CENTRO DE CIENCIAS DE BENASQUE PEDRO PASCUAL

Gravitational waves meet effective field theories

TESTING EFTS: AMPLITUDES, GRAVITATIONAL WAVES, AND CAUSALITY

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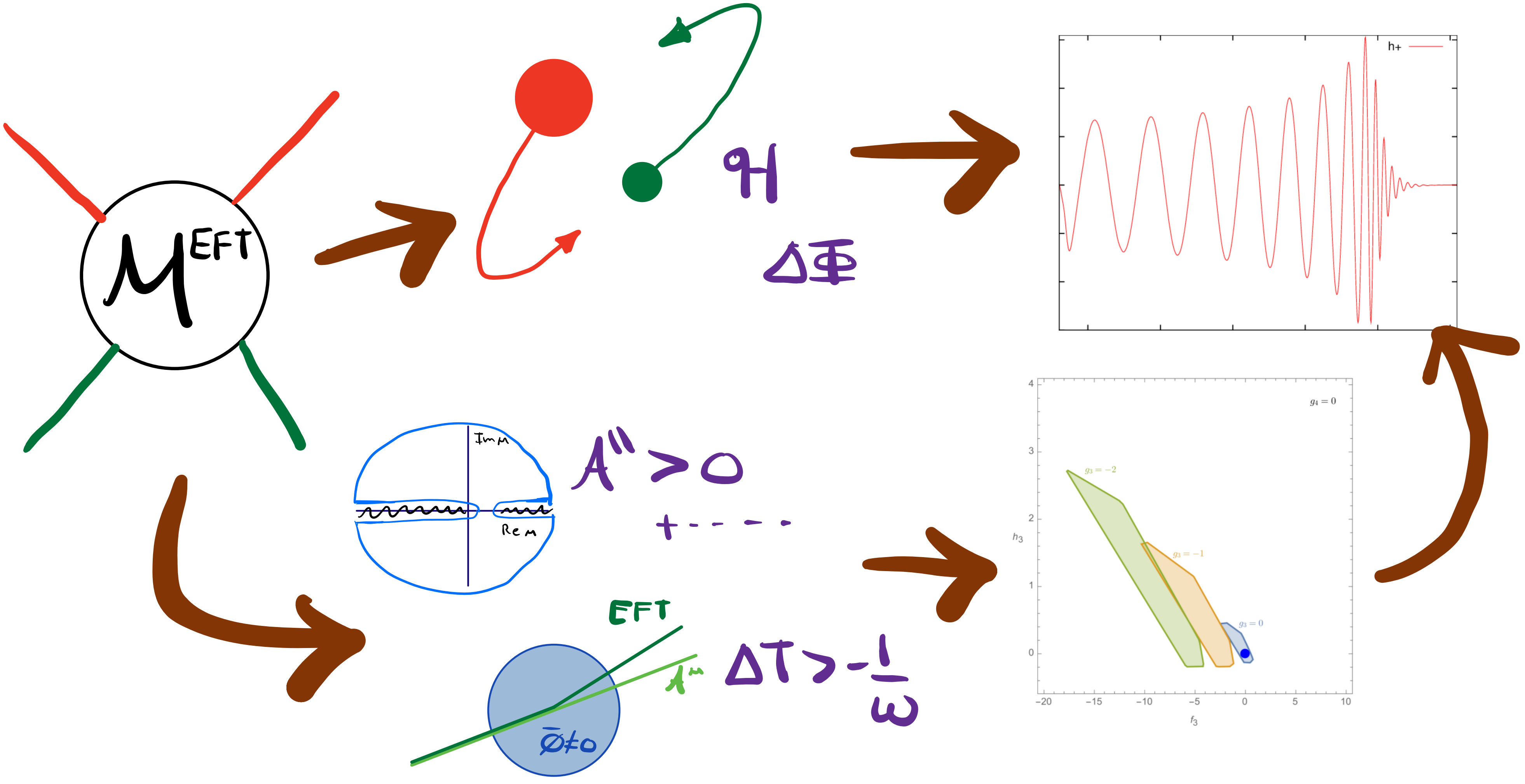
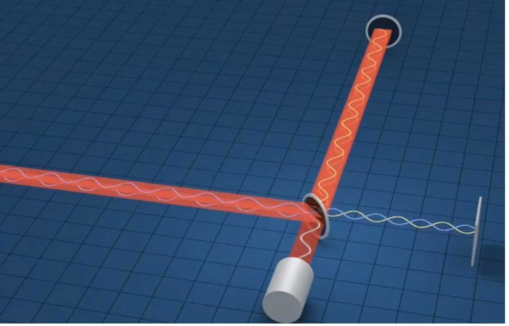
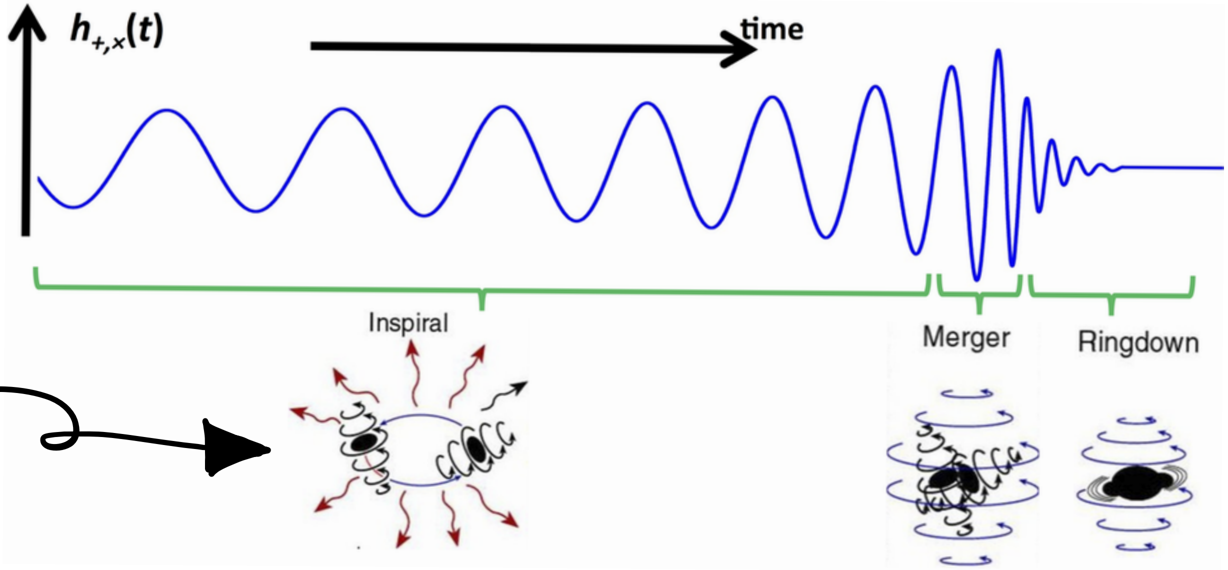


$\mathcal{M}^{EFT}$
$\mathcal{H}$
$\Delta\Phi$
h+
Im μ
Re μ
$A'' > 0$
$+ \ldots$
EFT
A''
$\bar{\phi} \neq 0$
$\Delta T > -\frac{1}{\omega}$
$g_4 = 0$
$g_3 = -2$
$g_3 = -1$
$g_3 = 0$
h_3
f_3
4
3
2
1
0
−20
−15
−10
−5
0
5
10

Motivation





Perturbative gravity

Can test GR extensions

PN

$$\frac{Gm}{r} \sim v^2 \ll 1$$

EFT methods

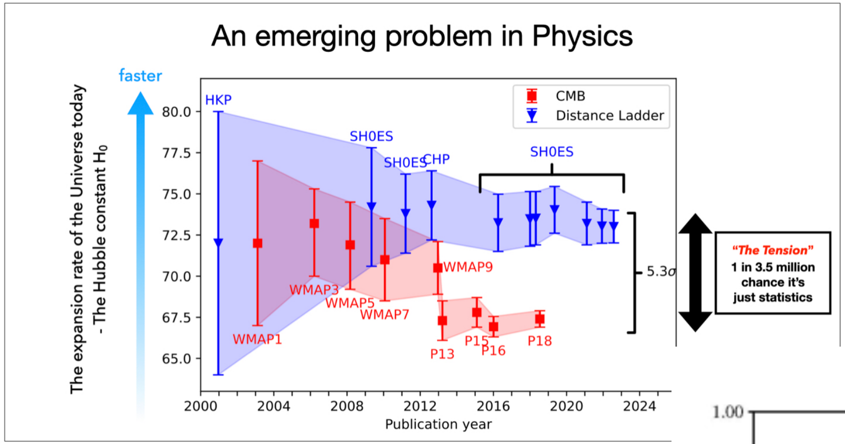
PM

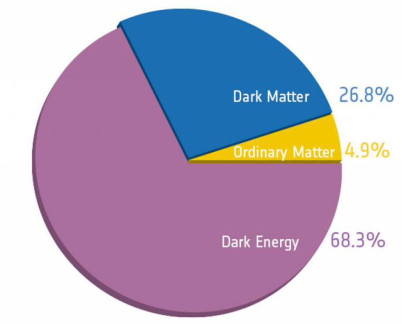
$$\frac{Gm}{r} \ll v^2 \sim 1$$ + perturbative QFT regime

WHY GO BEYOND GR?

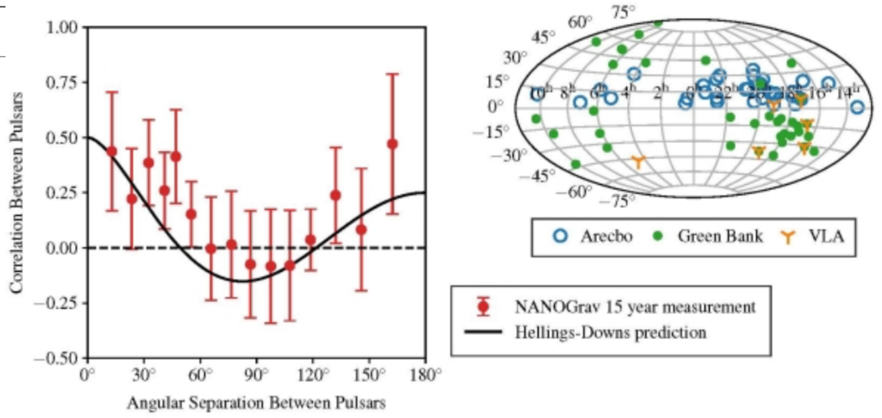
Missing explanation for dark matter and dark energy

Cosmological tensions





Observed stochastic gravitational wave background of unknown origin



Beyond GR

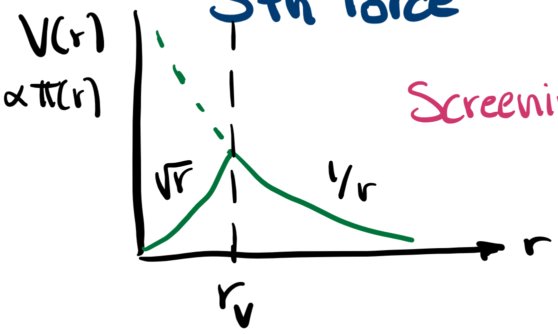
$h: \pm 2 \quad \pm 1 \quad 0$ ★

Generically arise in theories with:

- higher dim.
- higher spin states
- massive gravitons

Strong constraints from Solar System tests

5th Force



Screening

or

Weak coupling

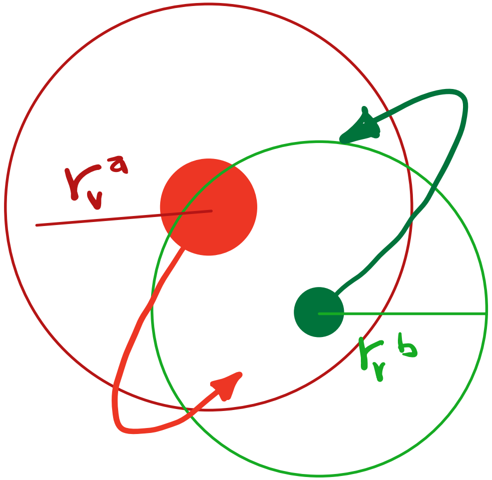
eg. Cubic Galileon

$$\mathcal{L} = -\frac{1}{2}(\partial\pi)^2 - \frac{c}{\Lambda^3}\Box\pi\,(\partial\pi)^2$$

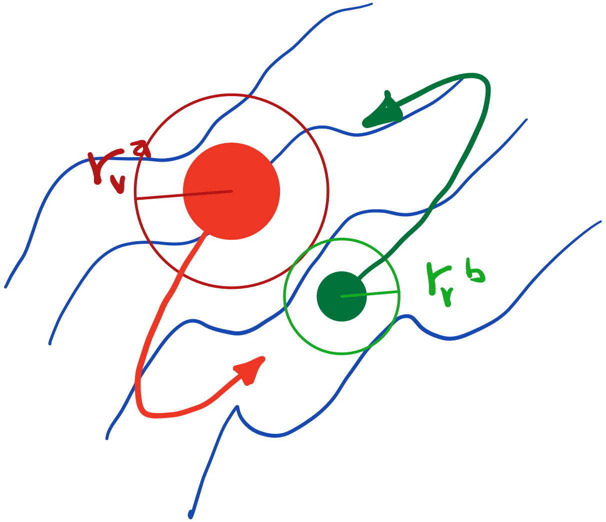
shift symmetry / soft limit

$$\delta\pi = c + b_\mu x^\mu \Rightarrow A \sim p^2$$

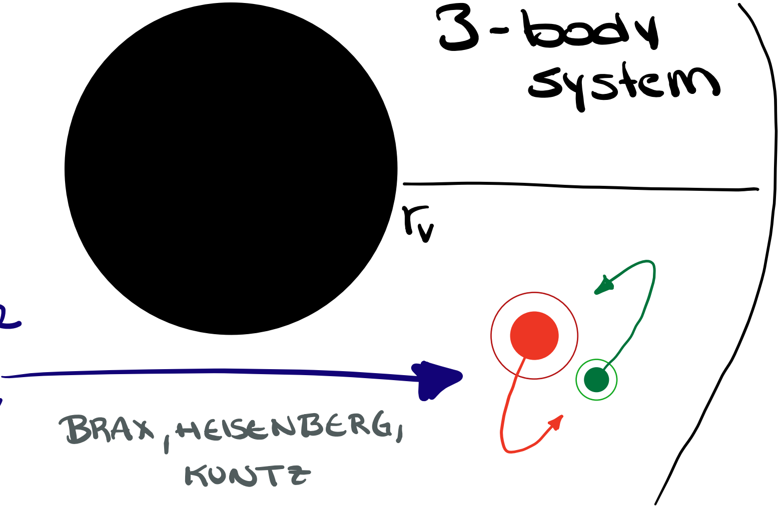
Expectation in vacuum



Redressed $r_V(g)$ in a background: $\pi = \pi_0 + \delta\pi$



3-body system



Perturbative regime

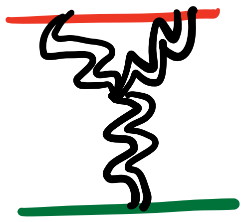
BRAX, HEISENBERG, KUNTZ

Classical GR from Amplitudes

1

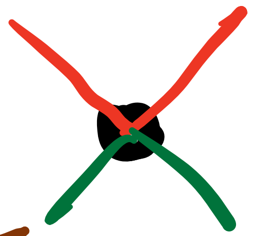
Post-Minkowskian expansion

Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng … Herrmann
Smirnov, Smirnov; Manohar, Ridgway, Shen;
(4PM → 5PM)



classical loops

2

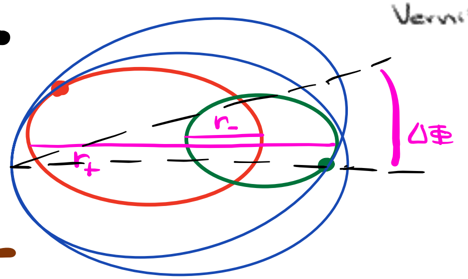


Hamiltonian

matching EFT's

3

ΔX → B2B



Other related approaches:

- **Worldline (QFT, EFT)**
  Levi, Yin, Morales, Kim
  Edison; Kälin, Porto, Neef,
  Liu, Dapla; Mogul, Plefka,
  Steinhoff, Jakobsen
  Sauer, Xu; Riva,
  Vernizzi, Mougiakakos
  …
- **Eikonal**
  Di Vecchia, Heissenberg,
  Russo, Veneziano,
  Georgoudis, Vazquez-
  Holm; …
- **HEFT**
  Brandhuber, Travaglini;
  Chen; Aoude, Haddad, Helset
  …

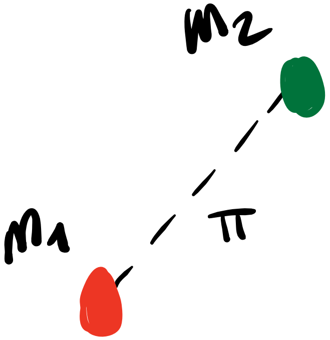
+ spin + radiation … See SNOWMASS 2204.05194

BINARY SYSTEMS BEYOND GR

2107.11384:
MCG, de Rham, Tolley

Amplitude methods beyond minimal couplings and GR

Test simple scenario: Cubic Galileon



Decoupling Limit $M_{Pl} \to \infty$, Λ fixed

Conformally coupled matter

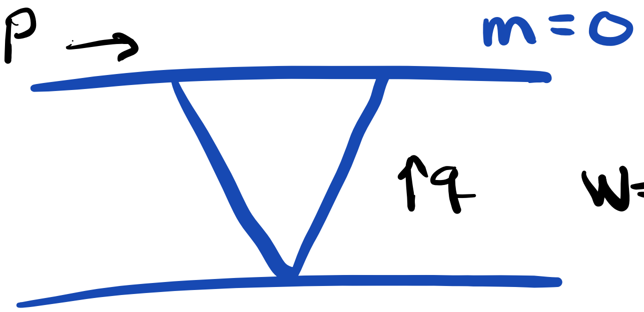
Spinless compact objects

CLASSICAL REGIME

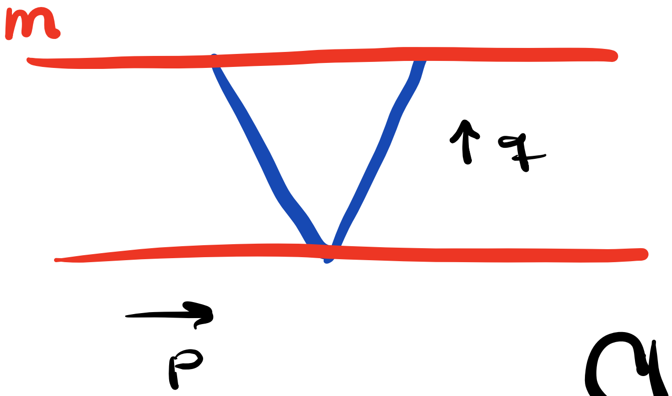
"Usual counting"

$$\hbar^{I-V+1} = \hbar^{L}$$

loop = quantum



$$W = \frac{\hbar}{i}\log Z \qquad I: \frac{i\hbar}{q^2 - i\epsilon} \qquad V: e^{\frac{i}{\hbar}\int \mathcal{L}_{int}}$$



$$I_m: \frac{i\hbar}{q^2 + \frac{m^2}{\hbar^2} - i\epsilon} \qquad \hbar\left(\sqrt{\frac{m^2}{q^2\hbar^2}}\right) \sim \hbar^0$$

Classical contribution from loops!

1 Identify Classical Contributions

Restore $\hbar$: $G \to \frac{G}{\hbar}$ $\quad q \to \hbar q$

Classical non-linearities

$$\epsilon^{NL} = \frac{h}{M_{pl}} \propto G m q \hbar^0 \ll 1$$

Post-Minkowskian expansion

Quantum corrections

$$\epsilon^{Q} = \frac{\partial^2}{M_{pl}^2} \propto q^2 G \hbar \ll \epsilon^{NL}$$

$$\epsilon^{Qm} = \frac{\partial}{m} \propto \frac{q}{m} \hbar \ll \epsilon^{NL}$$

Classical limit

Identify Classical Contributions

wave number $\rightarrow q \sim 1/b \leftarrow$ impact parameter

Classical non-linearities

$$\epsilon^{NL} = \frac{h}{M_{pl}} \propto \frac{r_s}{b} \ll 1$$

Post-Minkowskian expansion

Quantum corrections

$$\epsilon^{Q}/\epsilon^{NL} \propto \lambda_c/b \ll 1$$

$$\epsilon^{Qm}/\epsilon^{NL} \propto \lambda_c/r_s \ll 1$$

Classical limit

Classical non-linearities

$$\epsilon^{NL} = r_s q \hbar^0 \ll 1$$

Quantum corrections

$$\epsilon^{Q} = q^2 G \hbar \ll \epsilon^{NL}$$

$$\epsilon^{Qm} = \frac{q}{m} \hbar \ll \epsilon^{NL}$$

Post-Minkowskian expansion

$$\epsilon^{NL} = \frac{\partial\partial\pi}{\Lambda^3} = r_v q \hbar^0 \ll 1$$

$$= \left(\frac{gm}{M_{pl}}\right)^{1/3} \frac{c}{\Lambda}$$

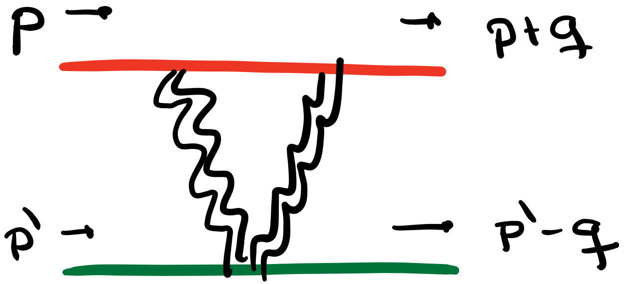
Classical limit

$$\epsilon^{Q} = \frac{\partial^2}{\Lambda^2} = \frac{q^2}{\Lambda^2} \hbar^{1/3} \ll \epsilon^{NL}$$

$$\partial \to \lambda_c / r_v \ll 1$$

Classical physics from loops

2PM



$$\mathcal{A} \sim \frac{G m_1^2 m_2^2}{q^2} (G m_1 q)$$

Non-analytic

Generic EFT classical expansion

$$\mathcal{A} \sim \frac{G m_1^2 m_2^2}{q^2} (r_s q)^n (r_v q)^{3m}$$

GR / Matter coupling

Galileon

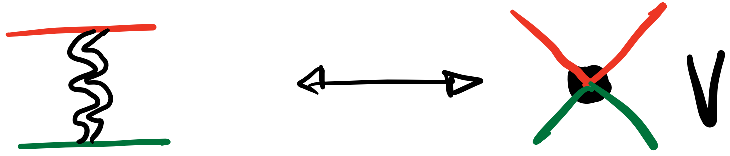
2 Matching EFTs

GR: Cheung, Rothstein, Solon

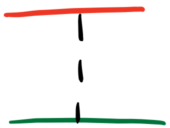
$$\frac{\mathcal{M}^{clas}}{4E_1E_2} = \mathcal{M}^{EFT}$$

$$\mathcal{L}^{EFT} \supset V\, \Phi_{\bar{p}}\, \Phi_{\bar{p}'}\, \Phi_{-\bar{p}}\, \Phi_{-\bar{p}'}$$

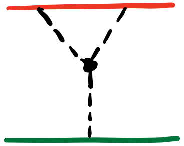
$\mathcal{O}(G)$



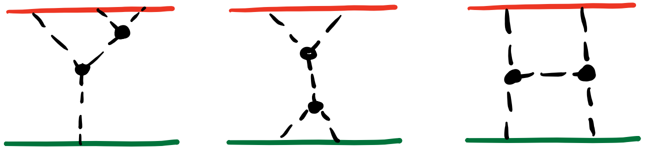
$\mathcal{O}(r_v^0)$



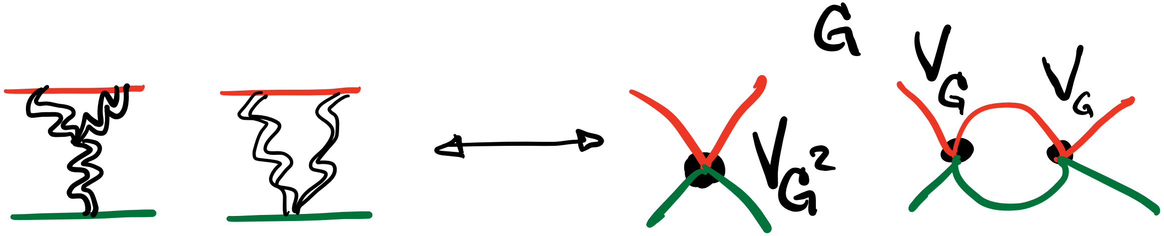
$\mathcal{O}(r_v^3)$



$\mathcal{O}(r_v^6)$



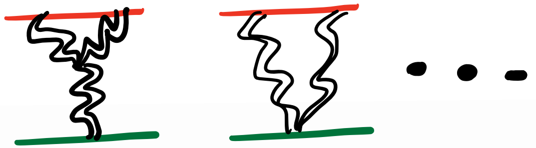
$\mathcal{O}(G^2)$



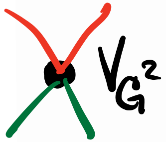
$\to \mathcal{H}$

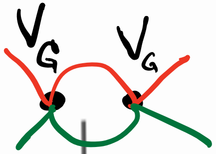
. . . . .

* Consider canonically normalized states for matching



$$V(\mathbf{p},\mathbf{p}+\mathbf{q})\big|_{\mathcal{O}(g^n)} = -\frac{1}{4E_1(\mathbf{p})E_2(\mathbf{p})}\mathcal{M}_4(\mathbf{p},\mathbf{p}+\mathbf{q})\Big|_{\mathcal{O}(g^n)}$$





$$-\frac{1}{4E_1(\mathbf{p})E_2(\mathbf{p})}\int_{\boldsymbol{k}}\frac{\mathcal{M}_4(\mathbf{p},\mathbf{k})\mathcal{M}_4(\mathbf{k},\mathbf{p}+\mathbf{q})}{4E_1(k)E_2(k)(E_p-E_k+i\epsilon)}\Bigg|_{\mathcal{O}(g^n)}$$

CLASSICAL POTENTIAL: $V(\bar{p},\bar{r}) = \cancel{\hbar^3}\int\frac{d^3\bar{q}}{(2\pi)^3}e^{-i\bar{q}\cdot\bar{r}}\,\frac{V(\bar{p},\bar{p}+\bar{q})}{\cancel{\hbar^3}}$

Same as Lippmann-Schwinger Eq. approach

Cristofoli, Bjerrum-Bohr, Damgaard, Vanhove

3 Scattering Angle

Conserved energy $\rightarrow$ $\mathcal{H} = E$ $\rightsquigarrow$ $|p|^2 = |p_\infty|^2 - V_{eff}(E, r)$

Implicit function theorem $= |p_\infty|^2 - \frac{1}{2E} \mathcal{M}_{cl}(|p_\infty|, r)$

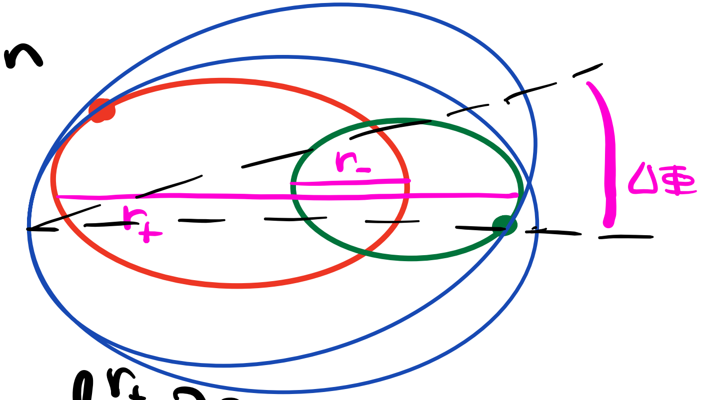
$$\boxed{\chi = \sum_{k=1}^{\infty} \frac{2b}{k!} \int_0^\infty du \left(\frac{d}{db^2}\right)^k \frac{\mathcal{M}_{cl}^k \, (u^2+b^2)^{k-1}}{(2E)^k \, |p_\infty|^{2k}}}$$

$\leftarrow$ for any EFT

Bjerrum-Bohr, Cristofoli, Damgaard; Kälin, Porto; Damour; ...

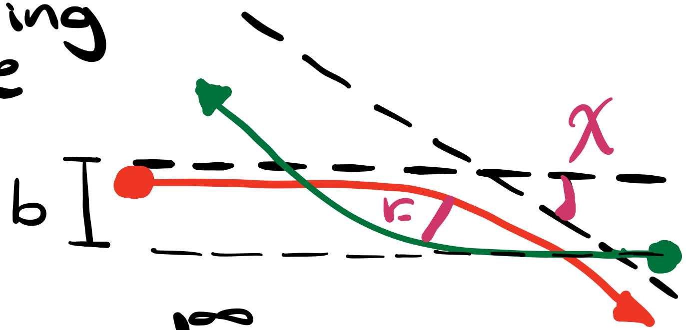
BINARY SYSTEMS AND SCATTERINGS: BOUNDED VS UNBOUNDED

Perihastron advance



$$\Delta\Phi = -2\int_{r_-}^{r_+} \frac{\partial p_r}{\partial J} - \pi$$

Scattering angle



$$\chi = -2\int_{r_-}^{\infty} \frac{\partial p_r}{\partial J} - \pi$$

$$\boxed{\Delta\Phi = \chi(J,E) + \chi(-J,E)}$$

Kälin, Porto

See also : van de Meent

* without spin and for aligned spins

New contribution to $\Delta\Phi$

$$\Delta\Phi = C_{Gr_v^3}\left(\frac{gGM^2}{J}\right)\left(\frac{r_v^3 |P_\infty|^2 M}{J^3}\right)$$ 1PM

$$+ C_{G^2}\left(\frac{gGM^2}{J}\right)^2$$ 2PM $+ \cdots$

+ gravitons contribution 2PM

Resummation?

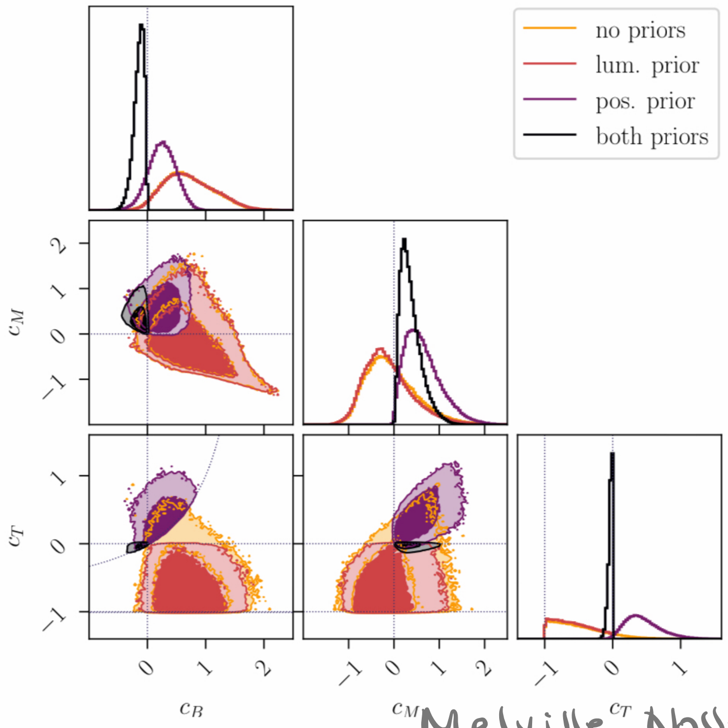
Access screened region.

Leading contribution depends on the size of $g\, C_{Gr_v^3}\left(\frac{r_v^3 |P_\infty|^2 M}{J^3}\right)$

Bounds on $C_{Gr_v^3}$?

Importance of bounds on Wilson coefs.

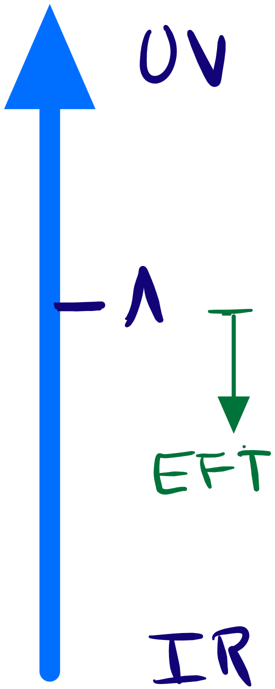
Theoretical priors can drastically change estimations of parameters



Melville, Noller

CAUSALITY BOUNDS ON EFTS

CAUSAL EFFECTIVE FIELD THEORIES



$$\mathcal{L} = \Lambda^4 \sum_n c_n \Lambda^{-n} \mathcal{O}_n$$

What are the allowed c_n?

1) UV = string theory $\rightarrow$ Swampland Conjectures

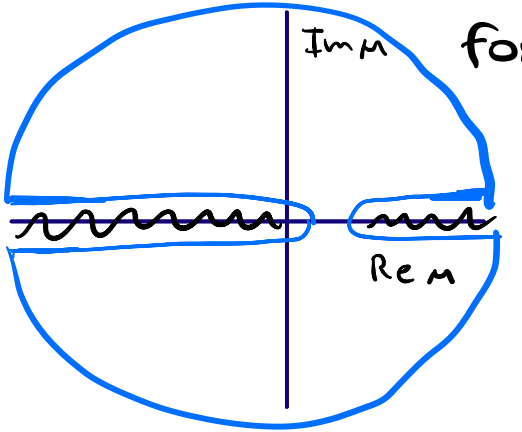
CAUSAL EFFECTIVE FIELD THEORIES

$$\mathcal{L} = \Lambda^4 \sum_n c_n \Lambda^{-n} \mathcal{O}_n$$

What are the allowed c_n?

2) UV = local, unitary, causal, Lorentz invariant → Positivity bounds

1 2 3 4



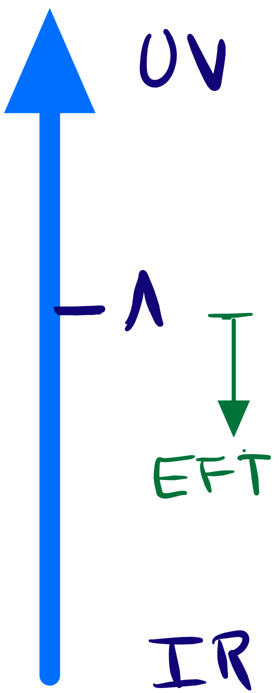
forward lim. ($t \to 0$)

$$A''(s) = \oint \frac{d\mu}{2\pi i} \frac{A(\mu)}{(\mu - s)^3} \overset{1,3}{=} \Big(\underbrace{\int_{-\infty}^{0} + \int_{4m^2}^{\infty}}_{\text{related by 4}} \Big) \frac{\text{Im}\, A}{(\mu - s)^3} \overset{2}{>} 0$$

IR

UV

CAUSAL EFFECTIVE FIELD THEORIES



$$\mathcal{L} = \Lambda^4 \sum_n c_n \Lambda^{-n} \mathcal{O}_n$$

What are the allowed c_n?

1) UV = string theory → Swampland

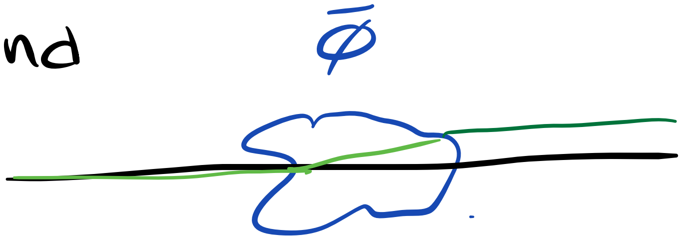
2) UV = local, unitary, causal, Lorentz invariant → Positivity bounds

3) Causal IR propagation → CAUSALITY BOUNDS

CAUSALITY

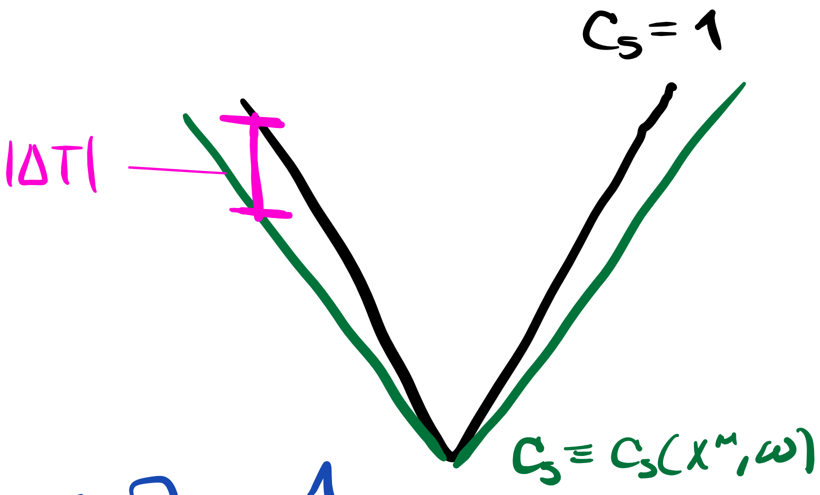
$$\boxed{G_R(x-y)=0 \quad \text{for} \quad (x-y)^2>0}$$

- Consider local propagation of information around a fixed background



Diagnose acausality by looking at time delay

$$\Delta T = -i\langle in|\,\hat{S}^\dagger \frac{\partial}{\partial\omega}\hat{S}\,|in\rangle$$



$$|\Delta T| \lesssim \lambda \sim \frac{1}{\omega}$$

Unresolvable

$$\hookrightarrow \boxed{\Delta T \gtrsim -\frac{1}{\omega}}$$

CAUSALITY + WKB + EFT

Consider $\phi = \bar{\phi} + \varphi$. Find ΔT experienced by φ.
Solve linearized φ eom using WKB approximation.

$$\boxed{\omega \Delta T \gtrsim -1}$$

$$\boxed{\underbrace{\frac{\lambda^{\text{background}}}{\lambda^{\text{perturbation}}}}_{\gg 1 \text{ WKB}} \underbrace{\int_{X \subset \mathbb{R}^{1+3}} \left(1 - c_s(\lambda^{\text{pert.}})\right)}_{= -\epsilon \text{ EFT} \to |\epsilon| \ll 1} \gtrsim -1}$$

CAUSALITY BOUNDS ON SCALAR EFTS

2207.03491:

MCG, de Rham, Pozsgay, Tolley

CAUSALITY INSIGHTS ON SCALAR EFTS

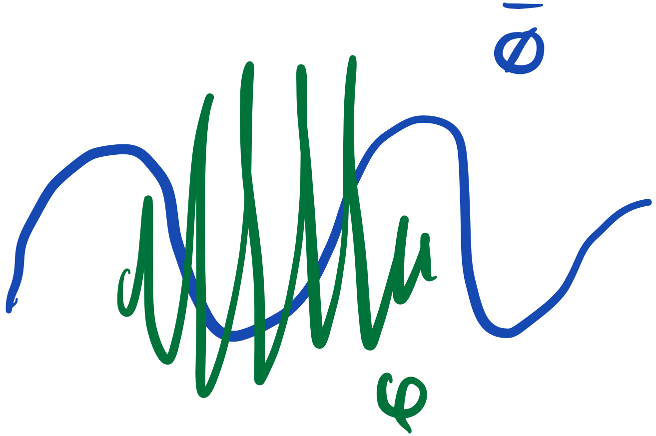
$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{g_8}{\Lambda^4}\left((\partial\phi)^2\right)^2$$

$$+ \frac{g_{10}}{\Lambda^6}(\partial\phi)^2\left((\partial\partial\phi)^2 - (\Box\phi)^2\right) + \frac{g_{12}}{\Lambda^8}\left((\partial\partial\phi)^2\right)^2 - g_{\text{matter}}\,\phi J$$

quartic galileon

external source

CAUSALITY INSIGHTS ON SCALAR EFTS



$\phi = \bar{\phi} + \varphi$ $\qquad \partial_\mu \varphi = i k_\mu \varphi$ plane waves

EOM + Disp. rel $\rightarrow c_s$

Adams et al

$$c_s^2 \simeq \left(1 - \# \frac{g_8}{\Lambda^4} \overbrace{\frac{(k\cdot\partial\bar{\phi})^2}{|k|^2}}^{>0} + \# \frac{g_{10}}{|k|^2\Lambda^6}\left((k_\mu \partial^\mu \partial_\rho \phi)^2 - \Box\phi \, (k_\mu k_\nu \partial^\mu \partial^\nu \phi) \right) - \# \frac{g_{12}}{\Lambda^8} \overbrace{\frac{(kk\cdot\partial\partial\phi)^2}{|k|^2}}^{>0} \right)$$

Work with

$$g_8 = \begin{cases} 1 \\ 0 \end{cases}$$

PROPAGATION AROUND SPHERICALLY SYMMETRIC BACKGROUNDS

Spherically-symmetric background $\bar{\phi} \equiv \Phi_0 f(r/r_0)$

Perturbations

$$\rightarrow \chi_\ell''(r/r_0) + \underbrace{(\omega r_0)^2}_{\gg 1 \ \text{WKB}} \underbrace{\frac{1}{c_s^2(\omega,r)}\left(1 - \frac{V_\ell^{eff}(r)}{(\omega r_0)^2}\right)}_{\equiv W_\ell(\omega,r) \ \ll 1 \ \text{EFT}} \chi_\ell(r/r_0) = 0$$

$\rightarrow$ Find $\delta \rightarrow \Delta T$

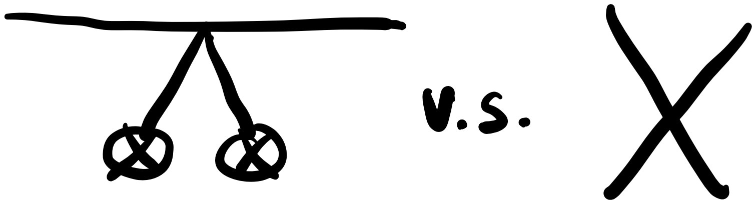
WKB : $\dfrac{\lambda_{bckgnd}}{\lambda_{pert}} = \omega r_0 \gg 1$

EFT : $\dfrac{\partial\phi}{\Lambda}, \ \dfrac{\partial^{p+1}\phi}{\Lambda^{p+2}} \ll 1$

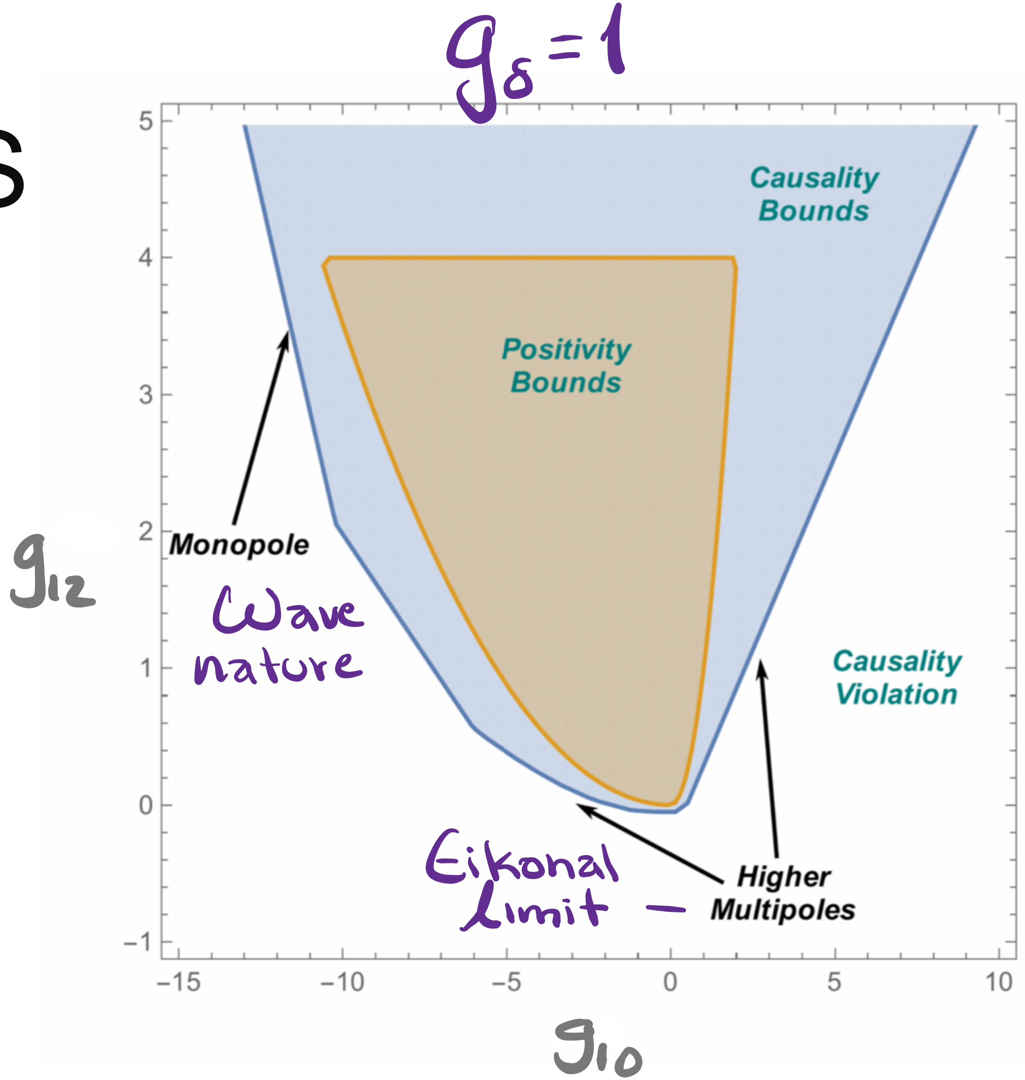
$$\frac{\Phi_0}{r_0\Lambda}, \ \frac{1}{r_0\Lambda} \sim \mathcal{O}(\epsilon) \qquad \frac{\omega}{r_0\Lambda^2} \sim \mathcal{O}\left(\epsilon\frac{\omega}{\Lambda}\right)$$

Note bound on ω $\longrightarrow$ $\epsilon \overset{\text{WKB}}{<} \dfrac{\omega}{\Lambda} \overset{\text{EFT}}{<} \dfrac{1}{\epsilon}$

CAUSALITY VS POSITIVITY



No upper bound on g_{12} from causality due to WKB technical issues.



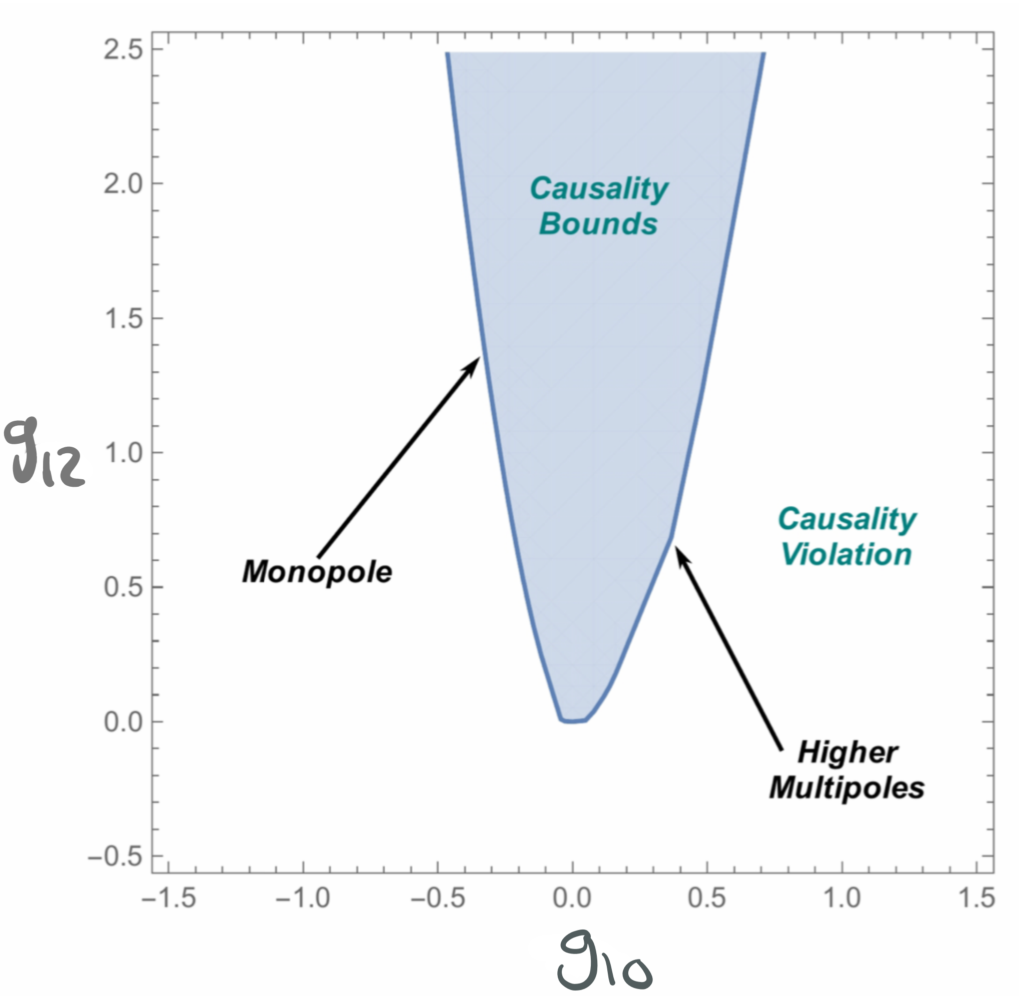
ADDITIONAL SYMMETRIES: GALILEONS

From positivity:

$g_8 = 0 \Rightarrow g_{10} = g_{12} = 0$

valid around $\langle \phi \rangle = 0$

$g_8 = 0$



CAUSALITY AND POSITIVITY BOUNDS ON PHOTON EFTS

2307.04784

MCG, de Rham, Jaitly, Pozsgay, Tokareva

PHOTON EFT

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &+\frac{c_1}{\Lambda^4}F^{\mu\nu}F_{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}+\frac{c_2}{\Lambda^4}F^{\mu\nu}F^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta} \\ &+\frac{c_3}{\Lambda^6}F^{\alpha\mu}F^{\nu\beta}\partial_\mu F_{\beta\gamma}\partial_\nu F_\alpha{}^\gamma+\frac{c_4}{\Lambda^6}F^{\alpha\mu}F^{\nu\beta}\partial_\beta F_{\mu\gamma}\partial^\gamma F_{\alpha\nu}+\frac{c_5}{\Lambda^6}F^{\alpha\mu}F^{\nu\beta}\partial_\beta F_{\nu\gamma}\partial^\gamma F_{\alpha\mu} \\ &+\frac{c_6}{\Lambda^8}F^{\mu\nu}\partial_\mu F_{\nu\rho}\partial^\rho\partial^\alpha F^{\beta\gamma}\partial_\alpha F_{\beta\gamma}+\frac{c_7}{\Lambda^8}F^\mu{}_\gamma\partial_\mu F_{\nu\rho}\partial^\nu F_{\alpha\beta}\partial^\rho\partial^\gamma F^{\alpha\beta} \\ &+\frac{c_8}{\Lambda^8}F^{\mu\gamma}\partial_\mu F_{\nu\rho}\partial^\rho\partial^\beta F_{\alpha\gamma}\partial^\alpha F^\nu{}_\beta\,.\end{aligned}$$

Dim. 8

F^4 : c_1, c_2

Dim. 10

$\partial^2 F^4$: c_3, c_4, c_5

Dim. 12

$\partial^4 F^4$: c_6, c_7, c_8

PHOTON EFT

$$\mathcal{A}^{++++} = f_2\left(s^2+t^2+u^2\right) + f_3 stu + f_4\left(s^2+t^2+u^2\right)^2$$
$$\mathcal{A}^{++--} = g_2 s^2 + g_3 s^3 + g_4 s^4 + g_4' s^2 tu$$
$$\mathcal{A}^{+++-} = h_3 stu$$

$+\mathcal{O}(s^5)$

Dim. 8

F^4 : f_2, g_2 $= 1, 0$

Dim. 10

$\partial^2 F^4$: f_3, g_3, h_3

Dim 12

$\partial^4 F^4$: f_4, g_4, g_4'

PROPAGATION AROUND SPHERICALLY SYMMETRIC BACKGROUNDS

$$A = \bar{A} + \delta A\,, \qquad \bar{A} = \Phi_0 f(r/r_0)\, dt$$

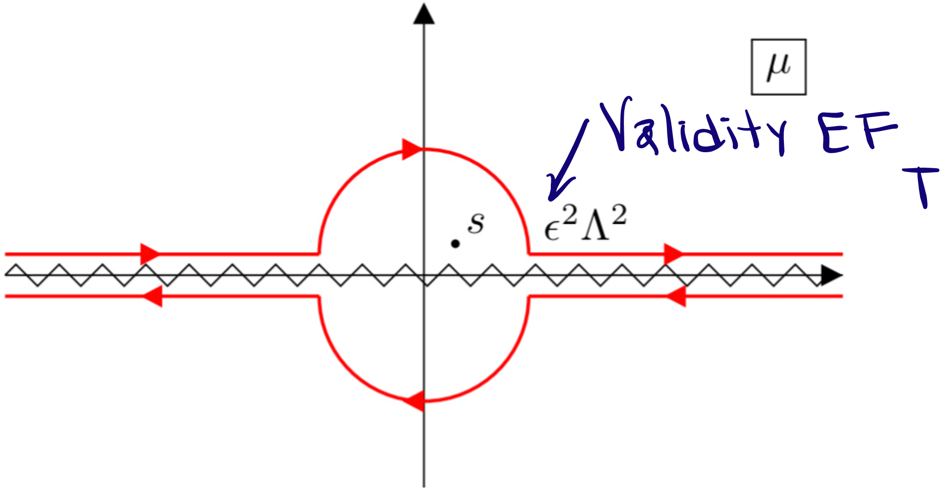
$$\chi_\ell^{even\,\prime\prime} + (\omega r_0)^2 W_\ell^{even}(\omega, r)\, \chi_\ell^{even} = 0$$

$$\chi_\ell^{odd\,\prime\prime} + (\omega r_0)^2 W_\ell^{odd}(\omega, r)\, \chi_\ell^{odd} = 0$$

$$\Rightarrow \quad \Delta T^{even} > -1/\omega$$

$$\Delta T^{odd} > -1/\omega$$

Positivity Bounds



Other explorations:
Henriksson, Mc Peak, Russo, Vichi; Häring, Hebbar, Karateev, Meineri, Penedones

$$\boxed{\frac{1}{2\pi \mathrm{i}}\int_\delta \frac{\mathcal{A}(\mu,t)}{(\mu-s)^3}\,\mathrm{d}\mu = \int_{\epsilon^2\Lambda^2}^{\infty}\frac{\mathrm{d}\mu}{\pi}\frac{\mathrm{Disc}_s\mathcal{A}_s(\mu,t)}{(\mu-s)^3} + \int_{\epsilon^2\Lambda^2-t}^{\infty}\frac{\mathrm{d}\mu}{\pi}\frac{\mathrm{Disc}_s\mathcal{A}_u(\mu,t)}{(\mu-u)^3}}$$

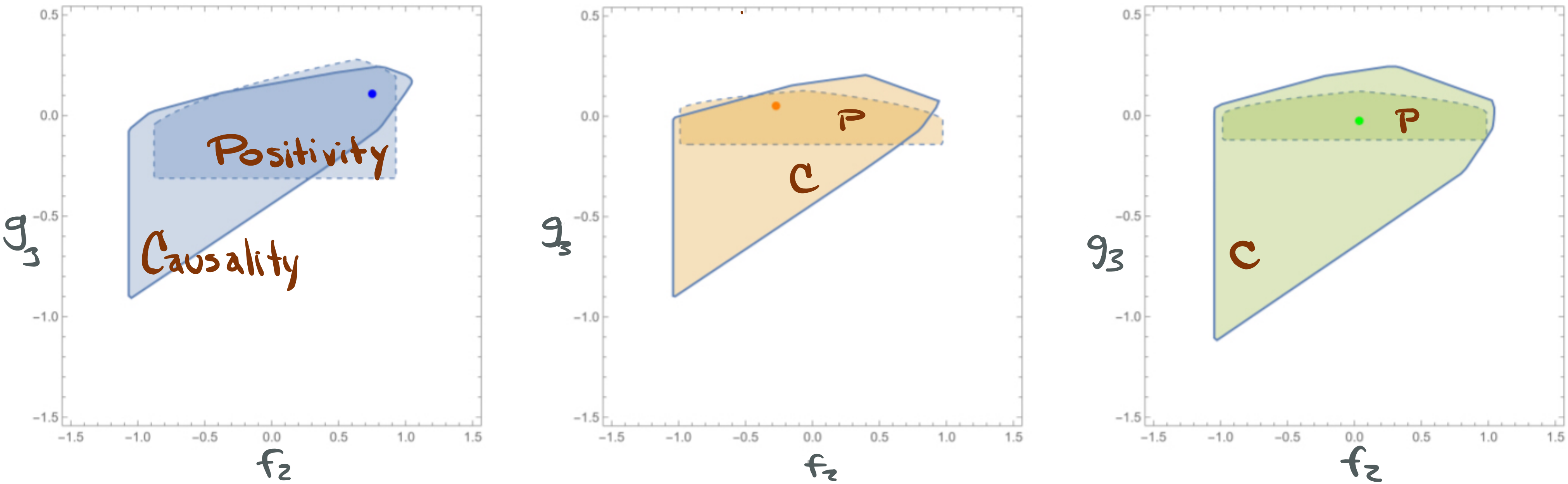
Simple example: forward limit $(t\to 0)$

$$\frac{1}{2\pi i}\int_\delta \frac{A(\mu,0)}{\mu^3}\,d\mu > 0$$

For $A(\mu) = g_2\mu^2 + \alpha\mu^4\log(\mu)$

$$\Rightarrow \quad g_2 + \frac{\alpha}{2}\epsilon^4 > 0$$

QED-like partial UV completions



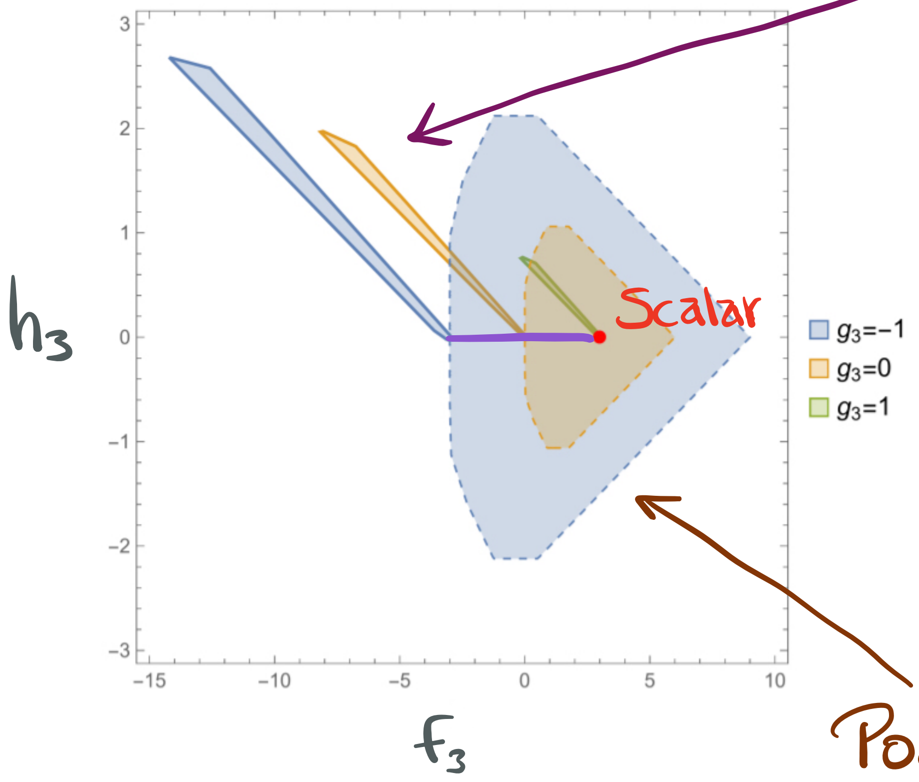
(a) Scalar (b) Spinor (c) Vector

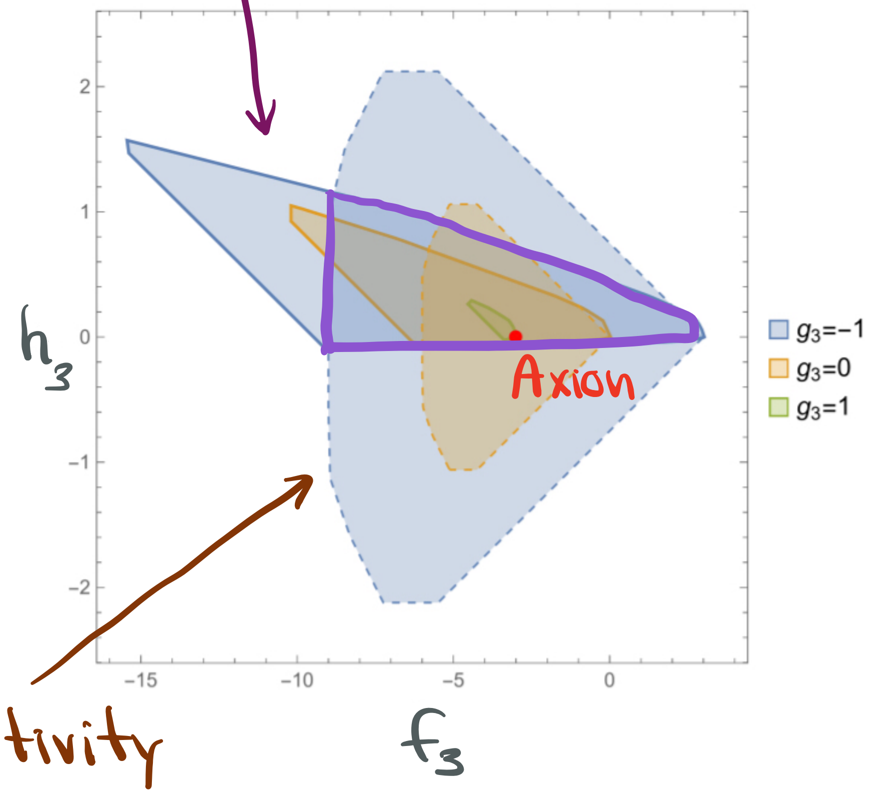
DIMENSION 10

$$\boxed{(f_3 + 3g_3) < X^{even} + \xi h_3}$$

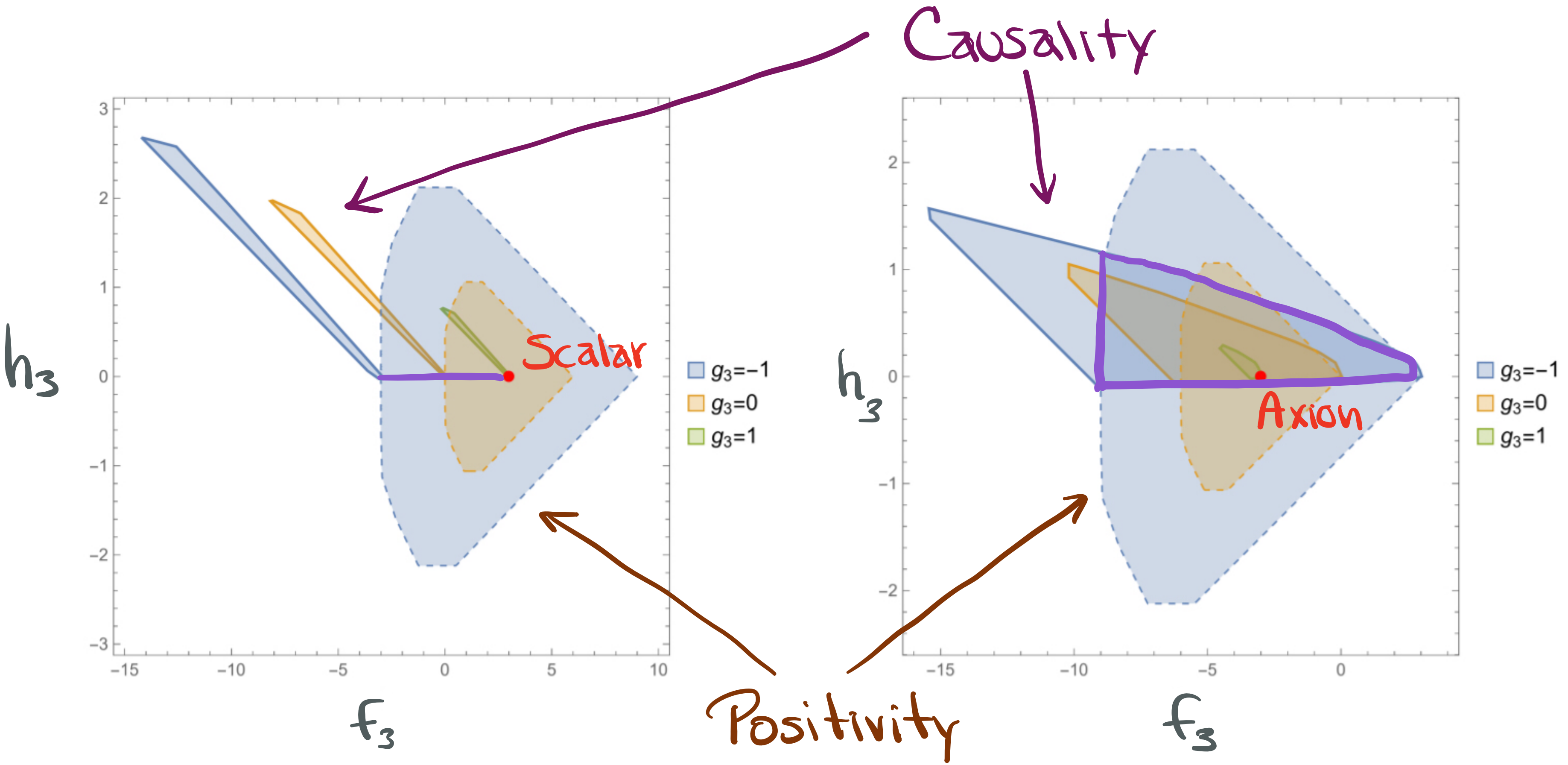
$$\boxed{-X^{odd} - \epsilon h_3 < f_3 - 3g_3 + 4h_3 < X^{odd} + \epsilon h_3}$$

Causality

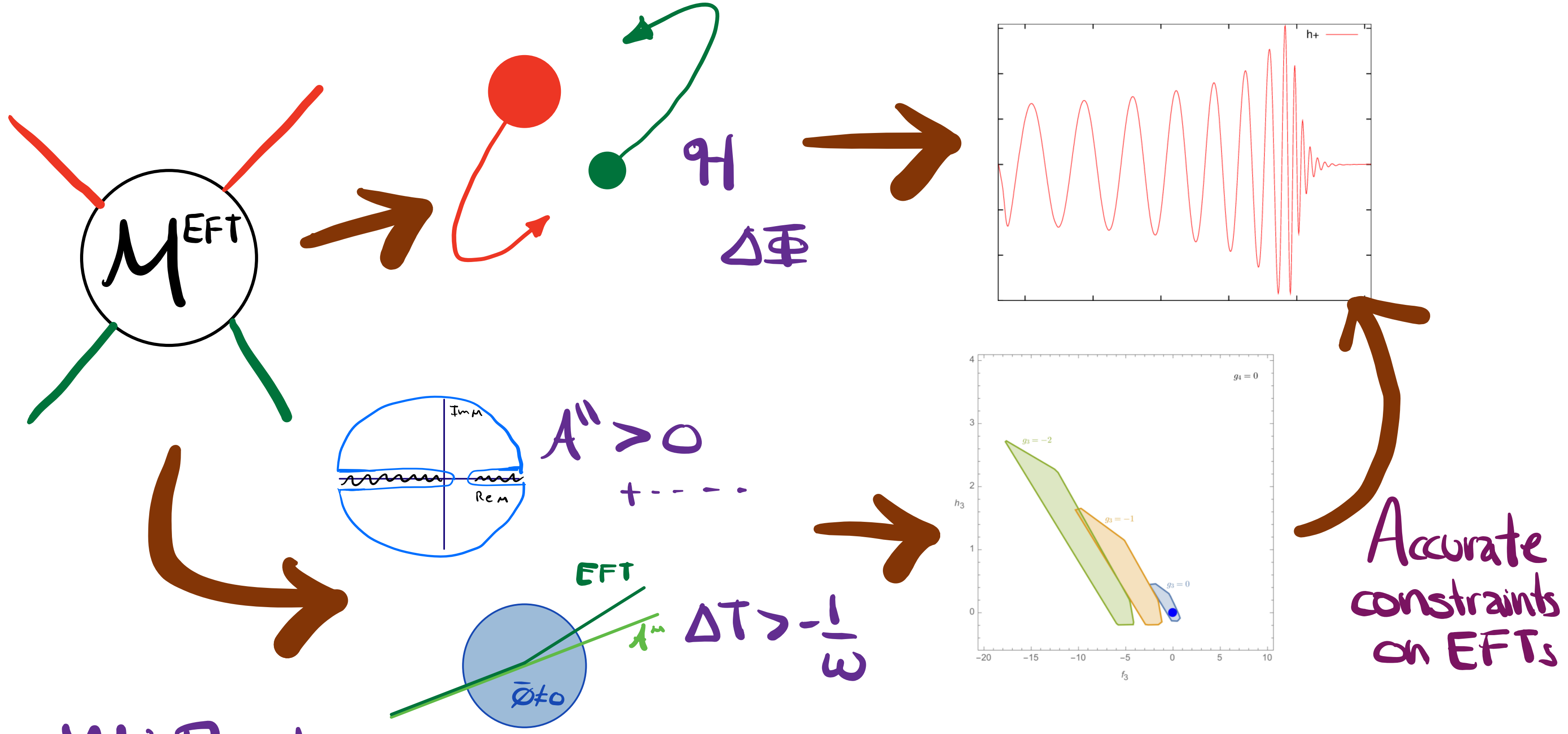




Positivity



Positivity and Causality test different regions of the parameter space, NOT in confict w/ each other



W.i.P. +

Future directions

- Test higher dimensional gravitational operators
- Cosmological backgrounds ✓ + EFT of inflation