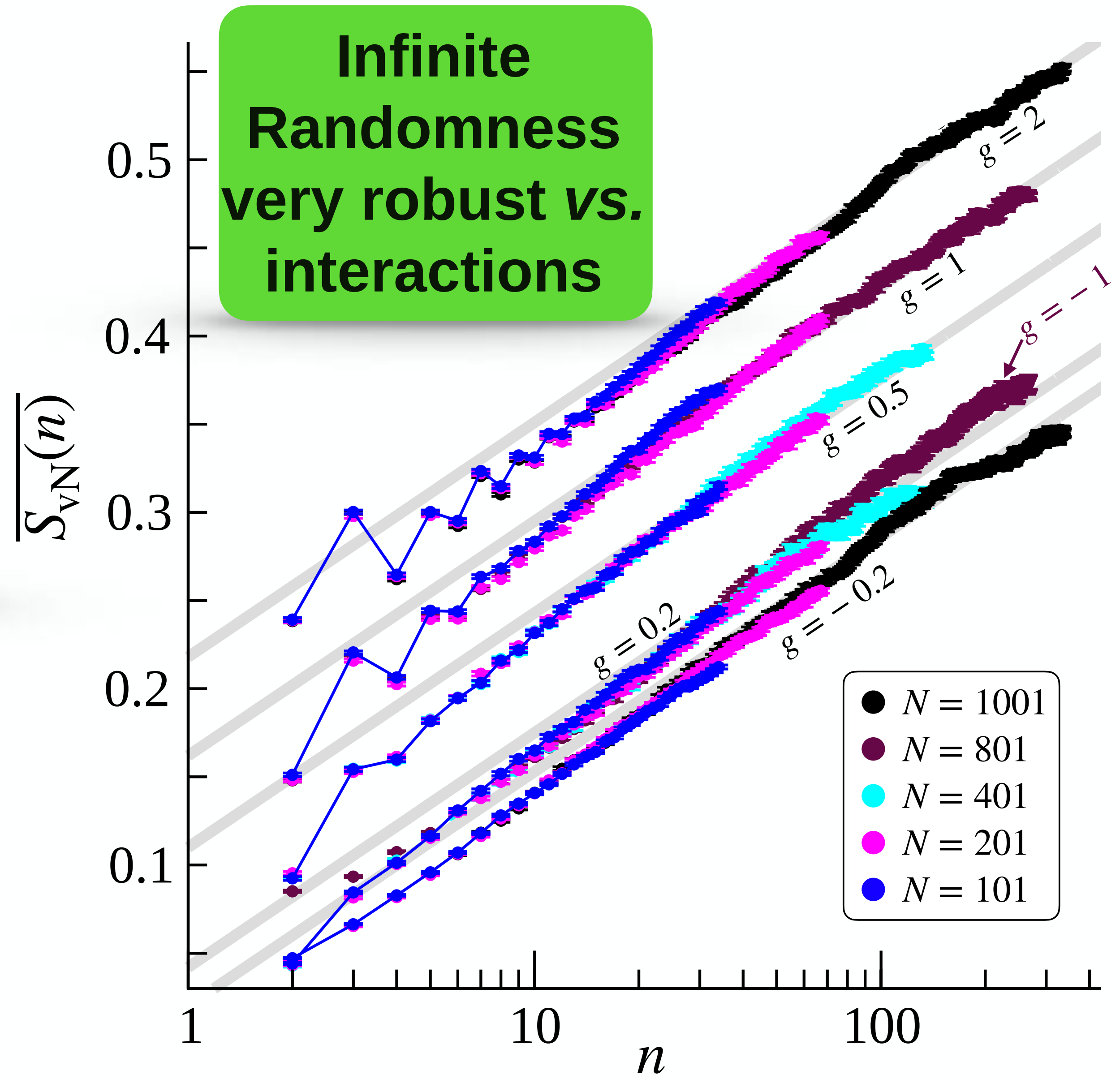
Resilient Infinite Randomness Criticality for a Disordered Chain of Interacting Majorana Fermions

Natalia Chepiga[1] and Nicolas Laflorencie[2]

arXiv:2305.10363



Infinite Randomness very robust *vs.* interactions



but this is zero temperature physics…

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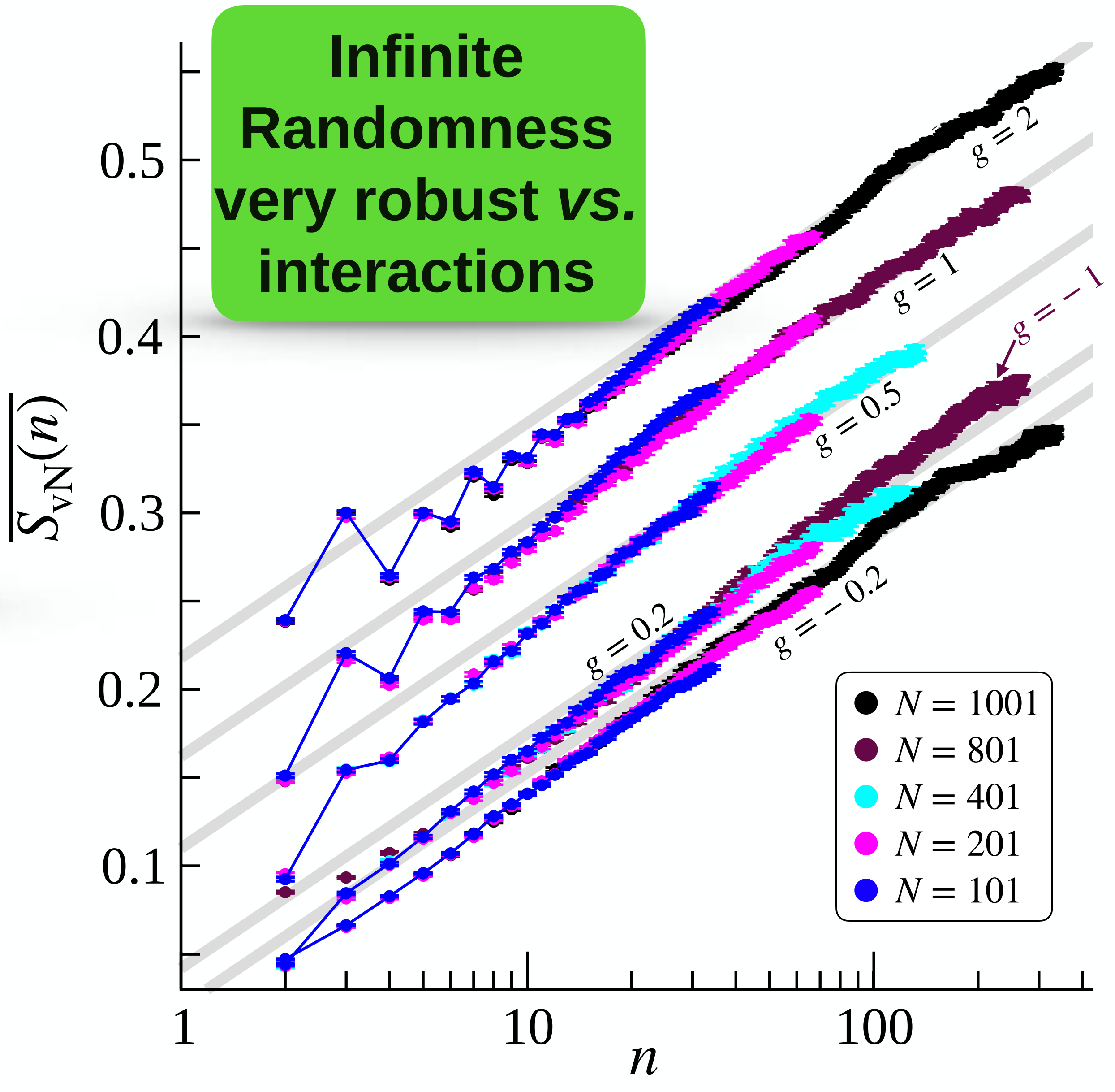

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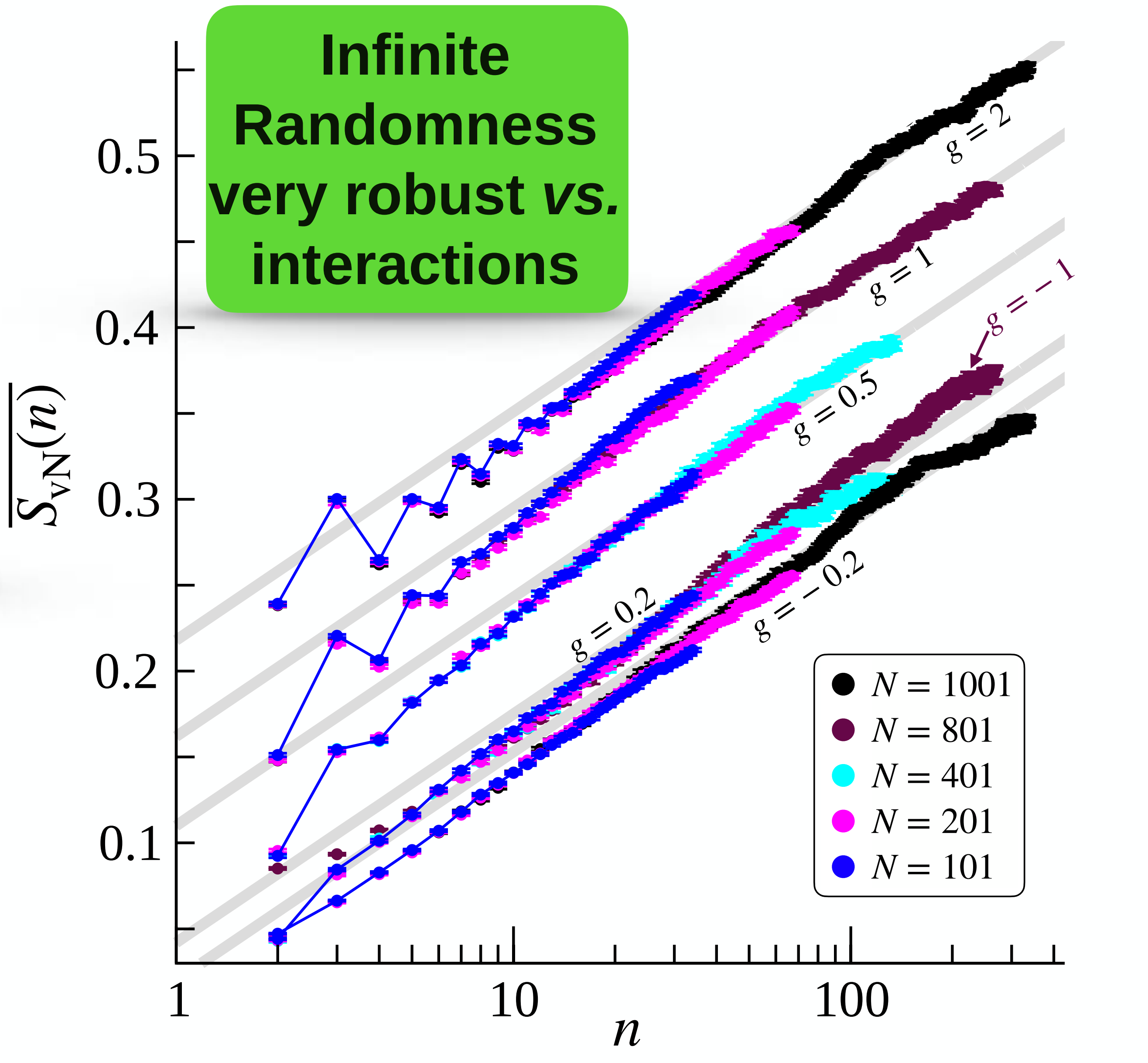
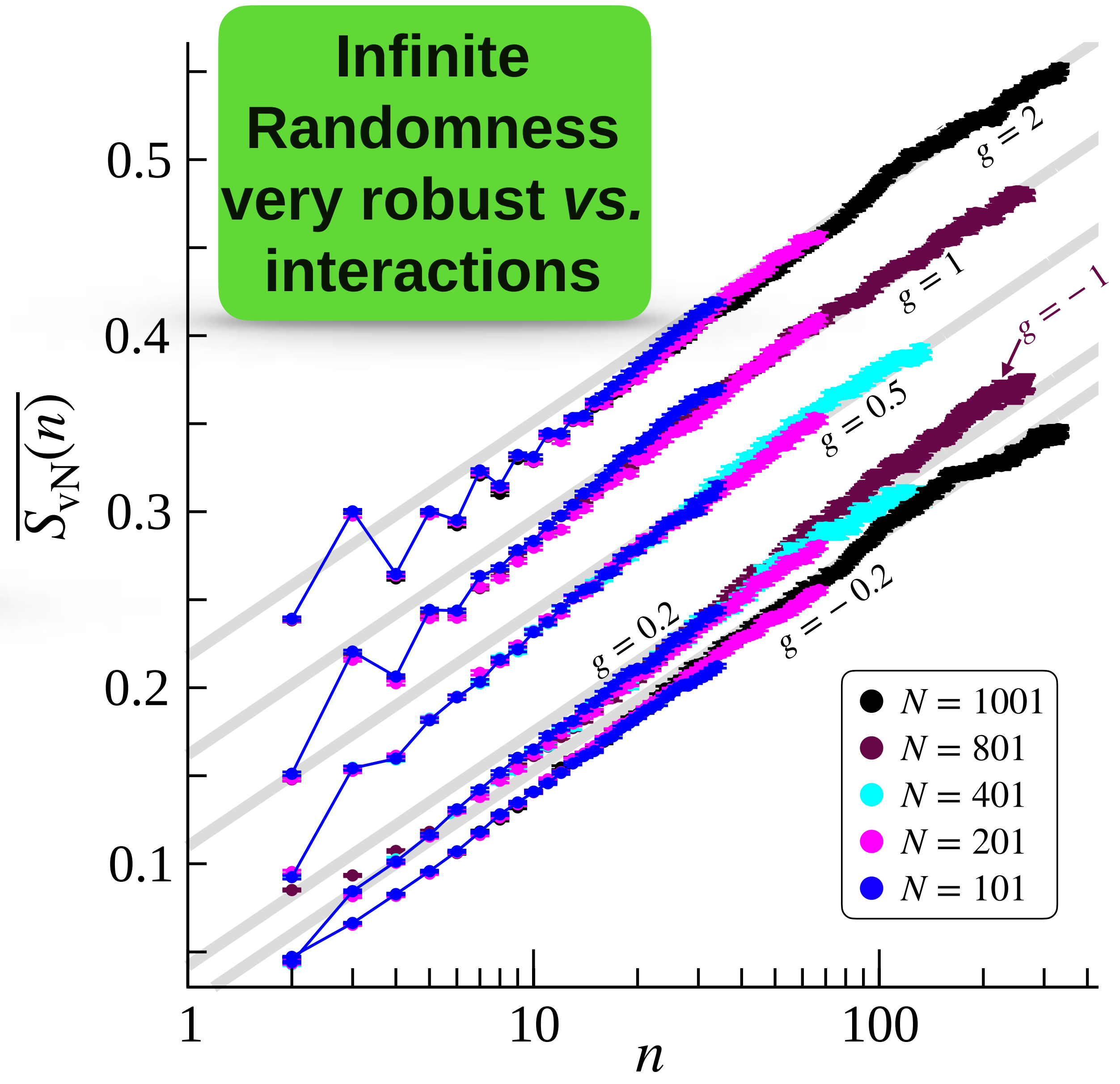


… so what happens at high energy?

$$\delta = \overline{\ln J} - \overline{\ln h}$$

but this is zero temperature physics…



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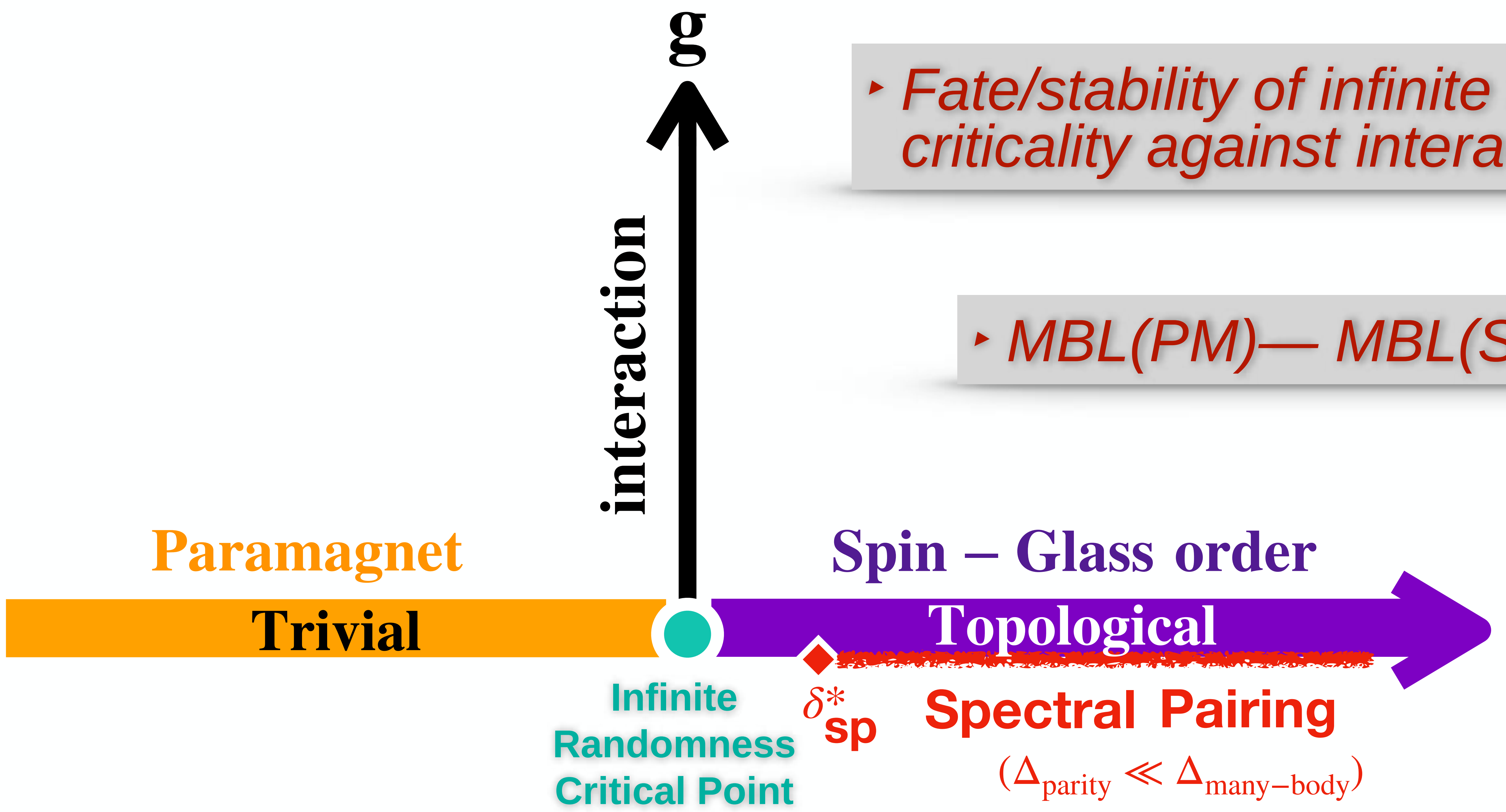
Resilient Infinite Randomness Criticality for a Disordered Chain of Interacting Majorana Fermions

Natalia Chepiga[1] and Nicolas Laflorencie[2]

arXiv:2305.10363

… so what happens at high energy?

- *Fate/stability of infinite randomness criticality against interaction?*
- *Spectral pairing (Strong Zero Mode) vs. interactions?*
- *MBL(PM)— MBL(SG) transition?*
- *Signatures of topological order?*



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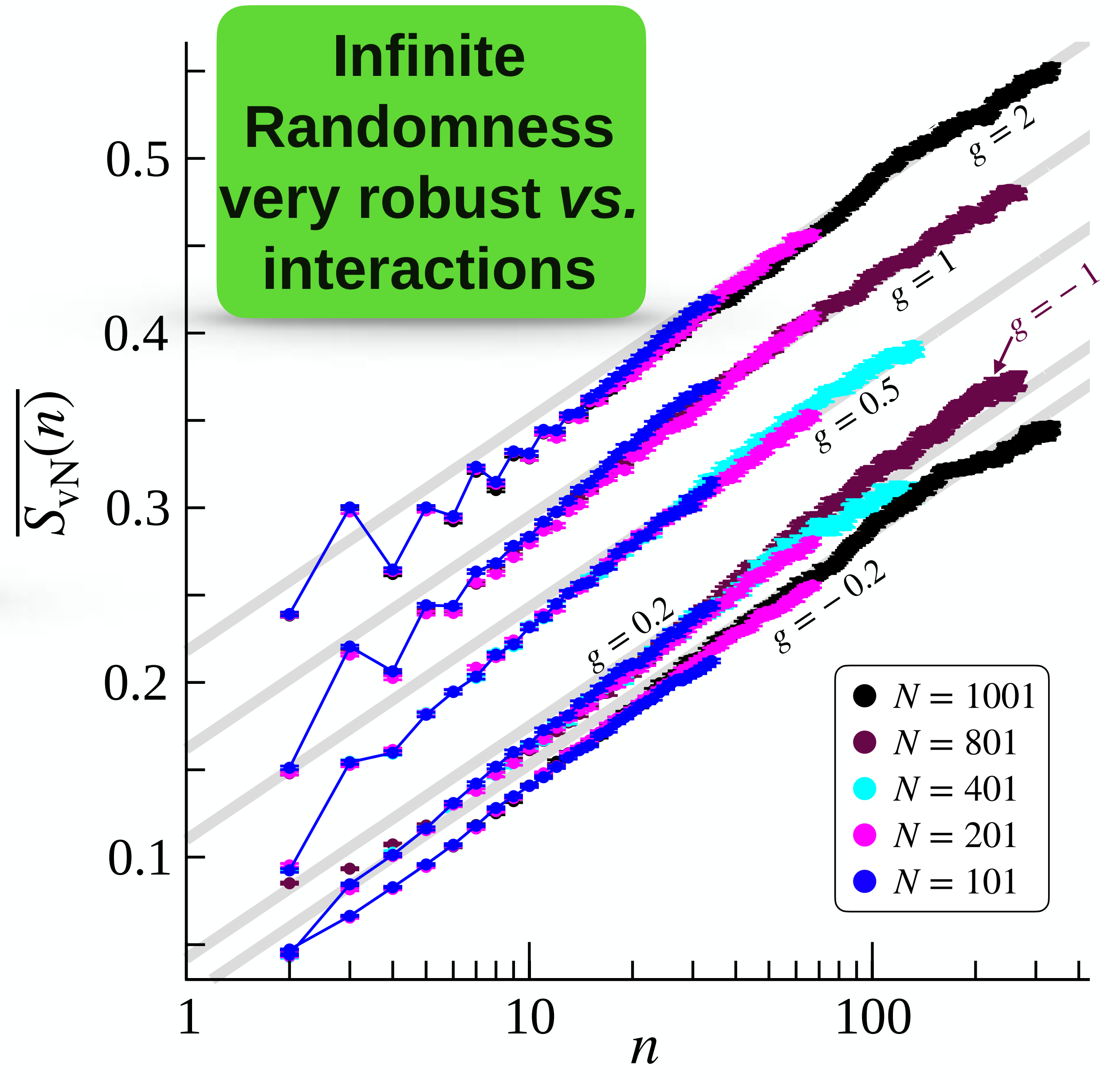
but this is zero temperature physics…



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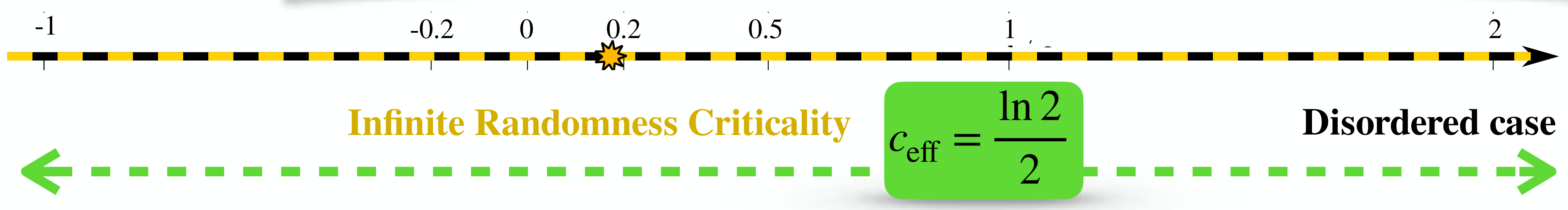

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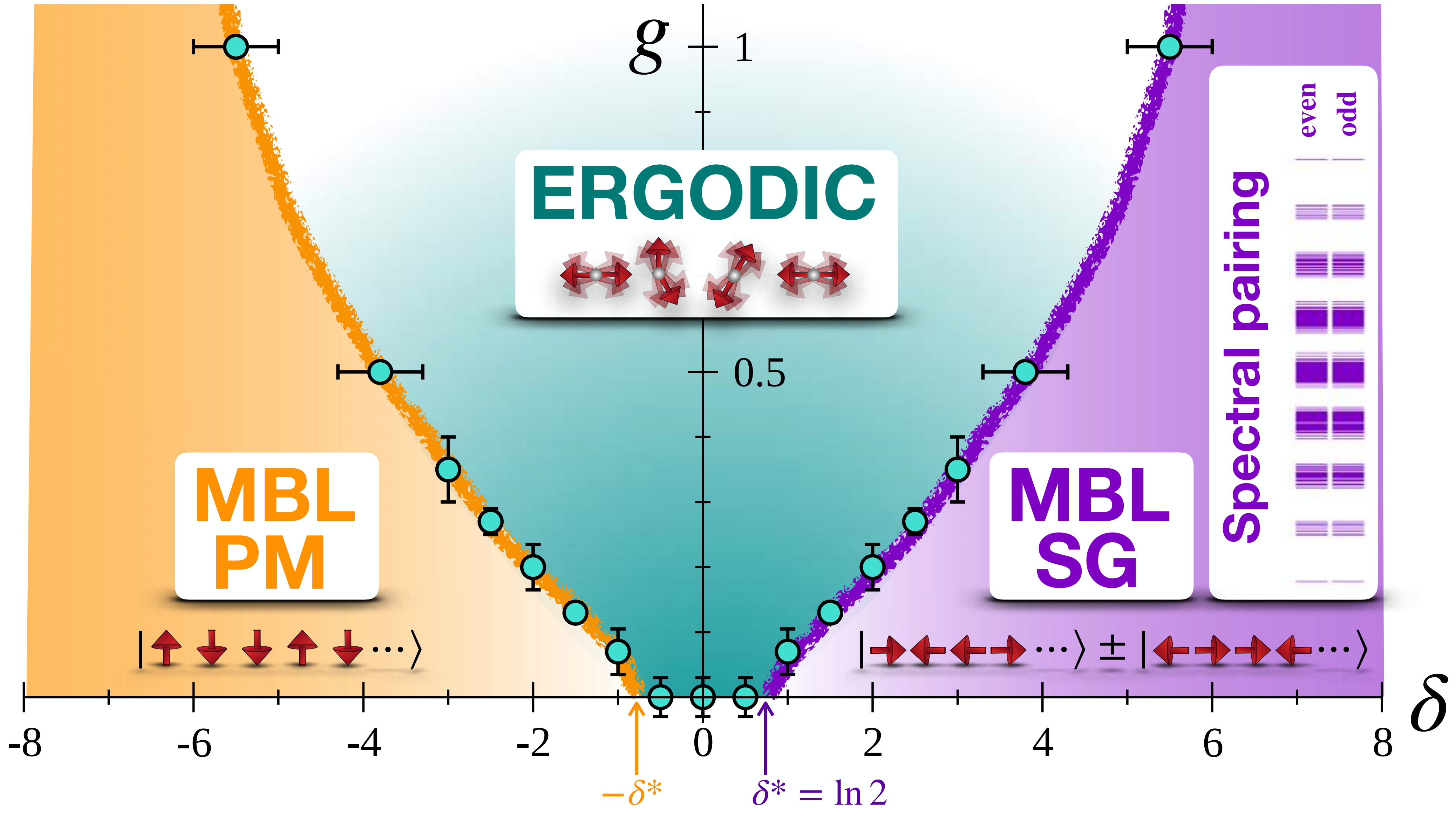


‣ *Shift-Invert Exact Diagonalization result at $T = \infty$*

PHYSICAL REVIEW RESEARCH **4**, L032016 (2022)

Letter

Topological order in random interacting Ising-Majorana chains stabilized by many-body localization

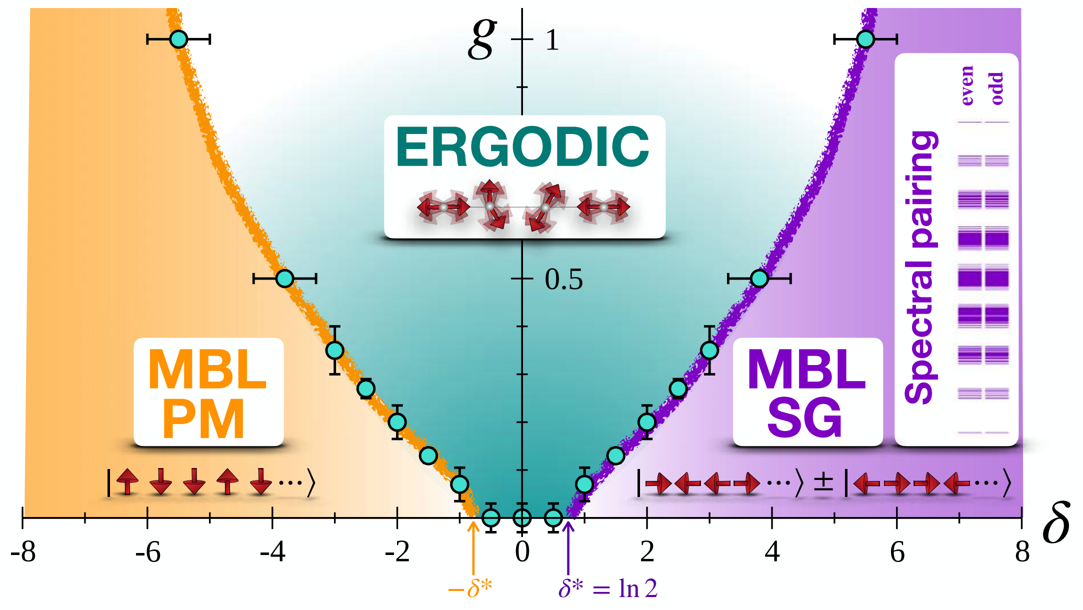
Nicolas Laflorencie,[1,2] Gabriel Lemarié,[1,3,4] and Nicolas Macé[1]

Interacting problem
Shift-Invert Exact Diagonalization

$$\mathcal{H} = -\sum_i \left(J_i \sigma_i^x \sigma_{i+1}^x + h_i \sigma_i^z\right) + g \sum_i \left(\sigma_i^x \sigma_{i+2}^x + \sigma_i^z \sigma_{i+1}^z\right)$$

- $L_{\max} = 18$

(harder than Heisenberg)



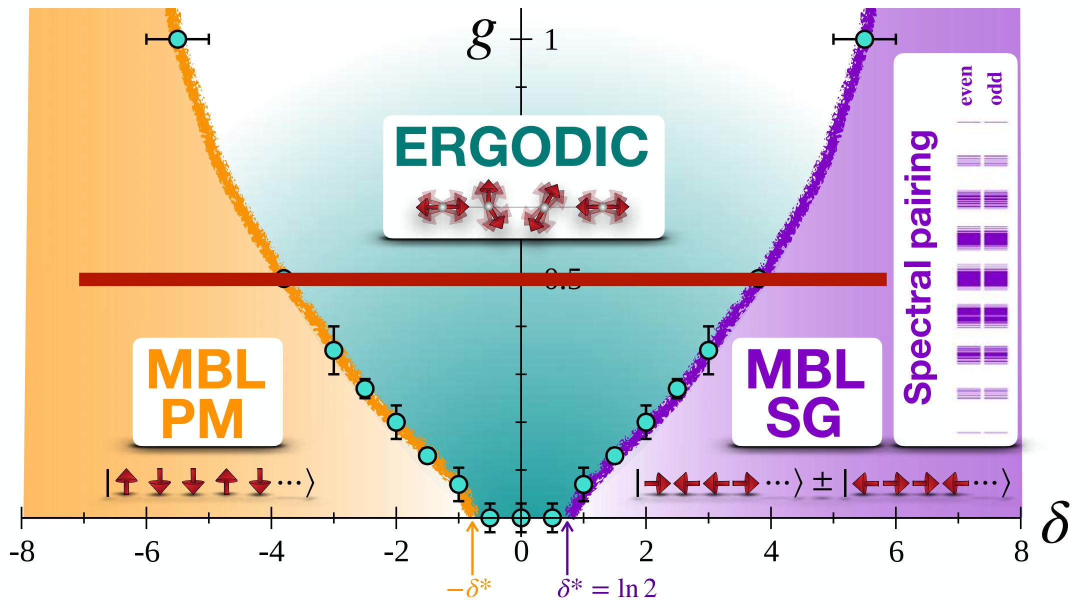
Interacting problem
Shift-Invert Exact Diagonalization

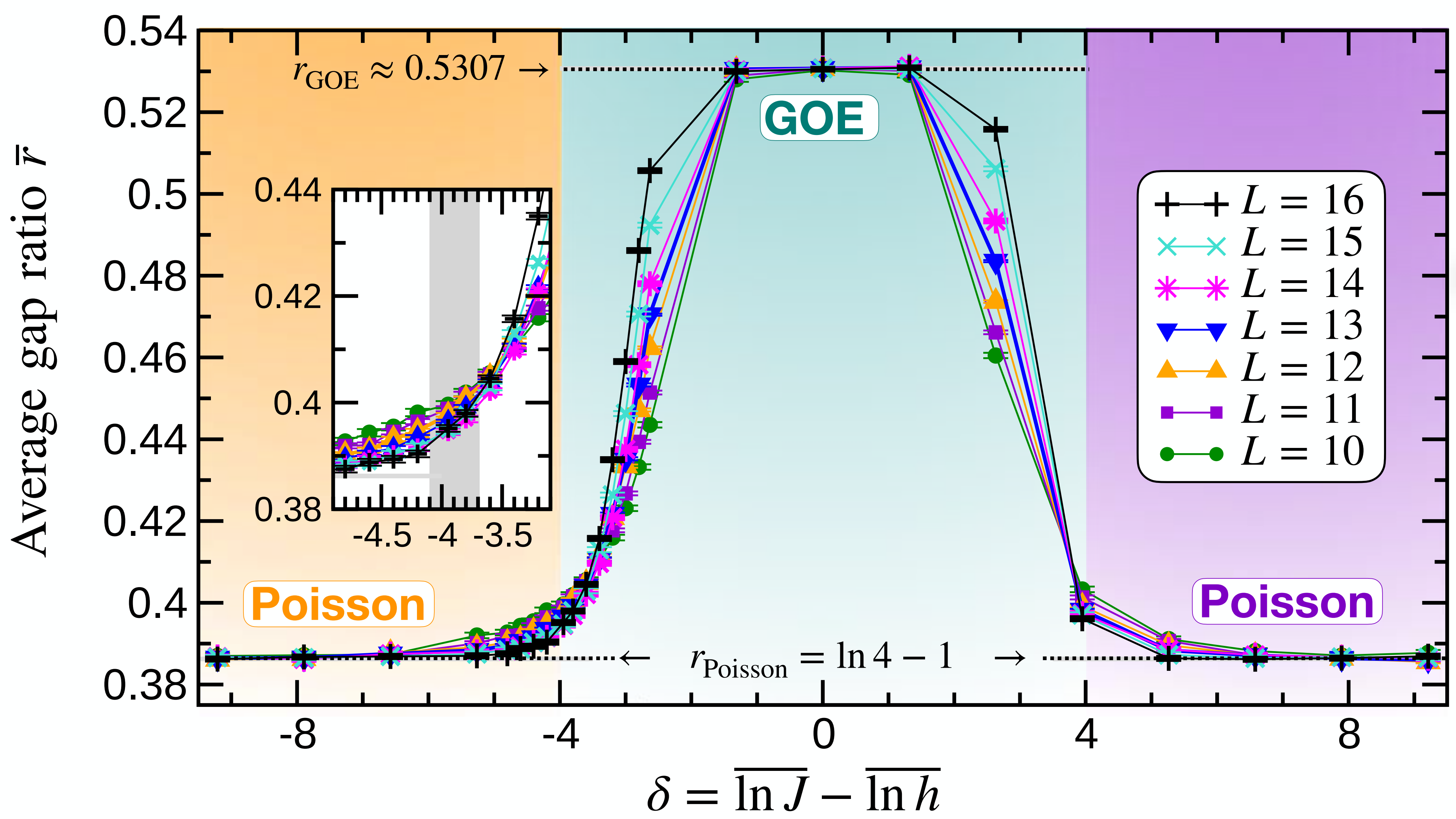
$$\mathscr{H} = -\sum_i \left(J_i\,\sigma_i^x\sigma_{i+1}^x + h_i\,\sigma_i^z\right) + g\sum_i\left(\sigma_i^x\sigma_{i+2}^x + \sigma_i^z\sigma_{i+1}^z\right)$$

- $L_{\max} = 18$
(harder than Heisenberg)

MBL PM — ERGODIC — MBL SG

g=0.5 *MBL — ergodic — MBL*





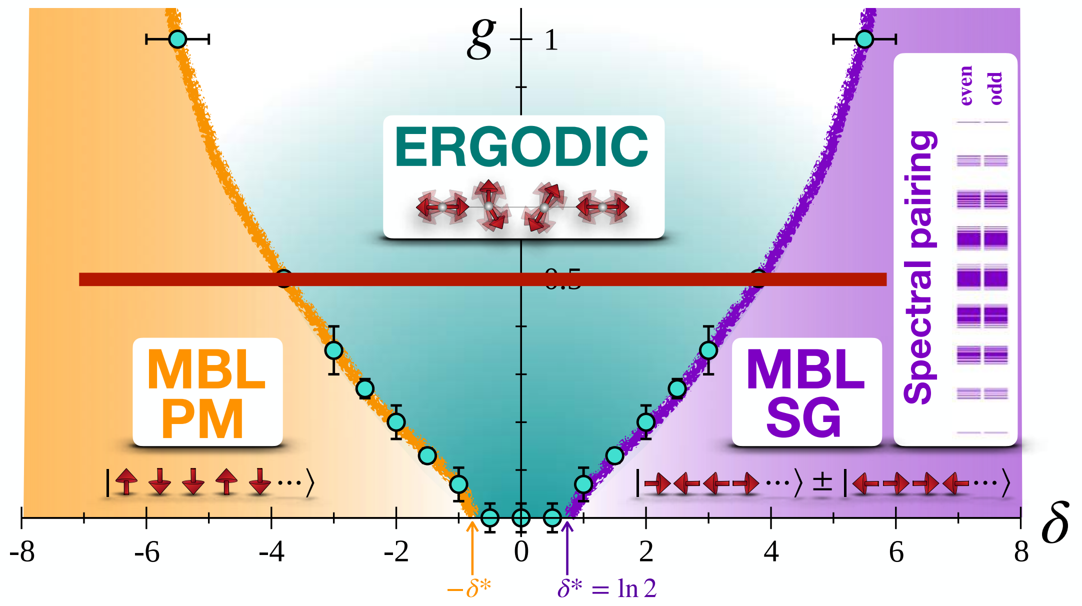
Spectral statistics

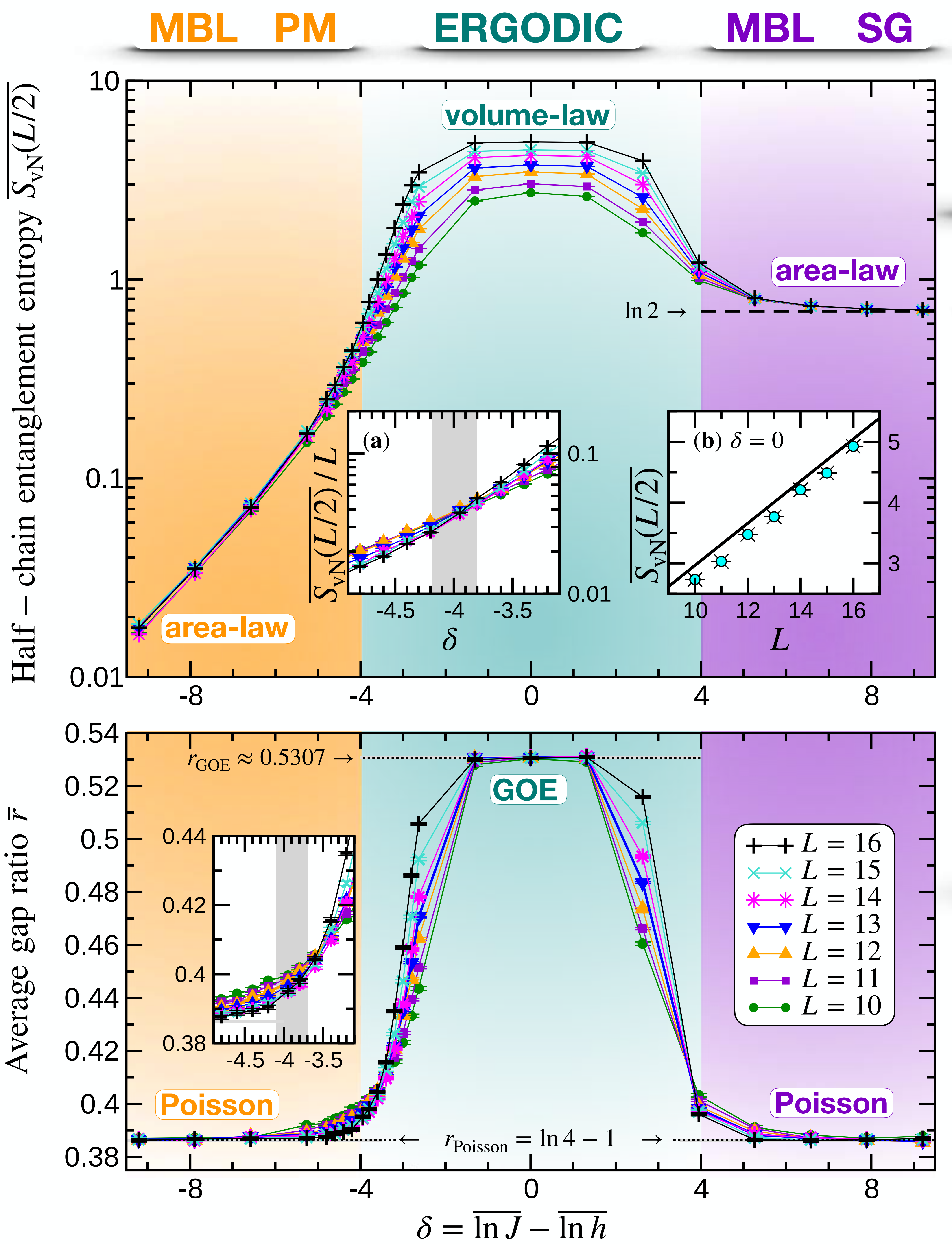
Interacting problem
Shift-Invert Exact Diagonalization

$$\mathscr{H} = -\sum_i \left(J_i\,\sigma_i^x\sigma_{i+1}^x + h_i\,\sigma_i^z\right) + g\sum_i \left(\sigma_i^x\sigma_{i+2}^x + \sigma_i^z\sigma_{i+1}^z\right)$$

- $L_{\max} = 18$ *(harder than Heisenberg)*

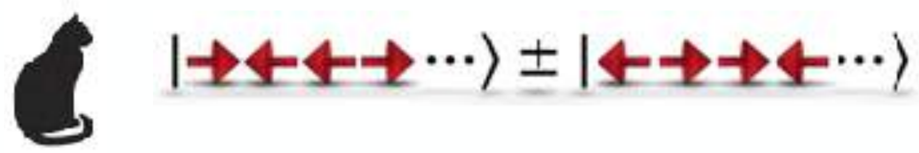
***g=0.5* MBL — ergodic — MBL**





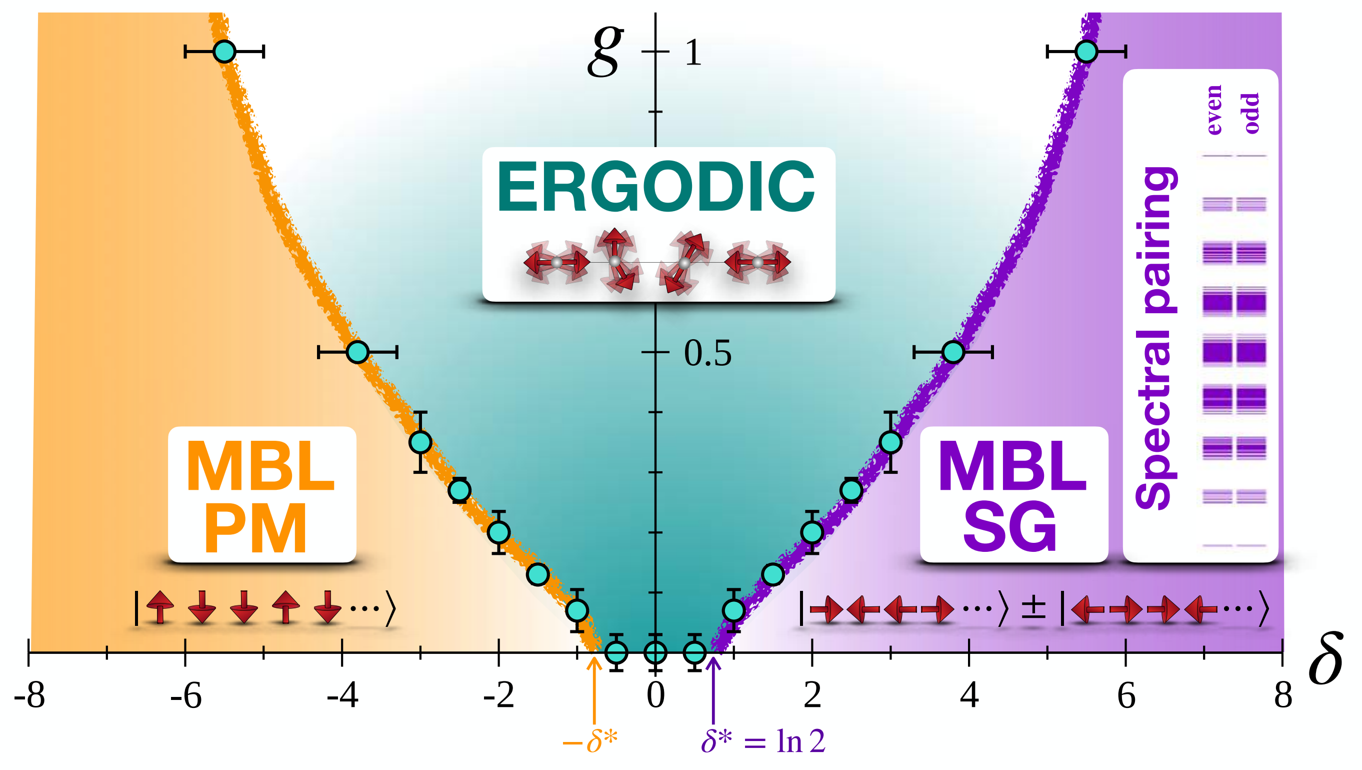
Entanglement Entropy

→ ln 2

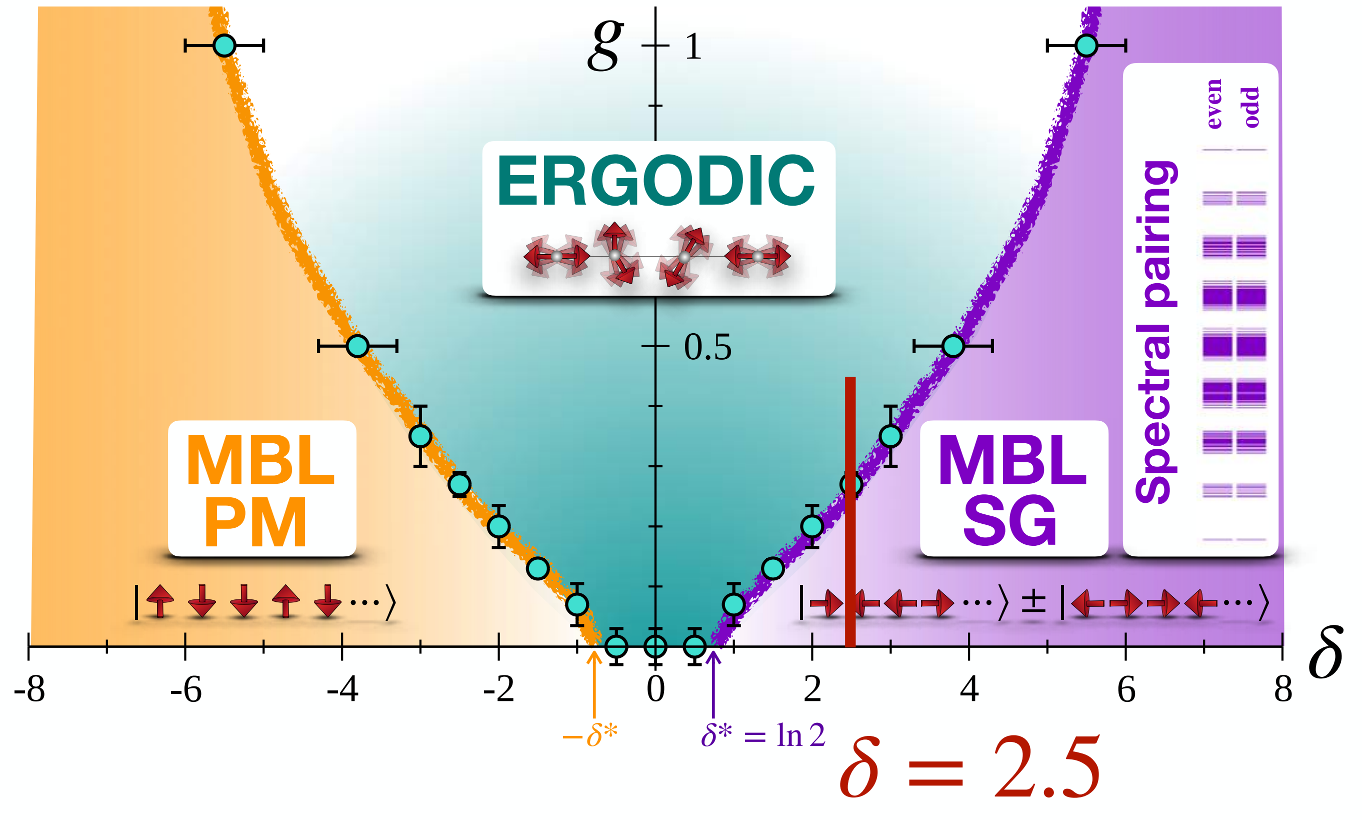


Spectral statistics

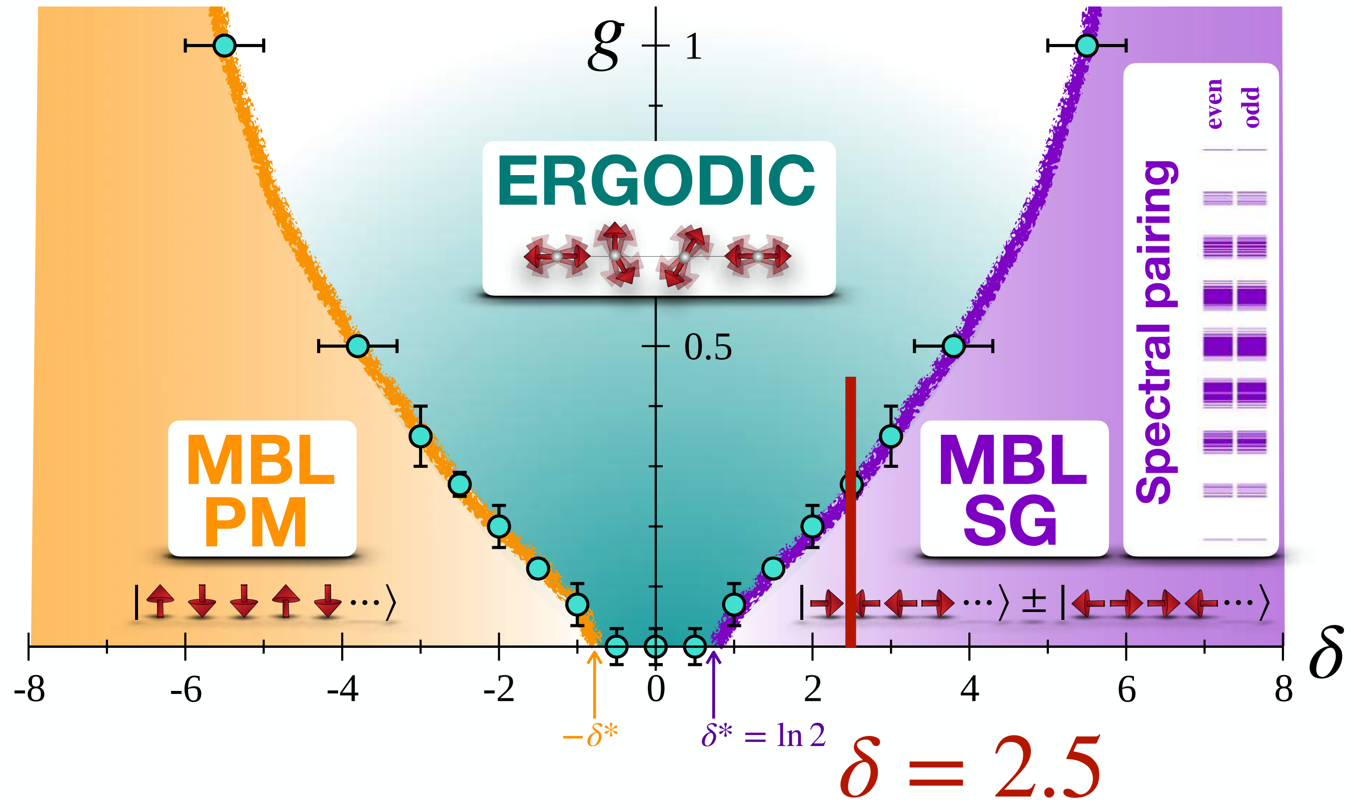
Spectroscopy of the MBL-SG



Spectroscopy of the MBL-SG

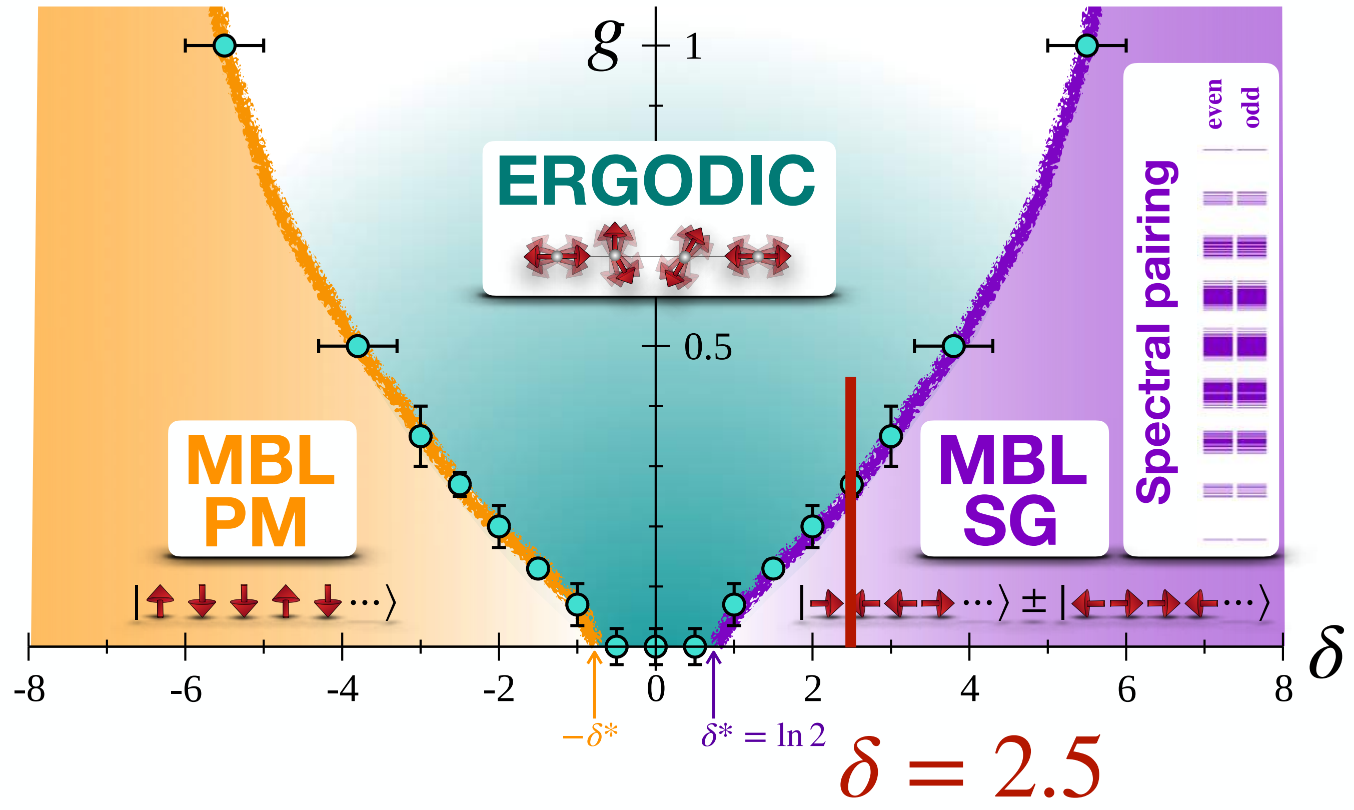


Spectroscopy of the MBL-SG



	Ergodic	MBL	MBL + paired spectrum
$\overline{r}_{\text{resolved}}$	0.5307 (GOE)	ln 4 − 1 (Poisson)	

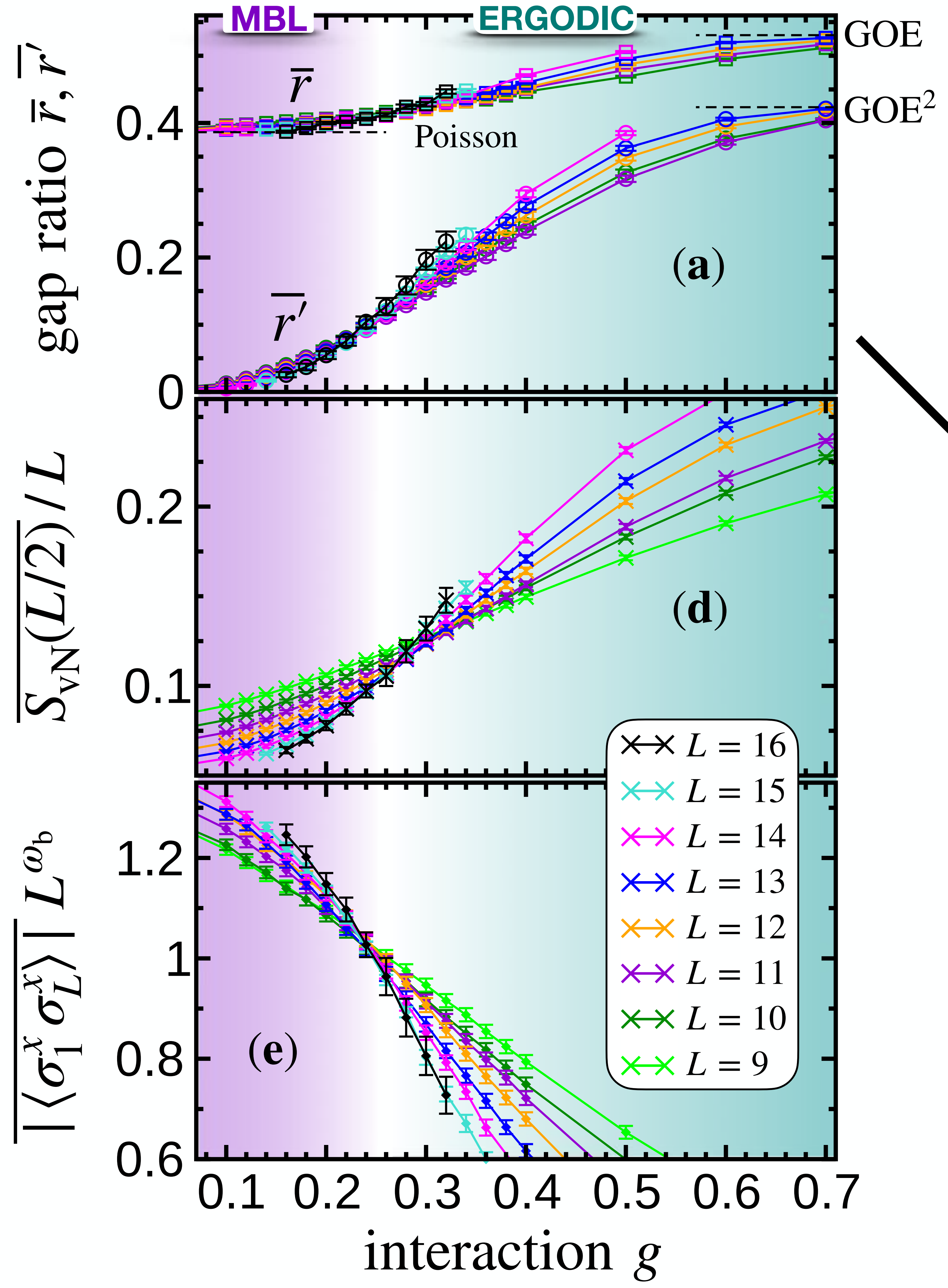
Spectroscopy of the MBL-SG

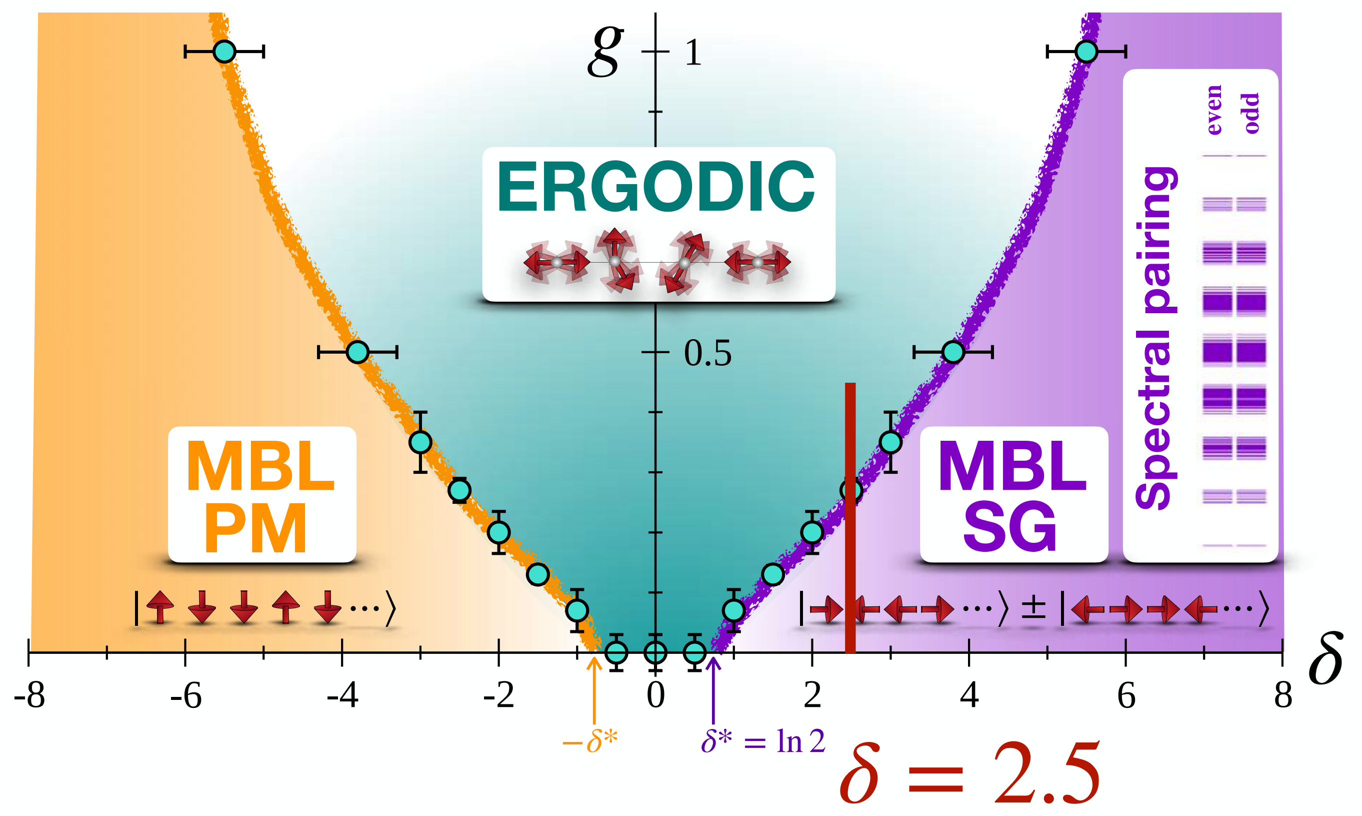


	Ergodic	MBL	MBL + paired spectrum
$\overline{r'}_{\text{mixed}}$	0.4234 (GOE^2)	$\ln 4 - 1$	$e^{L(s-\xi^{-1})} \to 0$
$\overline{r}_{\text{resolved}}$	0.5307 (GOE)	$\ln 4 - 1$ (Poisson)	

O. Giraud, N. Macé, E. Vernier, and F. Alet, Phys. Rev. X **12**, 011006 (2022)

Spectroscopy of the MBL-SG





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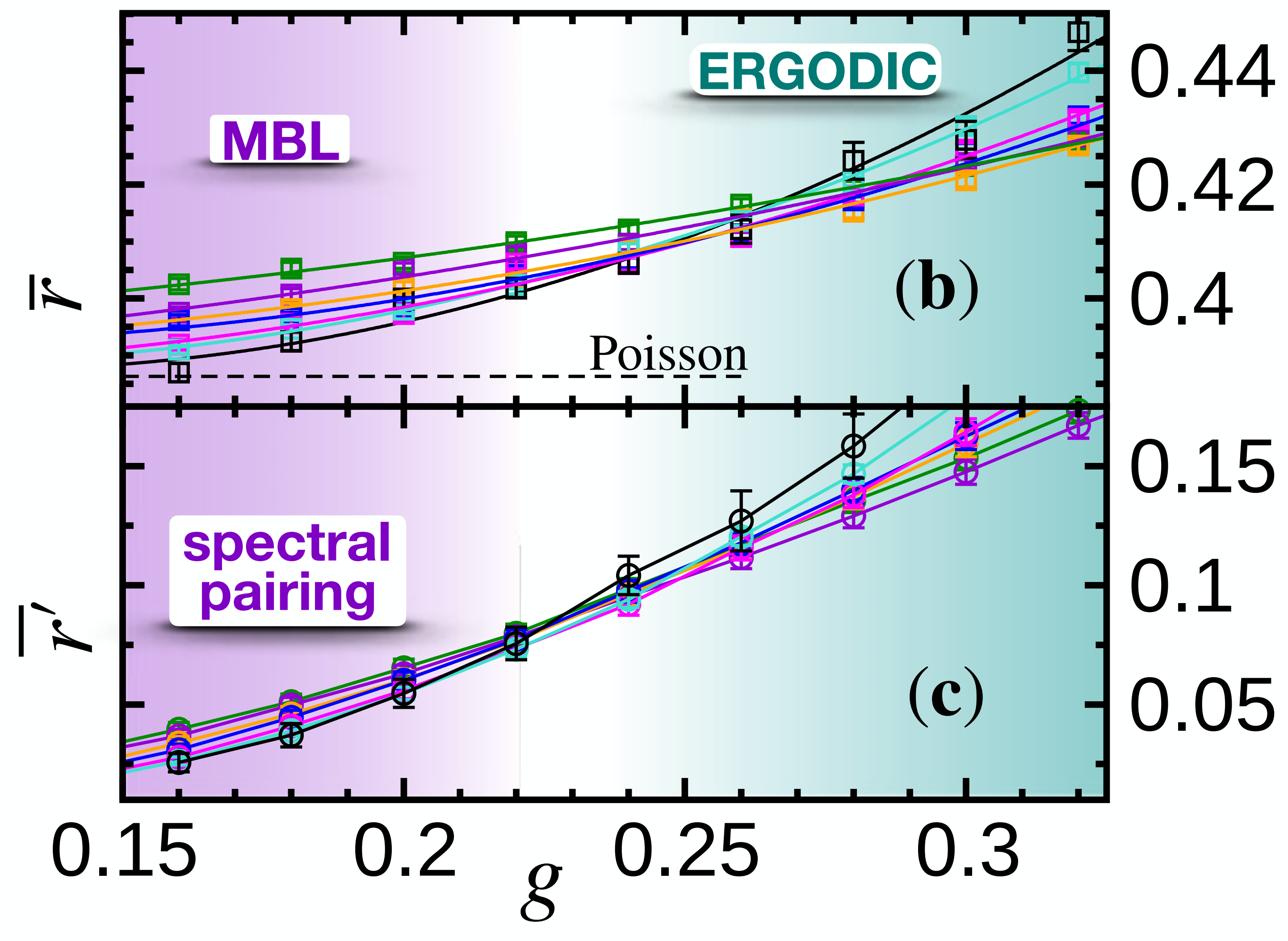
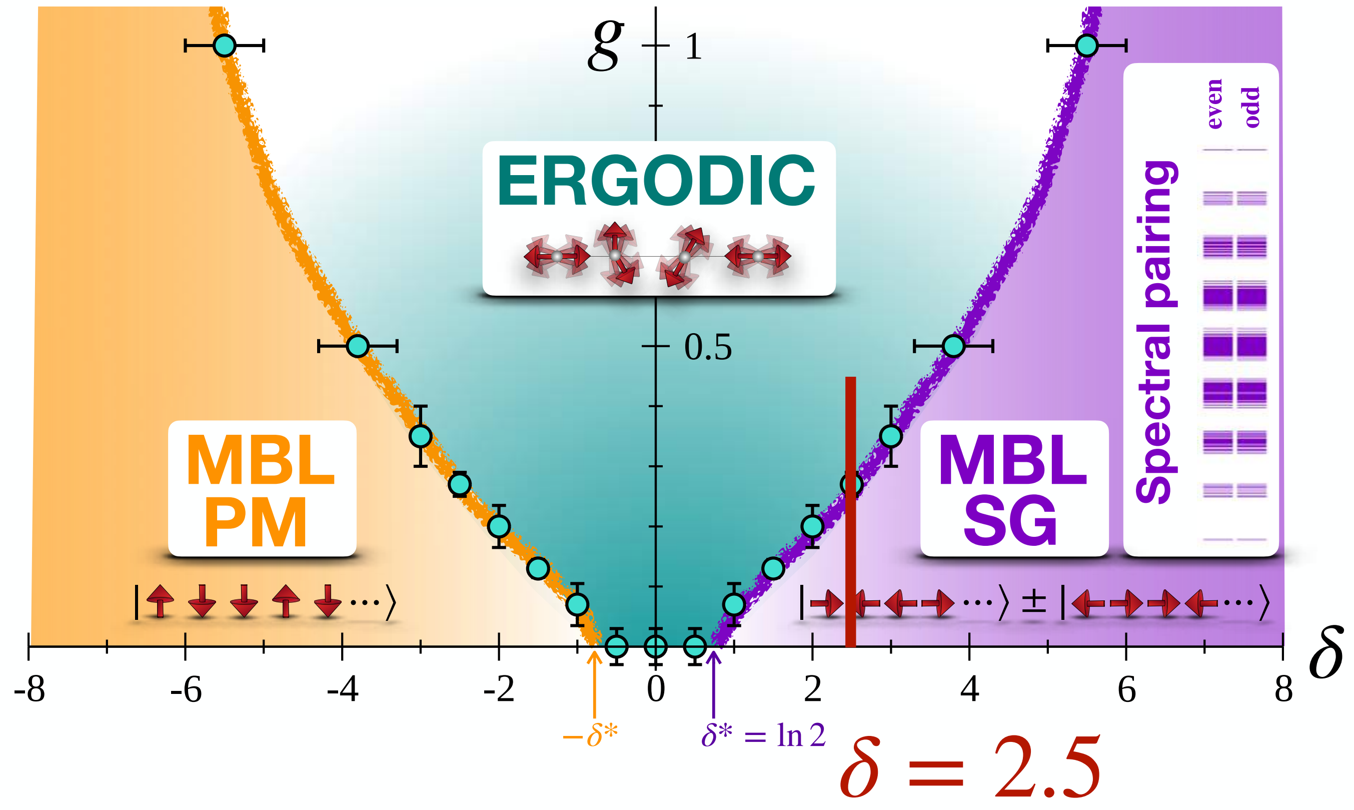
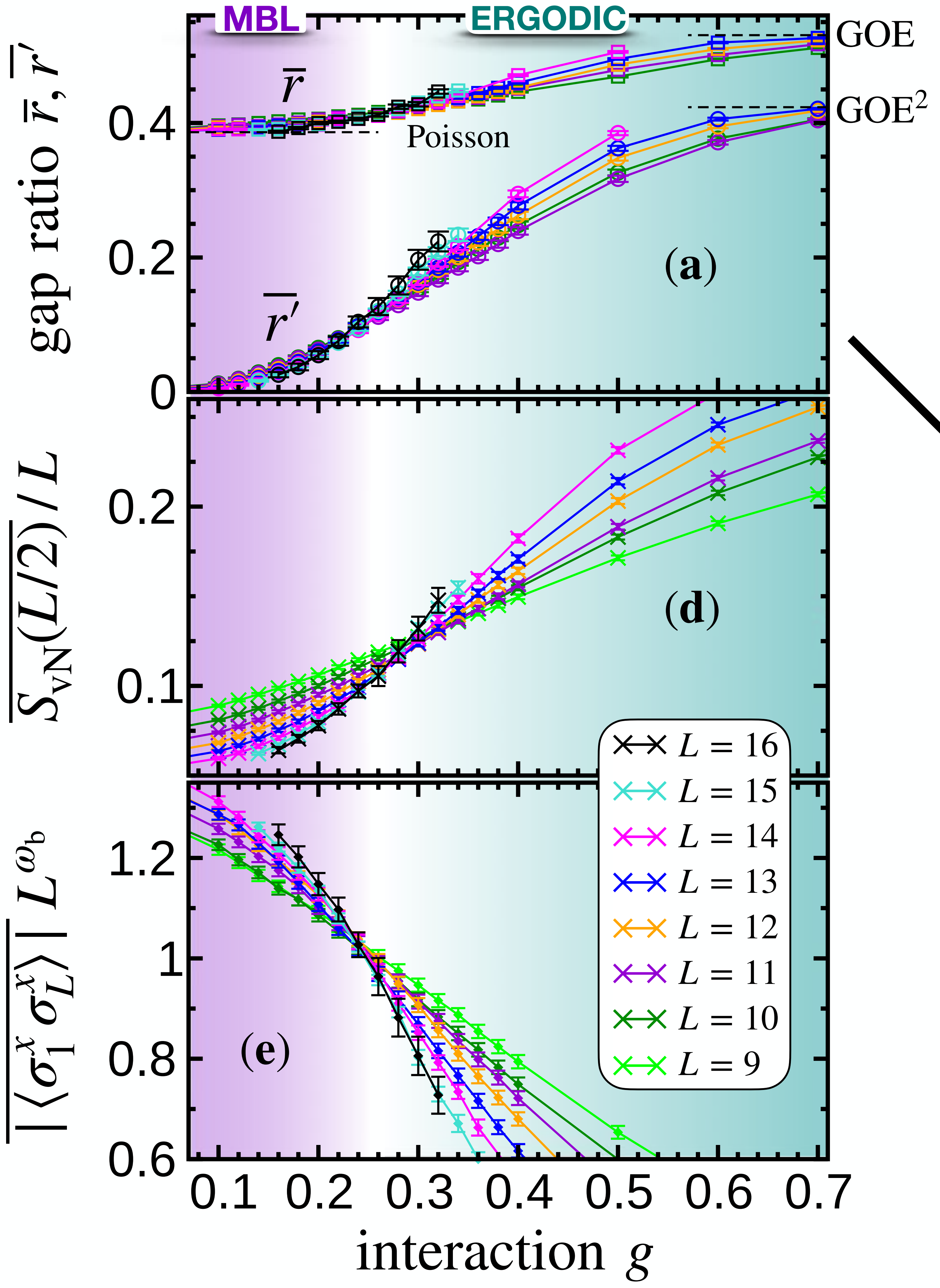


Spectroscopy of the MBL-SG





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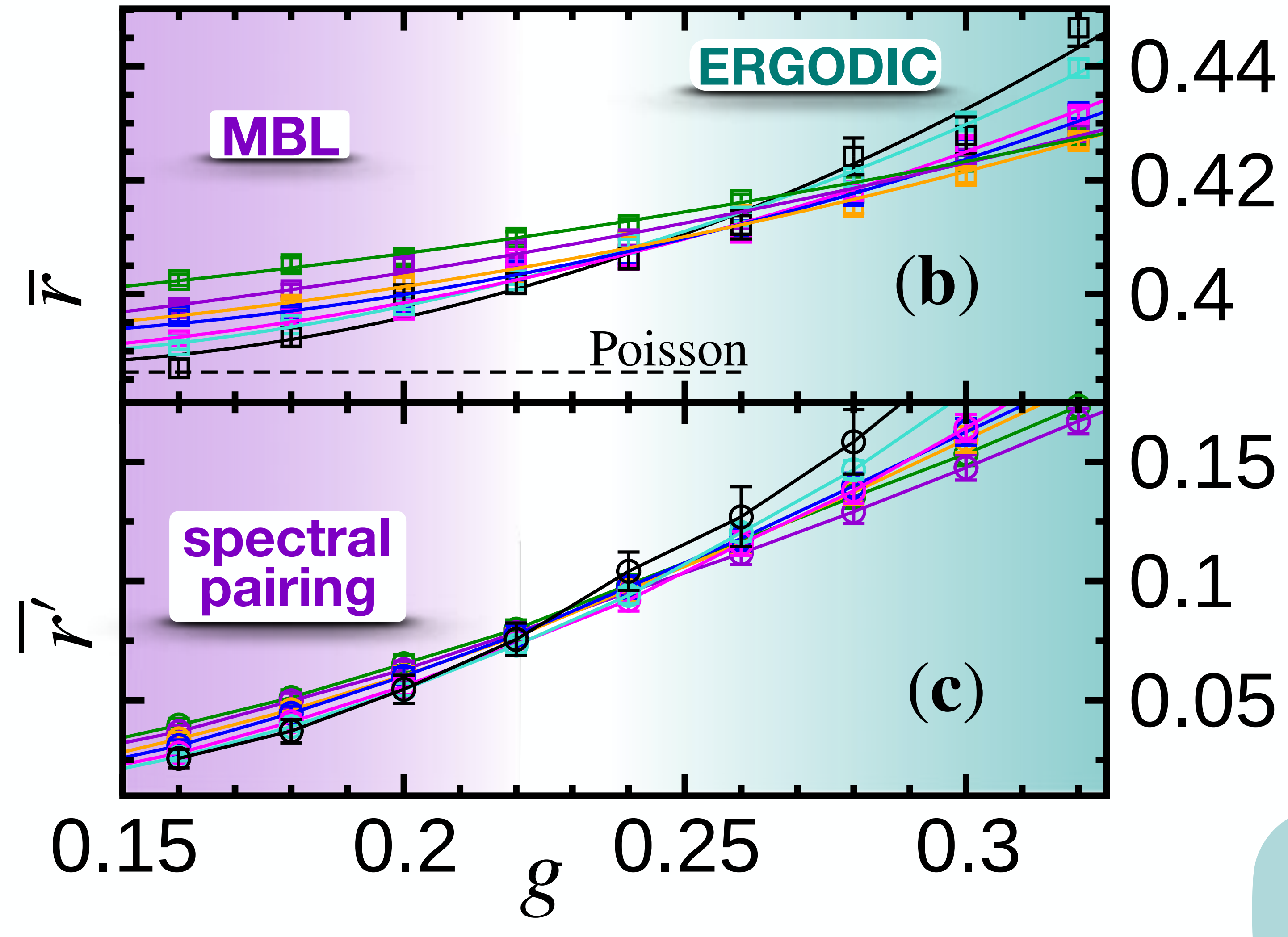
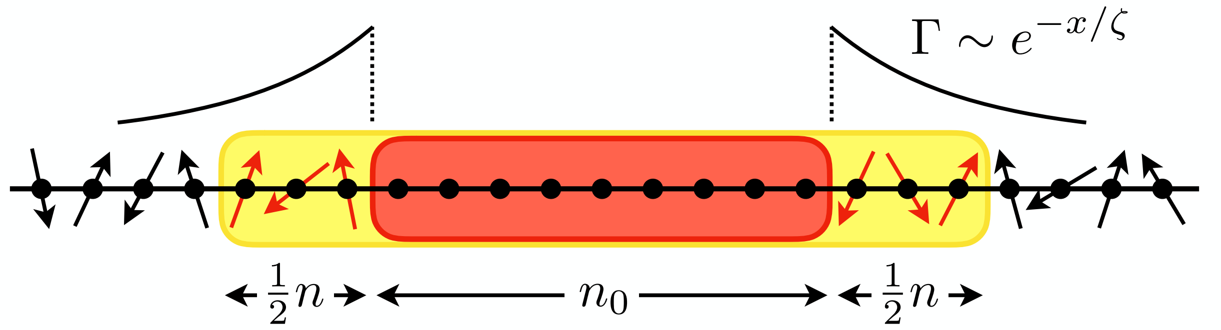


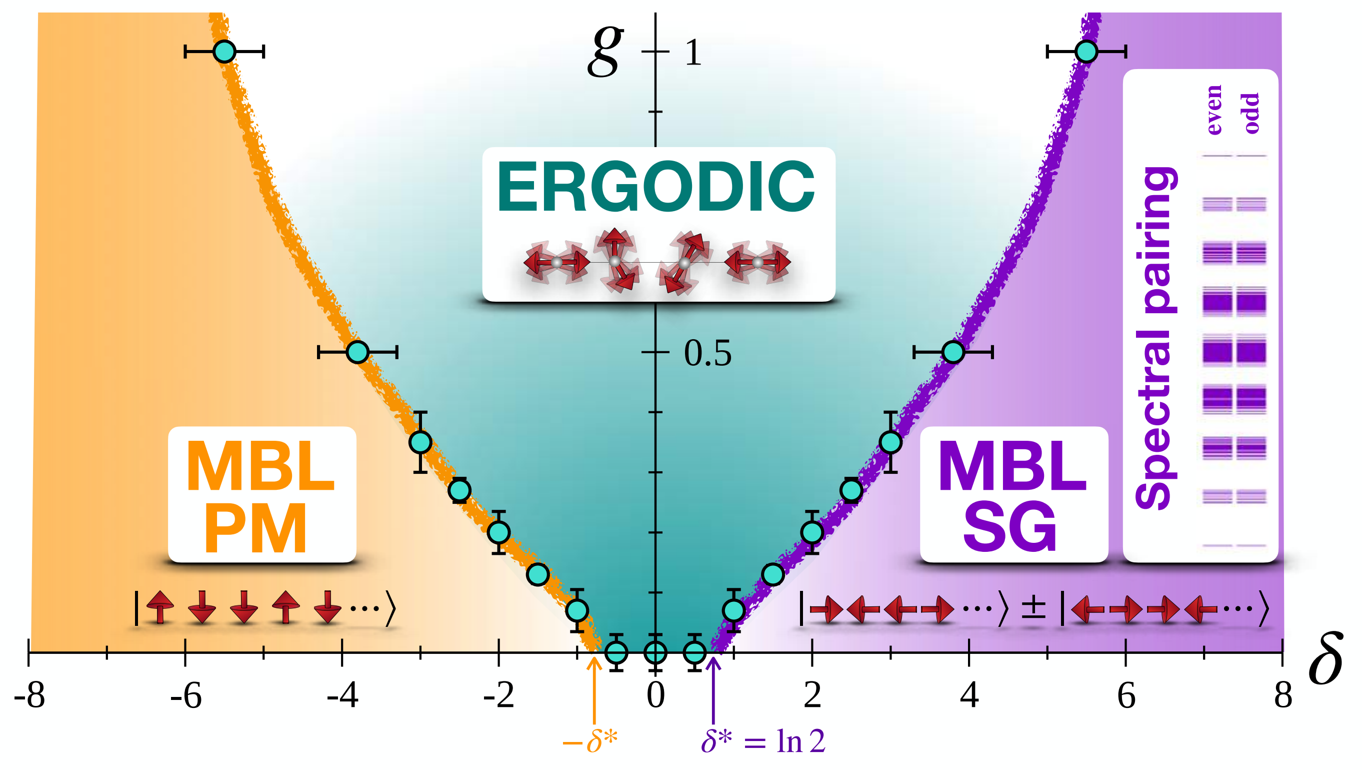
MBL-SG always associated with **spectral pairing**

$$\Delta_{\mathrm{parity}} \ll 2^{-L} \Rightarrow \xi_{\mathrm{typ}} < \frac{1}{\ln 2}$$

ERGODIC transition meets the **Avalanche criterion** $\xi^*_{\mathrm{typ}} = \frac{1}{\ln 2}$



Consequences for the weakly interacting regime

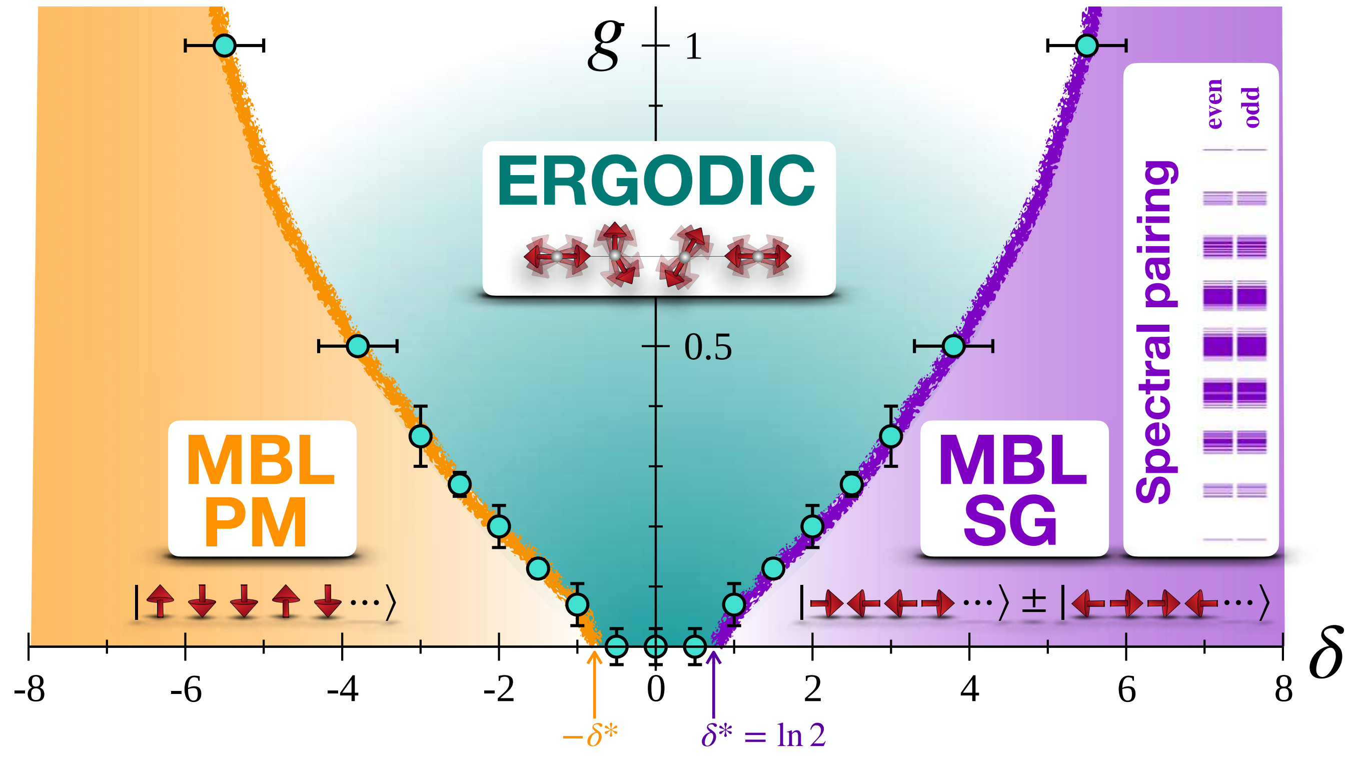


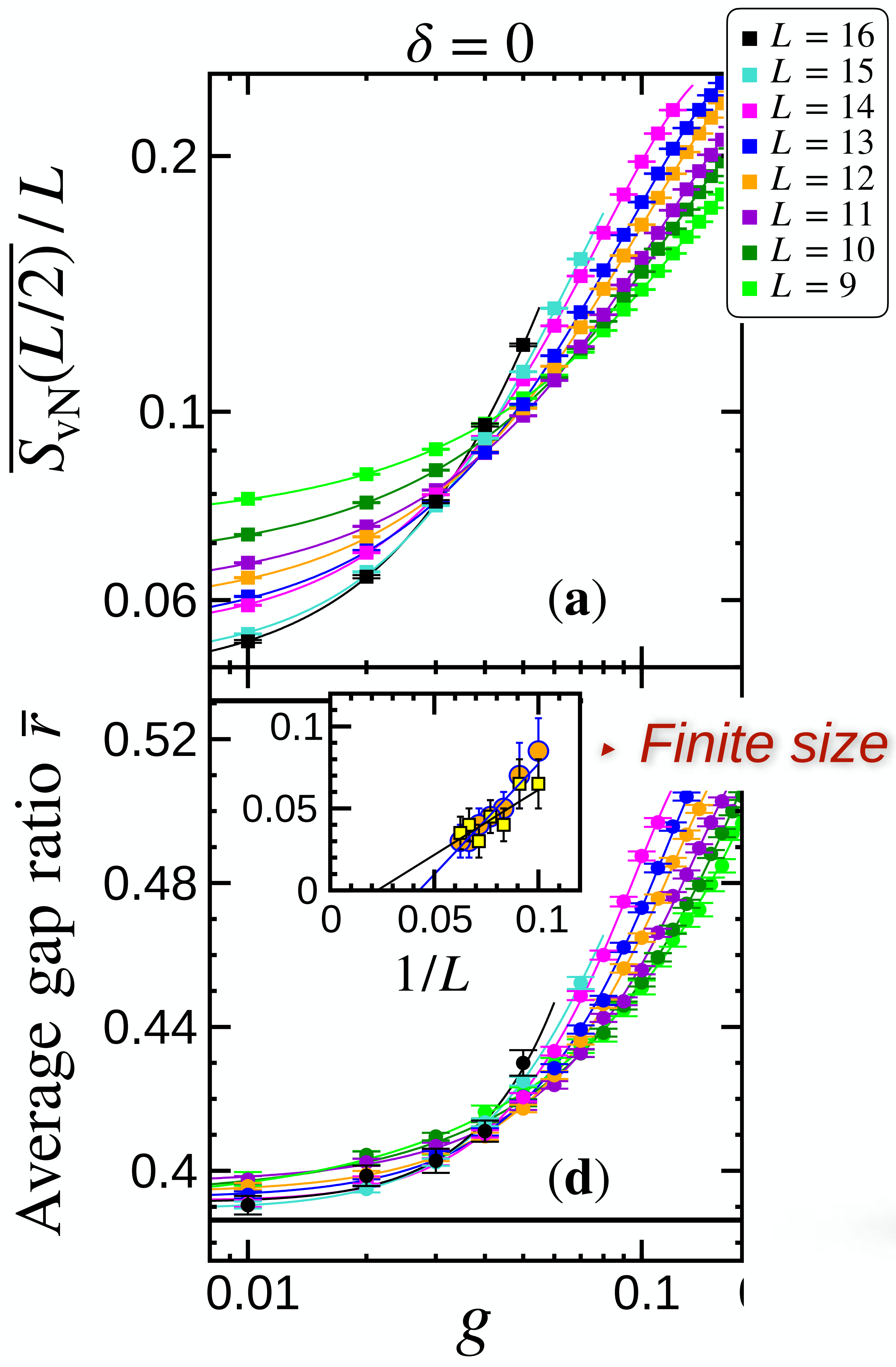
$$\xi_{g=0} = \frac{1}{|\delta|}$$

$$\text{If} \quad \xi_{g=0} > \frac{1}{\ln 2} \quad \Rightarrow \text{Interaction-driven } \underline{\text{ergodic instability}}$$

$(|\delta| < \delta^*_{\rm sp})$

Consequences for the weakly interacting regime

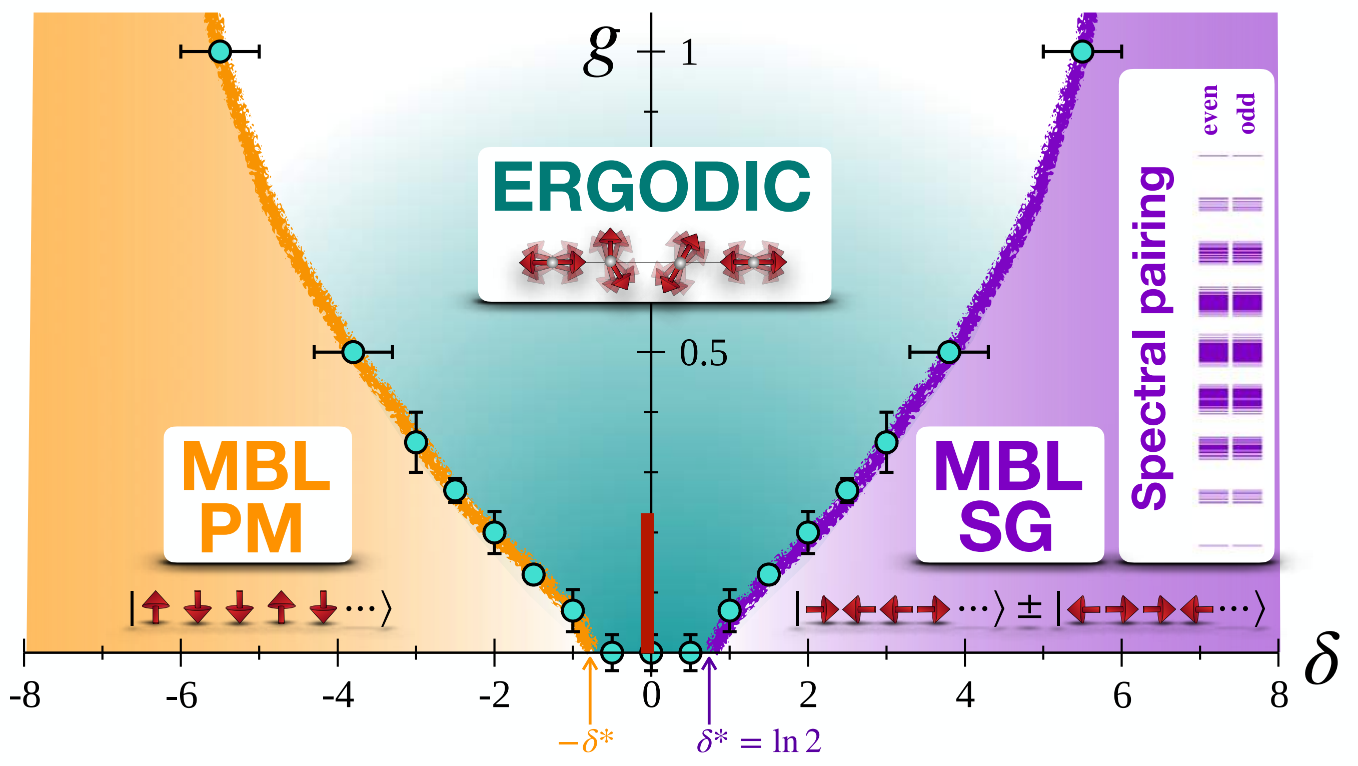


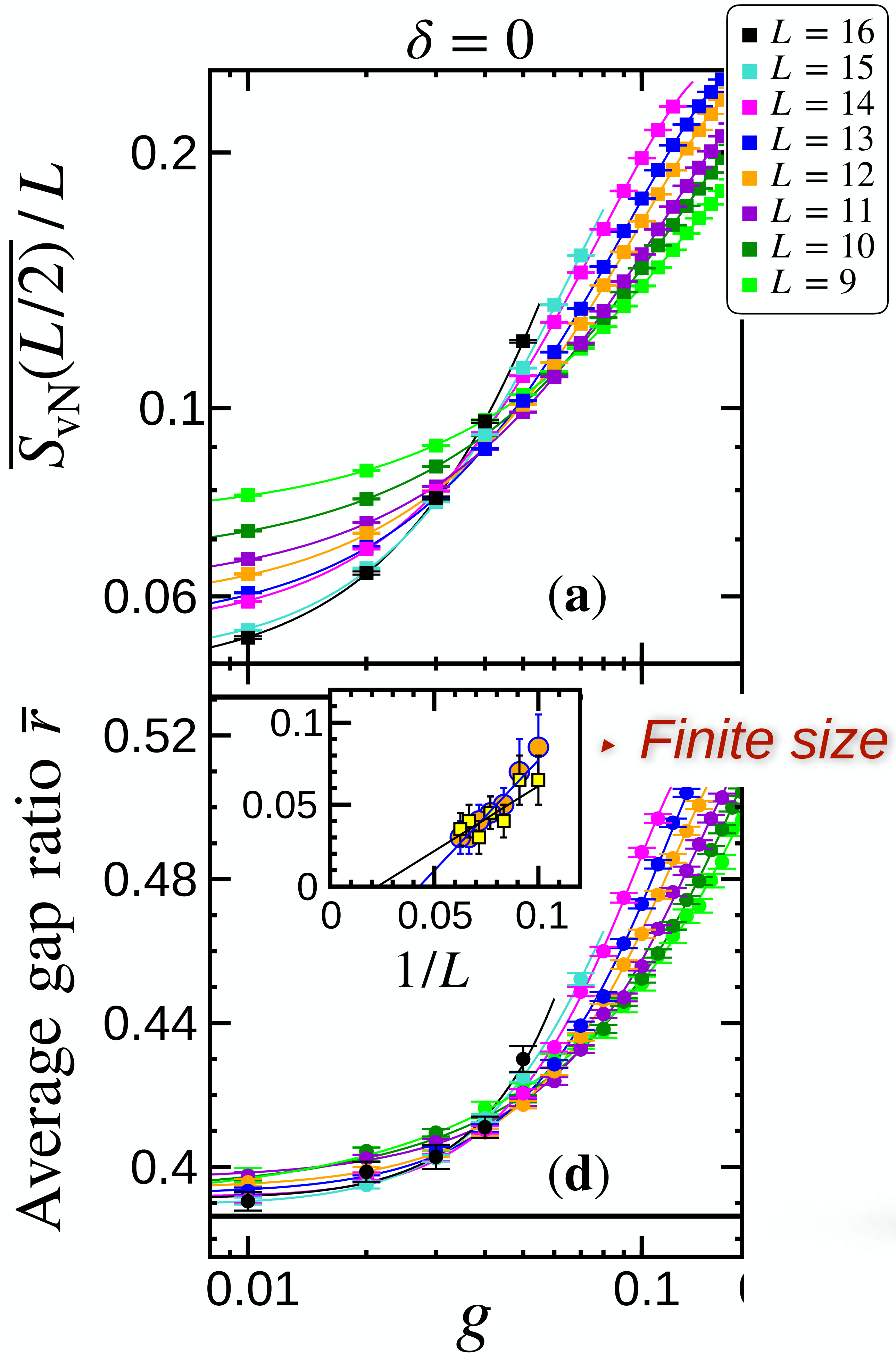


- *Finite size drift $g_c \to 0$*

Infinite Randomness Criticality immediately turns Ergodic

Consequences for the weakly interacting regime





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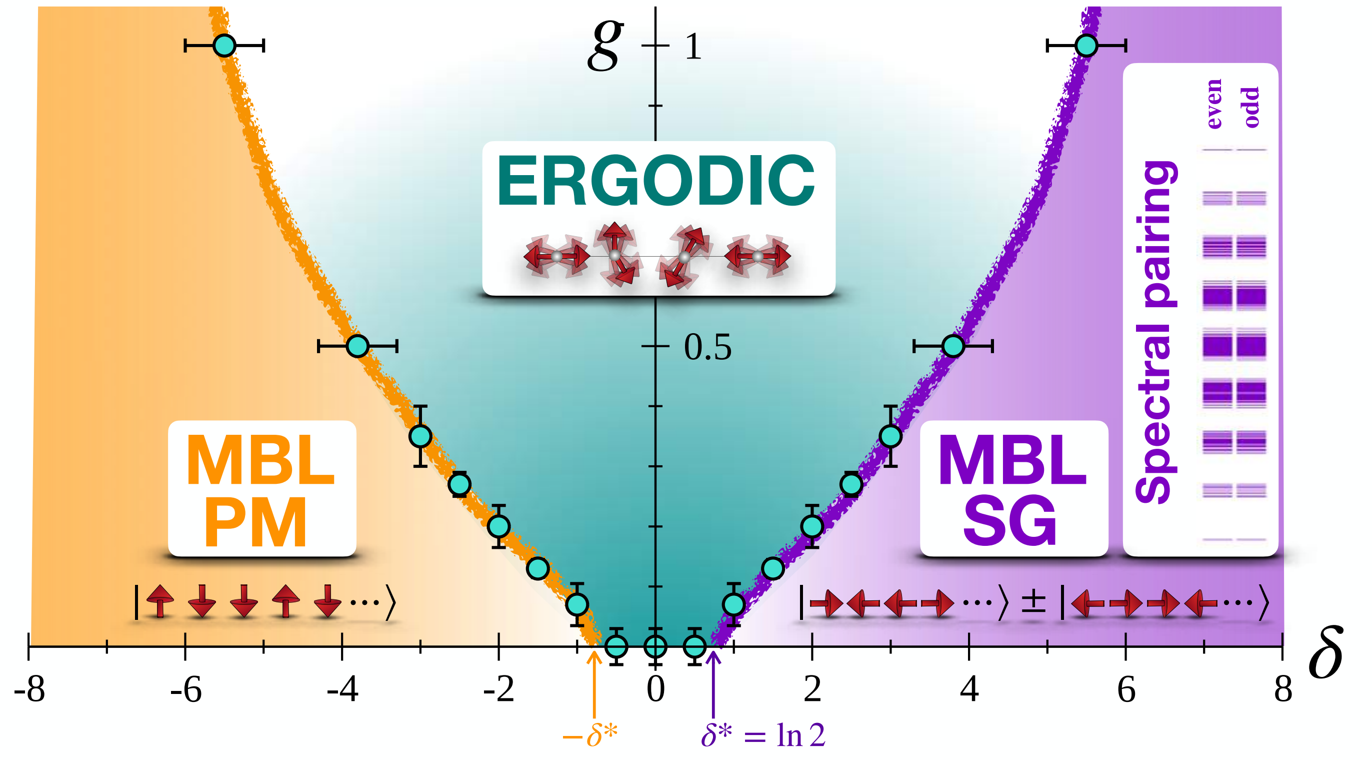
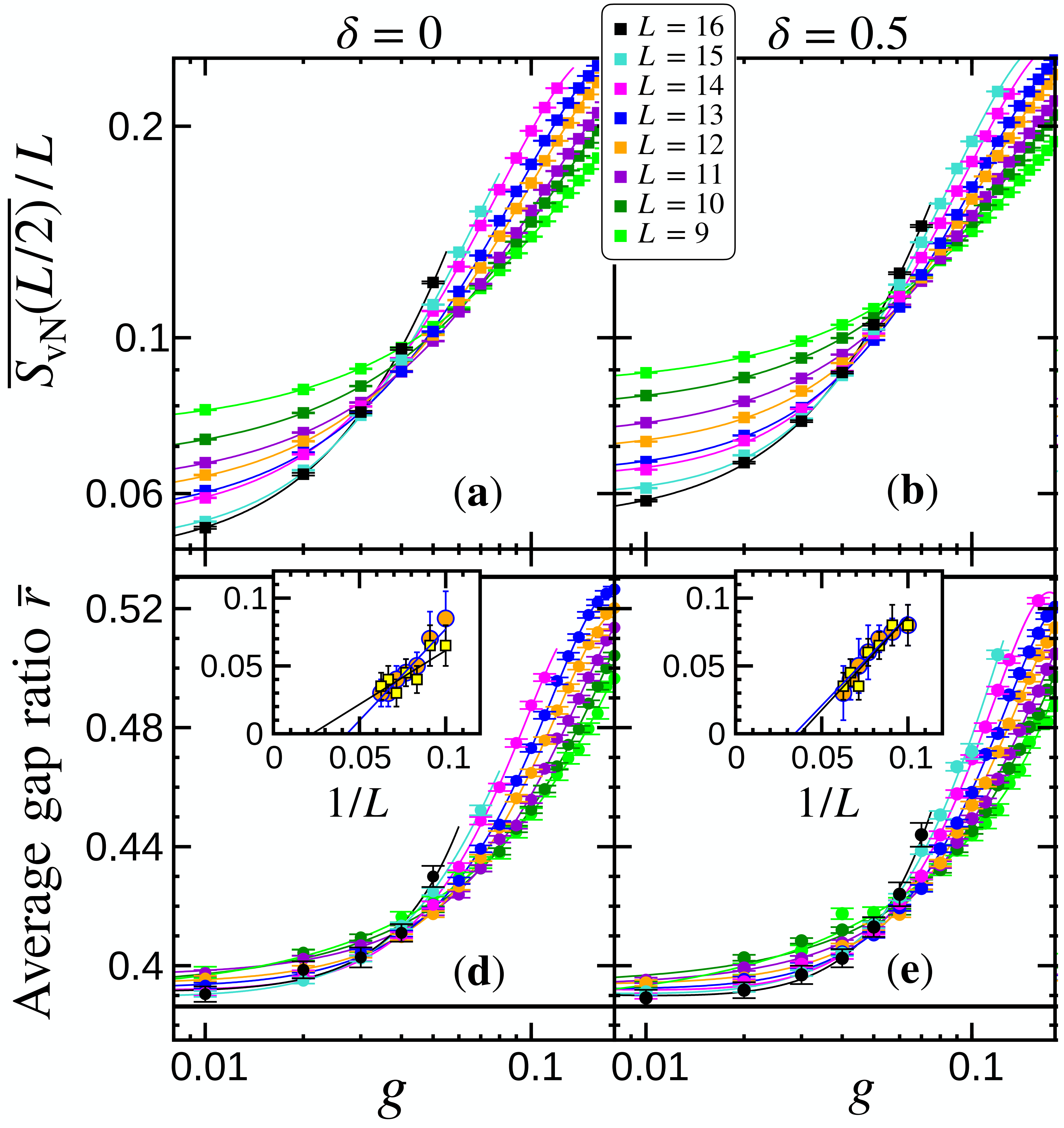
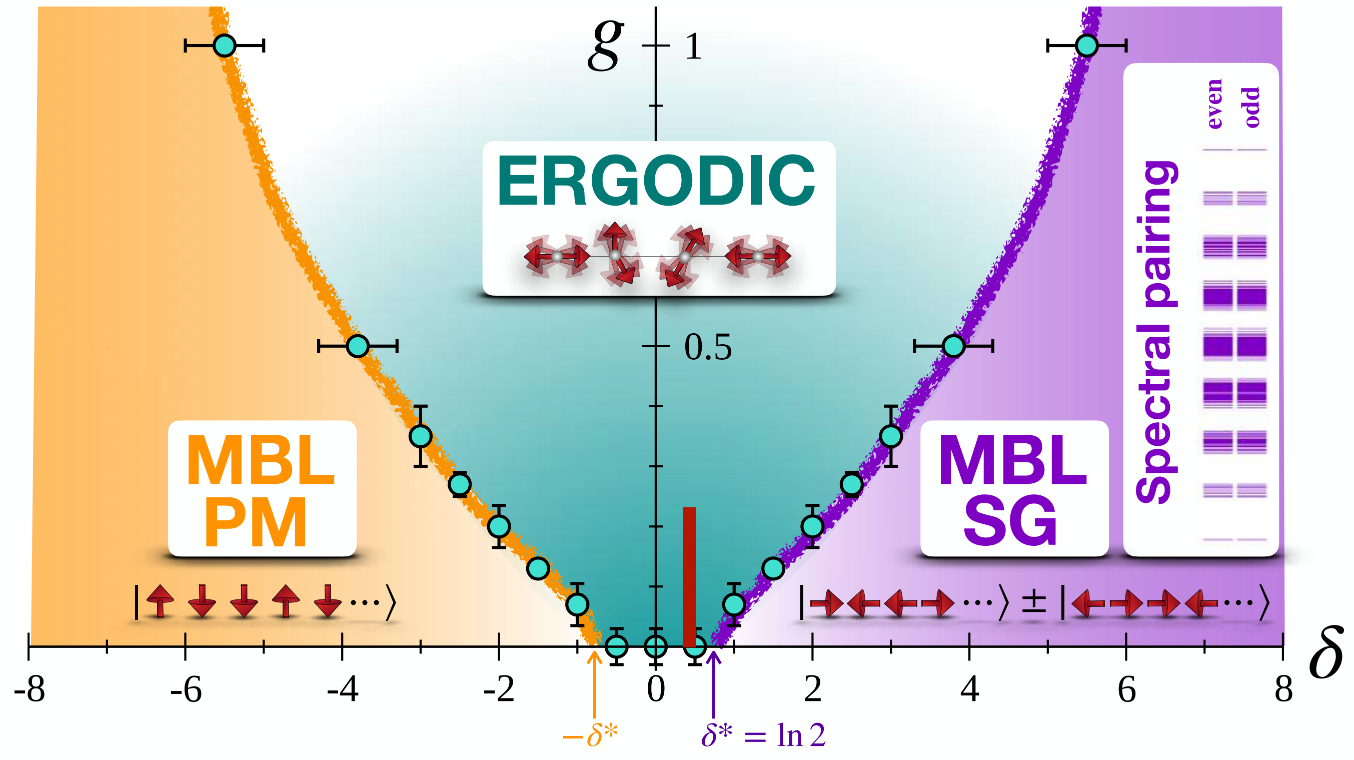
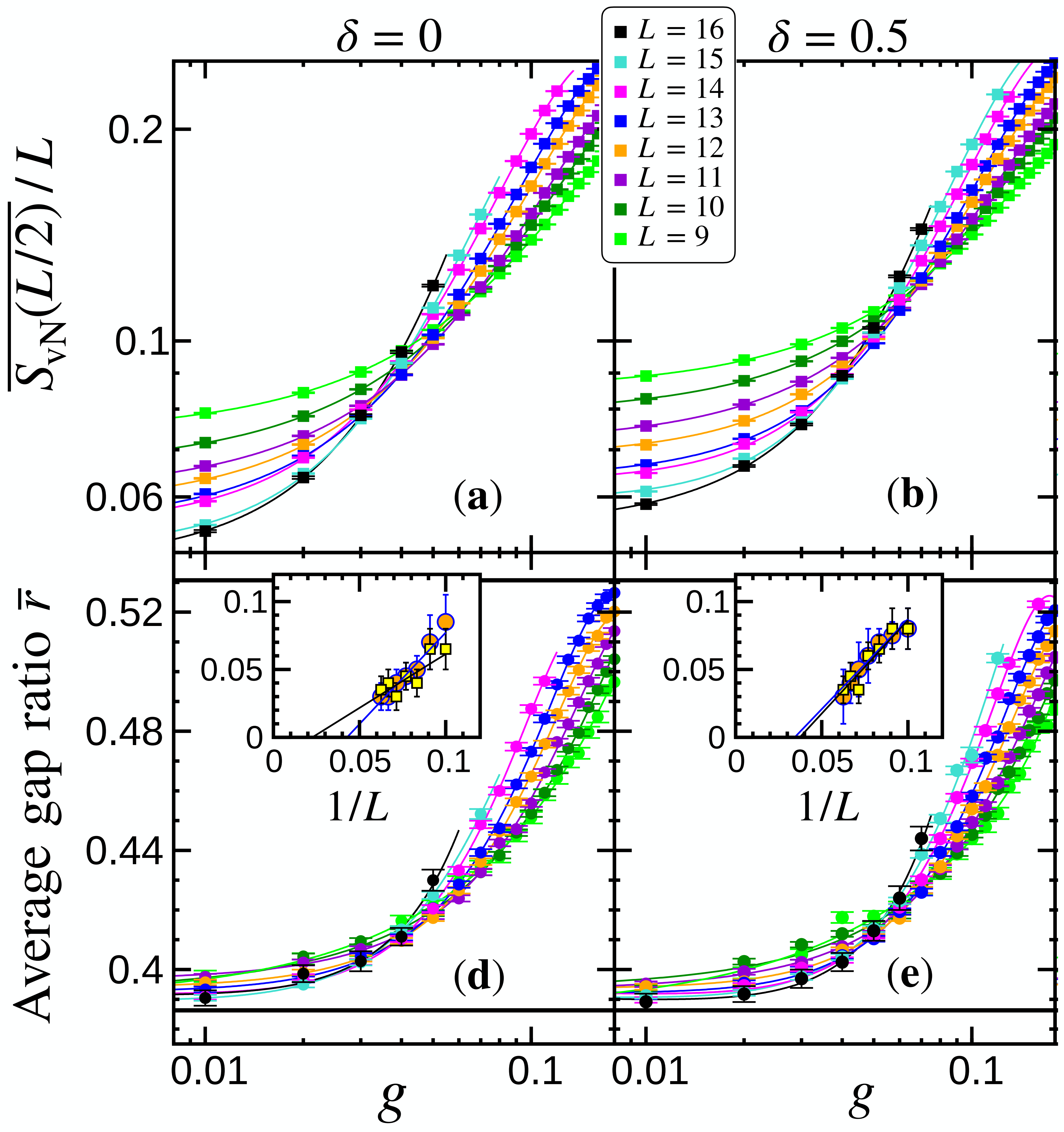




- $g_c \to 0$
- *Similar drift as* $\delta = 0$

Consequences for the weakly interacting regime





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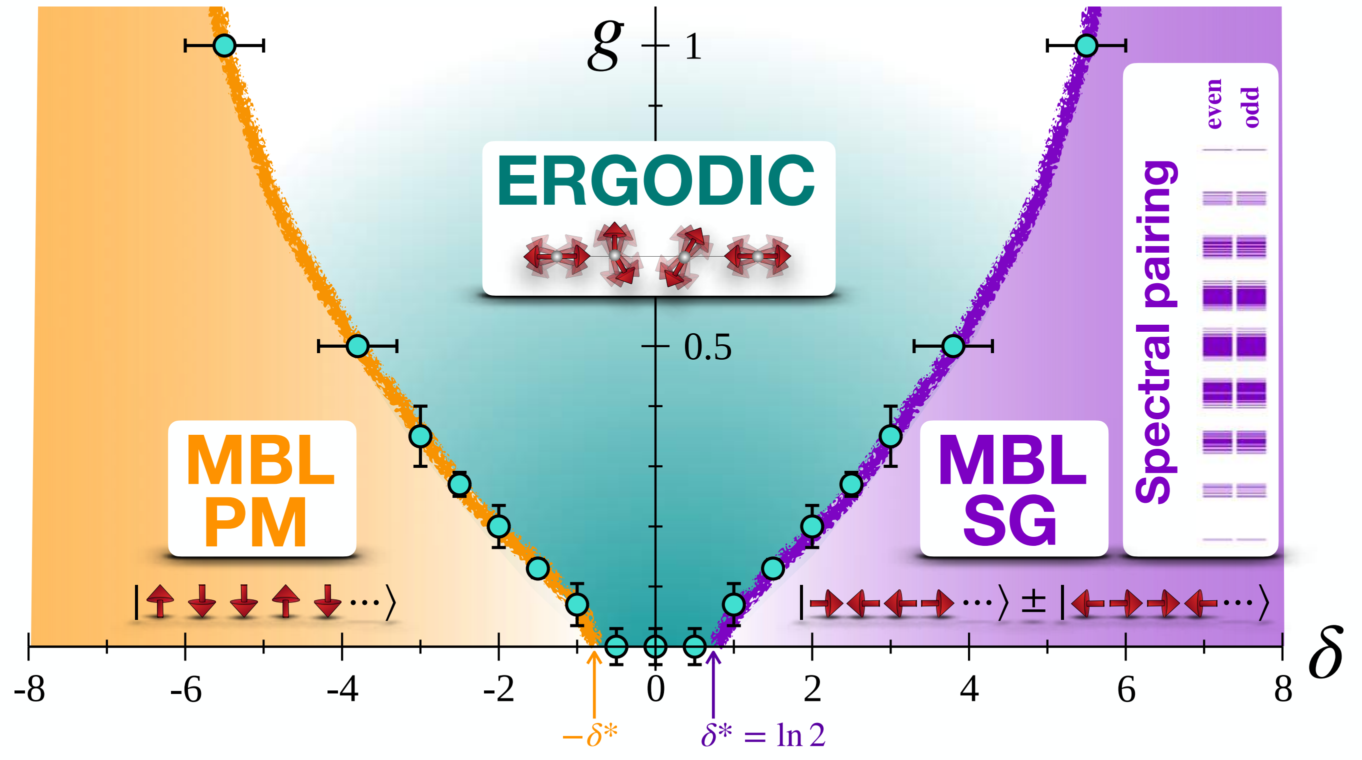
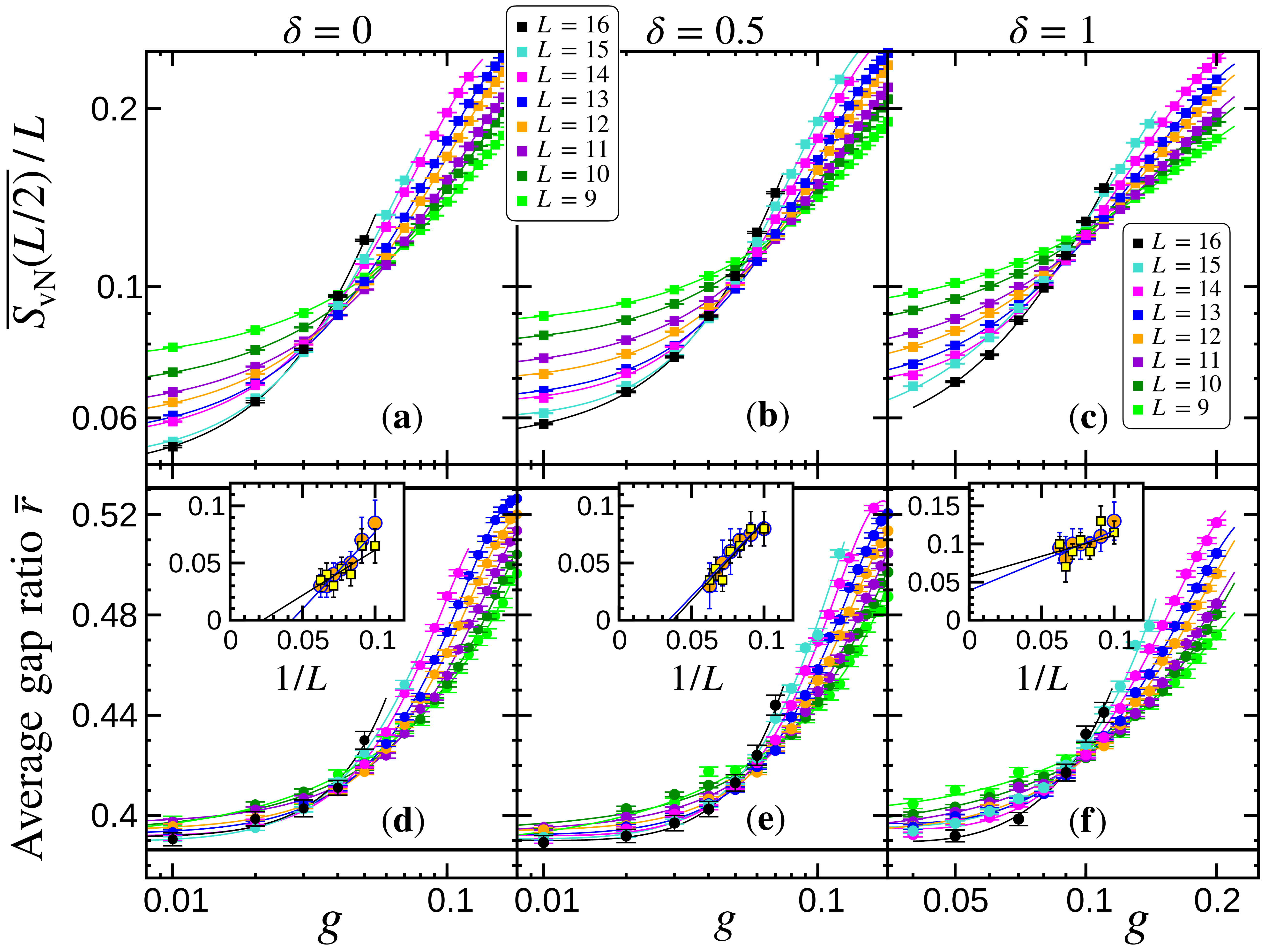
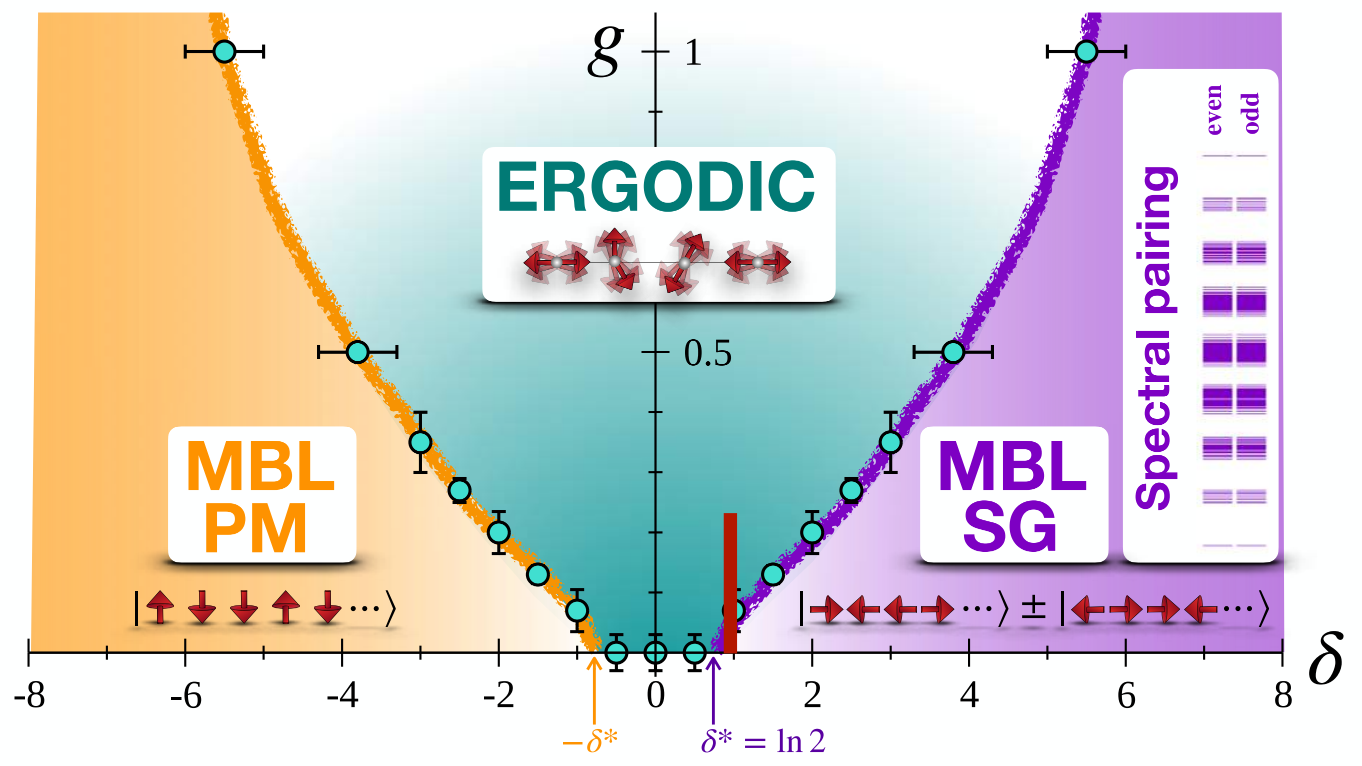
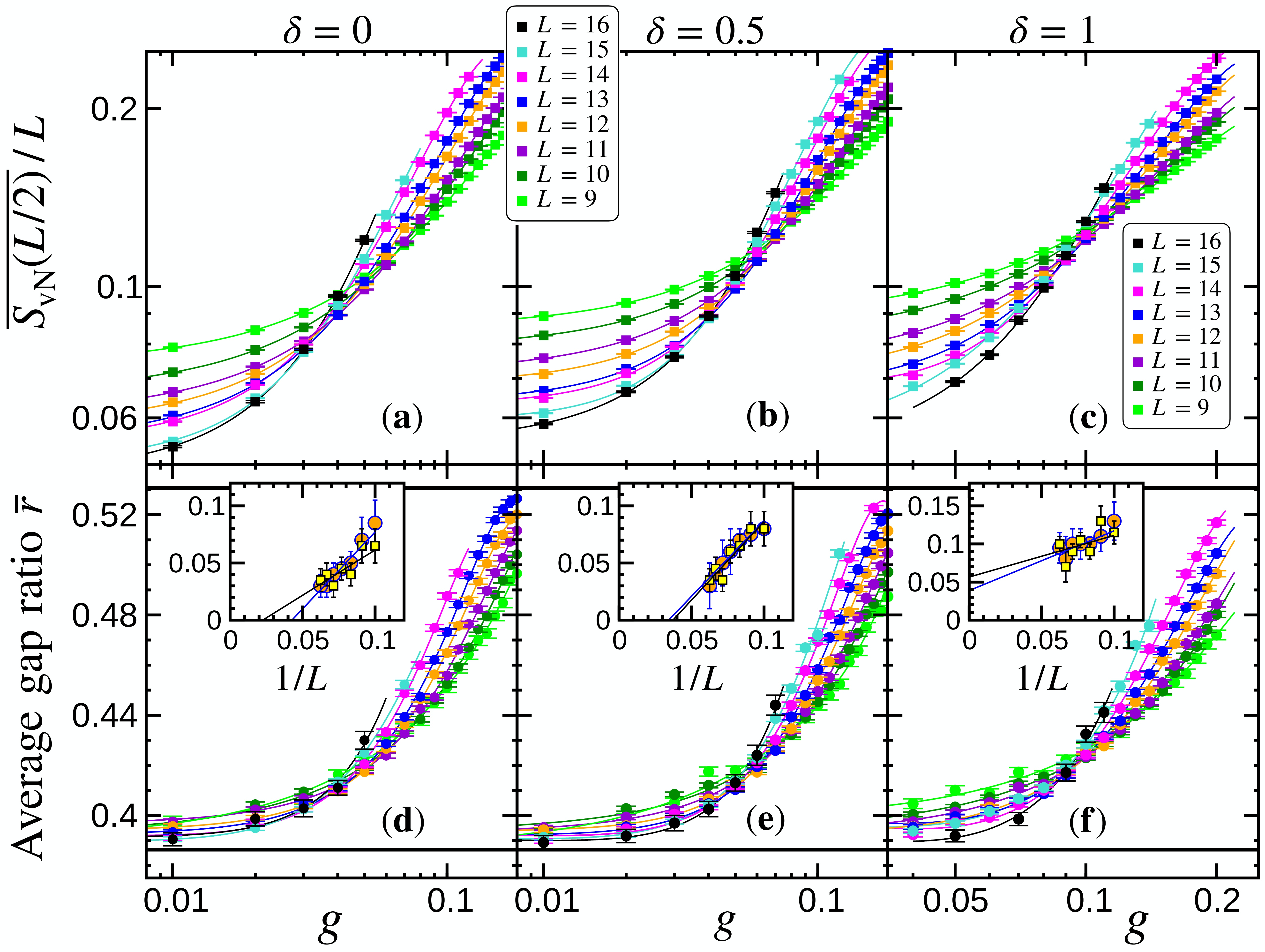
Consequences for the weakly interacting regime



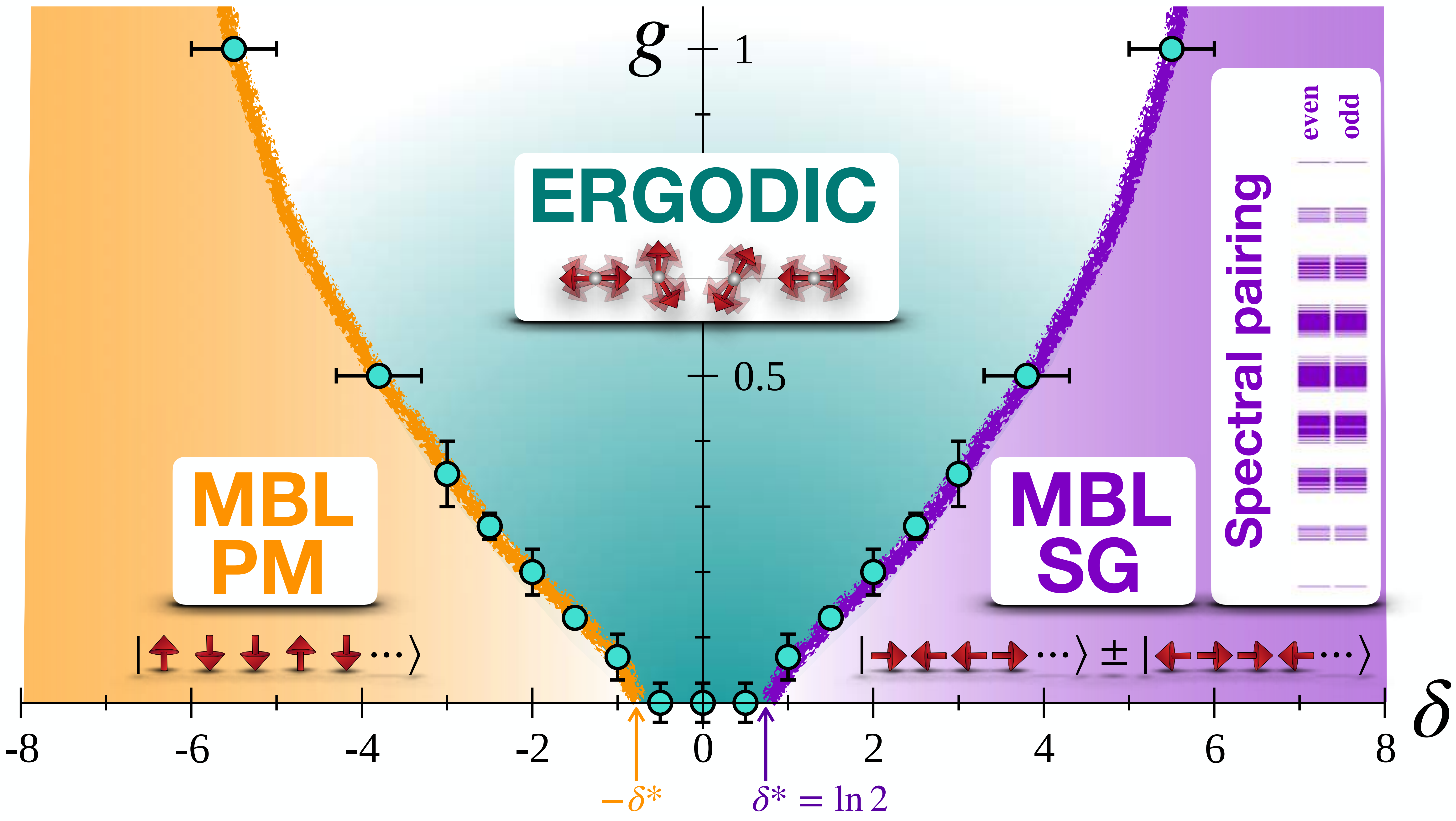


Consequences for the weakly interacting regime



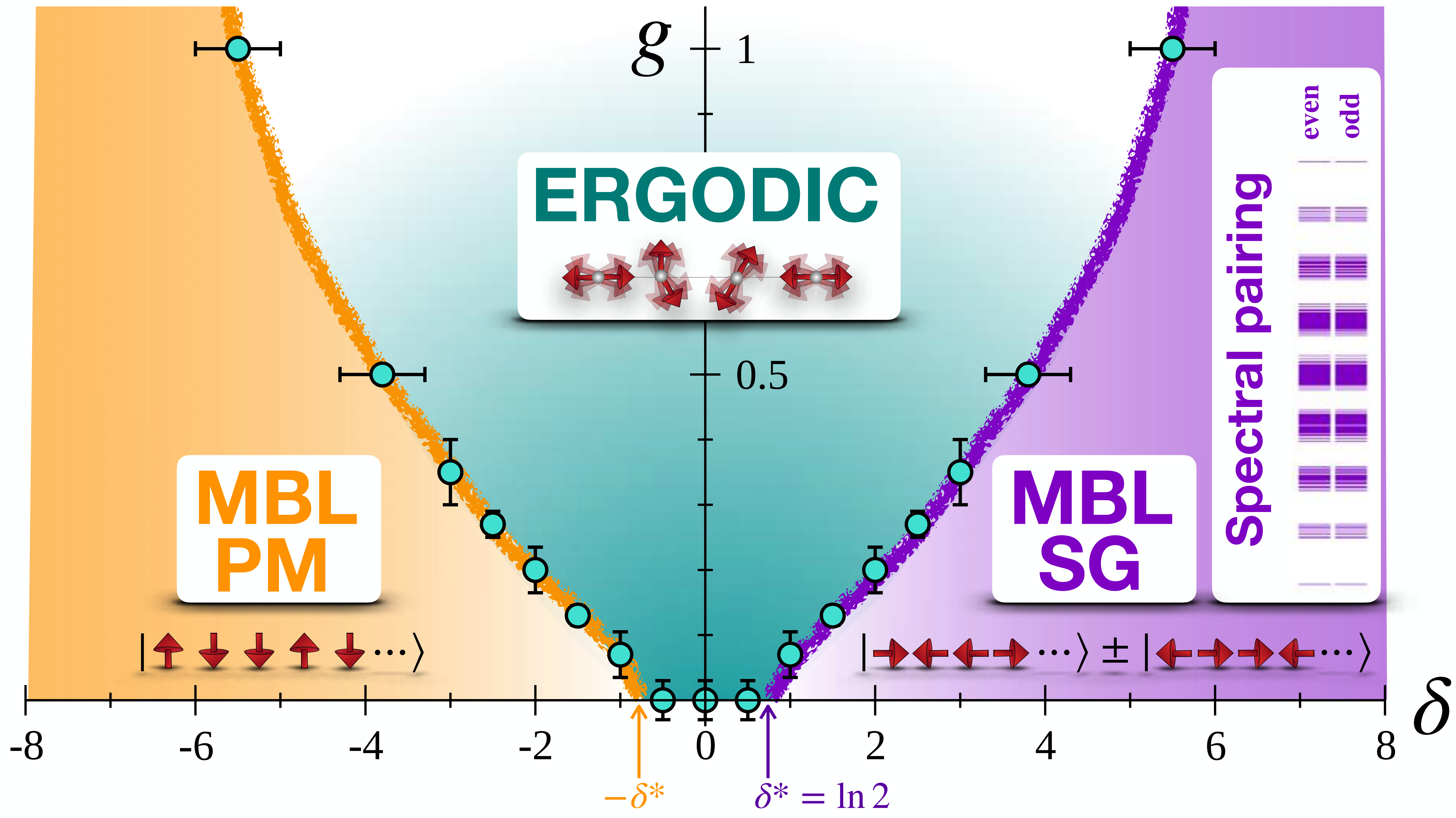


Summary



- Two topologically different MBL phases
  - Topological MBL-SG + spectral pairing
  - Featureless MBL Paramagnet
- Wide intervening ERGODIC regime
- Infinite Randomness unstable towards ergodicity
- Spectral-pairing transition = avalanche criterion

Summary



- Two topologically different MBL phases {
  - Topological MBL-SG + spectral pairing ▸ **Strong Zero Mode operator?**
  - Featureless MBL Paramagnet
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