

EXPERIMENTAL QUANTUM ANNEALING

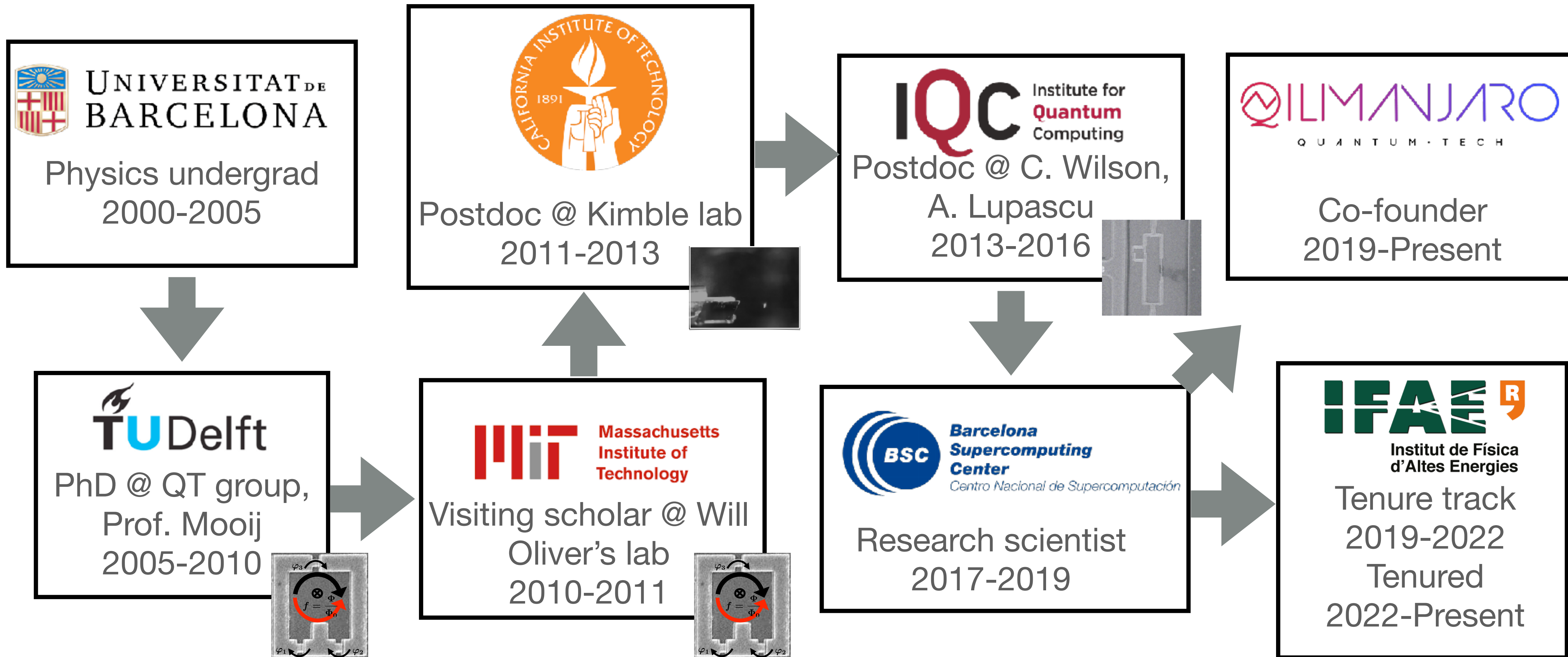
Pol Forn-Díaz

Institut de Física d'Altes Energies (IFAE), Barcelona
Quantum Computing Technology group: <https://qct.ifae.es/>

SPRING SCHOOL ON SUPERCONDUCTING QUBIT TECHNOLOGY
Benasque Center for Science, April 14th 2023



My career



QCT group

Team members

<https://qct.ifae.es>



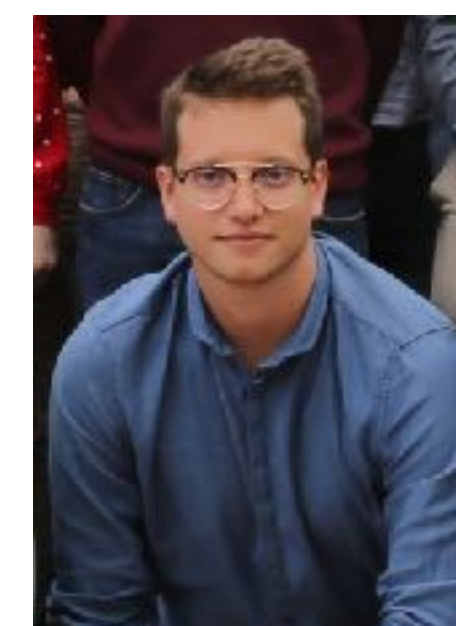
P. Forn-Díaz, PI



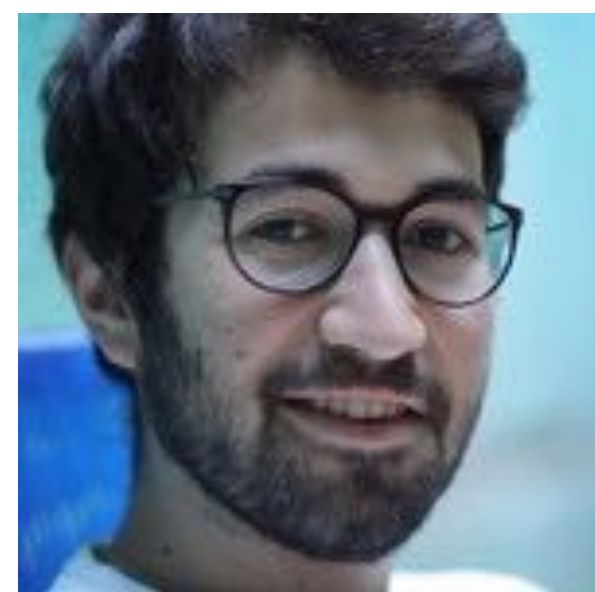
Manel Martínez, staff



Sara Martínez, PM
07/2020-07/2023



Elia Bertoldo,
Postdoc
02/2021-02/2023



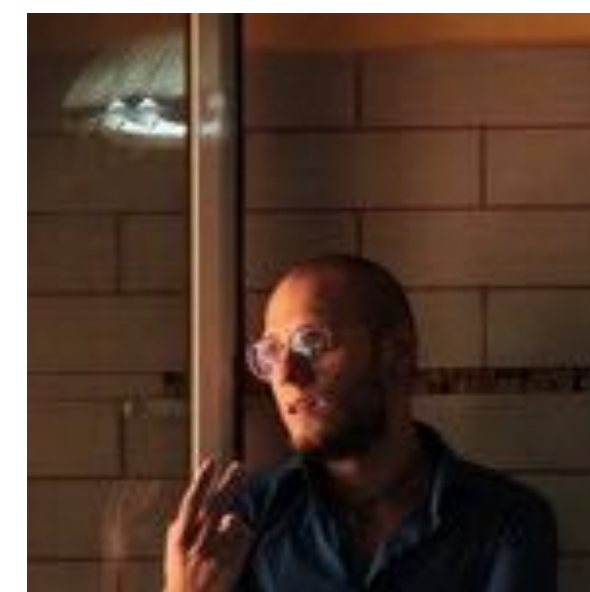
David López, PhD
11/2018-04/2023



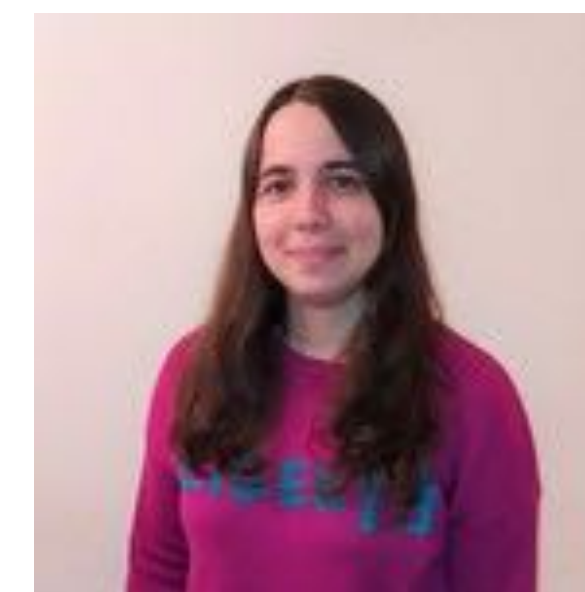
Fabian Zwiehoff, PhD
11/2019-11/2024



Boris Nedyalkov, PhD
10/2020-10/2023



Luca Cozzolino, PhD
11/2020-11/2024



Alba Torras, PhD
11/2020-11/2024

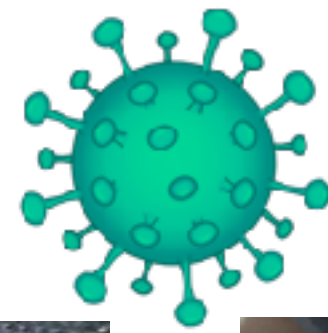
Alumni

Queralt Portell (TFG) 2021-2022
Ariadna del Pulgar (TFG) 2021-2022
Adrià Riera (TFG) 2020-2021
Raquel Garcia (TFG) 2020-2021
Olga Marco (TFG) 2019-2020
Ivan Alsina (TFG) 2019-2020
Alberto Lajara (TFM) 2019-2020
Barkay Guttel (TFM) 2019-2020
Santi Vallés (TFG) 2019-2020
Rafael Luque (TFM) 2019-2020
Adrià Grabulosa (TFG) 2018-2019

QCT group

Laboratory space

January 2020



August 2020

September 2020

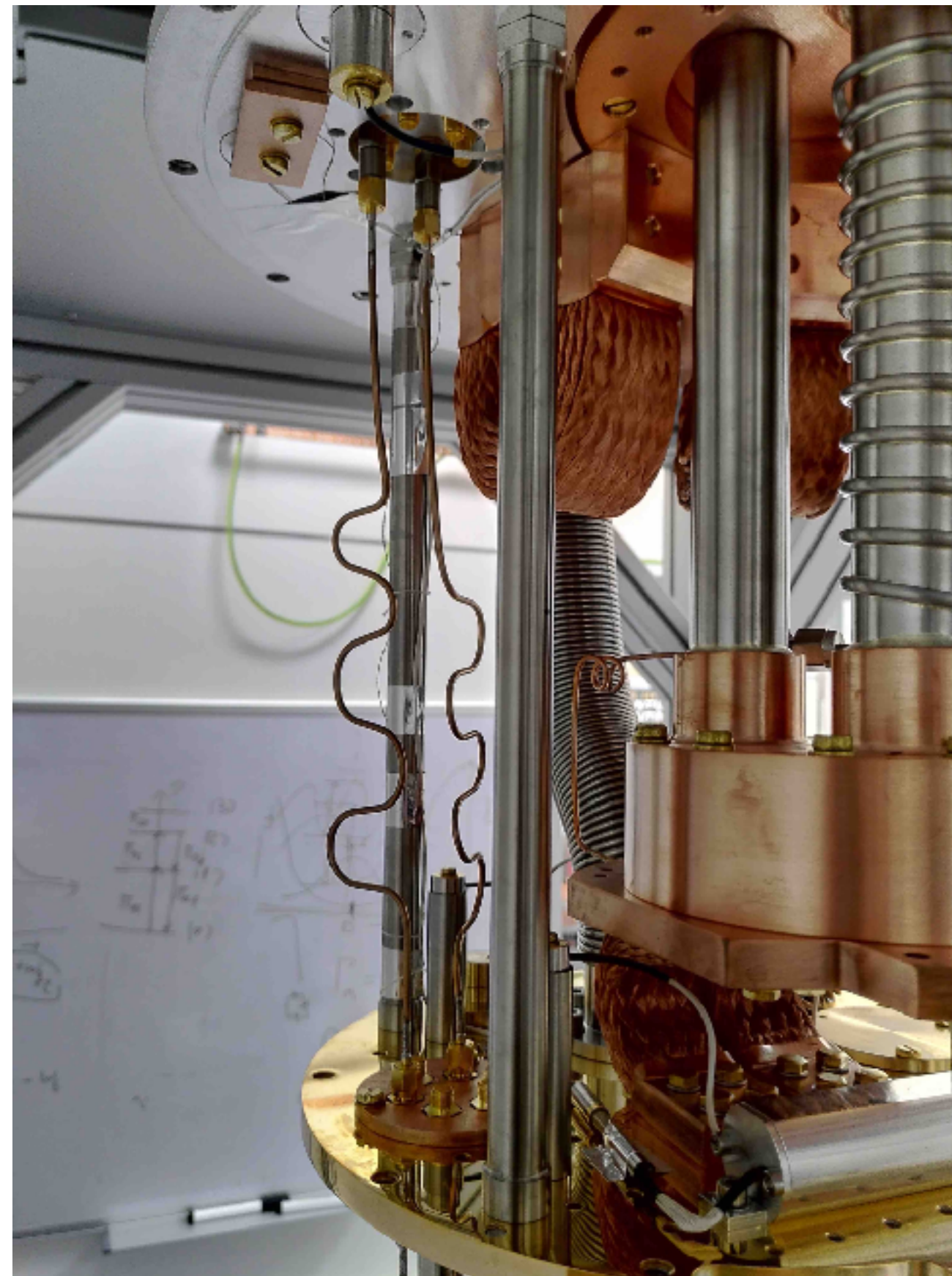
December 2020

<https://qct.ifae.es>



QCT group

Laboratory space: Dilution refrigerators



- Installed 2 dilution refrigerators, both functional attaining temperatures in the **10-20mK range**.
- All laboratory infrastructure is **in place**, including storage shelves, workstations, instrument racks
- Engine room in upper floor containing compressors and storage
- **1 DR financed by IFAE**, 1DR financed by Qilimanjaro
- 1 more DR financed by Qilimanjaro at new UAB lab
- Necessary instrumentation and components to control up to 5 qubit devices.

QCT group

IFAE cleanroom space

- Installed metal deposition facility inside IFAE's cleanroom for qubit device production.
- First device prototypes evaporated in April-May 2021.
- Evaporator financed by Qilimanjaro
- Upcoming wet bench will complement QCT's group fabrication infrastructure.
- Combined with CNM's cleanroom facilities, evaporator enables QCT group for own device production.



<https://www.qilimanjaro.tech/>



Plassys metal evaporator,
“qubit foundry”

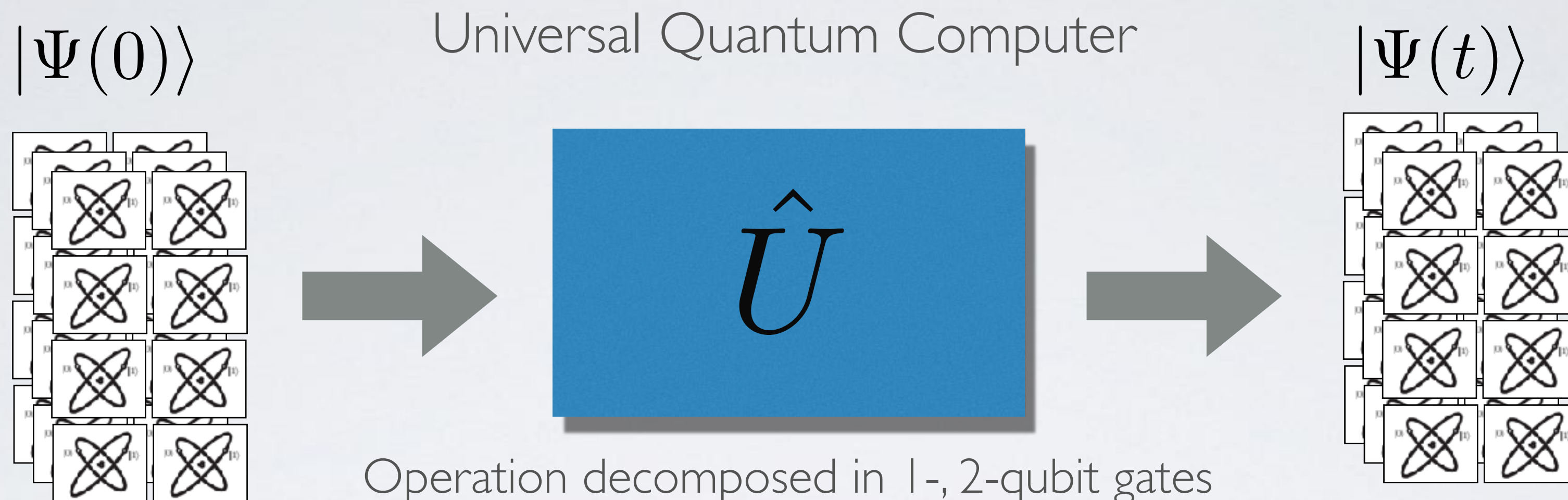
Fumehood for lift-off

Space for microscope

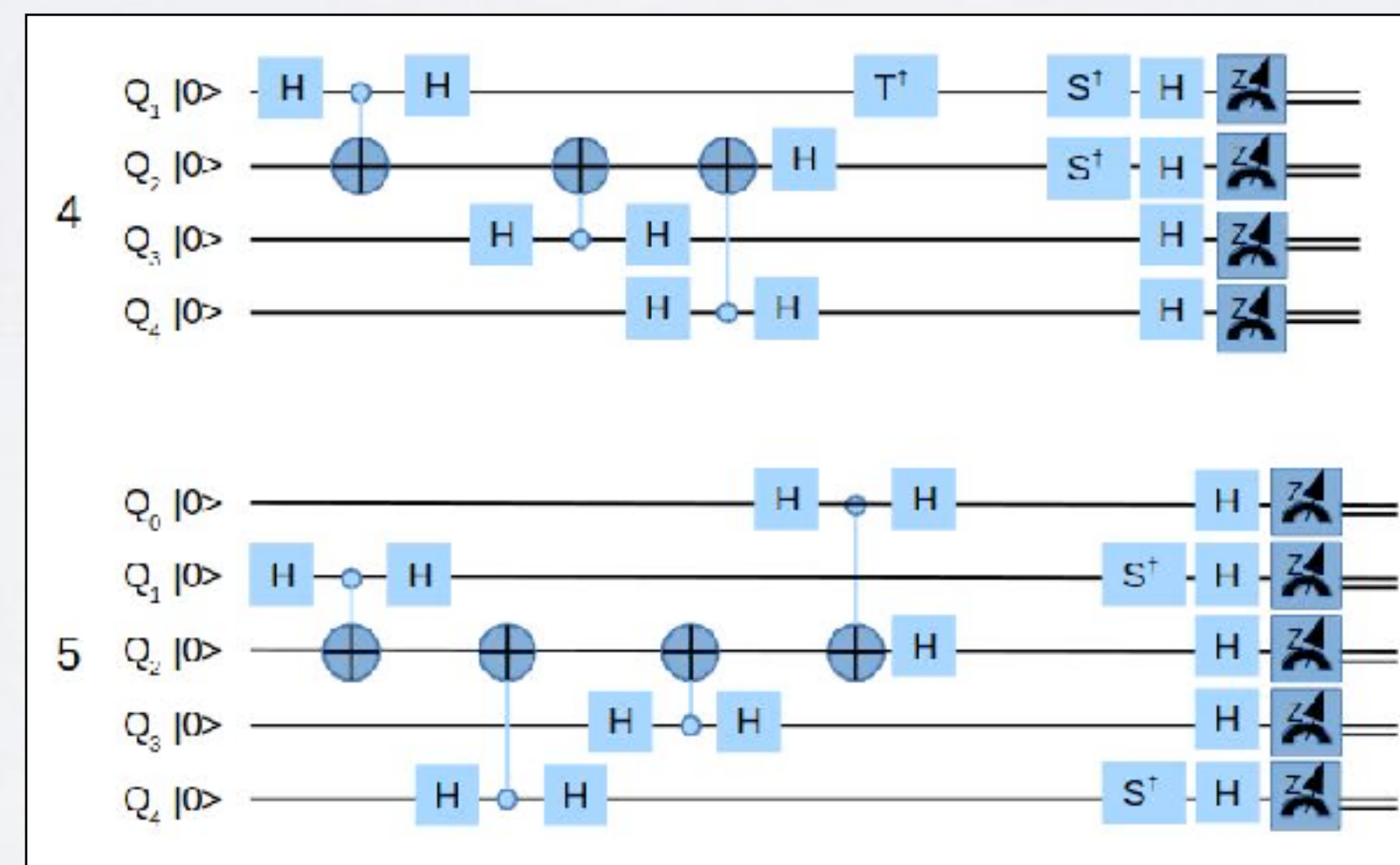
OUTLOOK

- Digital vs Adiabatic Quantum Computers
 - Adiabatic Quantum Computation
 - Quantum Annealing
 - Superconducting circuits with Josephson junctions
 - Commercial (incoherent) Quantum Annealers
 - Coherent Quantum Annealing
- break

DIGITAL VS ANALOG QC



PROS:
Applicable to any
quantum algorithm

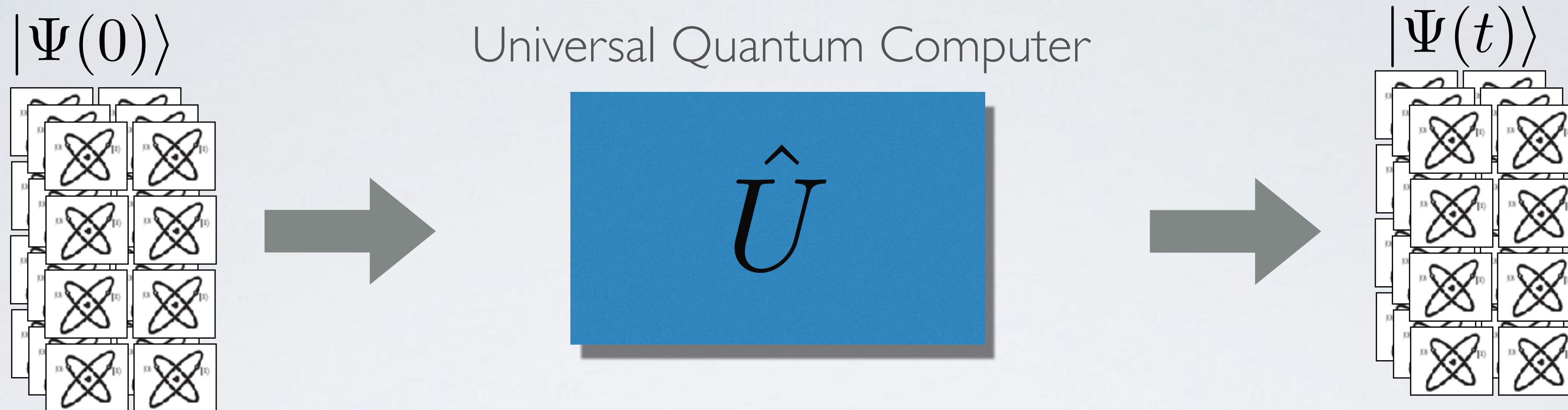


Quantum gates circuit

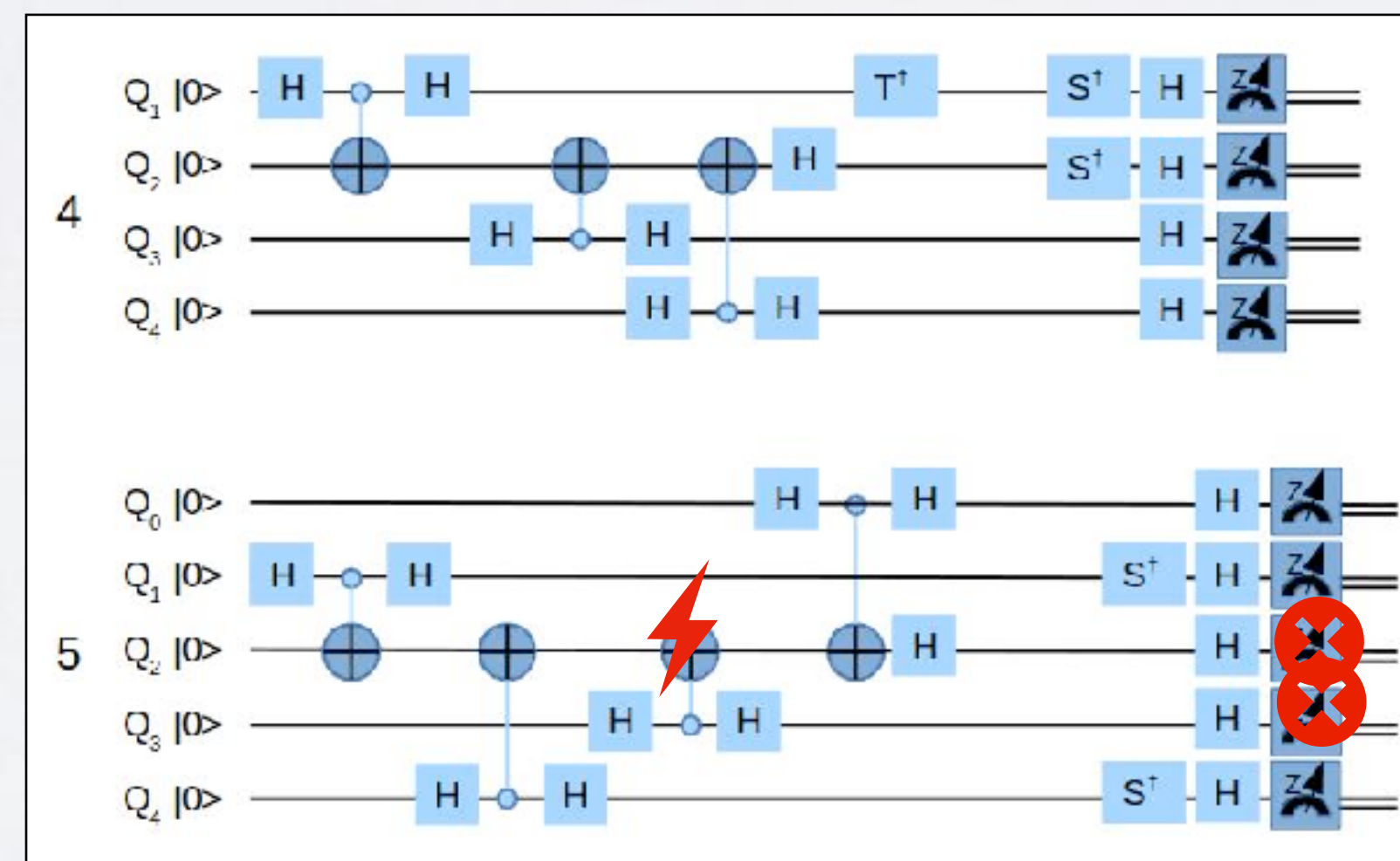
Processing 2^N
states simultaneously

CONS:
Requires quantum
error correction: $\sim 10^6$
Long-term

DIGITAL VS ANALOG QC



Operation decomposed in 1-, 2-qubit gates



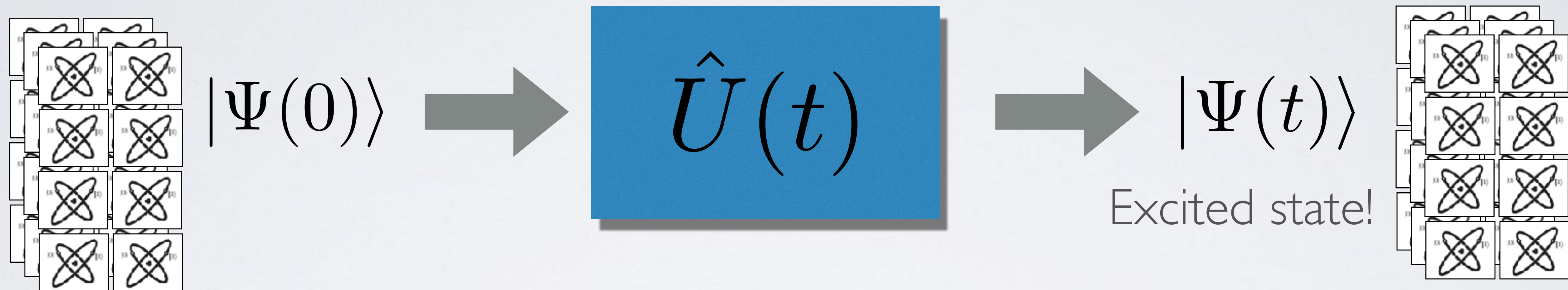
If error occurs, algorithm fails

Unless Quantum Error Correction is implemented, approach is unreliable

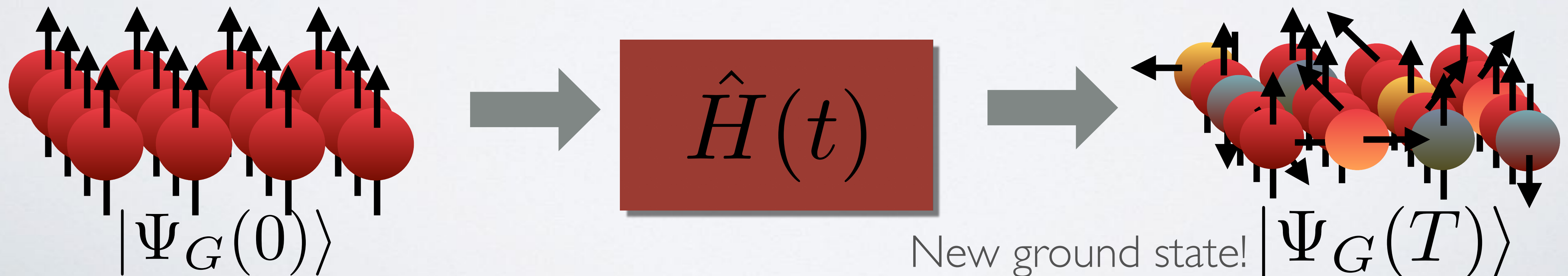
DIGITAL VS ANALOG QC

Digital quantum computers are most well-known. Alternative approach is by using analog quantum processors, also known as quantum annealers.

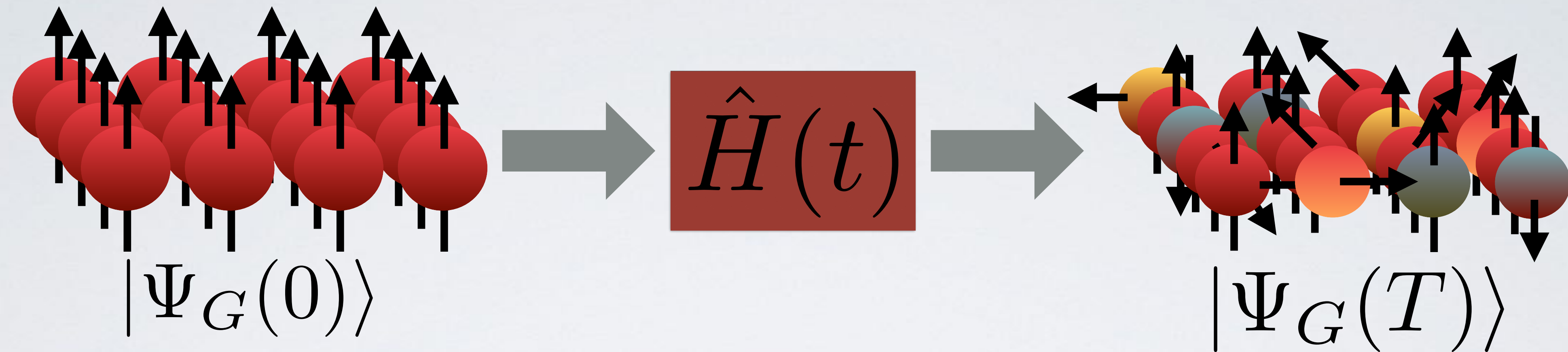
Digital: Evolution of quantum state by change of state:



Analogue: Evolution of quantum state by change of H. Also known as AQCC.



DIGITAL VS ANALOG QC



Adiabatic Quantum Computation is Equivalent to Standard Quantum Computation
Dorit Aharonov, Wim van Dam, Julia Kempe, Zeph Landau, Seth Lloyd, Oded Regev
SIAM Journal of Computing, Vol. 37, Issue 1, p. 166-194 (2007)
[arXiv:quant-ph/0405098](https://arxiv.org/abs/quant-ph/0405098)

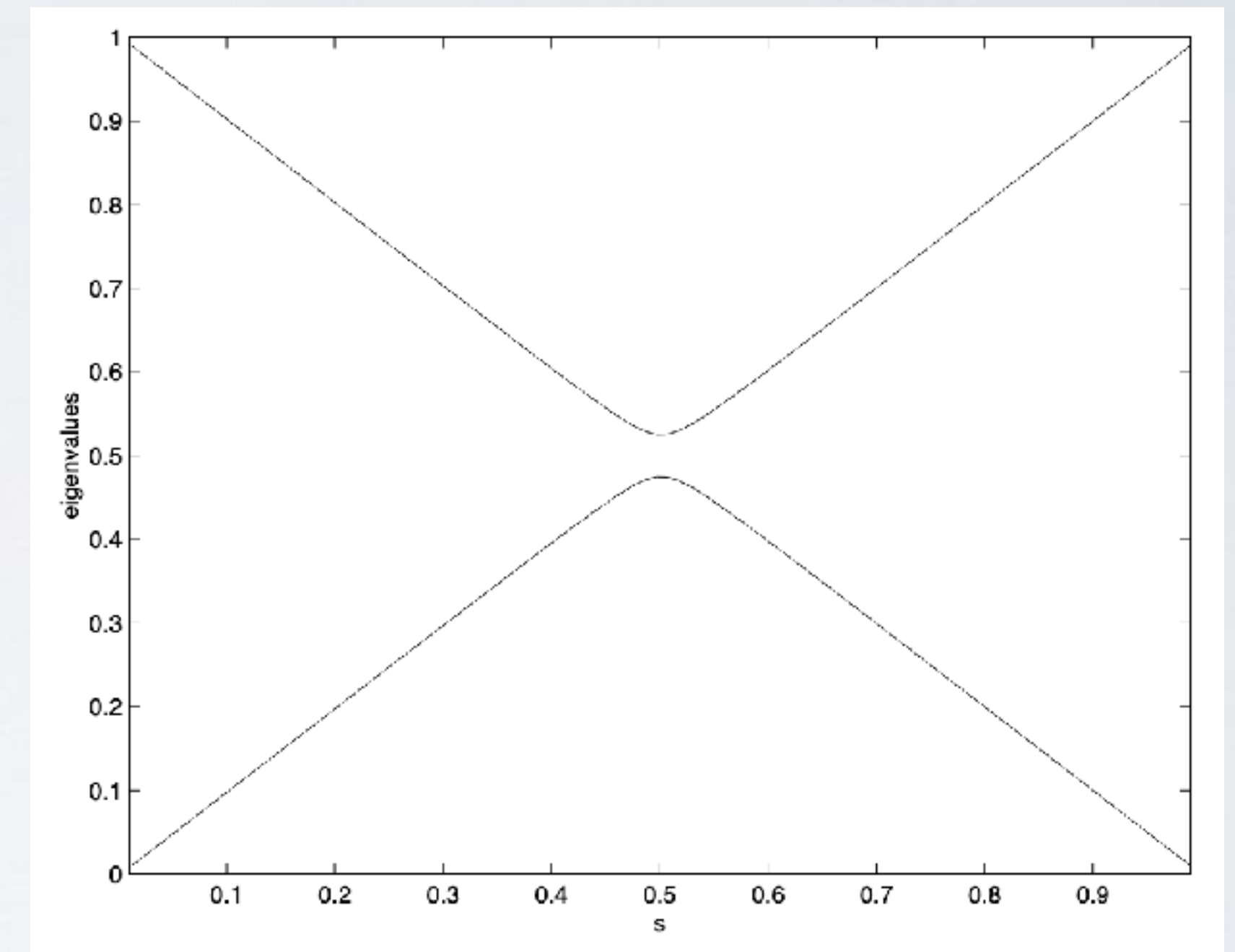
We know how to realize discrete sets of gate sets in digital QC.
How do we implement analog quantum computing protocols in practice?

ADIABATIC QUANTUM COMPUTATION

ADIABATIC QUANTUM COMPUTATION (AQC)

The adiabatic theorem

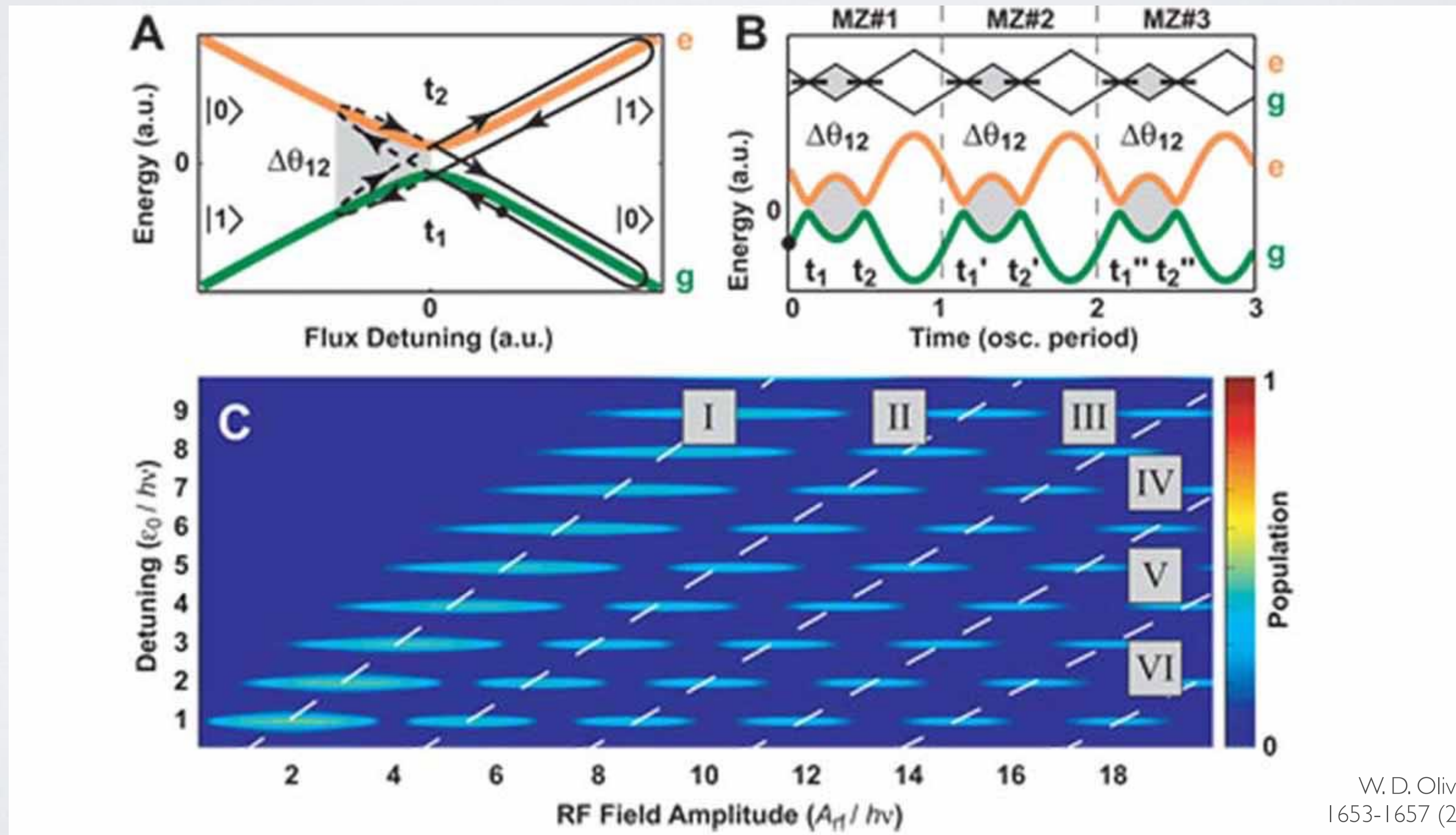
Blackboard



ADIABATIC QUANTUM COMPUTATION (AQC)

Equivalent picture: Landau-Zener tunneling

$$P_D = e^{-2\pi\Gamma} \quad \Gamma = \frac{\Delta^2}{4\hbar|\alpha|} \quad \alpha = \frac{\partial}{\partial t}(E_1 - E_0)$$



ADIABATIC QUANTUM COMPUTATION (AQC)

Practical example: 2-SAT problem

Blackboard

QUBO formulation

Blackboard

Classical Constraint	Equivalent Penalty
$x + y \leq 1$	$P(xy)$
$x + y \geq 1$	$P(1 - x - y + xy)$
$x + y = 1$	$P(1 - x - y + 2xy)$
$x \leq y$	$P(x - xy)$
$x_1 + x_2 + x_3 \leq 1$	$P(x_1x_2 + x_1x_3 + x_2x_3)$
$x = y$	$P(x + y - 2xy)$

Table of a few Known constraint/penalty pairs

A Tutorial on Formulating and Using QUBO Models
Fred Glover, Gary Kochenberger, Yu Du
arXiv:1811.11538

ADIABATIC QUANTUM COMPUTATION (AQC)

Quantum solution

Quantum solution: build Hamiltonian whose ground state energy is the minimum of f :

$$H_P(|x_i\rangle) = f(x_{i_C}, x_{j_C})(|x_i\rangle)$$

equivalence only in ground state $|x_i\rangle$

Finds equivalence between mathematical formulation of real problem with physics from a physical system.

Which physical system is suitable for QUBO problems?

ADIABATIC QUANTUM COMPUTATION (AQC)

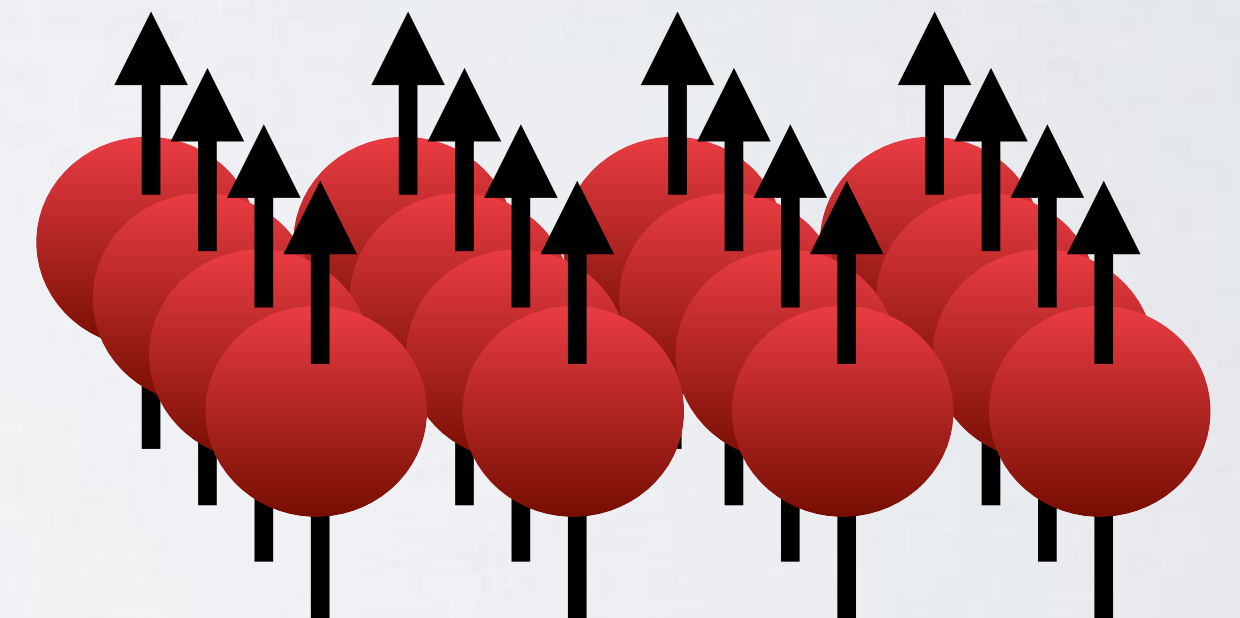
The problem Hamiltonian

Start with cost function $f(x_i) = \sum_{i,j} C_{ij} x_i x_j + \sum_i B_i x_i$

Replace bits x_i by spin 1/2 qubits $|z_i\rangle$ eigenstates of σ_i^z

$$\frac{1}{2}(1 - \sigma_i^z)|z_i\rangle = z_i|z_i\rangle \quad \sigma_i^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$H_P(|z_i\rangle) = \sum_{ij} J_{ij} \sigma_{z_i} \sigma_{z_j} + \sum_i h_i \sigma_{z_i}$$



Corresponds to the Ising model of interacting spins with on-site magnetic field

One-to-one correspondence with cost function f

ADIABATIC QUANTUM COMPUTATION (AQC)

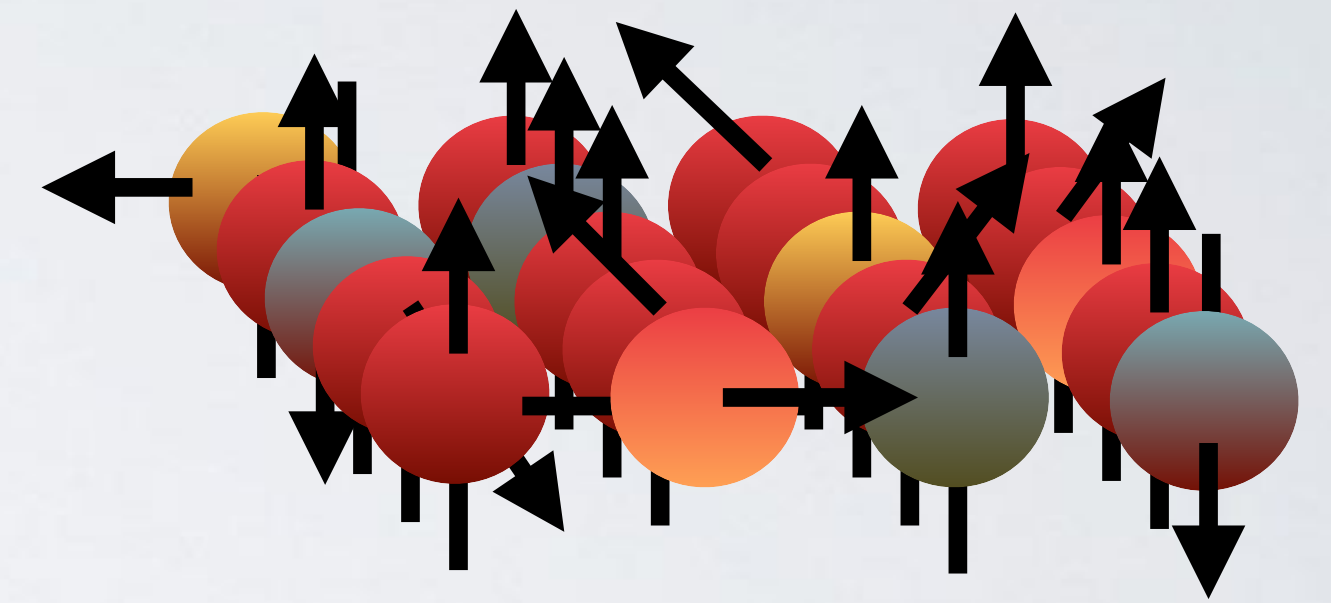
The problem Hamiltonian

Clause C now associated with operator $H_{P,C}$

$$H_{P,C}(|z_1\rangle|z_2\rangle \dots |z_N\rangle) = f(z_{i_C}, z_{j_C})|z_1\rangle|z_2\rangle \dots |z_N\rangle$$

Hamiltonian associated with all clauses, $H_P = \sum_C H_{P,C}$

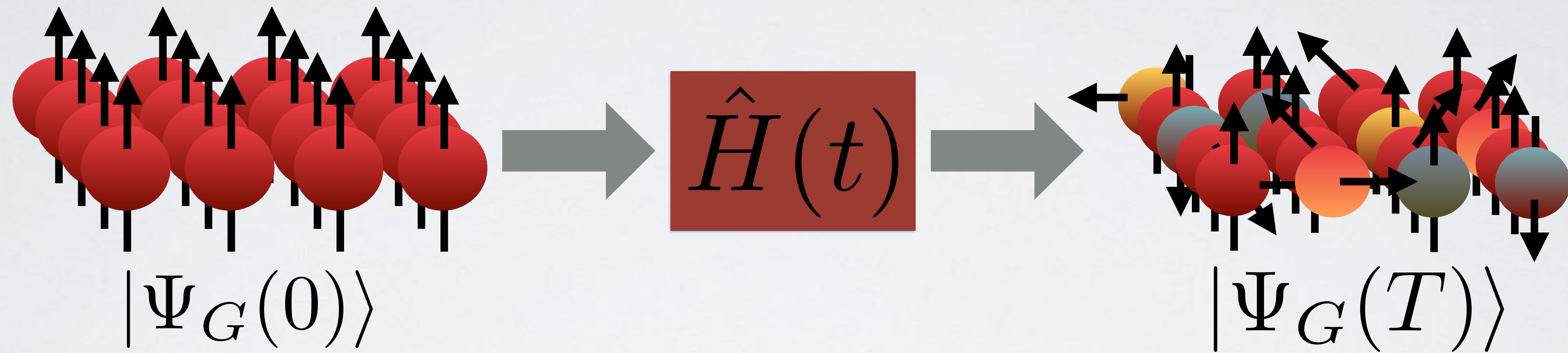
$H_P|\psi\rangle = 0$ if and only if $|\psi\rangle$ is a superposition of states of the form $|z_1\rangle|z_2\rangle \dots |z_N\rangle$ where $z_1, z_2, \dots, z_N$ satisfy all clauses.



Minimum of Hamiltonian eigenvalue corresponds to optimal configuration to minimize cost function

ADIABATIC QUANTUM COMPUTATION (AQC)

Recalling adiabatic evolution...



Energy of ground state wave function
maps to minimum of cost function f

E. Farhi et al., arxiv:quant-ph/0001106

E. Farhi et al., Science 292, 472 (2001)

T. Albash and D. Lidar, Rev. Mod. Phys. 90, 015002 (2018)

ADIABATIC QUANTUM COMPUTATION (AQC)

The initial/driving Hamiltonian

Ground state is **easy to find**.

$$H_{D_i} = \frac{1}{2}(1 - \sigma_i^x) \quad H_D = \sum_i H_{D_i}$$

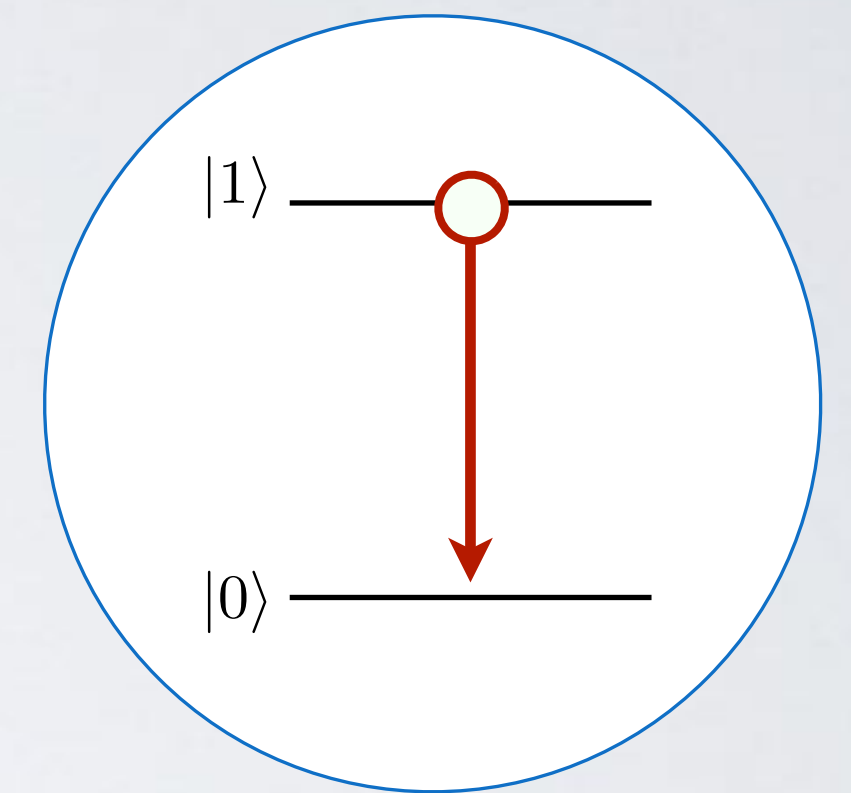
No interaction between qubits:

preparing ground state is equivalent to individual qubit ground state

Ground state is $|\Psi_{\text{GS}}(0)\rangle = |x_1 = 0\rangle |x_2 = 0\rangle \dots |x_N = 0\rangle$

$$|x_1 = 0\rangle |x_2 = 0\rangle \dots |x_N = 0\rangle = \frac{1}{2^{n/2}} \sum_{z_1} \sum_{z_2} \dots \sum_{z_n} |z_1\rangle |z_2\rangle \dots |z_N\rangle$$

In x-basis, initial state becomes a superposition of states in z-basis, known as the **computational basis. Samples all classical possibilities at once.**



Preparation easy by decay, measurement, etc.

ADIABATIC QUANTUM COMPUTATION (AQC)

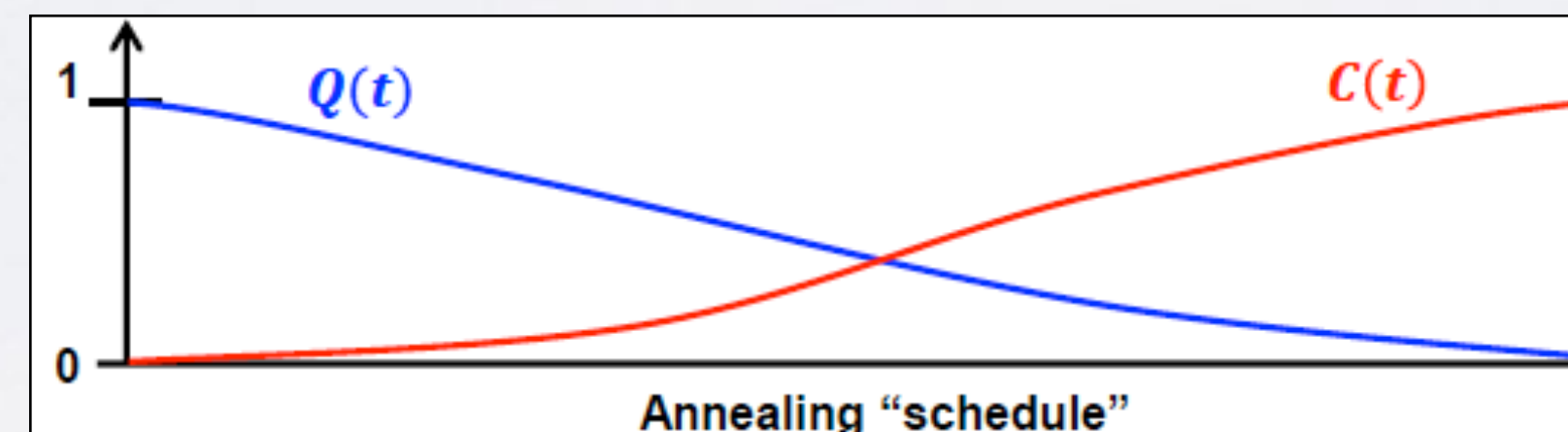
Adiabatic evolution

Simplest approach: **linear interpolation**:

For a single clause: $H_C(t/T) = (1 - t/T)H_{D,C} + (t/T)H_{P,C}$

Total Hamiltonian = sum of clause Hamiltonians $H(t/T) = \sum_C H_C(t/T)$

In general, nonlinear time evolution



$$\mathcal{H}(t) = \underbrace{Q(t)\mathcal{H}_D}_{\text{Trivial}} + \underbrace{C(t)\mathcal{H}_P}_{\text{Problem}}$$

Grover's algorithm in AQC:

Quantum Search by local adiabatic evolution
J. Roland, N. J. Cerf, Phys. Rev. A 65, 042308 (2002)

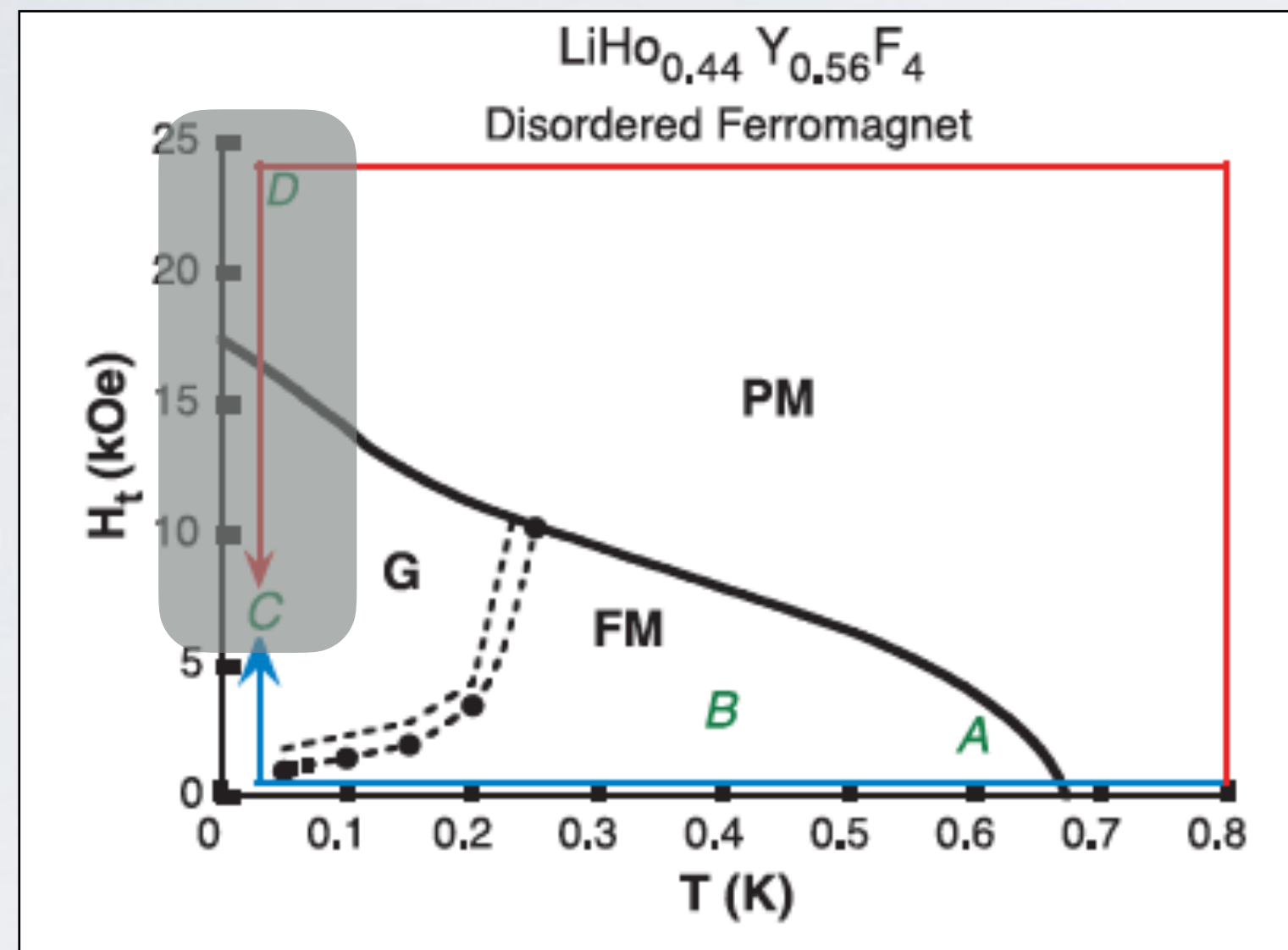
So far, only considered unitary evolution of closed system. In the real world: finite temperature, finite times.

Need open-system framework: Quantum Annealing

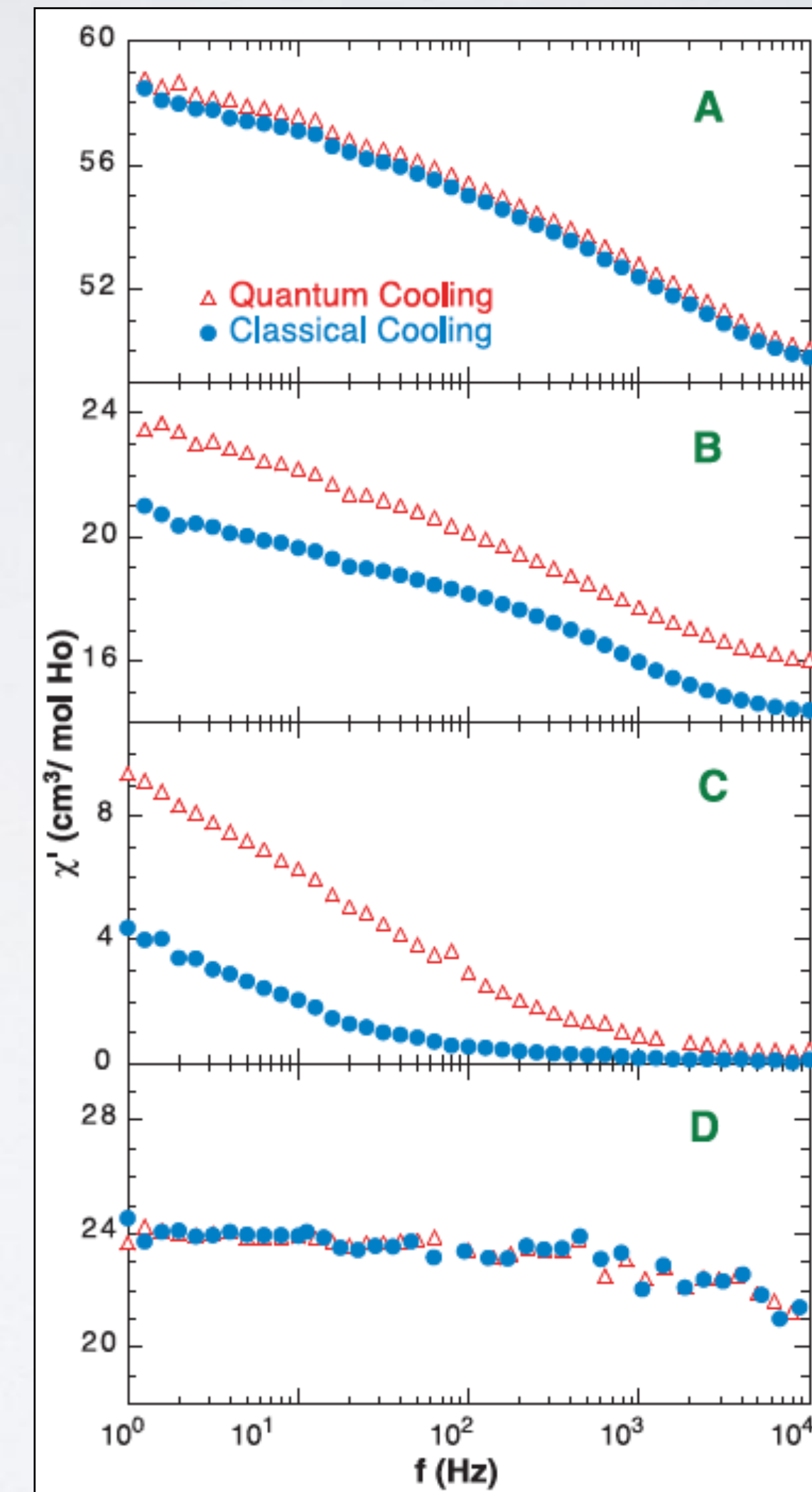
QUANTUM ANNEALING

WHERE IT ALL STARTED: MAGNETISM

Cooling a disordered ferromagnet in two paths: open system



Magnetic susceptibility shows signatures of cooling system to a different state



Guided phase transition between two ground states!

Motivated the idea to reach arbitrary ground states...

Brooke *et al.*, Nature **284**, 779 (1999)

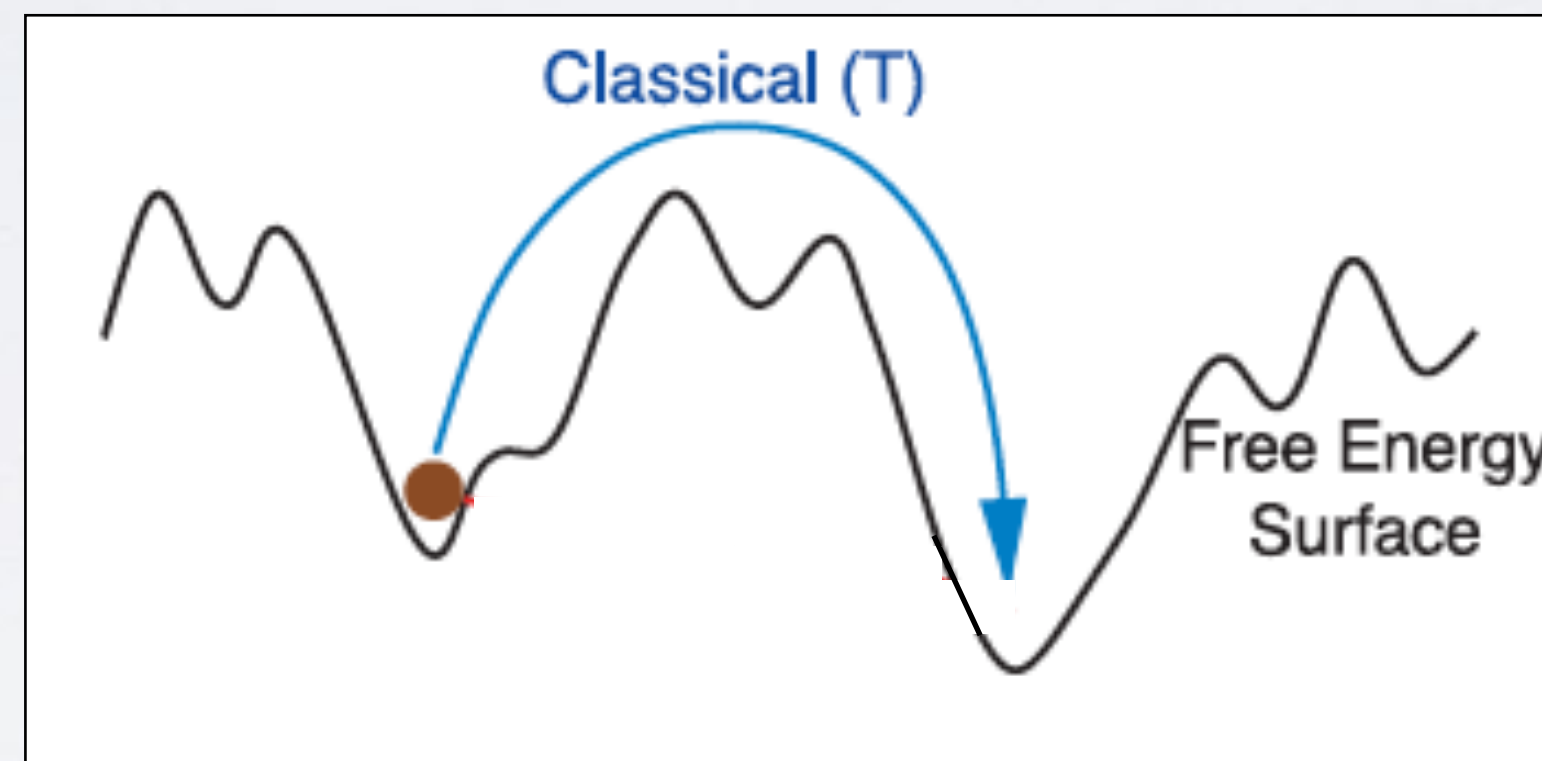
QUANTUM ANNEALING

Simulated annealing

Kirkpatrick, Gelatt, Vecchi, Science **220**, 671 (1983).

Connection between **statistical mechanics** and combinatorial **optimization**

Cost function: $f(x_i)$



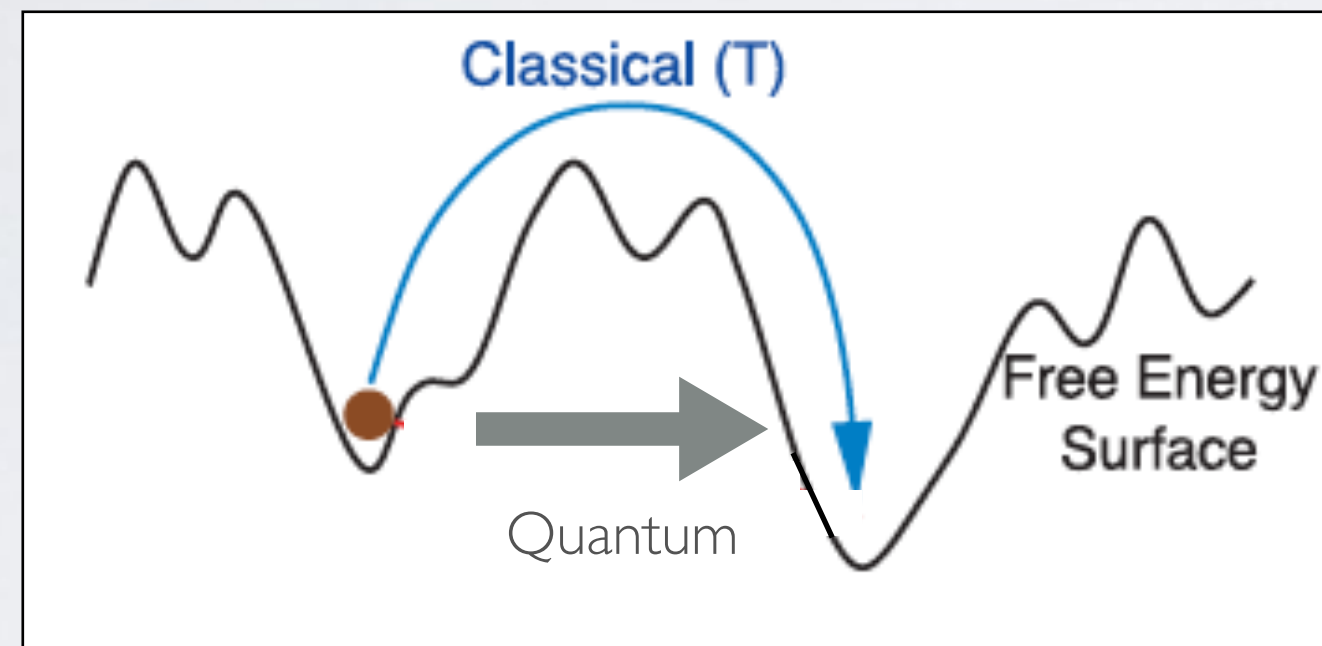
Thermal fluctuations help escape from local minima under annealing schedule by **decreasing temperature**

QUANTUM ANNEALING

Quantum annealing

Kadowaki, Nishimori, Phys. Rev. E **58**, 5355 (1998).

Ising model in a transverse magnet



$$\mathcal{H}_{IT} = - \sum_{i,j}^N J_{ij} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^z - \Gamma(t) \sum_i \sigma_i^x$$

Example of 2-SAT problem:

$$\mathcal{H}_{IT}(|x_1\rangle|x_2\rangle \dots |x_N\rangle) = f(x_{i_C}, x_{j_C})(|x_1\rangle|x_2\rangle \dots |x_N\rangle)$$

Many classical optimization problems can be cast into the classical Ising model

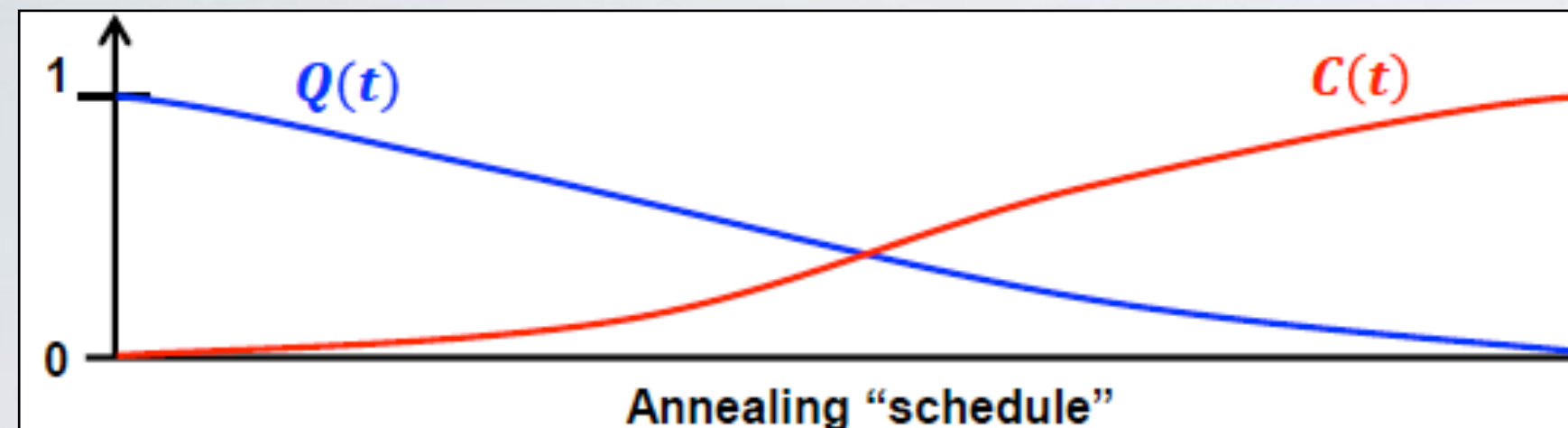
$$\Gamma \neq 0 \Rightarrow [\mathcal{H}_{IT}, \sigma_i^z] \neq 0$$

σ_i^z is not a conserved quantity, it **fluctuates**

Γ controls effective precession rate of spins!

We need spin 1/2-like particles to implement algorithm!

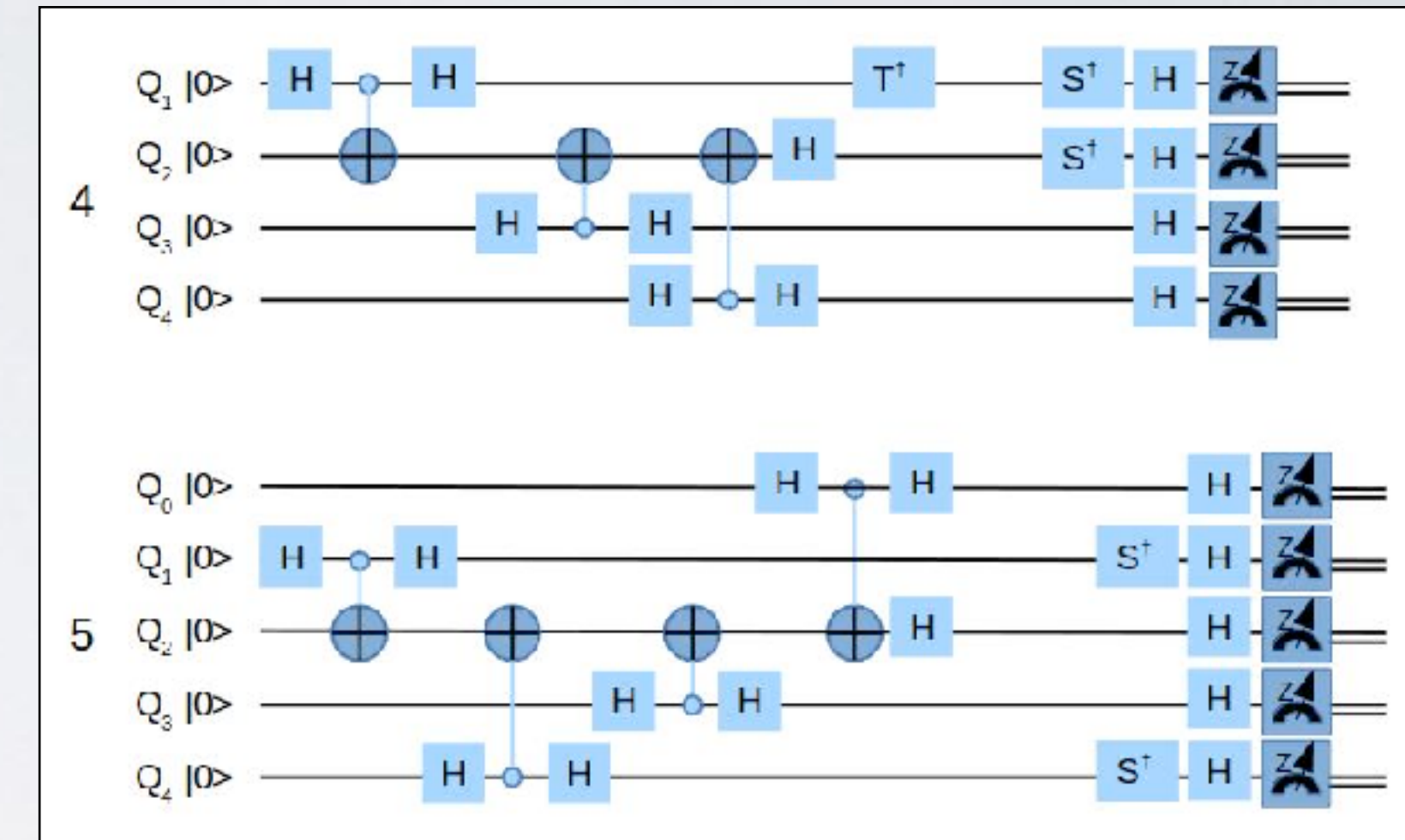
QUANTUM ANNEALING



$$\mathcal{H}(t) = \underbrace{Q(t)\mathcal{H}_D}_{\text{Trivial}} + \underbrace{C(t)\mathcal{H}_P}_{\text{Problem}}$$

Quantum Annealer	Gate-based Quantum processor
Not universal*	Universal**
Does not require QEC	Requires QEC
Scalable in the mid-term	Scalable in the long term

- * Ideas exist to make it universal
- ** There exist non-universal versions (NISQ), not clear they may show advantage



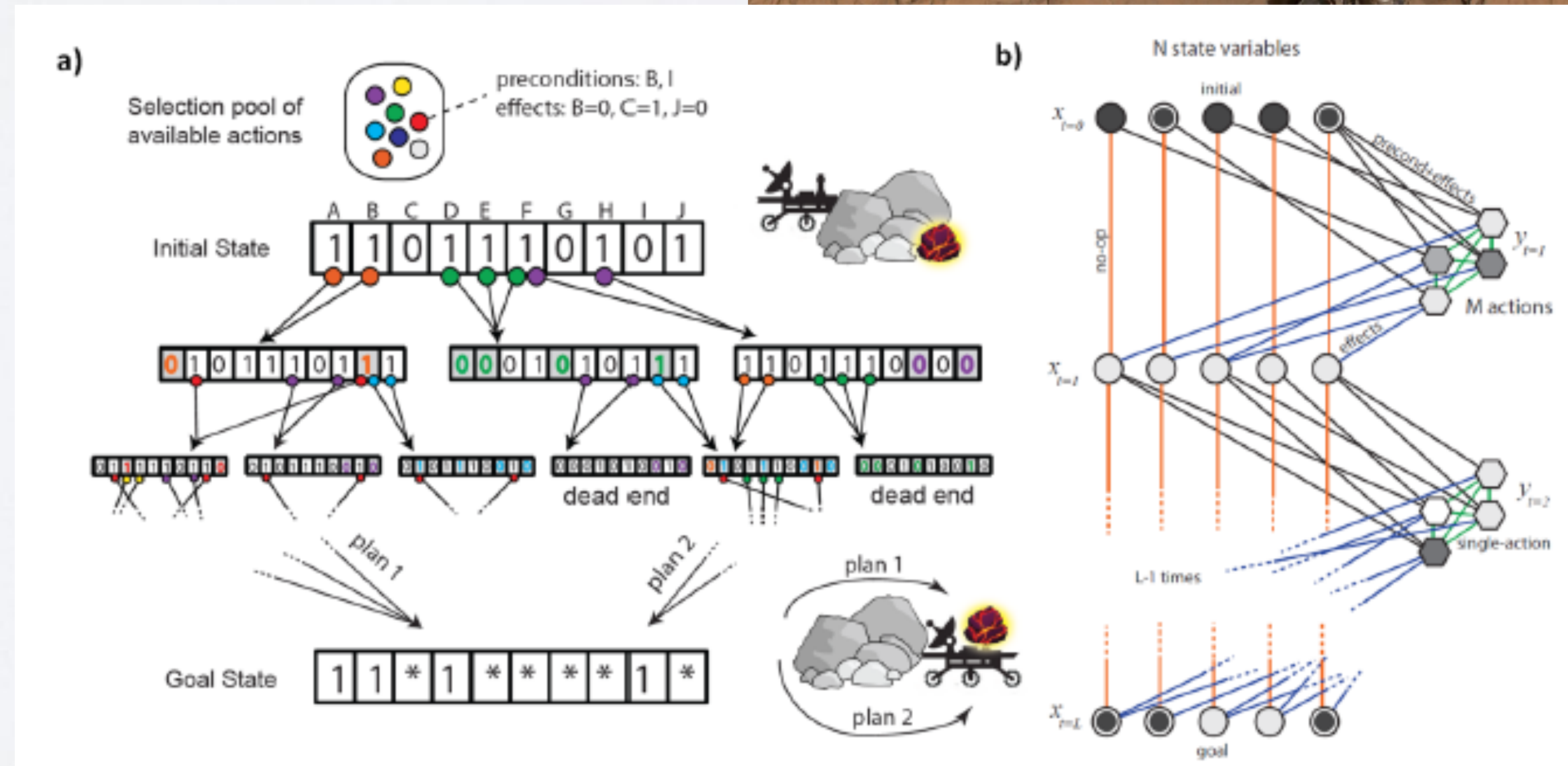
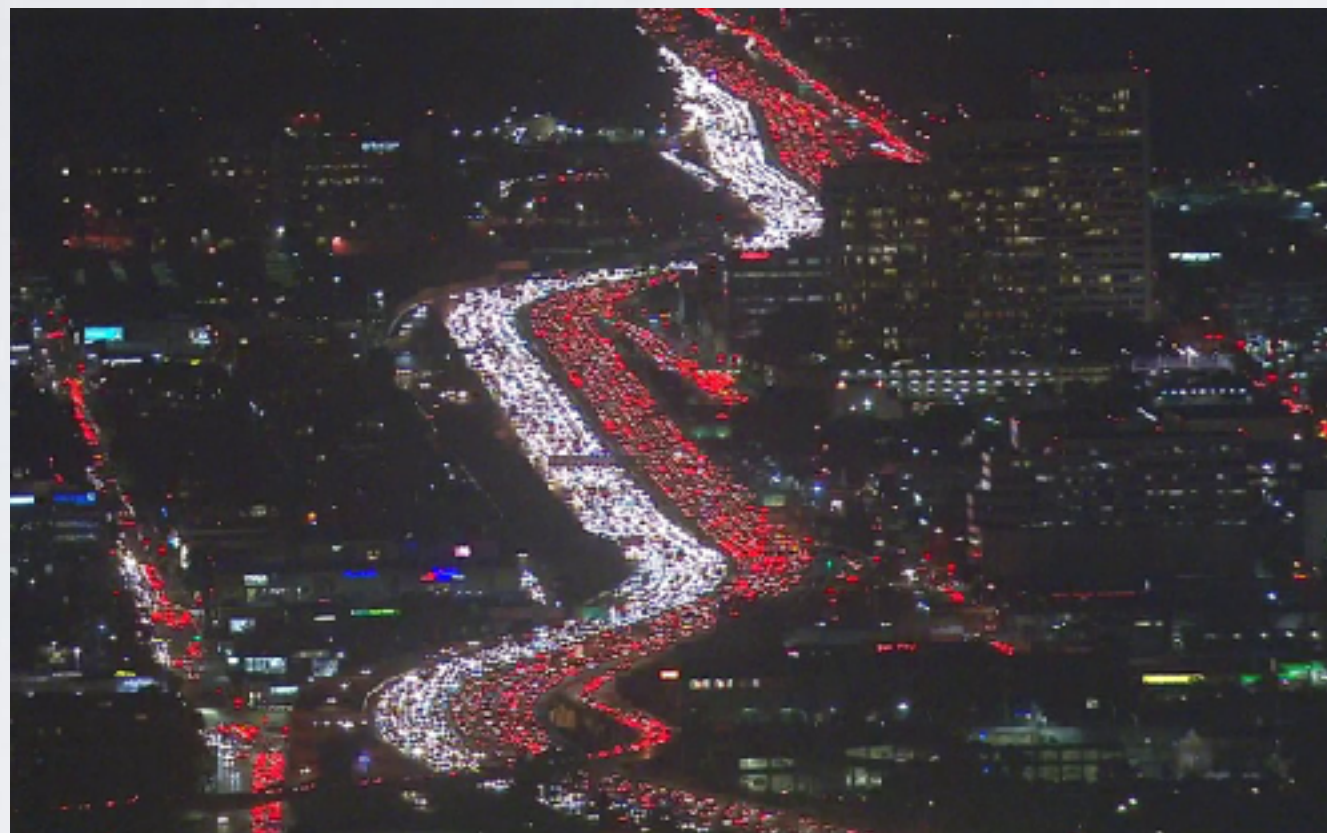
E. Farhi et al., arxiv:quant-ph/0001106

T. Albash and D. Lidar, Rev. Mod. Phys. 90, 015002 (2018)

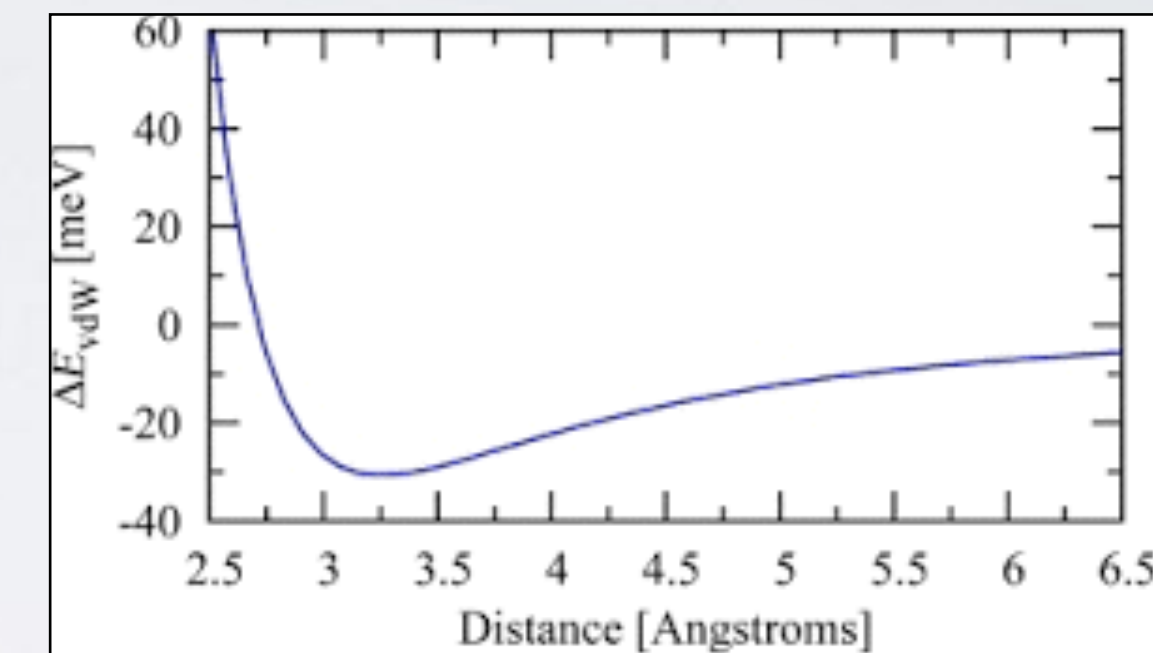
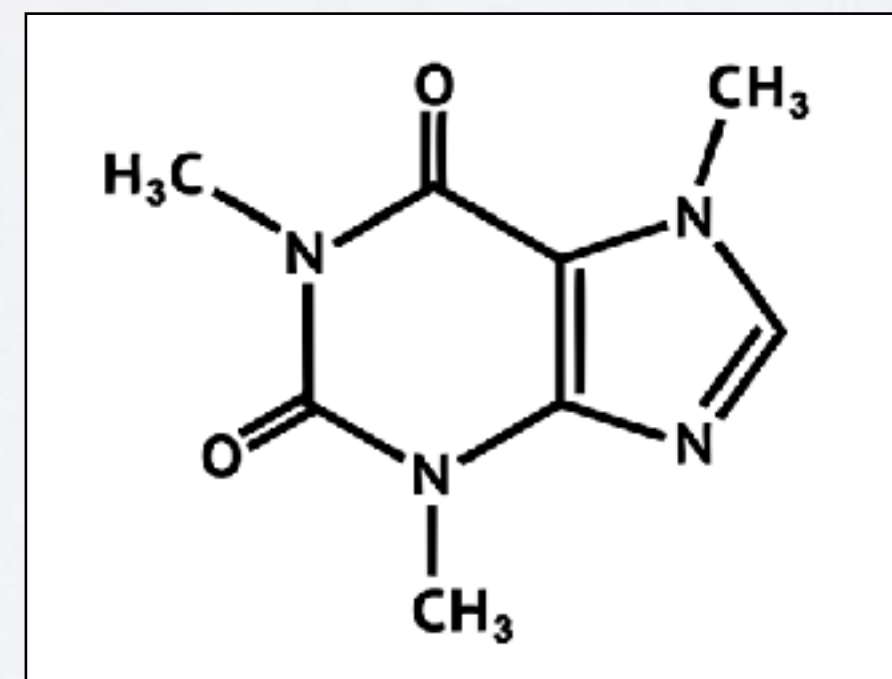
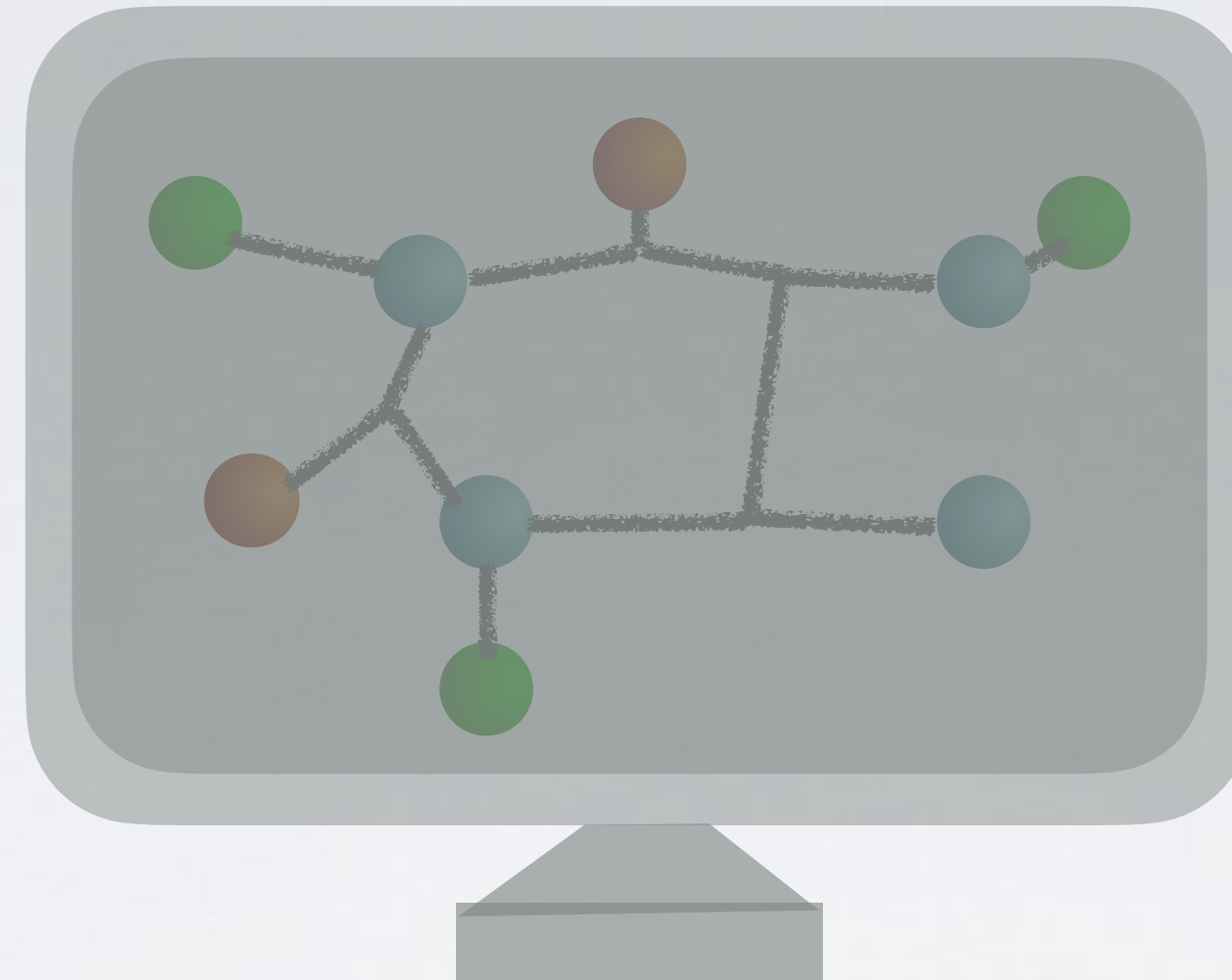
QUANTUM ANNEALING

Why do we care about small Quantum Annealers?

- Applications where small-sized quantum processors can outperform classical computers
 - 1. Optimization: traffic, navigation, scheduling, machine learning, etc.



QUANTUM ANNEALING

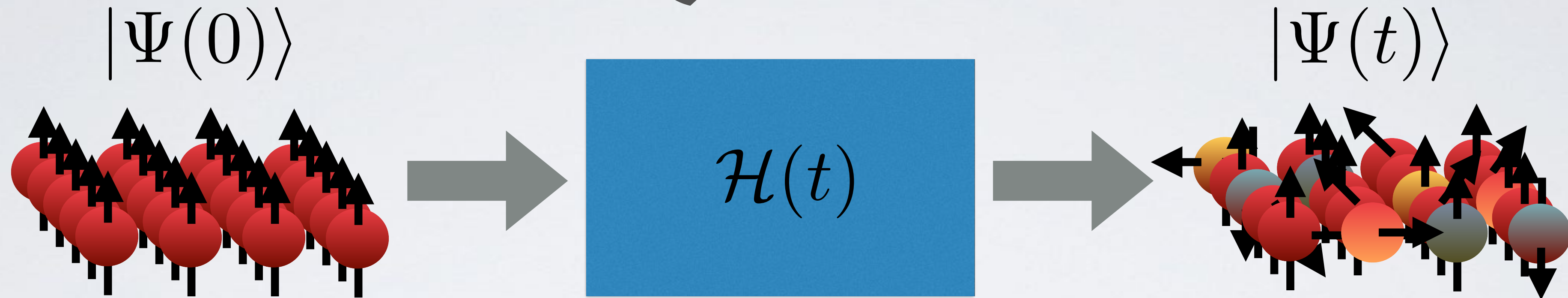


Using intrinsic properties of quantum systems can we map **and compute** complex problems?

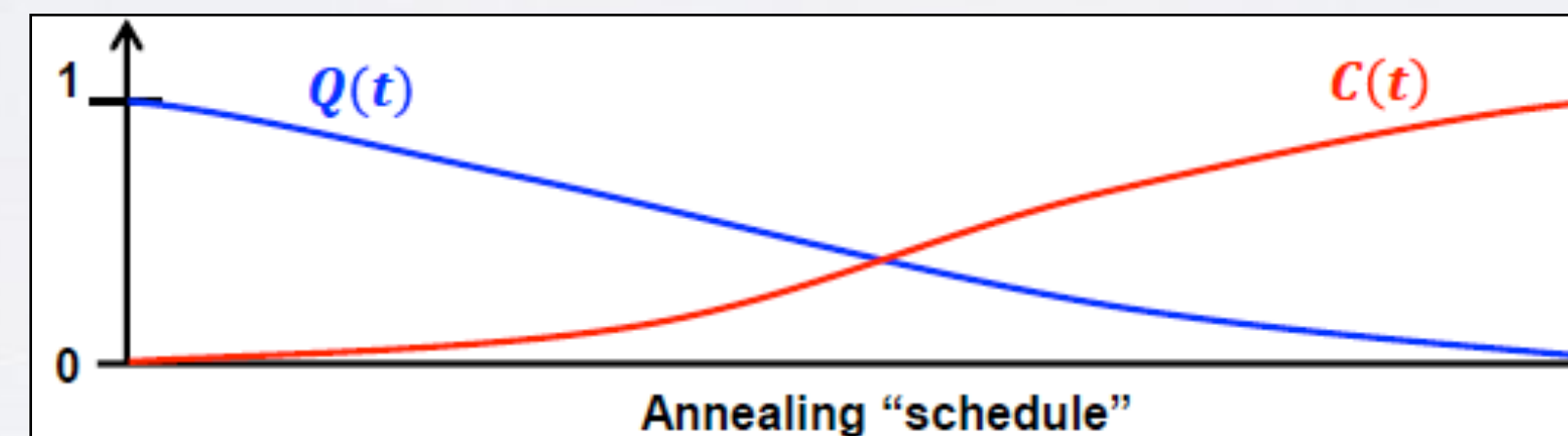
R. P. Feynman, *Simulating physics with computers*, *Int. J. Theor. Phys.* **21**, 467 (1982)

QUANTUM ANNEALING

Adiabatic Quantum Simulator



Initial state may be a superposition of each qubit's eigenstates



Final state contains the Hamiltonian of molecule/reaction in question. May **not** be ground state.

$$\mathcal{H}(t) = Q(t)\mathcal{H}_D + C(t)\mathcal{H}_P$$

Trivial

Problem

Requires system to hold quantum coherence throughout evolution. Must contain complex qubit-qubit interactions (beyond Ising): **Coherent Quantum Annealer**.

QUANTUM ANNEALING

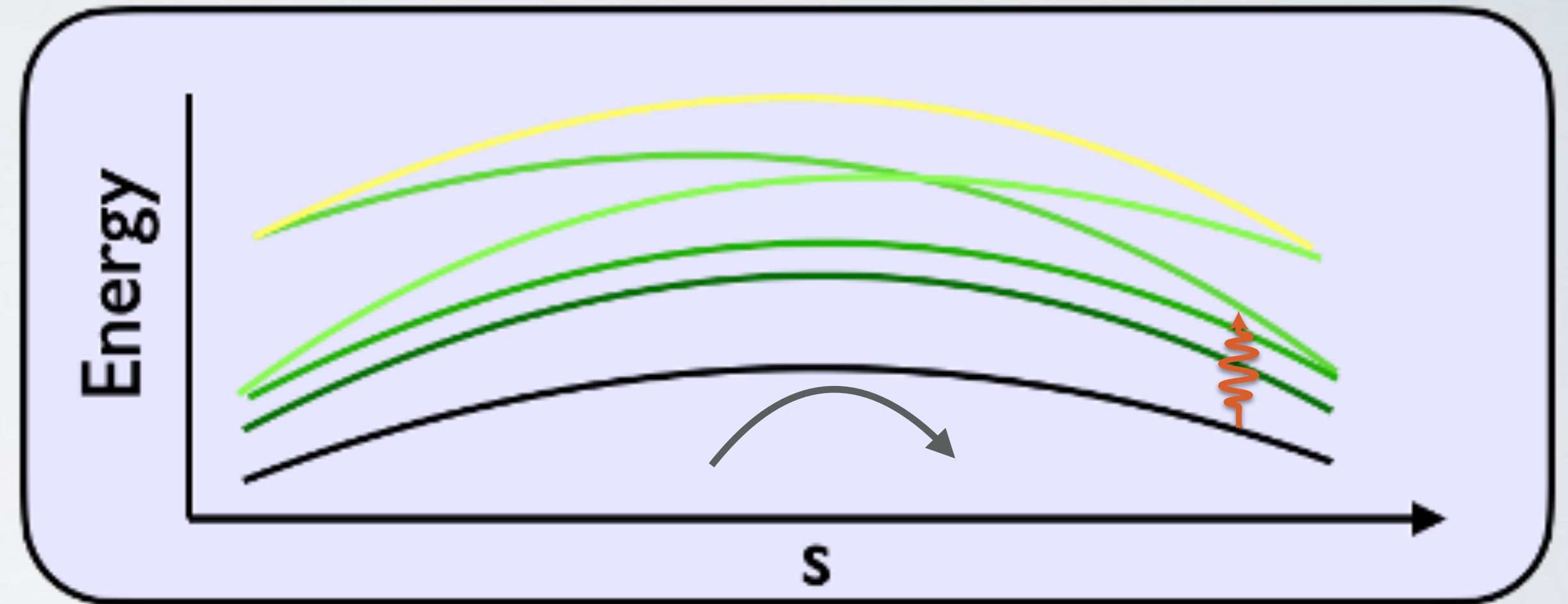
All very nice, but....

- The gap problem: $\Delta \sim e^{-kn}$
- Gap size + gap position (NP hard)
- Thermal noise / decoherence
- Quantum Monte Carlo
- ...

Some solutions exist, no general prescription:

- Reverse annealing: partial knowledge of solution
- Non-homogeneous annealing: qubit-specific schedule
- Non-stoquastic terms $\mathcal{H}(t) = (1 - t/T)\mathcal{H}_D + (t/T)\mathcal{H}_P + (t/T)(1 - t/T)\mathcal{H}_{NS}$

Much more work needed!



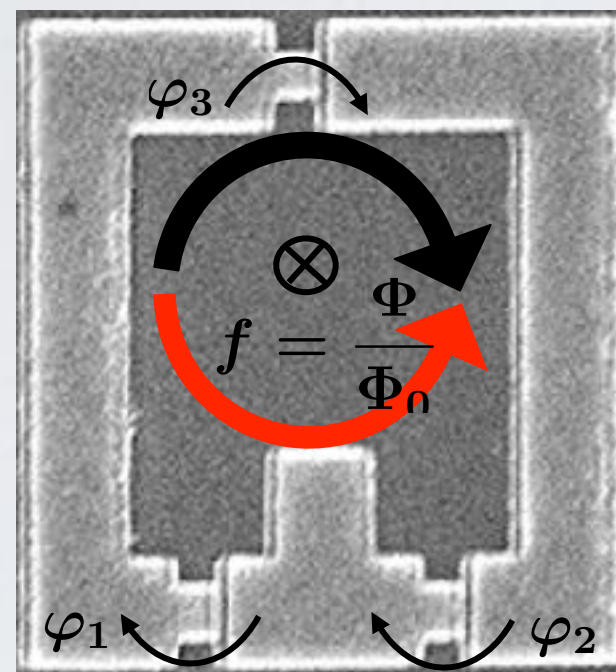
Perspectives of quantum annealing: Methods and implementations
P. Hauke, H. G. Katzgraber, W. Lechner, H. Nishimori, W. D. Oliver
Reports on Progress in Physics 83, 054401 (2020) | arXiv:1903.06559 (2019)

END OF PART I

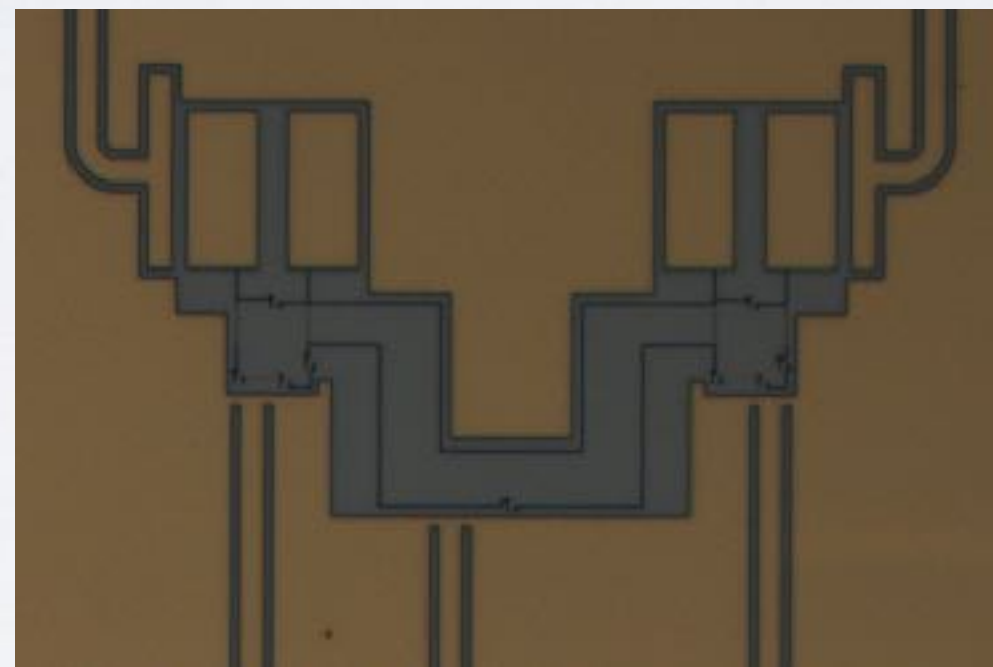
EXPERIMENTAL QA

Ingredients to build a quantum annealer:

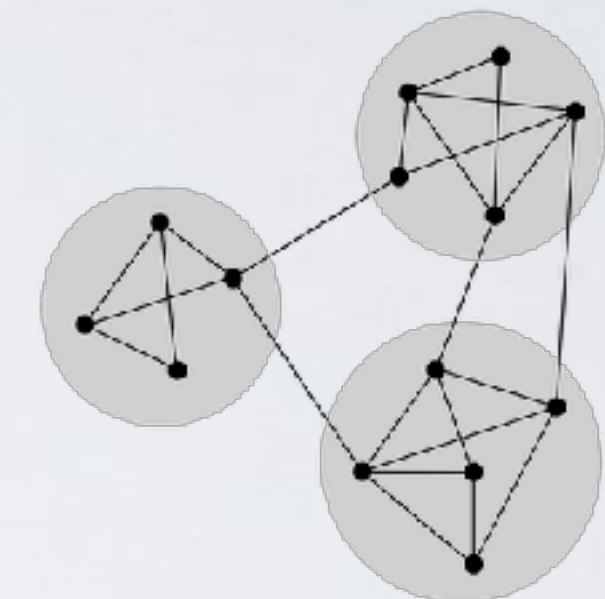
Spin-1/2-like Qubits



Qubit-qubit couplers



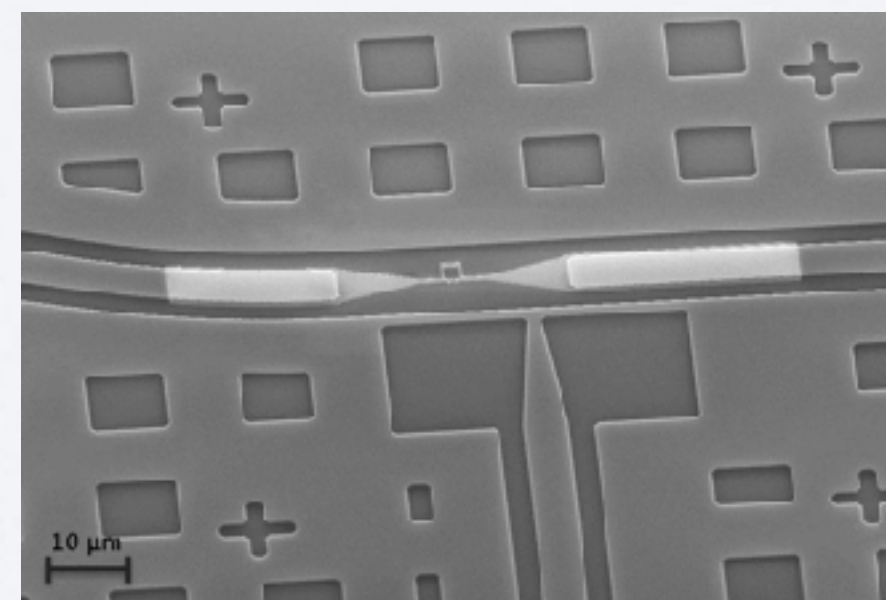
Qubit connectivity map



Readout



Individual qubit control



Classical control



SUPERCONDUCTING QUBITS WITH JOSEPHSON JUNCTIONS

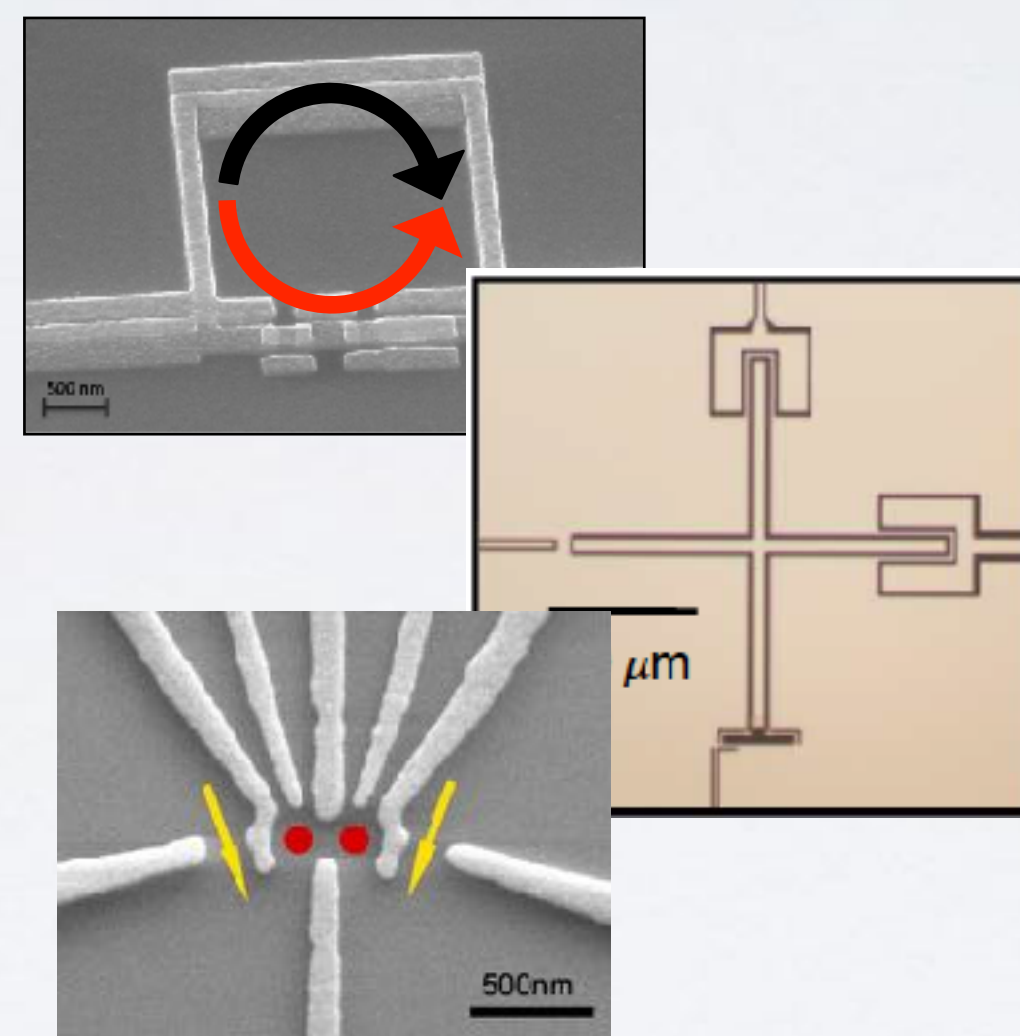
CIRCUIT QUANTIZATION

Microscopic



$$i\hbar \frac{\partial \psi(\vec{r}, t)}{\partial t} = \mathcal{H}(\vec{r}, t) \psi(\vec{r}, t)$$

Mesoscopic



Large number of particles

Artificial, man-made

Quantum collective degrees of freedom

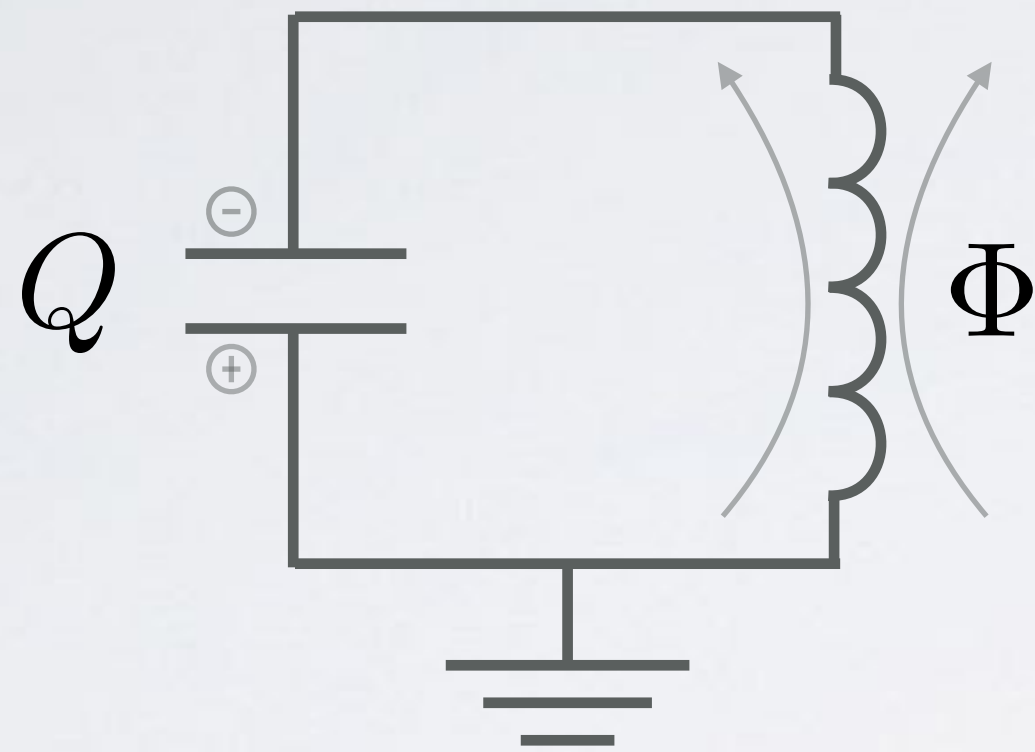
Macroscopic



$$\vec{F} = m\vec{a}$$

CIRCUIT QUANTIZATION

What corresponds to these collective degrees of freedom?



$$\mathcal{H} = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

$$\omega_{\text{LC}} = \frac{1}{\sqrt{LC}}$$

$$k_B T \ll \omega_{\text{LC}}$$

$$P_e = e^{-\frac{\hbar \omega_{\text{LC}}}{k_B T}} \approx 6 \times 10^{-6}$$

$$\langle \hat{\Phi}^2 \rangle = \frac{\hbar Z_0}{2} \coth \left(\frac{\hbar \omega_{\text{LC}}}{2k_B T} \right)$$

$$\begin{matrix} \Phi \\ Q \end{matrix} \rightarrow \begin{matrix} \hat{\Phi} \\ \hat{Q} \end{matrix}$$

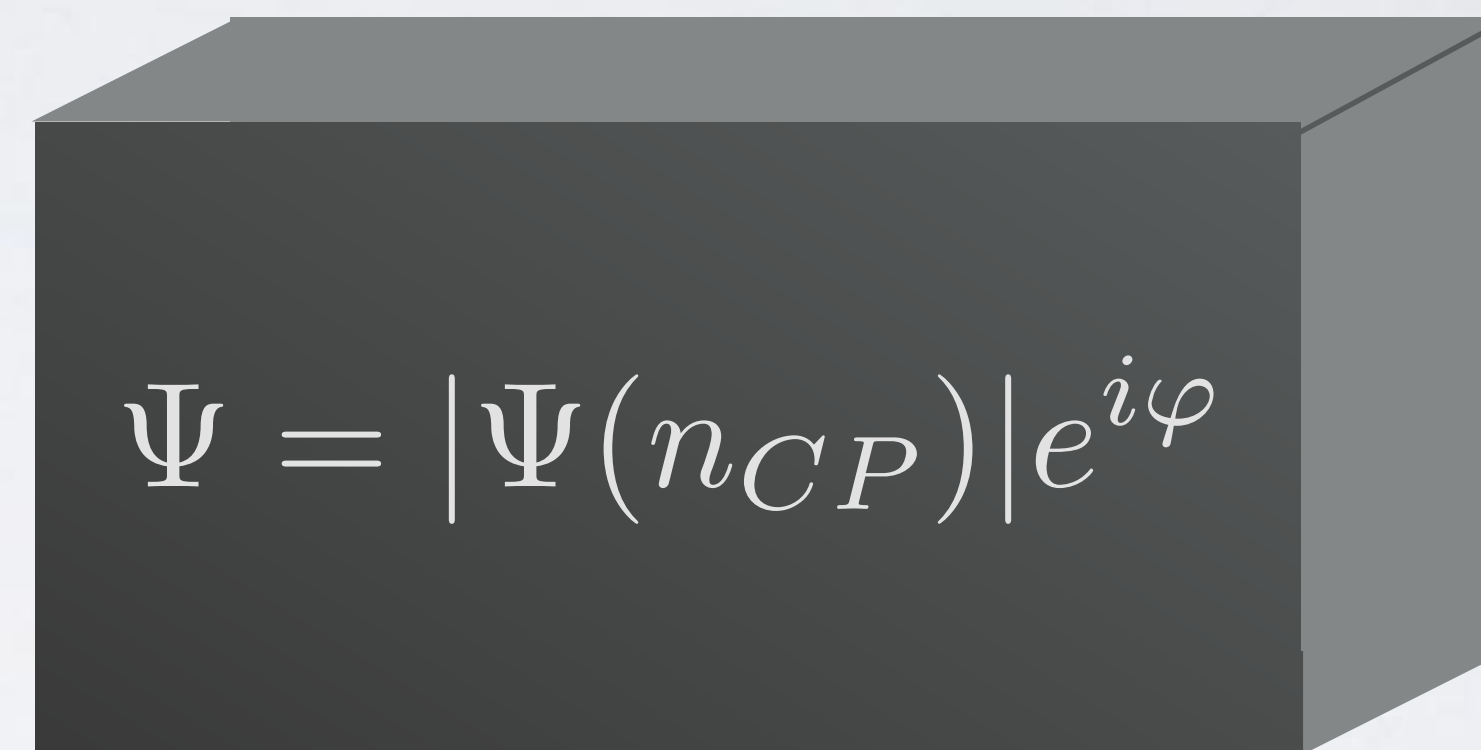
$$[\hat{\Phi}, \hat{Q}] = i\hbar$$

Quantum fluctuations of collective d.o.f.!

CIRCUIT QUANTIZATION

Superconductivity for qubits |0|

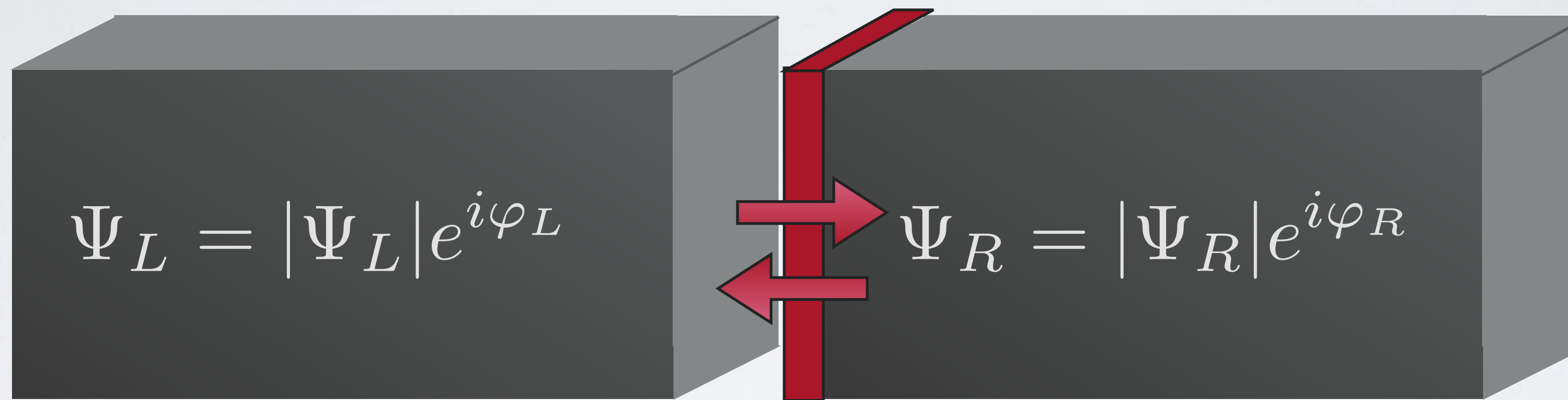
$$T < T_C$$


$$\Psi = |\Psi(n_{CP})|e^{i\varphi}$$

- Macroscopic quantum system with collective wave function
- No dissipation by charge transport
- Charge and phase conjugate variables: $[\hat{\varphi}, \hat{n}] = i \longleftrightarrow [\hat{x}, \hat{p}] = i\hbar$

CIRCUIT QUANTIZATION

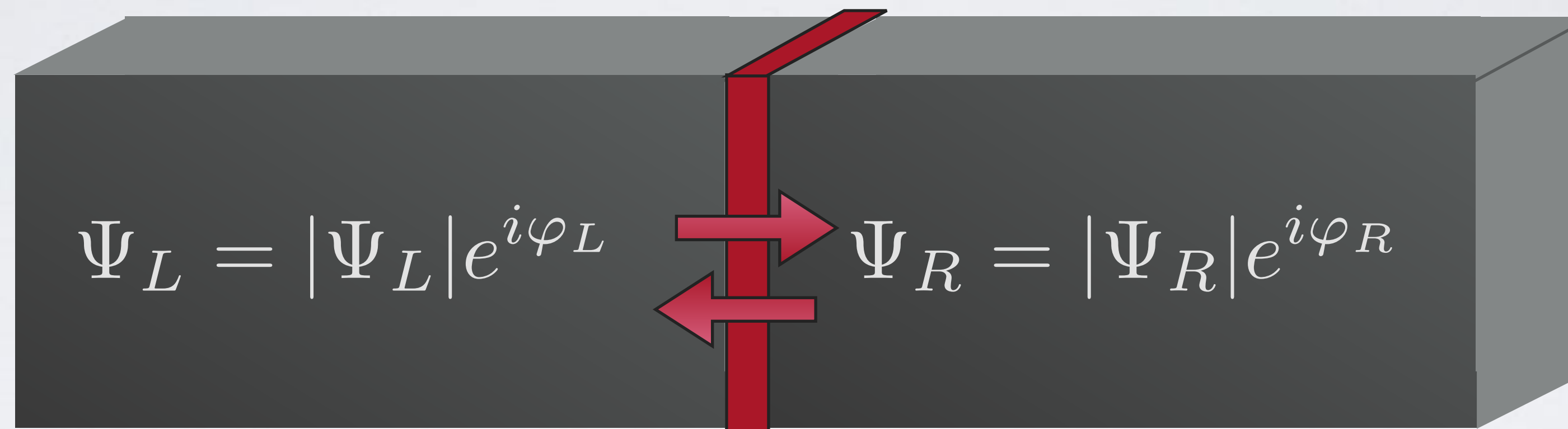
Superconductivity for qubits |0|



- Each block has its own wavefunction
- Perfectly isolated systems
- Insulating layer separates superconducting blocks
- Quantum tunneling is established between both sides leading to SUPERCURRENTS. It's a Josephson tunnel junction.

CIRCUIT QUANTIZATION

Superconductivity for qubits |0|



First Josephson relation:

$$I_{L \rightarrow R} = I_C \sin(\varphi_L - \varphi_R)$$

Critical
current

Flux quantum:

$$\Phi_0 = \frac{h}{2e} \sim 2.07 \times 10^{-15} \text{ Wb}$$

Second Josephson relation:

$$V = \frac{\Phi_0}{2\pi} \frac{d(\varphi_L - \varphi_R)}{dt}$$

Energy stored:

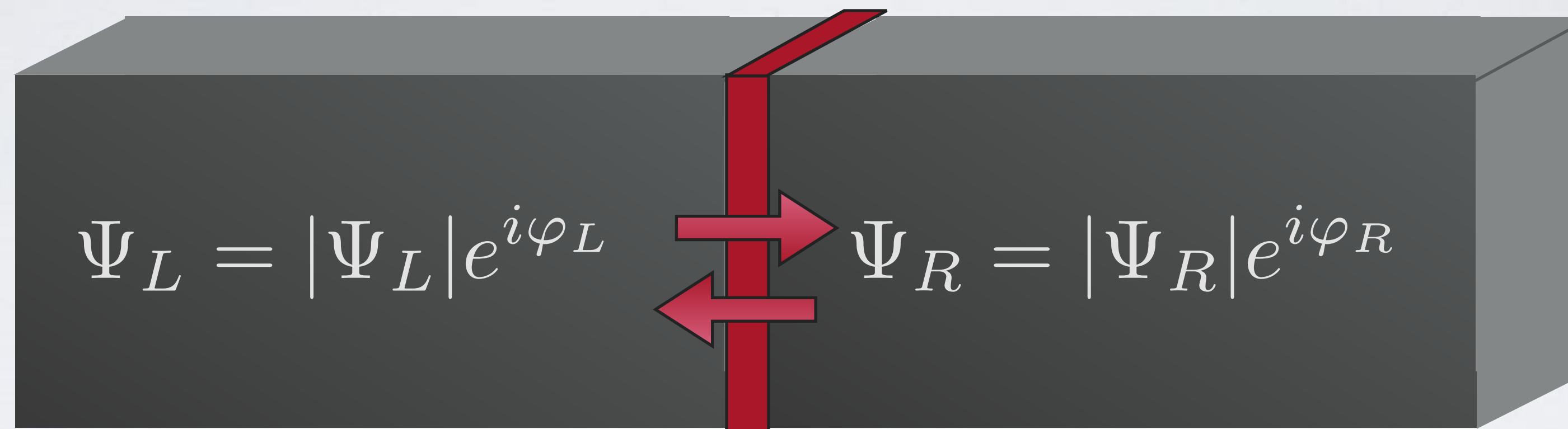
$$U = -E_J \cos(\varphi_L - \varphi_R)$$

Josephson energy:

$$E_J = I_C \Phi_0 / 2\pi$$

CIRCUIT QUANTIZATION

Superconductivity for qubits |0|



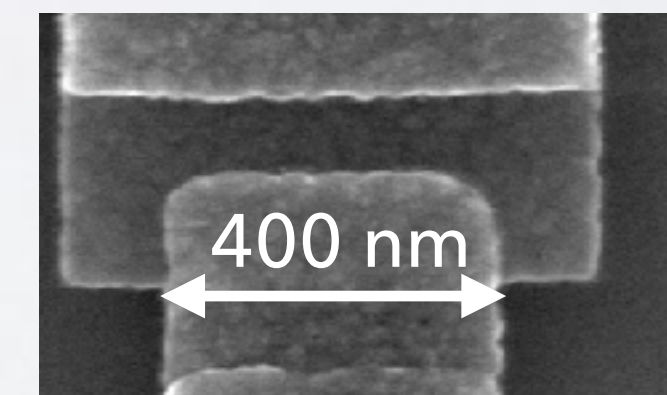
Constituent relation:

$$\frac{dI}{dt} = V \frac{2\pi I_C}{\Phi_0} \cos \varphi$$

$$\equiv 1/L_J(\varphi)$$

Josephson inductance

Electrical circuits symbol



SEM micrograph

- ▶ Nonlinear circuit element
- ▶ Lossless

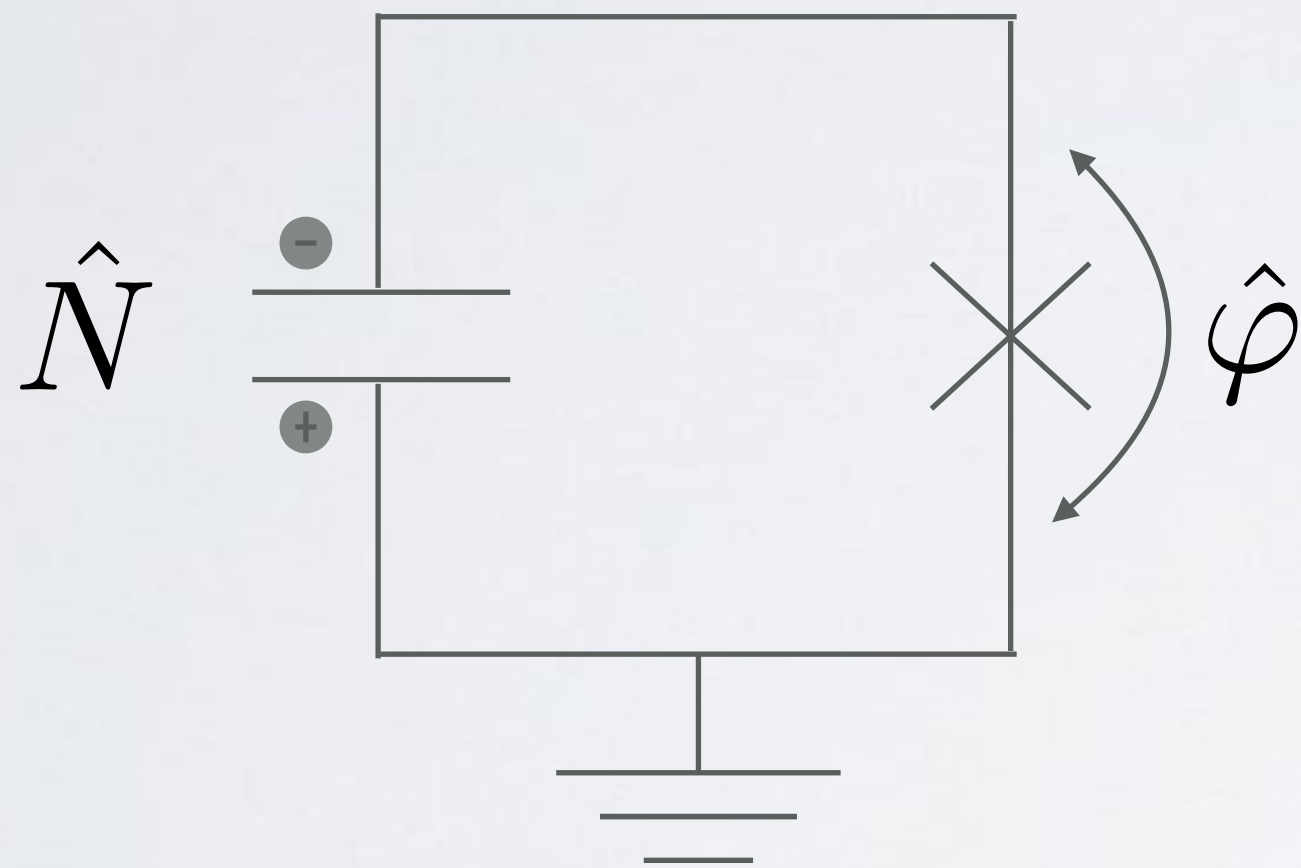
Orlando, Delin.
Foundation of applied superconductivity

SUPERCONDUCTING QUBITS

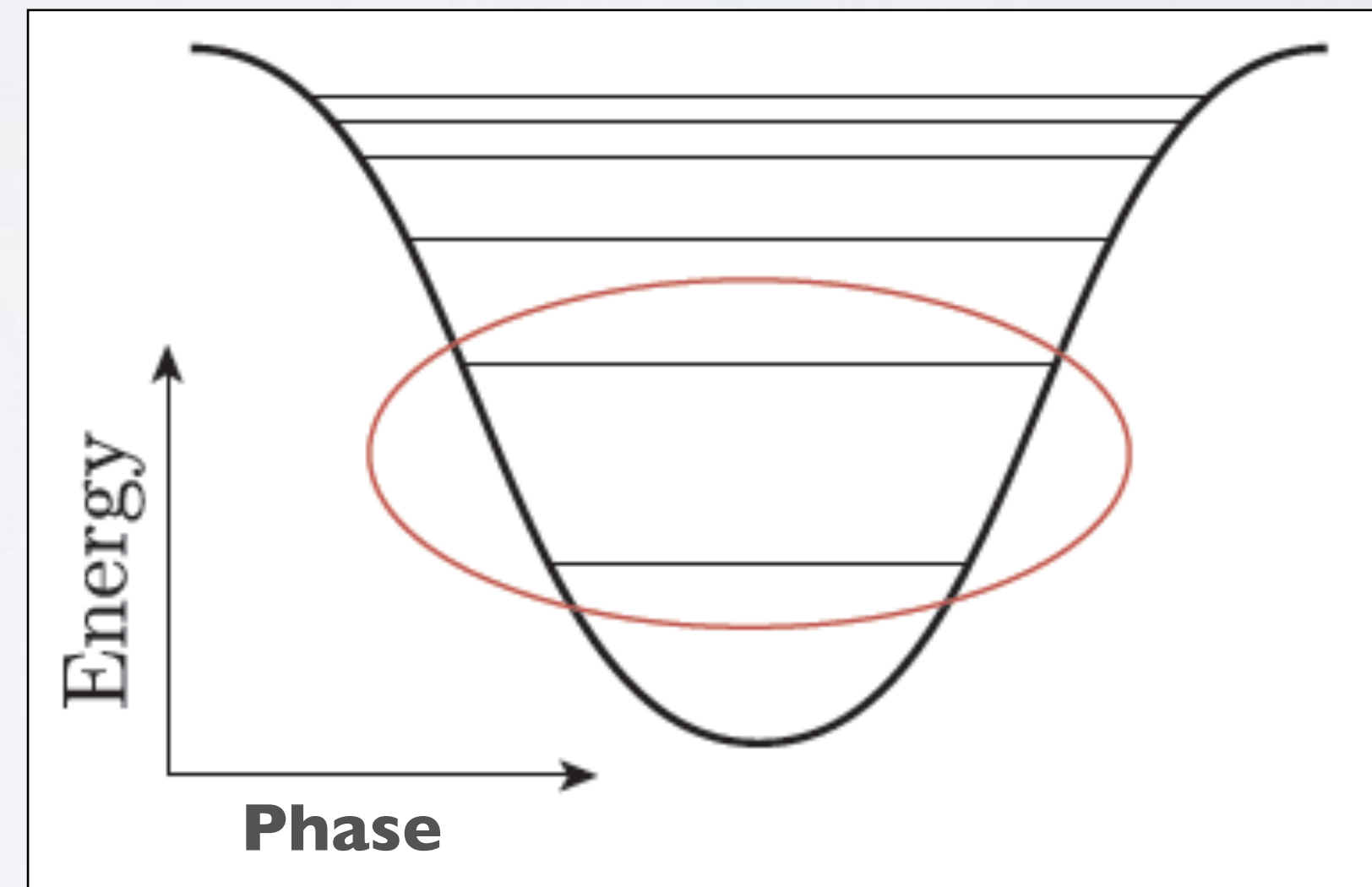
Can we use transmon qubits for annealing?

Directly, no. Hamiltonian does not map well into spin 1/2-like particle. It has no Zeeman-like splitting

$$\hat{\mathcal{H}} = E_C \hat{N}^2 - E_J \cos(\hat{\varphi})$$



Hydrogen atom of superconducting qubits

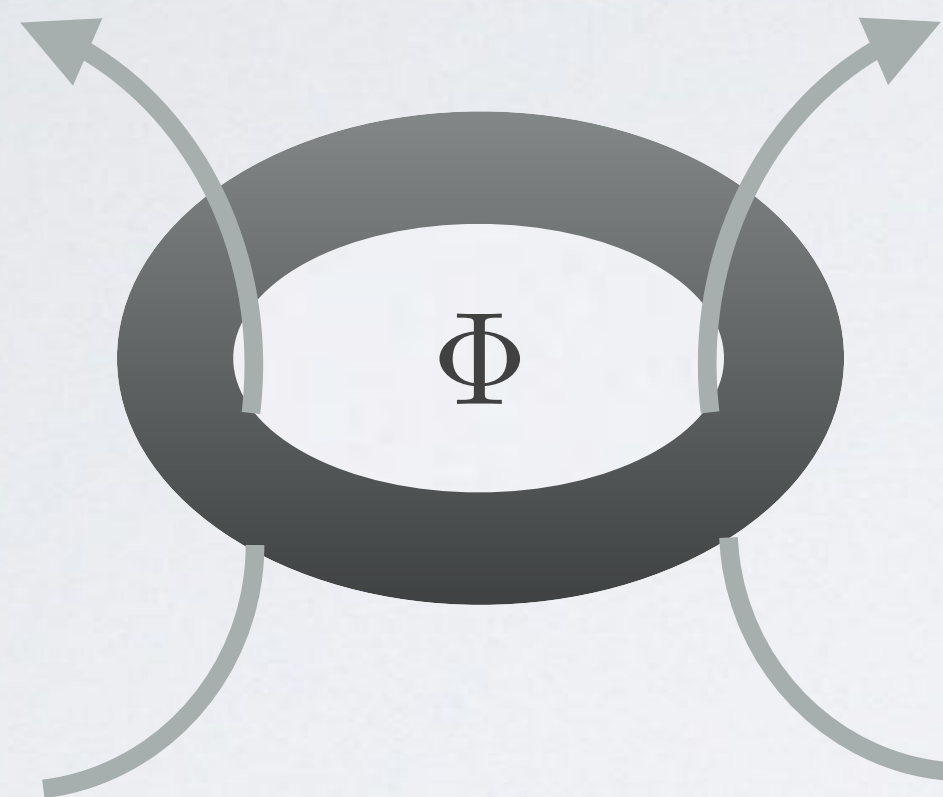


Harmonic-like potential energy: ground state is the vacuum state

Need more sophisticated qubit...

SUPERCONDUCTING QUBITS

Superconducting ring



$$\Psi = |\Psi(n_{CP})|e^{i\varphi}$$

$$\Delta\varphi = \varphi + 2\pi N$$

fixed upon cooldown:
Number of flux quanta trapped

If ring is very thick, supercurrent only runs on the surface, and the magnetic **flux becomes quantized** to a **flux quantum**:

$$\Phi = N\Phi_0$$

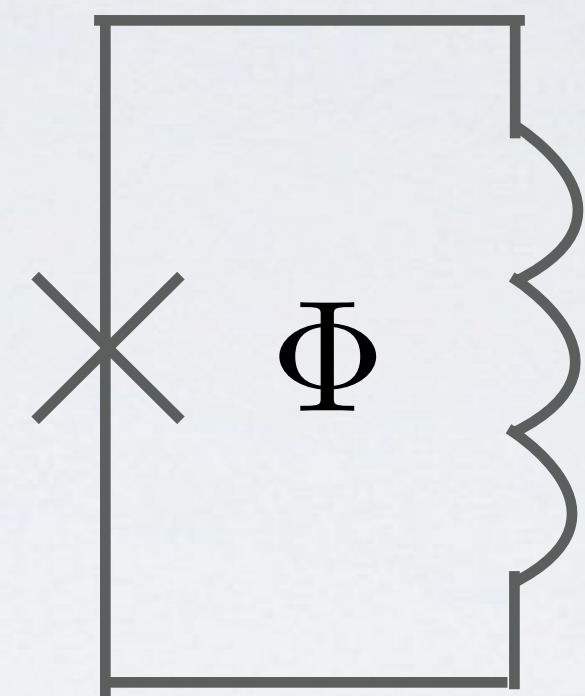
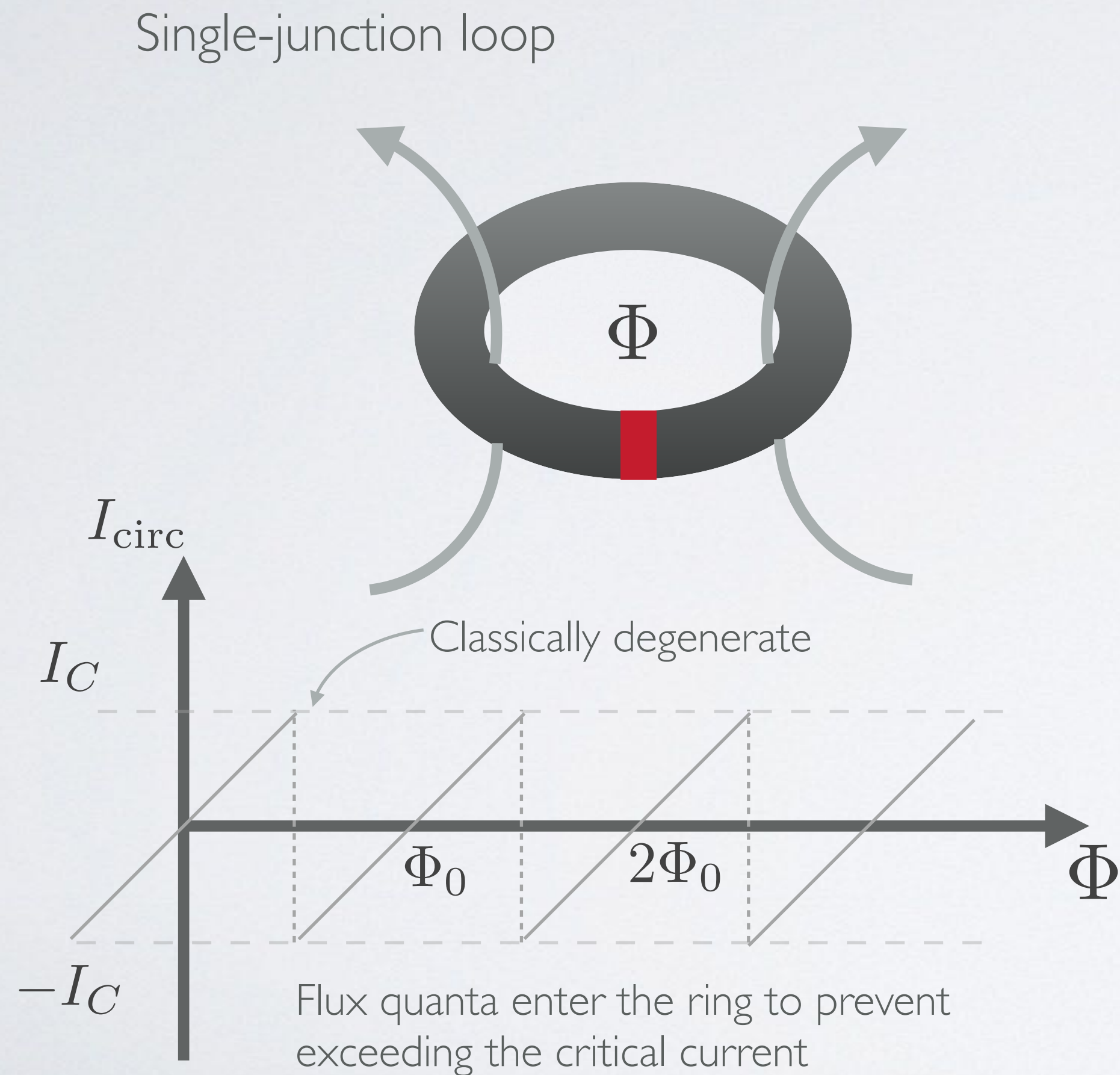
$$\Phi_0 = \frac{h}{2e} = 2.07 \times 10^{-15} \text{ Wb}$$

Earth's field: Check phone

Area for 1 flux quantum?

SUPERCONDUCTING QUBITS

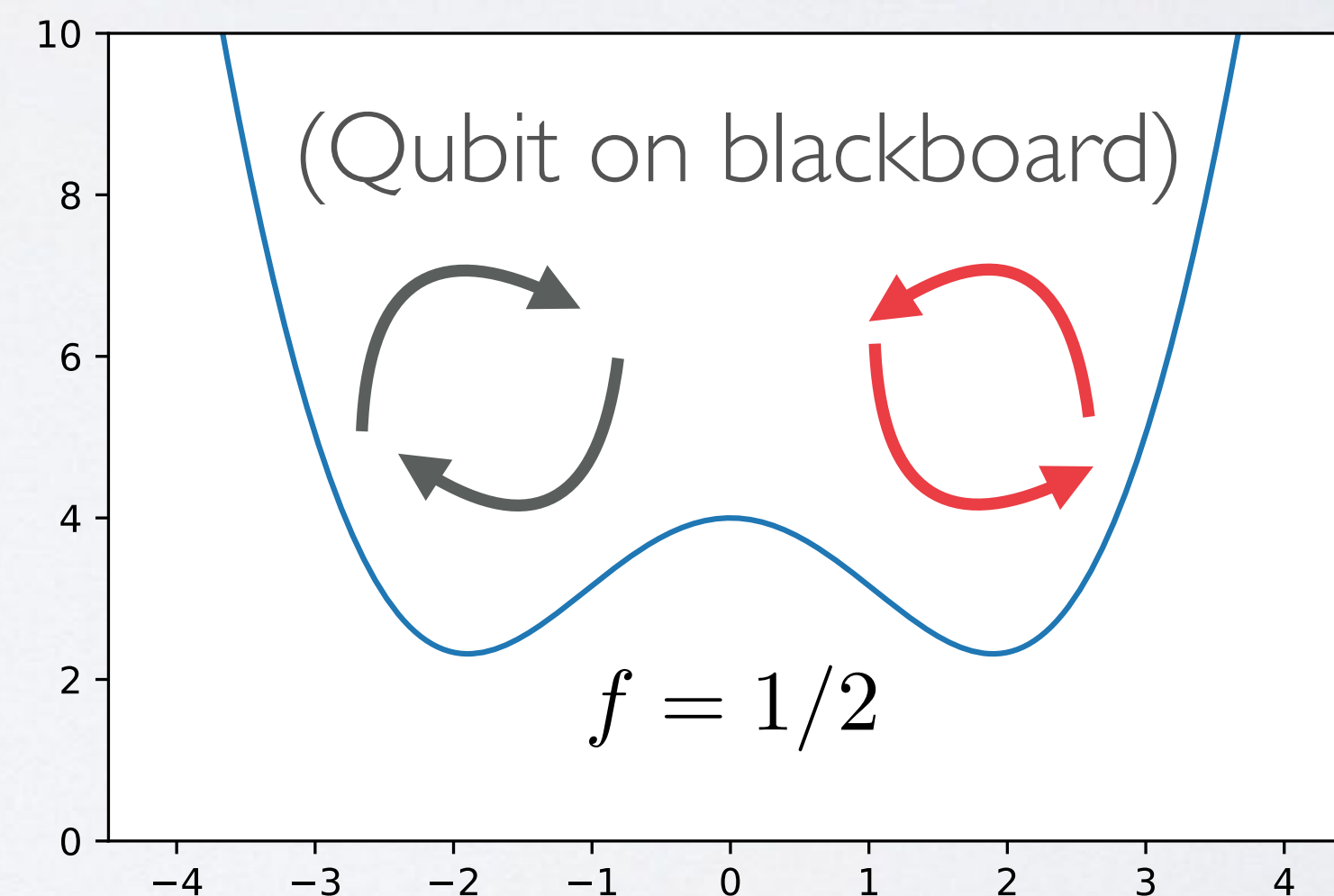
Hydrogen atom Flux qubit: rf-SQUID



$$\beta = \frac{L_g}{L_J} = \frac{E_J}{E_L} > 1$$

Bistable, good qubit regime. Condition requires **large loop inductance**.

Qubit potential energy $U = -E_J \cos \varphi + (E_L/2)(2\pi f - \varphi)^2$



$$f = \Phi/\Phi_0$$

$$E_L = \frac{1}{L} \left(\frac{\Phi_0}{2\pi} \right)^2$$

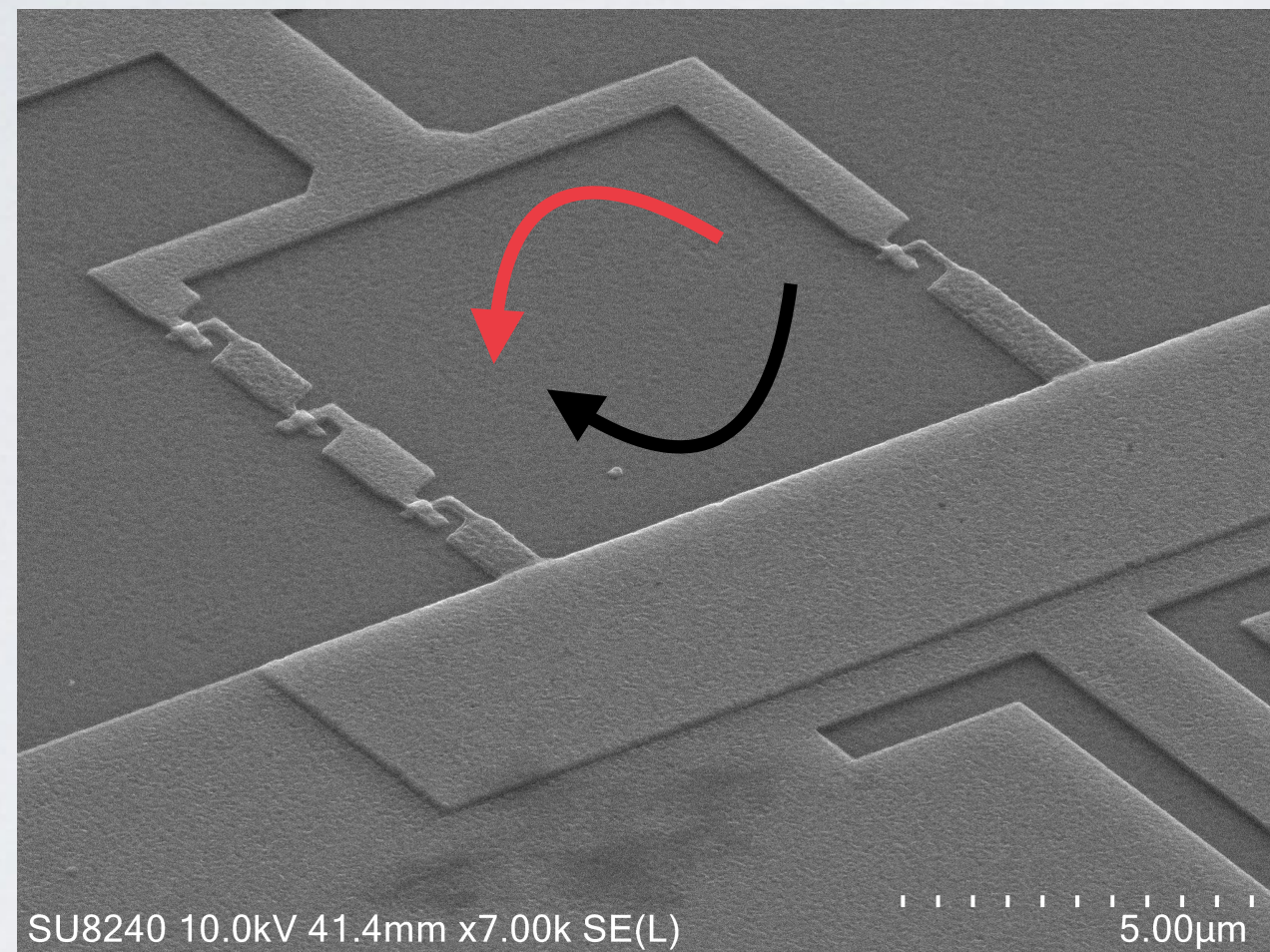
$$E_J = \frac{I_C \Phi_0}{2\pi}$$

Usually, large inductance implies large loops, too sensitive to flux noise...

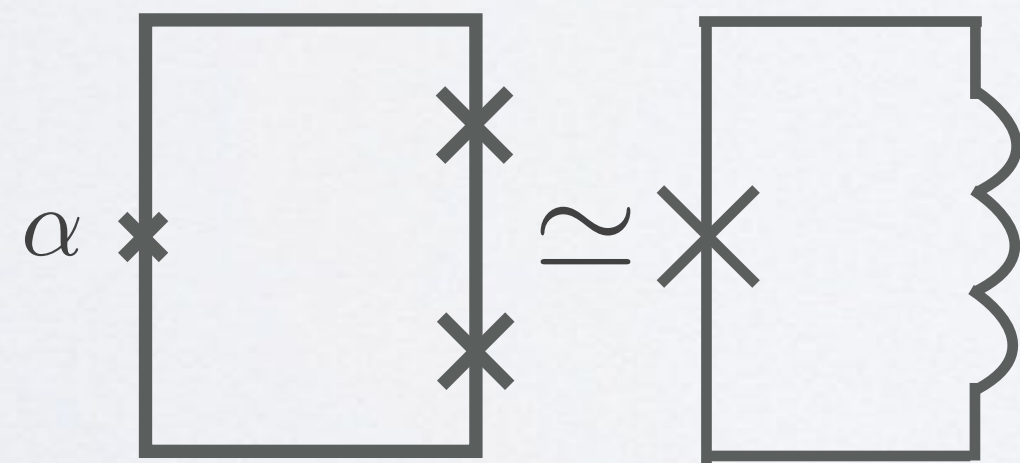
SUPERCONDUCTING QUBITS

Persistent current qubit: multiple junctions to reduce loop size

Aluminum loop with 3 Josephson junctions



Circuit diagram:

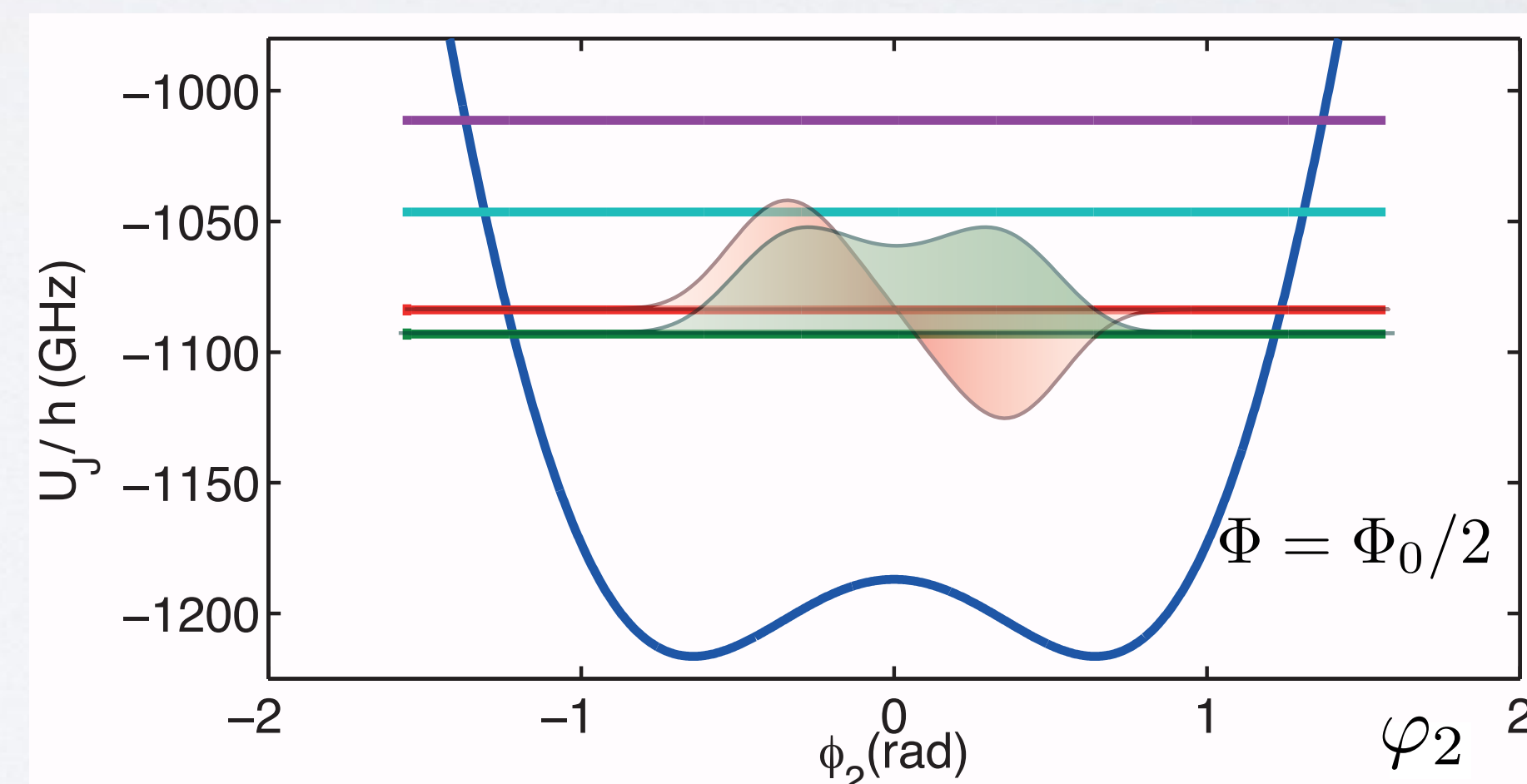


T. P. Orlando, et al. PRB 60 15398 (1999)

J. E. Mooij, et al. Science 285 1036 (1998)

Circuit Hamiltonian:

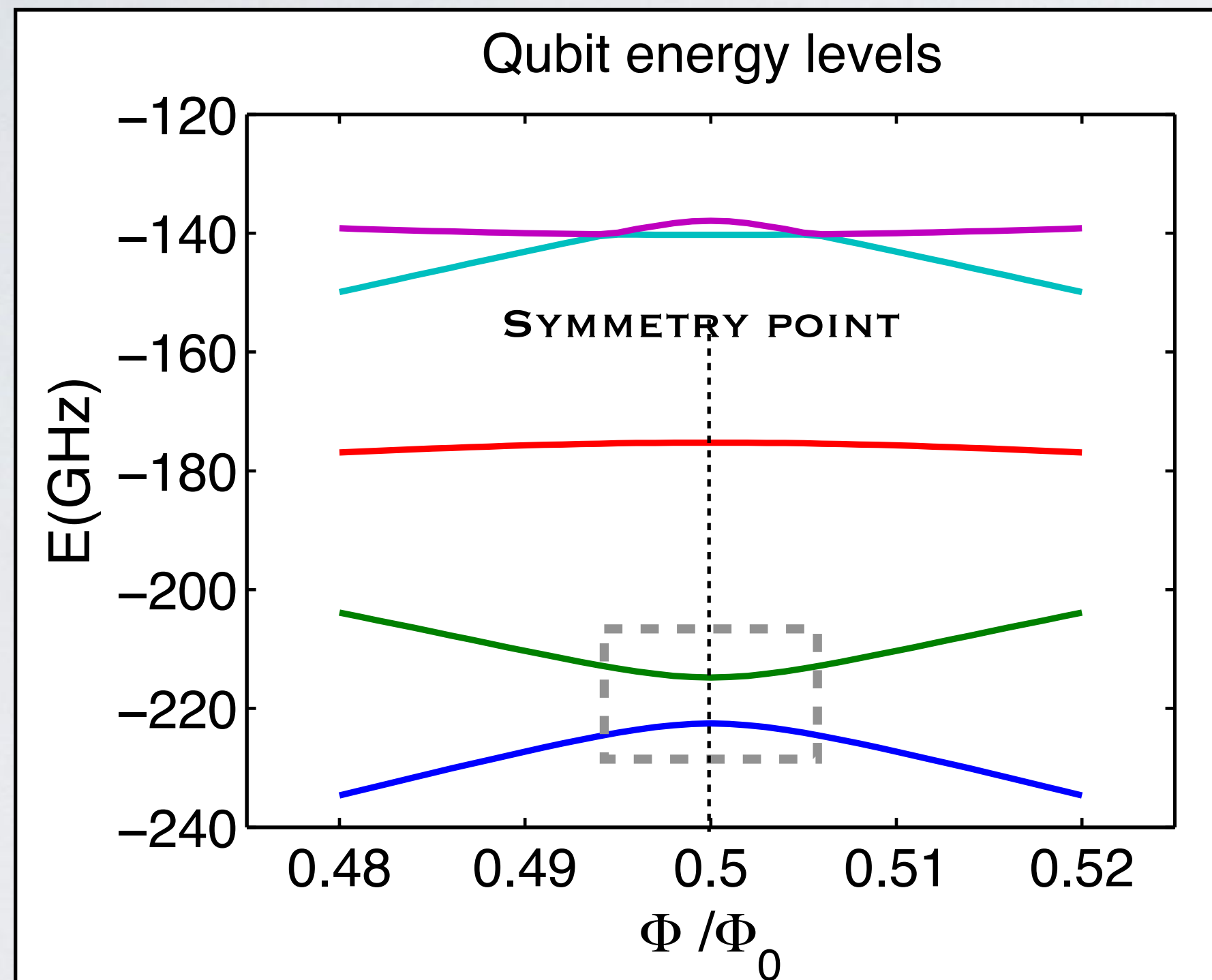
$$\mathcal{H}_{\text{PCQ}} = \frac{E_C}{1 + 2\alpha} [n_1^2 - 2\alpha n_1 n_2 + (1 + \alpha)n_2^2] + E_J [\cos \varphi_1 + \cos \varphi_2 + \alpha \cos(\varphi_1 + \varphi_2 + 2\pi\Phi/\Phi_0)]$$



The tunneling through or over the barrier couples states into superposition of phase, hence current states

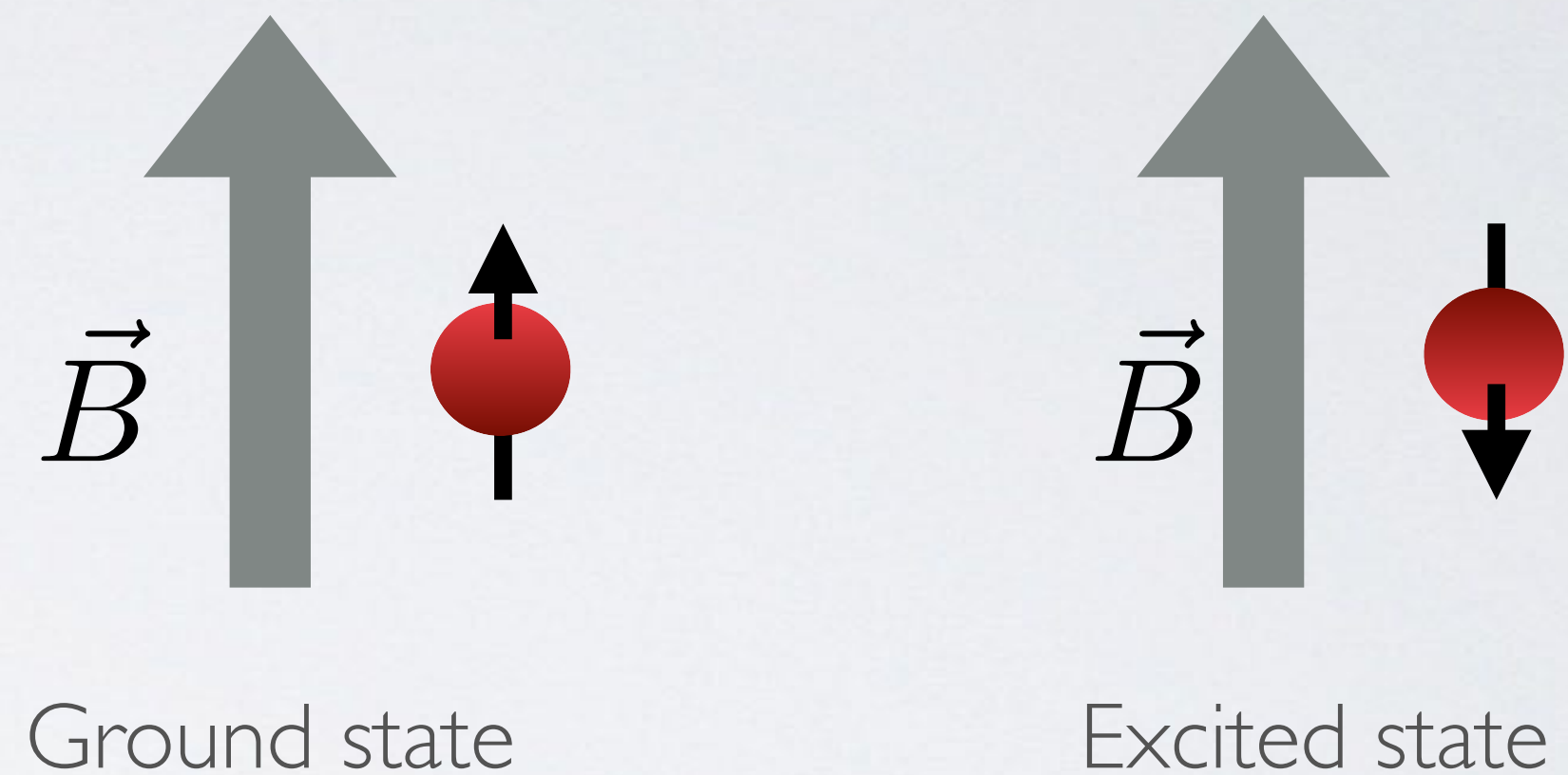
SUPERCONDUCTING QUBITS

Persistent current qubit



Ground state energy changes slope: persistent current flip
External field determines current orientation in ground state!

Qubit spectrum resembles that of a spin-1/2 particle in a magnetic field!



Zeeman energy:

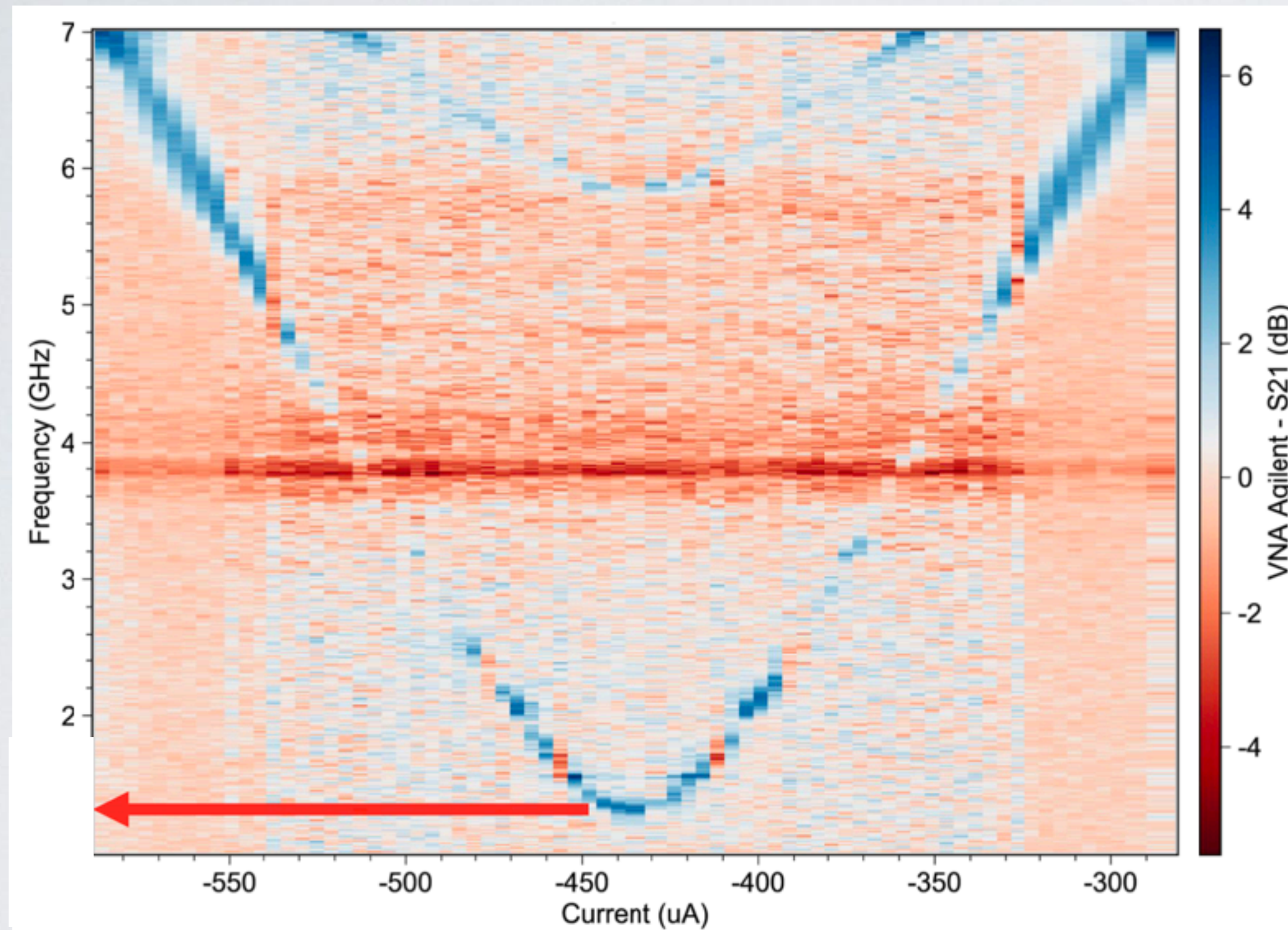
$$U_Z = -\vec{B} \cdot \vec{S}$$

Zero-field splitting (could be from spin-orbit coupling effects)

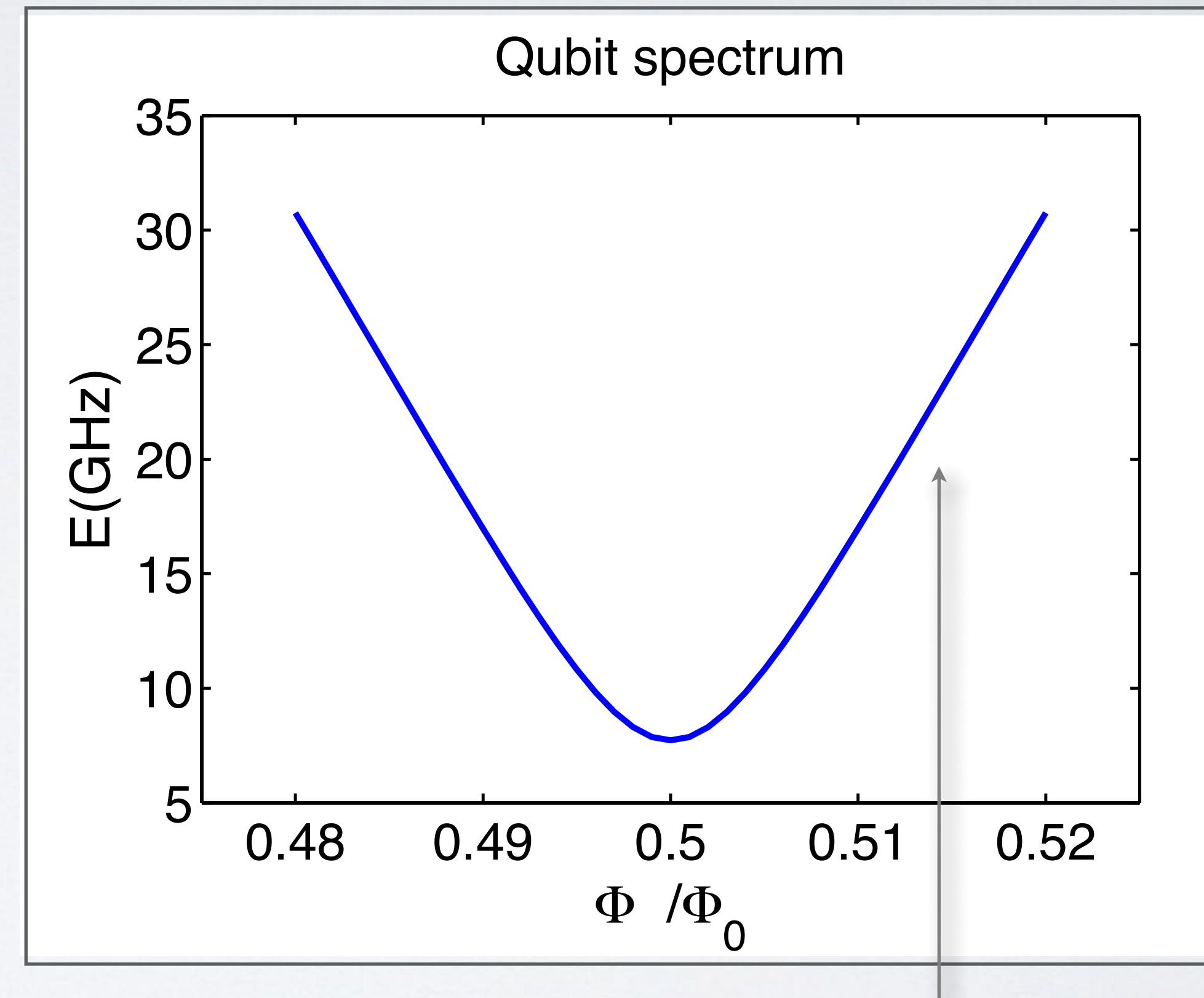
Flux qubit is a good emulator of spin-1/2 particle!

SUPERCONDUCTING QUBITS

Experimental verification at IFAE lab



Flux qubit spectrum

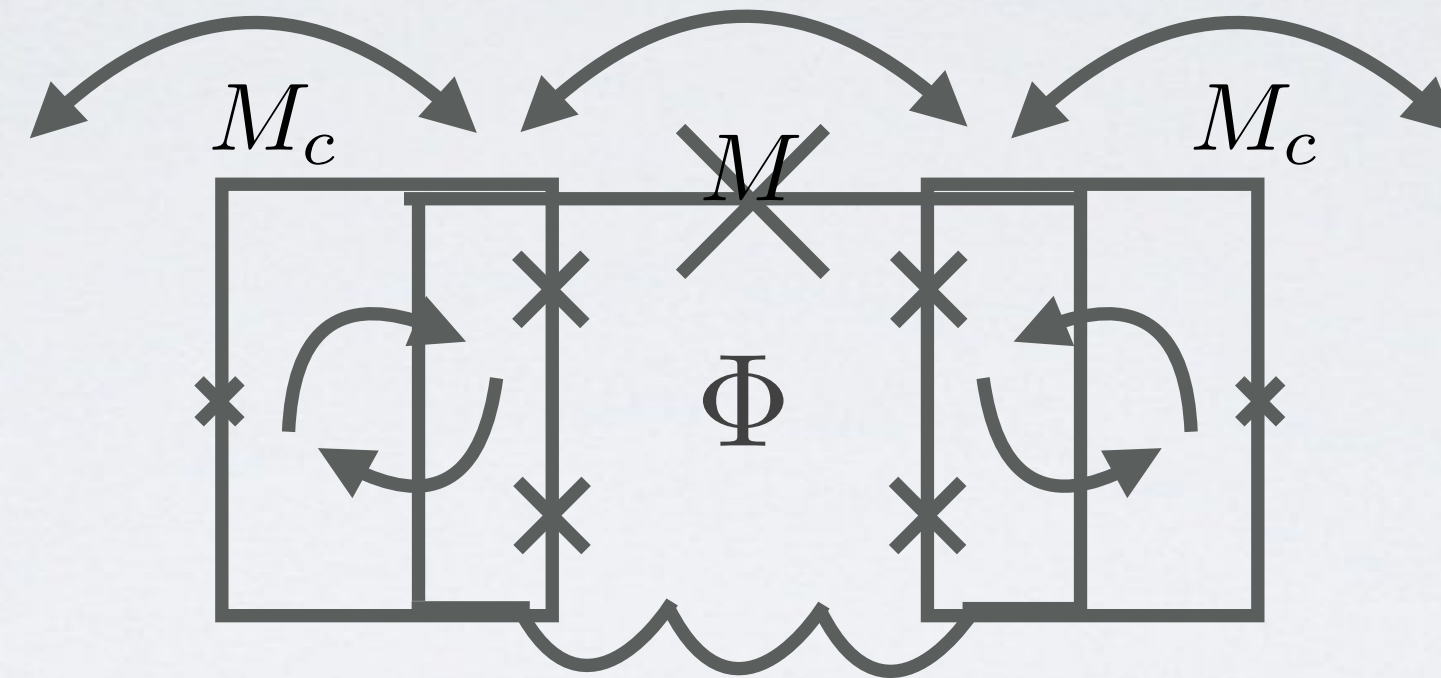


High sensitivity to magnetic flux*

*M. Bal, et al., Nat. Comm. 3, 1324 (2012)

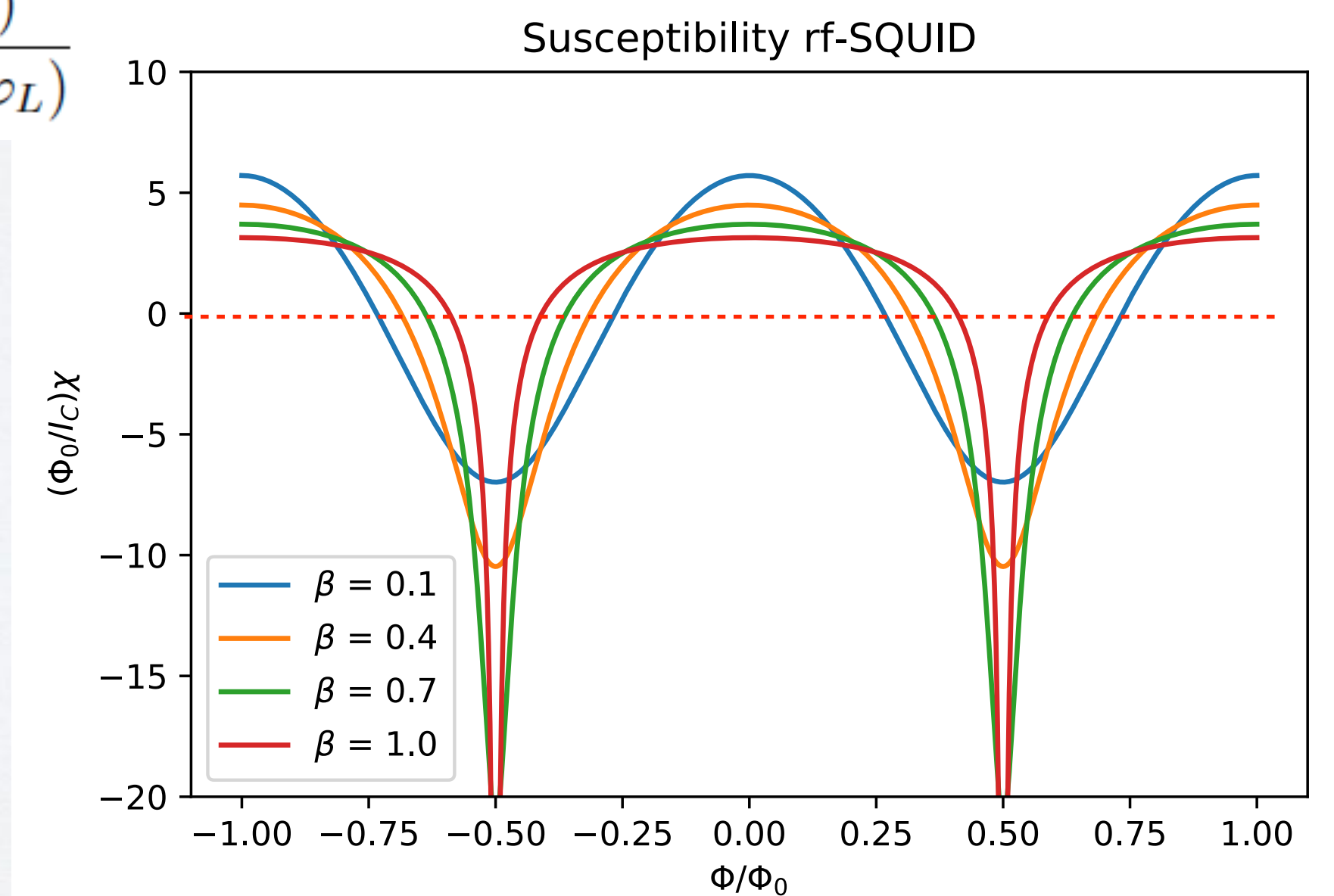
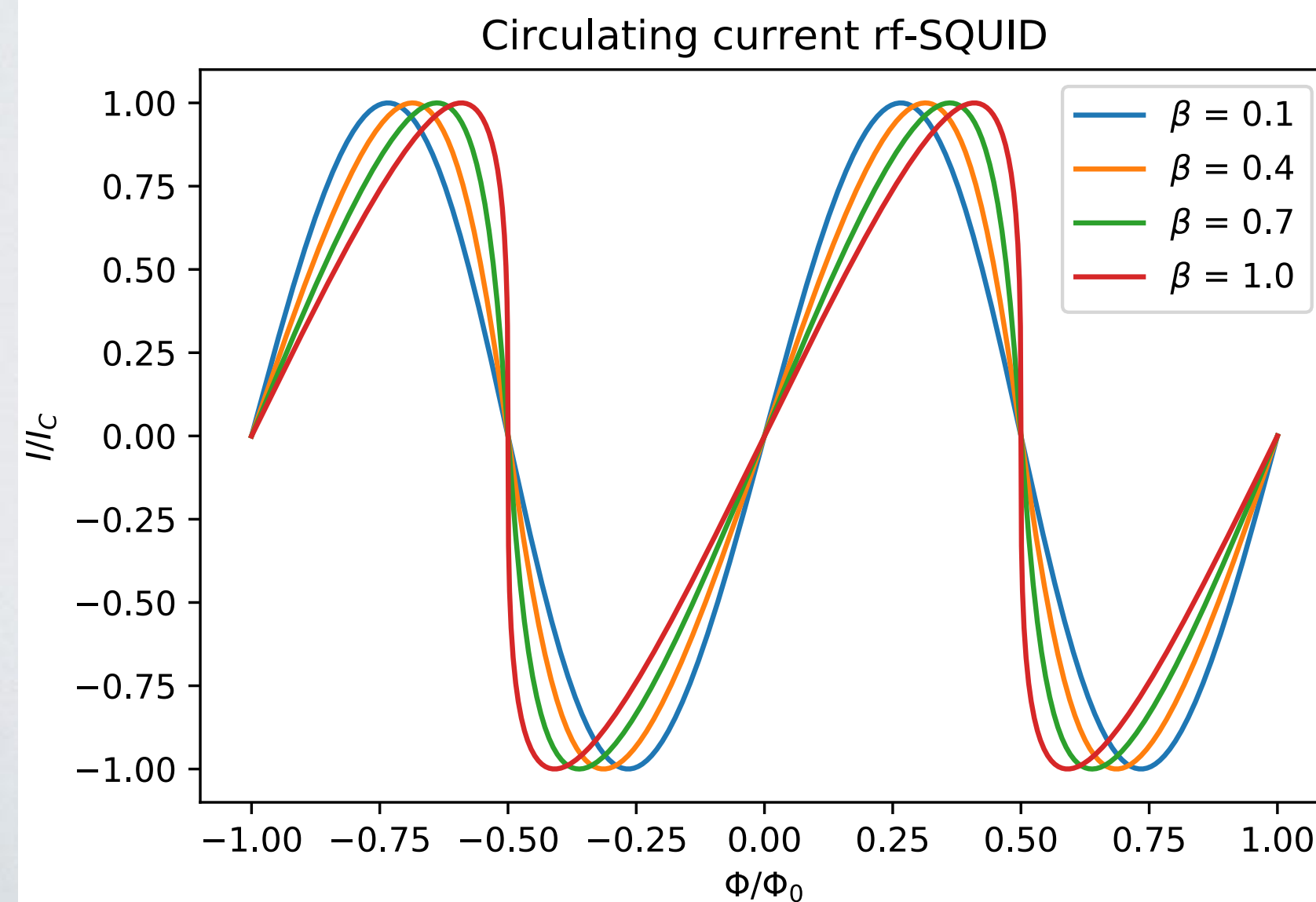
QUBIT-QUBIT COUPLING

rf-SQUID inductive coupler

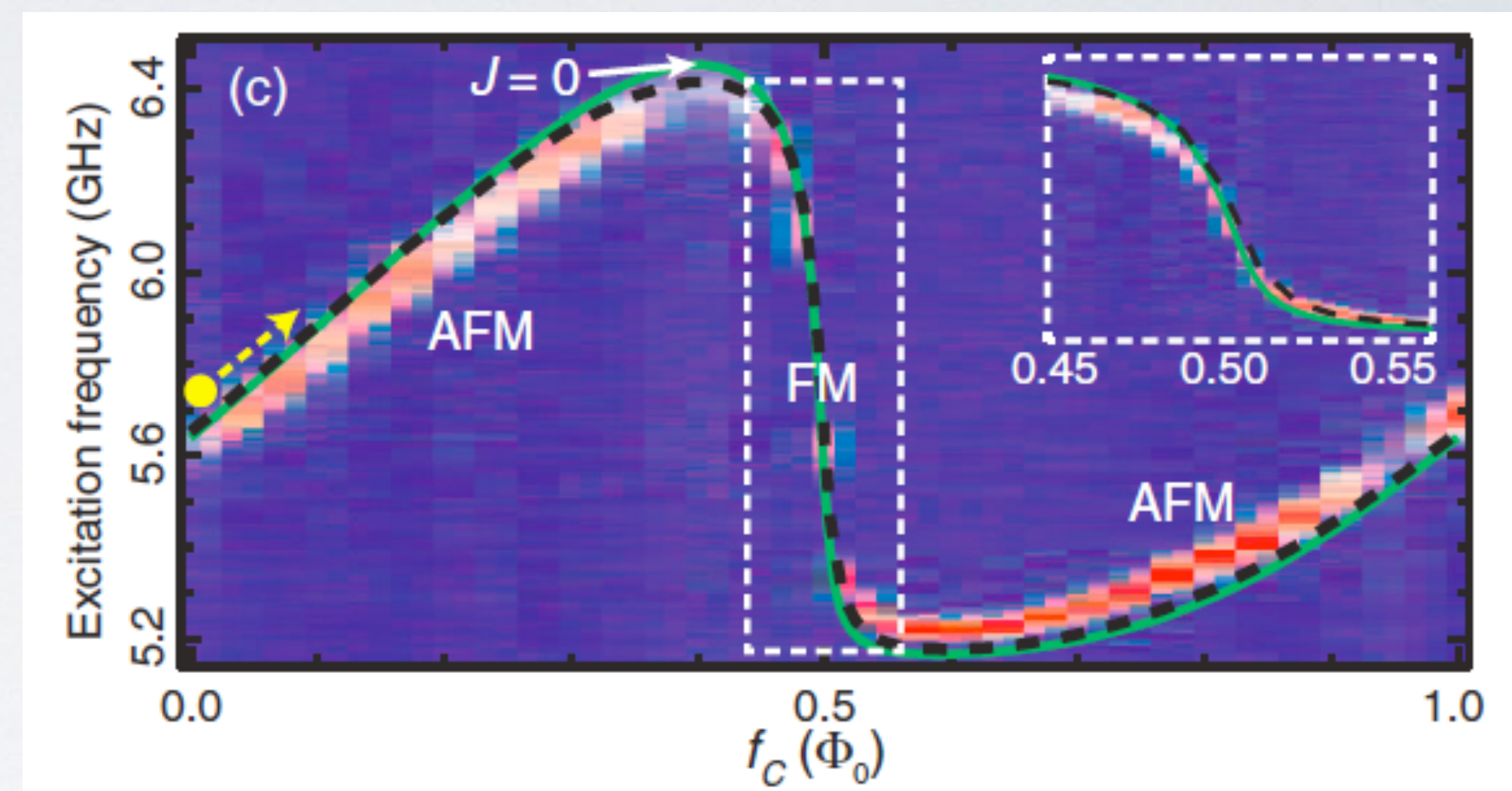
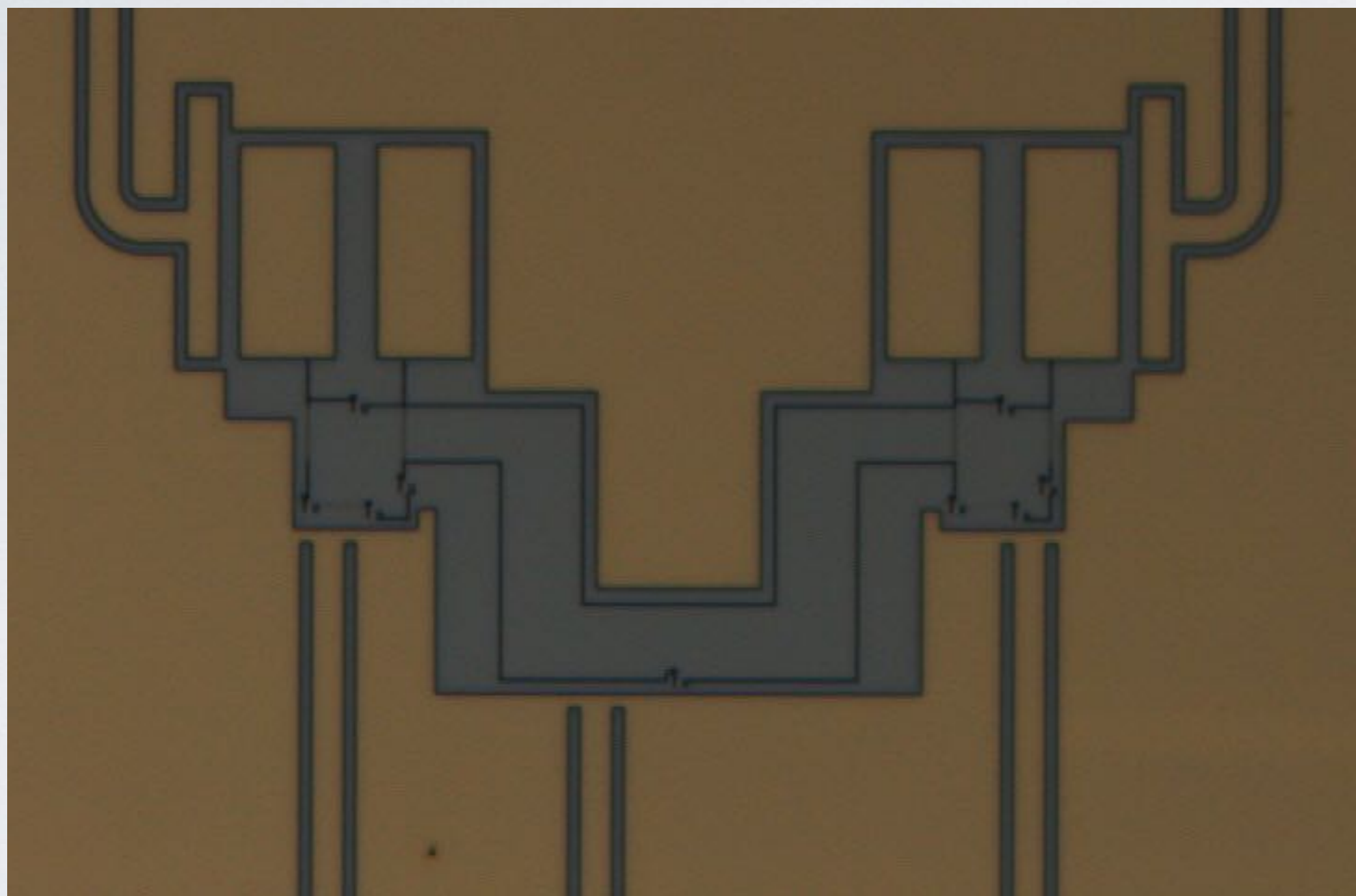


Circuit susceptibility / Effective inductance

$$\chi = L_{\text{eff}}^{-1} = \frac{1}{L_J} \frac{\beta \cos(2\pi f - \Delta\varphi_L)}{1 + \beta \cos(2\pi f - \Delta\varphi_L)}$$

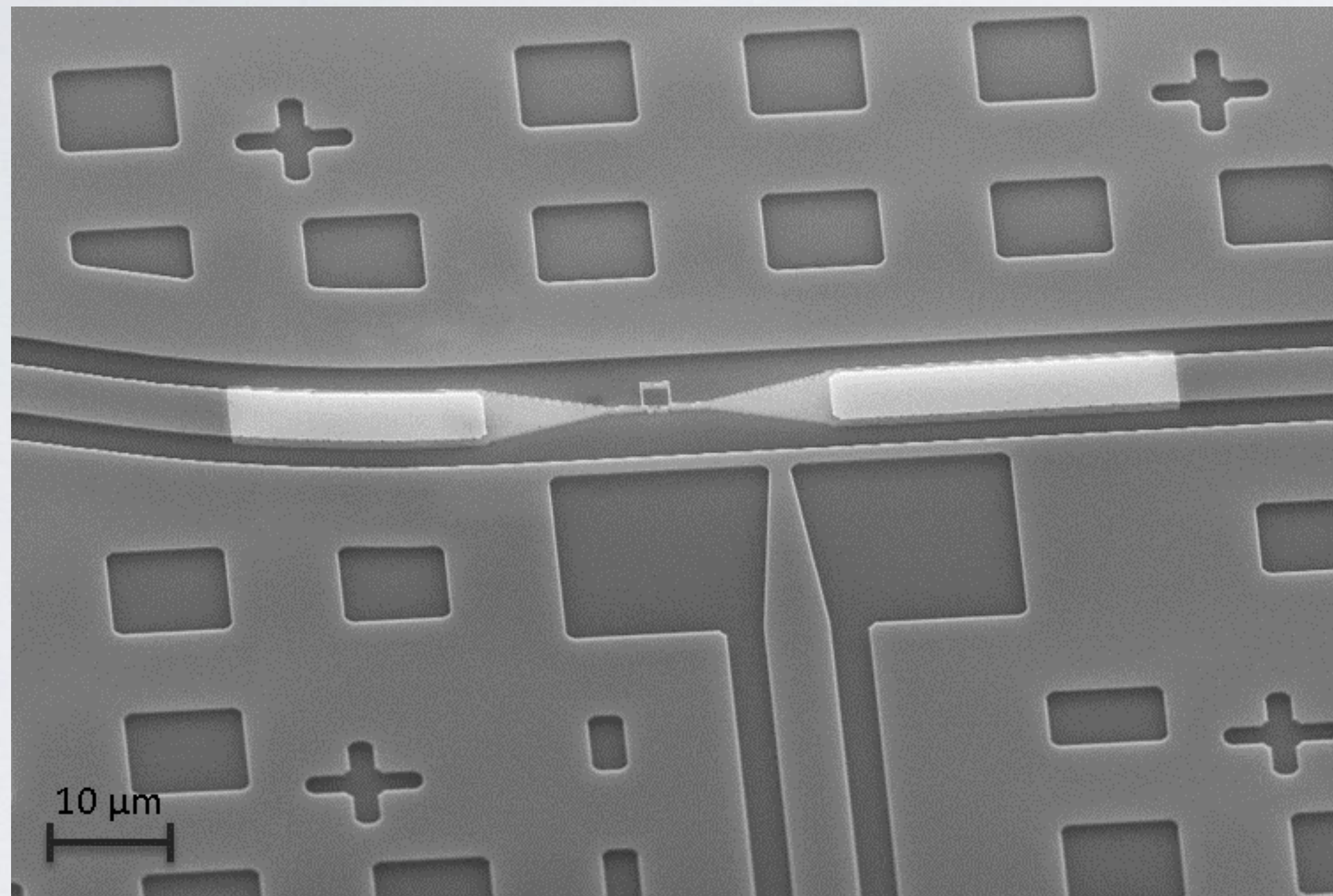


QUBIT-QUBIT COUPLING

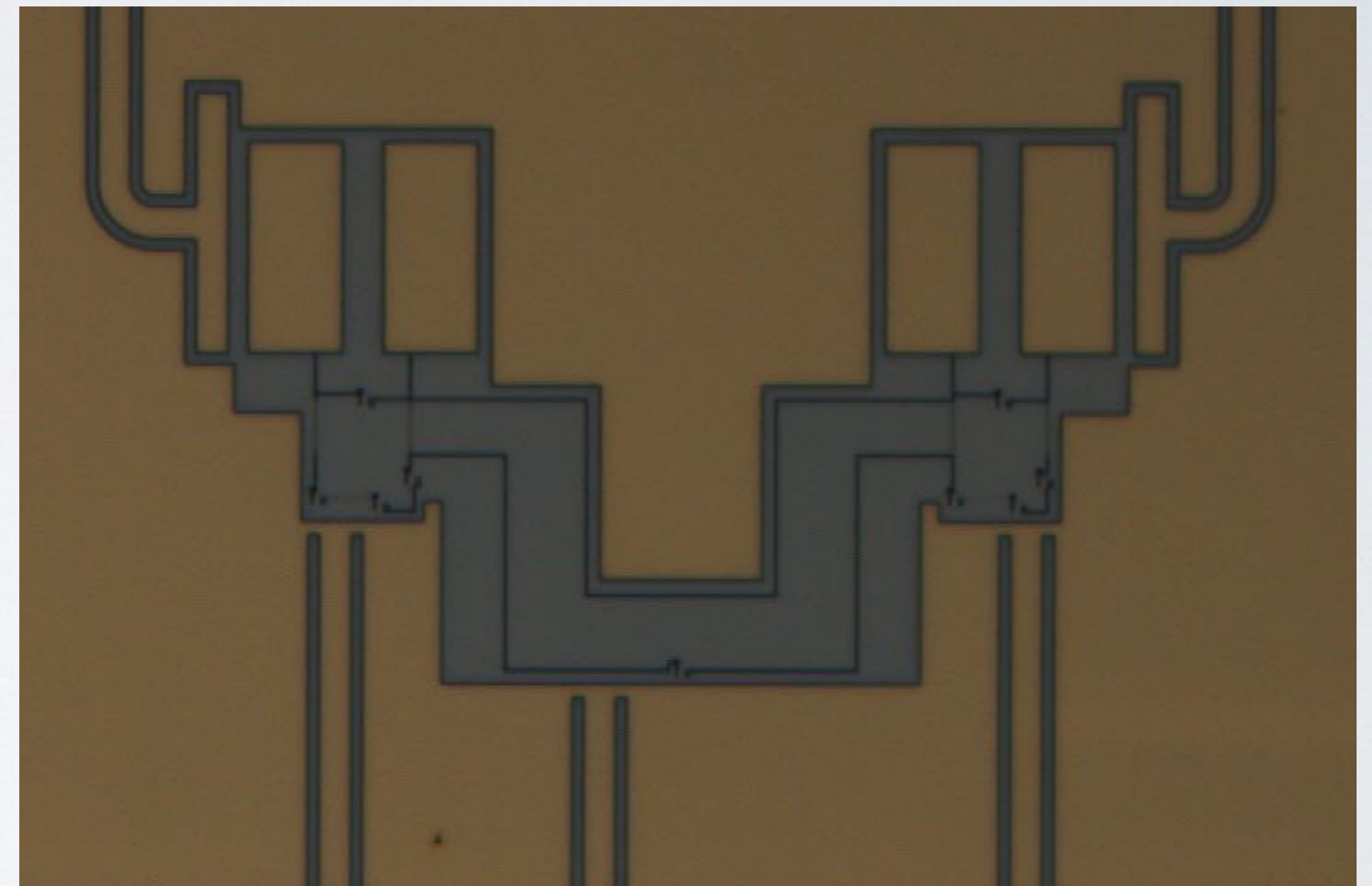


FLUX BIASING

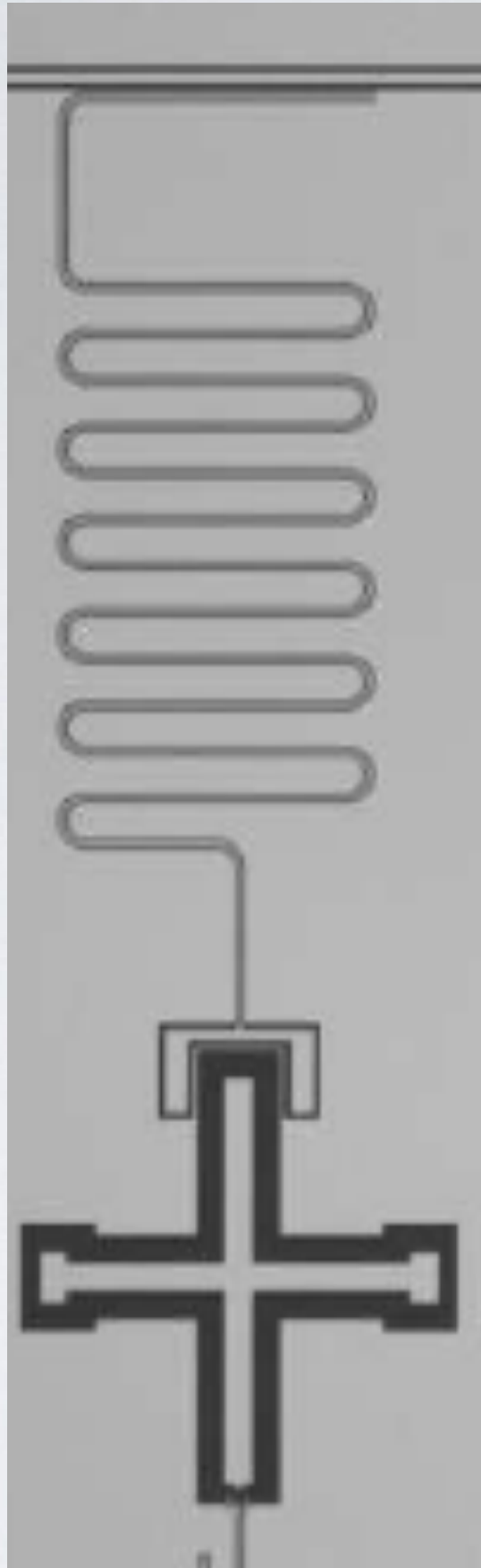
Single qubit flux control



Multi-qubit flux control



QUBIT STATE READOUT



Dispersive limit: $g \ll \Delta \equiv \omega_q - \omega_r$

$$\mathcal{H}_{\text{JC}}/\hbar \approx \left(\omega_r + \frac{g^2}{\Delta} \sigma_z \right) a^\dagger a + \frac{1}{2} \left(\omega_q + \frac{g^2}{\Delta} \right) \sigma_z$$

Stark shift

Lamb shift

- Resonator frequency depends on qubit state
- Nondestructive readout, as operators commute
- As there is no energy exchange: dispersive readout

(Flux readout on blackboard)

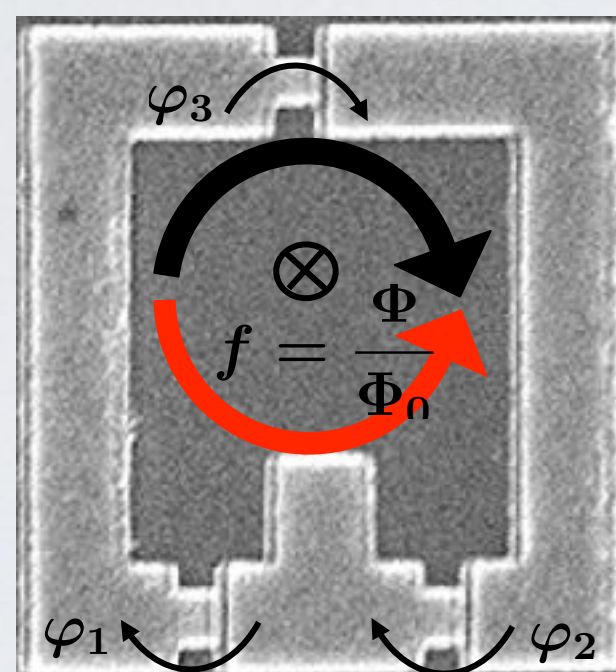
A. Blais et al., PRA 69, 062320 (2004)

R. Barends et al., Nature 508, 500 (2014)

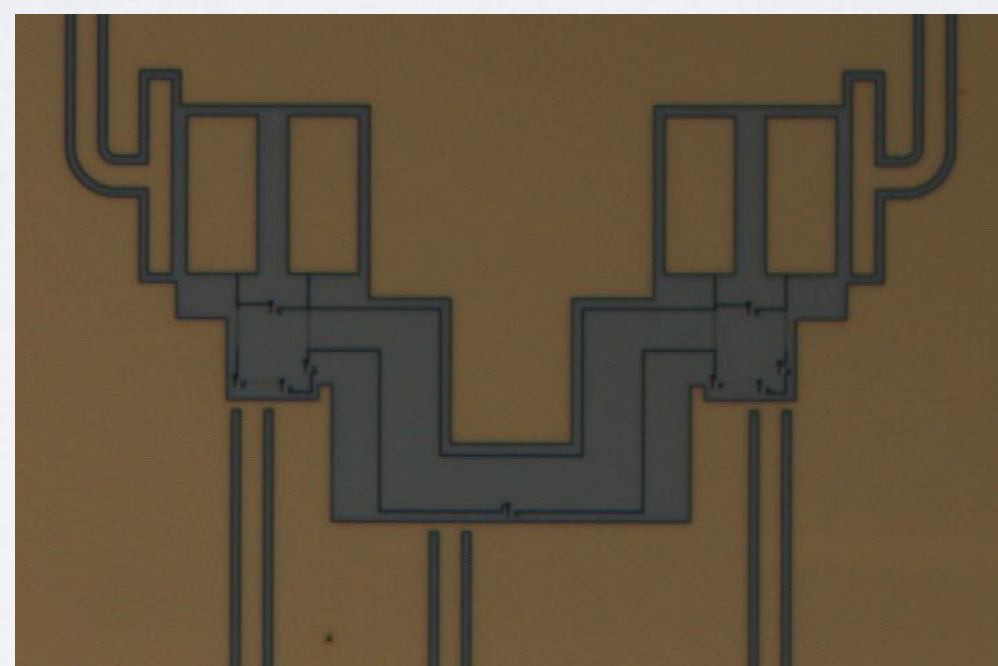
EXPERIMENTAL QA

Ingredients to build a quantum annealer:

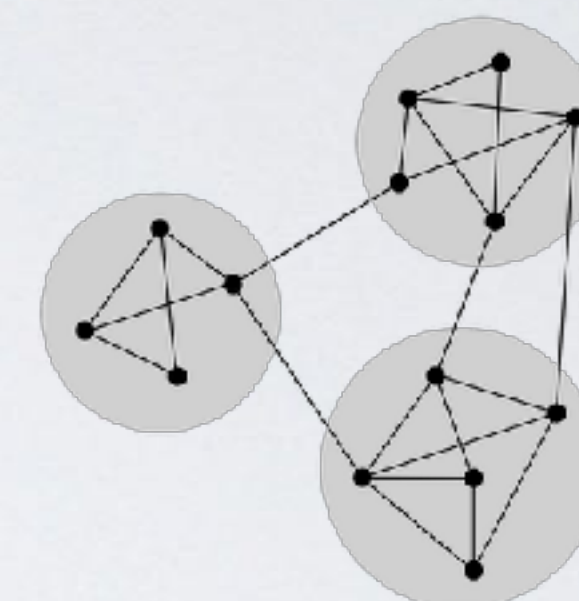
Spin-1/2-like Qubits



Couplers



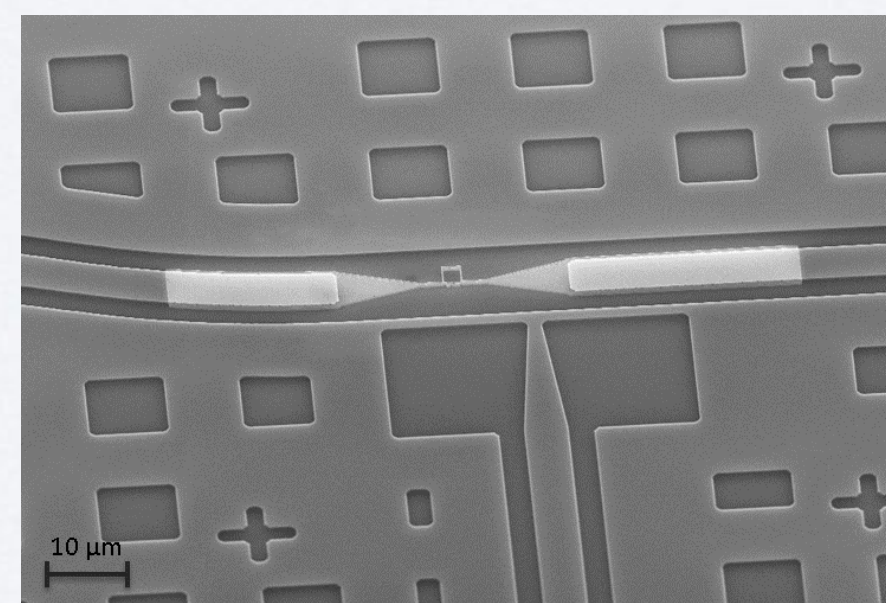
Connectivity map



Readout



Flux bias



Classical control



ACKNOWLEDGEMENTS

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The QCT group at IFAE
The Qilimanjaro team



THANK
YOU!



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Horizon 2020
European Union funding
for Research & Innovation

