



Aalto University  
School of Science

# Topological Photonics with Plasmonic Lattices

Päivi Törmä  
Aalto University

Quantum Nanophotonics, Benasque, Spain

13.3.2023

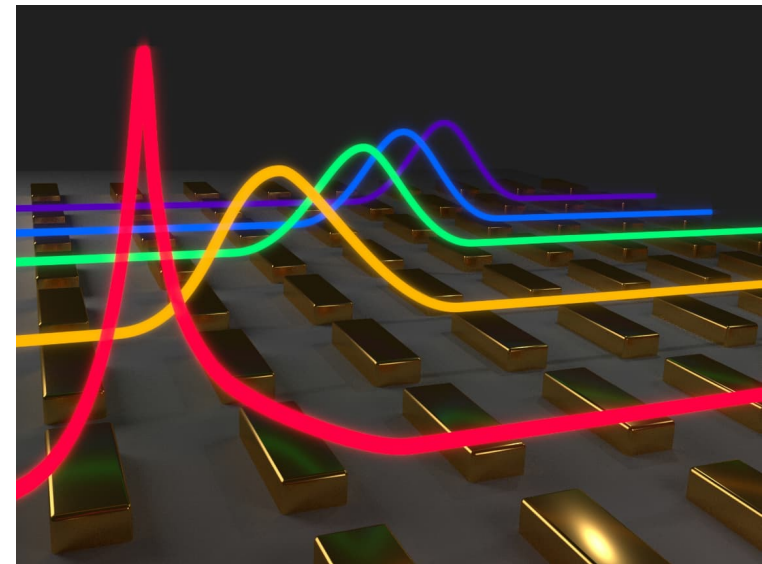


# Contents

## Bose-Einstein condensation (BEC) in a plasmonic lattice

- Background
- BEC at weak and strong coupling
- Spatial and temporal coherence
- Polarization textures

## Quasi-BIC mode lasing and topological transitions

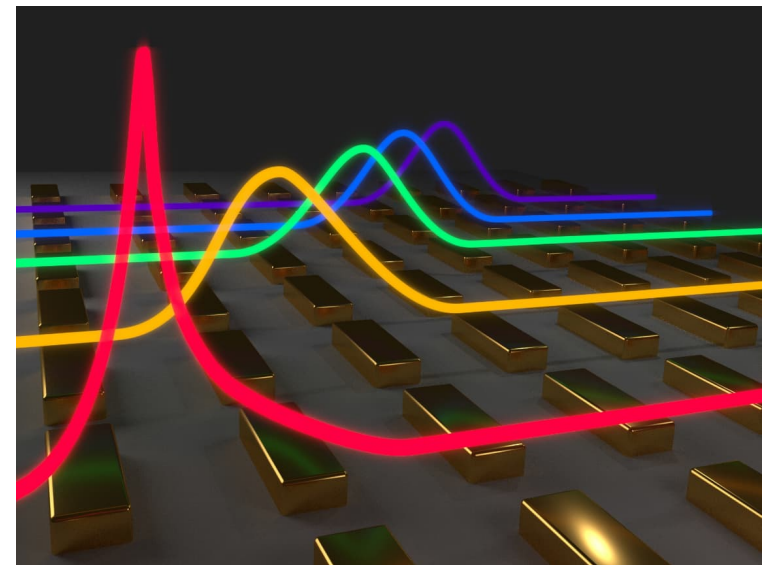


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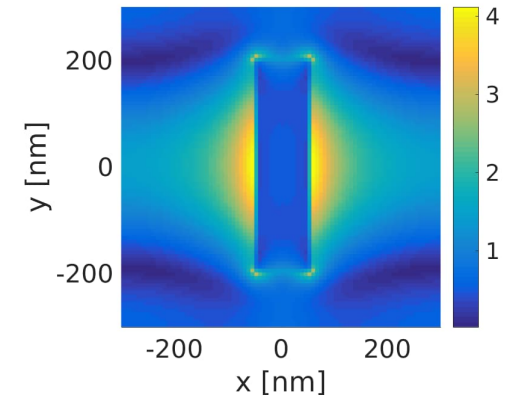
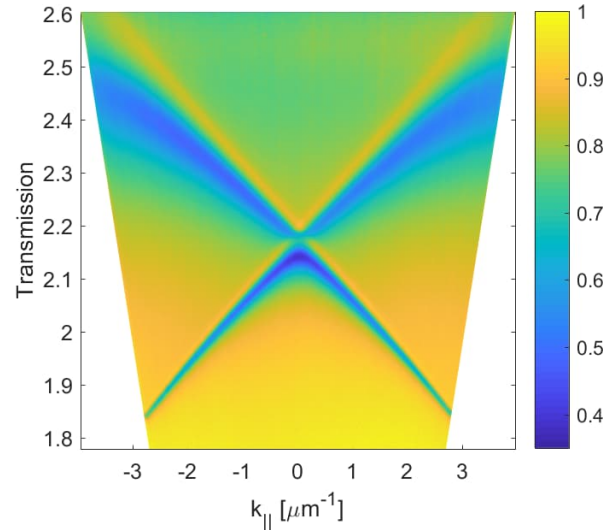
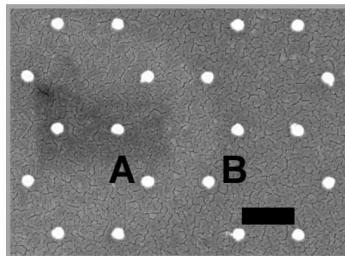
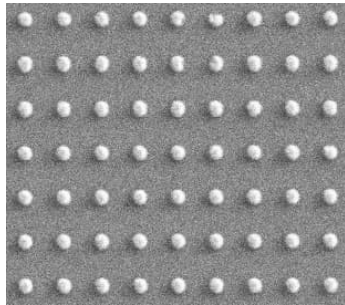
- **Background**
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## Quasi-BIC mode lasing and topological transitions



# Plasmonic lattices

## *light-matter interaction at new regimes*



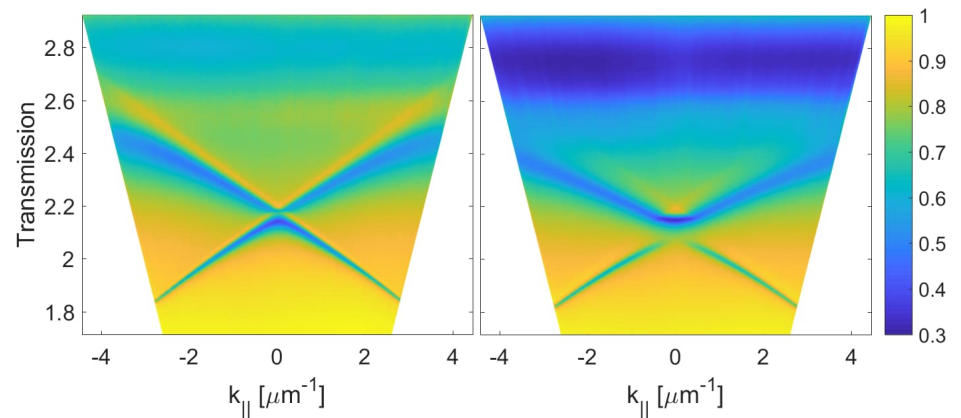
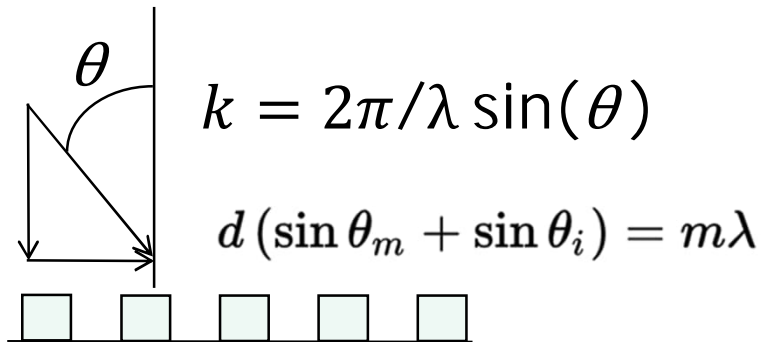
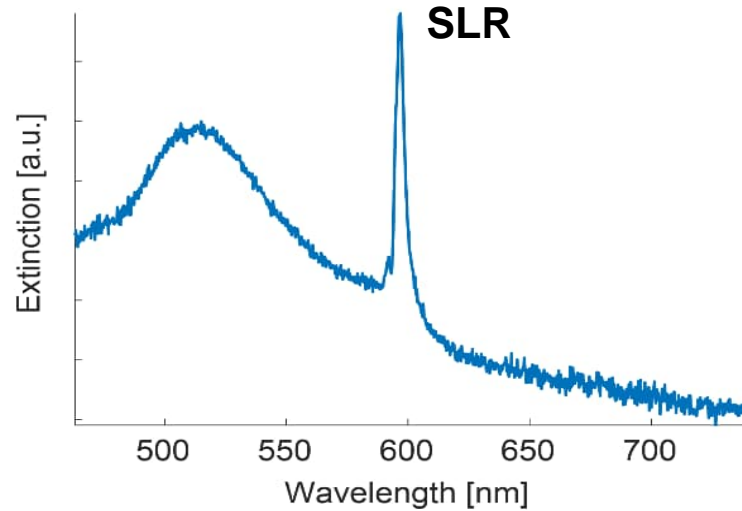
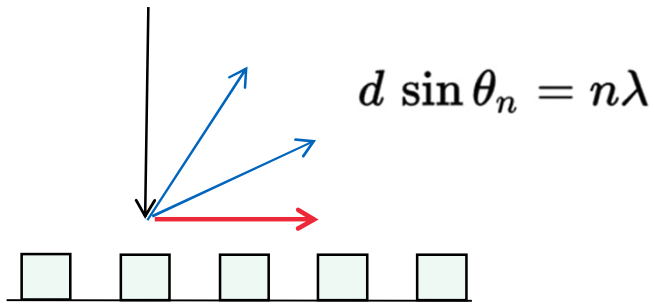
### Reviews

Garcia de Abajo, Rev. Mod. Phys. 2007

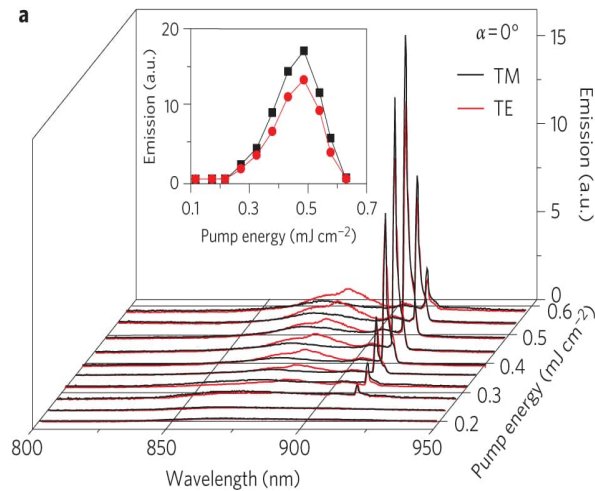
Wang, Ramezani, Väkeväinen, PT, Gomez-Rivas, Odom, Materials Today 2018

Kravets, Kabashin, Barnes, Grigorenko, Chemical Reviews 2018

# Surface lattice resonance (SLR)



# Nanoparticle arrays combined with organic molecules: lasing, strong coupling



Picture from  
Odom, Schatz, Nat. Nanotech. 2013

## Reviews

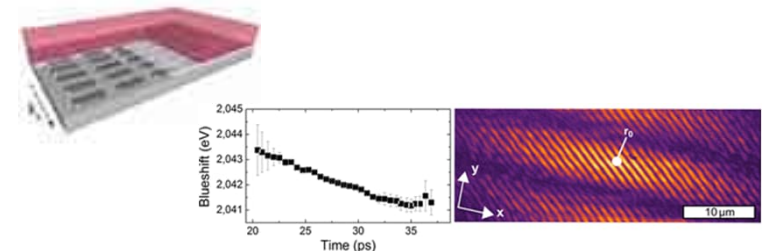
*Small lasers, the spaser concept:*

Hill and Gather, Nat. Phot. 8, 908 (2014)

*Focus on nanoparticle arrays:*

Wang et al., Chem. Rev. 118, 2865 (2018)

Wang et al., Materials Today 21, 303 (2018)



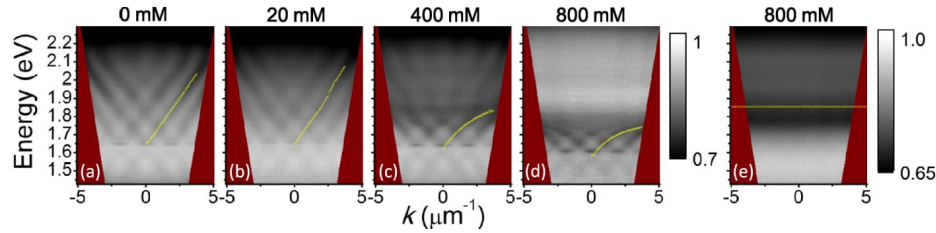
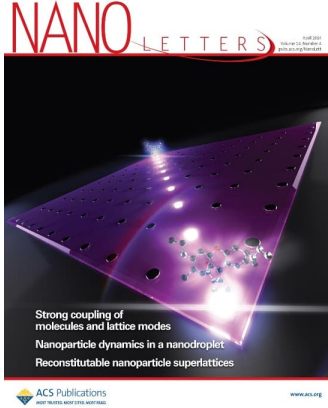
Polariton lasing/condensation

Ramezani, Halpin, Fernández-Domínguez, Feist, Rodriguez, Garcia-Vidal, Gomez-Rivas, Optica 2017,

Gomez-Rivas, Sanvitto groups

ACS Photonics 2018, Nano Letters 2019

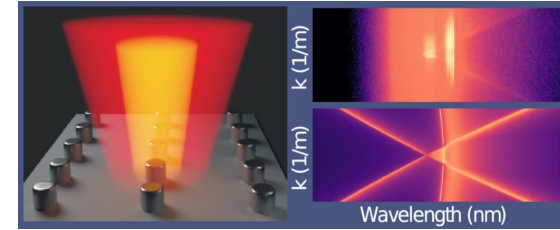
# Our previous SLR work



Spatial coherence at strong coupling  
Shi et al. Phys Rev Lett 2014

Strong coupling in a plasmonic lattice  
Väkeväinen et al. Nano Lett 2014

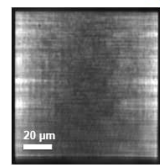
Strong coupling in dielectric particle array  
Heilmann et al. Nanophot. 2020



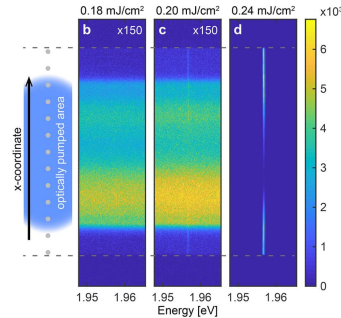
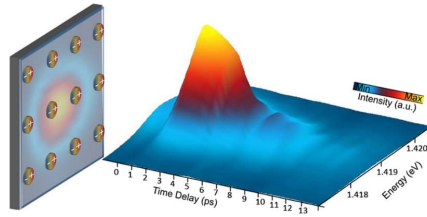
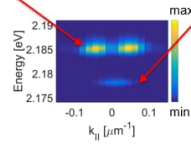
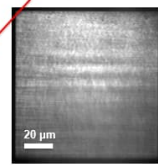
Lasing in Ni nanodisk arrays  
Pourjamal et al. ACS Nano 2019

Magnetoplasmonic lattices  
Kataja et al. Nature Comm 2015

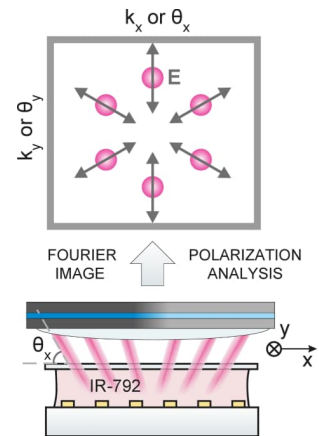
Higher energy mode "dark mode"



Lower energy mode "bright mode"



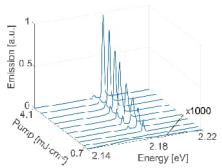
1D plasmonic lasing  
Rekola et al. ACS Phot 2018



K-point lasing in a honeycomb lattice  
Guo et al. Phys Rev Lett 2019

Lasing in dark (BIC) and bright modes  
Hakala et al. Nature Comm 2017

Ultrafast pulse generation  
Daskalakis et al. Nano Lett 2018

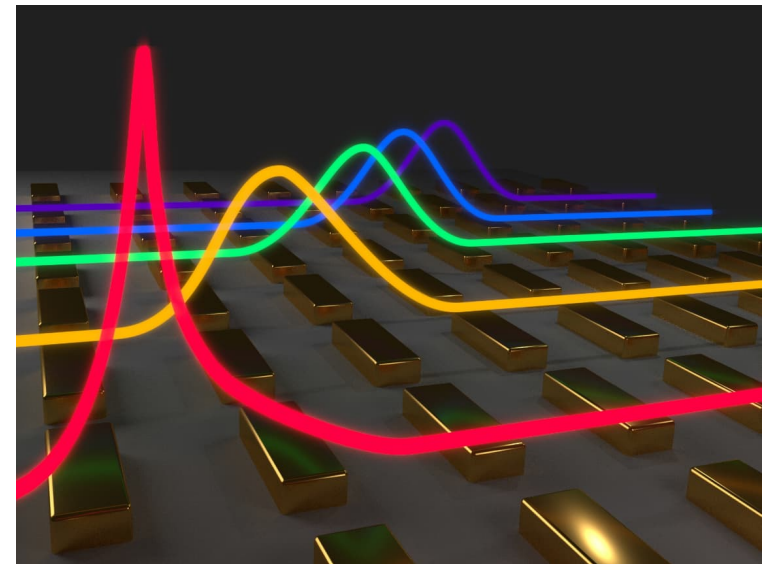


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## Quasi-BIC mode lasing and topological transitions





# Bose-Einstein condensation in a plasmonic lattice

A 3D visualization of a plasmonic lattice, represented by a grid of small, rectangular metallic blocks. Overlaid on this lattice are several colorful, glowing wave functions. A prominent red wave function shows a sharp peak on the left side of the lattice. Other waves in yellow, green, blue, and purple show smoother, oscillating patterns across the lattice. The background is dark, and the metallic blocks have a reflective, golden-brown surface.

Hakala, Moilanen, Väkeväinen, Guo, Martikainen, Daskalakis, Rekola, Julku, PT, Nature Physics 2018

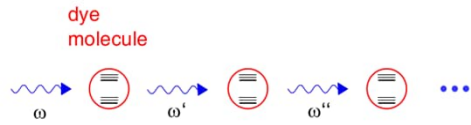
Väkeväinen, Moilanen, Necada, Hakala, Daskalakis, PT, Nature Communications 2020

Moilanen, Daskalakis, Taskinen, PT, PRL 2021

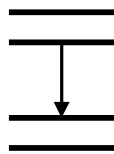
Taskinen, Kliuiev, Moilanen, PT, Nano Letters 2021

# Photon and/or exciton condensation

Thermalization by emission/absorption cycles



Relaxation by emission of a vibrational quantum



Coulomb relaxation/thermalization

Photon BEC

*Plasmonic BEC  
weak coupling  
Hakala et al.  
Nat.Phys.2018*

*Plasmonic BEC  
strong coupling  
Väkeväinen et al.  
Nat.Comm.2020*

Organic polariton condensate/lasing

Inorganic polariton condensate/BEC/lasing

Exciton BEC

Light

Polariton  $c_l|light\rangle + c_m|matter\rangle$

Matter

**Literature:** Byrnes, Kim, Yamamoto, Nat. Phys. 2014; Klaers, Schmitt, Vewinger, Weitz, Nature 2010; Keeling, Kena-Cohen, Ann. Rev. Phys. Chem. 2020

# Plasmonic BEC: weak coupling



Tommi Hakala



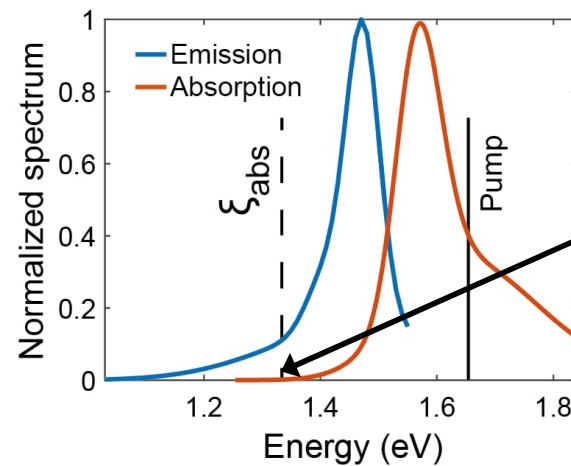
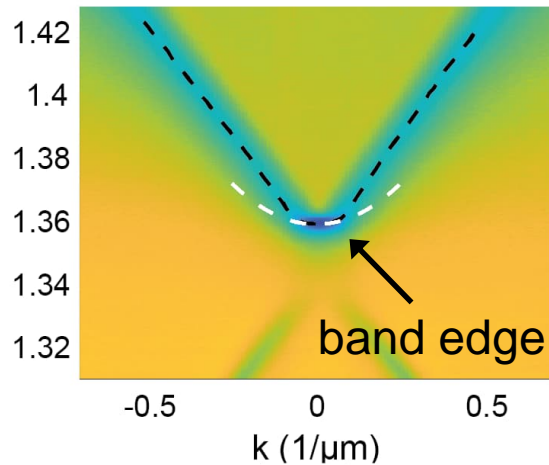
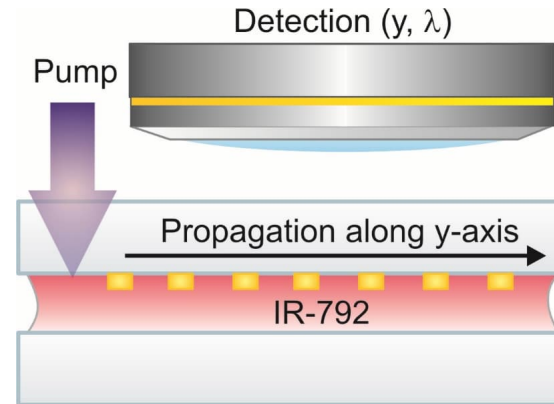
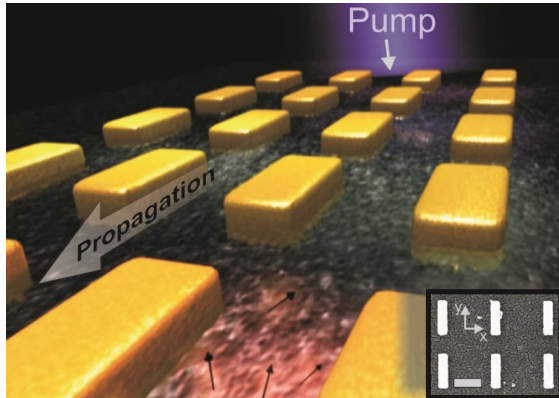
Antti Moilanen



Aaro Väkeväinen

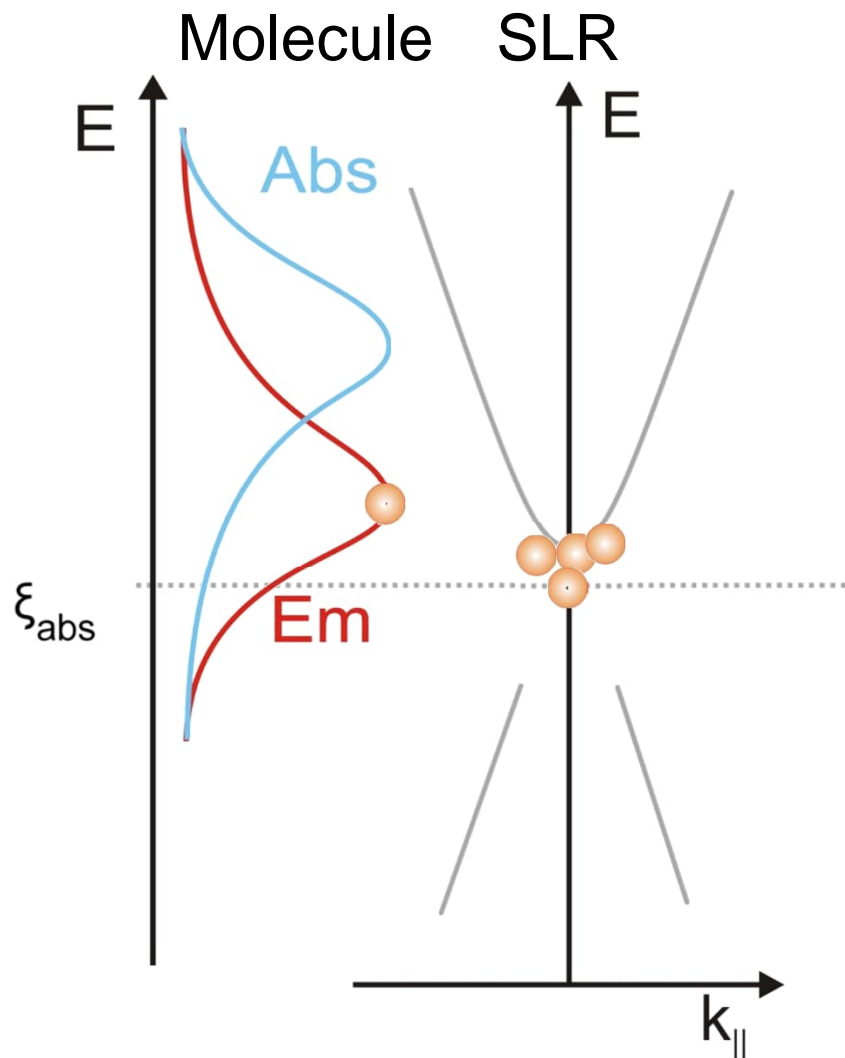
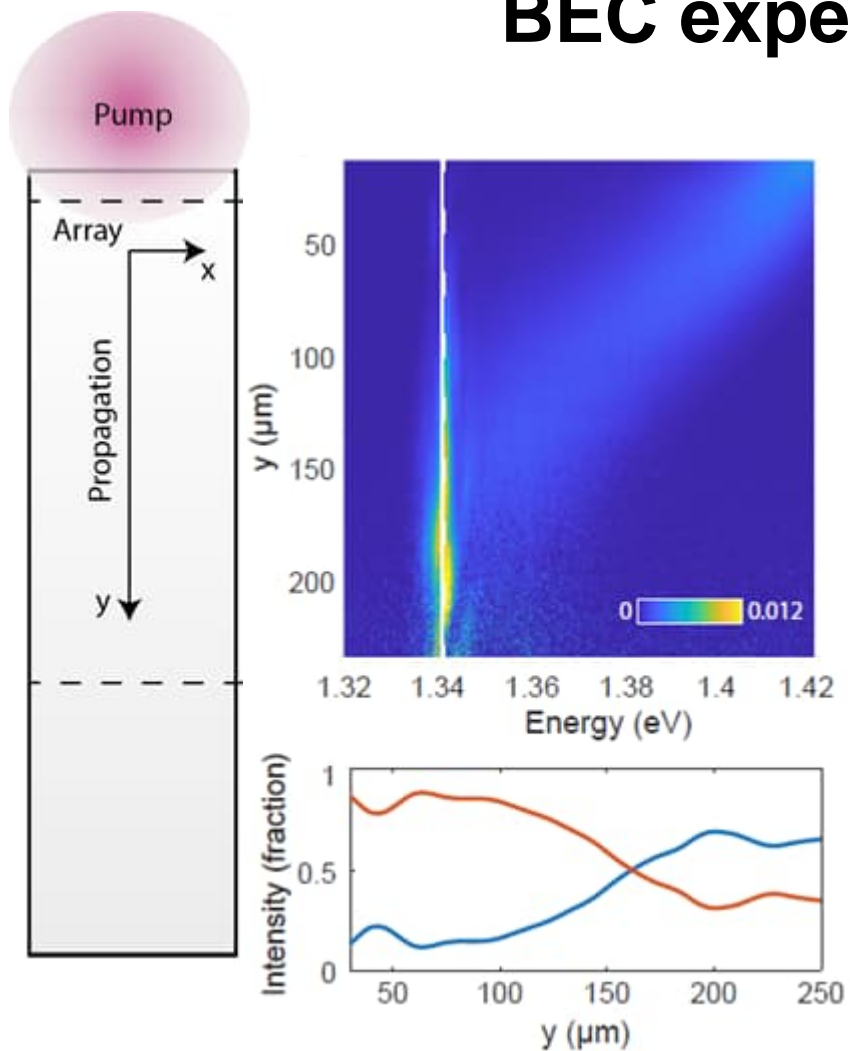
Hakala, Moilanen, Väkeväinen, Guo, Martikainen, Daskalakis, Rekola, Julku, PT, Nature Physics 2018

# Nanoparticle array + molecules (weak coupling)



Energy where absorption rate is effectively zero (i.e. smaller than the loss rate)

# BEC experiment



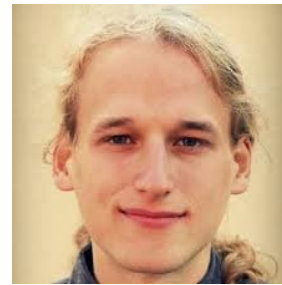
# Plasmonic BEC: strong coupling



Aaro Väkeväinen



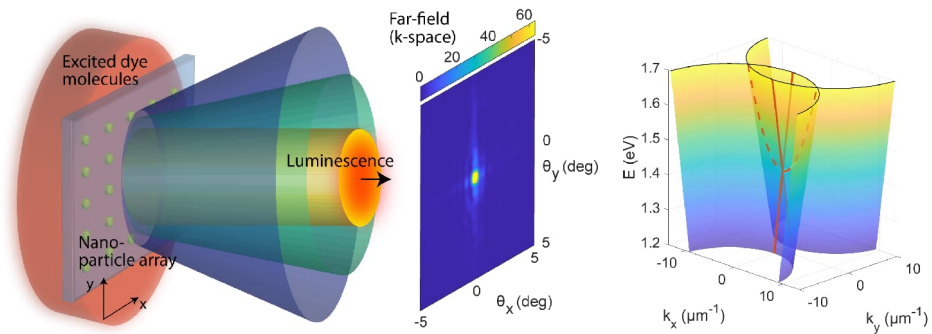
Antti Moilanen



Marek Necada

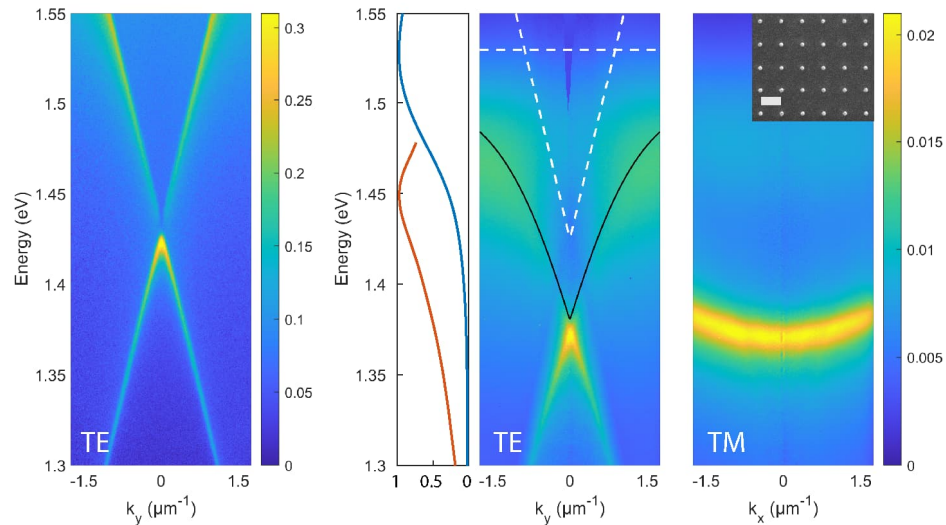
Väkeväinen, Moilanen, Necada, Hakala, Daskalakis, PT, Nature Communications 2020

# Sub-picosecond thermalization dynamics of the condensate

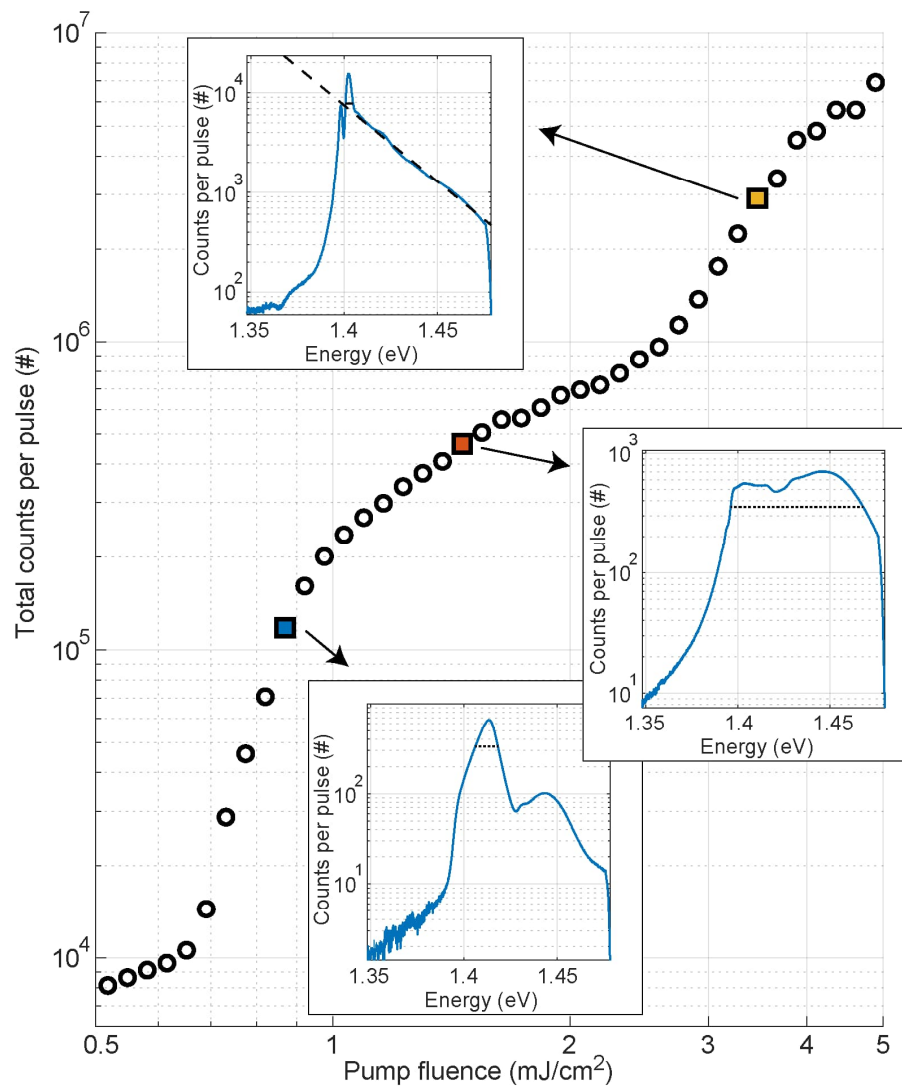


**Strong coupling regime:**  
**Polariton**  
= SLR excitation (“light”)  
+ dyes molecules (“matter”)

Pump over the whole sample

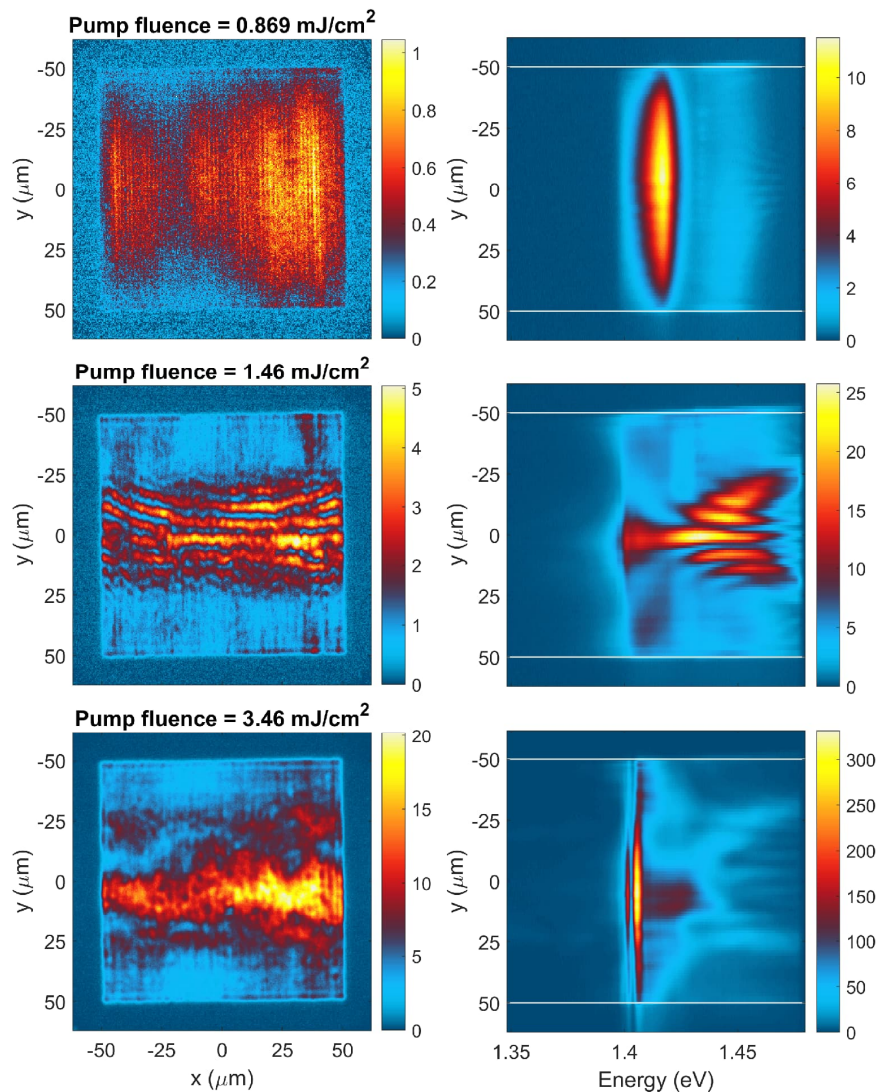


# Three regimes: polariton lasing, stimulated thermalization, BEC



Real space image

Spatially resolved spectrum



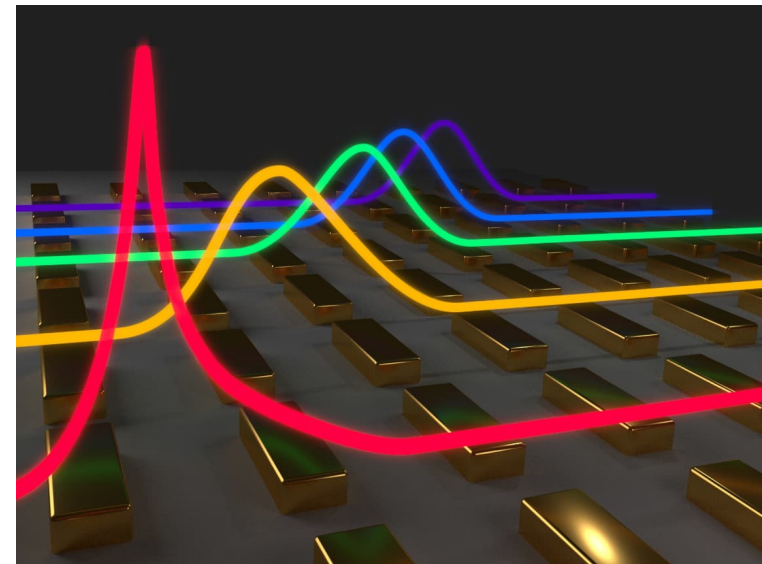


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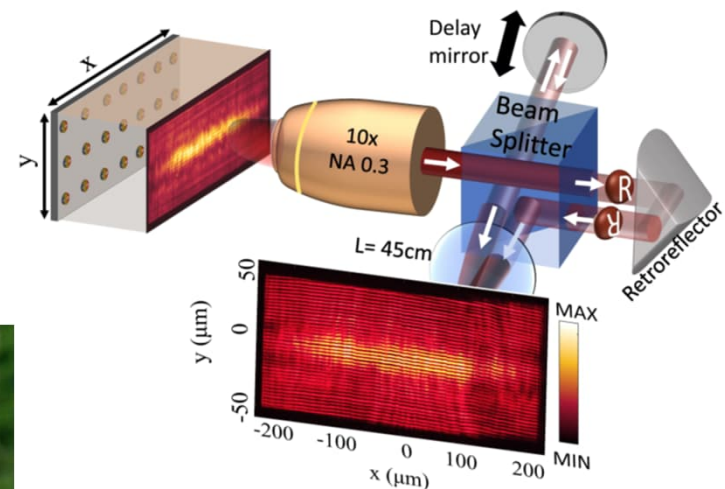
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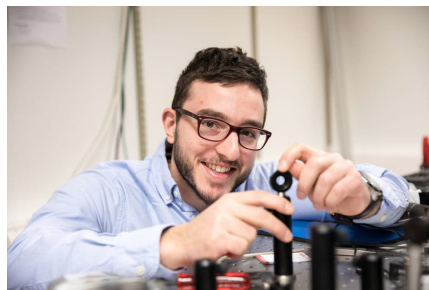
## Quasi-BIC mode lasing and topological transitions



# Spatial and temporal coherence of the strong coupling BEC



Antti Moilanen



Konstantinos Daskalakis



Jani Taskinen

Moilanen, Daskalakis, Taskinen, PT, PRL 2021

# Long-range order in 2D BECs

## Mermin-Wagner:

- thermal fluctuations prevent ODLRO in 2D at any finite T

## Berezinskii-Kosterlitz-Thouless (BKT):

- vortex-antivortex pairing
- quasi-long-range order
- algebraic decay:  $b < 0.25$

$$g^{(1)}(x) = ax^{-b}$$

*How about non-equilibrium condensates?*

## Non-equilibrium BKT

- algebraic decay; different exponents for spatial and temporal coherence[2,3]

$$b_s = 2b_t$$

## Kardar-Parisi-Zhang (KPZ)

- dynamical phase ordering; decay by stretched exponential with universal exponents[4,5,6]

$$g^{(1)}(x) = ae^{-(x/d)^\beta}$$

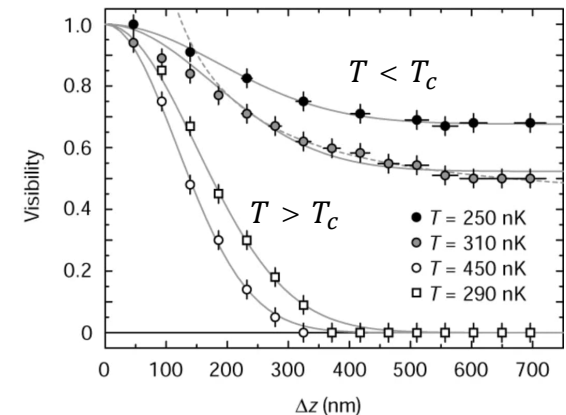
$$\beta_s = 0.78$$

$$\beta_t = 0.48$$

*BKT-KPZ crossover by system anisotropy[4,7]*

In 3D BEC at equilibrium, off-diagonal long-range order (ODLRO)

Spatial correlation function of a trapped Bose gas [1]:



[1] Bloch, Hänsch & Esslinger, Nature 403, 166–170 (2000)

[2] Szymanska, Keeling & Littlewood, PRB 75, 195331 (2007)

[3] Comaron, Carusotto, Szymanska & Proukakis, EPL 133, 17002 (2021)

[4] Altman, Sieberer, Chen, Diehl & Toner, PRX 5, 011017 (2015)

[5] Ferrier, Zamora, Dagvadorj & Szymanska, arXiv:2009.05177 (2020)

[6] Comaron, Dagvadorj, Zamora, Carusotto, Proukakis & Szymanska, PRL 121, 095302 (2018)

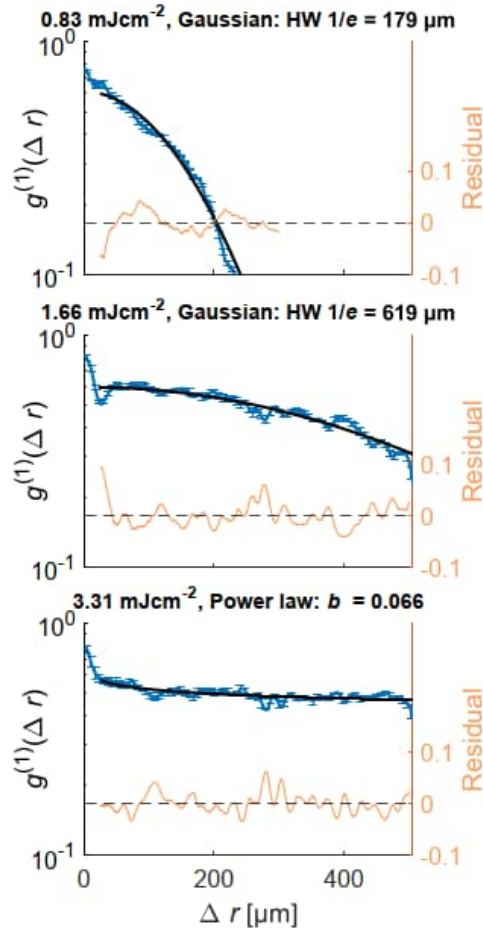
[7] Zamora, Sieberer, Dunnett, Diehl & Szymanska, PRX 7, 041006 (2017)

KPZ and polaritons in 1D: J. Bloch group, Nature 2022

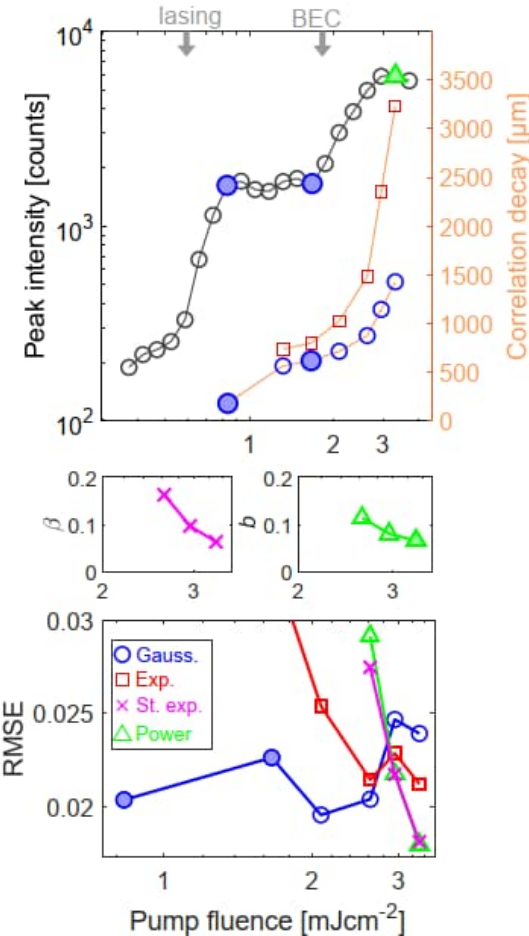
# Spatial coherence decay laws

Gaussian	$g^{(1)}(\Delta r) = ae^{-(\Delta r/d)^2}$
Exponential	$g^{(1)}(\Delta r) = ae^{-\Delta r/d}$
Str. Exponential	$g^{(1)}(\Delta r) = ae^{-(\frac{\Delta r}{d})^\beta}$
Power law	$g^{(1)}(\Delta r) = a(\Delta r)^b$

Below BEC threshold  
(polariton lasing):  
Gaussian decay



Above BEC  
threshold, power  
law ( $b_t = 0.066$ )  
fits well, stretched  
exponential gives  
 $\beta_t \sim 0.095$

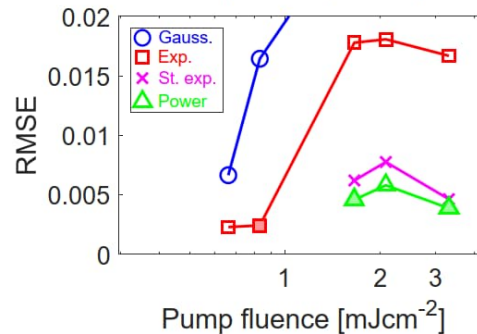
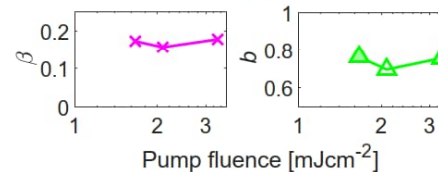
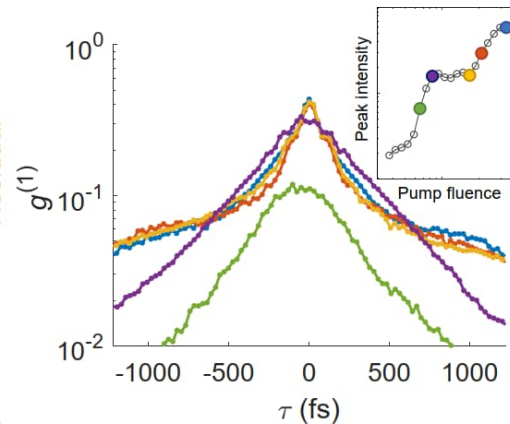
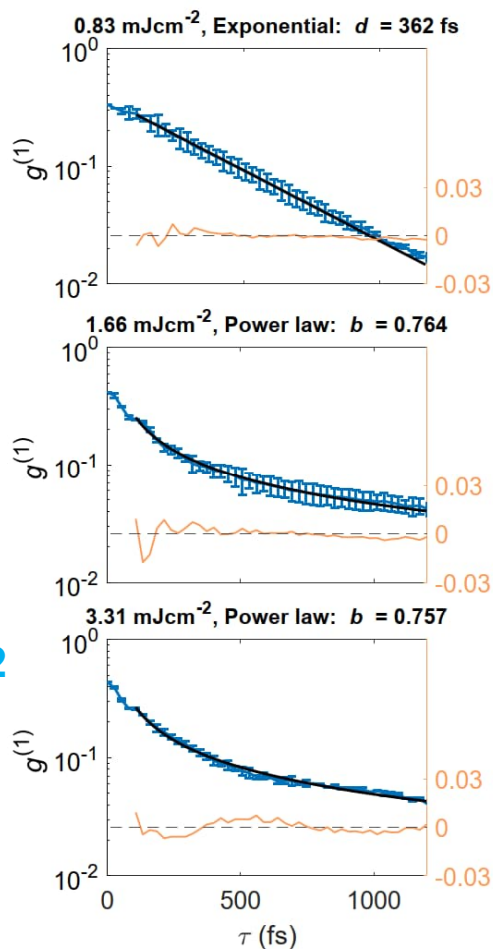


# Temporal coherence decay laws

Gaussian	$g^{(1)}(\Delta r) = ae^{-(\Delta r/d)^2}$
Exponential	$g^{(1)}(\Delta r) = ae^{-\Delta r/d}$
Str. Exponential	$g^{(1)}(\Delta r) = ae^{-(\frac{\Delta r}{a})^\beta}$
Power law	$g^{(1)}(\Delta r) = a(\Delta r)^b$

*Below BEC threshold (polariton lasing): exponential decay ( $d=366$  fs)*

*Above BEC threshold: power law ( $b_t = 0.756$ ) fits well, stretched exponential gives  $\beta_t \sim 0.2$*



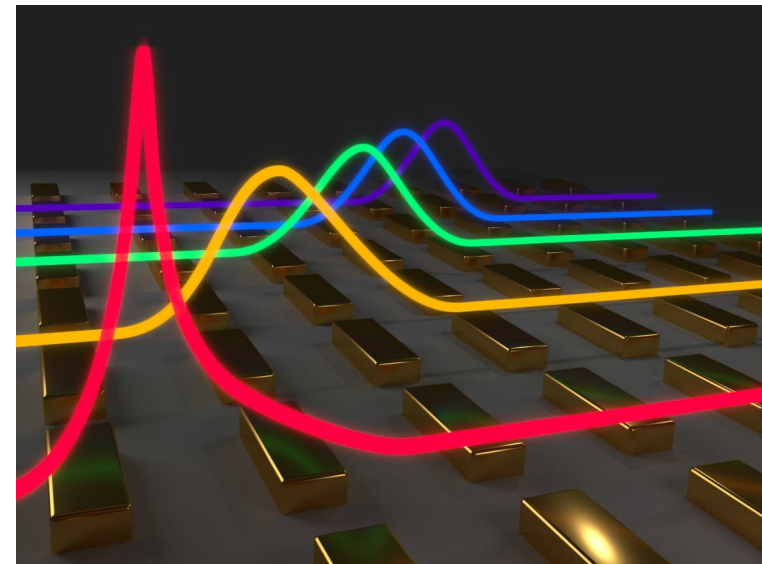
**Conclusion** BEC clearly different from (polariton) lasing; no KPZ; power law observed – BKT-type physics but with exponents not given by present theory  
 Theory of strongly coupled vibrational system interacting with many modes needed;  
 Arnardottir, Moilanen, Strashko, PT, Keeling, PRL 2020

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# Plasmonic BEC: polarization textures



Jani Taskinen



Pavel Kliuiev

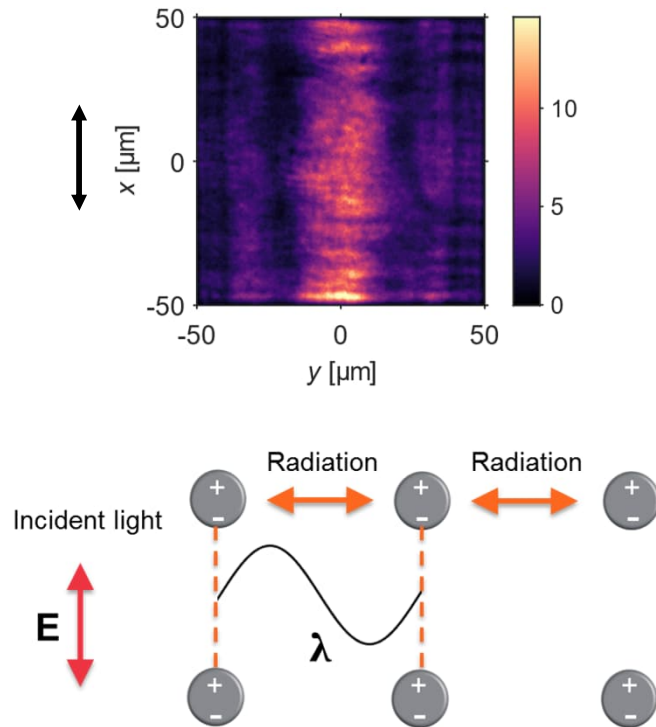


Antti Moilanen

Taskinen, Kliuiev, Moilanen, PT, Nano Letters 2021

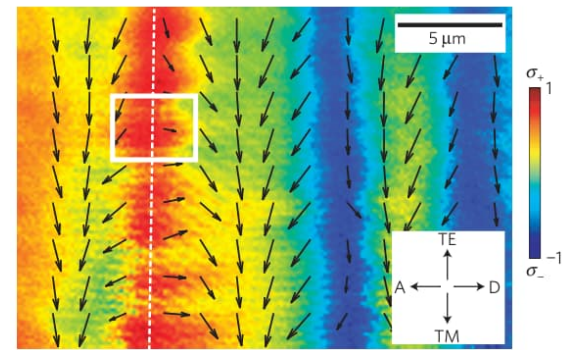
# Condensate of a vector field

So far, pump and emission x-polarized;  
essentially a scalar condensate



(Pseudo)spin textures in  
quantum or classical vector fields:

- Liquid Helium
- Atomic spinor BECs
- Semiconductor polariton condensates (at low  $T$ )
- Solid state magnetic systems
- Photonic crystals
- Metamaterials
- Liquid crystals

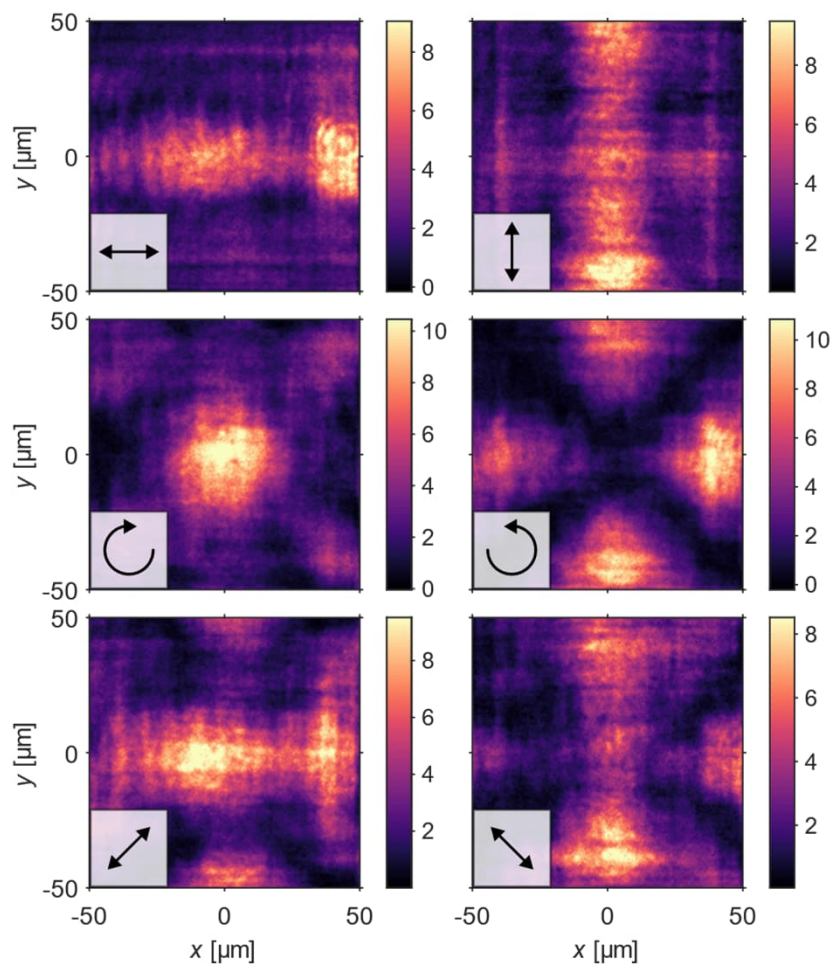


Hivet et al. Nat Phys 2020  
(Bloch, Bramati, Malpuech, Amo)

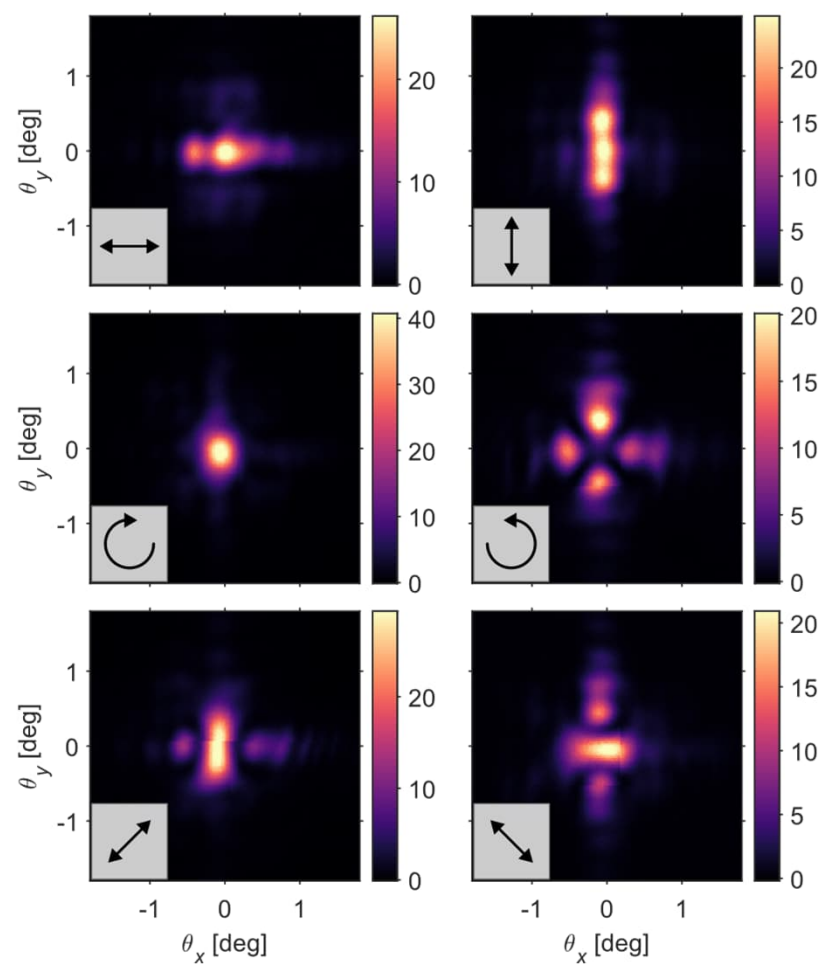




## Real space

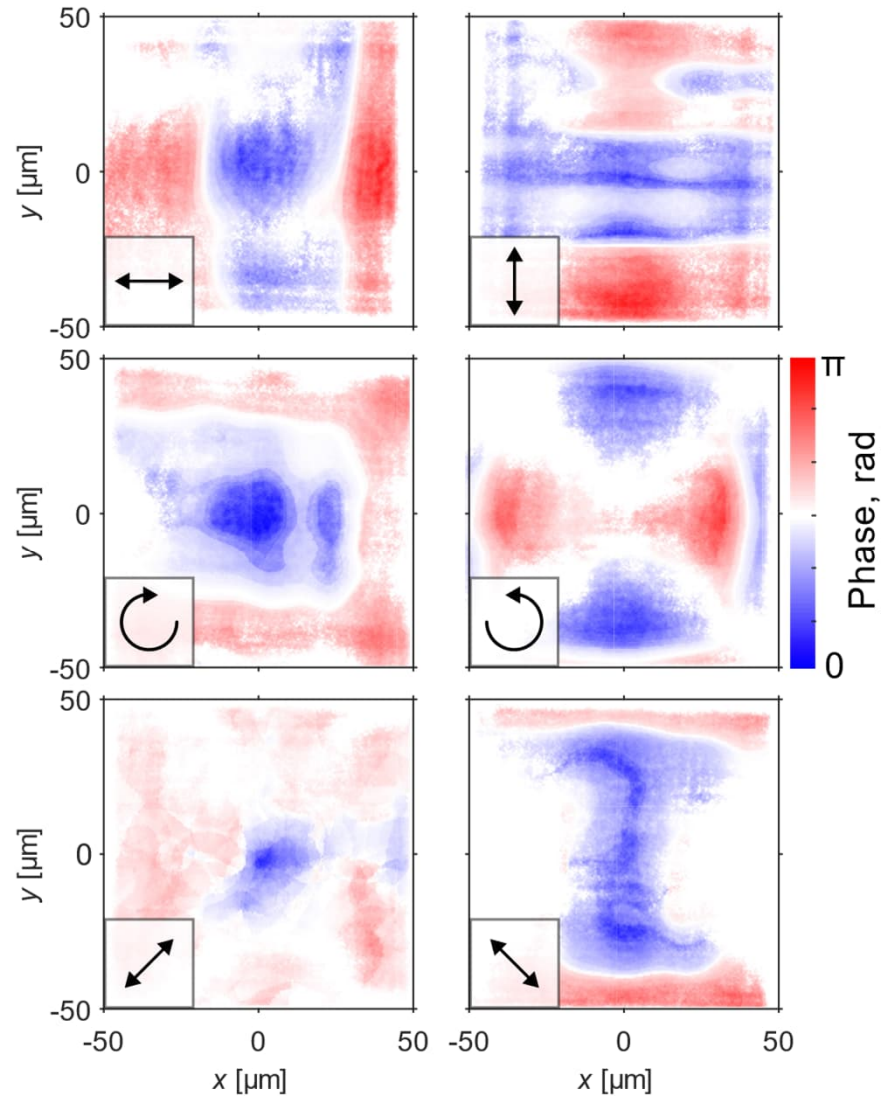


## $k$ -space

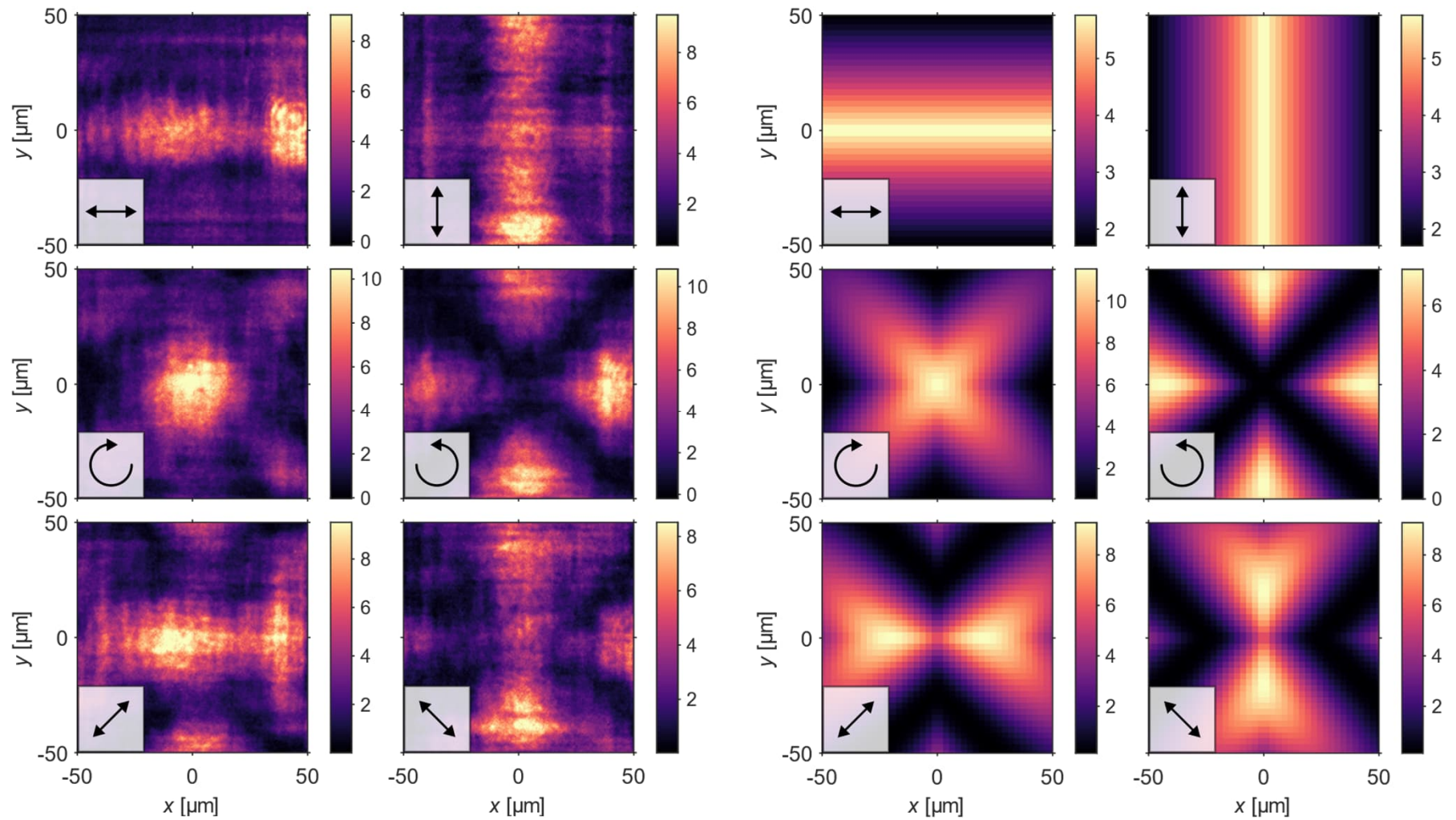


BEC phase by Gerchberg-Saxton algorithm

# Condensate phase determined for the first time by phase retrieval



# Polarization textures: experiment vs theory

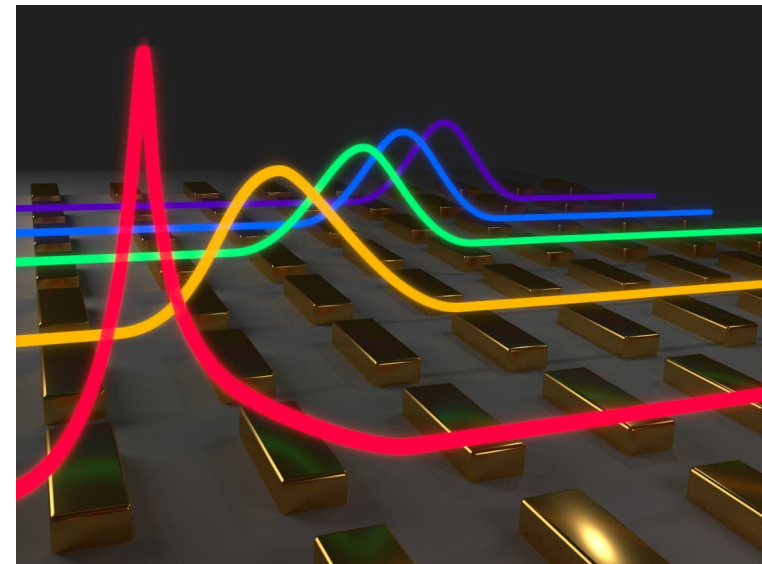


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# Quasi-BIC mode lasing

Heilmann, Salerno, Cuerda, Hakala, PT, ACS Photonics 2022

Salerno, Heilmann, Arjas, Aronen, Martikainen, PT, PRL 2022



Rebecca Heilmann



Grazia Salerno

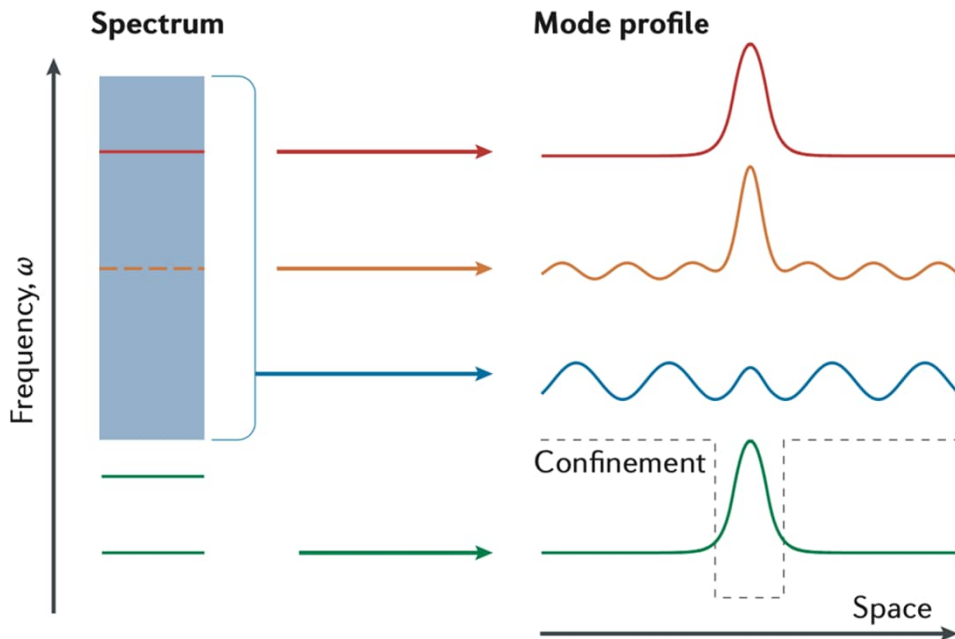


Javier Cuerda



Kristian Arjas

# Bound states in continuum (BICs)



BICs are *embedded eigenvalues*, decoupled from the continuum spectrum

BICs have infinite Q-factor, i.e. zero linewidth

BICs are totally invisible in the radiation field (hence dark mode)

Hsu *et al.*, *Nat Rev Mater* **1**, 16048 (2016)

Marinica *et al.*, *Phys. Rev. Lett.* **100**, 183902 (2008)

Hsu *et al.*, *Nature* **499**, 188–191 (2013)

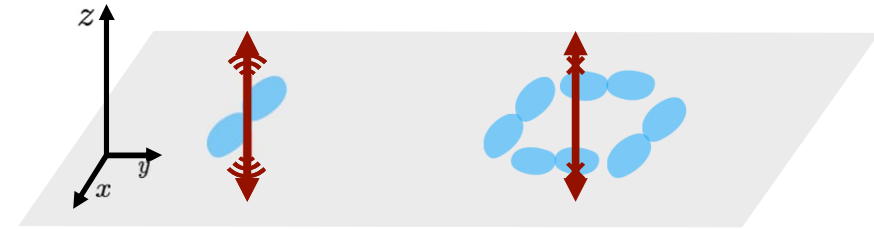
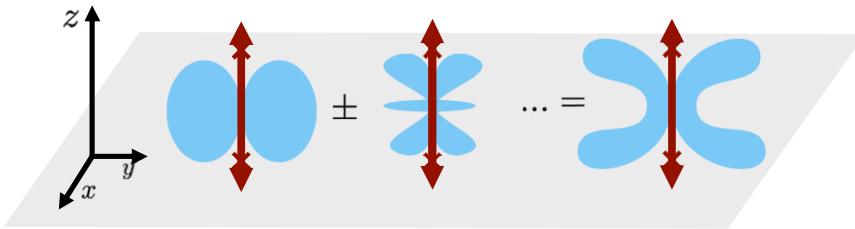
Doeleman *et al.*, *Nat. Phot.* **2**, 397 (2018)

Azzam & Kildishev, *Adv Opt Mater* **9**, 2001469 (2021)

# What type of BICs?

Realized in many systems, including waveguides, metasurfaces, plasmonic and photonic crystals.

They can be symmetry-protected, “accidental”, and even topological.

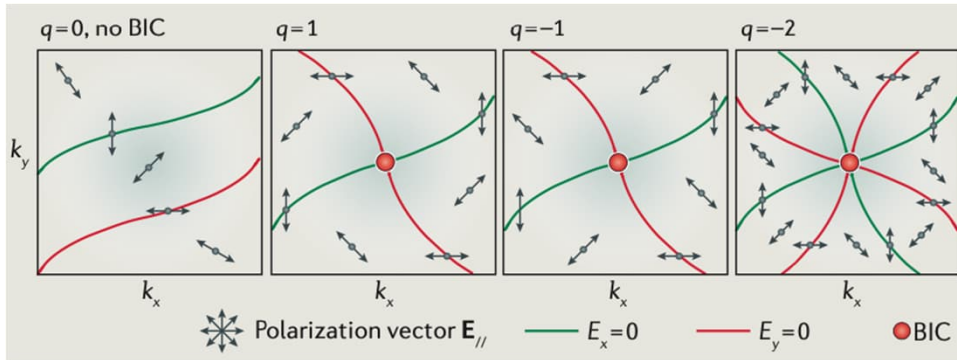


Marinica *et al.*, *Phys. Rev. Lett.* **100**, 183902 (2008)  
Hsu *et al.*, *Nature* **499**, 188–191 (2013)

Doeleman *et al.*, *Nat. Phot.* **2**, 397 (2018)  
Azzam & Kildishev, *Adv Opt Mater* **9**, 2001469 (2021)



# Topological BICs are polarization vortices



Topological BICs cannot radiate because **there is no way to assign a far-field polarization** that is consistent with neighbouring  $\mathbf{k}$  points.

Robust BICs are possible when there is vorticity in the polarization field, protected by the existence of a non-trivial topological invariant, the vortex charge:

$$q = \frac{1}{2\pi} \int d\mathbf{k} \cdot \nabla_{\mathbf{k}} \phi(\mathbf{k})$$

$$\Phi(\mathbf{k}) = \arg[\mathbf{p}(\mathbf{k}) \cdot \hat{x} + i\mathbf{p}(\mathbf{k}) \cdot \hat{y}]$$

$$\mathbf{p}(\mathbf{k}) = (\hat{x} \cdot \langle \mathbf{u}_{\mathbf{k}}(\mathbf{r}, z) \rangle) \hat{x} + (\hat{y} \cdot \langle \mathbf{u}_{\mathbf{k}}(\mathbf{r}, z) \rangle) \hat{y}$$

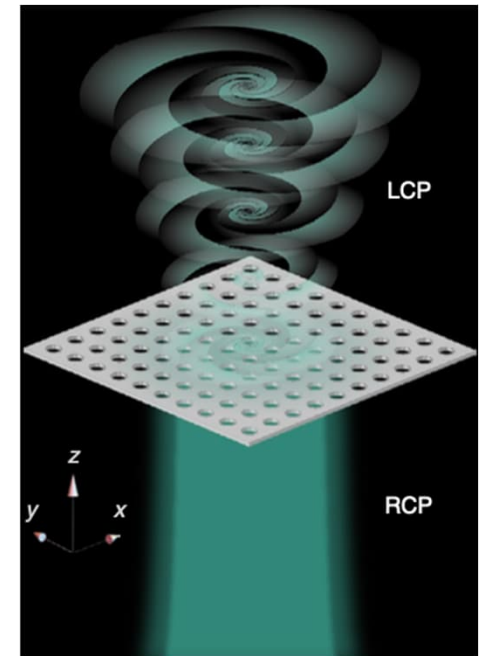
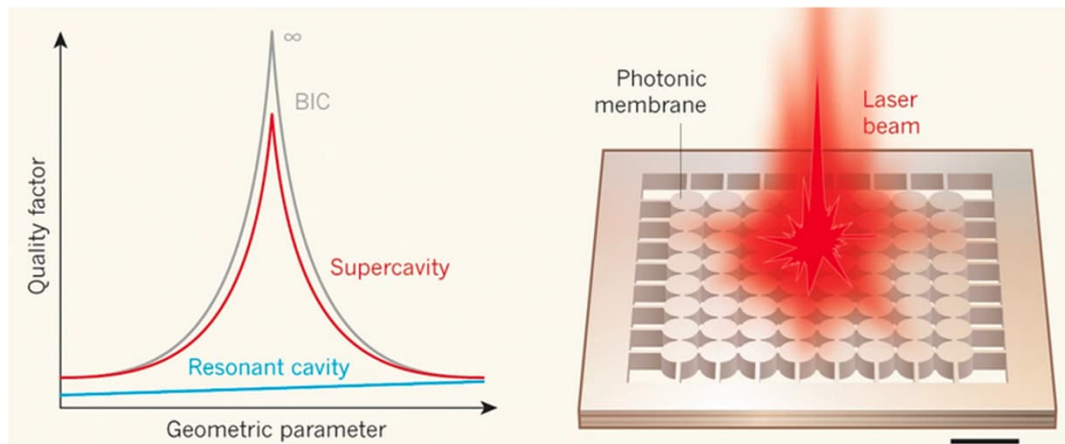
Zhen *et al.*, *Phys. Rev. Lett.* **113**, 257401 (2014)

Hsu *et al.*, *Nat Rev Mater* **1**, 16048 (2016)

# Lasing and vortex beams from BICs

High-Q BICs support lasing

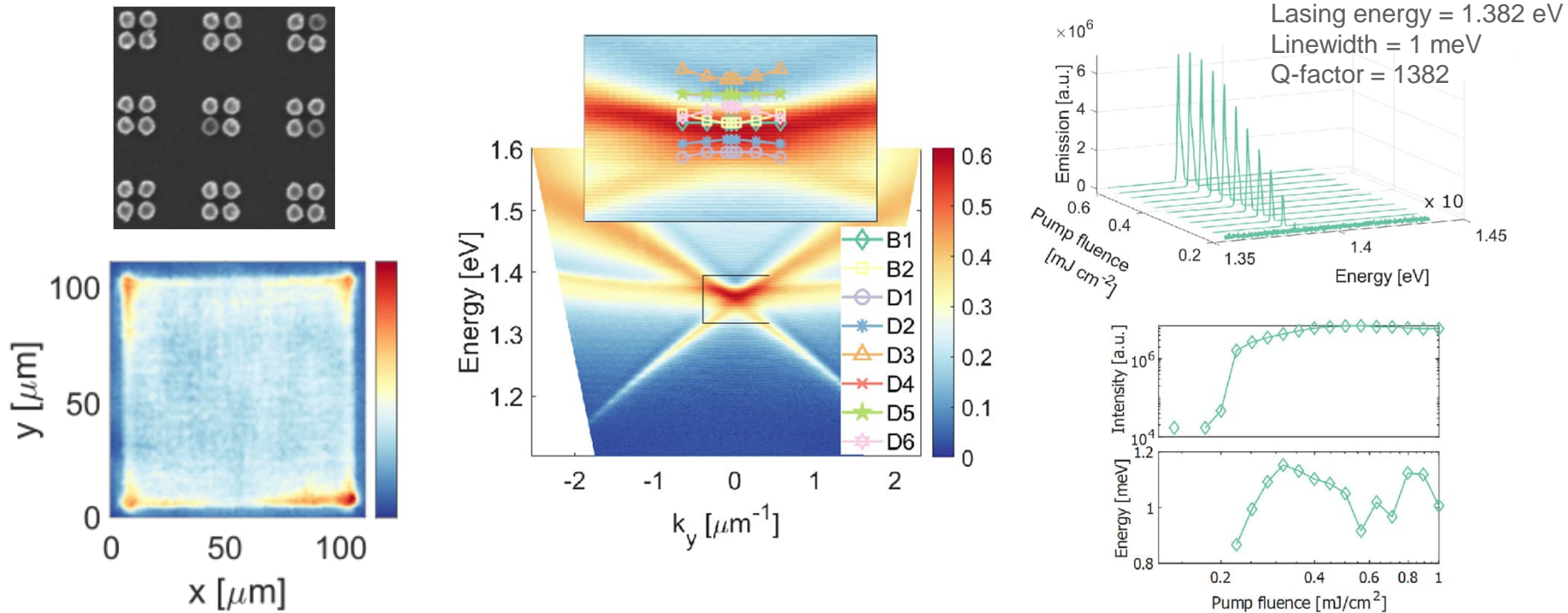
They directly offer large quantum numbers of optical angular momentum for the generation of optical vortex beam



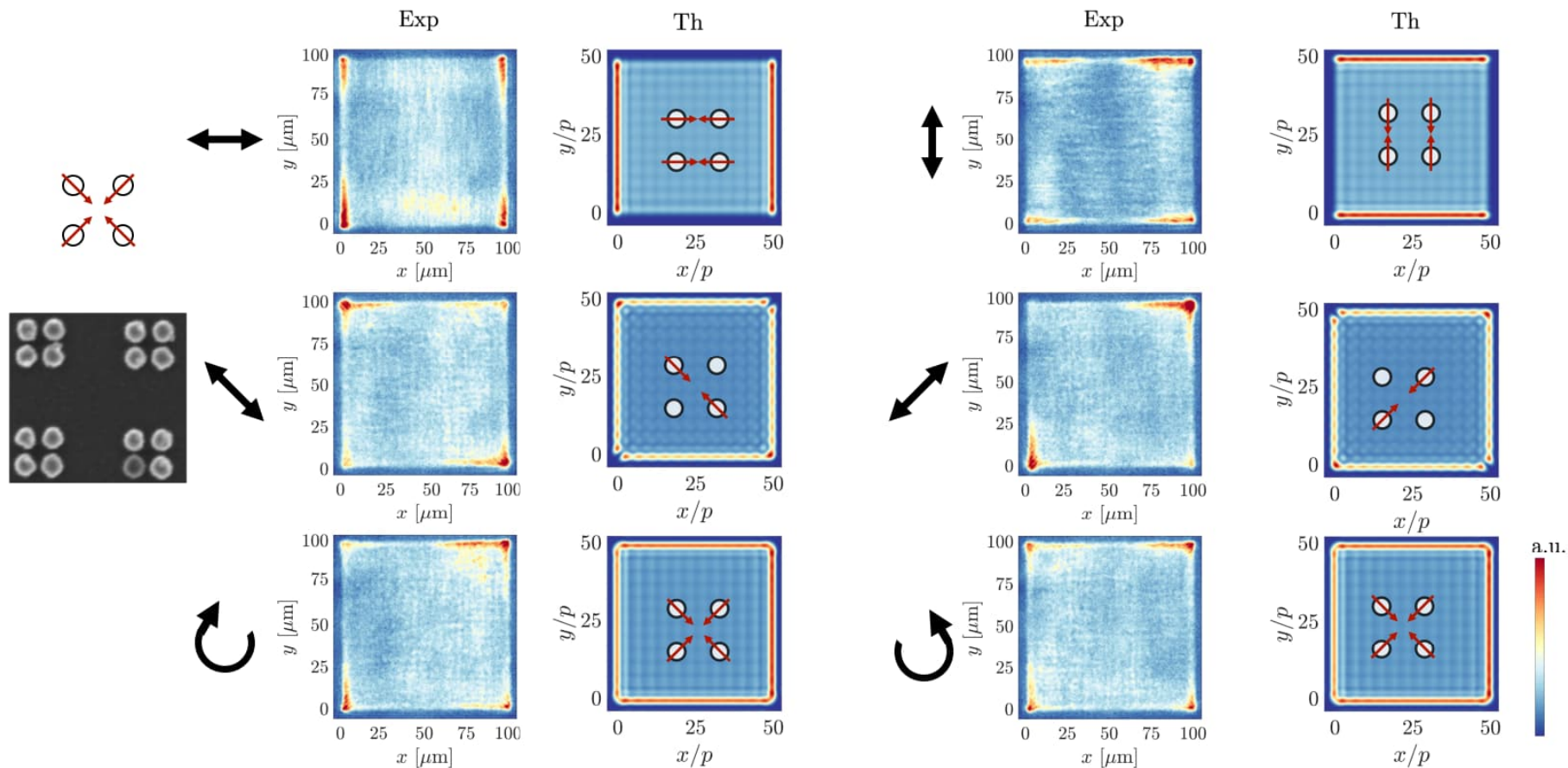
Rybin and Kivshar, *Nature* **541**, 164 (2017)  
Kodigala *et al.*, *Nature* **541**, 196 (2017)  
Ha *et al.*, *Nat. Nanotechnol.* **13**, 1042 (2018)  
Huang *et al.*, *Science* **367**, 1018 (2020)

Wang *et al.*, *Nat. Phot.* **14**, 623 (2020)  
Wu *et al.*, *New J. Phys.* **24**, 033002 (2022)  
Kang *et al.*, *Adv. Optical Mater.* **10**, 2101497 (2022)

# Lasing from a plasmonic lattice: 4-fold rotational symmetry



# Polarization-resolved images: the vortex

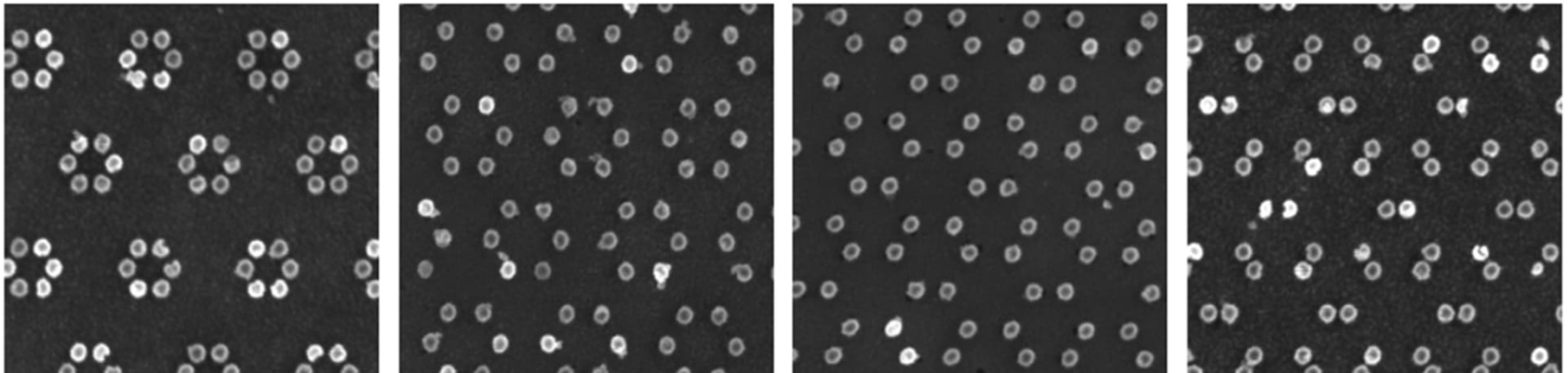


Heilmann *et al.*, *ACS Photonics* **9** 224 (2022)

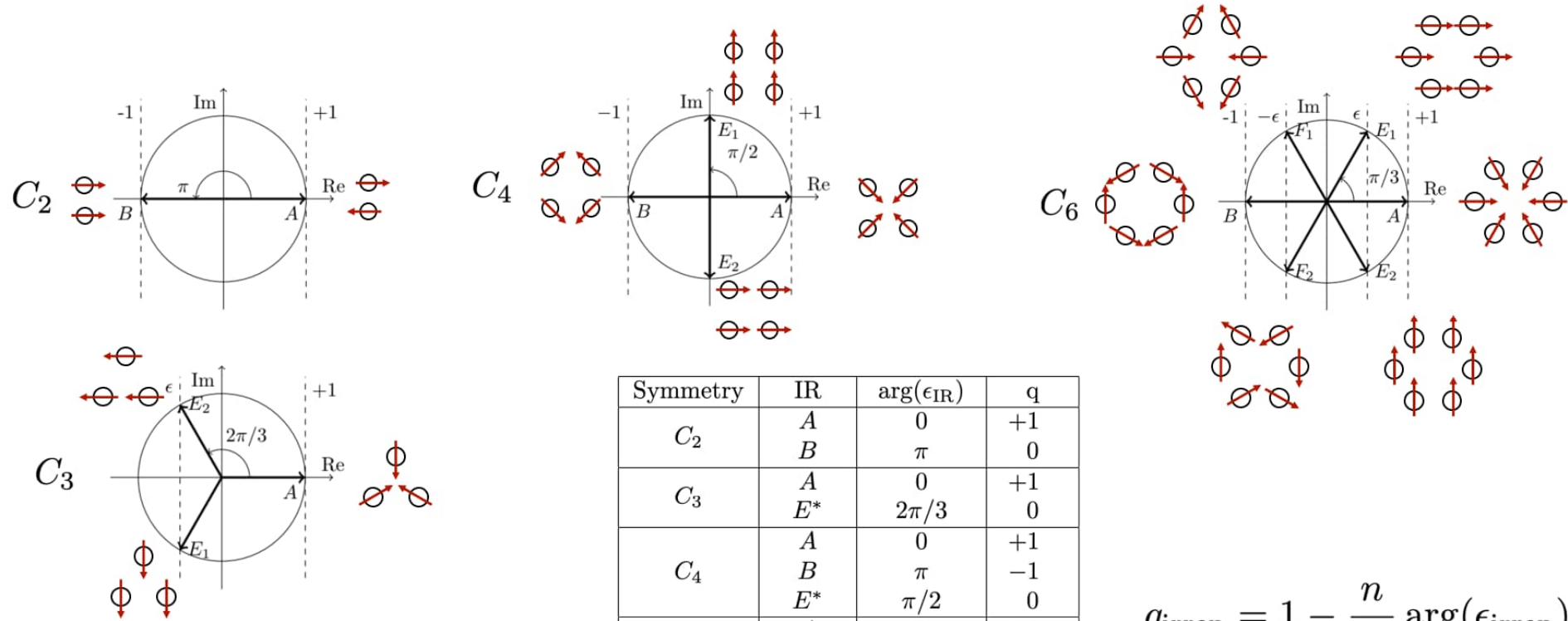
# Loss-induced topological transition

Salerno, Heilmann, Arjas, Aronen, Martikainen, PT, PRL 2022

Tuning interparticle distance in a hexamer nanoparticle array



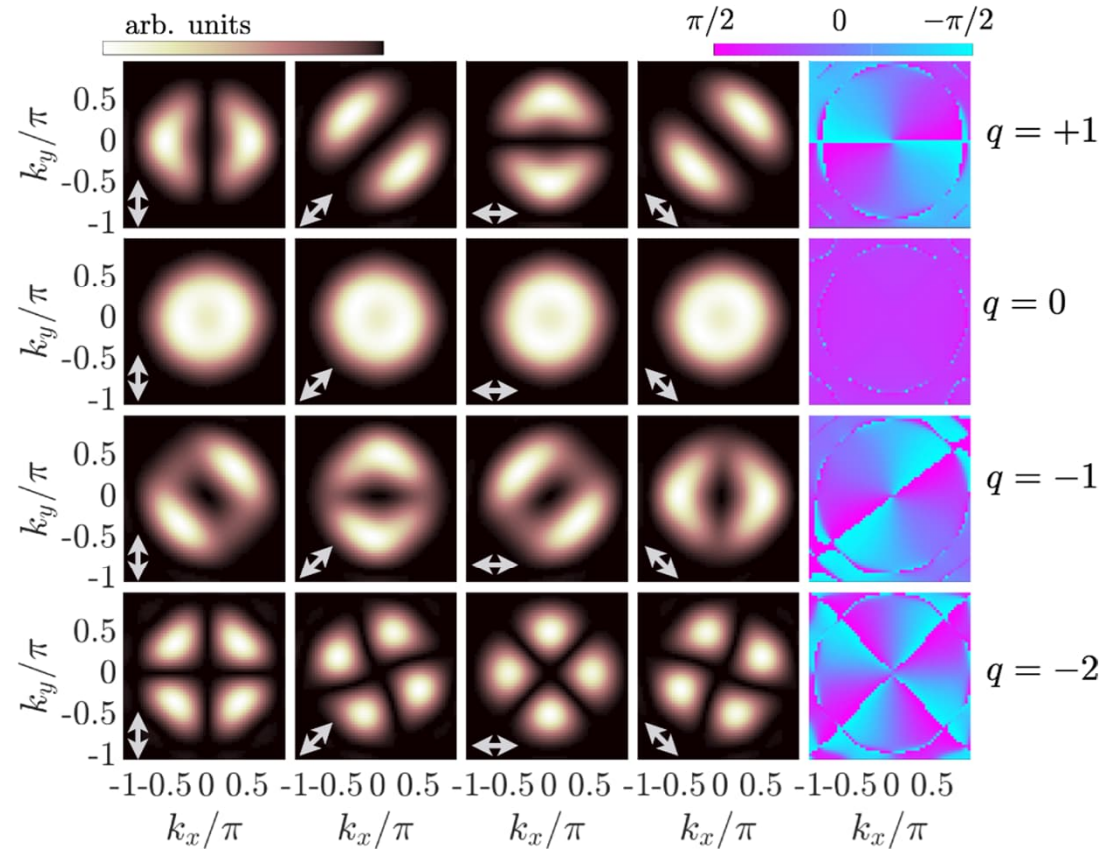
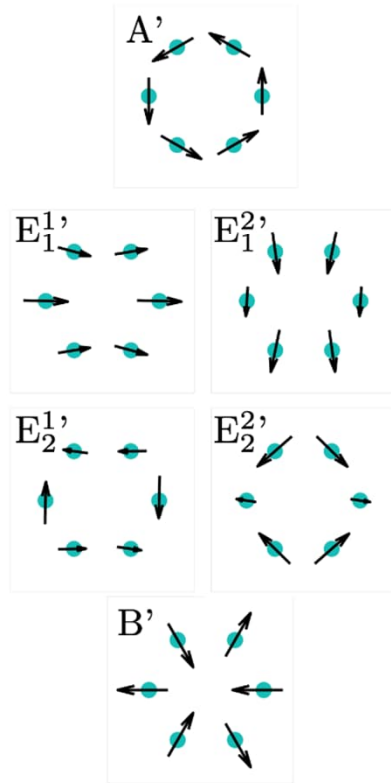
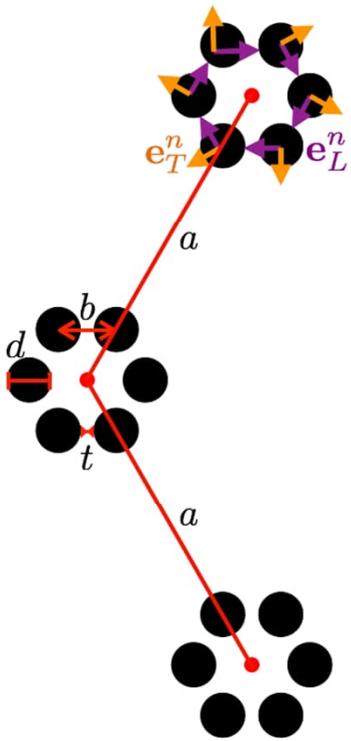
# Rotational symmetries and irreducible representations



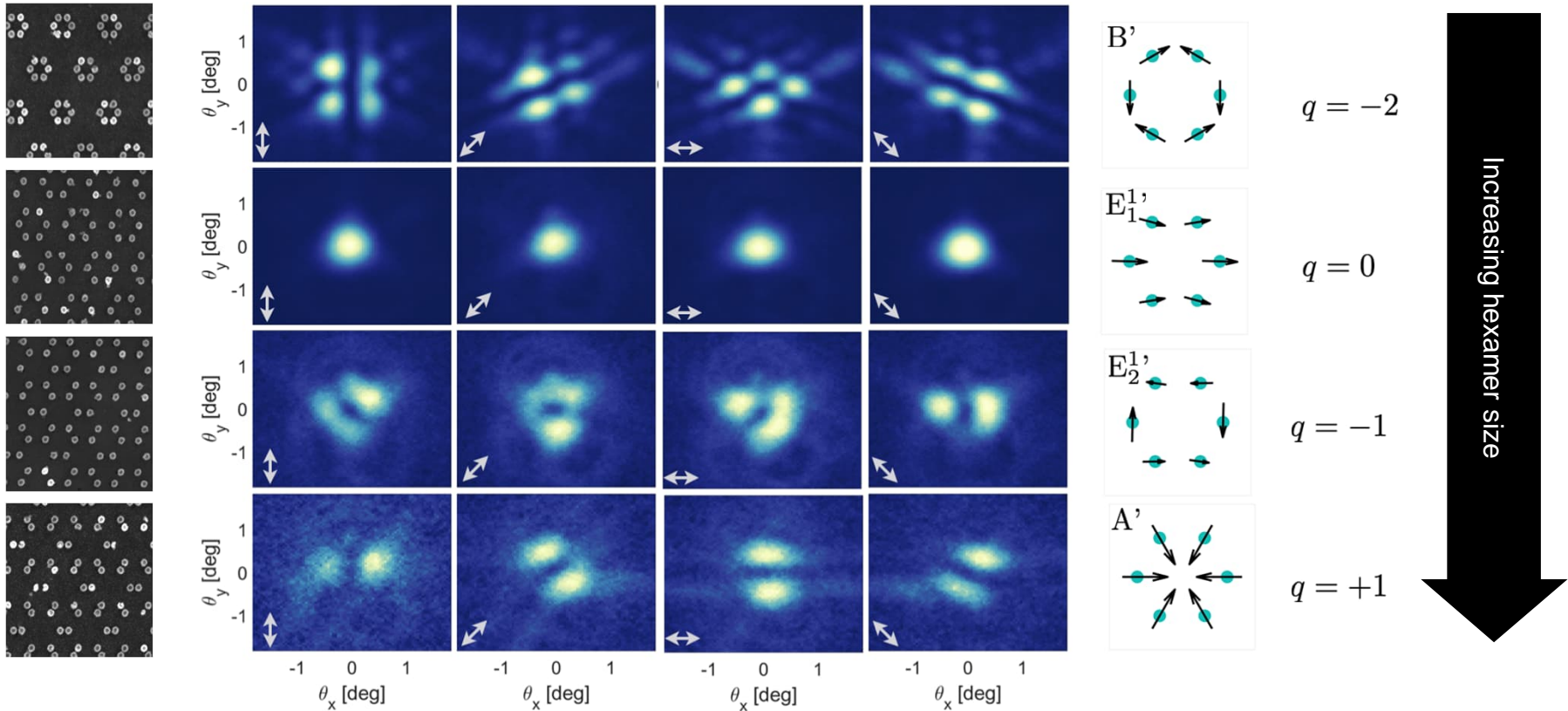
Symmetry	IR	$\arg(\epsilon_{\text{IR}})$	q
$C_2$	A	0	+1
	B	$\pi$	0
$C_3$	A	0	+1
	$E^*$	$2\pi/3$	0
$C_4$	A	0	+1
	B	$\pi$	-1
	$E^*$	$\pi/2$	0
$C_6$	A	0	+1
	B	$\pi$	-2
	$E_1^*$	$\pi/3$	0
	$E_2^*$	$2\pi/3$	-1

$$q_{\text{irrep}} = 1 - \frac{n}{2\pi} \arg(\epsilon_{\text{irrep}})$$

# Modes of a single hexamer: real and k-space



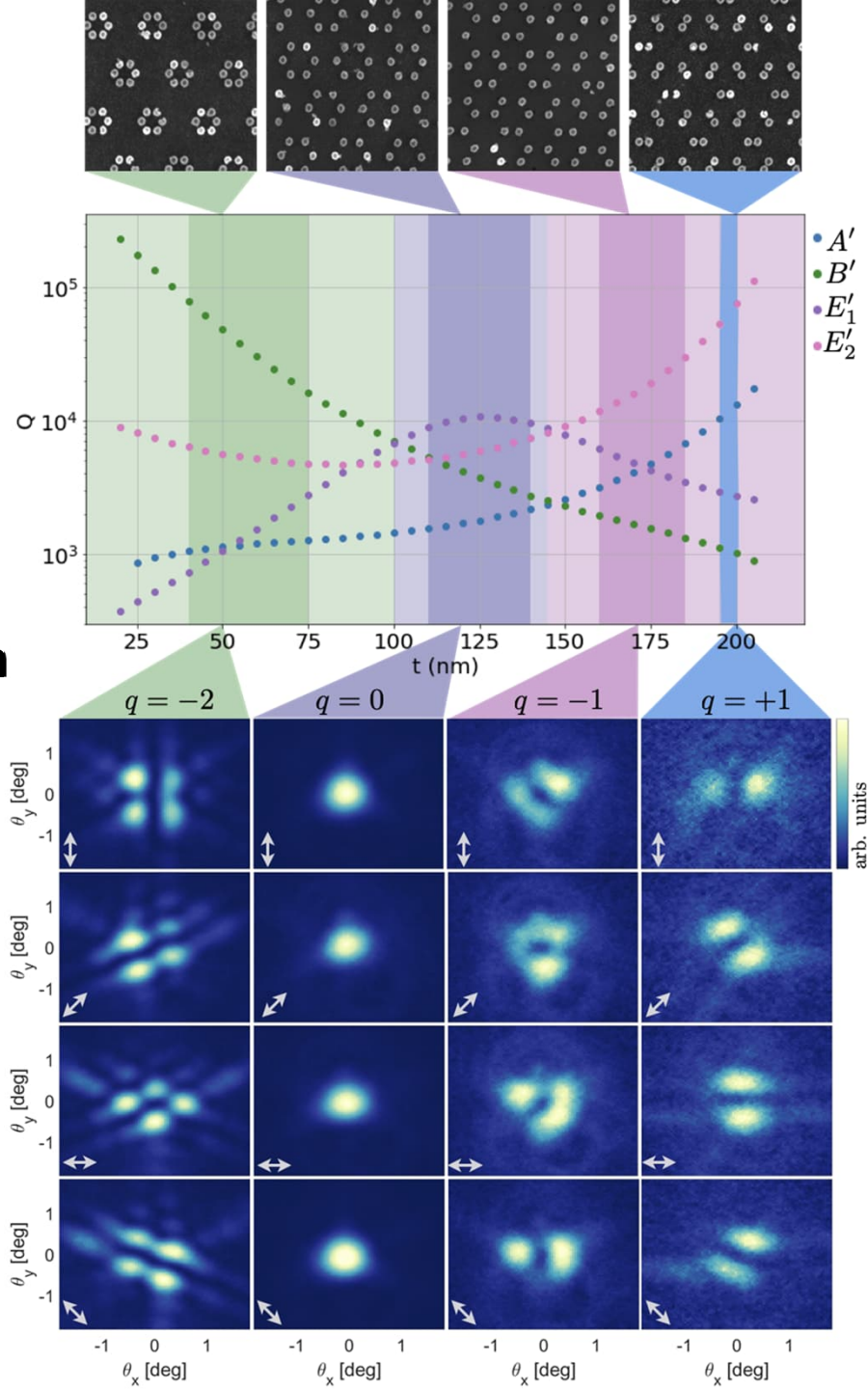
# Lasing mode changes with geometry: topological transition





T-matrix simulations:  
Q-factors of the modes vary

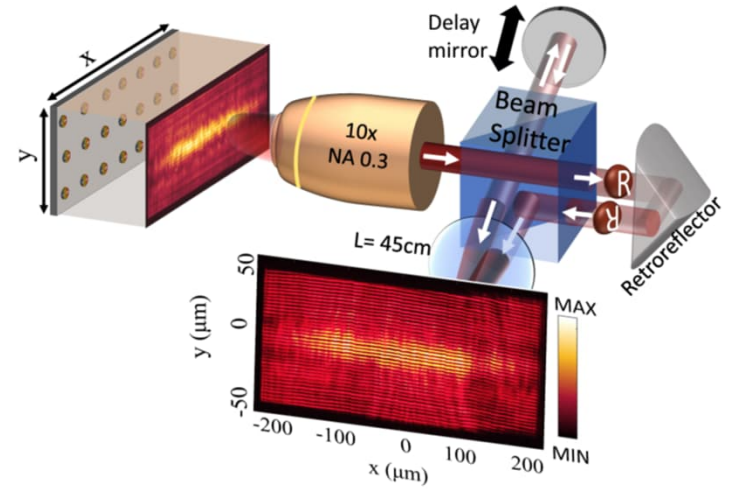
→ **Topological transitions driven  
by losses (gain)**



# Summary

Bose-Einstein condensation in a plasmonic lattice; spatial and temporal coherence show power law; polarization and phase textures

Quasi-BIC lasing with a quadrumer and hexamer lattice; topological transitions driven by losses



# Outlook

Interplay of quantum geometry, topology and interactions in photonic systems

