Superconducting Quantum Circuits

Interaction engineering

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GOBIERNO DE ESPAÑA — MINISTERIO DE CIENCIA E INNOVACIÓN

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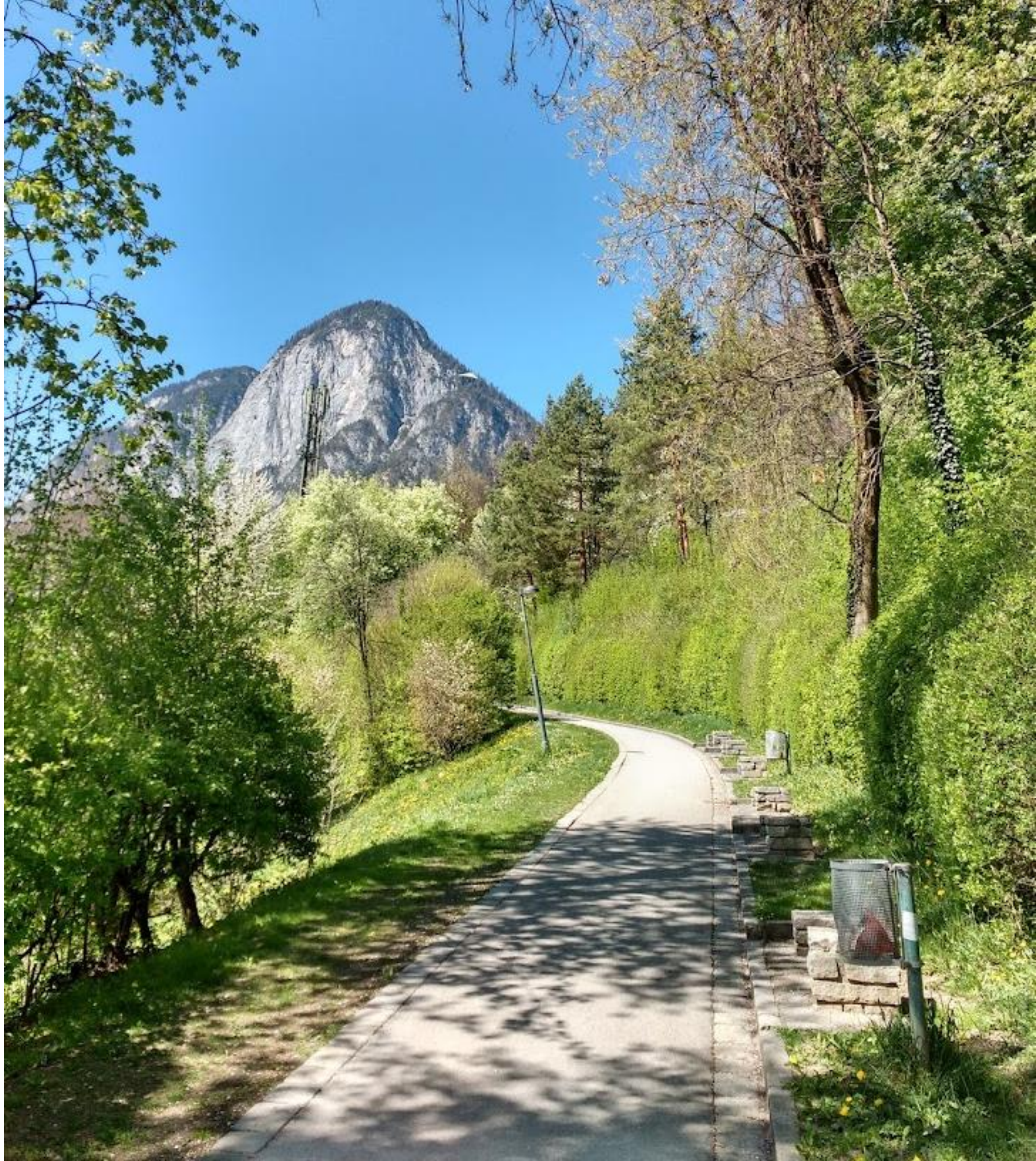
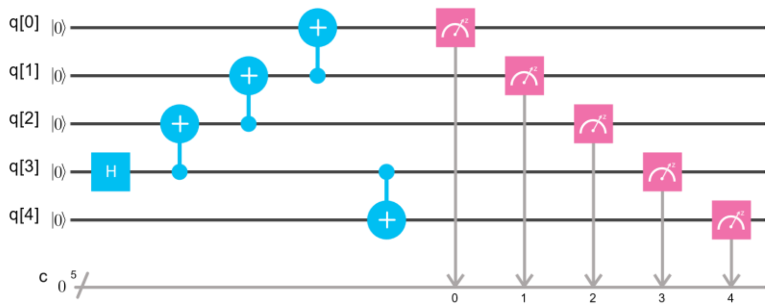
CSIC — Consejo Superior de Investigaciones Científicas

Outline

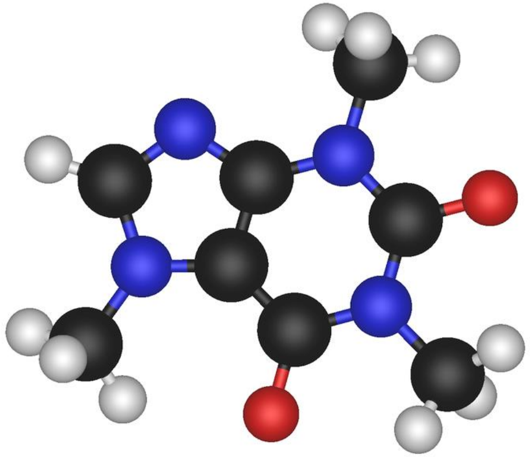
- Motivations / basic ideas
- Passive coupling
  - Dipolar coupling
  - Ultrastrong coupling
  - Mediated coupling
  - Tunable coupling
- Active coupling
  - Photon-assisted coupling
  - Parametric coupling
- Some applications



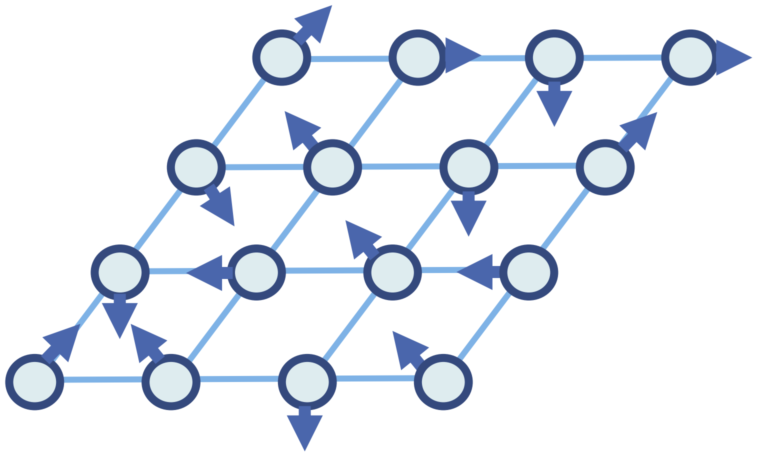
Coupling quantum systems



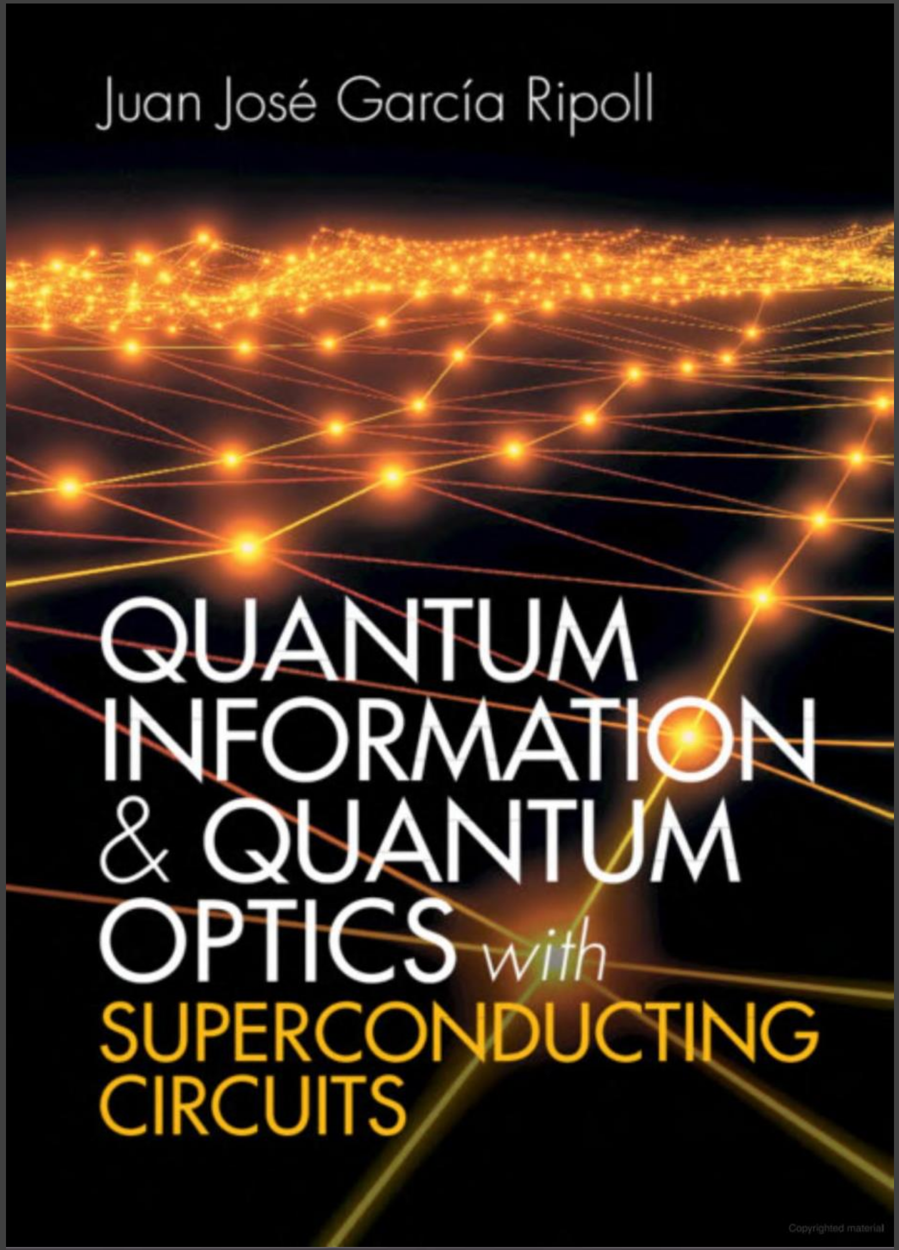
Arbitrary strength.
Tuneability to study e.g. phase transitions
Quantum simulation / quantum optimizers



Gate-based quantum computers
Moderate strength.
High tuneability: gates.



High strength.
Don't matter if fixed
Engineering new quantum systems / new physics

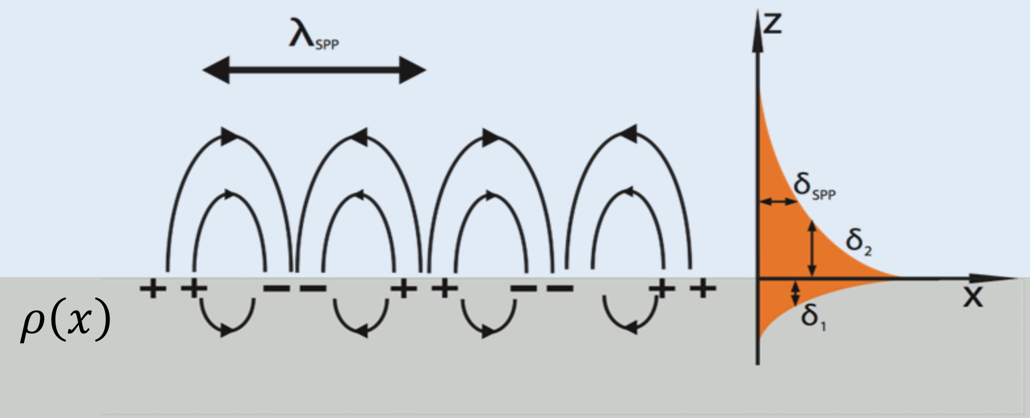


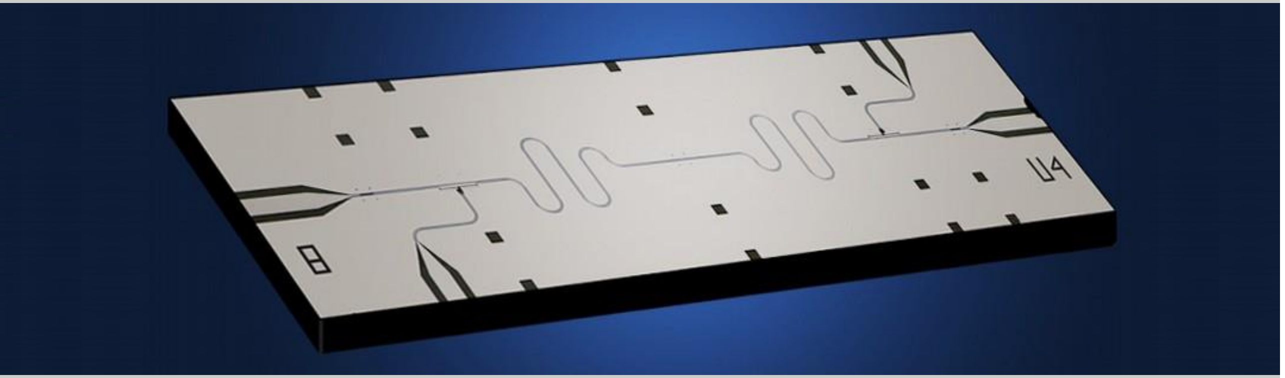
For more information, and to order, visit:
www.cambridge.org/9781107172913
and enter the code QIQOSC22 at the checkout

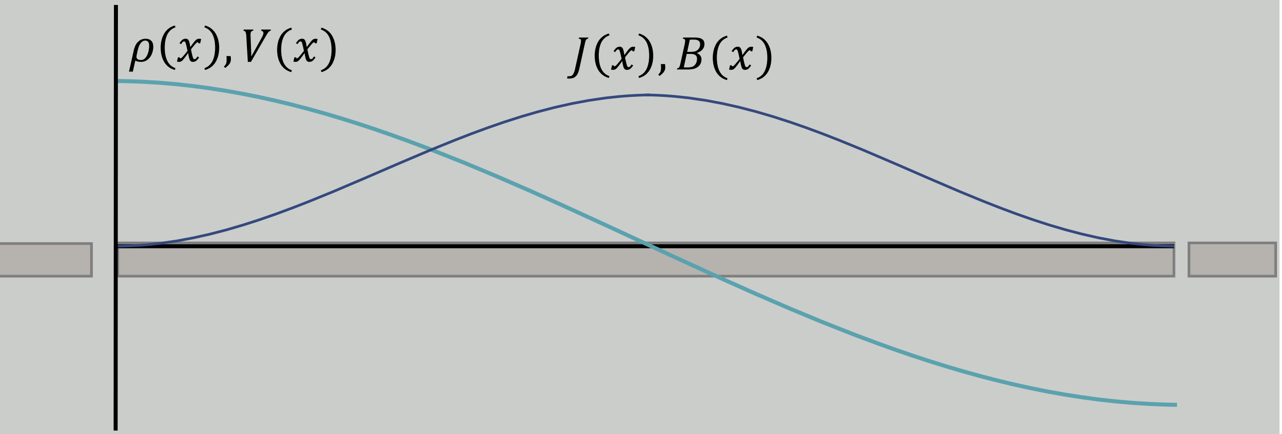
Material

- Complete, standalone lectures on superconducting circuits, quantum optics and quantum information applications.
- Emphasis on this lecture on state-of-the-art **literature** with references throughout the presentation.
- A copy of the book will be gifted at the end of the school.

Surface plasmons



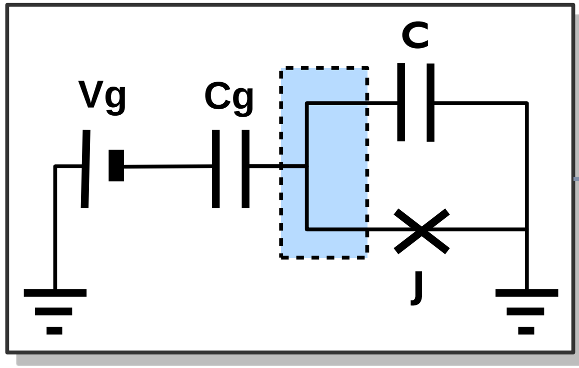




Excitations in the superconductor are surface waves of charge accompanied by E.M. field excitations (photons).

- Light: fast motion, close to speed of light.
- Matter: excitations **may interact** (Coulomb) and be confined to the superconductor (trapping).

Quantum circuits



Kirchoff theory

$$C_{ij}\dot{V}_j = \frac{1}{L_{ij}}(V_i - V_j) + \cdots$$

Flux variables

$$V = \frac{d\phi}{dt}$$

Lagrangian

$$S = \int_{t_0}^{t_f} L\,(\dot{\phi}, \phi)dt$$

Hamiltonian

$$H = H(q, \phi)$$
$$q = \partial\, L/\partial\, \dot{\phi}$$

Quantization

$$[\hat{\phi}, \hat{q}] = i\hbar$$

$$H = \frac{1}{C_g + C}q^2 + V_g q + \frac{1}{2L}\phi^2 + J\phi_0 \cos(\phi/\Phi_0)$$

Charge-like qubits

Effective Hamiltonian

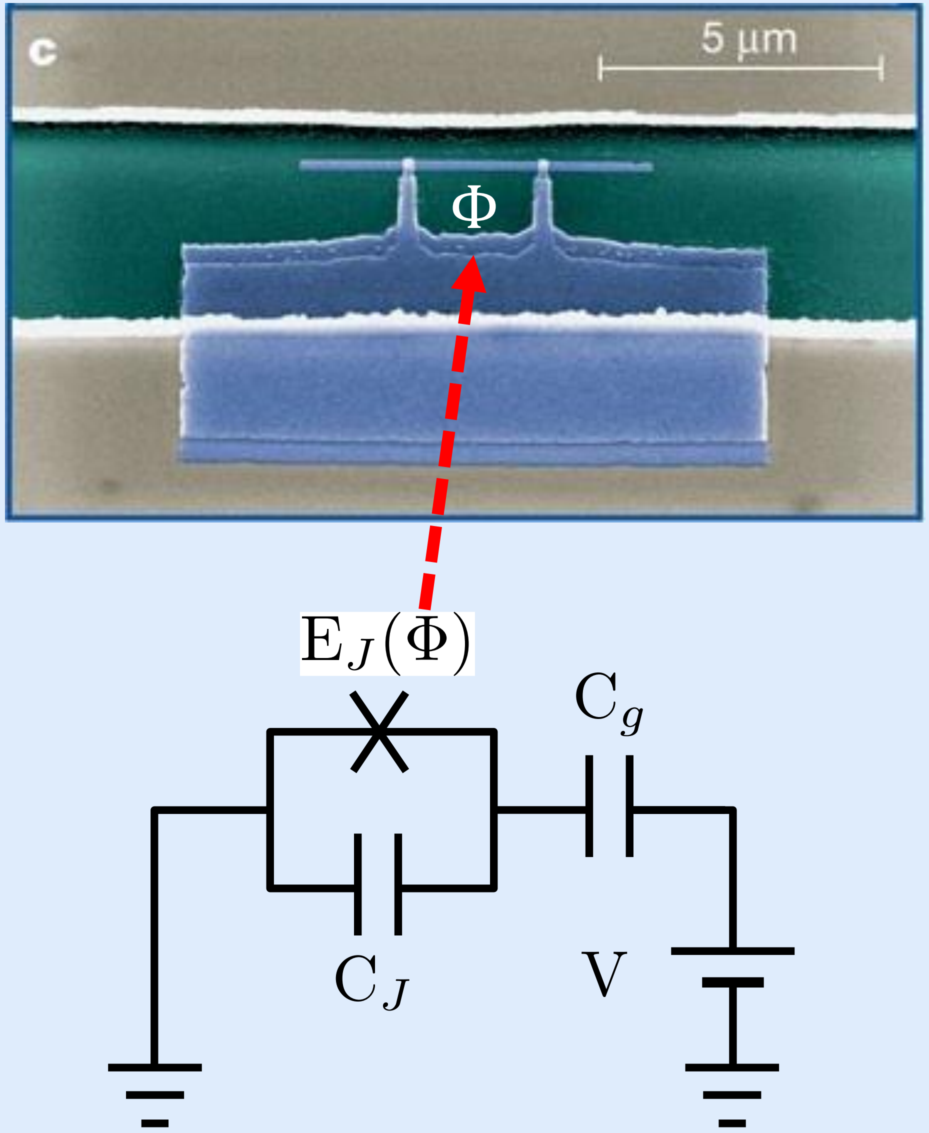
$$H = \frac{1}{2C}\hat{q}^2 - E_J(\Phi_{ext})\cos\left(\frac{2\pi\hat{\phi}}{\Phi_0}\right)$$

$$\simeq 4E_C\hat{n}^2 - E_J\cos(\hat{\varphi})$$

Two regimes

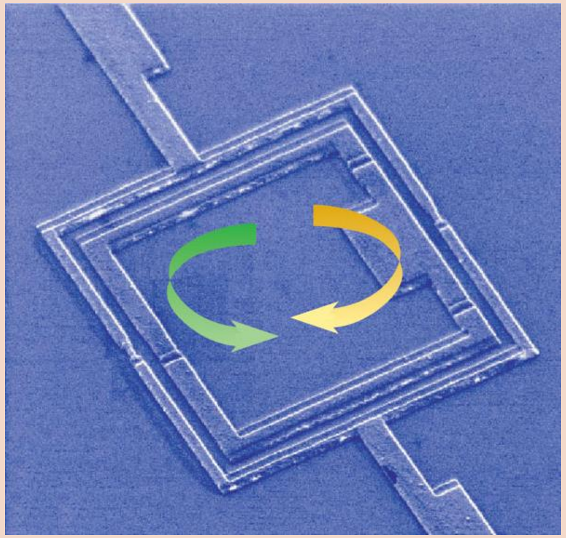
- $E_C \gg E_J$, charge qubit
- $E_C \ll E_J$, transmon

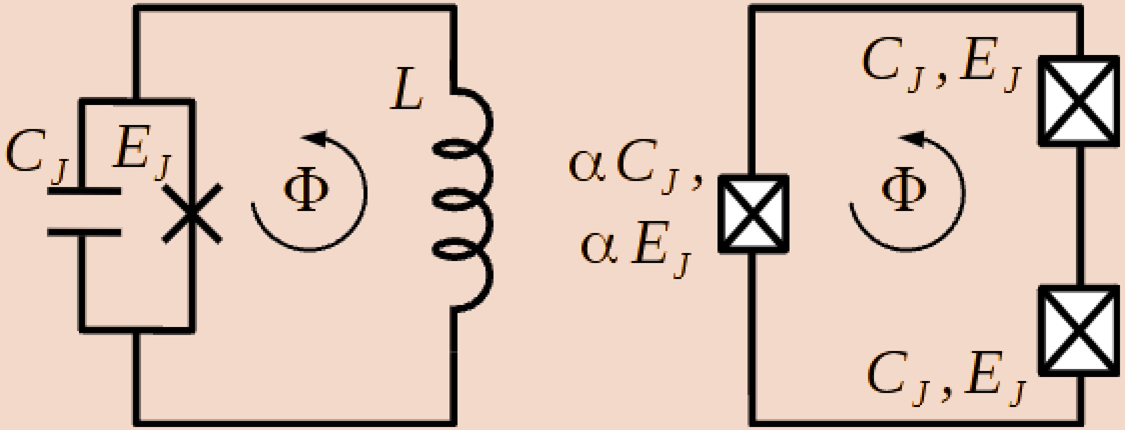
In both, charge d.o.f. most dominantly couples to world.



Flux-like qubits

Supercurrents on a JJ-enabled ring



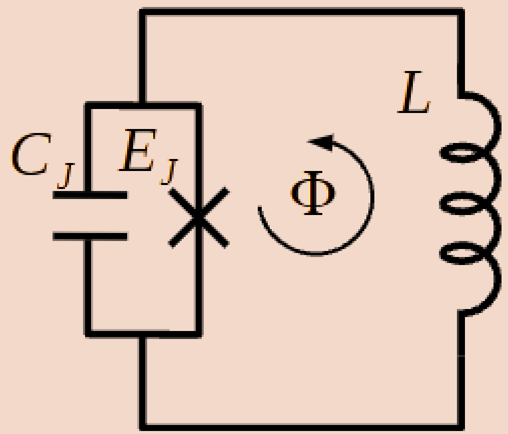


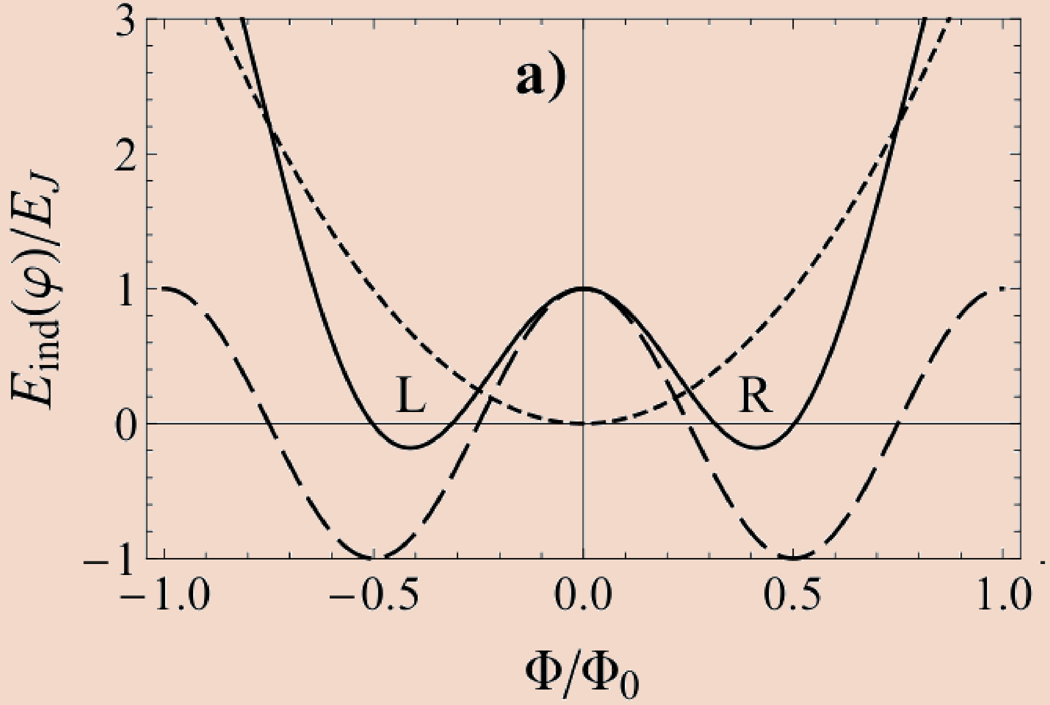
Flux-like qubits

Supercurrents on a JJ-enabled ring.
For instance, rf-squid:

$$H = \frac{1}{2C}\hat{q}^2 + \frac{\hat{\phi}^2}{2L} - E_J \cos\left(\frac{2\pi\hat{\phi}}{\Phi_0}\right)$$

Two supercurrent states form a qubit subspace.





Flux-like qubits

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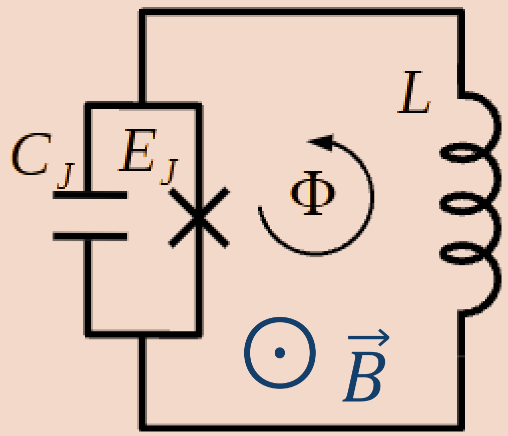
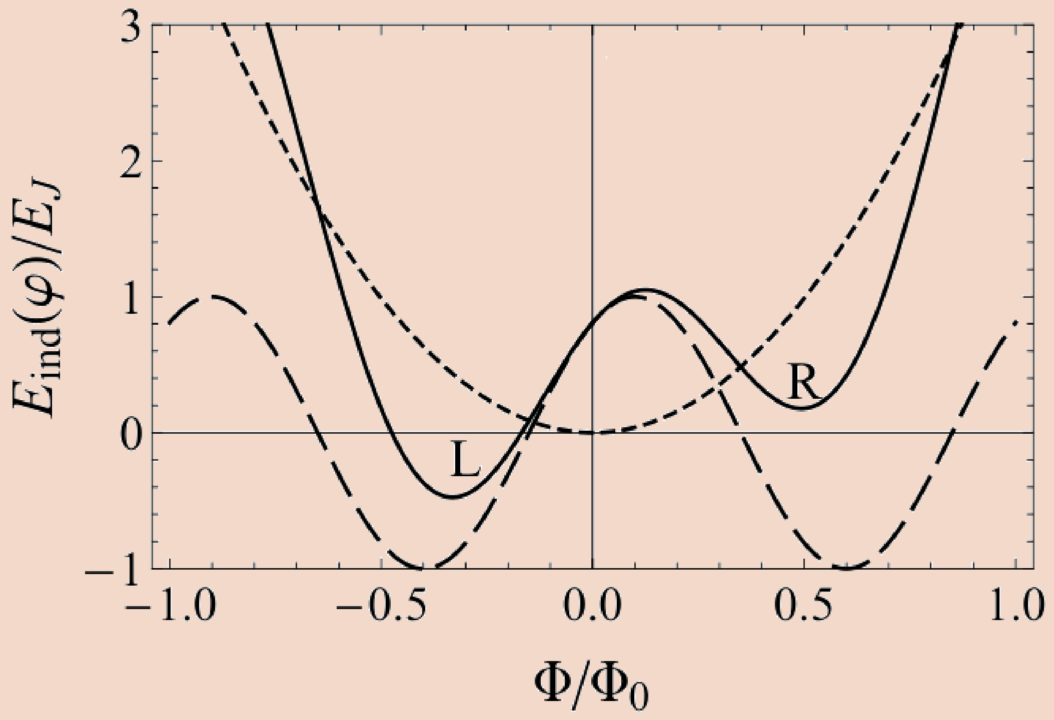
Two supercurrent states form a qubit subspace.

Those currents predominantly couple to external magnetic fields.





LC resonator

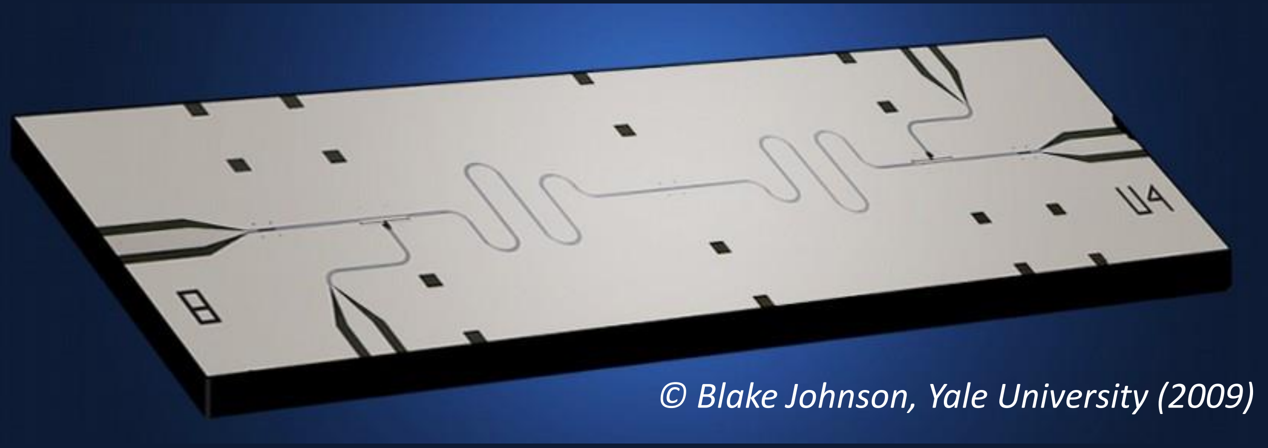
Effective Hamiltonian (V=0)

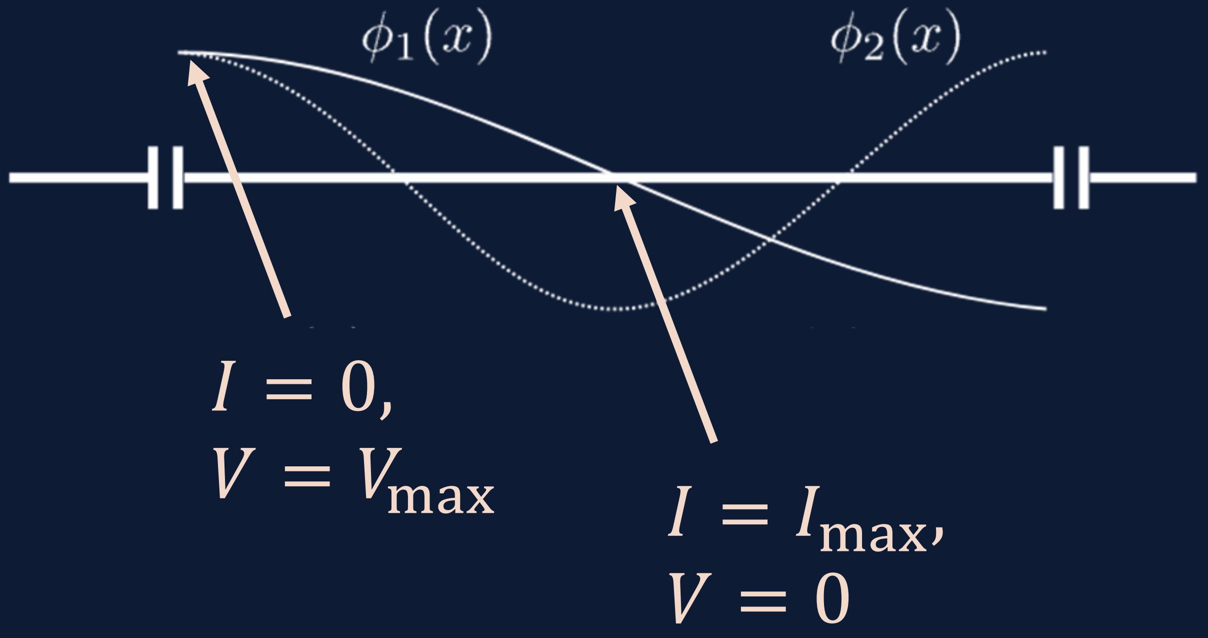
$$H = \sum_k \hbar\omega_k \left(\hat{a}_k^{+} a_k + \frac{1}{2} \right)$$

Best of both worlds

- Points where electric field are maxima, optimal coupling to charge
- Areas where currents dominate, optimal magnetic coupling.

But always linear!





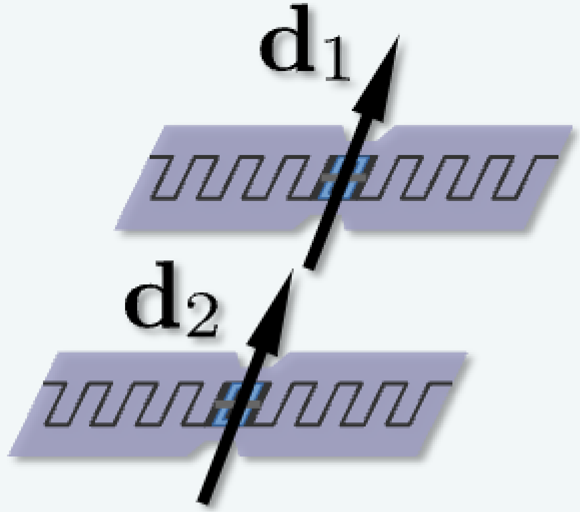
Dipolar coupling

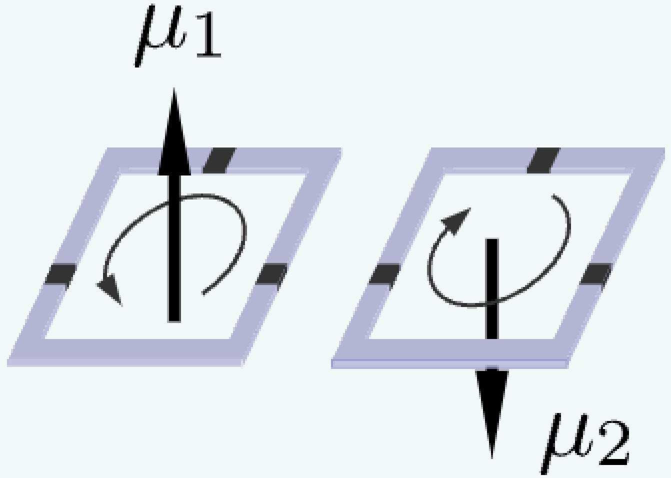
We identify "dipolar moment" observables for each qubit

- Electric dipole for transmon qubit, $d \sim \hat{q}$
- Magnetic dipole for flux qubits, , $d \propto |L\rangle\langle L| - |R\rangle\langle R| \sim \sigma_x$

Pose an effective interaction between them

$$H_{int} = \alpha \hat{d}_1 \hat{d}_2$$





Dipolar coupling

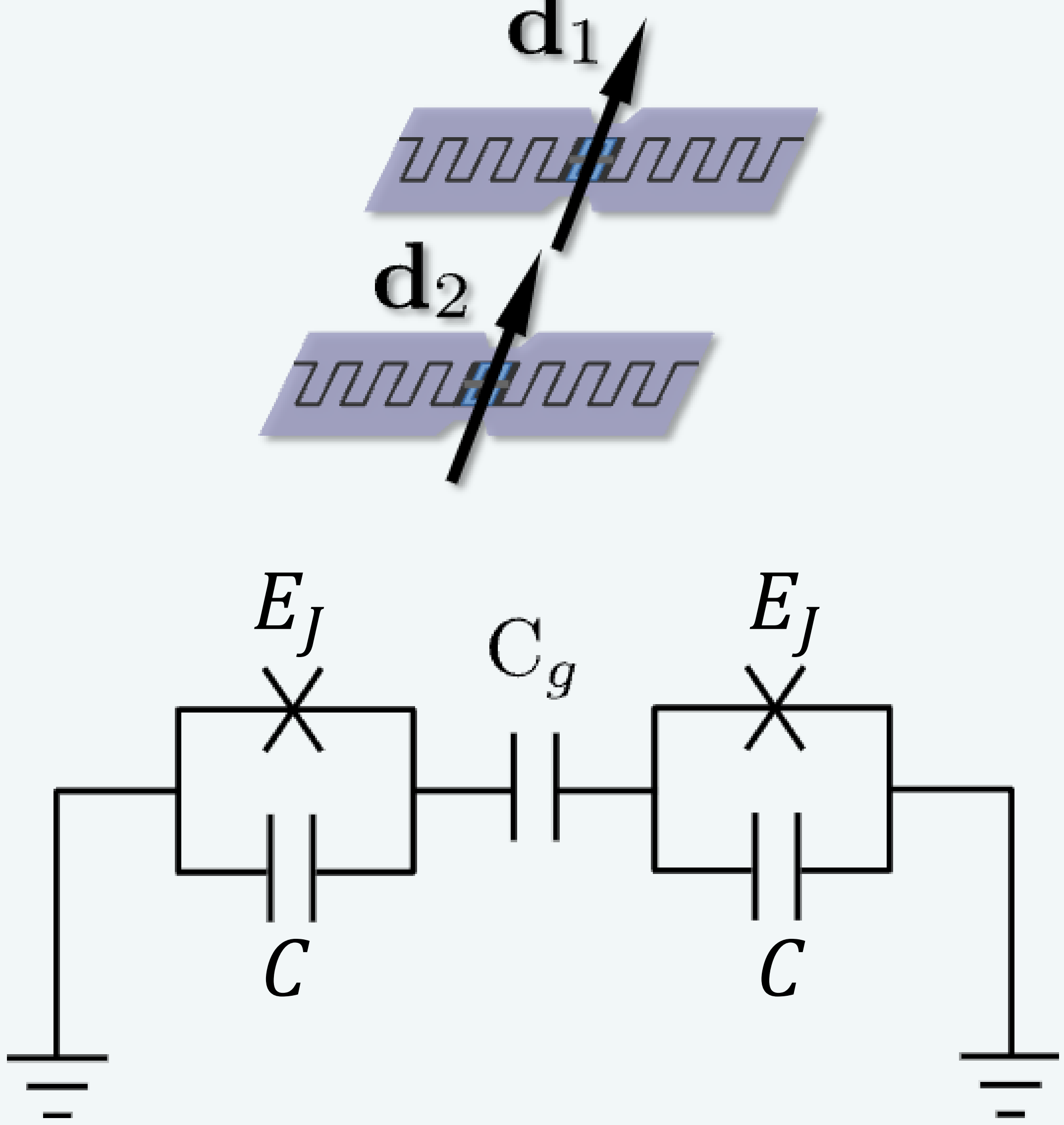
This may be justified from circuit theory. For transmon qubits,

$$H = \frac{1}{2}\vec{Q}^T C^{-1}\vec{Q} + \sum_i E_J \cos\left(\frac{\phi}{\varphi_0}\right)$$

With capacitance matrix

$$C = \begin{pmatrix} C + C_g & -C_g \\ -C_g & C + C_g \end{pmatrix}$$

Note how C_g appears **everywhere!**



Dipolar coupling

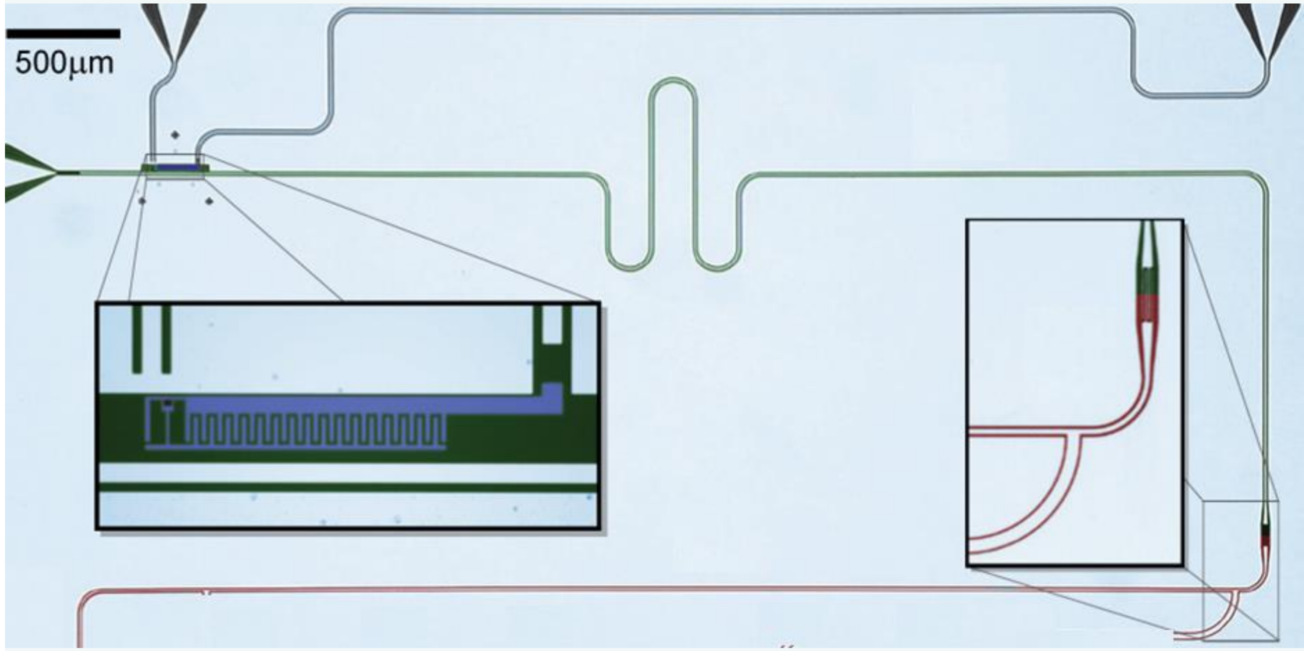
Same thing happens for a qubit in a waveguide resonator

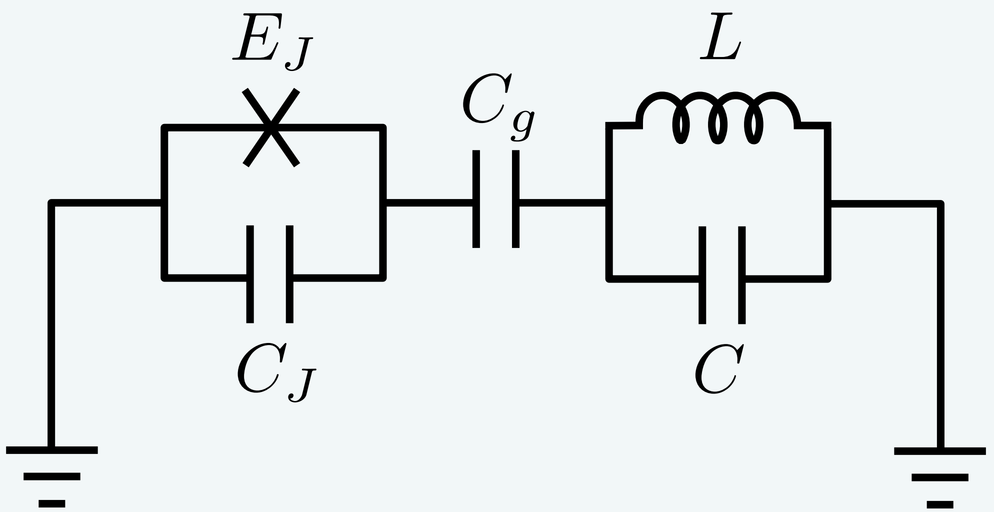
$$H = \hbar\omega_{01}\sigma^z + \hbar\omega\hat{a}^+\hat{a}$$
$$+g\sigma^x(\hat{a}^+ + \hat{a})$$

With the dipolar interpretation

$$g\sigma^x(\hat{a}^+ + \hat{a}) \sim \frac{C_g}{CC_J}\hat{q}_{qb}\hat{V}_{LC}$$

Access also to multiple modes.





Long-range cross-talk

Typical structure

$$L = \frac{1}{2}\dot{\vec{\phi}}^T C \dot{\vec{\phi}} - \frac{1}{2}\vec{\phi}^T L \vec{\phi} - V(\vec{\phi})$$

When computing the charge $\vec{q} = C\dot{\vec{\phi}}$ makes the inverse of the matrix appear

$$H = \frac{1}{2}\vec{q}^T C^{-1}\vec{q} + \frac{1}{2}\vec{\phi}^T L \vec{\phi} + V(\vec{\phi})$$

The inverse does not respect the structure of interactions in "C"

$$\text{C}= \begin{pmatrix} \ddots & \ddots & & & & & \\ \ddots & X & G & & & 0 & \\ & G & X & G & & & \\ & & G & X & G & & \\ & 0 & & G & X & G & \\ & & & & G & X & \ddots \\ & & & & & \ddots & \ddots \end{pmatrix}$$

$$C^{-1} = \begin{pmatrix} \ddots & \ddots & & & & & \\ \ddots & A & B & & & \neq 0 & \\ & B & A & B & & & \\ & & B & A & B & & \\ & & & B & A & B & \\ & \neq 0 & & & B & A & \ddots \\ & & & & & \ddots & \ddots \end{pmatrix}$$

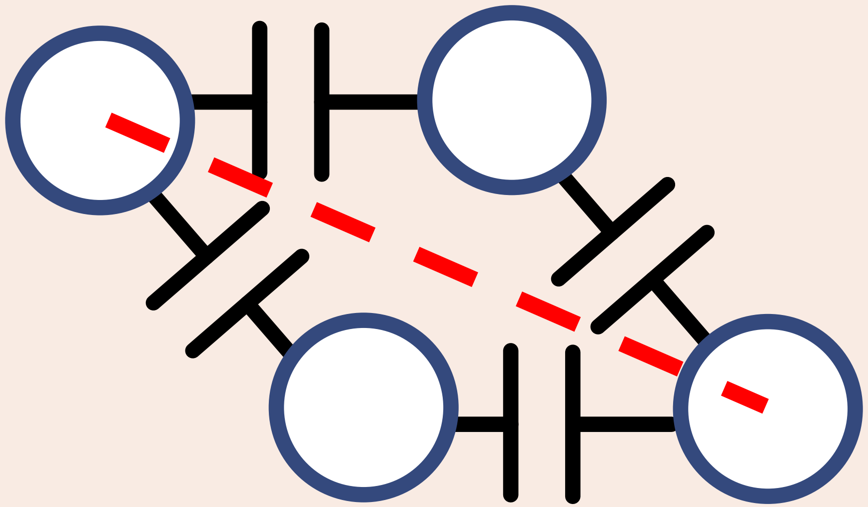
Unwanted cross-talk?

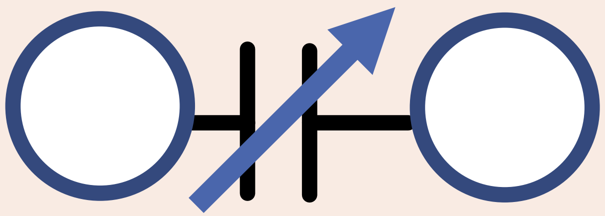
Challenges

- Non-perturbative, arbitrarily large couplings
- Consider renormalization effects due to couplings.
- Consider and ideally cancel spurious couplings

$$C_{ij}^{-1} \neq 0!!!$$

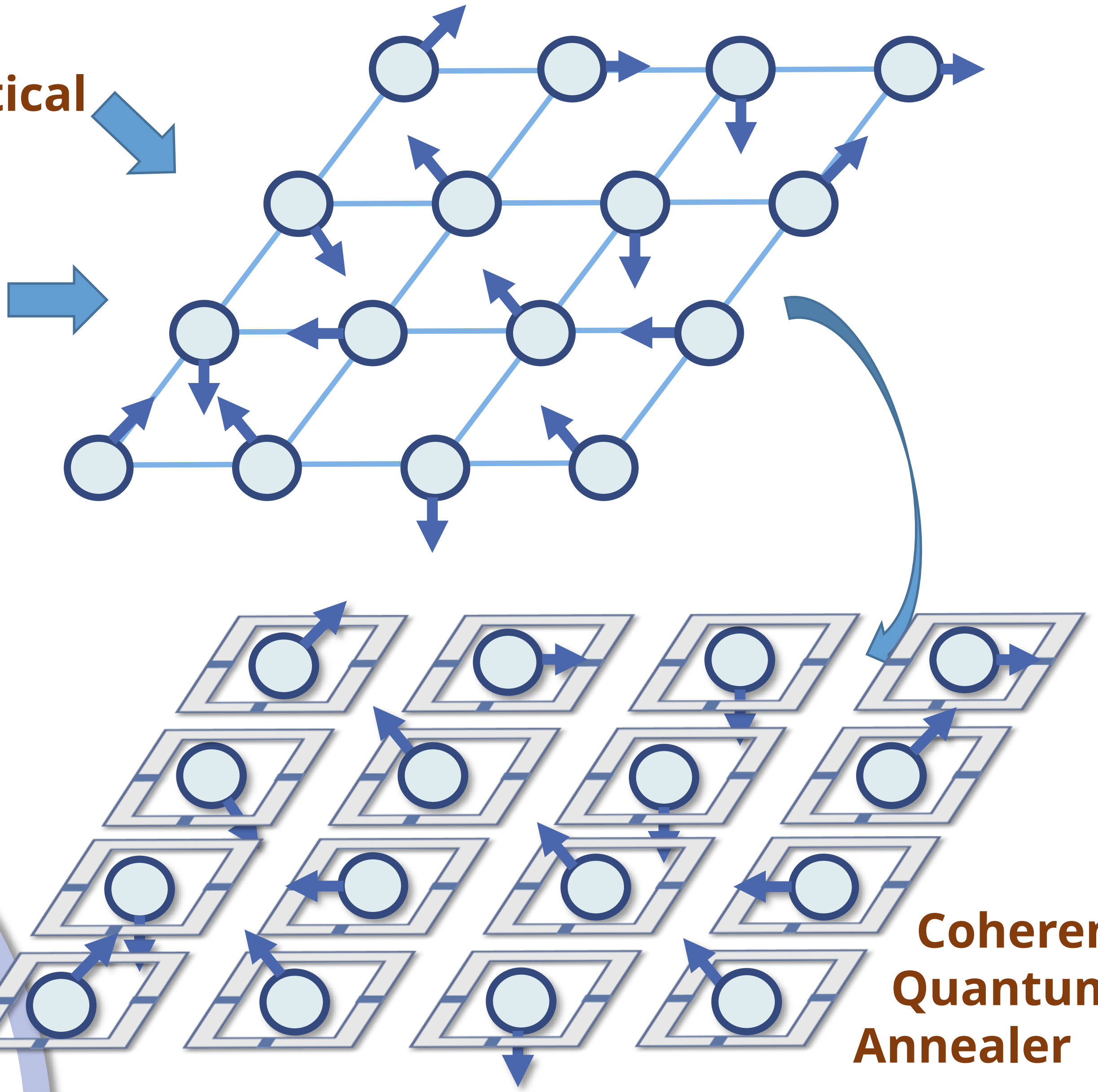
- Introduce some kind of tuneability / adjustments





Hard mathematical problems

Quantum matter models

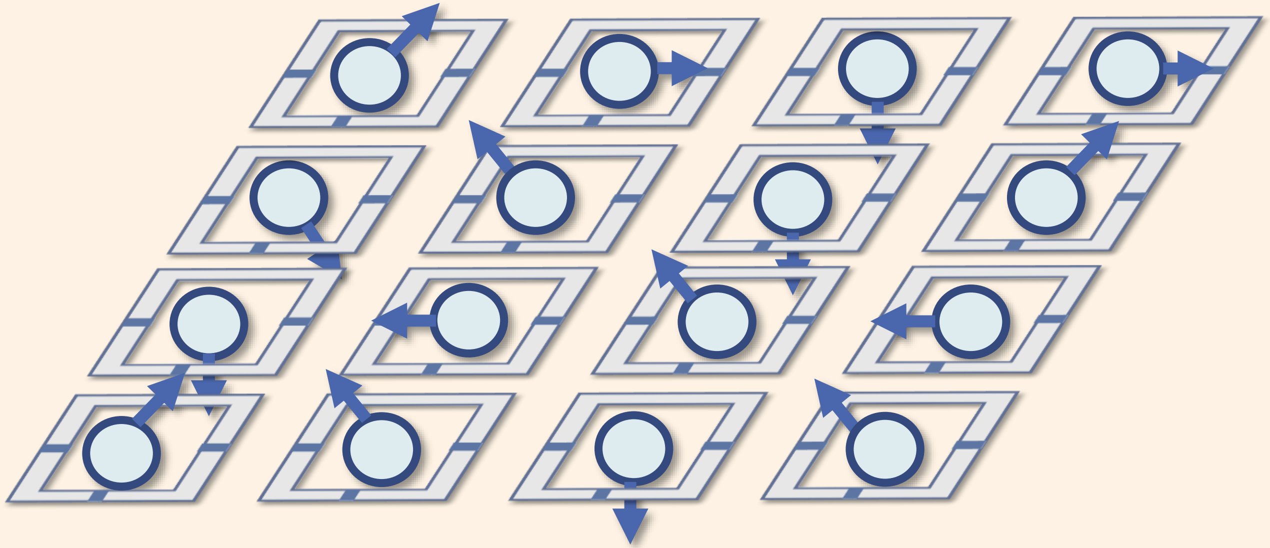


Coherent Quantum Annealer

Avaqus

http://avaqus.eu

Quantum annealer



Coherent
Quantum
Annealer

$$H \sim \sum_i \Delta_i \sigma_i^z + \sum_{ij} J_{ij}^{\alpha} \sigma_i^{\alpha} \sigma_j^{\alpha}$$

Requirements:

- High-coherence qubits / qudits
- Potentially strong interactions
- Tunable interactions for adiabatic preparation of ground states
- Possibly, interactions beyond Ising (non-stoquastic)
  - Greater physical interest
  - Greater computational complexity

Enhancing couplings

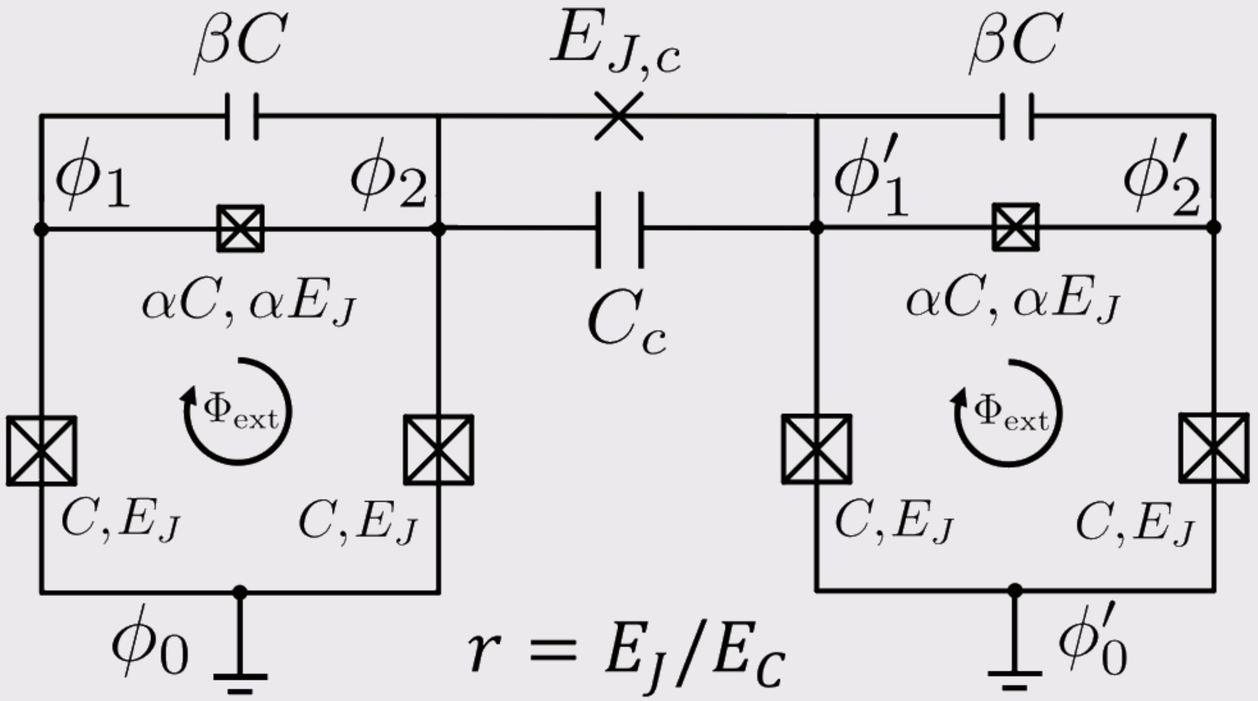


Explore these requirements using inductive and capacitive couplings between flux qubits.

- We can galvanically couple circuit elements for enhanced strength.
- Not limited to perturbative interactions.
- Different couplings may appear

$$\mathrm{H}_{int} = \sum_{\alpha} J_{ij}^{\alpha} \sigma_i^{\alpha} \sigma_j^{\alpha}$$

Not yet tunable!



M. Hita et al, PRAppl. 17, 014028 (2022)
M. Hita et al, Appl. Phys. Lett. (2021)

Change in paradigm

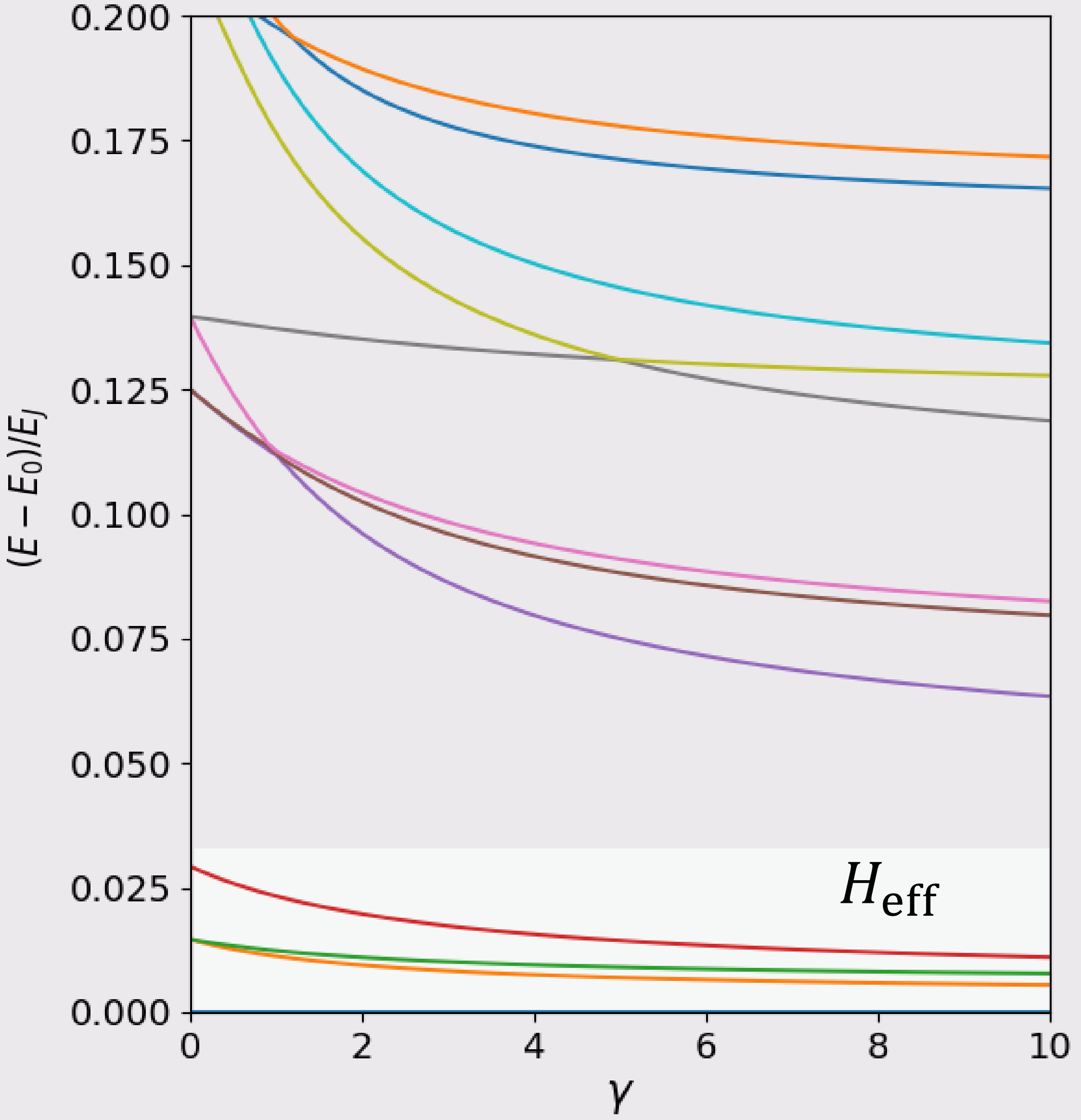
Before: interaction term can be **separated** and explained in the basis of the constituents

$$H \simeq H_A + H_B + \gamma H_{AB}$$

Now: interaction is very strong and changes both systems. We need to find an **effective basis** where coupling is explained, typically within a subspace

$$e^{-iS} H e^{iS} = H_{\text{eff}} + H_{extra}$$

$$H_{\text{eff}} = H_A(\gamma) + H_B(\gamma) + \gamma H_{AB}$$

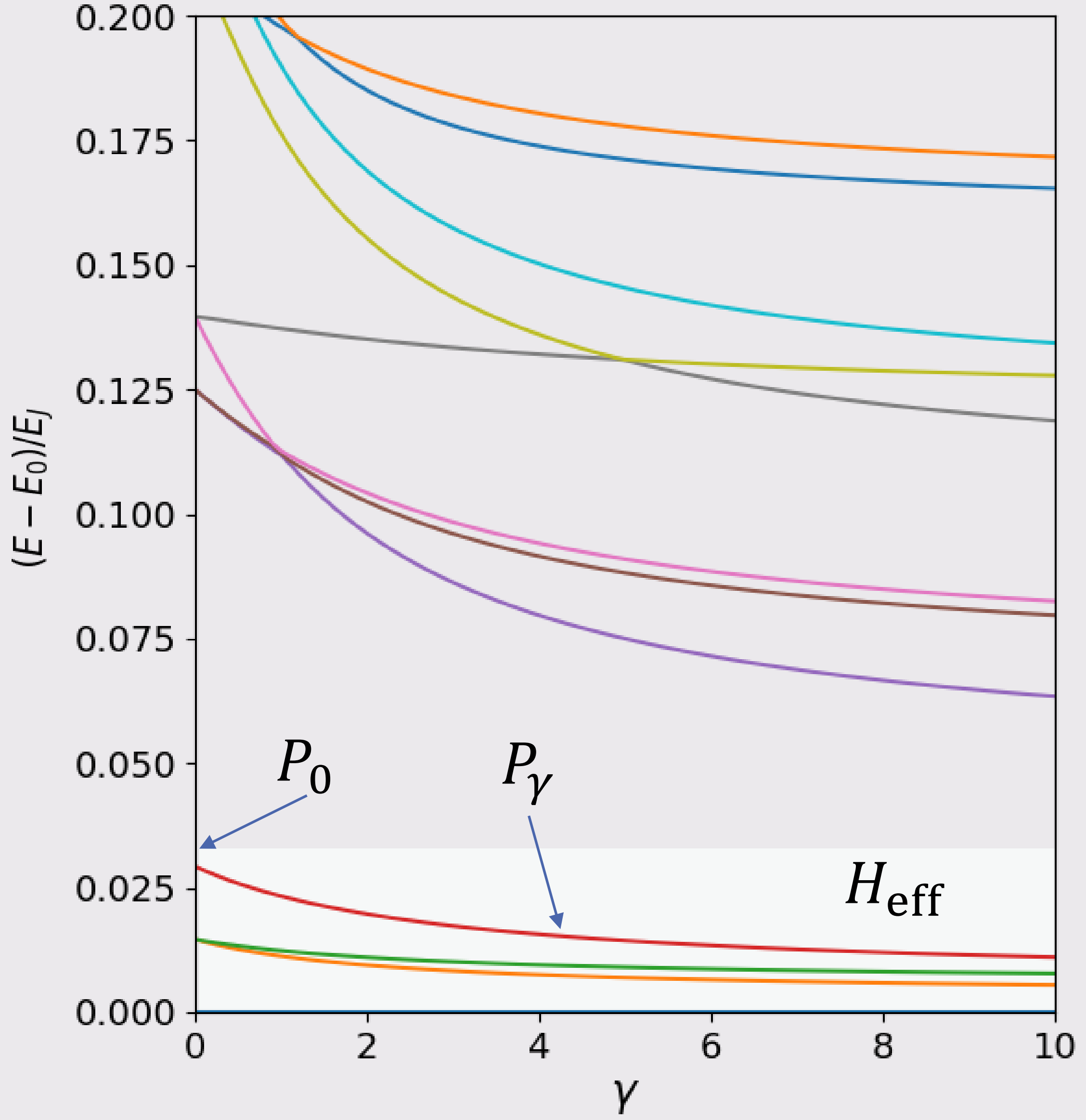


Schrieffer-Wolff

The unitary transformation is designed to “reinterpret” the computational basis in presence of the interaction

$$P(\gamma) = \sum_n |n_\gamma\rangle\langle n_\gamma| = e^{iS} P_0 e^{-iS}$$

Here P_0 is the projector onto the original computational subspace.



Schrieffer-Wolff

The transformed Hamiltonian is usually studied perturbatively

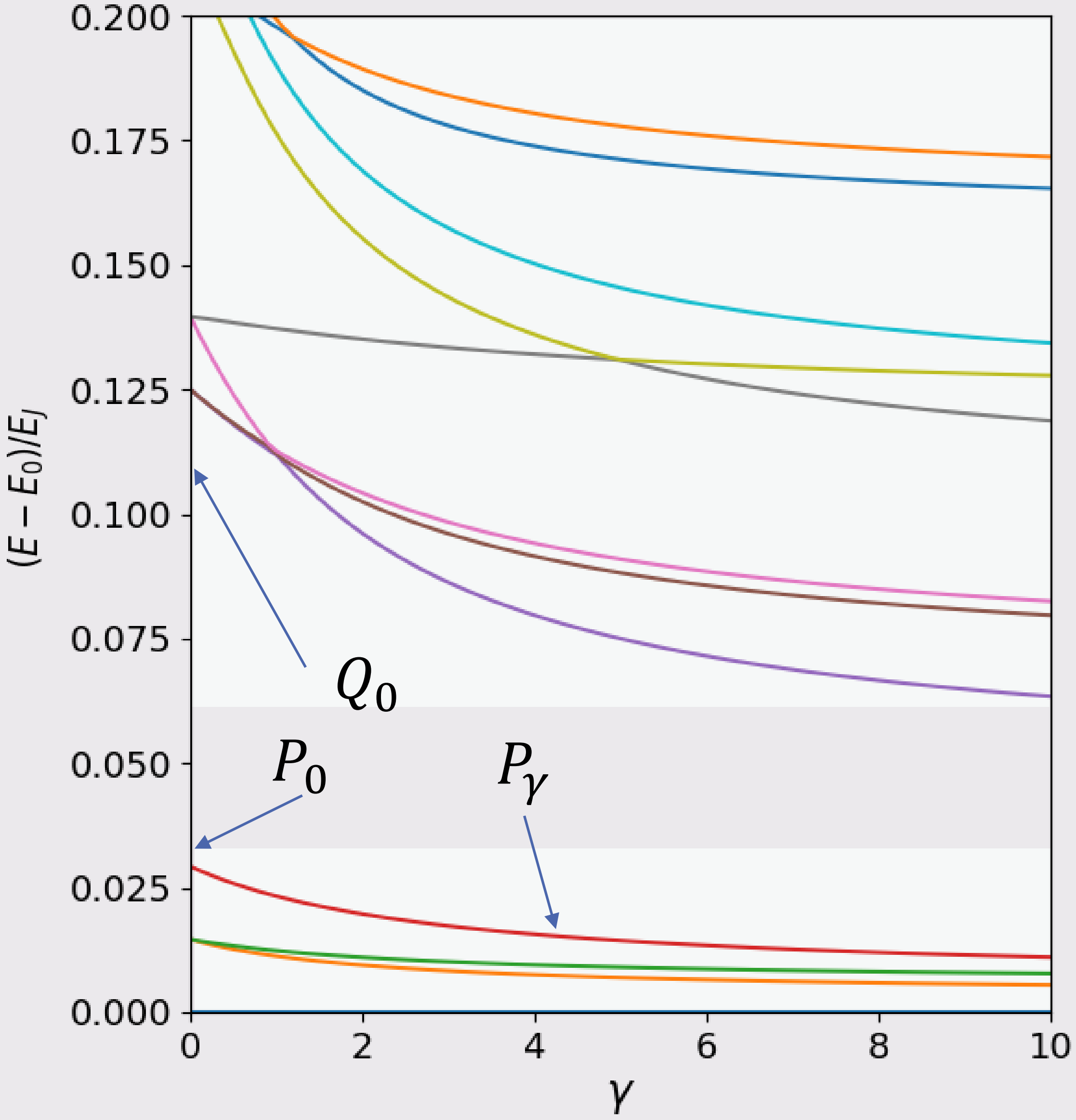
$$e^{\gamma S} H e^{-\gamma S}$$
$$= H_0 + \gamma V + [\gamma S, H_0] + \gamma^2 [S, V]$$
$$+ \frac{\gamma^2}{2} [S, [S, H_0]] + O(\gamma^3)$$

Generator “S” chosen to cancel terms that take us outside computational subspace.

To second order:

$$Q_0 [\gamma S, H_0] P_0 = -\gamma Q_0 V P_0$$

$$S = \sum_{ij \in P_0} \sum_{e \in Q_0} \frac{|i\rangle\langle i|V|e\rangle\langle e|}{E_i - E_e} - \mathrm{H.c.}$$



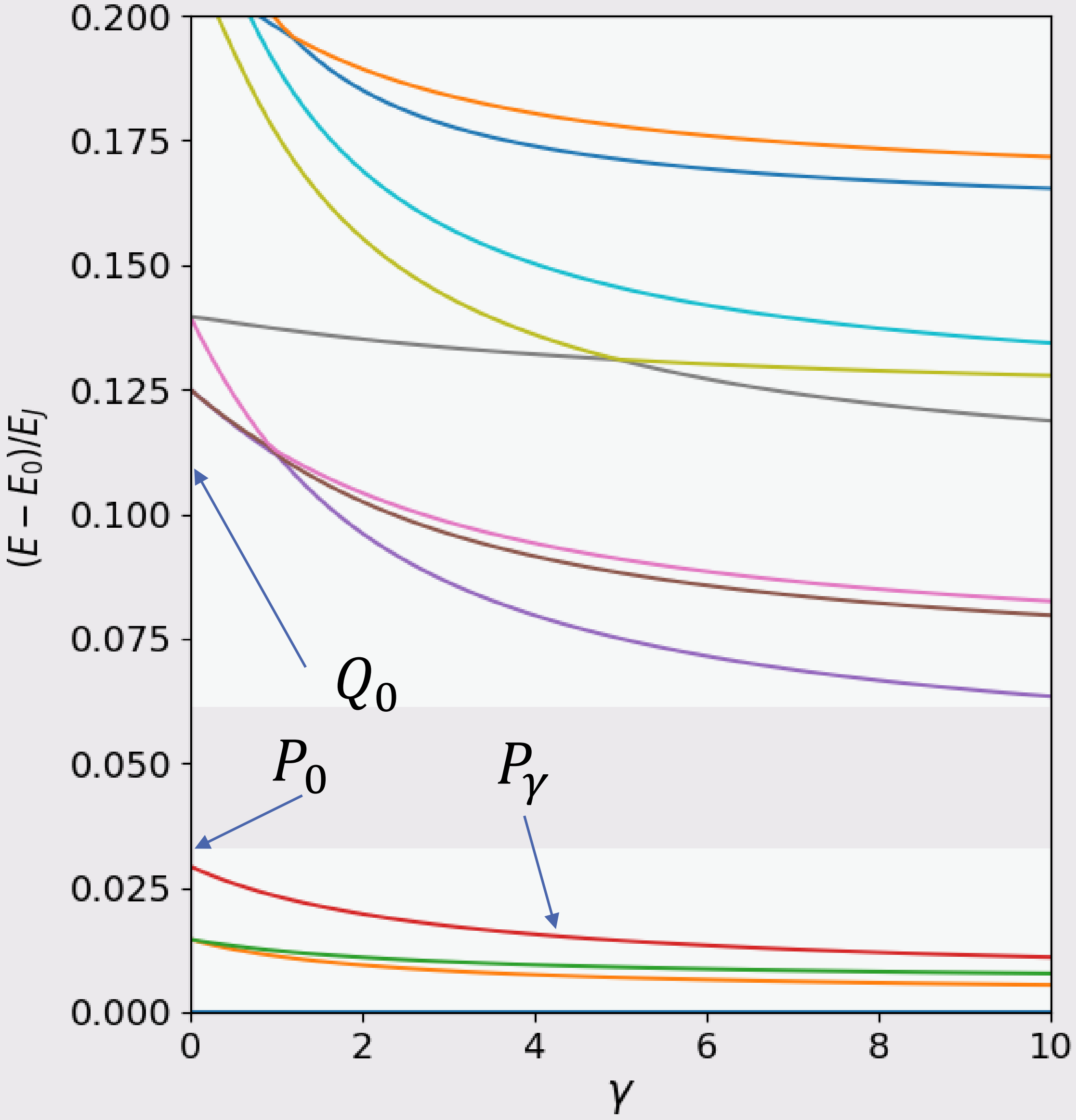
Schrieffer-Wolff

A non-perturbative study of SW by Bravy et al. establishes, through exact analytical continuation

$$U(\gamma) = \sqrt{(1 - P_0)(1 - P_\gamma)}$$
$$H_{\text{eff}} = P_0 U(\gamma)^+ H U(\gamma) P_0$$

If we can estimate P_γ somehow, we can get the effective model including all interactions.

Problem: How do we compute this square root in an infinite-dimensional space?



S. Bravyi et al, Annals of Physics (2011)
G. Consani, P. Warburton, NJP (2020)

Non-perturbative Schrieffer-Wolff

The Schrieffer-Wolff transformation connects states from different Hamiltonians in the computational subspace

$$P_0 U P_\gamma = \sum_{ij} a_{ij} |i_0\rangle\langle j_\gamma| := \hat{A}$$

We could use this to compute the effective Hamiltonian formally

$$H_{\text{eff}} = \sum_{ijk} |i_0\rangle a_{ij} \langle j_\gamma | H | j_\gamma \rangle a_{kj}^* \langle k_0|$$

But what are the elements of “A”?

Non-perturbative Schrieffer-Wolff

We can deduce "A" from a different operator

$$\hat{B} = P_\gamma P_0 = \sum_{ij} |i_\gamma\rangle\langle i_\gamma|j_0\rangle\langle j_0| = \sum_{ij} |i_\gamma\rangle b_{ij}\langle j_0|$$

Such that $\hat{A}\hat{B}\hat{A} = P_0 P_\gamma = B^+$

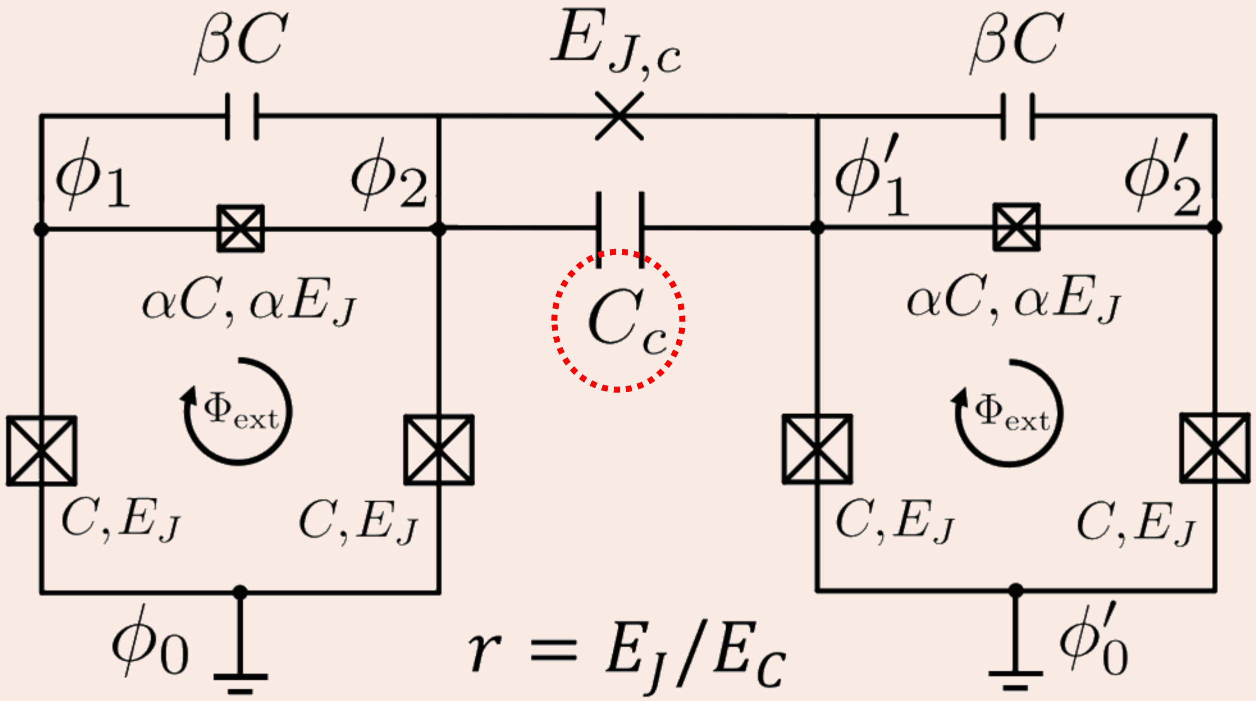
But what are the elements of "A"? We can find them through

$$aba = b^+$$

If the SVD of $b = \mathrm{w}\Sigma v^+$, then $b^+ = \mathrm{v}\Sigma w^+$, and $a = vw^+$ satisfies the equation.

3JJ-qb coupling

Nonperturbative study of inductive and capacitive couplings.



M. Hita et al, PRAppl. 17, 014028 (2022)
M. Hita et al, Appl. Phys. Lett. (2021)

3JJ-qb coupling

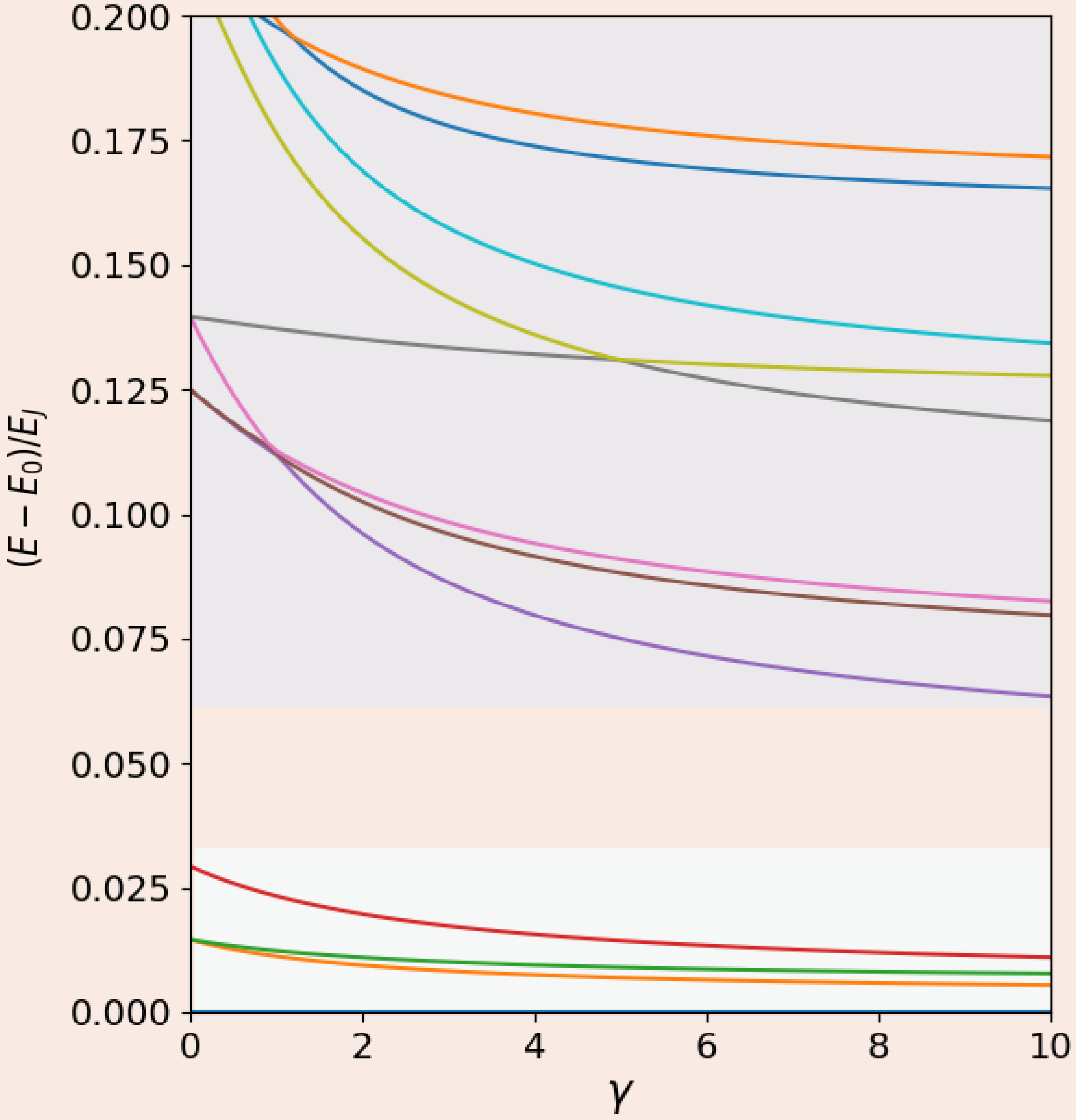
Nonperturbative study of inductive and capacitive couplings.

Three types of interaction

- YY direct capacitive interaction

$$\frac{g_{ZZ}}{\Delta} \sim \frac{C_q \varphi_\star^2}{C_{OD}} \frac{\Delta}{8E_C}$$

- ZZ capacitively mediated by excited states (or inductive).
- XX third order combination of both processes.



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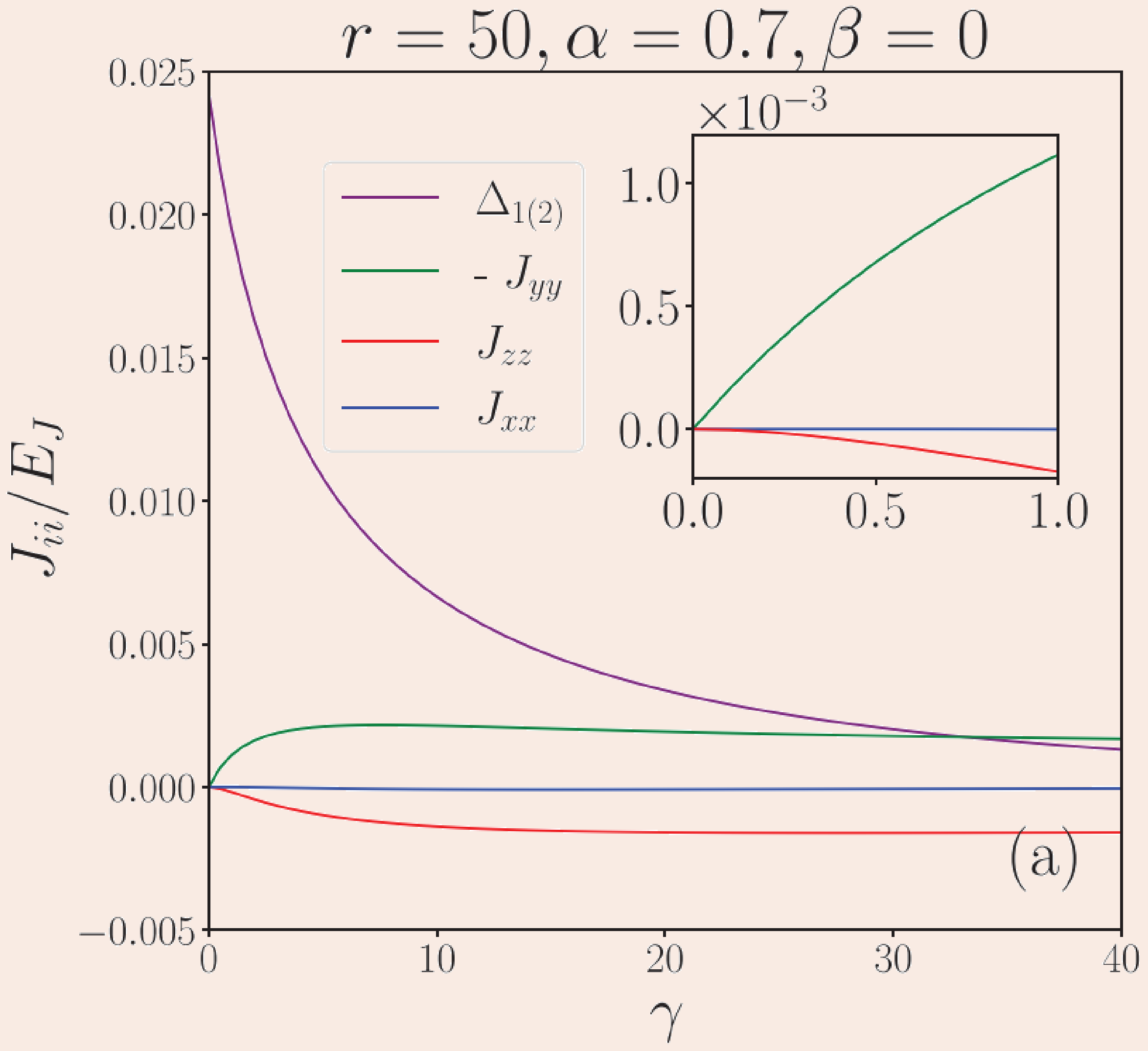


M. Hita et al, PRAppl. 17, 014028 (2022)
M. Hita et al, Appl. Phys. Lett. (2021)

3JJ-qb + cavity

Same analysis can be performed using a qubit and a cavity.

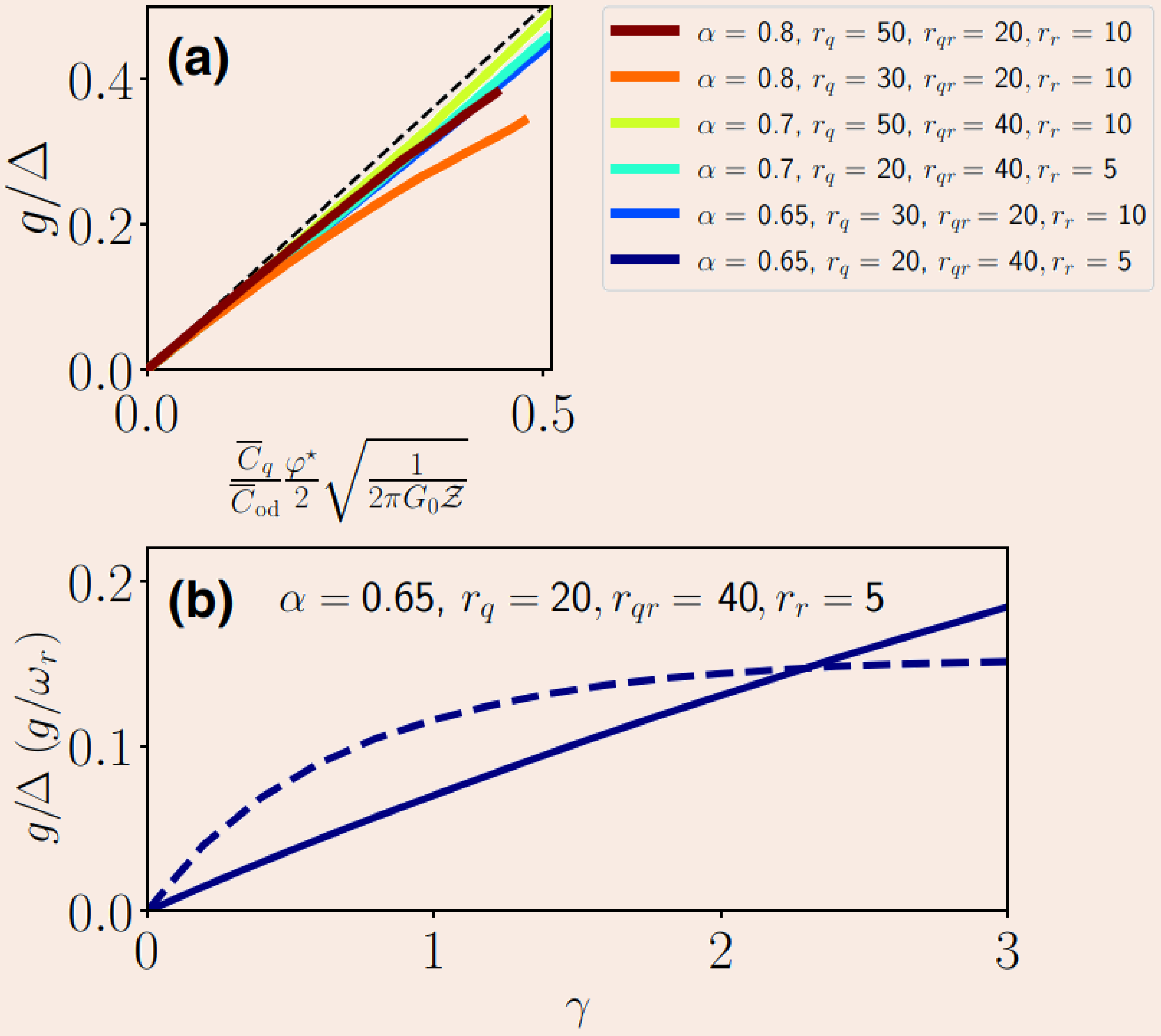
The study is simpler, because the resonator is harmonic

$$H = H_{qb} + H_{cav} + g_Y \sigma^y(\hat{a}^+ + \hat{a})$$

with coupling strength

$$g_Y \propto \frac{\varphi_\star}{\sqrt{Z}}$$

Higher order terms suppressed by resonator.

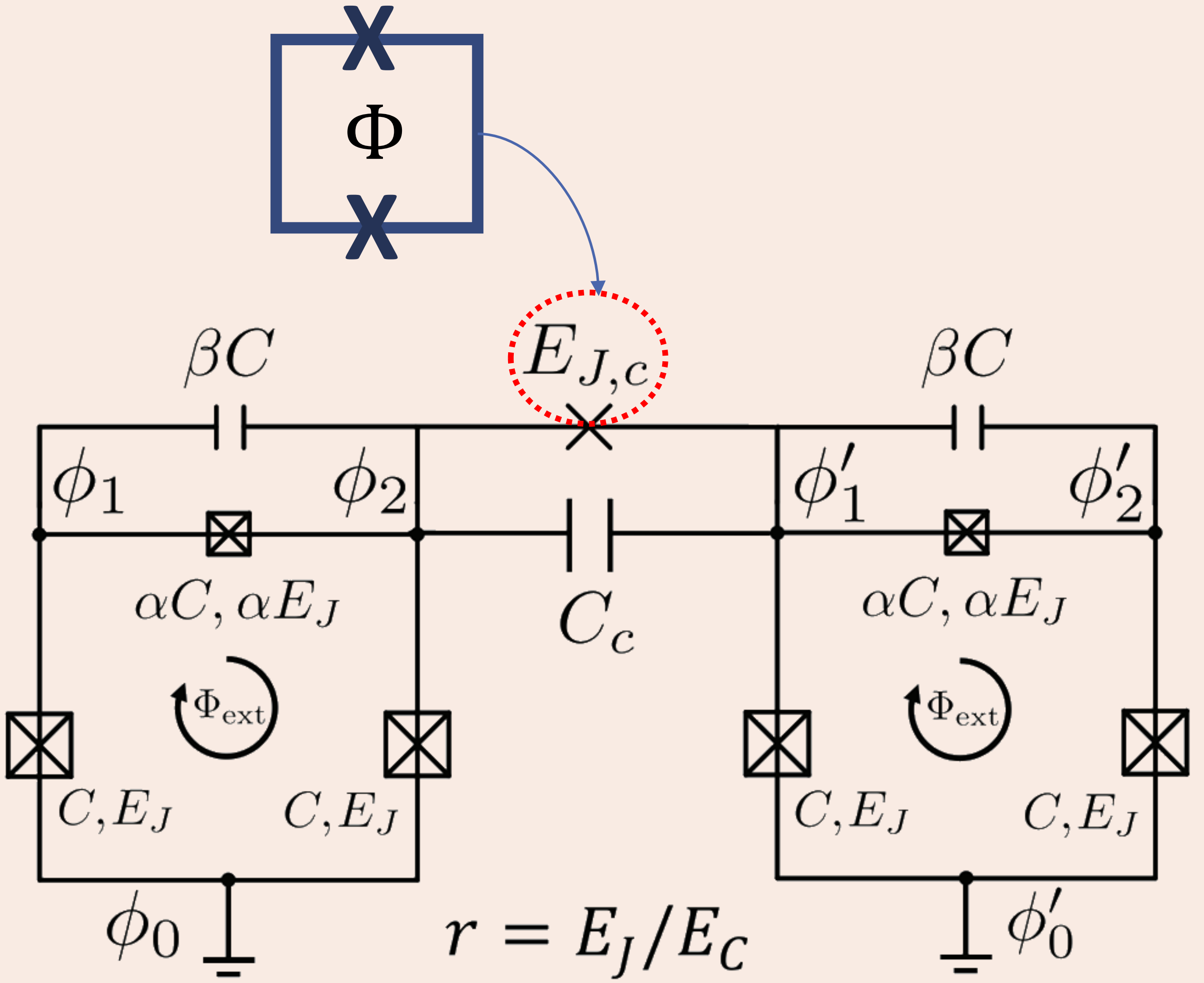


Coupling tunability

Not always obvious how to "tune" and not only "design" the coupling

- In the capacitive case, we could use "voltage" dividers (-> Martinis X-mons)
- In the inductive case, we can replace the junction by a SQUID

$$E_J = E_J \cos\left(\frac{2\pi\Phi}{\Phi_0}\right)$$



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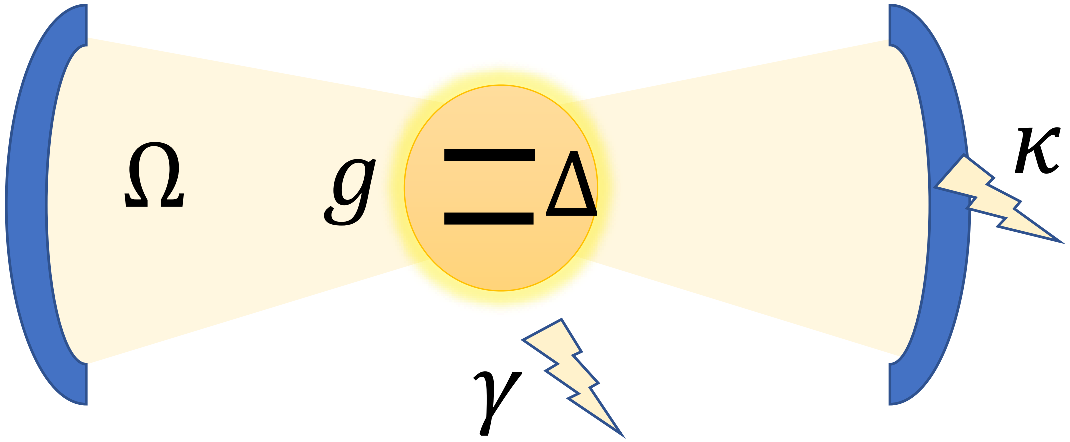
Application: quantum simulation

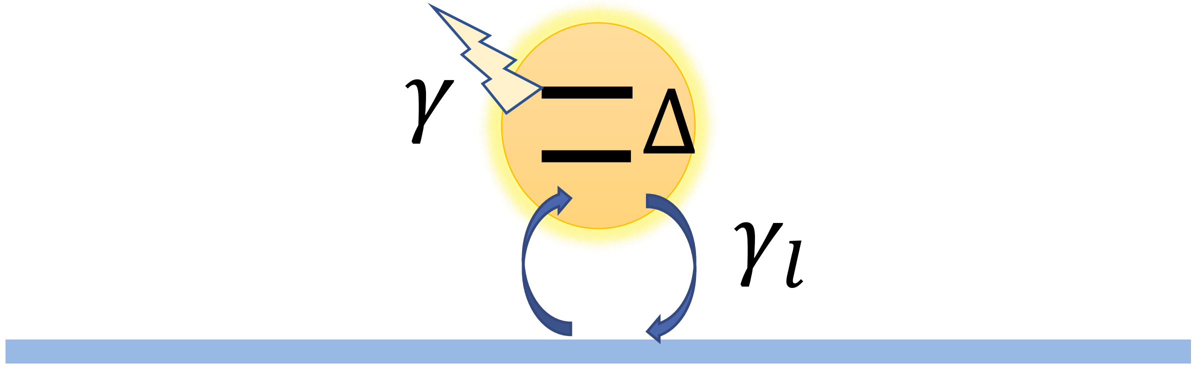
Cavity-QED



Rabi frequency comparable to light and matter timescales

$$g \simeq \Delta, \omega \gg \kappa, \gamma$$

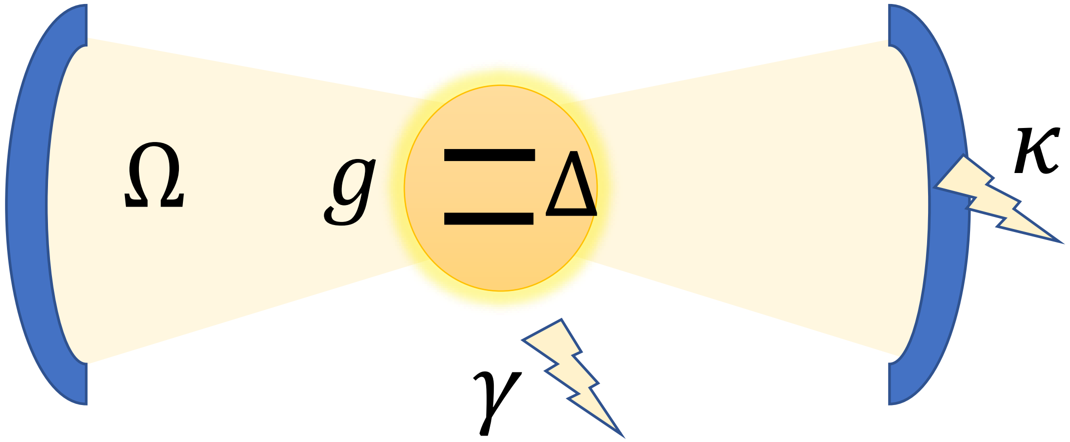
Open systems



Emission rate comparable to light and matter timescales

$$\gamma_l \simeq \Delta, \omega \gg \gamma$$

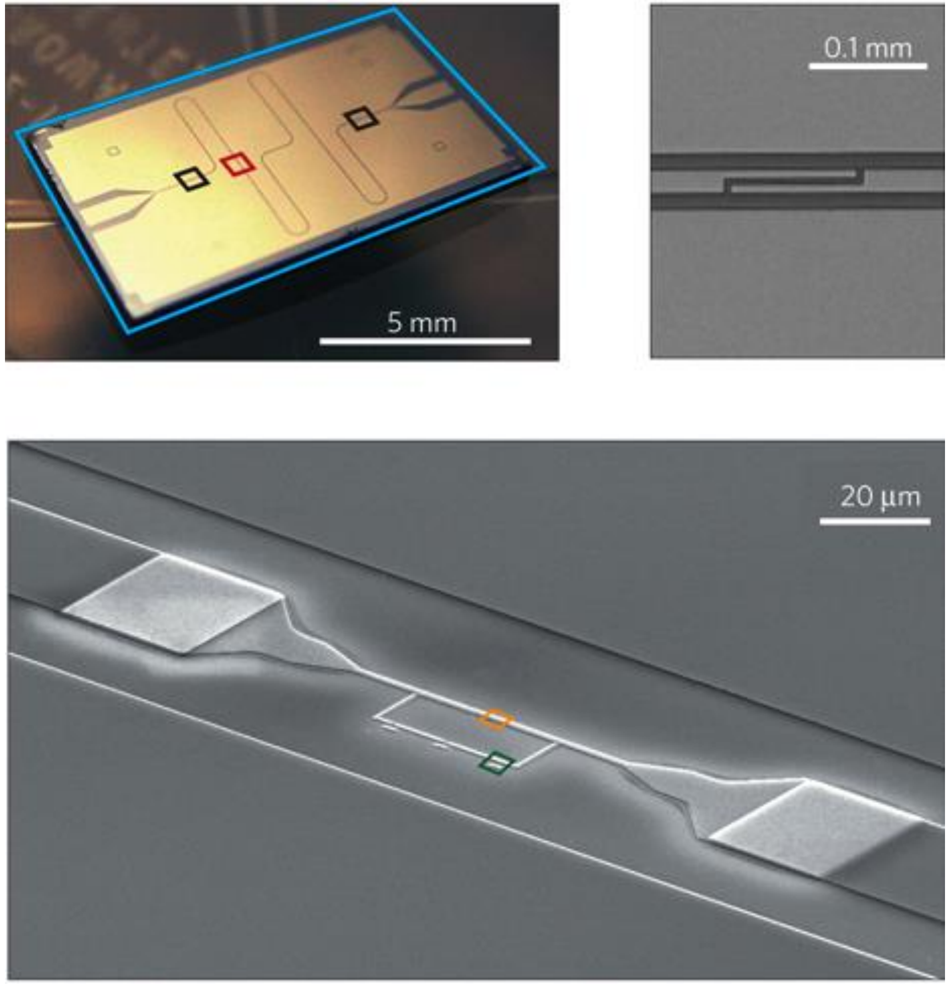
USC vs RWA

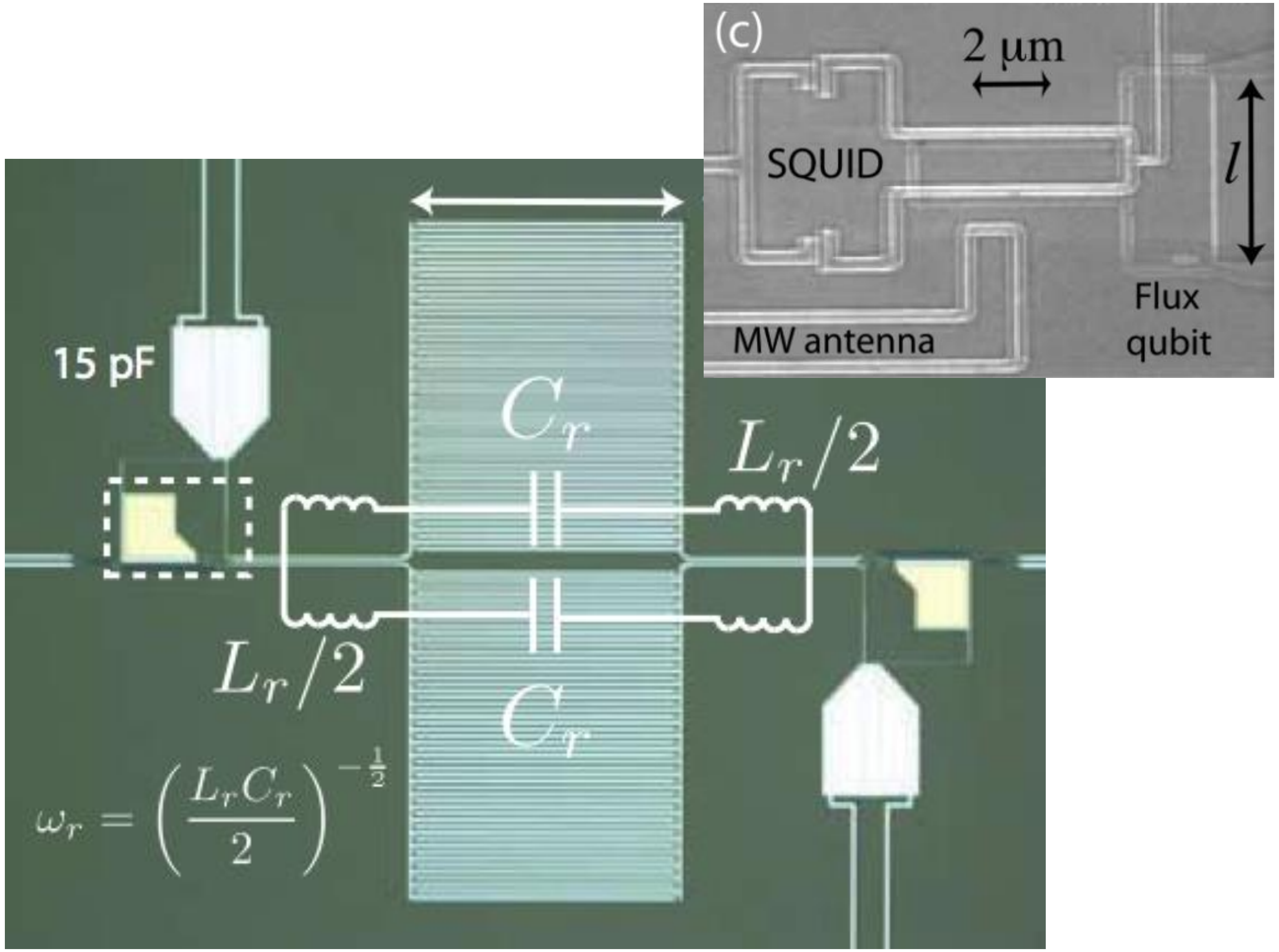


In both models, this limit questions the usual rotating wave approximation:

$$H = \frac{\Delta}{2}\sigma^z + \omega \hat{a}^+ \hat{a} + \begin{cases} g\sigma^x(\hat{a} + \hat{a}^+) \\ g(\sigma^+ \hat{a} + \sigma^- \hat{a}) \end{cases}$$

Ultrastrongly coupled circuit-QED

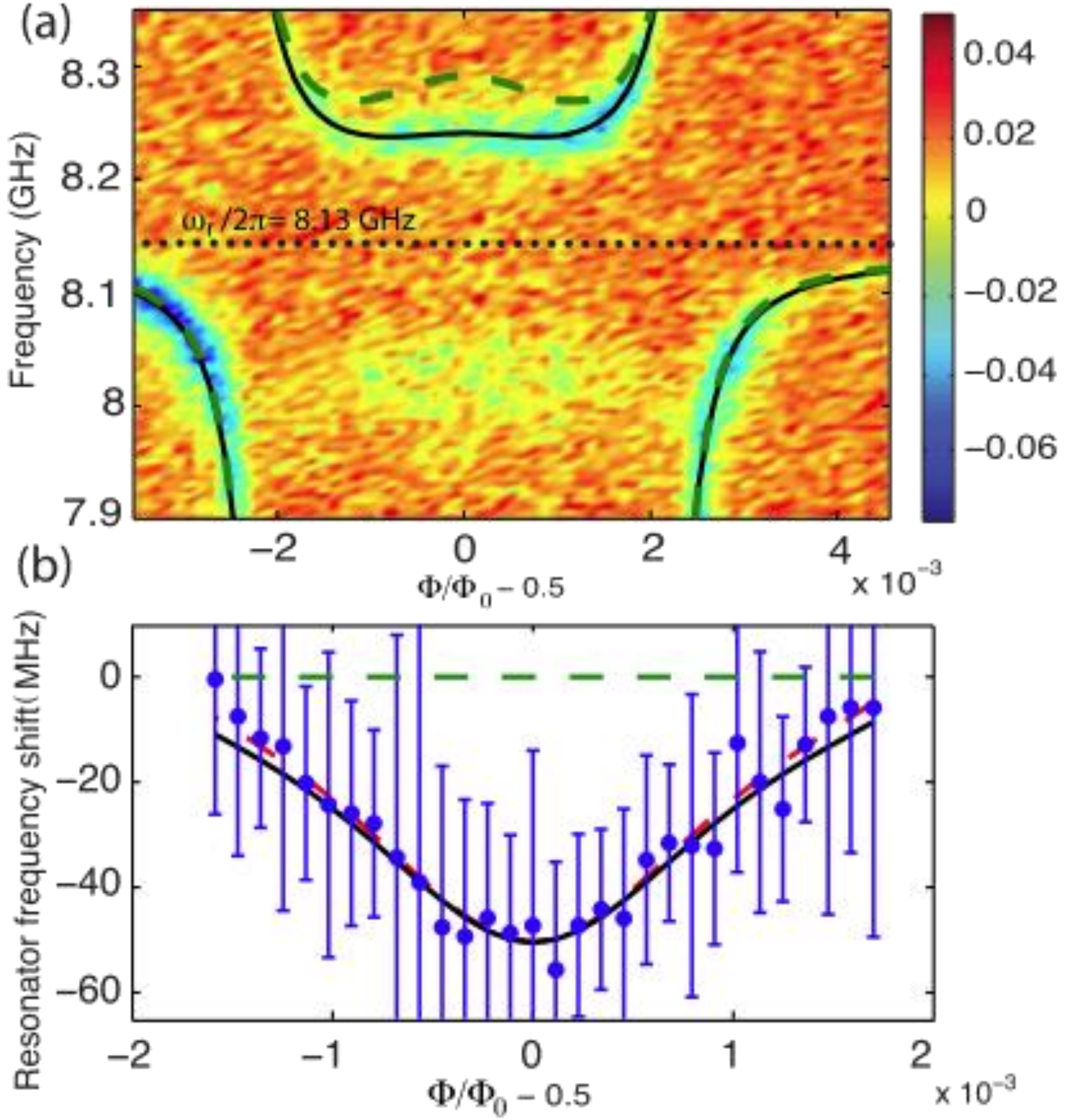


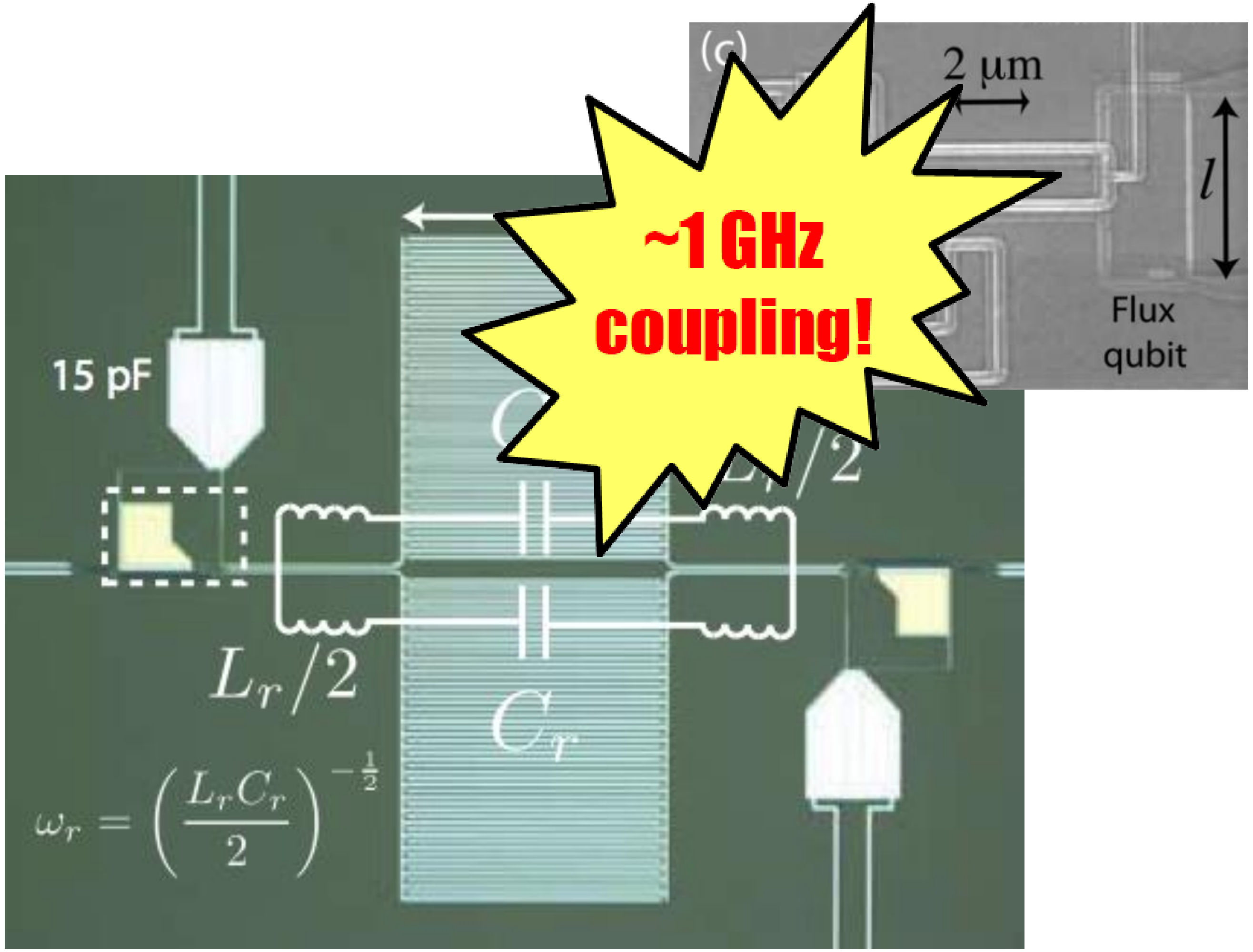


T. Niemczyk et al, Nature Physics ***6****,772–776 (2010)*

P. Forn-Díaz et al PRL ***105*** *237001 (2010)*

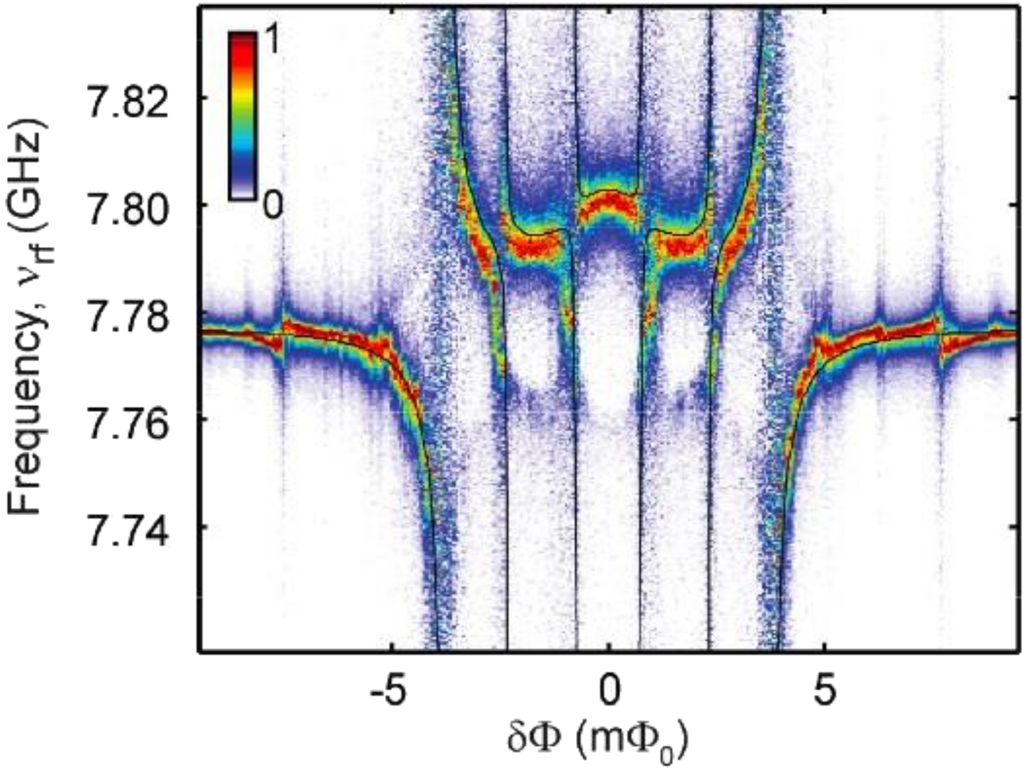
Very large Bloch-Siegert shift

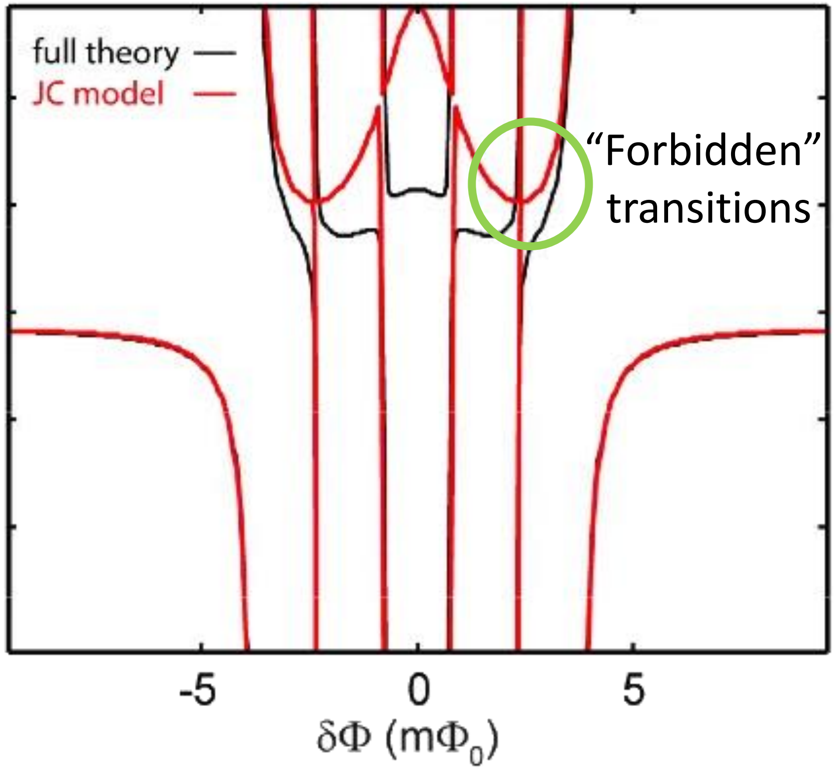




*P. Forn-Díaz et al PRL **105** 237001 (2010)*

Non-RWA spectra

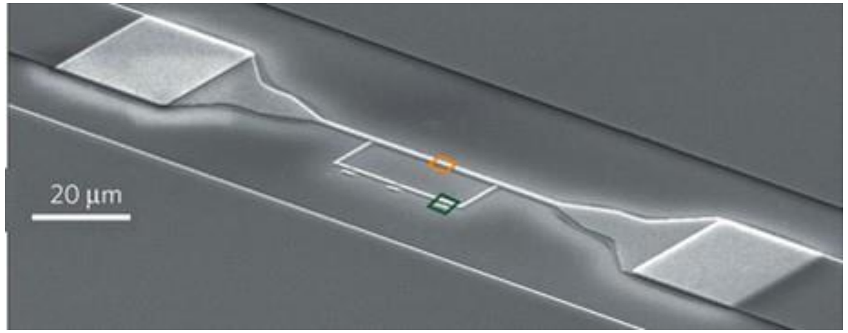




$$\frac{g}{\Delta} = 12\%$$

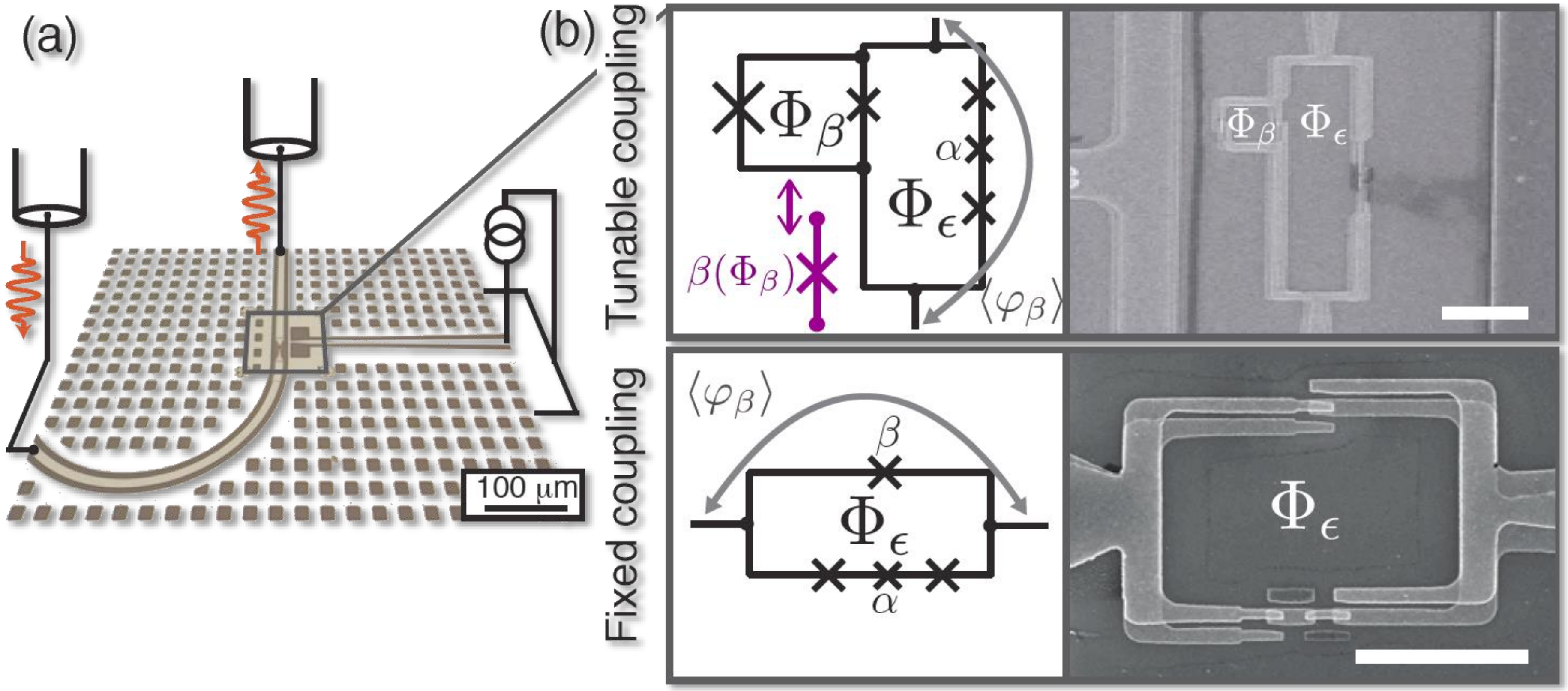
RWA $\quad g(\sigma^+ a + \sigma^- a^\dagger)$

Rabi $\quad g\sigma^x(a + a^\dagger)$



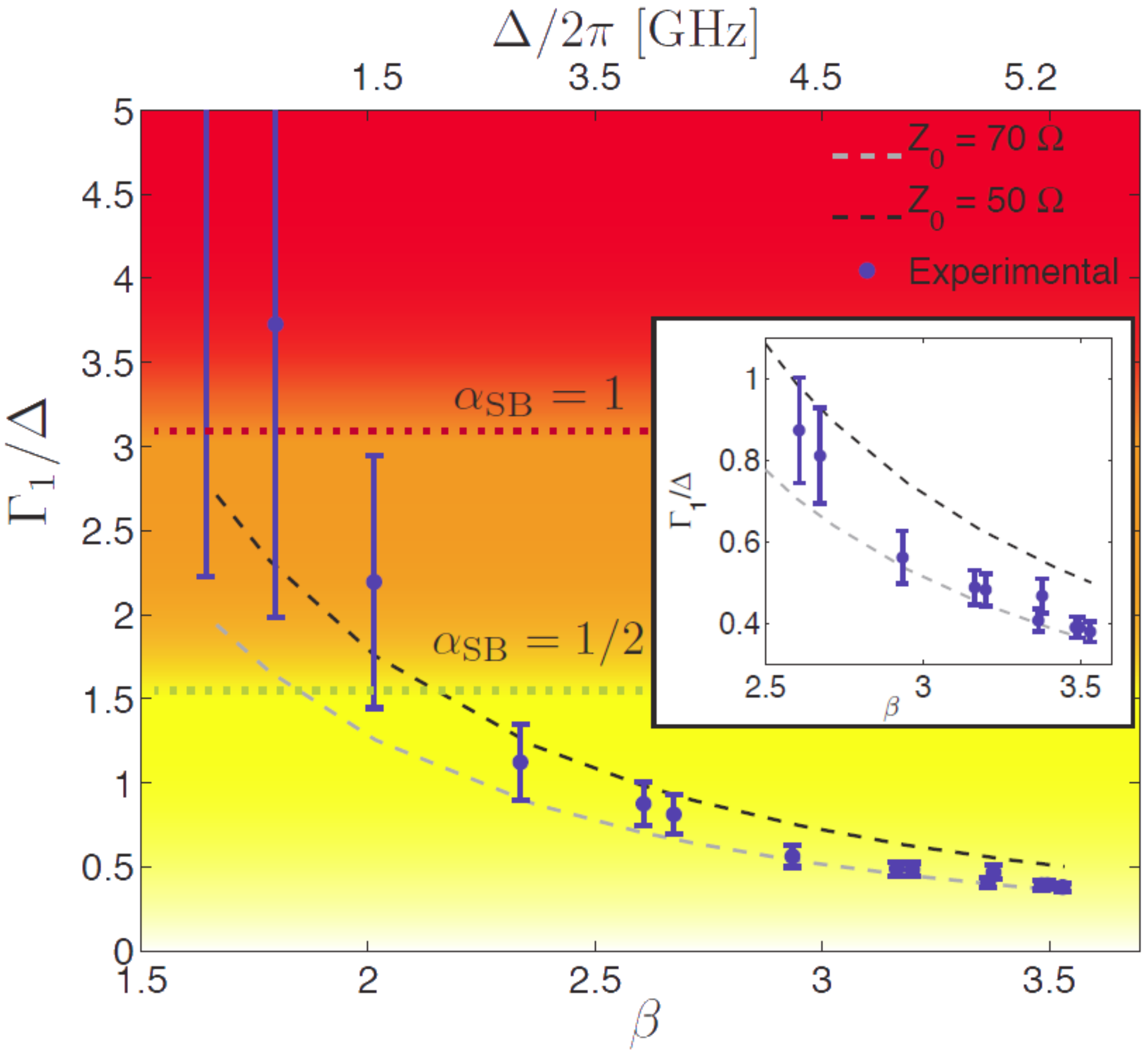
T. Niemczyk et al, Nature Physics ***6****,772–776 (2010)*

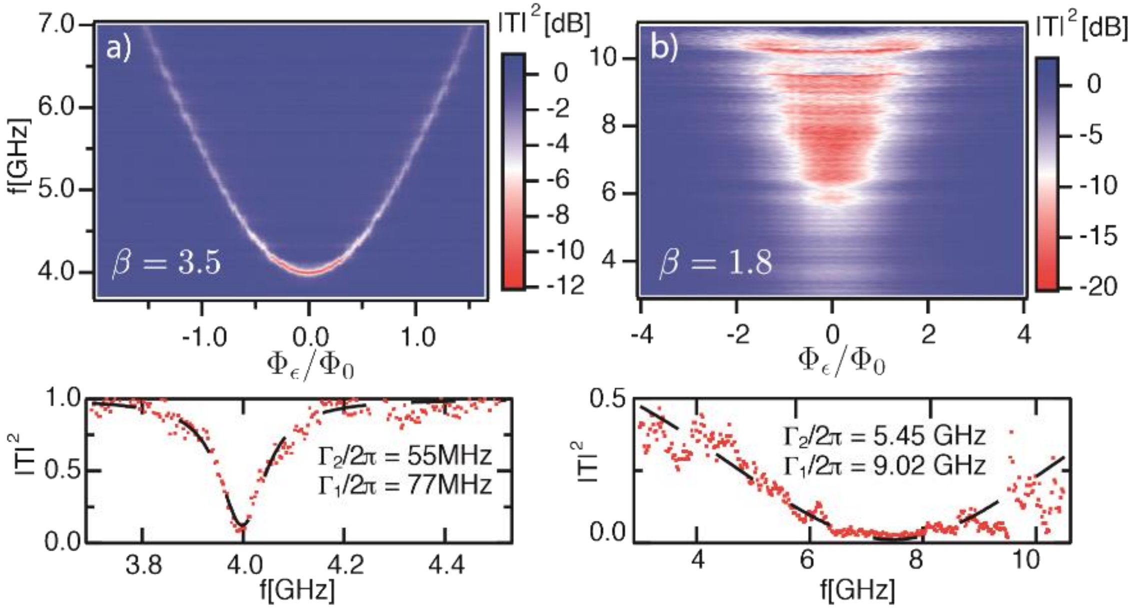
Ultrastrong coupling in open lines

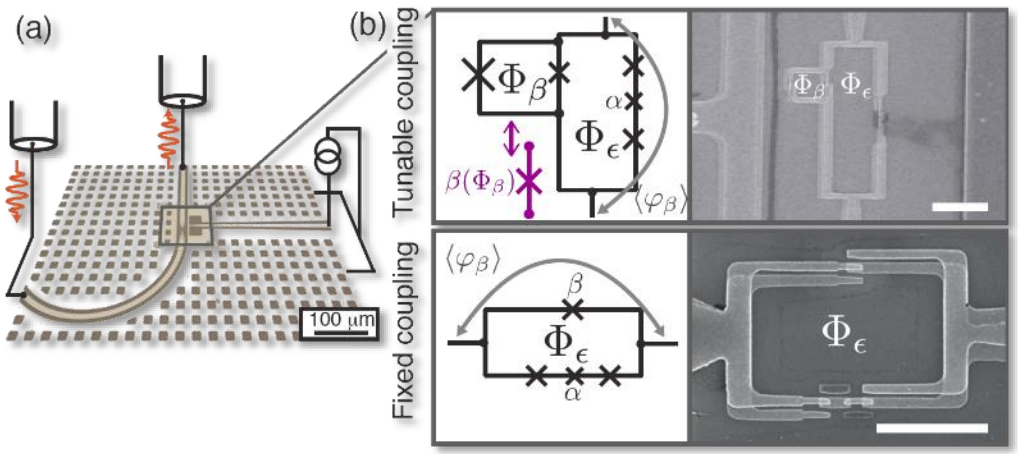


P. Forn Díaz et al, Nat. Phys. 13, 39 (2017)

Ultrastrong coupling in open lines







P. Forn Díaz et al, Nat. Phys. 13, 39 (2017)

Hybridization

Mediated interactions

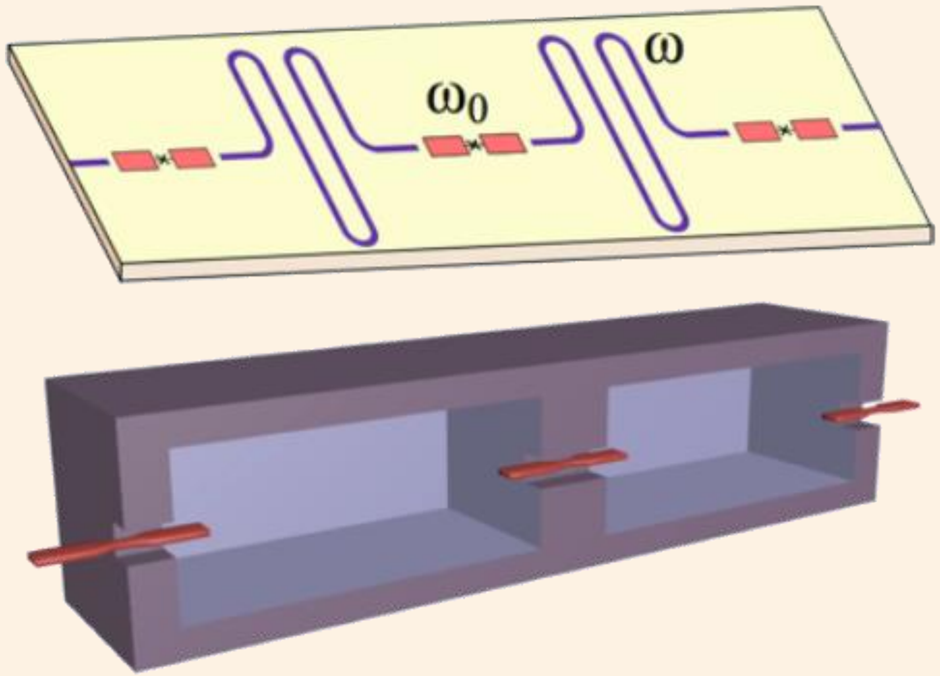
So far, coupling elements had no intrinsic dynamics.

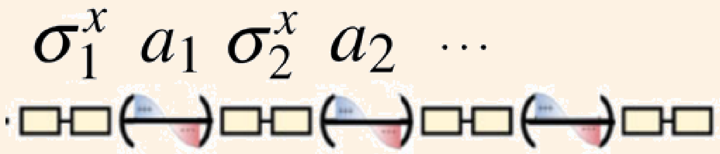
Let's now try connecting qubit with resonators

$$H = \sum_i \frac{\omega_0}{2}\sigma_i^z + \sum_n \omega \hat{a}_n^+ \hat{a}_n$$

$$+ \sum_{in} g_{in}\sigma_i^x(\hat{a}_n^+ + \hat{a})$$

Because of USC, we can afford large "g", comparable to ω_0



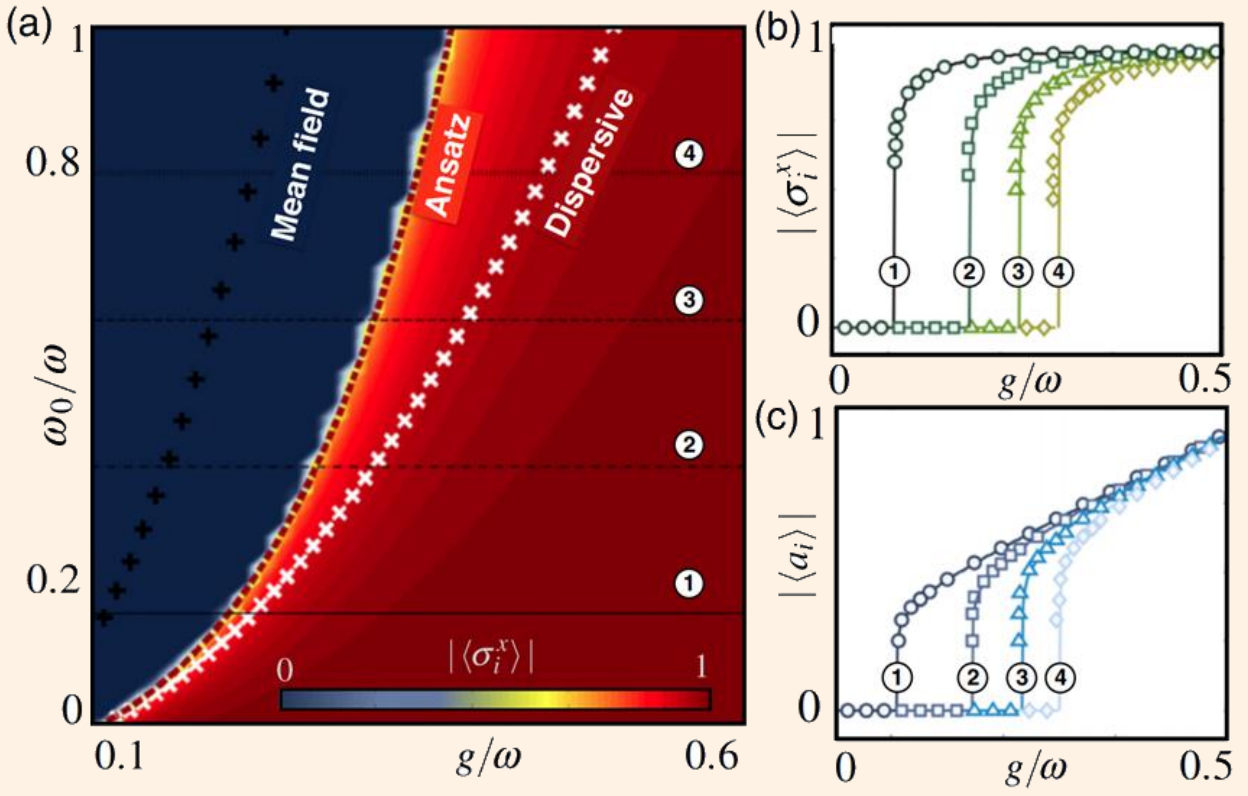


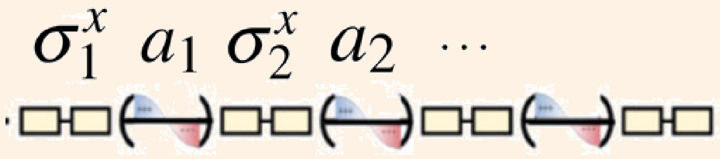
A. Kurcz et al, PRL 112, 180405 (2014)
M. Pino & JJGR NJP (2018); PRA (2020)

USC Quantum Ising

When doing so, the many-qubit system exhibits an Ising-type second order quantum phase transition

- Both qubit and cavity polarize in the magnetic phase.
- Energy and magnetization are not differentiable.
- The critical exponents are Ising.





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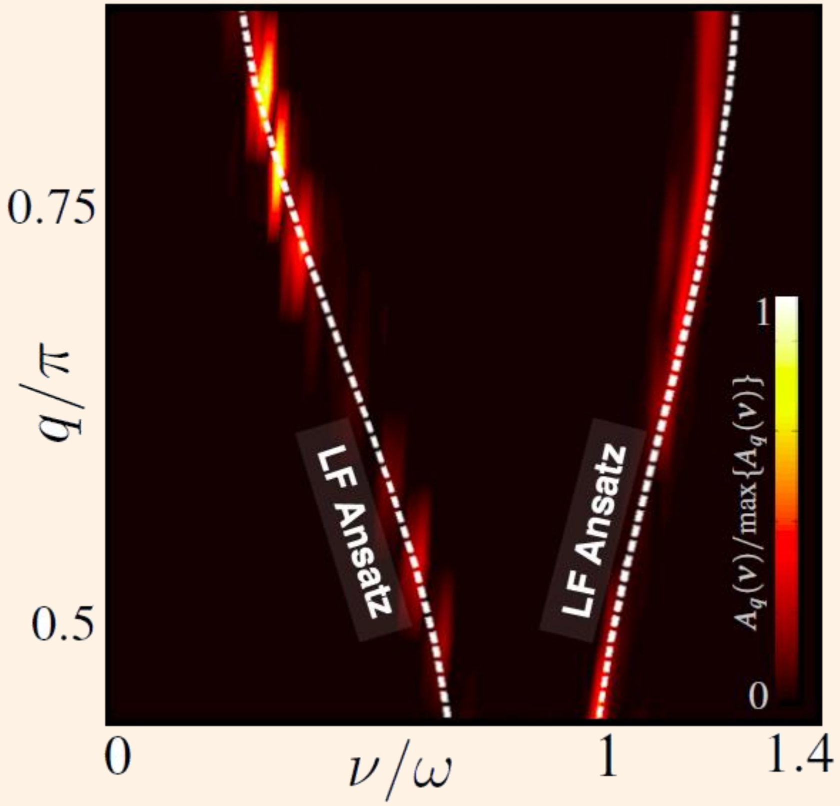
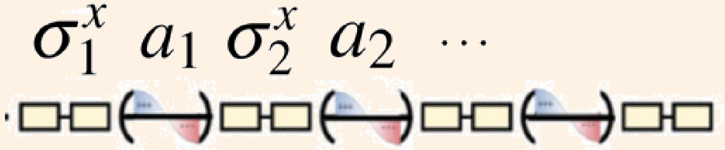
- Both qubit and cavity polarize in the magnetic phase.
- Energy and magnetization are not differentiable.
- The critical exponents are Ising.
- The low-energy excitation sector reproduces spin waves.





A. Kurcz et al, PRL 112, 180405 (2014)
M. Pino & JJGR NJP (2018); PRA (2020)

Polaron transformation

Initial Hamiltonian

$$H = \sum_i \frac{\omega_0}{2}\sigma_i^z + \sum_n \omega \hat{a}_n^+ \hat{a}_n$$

$$+ \sum_{in} g_{in}\sigma_i^x(\hat{a}_n^+ + \hat{a})$$

Collective displacement

$$U = \exp\left(-\sum_{in}\frac{g_{in}}{\omega_n}\sigma_i^x \hat{a}_n^+ + H.c.\right)$$

Effective model

$$H_{\text{eff}} = U^+ H U \simeq \sum_i \frac{\omega_0}{2}\sigma_i^z + \sum_{in}\frac{g_{in}g_{jn}}{\omega_n}\sigma_i^x\sigma_j^x + \cdots$$

Good variational ground state

$$|GS\rangle \simeq U^+|\downarrow\rangle^{\otimes N}|vac\rangle^{\otimes N}$$

The observables hybridize

$$\hat{a}_n \simeq \hat{a}_n - \sum \frac{g_{in}}{\omega_n}\sigma_i^x$$

When the qubits experience a magnetic polarization, the cavity also shows it

$$\langle\sigma^x\rangle \neq 0 \Rightarrow \langle\hat{a}_n\rangle \neq 0$$

The cavities act as a reservoir for “cooling” the magnetic system during adiabatic passages.

Adiabatic approxim.

A composite system, made of slow and rapidly evolving d.o.f.

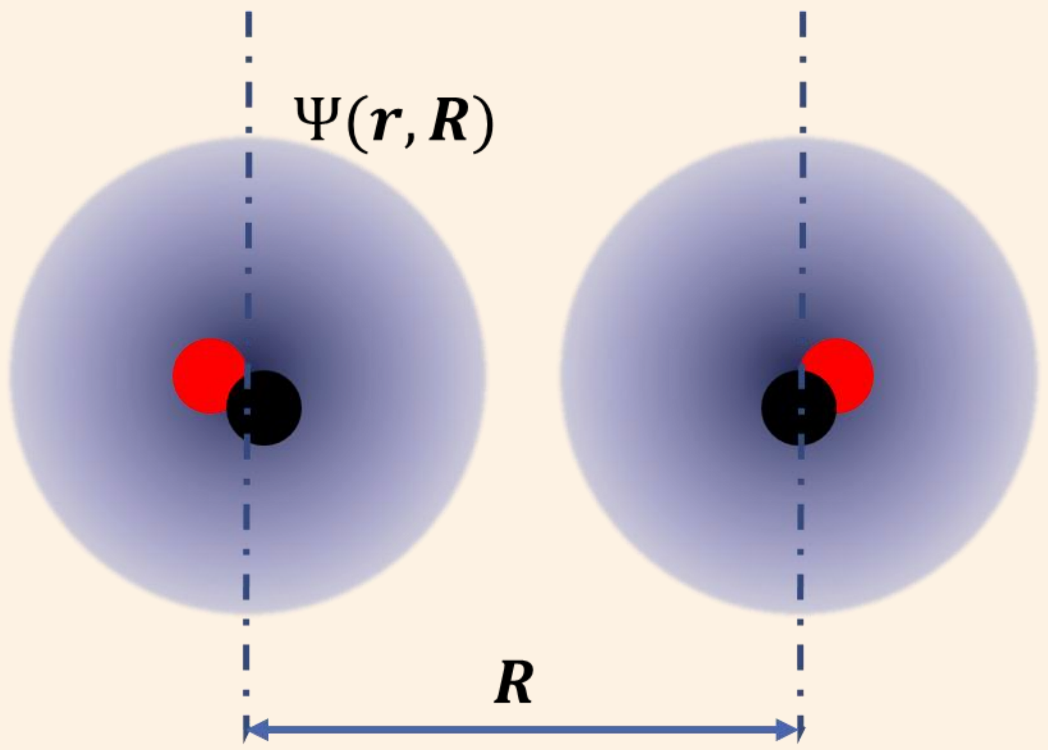
$$H_1(R_1) + H_2(R_2) + H_{\text{fast}}(r, R_1, R_2)$$

The fast subsystem rapidly adapts to the lowest energy configuration allowed by the slow d.o.f.

$$H = H_1(R_1) + H_2(R_2) + V(R_1, R_2)$$

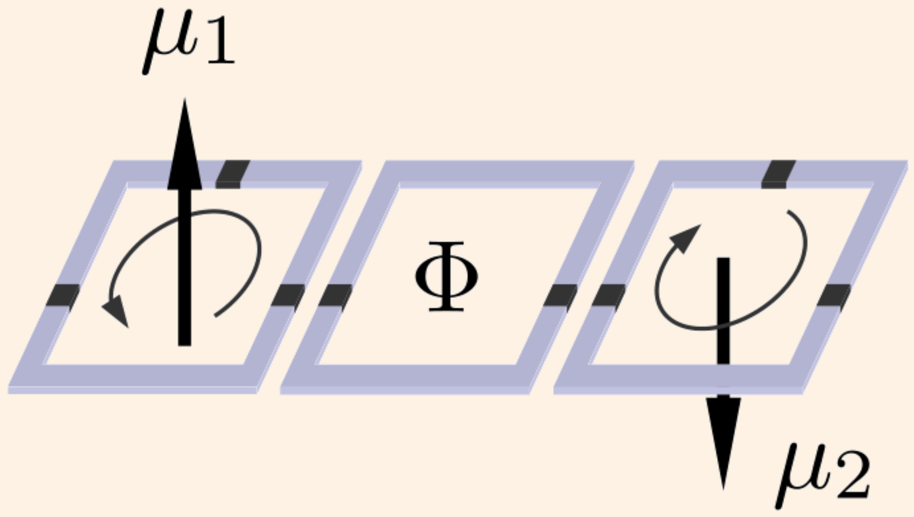
$$V = \min_r H_{\text{fast}}(r, R_1, R_2)$$

This is the Born-Oppenheimer or adiabatic approximation.



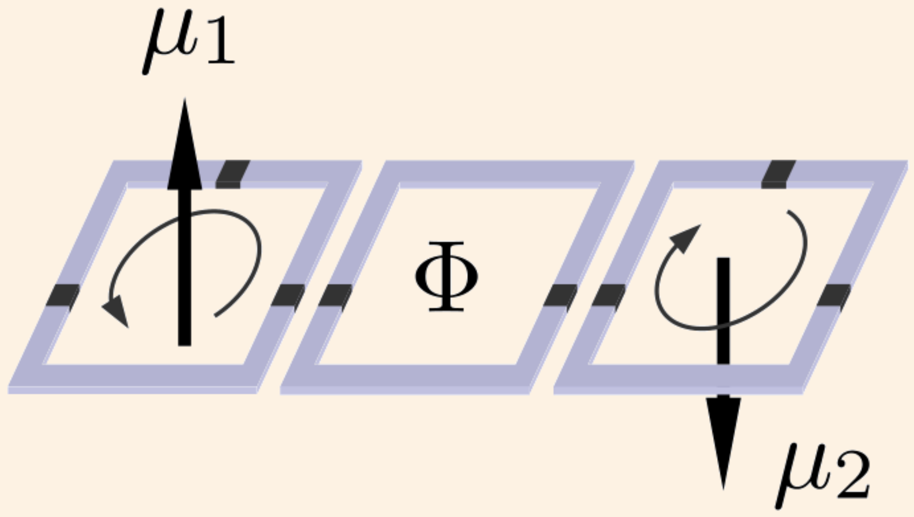
Example: dc-SQUID

Two flux qubits couple inductively through a superconducting loop.



Example: dc-SQUID

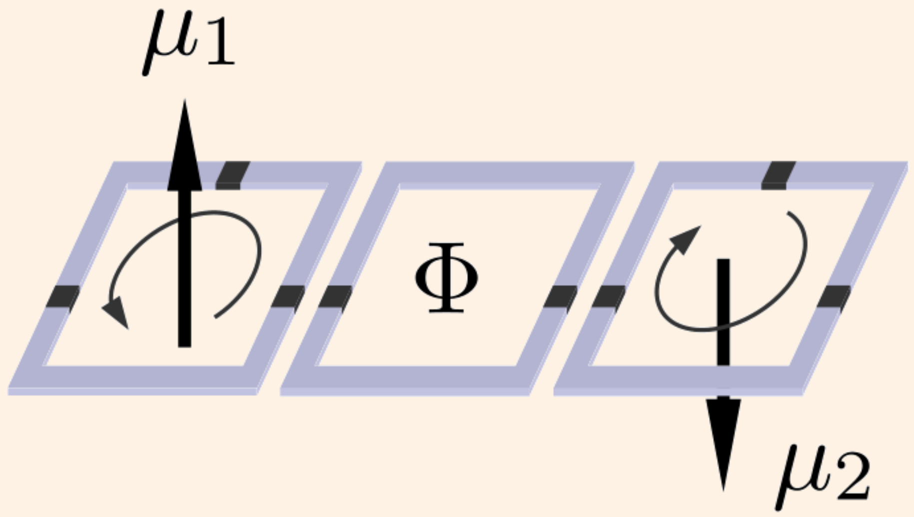
The SQUID has an effective ground-state energy

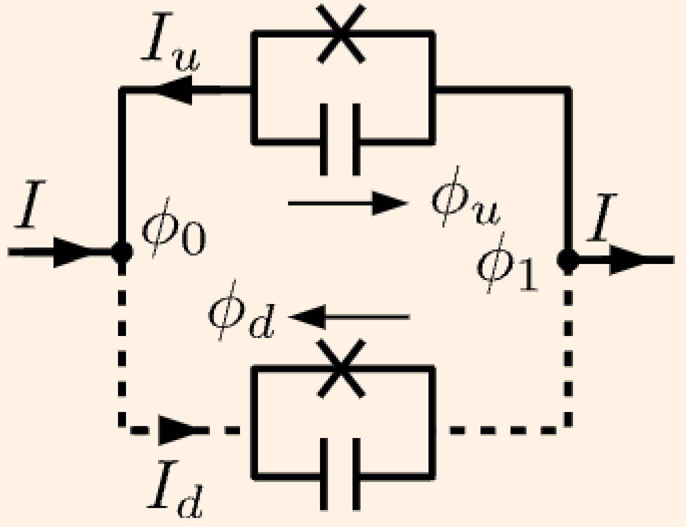


Example: dc-SQUID

Two flux qubits couple inductively through a superconducting loop.

The SQUID's dynamics is like a Josephson junction with tunable critical current





Example: dc-SQUID

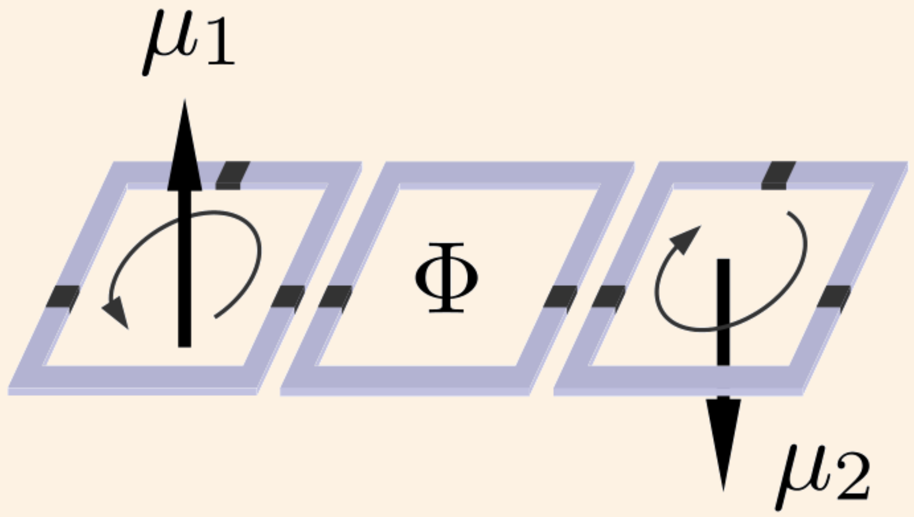
Two flux qubits couple inductively through a superconducting loop.

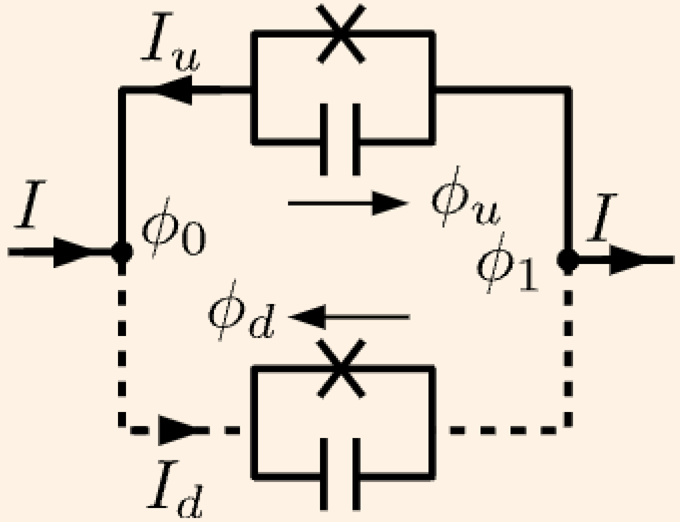
The SQUID's dynamics is like a Josephson junction with tunable critical current

$$L \sim \frac{1}{2}(2C_J)\ddot{\phi}^2 + E_J(\Phi)\cos\left(\frac{2\pi\phi}{\Phi_0}\right)$$

Alternatively

$$H = \frac{1}{2(2C_J)}\hat{q}^2 - E_J(\Phi)\cos\left(\frac{2\pi\phi}{\Phi_0}\right)$$





Example: dc-SQUID

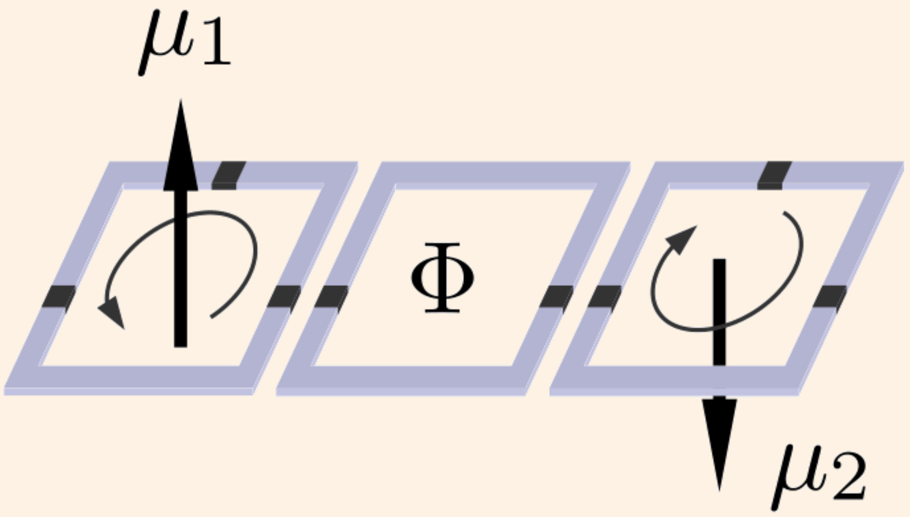
The SQUID has an effective ground state energy, which in the harmonic limit

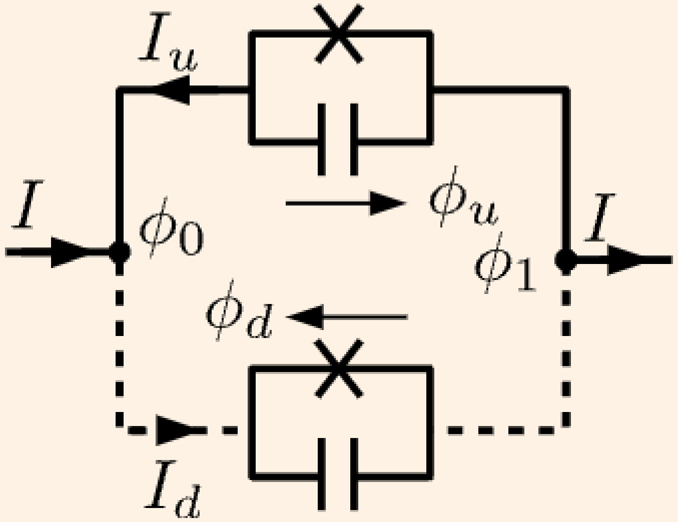
$$H \simeq \frac{1}{2(2C_J)}\hat{q}^2 + E_J(\Phi)\left(\frac{2\pi}{\Phi_0}\right)^2 \phi^2$$

Is approximately

$$E_{GS}(\Phi) \simeq \hbar\omega(\Phi)\left(0+\frac{1}{2}\right)$$

$$\omega(\Phi) = \left(2C_J L_J(\Phi)\right)^{1/2} = \left(\frac{2C_J\Phi_0}{2\pi I_c(\Phi)}\right)^{1/2}$$





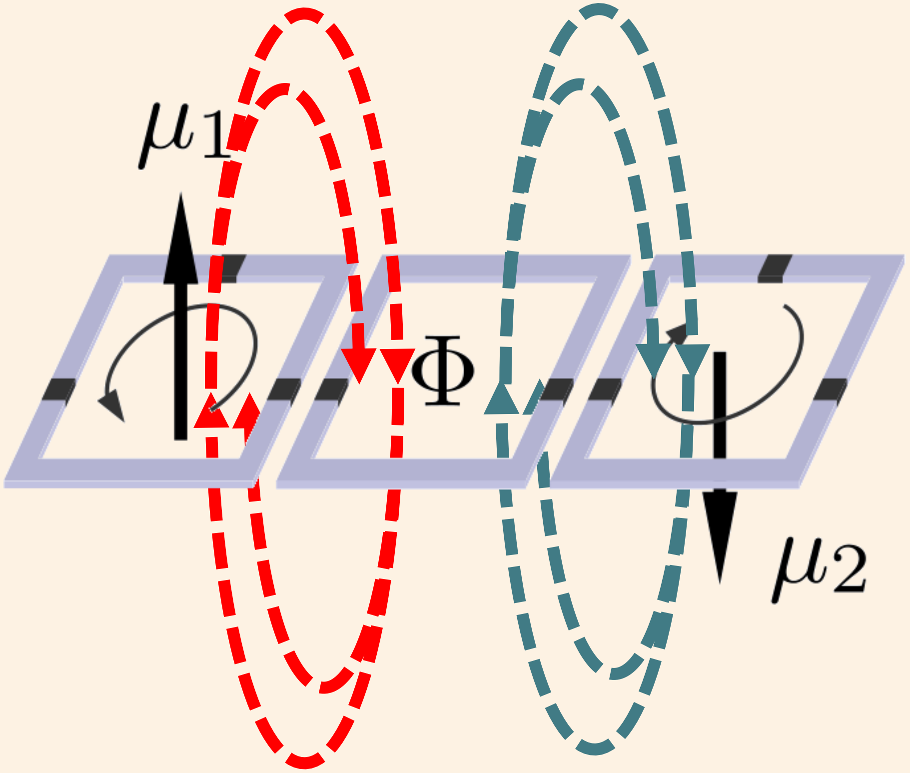
Example: dc-SQUID

Now, the two qubits contribute to the magnetic flux inside the SQUID, via their mutual inductance:

$$\Phi = \Phi_{\text{ext}} + \mathrm{M}_1\sigma_1^z + M_2\sigma_2^z$$

We can therefore expand

$$H_{\text{eff}} \simeq \frac{\Delta_1}{2}\sigma_1^z + \frac{\Delta_2}{2}\sigma_2^z + \\ +\partial_\Phi E_{GS}(\Phi)(M_1\sigma_1^z + M_2\sigma_2^z) \\ +\partial_\Phi^2 E_{GS}(\Phi) \times (M_1 M_2) \times \sigma_1^z\sigma_2^z$$



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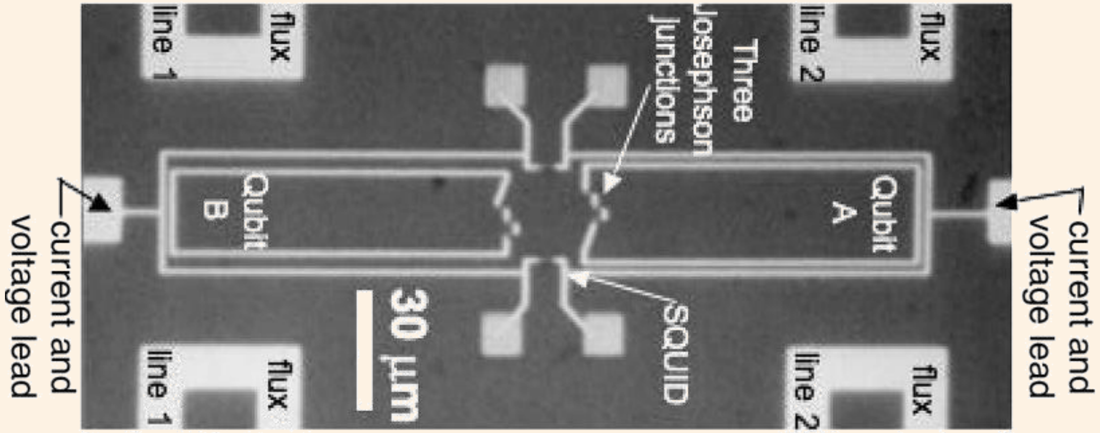
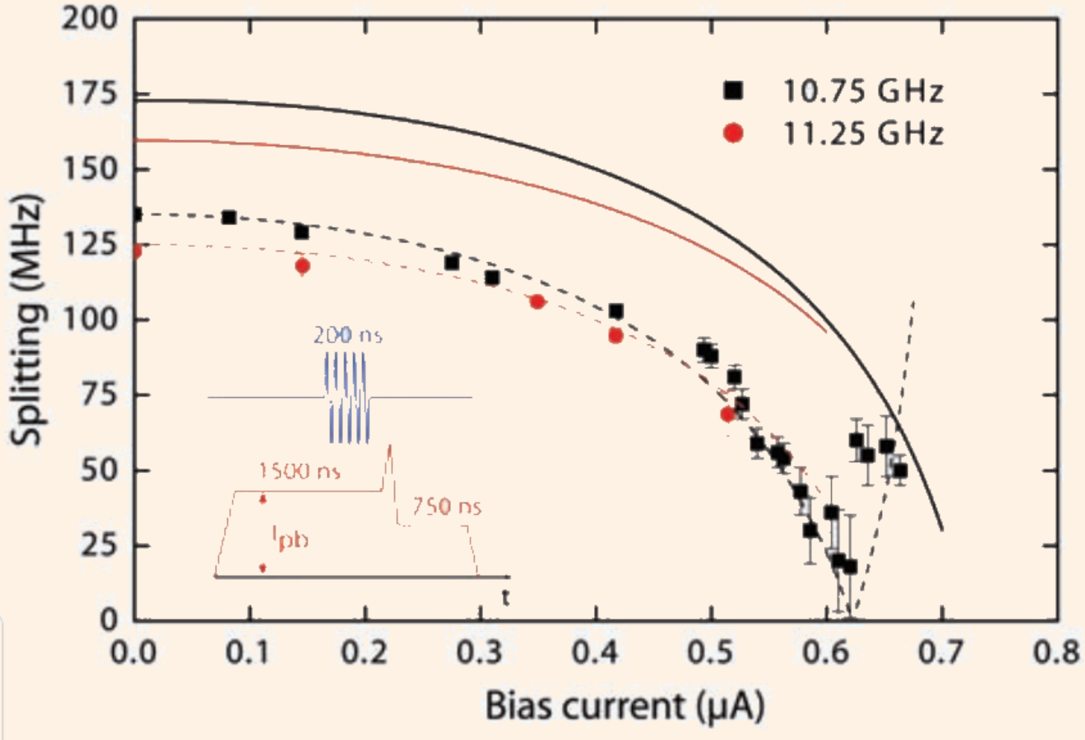
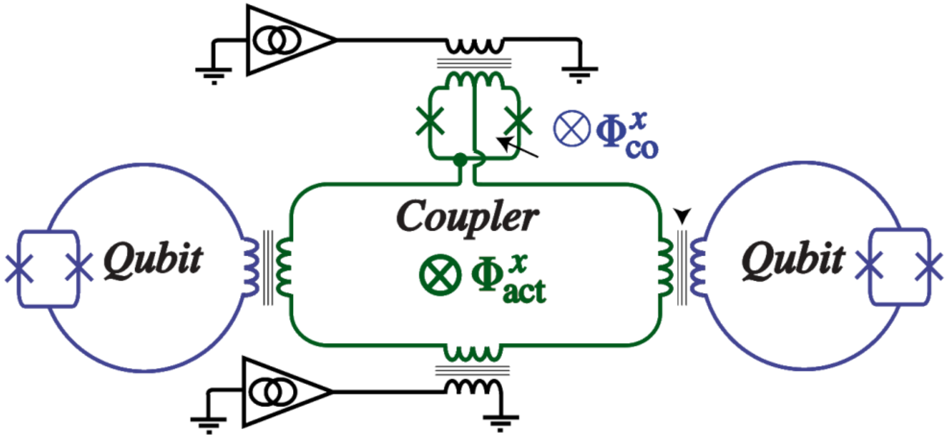
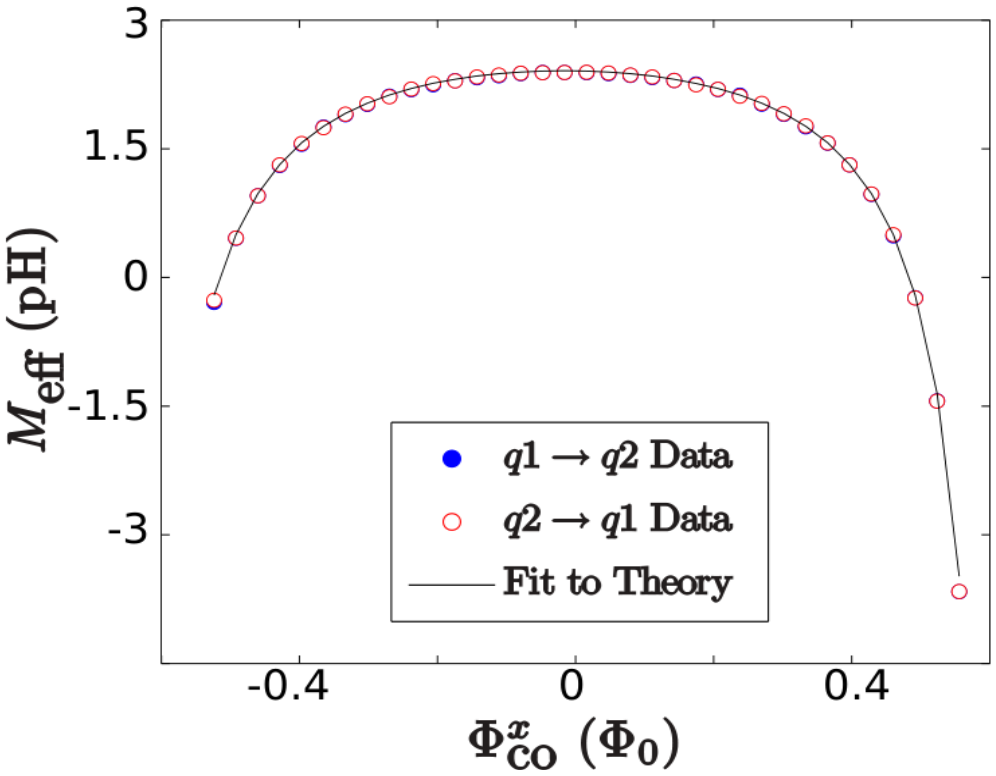
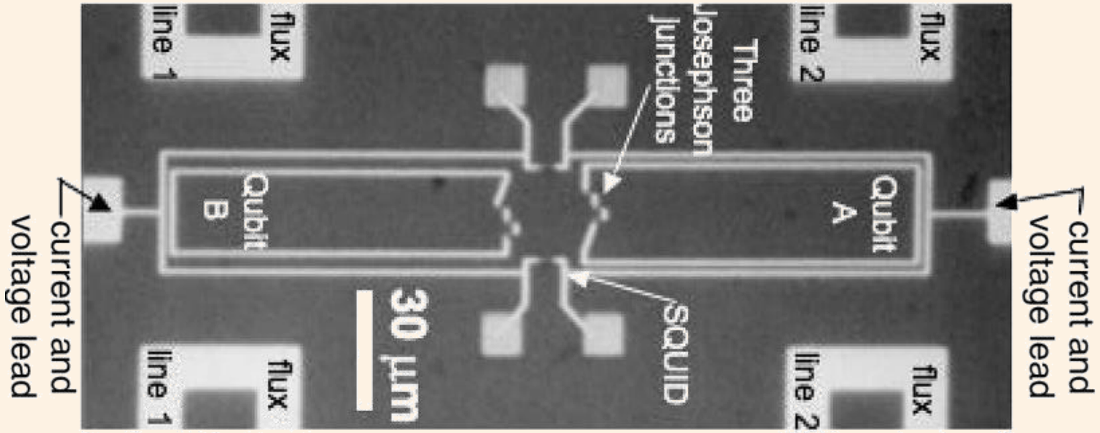


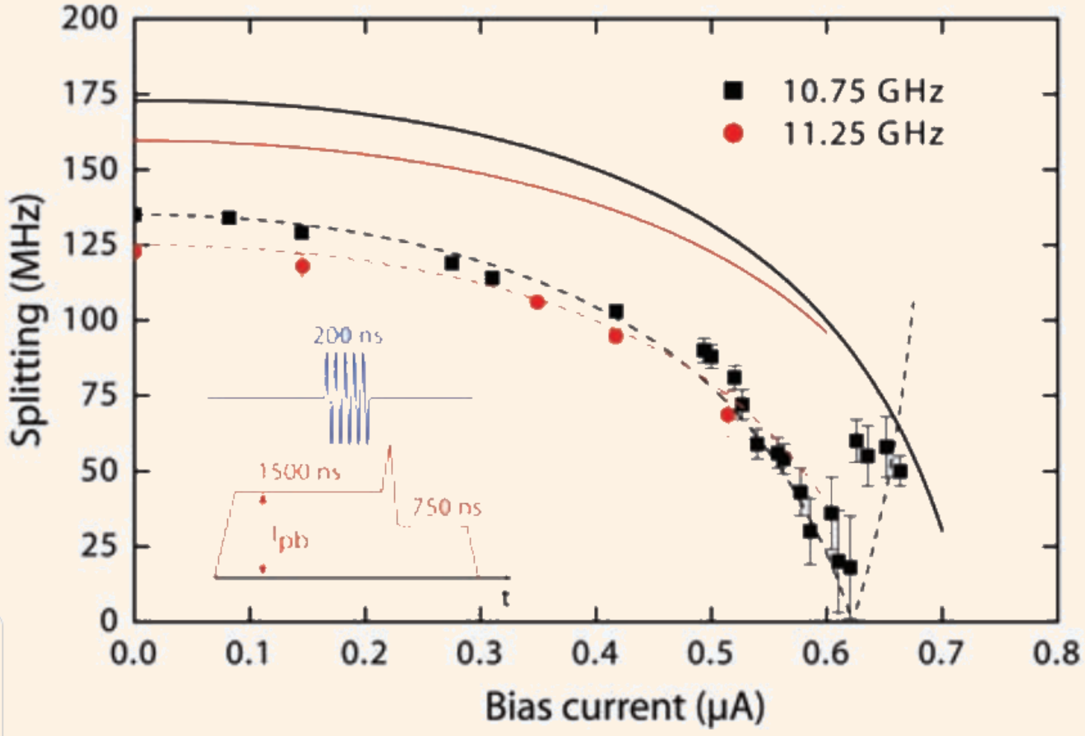


T. Hime et al, Science 314 (2006)









R. Harris et al PRB, 80(5), 052506 (2009)

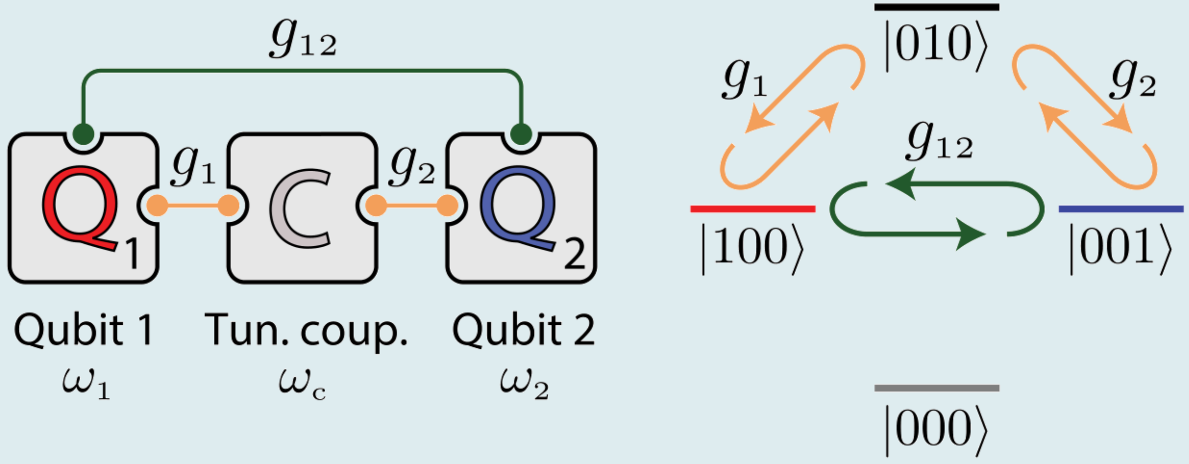
T. Hime et al, Science 314 (2006)

Excited state mediation

We can engineer a method of mediation to carry excitations via an off-resonant system.

- A and B are coupled to C
- A photon in A/B has not enough energy to excite C.
- Perturbation theory enables a coupling A <> B mediated by virtual excitations of C

It is a different type of "adiabatic" approximation, described by Schrieffer-Wolff transformations.



Fei Yan et al, PRAppl 10, 054062 (2018)

Excited state mediation

Original model

$$H_{\text{eff}} \simeq \frac{\omega_1}{2}\sigma_1^z + \frac{\omega_2}{2}\sigma_2^z + \frac{\omega_c}{2}\sigma_c^z$$
$$+\sum_i g_i \sigma_i^+ \sigma_c^- + g_{12}\sigma_2^+\sigma_1^- + \text{H.c.}$$

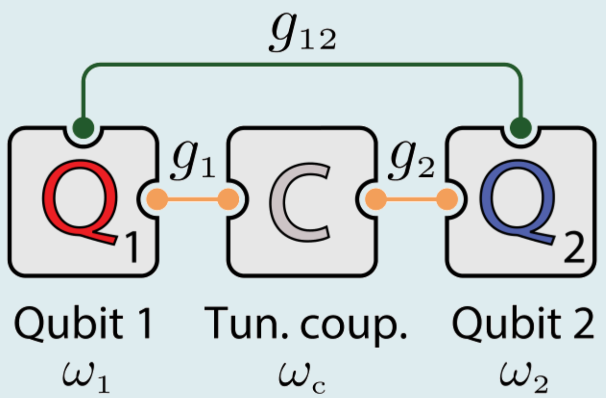
Perturbative parameter:

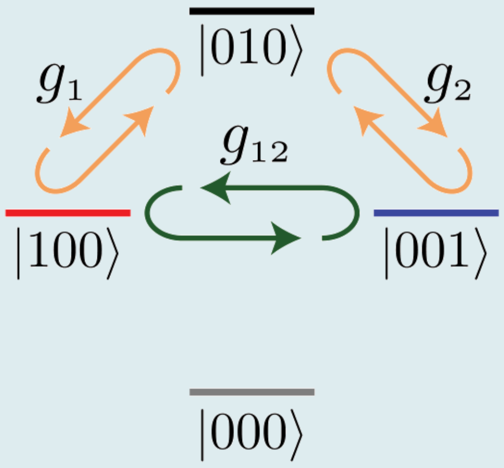
$$g_1, g_2 \ll |\omega_c - \omega_{1,2}| = \delta_{1,2}$$

We expect

$$H_{\text{eff}} \simeq H_1 + H_2 + g_{\text{eff}}(\sigma_1^+\sigma_2^- + \sigma_2^+\sigma_1)$$

with an effective exchange g_{eff}





Fei Yan et al, PRAppl 10, 054062 (2018)

Perturbative calculation

The transformed Hamiltonian is usually studied perturbatively

$$H_{\text{tot}} \simeq \underbrace{\frac{\omega_1}{2}\sigma_1^z + \frac{\omega_2}{2}\sigma_2^z + \frac{\omega_c}{2}\sigma_c^z}_{H_0} + \underbrace{\sum_i (g_i \sigma_i^+ \sigma_c^- + \text{H.c.})}_{V_{od}} + \underbrace{(g_{12}\sigma_2^+ \sigma_1^- + \text{H.c.})}_{V_d}$$

Generator “S” chosen to cancel terms that take outside computational subspace.

$$[S, H_0] = -V_{od}$$

$$[\sigma^{\pm}, \sigma^z] = \mp 2 \Rightarrow S = \sum_i \frac{g_i}{\omega_i - \omega_c}(\sigma_i^+ \sigma_c^- - \sigma_i^- \sigma_c^+)$$

Perturbative calculation

The transformed Hamiltonian is usually studied perturbatively

$$H_{\mathrm{eff}} \simeq H_0 + V_d + \frac{1}{2}[S, V]$$

Final model

$$H_{\mathrm{eff}} \simeq \frac{1}{2}\sum_i \left(\omega_i + \frac{g_i^2}{\delta_i}\right)\sigma_i^z + \left(\frac{1}{2}\frac{g_1 g_2}{\delta_1} + \frac{1}{2}\frac{g_1 g_2}{\delta_2} + g_{12}\right)(\sigma_2^+\sigma_1^- + \sigma_1^+\sigma_2^-)$$

With the detuning

$$\delta_i = \omega_c - \omega_i$$

With transmons

For transmons we know the couplings

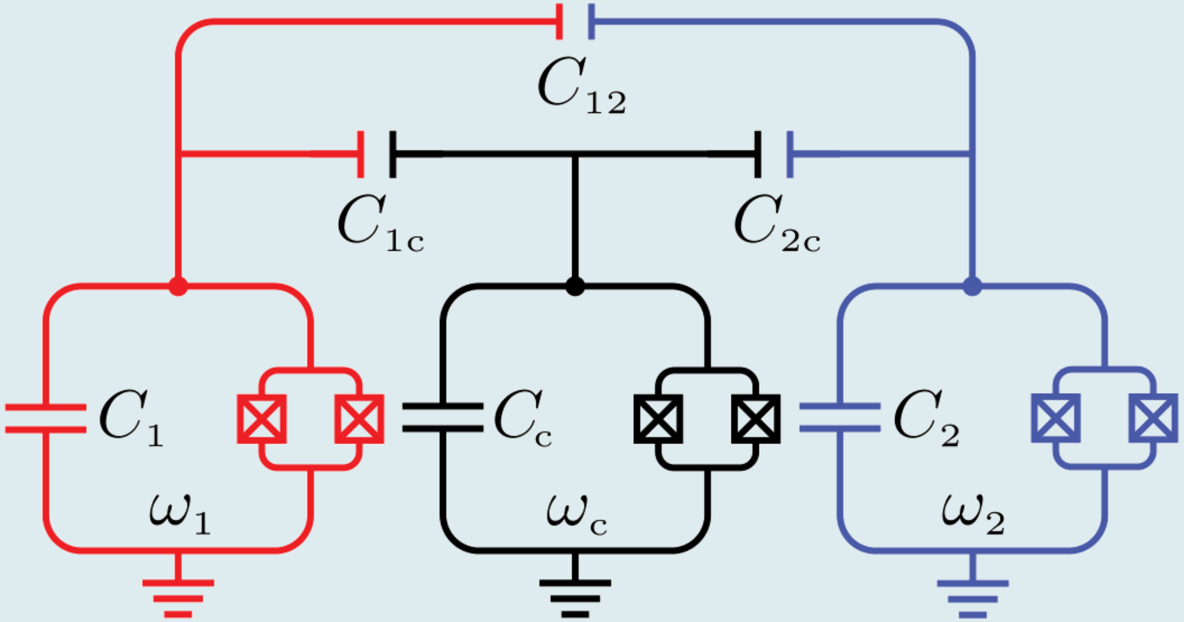
$$g_i \simeq \frac{1}{2}\frac{C_{jc}}{\sqrt{C_j C_c}}\sqrt{\omega_1 \omega_c},$$

$$g_{12} \simeq \frac{1}{2}\frac{C_{12}}{\sqrt{C_1 C_2}} + \frac{g_1 g_2}{\omega_c}$$

Interactions can be mediated by the 010 and 111 states

$$g_{\text{eff}} \simeq \left[1 + \eta\left(1 + \omega_c\left(\frac{1}{2\Delta} - \frac{1}{2\Sigma}\right)\right)\right]\frac{1}{2}\frac{C_{12}}{\sqrt{C_1 C_2}},$$

$$\frac{1}{\Delta} = \frac{1}{2}\sum_i \frac{1}{\omega_c - \omega_1}, \frac{1}{\Sigma} = \frac{1}{2}\sum_i \frac{1}{\omega_c + \omega_i}$$



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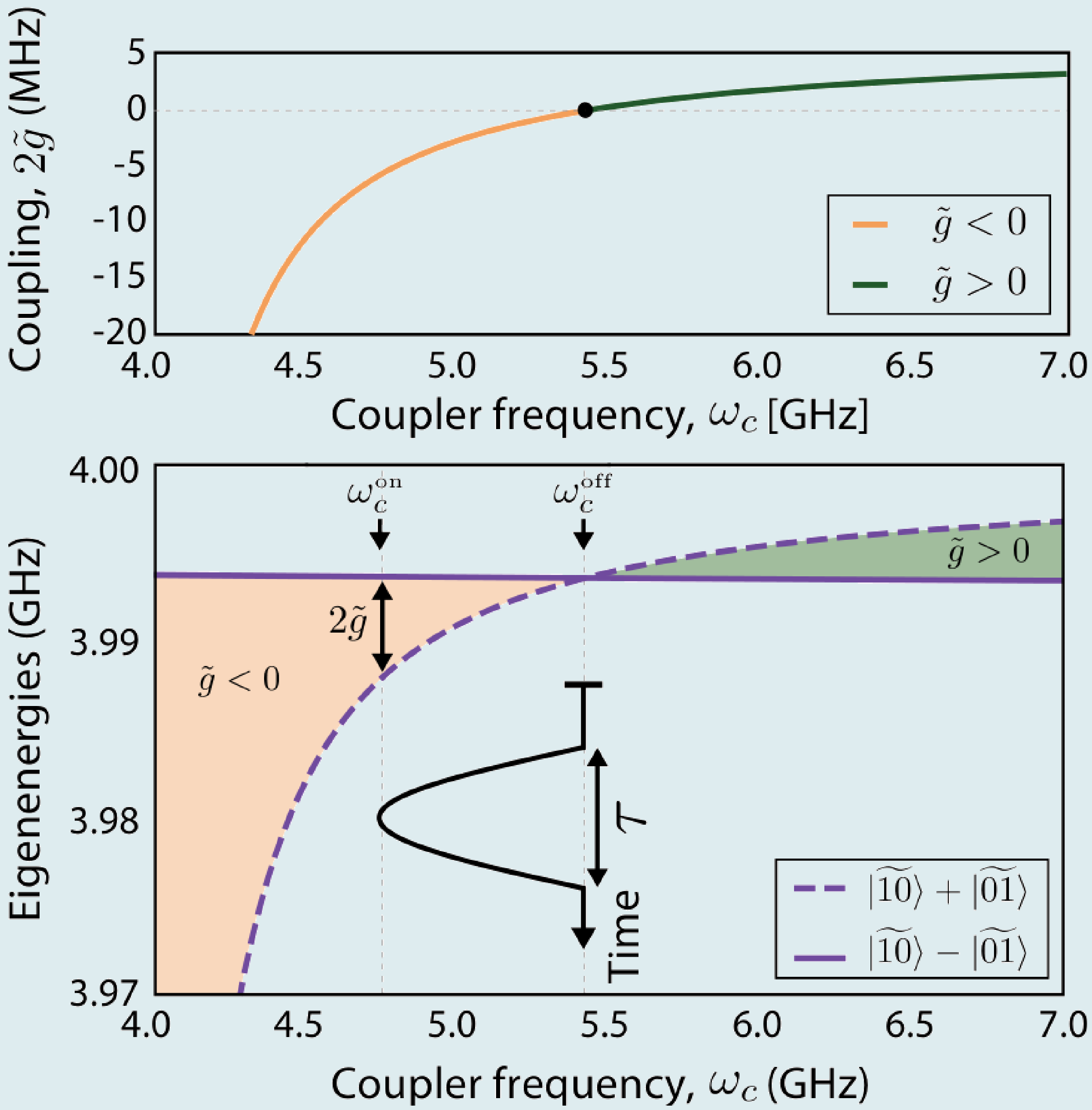


Fei Yan et al, PRAppl 10, 054062 (2018)

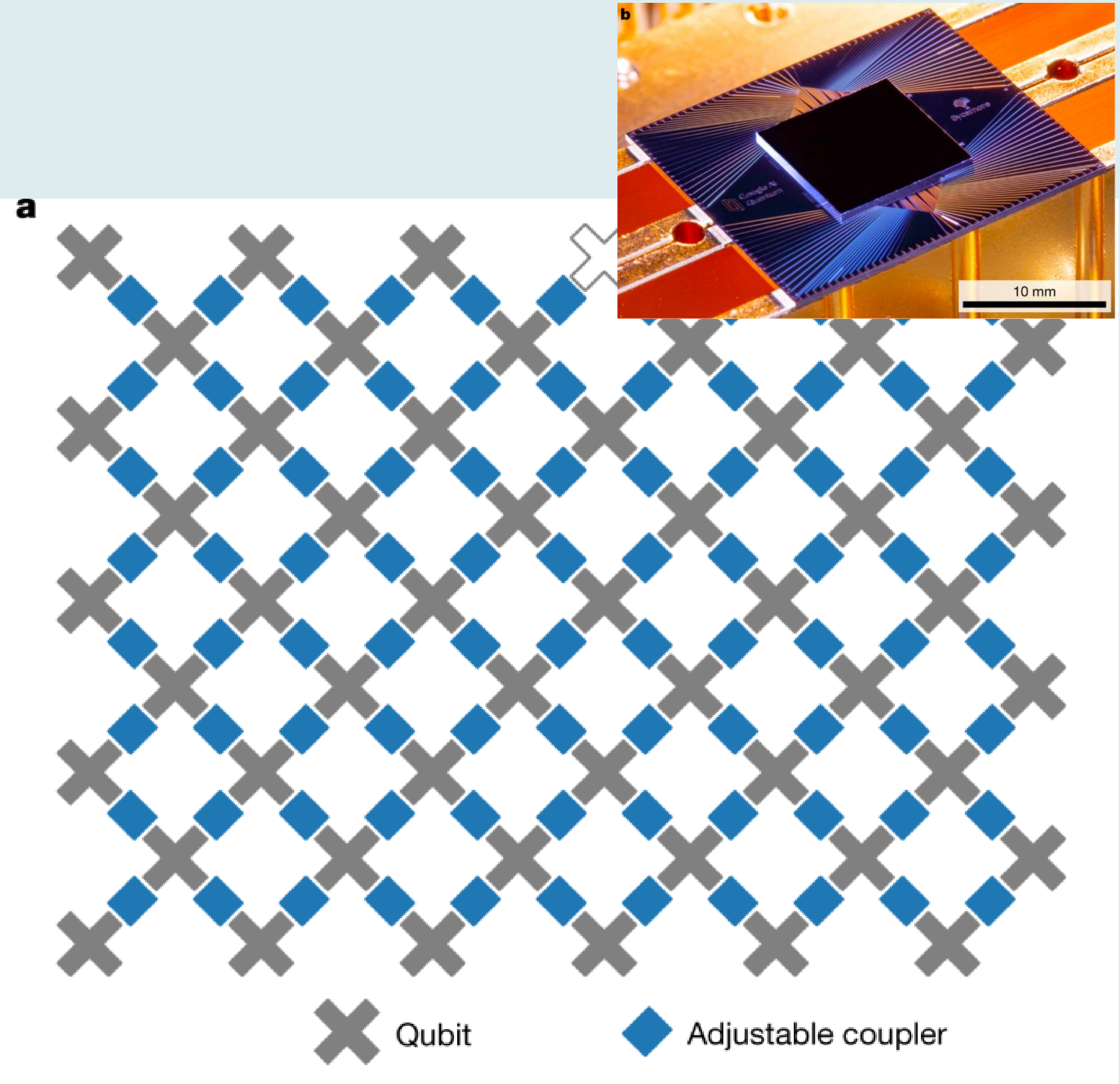
With transmons

Actually, also excited states 200 and similar must be considered

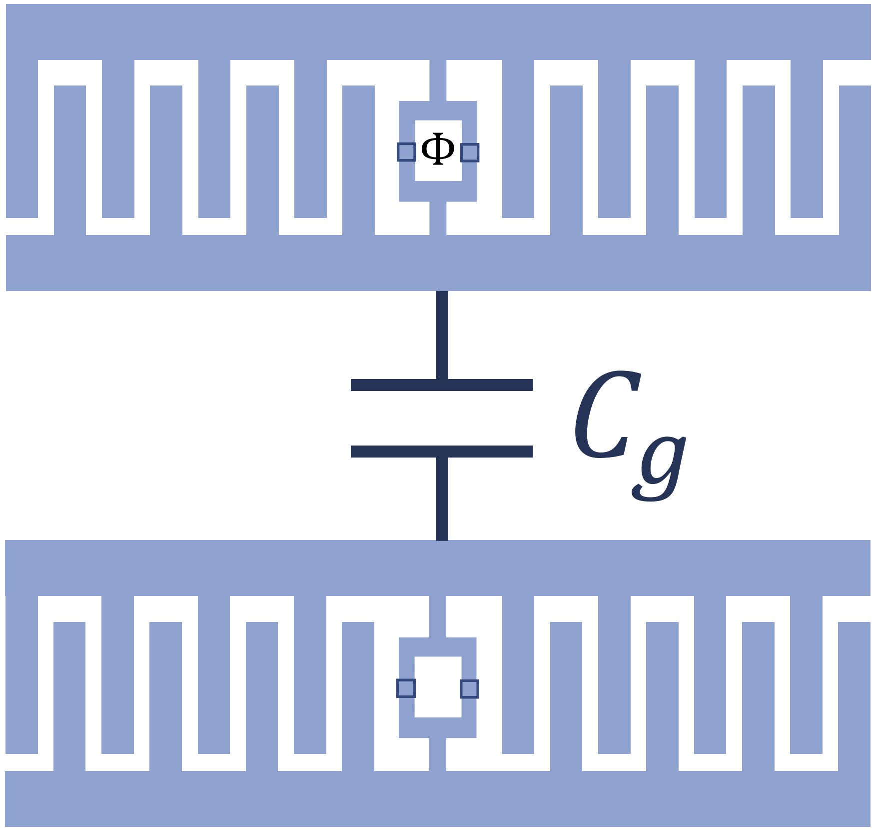
$$H = g_{12}(t)(\sigma_1^+\sigma_2^- + \sigma_2^+\sigma_1^-) +$$

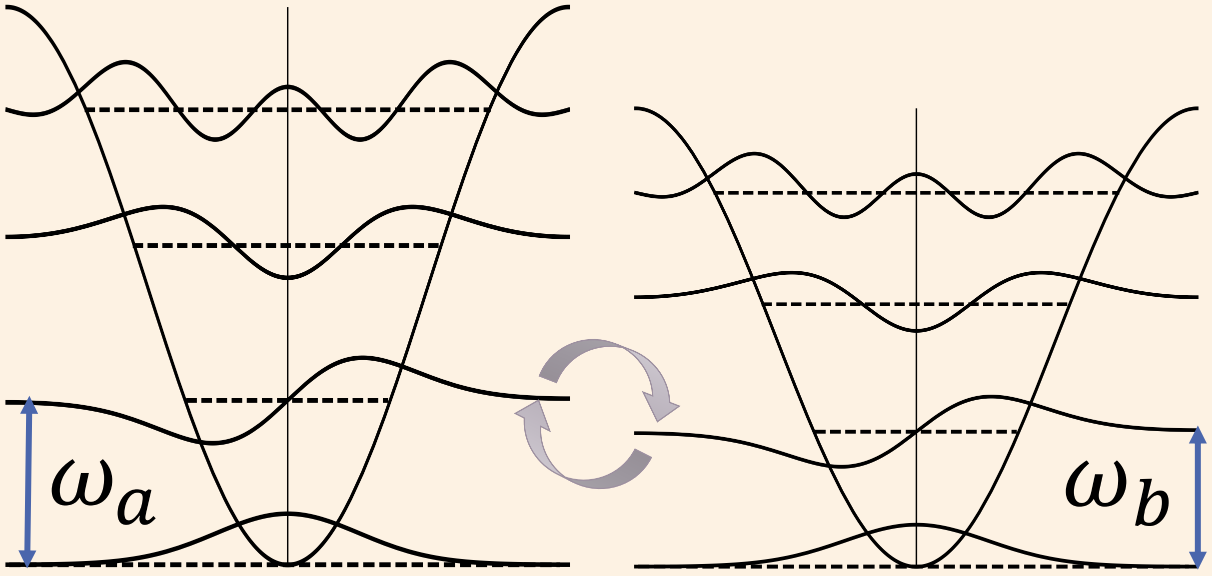
$$+\frac{g_{12}(t)^2}{\alpha}\sigma_1^z\sigma_2^z$$

This is a “fermionic”-like interaction that enables a broader family of quantum gates.

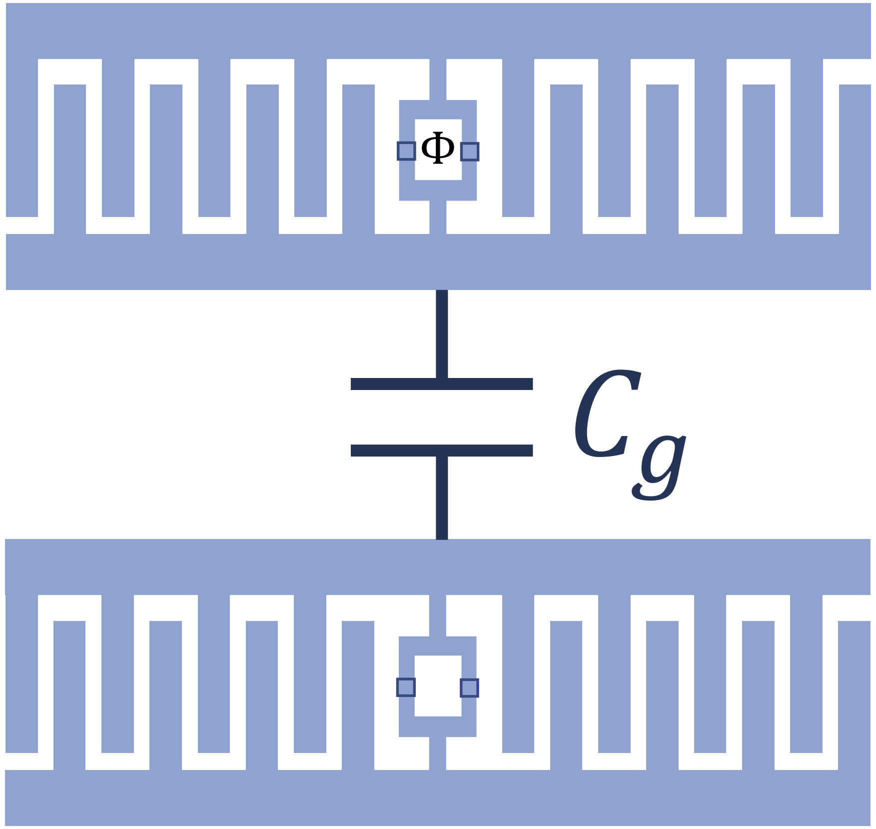


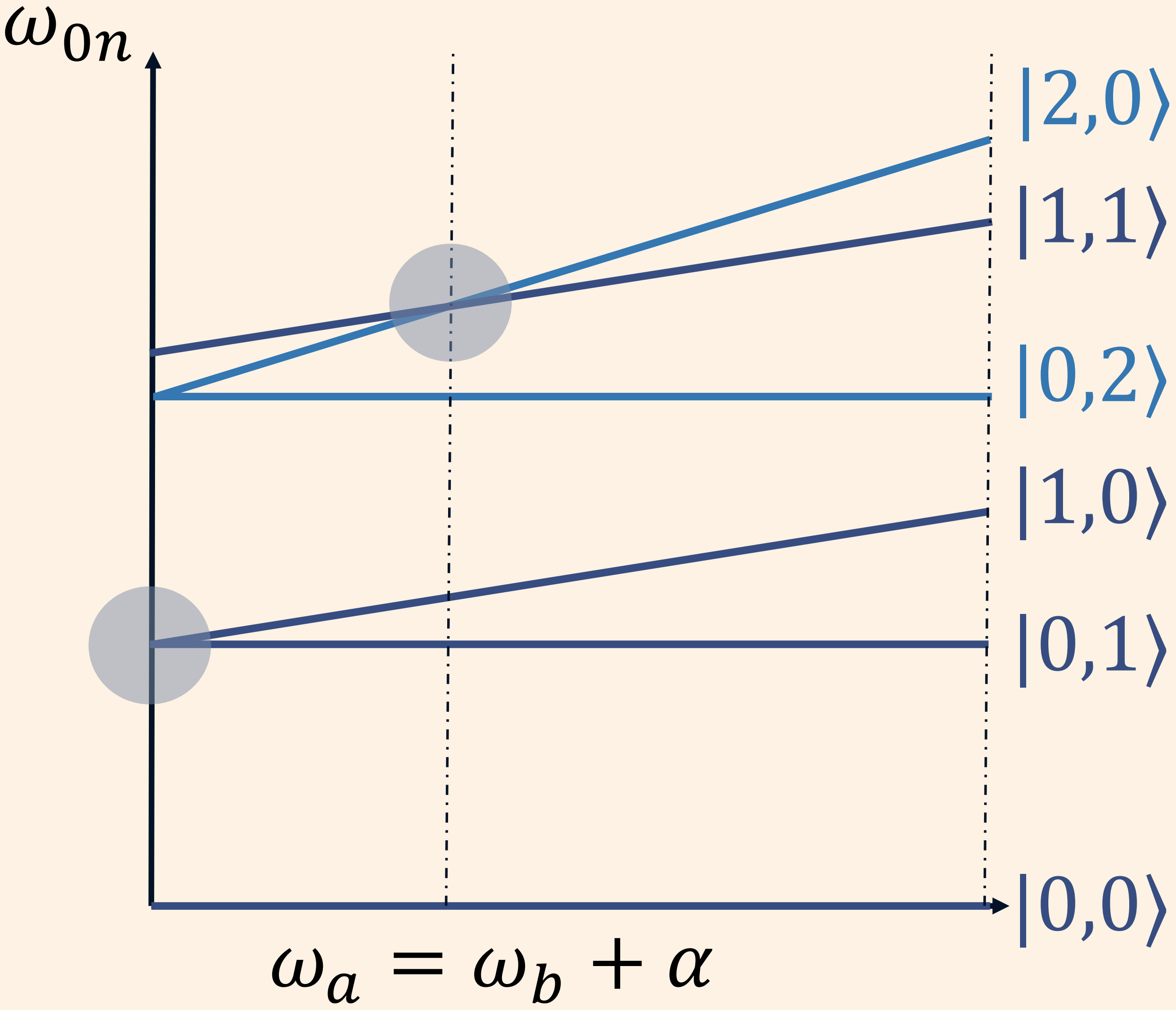
F. Arute et al Nature, 574 (2019)

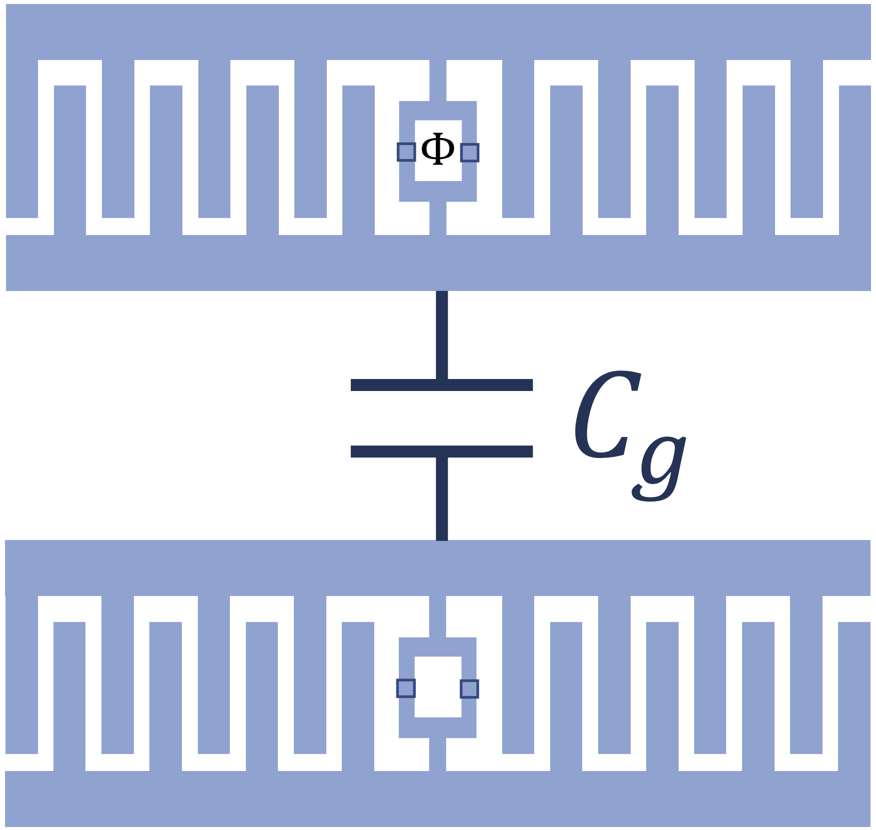


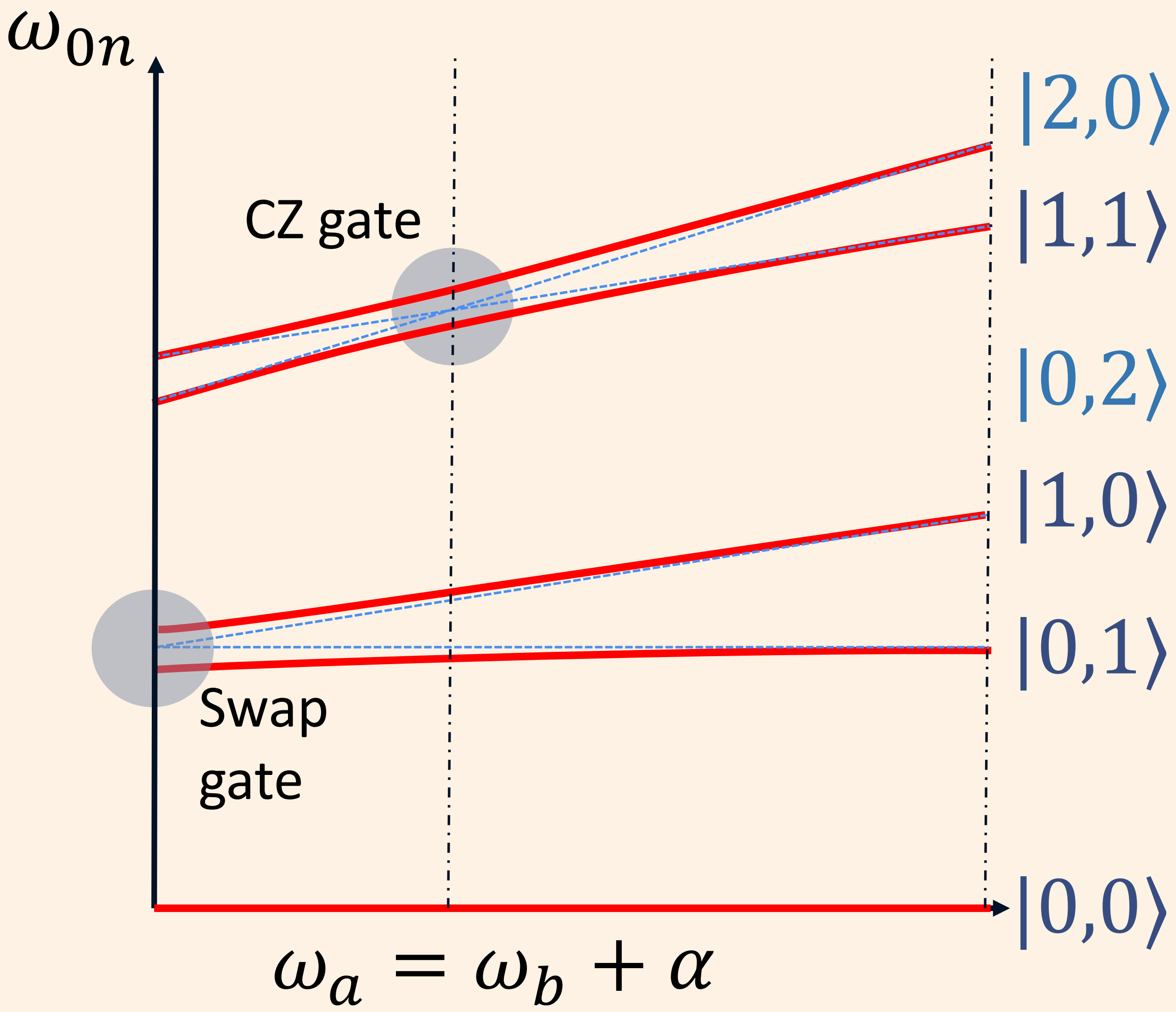


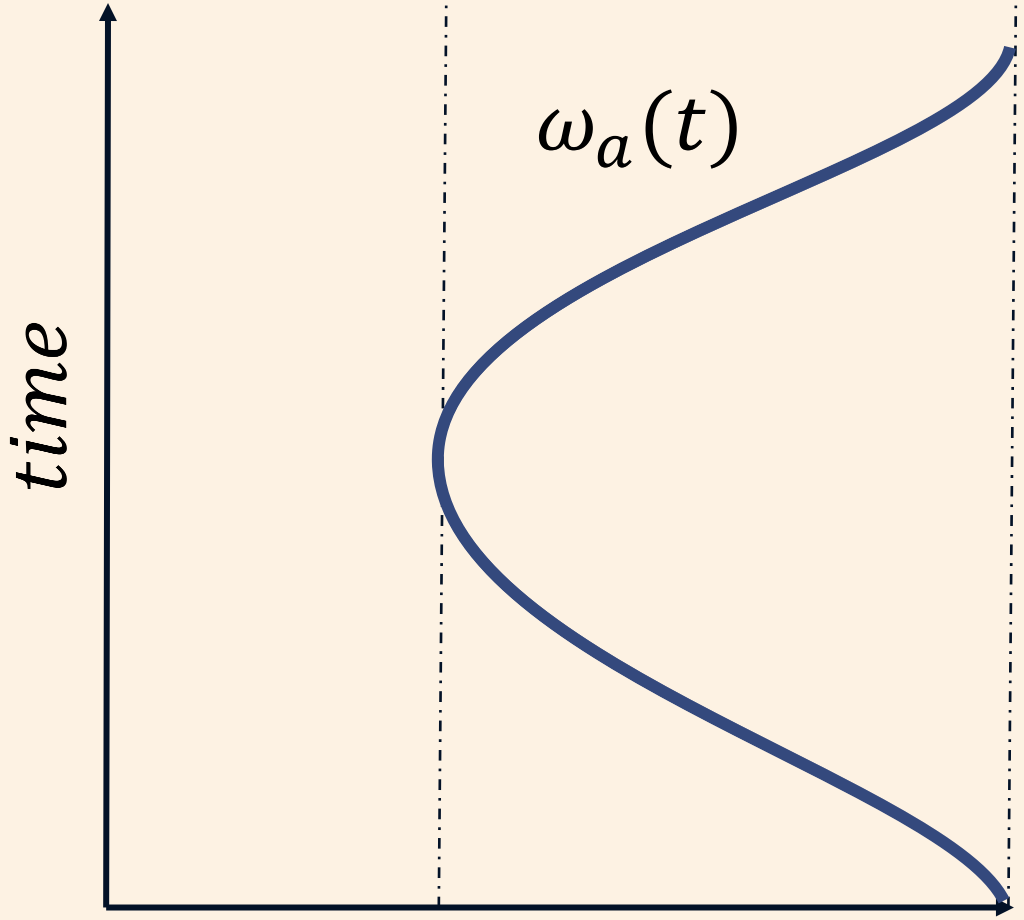
Interaction

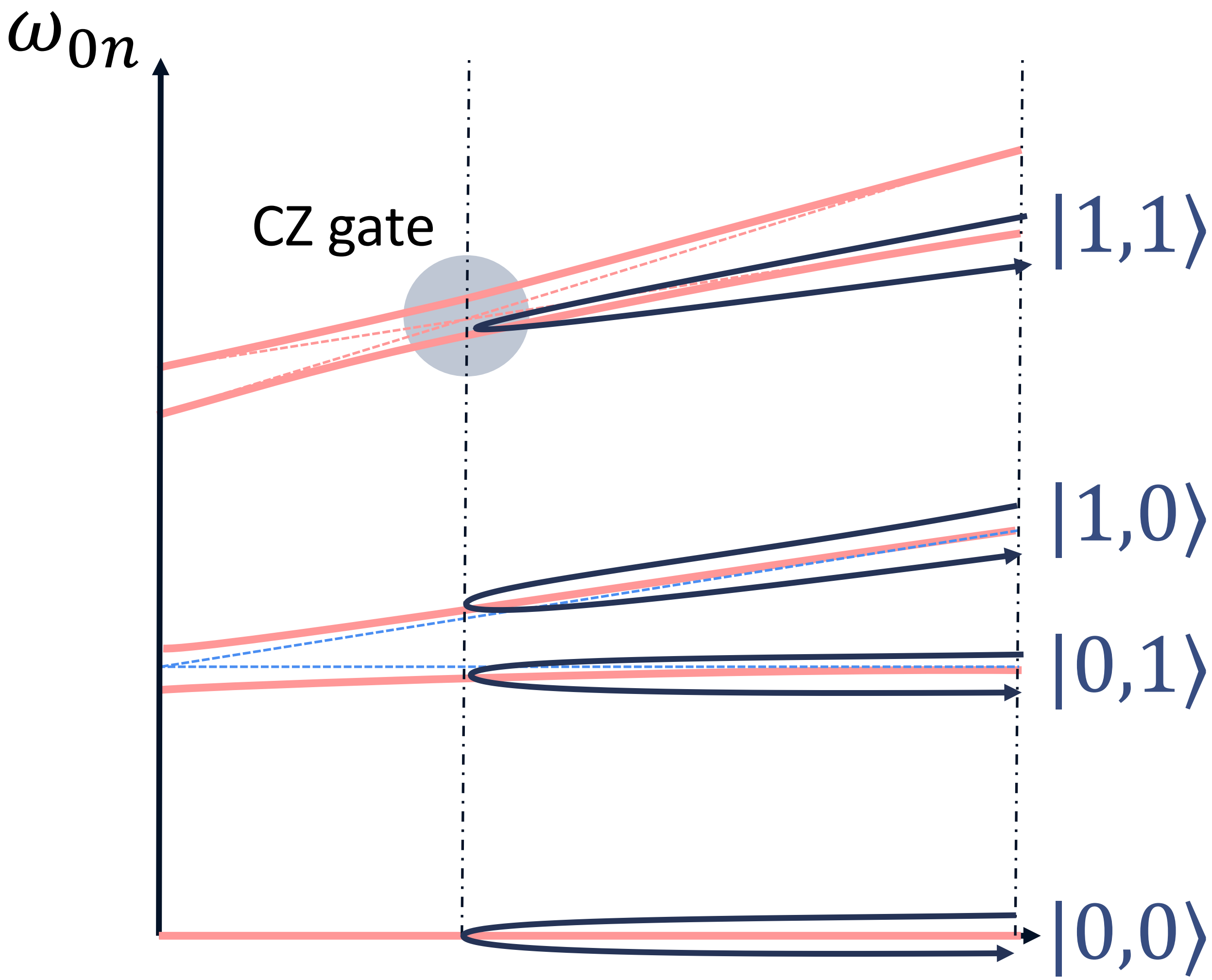

Φ
C_g


ω_{0n}
$|2,0\rangle$
$|1,1\rangle$
$|0,2\rangle$
$|1,0\rangle$
$|0,1\rangle$
$|0,0\rangle$
$\omega_a = \omega_b + \alpha$


Φ
C_g


ω_{0n}
CZ gate
Swap gate
$|2,0\rangle$
$|1,1\rangle$
$|0,2\rangle$
$|1,0\rangle$
$|0,1\rangle$
$|0,0\rangle$
$\omega_a = \omega_b + \alpha$





$$|11\rangle \rightarrow |11\rangle e^{i\phi_{12}}$$

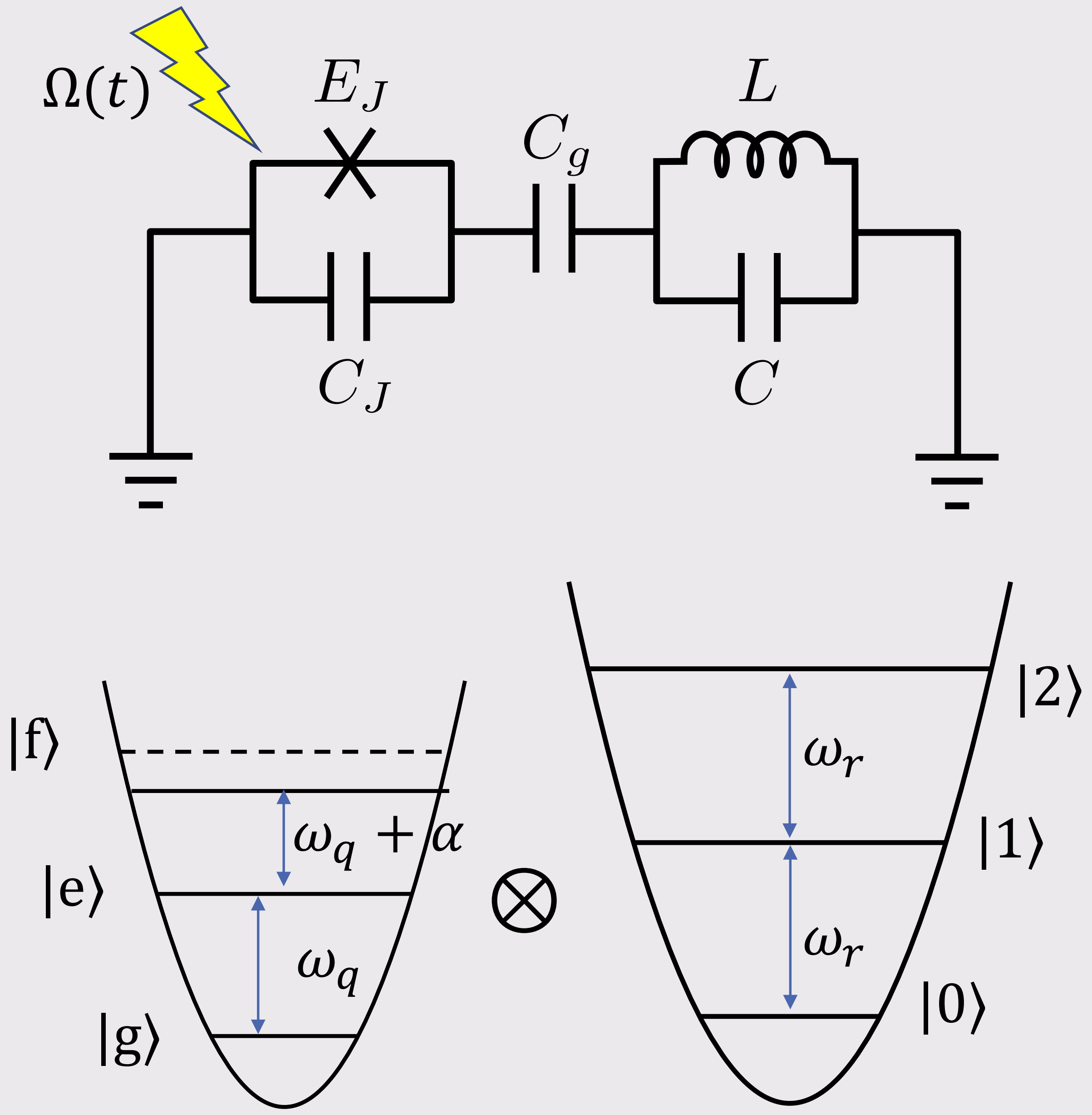
L. DiCarlo et al., Nature **460**, 240–244 (2009);
R. Barends et al., Nature **508**, 500–503 (2014);
M. Rol et al., PRL **123**, 120502 (2019);
C. K. Andersen et al, arXiv:1912.09410v1

Active coupling

So far, all coupling elements were passive: no external “drive” to activate them.

We will now study couplers that use **microwaves to activate** energetically forbidden transitions.

The goal will be to establish a link between the qubit and the resonator, for extracting qubits as photons.



M. Pechal PRX 4, 041010 (2014)

The model

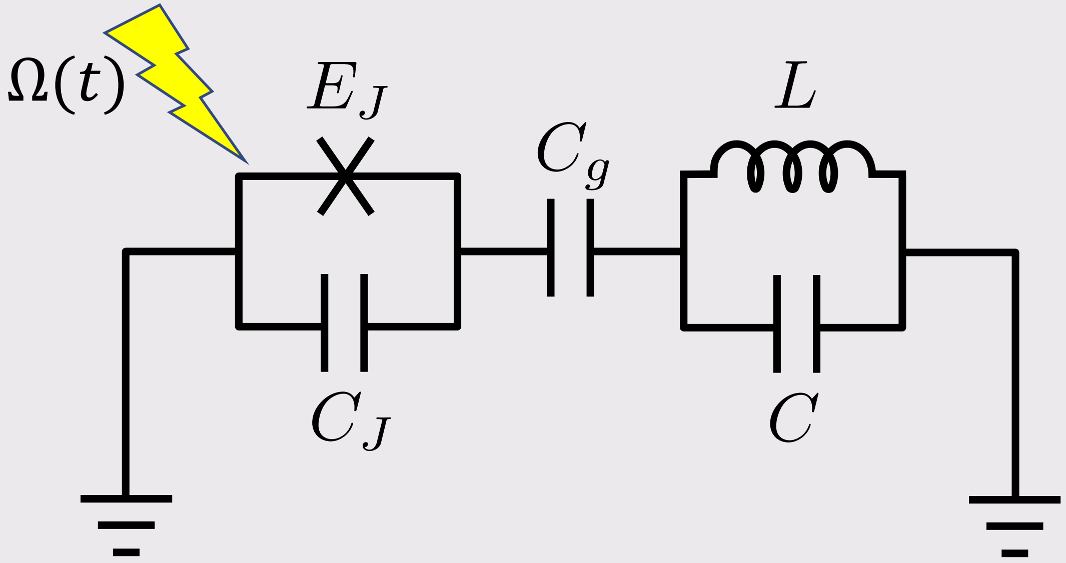
We model the qubit as an anharmonic oscillator

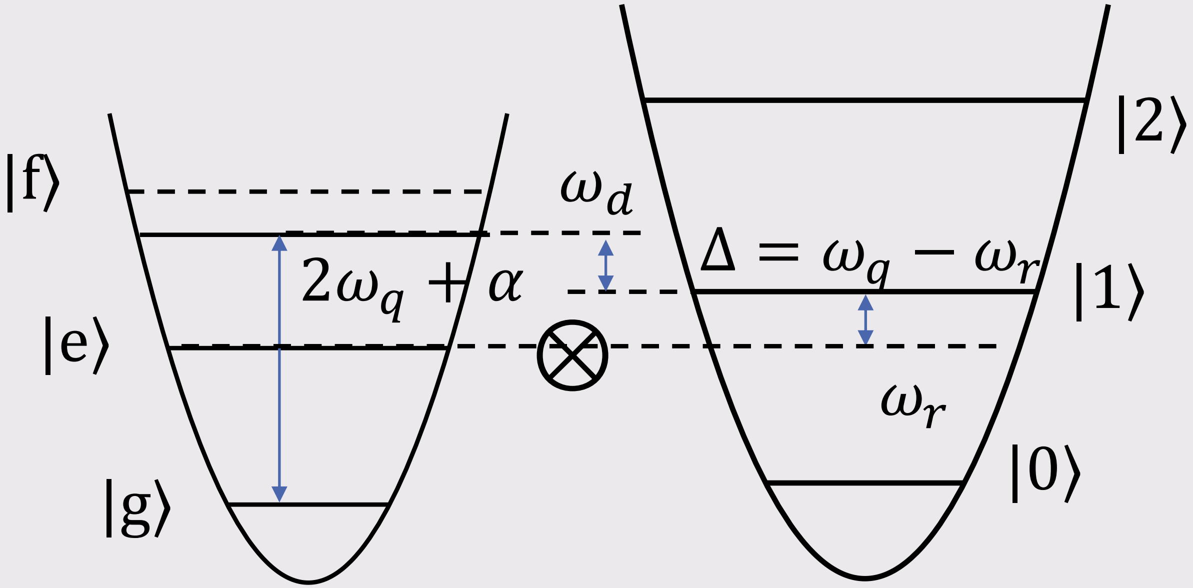
$$H = \omega_r \hat{a}^{+}\hat{a} + \omega_q \hat{b}^{+}\hat{b} + \frac{1}{2}\alpha \hat{b}^{+}\hat{b}^{+}\hat{b}\hat{b}$$
$$+g(\hat{b}^{+} + \hat{b})(\hat{a} + \hat{a}^{+})$$
$$+\Omega_0 \cos(\omega_d t + \phi)\left(\hat{b}^{+} + \hat{b}\right)$$

We can apply a RWA and move to a rotating frame which oscillates with frequency

$$\omega_d = 2\omega_q + \alpha - \omega_r$$

This energy bridges the gap between the “f” and “1” states.





M. Pechal PRX 4, 041010 (2014)

RWA

After RWA

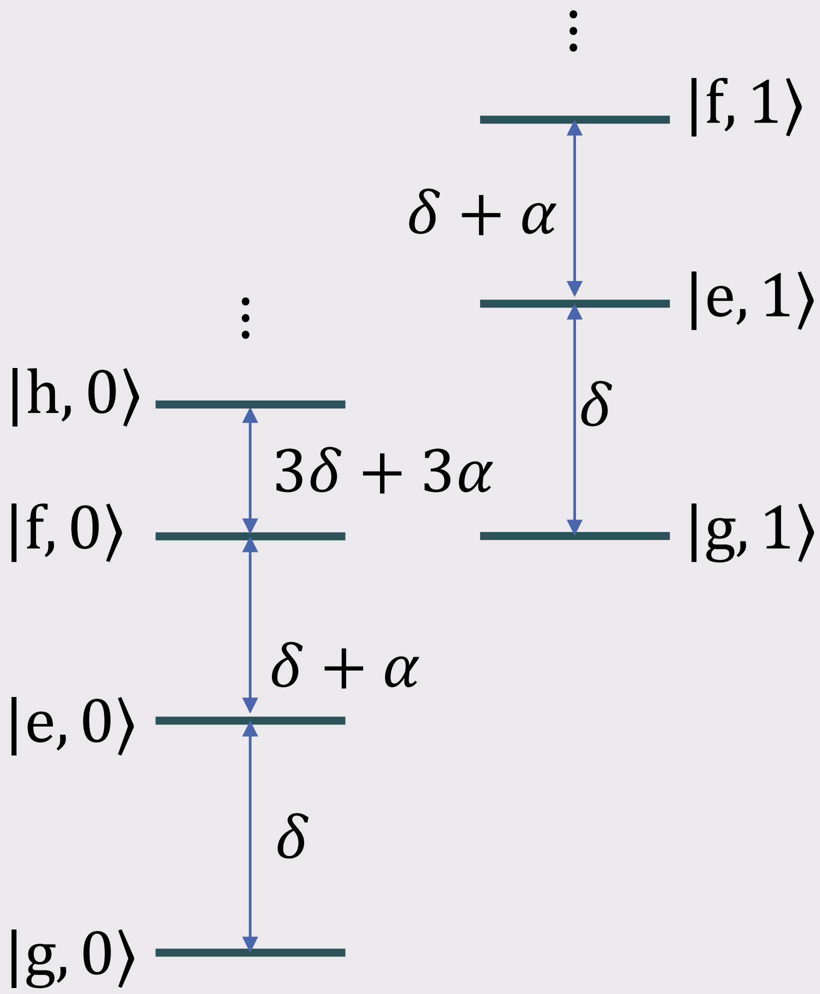
$$H = (2\delta + \alpha)\hat{a}^{+}\hat{a} + \delta\hat{b}^{+}\hat{b} + \frac{1}{2}\alpha\hat{b}^{+}\hat{b}^{+}\hat{b}\hat{b}$$

$$+g(\hat{b}^{+} + \hat{b})(\hat{a} + \hat{a}^{+})$$

$$+\Omega_0\left(e^{-i\phi}\hat{b}^{+} + e^{i\phi}\hat{b}\right)$$

with

$$\delta = \omega_r - \omega_q - \alpha$$



M. Pechal PRX 4, 041010 (2014)

RWA

After RWA

$$H = (2\delta + \alpha)\hat{a}^{+}\hat{a} + \delta\hat{b}^{+}\hat{b} + \frac{1}{2}\alpha\hat{b}^{+}\hat{b}^{+}\hat{b}\hat{b}$$
$$+g(\hat{b}^{+} + \hat{b})(\hat{a} + \hat{a}^{+})$$
$$+\Omega_0\left(e^{-i\phi}\hat{b}^{+} + e^{i\phi}\hat{b}\right)$$

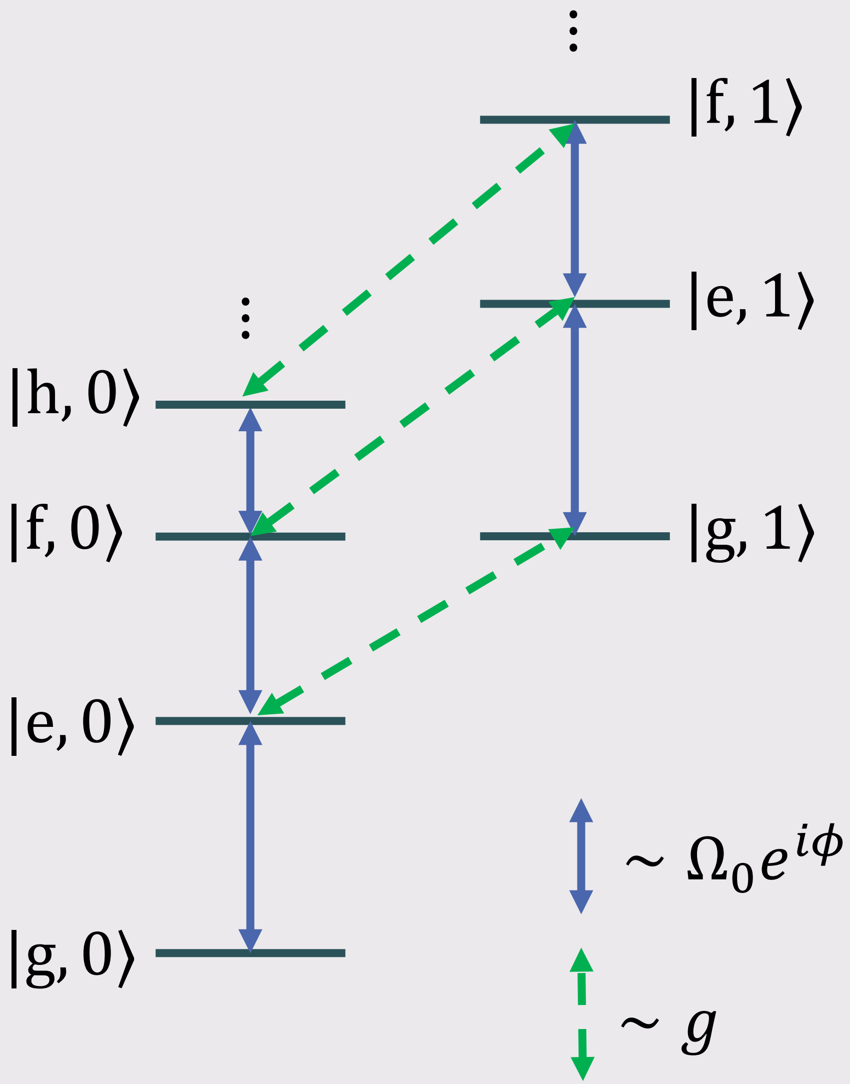
with

$$\delta = -\Delta - \alpha$$

The states $|\mathrm{f}, 0\rangle$ and $|g, 1\rangle$ are resonant and we expect them to be coupled by 2nd order processes

$$|f, 0\rangle \rightarrow |e, 0\rangle \rightarrow |g, 1\rangle$$
$$|f, 0\rangle \rightarrow |e, 1\rangle \rightarrow |g, 1\rangle$$

All other states experience Stark shifts.



M. Pechal PRX 4, 041010 (2014)

Final result

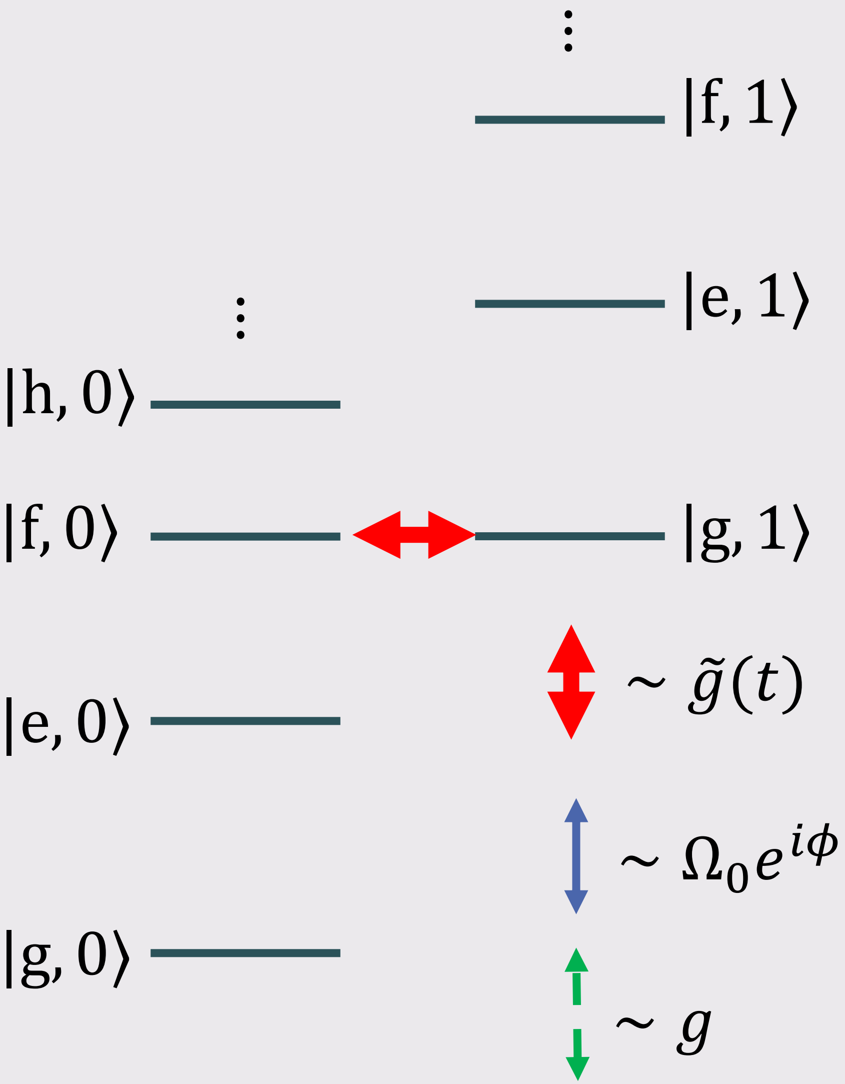
To 2nd order perturbation theory

$$H_{\mathrm{eff}} = \omega_f |f,0\rangle\langle f,0| + \tilde{g}(t)|f,0\rangle\langle g,1| + \mathrm{H.c.}$$

with

$$\tilde{g}(t) = \frac{1}{\sqrt{2}}\frac{g\alpha}{\delta(\delta+\alpha)}\Omega_0 e^{i\phi}$$

This coupling can be used to transfer the $|f,0\rangle$ state to a single-photon state $|g,1\rangle$.



M. Pechal PRX 4, 041010 (2014)

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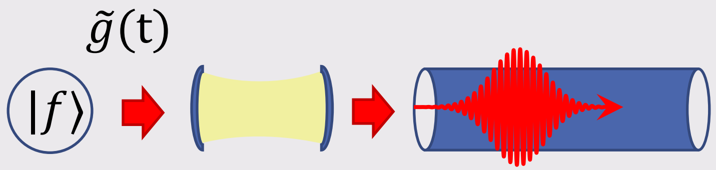
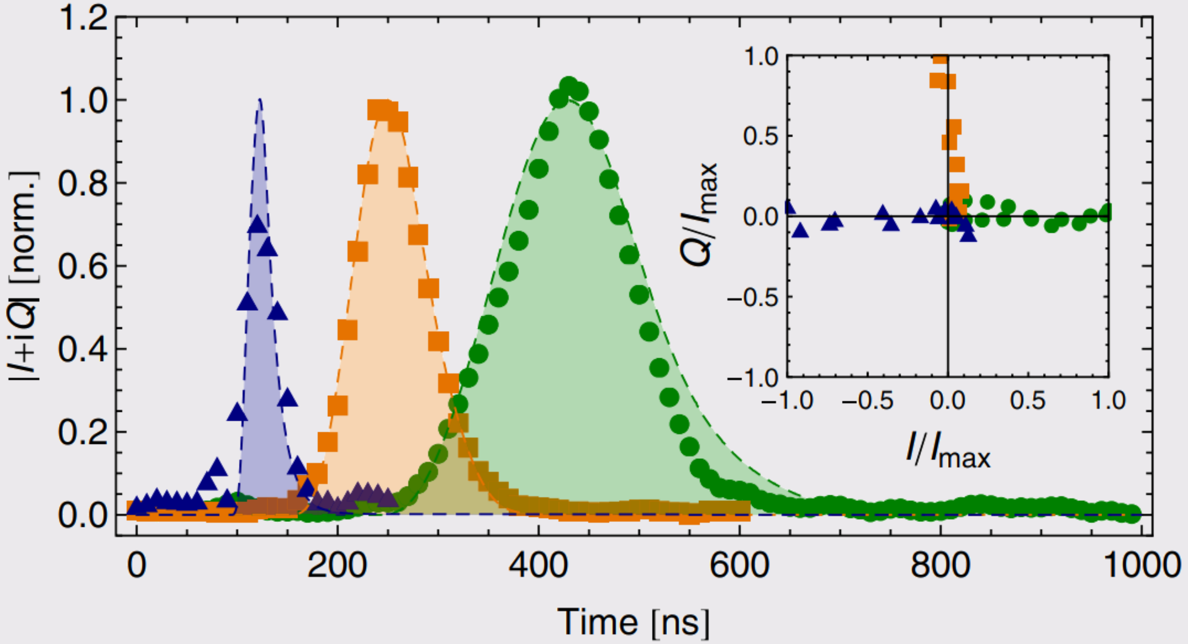
This coupling can be used to transfer the $|f,0\rangle$ state to a single-photon state $|g,1\rangle$.

This photon is released with amplitude $\propto$ $\Omega_0(t)$ and phase $\propto e^{i\phi}$ if these are slowly varying.





M. Pechal PRX 4, 041010 (2014)

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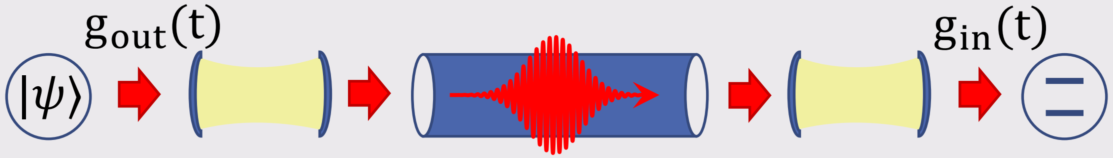
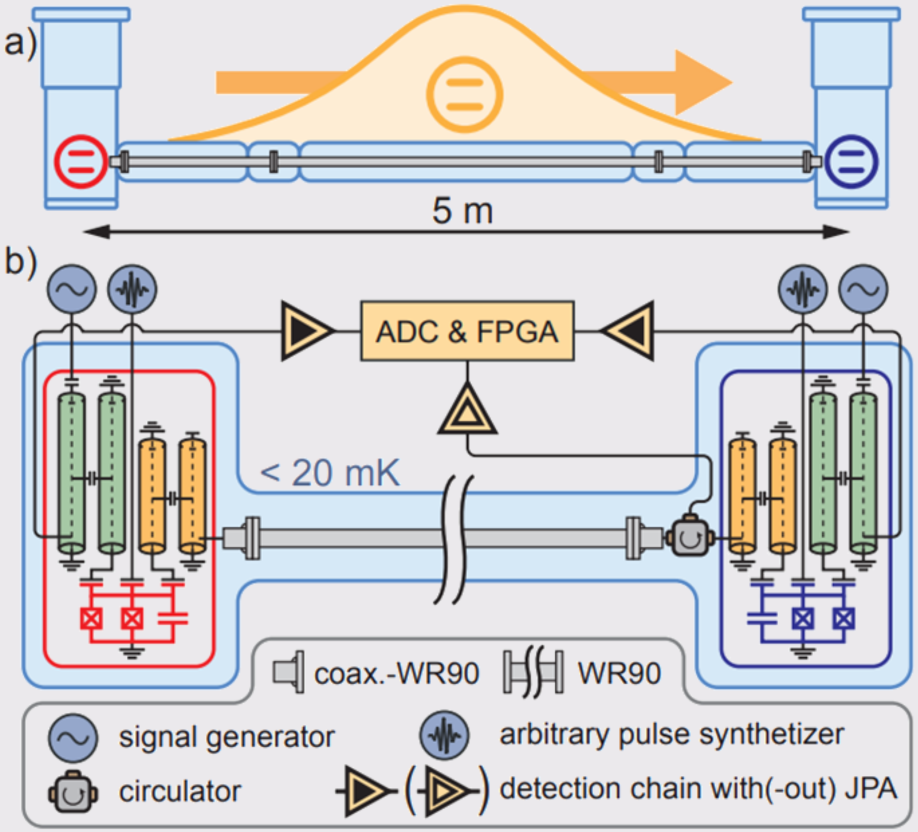
This coupling can be used to transfer the $|f,0\rangle$ state to a single-photon state $|g,1\rangle$.

This photon is released with amplitude $\propto \Omega_0(t)$ and phase $\propto e^{i\phi}$ if these are slowly varying.

Placing a similar control at the other end of a waveguide, we can engineer perfect state transfer.



$$|\psi\rangle|g\rangle|vac\rangle \rightarrow |g\rangle|\psi\rangle|vac\rangle$$



K. Reuer et al arXiv:2106.03481

P. Magnar et al PRL 125, 260502 (2020

Quantum links

Parametric driving

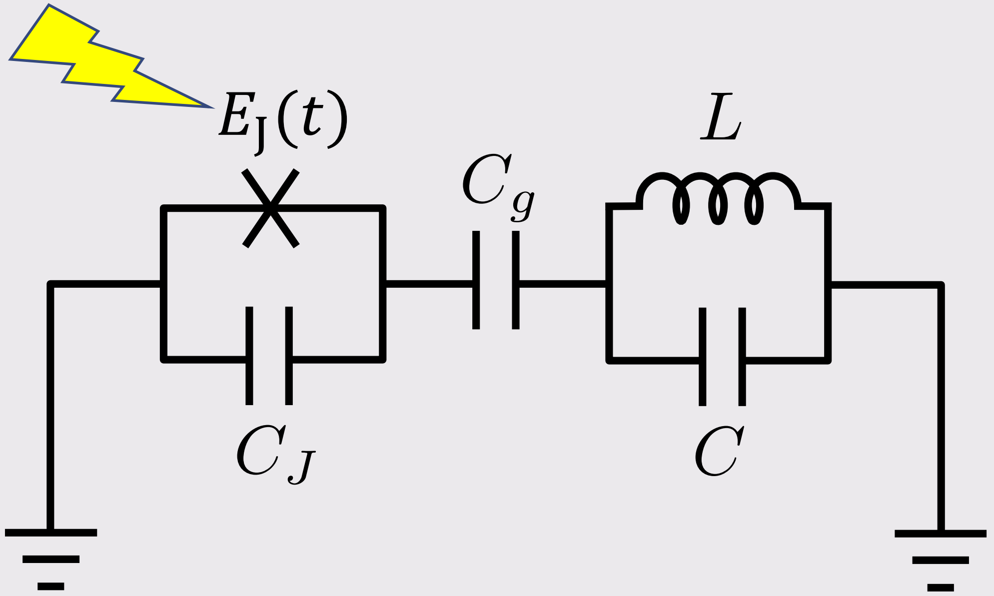
Microwave drives are not the only approach to supply energy.

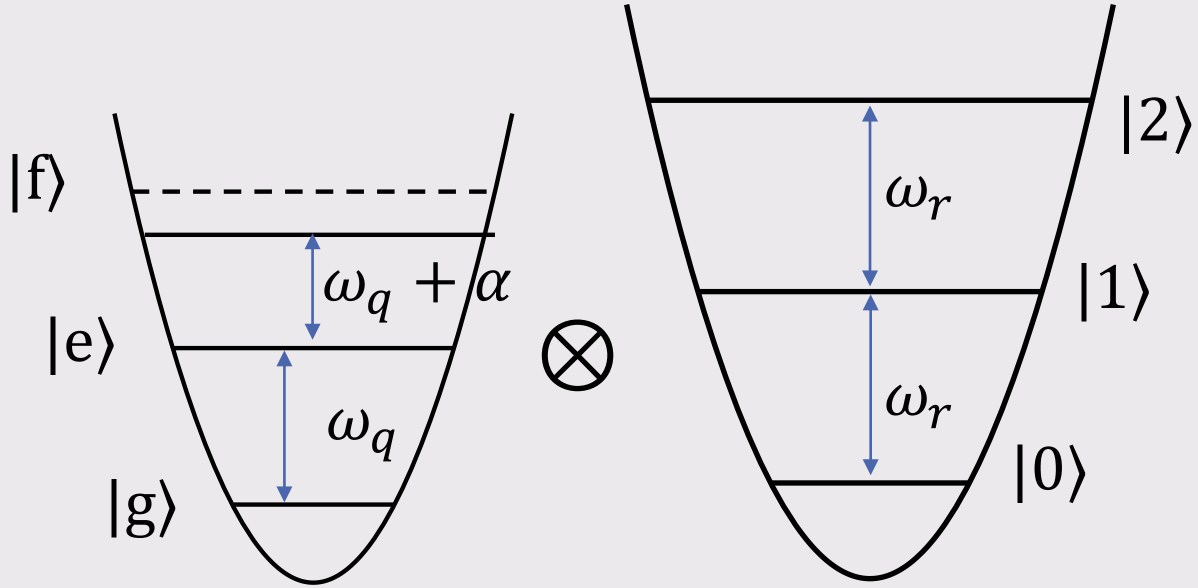
We can also induce parametric resonances by “shaking” the intrinsic energies of one system

$$\omega_q(t) = \omega_q + \epsilon\cos(\omega_d t + \phi)$$

In the usual model

$$H = \omega_q(t)\hat{a}^+\hat{a} + \omega_r\hat{b}^+\hat{b}$$
$$+g(\hat{a} + \hat{a}^+)(\hat{b}^+ + \hat{b}) + \cdots$$





Change of frame

Starting point

$$H = [\omega_0 + \epsilon \cos(\omega_d t + \theta)]\hat{a}^+\hat{a} + \omega_r \hat{b}^+\hat{b} + g(\hat{a} + \hat{a}^+)\left(\hat{b}^+ + \hat{b}\right) + \frac{1}{2}\hat{a}^+\hat{a}^+\hat{a}\hat{a}$$

Rotating frame

$$U(t) = \exp(i\phi(t)\hat{a}^+\hat{a})$$

Effective dynamics

$$i\partial_t\psi = [U(t)^+ H U(t) - iU(t)^+\partial_t U(t)]\psi$$

We choose

$$U(t) = \exp(i\varepsilon \sin(\omega_d t + \theta)\,/\omega_d)$$

Change of frame

Effective model

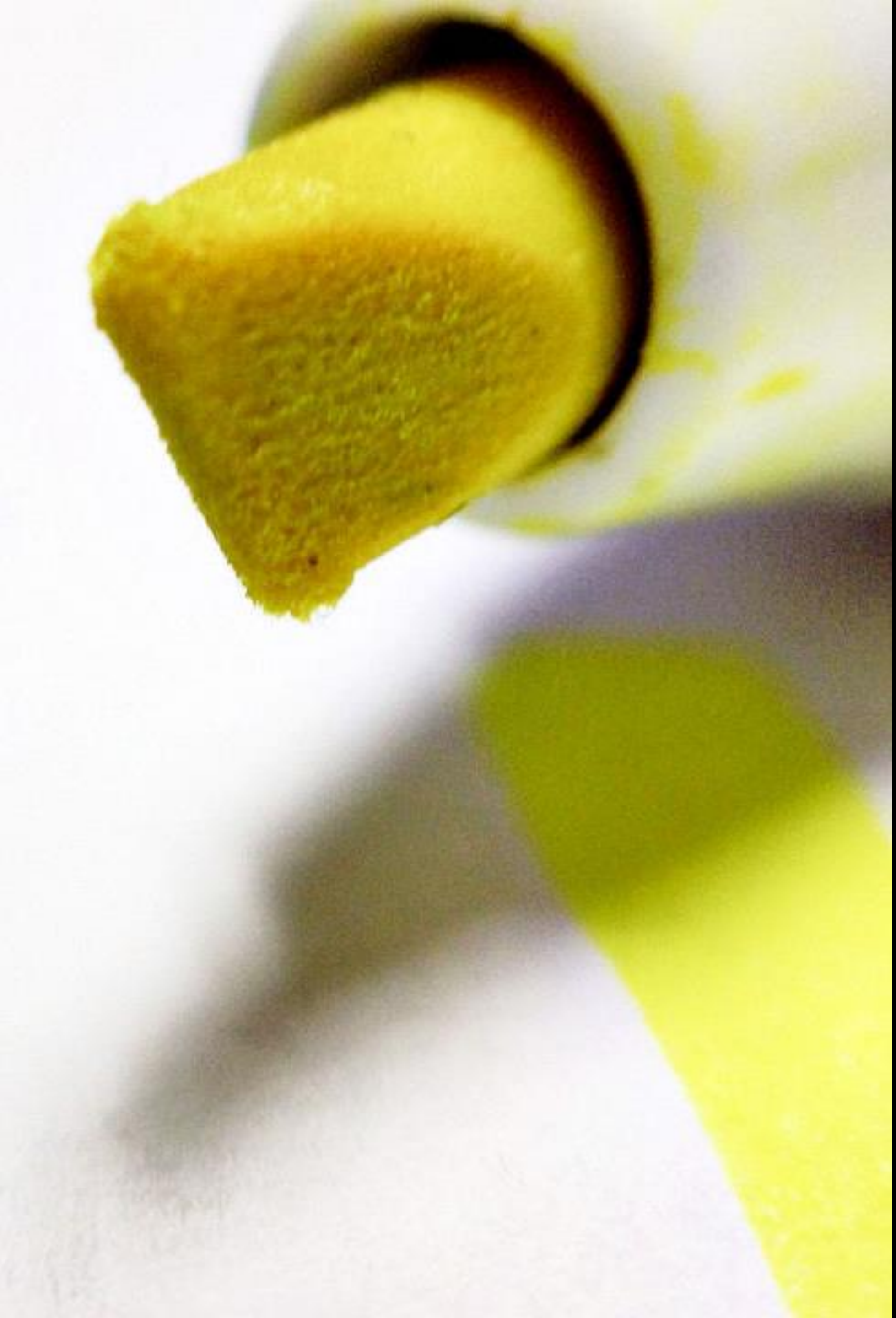
$$H = \omega_0 \hat{a}^+ \hat{a} + \omega_r \hat{b}^+ \hat{b} + g\left(e^{-i\phi}\hat{a} + e^{i\phi}\hat{a}^+\right)\left(\hat{b}^+ + \hat{b}\right) + \frac{1}{2}\hat{a}^+\hat{a}^+\hat{a}\hat{a}$$

$$\exp\left(i\phi(t)\right) = \sum_n e^{in\omega_d t + i\theta} J_n\left(\frac{\varepsilon}{\omega_d}\right) \quad U(t) = \exp(i\phi(t)\hat{a}^+\hat{a})$$

When $|\omega_0 - \omega_r| \gg |g|$, we can enhance selected non-rotating terms

$$\omega_d + \omega_0 - \omega_r = 0 \;\Rightarrow\; g e^{-i\theta} J_{-1}\left(\frac{\varepsilon}{\omega_d}\right)\hat{a}^+\hat{b},$$

$$-\omega_d + \omega_0 + \omega_r = 0 \;\Rightarrow\; g e^{i\theta} J_1\left(\frac{\varepsilon}{\omega_d}\right)\hat{a}^+\hat{b}^+, \text{etc}$$



Main ideas

- Interest in engineering couplers
  - Change form, change intensity.
- Two big families of approaches
  - Passive couplers
    - Adiabatic principles
    - Perturbative approaches
  - Active couplers
    - Parametric driving
    - Energy compensation
- Other considerations: dephasing & decoherence

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Developing quantum science and technology in Spain

QTEP is the quantum ecosystem created by CSIC to advance the field of quantum technologies, fostering training, research and innovation in a new generation of secure communication devices, computers and sensors based on quantum phenomena, such as entanglement or superposition.

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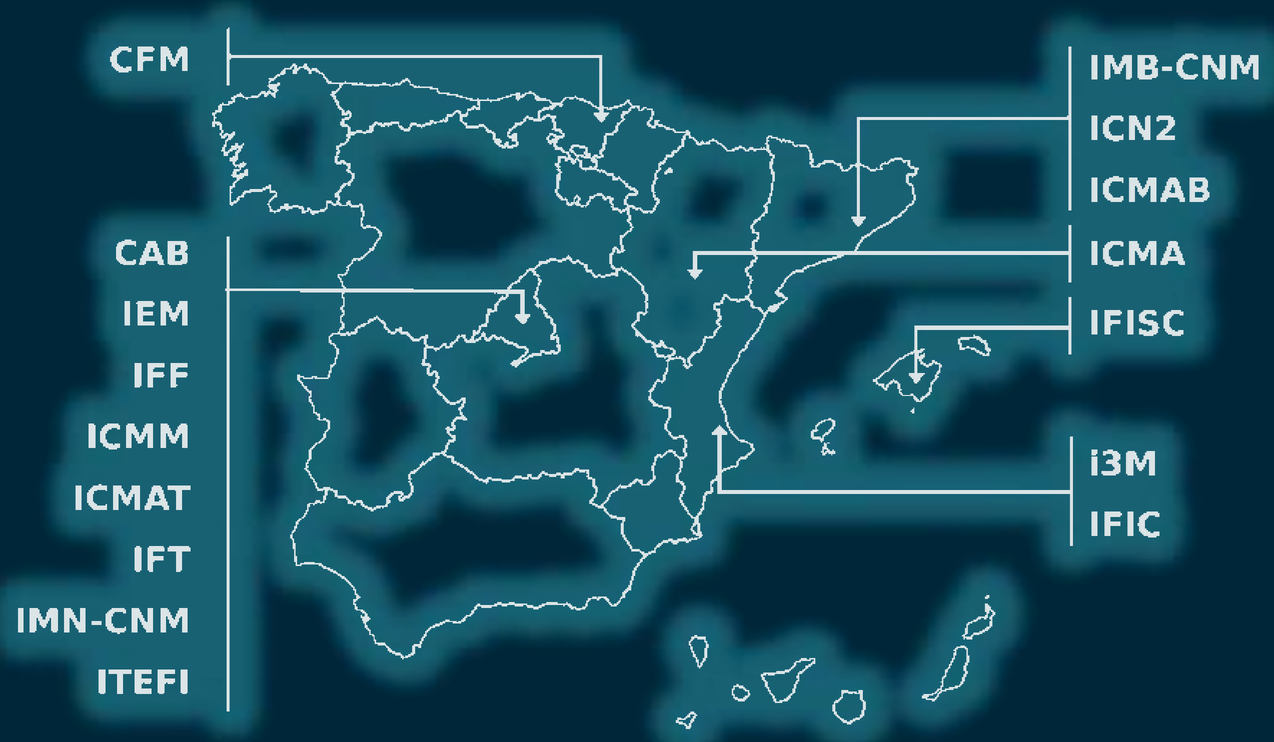

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