Time and computation in quantum gravity

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Joint with David Pérez-García + Aleksander Kubicki

(Upcoming / discussions with DPG, Michelle Xu, Henry Lin)

Time in gravity vs QM.

- Quantum mechanics:

$$|\psi(t)\rangle = e^{iHt}|\psi_0\rangle$$

↳ time runs forever

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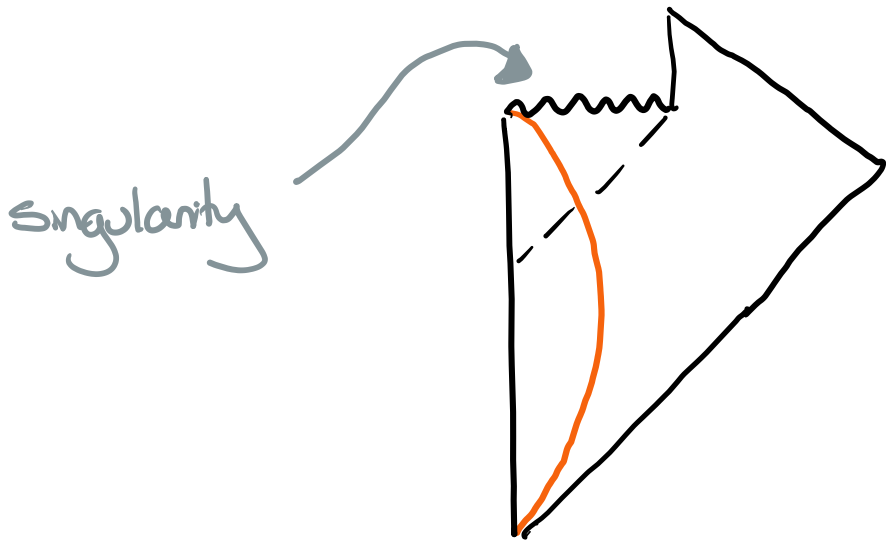

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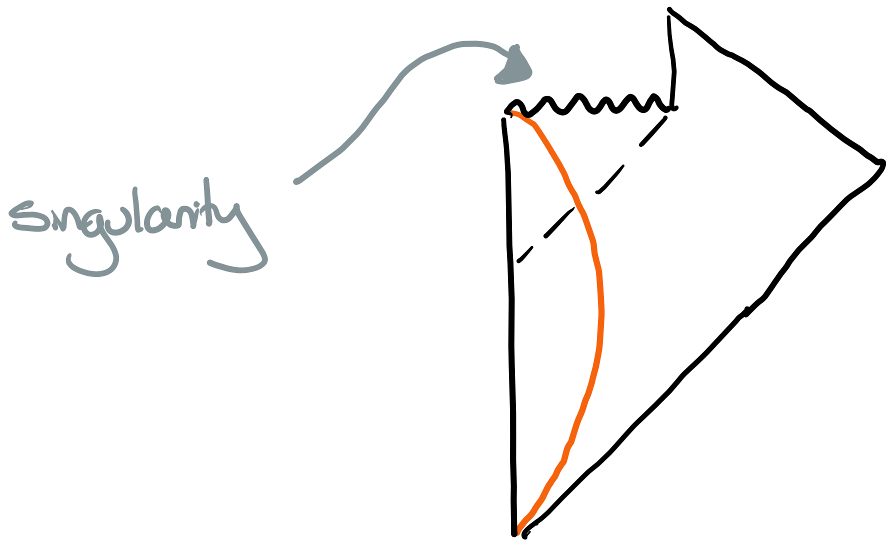

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How do we reconcile these two views on time?

Setting + tools

$$\boxed{\text{How do we reconcile these two views on time?}}$$

- As a start, we'll study this in the AdS/CFT correspondence

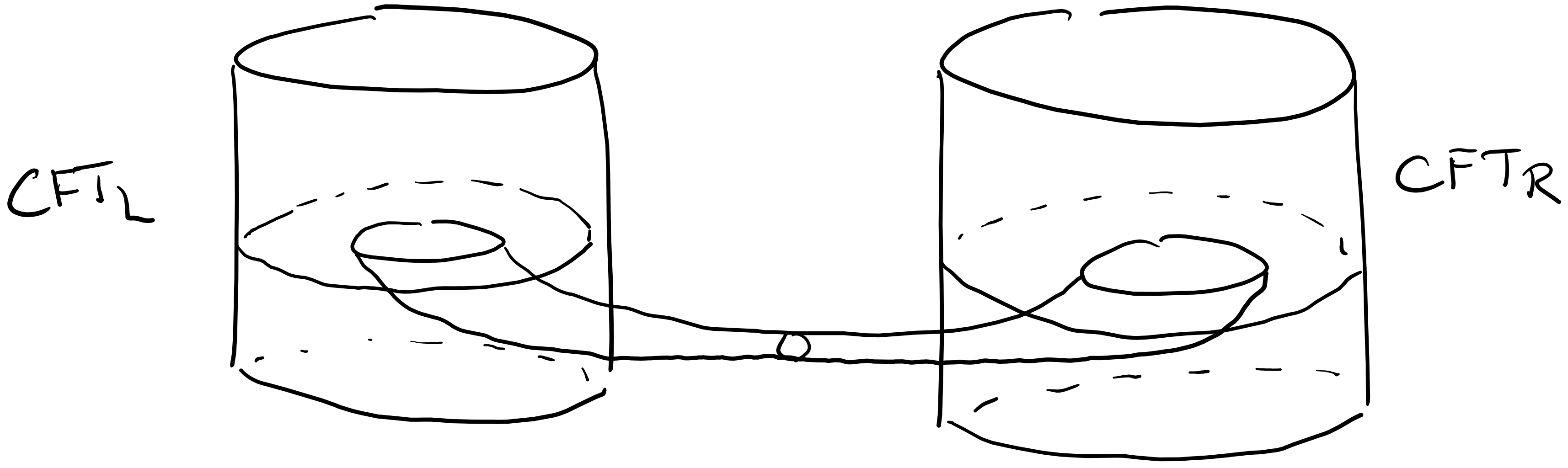
  ↳ AdS/CFT applies to wrong universe, etc. but still has singularities recorded into QM

- Draw on tools from quantum info (e.g. quantum processors) and complexity theory (e.g. Komolgorov complexity)

Some AdS/CFT

Two-sided black hole

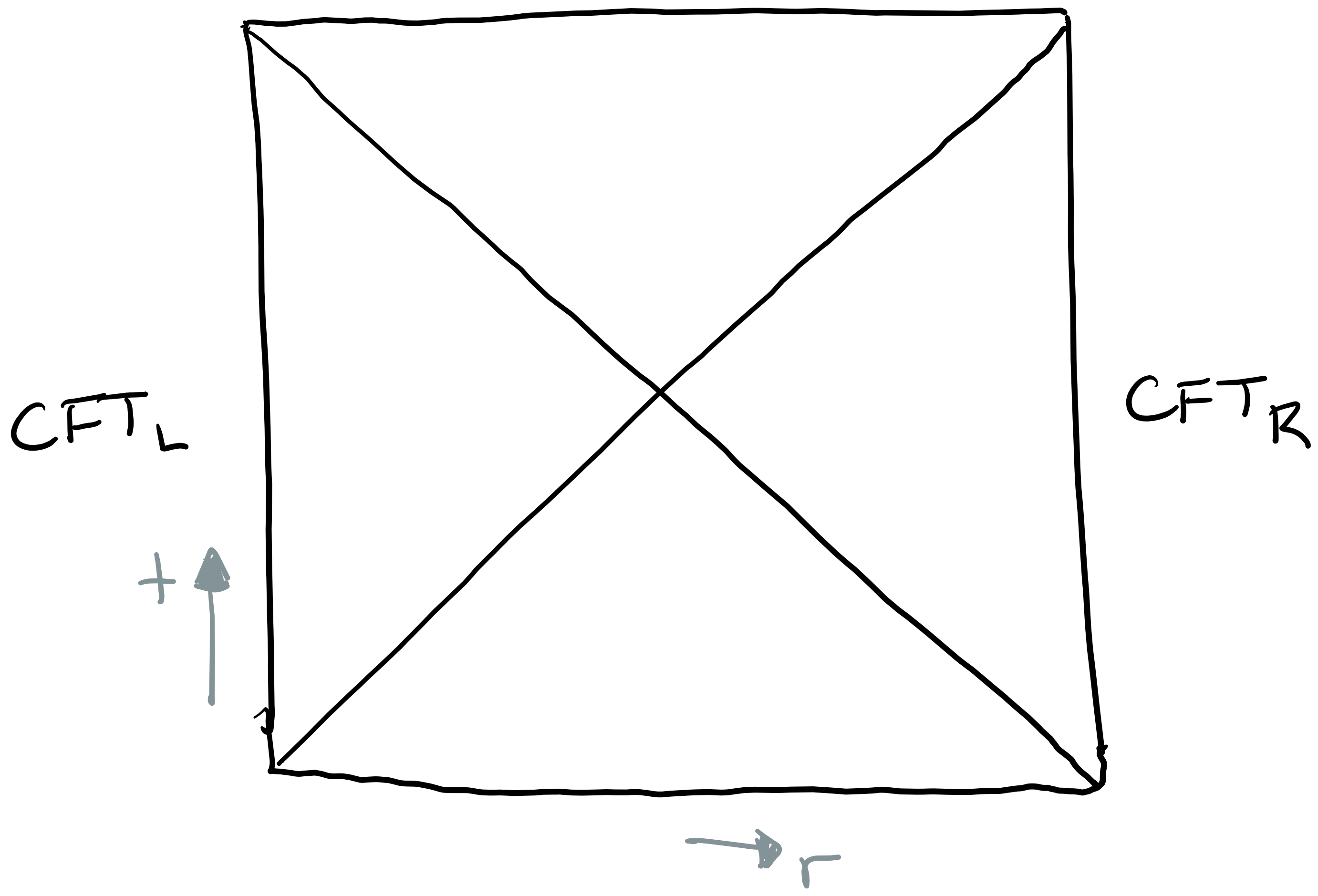
- In AdS/CFT, the boundary state determines the bulk geometry.
- Considering two CFT's, and choosing an appropriate entangled state, we obtain a two-sided black hole geometry:



(a cartoon)

Penrose diagram:

- The Penrose diagram captures causal aspects of the bulk geometry:



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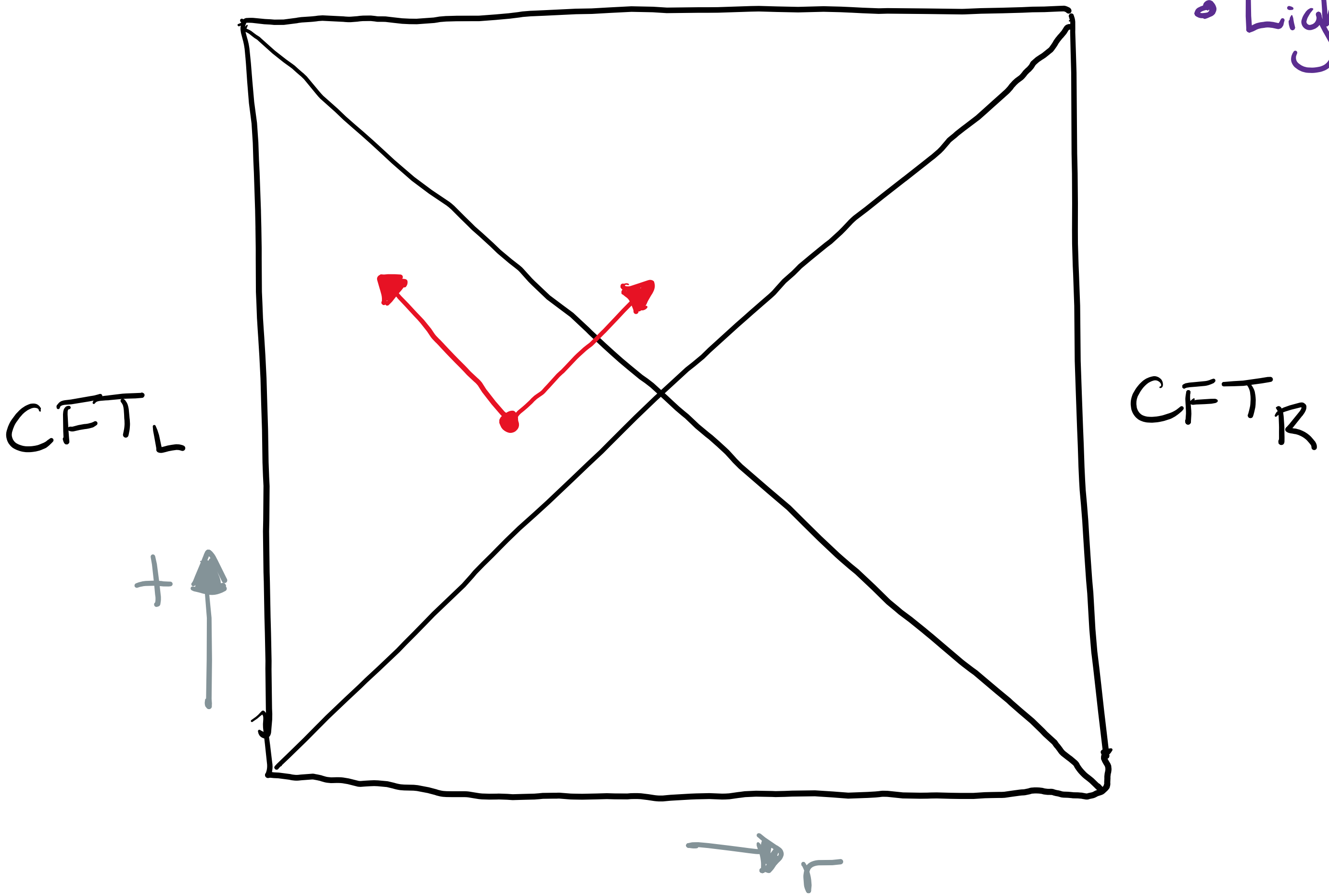

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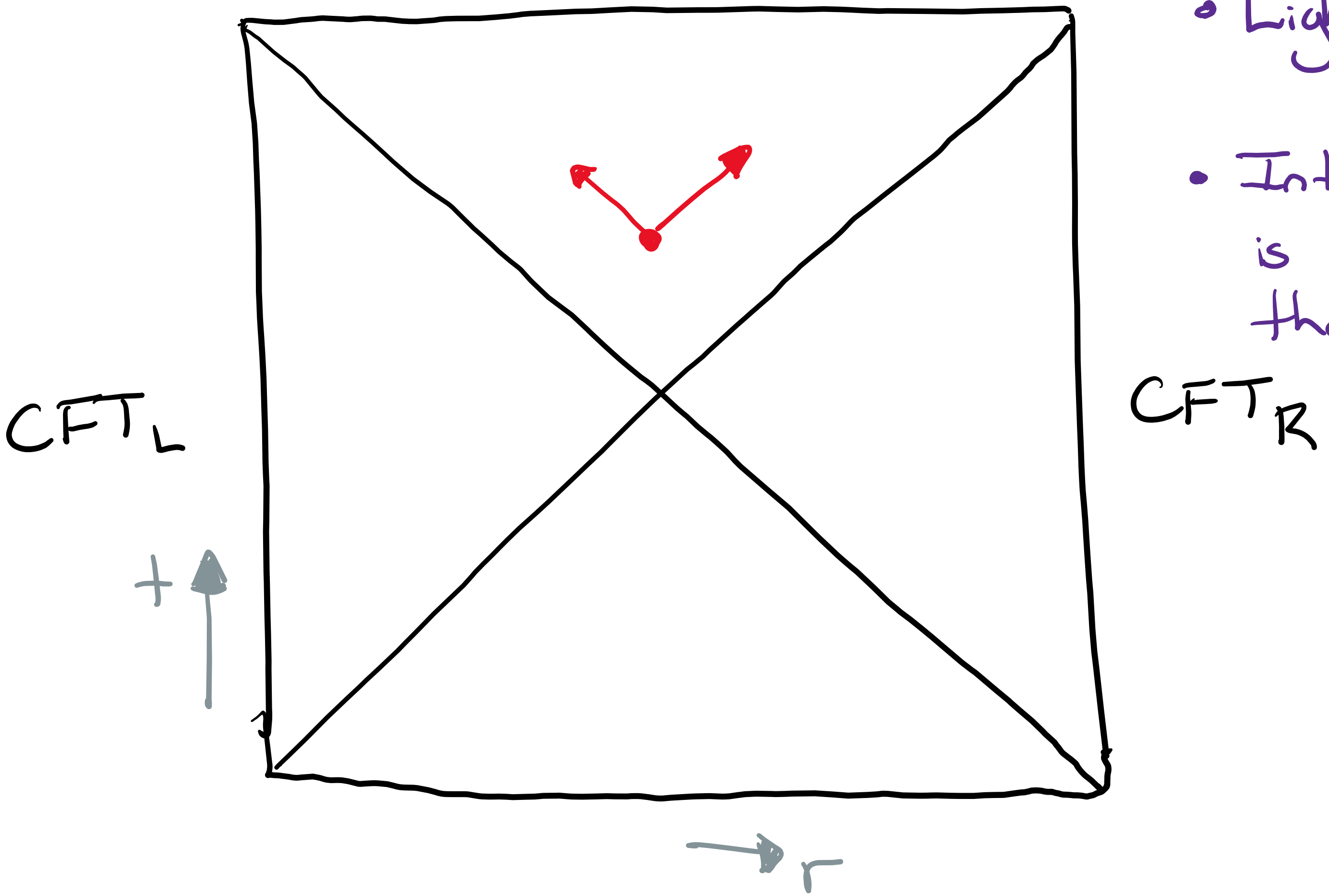

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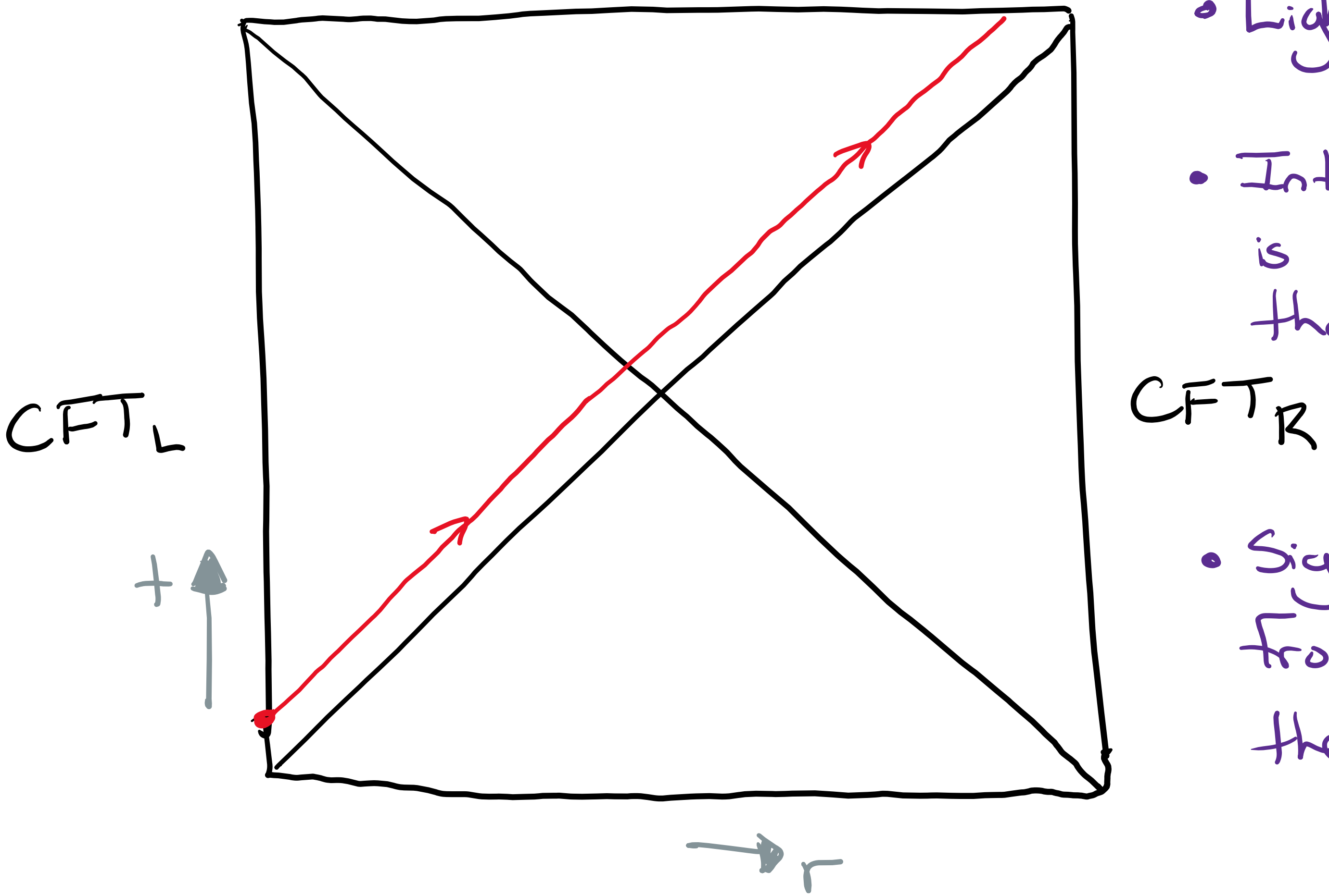

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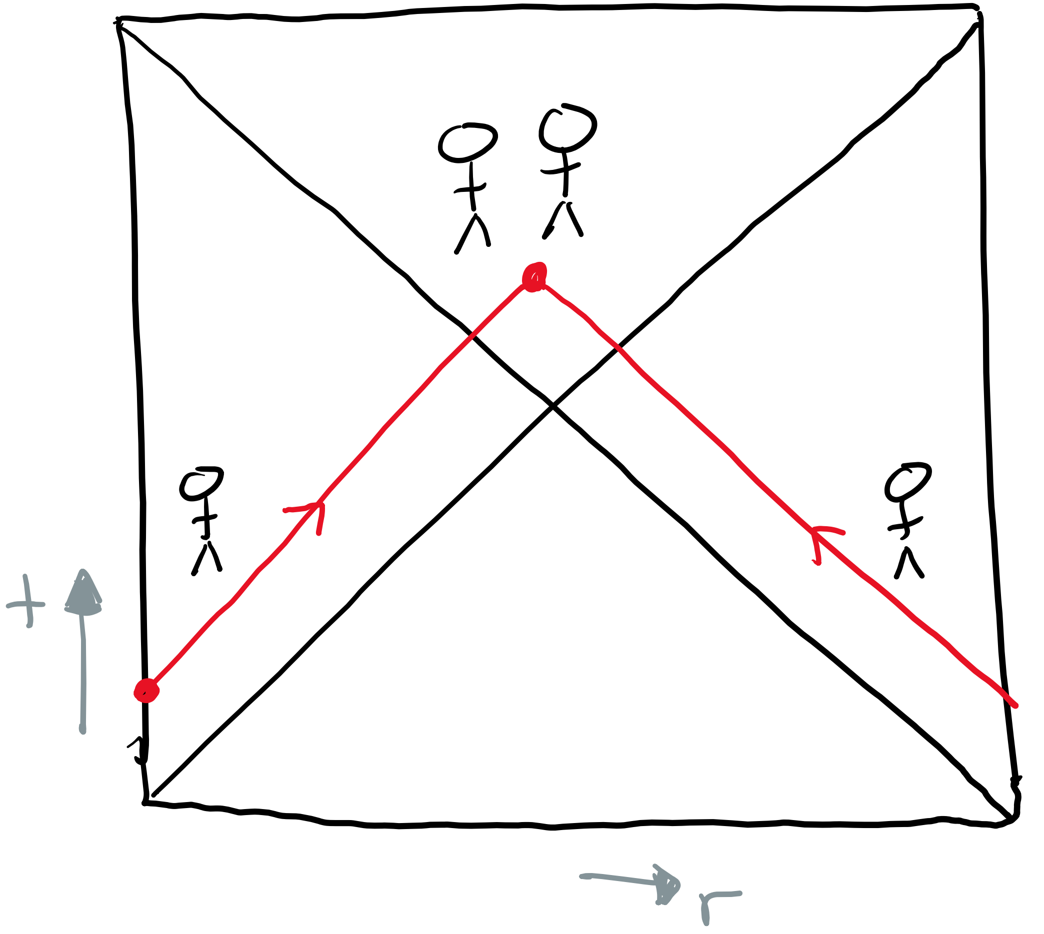
- Interior of black hole is portion of spacetime that cannot signal either boundary
- Signals cannot travel from one CFT to the other

Penrose diagram:

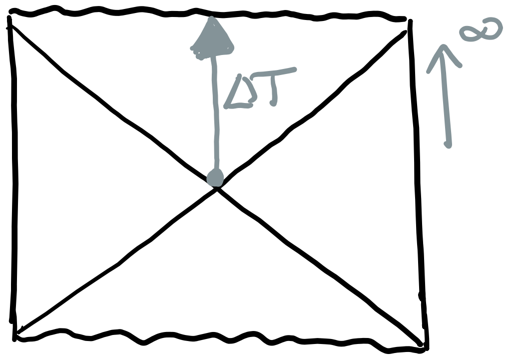
- Causal structure does allow signals/observers to meet behind the horizon.



Approach to understanding time
inside the black hole.

Time inside a black hole

- Time inside the hole is finite



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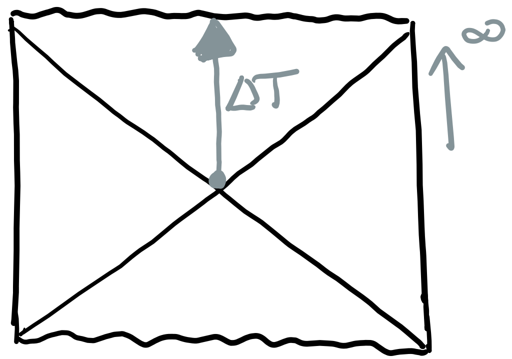

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Basic idea: CFT's place computational constraints on what can happen inside the BH. Bulk description realizes these constraints geometrically, by featuring an end to time

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Idea #1: Interior is supported by CFT entanglement. Maybe finite entanglement constrains computations that can happen there?

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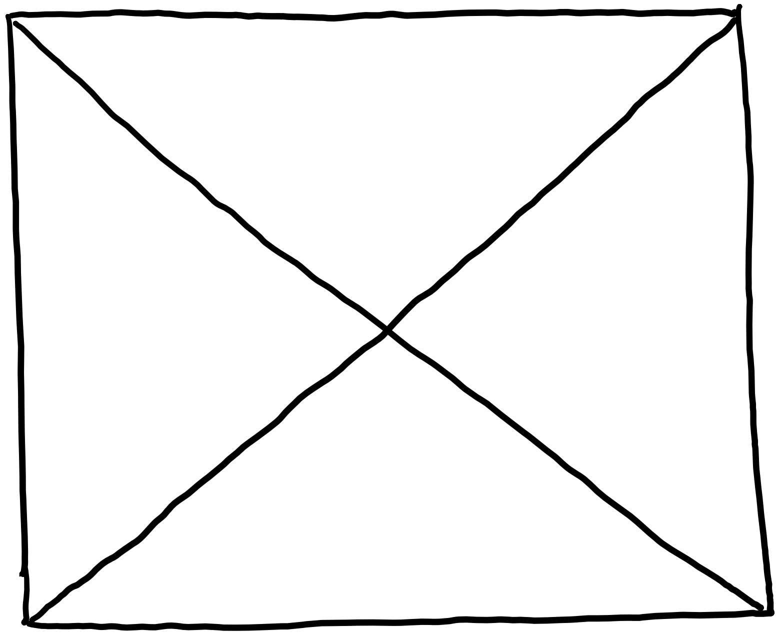
Idea #2: Something else?

↳ Rest of this talk.

Set-up : computation inside a black hole

Set-up

- Consider a two-sided black hole, dual to thermofield double state.



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$$|\psi\rangle_A \rightarrow U_A|\psi\rangle_A$$

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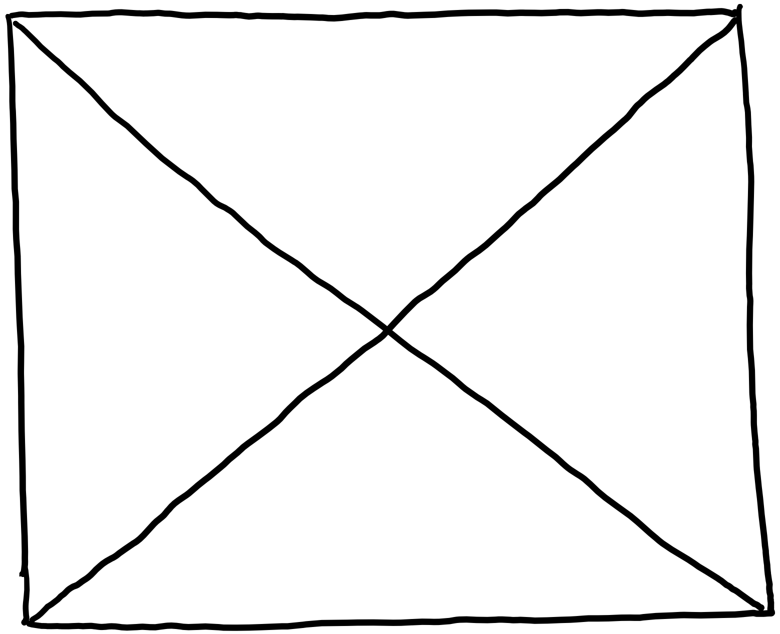

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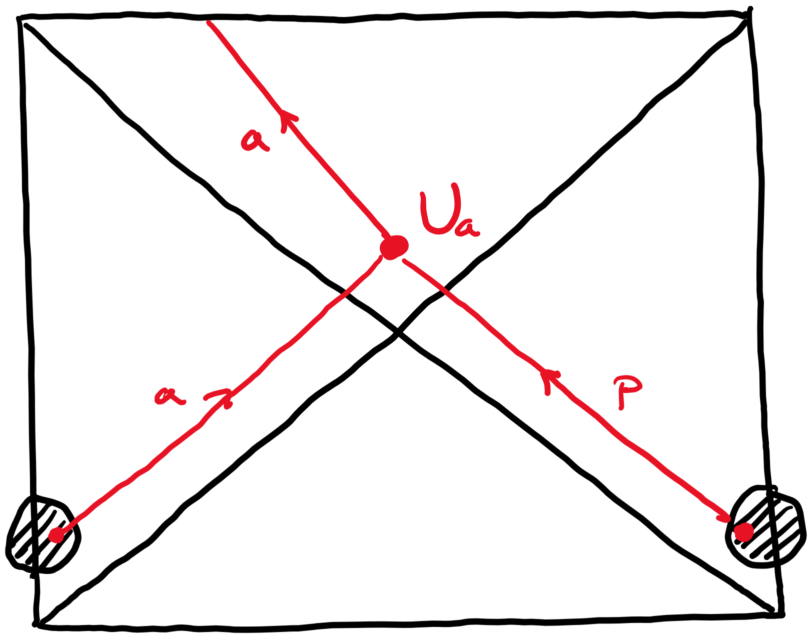

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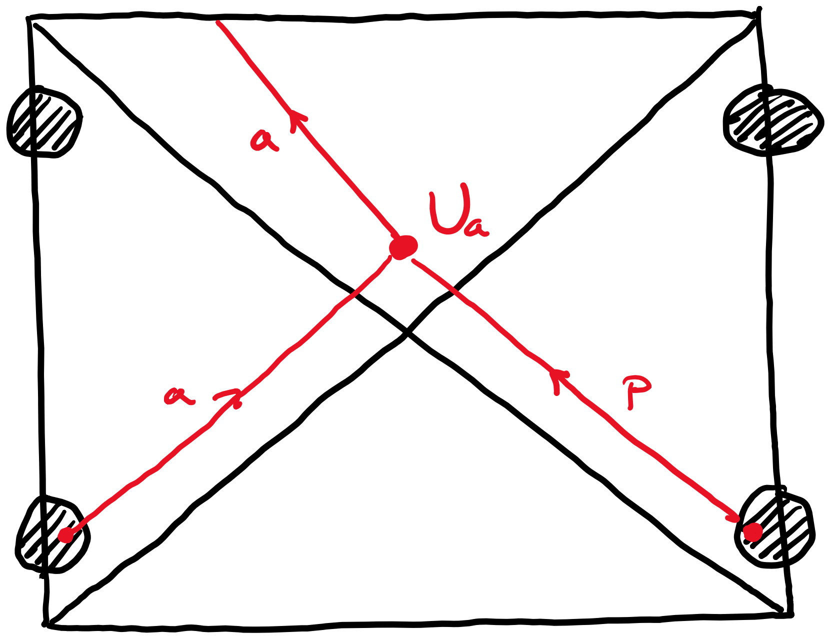
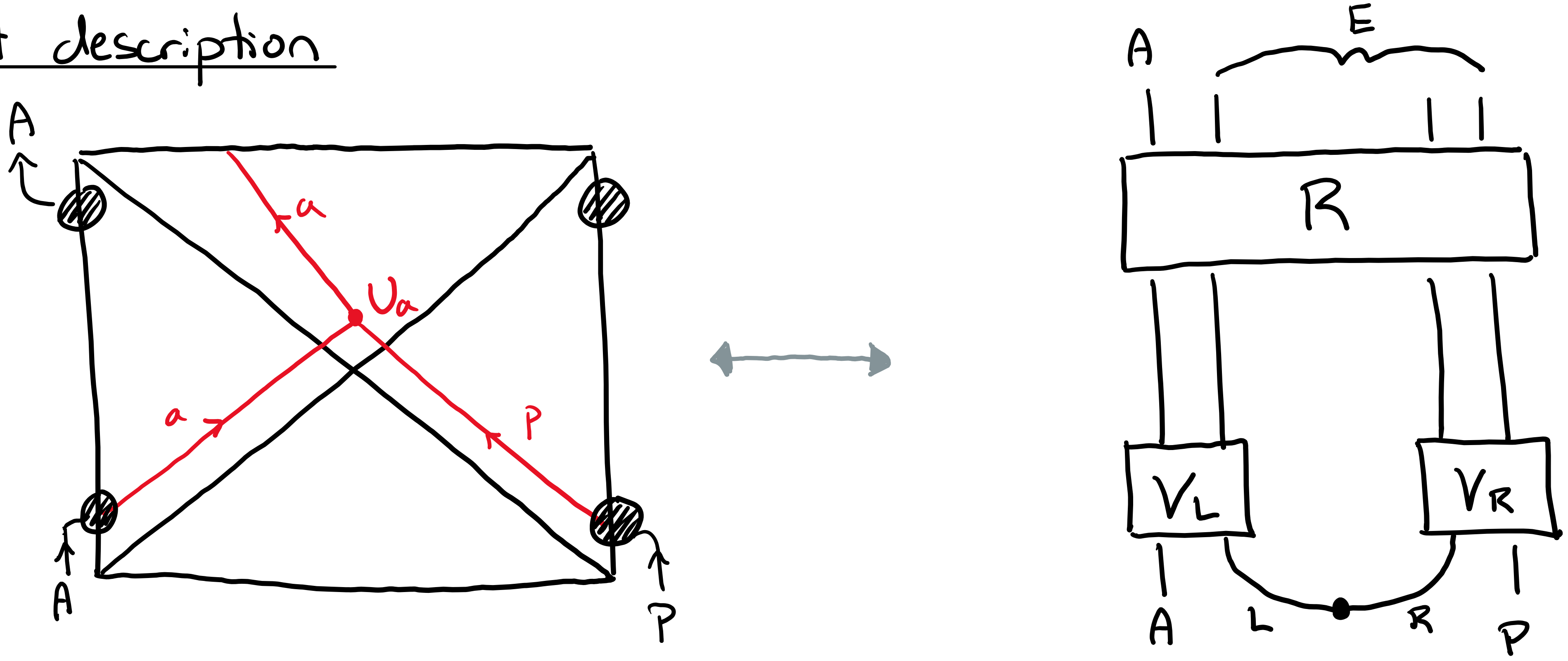


- Record A, P into bulk degrees of freedom a, p, which will fall into the black hole

- At later time, act jointly on L, R to recover a subsystem, and record back into A

$\boxed{\text{Interested in constraining the allowed } U_a}$

Circuit description



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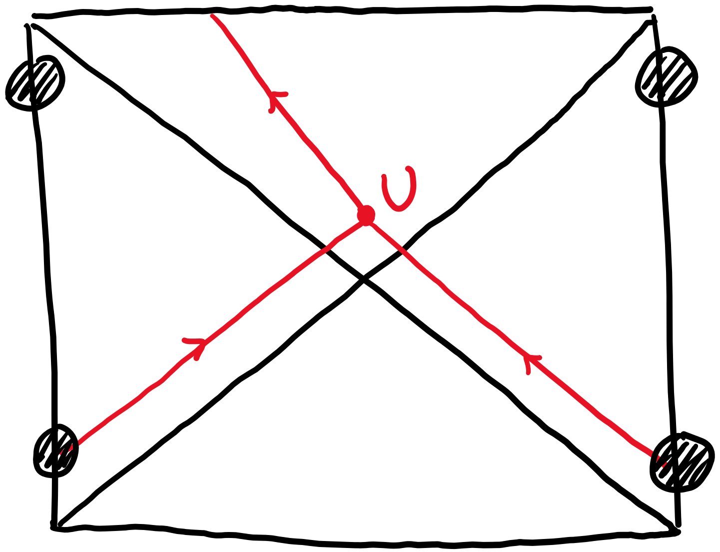

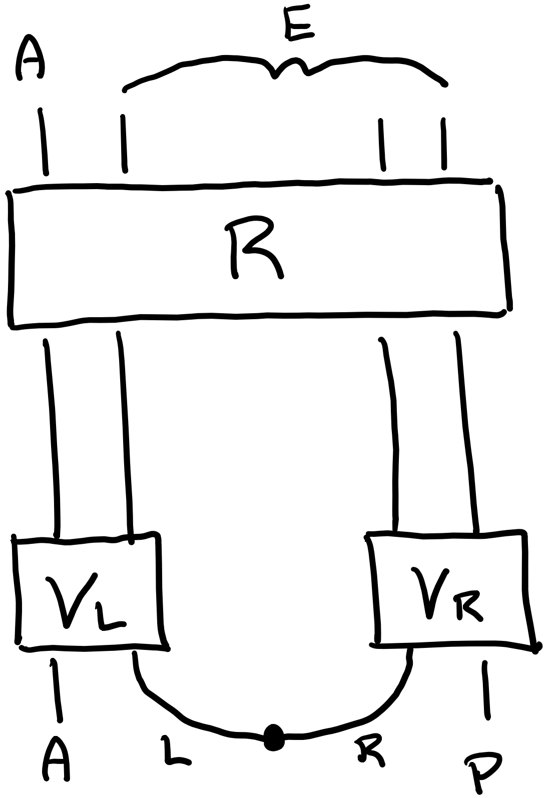

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  ↳ Want to use this to constrain U_a

Recovery map

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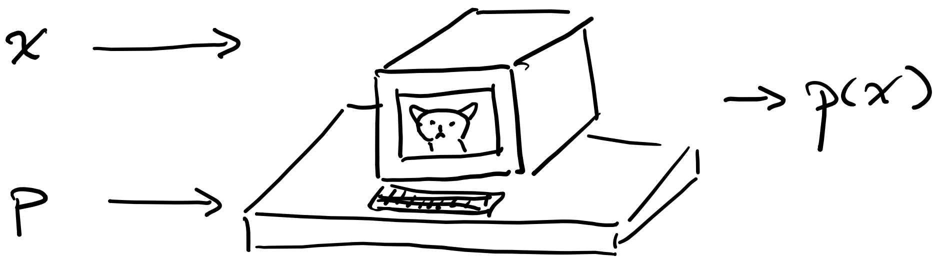
Also important: R is "easy" to describe

QI tools

Quantum processors

- A computer takes in a program and some data, then applies the program to the data



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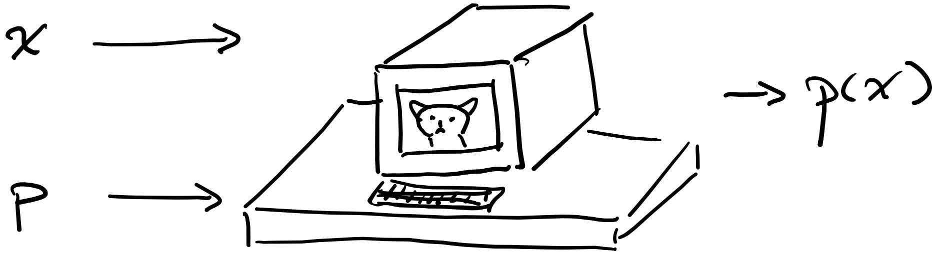

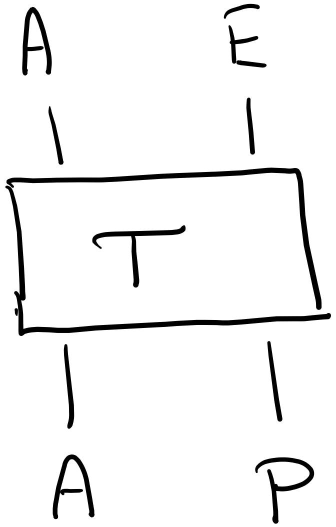

- A "quantum processor" is one way to view this in the quantum setting:

$$T\left(|\psi\rangle_A |\varphi_U\rangle_P\right) = \left(U|\psi\rangle_A\right) |\varphi_U'\rangle_E$$

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- We'd like to characterize how well T applies a particular U_A^{ε}:

$$|\Phi_\varepsilon\rangle_{A\bar{A}} = U_A^{\varepsilon} |\Phi^+\rangle_{A\bar{A}} \qquad \text{(correct state)}$$

$$p(T,\varepsilon) = \sup_{|\varphi_\varepsilon\rangle_P} \underbrace{\langle \Phi_\varepsilon | \, \mathrm{tr}_E\left(T \left(|\Phi^+\rangle\langle\Phi^+| \otimes |\varphi_\varepsilon\rangle\langle\varphi_\varepsilon| \right) T^\dagger \right) |\Phi_\varepsilon\rangle}_{\text{overlap of correct state with output of } T}$$

Bounds on quantum processors

- All processors $T: \mathcal{H}_A \otimes \mathcal{H}_P \to \mathcal{H}_A \otimes \mathcal{H}_E$ satisfy:

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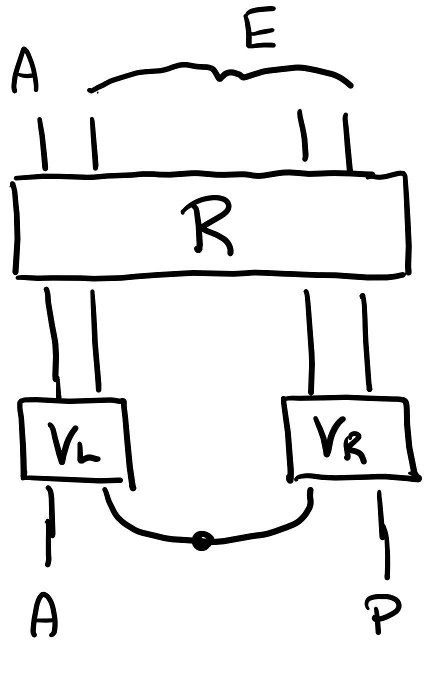
- Weirdly, this explanation doesn't suffice for some U^ε...

Back to our black hole...

Processor bounds on black holes

- Returning to the black hole setting, take:

$$n_P \ll S_{bh}\,, \qquad \log(S_{bh}) \leq n_A \ll S_{bh}$$



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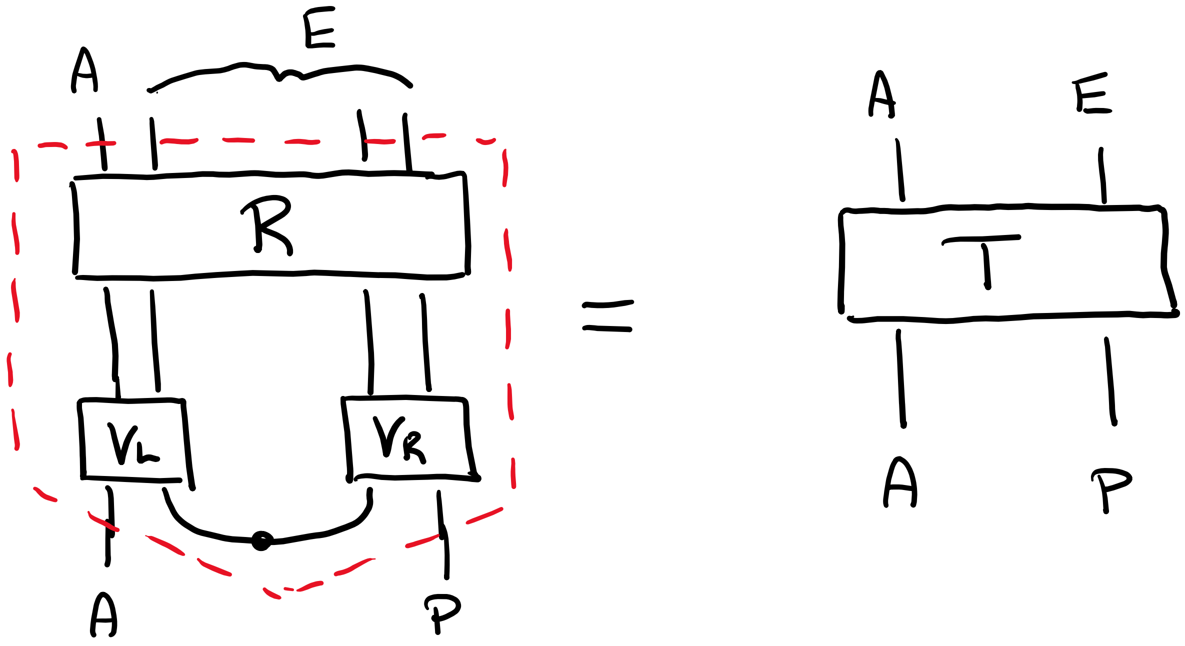

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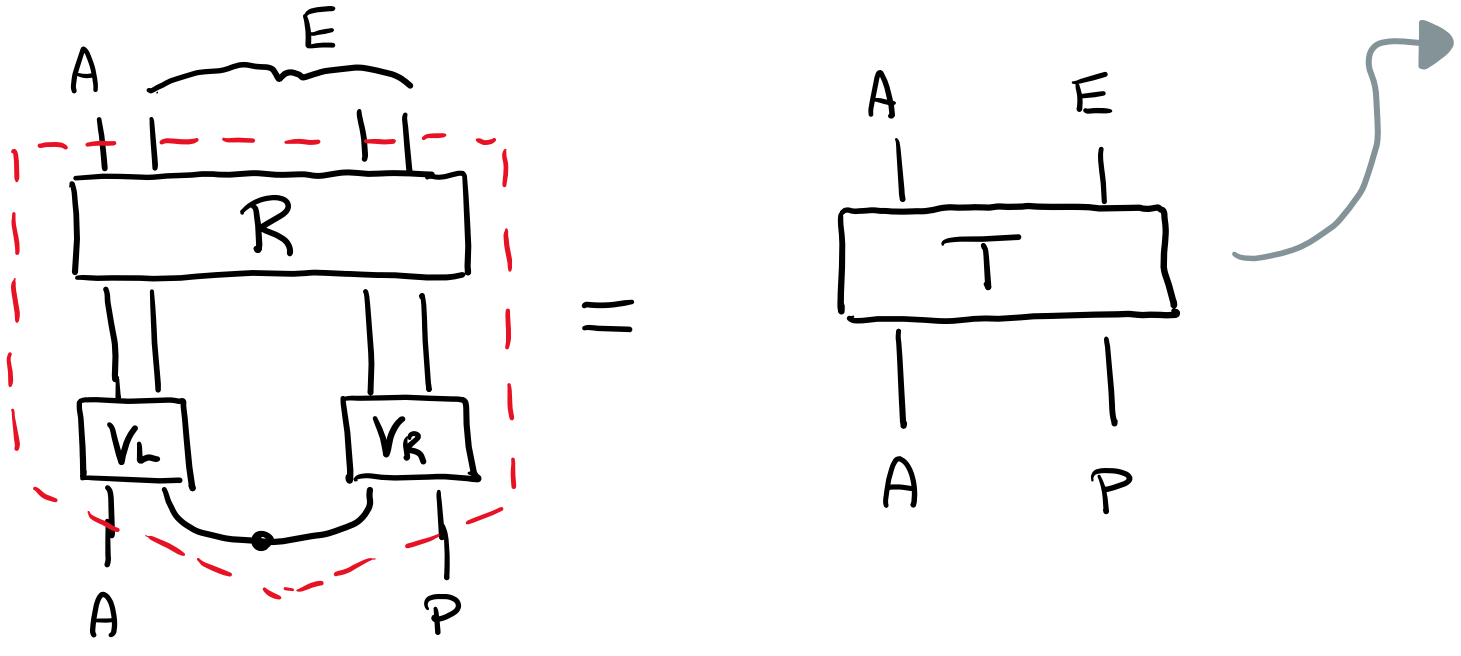

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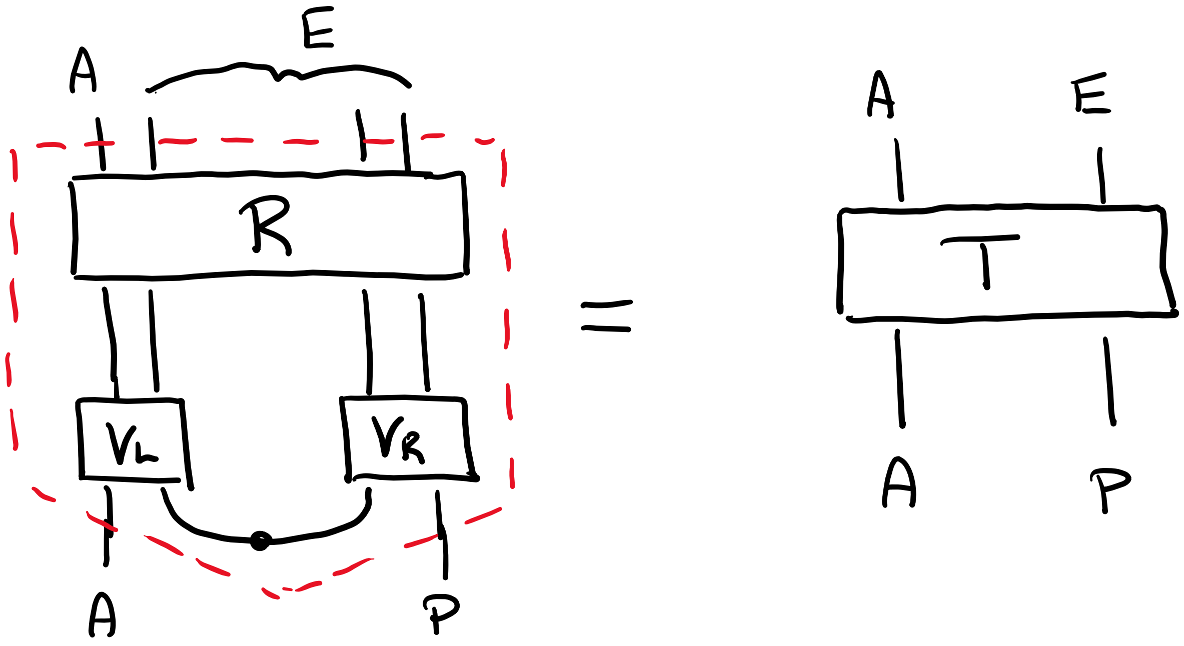

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Means: most unitaries U^{ε} cannot be implemented inside the black hole

Computationally forbidden unitaries

- So far this isn't very interesting:

$$\log(S_{bh}) \leq n_A \quad ; \quad n_p \ll S_{bh}$$

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- Not all strings of length K require K bits to describe:

$$d(\text{random string}) = \text{length of string}$$

$$d(000\ldots0) = d(\text{"all zero string of length K"}) = \log K$$

- Claim there are U^ε with short descriptive complexity that are forbidden.

Computationally forbidden unitaries

- Interpreting $\mathcal{E}$ as a binary number, get a natural ordering on the $U^{\mathcal{E}}$

Computationally forbidden unitaries

- Interpreting ε as a binary number, get a natural ordering on the U^ε

Define: U^{ε_0} = "the first U^ε such that $p(T,\varepsilon) < \delta$"

ensures uniqueness

$\mathbb{E}_\varepsilon\, p(T,\varepsilon) \leq c\,\frac{n_p}{2^{n_A}}$

guarantees this exists.

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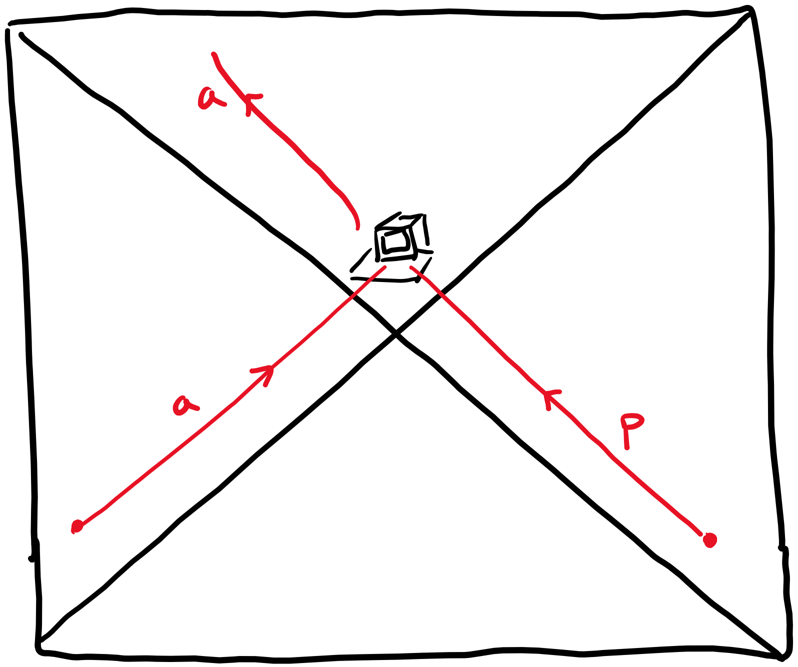
$$\boxed{d(U^{\varepsilon_0}) = O(\log S_{bh})}$$

Recap

- We prepare our computer, and record into it the (short) description of $U^{\mathcal{E}_0}$
- A universal computer could, given enough steps/memory, compute $\mathcal{E}_0$ and apply $U^{\mathcal{E}_0}$



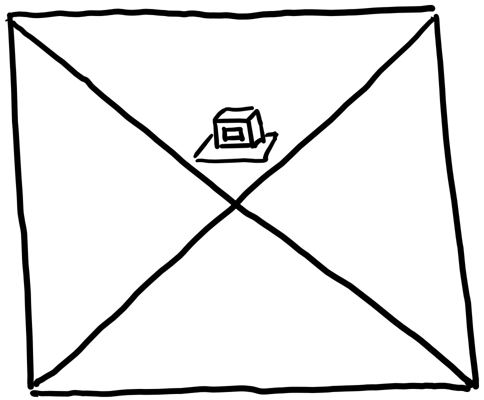
- Our computer in the bulk cannot run this computation, no matter how it is built or operates.

Summary

Summary of our construction

Inside view:

A universal computer, can run any program



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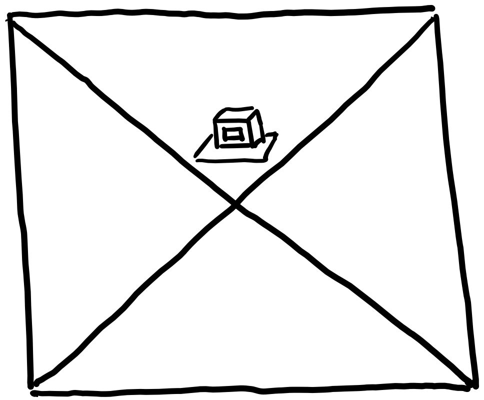

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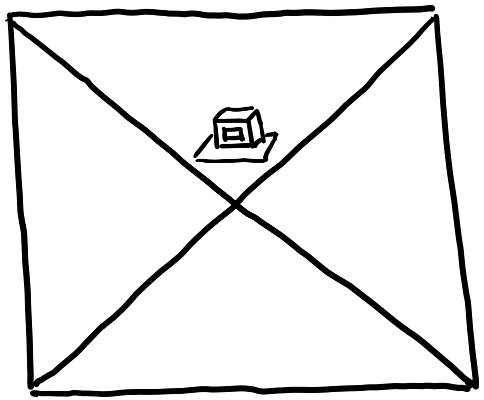

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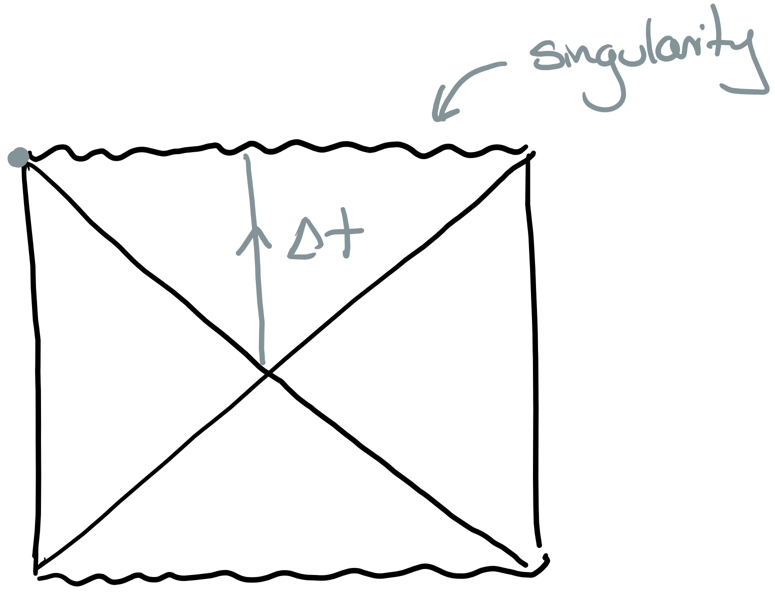
Comment: To find forbidden computations, feed computer a description of T, which describes the spacetime computer lives in

→ Similar to "diagonalization"

Returning to time inside the black hole

- Assuming we can build a universal computer inside the black hole, if we had ∞ time could do any computation.



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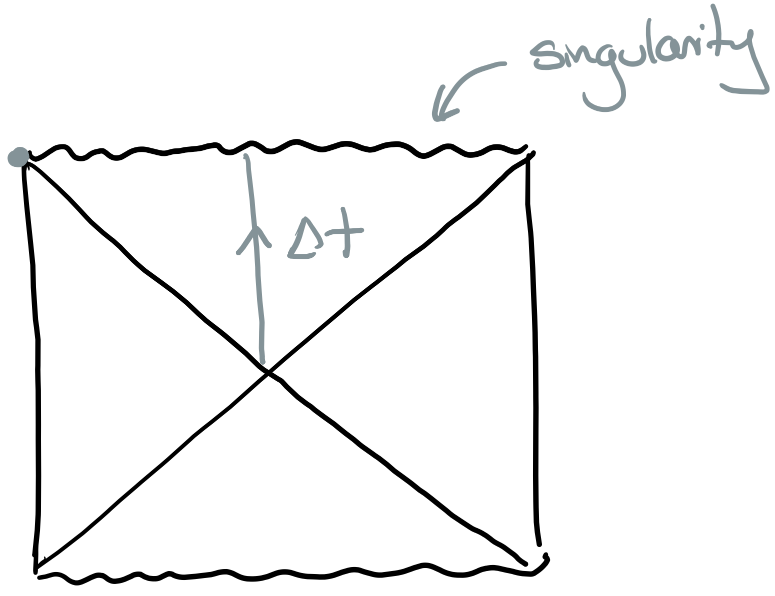

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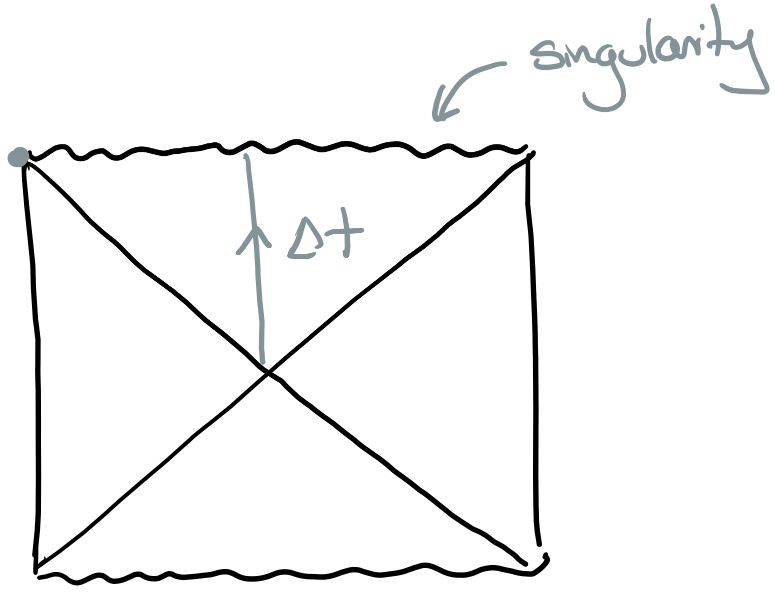

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- Future direction: If have forbidden computation that takes time T, black hole must have $\Delta t < T$.

Can we prove bounds on Δt in General relativity?