Chiral spin liquids from bosonic iPEPS to fermionic iPEPS

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Laboratoire de Physique Théorique, Toulouse

Entanglement in Strongly Correlated Systems, Benasque, 2023/8/17

SN, Juraj Hasik, Ji-Yao Chen, Didier Poilblanc, Phys. Rev. B 106, 245119 (2022).
SN, Jheng-Wei Li, Ji-Yao Chen and Didier Poilblanc, arXiv: 2306.10457.

Acknowledgement: Hong-Hao Tu

Outline

- Introduction
  - Chiral spin liquids
  - iPEPS
- Chiral spin liquids with bosonic iPEPS on the kagome lattice
- Chiral spin liquids with projected Gaussian fermionic iPEPS

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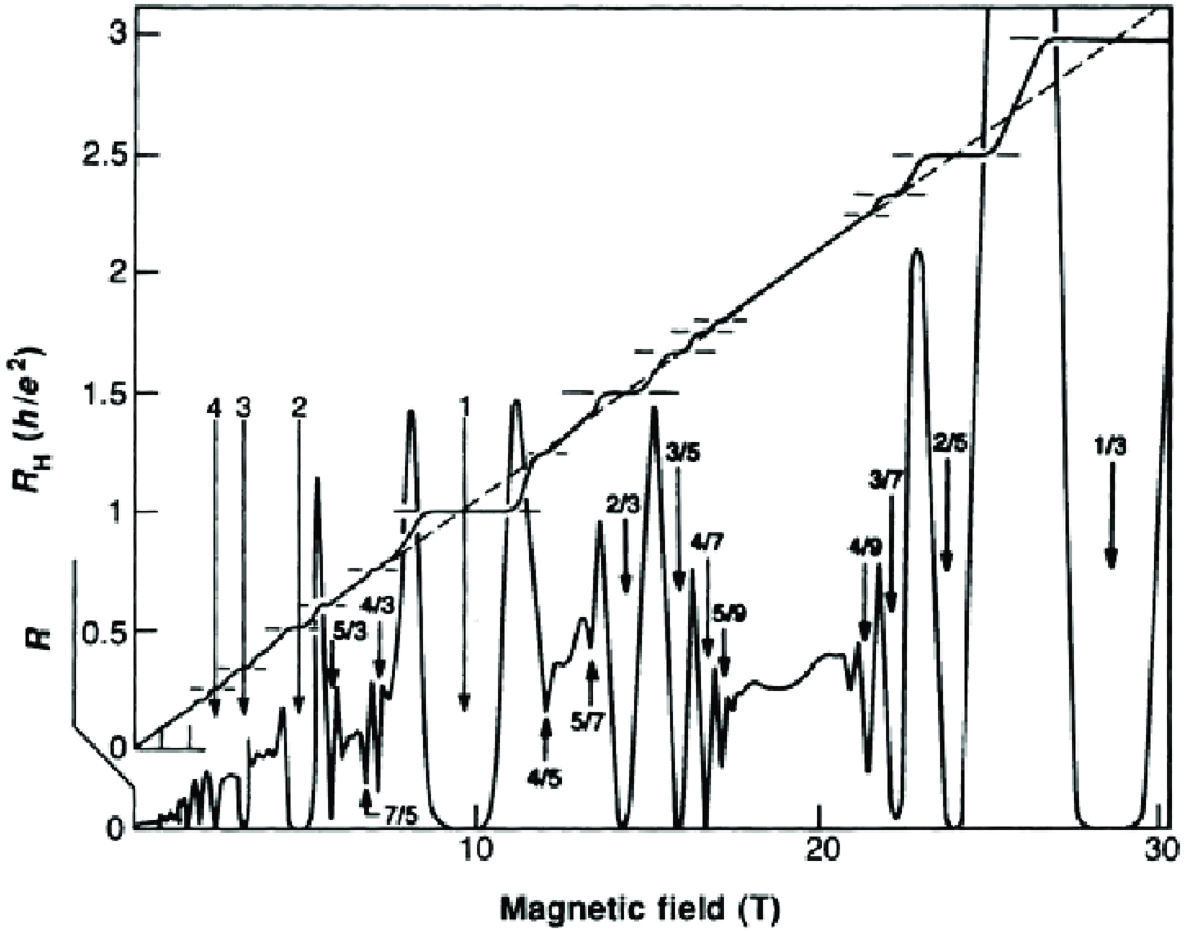
Introduction

Fractional quantum Hall states

Quantum matter beyond Landau-Ginzburg paradigm

Discovery: D. C. Tsui, H. L. Stormer, & A. C. Gossard, PRL (1982)

First theory: R. B. Laughlin PRL (1983)



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Topological order

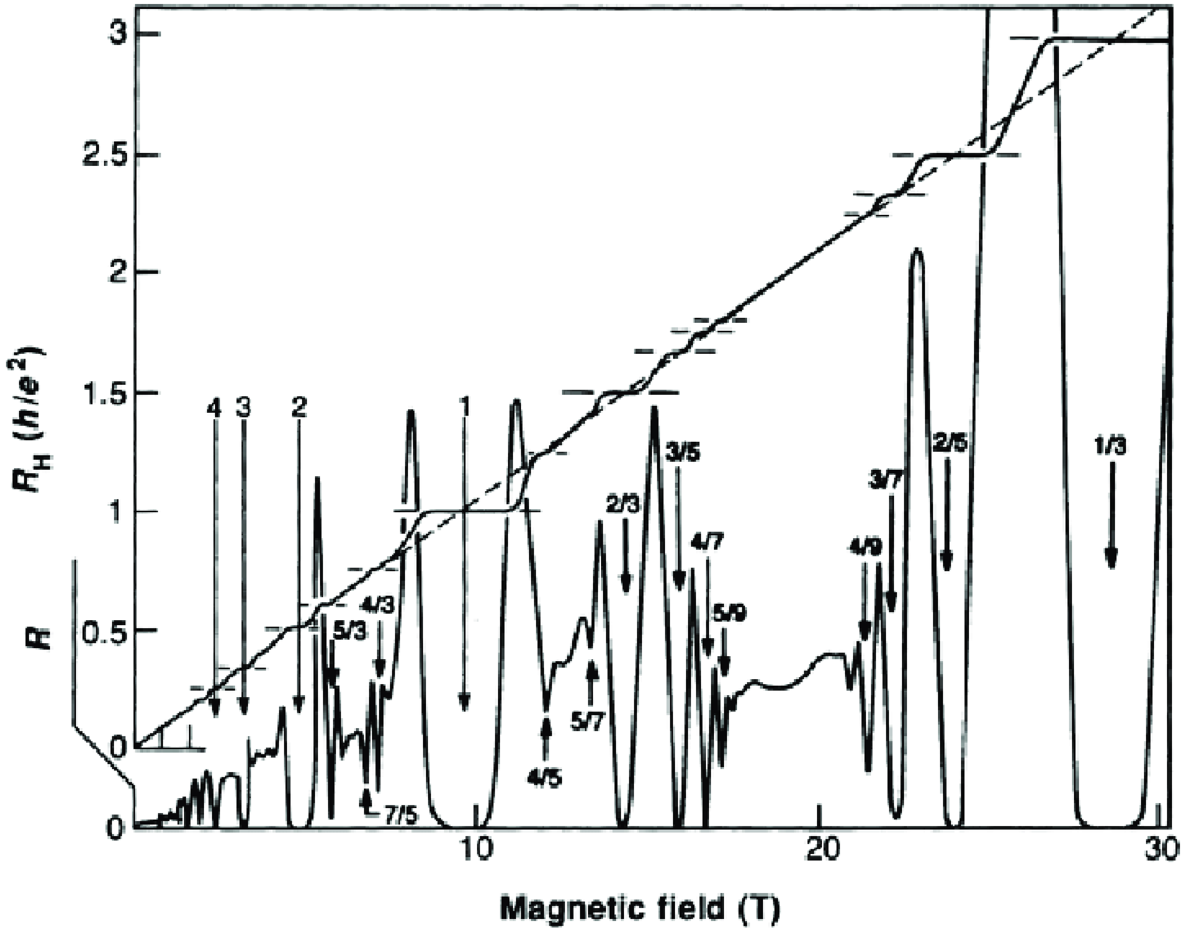
- Fractionally charged quasiparticles

Goldman, Su, Science (1995)
Saminadayar, Glattli, Jin, and Etienne, PRL (1997)
de-Picciotto et al, Nature (1997)
Martin et al, Science (2004)

- Anyonic exchange statistics

J. Nakamura et al, Nat. Phys. (2020)

Nobel Prize 1998
Laughlin, Störmer, Tsui



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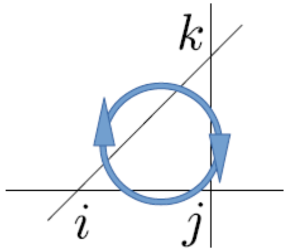
Introduction

Chiral spin liquids (CSL): bosonic variant of FQH states

- Violate P, T but preserves PT: $\langle s_i \cdot (s_j \times s_k) \rangle \neq 0$



Kalmeyer, Laughlin, PRL (1987); Wen, Wilczek, Zee, PRL (1989)

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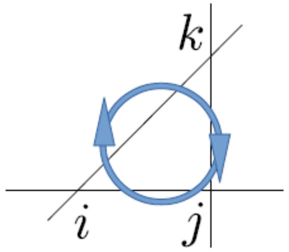
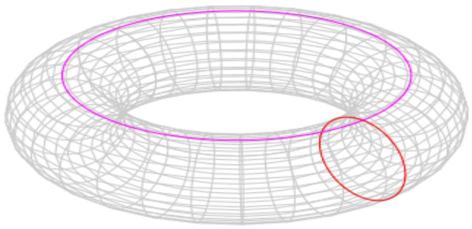


- Ground state degeneracy on torus



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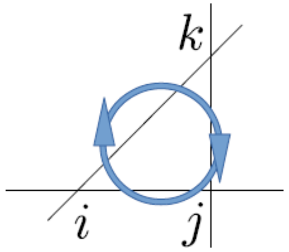

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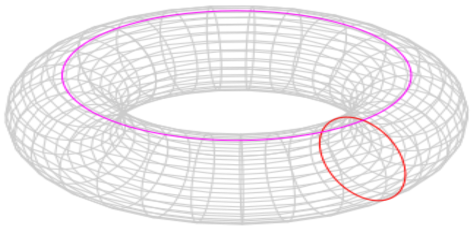
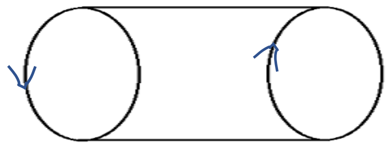


- Bulk-boundary correspondence (chiral edge states)



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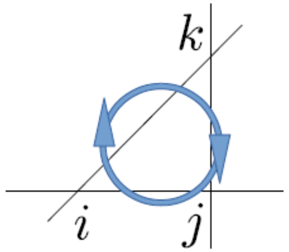

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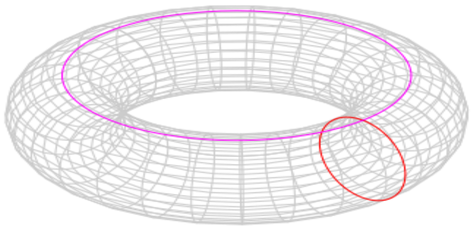

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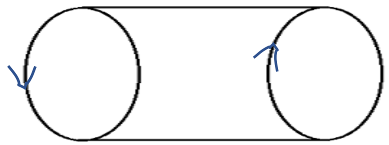

- Li-Haldane conjecture (entanglement spectrum)

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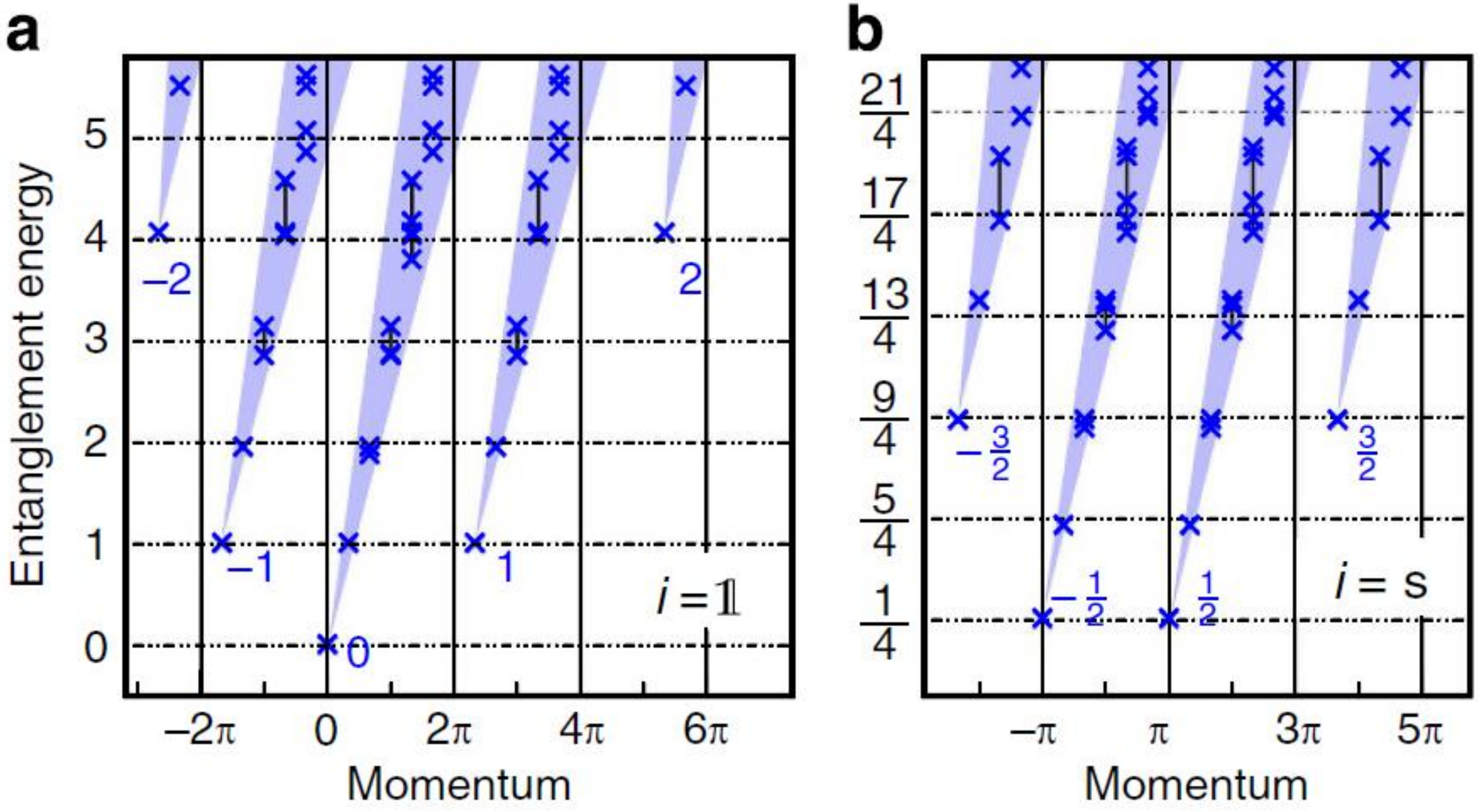

Bauer et al, Nat. comm. (2014)

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Chiral spin liquids (CSL): bosonic variant of FQH states

A typical example: $\mathrm{SU}(2)_1$ CSL

Kalmeyer-Laughlin wave function

$$\left[\prod_{N\geqslant i>j\geqslant 1}(z_i - z_j)^n\right]\prod_{k=1}^{N}\exp(-\mid z_k\mid^2)$$

$$\mathrm{n} = 2$$

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Parton wave function

$$\psi_{spin}(\{x_i\}) = P_G[\psi_\uparrow(\{x_i\}) \otimes \psi_\downarrow(\{x_i\})]$$

$\psi_s(x_i)$: free fermion state with $C = 1$

Gutzwiller projector $P_G = \prod_i (n_{i,\uparrow} - n_{i,\downarrow})^2$

Introduction

Spin models for Chiral Spin Liquids

Parent Hamiltonian construction

D. F. Schroeter, E. Kapit, R. Thomale, and M. Greiter, PRL (2007)

R. Thomale, E. Kapit, D. F. Schroeter, and M. Greiter, PRB (2009)

A. E. B. Nielsen, G. Sierra, and J. I. Cirac, Nat. comm. (2013)

I. Glasser, J. I. Cirac, G. Sierra, and A. E. B. Nielsen, NJP (2015)

B. Jaworowski, A. E. B. Nielsen, PRB, (2022)

Models with scalar chirality or extended interactions

Bauer et al, Nat. comm. (2014)

S.-S. Gong, W. Zhu, and D. Sheng, Sci. rep. (2014)

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Challenge: triangular lattice?

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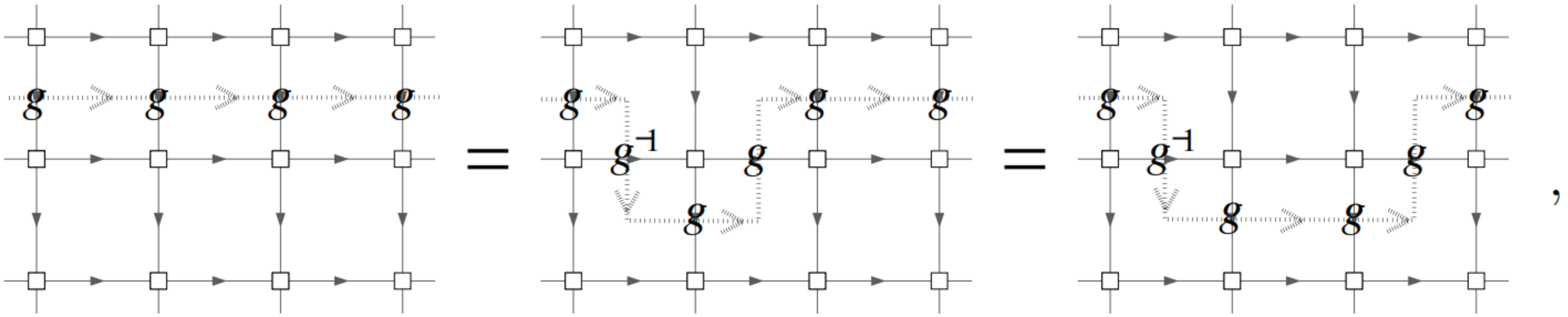
Why infinite projected entangled pair states (iPEPS)?

F. Verstraete and J. I. Cirac, 2006

Norbert et al, 2010

- A single tensor describes the state on a lattice of arbitrary size



Topological order encoded in the virtual gauge symmetry

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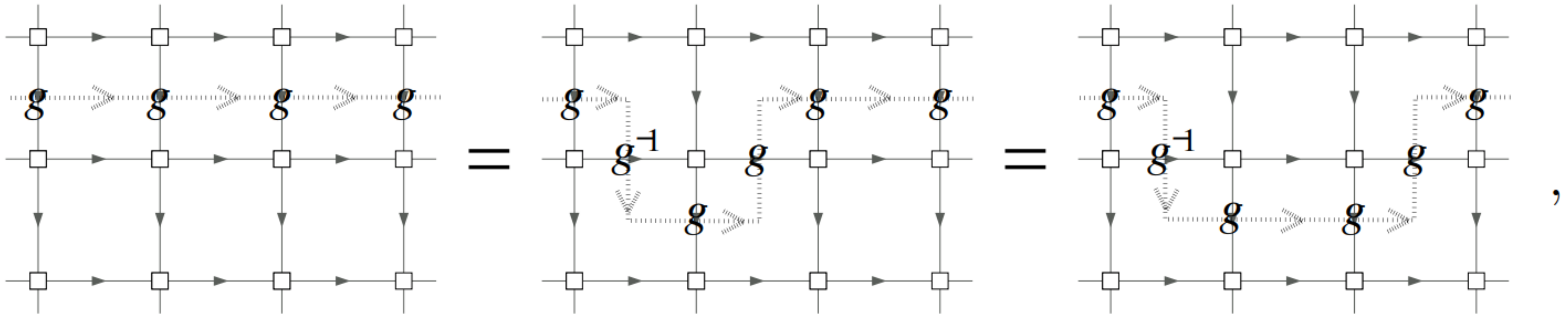

Topological order encoded in the virtual gauge symmetry

- Variational ansatz for numerical simulation on infinite lattice

Contraction

Optimization

T. Nishino (2006); J. Jordan et al, PRL (2008); Román Orús and Guifré Vidal, PRB (2009); P. Corboz, PRL (2014), PRB (2016); Liao et al., PRX (2019); Vanderstraeten et al, PRB (2022)

Introduction

Topological obstruction

Free fermion system (proved)

① Chern insulators have no localized Wannier functions

② Gaussian fermionic PEPS (GfPEPS) composed of local tensors

⇒ No-go theorem T. B. Wahl et al, 2013; J.Dubail and N. Read, 2015;

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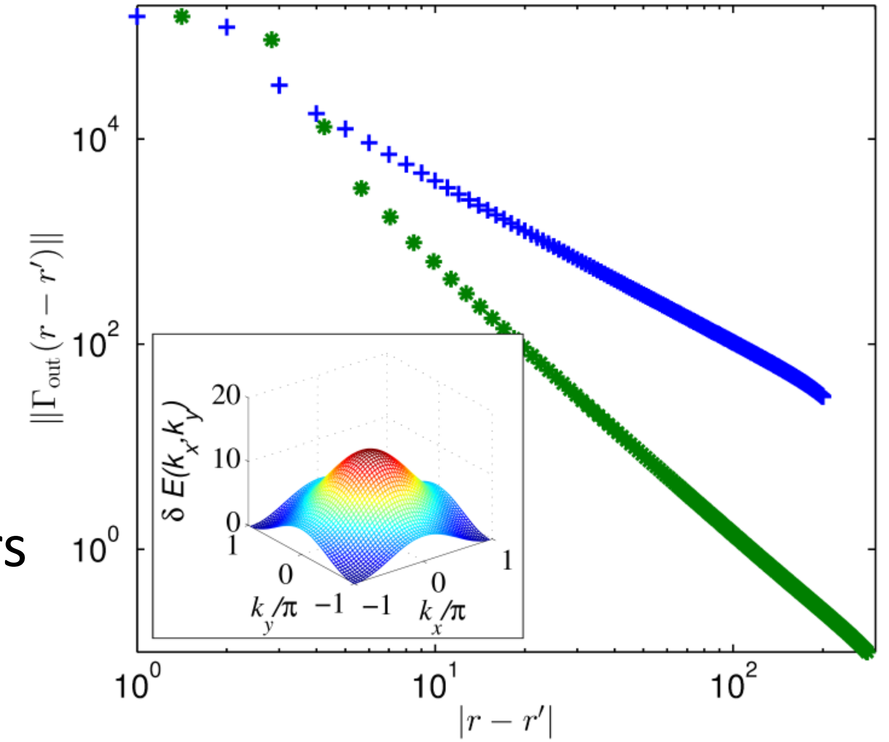

Power-law decay correlation

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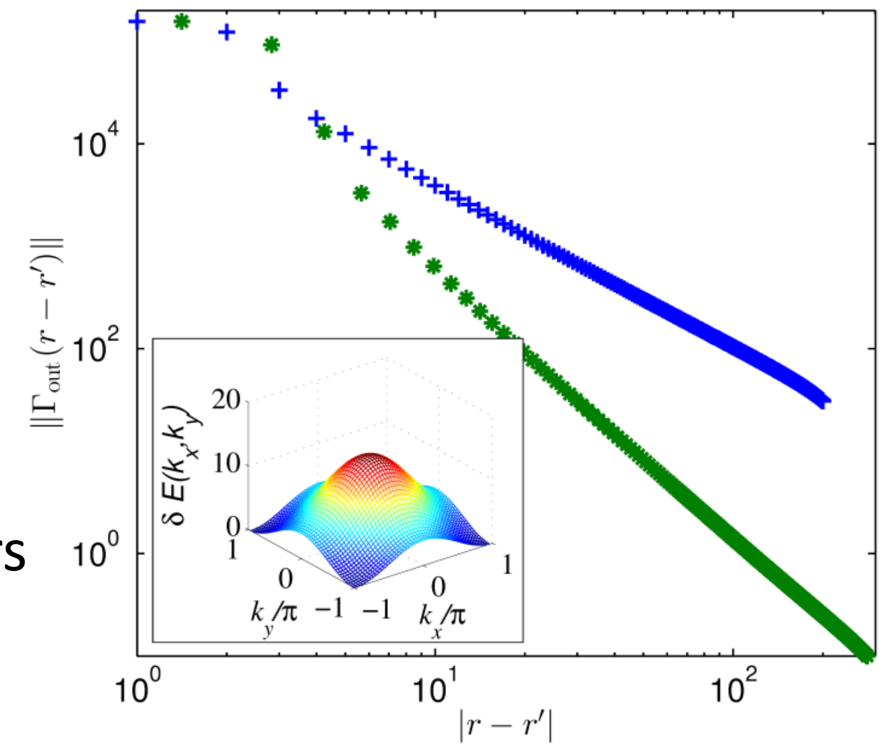


Power-law decay correlation

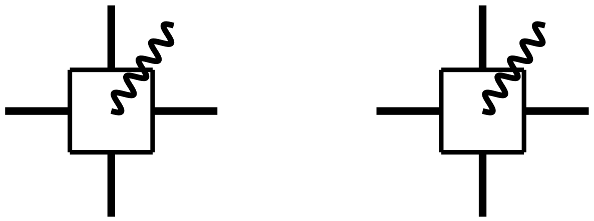
Interacting (spin) system (universal?)

S. Yang et al, 2015; D. Poilblanc et al, 2015;

Introduction

iPEPS investigations of CSLs in spin models

Method: symmetry constraints & variational optimization



A_1 $+$ iA_2

A_1, A_2: IRREPS of C_{4v};

PT symmetric

D. Poilblanc, PRB (2017); J.-Y. Chen et al, PRB (2018), (2020), (2021); J. Hasik et al, PRL (2022).

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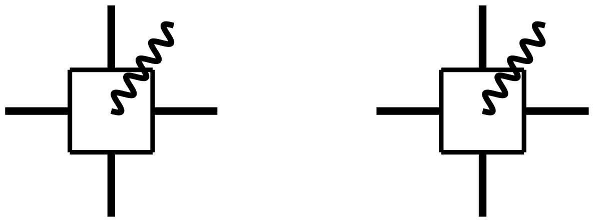

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$A_1 + iA_2$

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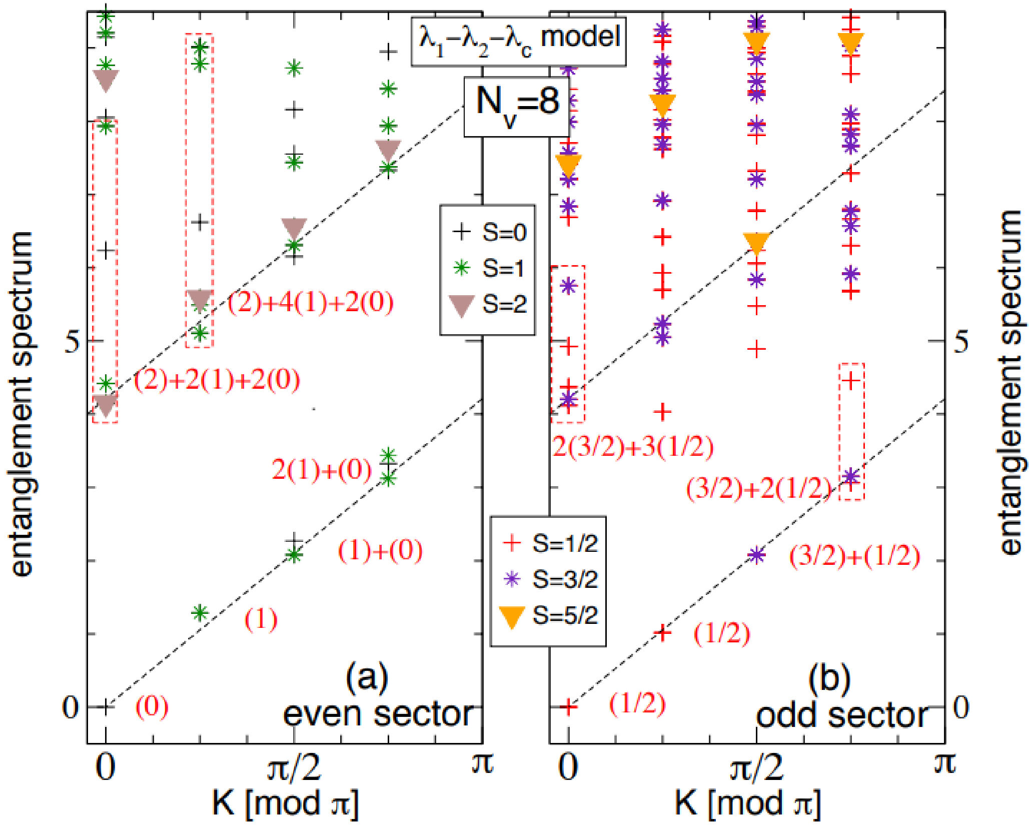

D. Poilblanc, PRB (2017);

Characterization

- Expected level counting in the entanglement spectrum predicted by CFT

D. Poilblanc, PRB (2017); J.-Y. Chen et al, PRB (2018), (2020), (2021); J. Hasik et al, PRL (2022).

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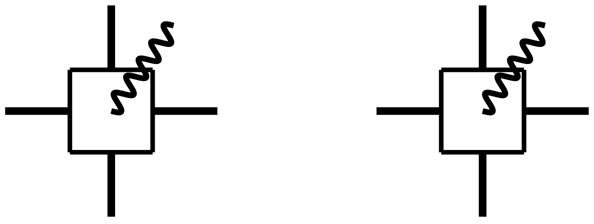

$A_1 + iA_2$

A_1, A_2: IRREPS of C_{4v};

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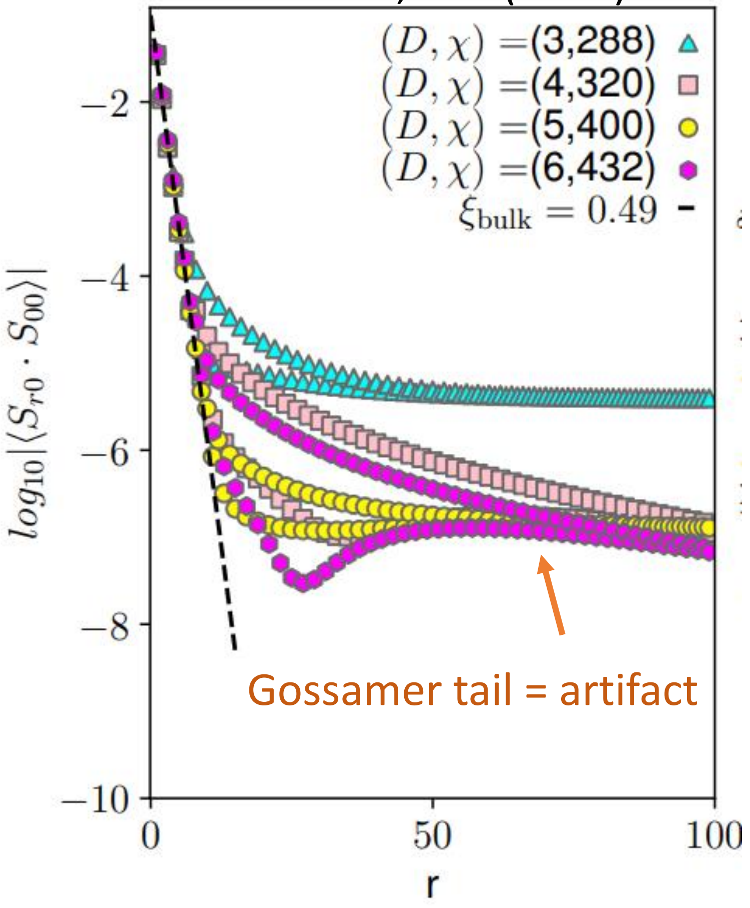
Characterization

- Expected level counting in the entanglement spectrum predicted by CFT
- Artifact in the correlation functions

J. Hasik et al, PRL (2022).



Short and long distance correlations

D. Poilblanc, PRB (2017); J.-Y. Chen et al, PRB (2018), (2020), (2021); J. Hasik et al, PRL (2022).

Introduction

Questions

- Can we apply iPEPS to study CSLs in generic models?

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- Can we apply iPEPS to study CSLs in generic models?

- Is the artifact of chiral iPEPS universal? **How faithful is iPEPS representation**?

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CSL with bosonic iPESS on the kagome lattice

Spin-1/2 Kagome lattice models with CSL

$$H = J_1 \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_\chi \sum_{i,j,k \in \triangle} \mathbf{S}_i \cdot (\mathbf{S}_j \times \mathbf{S}_k),$$

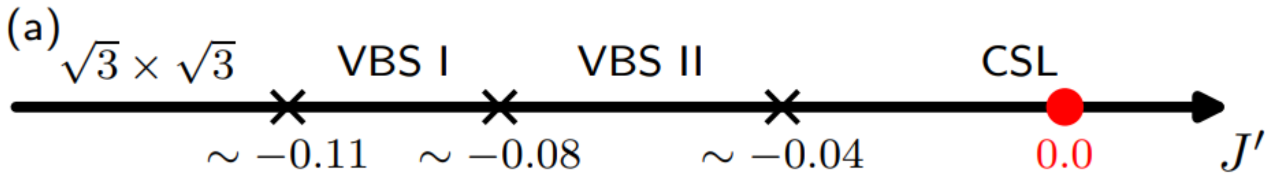
Bauer et al, Nat. comm. (2014)

$$H = J \sum_{\langle ij \rangle_1} \mathbf{S}_i \cdot \mathbf{S}_j + J' \left(\sum_{\langle ij \rangle_2} \mathbf{S}_i \cdot \mathbf{S}_j + \sum_{\langle ij \rangle_3} \mathbf{S}_i \cdot \mathbf{S}_j \right),$$

S.-S. Gong et al, Sci. rep. (2014)

Possible CSL ground state at Heisenberg point? [DMRG]

R.-Y. Sun et al, arXiv:2203.07321



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Spin-1/2 Kagome lattice models with CSL

$$H = J_H \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_\chi \sum_{i,j,k \in \triangle} \mathbf{S}_i \cdot (\mathbf{S}_j \times \mathbf{S}_k),$$

Bauer et al, Nat. comm. (2014)

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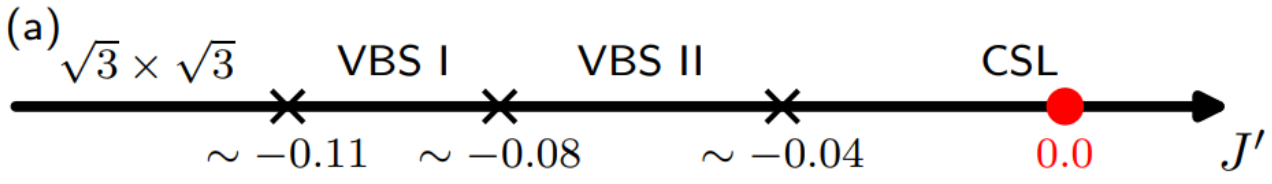
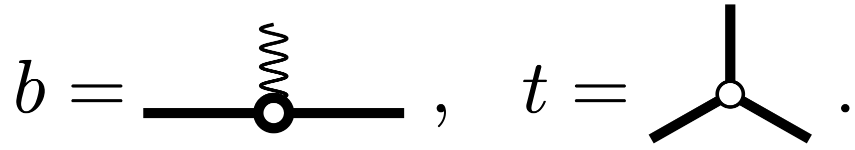


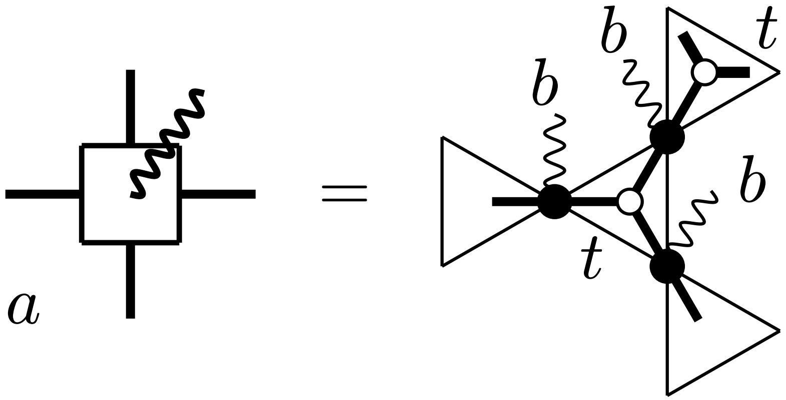
Motivation I

Can we get insights on this problem from iPEPS method?

CSL with bosonic iPESS on the kagome lattice

Infinite projected entangled simplex states (IPESS)





A unit-cell tensor a on the kagome lattice

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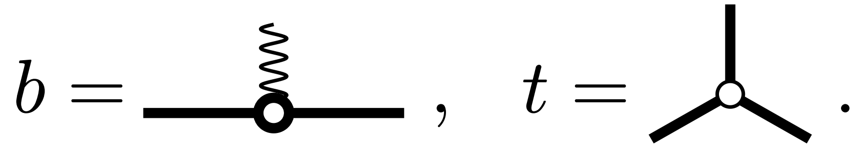
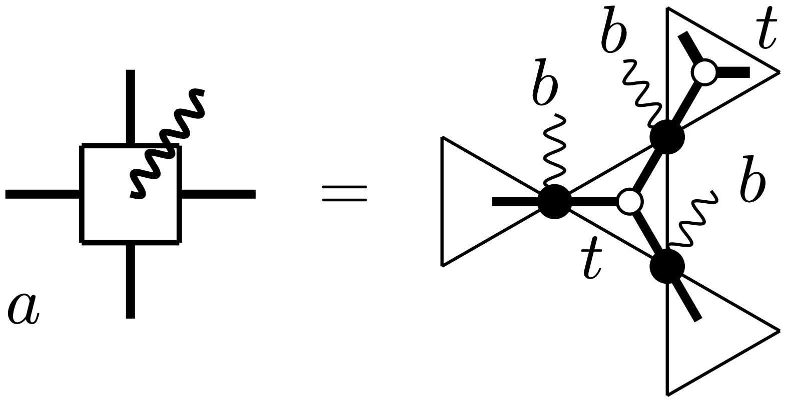
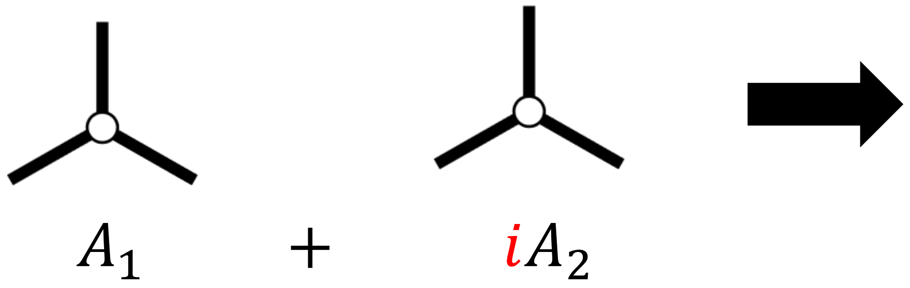




A unit-cell tensor a on the kagome lattice

Motivation II

Can iPESS represent CSLs on non-bipartite lattice?



Non-chiral RVB state!

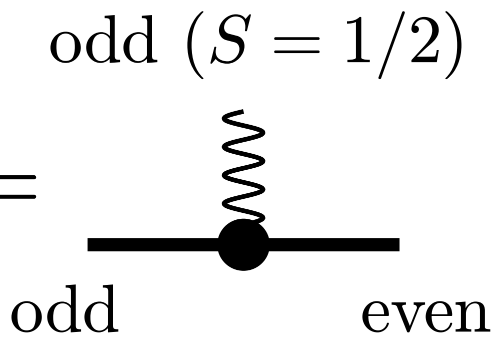
SU(2) Virtual space: $\mathrm{V} = 0 \oplus 1/2$

Straightforward extension fails at $D = 3$. Why?

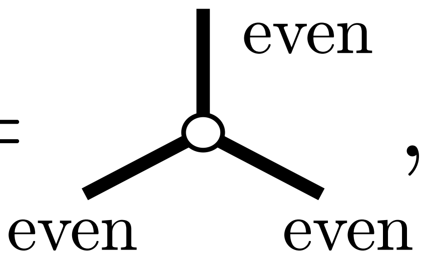
CSL with bosonic iPESS on the kagome lattice

Symmetric tensors

SU(2) symmetry: fusion rule

$b =$ 

,

$t^{\mathrm{I}} =$ 

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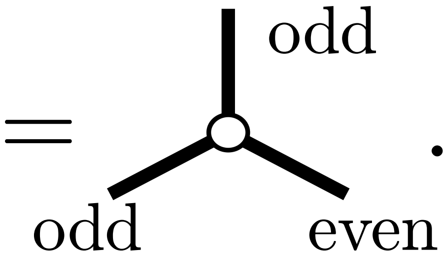
$t^{\mathrm{II}} =$ 

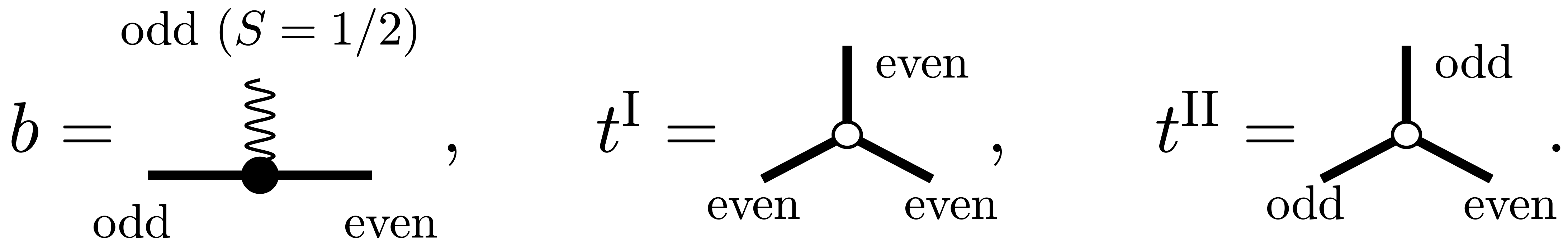
.

Odd: half-integer spin;
even: integer spin.

CSL with bosonic iPESS on the kagome lattice

Symmetric tensors

SU(2) symmetry: fusion rule



Odd: half-integer spin;
even: integer spin.

Point group Symmetrization

- C_2 on b tensor: b_A, b_B (equivalent)
- C_{3v} on t tensor: $t^{\mathrm{I}}_{A_1}, t^{\mathrm{II}}_{A_1}, t^{\mathrm{I}}_{A_2}, t^{\mathrm{II}}_{A_2}$

CSL with bosonic iPESS on the kagome lattice

Symmetric tensors

D^*	D	Virtual space	b_A	[redacted]	$t^{\mathrm{I}}_{A_1}$	$t^{\mathrm{II}}_{A_1}$	$t^{\mathrm{I}}_{A_2}$	$t^{\mathrm{II}}_{A_2}$
2	3	$0 \oplus \frac{1}{2}$	1		1	0	0	1
3	6	$0 \oplus \frac{1}{2} \oplus 1$	2		1	2	1	1
4	8	$0 \oplus \frac{1}{2} \oplus 1 \oplus \frac{1}{2}$	4		2	4	1	4
5	12	$0 \oplus \frac{1}{2} \oplus 1 \oplus \frac{1}{2} \oplus \frac{3}{2}$	5		2	7	1	7

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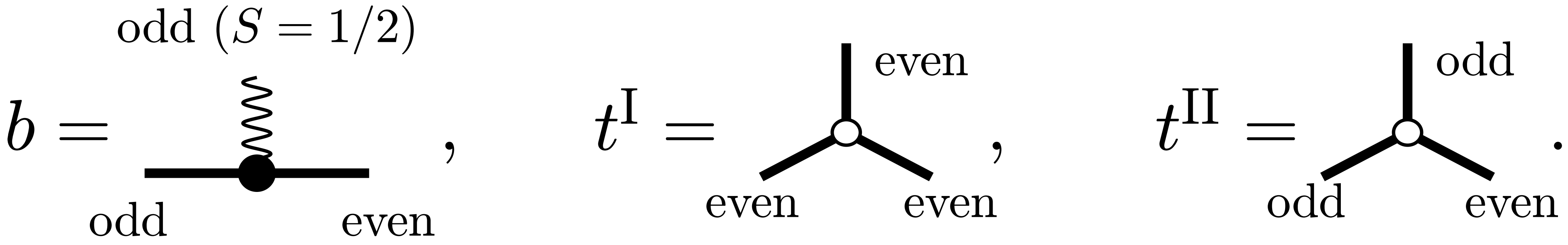
3	6	$0 \oplus \frac{1}{2} \oplus 1$	2		1	2	1	1
4	8	$0 \oplus \frac{1}{2} \oplus 1 \oplus \frac{1}{2}$	4		2	4	1	4
5	12	$0 \oplus \frac{1}{2} \oplus 1 \oplus \frac{1}{2} \oplus \frac{3}{2}$	5		2	7	1	7

- Ansatz: linear combination of symmetric tensors
- For D=3: only $t = t^{\mathrm{I}}_{A_1} + \mathrm{e}^{i\phi} t^{\mathrm{II}}_{A_2}$ allowed

CSL with bosonic iPESS on the kagome lattice

A tensor conservation law in iPESS

① Let $t = t^{\mathrm{I}} + e^{i\phi} t^{\mathrm{II}}$ be the t tensor on a lattice of size N



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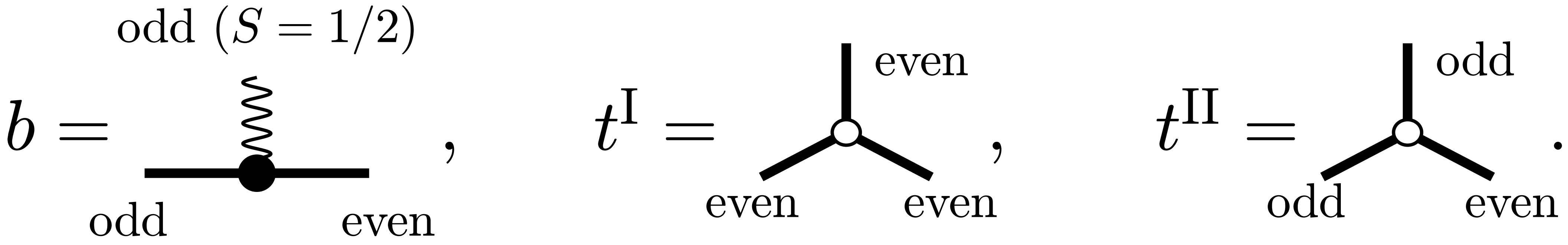
A tensor conservation law in iPESS

① Let $t = t^{\mathrm{I}} + e^{i\phi} t^{\mathrm{II}}$ be the t tensor on a lattice of size N

② $\mathrm{N} = \mathrm{N}_b = \mathrm{N}_{even} = \mathrm{N}_{odd}$, # of b tensors, even/odd bonds



CSL with bosonic iPESS on the kagome lattice

A tensor conservation law in iPESS

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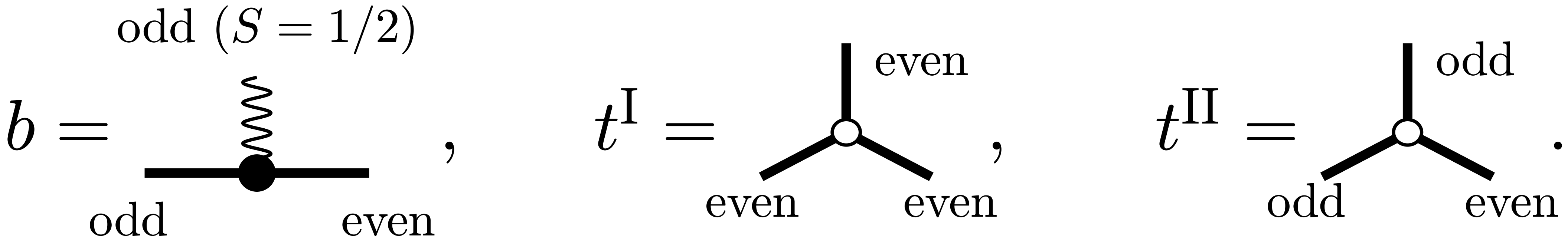
Count number of virtual bonds from either b tensor or t tensor



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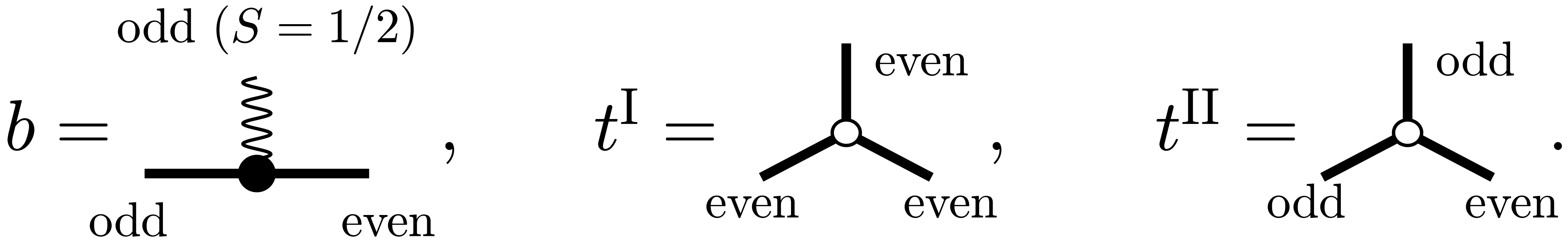

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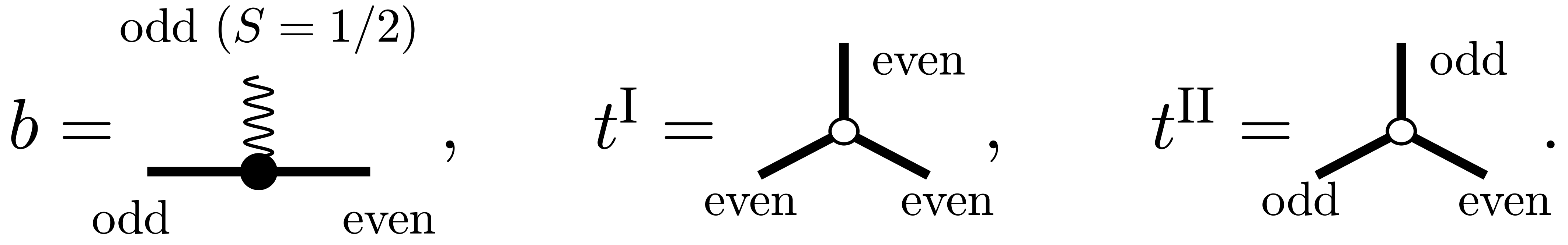
⇒ $\mathrm{N}_{t^{\mathrm{I}}}$ and $\mathrm{N}_{t^{\mathrm{II}}}$ are fixed in the global tensor product state



CSL with bosonic iPESS on the kagome lattice

Non-chiral and chiral Ansatze

Non-chiral ansatze:

① Pure A_1 or pure A_2 IRREP;

② $\mathrm{t}^{\mathrm{I}}_{\mathrm{A}_1} + e^{i\phi} t^{\mathrm{II}}_{\mathrm{A}_2}$ or $t^{\mathrm{II}}_{\mathrm{A}_1} + e^{i\phi} t^{\mathrm{I}}_{\mathrm{A}_2}$

CSL with bosonic iPESS on the kagome lattice

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Chiral ansatz:

$(t^{\mathrm{I}}_{\mathrm{A}_1} + t^{\mathrm{II}}_{\mathrm{A}_1}) + i(t^{\mathrm{I}}_{\mathrm{A}_2} + t^{\mathrm{II}}_{\mathrm{A}_2})$

CSL with bosonic iPESS on the kagome lattice

Non-chiral and chiral Ansatze

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Chiral ansatz:

$$(t^{\mathrm{I}}_{\mathrm{A}_1} + t^{\mathrm{II}}_{\mathrm{A}_1}) + i(t^{\mathrm{I}}_{\mathrm{A}_2} + t^{\mathrm{II}}_{\mathrm{A}_2})$$

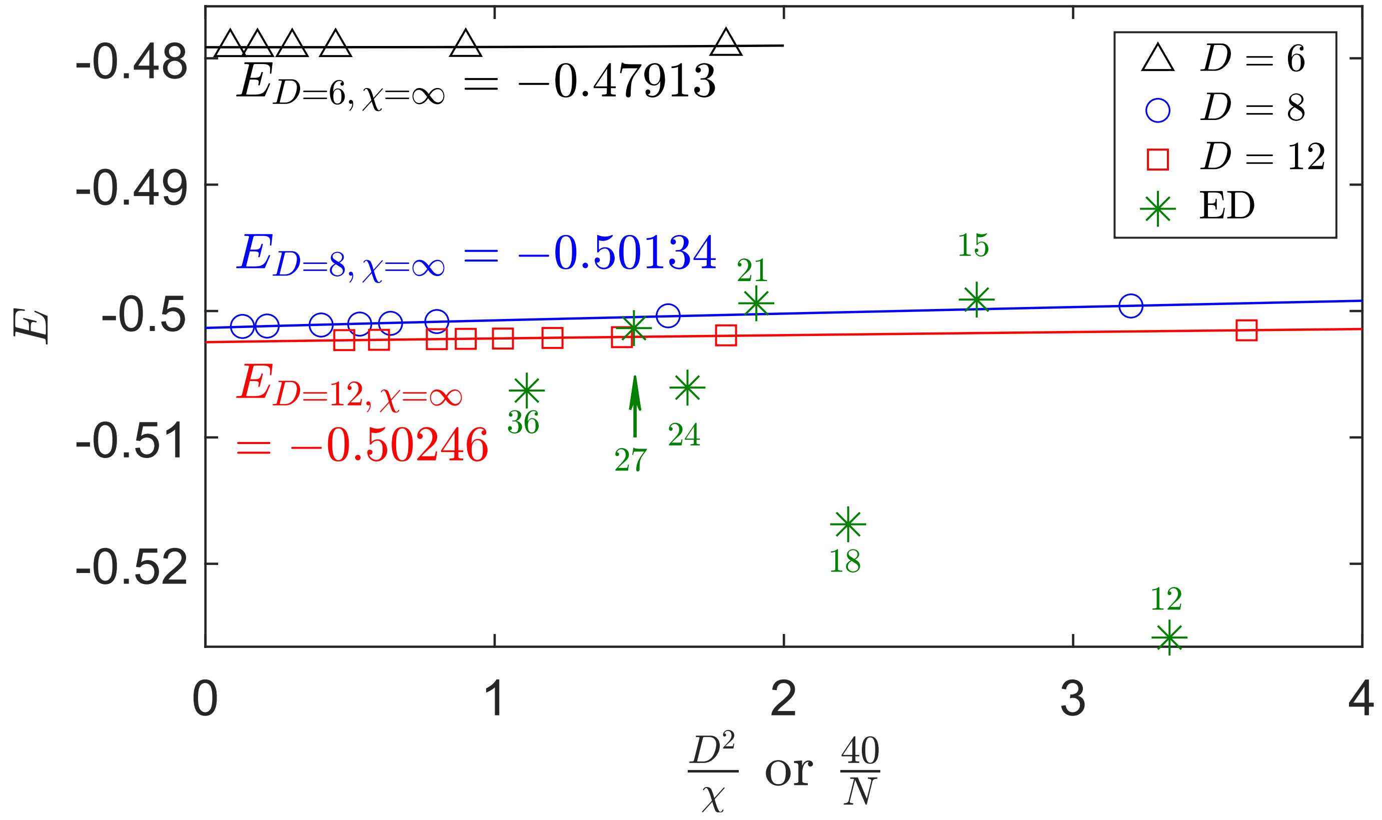
Non-chiral ansatz is a subset of chiral ansatz

CSL with bosonic iPESS on the kagome lattice

Numerical results: CSL

$$H = J_1 \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_\chi \sum_{i,j,k \in \triangle} \mathbf{S}_i \cdot (\mathbf{S}_j \times \mathbf{S}_k),$$

Variationally optimized at $J_\chi / J_1 = tan(0.2\pi)$



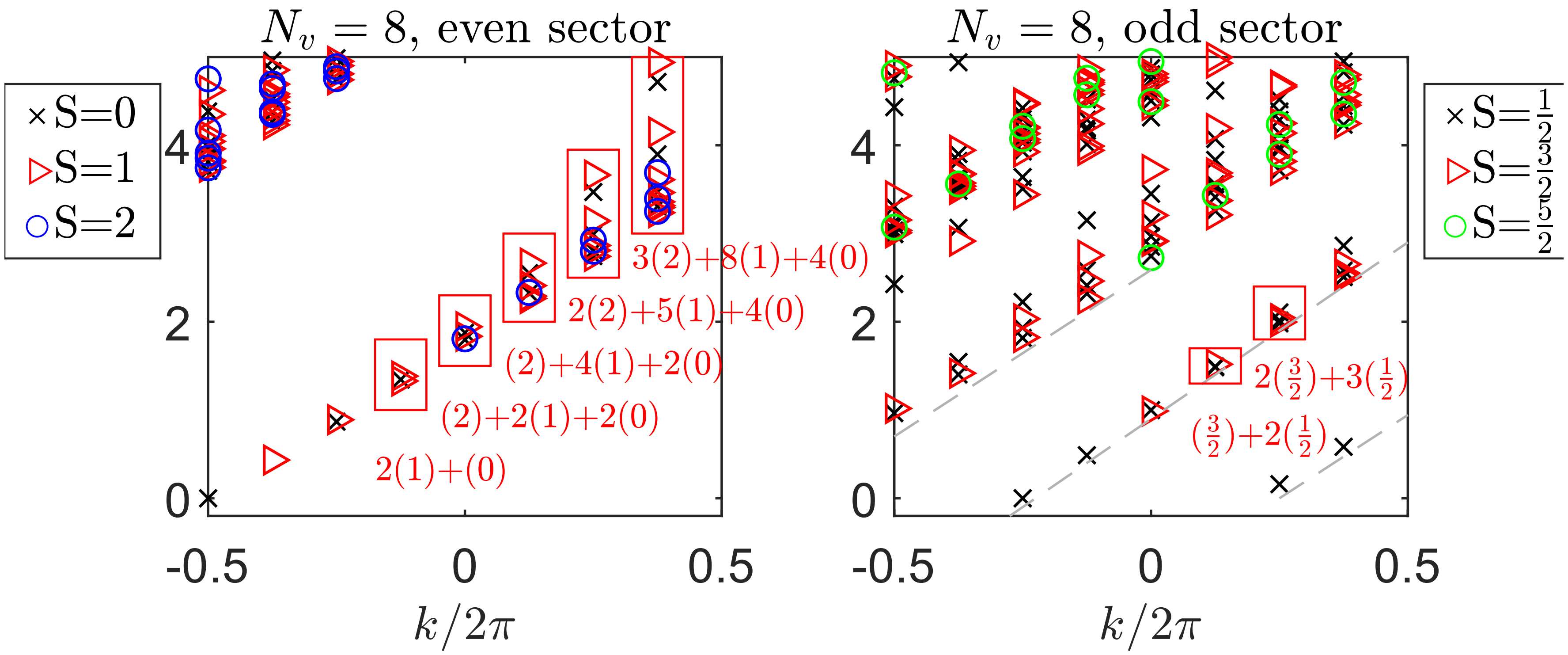
energies evaluated at CTMRG bond dimension χ

The variational energy becomes good for $D \geq 8$

CSL with bosonic iPESS on the kagome lattice

Numerical results: CSL

Variationally optimized at $J_\chi/J_1 = tan(0.2\pi)$



entanglement spectrum on cylinder with $D = 8$;

Level counting matches $\mathrm{SU}(2)_1$ CFT prediction for $D \geq 8$

CSL with bosonic iPESS on the kagome lattice

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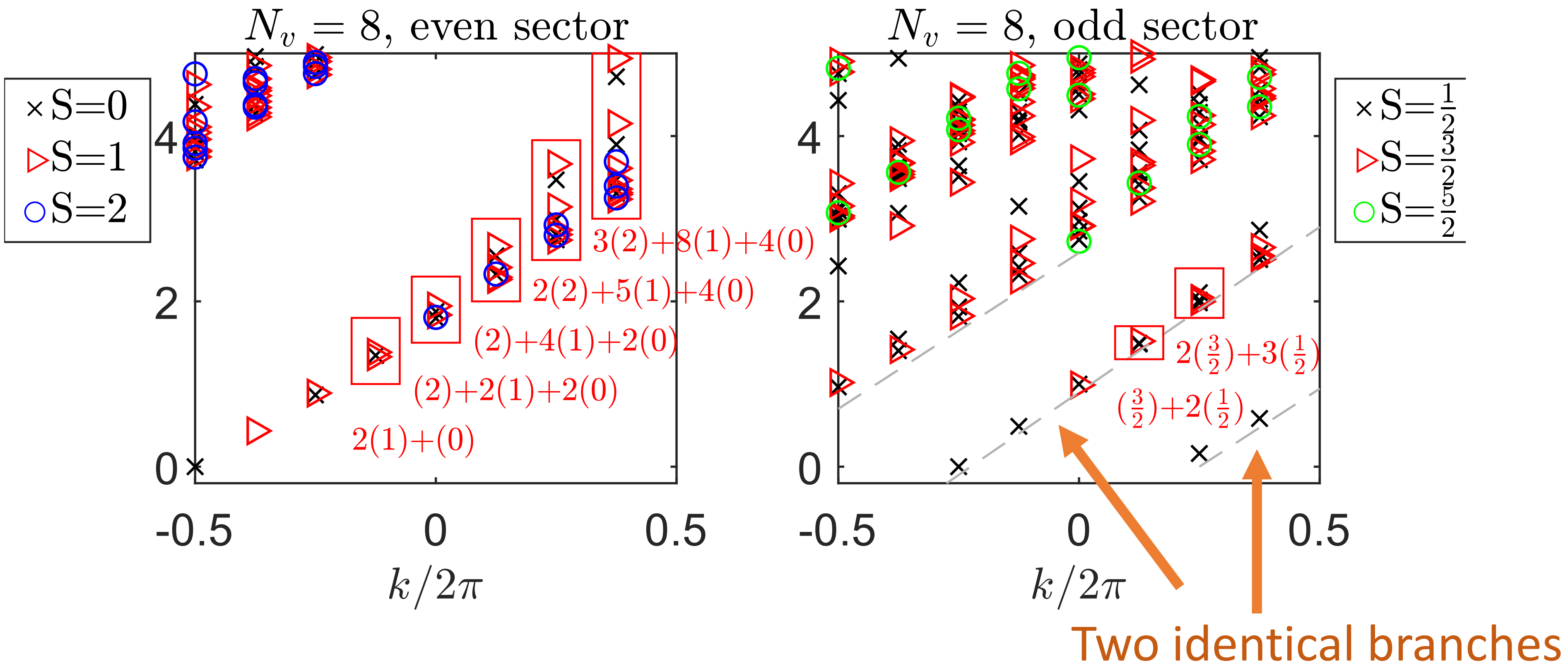

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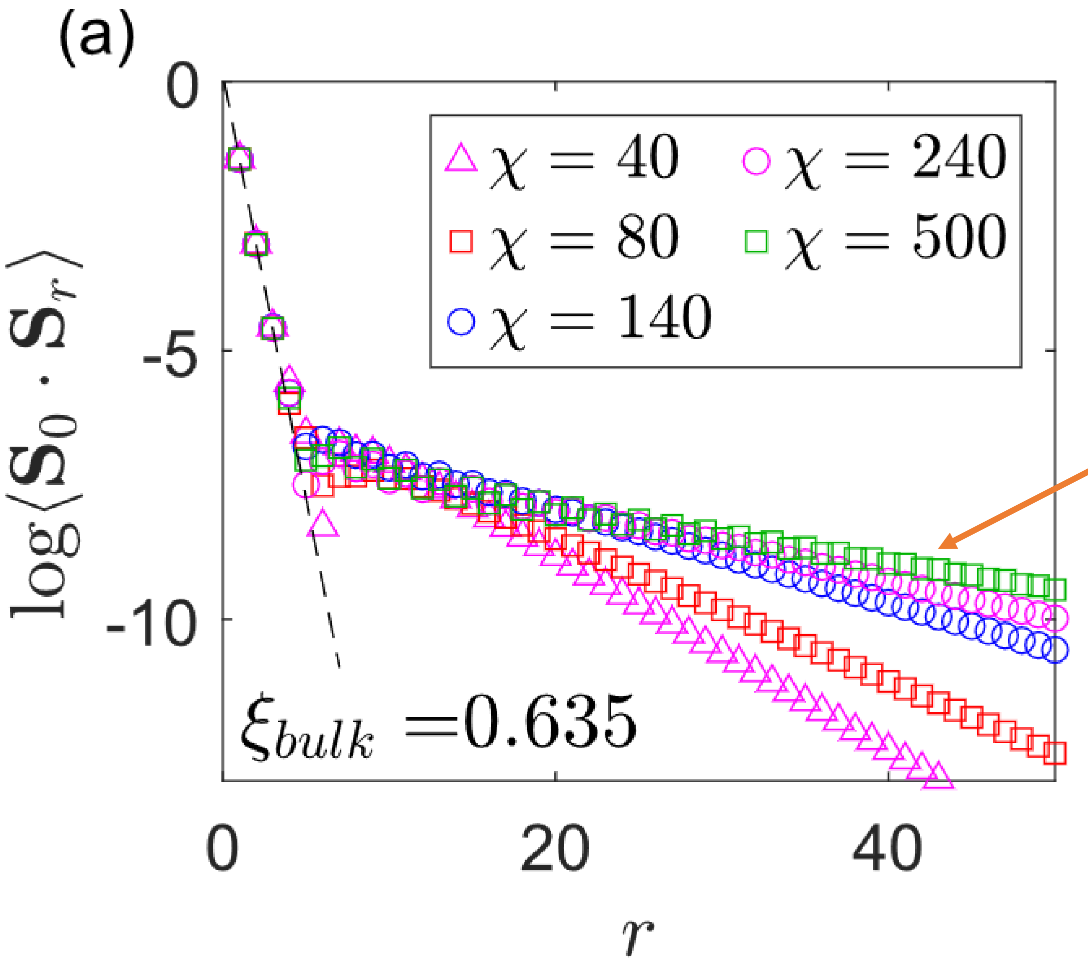


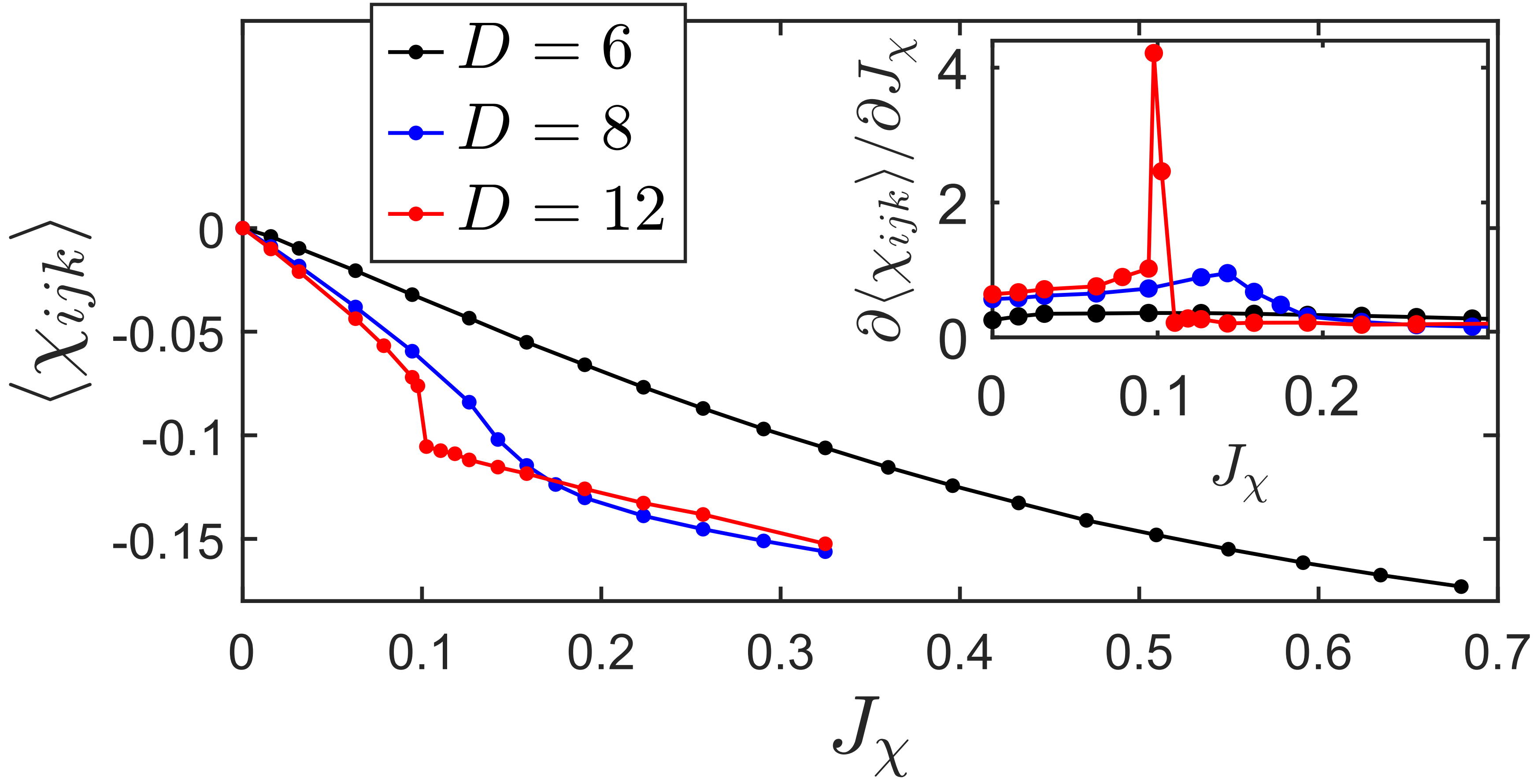
Bulk correlations at $D = 8$;
χ: CTMRG bond dimension

Short distance: physical; long distance: artifact

CSL with bosonic iPESS on the kagome lattice

Numerical results: the transition and the Heisenberg point

Optimized with chiral ansatz



- First order transition [DMRG, R. Haghshenas et al, PRB (2019)]

CSL with bosonic iPESS on the kagome lattice

Numerical results: the transition and the Heisenberg point

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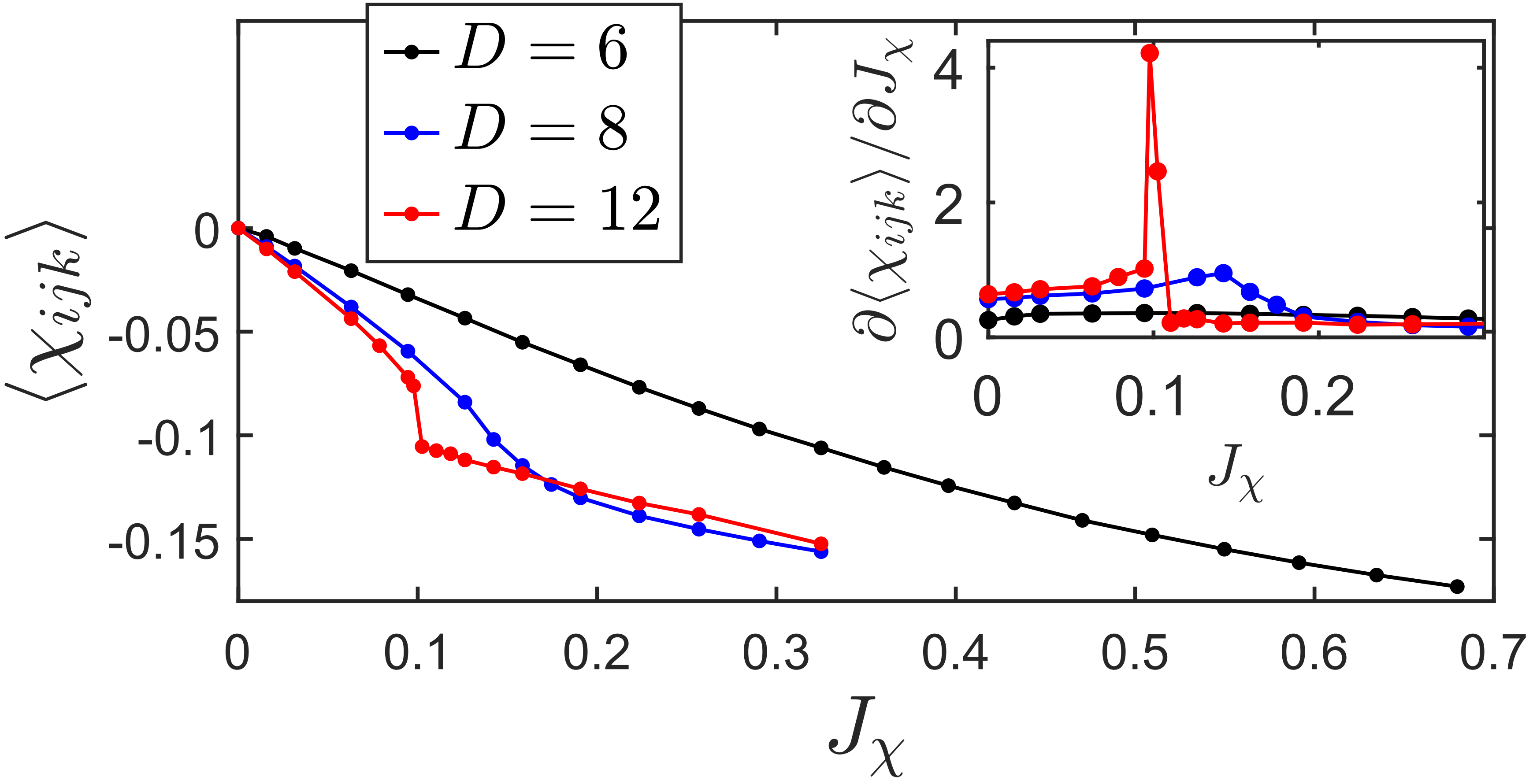
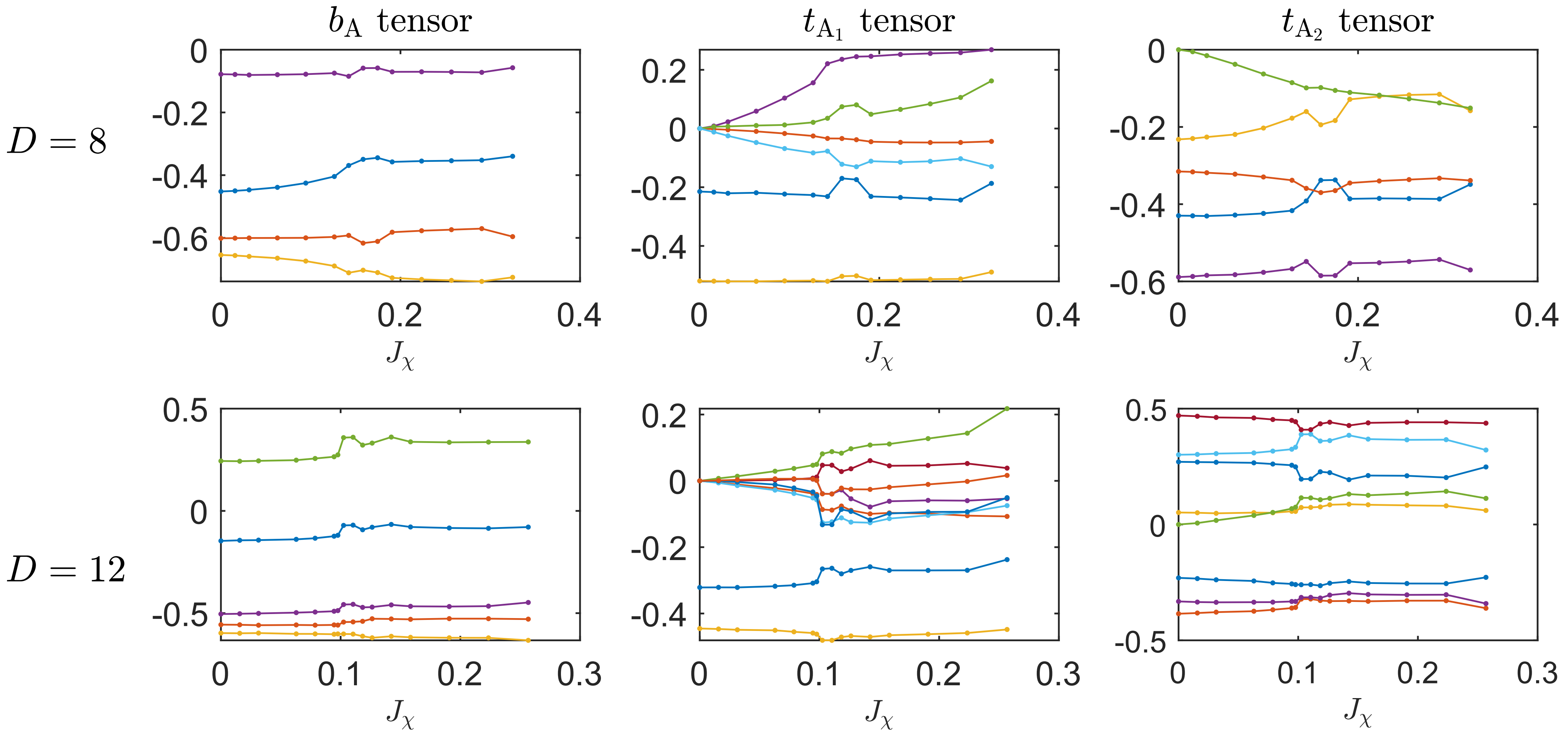


- First order transition [DMRG, R. Haghshenas et al, PRB (2019)]
- J_χ^c decreases for larger D;

CSL with bosonic iPESS on the kagome lattice

Numerical results: the transition and the Heisenberg point

Optimized with chiral ansatz



CSL with bosonic iPESS on the kagome lattice

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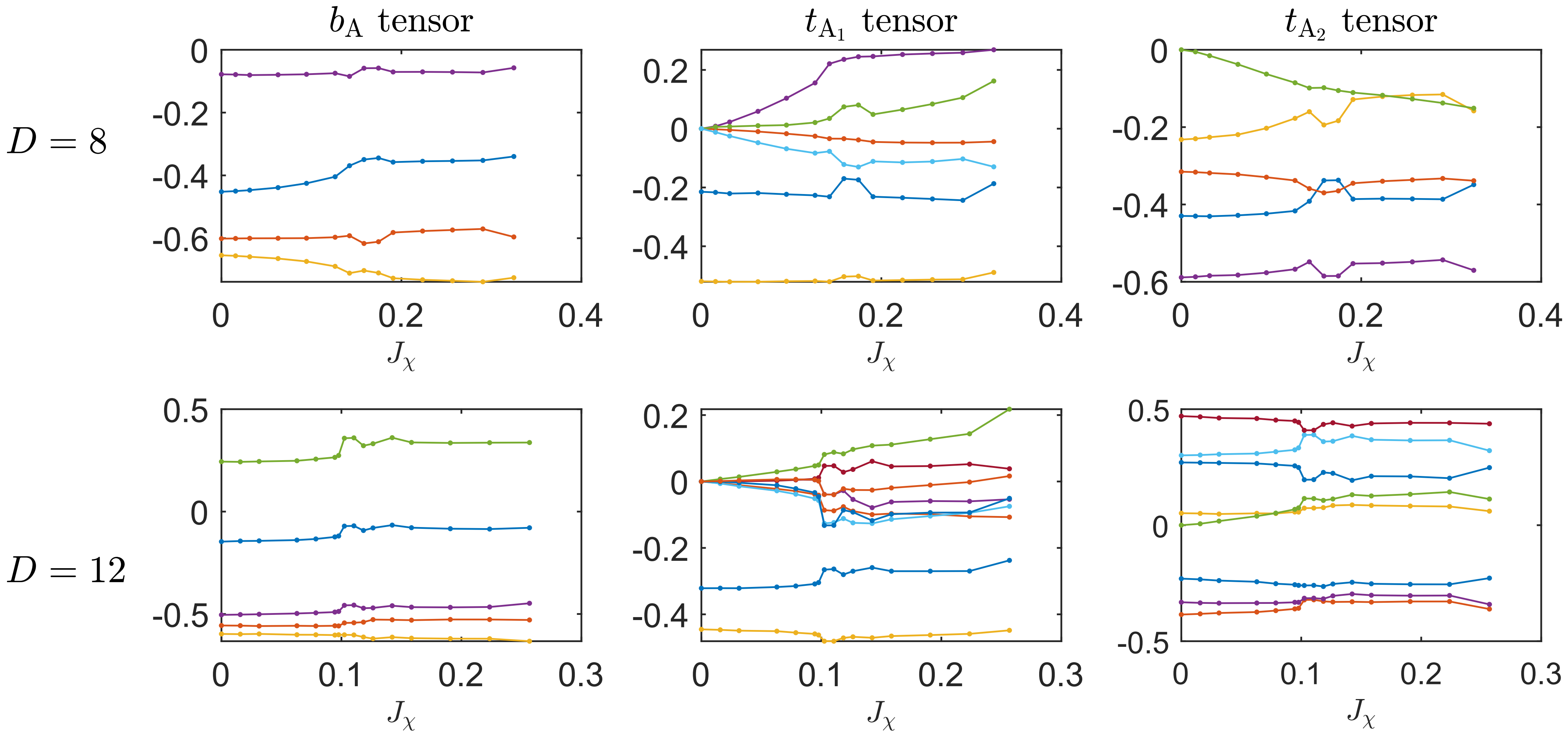

- Variational parameters flow to the non-chiral ansatz at $J_\chi = 0$; no evidence for spontaneous time reversal symmetry breaking.

CSL with bosonic iPESS on the kagome lattice

Summary

- Kagome CSL can be represented by iPESS

CSL with bosonic iPESS on the kagome lattice

Summary

- Kagome CSL can be represented by iPESS
- No evidence of spontaneous time reversal symmetry breaking at the Heisenberg point

Outline

- Introduction
  - Chiral spin liquids
  - iPEPS
- Chiral spin liquids with bosonic iPEPS on the kagome lattice
- Chiral spin liquids with projected Gaussian fermionic iPEPS

SN, Jheng-Wei Li, Ji-Yao Chen and Didier Poilblanc, arXiv: 2306.10457.

CSL with Gutzwiller projected Gaussian fermionic iPEPS

Motivation (why fermionic PEPS?)

- Topology is not guaranteed by symmetry constraints in bosonic iPEPS. A universal way to construct generic CSL ansatz?

CSL with Gutzwiller projected Gaussian fermionic iPEPS

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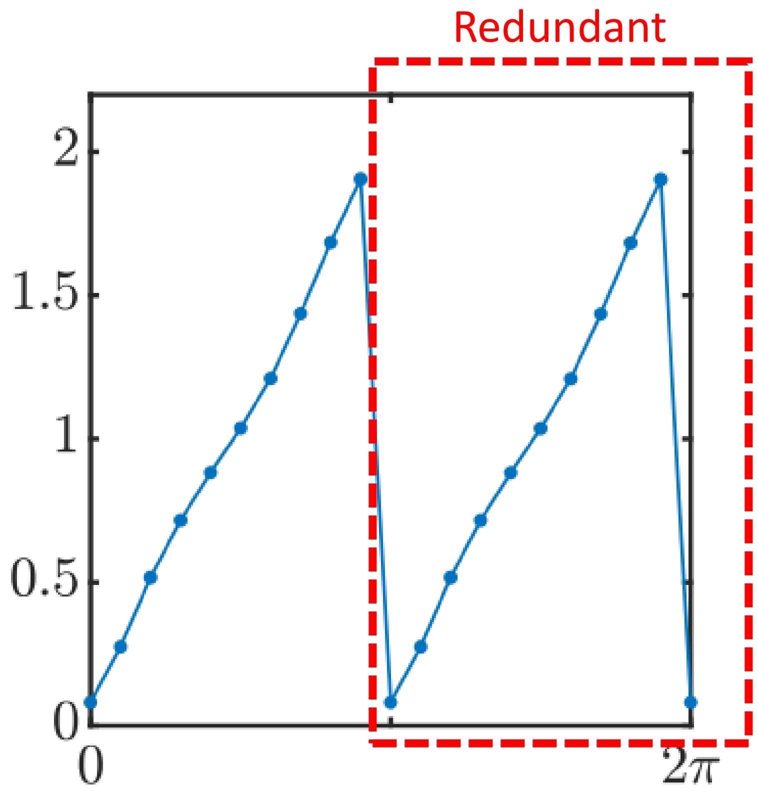
- Topology is not guaranteed by symmetry constraints in bosonic iPEPS. A universal way to construct generic CSL ansatz?

- Subtle issues in bosonic chiral iPEPS
  - ➢ Redundant identical chiral branch ($SU(N)$, non-Abelian)
  - ➢ Minimally entangled states (non-Abelian)



J. Hasik et al, PRL (2022).

D. Poilblanc, PRB (2017); J.-Y. Chen et al, PRB (2018), (2020), (2021); J. Hasik et al, PRL (2022). A Hackenbroich et al, PRB (2018);

CSL with Gutzwiller projected Gaussian fermionic iPEPS

Ingredients

- Parton ansatz: Gutzwiller projected layered free fermions [1]

$$\psi_{spin}(\{x_i\}) = P_G[\psi_\uparrow(\{x_i\}) \otimes \psi_\downarrow(\{x_i\})]$$

$$P_G = \prod_i \left(n_{i,\uparrow} - n_{i,\downarrow}\right)^2$$

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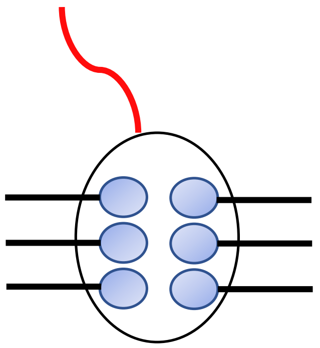
- Gaussian fermionic iPEPS (GfPEPS) for free fermion states [2]
  - ➢ Virtual particle: fermions;
  - ➢ $D = 2^M$, M is # of virtual modes.



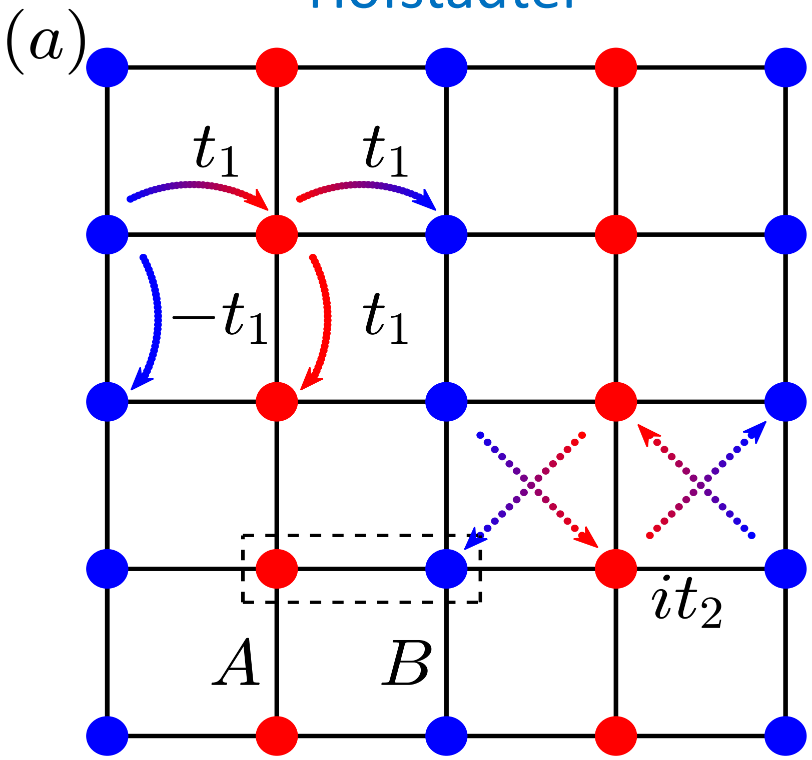
$M = 3$

[1] A parton construction without $SU(2)$ symmetry: S. Yang et al, PRL (2015);
[2] Variational optimization: T. B. Wahl et al, PRL (2013); Q. Mortier et al, PRL (2022); J.-W. Li, et al, PRB (2023).

CSL with Gutzwiller projected Gaussian fermionic iPEPS

Setup

Hofstadter

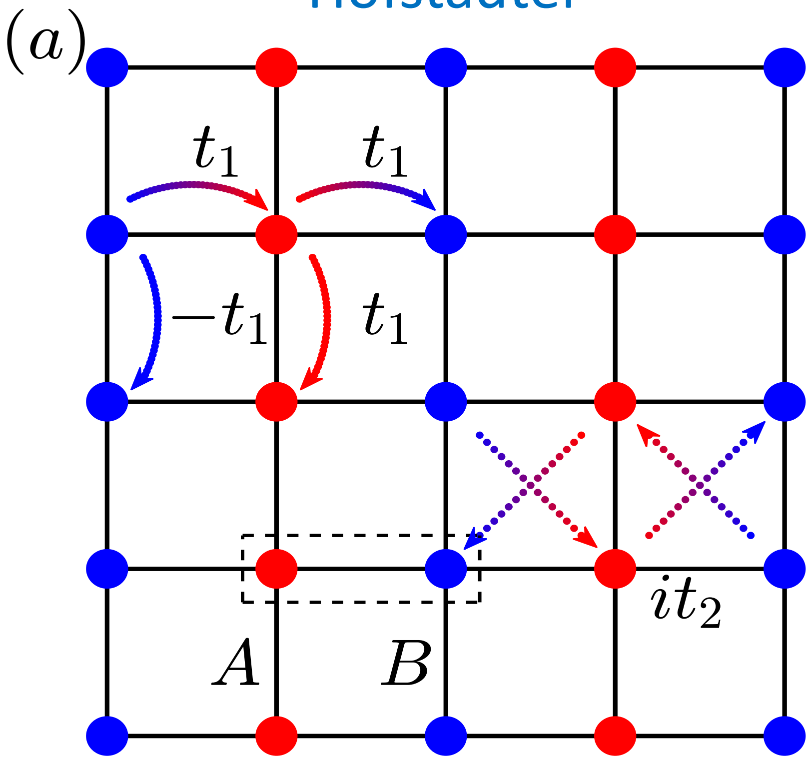


(a) Free fermion (mean-field) model for Chern insulators

CSL with Gutzwiller projected Gaussian fermionic iPEPS

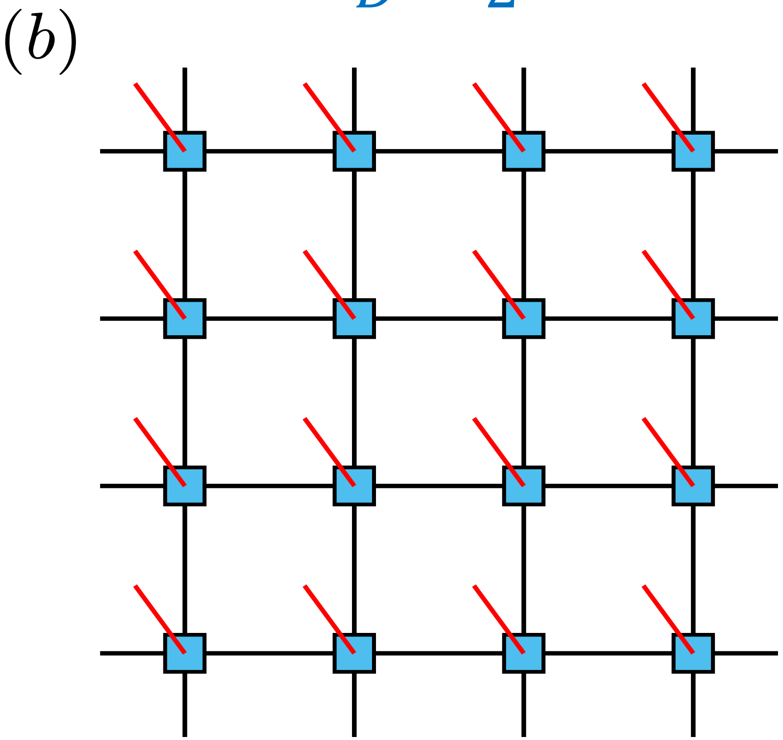
Setup

Hofstadter



(a) Free fermion (mean-field) model for Chern insulators

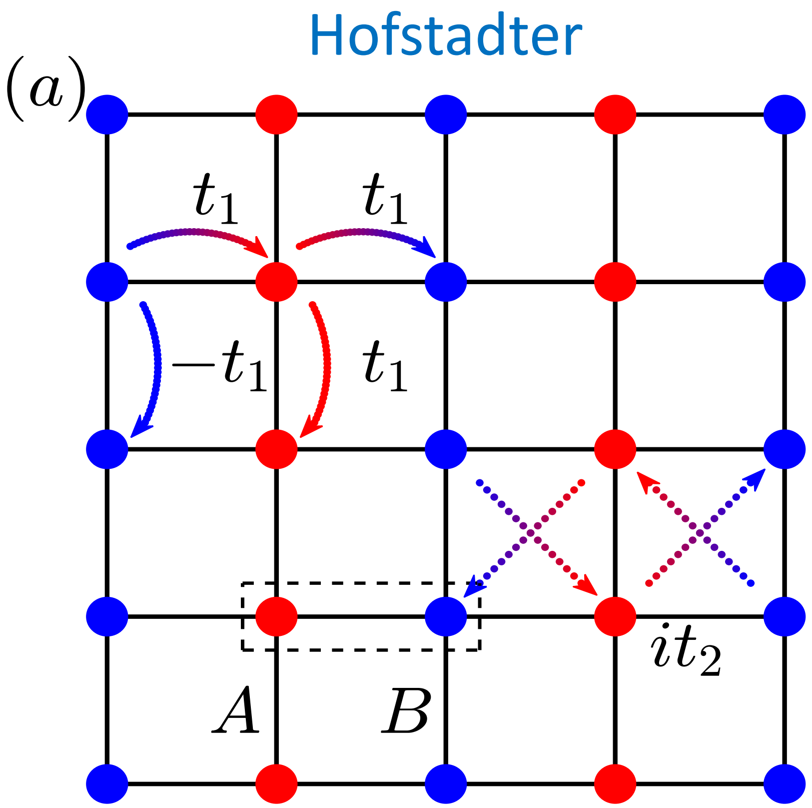
$D = 2^M$

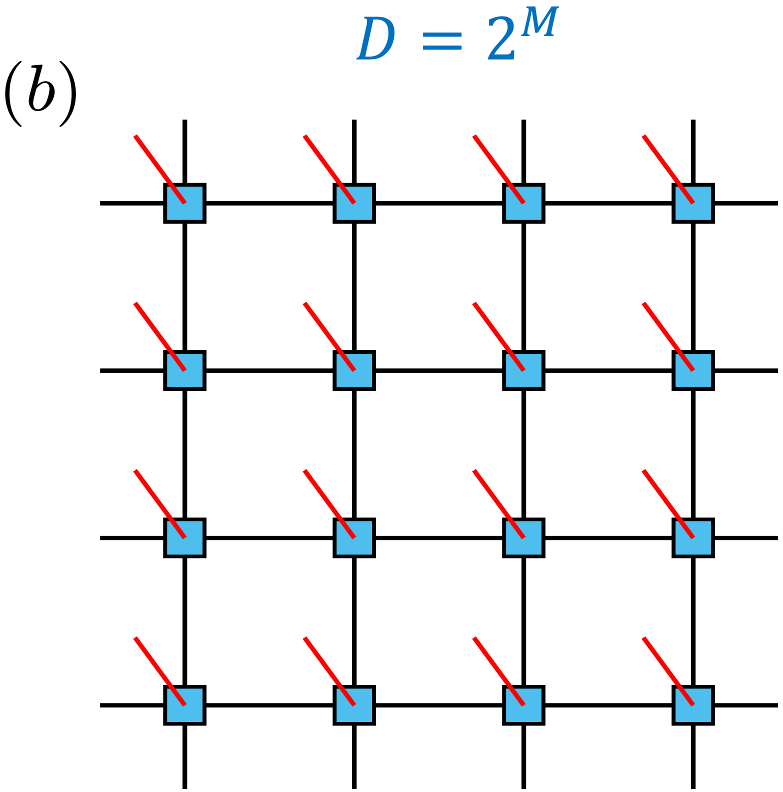


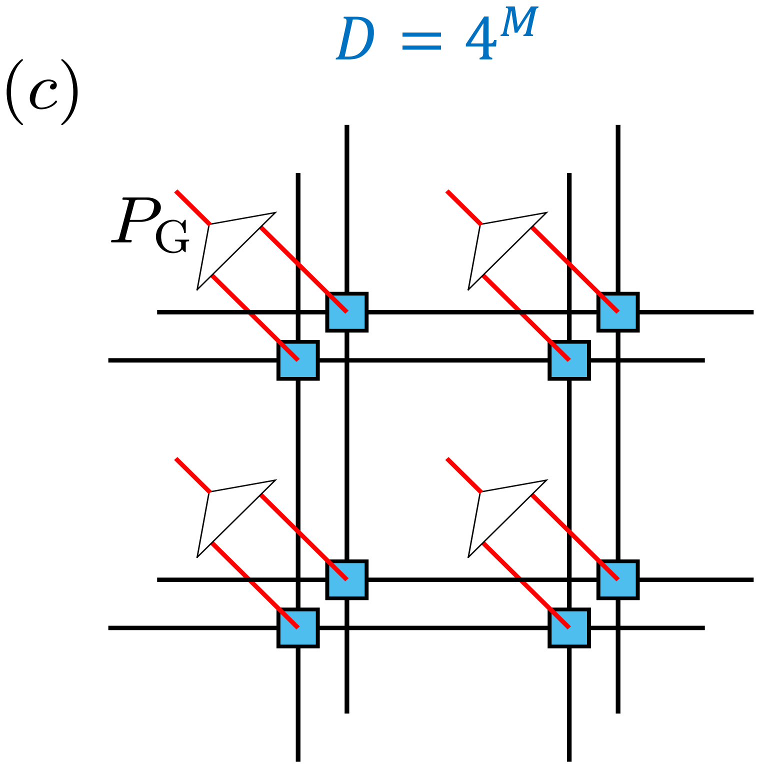
(b) Optimized GfPEPS

CSL with Gutzwiller projected Gaussian fermionic iPEPS

Setup







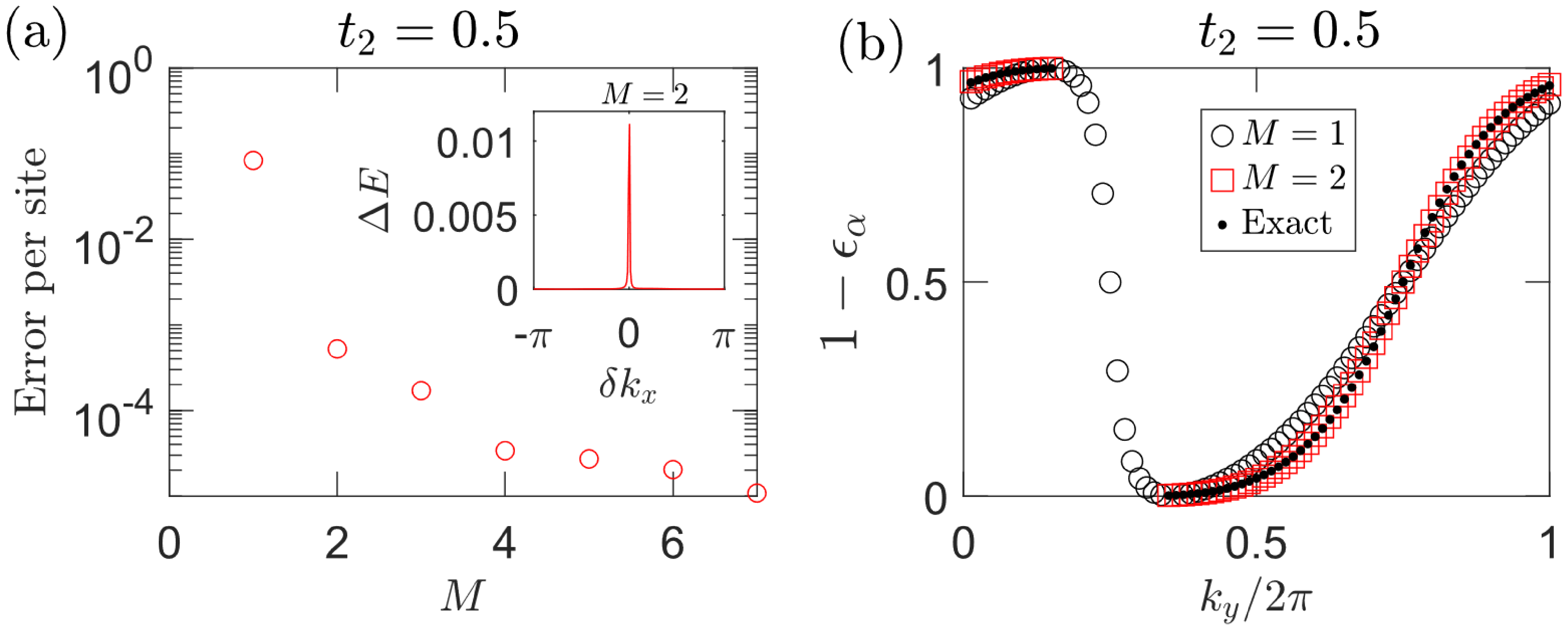
(a) Free fermion (mean-field) model for Chern insulators

(b) Optimized GfPEPS

(c) Gutzwiller projected layered GfPEPS for spin state

CSL with Gutzwiller projected Gaussian fermionic iPEPS

GfPEPS for Chern insulators: Hofstadter model, Chern number $C = 1$

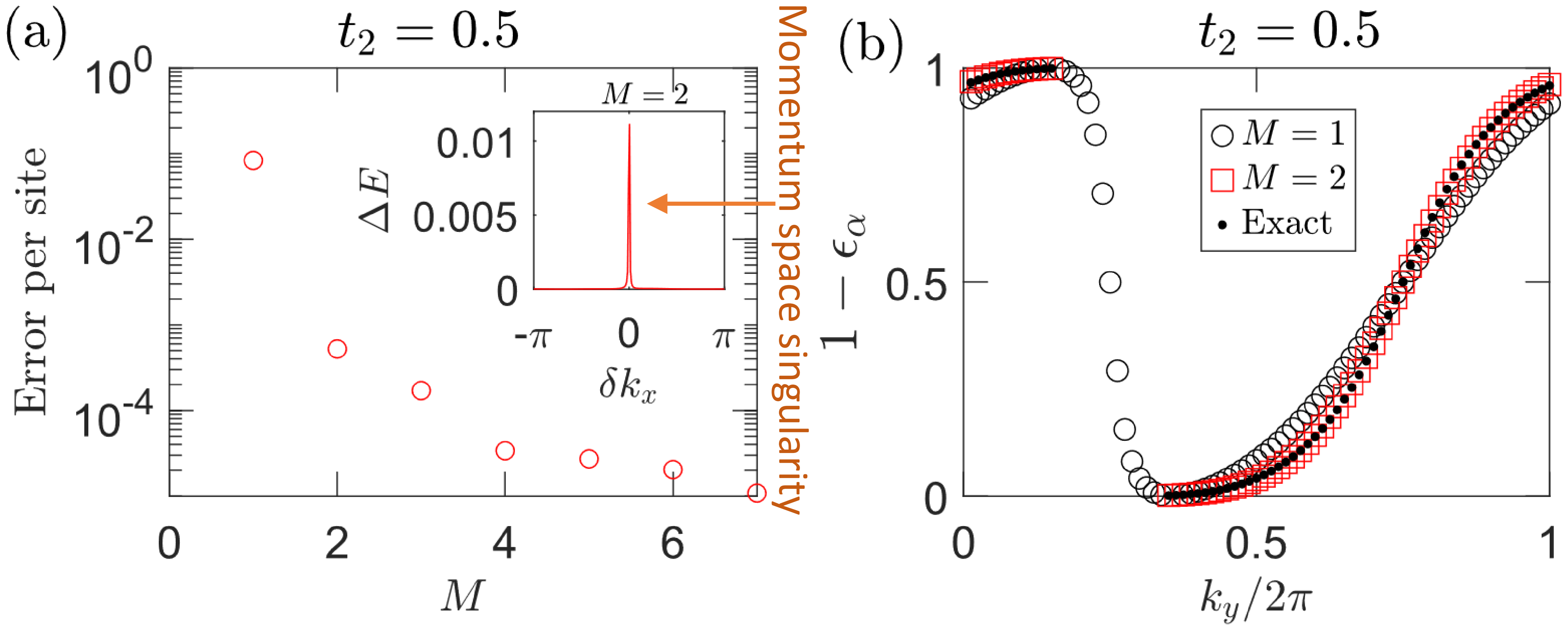


(a) Variational energy; (b) entanglement spectrum

States become chiral for $M \geq 2$

CSL with Gutzwiller projected Gaussian fermionic iPEPS

GfPEPS for Chern insulators: Hofstadter model, Chern number $C = 1$



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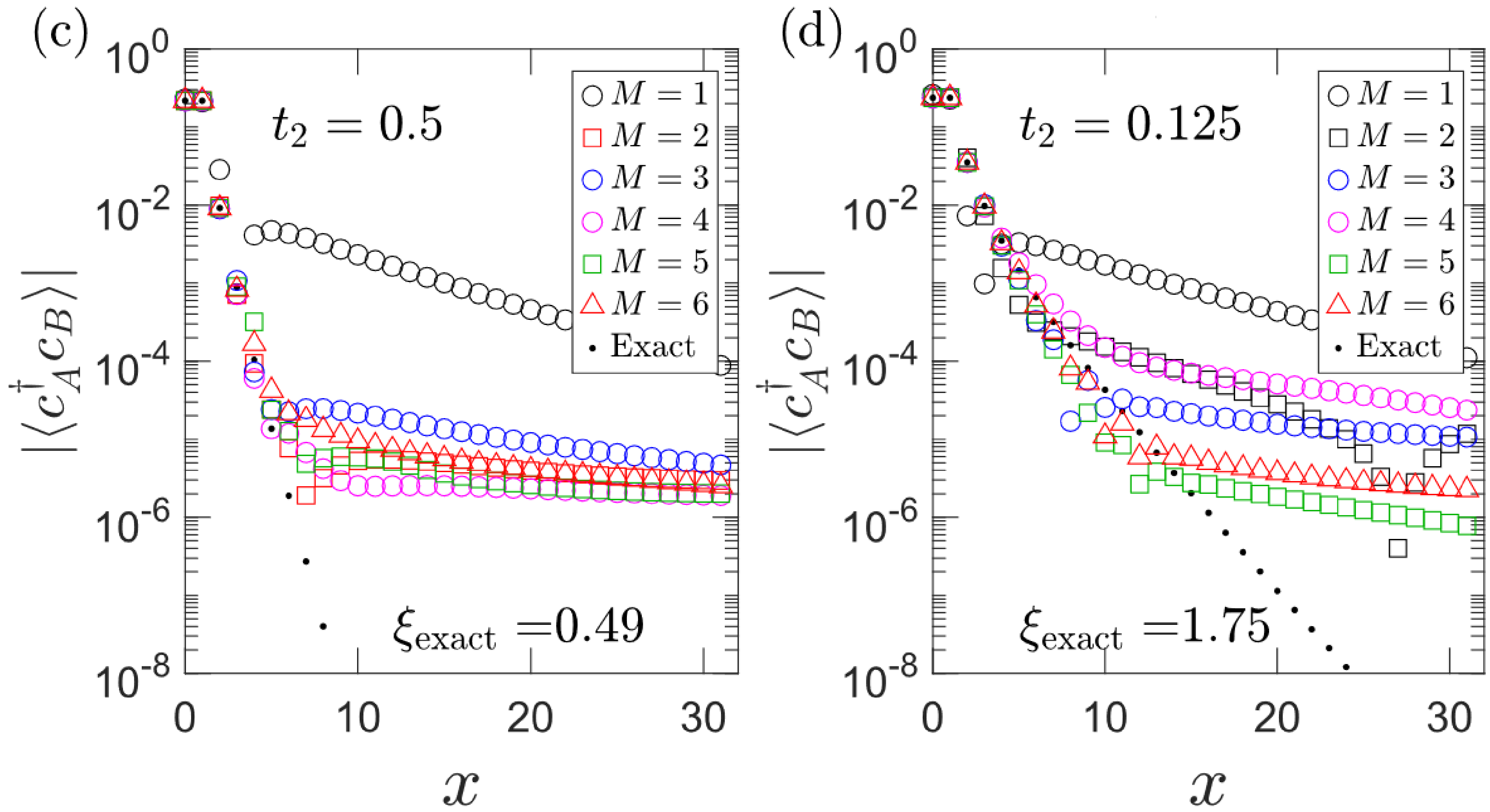

Correlation functions for different t_2 (bulk gap).

Finite M effects: no practical limitation

CSL with Gutzwiller projected Gaussian fermionic iPEPS

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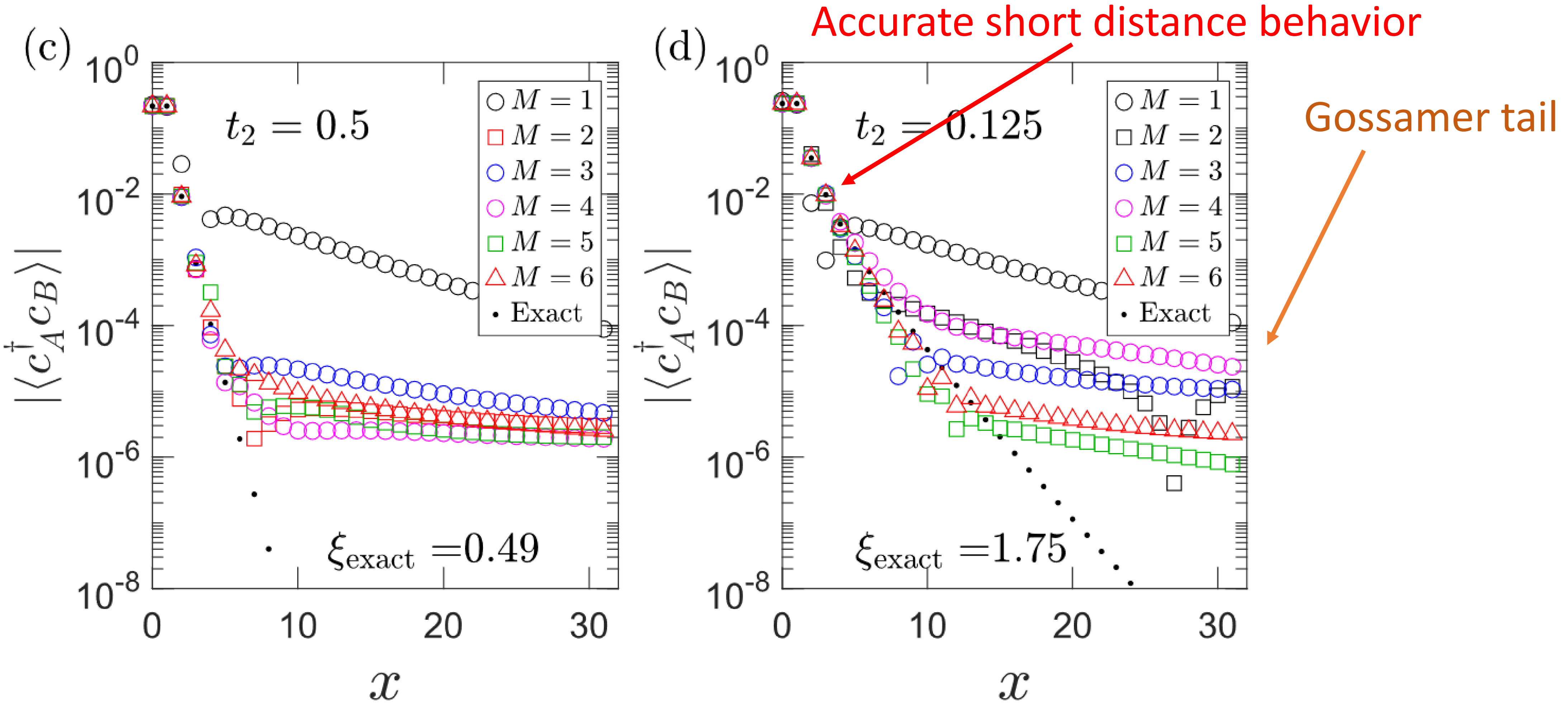

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CSL with Gutzwiller projected Gaussian fermionic iPEPS

What we learned and expect?

- GfPEPS represents Chern insulators faithfully with finite M

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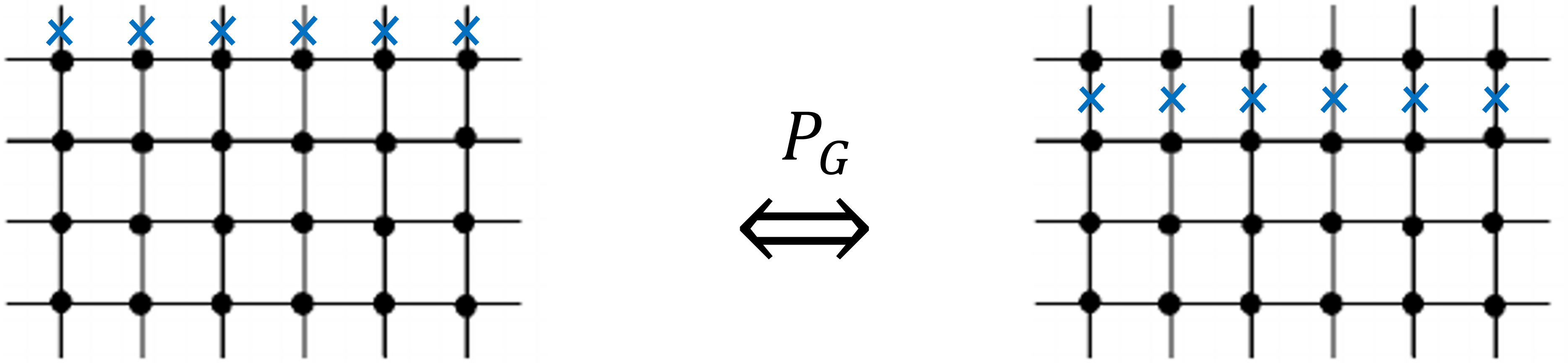
- GfPEPS represents Chern insulators faithfully with finite M
- We expect the projected spin state should exhibit expected number of branches
- We can take $M = 2$ for constructing CSL ansatz

CSL with Gutzwiller projected Gaussian fermionic iPEPS

Gutzwiller projected spin state: topological sectors from parton



- Unprojected states related by gauge transformation are equivalent after projection

CSL with Gutzwiller projected Gaussian fermionic iPEPS

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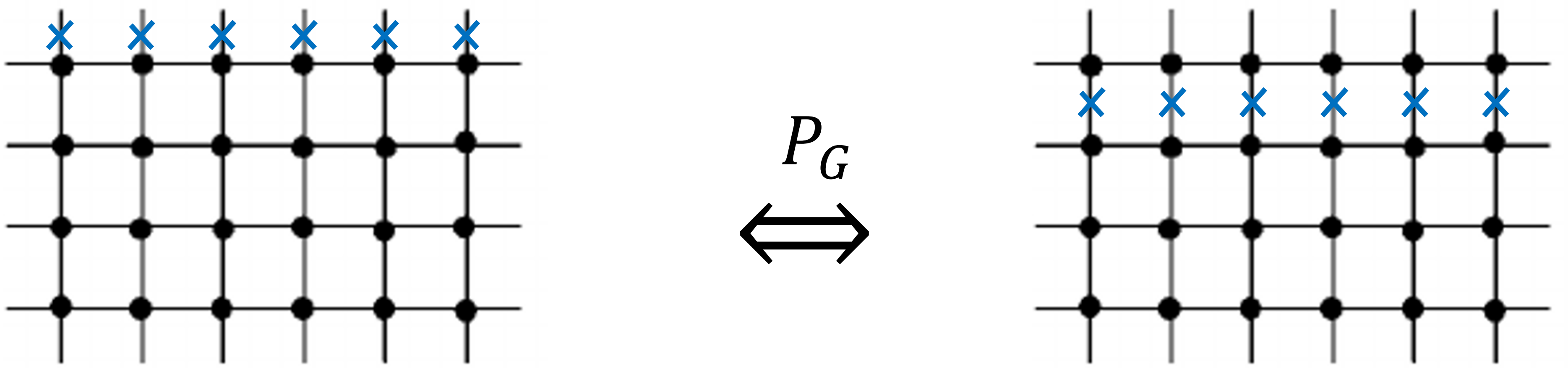

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- Anti-periodic boundary condition ⟺ flux insertion in virtual space

CSL with Gutzwiller projected Gaussian fermionic iPEPS

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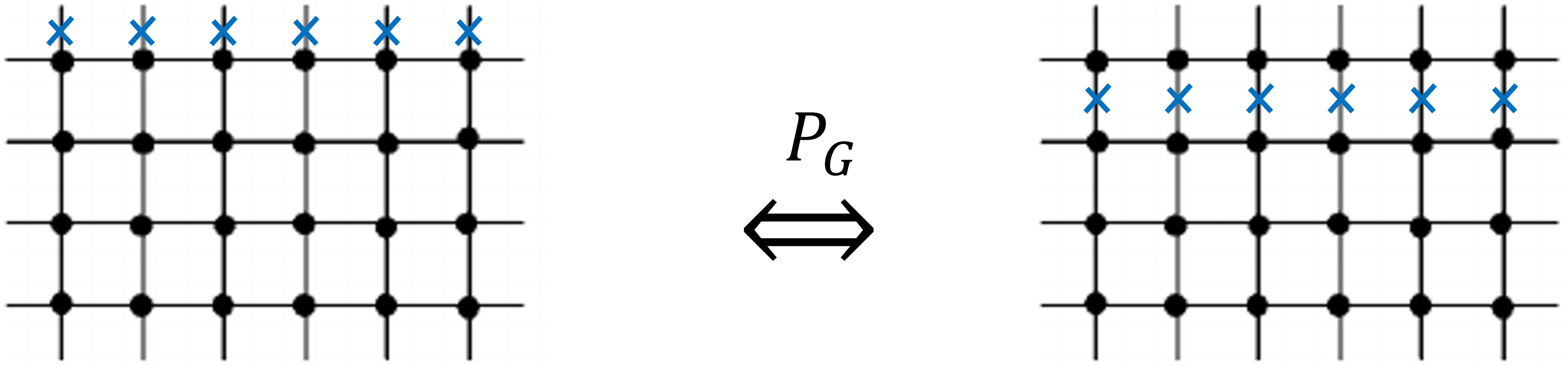
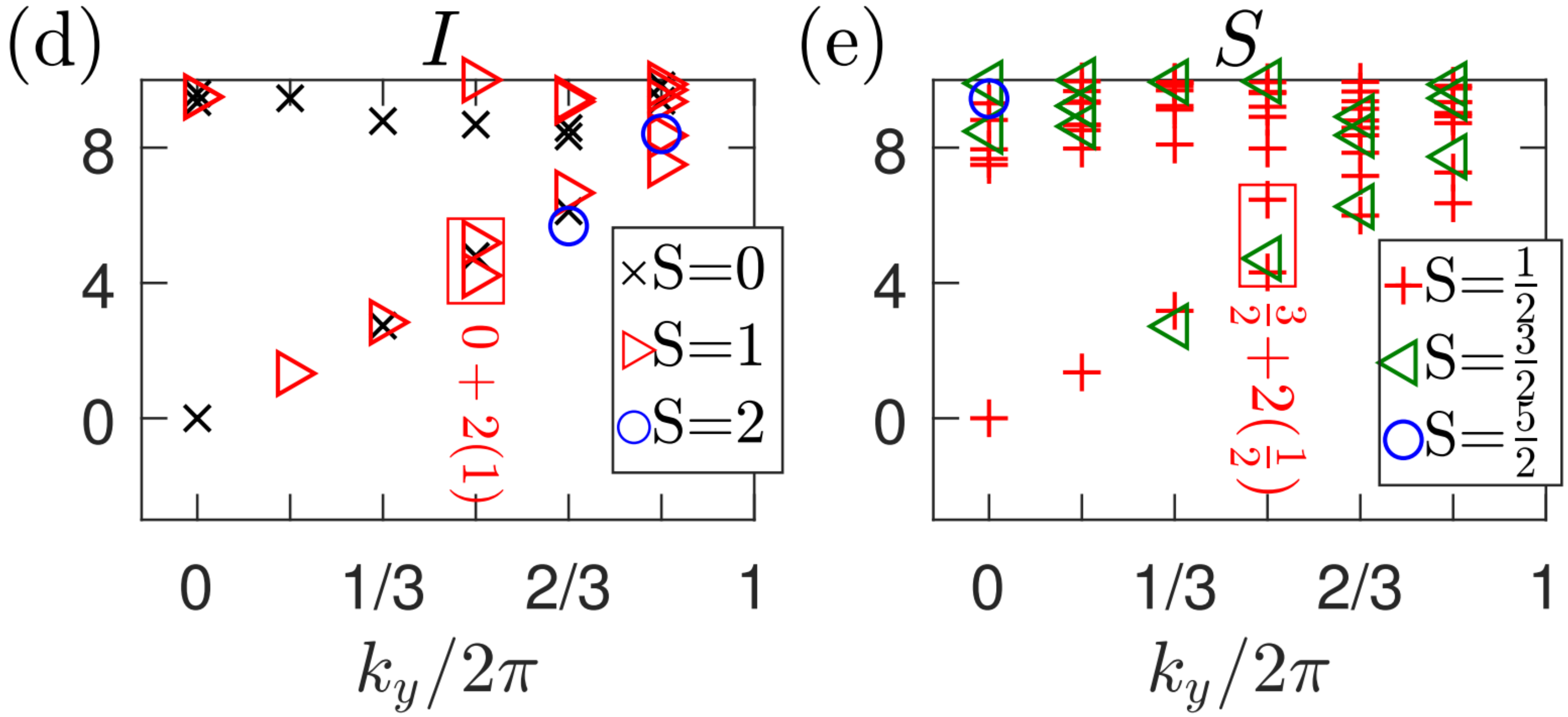


- Unprojected states related by gauge transformation are equivalent after projection
- Anti-periodic boundary condition ⟺ flux insertion in virtual space
- All minimally entangled states (MES) can be constructed

MES construction: Y. Zhang et al, PRB (2012); H.-H. Tu et al, PRB (2013)

CSL with Gutzwiller projected Gaussian fermionic iPEPS

Gutzwiller projected spin state: $C = 1 \Rightarrow$ Abelian $\mathrm{SU}(2)_1$ CSL



- Level counting matches $\mathrm{SU}(2)_1$ CFT with exactly one branch in each sector

CSL with Gutzwiller projected Gaussian fermionic iPEPS

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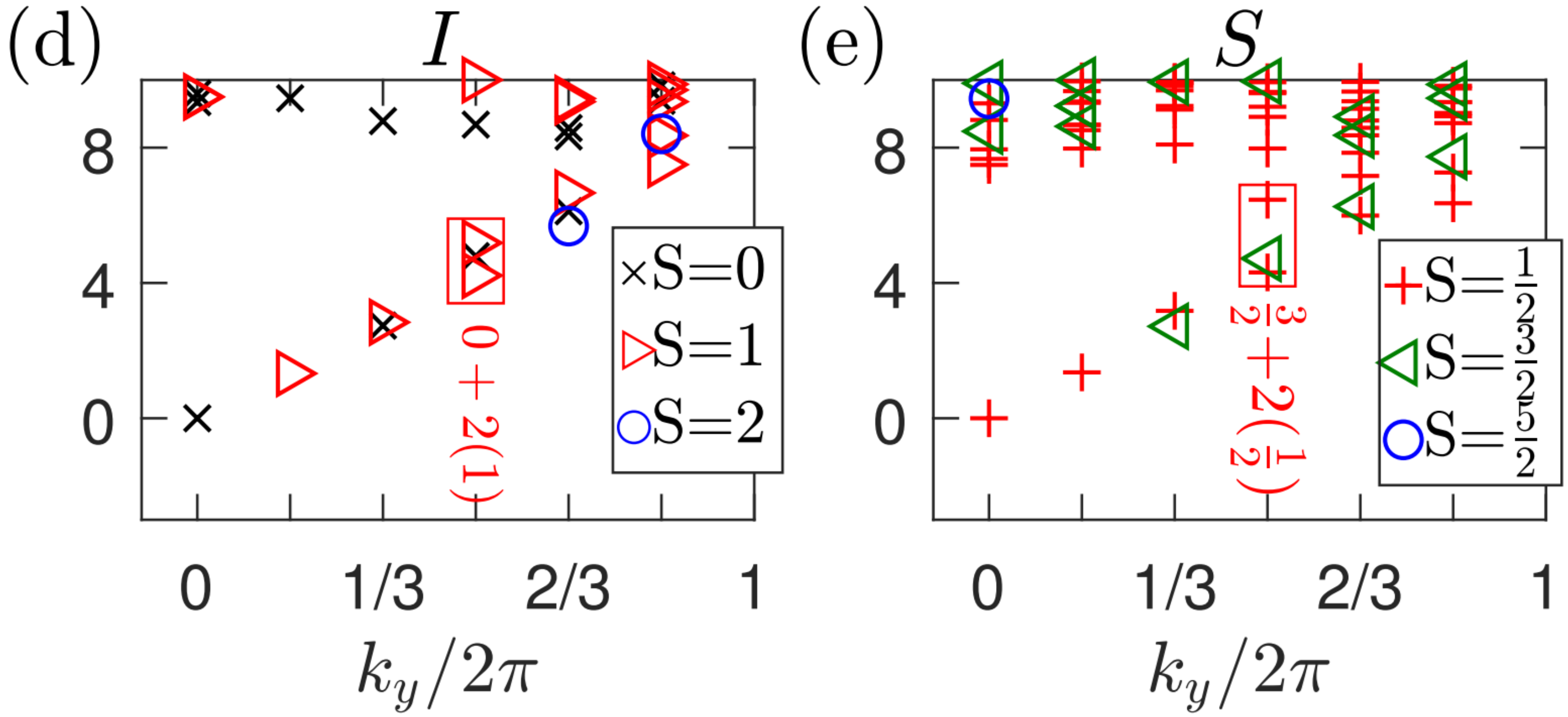

Main progress

- Level counting matches $\mathrm{SU}(2)_1$ CFT with exactly one branch in each sector

CSL with Gutzwiller projected Gaussian fermionic iPEPS

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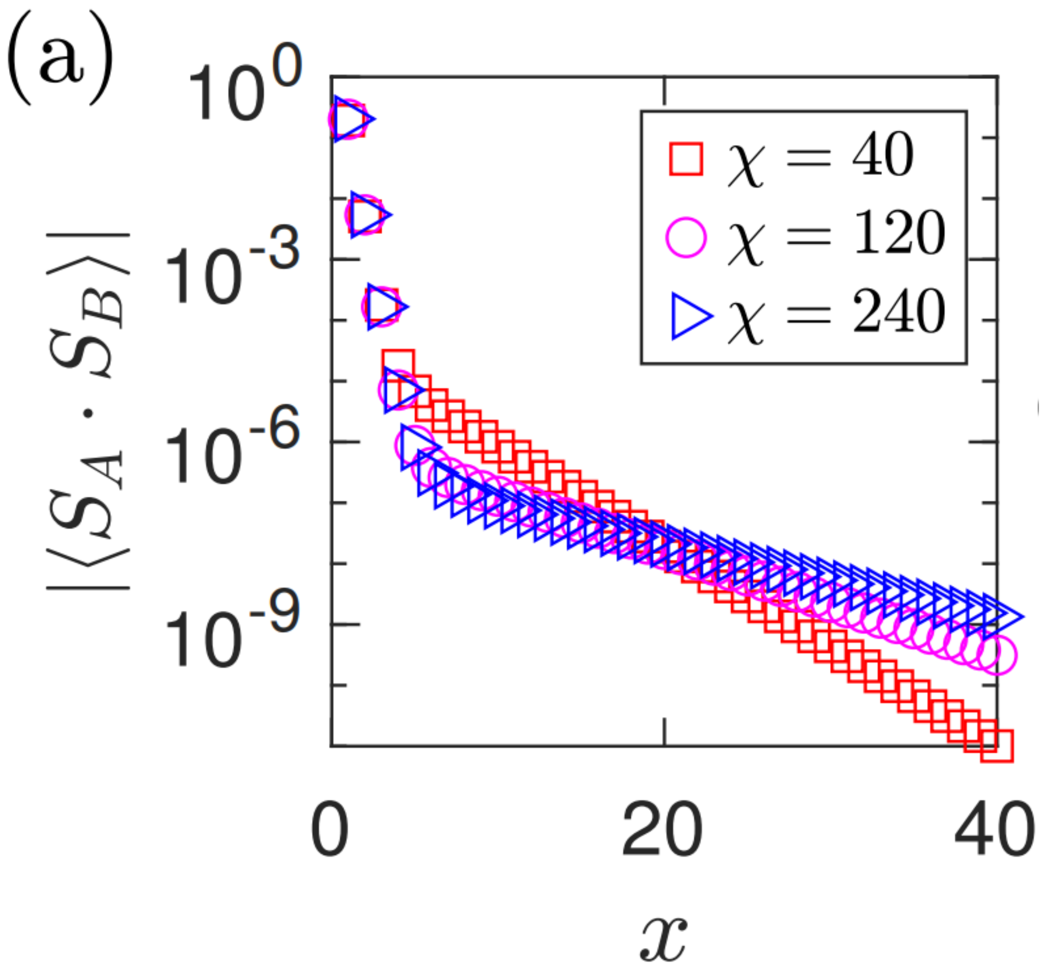

Spin-spin correlation with $M = 2$;
χ: CTMRG bond dimension

Gossamer tail

- Features of correlation functions are retained after projection

CSL with Gutzwiller projected Gaussian fermionic iPEPS

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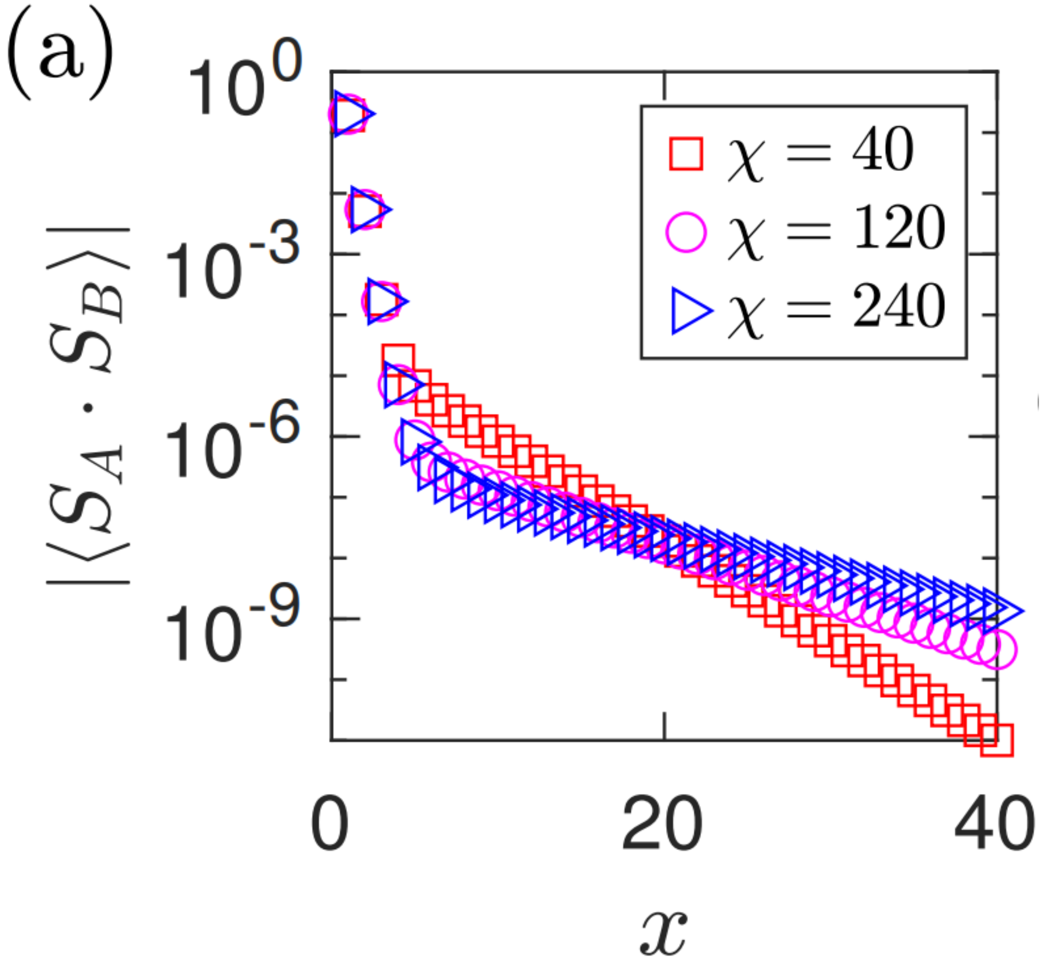

Spin-spin correlation with $M = 2$;
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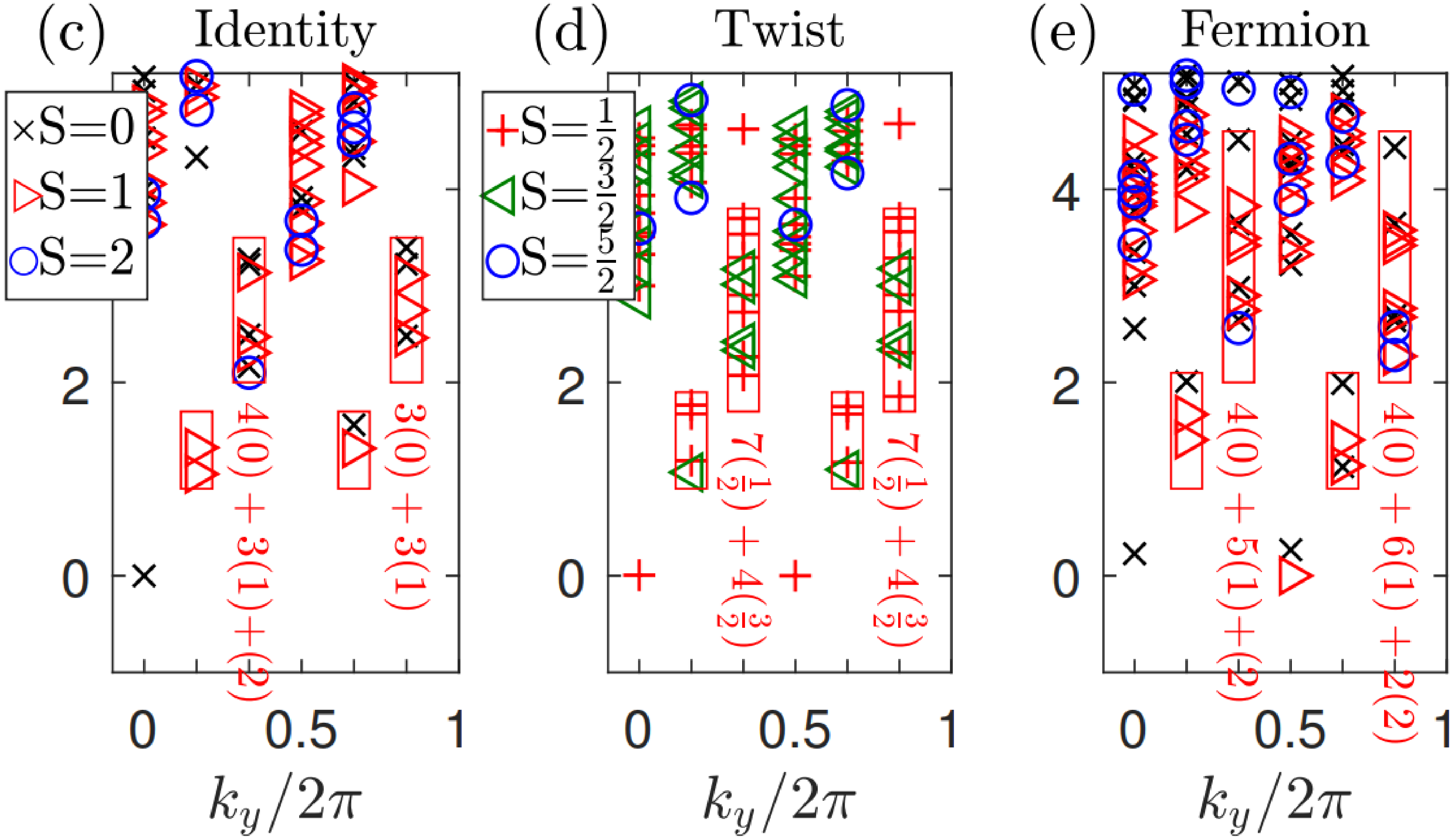
Gossamer tail

- Features of correlation functions are retained after projection
- Finite M effect?

CSL with Gutzwiller projected Gaussian fermionic iPEPS

Gutzwiller projected spin state: $C = 2 \Rightarrow$ non−Abelian $SO(5)_1$ CSL



- Level counting matches $SO(5)_1$ CFT with exact branch number in each sector

Y. Zhang et al, PRB (2014); Y.-H. Wu et al, PRB (2022)

CSL with Gutzwiller projected Gaussian fermionic iPEPS

Summary

- Fermionic iPEPS provides a faithful description of CSL edge spectra

CSL with Gutzwiller projected Gaussian fermionic iPEPS

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Attention: states with different entanglement properties could have similar variational energy!

CSL with Gutzwiller projected Gaussian fermionic iPEPS

Summary

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Outlook:

Use fermionic (virtual particle) iPEPS variational ansatz to simulate CSLs in spin models?

Conclusions

- iPEPS can represent generic CSLs, with faithful description of edge spectra;

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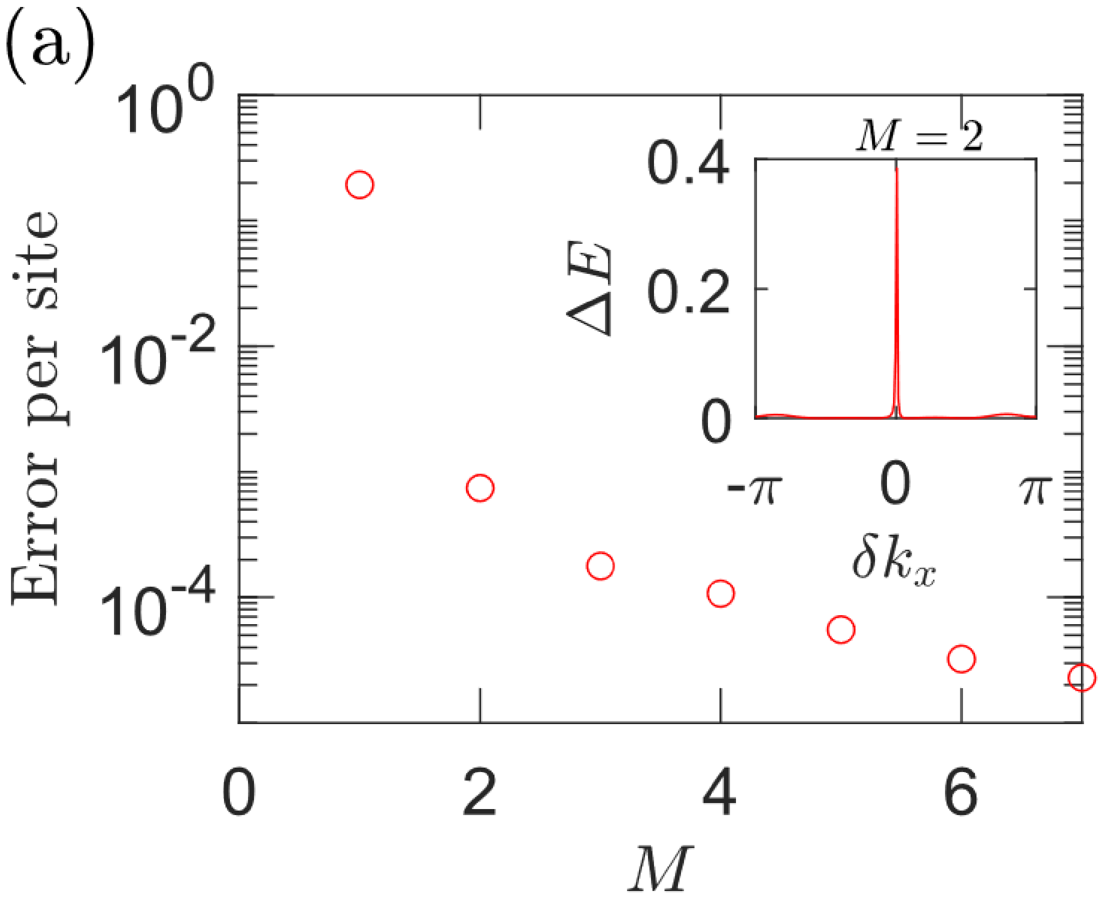
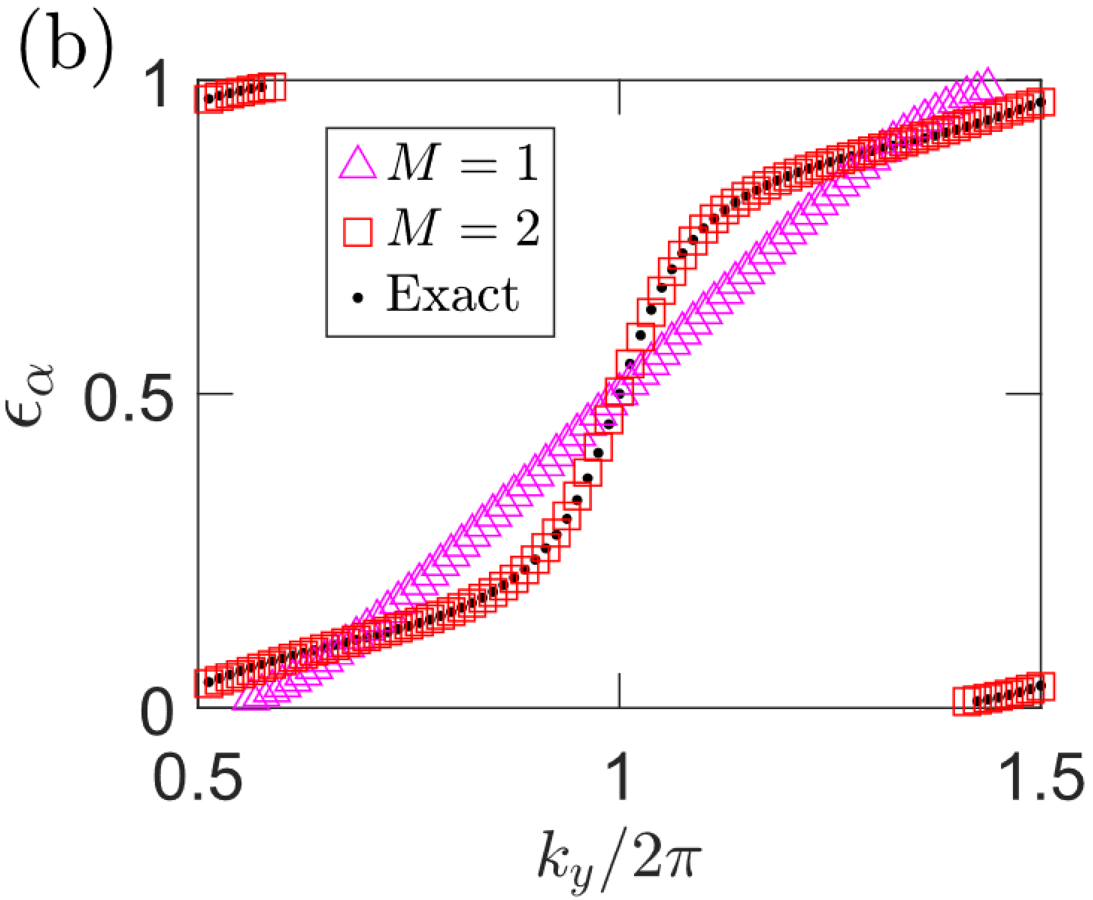
Thank you for your attention!

Appendix

CSL with Gutzwiller projected Gaussian fermionic iPEPS

GfPEPS for Chern insulators: scenario II Qi-Wu-Zhang model, Chern number $C = 1$





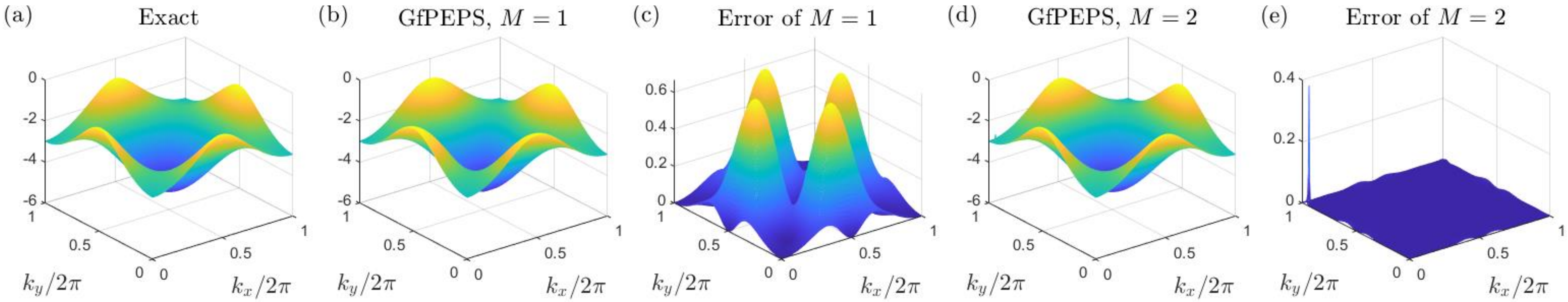
(a) Variational energy; (b) entanglement spectrum

States are chiral for $M \geq 1$, but dispersion becomes accurate for $M \geq 2$.

CSL with Gutzwiller projected Gaussian fermionic iPEPS

GfPEPS for Chern insulators: scenario II

Qi-Wu-Zhang model, Chern number $C = 1$



Single-particle energy in momentum space

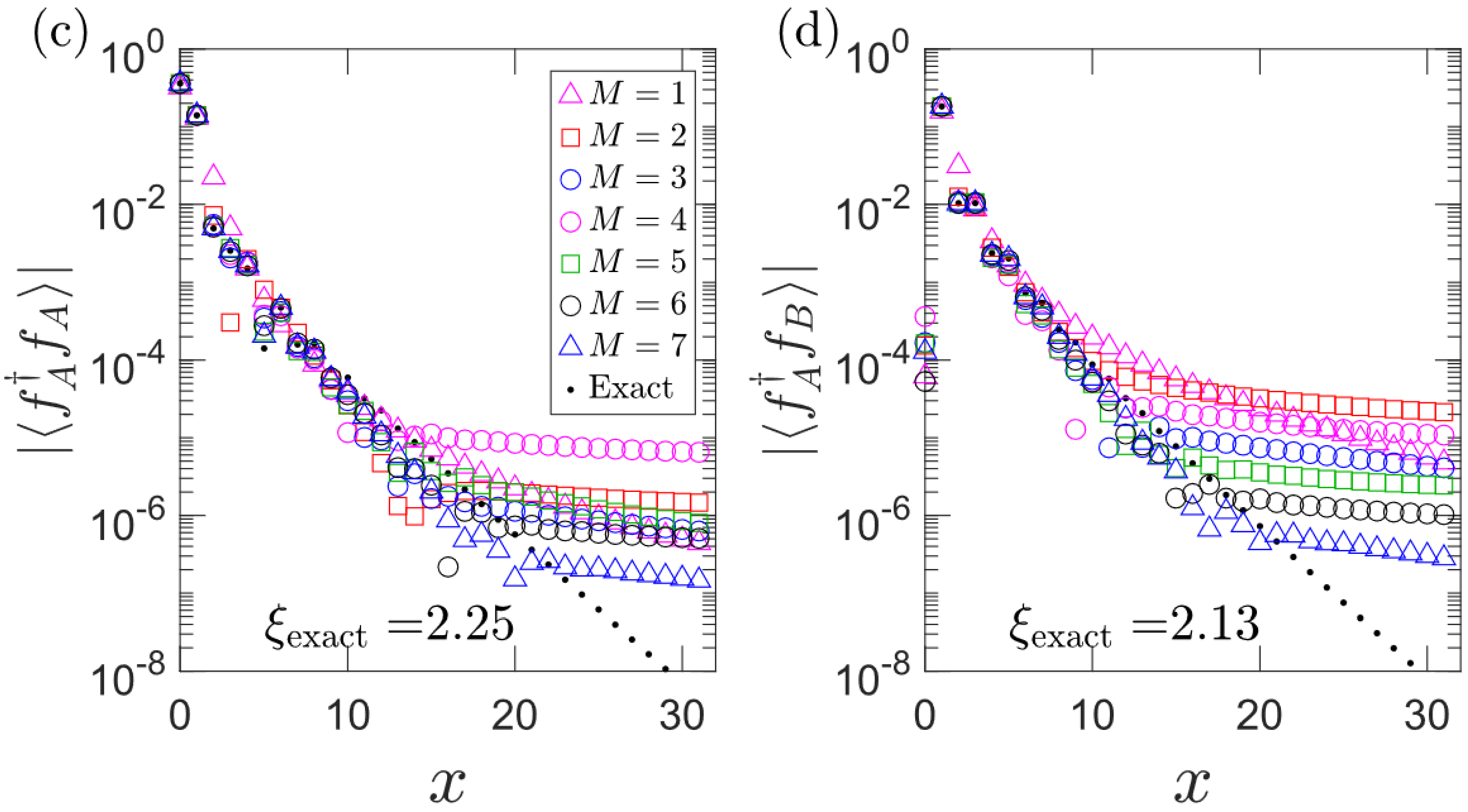
$M = 1$, smooth band, power-law decay correlations; similar to [T. B. Wahl et al, PRL (2013)]

$M \geq 2$, sharp singularity, crossover behavior in correlations.

CSL with Gutzwiller projected Gaussian fermionic iPEPS

GfPEPS for Chern insulators: scenario II

Qi-Wu-Zhang model, Chern number $C = 1$



Correlation functions.

Similar finite M effects