Beyond General Relativity and the strongly gravitating/dynamical regime

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CIFAR

Based on:

- Cayuso R-Franca-Figueras-LL. 2303.07246 [gr-qc]
- Cayuso R-LL. 2005.13720 [gr-qc]
- Allwright-LL. 1808.07897 [gr-qc]
- Cayuso J- Ortiz-LL. 1706.07421 [gr-qc]

Also some relevant… (non-linear/backreacting studies)

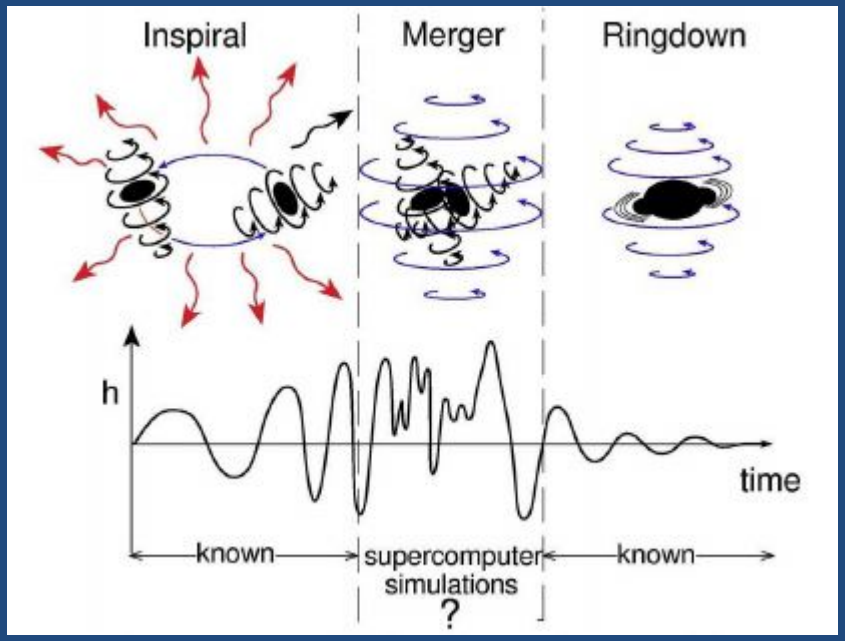
- *Held-Lim: 2306.04725*
- *Corman-East. 2210.09235 [gr-qc]*
- *Arestelo,Klough,Figueras. 2208.14470 [gr-qc]*
- Franchini-Bezares-Barausse-LL. 2206.00014 [gr-qc]
- Bernard-Luna-LL. 1904.12866 [gr-qc]
- Ripley-Pretorius. 1902.01468 [gr-qc]
- *Barausse-Palenzuela-Ponce-LL. 1212.5053 [gr-qc] (+ following works)*

Data Analysis side

- Dideron-Mukherjee-LL. 2209.14321 [gr-qc]

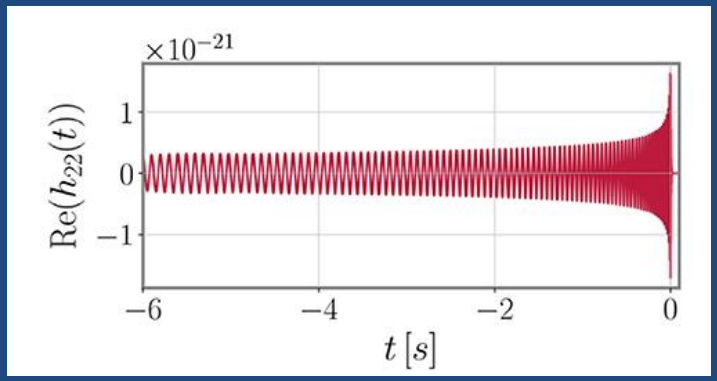
Outline

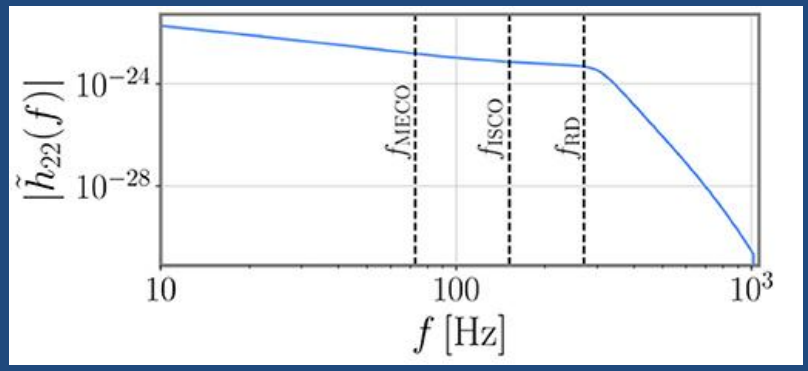
- Motivation
- Dissecting a gravitational wave train



- Beyond GR?
  - Beyond ‘phenomenology/wishful models/calculations’?
- Analysis?

Current types of tests



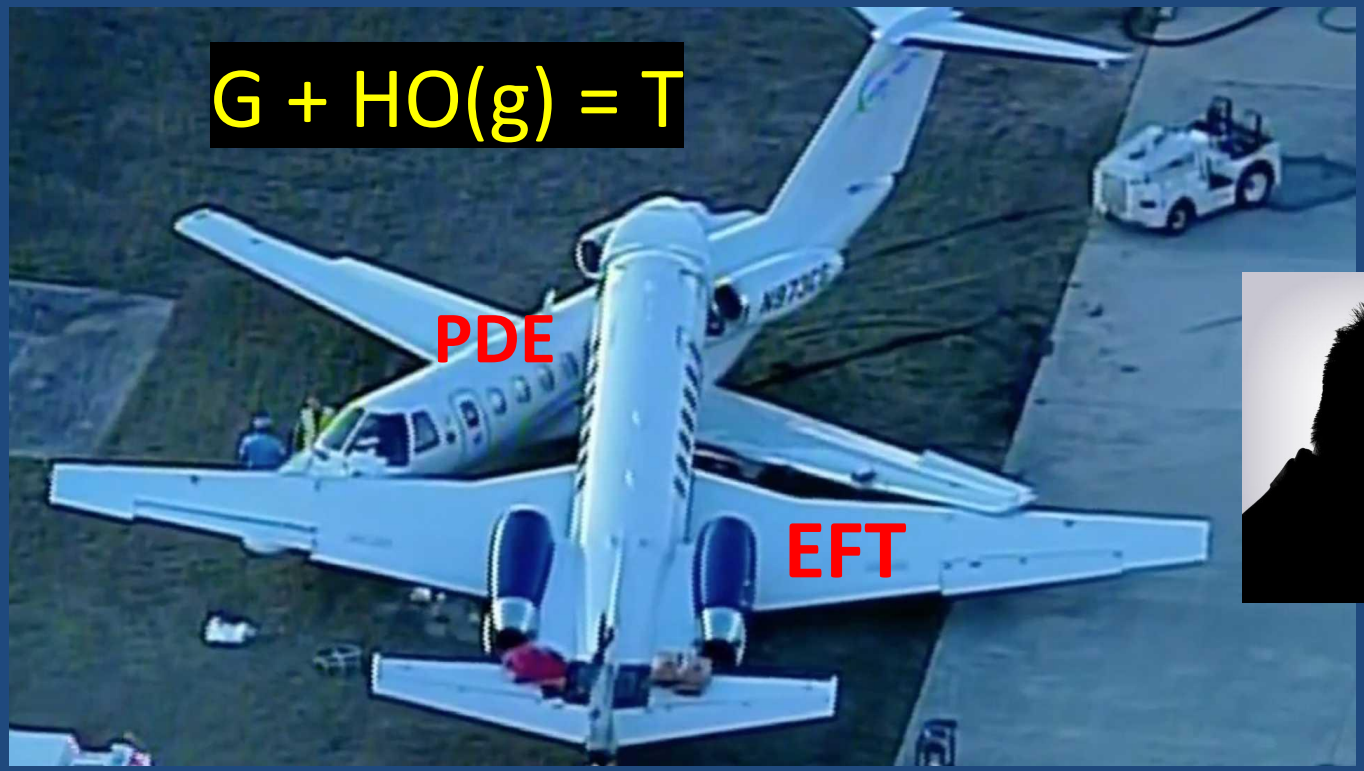


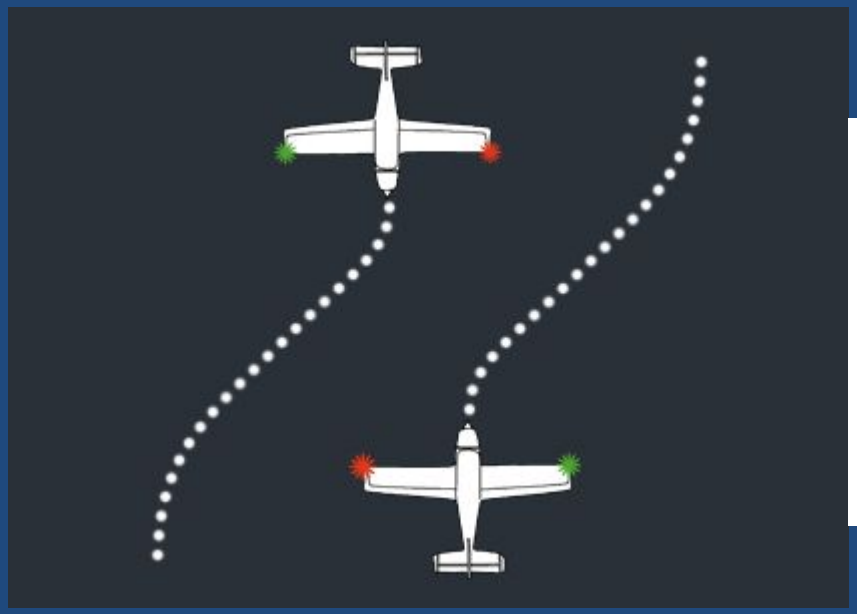
[Schmidt]

- *Null tests of GR* ⟶ consistency with GR vs not. Do we know what to expect in GR completely?

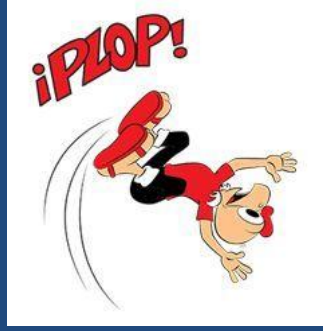
- *Parameterized tests of GR* ⟶build deviations in inspiral, merger, RD (pPN,pPE,RD,deformed match). But, stages are not independent
  - Go solely on each stage with smoking guns? (polarizations, dipolar radn, QNMs, echoes). Do we know what to expect?

- *Full waveforms* in specific theories. Can this be done self-consistently? ⟶ in most cases not without further steps


G + HO(g) = T
PDE
EFT



Warning for EFT: Already in GR



GR is rich!, theorems of stability of Minkowski and singularities hint at a rich phenomenology.

- way out of the latter is a BH. Would it be the same in beyond GR (and why)?

$$G_{ab} = S(g)_{ab} + T_{ab}$$

- Assumptions of special symmetries and linearization studies, not necessarily justified (linearization stability?)

$$G^L_{ab} = S^L(g^B)_{ab} + T_{ab}$$

Beyond GR?

Options?

- *Model Building:* specific theories built from key assumptions of new physics. E.g. Brans-Dicke, Horndenski, dCS, Einstein-Aether…

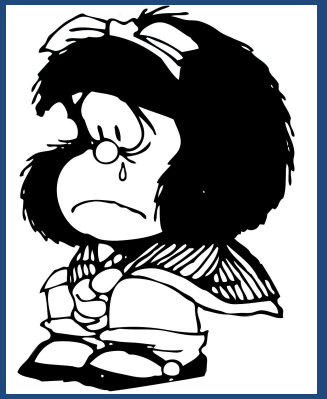
- *Effective Field Theories (EFTs):* no need for 'new' degrees of freedom (as they are integrated out), and new phenomena arises through short scale interactions organized in higher derivatives

- *In all cases, a richer structure of underlying PDEs, and tempting to think 'corrections are small → can deal with things easily'*


¡¡¡Paren el mundo,
que me quiero bajar!!!

Simple analogy of a potential problem 0

- Secular effects….
  - harmonic oscillator (and reduction of order)



$$\ddot{x} + x + \epsilon x^3 = 0$$

$$\text{with } x = x_0 + \epsilon x_1 + \ldots$$

$$\ddot{x}_1 + x_1 = -(1/4)\cos(3t) - (3/4)\cos(t)$$

$$x(t) = \cos(t) + \epsilon((1/32)(\cos(3t) - \cos(t)) - (3/8)t\cos(t))$$

Resum...

$$x = \cos([1 + (3/8)\epsilon + \ldots]t) + \epsilon/32\cos(3[1 + 3\epsilon/8 + \ldots]t)$$

Simple analogy of a potential problem 1

- Lorentz-Abraham-Dirac equation

$$\frac{du^\mu}{d\tau} = \frac{q}{mc} F^{\mu\nu} u_\nu + \frac{2q^2}{3mc^3}\left(\frac{d^2u^\mu}{d\tau^2} + \frac{a^\nu a_\nu}{c^2} u^\mu\right),$$

- 2nd order ODE? 3rd order?, timescales?
- Spurious solutions/runaway behavior
  - reduction of order?
  - secular effects?

Simple analogy of a potential problem 2

- Einstein equations ⟶ linearly degenerate
- propagation speed of perturbations (largely) independent of state of the field

- Beyond GR?
- $u_{,tt} = (u + u_{,i}\, u_{,i})\, u_{,xx}$ [e.g. in Horndenski]
- $u_{,tt} = (u + u_{,ii})\, u_{,xx}$ [e.g. in Horndenski]

Consequence? Loss of hyperbolicity (--> elliptic region) or *even worse*

Simple analogy of a potential problem 3

- Consider the following systems (e.g. heat eqn -> hyperbolic case [Geroch])

$$u_{,t} = kq_{,x}$$
$$q = u_{,x}$$

$$u_{,t} = kq_{,x}$$
$$\tau q_{,t} = \left(-q + u_{,x}\right)$$

Simple analogy of a potential problem 4

- Consider the following systems (e.g. ADM formulation)

$$u_{,t} = u_{,x} + v_{,x}$$
$$v_{,t} = v_{,x}$$

$$u_{,t} = u_{,x} + \Pi$$
$$v_{,t} = v_{,x}$$
$$\tau \Pi_{,t} = -\Pi + v_{,x}$$

And so what to do?

- Exploit further identities, promoting curvature scalar/tensor as new independent variables [Noakes -> Held-Lim]
  - Not always possible even with 2nd order corrections

- Explore, if at a specific theory, conditions could be chosen to at least locally establish ‘well posedness’ [Kovacs-Reall -> Corman-East, Figueras]
  - Higher derivatives get in the way in many cases

- *Agnostically ‘fix’ equations, ensure hyperbolicity, account for full back-reaction within reasonable scales [$>L_c$] not requiring further structure.*

EFT route

- Higher energies degrees of freedom: 'integrated out', their role appear as higher order contributions from low energy variables
  - E.g. Euler equations $\longrightarrow$ Navier-Stokes equations (viscous contribution, transport coefficient η)

- For gravity -> action ~ $R + \lambda (R)^p$
  - Generically introducing mathematical pathologies (even going beyond math PDE classifications)
  - Further, assessment of mathematical soundness clashes with EFT 'wavelength bounds'

- Application [Cayuso R,LL]

$$S_{\rm eff} = \int d^4x\sqrt{-g}\,2M_{\rm pl}^2\left(R - \frac{\mathcal{C}^2}{\Lambda^6} - \frac{\tilde{\mathcal{C}}^2}{\tilde{\Lambda}^6} - \frac{\tilde{\mathcal{C}}\mathcal{C}}{\Lambda_-^6}\right)$$

$$\mathcal{C} \equiv R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}\,, \quad \tilde{\mathcal{C}} \equiv R_{\alpha\beta\gamma\delta}\tilde{R}^{\alpha\beta\gamma\delta}\,,$$

[Endlich,Gorbenko,Huang,Senatore]

- EOMS -> G_{ab} $[g/L^2] \sim \lambda\, F(g^3/L^8)$
- No generic rotating BH solns known (only slowly rotating)
- Inspiral? corrections induce ‘structure’ on the BH which can be captured by tidal “Love numbers” (entering at 5PostNewtonian order ⟶ dephasing wrt to GR waveforms)
- Merger? ringdown?

[Cayuso, Franca, Figueras, LL ‘23]

some ugly details R_{ab} -½ $R\, g_{ab}$ =H_{ab}

$$H_{\mu\nu} = \mathcal{C}\Big[\Box R_{\mu\nu} - \tfrac{1}{2}\nabla_\mu\nabla_\nu R - \tfrac{1}{16}\mathcal{C}\, g_{\mu\nu} - R_{\mu\lambda}R^{\lambda}{}_{\nu} + R^{\alpha\beta}R_{\mu\alpha\nu\beta} + \tfrac{1}{2}R_{\mu\sigma\rho\lambda}R_{\nu}{}^{\sigma\rho\lambda}\Big] + 2(\nabla^\alpha\mathcal{C})\big[\nabla_\alpha R_{\mu\nu} - \nabla_{(\mu}R_{\nu)\alpha}\big] + R_{\mu}{}^{\alpha}{}_{\nu}{}^{\beta}\nabla_\alpha\nabla_\beta\mathcal{C}\,. \quad (2)$$

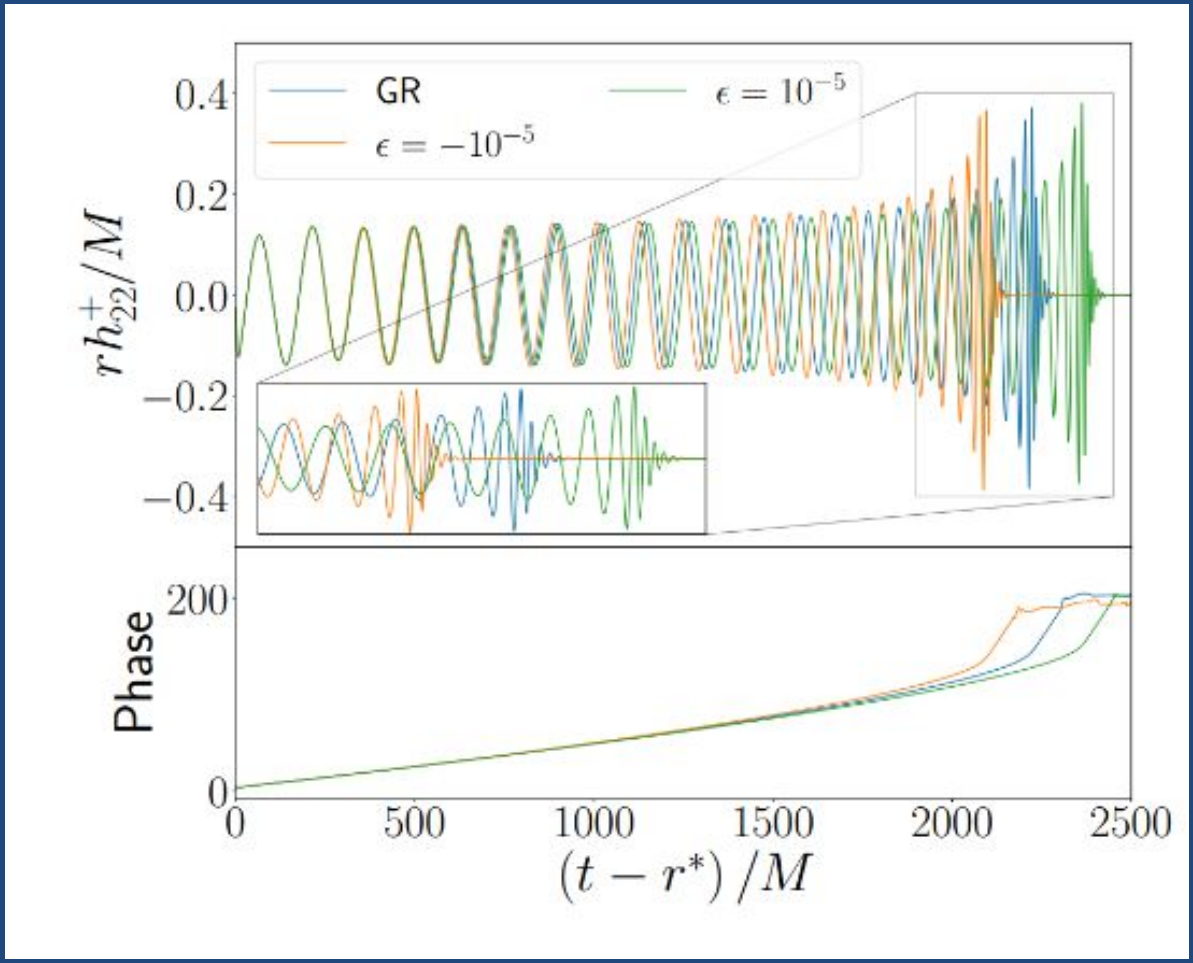
$$G_{\mu\nu} = \epsilon\left(4\mathcal{C}\,W_{\mu}{}^{\alpha\beta\gamma}W_{\nu\alpha\beta\gamma} - \tfrac{g_{\mu\nu}}{2}\mathcal{C}^2 + 8\,W_{\mu}{}^{\alpha}{}_{\nu}{}^{\beta}\nabla_\alpha\nabla_\beta\mathcal{C}\right), \quad (3)$$

Must deal with… $\Box(\phi) = \lambda\partial_t^4\phi \simeq \lambda\partial_t^2(\partial_x^2\phi)$

inspired by Israel-Stewart ‘fixing’ of relativistic hydrodynamics…

$$\Box\phi = -\epsilon\,\partial_t^2\hat{C}\,,$$
$$\tau\partial_0\hat{C} + \sigma(\partial_t^2 - 2\beta^i\partial_{ti} + \beta^i\beta^j\partial_{ij})\hat{C} = C(\phi) - \hat{C}\,,$$

- Add a further variable $\hat{C}$, with its own equation that drives it to C , within some timescale τ.
  - ‘Restores’ a d.o.f. which had been integrated out
  - Controls high frequencies, ensuring solution is well behaved



Waveform characteristics

- inspiral: tidal effects scaling as m_i^{-p}
  - delay or advance wrt GR depending on coupling sign
- merger: smooth transition to single BH. Amplitude ~ amplitude in GR
- ringdown: deviations in both oscillatory frequency and decay rate, modulated as $(M_T)^{-p}$

- Interestingly, transition to 'final fate' is rapid, evolving towards axisymmetric ('less hairy') BH [Reall+] without any significant excitation of higher modes

- Smoking guns? $\longrightarrow$ dependence of mass and operator order p (among events); reduction of non-GR effects pre/post merger (per event)

Switching to detection/analysis

…ultimately, all templates will be ‘wrong’...

Systematics in:

- Known but unmodeled physics
- Accuracy of models
- Unknown physics
- insufficient templates

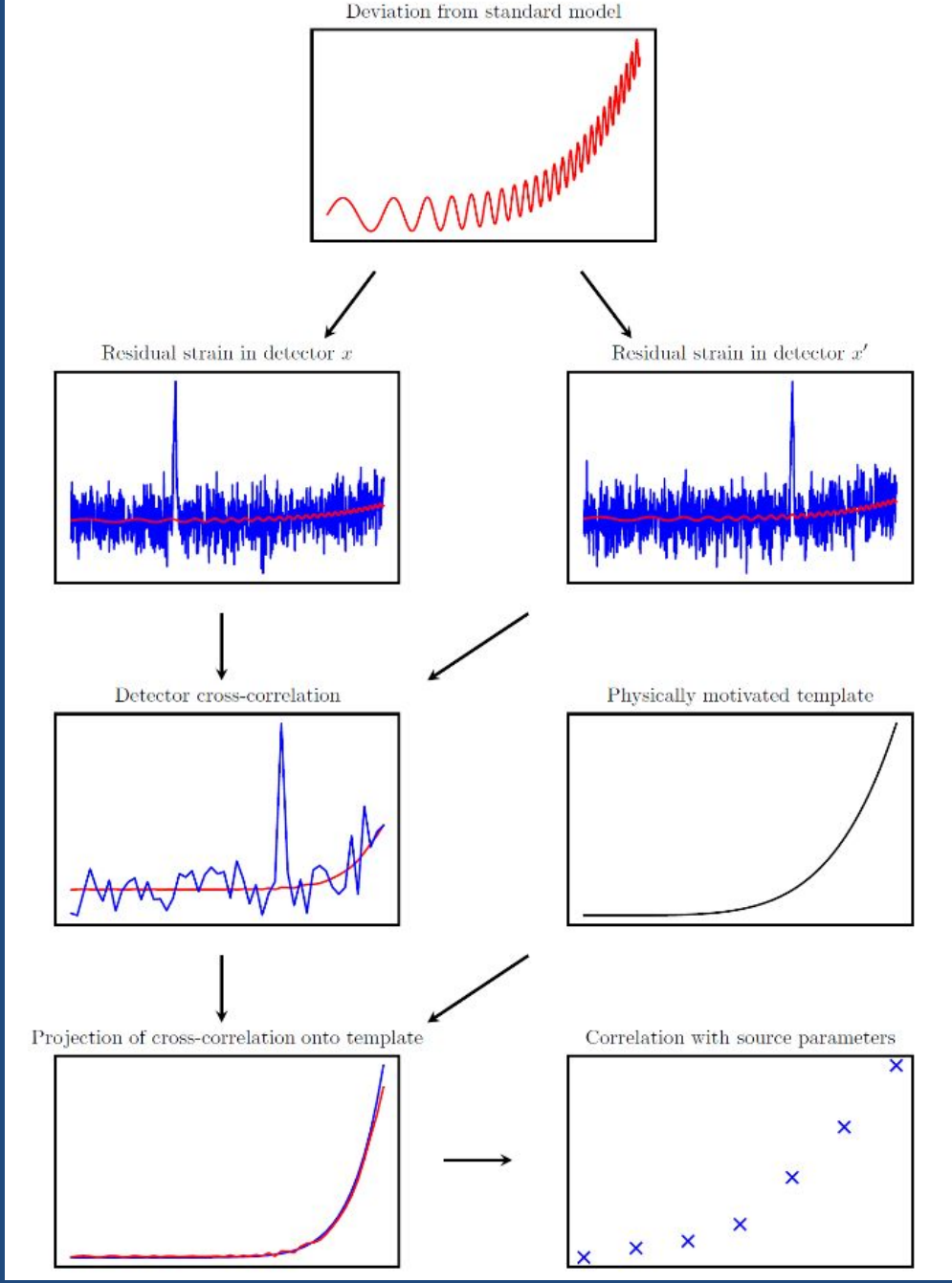
Can we devise a way to tell underlying features in the residual being noise or physics and extract it?

- ‘agnostic’ analysis of signals/residual (e.g. bayeswave; coherent spline [Edelman+]...),

Cross-correlation of residuals in power: ***SCoRe***

[Dideron,Mukherjee,LL '22]

- (A) Cross-correlation of residual wrt to best fit templates. *Is there anything real?*
- (B) Choice of a residual template. *Informative features to search for?*
- (C) Projection on a template
- (D) Inference using a Bayesian framework. *Evidence for a particular feature?*



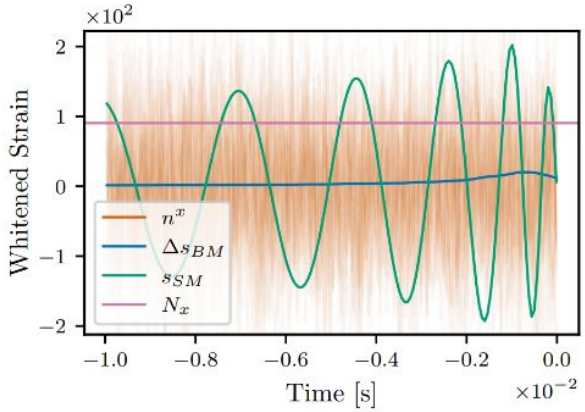


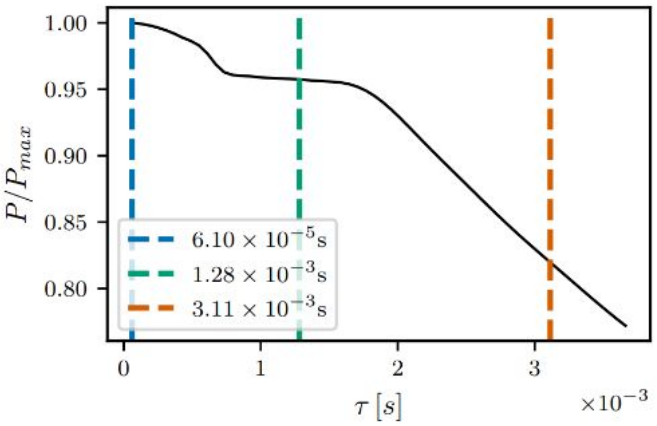
FIG. 2. Example toy model data. We use the PyCBC software package to generate $s_{\rm SM}$, the waveform for a circular, non-spinning, equal mass BBH, with individual masses both equal to $5M_\odot$. This is plotted in green. We add onto it a BM signature, $\Delta s_{\rm BM}$ (blue line), that is proportional to the change in the orbital frequency logarithm ($\tilde{\alpha}_1 = 0.05$). It is normalized so that, for $\tilde{\alpha}_1 = 1$, the maximum amplitude reaches to the noise auto-correlation, N_x (purple line). Realisations of Gaussian, stationary noise are added to $s_{\rm SM}+\Delta s_{\rm BM}$ to obtain different events. Some examples of noise realisations are plotted in orange.

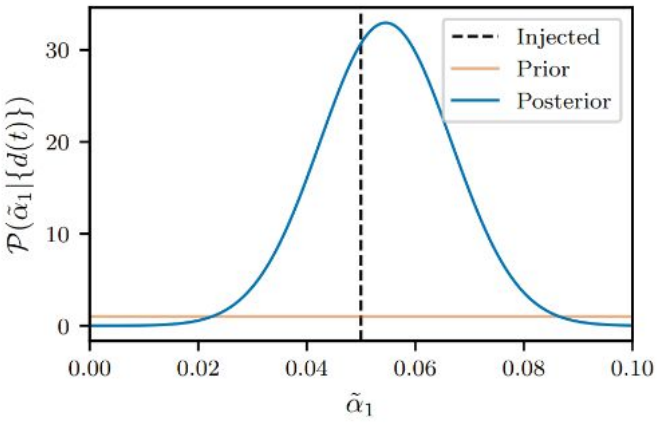
Fitting power `basis'

$$S^{xx'}_{\theta_{\rm MLE}}(t) = \sum_{i=1}^{i=n} \alpha_i(\theta_{\rm MLE}, t) Z_i(f(t)),$$

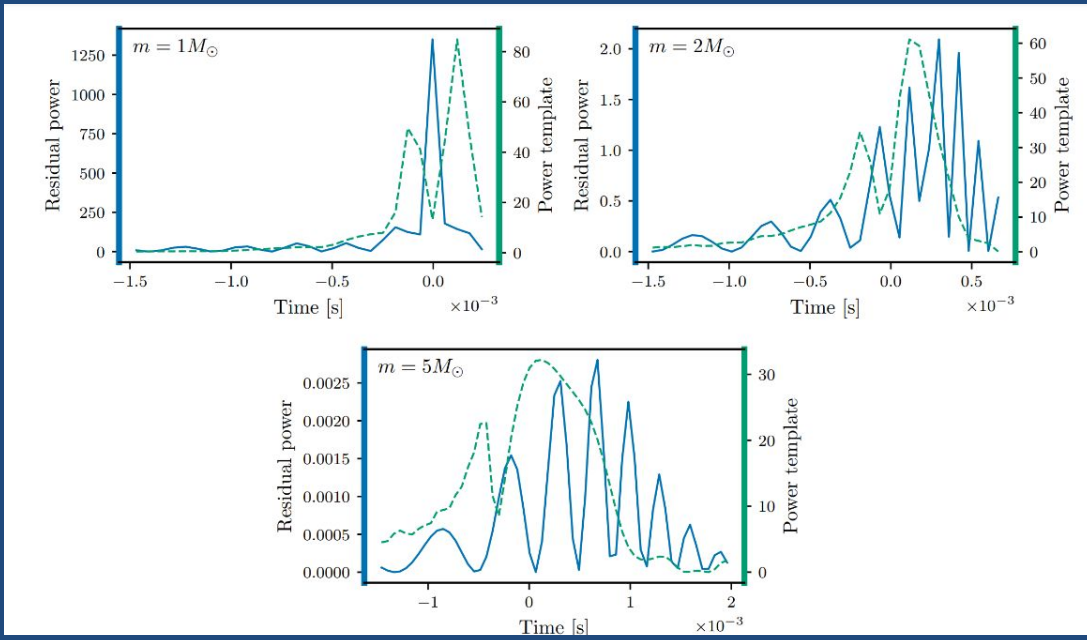
One option…

$$Z_{i>0}(t) = \left\langle \left(\frac{d^i \ln f(t)}{dt^i} \right)^2 \right\rangle,$$

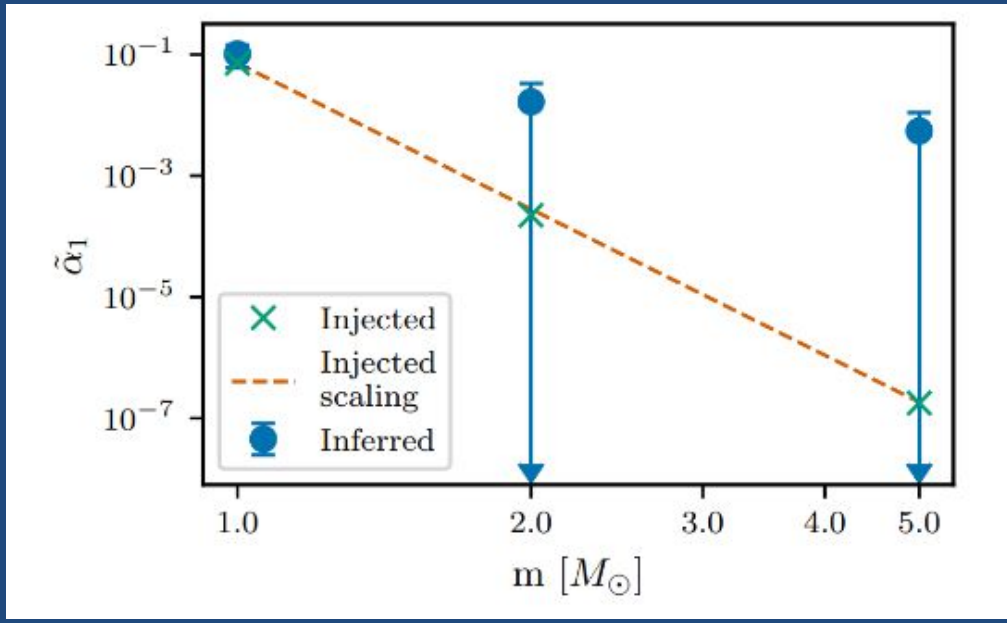




- Take a full gravitational wave train, phenomenologically constructed following lessons from EFT-grav. With 4th order operator



- Searching for residual power scaling as m^{-8}
- Reasonable inference (but with 500 events, expected O4 sensitivity)



Wrapping up

- Signals in GR, understood ‘reasonably well’ . Though (i) still corners under-explored [spins, mass ratio, eccentricity, (ii) efficient & faithful encoding bringing new challenges/opportunities

- For beyond GR, difficulties at the ground level to explore the relevant regime. *Introduced/validated a method to push through.*