Identifying unbound strong bunching and the breakdown of the Rotating Wave Approximation in the quantum Rabi model

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"miko" + *why*

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arXiv:2211.13249 (2022)

Can measuring the statistics of light herald the breakdown of the JC model?

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Alvaro Nodar

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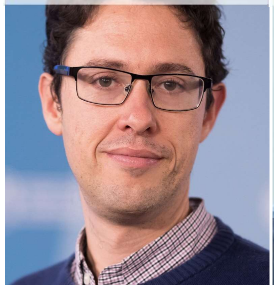


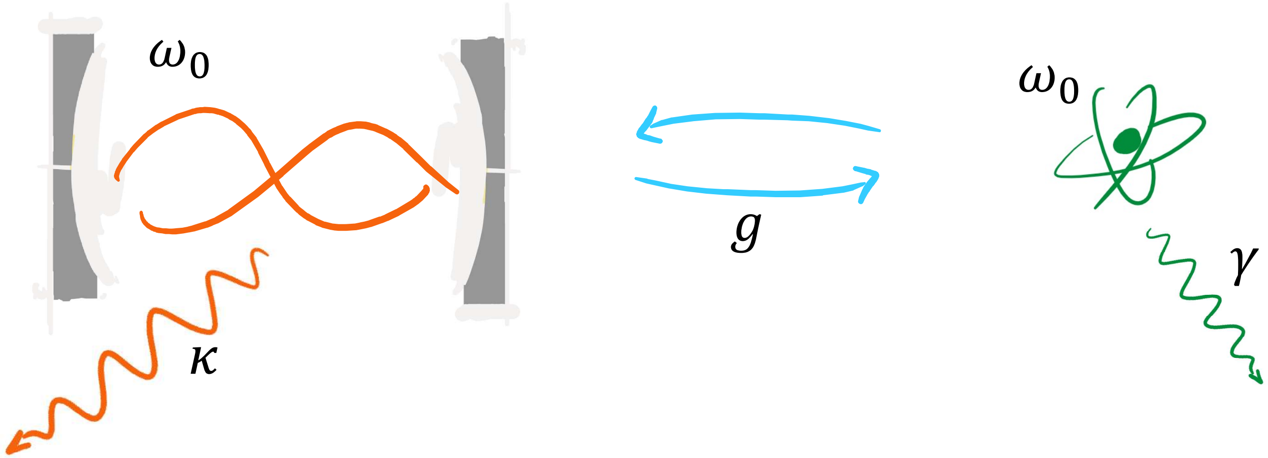
Javier Aizpurua

Unai Muniain

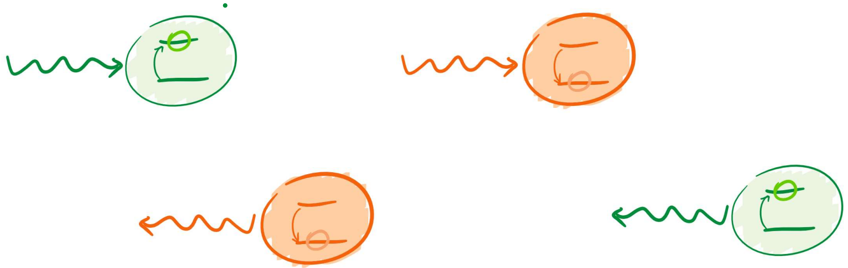
University of the Basque Country

$$\hat{\boldsymbol{E}}(\boldsymbol{r}_\sigma) = \boldsymbol{E}(\boldsymbol{r}_\sigma) i(\hat{a}^\dagger - \hat{a})$$

$$\hat{\boldsymbol{d}} = \boldsymbol{d}(\hat{\sigma} + \hat{\sigma}^\dagger)$$

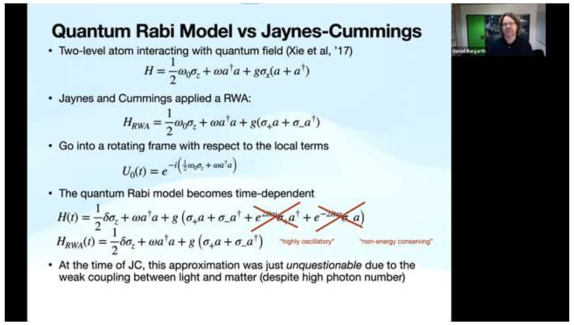


$$\hat{H}_R = -\hat{\boldsymbol{d}} \cdot \hat{\boldsymbol{E}}(\boldsymbol{r}_\sigma) = ig\left(\hat{a}^\dagger\hat{\sigma} - \hat{a}\hat{\sigma}^\dagger\right) + ig(\hat{a}^\dagger\hat{\sigma}^\dagger - \hat{a}\hat{\sigma})$$



Jaynes and Cummings, Proceedings of the IEEE 51, 89 (1963).

which justification for the *rotating wave approximation (RWA)* did you learn in school?



Australian Institute of Physics seminar by Daniel Burgarth (MQ/FAU) @ YouTube

$\hat{H}_R = g\left(\hat{a} + \hat{a}^\dagger\right)\hat{\sigma}_x$ for Fock states RWA introduces error $O(\sqrt{n}g/\omega_0)$

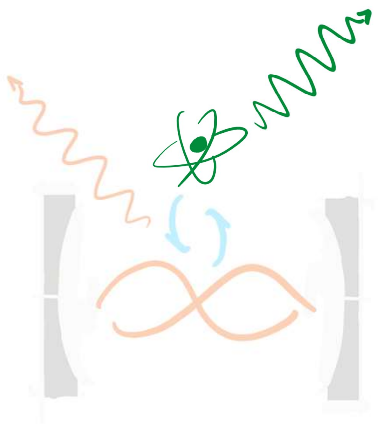
arXiv:2301.02269

arXiv:2111.08961

coupling g

$g \ll \kappa$

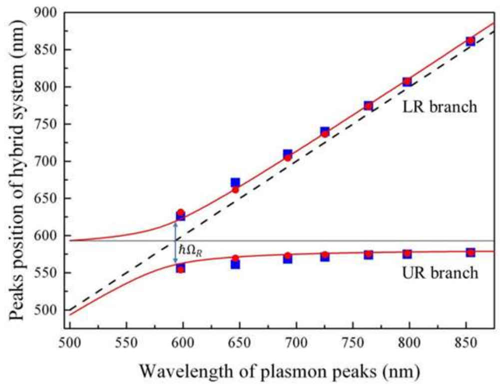
$\hat{H}_{JC}$



$$\gamma \to \gamma \times \frac{4g^2}{\kappa\gamma}$$

$\kappa \lesssim g \ll \omega_0$

$\hat{H}_{JC}$

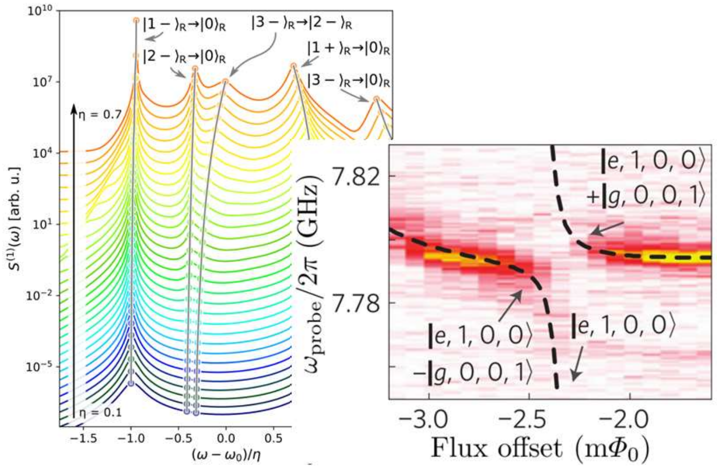


Melnikau, et al., Phys. Chem. Lett. 7, 354–362 (2016)

$$\omega_{n\pm} = \omega_a \pm g\sqrt{n+1}$$

$\omega_0/10 \lesssim g$

$\hat{H}_R$



Niemczyk et al., Nat. Physics 6, 772 (2010).

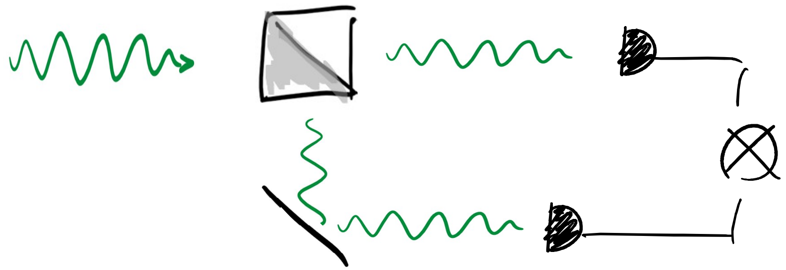
further mode splitting $\omega_{n\pm}$

other signatures?

$$\hat{H}_R^2|0,g\rangle = g^2|0,g\rangle + g^2|2,g\rangle$$

1/ fix the Rabi Hamiltonian, add *measurement*,

2/ calculate/measure the intensity correlations



$$g^{(2)}(\tau) = \frac{\langle I_1(t+\tau) I_2(t)\rangle}{\langle I_1(t+\tau)\rangle\langle I_2(t)\rangle}$$

VOLUME 10, NUMBER 3 PHYSICAL REVIEW LETTERS FEBRUARY 1963

PHOTON CORRELATIONS*

Roy J. Glauber
Lyman Laboratory, Harvard University, Cambridge, Massachusetts
(Received 27 December 1962)

1/ rewrite the Hamiltonian, add *measurement*,

2/ calculate/measure the intensity correlations

vector potential $\propto a^\dagger + a$

electric field $\propto i(a^\dagger - a)$

rewrite the Quantum Rabi Hamiltonian (in the Coulomb gauge):

$$\hat{H}_{QR} = \hbar\omega_0 a^\dagger a + \frac{1}{2}\hbar\omega_0\left(\sigma_z \cos\left[2\frac{g}{\omega_0}(a^\dagger + a)\right] + \sigma_y \sin\left[2\frac{g}{\omega_0}(a^\dagger + a)\right]\right)$$

Di Stefano et al., Nat. Physics 15, 803 (2019);
Settineri et al., Phys. Rev. Res. 3, 023079 (2021);
Salmon et al., Nanophotonics 11, 1573 (2022);
+ many others!

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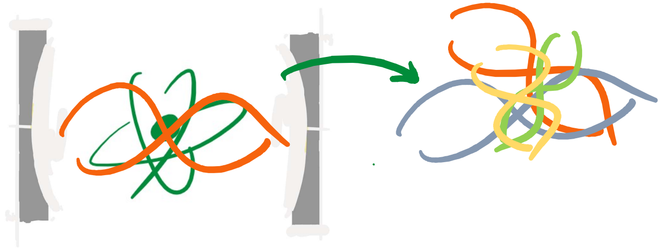

dressed operators $x_a = \sum_{\substack{\nu,\mu; \\ \omega_\nu > \omega_\mu}} |\mu\rangle_R \langle\mu| i(a^\dagger - a) |\nu\rangle_R \langle\nu|$ $\quad x_\sigma = \sum_{\substack{\nu,\mu; \\ \omega_\nu > \omega_\mu}} |\mu\rangle_R \langle\mu| (\sigma^\dagger + \sigma) |\nu\rangle_R \langle\nu|$

vector potential $\propto a^\dagger + a$

electric field $\propto i(a^\dagger - a)$

master equation

$$\partial_t \rho(t) = -\frac{i}{\hbar}[\hat{H}_{QR}, \rho(t)]$$ hermitian evolution

$$+\frac{\gamma}{2} D_{x_\sigma}[\rho(t)] + \frac{\kappa}{2} D_{x_a}[\rho(t)]$$ TLS and cavity decay

$$+\frac{\Gamma}{2} D_{x_\sigma^\dagger}[\rho(t)]$$ incoherent TLS pumping

Salmon et al., Nanophotonics 11, 1573 (2022).

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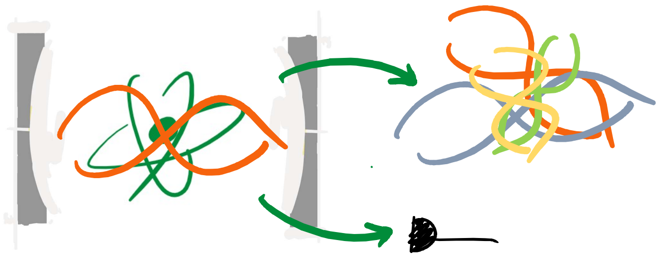
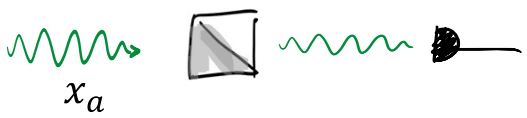


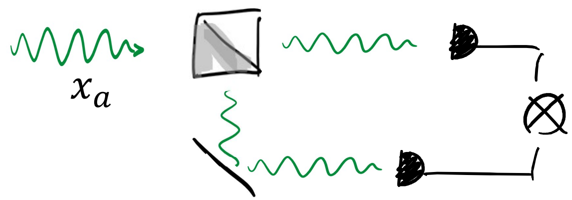
dressed operators

$$x_a = \sum_{\substack{\nu,\mu;\\ \omega_\nu > \omega_\mu}} |\mu\rangle_R\langle\mu| i\left(a^\dagger - a\right)|\nu\rangle_R\langle\nu| \qquad x_\sigma = \sum_{\substack{\nu,\mu;\\ \omega_\nu > \omega_\mu}} |\mu\rangle_R\langle\mu|\left(\sigma^\dagger + \sigma\right)|\nu\rangle_R\langle\nu|$$

$$S(\omega) = \int_{-\infty}^{\infty} \mathrm{d}\tau\, e^{-i\omega\tau} \left\langle x_a^\dagger(\tau) x_a(0)\right\rangle_{ss}$$

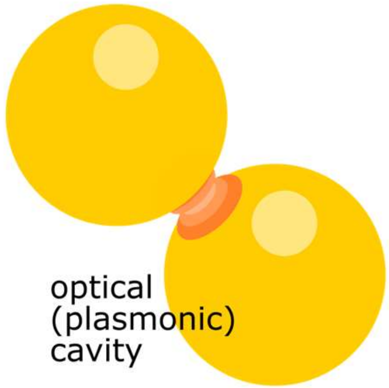


$$g^{(2)}(0) = \frac{\left\langle x_a^\dagger x_a^\dagger x_a x_a\right\rangle_{ss}}{\left\langle x_a^\dagger x_a\right\rangle_{ss}^2}$$

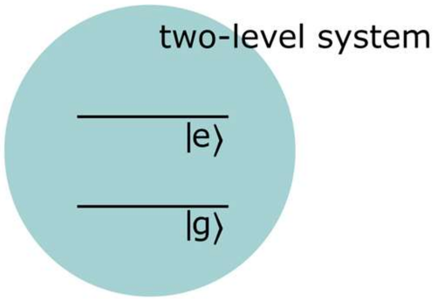


Setup

plasmonic single low-Q mode cavity,



TLS



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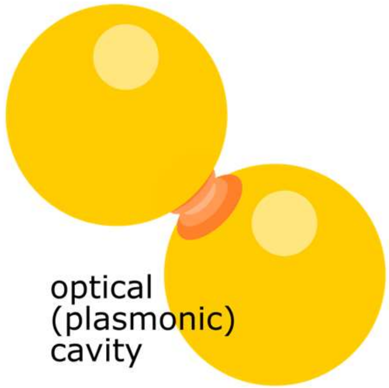
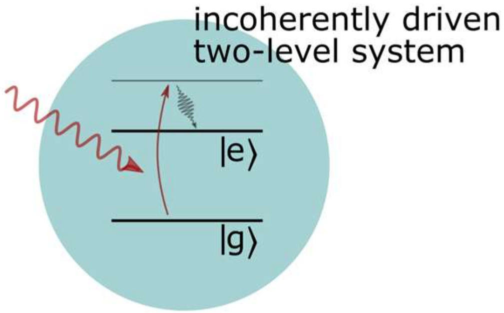


TLS incoherently pumped at rate Γ.



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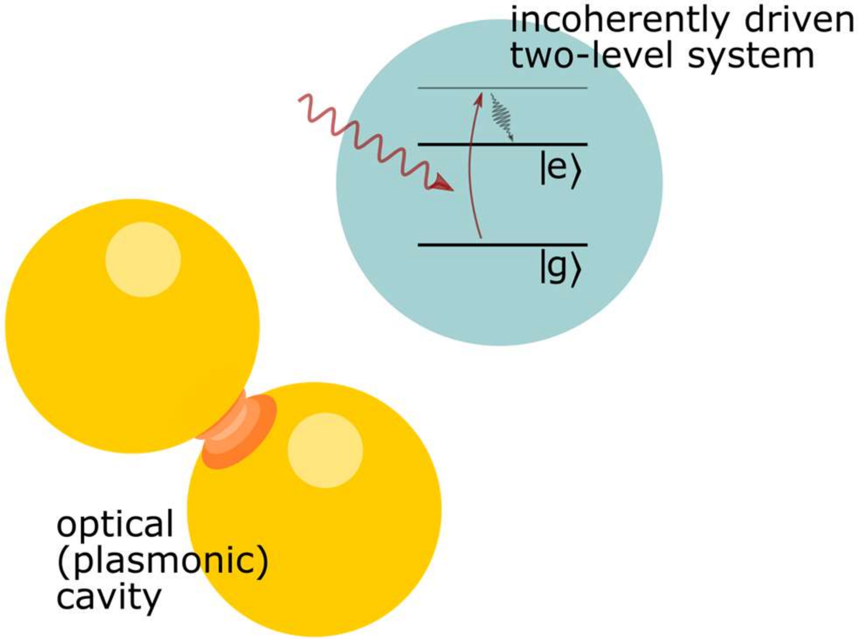

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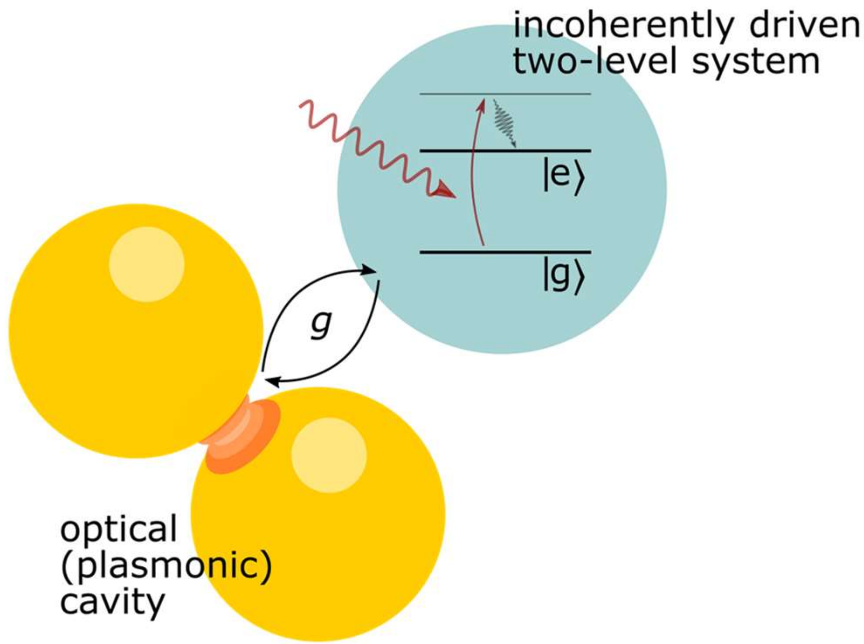

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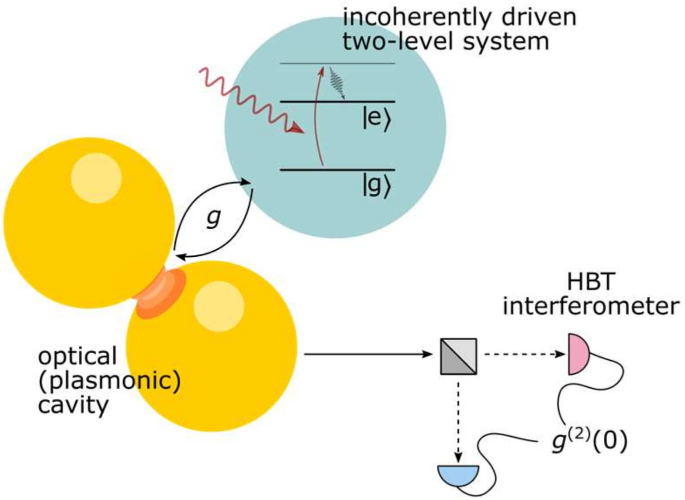

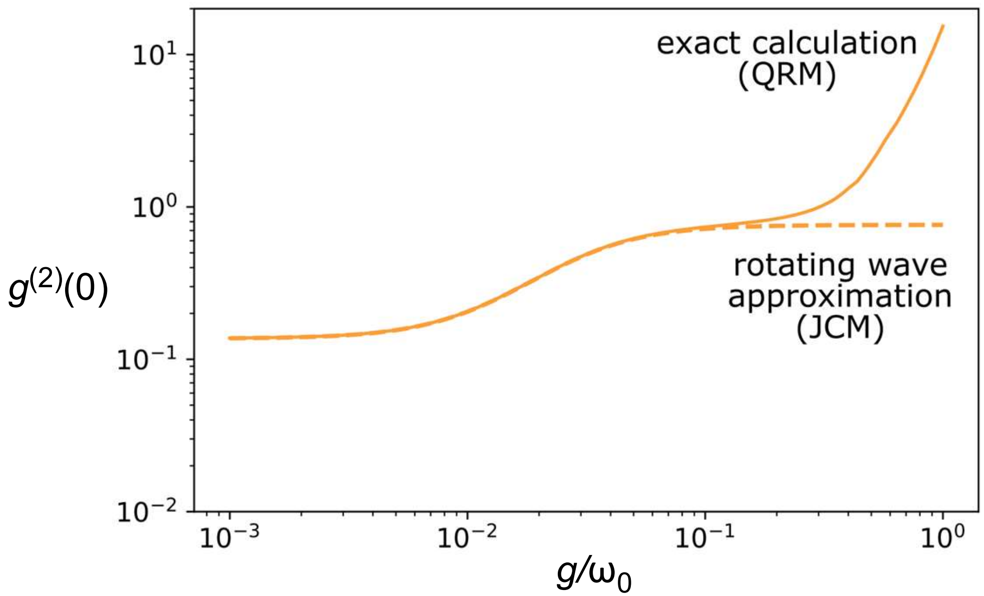

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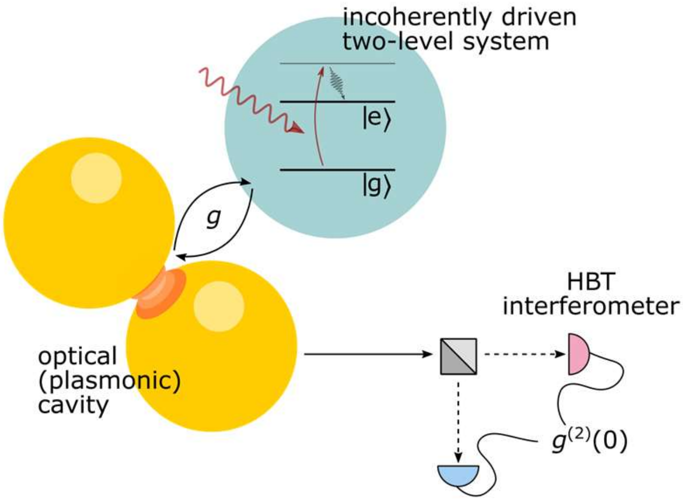
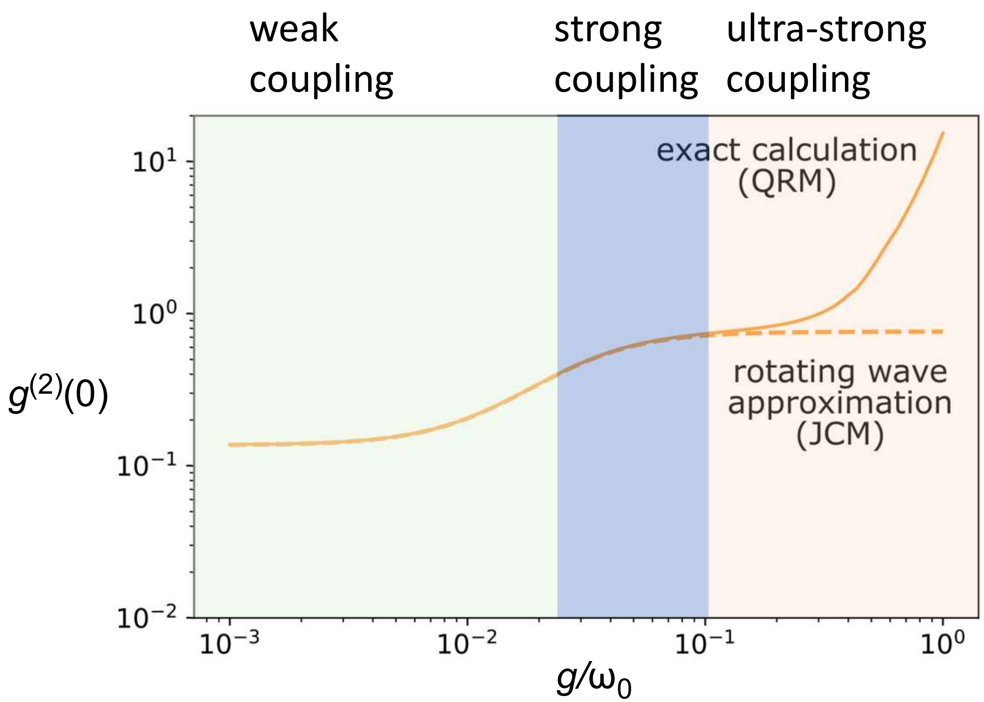
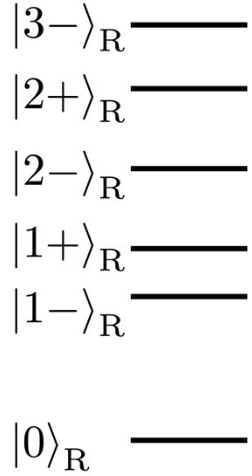




i. intensity correlation is expressed through the dressed operators

$$g^{(2)}(0) = \frac{\left\langle x_a^\dagger x_a^\dagger x_a x_a \right\rangle_{ss}}{\left\langle x_a^\dagger x_a \right\rangle_{ss}^2}$$

where $x_a = \sum_{\substack{\nu,\mu; \\ \omega_\nu > \omega_\mu}} |\mu\rangle_R \langle\mu| i\left(a^\dagger - a\right) |\nu\rangle_R \langle\nu|$



ii. approximate the steady state as the statistical mixture of Rabi polaritons,

$$\rho_{ss} \approx \sum_\nu R_\nu \, |\nu\rangle_R \langle\nu|$$

$$g^{(2)}(0) = \frac{\sum_{\nu,\mu} R_\nu \, |\langle\mu| x_a x_a |\nu\rangle_R|^2}{\left(\sum_{\nu,\mu} R_\nu \, |\langle\mu| x_a |\nu\rangle_R|^2\right)^2}$$

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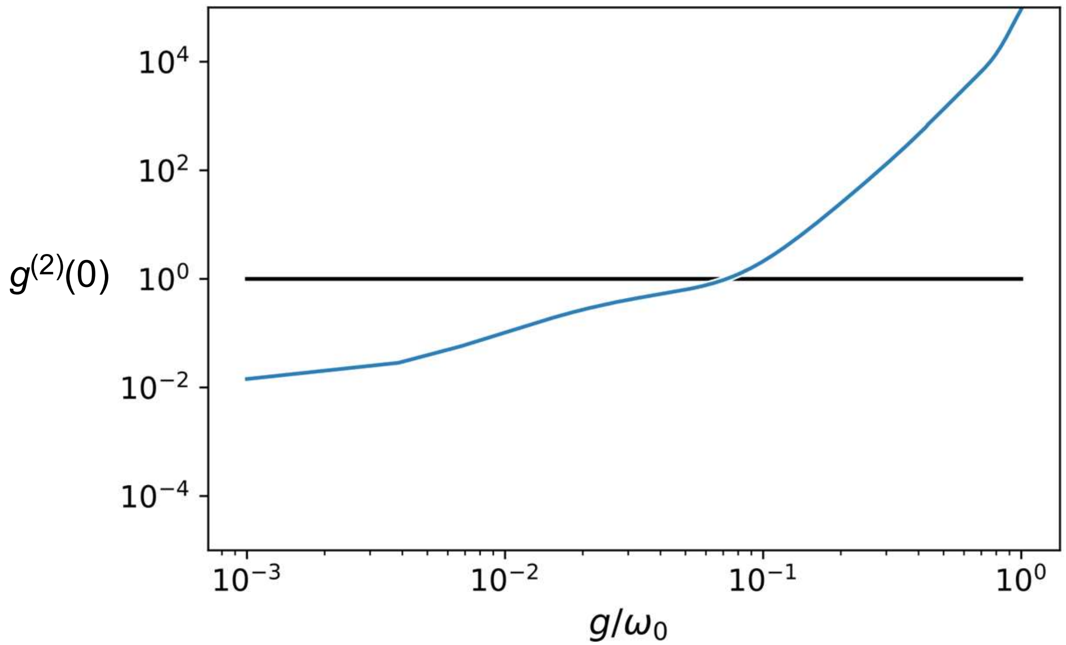

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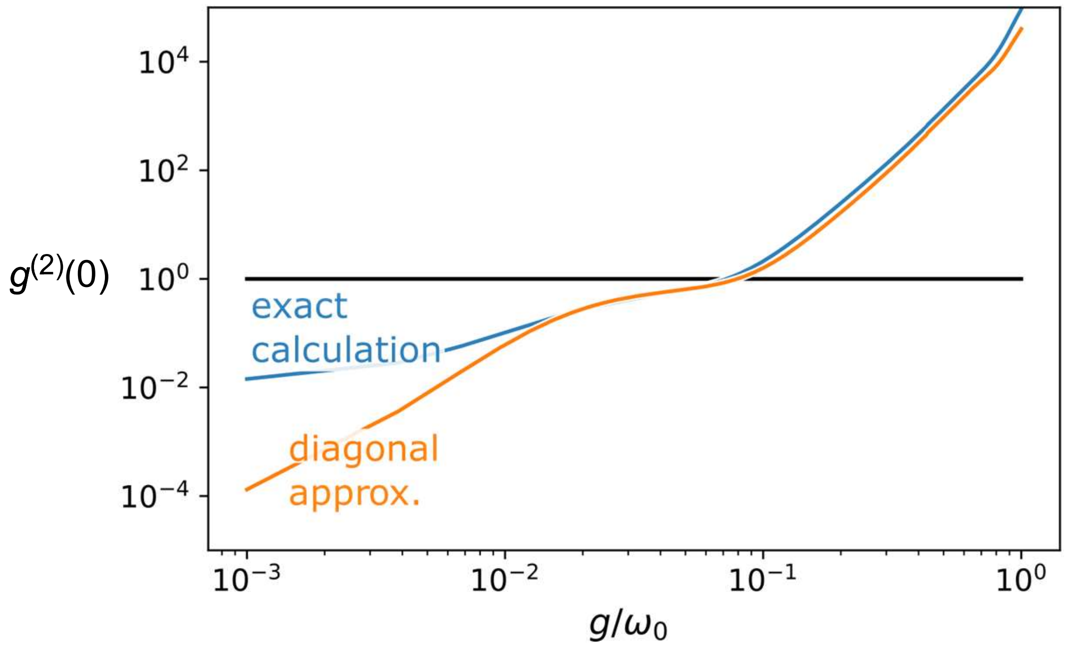
$$\approx \frac{\sum_{\nu,\mu} R_\nu \, |\langle \mu | x_a x_a | \nu \rangle_R|^2}{\left(\sum_{\nu,\mu} R_\nu \, |\langle \mu | x_a | \nu \rangle_R|^2\right)^2}$$



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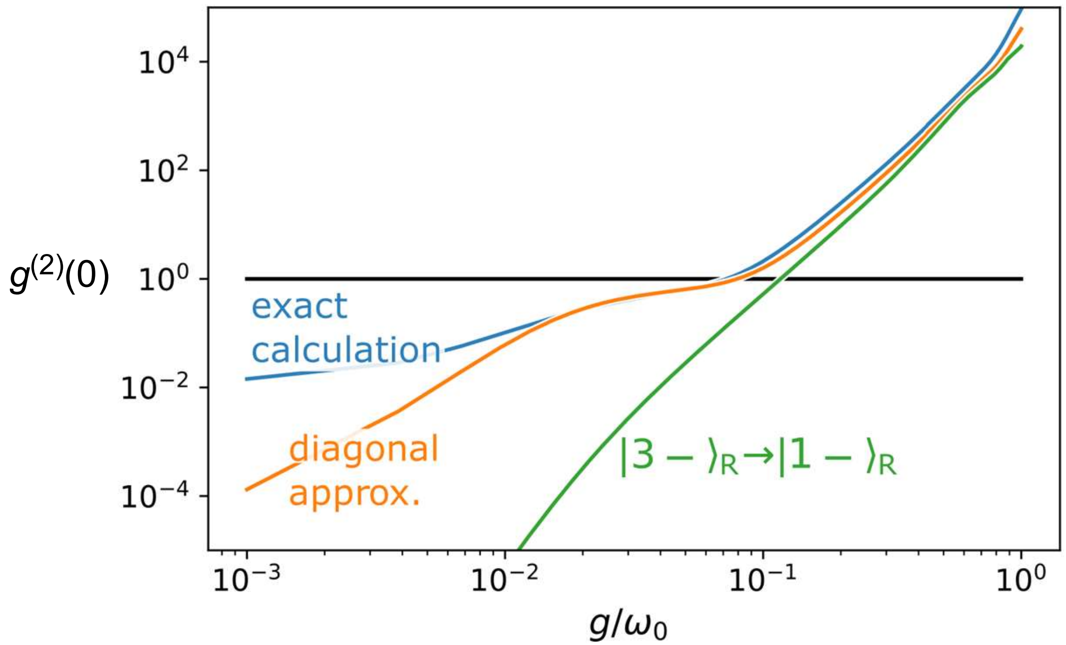
$$\approx \frac{R_{3-} \left|\langle 1 - | x_a x_a | 3 - \rangle_R\right|^2}{\left(\sum_{\nu,\mu} R_\nu \left|\langle \mu | x_a | \nu \rangle_R\right|^2\right)^2}$$



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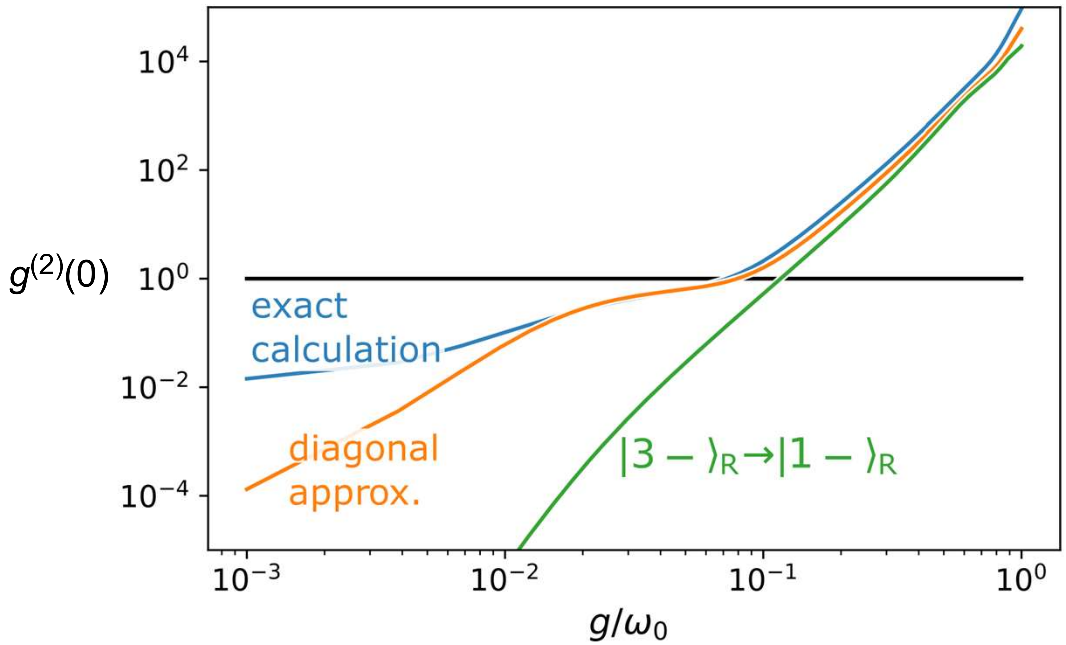
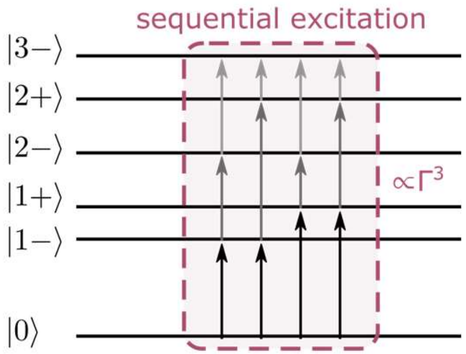


what's so special about $|3 -\rangle_R$?

- decays to $|1 -\rangle_R$, emitting a pair of photons
  $${}_R\langle 1 - | x_a x_a | 3 - \rangle_R \neq 0$$
- efficiently populated from the ground state (large R_{3-})

in JC model, pumping of higher polaritons is a sequential process $\propto \Gamma^3$



$$\frac{\Gamma}{2} D_{\sigma^\dagger}[\rho]$$

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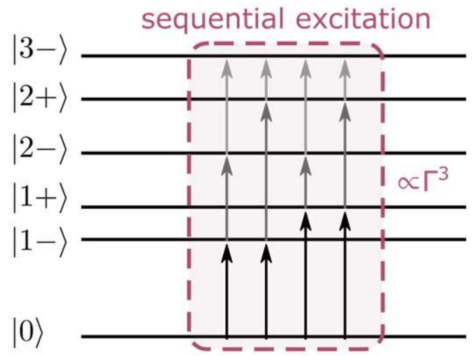

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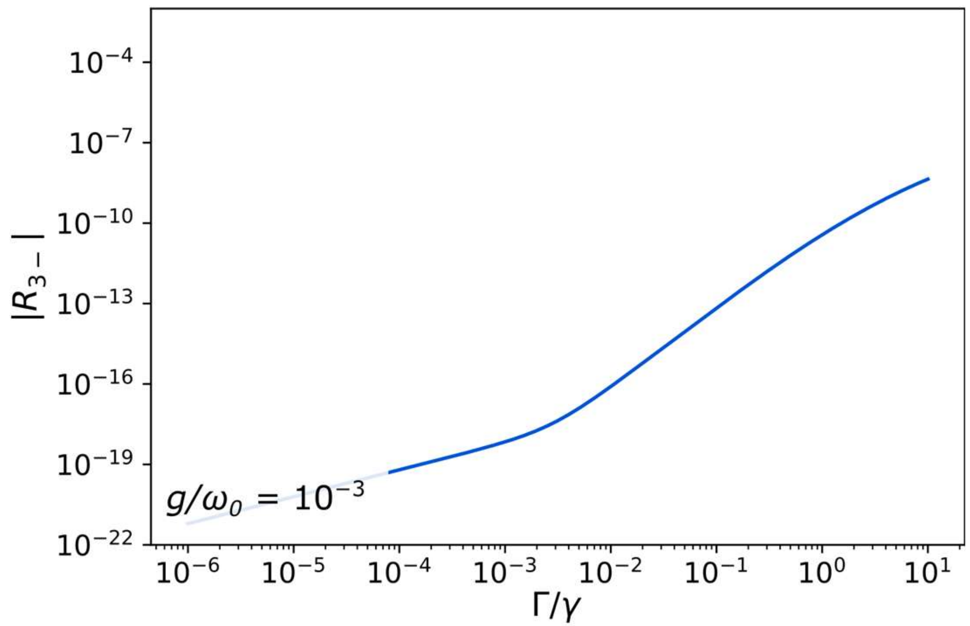

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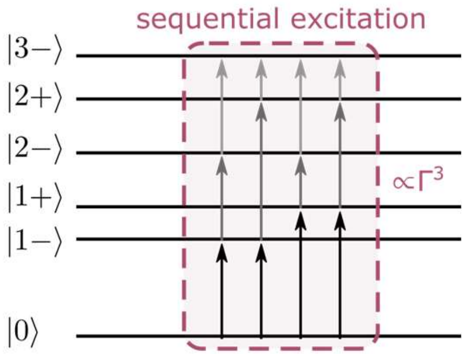

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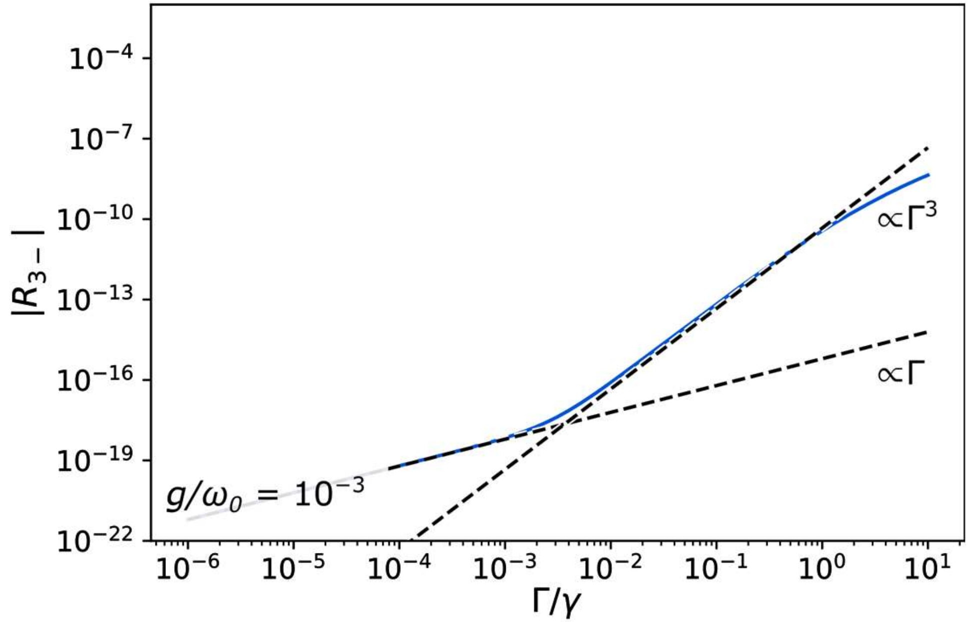

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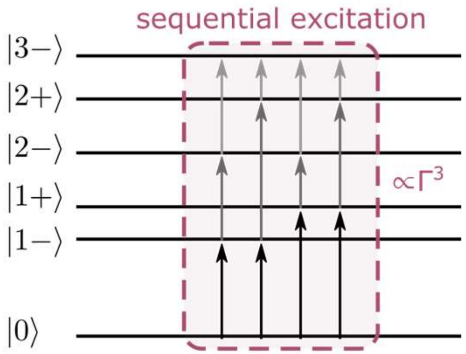

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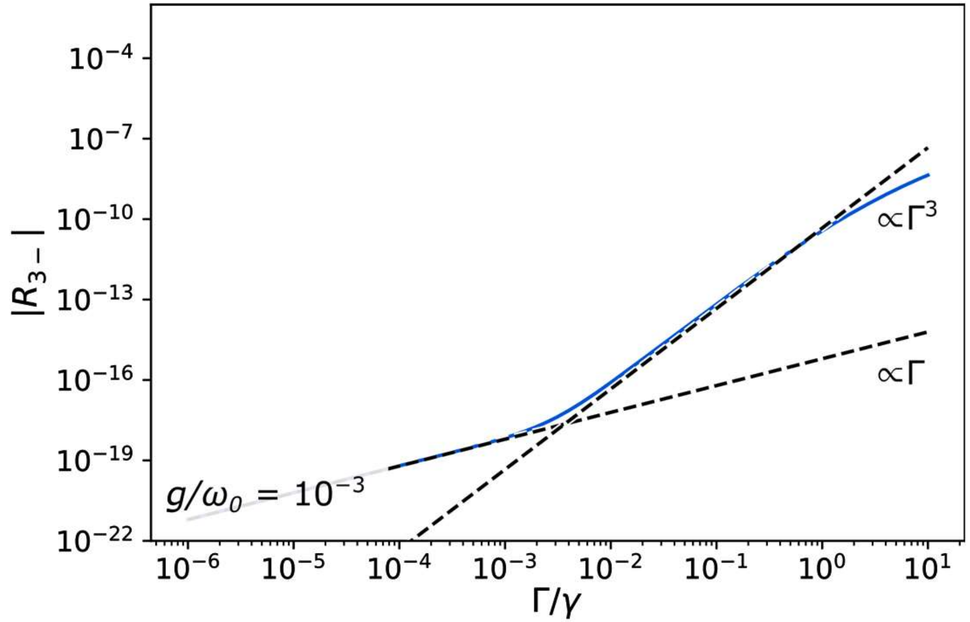
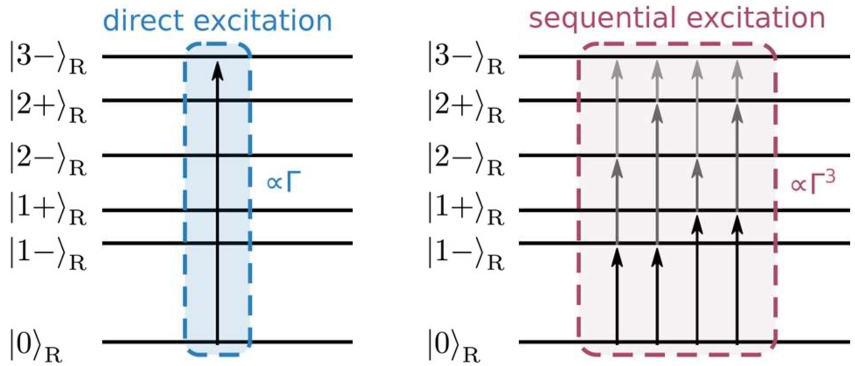


in QR model, state $|3-\rangle_R$ has a non-zero contribution from $x_a^\dagger|0\rangle_R$, and can be driven directly $\propto \Gamma$

$$\frac{\Gamma}{2} D_{x_\sigma^\dagger}[\rho]$$



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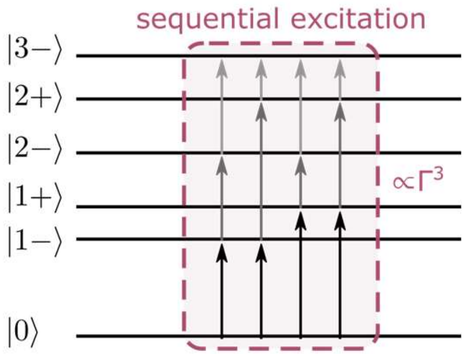

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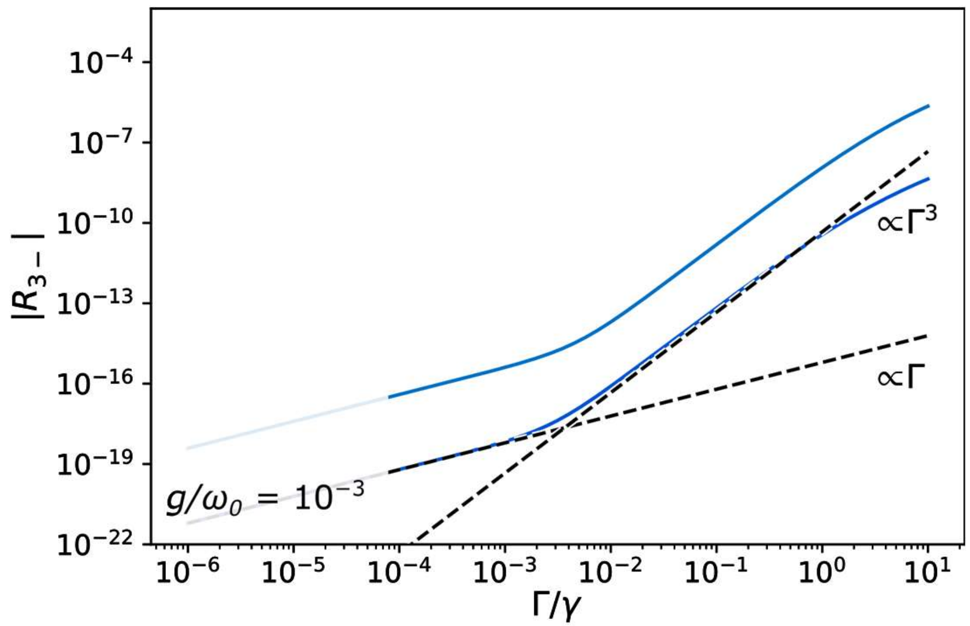

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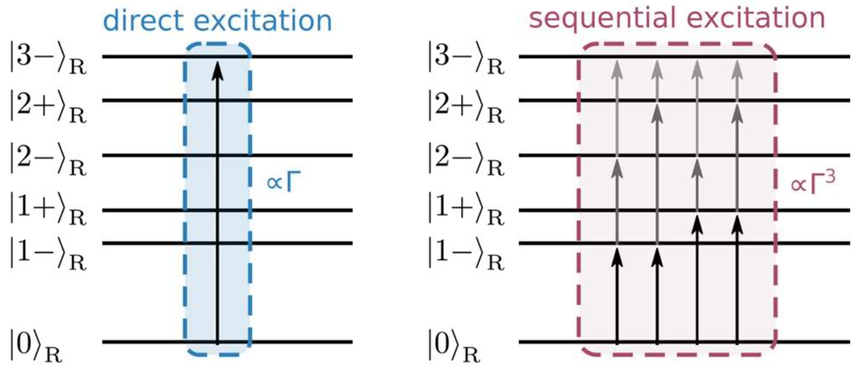

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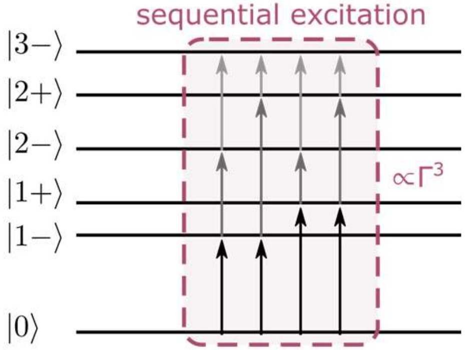

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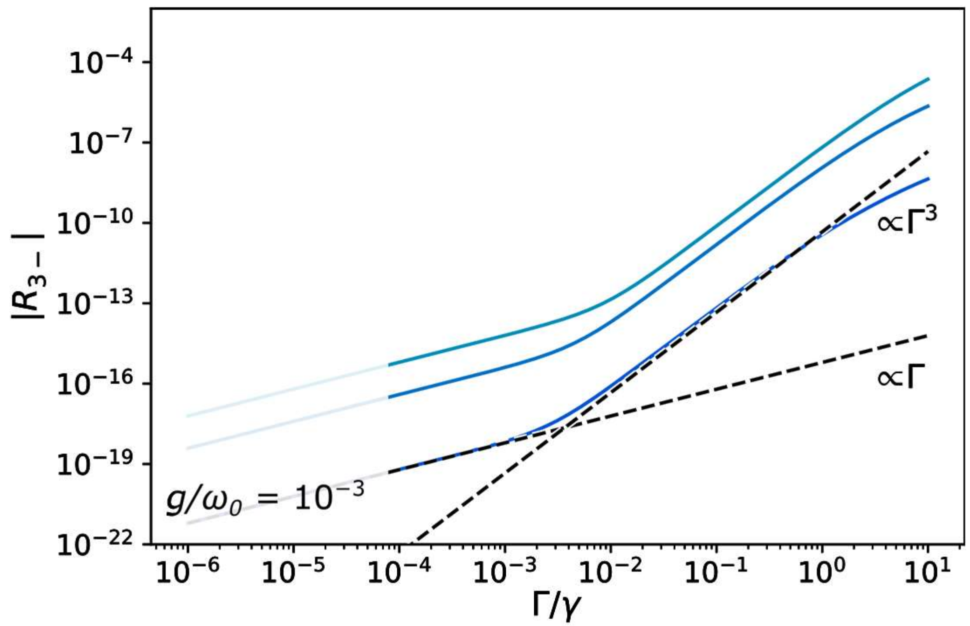

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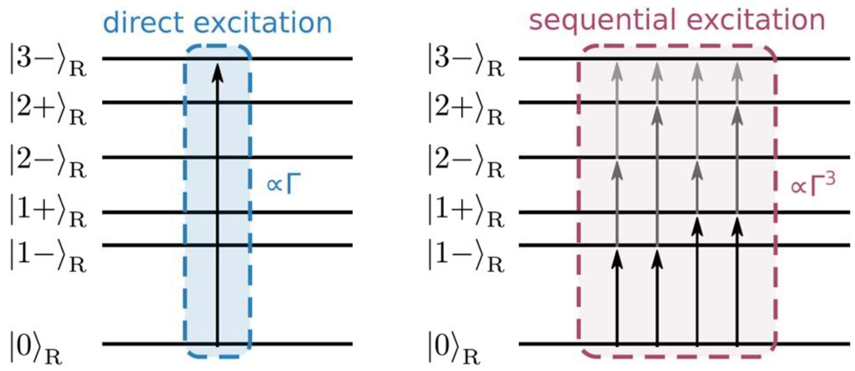

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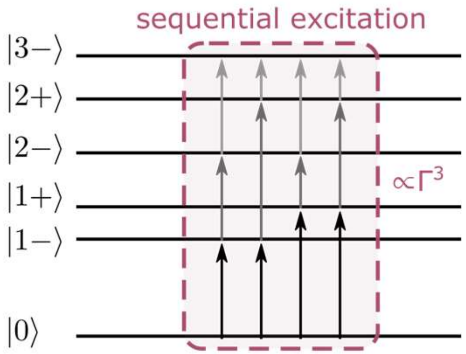

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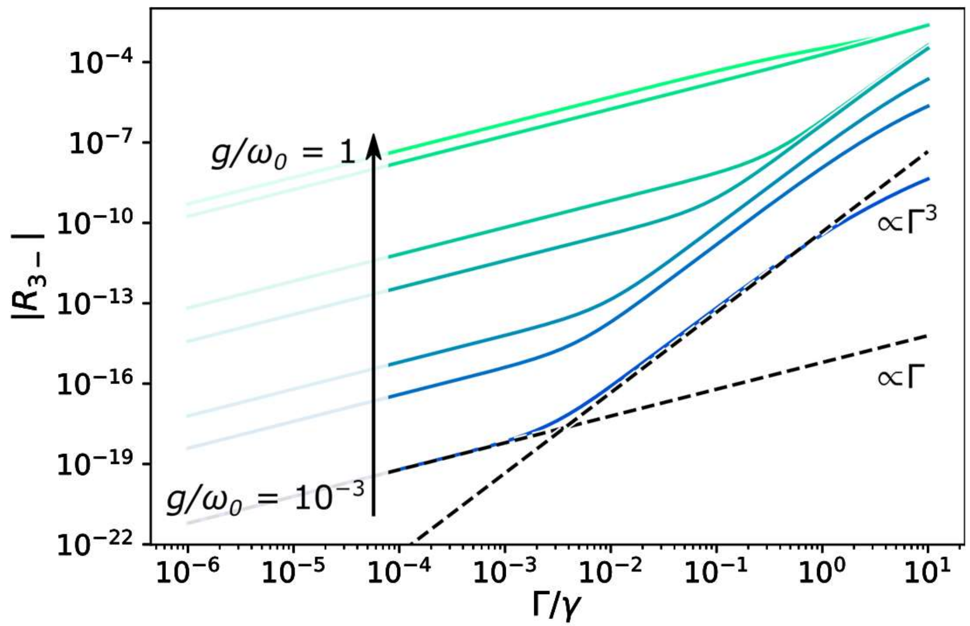

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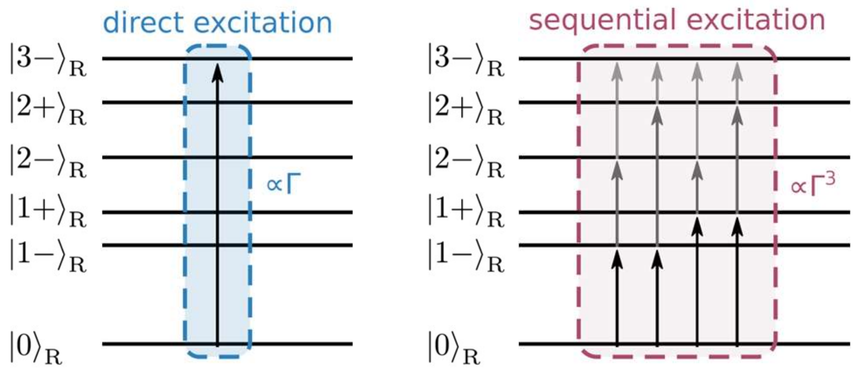
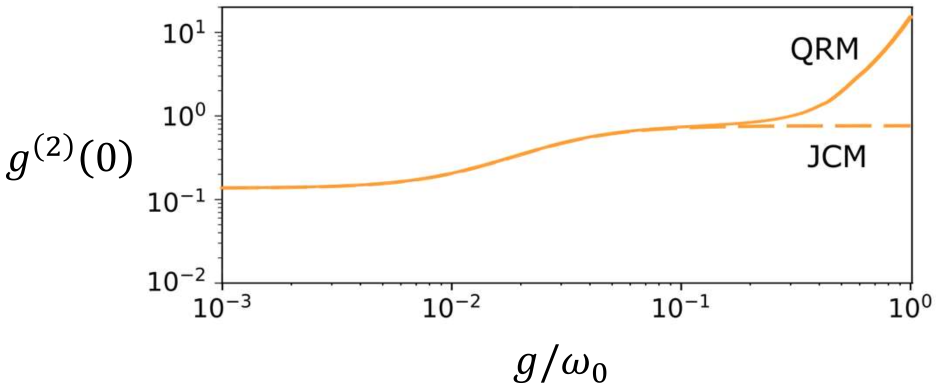
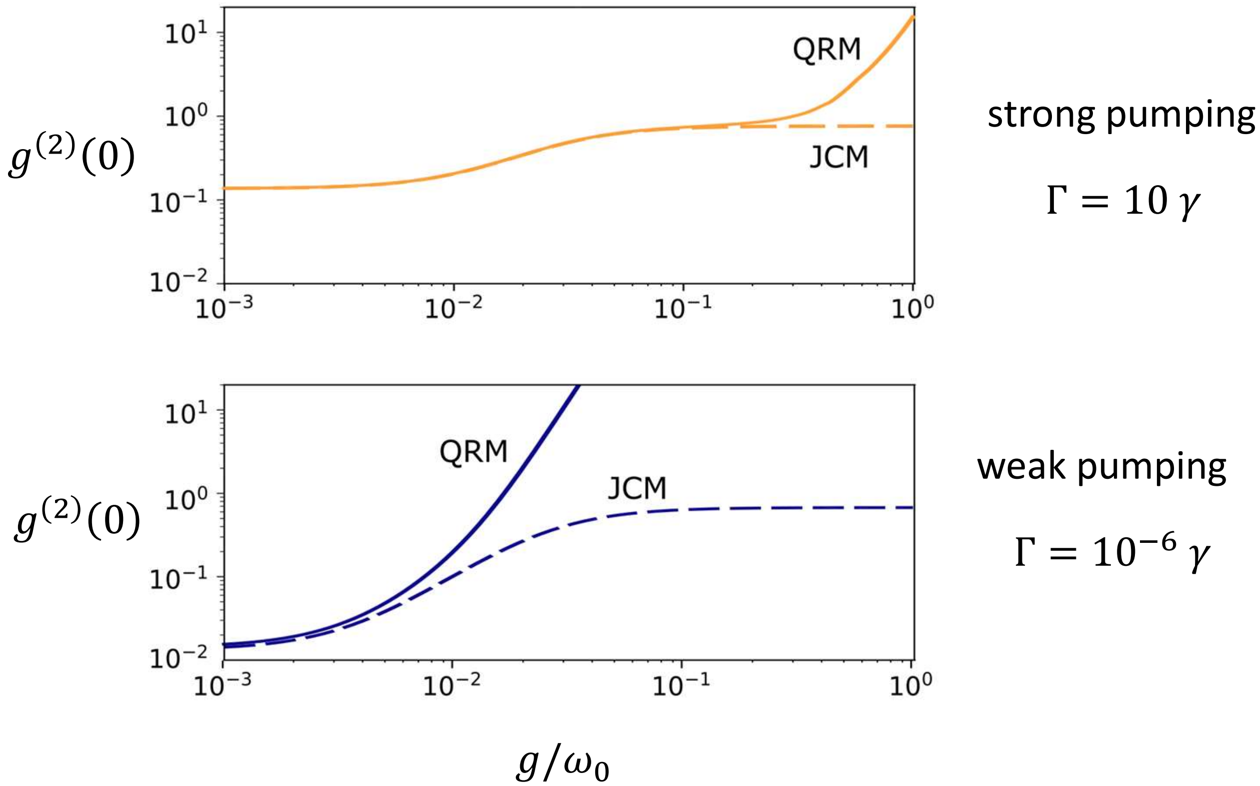


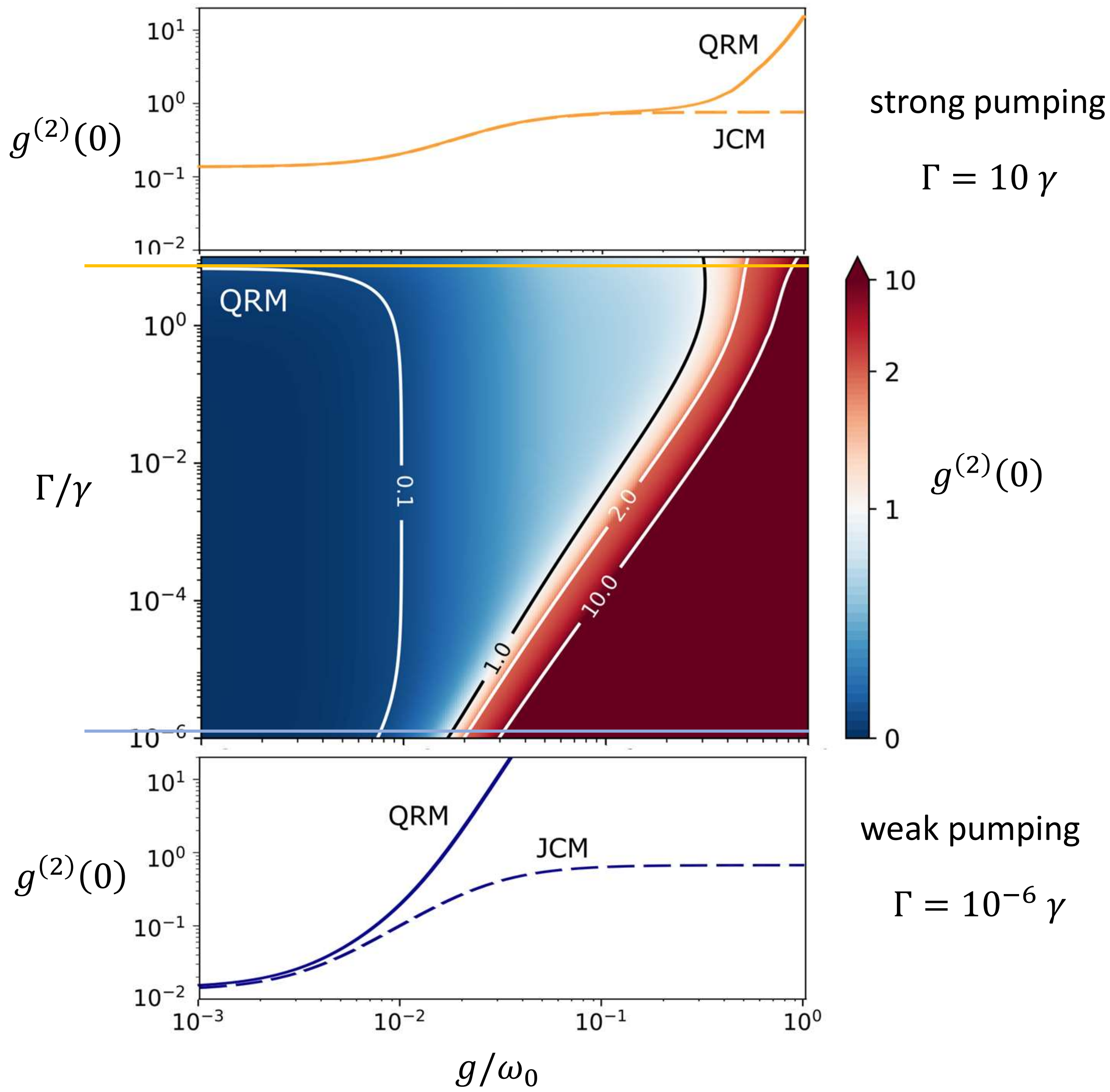


strong pumping

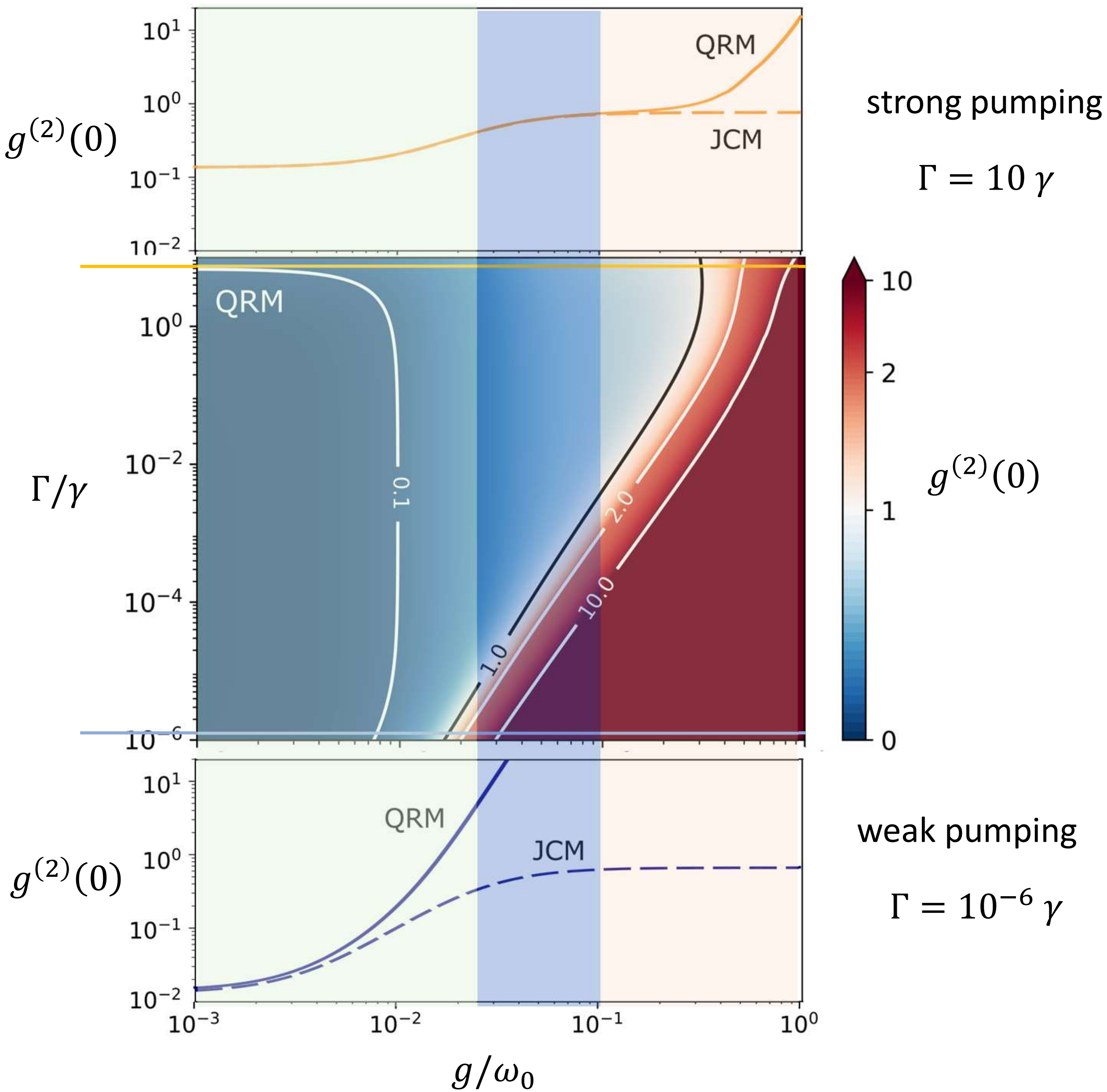
$\Gamma = 10\,\gamma$



QRM
JCM
strong pumping
$\Gamma = 10\,\gamma$
QRM
JCM
weak pumping
$\Gamma = 10^{-6}\,\gamma$
$g^{(2)}(0)$
g/ω_0
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}



$g^{(2)}(0)$
QRM
JCM
strong pumping
$\Gamma = 10\,\gamma$
Γ/γ
QRM
0.1
1.0
2.0
10.0
$g^{(2)}(0)$
weak pumping
$\Gamma = 10^{-6}\,\gamma$
g/ω_0



QRM
JCM
strong pumping
$\Gamma = 10\,\gamma$
$g^{(2)}(0)$
QRM
0.1
1.0
2.0
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Γ/γ
$g^{(2)}(0)$
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$\Gamma = 10^{-6}\,\gamma$
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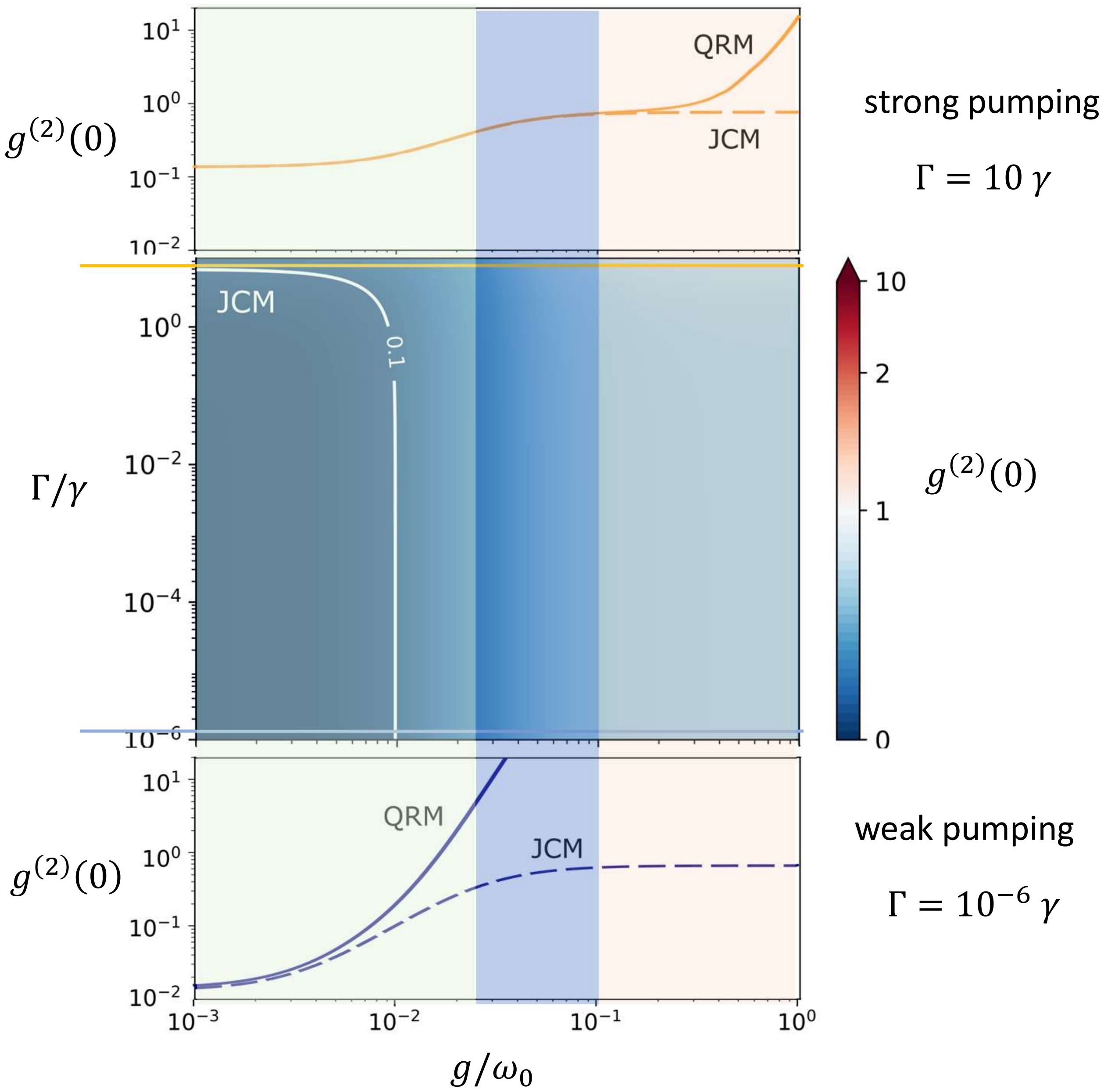
g/ω_0



QRM
JCM
strong pumping
Γ = 10 γ
g^(2)(0)
Γ/γ
0.1
weak pumping
Γ = 10^−6 γ
g/ω0
10
2
1
0

what breaks down?

i/ Coulomb gauge *quantum Rabi Hamiltonian*:

$$\hat{H}_{QR} = \hbar\omega_0 a^\dagger a + \frac{1}{2}\hbar\omega_0\left(\sigma_z \cos\left[2\frac{g}{\omega_0}(a^\dagger + a)\right] + \sigma_y \sin\left[2\frac{g}{\omega_0}(a^\dagger + a)\right]\right)$$

ii/ interaction with environment dressed operators:

$$x_a, x_\sigma$$

for $g/\omega_0 \ll 1$ $\quad \hat{H}_{QR} \approx \hbar\omega_0 a^\dagger a + \hbar\omega_0 \sigma^\dagger\sigma + i\hbar g(\hat{\sigma} - \hat{\sigma}^\dagger)(a^\dagger + a) = \hat{H}_R$ $\quad g^{(2)}(0) \gg 1$

rotating wave approximation and bare operators

$$\hat{H}_{JC} = \hbar\omega_0 a^\dagger a + \hbar\omega_0 \sigma^\dagger\sigma + ig(\hat{a}\hat{\sigma}^\dagger - \hat{a}^\dagger\hat{\sigma})$$ $\quad g^{(2)}(0) < 1$

Summary

- intensity correlations are a good (better?) probe of the Rabi Hamiltonian effects,
- $g^2(0)$ can herald the breakdown of the RWA even in the WC regime!

Where from here?

- experimental verification?
- revisiting theoretical results of $g^2(0)$ calculated with JC,
- more direct proofs of the role of squeezing.

Thank you!

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