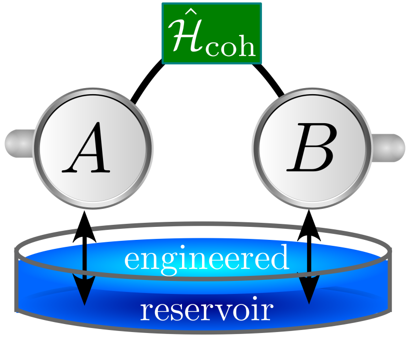
Engineering Nonreciprocal Devices



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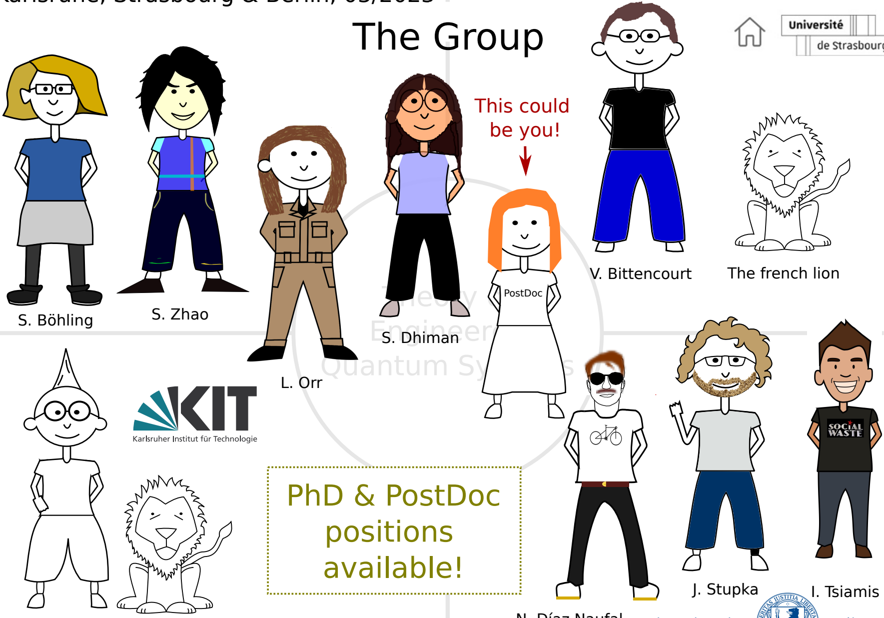
Spring School Benasque, April 13th 2023

Université de Strasbourg

Karlsruhe, Strasbourg & Berlin, 03/2023

The Group



Université de Strasbourg

Karlsruher Institut für Technologie

PhD & PostDoc positions available!

Freie Universität Berlin

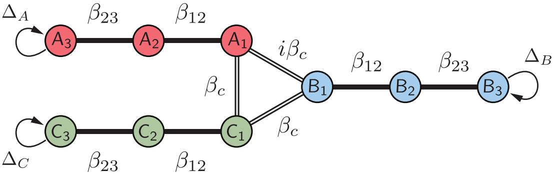
A brief disclaimer...

$\hbar = 1$

- no actual 'engineering' is going to happen
- there will be 'blobs' (like a lot..)
- lecturer-biased perspective

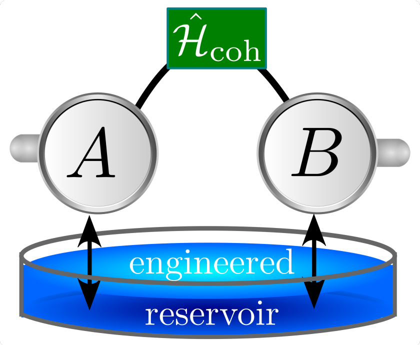
a range of other perspectives, e.g.

Graph-based Analysis in Coupled-Mode Systems
[L. Ranzani and J. Aumentado, New J. Phys. 2015]



Synthesis of Parametrically Coupled Networks
[O. Naaman and J. Aumentado, PRX Quantum 2022]

Engineering Nonreciprocal Devices

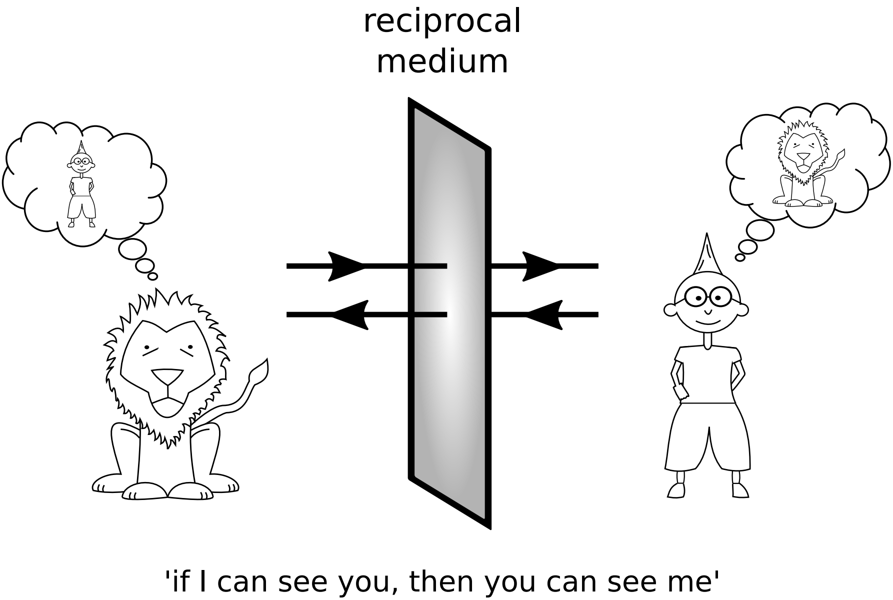


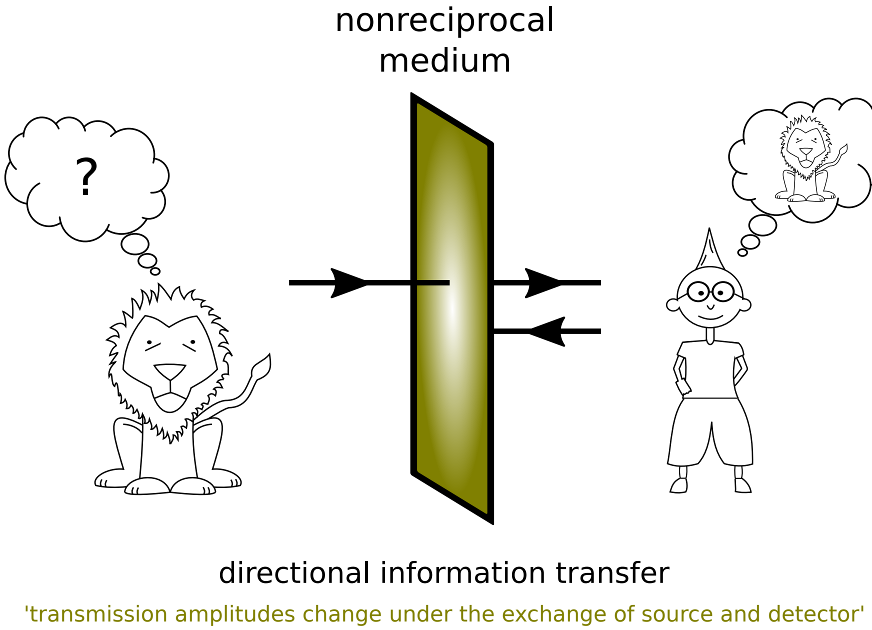
[A. Metelmann and A.A. Clerk, PRX 2015]

Lecture Notes of the Les Houches Summer School 2019, Quantum Information Machines

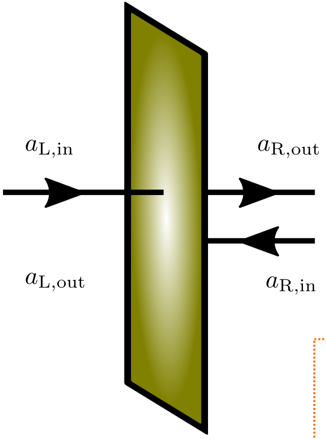
THE UNIVERSITY OF CHICAGO

[A.A. Clerk, SciPost Phys. Lect. Notes 44 (2022)]


reciprocal
medium
'if I can see you, then you can see me'


nonreciprocal
medium
?
directional information transfer
'transmission amplitudes change under the exchange of source and detector'

nonreciprocal medium



asymmetric scattering matrix

$$|s_{12}| \neq |s_{21}|$$

$$\begin{bmatrix} a_{\mathrm{L,out}} \\ a_{\mathrm{R,out}} \end{bmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{bmatrix} a_{\mathrm{L,in}} \\ a_{\mathrm{R,in}} \end{bmatrix} + f$$

unidirectional transmission from left to right

ToDo: Lorentz reciprocity theorem

violated for real s-matrix if: $S \neq S^T$

[D. Jalas et. al. Nat Photonics, 2013]

What happens for complex s-matrix?

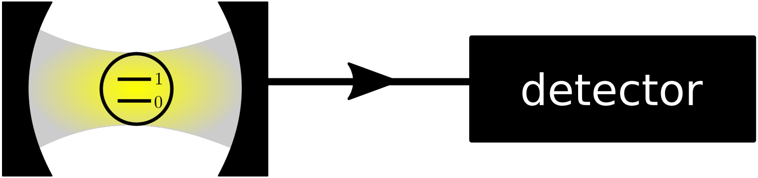
Quantum information processing

IBM 50 qubit system

[picture IBM - idea credit to John Teufel]

Quantum information processing

e.g. qubit in a cavity



aim: read-out system by protecting the source

⟶ requires nonreciprocal devices to control signal propagation

conventional microwave isolator

[MECA electronics]

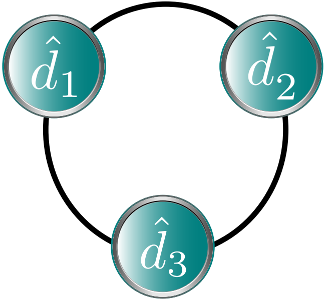
disadvantages of conventional devices:

(based on magneto-optical effect)

- bulky and no on-chip implementations
- require large magnetic fields
- plagued by losses
- no amplication of the signal

aim: new strategies for nonreciprocity in photonic devices

Let's start simple... breaking time-reversal symmetry



hopping among three coupled modes

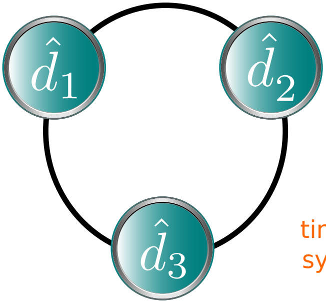
$$\hat{H} = Je^{i\varphi}\left(\hat{d}_1\hat{d}_2^\dagger + \hat{d}_2\hat{d}_3^\dagger + \hat{d}_3\hat{d}_1^\dagger + h.c.\right)$$

When is or is not time-reversal symmetry intact?

Intact if Gauge trafo makes $\hat{H}$ real-valued: $\hat{d}_n \rightarrow e^{i\varphi_n}\,\hat{d}_n$

more details for TRS: [J. Koch et al. PRA 82, 043811, 2010]

Let's start simple... breaking time-reversal symmetry



$$
\begin{array}{rl}
 & \varphi + \varphi_1 - \varphi_2 = n_1\pi \\
 & \varphi + \varphi_2 - \varphi_3 = n_2\pi \\
+ & \varphi + \varphi_3 - \varphi_1 = n_3\pi \\
\hline
 & 3\varphi = n\pi
\end{array}
$$

time-reversal symmetry is intact if: $\rightarrow \boxed{\varphi = \frac{n}{3}\pi, \;\; n \in Z}$

hopping among three coupled modes

$$\hat{H} = Je^{i\varphi}\left(\hat{d}_1\hat{d}_2^\dagger + \hat{d}_2\hat{d}_3^\dagger + \hat{d}_3\hat{d}_1^\dagger + h.c.\right)$$

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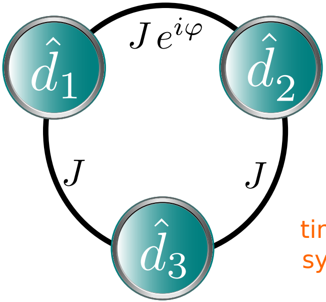
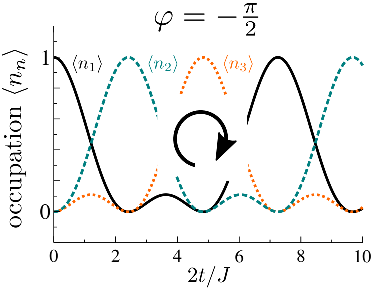
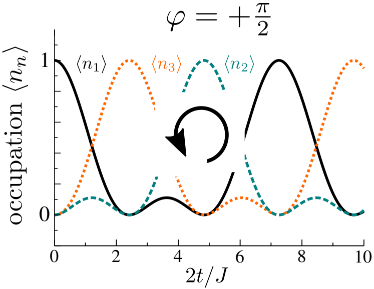
Let's start simple... breaking time-reversal symmetry



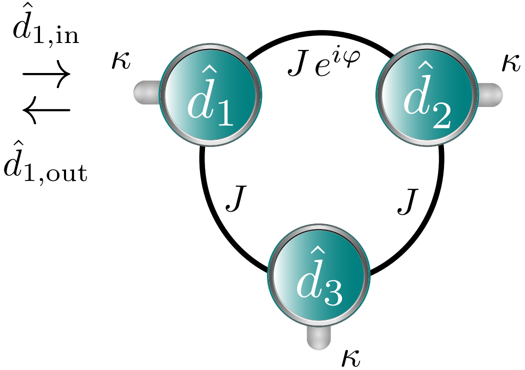
$$\begin{aligned}
\varphi + \varphi_1 - \varphi_2 &= n_1\pi \\
\varphi + \varphi_2 - \varphi_3 &= n_2\pi \\
+\quad \varphi + \varphi_3 - \varphi_1 &= n_3\pi \\
\hline
3\varphi &= n\pi
\end{aligned}$$

time-reversal symmetry is intact if: $\rightarrow \boxed{\varphi = \frac{n}{3}\pi,\ \ n \in Z}$





Adding input-output ports



quantum Langevin equations:

$$\frac{d}{dt}\hat{d}_n = i\left[\hat{H},\hat{d}_n\right] - \frac{\kappa}{2}\hat{d}_n - \sqrt{\kappa}\hat{d}_{n,\mathrm{in}}$$

input-output relation

$$\boxed{\hat{d}_{n,\mathrm{out}} = \hat{d}_{n,\mathrm{in}} + \sqrt{\kappa}\hat{d}_n}$$

[C. W. Gardiner and M. J. Collett, PRA 1985]

Adding input-output ports

$$\mathbf{D}_{\text{in}}$$

$$\frac{d}{dt}\hat{d}_1 = -iJe^{-i\varphi}\hat{d}_2 - iJ\hat{d}_3 - \frac{\kappa}{2}\hat{d}_1 - \sqrt{\kappa}\,\hat{d}_{1,\text{in}}$$

$$\frac{d}{dt}\hat{d}_2 = -iJe^{+i\varphi}\hat{d}_1 - iJ\hat{d}_3 - \frac{\kappa}{2}\hat{d}_2 - \sqrt{\kappa}\,\hat{d}_{2,\text{in}}$$

$$\frac{d}{dt}\hat{d}_3 = -iJ\hat{d}_1 - iJ\hat{d}_2 - \frac{\kappa}{2}\hat{d}_3 - \sqrt{\kappa}\,\hat{d}_{3,\text{in}}$$

coherent dynamics

diss. & fluctuations

$$\boxed{\hat{d}_{n,\text{out}} = \hat{d}_{n,\text{in}} + \sqrt{\kappa}\hat{d}_n} \quad \Rightarrow \mathbf{D}_{\text{out}} = \mathbf{s}\,\mathbf{D}_{\text{in}}$$

$$\mathcal{C} = \frac{4J^2}{\kappa^2}$$

$$\varphi = \pm\frac{\pi}{2}$$

$$\mathbf{s} = \frac{2}{3\mathcal{C}+1}\begin{pmatrix} \frac{1}{2}(1-\mathcal{C}) & \pm\sqrt{\mathcal{C}}+\mathcal{C} & \mp i\mathcal{C}+i\sqrt{\mathcal{C}} \\ \mp\sqrt{\mathcal{C}}+\mathcal{C} & \frac{1}{2}(1-\mathcal{C}) & \pm i\mathcal{C}+i\sqrt{\mathcal{C}} \\ \pm i\mathcal{C}+i\sqrt{\mathcal{C}} & \mp i\mathcal{C}+i\sqrt{\mathcal{C}} & \frac{1}{2}(1-\mathcal{C}) \end{pmatrix}$$

Adding input-output ports

$$\mathbf{D}_{\text{in}}$$

$$\frac{d}{dt}\hat{d}_1 = -iJe^{-i\varphi}\hat{d}_2 - iJ\hat{d}_3 - \frac{\kappa}{2}\hat{d}_1 - \sqrt{\kappa}\;\hat{d}_{1,\text{in}}$$

$$\frac{d}{dt}\hat{d}_2 = -iJe^{+i\varphi}\hat{d}_1 - iJ\hat{d}_3 - \frac{\kappa}{2}\hat{d}_2 - \sqrt{\kappa}\;\hat{d}_{2,\text{in}}$$

$$\frac{d}{dt}\hat{d}_3 = -iJ\qquad\hat{d}_1 - iJ\hat{d}_2 - \frac{\kappa}{2}\hat{d}_3 - \sqrt{\kappa}\;\hat{d}_{3,\text{in}}$$

coherent dynamics

diss. & fluctuations

$$\mathcal{C} = \frac{4J^2}{\kappa^2}$$

$$\boxed{\hat{d}_{n,\text{out}} = \hat{d}_{n,\text{in}} + \sqrt{\kappa}\hat{d}_n} \quad \Rightarrow \mathbf{D}_{\text{out}} = \mathbf{s}\;\mathbf{D}_{\text{in}}$$

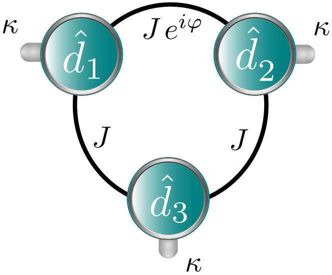
$$\boxed{\begin{array}{c}\varphi = \pm\frac{\pi}{2} \\ \mathcal{C} = 1\end{array}}$$

$$\varphi = -\frac{\pi}{2} \qquad \mathbf{s} = \begin{pmatrix} 0 & 0 & i \\ 1 & 0 & 0 \\ 0 & i & 0 \end{pmatrix}$$

$$\varphi = +\frac{\pi}{2} \qquad \mathbf{s} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & i \\ i & 0 & 0 \end{pmatrix}$$

In summary

$$\hat{H} = J\left(\hat{d}_1\hat{d}_2^\dagger\, e^{i\varphi} + \hat{d}_2\hat{d}_3^\dagger\, + \hat{d}_3\hat{d}_1^\dagger + h.c.\right)$$



two conditions:

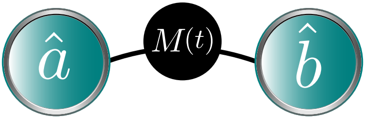
$$\boxed{\begin{array}{c}\varphi = \pm\frac{\pi}{2} \\ \mathcal{C} = 1\end{array}}$$

$$\mathcal{C} = \frac{4J^2}{\kappa^2}$$

$$\varphi = -\frac{\pi}{2} \qquad\qquad \varphi = +\frac{\pi}{2}$$

$$\mathbf{s} = \begin{pmatrix} 0 & 0 & i \\ 1 & & 0 \\ 0 & i & 0 \end{pmatrix} \qquad \mathbf{s} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & & \\ i & 0 & 0 \end{pmatrix}$$

Example: two coupled oscillators



$$\hat{\mathcal{H}}' = M(t)\left(\hat{a}\hat{b}^\dagger e^{-i(\omega_A - \omega_B)t} + \hat{a}\hat{b}e^{-i(\omega_A+\omega_B)t} + h.c.\right)$$

periodic modulation

$$M(t) = 2\lambda\cos(\omega_P t + \phi)$$

modulation at frequency difference: $\omega_P = \omega_A - \omega_B$

frequency conversion

$$\hat{\mathcal{H}}' = \lambda\left(\hat{a}\hat{b}^\dagger e^{+i\phi} + \hat{a}^\dagger\hat{b}e^{-i\phi}\right) + \hat{\mathcal{H}}'_{\mathrm{CR}}(t)$$

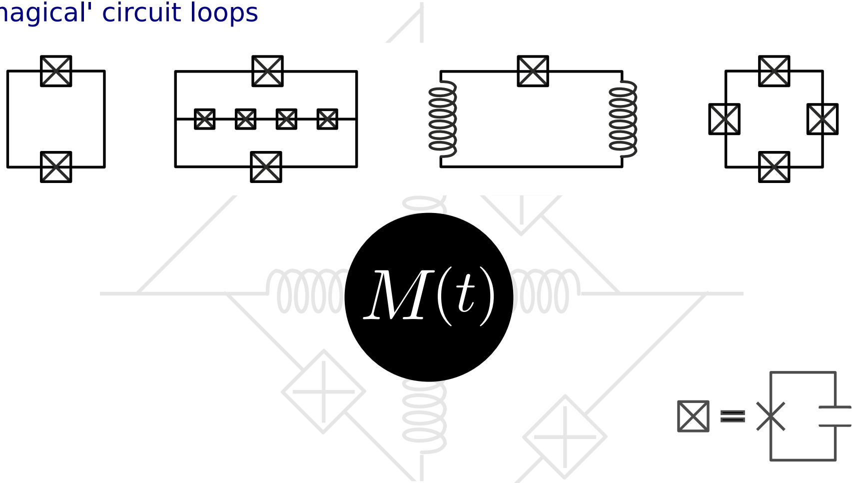
modulation at frequency sum: $\omega_P = \omega_A + \omega_B$

parametric amplification

$$\hat{\mathcal{H}}' = \lambda\left(\hat{a}\hat{b}e^{+i\phi} + \hat{a}^\dagger\hat{b}^\dagger e^{-i\phi}\right) + \hat{\mathcal{H}}''_{\mathrm{CR}}(t)$$

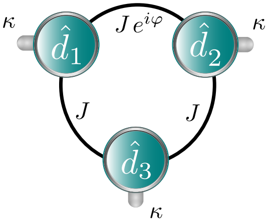
Parametric Couplings in Superconducting Circuits

'magical' circuit loops



- circuit loops intersected with Josephson junctions
- tunable couplers via external/internal modulation
- couplings between linear and nonlinear resonators

Back to the circulator...



$$\frac{d}{dt}\hat{d}_1 = -iJe^{-i\varphi}\hat{d}_2 - iJ\hat{d}_3 - \frac{\kappa}{2}\hat{d}_1 - \sqrt{\kappa}\;\hat{d}_{1,\mathrm{in}}$$

$$\frac{d}{dt}\hat{d}_2 = -iJe^{+i\varphi}\hat{d}_1 - iJ\hat{d}_3 - \frac{\kappa}{2}\hat{d}_2 - \sqrt{\kappa}\;\hat{d}_{2,\mathrm{in}}$$

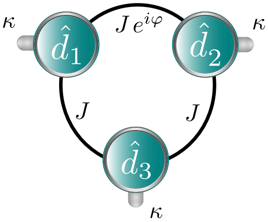
$$\frac{d}{dt}\hat{d}_3 = -iJ\qquad\hat{d}_1 - iJ\hat{d}_2 - \frac{\kappa_3}{2}\hat{d}_3 - \sqrt{\kappa_3}\,\hat{d}_{3,\mathrm{in}}$$

coherent dynamics

diss. & fluctuations

$$\kappa_3 \longrightarrow \infty$$

Back to the circulator...



$$\frac{d}{dt}\hat{d}_1 = -iJe^{-i\varphi}\hat{d}_2 - iJ\hat{d}_3 - \frac{\kappa}{2}\hat{d}_1 - \sqrt{\kappa}\ \hat{d}_{1,\mathrm{in}}$$

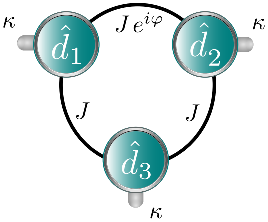
$$\frac{d}{dt}\hat{d}_2 = -iJe^{+i\varphi}\hat{d}_1 - iJ\hat{d}_3 - \frac{\kappa}{2}\hat{d}_2 - \sqrt{\kappa}\ \hat{d}_{2,\mathrm{in}}$$

adiabatic elimination of mode 3:

$$\hat{d}_3 = -i\frac{2J}{\kappa_3}\hat{d}_1 - i\frac{2J}{\kappa_3}\hat{d}_2 - \frac{2}{\sqrt{\kappa_3}}\hat{d}_{3,\mathrm{in}}$$

$$\Gamma = \frac{4J^2}{\kappa_3} \quad \Rightarrow \quad \boxed{J\hat{d}_3 = -i\frac{\Gamma}{2}\hat{d}_1 - i\frac{\Gamma}{2}\hat{d}_2 - \sqrt{\Gamma}\hat{d}_{3,\mathrm{in}}}$$

Back to the circulator...



coherent coupling

$$\frac{d}{dt}\hat{d}_1 = -\left[iJe^{-i\varphi} + \frac{\Gamma}{2}\right]\hat{d}_2 - \frac{\kappa+\Gamma}{2}\hat{d}_1 - \sqrt{\kappa}\hat{d}_{1,\mathrm{in}} + i\sqrt{\Gamma}\hat{d}_{3,\mathrm{in}}$$

$$\frac{d}{dt}\hat{d}_2 = -\left[iJe^{+i\varphi} + \frac{\Gamma}{2}\right]\hat{d}_1 - \frac{\kappa+\Gamma}{2}\hat{d}_2 - \sqrt{\kappa}\hat{d}_{2,\mathrm{in}} + i\sqrt{\Gamma}\hat{d}_{3,\mathrm{in}}$$

dissipative coupling

modified diss. & fluctuations

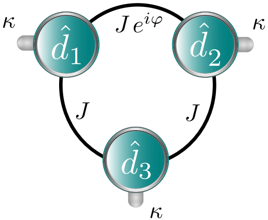
$\varphi = \pm\frac{\pi}{2}$

$\mathcal{C} = 1$

$\Rightarrow J = \frac{\Gamma}{2}$

$$\Gamma = \frac{4J^2}{\kappa_3}$$

Back to the circulator...



$$\varphi = -\frac{\pi}{2}$$

$$\frac{d}{dt}\hat{d}_1 = \quad - \frac{\kappa+\Gamma}{2}\hat{d}_1 - \sqrt{\kappa}\hat{d}_{1,\mathrm{in}} + i\sqrt{\Gamma}\hat{d}_{3,\mathrm{in}}$$

$$\frac{d}{dt}\hat{d}_2 = \quad -\Gamma \ \hat{d}_1 - \frac{\kappa+\Gamma}{2}\hat{d}_2 - \sqrt{\kappa}\hat{d}_{2,\mathrm{in}} + i\sqrt{\Gamma}\hat{d}_{3,\mathrm{in}}$$

$\varphi = \pm\frac{\pi}{2}$

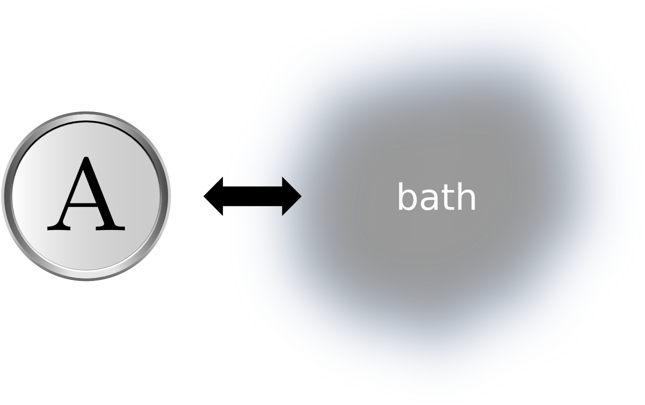
$\mathcal{C} = 1$

$\Rightarrow J = \frac{\Gamma}{2}$

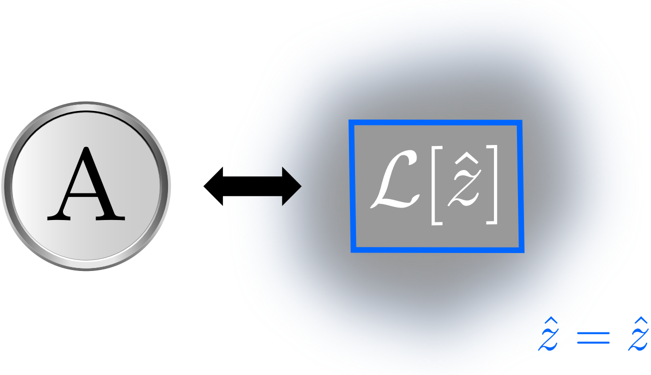
nonreciprocal interaction:
mode 2 is driven by mode 1
but not vice versa

$$\Gamma = \frac{4J^2}{\kappa_3}$$

'conventional' dissipation is uncontrolled



'conventional' dissipation is uncontrolled

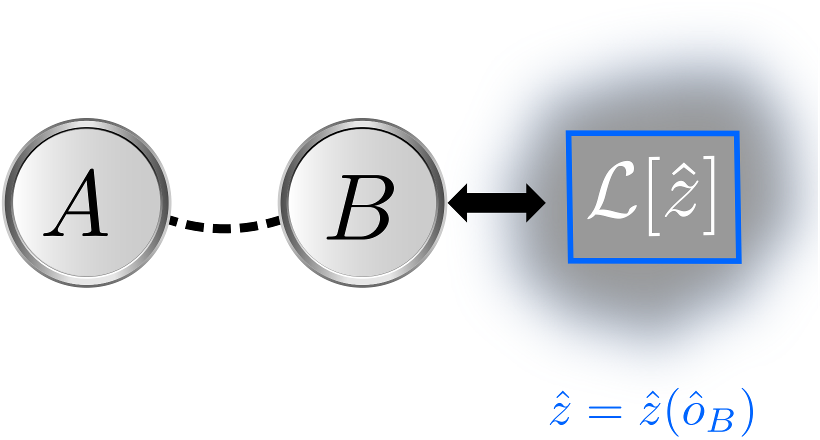


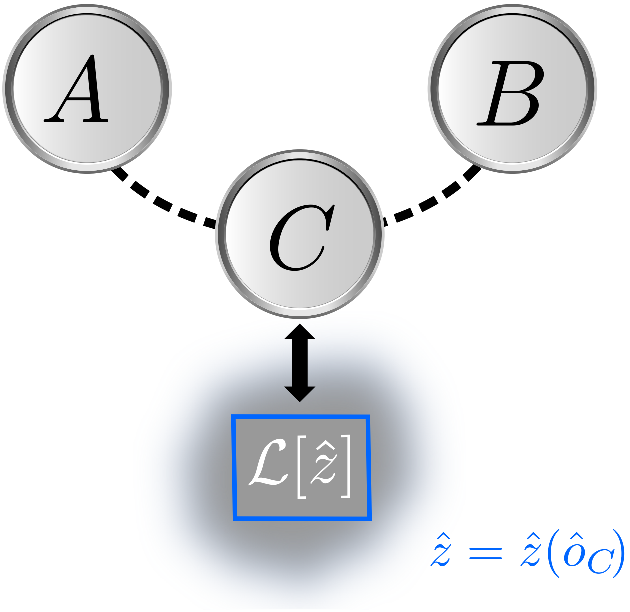
$$\hat{z} = \hat{z}(\hat{o}_A)$$

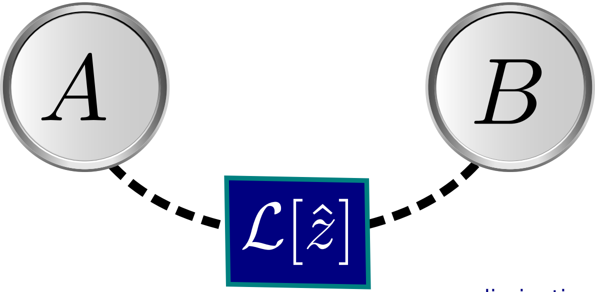
$$\frac{d}{dt}\hat{\rho} = -i\left[\hat{\mathcal{H}}_{\mathrm{A}}, \hat{\rho}\right] + \mathcal{L}\left[\hat{z}\right]\hat{\rho}$$

$$\mathcal{L}\left[\hat{o}\right]\hat{\rho} = \hat{o}\hat{\rho}\hat{o}^\dagger - \frac{1}{2}\hat{o}^\dagger\hat{o}\hat{\rho} - \frac{1}{2}\hat{\rho}\hat{o}^\dagger\hat{o}$$

engineered dissipation is controlled




A
B
C
$\mathcal{L}[\hat{z}]$
$\hat{z} = \hat{z}(\hat{o}_C)$



dissipative coupling

$$\hat{z} = \hat{z}(\hat{o}_A, \hat{o}_B)$$

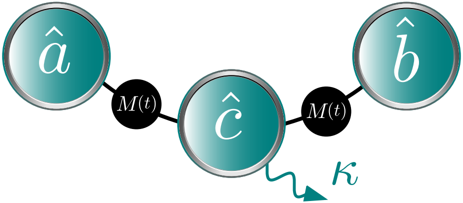
$$\frac{d}{dt}\hat{\rho} = -i\left[\hat{\mathcal{H}}, \hat{\rho}\right] + \mathcal{L}\left[\hat{z}\right]\hat{\rho}$$

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}(\hat{o}_A, \hat{o}_B)$$

coherent coupling

$$\mathcal{L}\left[\hat{o}\right]\hat{\rho} = \hat{o}\hat{\rho}\hat{o}^\dagger - \frac{1}{2}\hat{o}^\dagger\hat{o}\hat{\rho} - \frac{1}{2}\hat{\rho}\hat{o}^\dagger\hat{o}$$

Example: two indirectly coupled oscillators



e.g. each coupling modulated at frequency difference:

$$\hat{\mathcal{H}} = \Delta \hat{c}^\dagger \hat{c} + \left[\lambda \hat{c}^\dagger \left(\hat{a} + \hat{b}\right) + h.c.\right] + \hat{\mathcal{H}}_{\mathrm{diss}}$$

adiabatic elimination of mode C:

$$\mathcal{L}\left[\hat{o}\right]\hat{\rho} = \hat{o}\hat{\rho}\hat{o}^\dagger - \frac{1}{2}\hat{o}^\dagger\hat{o}\hat{\rho} - \frac{1}{2}\hat{\rho}\hat{o}^\dagger\hat{o}$$

$$\frac{d}{dt}\hat{\rho} = i\left[\Lambda\left(\hat{a}^\dagger + \hat{b}^\dagger\right)\left(\hat{a} + \hat{b}\right), \hat{\rho}\right] + \Gamma\mathcal{L}\left[\hat{a} + \hat{b}\right]\hat{\rho}$$

coherent coupling

$$\Lambda = \frac{\Delta\lambda^2}{\left(\Delta^2 + \frac{\kappa^2}{4}\right)}$$

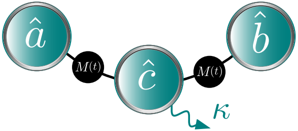
dissipative coupling

$$\Gamma = \frac{\kappa\lambda^2}{\left(\Delta^2 + \frac{\kappa^2}{4}\right)}$$

What is the difference?

decomposition of master equation:

$$\Gamma = \frac{\kappa\lambda^2}{\left(\Delta^2+\frac{\kappa^2}{4}\right)} \qquad \Lambda = \frac{\Delta\lambda^2}{\left(\Delta^2+\frac{\kappa^2}{4}\right)}$$



local processes

$$\frac{d}{dt}\hat{\rho} = \boxed{-i\Lambda\left[\hat{a}^\dagger\hat{a}+\hat{b}^\dagger\hat{b},\hat{\rho}\right]+\Gamma\mathcal{L}\left[\hat{a}\right]\hat{\rho}+\Gamma\mathcal{L}\left[\hat{b}\right]\hat{\rho}}$$

$$\boxed{-i\Lambda\left[\hat{a}\hat{b}^\dagger+\hat{a}^\dagger\hat{b},\hat{\rho}\right]+\Gamma\left[\hat{a}\hat{\rho}\hat{b}^\dagger-\frac{1}{2}\left\{\hat{a}\hat{b}^\dagger,\hat{\rho}\right\}+h.c\right]}$$

non-local processes

local processes

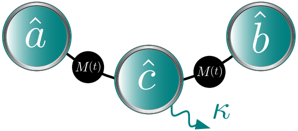
- Λ: frequency shift
- Γ: damping

non-local processes

- Λ: coherent hopping
- Γ: dissipative hopping

$$\mathcal{L}\left[\hat{o}\right]\hat{\rho} = \hat{o}\hat{\rho}\hat{o}^\dagger - \frac{1}{2}\hat{o}^\dagger\hat{o}\hat{\rho} - \frac{1}{2}\hat{\rho}\hat{o}^\dagger\hat{o}$$

What is the difference?



decomposition of master equation:

$$\Gamma = \frac{\kappa\lambda^2}{\left(\Delta^2+\frac{\kappa^2}{4}\right)} \qquad \Lambda = \frac{\Delta\lambda^2}{\left(\Delta^2+\frac{\kappa^2}{4}\right)}$$

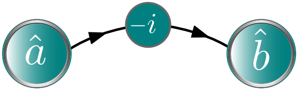
local processes

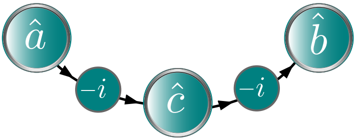
$$\frac{d}{dt}\hat{\rho} = -i\Lambda\left[\hat{a}^\dagger\hat{a}+\hat{b}^\dagger\hat{b},\hat{\rho}\right]+\Gamma\mathcal{L}\left[\hat{a}\right]\hat{\rho}+\Gamma\mathcal{L}\left[\hat{b}\right]\hat{\rho}$$

$$-i\Lambda\left[\hat{a}\hat{b}^\dagger+\hat{a}^\dagger\hat{b},\hat{\rho}\right]+\Gamma\left[\hat{a}\hat{\rho}\hat{b}^\dagger-\frac{1}{2}\left\{\hat{a}\hat{b}^\dagger,\hat{\rho}\right\}+h.c\right]$$

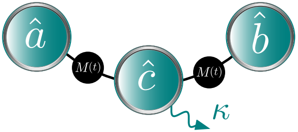
non-local processes

- Λ: coherent hopping
- Γ: dissipative hopping





What is the difference?



decomposition of master equation:

$$\Gamma = \frac{\kappa\lambda^2}{\left(\Delta^2+\frac{\kappa^2}{4}\right)} \qquad \Lambda = \frac{\Delta\lambda^2}{\left(\Delta^2+\frac{\kappa^2}{4}\right)}$$

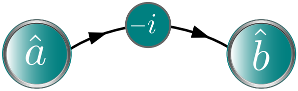
local processes

$$\frac{d}{dt}\hat{\rho} = \boxed{-i\Lambda\left[\hat{a}^\dagger\hat{a}+\hat{b}^\dagger\hat{b},\hat{\rho}\right]+\Gamma\mathcal{L}\left[\hat{a}\right]\hat{\rho}+\Gamma\mathcal{L}\left[\hat{b}\right]\hat{\rho}}$$

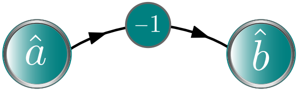
$$\boxed{-i\Lambda\left[\hat{a}\hat{b}^\dagger+\hat{a}^\dagger\hat{b},\hat{\rho}\right]+\Gamma\left[\hat{a}\hat{\rho}\hat{b}^\dagger-\frac{1}{2}\left\{\hat{a}\hat{b}^\dagger,\hat{\rho}\right\}+h.c\right]}$$

non-local processes

- Λ: coherent hopping

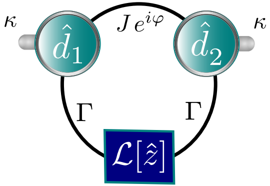


- Γ: dissipative hopping



- Γ vs. Λ: same process with local damping & phase $\pm\pi$

Again back to the circulator...



$$\hat{H} = J\left(\hat{d}_1\hat{d}_2^\dagger\, e^{i\varphi} + \hat{d}_2\hat{d}_3^\dagger + \hat{d}_3\hat{d}_1^\dagger + h.c.\right)$$

adiabatic elimination of mode 3:

$$\frac{d}{dt}\hat{\rho} = -i\left[J\left(\hat{d}_1\hat{d}_2^\dagger e^{i\varphi} + \hat{d}_1^\dagger\hat{d}_2^\dagger e^{-i\varphi}\right), \hat{\rho}\right] + \Gamma\mathcal{L}\left[\hat{d}_1 + \hat{d}_2\right]\hat{\rho}$$

coherent coupling dissipative coupling

equations of motion for expectation values:

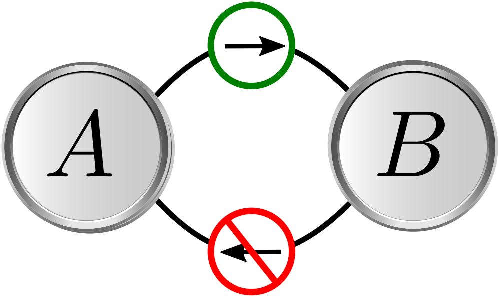
$$\frac{d}{dt}\langle\hat{d}_1\rangle = -\left[iJe^{-i\varphi} + \frac{\Gamma}{2}\right]\langle\hat{d}_2\rangle - \frac{\kappa+\Gamma}{2}\langle\hat{d}_1\rangle$$

$$\frac{d}{dt}\langle\hat{d}_2\rangle = -\left[iJe^{+i\varphi} + \frac{\Gamma}{2}\right]\langle\hat{d}_1\rangle - \frac{\kappa+\Gamma}{2}\langle\hat{d}_2\rangle$$

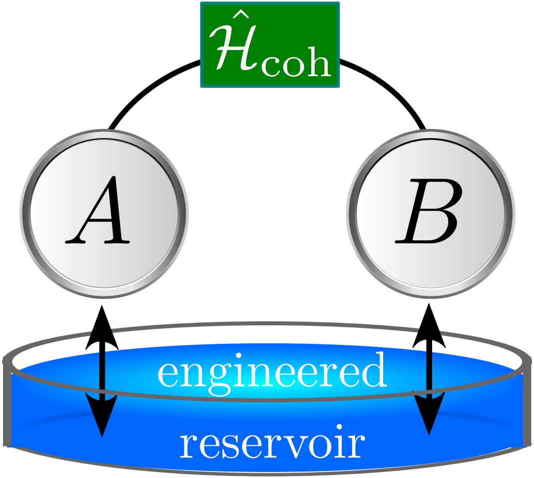
$$\boxed{\begin{array}{c}\varphi = \pm\frac{\pi}{2} \\ J = \frac{\Gamma}{2}\end{array}}$$

engineering nonreciprocity

'balancing a coherent interaction with the appropriate dissipative interaction'



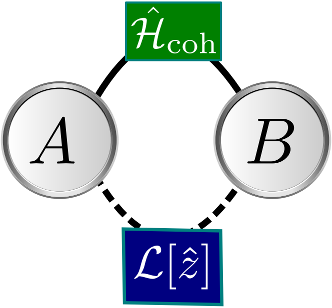
[A. Metelmann & A.A. Clerk, PRX 2015]



nonreciprocity via balancing a coherent interaction
with the corresponding dissipative interaction

[A. Metelmann & A.A. Clerk, PRX 2015]

Markovian limit



system modeled via a Lindblad master equation:

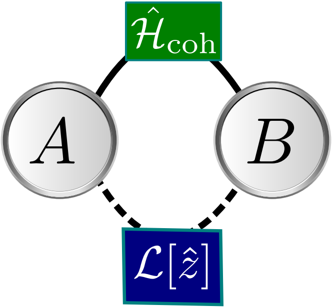
$$\frac{d}{dt}\hat{\rho} = -i\,\frac{\lambda}{2}\left[\hat{A}\hat{B} + \hat{A}^{\dagger}\hat{B}^{\dagger}, \hat{\rho}\right] + \Gamma\mathcal{L}\left[\hat{A} + e^{i\varphi}\hat{B}^{\dagger}\right]\hat{\rho}$$

coherent coupling

dissipative coupling

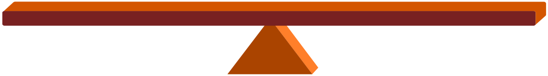
$$\mathcal{L}[\hat{o}]\hat{\rho} = \hat{o}\hat{\rho}\hat{o}^{\dagger} - \frac{1}{2}\hat{o}^{\dagger}\hat{o}\hat{\rho} - \frac{1}{2}\hat{\rho}\hat{o}^{\dagger}\hat{o}$$

Markovian limit



nonreciprocity via balancing the coherent and dissipative interaction:

$$\frac{d}{dt}\hat{\rho} = -i\,\frac{\lambda}{2}\left[\hat{A}\hat{B} + \hat{A}^\dagger\hat{B}^\dagger, \hat{\rho}\right] + \Gamma\mathcal{L}\left[\hat{A} + e^{i\varphi}\hat{B}^\dagger\right]\hat{\rho}$$

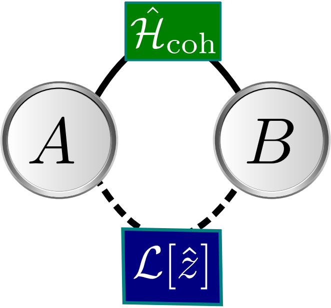


$$\lambda = \Gamma$$
$$\varphi = \pm\frac{\pi}{2}$$

determines the direction of the interaction

$$\mathcal{L}[\hat{o}]\hat{\rho} = \hat{o}\hat{\rho}\hat{o}^\dagger - \frac{1}{2}\hat{o}^\dagger\hat{o}\hat{\rho} - \frac{1}{2}\hat{\rho}\hat{o}^\dagger\hat{o}$$

Markovian limit



dynamics of expectation values of arbitrary system operators:

$$\frac{d}{dt}\langle\hat{O}_A\rangle = \Gamma\langle\hat{O}_A\mathcal{L}\left[\hat{A}\right]\rangle$$

$$\frac{d}{dt}\langle\hat{O}_B\rangle = \Gamma\langle\hat{O}_B\mathcal{L}\left[\hat{B}^\dagger\right]\rangle - i\Gamma\left[\langle\left[\hat{O}_B,\hat{B}\right]\hat{A}\rangle + \langle\left[\hat{O}_B,\hat{B}^\dagger\right]\hat{A}^\dagger\rangle\right]$$

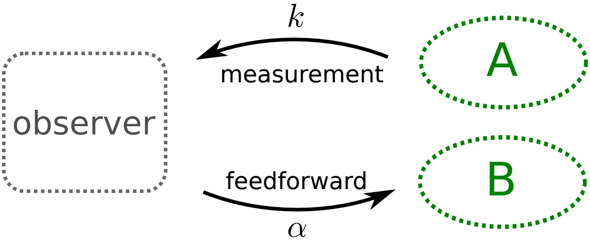
$$\lambda = \Gamma$$
$$\varphi = \pm\frac{\pi}{2}$$

determines the direction of the interaction

nonreciprocal interaction:
system B is driven by system A
but not vice versa

Nonreciprocity via reservoir engineering

related to measurement based feed-forward approach



$$\frac{d}{dt}\hat{\rho} = \frac{k}{4}\mathcal{L}\left[\hat{A}\right]\hat{\rho} + \alpha^2\mathcal{L}\left[\hat{B}\right]\hat{\rho} - i\alpha\frac{\sqrt{k}}{2}\left[\hat{B}, \hat{A}\hat{\rho} + \hat{\rho}\hat{A}\right]$$

mapping $\alpha = \eta^2\Gamma$ and $k = 4\Gamma$

$$\boxed{\frac{d}{dt}\hat{\rho} = -i\Gamma\left[\hat{A}\hat{B}, \hat{\rho}\right] + \Gamma\mathcal{L}\left[\hat{A} - i\eta\hat{B}\right]}$$

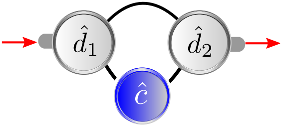
coherent interaction

dissipative interaction

[A. Metelmann & A.A. Clerk, PRA 2017]

Examples of two-port realizations

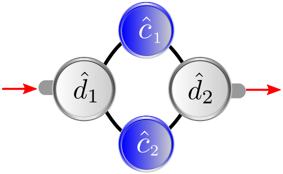
based on parametric modulation of coupled modes



$\hat{\mathcal{H}}_{\text{coh}}$ → direct coupling

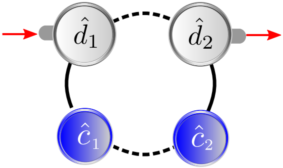
$\mathcal{L}[\hat{z}]$ → auxiliary mode

[Sliwa et al., PRX 2015]
[Lecocq et al., PRApp 2017]



$\left.\begin{array}{l}\hat{\mathcal{H}}_{\text{coh}} \\ \mathcal{L}[\hat{z}]\end{array}\right\}$ via two auxiliary modes

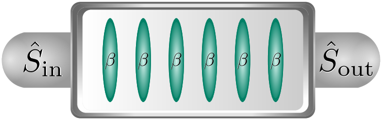
[Bernier et al., Nat Comm 2017]
[Peterson et al., PRX 2017]
[Barzanjeh et al., Nat Comm 2017]



$\left.\begin{array}{l}\hat{\mathcal{H}}_{\text{coh}} \\ \mathcal{L}[\hat{z}]\end{array}\right\}$ direct coupling & via two auxiliary modes

[Fang et al., Nat Phys 2017]

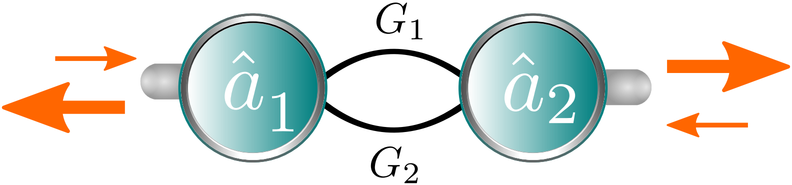
See also: *Graph-based anlysis in coupled-mode systems* [L. Ranzani & J. Aumentado, NJP, 2015]



The Nonreciprocal Bogoliubov Amplifier

-- breaking the gain-bandwidth limit

Bogoliubov amplifier



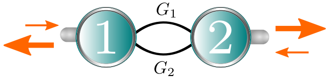
$$\hat{\mathcal{H}} = G_1 \left[\hat{a}_1^\dagger \hat{a}_2^\dagger + \hat{a}_1 \hat{a}_2\right] + G_2 \left[\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_1 \hat{a}_2^\dagger\right]$$

parametric amplifier

frequency conversion

Bogoliubov amplifiers

Phase-sensitive quantum-limited amplifier

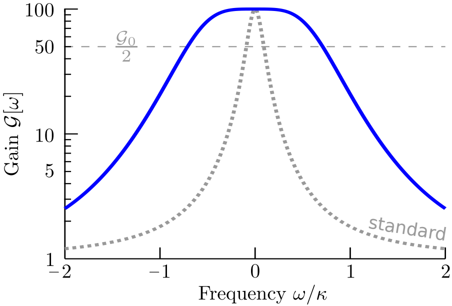


$$\hat{\mathcal{H}} = G_1 \left[\hat{a}_1^\dagger \hat{a}_2^\dagger + \hat{a}_1 \hat{a}_2\right] + G_2 \left[\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_1 \hat{a}_2^\dagger\right]$$

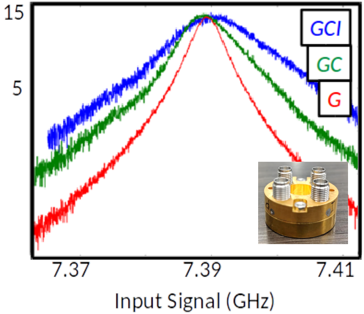
parametric amplifier

frequency conversion

No Gain-Bandwidth limit!



implemented by the
Hatridge group U Pittsburgh

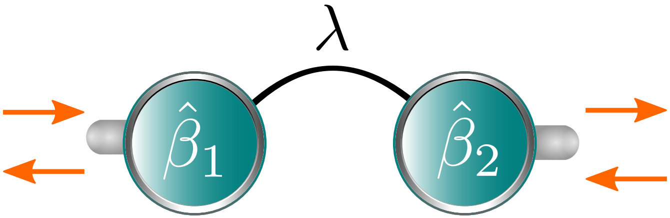


[A. Metelmann, O. Lanes, A. McDonald, M. Hatridge, A. Clerk, iarXiv:2208.00024 (2022)]

Bogoliubov amplifier

$$\hat{\beta}_n = \cosh r \ \hat{a}_n + \sinh r \ \hat{a}_n^\dagger$$

$$\tanh 2r = G_1/G_2$$

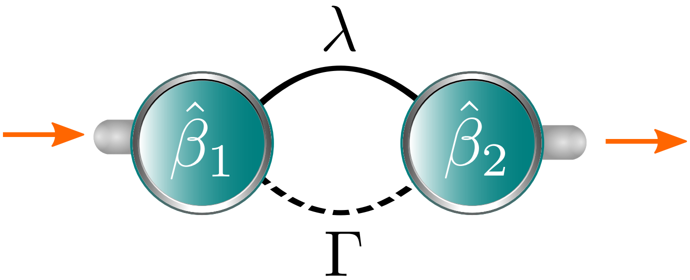


$$\hat{\mathcal{H}} = \lambda \ \left(\hat{\beta}_1^\dagger \hat{\beta}_2 + \hat{\beta}_1 \hat{\beta}_2^\dagger\right)$$

reciprocal: coherent hopping-interaction

$$\lambda = \sqrt{G_2^2 - G_1^2}$$

Nonreciprocal Bogoliubov amplifier

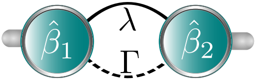


$$\frac{d}{dt}\hat{\rho} = -i\lambda\left[\left(\hat{\beta}_1^\dagger\hat{\beta}_2 + \hat{\beta}_1\hat{\beta}_2^\dagger\right), \hat{\rho}\right] + \Gamma\mathcal{L}\left[\hat{\beta}_1 + e^{i\phi}\hat{\beta}_2\right]\hat{\rho}$$

nonreciprocal: coherent & dissipative hopping

Nonreciprocal Bogoliubov amplifier

coherent & dissipative hopping



$$\frac{d}{dt}\hat{\rho} = -i\lambda\left[\left(\hat{\beta}_1^\dagger\hat{\beta}_2 + \hat{\beta}_1\hat{\beta}_2^\dagger\right), \hat{\rho}\right] + \Gamma\mathcal{L}\left[\hat{\beta}_1 + e^{i\phi}\hat{\beta}_2\right]\hat{\rho}$$

dynamics of the operators expectation values:

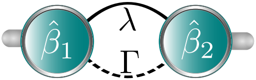
$$\frac{d}{dt}\langle\hat{\beta}_1\rangle = -\frac{\Gamma}{2}\langle\hat{\beta}_1\rangle - i\left[\lambda - i\frac{\Gamma}{2}e^{+i\phi}\right]\langle\hat{\beta}_2\rangle$$

$$\frac{d}{dt}\langle\hat{\beta}_2\rangle = -\frac{\Gamma}{2}\langle\hat{\beta}_2\rangle - i\left[\lambda - i\frac{\Gamma}{2}e^{-i\phi}\right]\langle\hat{\beta}_1\rangle$$

- dissipative hopping: local damping and coupling
- directionality conditions: $\lambda = \frac{\Gamma}{2} \qquad \phi = \pm\frac{\pi}{2}$

Nonreciprocal Bogoliubov amplifier

coherent & dissipative hopping



$$\frac{d}{dt}\hat{\rho} = -i\lambda\left[\left(\hat{\beta}_1^\dagger\hat{\beta}_2 + \hat{\beta}_1\hat{\beta}_2^\dagger\right), \hat{\rho}\right] + \Gamma\mathcal{L}\left[\hat{\beta}_1 + e^{i\phi}\hat{\beta}_2\right]\hat{\rho}$$

dynamics of the operators expectation values:

$$\phi = -\frac{\pi}{2}$$

$$\frac{d}{dt}\langle\hat{\beta}_1\rangle = -\frac{\Gamma}{2}\langle\hat{\beta}_1\rangle$$

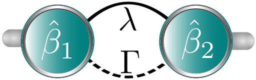
$$\frac{d}{dt}\langle\hat{\beta}_2\rangle = -\frac{\Gamma}{2}\langle\hat{\beta}_2\rangle - i\,\Gamma\,\langle\hat{\beta}_1\rangle$$

- dissipative hopping: local damping and coupling
- directionality conditions: $\lambda = \frac{\Gamma}{2}$ $\quad \phi = \pm\frac{\pi}{2}$

nonreciprocal interaction:

system 2 is driven by system 1 but not vice versa

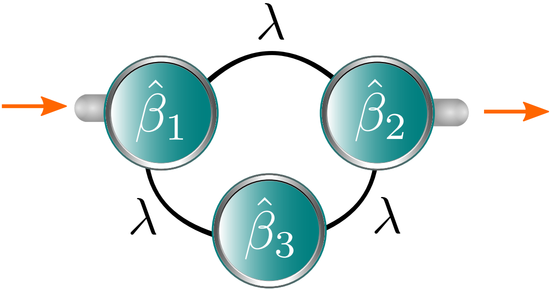
Nonreciprocal Bogoliubov amplifier



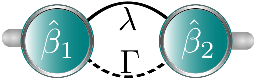
$$\frac{d}{dt}\hat{\rho} = -i\lambda\left[\left(\hat{\beta}_1^\dagger\hat{\beta}_2 + \hat{\beta}_1\hat{\beta}_2^\dagger\right), \hat{\rho}\right] + \Gamma\mathcal{L}\left[\hat{\beta}_1 + e^{i\phi}\hat{\beta}_2\right]\hat{\rho}$$

- dissipator realized via coupling to auxiliary mode

$$\hat{\mathcal{H}} = \lambda\left(\hat{\beta}_1^\dagger\hat{\beta}_2 + \hat{\beta}_3^\dagger\hat{\beta}_1 + \hat{\beta}_3^\dagger\hat{\beta}_2\, e^{i\phi} + h.c.\right)$$



Nonreciprocal Bogoliubov amplifier



$$\frac{d}{dt}\hat{\rho} = -i\lambda\left[\left(\hat{\beta}_1^\dagger\hat{\beta}_2 + \hat{\beta}_1\hat{\beta}_2^\dagger\right), \hat{\rho}\right] + \Gamma\mathcal{L}\left[\hat{\beta}_1 + e^{i\phi}\hat{\beta}_2\right]\hat{\rho}$$

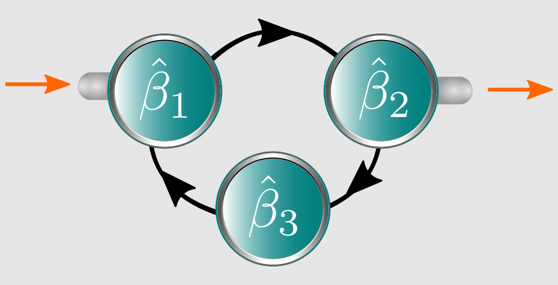
- dissipator realized via coupling to auxiliary mode

$$\hat{\mathcal{H}} = \lambda\left(\hat{\beta}_1^\dagger\hat{\beta}_2 + \hat{\beta}_3^\dagger\hat{\beta}_1 + \hat{\beta}_3^\dagger\hat{\beta}_2\, e^{i\phi} + h.c.\right)$$

- output with directionality conditions: $\lambda = \frac{\Gamma}{2} = \frac{\kappa}{2} \quad \phi = -\frac{\pi}{2}$

$$\begin{pmatrix} \hat{\beta}_{1,\mathrm{out}}[0] \\ \hat{\beta}_{2,\mathrm{out}}[0] \\ \hat{\beta}_{3,\mathrm{out}}[0] \end{pmatrix} = \begin{pmatrix} 0 & 0 & i \\ i & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \hat{\beta}_{1,\mathrm{in}}[0] \\ \hat{\beta}_{2,\mathrm{in}}[0] \\ \hat{\beta}_{3,\mathrm{in}}[0] \end{pmatrix}$$

Nonreciprocal Bogoliubov amplifier



$\frac{d}{dt}\hat\rho = -$ … $+ e^{i\phi}\hat\beta_2\Big]\,\hat\rho$

- dissipato … de

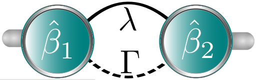
$\hat{\mathcal{H}}$ … $.c.\Big)$

- output w … $\phi = -\frac{\pi}{2}$

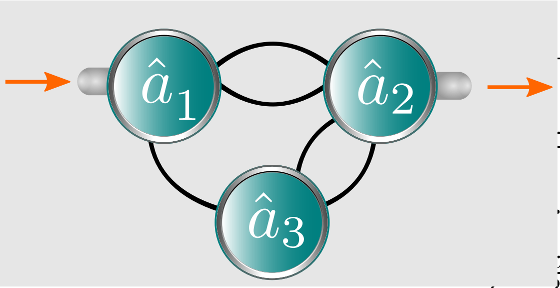
$$\begin{pmatrix} \hat\beta_{1,\mathrm{out}}[0] \\ \hat\beta_{2,\mathrm{out}}[0] \\ \hat\beta_{3,\mathrm{out}}[0] \end{pmatrix} = \begin{pmatrix} 0 & 0 & i \\ i & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \hat\beta_{1,\mathrm{in}}[0] \\ \hat\beta_{2,\mathrm{in}}[0] \\ \hat\beta_{3,\mathrm{in}}[0] \end{pmatrix}$$

➤➤ circulator in Bogoliubov basis

Nonreciprocal Bogoliubov amplifier



$\frac{d}{dt}\hat{\rho} = -i$ … $+ e^{i\phi}\hat{\beta}_2\Big]\,\hat{\rho}$



- dissipato… de

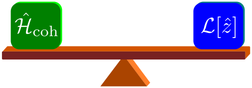
$\hat{\mathcal{H}} =$ … $.c.)$

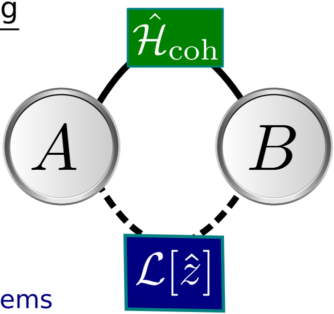
- output w… $\frac{}{2}$ … $\frac{}{2}$ $\quad \phi = -\frac{\pi}{2}$

$$\begin{pmatrix} \hat{X}_{1,\mathrm{out}}[0] \\ \hat{P}_{1,\mathrm{out}}[0] \\ \hat{X}_{2,\mathrm{out}}[0] \\ \hat{P}_{2,\mathrm{out}}[0] \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{\mathcal{G}}} & 0 & 0 \\ \sqrt{\mathcal{G}} & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \hat{X}_{1,\mathrm{in}}[0] \\ \hat{P}_{1,\mathrm{in}}[0] \\ \hat{X}_{2,\mathrm{in}}[0] \\ \hat{P}_{2,\mathrm{in}}[0] \end{pmatrix}$$

- nonreciprocal amplifier in the original basis

Nonreciprocity via reservoir engineering





nonreciprocity via balancing
the coherent and dissipative interaction

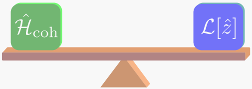
→ purely general recipe to construct a directional interaction between two systems

- *Nonreciprocal Photon Transmission and Amplification via Reservoir Engineering*
  [A. Metelmann & A.A. Clerk, PRX 2015]
- *Nonreciprocal Quantum Interactions and Devices via Autonomous Feedforward*
  [A. Metelmann & A.A. Clerk, PRA 2017]
- *Minimal Models for Nonreciprocal Amplification Using Biharmonic Drives*
  [A. Kamal & A. Metelmann, PRApplied 2017]

beyond signal processing:

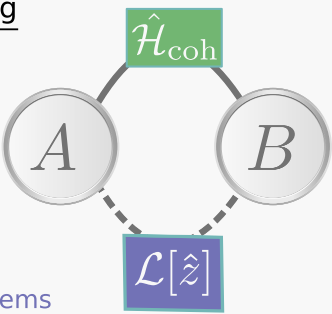
- *High-purity Entanglement of Hot Propagating Modes using Nonreciprocity*
  [Lindsay Orr, Saeed A. Khan, Nils Buchholz, Shlomi Kotler, A. Metelmann, arXiv:2209.06847 (2022)]
- *Embedding of Time-Delayed Quantum Feedback in a Nonreciprocal Array*
  [Xin H. H. Zhang, S. H. L. Klapp, A. Metelmann, arXiv 2204.02367 (2022)]

Nonreciprocity via reservoir engineering



nonreciprocity via balancing
the coherent and dissipative interaction

→ purely general recipe to construct a
directional interaction between two systems



Nonreciprocal Signal Routing in an Active Quantum Network
[A. Metelmann & H.E. Türeci, PRA 2017]