Conformal Field Theory, spin chains and fractional quantum Hall states

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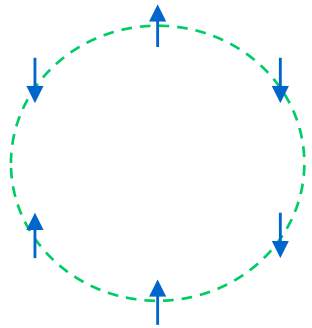
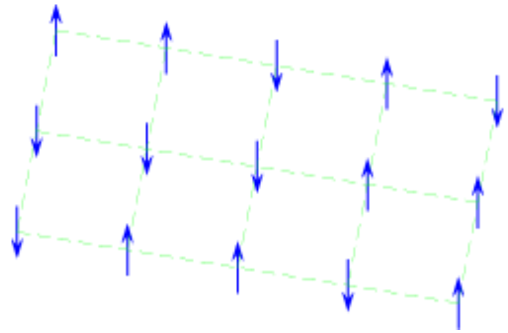
Workshop Entanglement in Strongly Correlated Systems
Centro de Ciencias de Benasque, August 7th 2023





Aim: To study interesting phenomena in quantum many-body systems.

Examples: magnetism in spin chains, quantum Hall effect, critical systems, topological phases, etc

Method: Use Tensor Networks and Quantum Field Theory to construct simple models with particular physical properties.

↓

Wave functions and/or Hamiltonians

Why?: Fundamental understanding of how the phenomena can arise.
Experimental simulations under well-controlled conditions.
Practical applications.

Plan of the talk

- Brief history of Tensor Networks

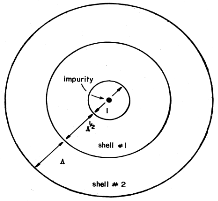
- A primer on Matrix Product States (MPS)

- infinite MPS vs CFT

- Application to the spin chains

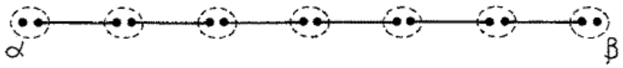
- Application to Fractional Quantum Hall

- Field Tensor Networks



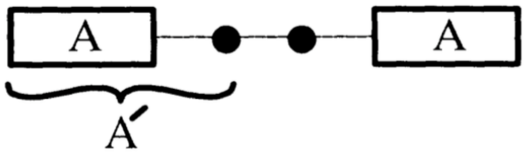
Wilson numerical RG (1975)

AKLT (1987)

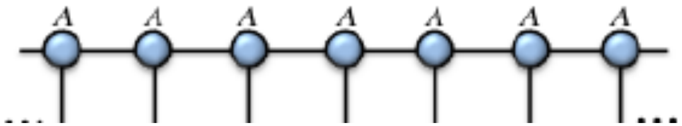


Finite correlated states

Fannes, Nachtergaele, Werner (1992)

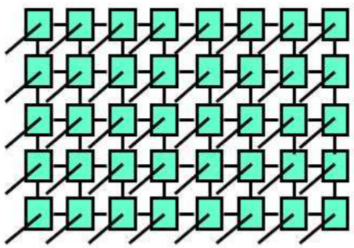


DMRG, White (1992)



MPS

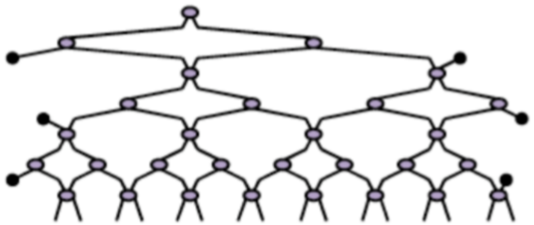
Östlund, Rommer (1995)



Verstraete, Cirac (2004)

PEPS

Vidal (2005)

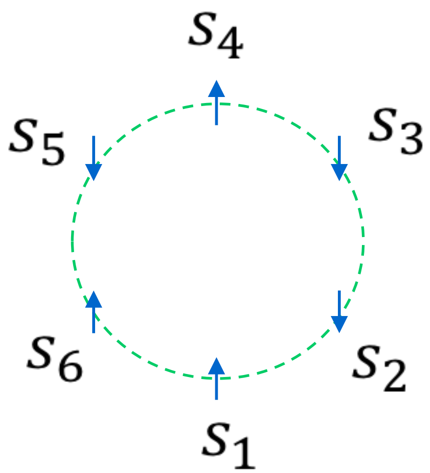


MERA

TN

MPS

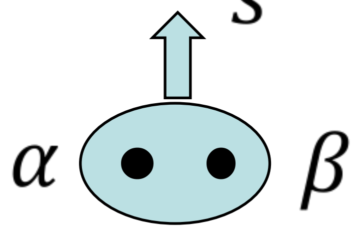
Matrix Product State (MPS)



$s = 1,\ldots,d$

$$|\psi\rangle = \sum_{s_1,\cdots,s_N} \psi(s_1,\cdots,s_N)\,|s_1,\cdots,s_N\rangle$$

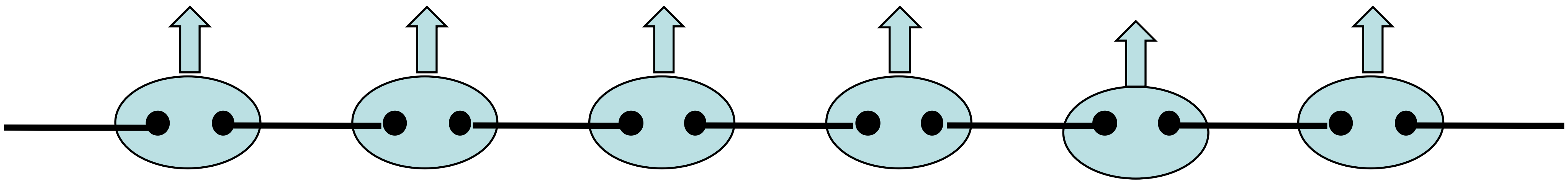
$A_{\alpha\beta}(s) =$



$\alpha,\beta = 1,\ldots,D$

bond dimension

$$\psi(s_1,\cdots,s_N) = Tr\,(A(s_1)\cdots A(s_N))$$



Affleck Kennedy Lieb Tasaki state (AKLT)

spin 1 chain $\qquad d = 3$

$A_{ab}(s) = \sigma^{s}_{a,b} \qquad D = 2$

Open chain: effective spins ½ at the edges

Affleck Kennedy Lieb Tasaki state (AKLT)

Parent Hamiltonian $$H = \sum_{n=1}^{N} \vec{S}_n \cdot \vec{S}_{n+1} + \frac{1}{3}(\vec{S}_n \cdot \vec{S}_{n+1})^2$$

Spin-spin correlator $$\left\langle S_0^a \, S_r^b \right\rangle = \delta^{ab} (-1)^r \frac{4}{3} 3^{-r}$$

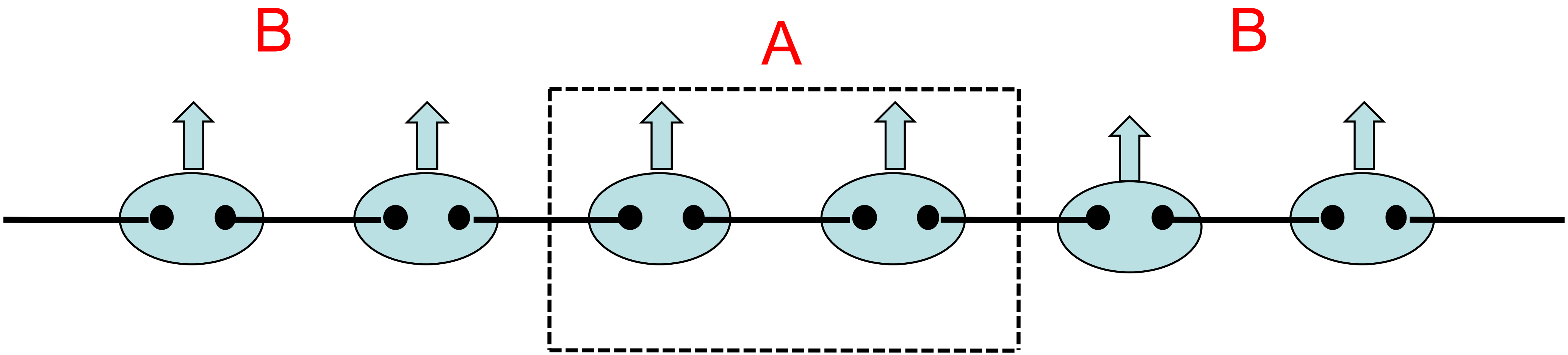
Finite energy gap $$\Delta = E_1 - E_0 > 0$$

Symmetry Protected Topological state (SPT)

Haldane phase

Entanglement entropy of MPS

$$\rho_A = Tr_B \, |\psi\rangle\langle\psi|$$



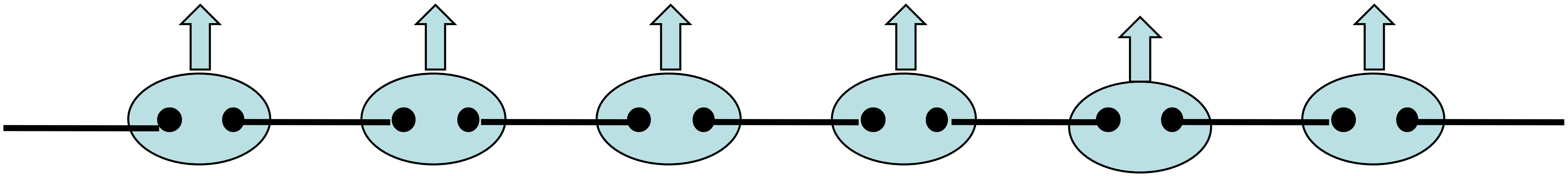
$$S_A = -\,Tr_A \, \rho_A \log \rho_A$$

$$S_A \leq 2\log D$$

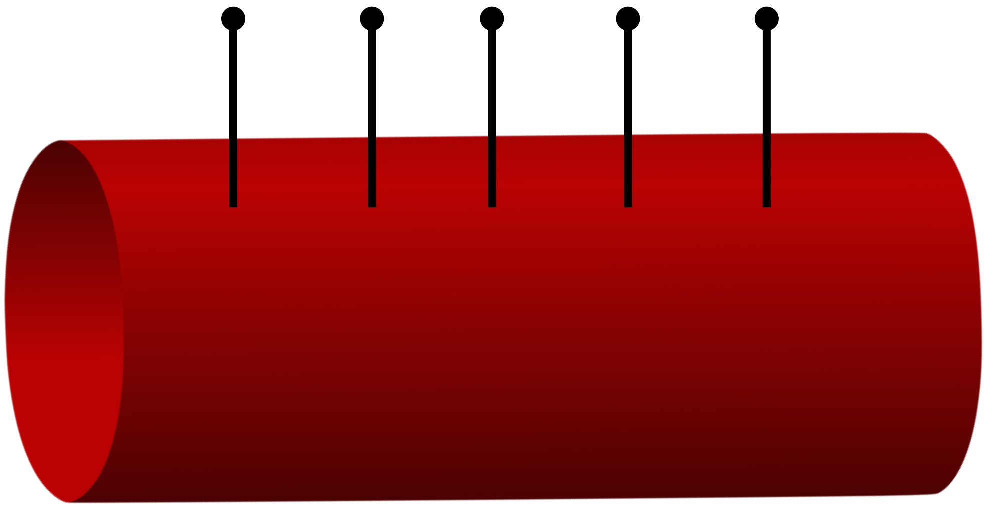
Area law in 1D

iMPS

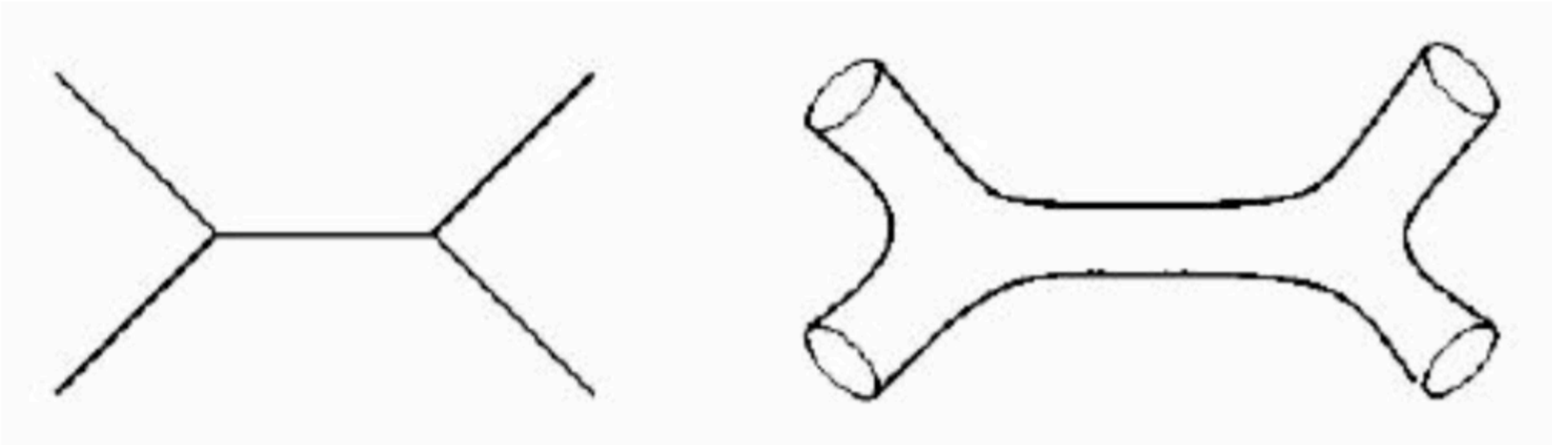
Matrix Product State (MPS)



infinite Matrix Product State (iMPS)



iMPS = “string inspired” MPS



where the “string” mediator of entanglement

MPS satisfies area law $S_A \leq 2 \log D$

Critical 1D systems described by CFT: log violation

$$S_A \approx \frac{c}{3} \log |A| + c_1$$

Holzhey, Larsen, Wilczek, 1994,
Vidal, Latorre, Rico, Kitaev, 2003
Calabrese, Cardy, 2004

One needs very large matrices to describe critical systems

$$N \propto D^{\kappa}, \quad \kappa = \kappa(c)$$

Tagliacozzo, de Oliveira, Iblisdir, Latorre, 2007
Pollmann, Mukerjee, Turner, Moore, 2008

An alternative to overcome this problem is the iMPS

Proposal

Cirac, GS, 2010

Use primary fields of a CFT as MPS “matrices”

MPS: $A(s) : D \times D$ matrix

iMPS: $A_z(s)$: primary field of a CFT

MPS: $\psi(s_1, \cdots, s_N) = Tr\,(A(s_1) \cdots A(s_N))$

iMPS: $\psi(s_1, s_2, \ldots, s_N) = \langle 0 | A_{z_1}(s_1) A_{z_2}(s_2) \cdots A_{z_N}(s_N) | 0 \rangle$

Similar to CFT ansatzs for Fractional Quantum Hall systems (Moore and Read, 1991)

iMPS - XXZ

iMPS and CFT

Consider a chiral massless boson $\varphi(z)$

$$A_z(s) = \chi_s :e^{is\sqrt{\alpha}\varphi(z)}: \quad \chi_s = \pm 1 \qquad s = \pm 1$$

$$\psi(s_1, s_2, \ldots, s_N) = \prod_i \chi_{s_i} \prod_{i<j} (z_i - z_j)^{\alpha s_i s_j} \times \delta(\sum_i s_i)$$

$$S^z_{tot} = \frac{1}{2}\sum_{i=1}^{N} s_i = 0, \quad N: even$$

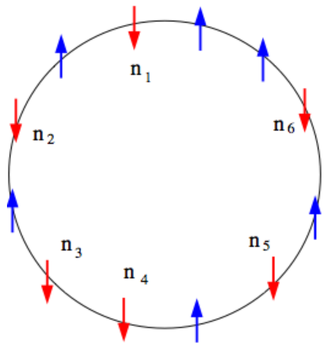
$\alpha, \; z_n, \chi_{s_n}$ are variational parameters obtained by minimization of the GS energy and imposing the symmetries of a Hamiltonian

XXZ model of a spin 1/2 chain

$$H_{XXZ} = \sum_{1=1}^{N} S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z \qquad S_i^a = \frac{1}{2}\sigma_i^a$$

Ferromagnetic   Critical   Antiferromagnetic

-1 0 1 Δ

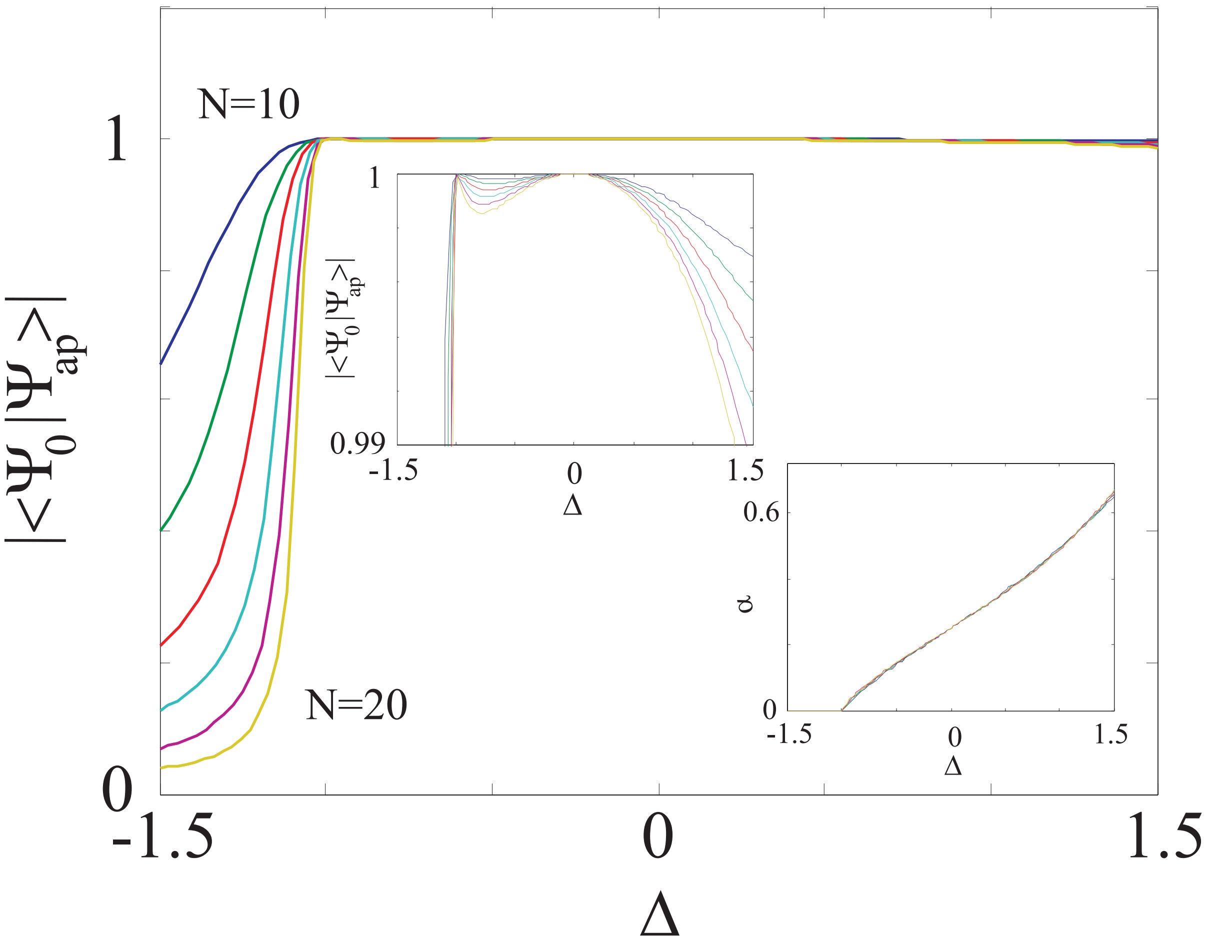


Translational invariance -> $z_n = e^{2\pi i n/N}, \quad n = 1,\ldots,N$

Marshall sign rule -> $\prod_i \chi_{s_i} = \prod_{i:odd} s_i$

Minimize the energy -> $\alpha = f(\Delta)$

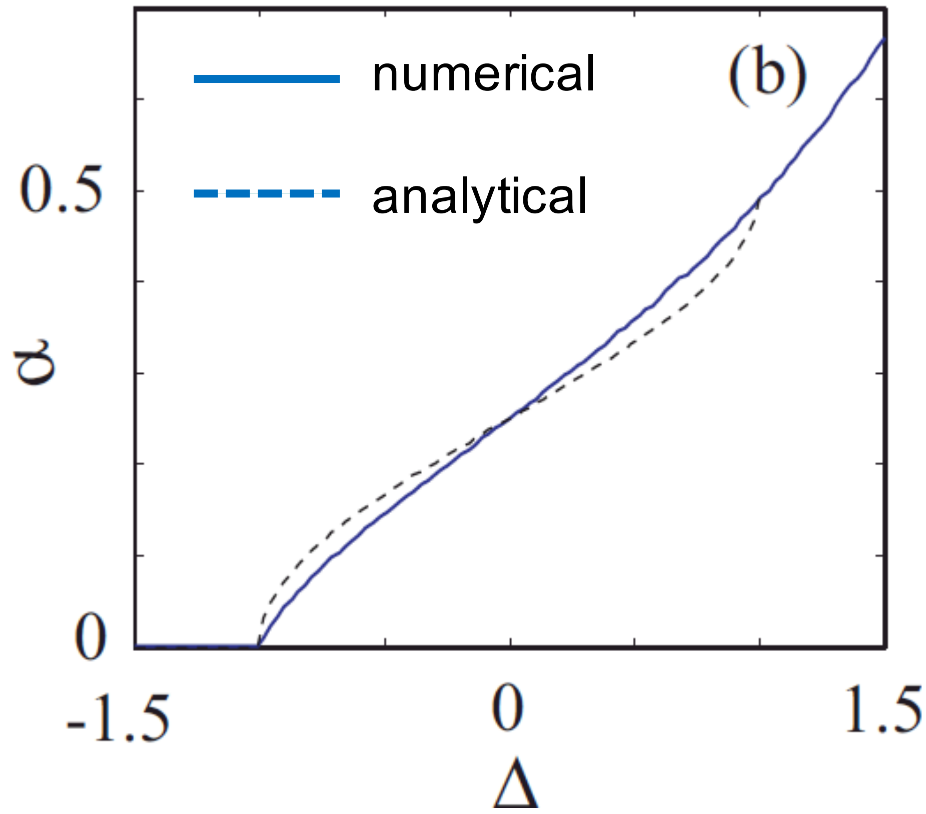
Overlap of the exact and the CFT wave functions



$$\Delta = 0 \rightarrow |\langle \Psi_0 | \Psi_{ap} \rangle| = 1$$

$$\Delta = 1 \rightarrow |\langle \Psi_0 | \Psi_{ap} \rangle| = 0.99..$$

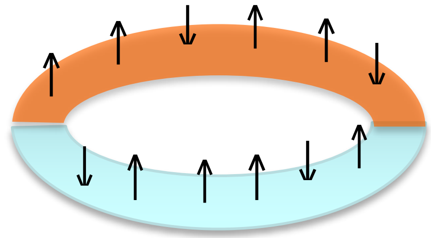
The parameter α $(N = 20)$



$$\Delta = -cos(2\pi\alpha)$$

$$-1 < \Delta \leq 1 \leftrightarrow 0 < \alpha \leq \frac{1}{2}$$

Entanglement properties



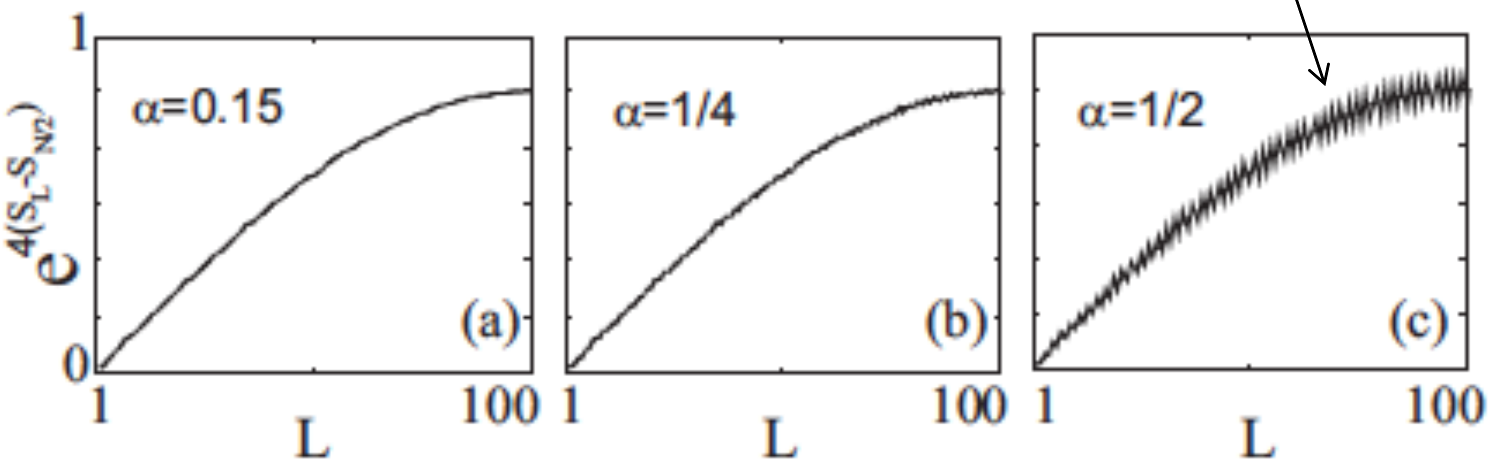
Renyi entropy $S_L = -\log Tr\,\rho_L^2$

CFT prediction $S_L = \frac{c}{4}\log\left(\frac{N}{\pi}\sin\frac{\pi L}{N}\right) + c'$

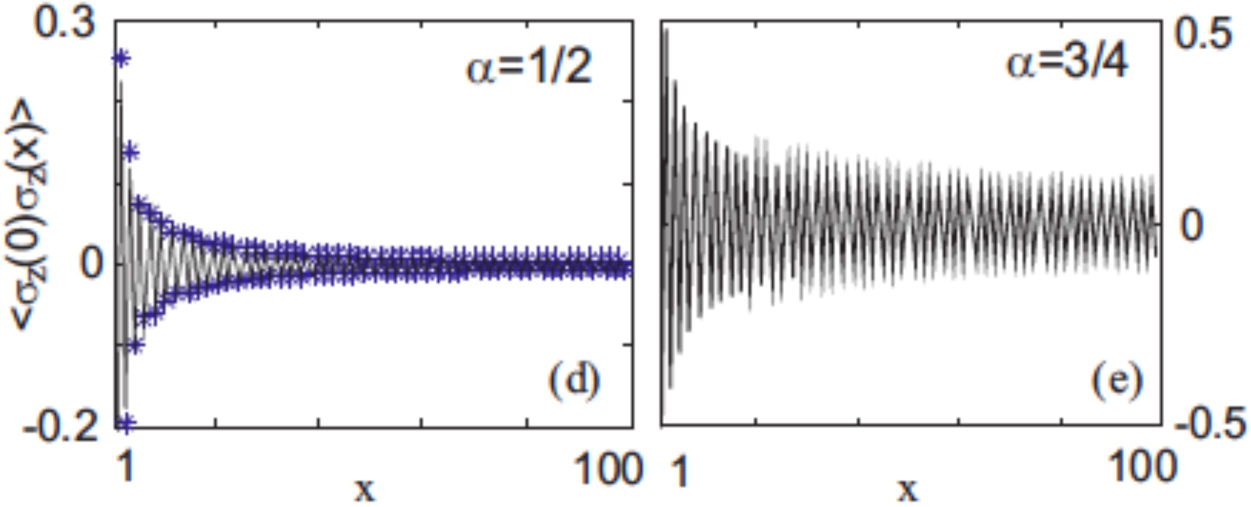
One finds $c = 1 \quad for \quad 0 < \alpha \leq \frac{1}{2}$

Fluctuations depend on α

$N = 200$

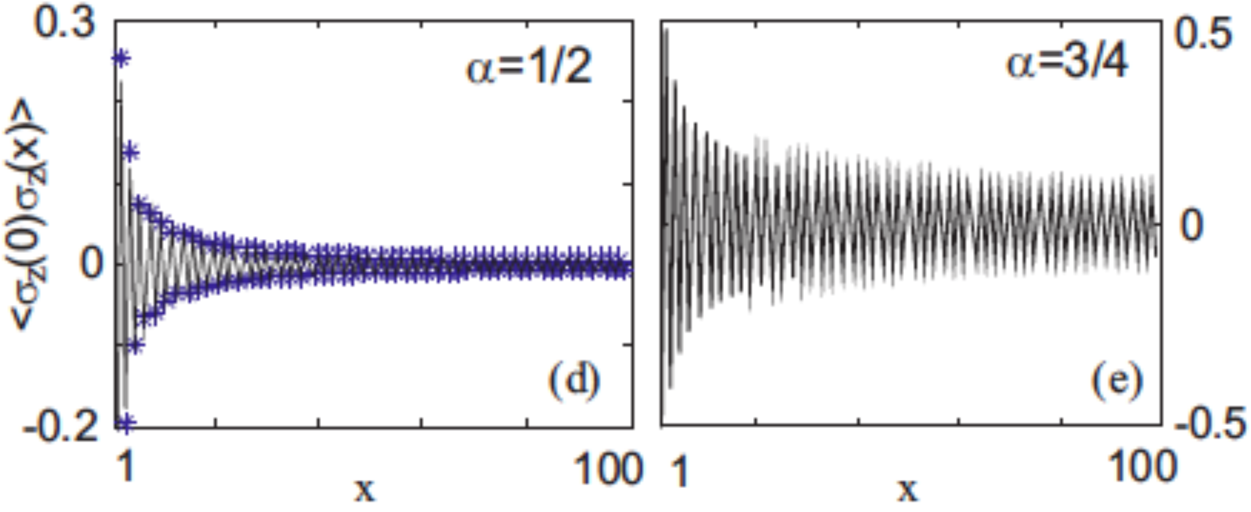


Spin-spin correlators



Algebraic decay for $0 < \alpha \leq \frac{1}{2}$ long range order for $\alpha > \frac{1}{2}$

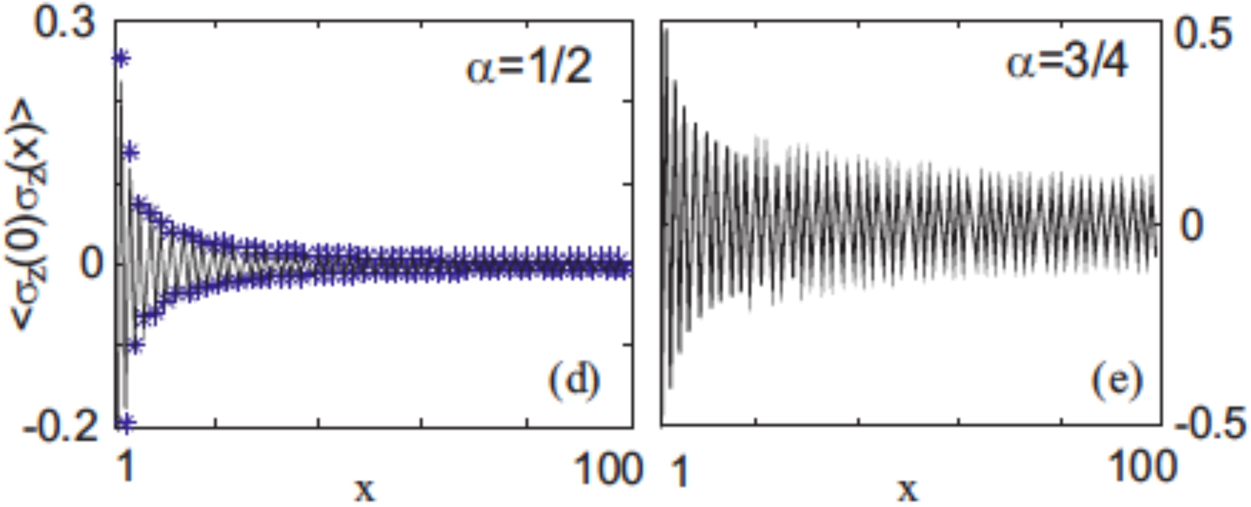
Spin-spin correlators



Algebraic decay for $0 < \alpha \leq \frac{1}{2}$ long range order for $\alpha > \frac{1}{2}$

$$\alpha = \frac{1}{2} \rightarrow \left\langle S_n^a S_0^b \right\rangle \cong \delta_{a,b} \left(\frac{(-1)^n}{8n} - \frac{1}{4\pi^2 n} \right) \quad as \quad n >> 1$$

Spin-spin correlators



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Correlator for Heisenberg model

universal term

$$\Delta = 1 \rightarrow \langle S_n^a S_0^b \rangle \cong \delta_{a,b} \left(C \frac{(-1)^n (\log n)^{1/2}}{n} - \frac{1}{4\pi^2 n} \right) \quad as \quad n >> 1$$

Luttinger liquid of XXZ $-1 < \Delta \leq 1$

$$H_{XXZ} = \sum_{1=1}^{N} S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z$$

$\downarrow$ bosonization

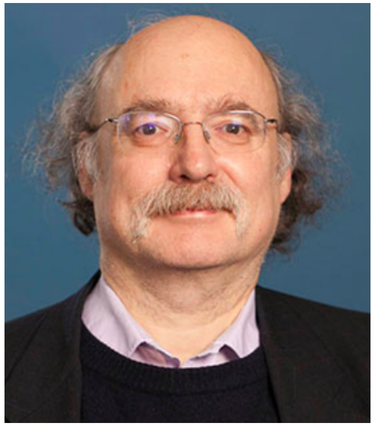
$$H_{XXZ}^{cont} = \frac{v}{2} \int dx \left[K \left(\partial_x \theta(x) \right)^2 + K^{-1} \left(\partial_x \phi(x) \right)^2 \right]$$

$$\Delta = -\cos(2\pi\alpha) \rightarrow K = \frac{1}{4\alpha},$$

$$\alpha = \frac{1}{4} \leftrightarrow K = 1 \leftrightarrow \Delta = 0 \qquad \alpha = \frac{1}{2} \leftrightarrow K = \frac{1}{2} \leftrightarrow \Delta = 1$$

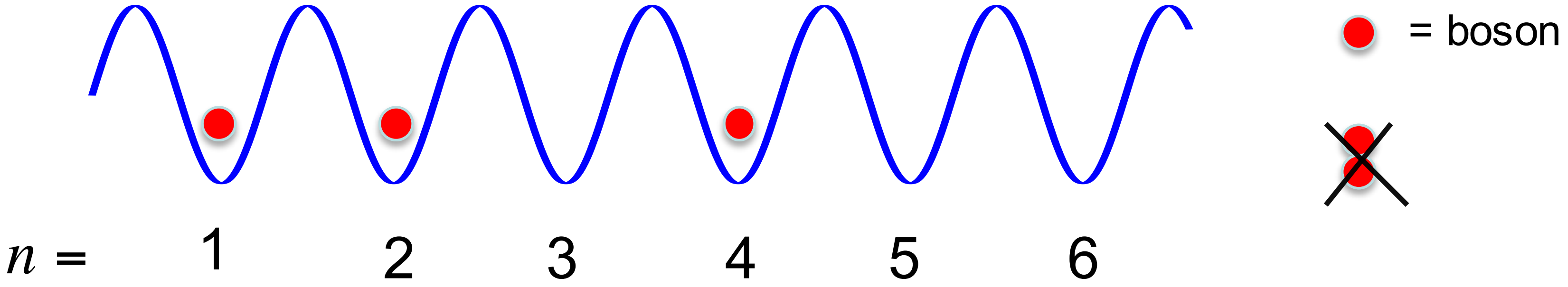
$\alpha = 1/2$

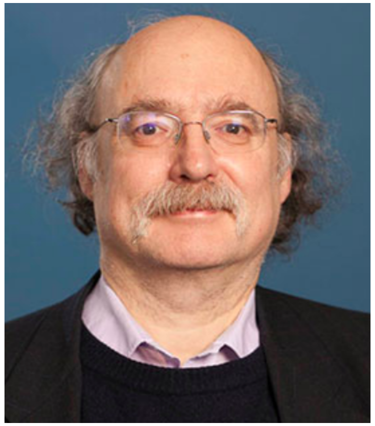
Haldane-Shastry



The Haldane-Shastry model (1988)

1D lattice of hard core bosons





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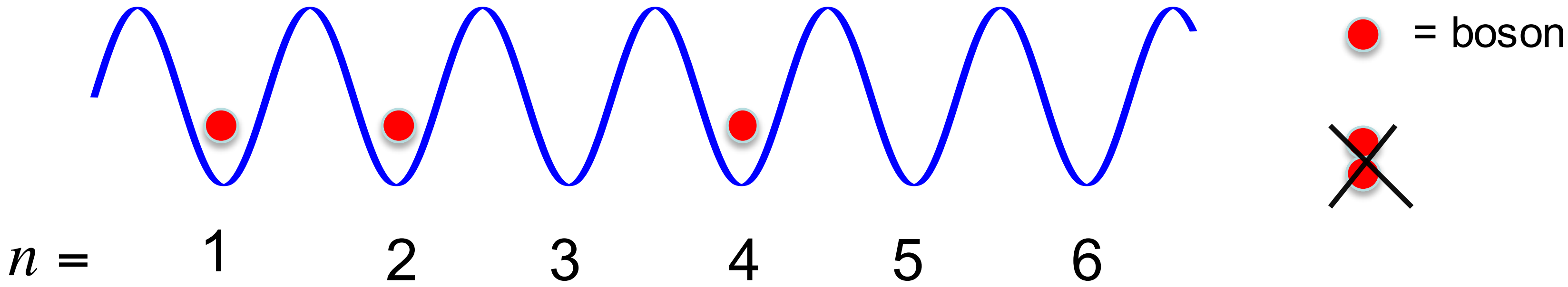
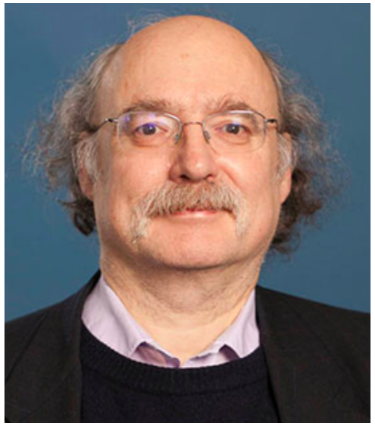


If the site n is occupied $\longrightarrow$ $z_n = e^{2\pi i n/N}$, $n = 1,\ldots,N$

many body state $|\psi\rangle = \sum_{n_1<n_2<\ldots<n_{N/2}} \psi(n_1,n_2,\ldots,n_{N/2})\,|n_1,n_2,\ldots,n_{N/2}\rangle$



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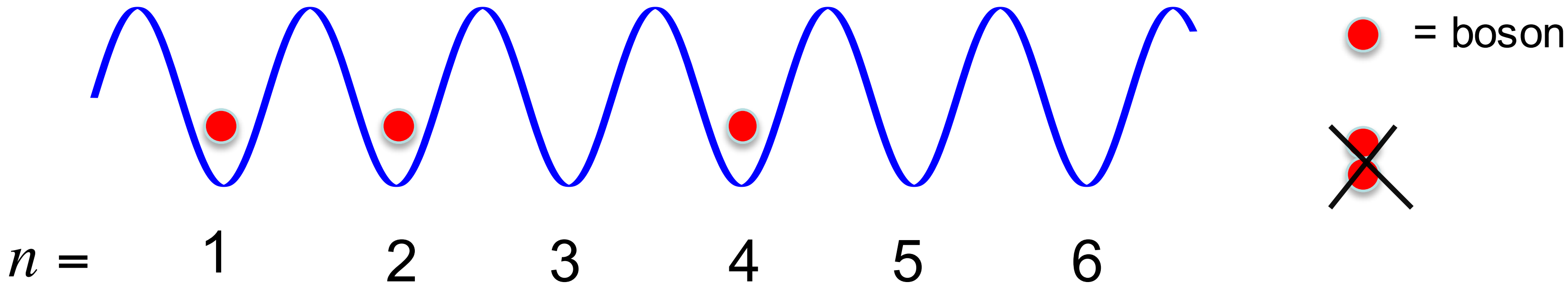

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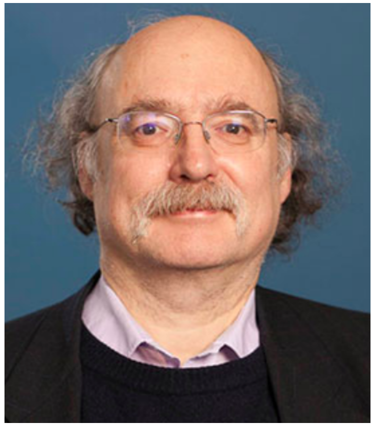
$$\psi_{HS}(n_1, \ldots, n_{N/2}) = \prod_i z_{n_i} \prod_{i<j} (z_{n_i} - z_{n_j})^2$$



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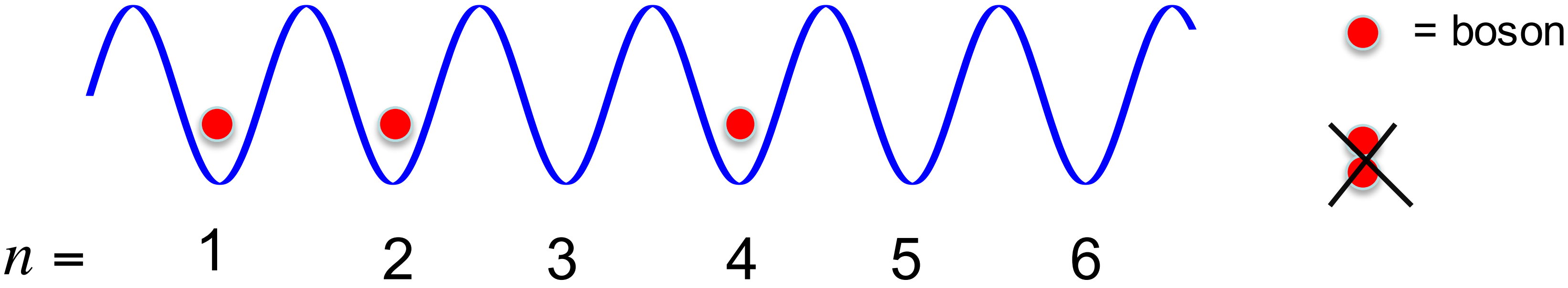

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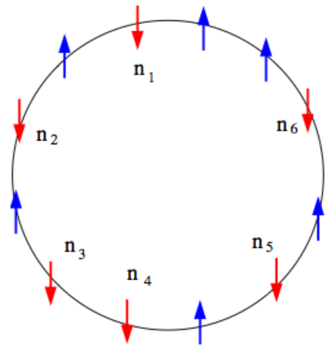
$$\psi_{HS}(n_1, \ldots, n_{N/2}) = \prod_i z_{n_i} \prod_{i<j} (z_{n_i} - z_{n_j})^2$$

Constructed using the Gutzwiller projection of the Fermi state at half filling

Map: hard core boson to spin 1/2

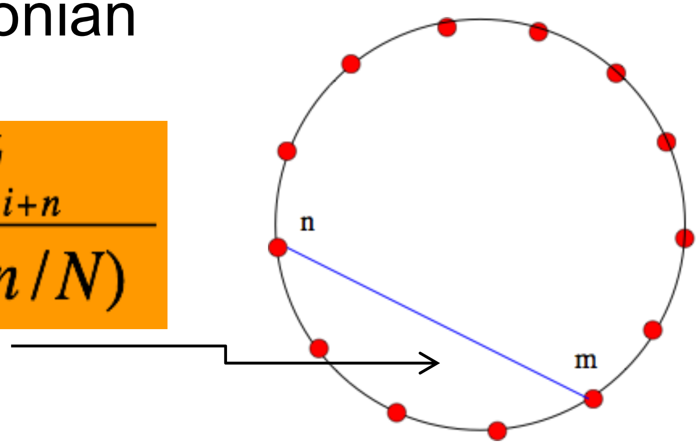
$$\textit{empty site}\ |0\rangle \leftrightarrow |\uparrow\rangle\ \textit{spin up}$$

$$\textit{occupied site}\ |1\rangle \leftrightarrow |\downarrow\rangle\ \textit{spin down}$$



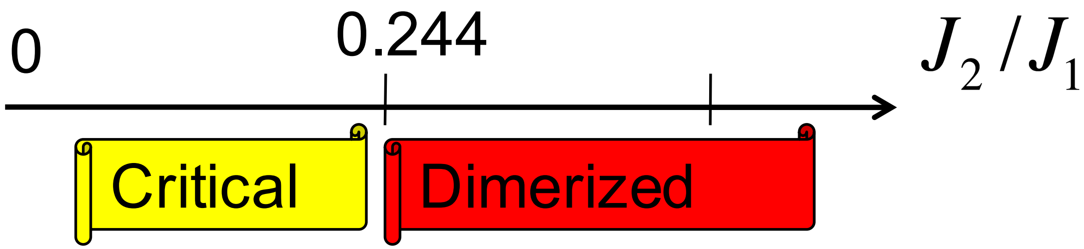
$|\psi_{HS}\rangle$ is the ground state of the Hamiltonian

$$H \propto -\sum_{n<m} \frac{z_n z_m}{(z_n - z_m)^2} \vec{S}_n \cdot \vec{S}_m = C \sum_{i,n} \frac{\vec{S}_i \cdot \vec{S}_{i+n}}{\sin^2(\pi n / N)}$$

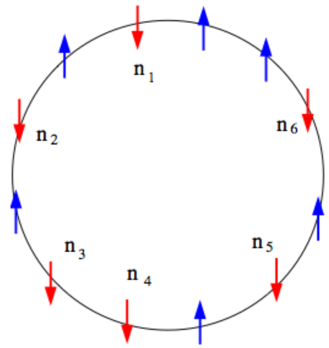


N>>1

$$H = \sum_i \vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{4} \vec{S}_i \cdot \vec{S}_{i+2} + \ldots$$



Relation between HS and iMPS



Take $$\alpha = \frac{1}{2}, \; z_n = e^{2\pi i n/N}$$

Using the hard core boson – spin map

$$\psi_{HS}(n_1, \ldots, n_{N/2}) \propto \psi_{CFT}(s_1, \ldots, s_N)$$

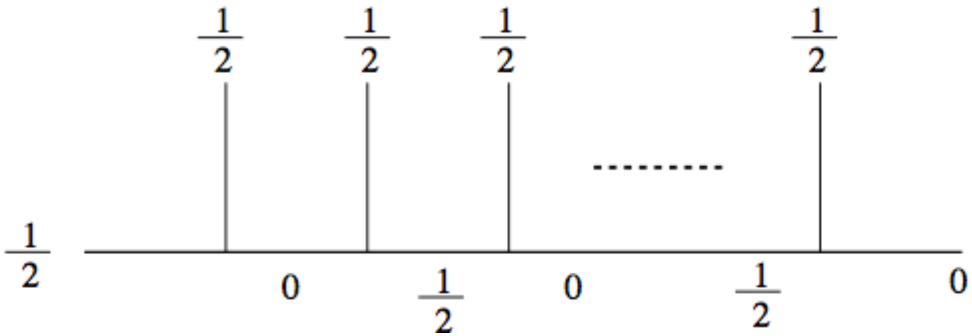
$$\prod_i z_{n_i} \prod_{1 \le i < j \le N/2} (z_{n_i} - z_{n_j})^2 \propto \prod_{i=odd} s_i \prod_{1 \le i < j \le N} (z_i - z_j)^{s_i s_j /2}$$

Connection: iMPS and the WZW $SU(2)@k=1$

$$A_z(s) \propto e^{is/\sqrt{2}\,\varphi(z)} \rightarrow h = \frac{1}{4}$$ primary field $\phi_{1/2}(z)$

fusion rule: $\phi_{1/2} \times \phi_{1/2} = \phi_0$

$$\psi_{CFT}(s_1, \ldots, s_N) =$$



The HS wave function is a conformal block
(chiral correlator)

Derivation of the parent Hamiltonian from CFT

Knizhnik-Zamolodchikov eq for $SU(2)_k$

$$\frac{k+2}{2}\frac{\partial}{\partial z_i}\psi(z_1,\ldots,z_N) = \sum_{j\neq i}^{N}\frac{\vec{S}_i\cdot\vec{S}_j}{z_i - z_j}\psi(z_1,\ldots,z_N)$$

Generalized Haldane-Shastry Hamiltonian

$$H = -\sum_{n\neq m}\left(\frac{z_n z_m}{(z_n - z_m)^2} + \frac{1}{12}w_{n,m}(c_n - c_m)\right)\vec{S}_n\cdot\vec{S}_m$$

Reduces to HS if $z_n = e^{2\pi i n/N}$ $\quad w_{ij} = \frac{z_i + z_j}{z_i - z_j}$

$$c_n = \sum_m w_{nm}, \quad if\ z_n = e^{2\pi i n/N} \rightarrow c_n = 0$$

Equations for spin correlators

One gets a linear system of equations for spin-spin correlators

$$w_{ij}\left\langle t_i^a\, t_j^a\right\rangle + \sum_{k(\neq i,j)} w_{ik}\left\langle t_j^a\, t_k^a\right\rangle + \frac{3}{4} w_{ij} = 0, \quad i \neq j$$

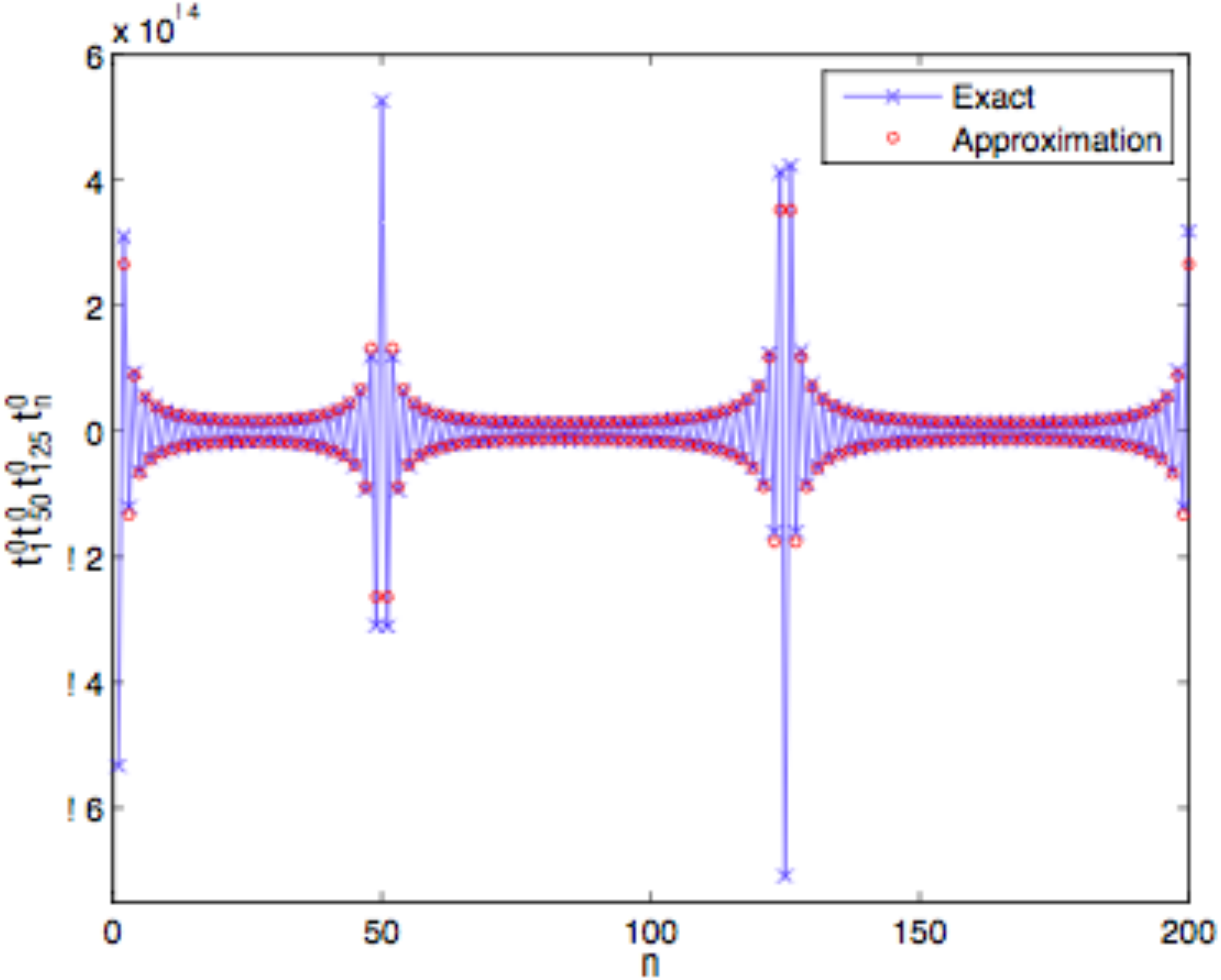
In the uniform case we recover the Gebhard-Vollhardt result

$$\left\langle t_n^a\, t_0^b\right\rangle = (-1)^n \delta_{ab} \frac{Si(\pi n)}{4\pi n}, \qquad Si(z) = \int_0^z dt \frac{\sin t}{t}$$

But we also find an exact formula for finite N

$$\left\langle t_n^a\, t_0^b\right\rangle = (-1)^n \delta_{ab} \frac{(-1)^n}{4N\sin(\pi n/N)} \sum_{m=1}^{N/2} \frac{\sin(2\pi n(m-1/2)/N)}{m-1/2}$$

Four point spin correlator $\left\langle t_1^0\, t_{50}^0\, t_{125}^0\, t_n^0 \right\rangle \quad n = 1, \cdots, 200$



iMPS = chiral correlators of CFT

Nielsen,Cirac, GS, 2011

Examples: SU(2)@k WZW model with k=1,2,…

$$\psi_p\left(z_1,m_1,,\ldots,z_N,m_N\right)=\left\langle \phi_{j_1 m_1}(z_1)\,\phi_{j_2 m_2}(z_2)\cdots\phi_{j_N m_N}(z_N)\right\rangle_p$$

↘ label the wave functions given by the fusion rules

↓

degenerate ground states

Parent Hamiltonians can be constructed using the *null vectors* of the primary fields : representation theory of Kac-Moody algebras

SU(2)@k=2

Primary fields: $\phi_0,\ \phi_{1/2},\ \phi_1 \quad h_{1/2} = \frac{3}{16}, h_1 = \frac{1}{2}$

Fusion rule: $\phi_1 \times \phi_1 = \phi_0$

$$\psi_{s_1 \ldots s_N} = \left\langle \phi_{s_1}(z_1) \cdots \phi_{s_N}(z_N) \right\rangle, \quad s_i = 0, \pm 1$$

$$H = -\frac{4}{3}\sum_{i \neq j} w_{ij}^2 - \frac{1}{3}\sum_{i \neq j}\left(w_{ij}^2 + 2\sum_{k(\neq i,j)} w_{ki} w_{kj} \right) t_i^a t_j^a + \frac{1}{6}\sum_{i \neq j} w_{ij}^2 \left(t_i^a t_j^a \right)^2 + \frac{1}{6}\sum_{i \neq j \neq k} w_{ij} w_{ik} t_i^a t_j^a t_i^b t_k^b$$

(See also M. Greiter et al for a s=1 Hamiltonian)

Spin 1 wave function

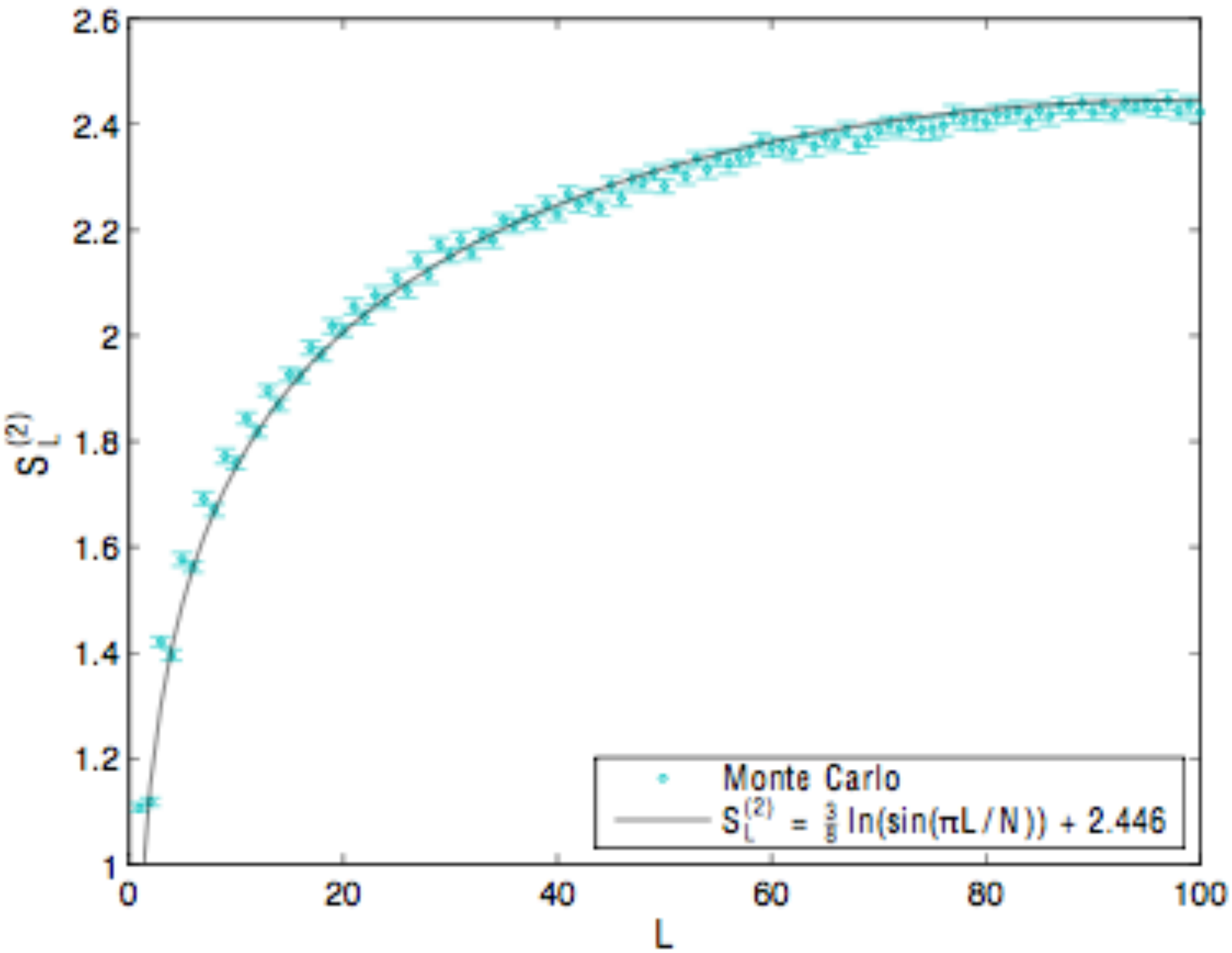
SU(2)@k=2 = Boson + Ising (c= 3/2 = 1+ 1/2)

Primary spin 1 fields (h=1/2)

$$\phi_{\pm 1}(z_j) = e^{\pm i\varphi(z_j)}, \quad \phi_0(z_j) = (-1)^j \chi(z_j)$$ Majorana fermion

$$\psi_{s_1 \cdots s_N} = (-1)^{\sum_{i:\,odd} s_i} \prod_{i<j} \left(z_i - z_j\right)^{s_i s_j} Pf_0 \frac{1}{z_i^0 - z_j^0}, \quad \sum_i s_i = 0, \quad N : even$$

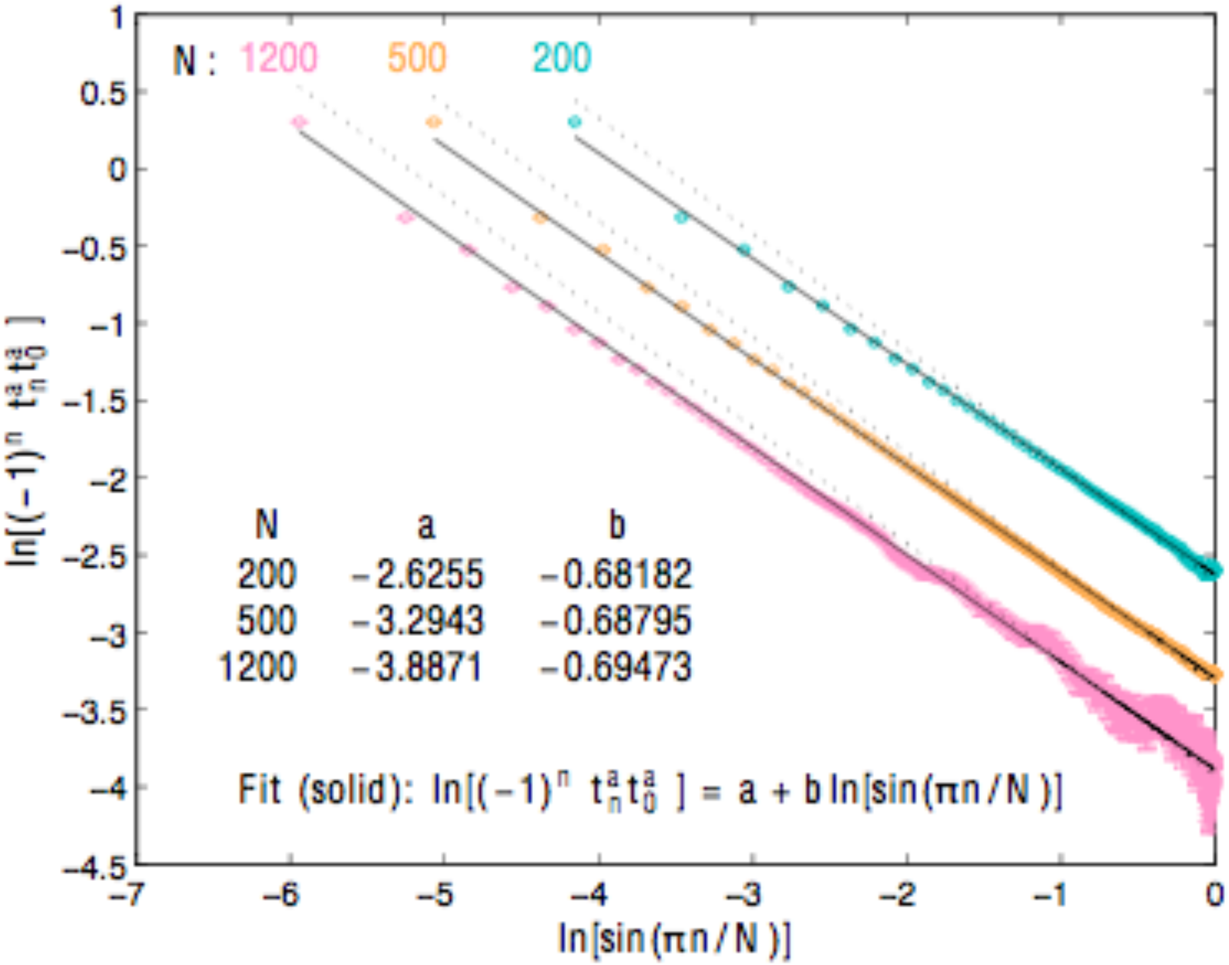
Renyi entropy $S_L = -\log Tr\, \rho_L^2$



c=3/2

Spin-spin correlator

CFT prediction $$\langle t_n^a t_0^a \rangle \approx (-1)^n \left(\sin \frac{\pi n}{N} \right)^b, \quad b = -\frac{3}{4}$$



Suggest existence of log corrections (Narajan and Shastry)

Ansatzs for excitations

Herwerth et al

	CFT		Spin system
Ground state	$\vert 0\rangle$	$\leftrightarrow$	$\langle 0\vert \Phi_{\mathbf{s}}(\mathbf{z})\vert 0\rangle$
	$\downarrow$		$\downarrow$
Excited states	$(J_{-1}^{a_k}\dots J_{-1}^{a_1})(0)\vert 0\rangle$	$\leftrightarrow$	$\langle 0\vert \Phi_{\mathbf{s}}(\mathbf{z})(J_{-1}^{a_k}\dots J_{-1}^{a_1})(0)\vert 0\rangle$

In 1D one recovers the exacts results by Haldane, Calogero-Sutherland model

iMPS in 2D

Fractional Quantum Hall

Laughlin wave function (1983)

Wave function for the ground state at filling fraction $\nu = 1/m$

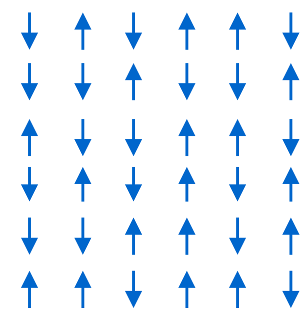
m is odd (even) for fermions (bosons)

$$\psi_L\left(z_1, z_2, \ldots, z_M\right) = \prod_{i<j} (z_i - z_j)^m \, e^{-\frac{1}{4}\sum_j |z_j|^2}$$

$$z_j = x_j + i\, y_j$$

Kalmeyer-Laughlin wave function (1987)

Bosonic wave function at filling fraction 1/2



$$\psi_{KL}(z_1,\cdots z_{N/2}) = \prod_n \chi(z_n) \prod_{n<m} (z_n - z_m)^2 \, e^{-\sum_q |z_q|^2/4}$$

- Hard-core bosons located on a square lattice $z_a = n_a + i\, m_a$

- $\prod_n \chi(z_n)$ -> spin singlet state

- KL state describes a chiral spin liquid : “topological matter”
  - -- gap in the bulk
  - -- gapless edge excitations
  - -- degenerate GS on the torus
  - -- indistinguisable by local observables
  - -- anyonic excitations (abelian)
  - -- topological entanglement entropy

Note: if we take $|z_a| = 1 \quad \rightarrow \quad \psi_{KL} = \psi_{HS}$

Relation with SU(2)@k=1 model

Take the z's in the square lattice

$$\psi_{CFT} = \left\langle \phi_{s_1}(z_1)\cdots\phi_{s_N}(z_N)\right\rangle = \prod_n (-1)^{(n-1)(s_n+1)/2} \prod_{i<j}(z_i - z_j)^{s_i s_j/2}$$

On large lattices $\psi_{CFT} \rightarrow \psi_{KL}$

including the gaussian factor $e^{-\sum_q |z_q|^2/4}$

No need to add this factor by hand. Charge neutrality is automatically guaranteed in the spin variables.

Parent Hamiltonian

Sum of two body and three body terms

$$H = \frac{1}{2}\sum_{i\neq j} |w_{ij}|^2 + \frac{2}{3}\sum_{i\neq j} |w_{ij}|^2 S_i^a S_j^a + \frac{2}{3}\sum_{i\neq j\neq k} w_{ij}^* w_{ik} S_j^a S_k^a$$

$$-\frac{2i}{3}\sum_{i\neq j\neq k} w_{ij}^* w_{ik}\, \varepsilon^{abc} S_i^a S_j^b S_k^c,$$

Breaks time reversal

$$w_{ij} = \frac{g(z_i)}{z_i - z_j} + h(z_i)$$

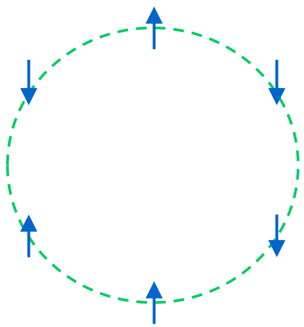
g(z), h(z): generic

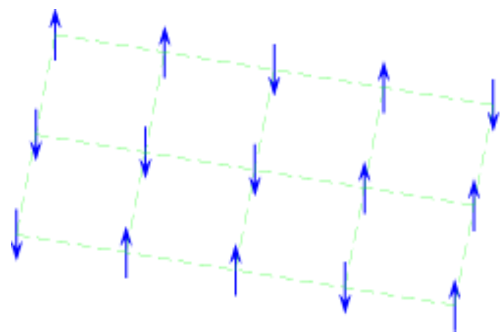
Problem : it is long range $\propto \log|z_i - z_j|$

If $z_n = e^{2\pi i n/N}$ $\quad w_{ij} = \dfrac{2z_i}{z_i - z_j} - 1$ -> HS Hamiltonian conformal block

$$z_n = e^{2\pi i n / N}$$

$$z = n + i m$$





The Haldane-Shastry state

$\forall N$

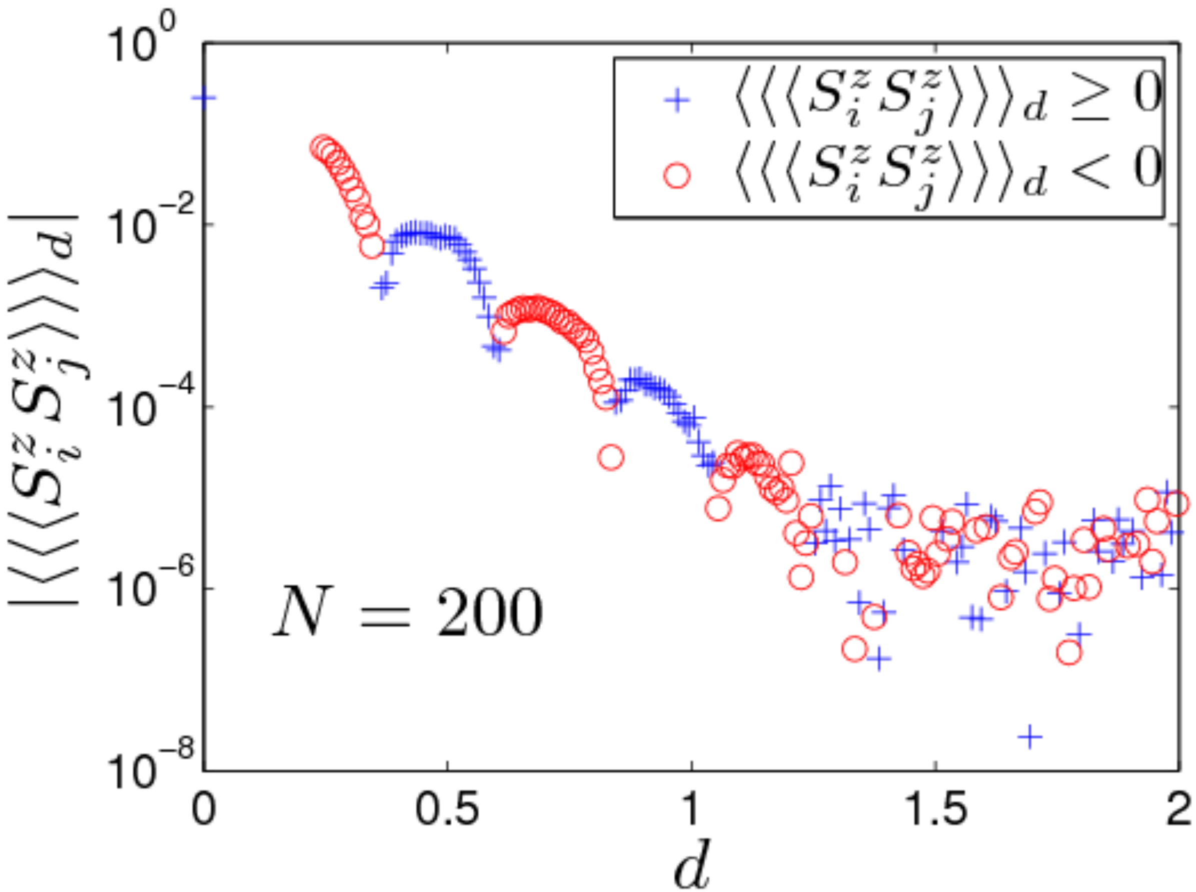
The Kalmeyer-Laughlin state

$N \to \infty$

Spin-spin correlation function

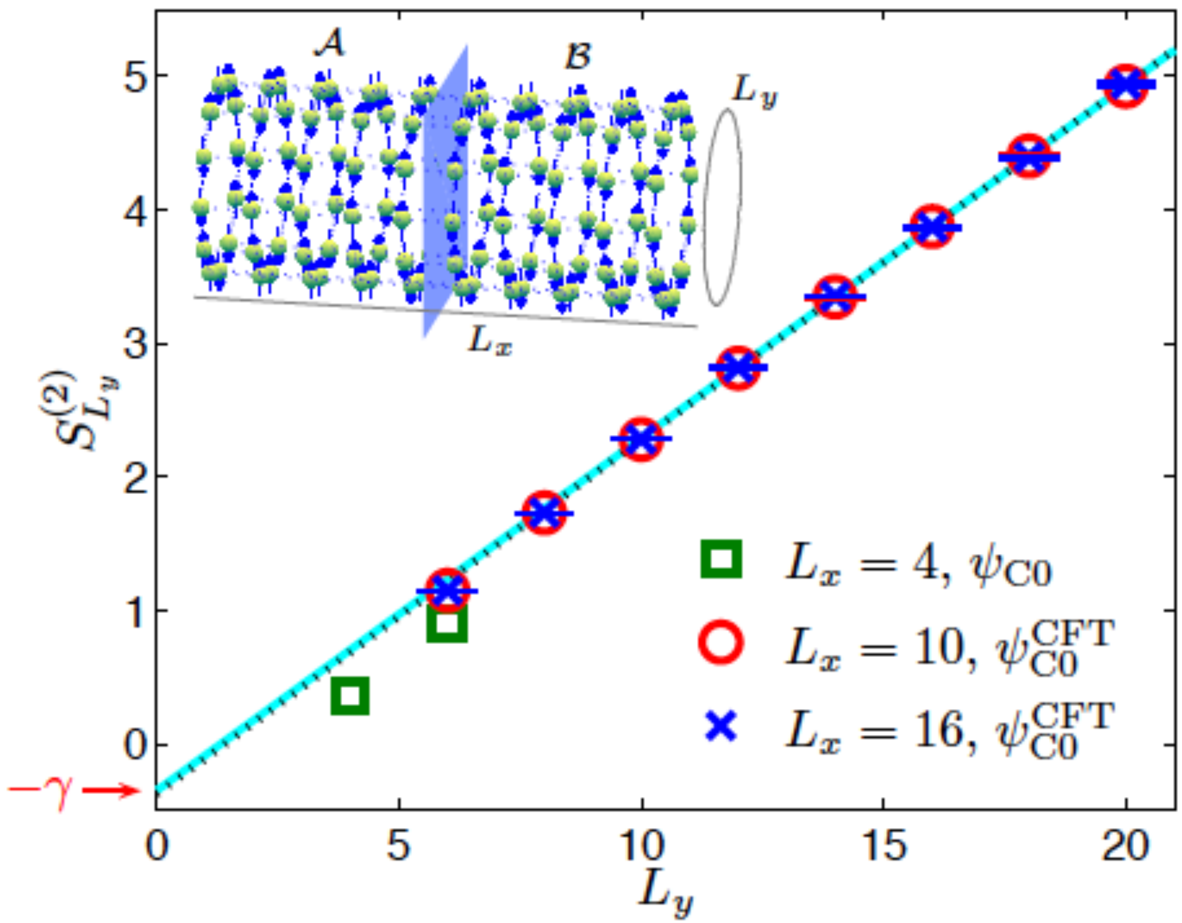
$$\langle S_i^z S_j^z \rangle = \frac{\sum_{s_1,\ldots,s_N} s_i s_j |\psi_{s_1,\ldots,s_N}(z_1,\ldots,z_N)|^2}{4\sum_{s_1,\ldots,s_N} |\psi_{s_1,\ldots,s_N}(z_1,\ldots,z_N)|^2}.$$

200 spins on the sphere



Entanglement entropy of the cylinder

$$S^{(2)} = -\ln(Tr\,\rho_A^2) \approx c\,P - \gamma$$



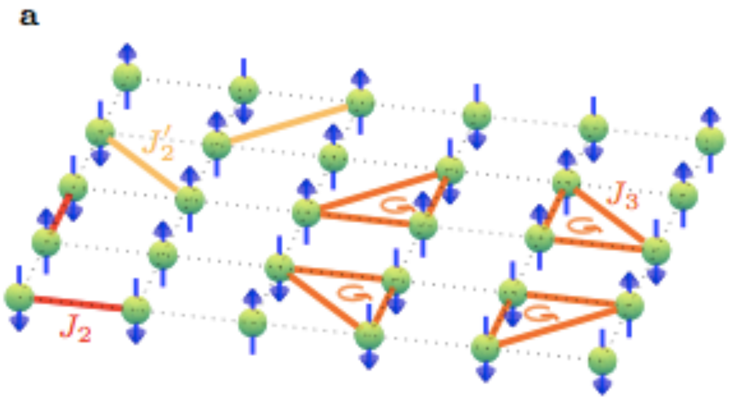
$$\gamma = 0.374 \approx \frac{1}{2}\log 2 = 0.347$$

Topological entanglement entropy

Kitaev, Preskill, 2006
Levin, Wen, 2006

Truncated Hamiltonian for the KL state

$$H = 2J_2 \sum_{\langle n,m \rangle} \vec{S}_n \cdot \vec{S}_m + 2J'_2 \sum_{\langle\langle n,m \rangle\rangle} \vec{S}_n \cdot \vec{S}_m - 4J_3 \sum_{\langle n,m,p \rangle} \vec{S}_n \cdot \left(\vec{S}_m \times \vec{S}_p \right)$$



$J_2 = 1, J'_2 = 0, J_3 = 1/2$

Overlap

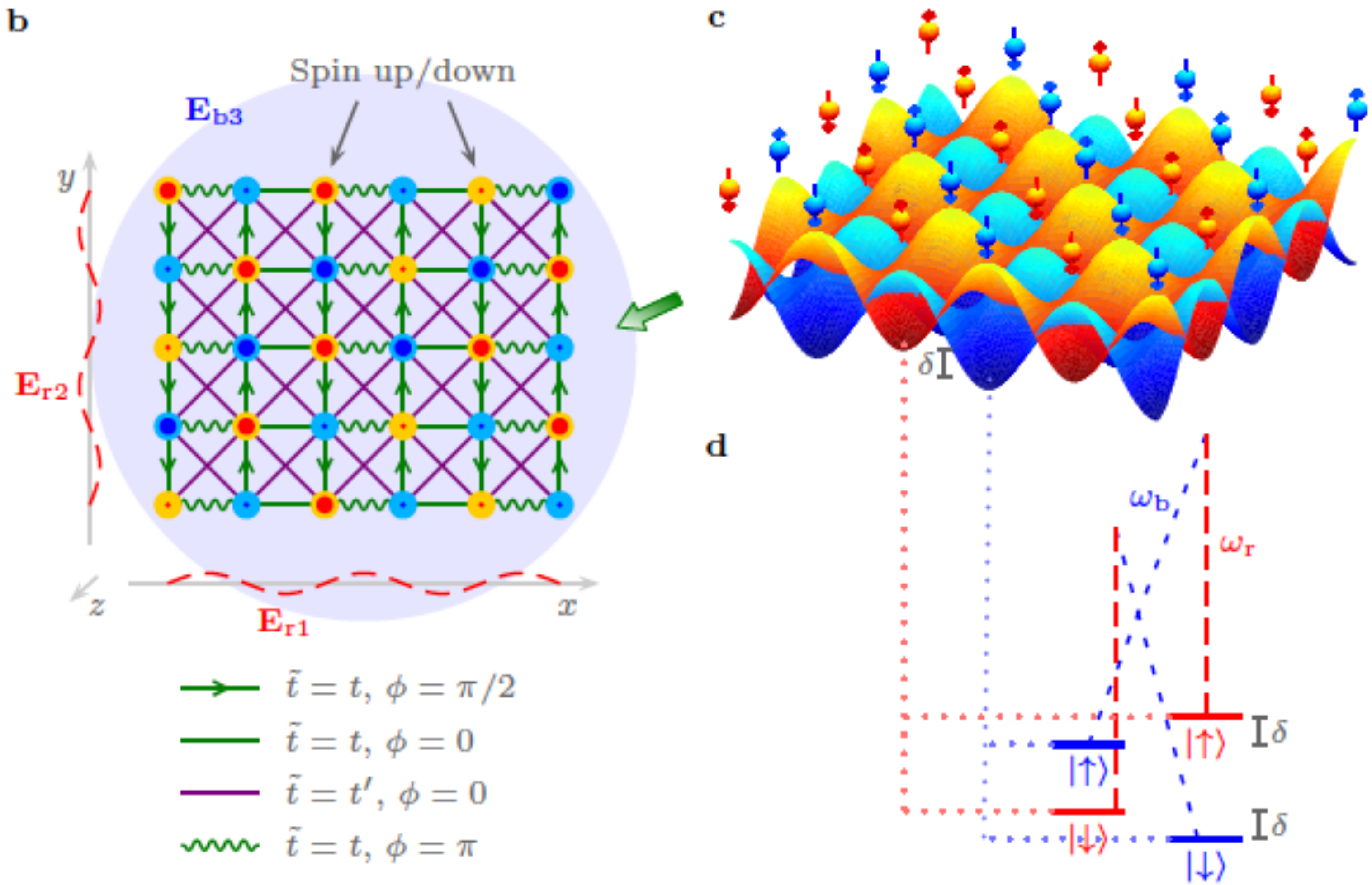
N	$L_x \times L_y$	Plane $\lvert\langle\psi_{\rm P0}\lvert\psi_{\rm P0}^{\rm CFT}\rangle\rvert$	Torus $\lvert\langle\psi_{\rm T0}\lvert\psi_{\rm T0}^{\rm CFT}\rangle\rvert$	$\lvert\langle\psi_{\rm T1}\lvert\psi_{\rm T1}^{\rm CFT}\rangle\rvert$
12	4×3	0.9860	0.9818	0.9533
16	4×4	0.9812	0.9747	0.9572
20	4×5	0.9728	0.9655	0.9200
30	6×5	-	0.9258	0.9361

$\dim \approx 1.1 \times 10^9$

Derivation from a Fermi-Hubbard Hamiltonian

$$H_{FH} = \sum_{\sigma} H_{kin,\sigma} + U \sum_{n} a^*_{n\uparrow} a_{n\uparrow} a^*_{n\downarrow} a_{n\downarrow}$$

Couplings: U, t , t'

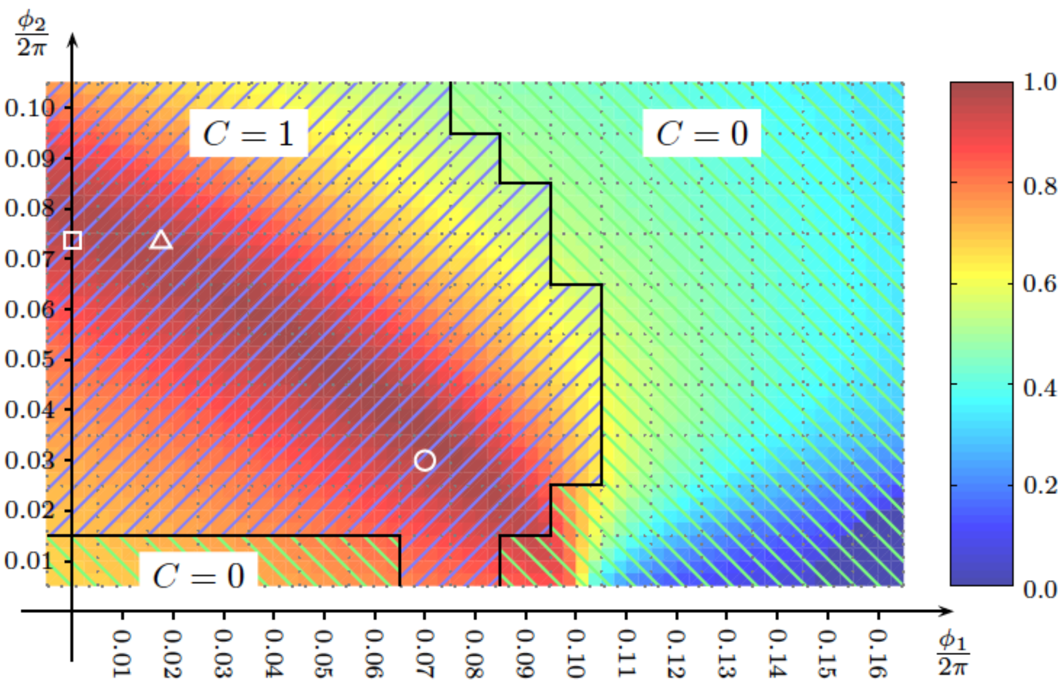


U>>t, t' -> Mott insulating regime: each site occupied by single fermion

$$J_2 = \frac{2t^2}{U}, \quad J'_2 = \frac{2t'^2}{U}, \quad J_3 = \frac{6t^2 t'}{U^2}$$

C= Chern number

overlap of CFT/exact wf's



FTNS

1D Field Tensor Network States

Nielsen, Herwerth, Cirac, Tilloy, Gasull, GS, 2020, 2022

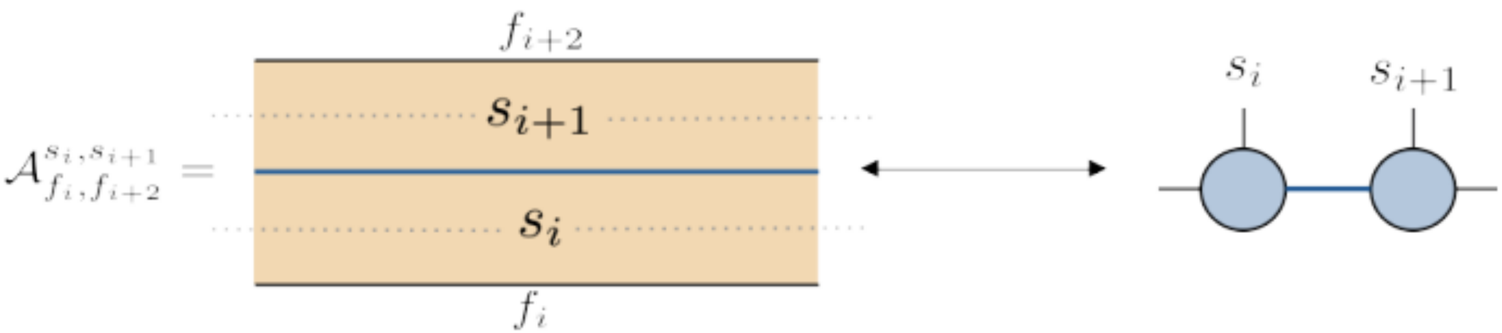
$$|\psi\rangle = \sum_{s_1 \ldots s_N = 1}^{d} c_{s_1,\ldots,s_N} |s_1 \ldots s_N\rangle$$

$$c_{s_1,\ldots,s_N} = \sum_{n_1,\ldots,n_N=1}^{D} A^{s_1}_{n_1,n_2} \ldots A^{s_N}_{n_N,n_1}$$

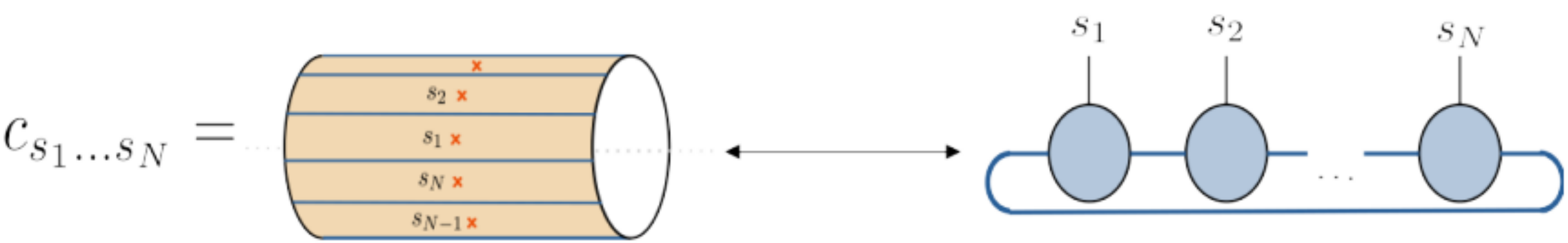
$$c_{s_1,\ldots,s_N} = \int \mathcal{D}[f_1] \ldots \mathcal{D}[f_N] \mathcal{A}^{s_1}_{f_1,f_2} \ldots \mathcal{A}^{s_N}_{f_N,f_1}$$

$$f_i : \mathbb{R} \to \mathbb{R} \in \mathbb{L}^2(\mathbb{R}) \cup \mathbb{K}$$

Sewing condition



Closing condition



FTN representation of CFT correlators

$$c_{s_1,\ldots,s_N} \propto \delta_{\sum_n s_n,0} \prod_n \chi_{s_n} \prod_{n>m} \{\sin[(x_n - x_m)/N]\}^{2q^2 s_n s_m}$$

φ is a real scalar field

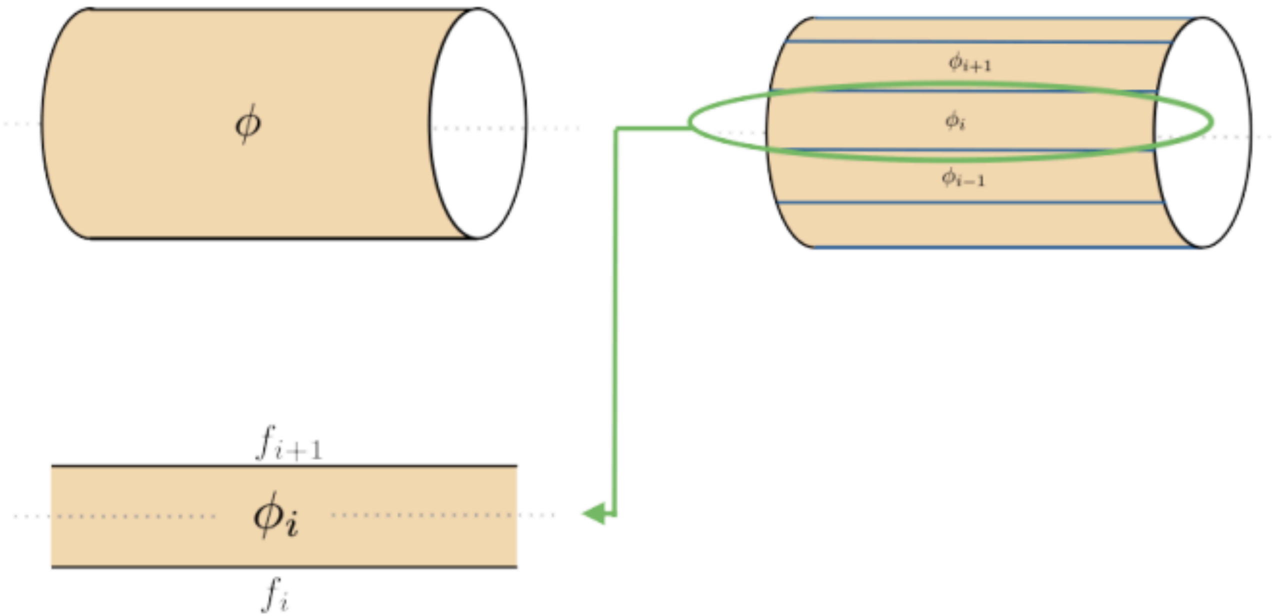
$$S = \frac{1}{8\pi} \int_0^{\pi N} dx \int_{-\infty}^{\infty} dt \{[\partial_x \varphi(x,t)]^2 + [\partial_t \varphi(x,t)]^2\}$$

$$c_{s_1,\ldots,s_N} \propto \int D[\varphi] e^{-S} e^{iq \sum_{n=1}^{N} s_n \varphi(\mathbf{r}_n)} \prod_n \chi_{s_n}$$

$$c_{s_1,\ldots,s_N} \propto \langle \chi_{s_1} : e^{iqs_1 \varphi(\mathbf{r}_1)} : \ldots \chi_{s_N} : e^{iqs_N \varphi(\mathbf{r}_N)} : \rangle_0$$

Slicing the path integral

$$\int \mathcal{D}[\phi] = \int \mathcal{D}[f_1]...\mathcal{D}[f_N] \int' \mathcal{D}[\phi_1]... \int' \mathcal{D}[\phi_N]$$



$$\phi(z_i^+) = f_{i+1}(x)$$
$$\phi(z_i^-) = f_i(x),$$

$$\mathcal{A}^{s_i}_{f_i,f_{i+1}} = \int' \mathcal{D}[\phi_i] e^{-S} e^{-is_i\sqrt{\alpha}\phi_i(z_i)} \chi_{s_i}$$

FTN amplitudes in 1D

$$\mathcal{A}_{\Delta_i}[f_0, f_+, f_-, \{z_i, s_i\}] = e^{f_0 \frac{s_i}{\sqrt{2}}} e^{-R_{\Delta_i}[f_+, f_-, \{z_i, s_i\}]}$$

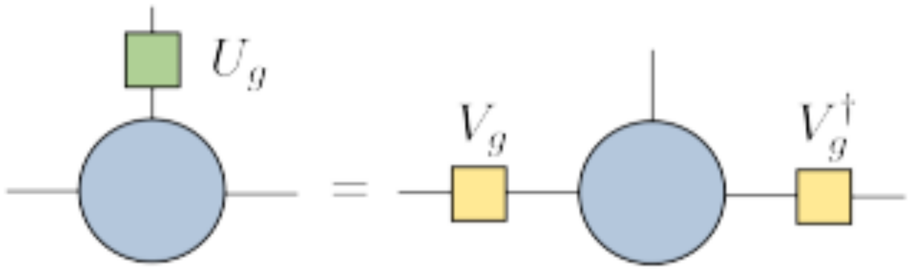
Momentum space representation

$$R_{\Delta_i}[f_+, f_-, \{z_i, s_i\}] = \frac{s_i^2}{4} \log \Delta_i$$
$$+ \frac{1}{2} \int_0^\infty \mathrm{d}k \begin{pmatrix} \hat{f}_+(k) & \hat{f}_-(k) \end{pmatrix} \begin{pmatrix} \omega_{+,\Delta_i}(k) & \omega_{-,\Delta_i}(k) \\ \omega_{-,\Delta_i}(k) & \omega_{+,\Delta_i}(k) \end{pmatrix} \begin{pmatrix} \hat{f}_+^*(k) \\ \hat{f}_-^*(k) \end{pmatrix}$$
$$- \frac{i}{2\sqrt{2}} s_i \int_{\mathbb{R}} \mathrm{d}k \frac{e^{ikz_i}}{\sinh(\pi k \Delta_i)} \left(e^{\pi k b_i} \hat{f}_+(k) - e^{\pi k a_i} \hat{f}_-(k) \right),$$

$$\omega_{+,\Delta_i} = k \coth(\pi k \Delta_i) \text{ and } \omega_{-,\Delta_i} = -k \,\mathrm{sech}(\pi k \Delta_i)$$

Symmetries for MPS and FTNS

Gasull, Tilloy, Cirac, GS

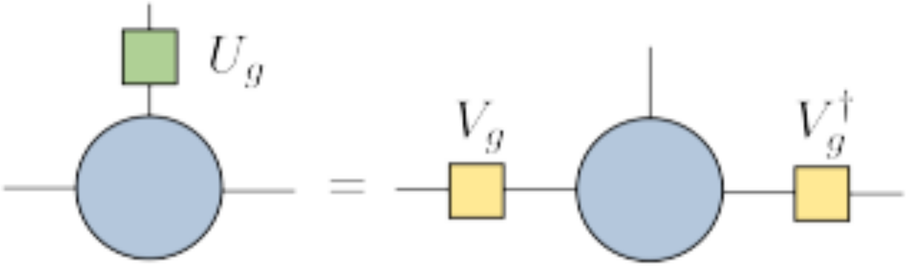


proyective irreps

$$V_g V_h = e^{i\omega(g,h)} V_{gh}$$

Symmetries for MPS and FTNS

Gasull, Tilloy, Cirac, GS



proyective irreps

$$V_g V_h = e^{i\omega(g,h)} V_{gh}$$

Conformal currents

$SU(2)_1$

$$\mathrm{H}(z) =: i\partial\phi(z) :,$$

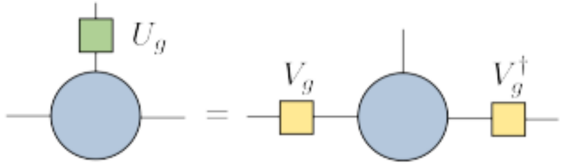
$$\mathrm{E}^{\pm}(z) =: e^{\pm i\sqrt{2}\phi(z)} :$$

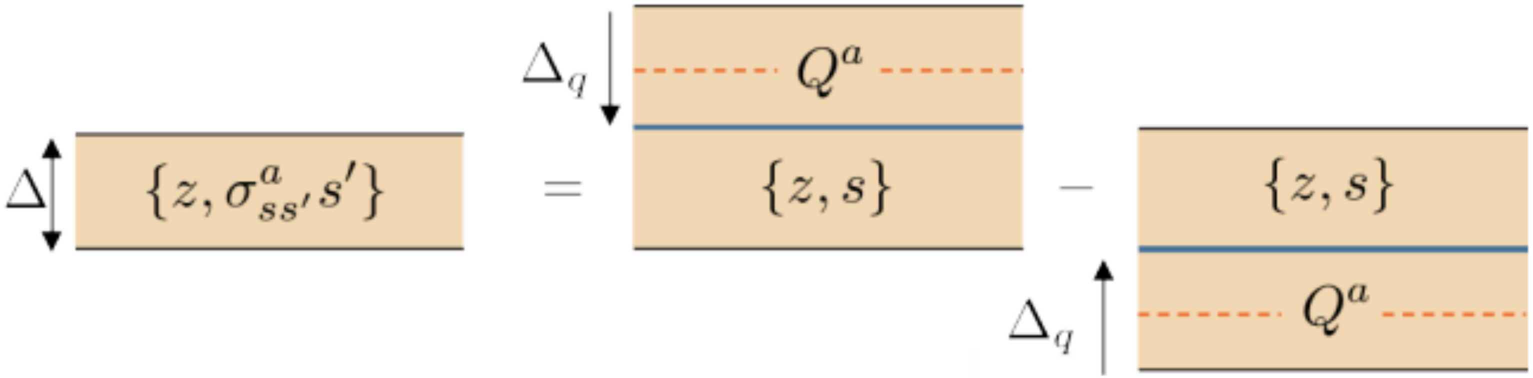
$$Q = \frac{1}{2\pi i}\oint \mathrm{d}z J(z)$$

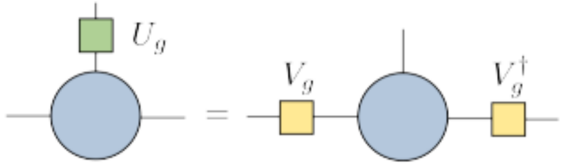
Acting on functionals they are defined as

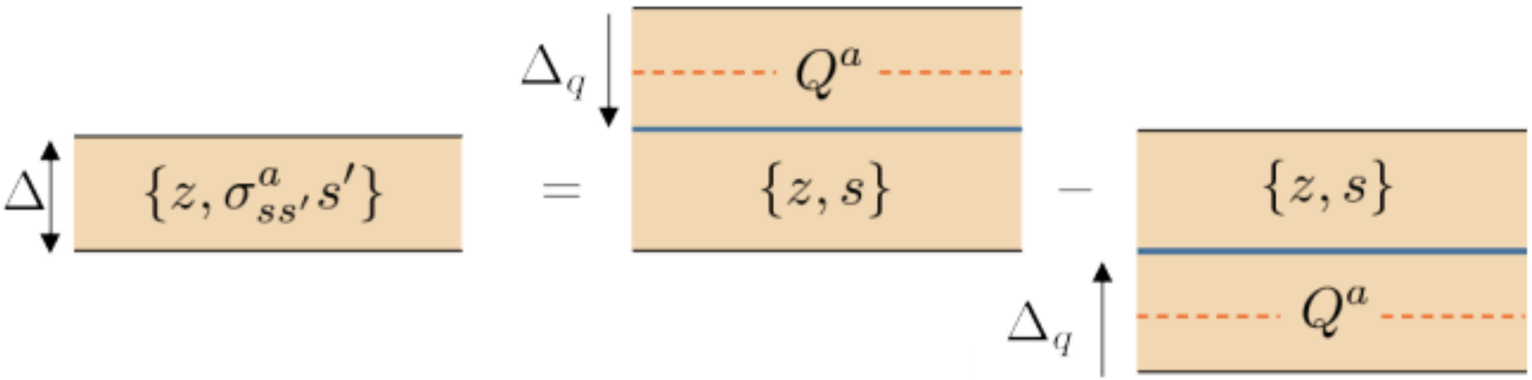
$$H(z)_\Delta = \sqrt{2}\lim_{q\to 0}\frac{1}{q}\partial_z \mathcal{A}_\Delta\left[f_+, f_-, \{z, q\}\right]$$

$$E^{\pm}(z)_\Delta = -\frac{\mu}{2} A_\Delta[f_0, f_+, f_-, \left\{z, \pm\sqrt{2}\right\}]$$

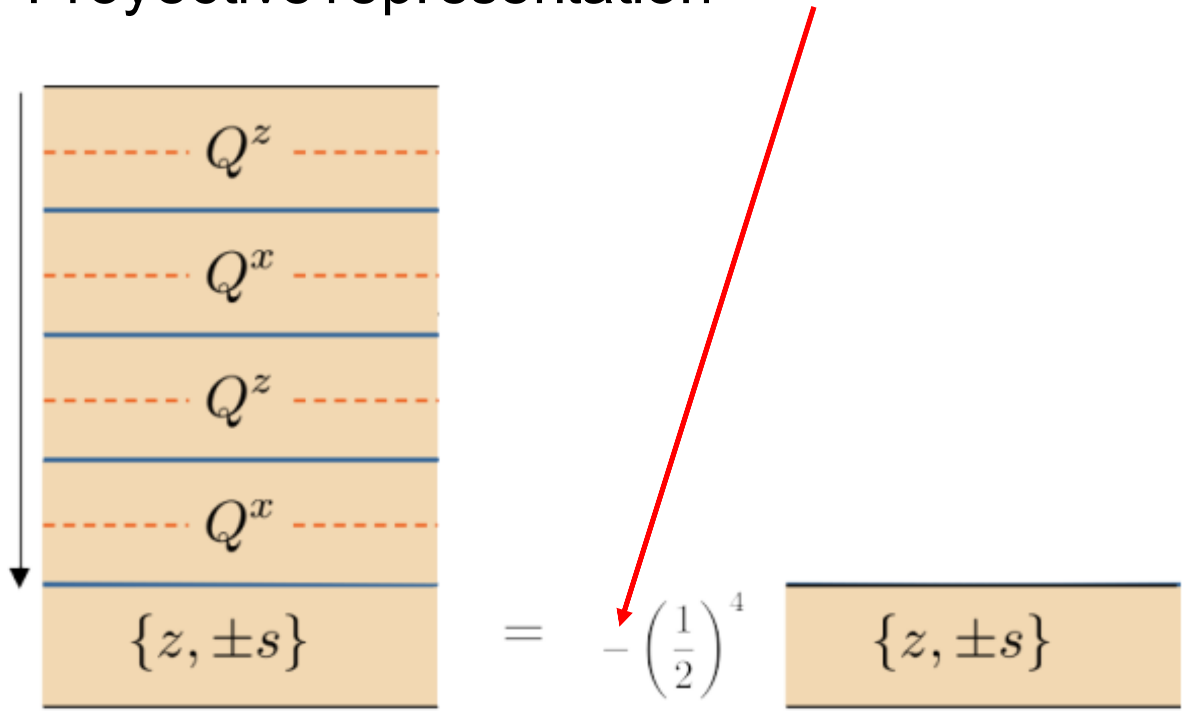

U_g
=
V_g
$V_g^\dagger$


Δ
$\{z, \sigma^a_{ss'} s'\}$
=
Δ_q
Q^a
$\{z, s\}$
−
$\{z, s\}$
Δ_q
Q^a

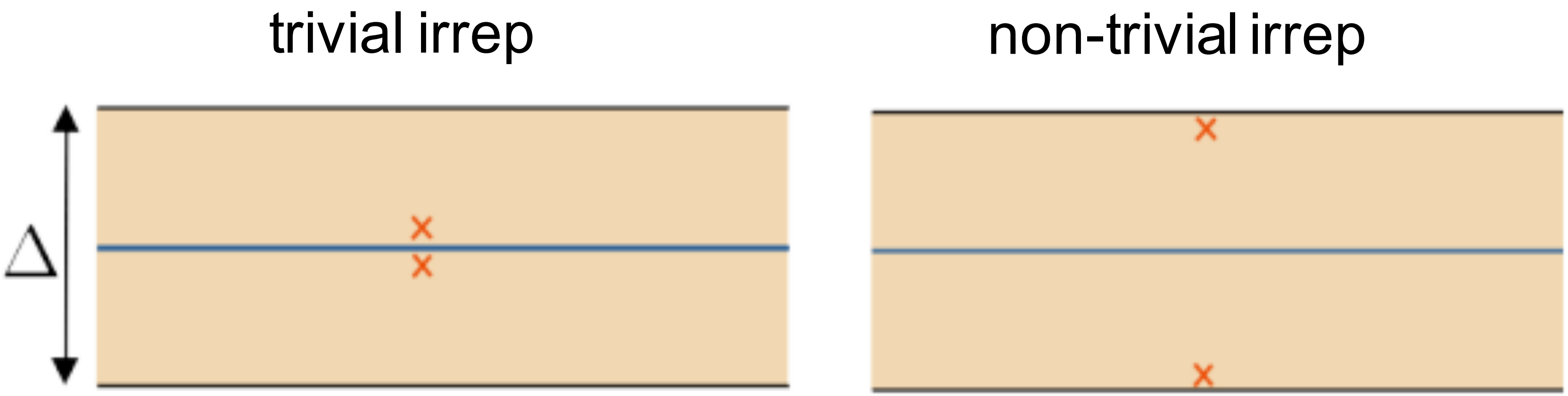




Proyective representation

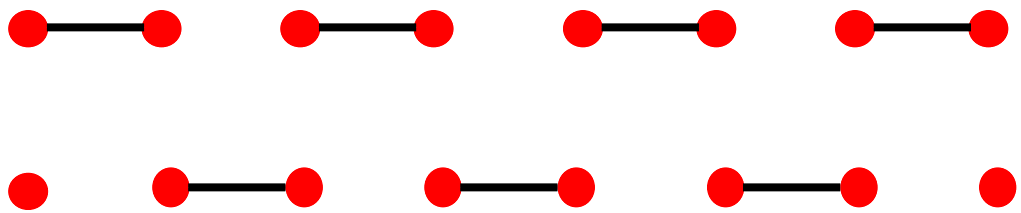


SPT ground states and the Majundar-Ghosh states

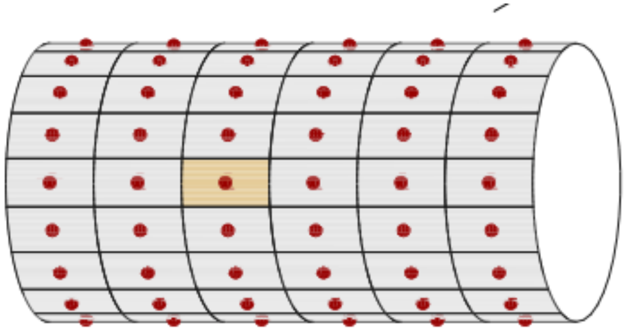


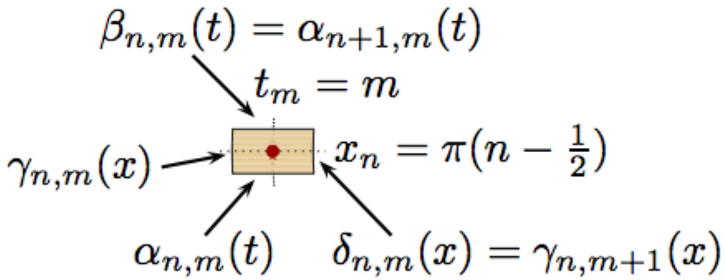
$$\mathcal{H}_{J_1,J_2} = \sum_{i=1}^{N} \left(J_1 \vec{S}_i \cdot \vec{S}_{i+1} + J_2 \vec{S}_i \cdot \vec{S}_{i+2} \right)$$

$$\frac{J_2}{J_1} = 1/2$$



2D Field Tensor Network States





$$A^s_{\alpha_{n,m},\beta_{n,m},\gamma_{n,m},\delta_{n,m}} = \int' D[\varphi] e^{-S[\varphi]} e^{iqs\varphi(x_n,t_m)}$$

$$\begin{aligned} \varphi(x_n - \delta, t) &= \alpha_{n,m}(t), \\ \varphi(x_n + \delta, t) &= \beta_{n,m}(t), \\ \varphi(x, t_m - \delta') &= \gamma_{n,m}(x), \\ \varphi(x, t_m + \delta') &= \delta_{n,m}(x). \end{aligned}$$

Infinite dimensional PEPS

Conjecture: sewing the amplitudes we get

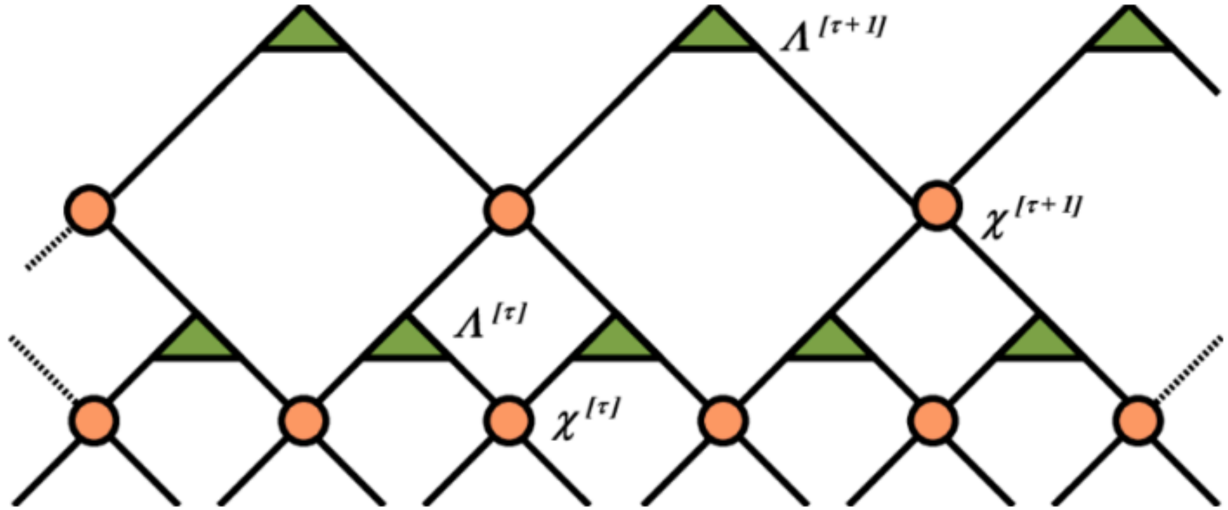
$$c_{s_1,\ldots,s_N} \propto \delta_{\sum_n s_n,0} \prod_n \chi_{s_n} \prod_{n>m} (z_n - z_m)^{q^2 s_n s_m}$$

When $q = 1/\sqrt{2}$, Kalmeyer-Laughlin wave function for $N \to \infty$

The chiral 2D FTN could through light into the “no-go” theorem concerning the non-existence of a PEPS with finite bond dimension for the chiral topological states.

FTN and MERA

work in progress



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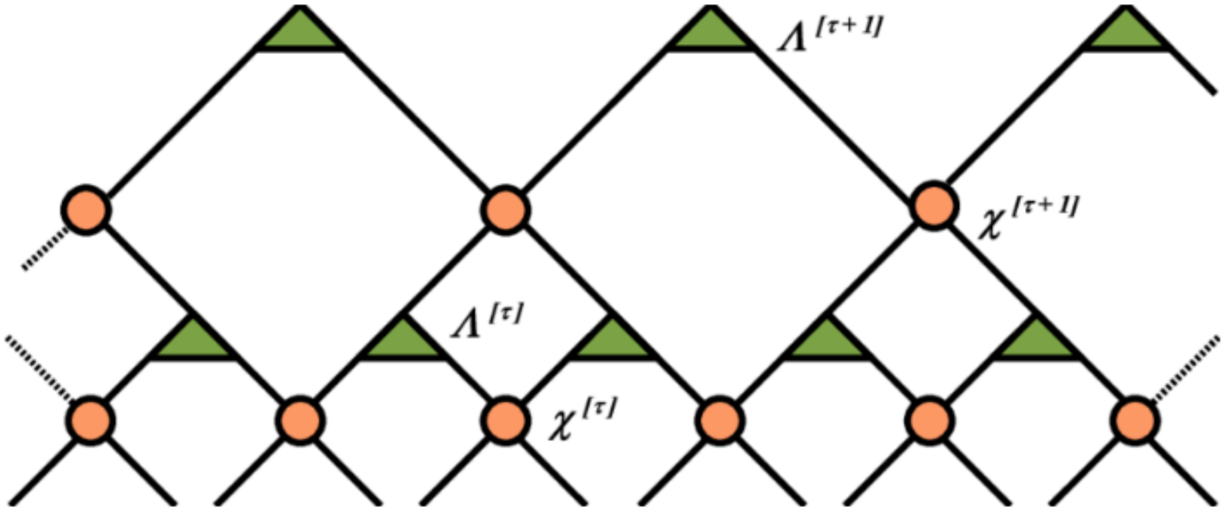
FTN and MERA

work in progress



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FTN and MERA

work in progress

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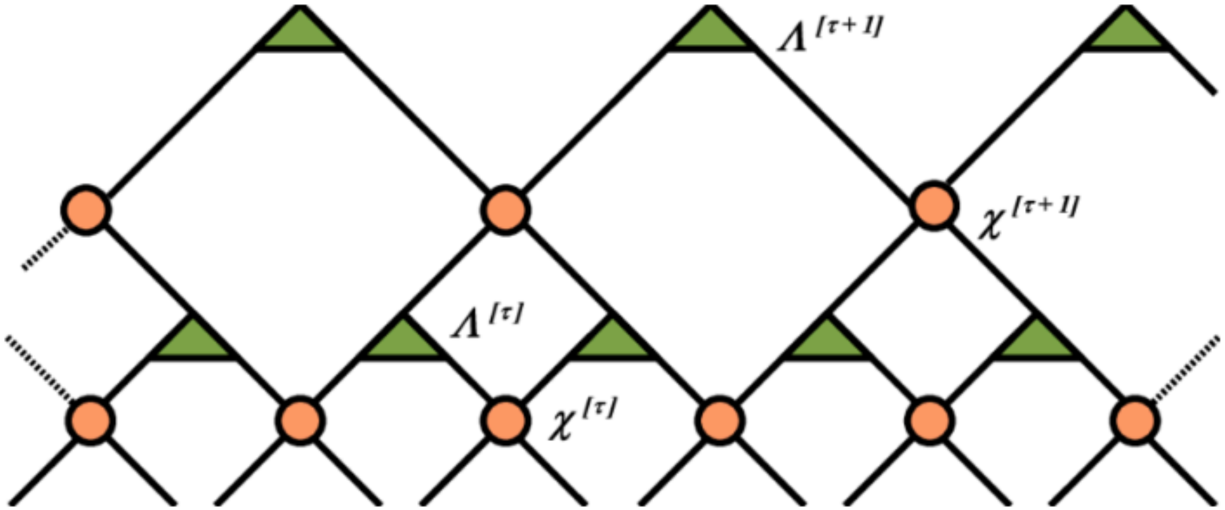

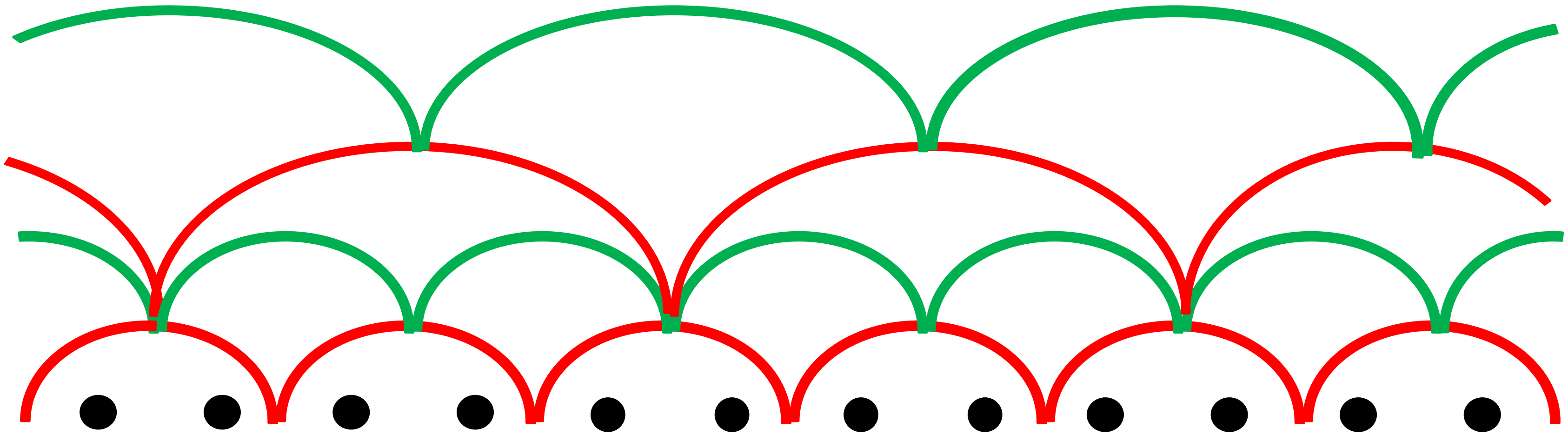

Conclusions

- Quantum Field Theory provides natural generalizations of Tensor Networks where the auxiliary space becomes infinite dimensional.

- This extension allows to described critical systems in 1D and FQH systems in 2D that are intimately related via the edge-bulk correspondence.

- FTN may provide a new way to study the topological properties of many body systems.

Collaborators

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Thank You

References

- Infinite matrix product states, Conformal Field Theory and the Haldane-Shastry model,
  J. I. Cirac and G. Sierra, Phys. Rev. B **81**, 104431 (2010).

- Quantum spin Hamiltonians for the SU(2)k WZW model,
  A. E. B. Nielsen, J. I. Cirac, G. Sierra, J. Stat. Mech. P11014 (2011).

- Laughlin Spin-Liquid States on Lattices obtained from Conformal Field Theory.
  A. E. B. Nielsen, J. I. Cirac, G. Sierra, Phys. Rev. Lett. **108**, 257206 (2012).

- Fractional quantum Hall states in lattices: Local models and physical implementation,
  A. E. B. Nielsen, G. Sierra, J. I. Cirac; Nature Communications **4**, 2864 (2013).

- Lattice Laughlin States of Bosons and Fermions at Filling Fractions 1/q,
  Hong-Hao Tu, A. E. B. Nielsen, J. I. Cirac, G. Sierra; New J. Phys. **16**, 033025 (2014).

- Exact parent Hamiltonians of bosonic and fermionic Moore-Read states on
  lattices and local models,
  I.Glasser, J.I. Cirac, G. Sierra, A.E. B. Nielsen, New J. Phys. **17**, 082001 (2015).

- Edge states for the Kalmeyer-Laughlin wave function,
  B. Herwerth, G. Sierra, Hong-Hao Tu, J. I. Cirac, A. E. B. Nielsen,
  Phys. Rev. B **92**, 245111 (2015).

- Quantum spin models for the SU(n)_1 Wess–Zumino–Witten model, H.-H. Tu, A. E. B. Nielsen, and G. Sierra, Nucl. Phys. B **886** (2014) 328.

- Excited States in Spin Chains from Conformal Blocks, Benedikt Herwerth, Germán Sierra, Hong-Hao Tu, Anne E. B. Nielsen, Phys. Rev. B **91**, 235121 (2015).

- Infinite matrix product states, boundary conformal field theory, and the open Haldane-Shastry model, H.-H. Tu and G. Sierra, Phys. Rev. B **92** (2015) 041119.

- Lattice effects on Laughlin wave functions and parent Hamiltonians", Ivan Glasser, J. Ignacio Cirac, Germán Sierra, Anne E. B. Nielsen, Phys. Rev. B **94**, 245104 (2016).

- Non-Abelian quasiholes in lattice Moore-Read states and parent Hamiltonians, Sourav Manna, Julia Wildeboer, Germán Sierra, Anne E. B. Nielsen; Phys. Rev. B **98**, 165147 (2018).

- Field Tensor Network States, A. E. B. Nielsen, B. Herwerth,J. I. Cirac, G. Sierra, Phys. Rev. B **103**, 155130 (2021).

- Symmetries and field tensor network states, A. Gasull,1, A. Tilloy, J. I. Cirac, and G. Sierra, Phys. Rev. B **107**, 155102 (2023).