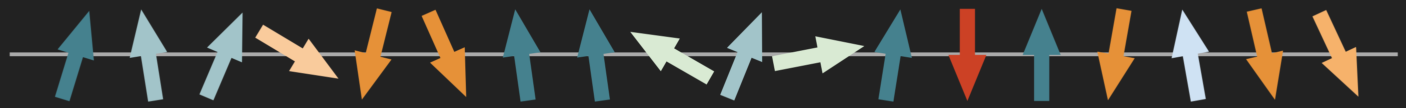
EXTREME VALUE THEORY AND LOCALIZATION IN RANDOM SPIN CHAINS



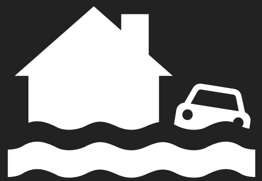
Jeanne Colbois, Nicolas Laflorencie
LPT Toulouse

arXiv:2305.10574

Entanglement in strongly correlated systems

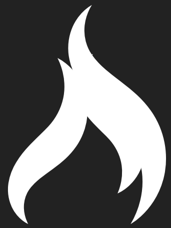
Benasque, 2023

EXTREME VALUE THEORY


Extreme floods


Market risks


Athletic records


Large wildfires

E. J. Gumbel, *Statistics of Extremes*, Dover, (1958, 2004)
S. N. Majumdar, A. Pal, G. Schehr, Physics Reports, **840**, 1 (2020)

EXTREME VALUE THEORY

Extreme floods

Market risks

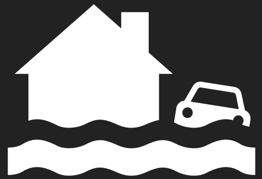
Athletic records

Large wildfires

Condensed matter

E. J. Gumbel, *Statistics of Extremes*, Dover, (1958, 2004)
S. N. Majumdar, A. Pal, G. Schehr, Physics Reports, **840**, 1 (2020)

EXTREME VALUE THEORY



Extreme floods

Market risks

Athletic records

Large wildfires

Condensed matter

Disordered spin chains

R. Juhász, Y,C. Lin, and F, Iglói, “S Phys. Rev. B 73, 224206 (2006)

N. Pancotti, M. Knap, D. A. Huse, J. I. Cirac, and M. C. Bañuls, Phys. Rev. B 97, 094206 (2018)

I. A. Kovács, T.Pető, and F.Iglói, Phys. Rev. Res. 3, 033140 (2021)

W.-H. Kao and N, B. Perkins, Phys. Rev. B 106, L100402 (2022)

J. C., N. Laflorencie, arXiv:2305.10574

E. J. Gumbel, *Statistics of Extremes*, Dover, (1958, 2004)

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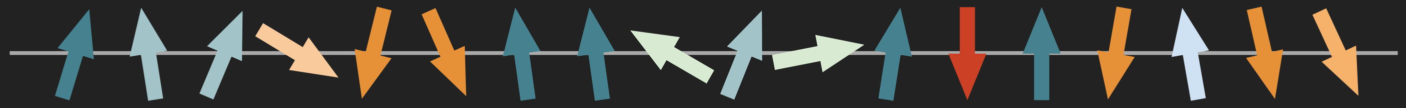
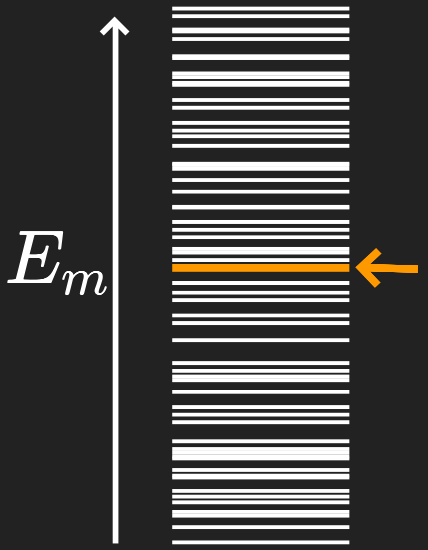
"TRAILER"



Spin-1/2 $W = h =$ disorder strength for random fields



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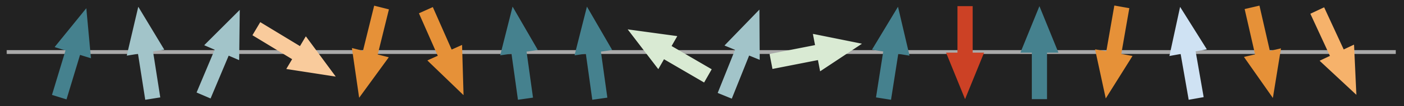

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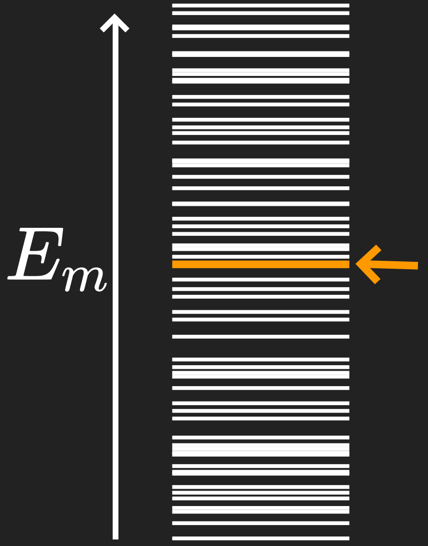

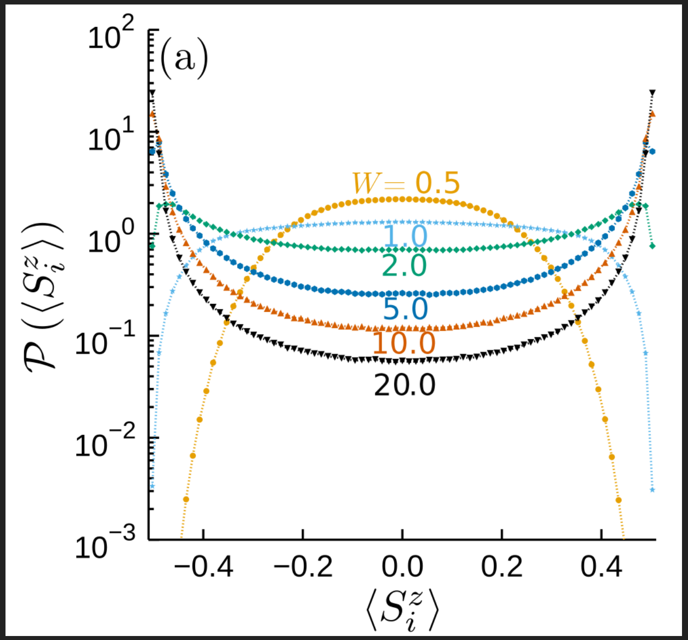

J. C., N. Laflorencie, arXiv:2305.10574

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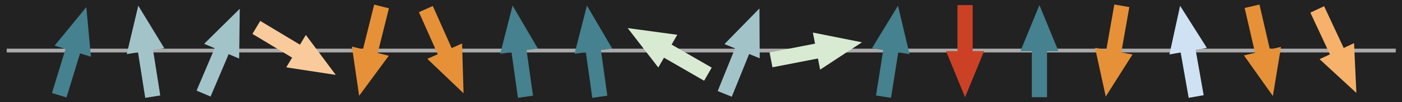

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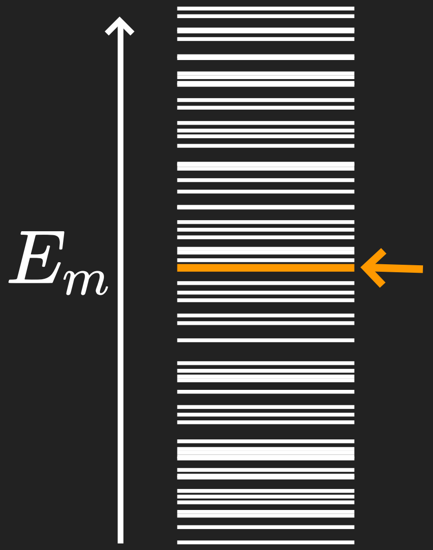

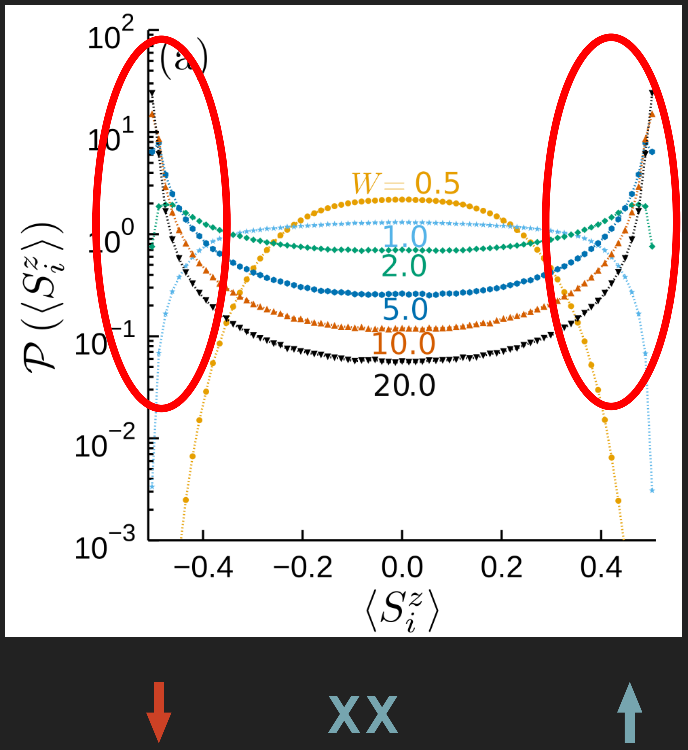

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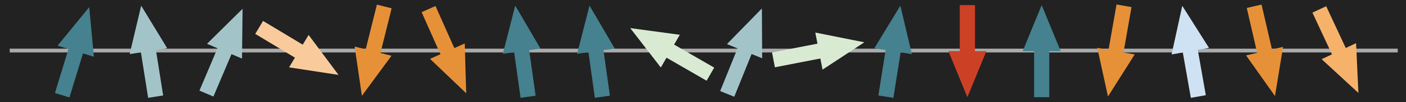

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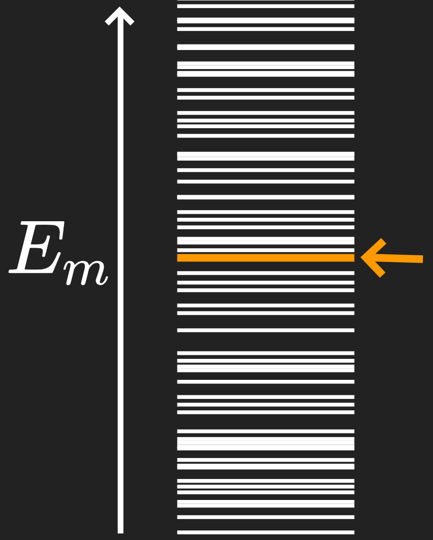
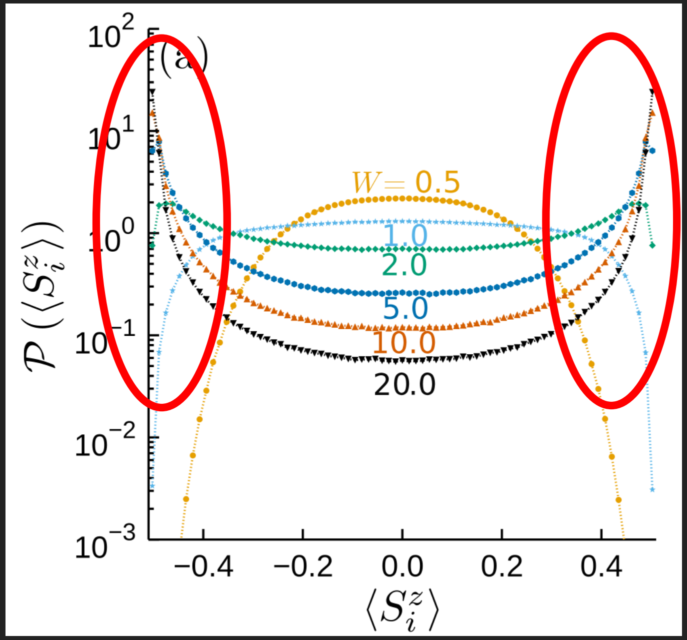
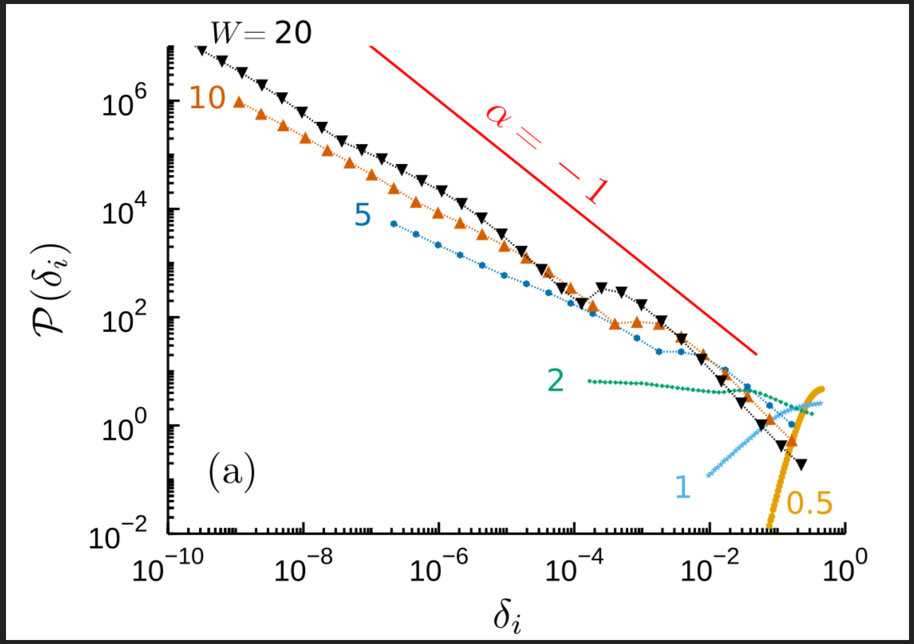




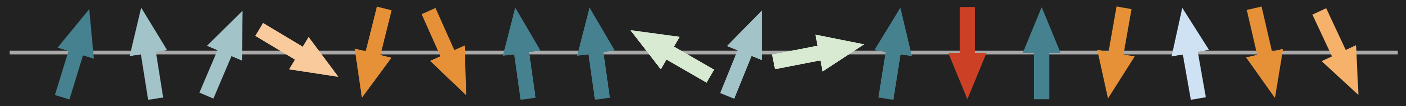


XX

$$\delta_i = 1/2 - |\langle S_i^z \rangle|$$

J. C., N. Laflorencie, arXiv:2305.10574

"TRAILER"



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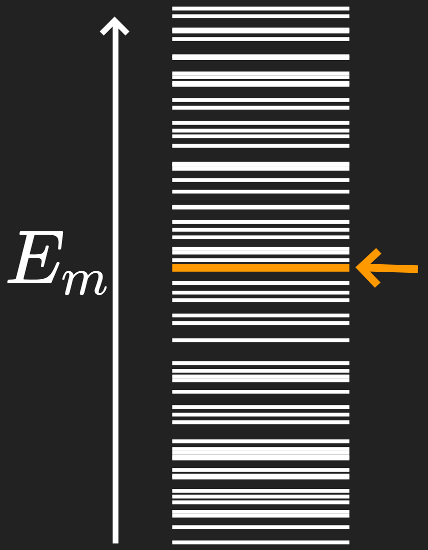
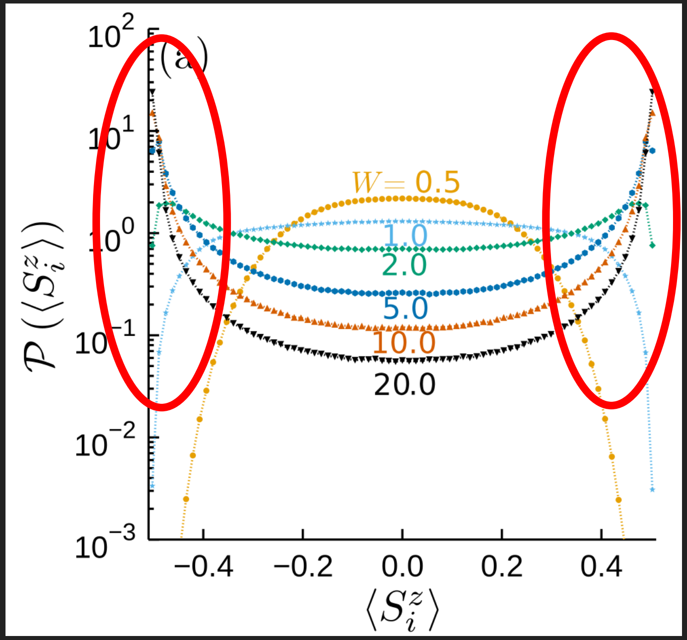
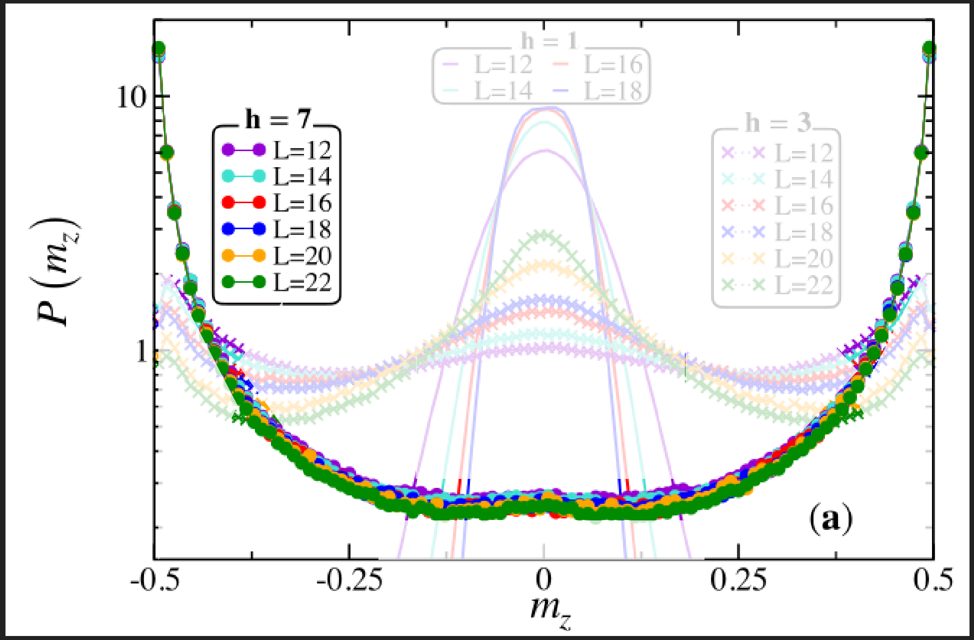






XX

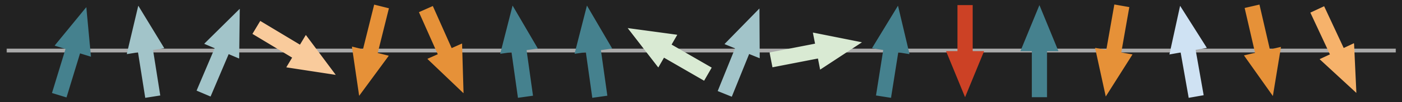
Heisenberg (XXX)

J. C., N. Laflorencie, arXiv:2305.10574

N. Laflorencie, N. Macé, G. Lemarié, PRR **2**, 042033(R) (2020)

V. Khemani, F. Pollmann, and S. L. Sondhi, PRL **116**, 247204 (2016)

"TRAILER"



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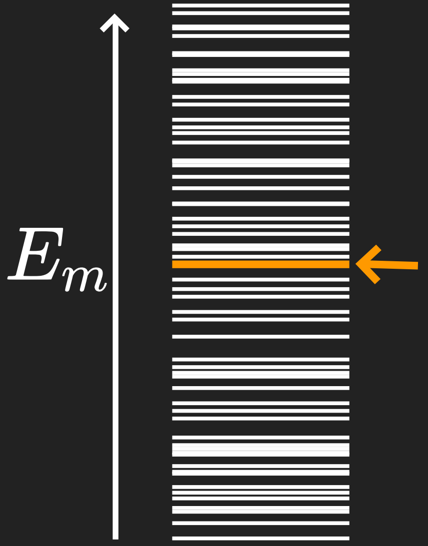

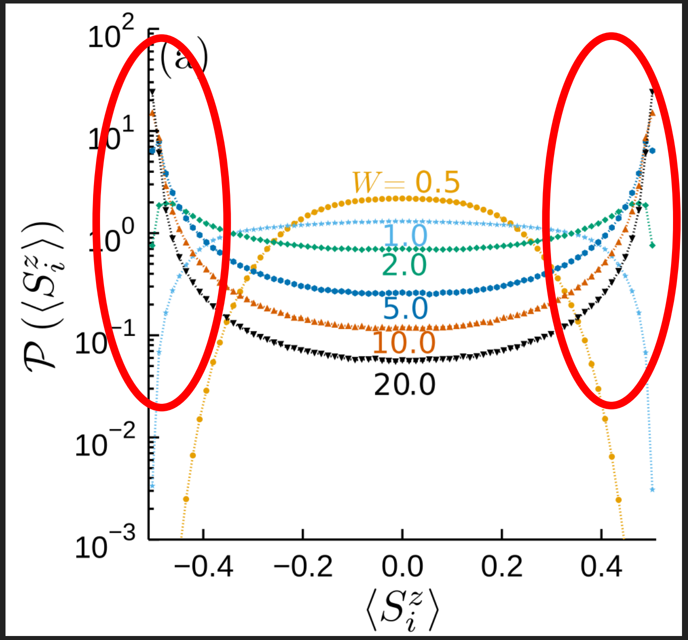

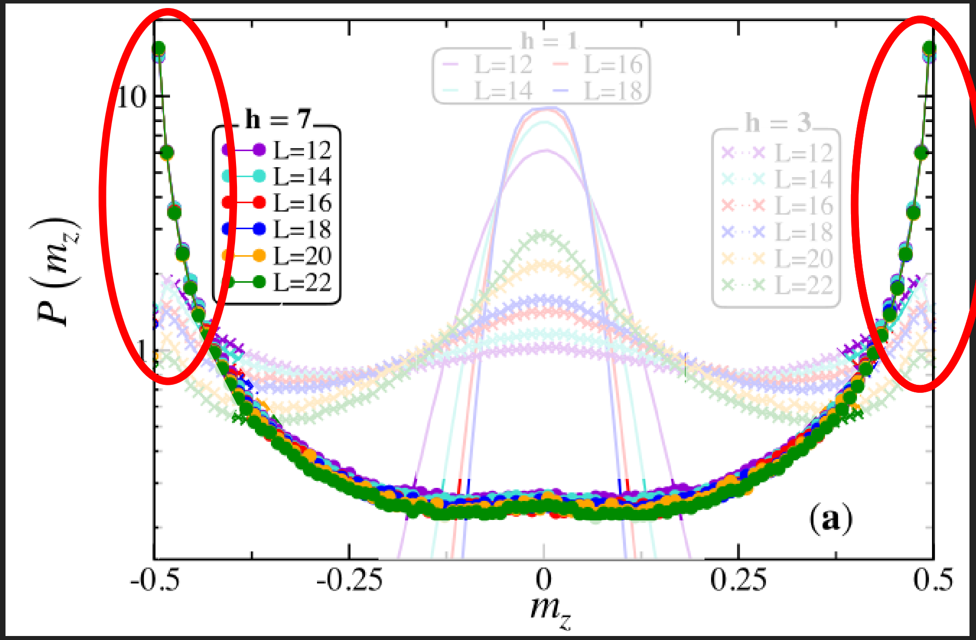

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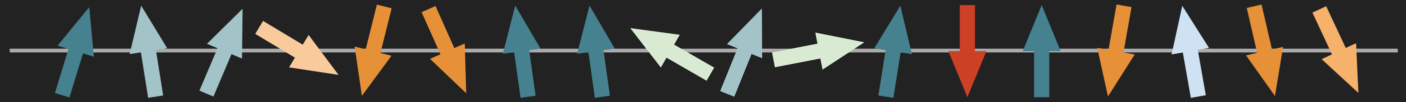

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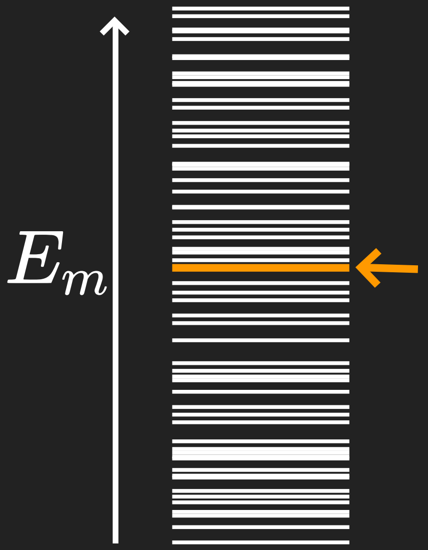

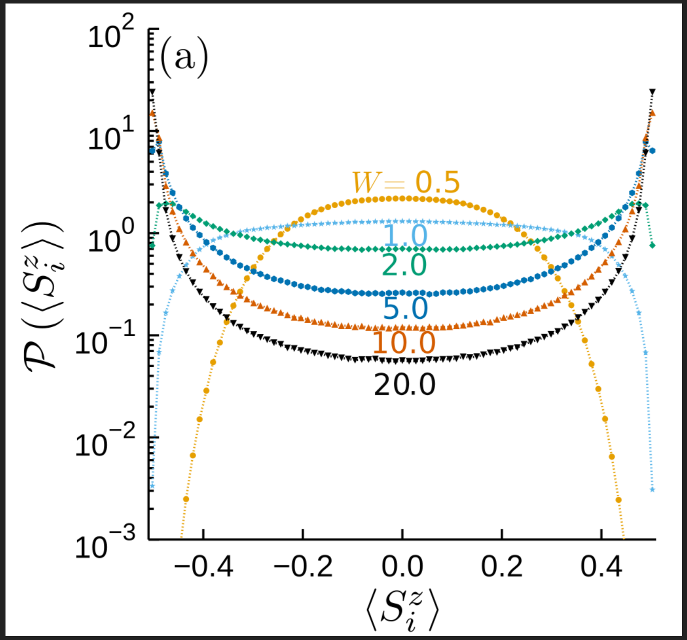

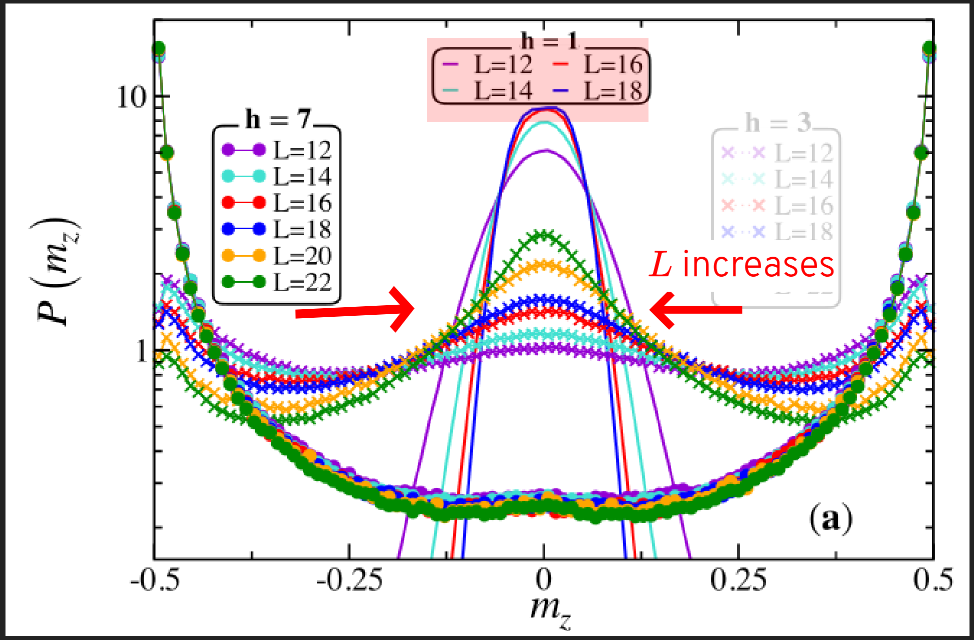

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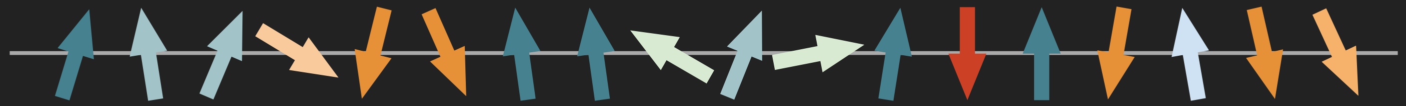

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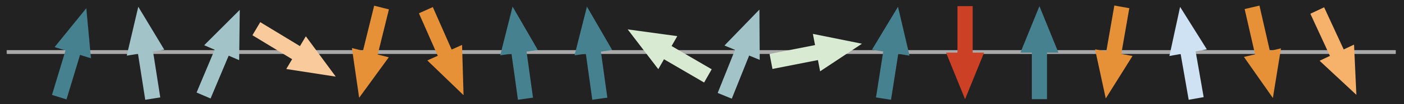

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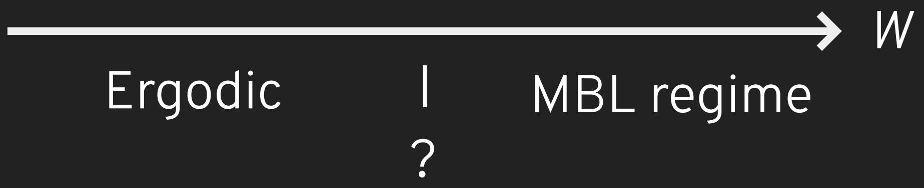


Toy model?

Quantitative description?

Consequences?

J. C., N. Laflorencie, arXiv:2305.10574

SCOPE

1. Spin chains in random field and localization

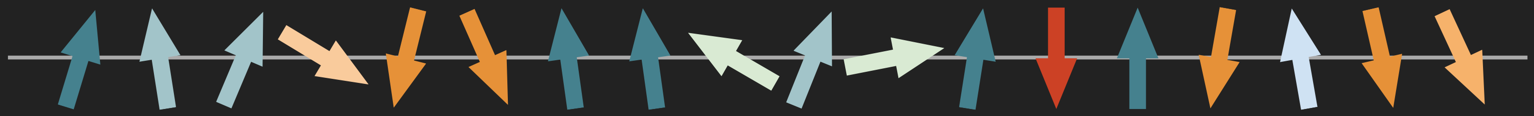
2. Toy model : chain breaking!

3. Quantitative analysis: extreme value theory

SPIN CHAINS IN RANDOM FIELD AND LOCALIZATION

SPIN-1/2 CHAIN IN A RANDOM FIELD

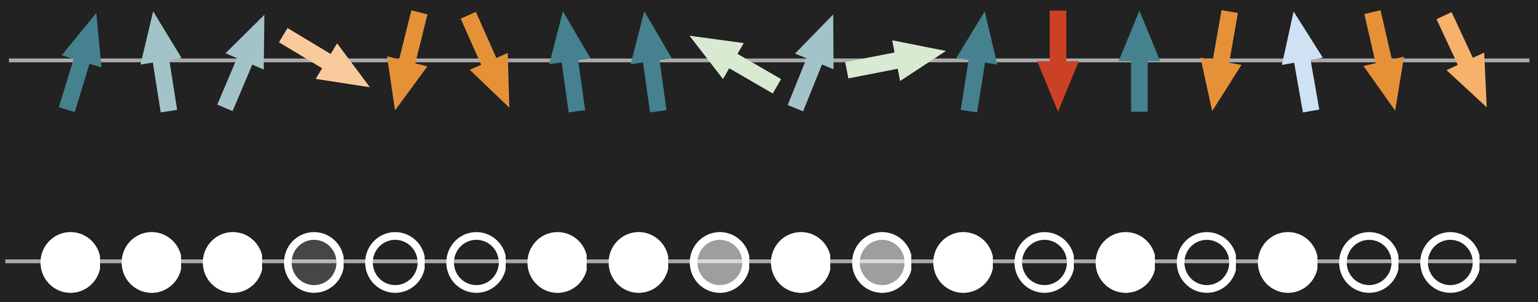
$$\mathcal{H} = \sum_i \frac{J}{2}\left(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+ + 2\Delta S_i^z S_{i+1}^z\right) - \sum_i h_i S_i^z \qquad S^{x,y,z} = \frac{1}{2}\sigma^{x,y,z}$$



SPIN-1/2 CHAIN IN A RANDOM FIELD

$$\mathcal{H} = \sum_i \frac{J}{2}\left(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+ + 2\Delta S_i^z S_{i+1}^z\right) - \sum_i h_i S_i^z$$

$$S^{x,y,z} = \frac{1}{2}\sigma^{x,y,z}$$



Jordan-Wigner

$$\mathcal{H}_f = \sum_i \left[\frac{J}{2}\left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i + 2\Delta n_i n_{i+1}\right) - h_i n_i\right]$$

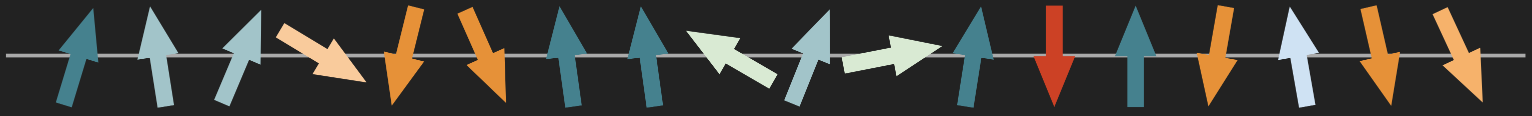
Spinless fermions

P. Jordan and E. Wigner, Z. Physik **47**, 631–651 (1928)

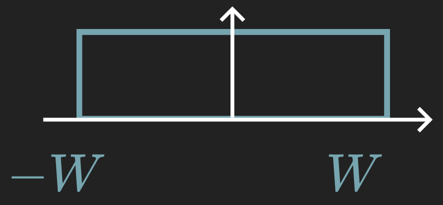
SPIN-1/2 CHAIN IN A RANDOM FIELD

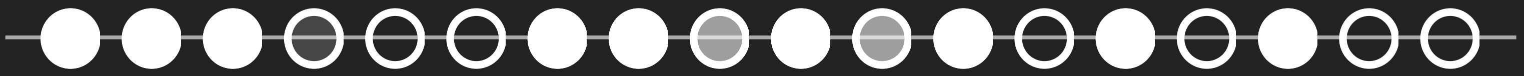
$$\mathcal{H} = \sum_i \frac{J}{2}\left(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+ + 2\Delta S_i^z S_{i+1}^z\right) - \sum_i h_i S_i^z \qquad S^{x,y,z} = \frac{1}{2}\sigma^{x,y,z}$$

Spin-flip Ising interaction Magnetic field



$\mathcal{P}(h_i) =$





Attraction/ repulsion

$$\mathcal{H}_f = \sum_i \left[\frac{J}{2}\left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i + 2\Delta n_i n_{i+1}\right) - h_i n_i\right]$$

Jump On-site energy

P. Jordan and E. Wigner, Z. Physik **47**, 631–651 (1928)

ANDERSON LOCALIZATION

$$\mathcal{H} = \sum_i \frac{J}{2}\left(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+ + 2\Delta S_i^z S_{i+1}^z\right) - \sum_i h_i S_i^z$$

1 particle

Spin-flip

Ising interaction

Magnetic field

ANDERSON LOCALIZATION

$$\mathcal{H}_f = \sum_i \left[\frac{J}{2} \left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i \right) - h_i n_i \right]$$

1 particle

ANDERSON LOCALIZATION

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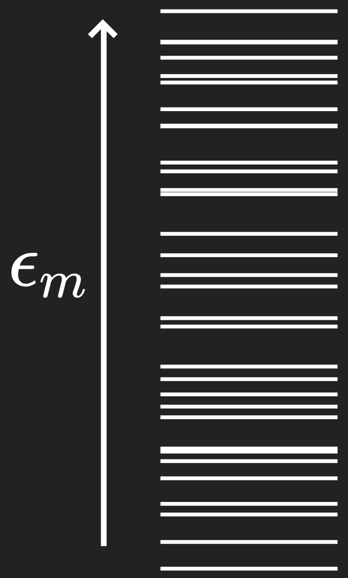

$$\mathcal{H}_f = \sum_m \epsilon_m b_m^\dagger b_m$$

P. W. Anderson, Phys. Rev. **109,** 1492 (1958)

B. A. Van Tiggelen, In: J. P. Fouque (eds), *Diffuse Waves in Complex Media*, NATO Science Series, **531,** Springer, Dordrecht, (1999)

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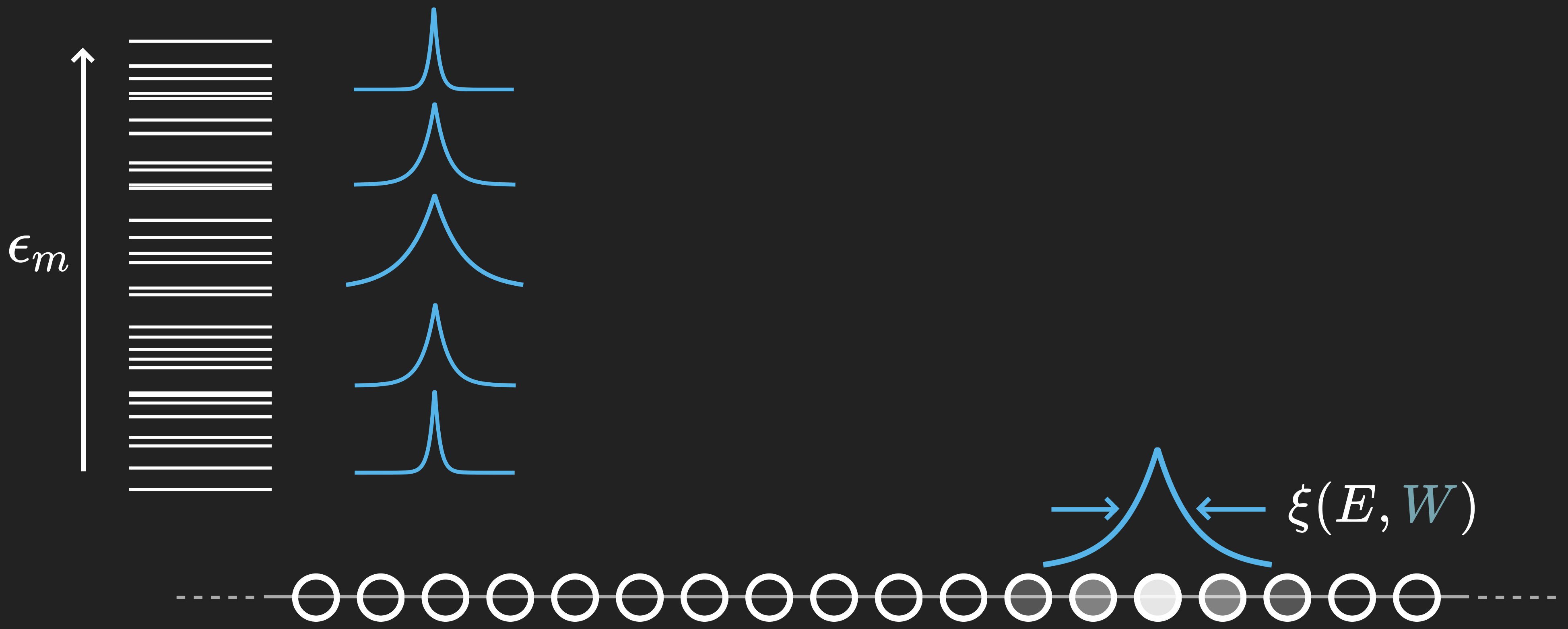

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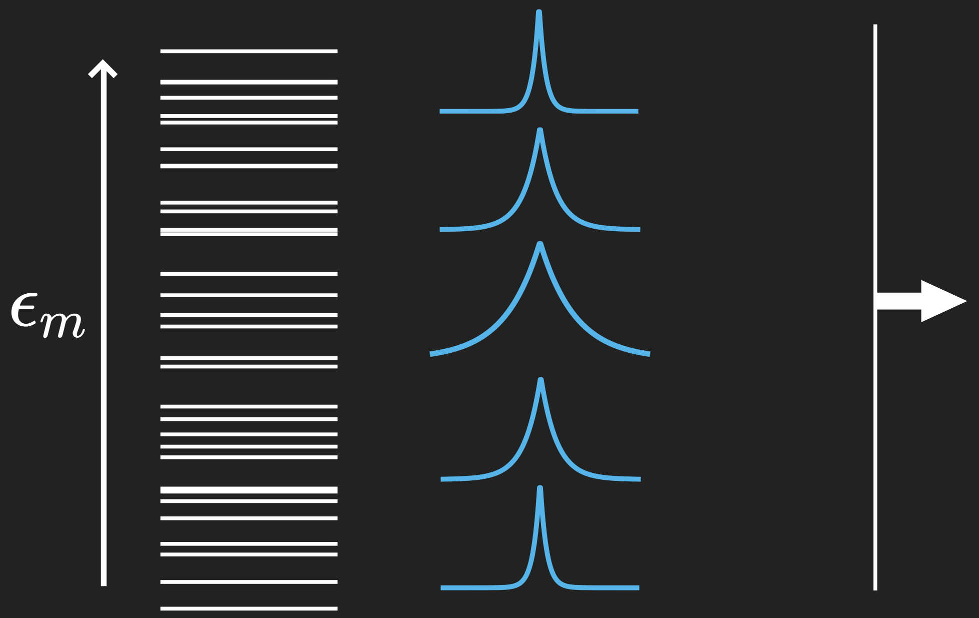

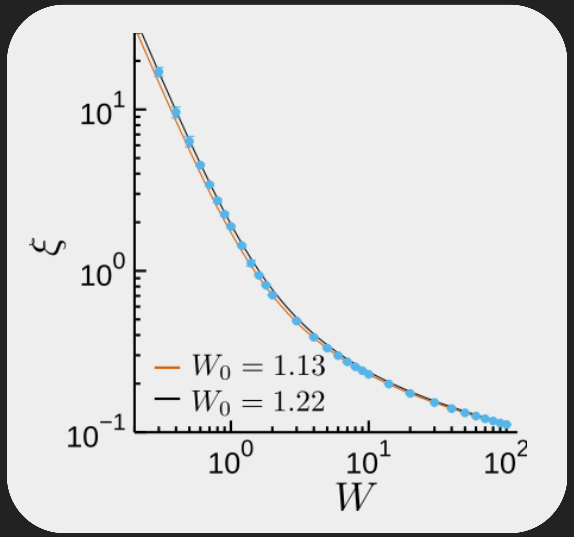

$$\xi = \frac{1}{\ln\left(1 + \left(\frac{W}{W_0}\right)^2\right)}$$

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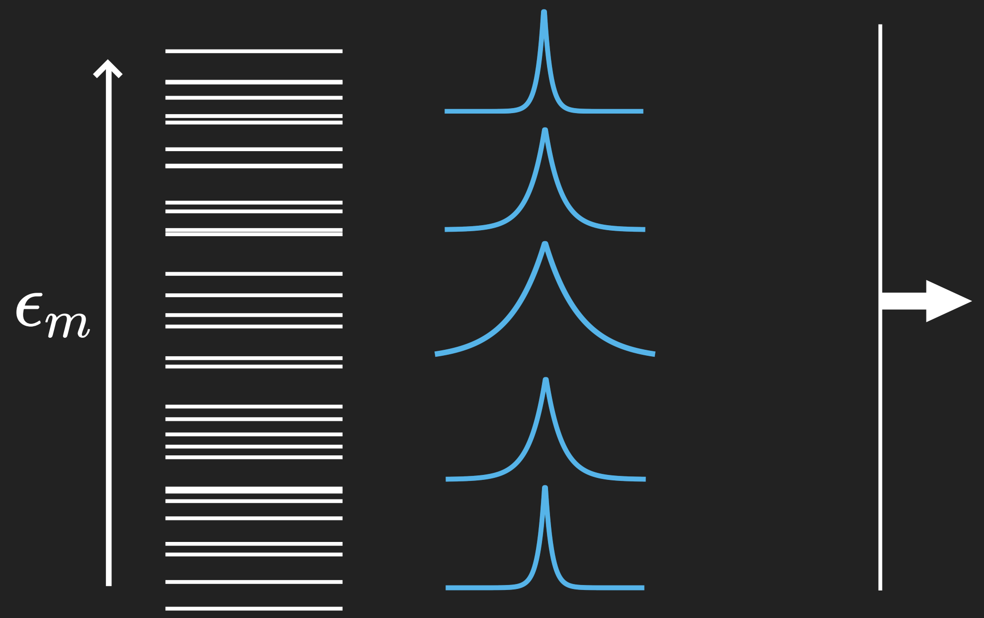
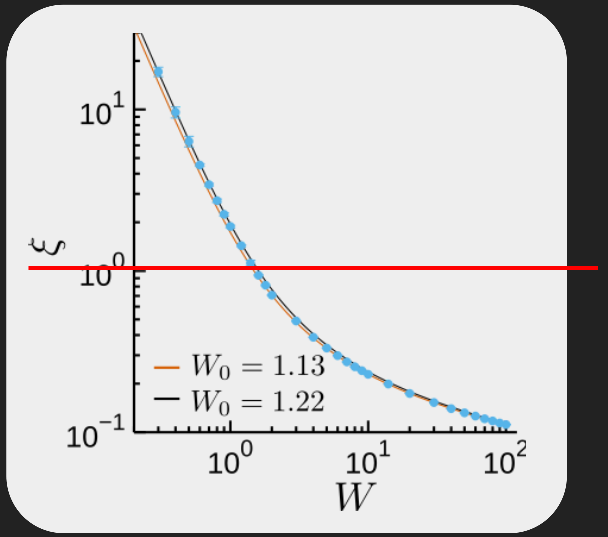
1 particle





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$W \gg 4$

$\xi \ll 1$

P. W. Anderson, Phys. Rev. **109,** 1492 (1958)

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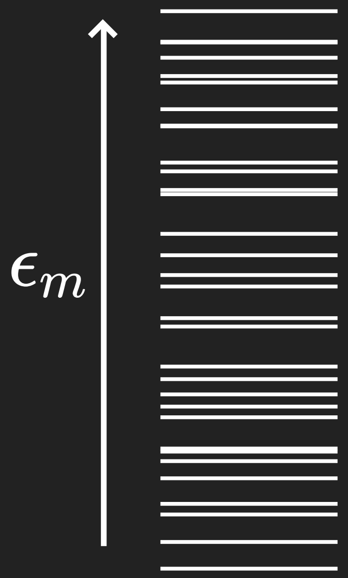

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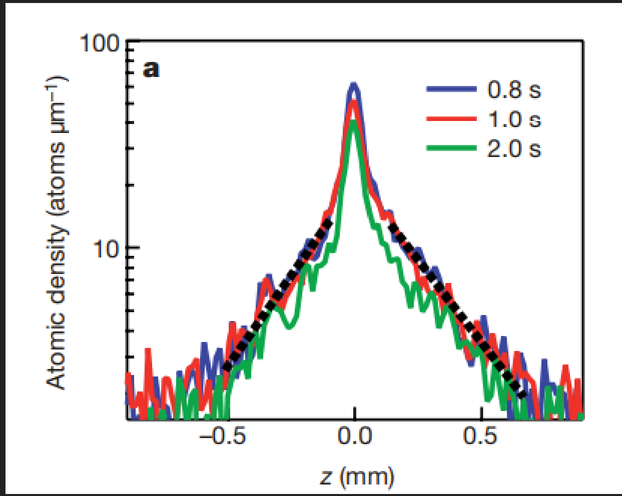
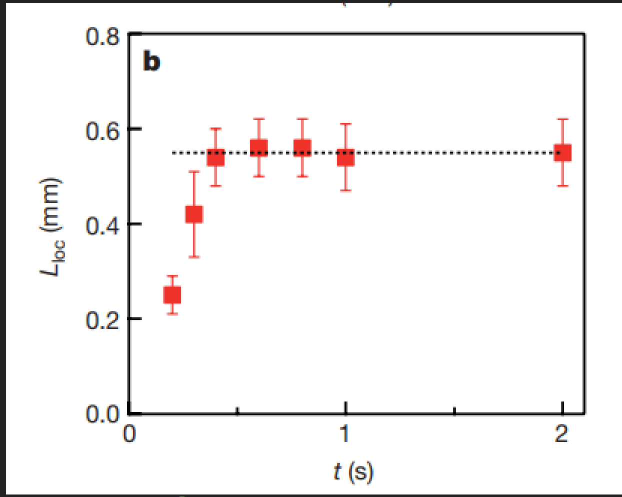


Billy J, et al. Direct observation of Anderson localization of matter waves in a controlled disorder. Nature. 2008



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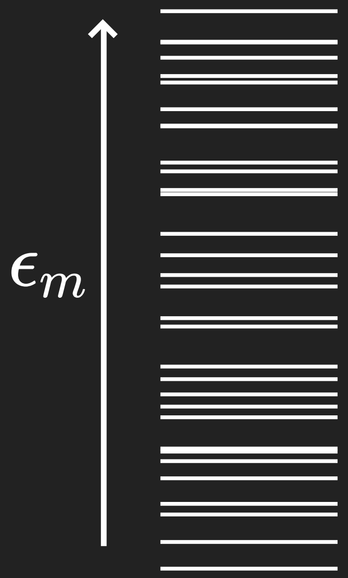
MANY-BODY ANDERSON INSULATOR

$$\mathcal{H}_f = \sum_i \left[\frac{J}{2} \left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i \right) - h_i n_i \right]$$

$L/2$ fermions

$S_z = 0$



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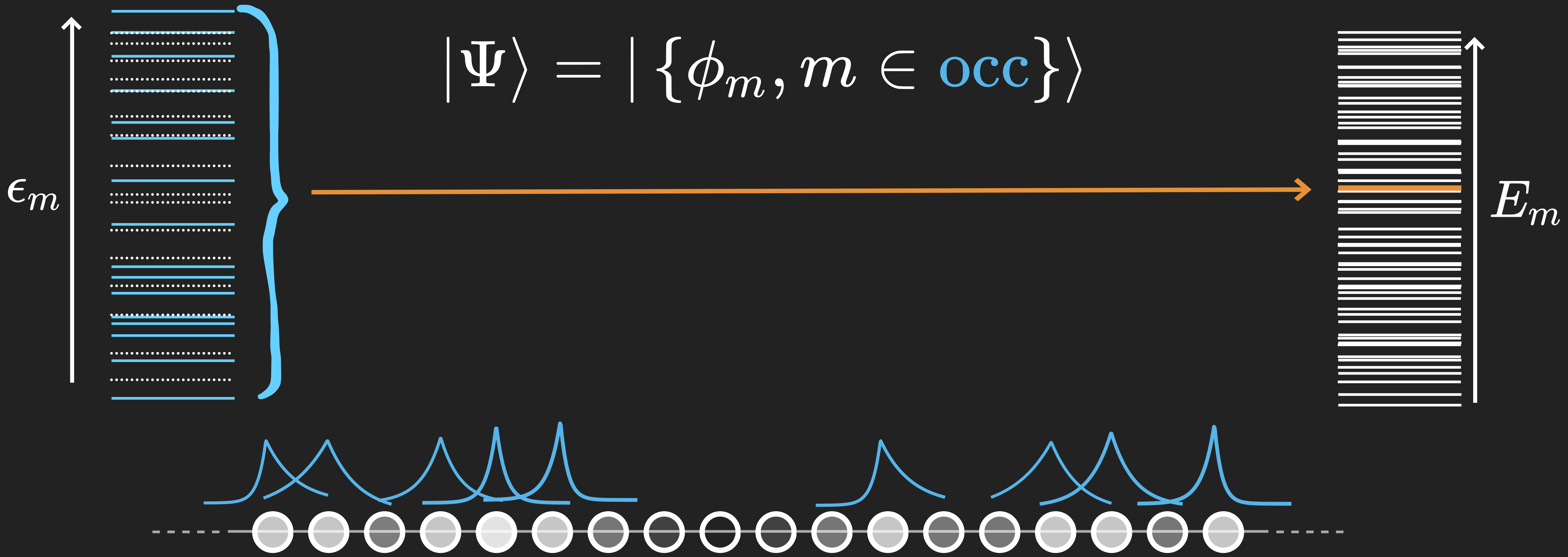
$L/2$ fermions

$S_z = 0$

$$|\Psi\rangle = |\{\phi_m, m \in \mathbf{occ}\}\rangle$$



HEISENBERG: INTRODUCING INTERACTIONS

$$\mathcal{H} = \sum_i \frac{J}{2}\left(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+ + 2\Delta S_i^z S_{i+1}^z\right) - \sum_i h_i S_i^z$$

$$\mathcal{H}_f = \sum_i \left[\frac{J}{2}\left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i + 2\Delta n_i n_{i+1}\right) - h_i n_i\right]$$

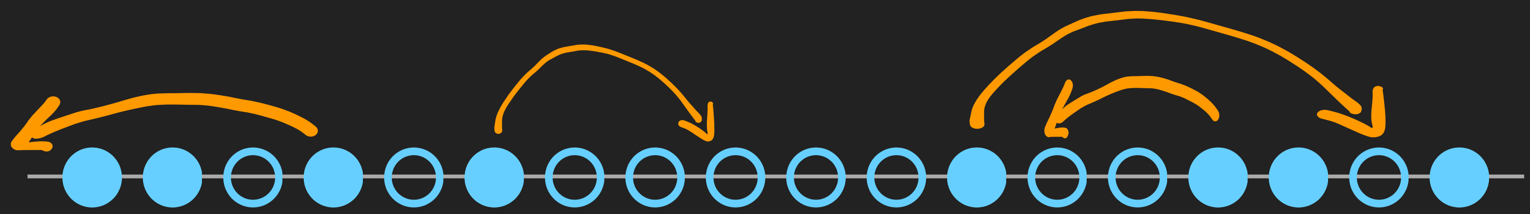
Fate of isolated quantum **many-body** systems ?

HEISENBERG: INTRODUCING INTERACTIONS

$$\mathcal{H} = \sum_i \frac{J}{2}\left(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+ + 2\Delta S_i^z S_{i+1}^z\right) - \sum_i h_i S_i^z$$

In the Anderson basis:

$$\mathcal{H} = \sum_m \epsilon_m b_m^\dagger b_m + \sum_{j,k,l,m} V_{j,k,l,m} b_j^\dagger b_k^\dagger b_l b_m$$



Anderson orbitals m

$$\mathcal{H}_f = \sum_i \left[\frac{J}{2}\left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i + 2\Delta n_i n_{i+1}\right) - h_i n_i\right]$$

Fate of isolated quantum **many-body** systems ?

HEISENBERG: INTRODUCING INTERACTIONS

$$\mathcal{H} = \sum_i \frac{J}{2}\left(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+ + 2\Delta S_i^z S_{i+1}^z\right) - \sum_i h_i S_i^z$$

In the Anderson basis:

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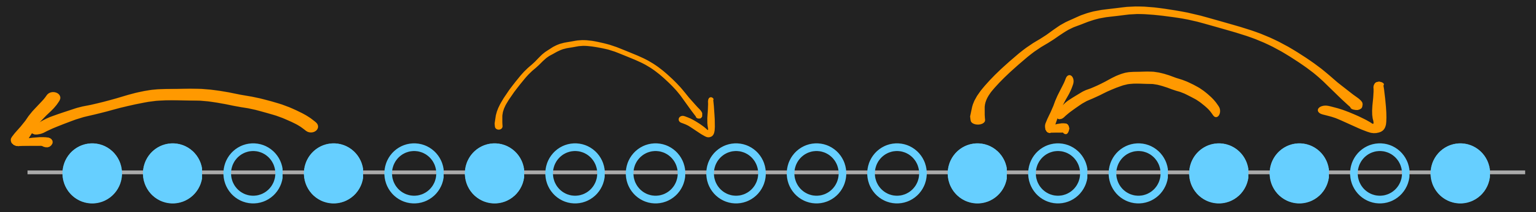


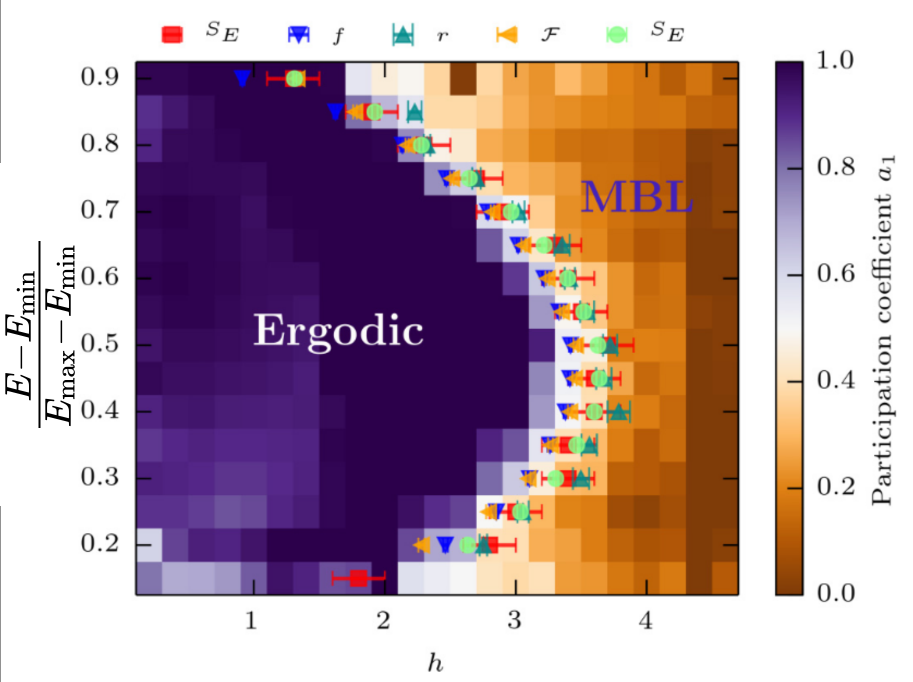
Anderson orbitals m

Challenge : high-energy eigenstate, lack of symmetries. Shift-invert ED $\rightarrow L = 22$

$$\mathcal{H}_f = \sum_i \left[\frac{J}{2}\left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i + 2\Delta n_i n_{i+1}\right) - h_i n_i\right]$$

Fate of isolated quantum **many-body** systems ?

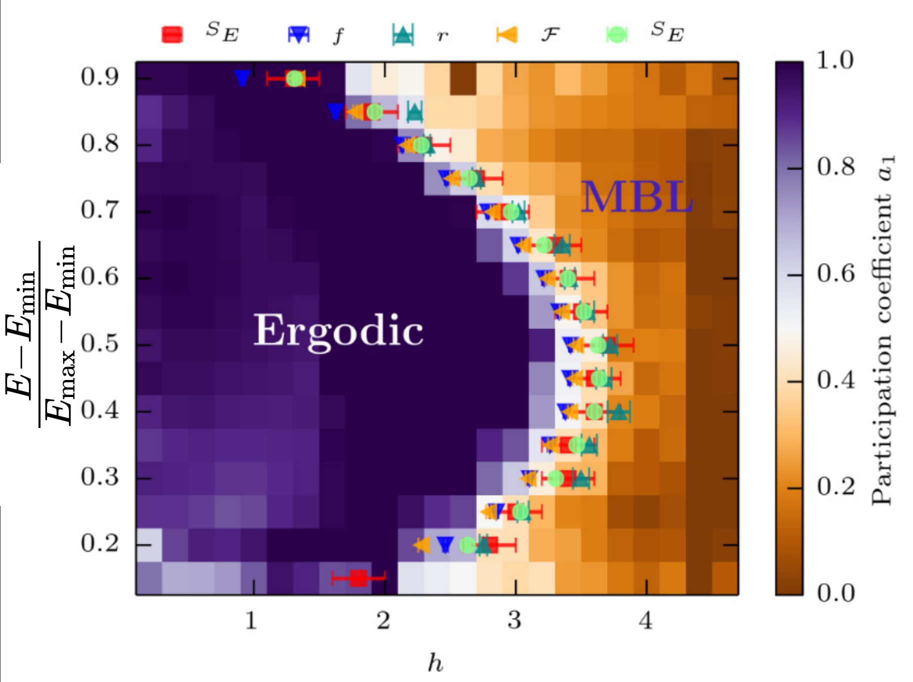
HEISENBERG: ERGODIC TO MBL



D. J. Luitz, N. Laflorencie, F. Alet, PRB **91**, 081103(R) (2015)

Fate of isolated quantum **many-body** systems ?

HEISENBERG: ERGODIC TO MBL



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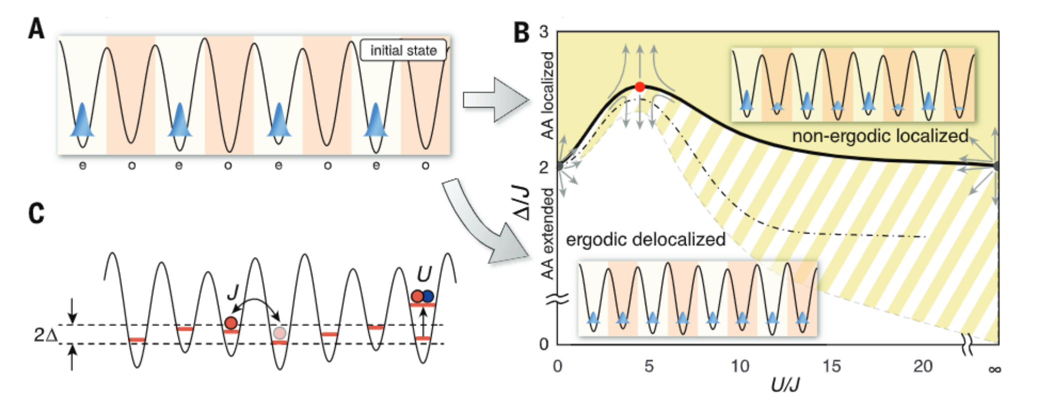
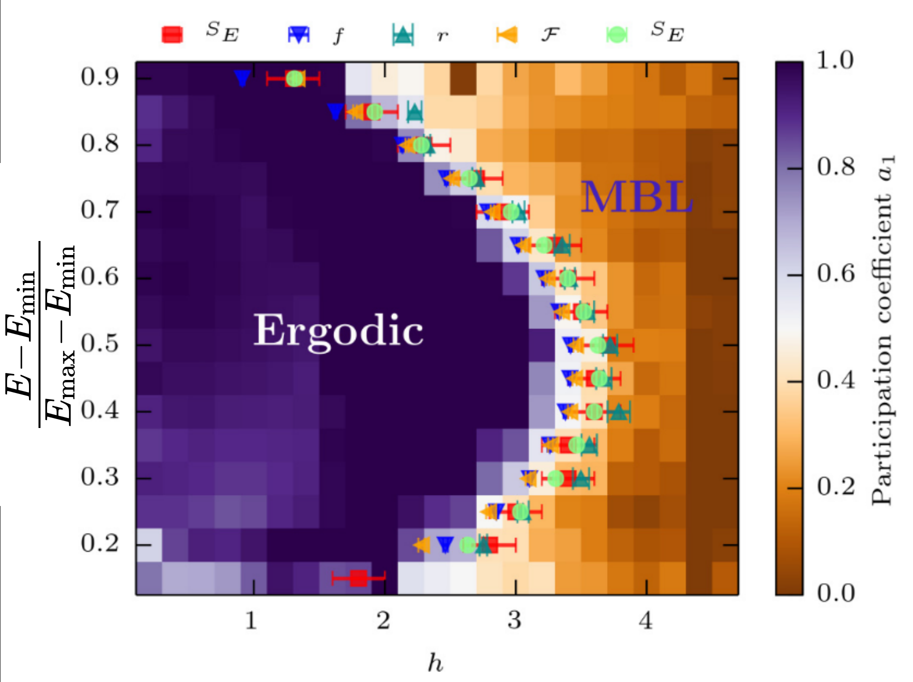


M. Schreiber *et al.* (I. Bloch) , Science **349**, 842 (2015)

Fate of isolated quantum **many-body** systems ?

HEISENBERG: ERGODIC TO MBL



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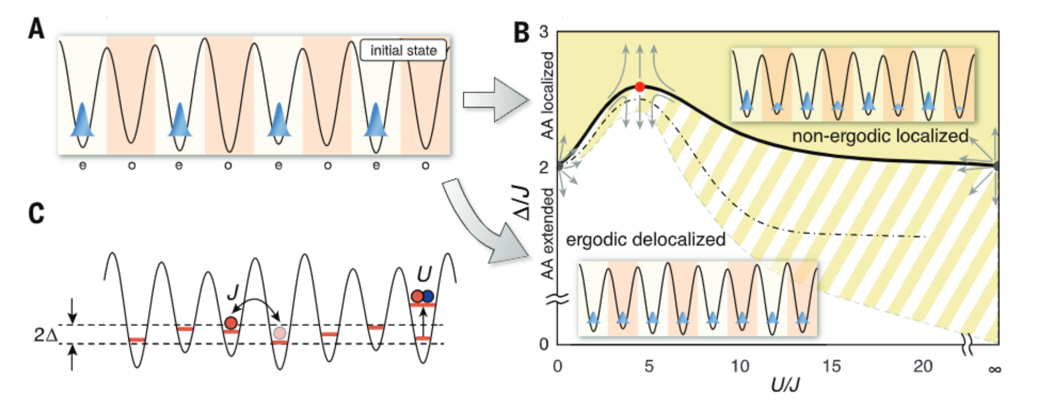


M. Schreiber *et al.* (I. Bloch) , Science **349**, 842 (2015)

2016 Finite L → W

Ergodic | MBL

2023 → W

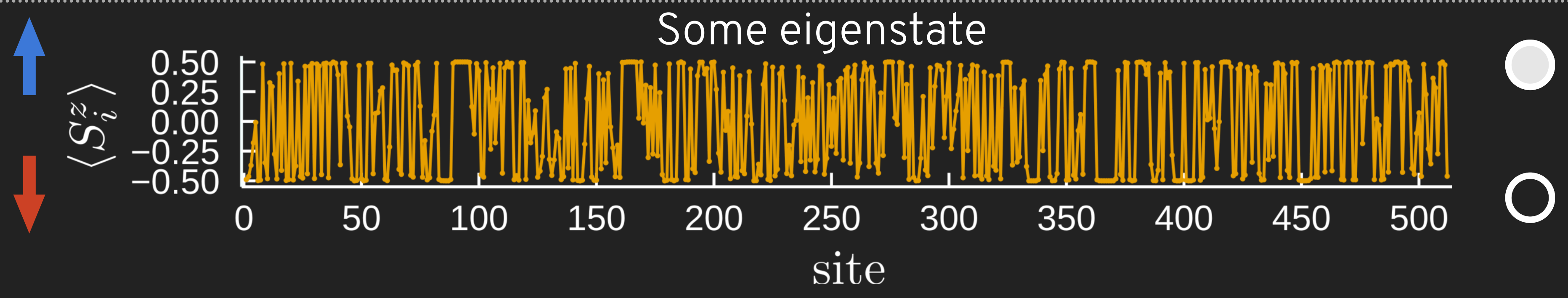
Ergodic — Prethermal regime? — MBL phase/ regimes?

Fate of isolated quantum **many-body** systems ?

ED ON ONE SAMPLE - XX (ANDERSON) CHAIN

J. C., N. Laflorencie, arXiv:2305.10574

ED ON ONE SAMPLE - XX (ANDERSON) CHAIN



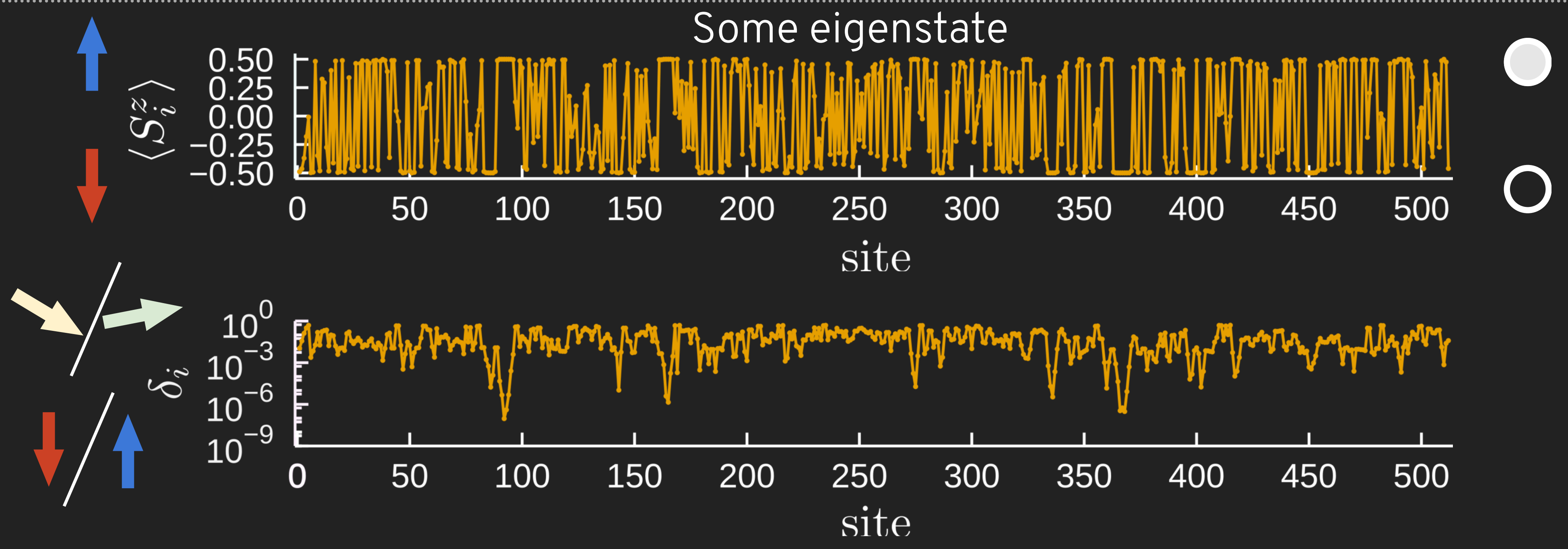
J. C., N. Laflorencie, arXiv:2305.10574

ED ON ONE SAMPLE - XX (ANDERSON) CHAIN

$$\delta_i = 1/2 - |\langle S_i^z \rangle|$$

J. C., N. Laflorencie, arXiv:2305.10574

ED ON ONE SAMPLE - XX (ANDERSON) CHAIN



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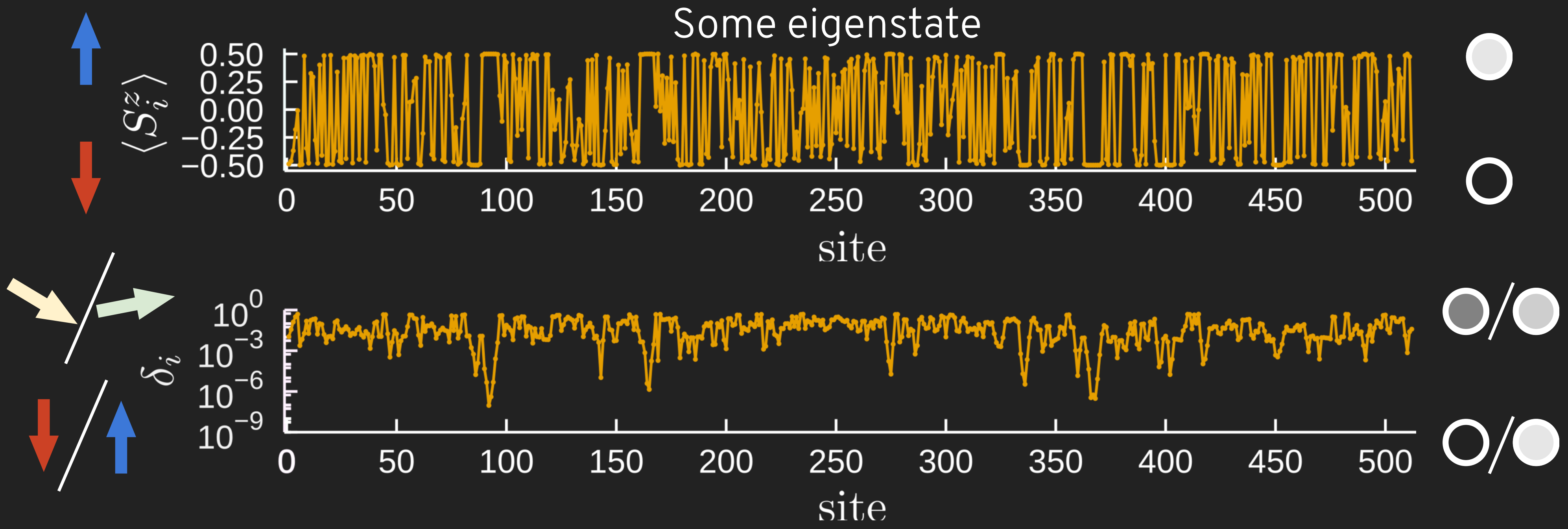
ED ON ONE SAMPLE - XX (ANDERSON) CHAIN



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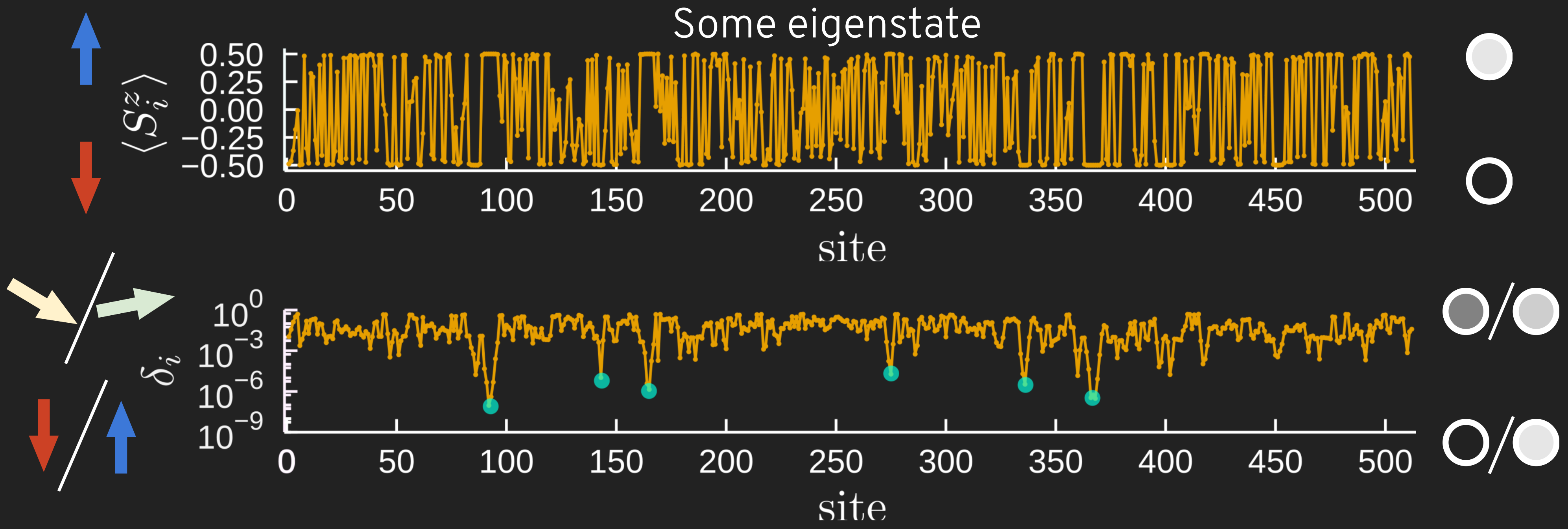

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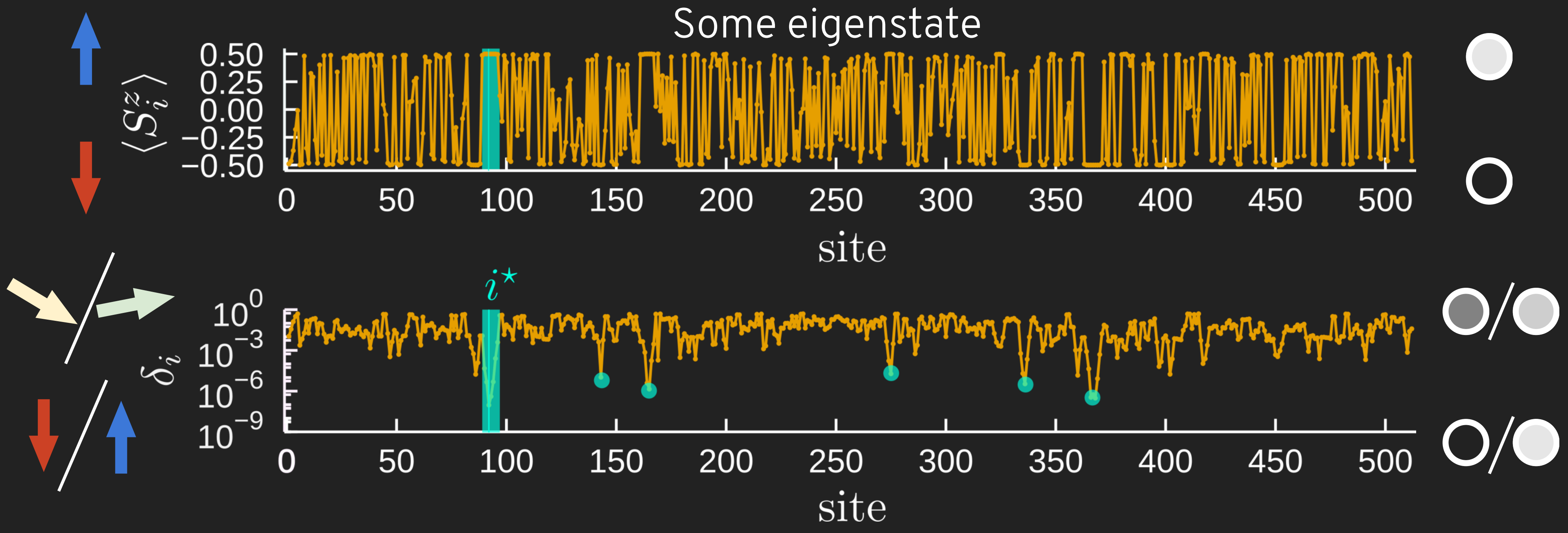
SPIN FREEZING



J. C., N. Laflorencie, arXiv:2305.10574

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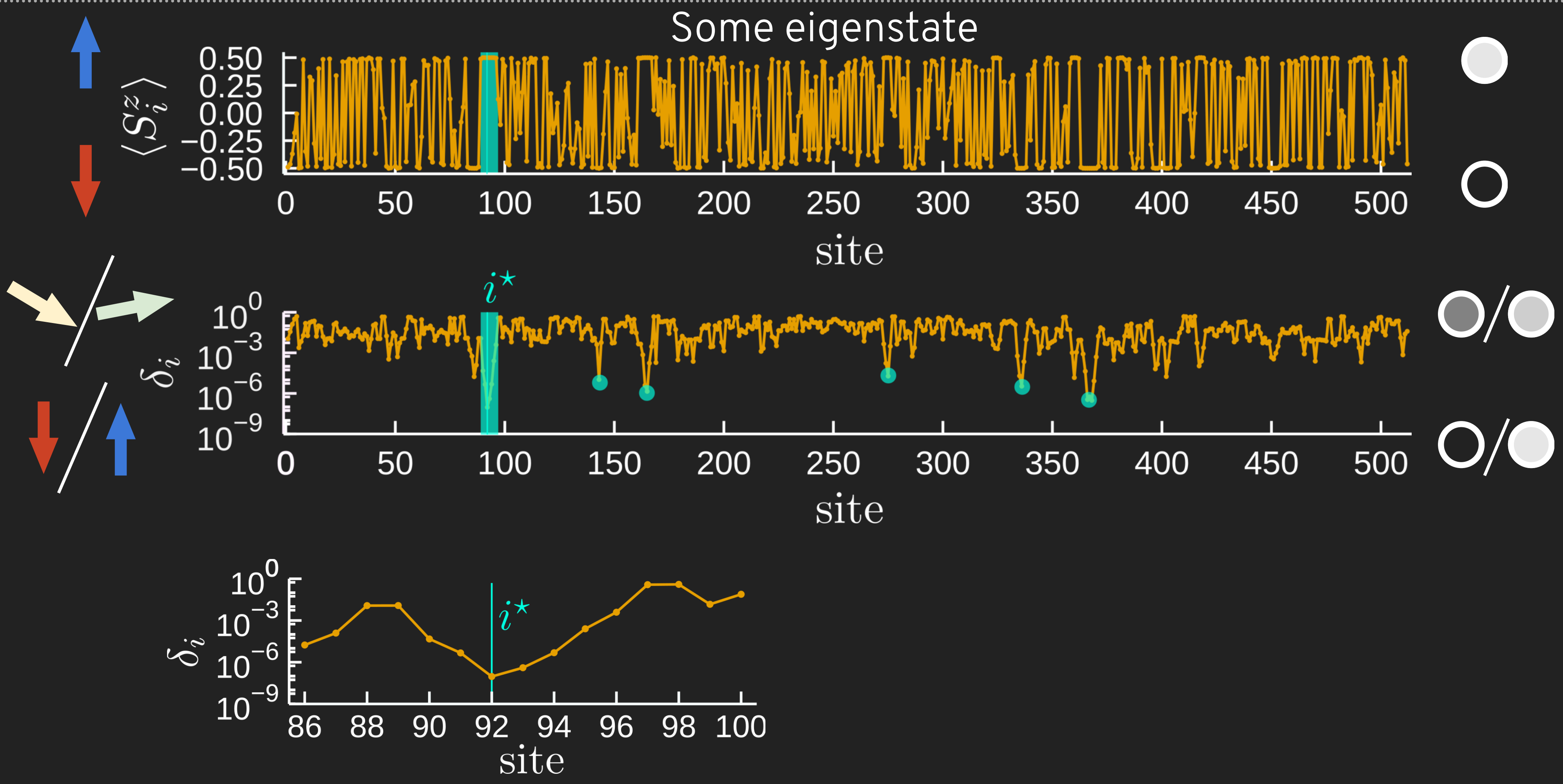
SPIN FREEZING



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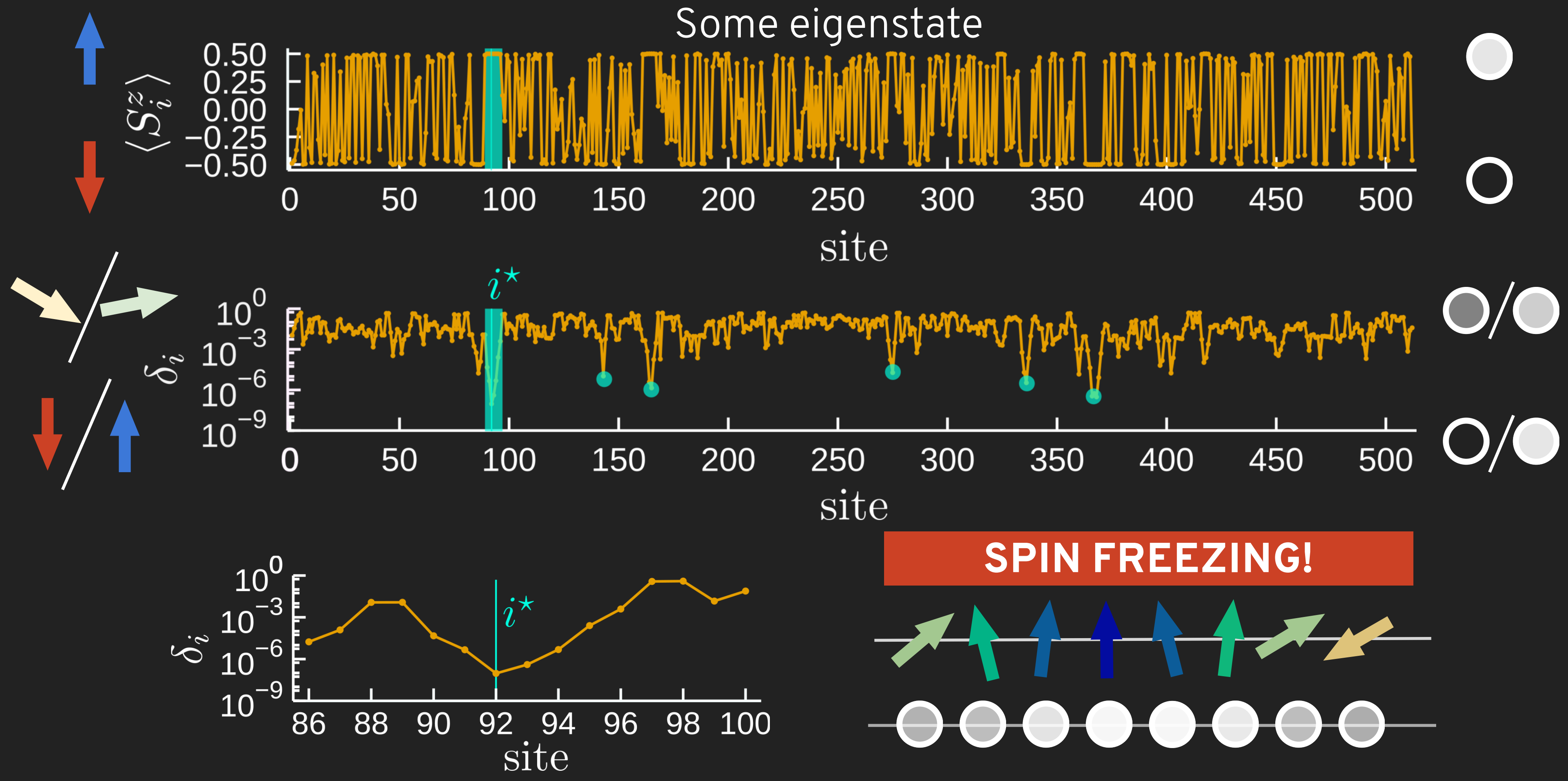
SPIN FREEZING



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SPIN FREEZING

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$$\delta_i = 1/2 - |\langle S_i^z \rangle|$$

TOY MODEL : CHAIN BREAKING!

TOY MODEL

Dupont, Macé, Laflorencie, PRB **100**, 134201, (2019)
Laflorencie, Lemarié, Macé, PRR **2**, 042033(R), (2020)
JC, N. Laflorencie, arXiv:2305.10574

TOY MODEL

$$|\phi_m(i)|^2 \propto \exp\left(-\frac{|i-i_0^m|}{\xi}\right)$$

Dupont, Macé, Laflorence, PRB **100**, 134201, (2019)
Laflorencie, Lemarié, Macé, PRR **2**, 042033(R), (2020)
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TOY MODEL

$$|\phi_m(i)|^2 \propto \exp\left(-\frac{|i-i_0^m|}{\xi}\right)$$

$$\Rightarrow \langle n_i \rangle = \langle S_i^z \rangle + 1/2 = \sum_{m\in \mathrm{occ}} |\phi_m(i)|^2$$

Dupont, Macé, Laflorencie, PRB **100**, 134201, (2019)
Laflorencie, Lemarié, Macé, PRR **2**, 042033(R), (2020)
JC, N. Laflorencie, arXiv:2305.10574

MINIMAL DEVIATION?

$$\delta_i = 1/2 - |\langle n_i \rangle - 1/2|$$

$$|\phi_m(i)|^2 \propto \exp\left(-\frac{|i - i_0^m|}{\xi}\right)$$

Dupont, Macé, Laflorencie, PRB **100**, 134201, (2019)
Laflorencie, Lemarié, Macé, PRR **2**, 042033(R), (2020)
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MINIMAL DEVIATION?

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$\bigcirc$: $\delta_i = \langle n_i \rangle$

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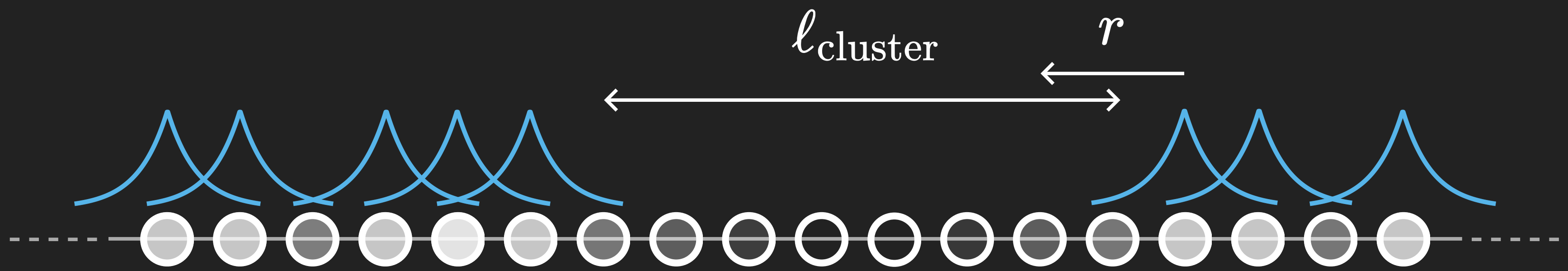

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MINIMAL DEVIATION?

$$\delta_i = 1/2 - |\langle n_i \rangle - 1/2|$$

$$\bigcirc : \quad \delta_i = \langle n_i \rangle \approx e^{-\frac{r}{\xi}} + \dots$$

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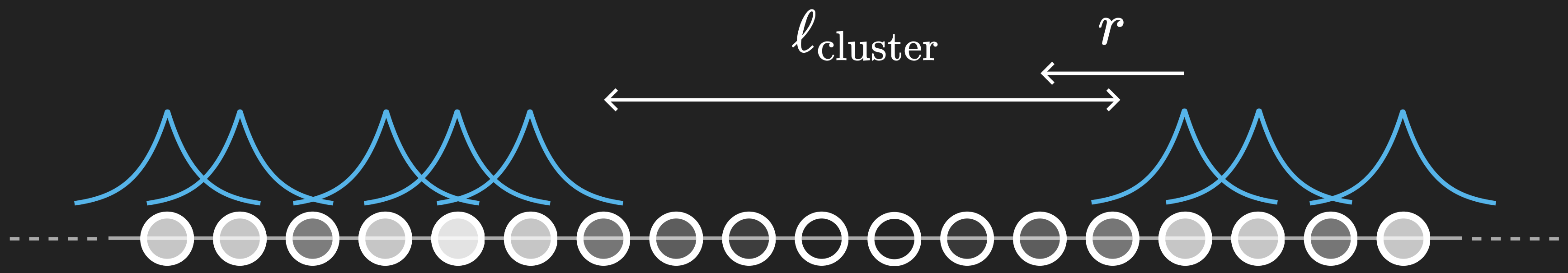

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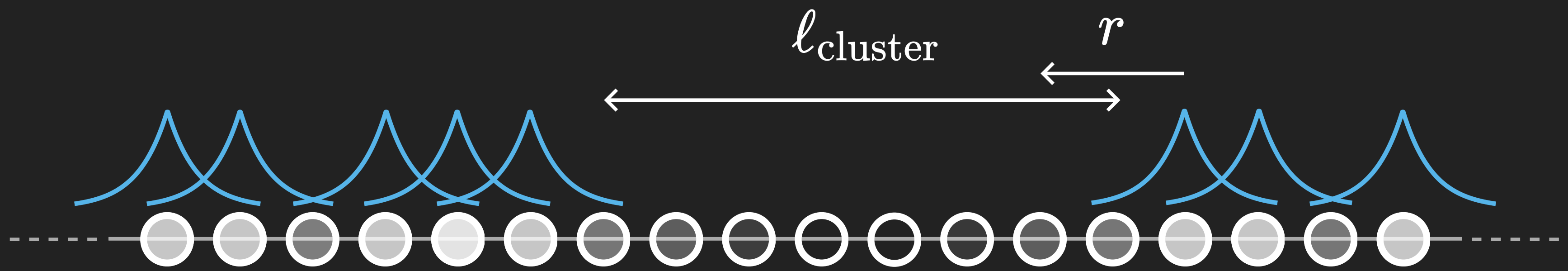

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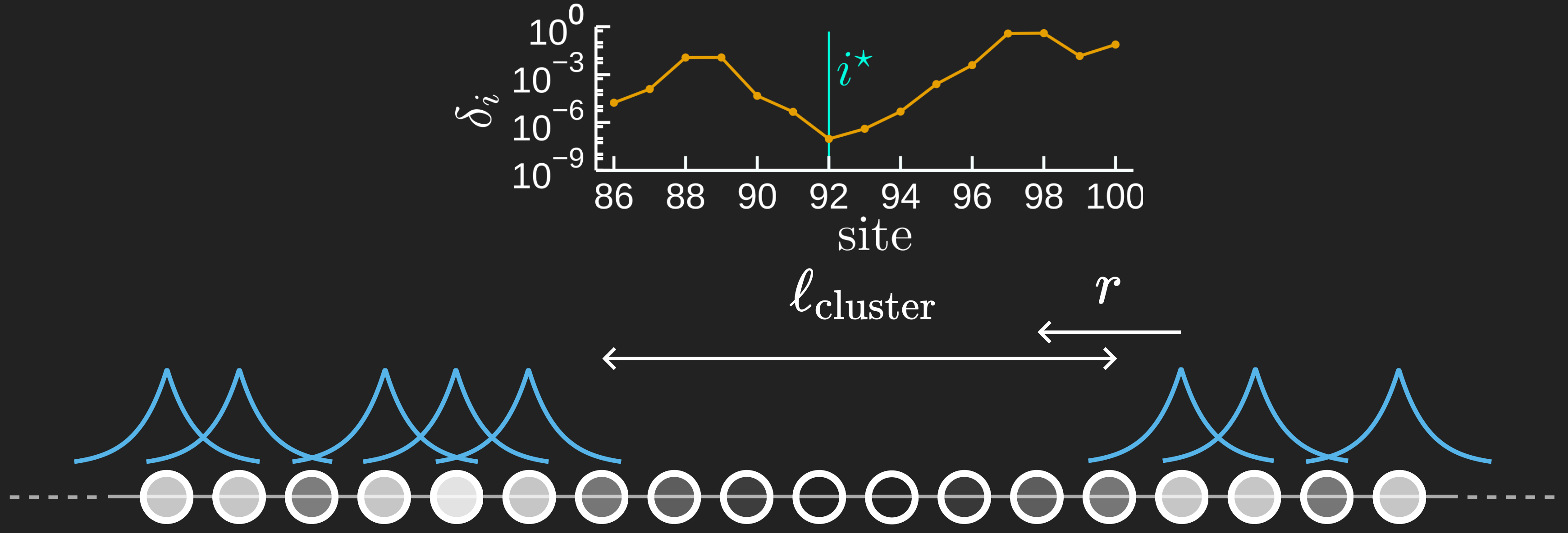

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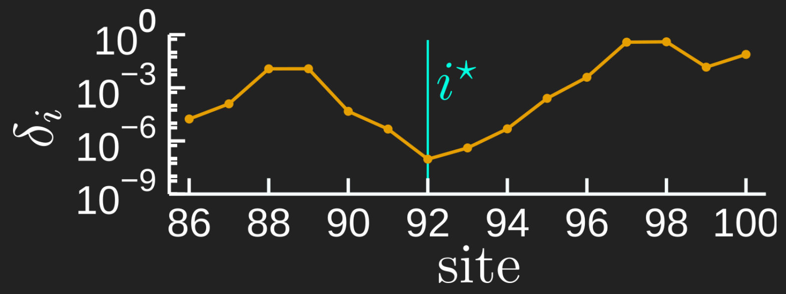

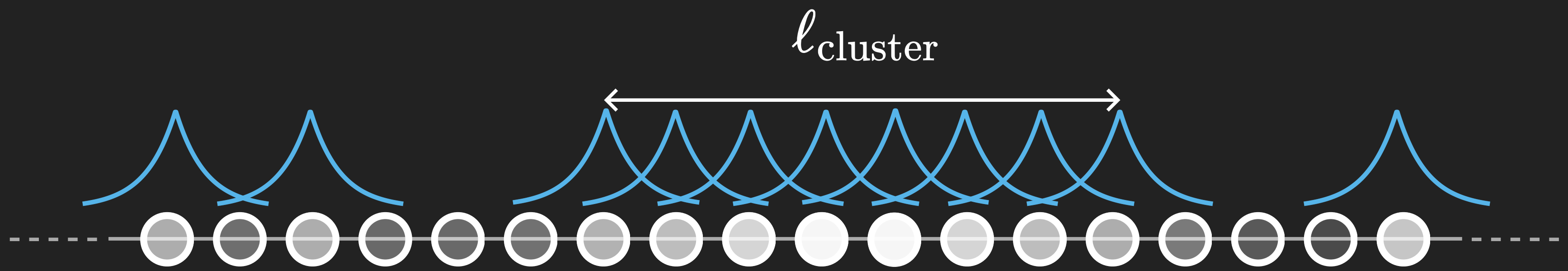

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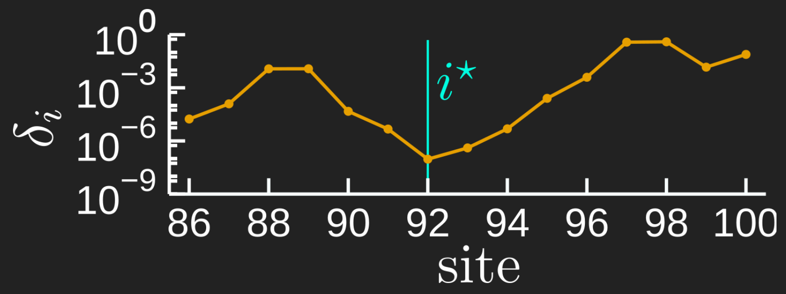

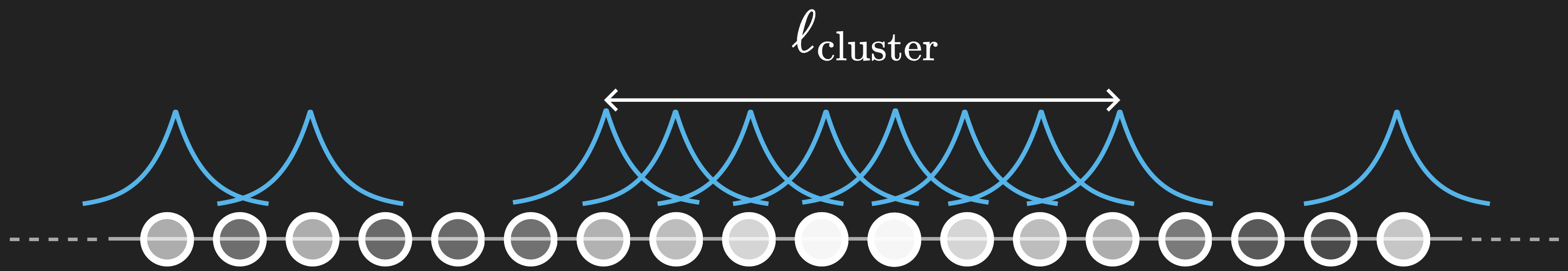

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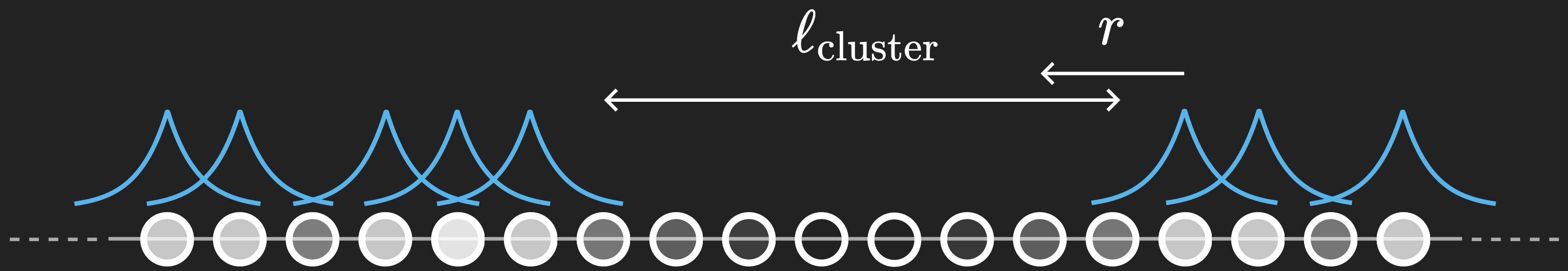
MINIMAL DEVIATION?

$$\delta_{\text{min}}^{\text{typ}} \approx e^{-\overline{\frac{\ell_{\text{cluster}}}{2\xi}}}$$



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Laflorencie, Lemarié, Macé, PRR **2**, 042033(R), (2020)
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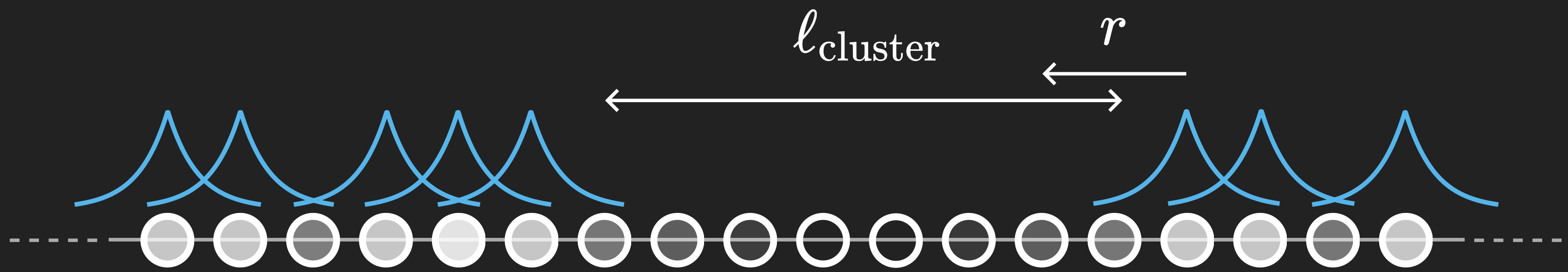
MINIMAL DEVIATION?

$$\delta_{\min}^{\mathrm{typ}} \approx e^{-\frac{\overline{\ell_{\mathrm{cluster}}}}{2\xi}}$$

$$\overline{\ell_{\mathrm{cluster}}} \approx \frac{\ln L}{\ln 2}$$



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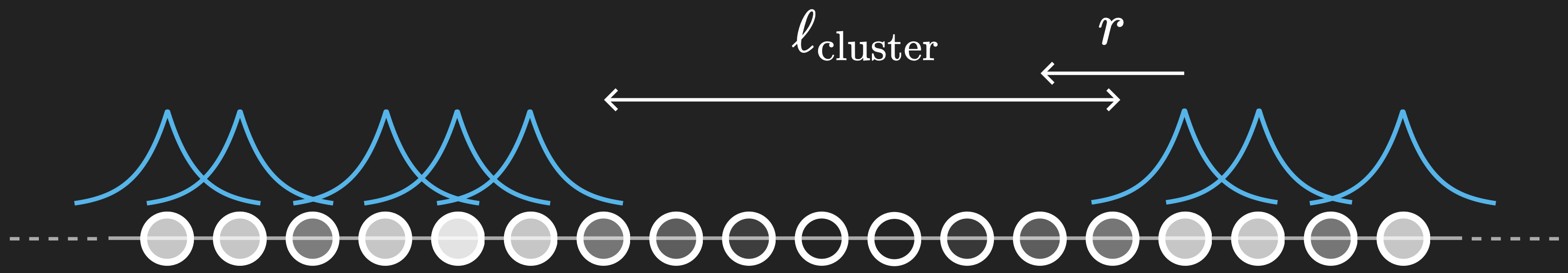
MINIMAL DEVIATION?

$$\delta_{\min}^{\text{typ}} \approx e^{-\frac{\overline{\ell_{\text{cluster}}}}{2\xi}}$$

$$\overline{\ell_{\text{cluster}}} \approx \frac{\ln L}{\ln 2}$$

$$\delta_{\min}^{\text{typ}} \approx L^{-\frac{1}{2\xi \ln 2}}$$



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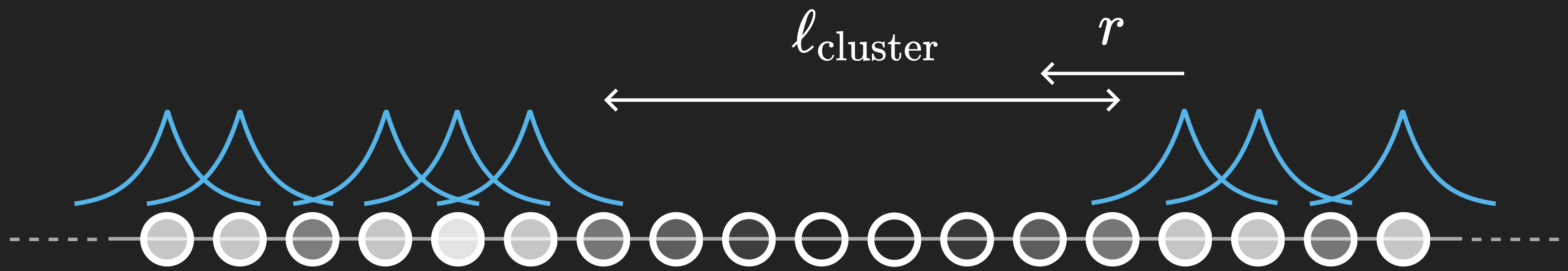

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EXPONENT : TOY MODEL

$$\delta_{\min}^{\text{typ}} \approx L^{-\frac{1}{2\xi \ln 2}}$$

JC, N. Laflorencie, arXiv:2305.10574

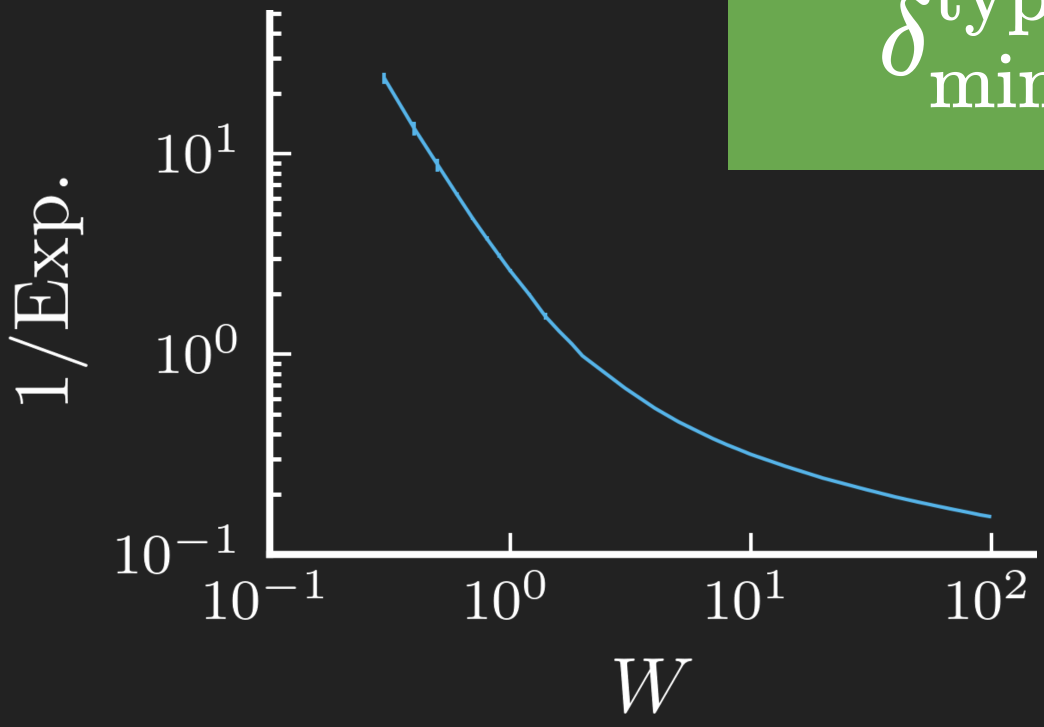
EXPONENT : TOY MODEL

$$\delta_{\min}^{\text{typ}} \approx L^{-\frac{1}{2\xi \ln 2}}$$

$$\xi = \frac{1}{\ln\left(1+\left(\frac{W}{W_0}\right)^2\right)}$$

JC, N. Laflorencie, arXiv:2305.10574

EXPONENT : TOY MODEL



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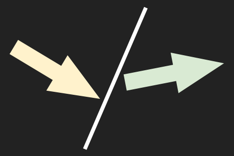
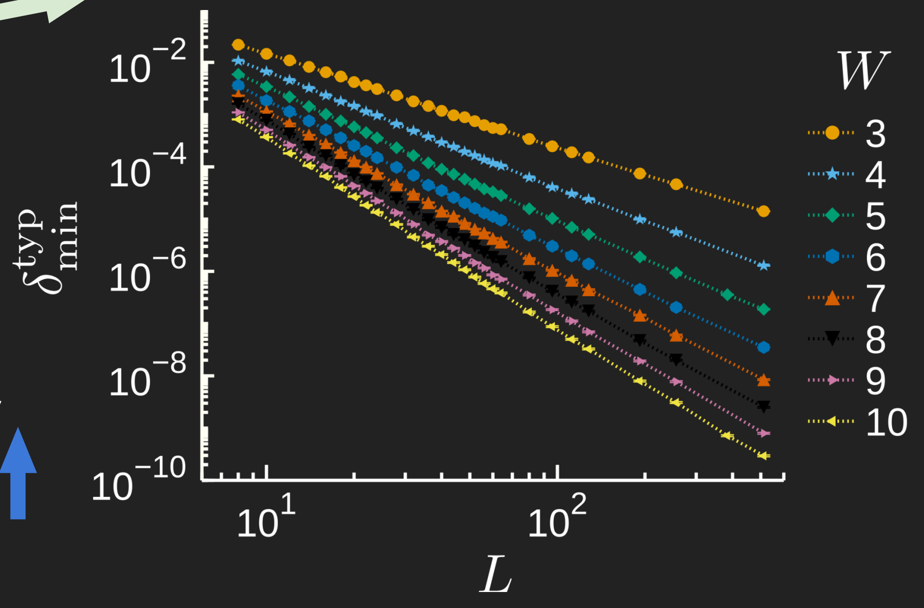
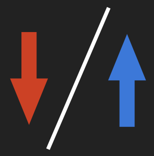
JC, N. Laflorencie, arXiv:2305.10574

CHAIN BREAKING



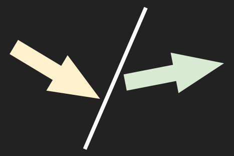


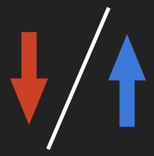


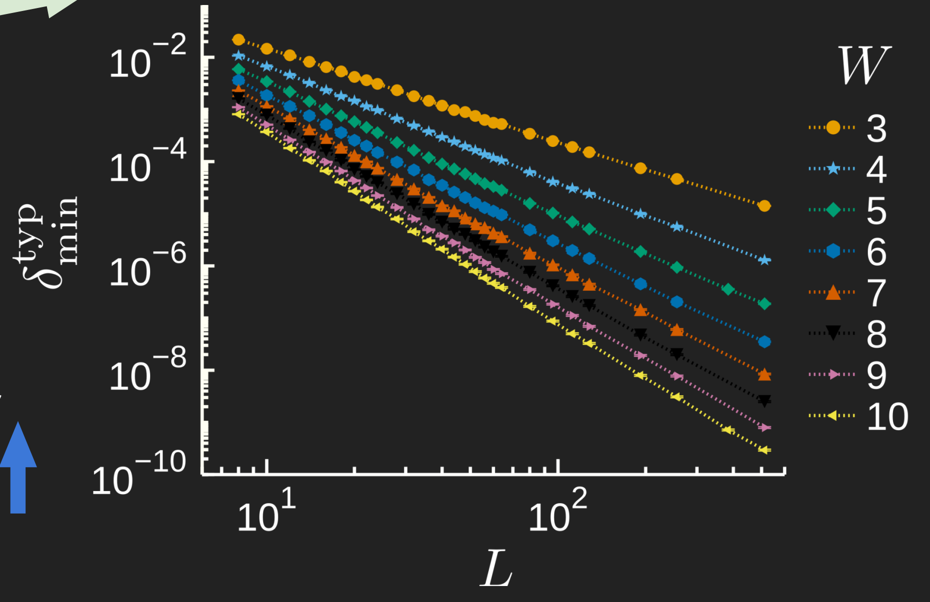
$$\delta_{\min}^{\mathrm{typ}} = e^{\overline{\ln \delta_{\min}}} \approx L^{-\gamma(W)}$$

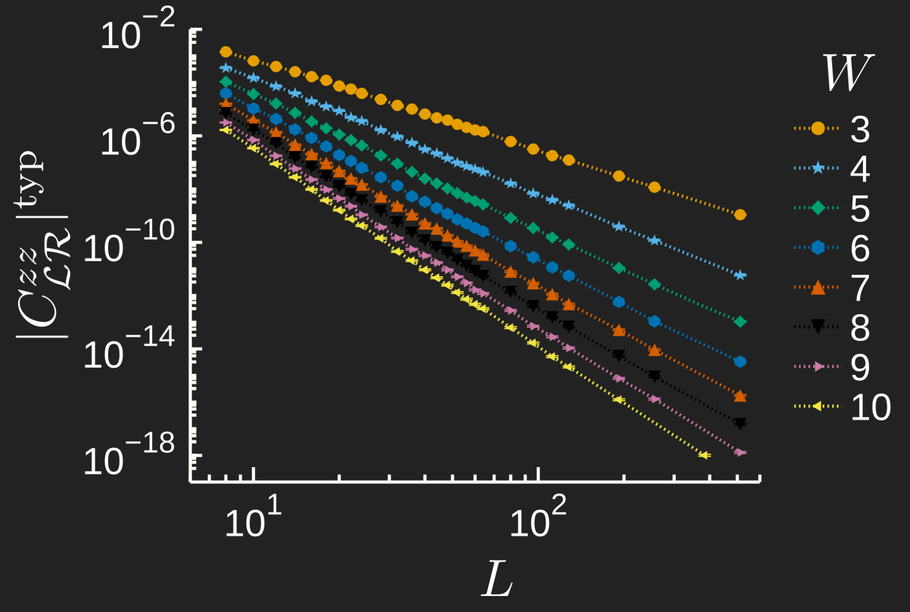
JC, N. Laflorencie, arXiv:2305.10574

CHAIN BREAKING

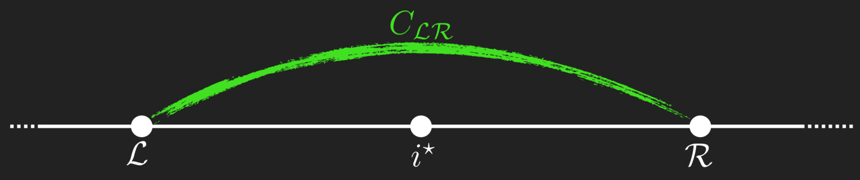






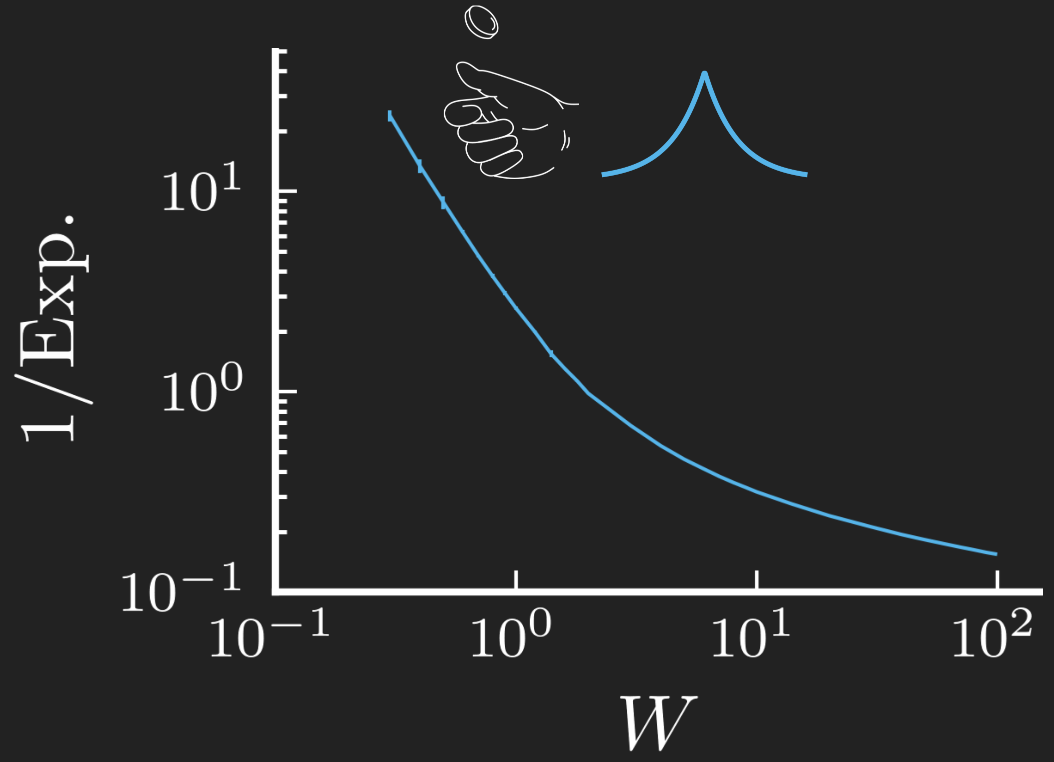


$$\delta_{\min}^{\rm typ} = e^{\overline{\ln \delta_{\min}}} \approx L^{-\gamma(W)}$$



JC, N. Laflorencie, arXiv:2305.10574

EXPONENTS: XX CHAIN



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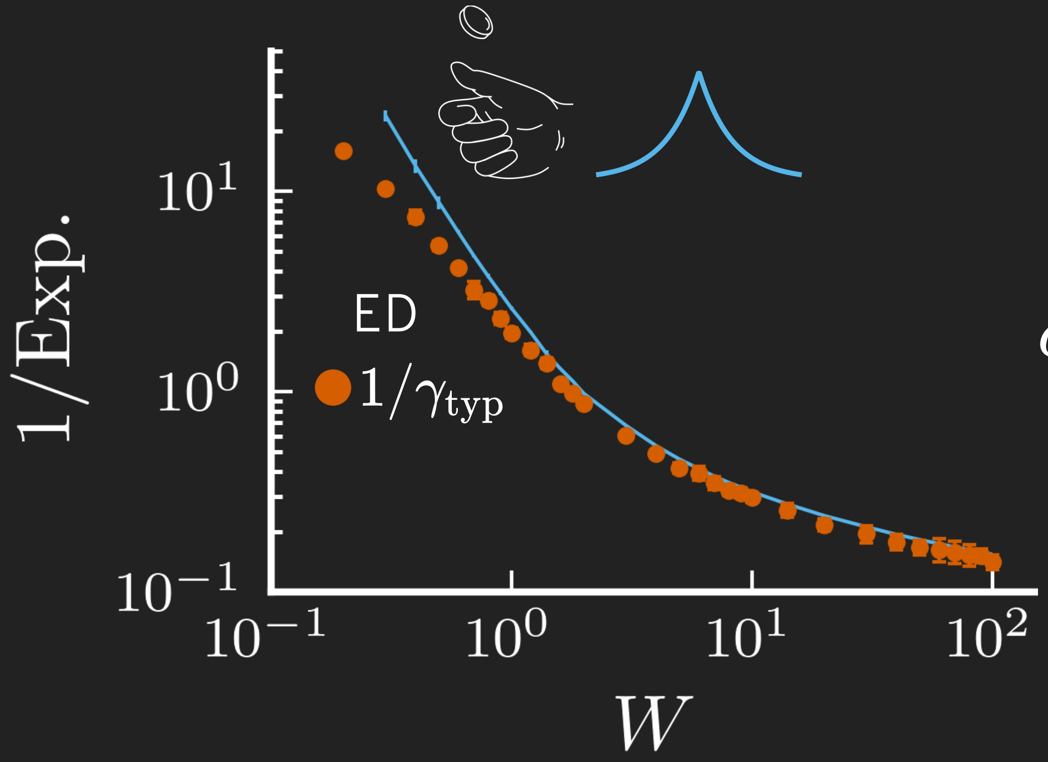
EXPONENTS: XX CHAIN



$$\delta_{\min}^{\text{typ}} \approx L^{-\gamma_{\text{typ}}(W)}$$

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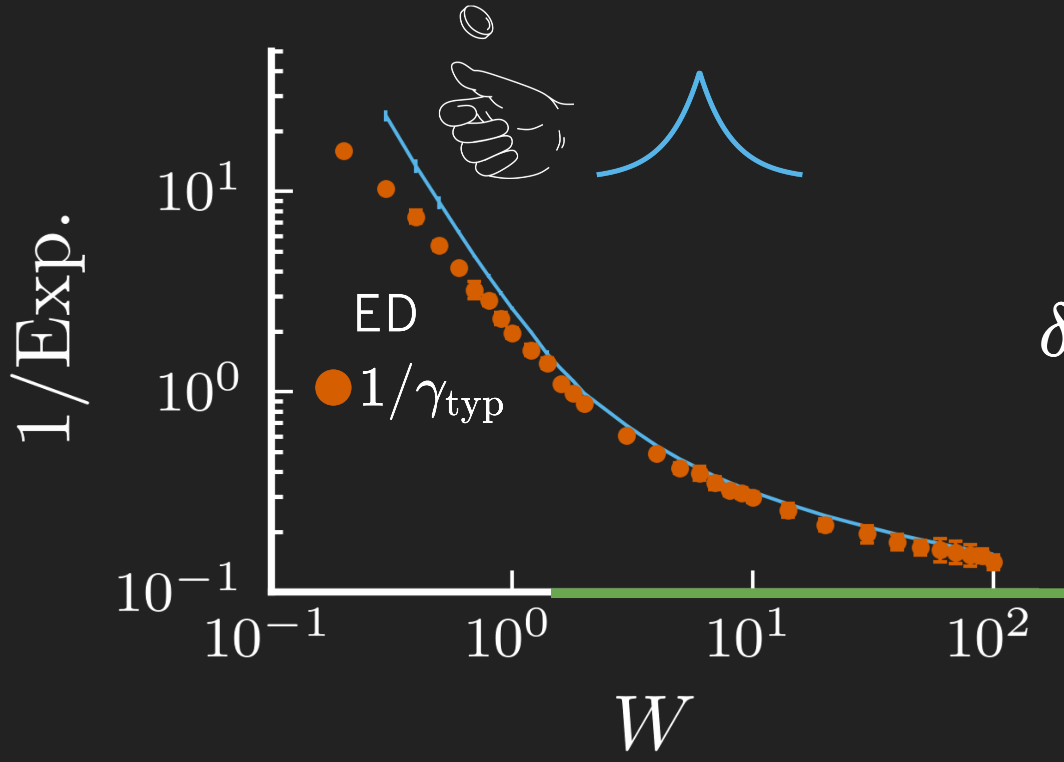
EXPONENTS: XX CHAIN

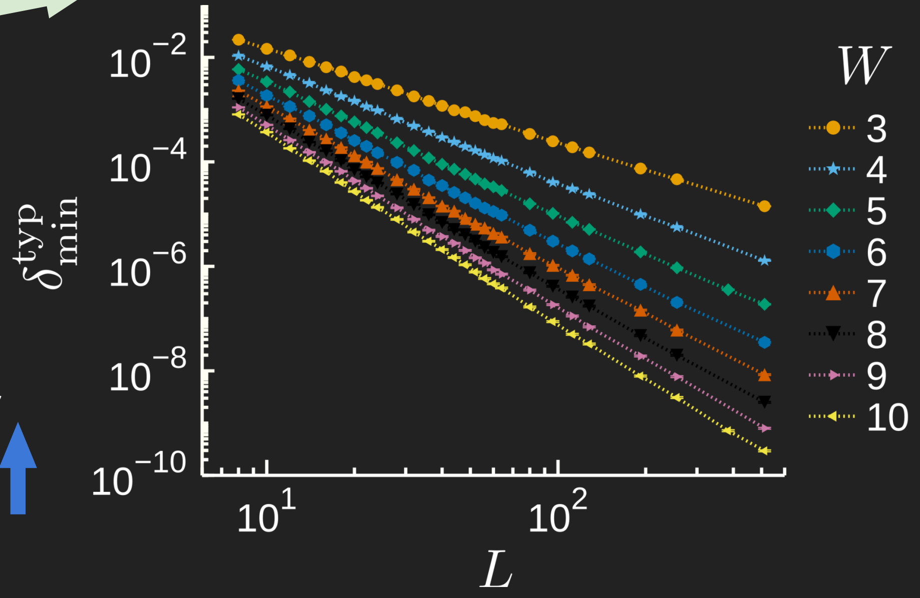


$$\delta_{min}^{typ} \approx L^{-\gamma_{typ}(W)}$$

Excellent agreement ED - Toy model !

JC, N. Laflorencie, arXiv:2305.10574

CHAIN BREAKING



$$\delta_{\rm min}^{\rm typ} \approx e^{-\overline{\frac{\ell_{\rm cluster}}{2\xi}}}$$

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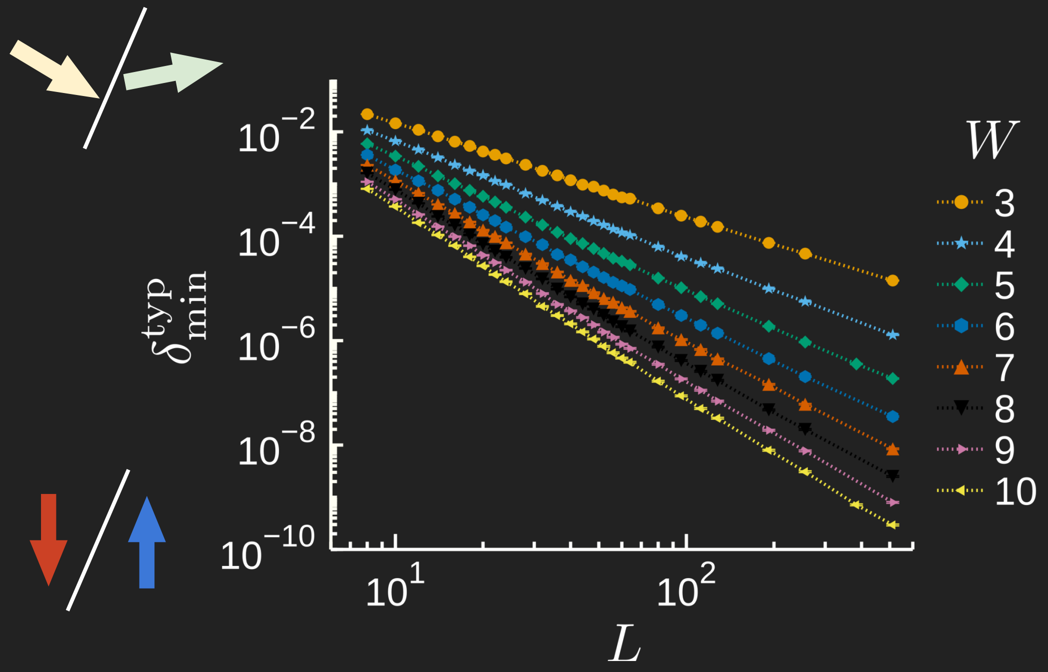
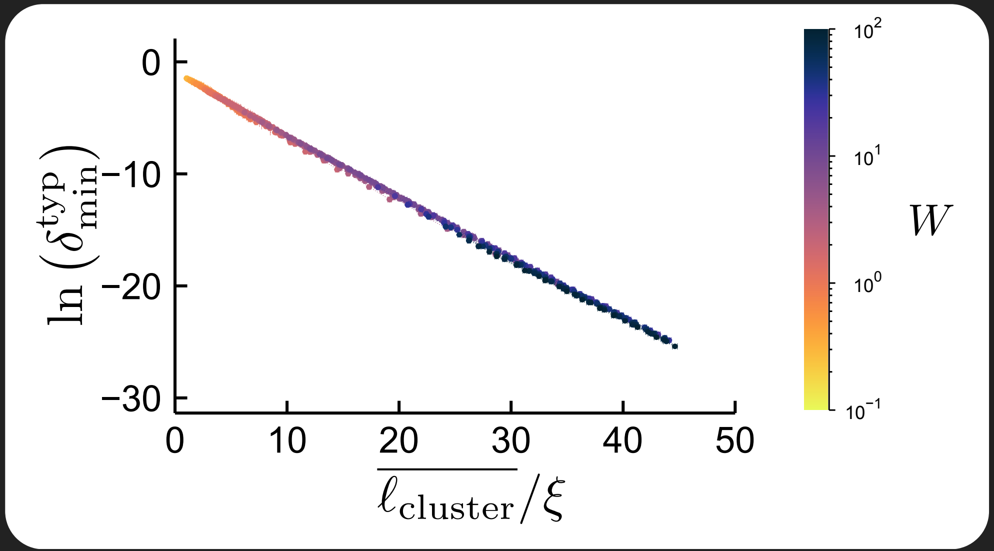
CHAIN BREAKING





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$$L \gg \xi$$

JC, N. Laflorencie, arXiv:2305.10574

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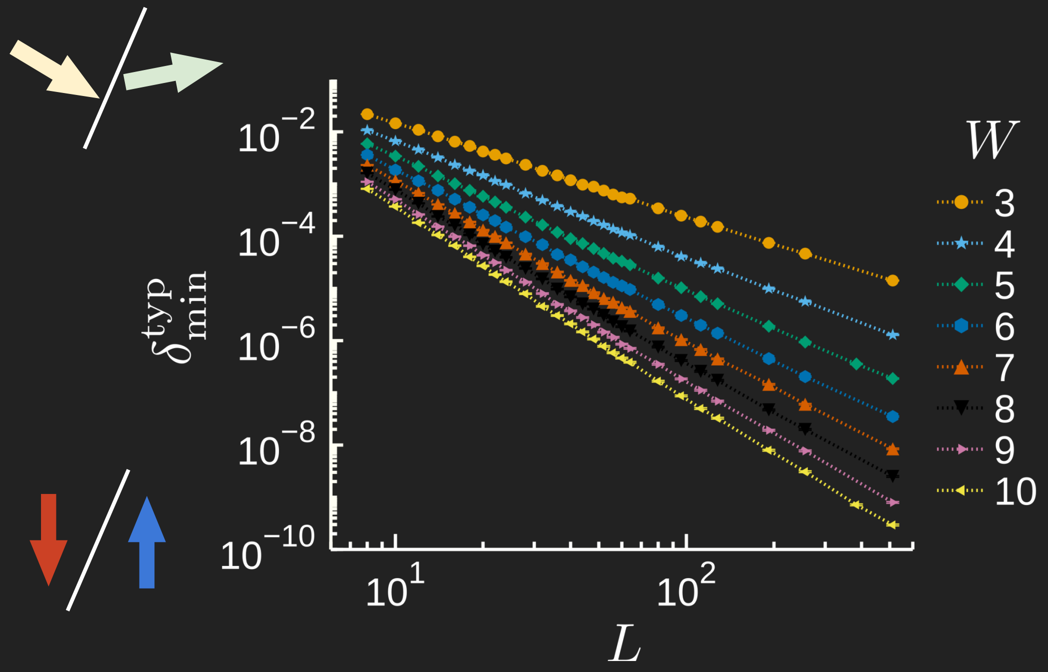

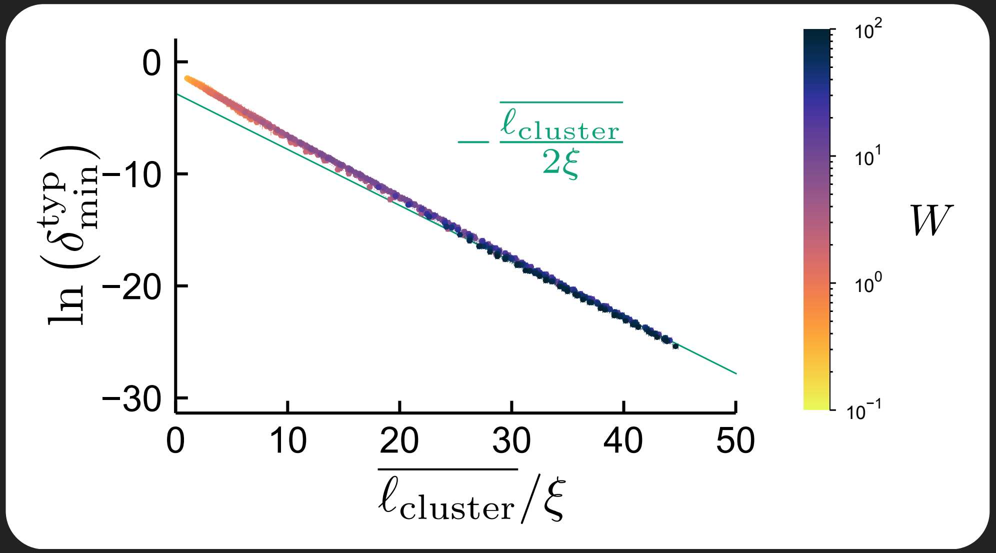

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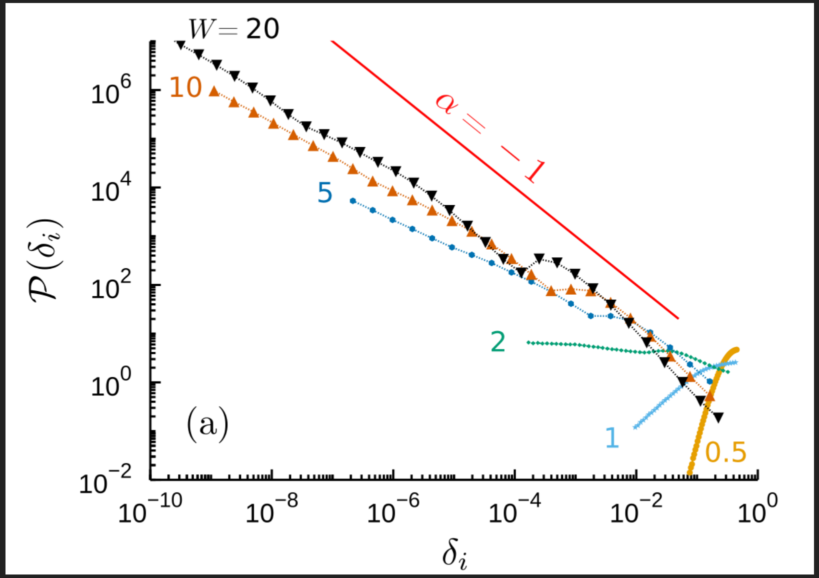

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QUANTITATIVE DESCRIPTION: EXTREME VALUE THEORY

TAILS AND EXTREMES

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TAILS AND EXTREMES

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Power-law tail	Fréchet law
Exponential tail Gaussian tail	Gumbel law

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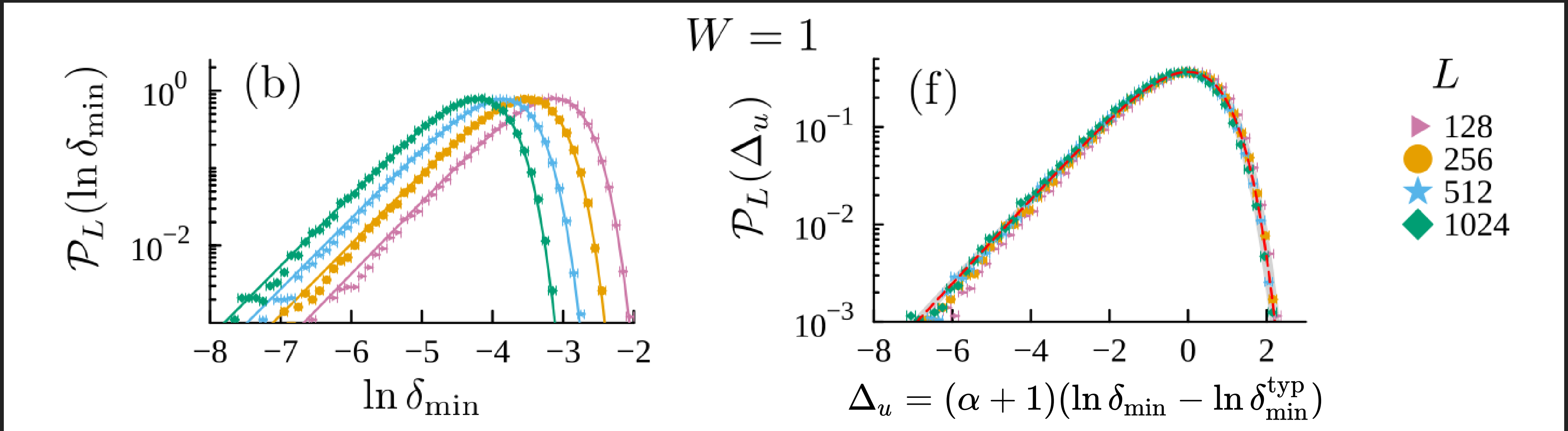

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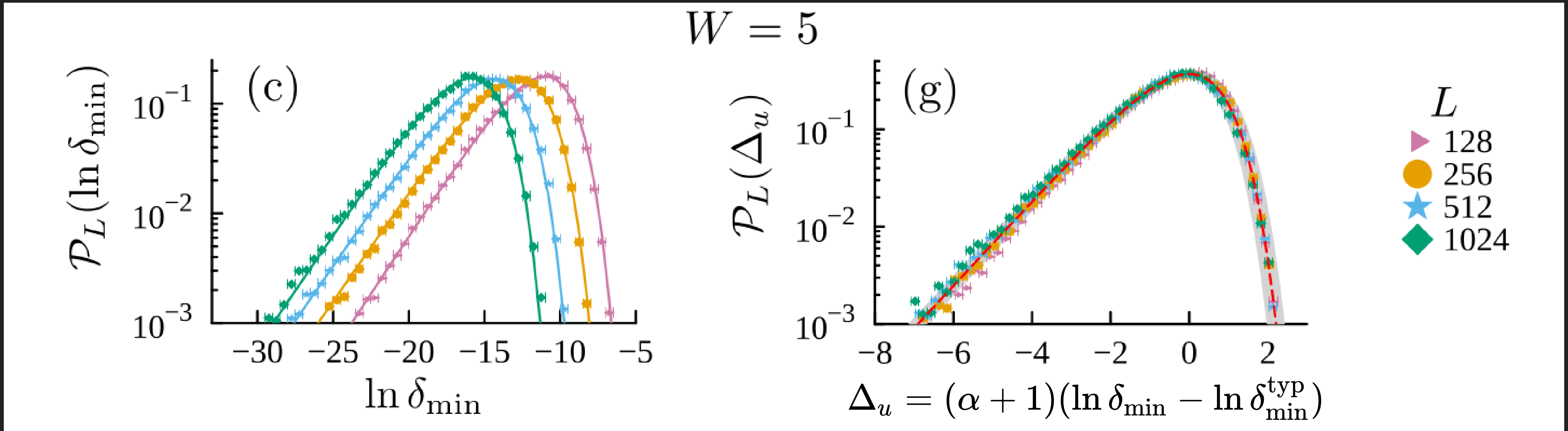

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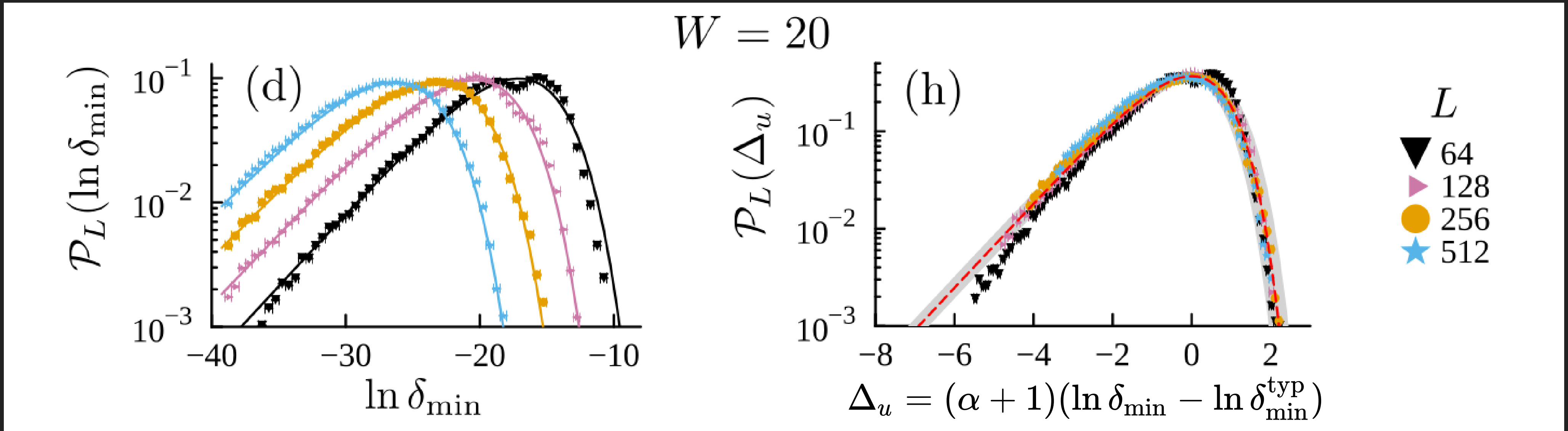

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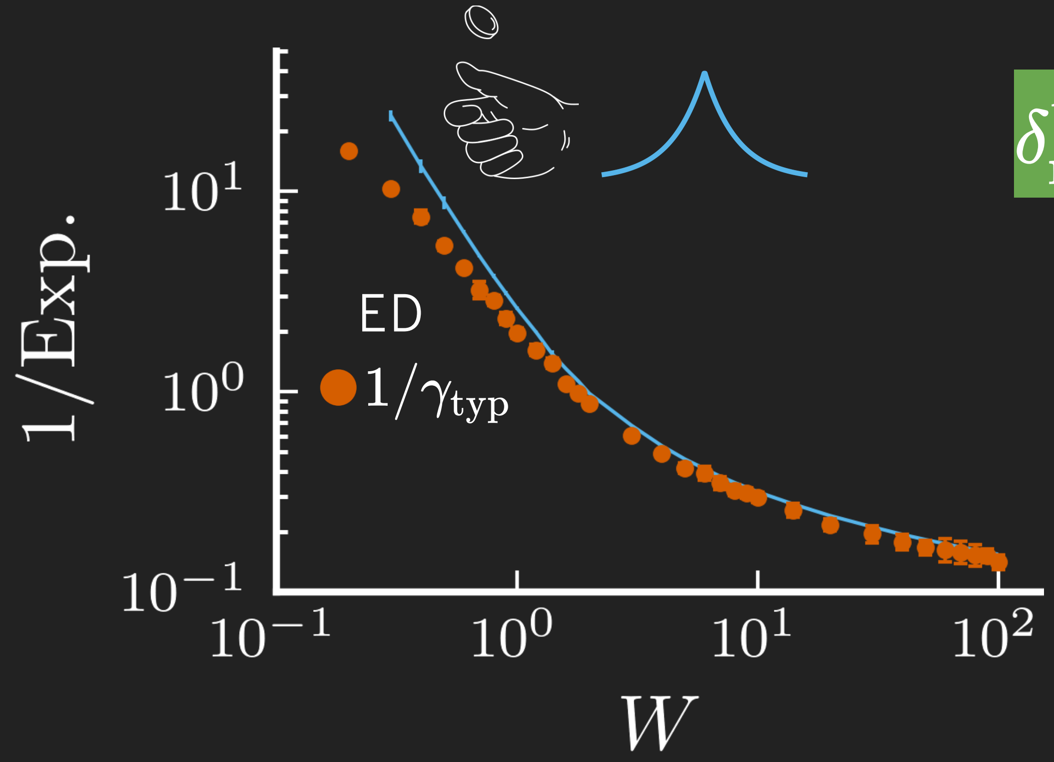

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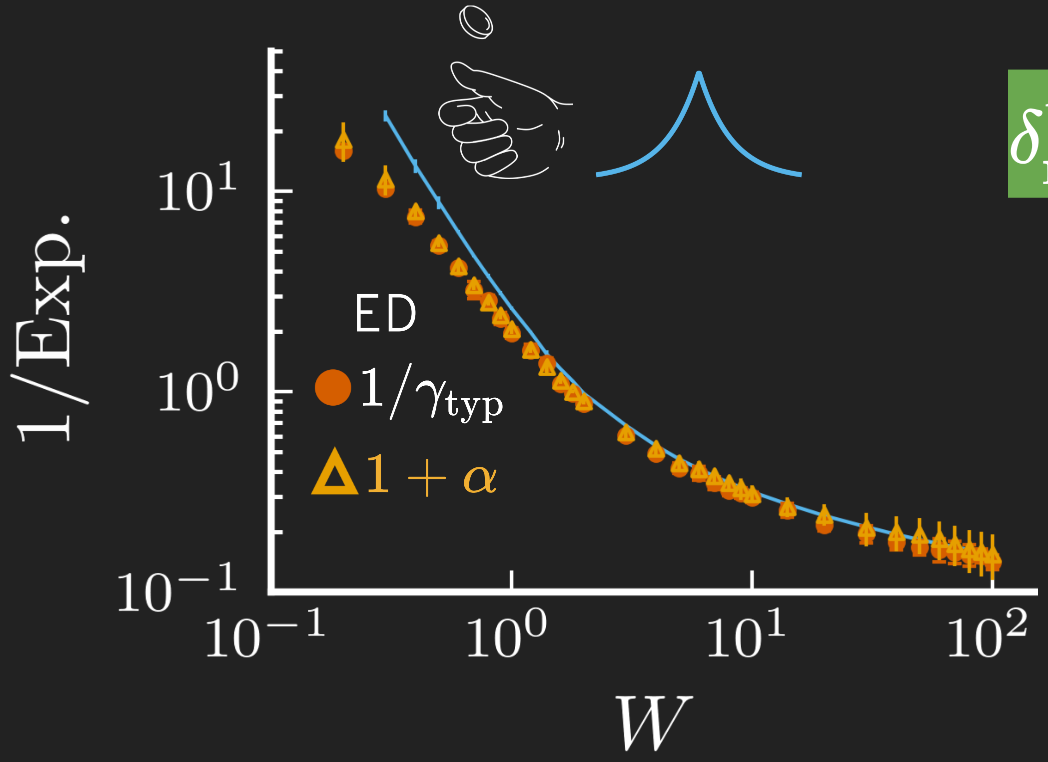

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EFFECT OF INTERACTIONS?

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SciPost Physics 5, 045 (2018)
Laflorencie, Lemarié, Macé, PRR **2**, 042033(R), (2020)
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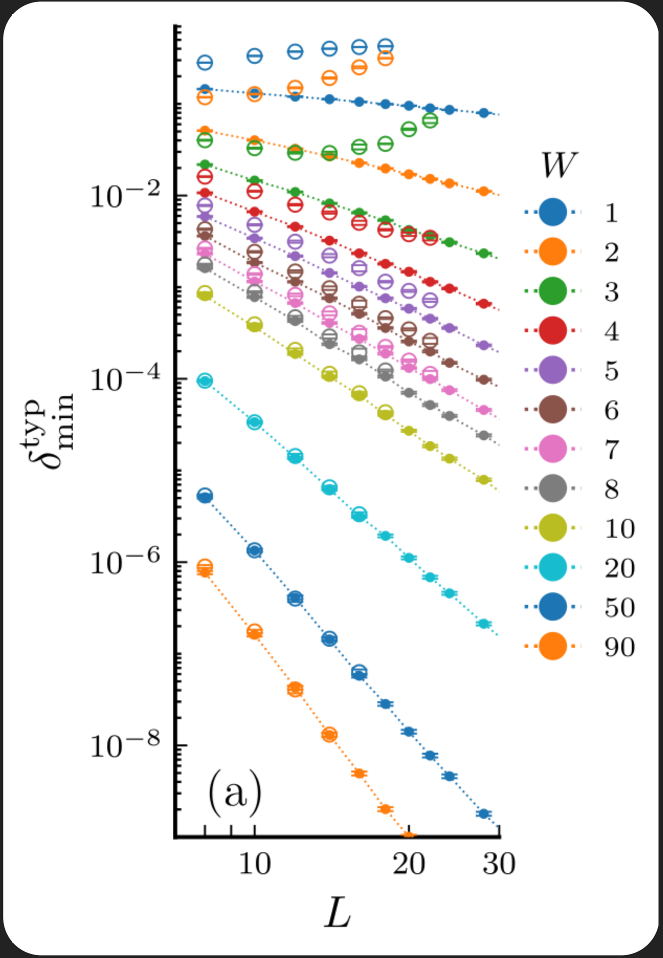
Challenge : high-energy eigenstate, lack of symmetries. Shift-invert ED $\rightarrow L = 22$



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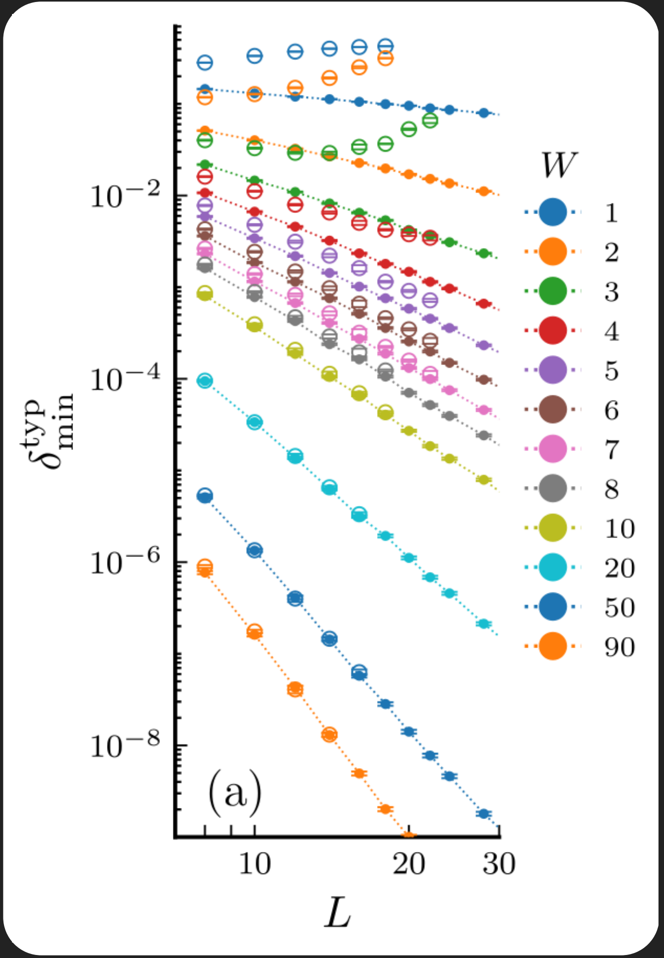
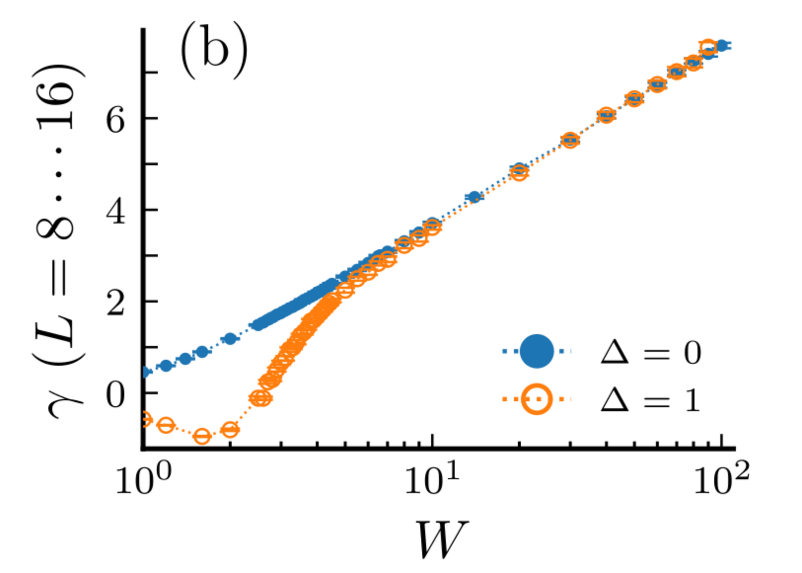


disorder increases



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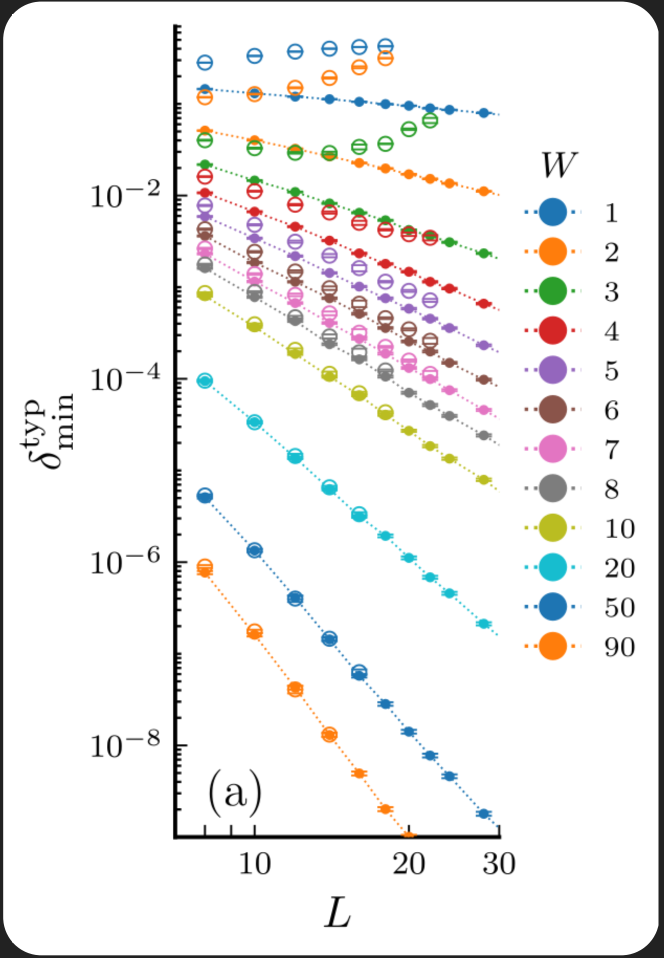

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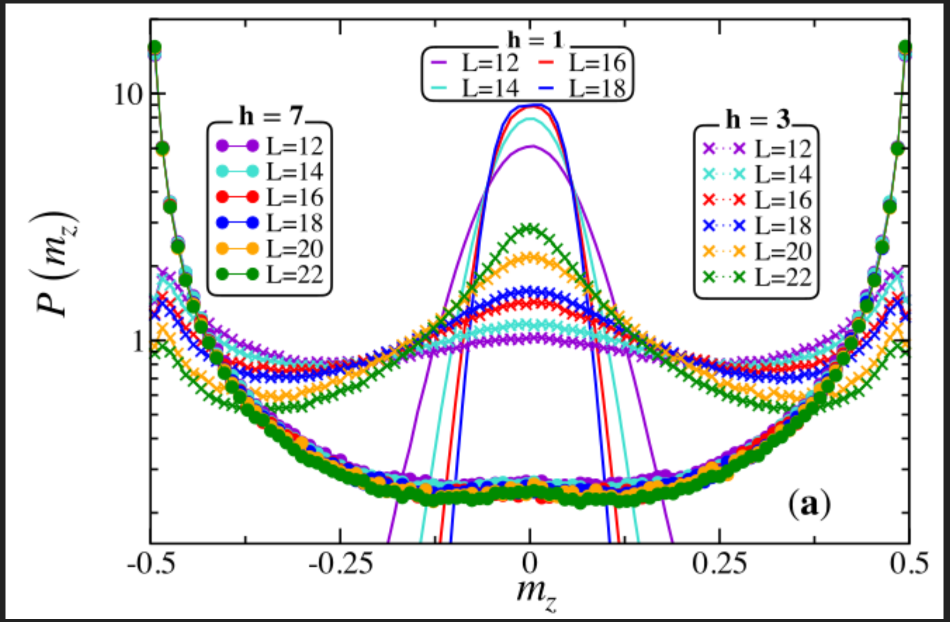

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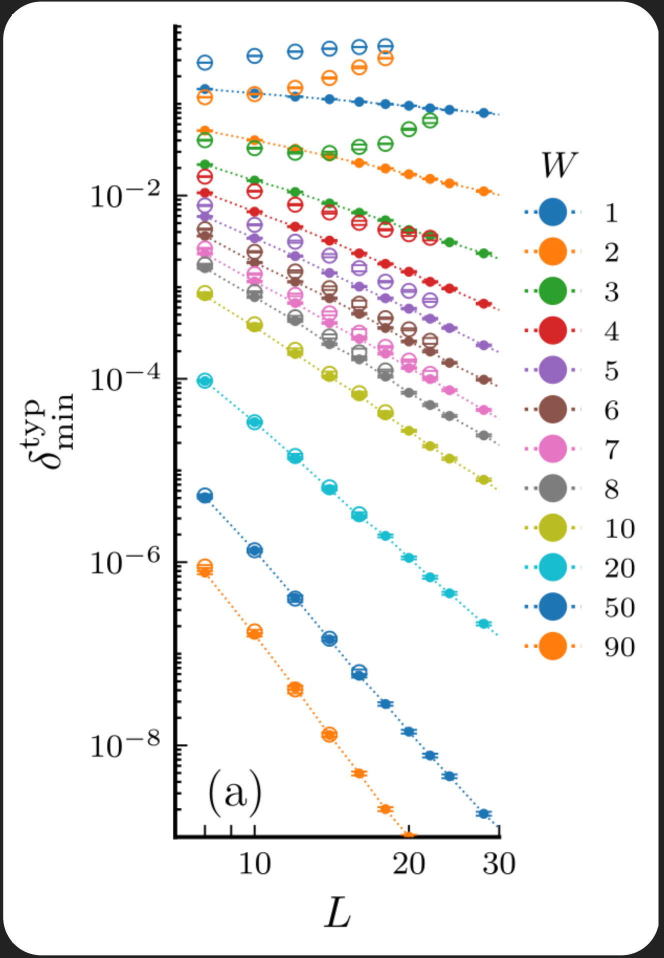

disorder increases

Two limits captured

Can we say more?

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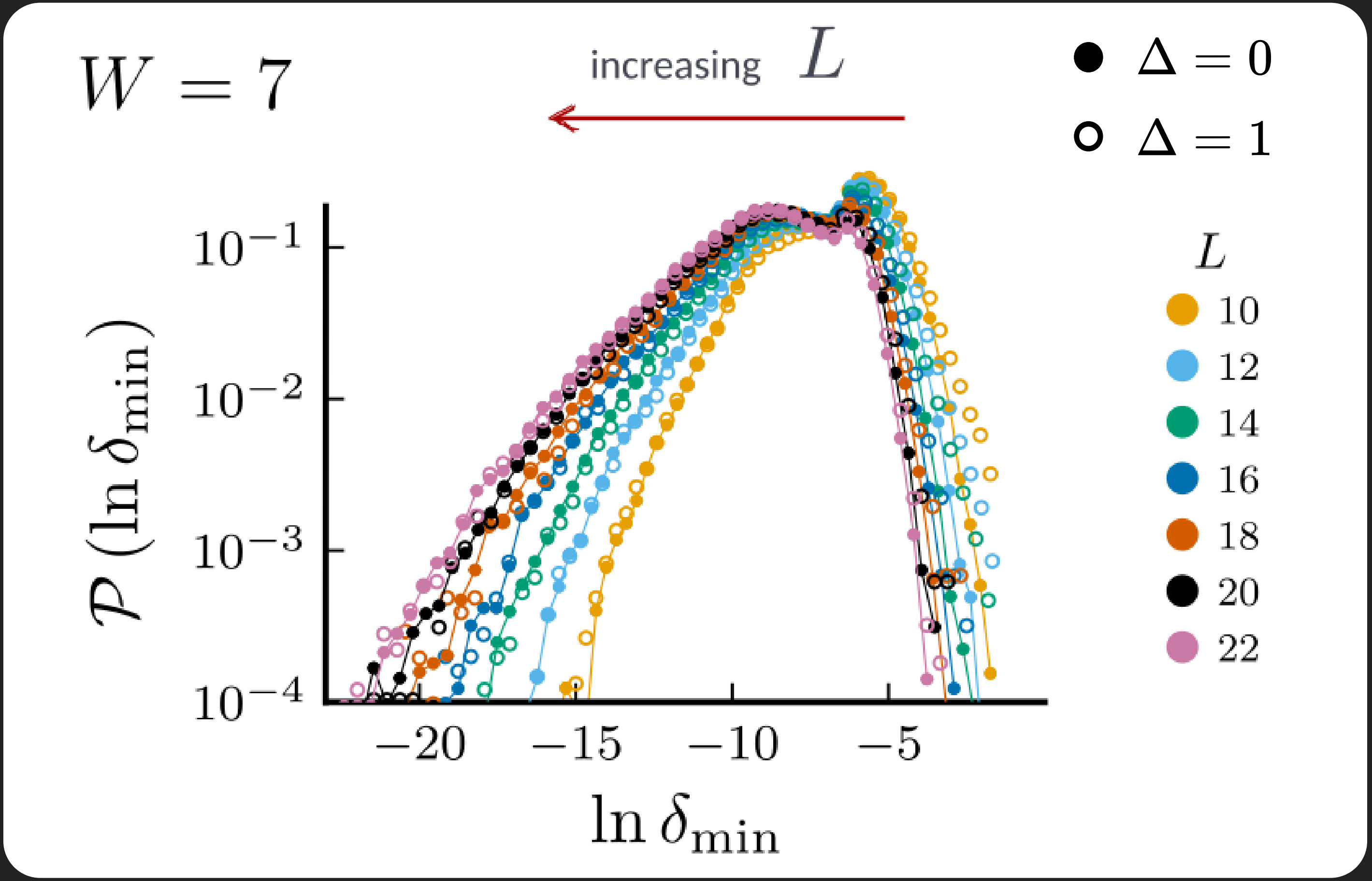

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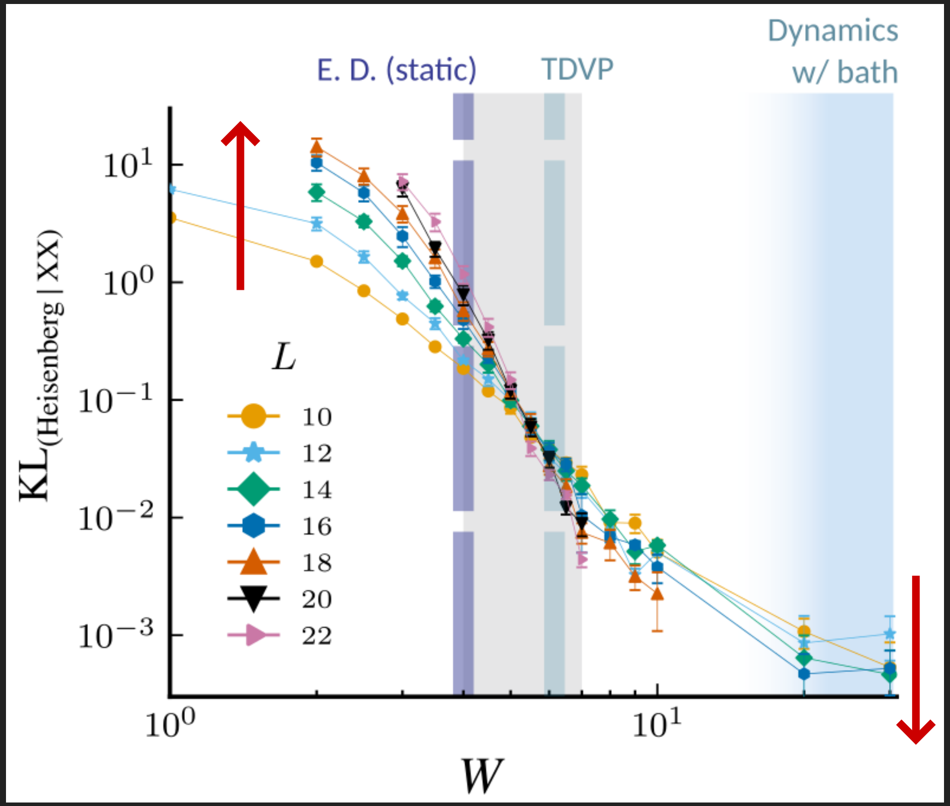

Kullback-Leibler divergence :

$$\mathrm{KL}(p|q) = \sum_i q_i \ln \frac{q_i}{p_i}$$

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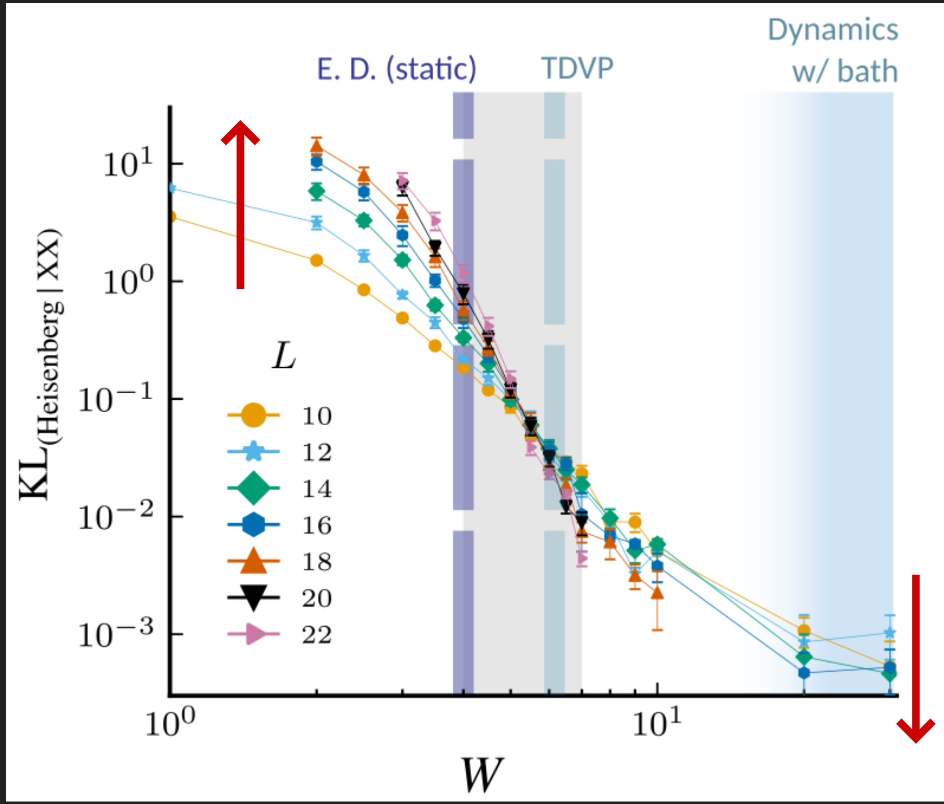

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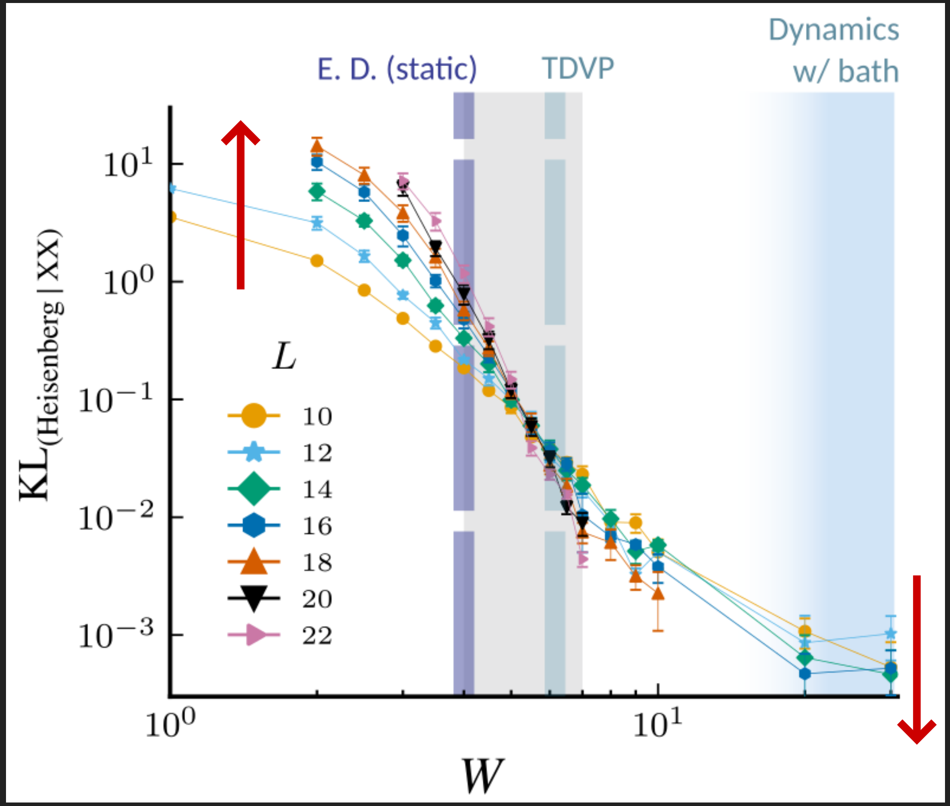

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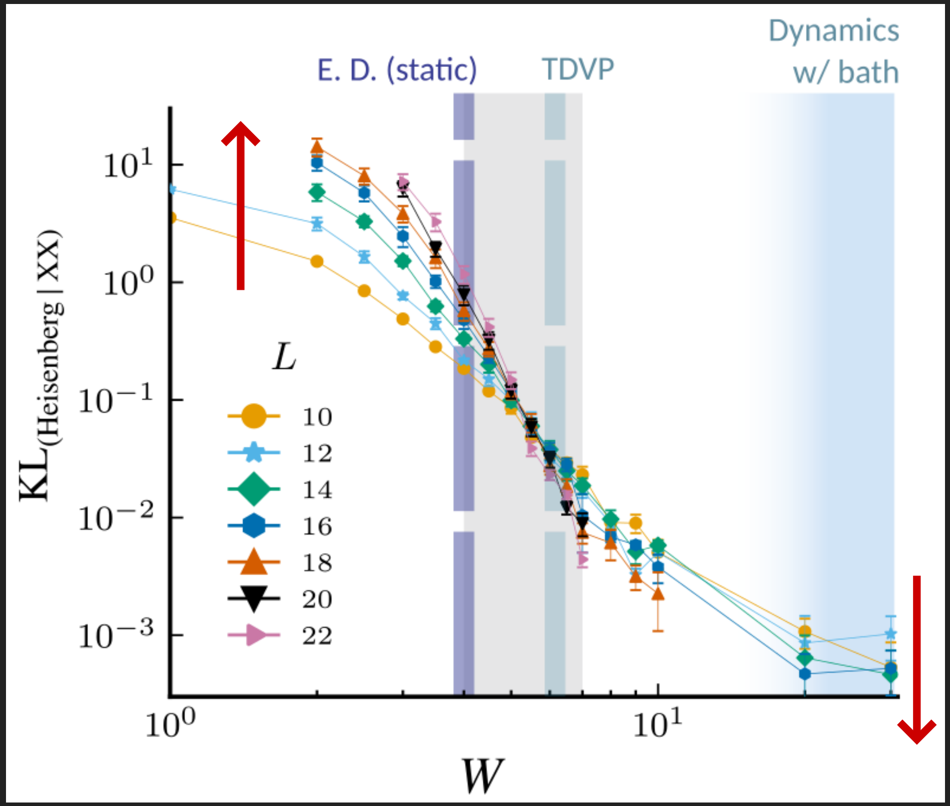

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TAKE HOME MESSAGE

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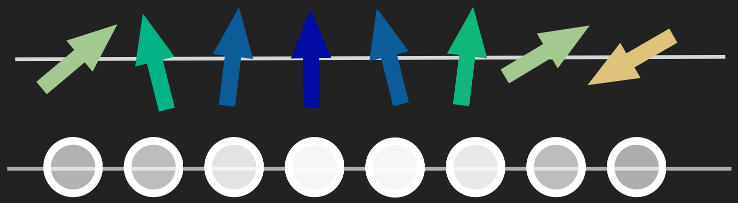
SPIN FREEZING!



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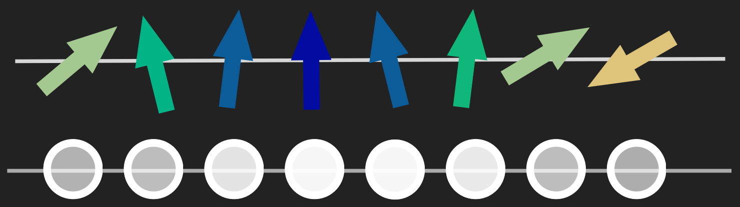
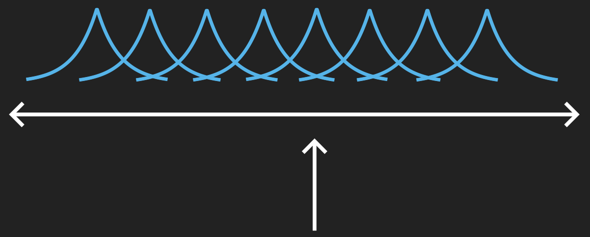


XX chain :

- controlled by largest cluster of occupied orbitals



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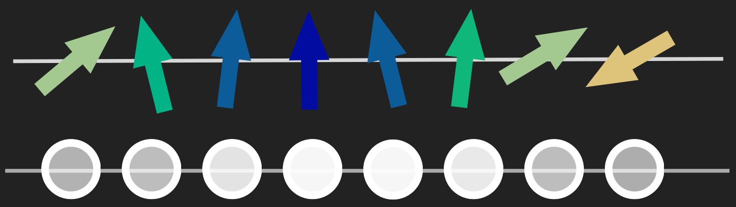

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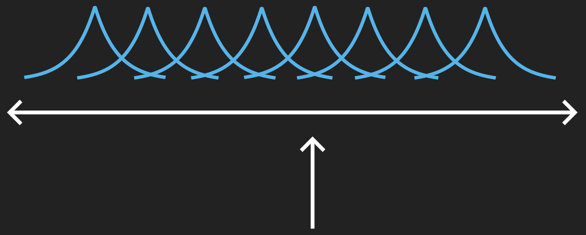

- Excellent fits & collapses with a Fréchet Law

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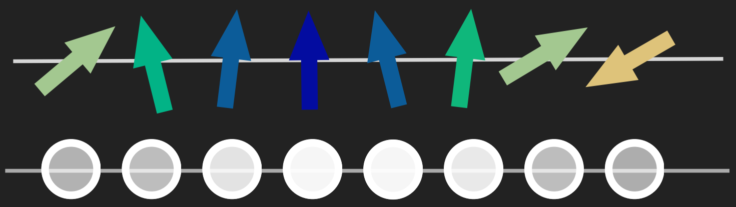

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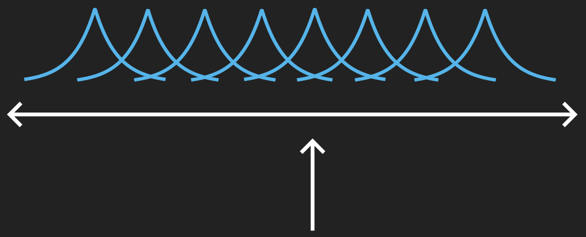

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Heisenberg chain at strong disorder:

Chain breaks!

→ W

No chain breaks | Chain breaks

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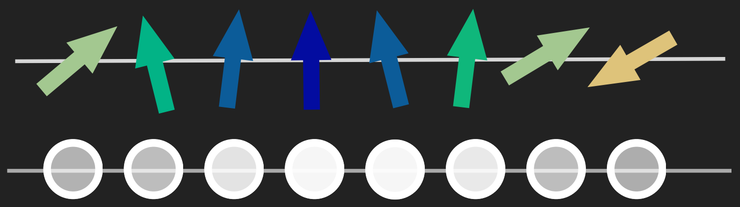

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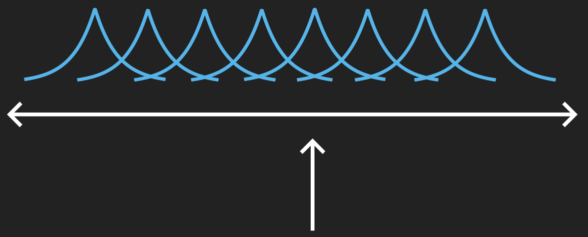

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Extreme value transition characterized by the **KL divergence.**

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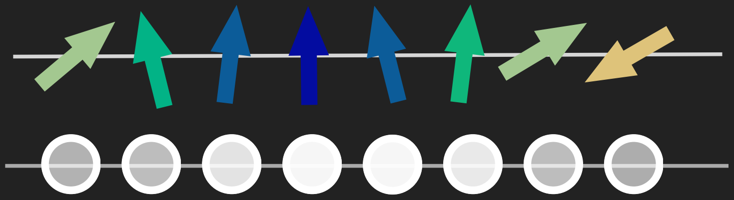

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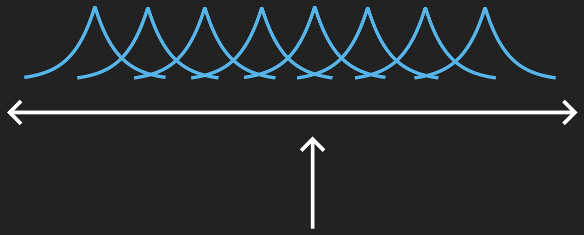

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