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Quantum measurement

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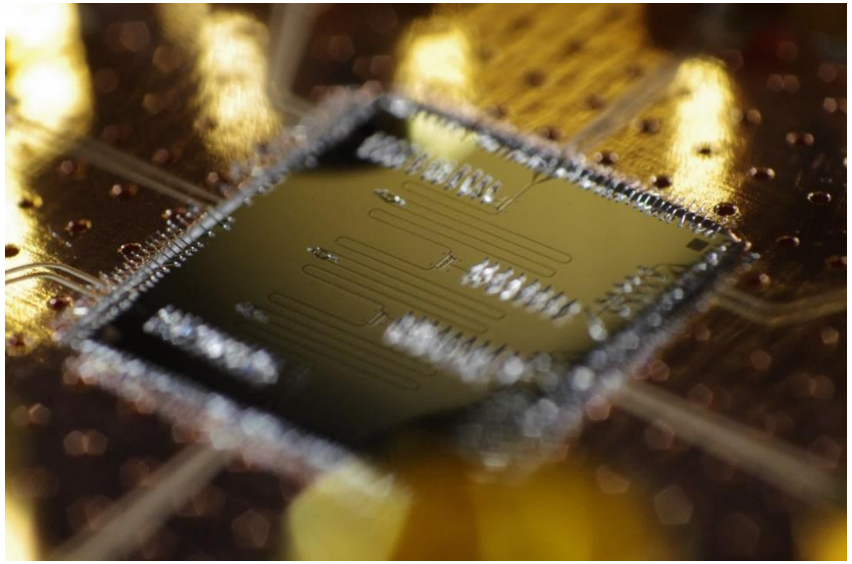

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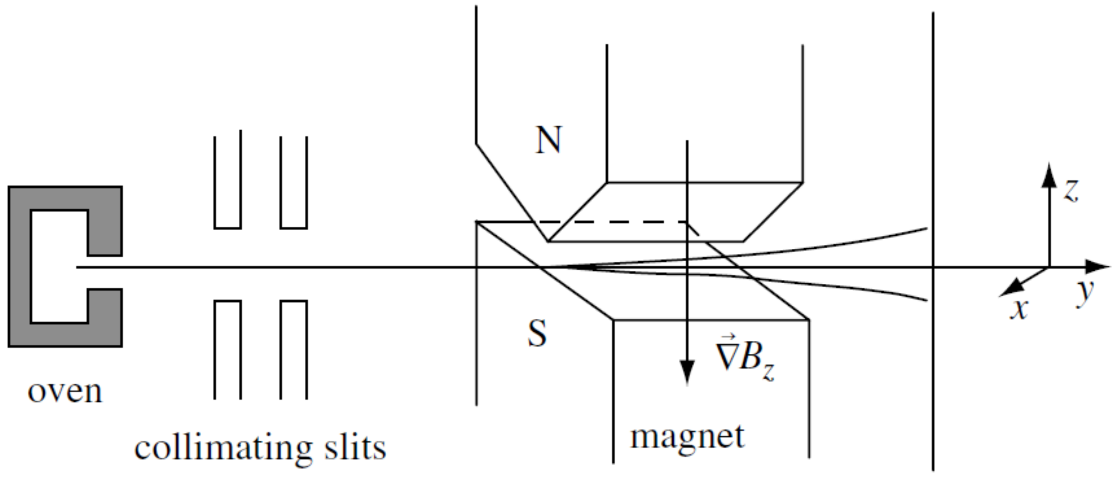
Outline:

1. Quantum projective measurement: Stern-Gerlach experiment
2. Circuit-QED and dispersive measurement
3. Transmon qutrit measurement and protection from wave-function collapse
4. Quantum rifling and protection from collapse on demand using qubit manipulation
5. Continuous measurement and quantum trajectories in different regimes
6. Some extra remarks

Quantum projective measurement: Stern-Gerlach experiment

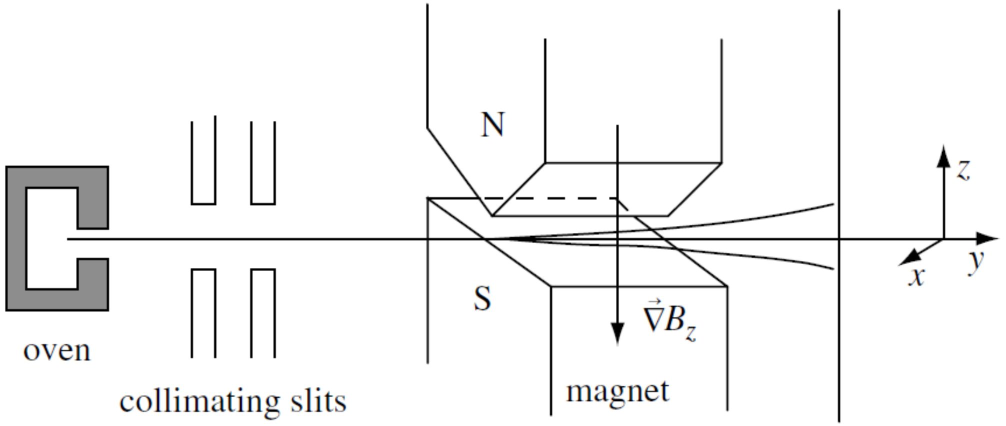
How to measure spin: Stern-Gerlach experiment

Uses neutral silver atoms (a good representation of a spin of an electron).



- Magnetic field gradient along z-axis
- Induces the force proportional to the magnetic moment $F_z = \mu_z \frac{\partial B_z}{\partial z}$
- Particles deflect depending on the value of z-component of their magnetic moment

How to measure spin: Stern-Gerlach experiment

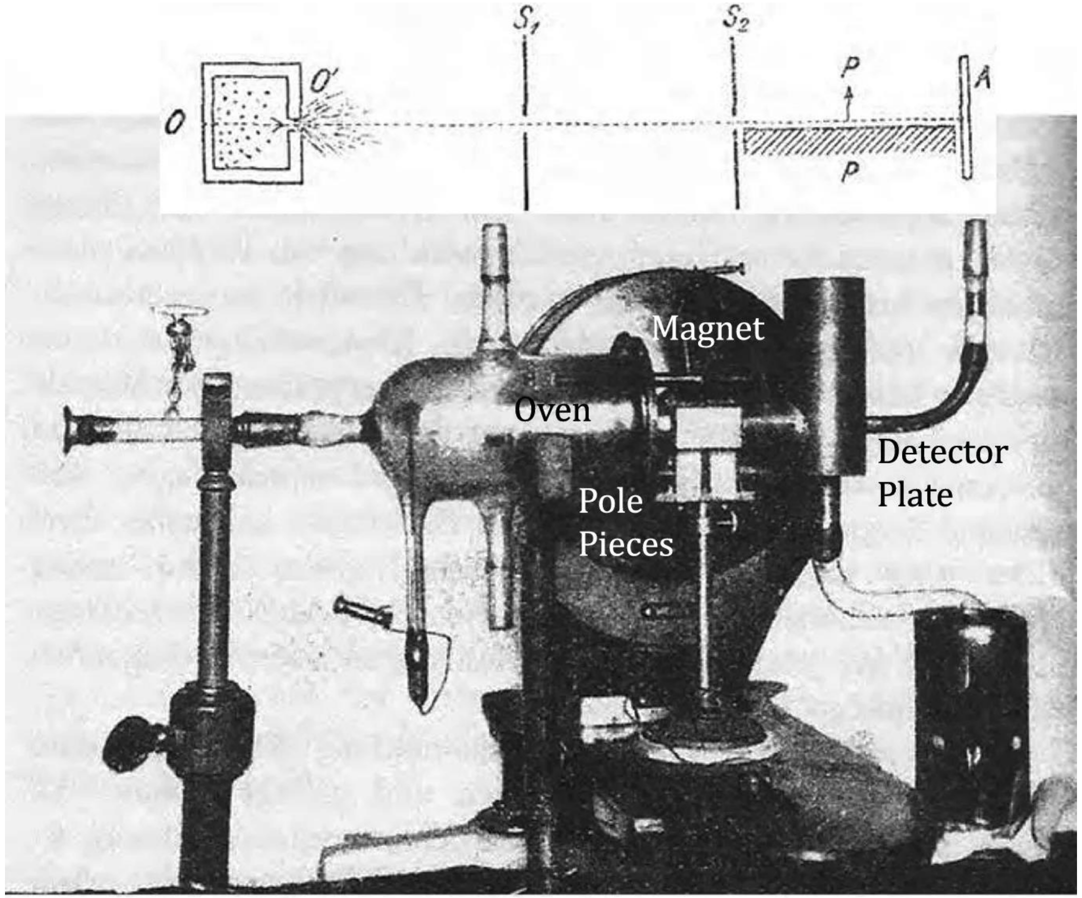


Assume that the magnetic moments of the particles are fully unpolarized.

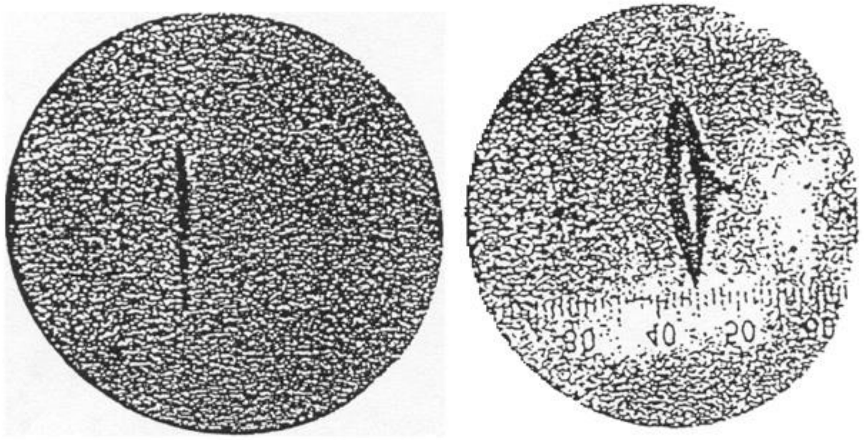
If colour represents the density of particles hitting the screen what kind of pattern will you expect to observe if particles were classical?

Z-component of magnetic moment μ_z takes only two values corresponding to $S_z = \pm\frac{\hbar}{2}$!

Stern-Gerlach experiment: history



Credit: Scientific American



Gerlach, W. and O. Stern, 1922a. “Der experimentelle Nachweis der Richtungsquantelung”, *Zeitschrift fur Physik*, 9: 349–352.

Quantum projective measurement: postulates of QM

Consider a measurement of a physical quantity Λ:

$$\Lambda = \sum_{\lambda} \lambda \, \hat{\Pi}_{\lambda}$$

If $\{\lambda\}$ is non-degenerate than $\hat{\Pi}_{\lambda} = |\lambda\rangle\langle\lambda|$ (*von Neumann measurement*).

The result of the measurement is one of the eigenvalues of λ with probability

$$p_{\lambda} = \mathrm{Tr}[\rho \hat{\Pi}_{\lambda}]$$

After measurement:

$$\rho_{\lambda} = \hat{\Pi}_{\lambda} \rho \hat{\Pi}_{\lambda} / p_{\lambda}$$

Known as *projection postulate, state collapse* or *state reduction.*

If one makes the measurement but ignores the result then the state after the measurement is

$$\rho_M = \sum_{\lambda} \hat{\Pi}_{\lambda} \rho \hat{\Pi}_{\lambda}$$

Quantum projective measurement: postulates of QM

Remarks:

- for a degenerate eigenvalue a_n the system would be projected to a subspace with dimensionality equal to the multiplicity of the eigenvalue (more details the section about ***qutrit measurement***).
- The postulate is not independent and can be derived from using other postulates and the detailed model of the system interacting with the measurement apparatus (see next section on ***dispersive readout***)
- Note: from operational point of view it is often sufficient to think about it as an instant collapse (see next).

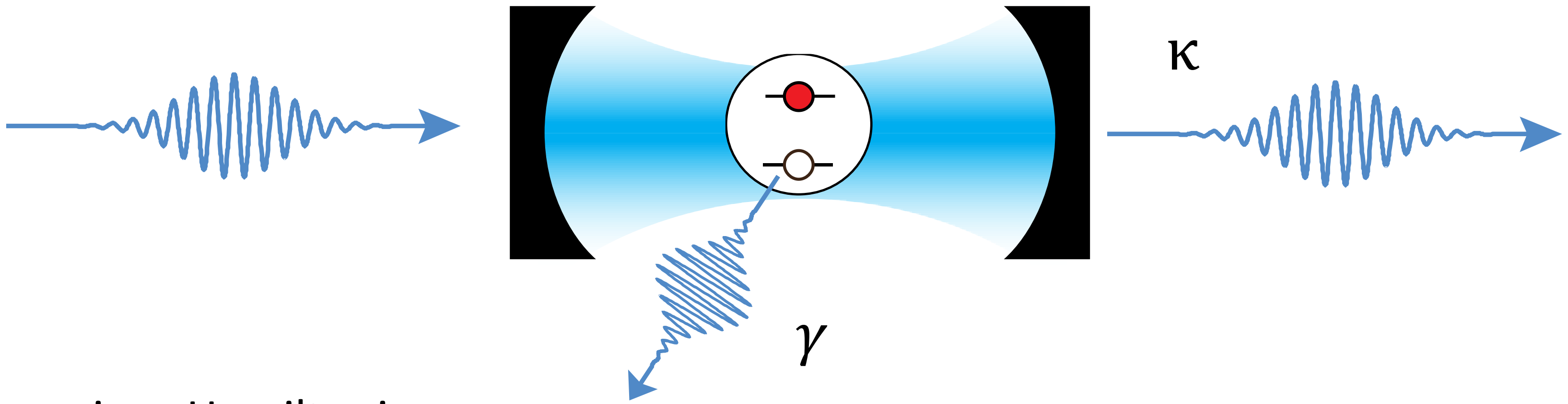
Circuit-QED and dispersive measurement

Cavity and circuit QED: recap

Cavity Quantum Electrodynamics

Enclose atom and photon in a cavity

Cavity QED: light-matter interaction at single atom-photon level, interaction enhanced by placing atoms inside high quality photonic cavities



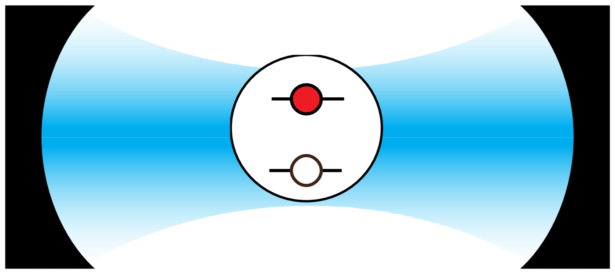
Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+) + H_\kappa + H_\gamma$$

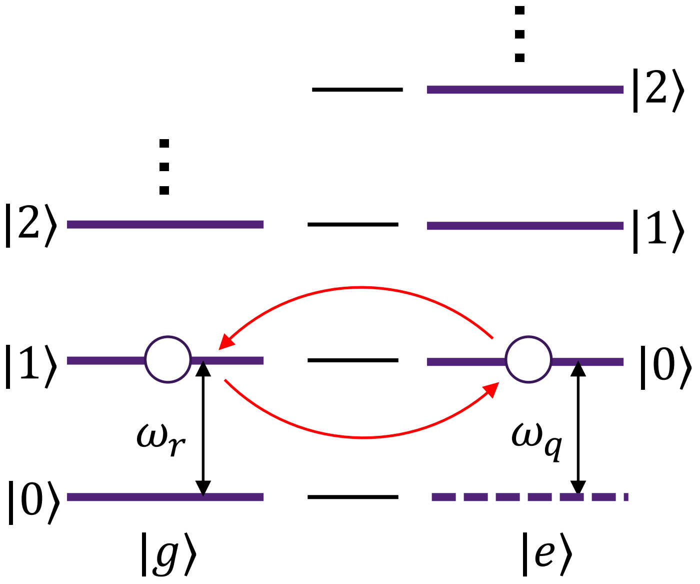
strong coupling limit: $g = dE_0/\hbar > \gamma,\ \kappa$

[D. Walls, G. Milburn, Quantum Optics (Springer-Verlag, Berlin, 1994)]

James-Cumming Hamiltonian



$$H = \hbar\omega_0(a^\dagger a + 1/2) - \frac{\hbar\omega_q}{2}\sigma_z$$
$$+\hbar g(a^\dagger\sigma^- + a\sigma^+)$$



Switch on the interaction:

- Conserved the number of excitations
- Couples $|g,n\rangle$ with $|e,n-1\rangle$

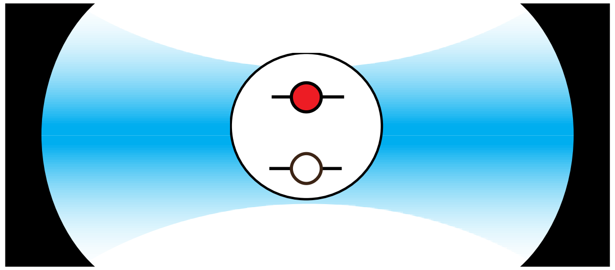
The Hamiltonian decouples into matrices:

$$H^{(n)} = \hbar\begin{pmatrix} 0 & \sqrt{n}\,g \\ \sqrt{n}g & 0 \end{pmatrix},$$

written in the basis of $\{|g,n\rangle, |e,n-1\rangle\}$

-> can be solved exactly!

James-Cumming Hamiltonian

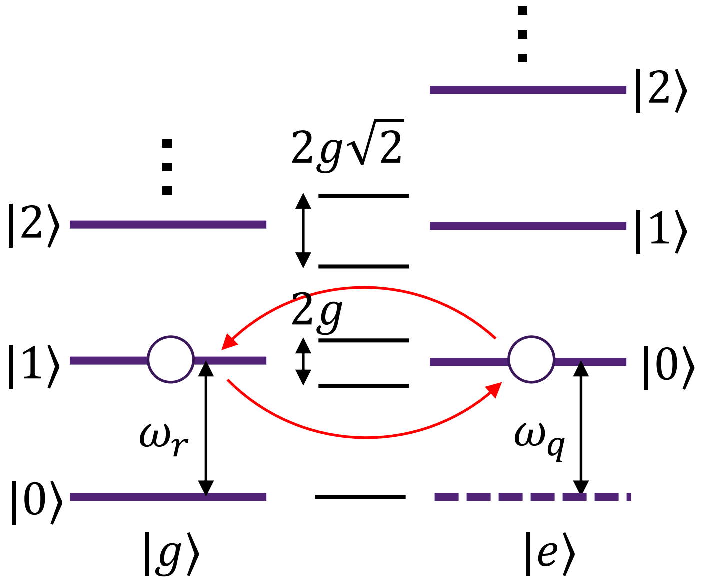


$$H = \hbar\omega_0(a^\dagger a + 1/2) - \frac{\hbar\omega_q}{2}\sigma_z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

The eigenstates are dressed states

$$|\pm\rangle = \left(\frac{1}{\sqrt{2}}\right)(|g,n\rangle \pm |e,n-1\rangle)$$

-> level spitting

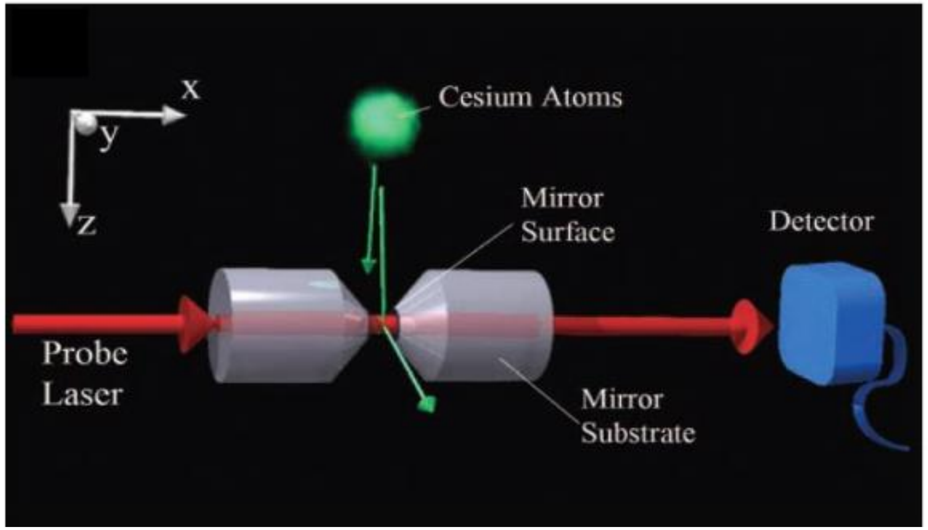


Level splitting between $|g,0\rangle$ and $|e,1\rangle$ is called the vacuum Rabi splitting and is manifestation of coherent interaction at single quantum level -> strong coupling regime

Note: normal splitting of two oscillators is purely classical regime. Scaling of the splitting with $\sqrt{n}$ is important to demonstrate quantum nature

Signature of strong coupling for natural atoms

$$H = \hbar\omega_0(a^\dagger a + 1/2) + \frac{\hbar\omega_q}{2}\sigma_z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

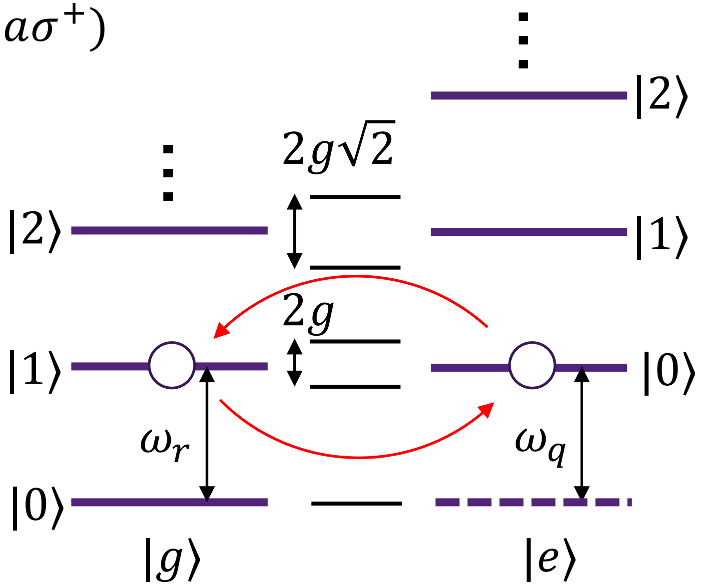


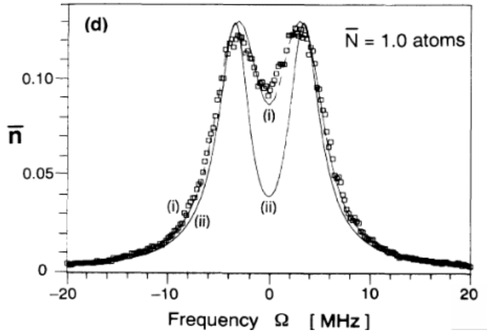
Alkali atoms: R.J. Thompson, G. Rempe, and H.J. Kimble, Phys. Rev. Lett. 68, 1132 (1992)

Rydberg Atoms: J. M. Raimond, M. Brune, and S. Haroche, Rev. Mod. Phys., 73, 565 (2004)

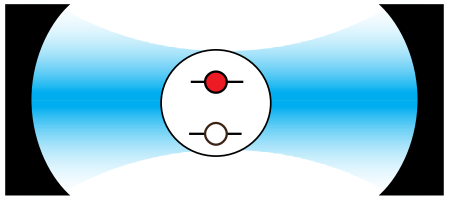
Highly demanding experiments!

Jaynes-Cumming Ladder





Cavity vs. Circuit QED



Coherent quantum mechanics with individual photons and atoms: Quantum photonics talk by *Tsuyoshi Yamamoto*

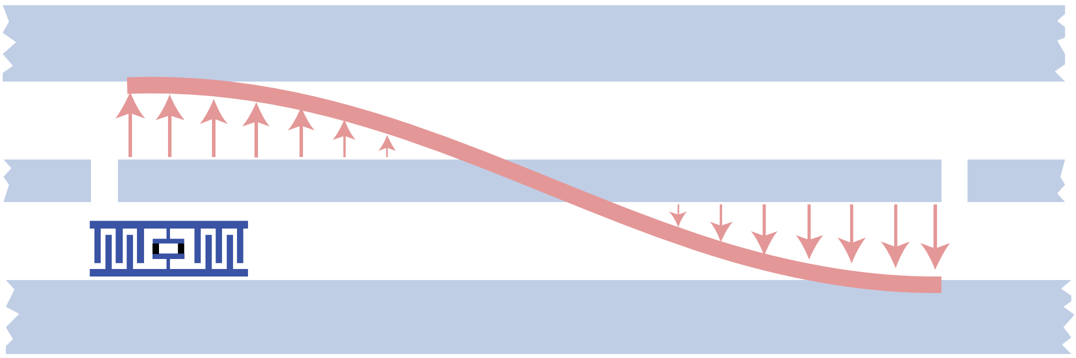
in superconducting circuits:

circuit quantum electrodynamics

Cavity =>
microwave transmission
line resonator

Mirrors => capacitors, or walls

Atoms => superconducting qubit



Strong E-field confinement + Large qubit dipole
=> v. strong coupling

A. Blais, *et al.* , *PRA* **69,** 062320 (2004)

Typical parameters

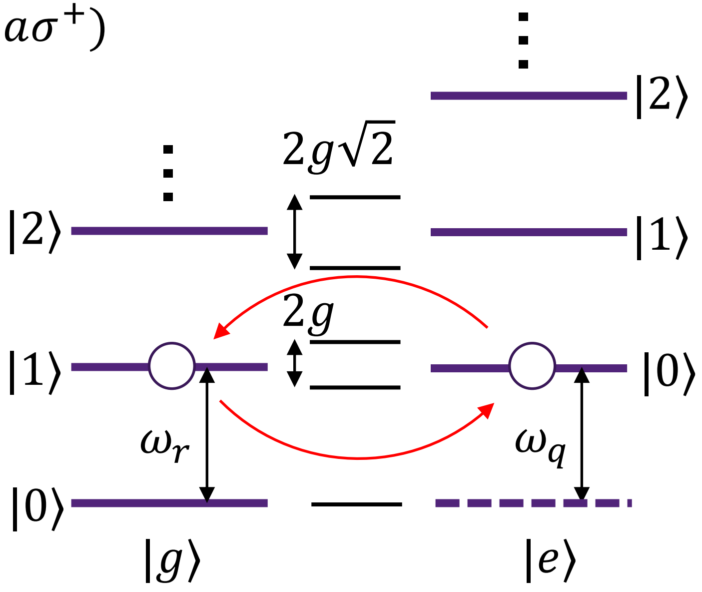
$$H = \hbar\omega_0(a^\dagger a + 1/2) + \frac{\hbar\omega_q}{2}\sigma_z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

Typical coupling strength for superconducting circuits:

$$g \sim < 300\ \mathrm{MHz}$$

Decay rate of a photon and a qubit

$$\kappa, \Gamma \sim 10\ \mathrm{kHz} - 10\ \mathrm{MHz}$$



Jaynes-Cumming Ladder

Strong-coupling regime $g \gg \kappa, \Gamma$ is easily achievable

Dispersive regime and measurement

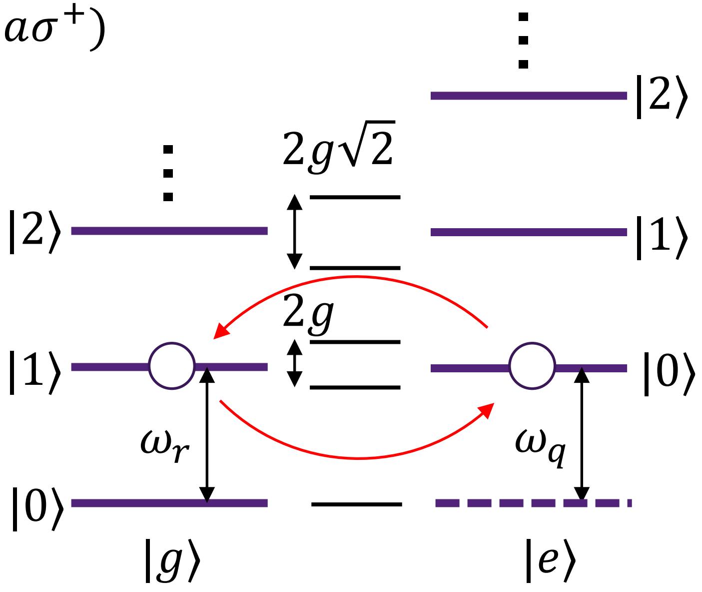
James-Cumming Hamiltonian

$$H = \hbar\omega_0(a^\dagger a + 1/2) + \frac{\hbar\omega_q}{2}\sigma_z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

In the strong-coupling regime $g \gg \kappa, \Gamma$

vacuum Rabi splitting for the resonant case

$$\omega_0 \simeq \omega_q$$



Jaynes-Cumming Ladder

What happens if the qubit and the resonator are detuned

$$\Delta = |\omega_r - \omega_q| \gg g?$$

Dispersive regime

$$H = \hbar\omega_0\, a^\dagger a + \frac{\hbar\omega_q}{2}\sigma_z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

Transform the Hamiltonian itself: $H' = UHU^\dagger$ with

$U = \frac{g}{\Delta}(a^\dagger\sigma^- - a\sigma^+)$ and keep the leading term in $\frac{g}{\Delta}$:

$$H' = \hbar\omega_0\, a^\dagger a + \frac{\hbar\omega_q}{2}\sigma_z + \hbar\chi a^\dagger a\sigma_z,$$

where $\chi = \frac{g^2}{\Delta}$ is called the dispersive shift.

Can combine the last term with the qubit

qubit frequency depends on number of photons

$$H' = \hbar\omega_0\, a^\dagger a + \frac{\hbar(\omega_q + \chi a^\dagger a)}{2}\sigma_z$$

Or with resonator

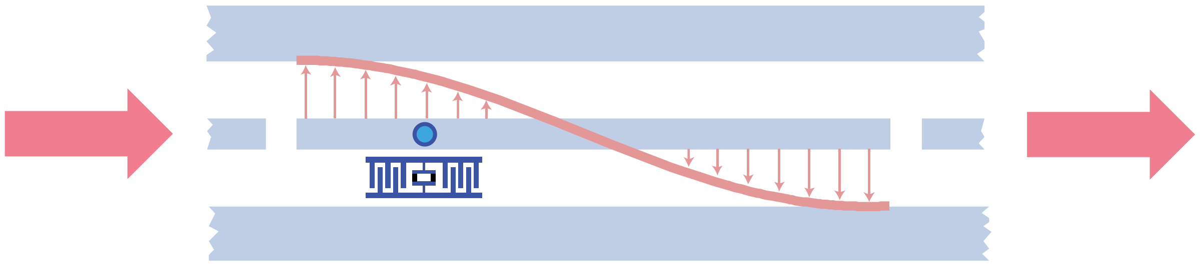
State dependent resonator frequency

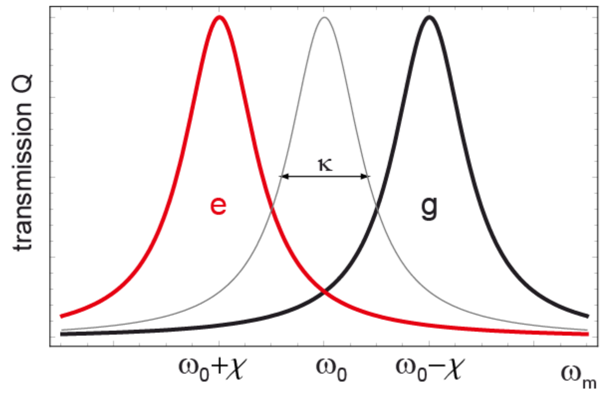
$$H' = \hbar\,(\omega_0 + \chi\sigma_z)a^\dagger a + \frac{\hbar\omega_q}{2}\sigma_z$$

[Blais *et al.*, PRA **69** (2004)]

Dispersive readout

$$\Delta = |\omega_r - \omega_q| \ll g$$





State-dependent frequency shift

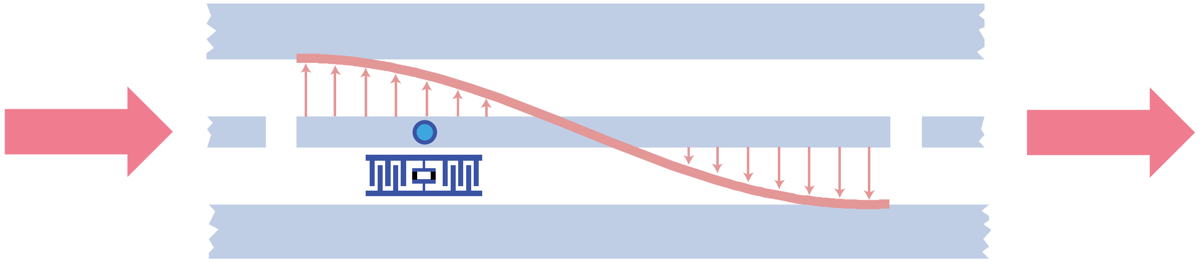
$$H = \hbar\,(\omega_0 + \chi\sigma_z)a^\dagger a + \frac{\hbar\omega_q}{2}\sigma_z$$

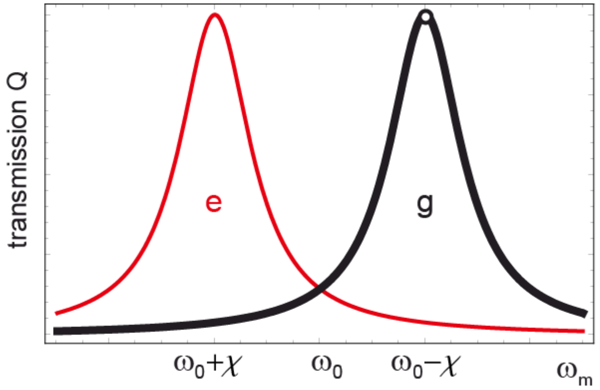
[Blais *et al.*, PRA **69** (2004)]

Transmission measurement can be used to determine the qubit state!

Circuit QED – read out of qubit state

transmission measurement to determine qubit state:

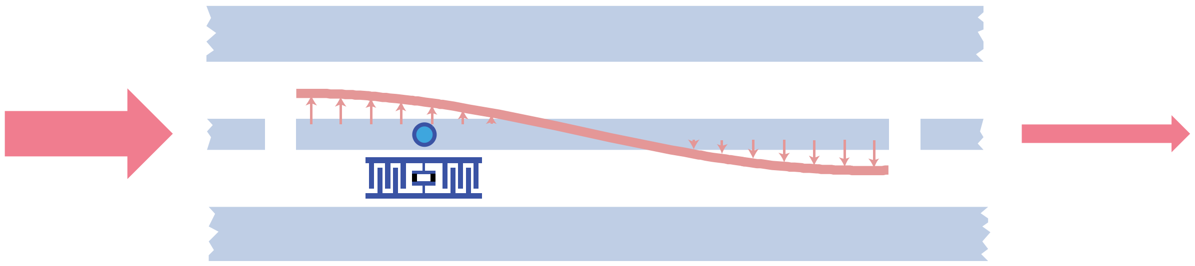


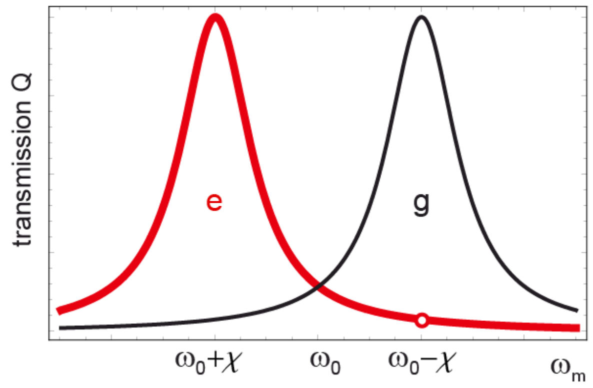


qubit in ground state

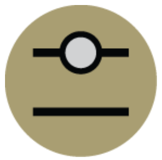
Circuit QED – read out of qubit state

transmission measurement to determine qubit state:





qubit in excited state

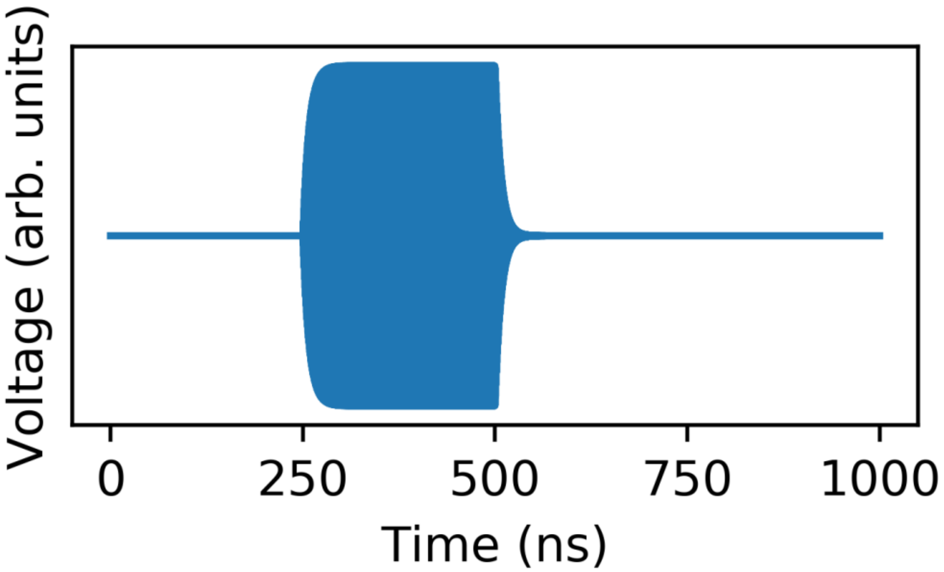


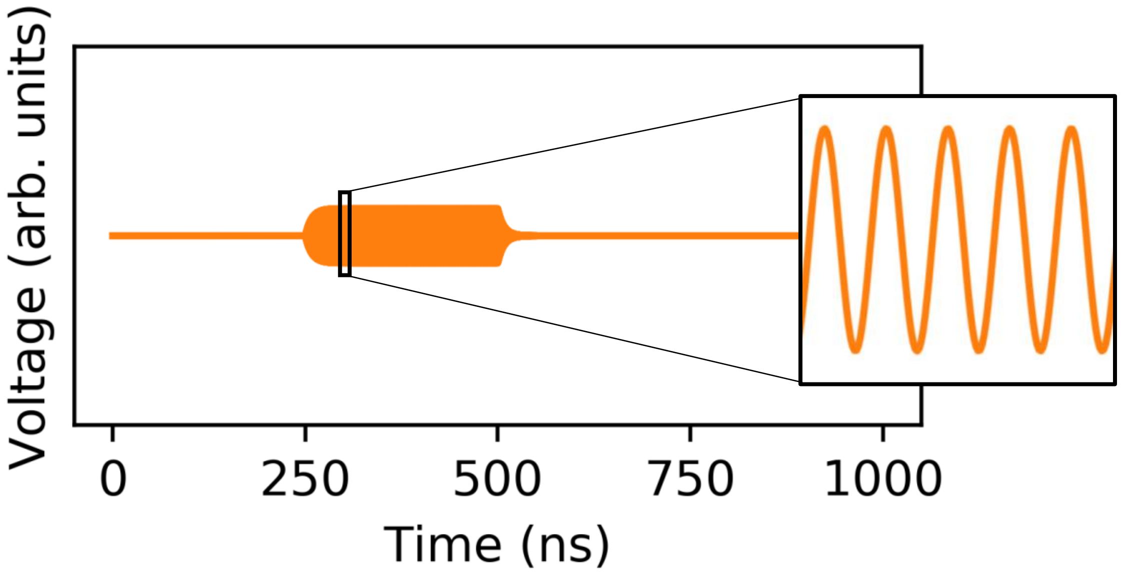
Dispersive read out in time domain

$|0\rangle$

Signal
7 GHz

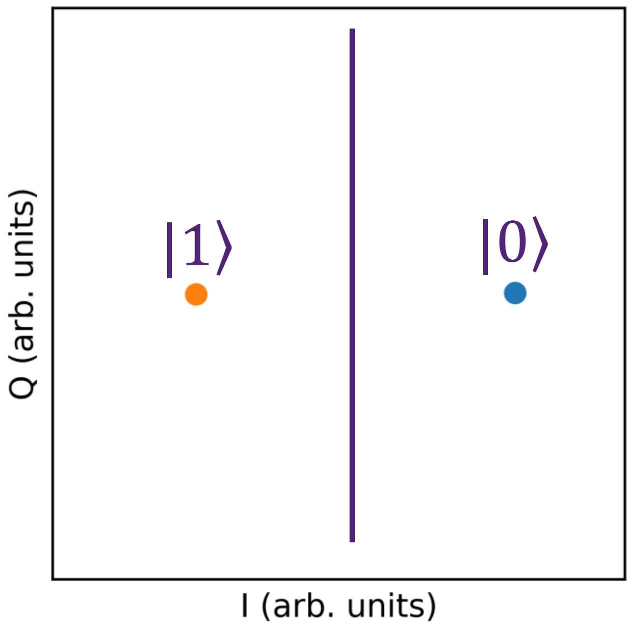
$|1\rangle$





Conventional data processing

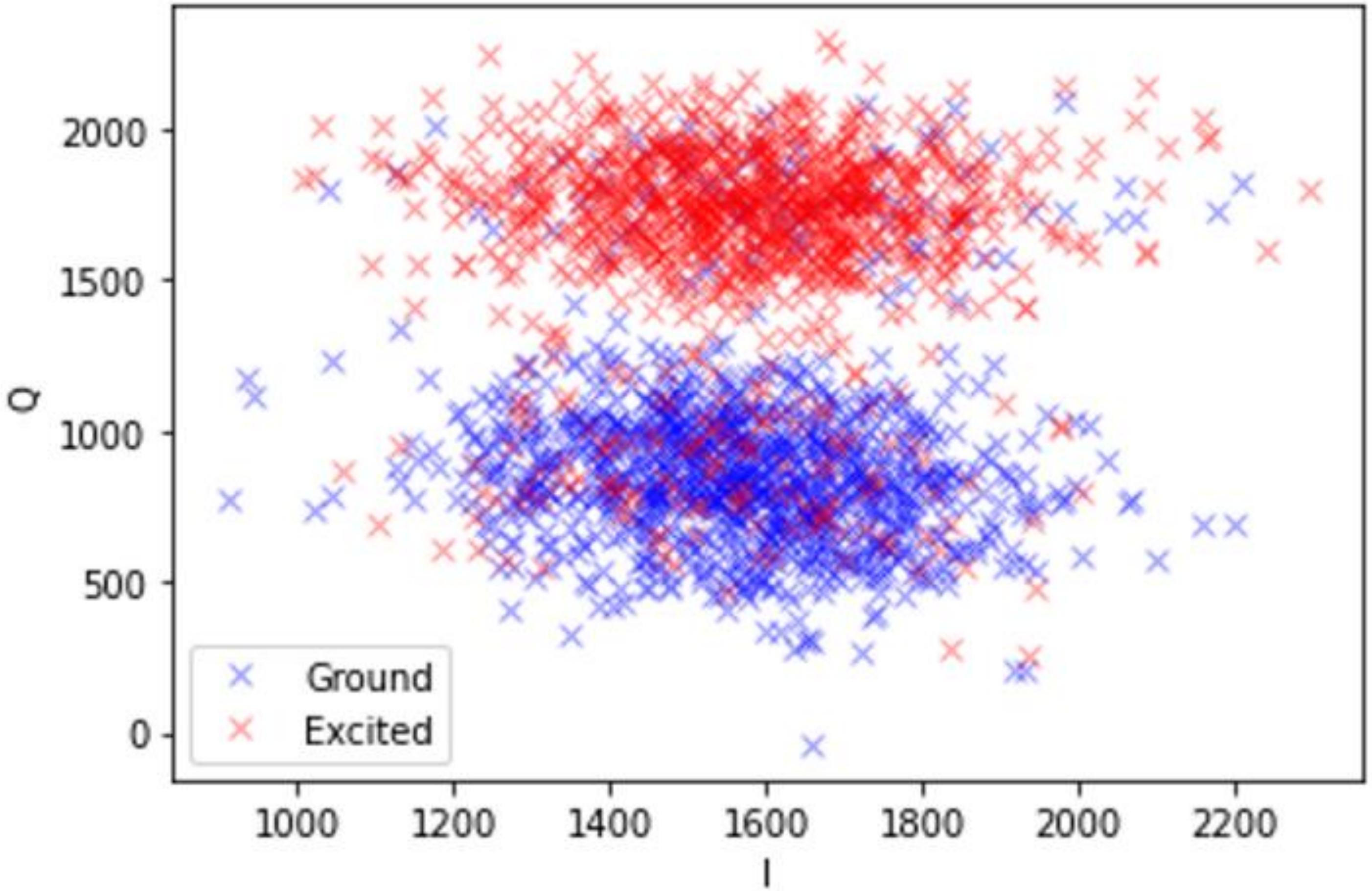
$$\text{Signal} \rightarrow \text{ADC} \rightarrow \text{DDC} \begin{array}{l} \nearrow \text{FIR} \rightarrow \text{Sum} \\ \searrow \text{FIR} \rightarrow \text{Sum} \end{array}$$



Assignment fidelity:

$$\mathcal{F}_a = (1/N) \sum_{i=1}^{N} P(i|i)$$

Single shot readout



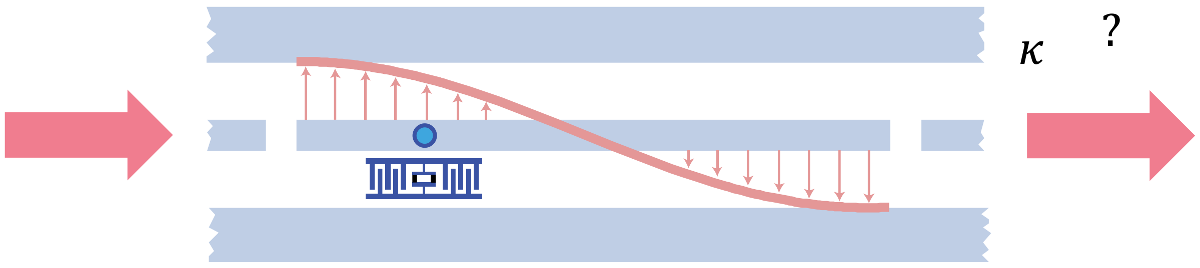
Noise:

- Pure quantum noise for an ideal quantum-limited amplifier
- More noise will appear from losses and additional amplification stages
- Quantum noise can be in principle removed if using single quadrature and phase-sensitive quantum amplifier

Dispersive measurement in more detail. Effect on qubit and cavity

Dispersive readout: more details

$$H = \hbar\,(\omega_0 + \chi\sigma_z)a^\dagger a + \frac{\hbar\omega_q}{2}\sigma_z + \hbar(E_d(t)e^{-i\omega_d t}a^+ + h.c.)$$



Before the measurement:

- $E_d(0) = 0$ and $|\psi(0)\rangle = (a|0\rangle + b|1\rangle) \otimes |0\rangle$

After the start of the measurement:

- Switch on cavity drive at $t > 0$, $E_d(t > 0) = E_d$
- The output field from the cavity is $a_{out} = \sqrt{\kappa}\,a - \eta$, where η is the vacuum field.

Need to solve for the cavity field a

Dispersive readout: short-time evolution

Consider ***short time*** and neglect all dissipation (for both cavity and qubit). Evolution is unitary under the dispersive Hamiltonian:

$$H = \hbar\,(\omega_0 + \chi\sigma_z)a^\dagger a + \frac{\hbar\omega_q}{2}\sigma_z + \hbar(E_d e^{-i\omega_d t}a^+ + h.c.)$$

It is convenient to move to the ***rotating frame*** and ***interaction picture*** for the qubit. Transform $H' = UHU^\dagger$ with $U = \hbar\omega_0\sigma_z + \hbar\omega_d\, a^\dagger a\, t$

$$H' = \hbar\delta\omega a^\dagger a + \hbar\chi\sigma_z a^\dagger a + \hbar(E_d a^+ + h.c.),\ \text{where } \delta\omega \equiv \omega_d - \omega_0$$

H' is time-independent and

$$|\psi(t)\rangle = e^{-\frac{iH'}{\hbar}t}\,|\psi(0)\rangle$$

We then note that H' is ***diagonal*** in the qubit basis and the initial state is ***factorised***

$|\psi(0)\rangle = (a|0\rangle + b|1\rangle) \otimes |vac\rangle$

Dispersive readout: short-time evolution

In this case

$$|\psi(t)\rangle = e^{-\frac{iH'}{\hbar}t}(a|0\rangle + b|1\rangle) \otimes |vac\rangle = a|0\rangle \otimes |\psi_0\rangle + b|1\rangle \otimes |\psi_1\rangle.$$

Here the cavity states

$$|\psi_n\rangle = e^{-\frac{i\langle n|H'|n\rangle}{\hbar}t}|vac\rangle \text{ with } n = 0,1$$

The evolution leads to entanglement between the cavity and the qubit!

But what are these cavity states?

Dispersive readout: cavity states

For a more complete picture consider the Master equation:

$$\dot{\rho} = -i[H,\rho] + \gamma D[\sigma^-]\rho + \gamma_\phi D[\sigma_z]\rho + \kappa D[a]\rho \equiv \mathcal{L}\rho$$

Here:

- γ is the relaxation and γ_ϕ is the pure dephasing rates for the qubit
- κ is cavity decay rate
- $D[A]\rho = A\rho A^+ - \frac{1}{2}(A^+ A\rho + \rho A^+ A)$

Let‘s neglect qubit relaxation (and dephasing) -> regime relevant for strong projective measurement and QC

Dispersive readout: cavity states

If we neglect qubit relaxation the superoperator is ***unitary*** and ***diagonal*** for the qubit and the form of the final state is still the same:

$$|\psi(t)\rangle = a|0\rangle \otimes |\psi_0\rangle + b|1\rangle \otimes |\psi_1\rangle$$

Here $\left|\psi_{0,1}\right\rangle$ are solutions of :

$$\dot{\rho}_c = \pm i\left[\chi a^\dagger a + (E_d a^+ + h.c.), \rho_c\right] + \kappa D[a]\rho_c,$$

Here we took $\delta\omega = 0$ for convinience.

Investigate the nature the states by testing the master equation.

Dispersive readout: cavity states

As an exercise one can prove that if we start from a ***coherent state*** we stay in the ***coherent state*** under the given Master equation.

Thus, starting from $|\psi_c(t)\rangle = |vac\rangle$ which is the ***coherent state:***

$$\rho_c(t) = |\alpha(t)\rangle\langle\alpha(t)|.$$

Here $\alpha(t)$ is the solution for $\alpha \equiv \langle a \rangle \equiv \mathrm{Tr}[\rho_c a]$ which one can obtain from the Master equation as

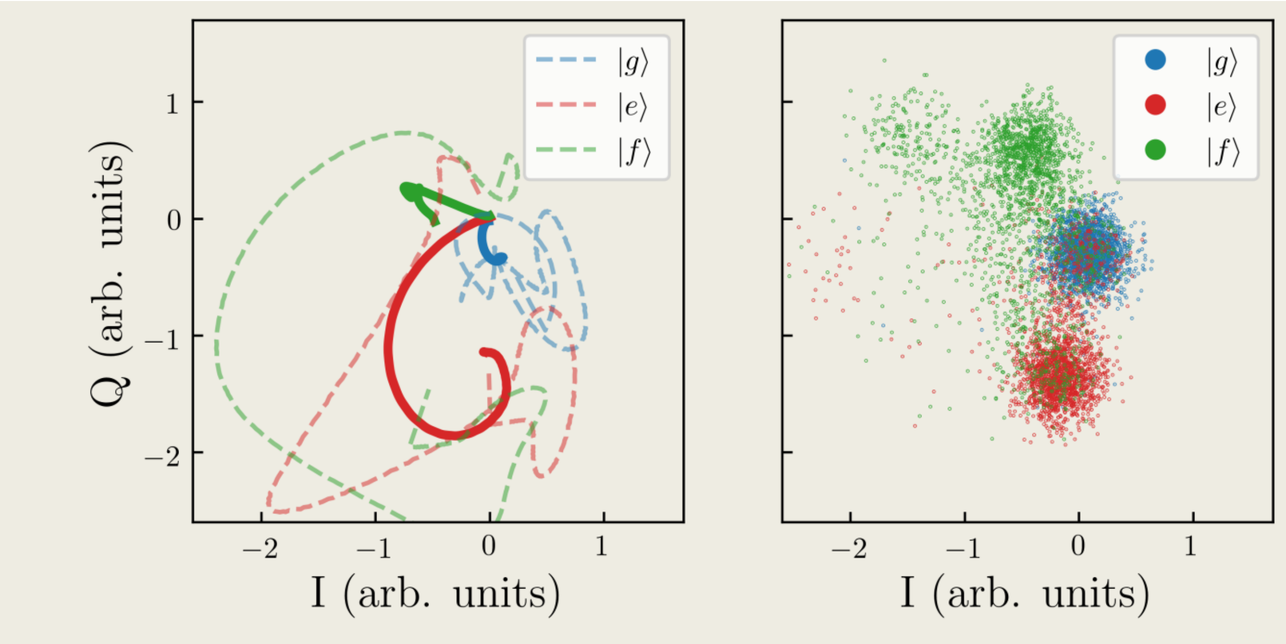
$$\dot{\alpha} = \pm i\chi\alpha - \frac{\kappa}{2}\alpha + E_d,$$

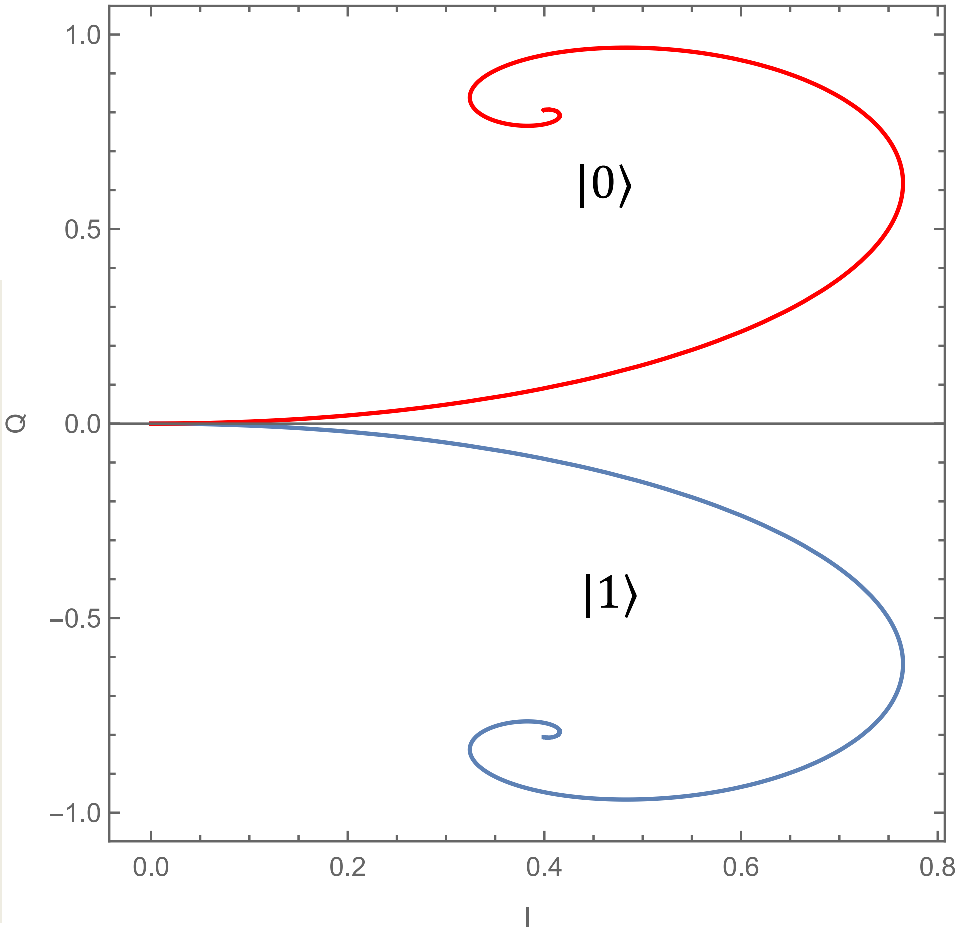
where we used the commutation relation $[a, a^+] = 1$.

Dispersive readout: cavity states

One can easily solve $\dot{\alpha} = \pm i\chi\alpha - \frac{\kappa}{2}\alpha + E_d$

Experimental data for qutrit





Can use NN for better single shot readout fidelity

See e.g. R. Navarathna et.al. Appl. Phys. Lett. **119**, 114003 (2021);

Readout: qubit side

Consider a qubit coupled to a driven readout cavity

$$H = \hbar\omega_0 \, a^\dagger a + \frac{\hbar\omega_q}{2}\sigma_z + \hbar\chi a^\dagger a \sigma_z + E(a + a^+)$$

$$\dot{\rho} = -i[H, \rho] + \gamma D[\sigma^-]\rho + \kappa D[a]\rho$$

Here:

- γ is qubit relaxation rate (neglect qubit dephasing for now)
- κ is cavity decay rate
- $D[A]\rho = A\rho A^+ - \frac{1}{2}(A^+ A\rho + \rho A^+ A)$

Redout: qubit side

Consider a qubit coupled to a driven readout cavity: $H = \hbar\delta\omega a^\dagger a + \hbar\chi a^\dagger a\sigma_z + E(a + a^+)$

$$\dot{\rho} = -i[H,\rho] + \gamma D[\sigma^-]\rho + \kappa D[a]\rho$$

For typical experiments $\kappa \ll \gamma$ -> can use adiabatic elimination of the cavity to obtain dynamics for the qubit:

$$\dot{\rho_Q} = -i\left[\Delta\sigma_z, \rho_Q\right] + \gamma D[\sigma^-]\rho_Q + \Gamma D[\sigma_z]\rho_z$$

Here:

- assumed that cavity driven by coherent state with an amplitude $\alpha_0 = -2iE/\kappa$ ($n_0 = |\alpha_0|^2$).
- $\Delta = \chi n_0$ - a/c Stark shift due to cavity driving
- $\Gamma = 4\left(\frac{\chi}{\kappa}\right)^2 n_0$ is dephasing due to measurement

Γ describes the collapse of the wavefunction due to measurement

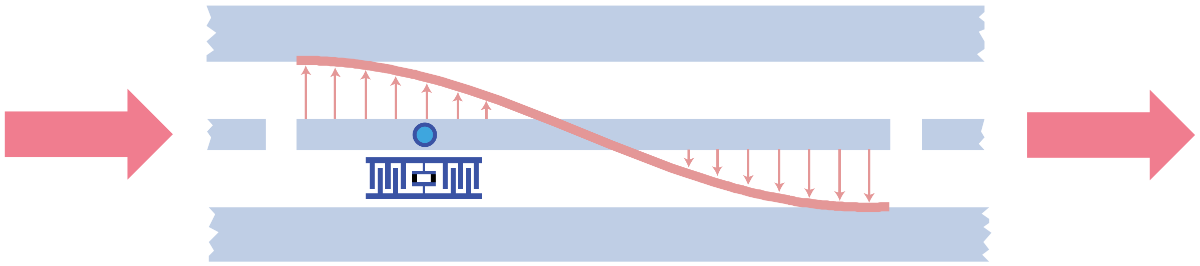
Dispersive readout: summary

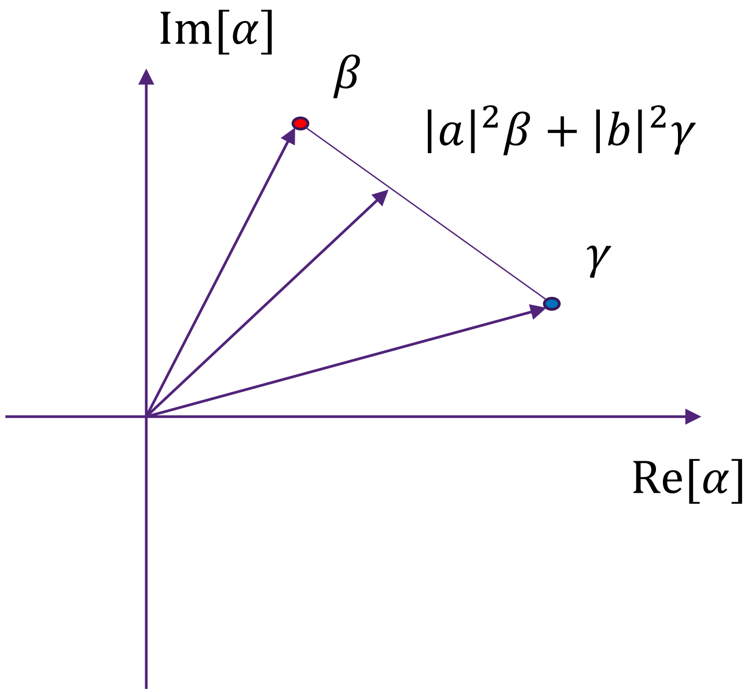
State-dependent frequency shift

$$H = \hbar\,(\omega_0 + \chi\sigma_z)a^\dagger a + \frac{\hbar\omega_q}{2}\sigma_z + (E_0 e^{-i\omega t}a^+ + h.c.)$$

$$|\psi_{\text{in}}\rangle = (a|g\rangle + b|e\rangle)\,|\alpha\rangle$$

$$|\psi_{\text{out}}\rangle = a|g\rangle|\beta\rangle + b|e\rangle|\gamma\,\rangle + \eta_{\text{noise}}$$





If trace out qubit:

- $\rho_C \equiv \mathrm{Tr}[\rho] = |a|^2|\beta\rangle\langle\beta| + |b|^2|\gamma\rangle\langle\gamma|$
- $\langle a\rangle \equiv \mathrm{Tr}[a\rho_C] = |a|^2\beta + |b|^2\gamma$

If trace out cavity:

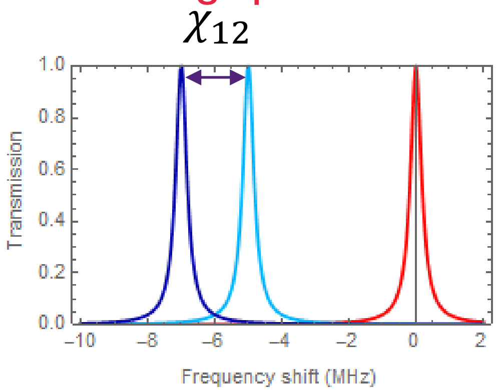
- $\rho_Q \equiv \mathrm{Tr}[\rho] = |a|^2|g\rangle\langle g| + |b|^2|e\rangle\langle e|$ if $\langle\alpha|\beta\rangle = 0$ -> strong projective measurement
- If $\langle\alpha|\beta\rangle \neq 0$ – partial or weak measurement (some qubit coherence remain)

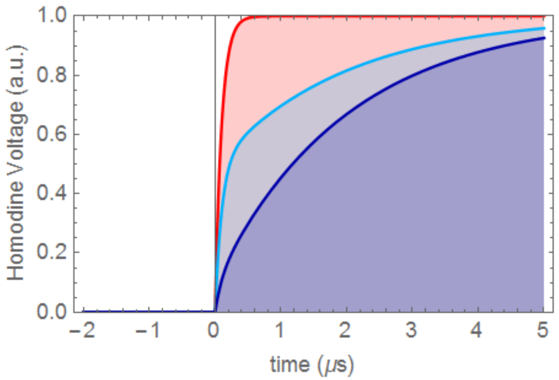
Transmon qutrit measurement and protection from wave-function collapse

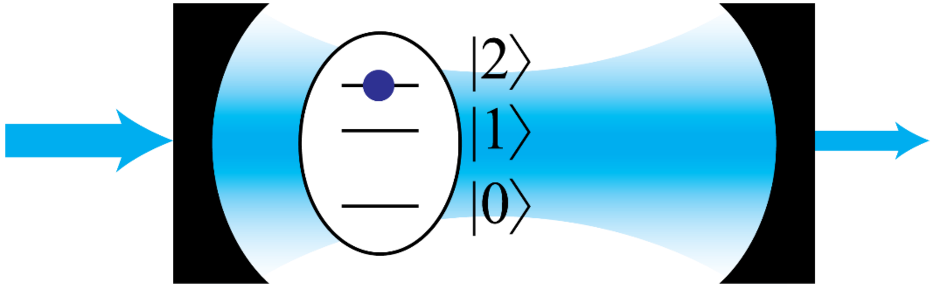
Degenerate qutrit measurement

Dispersive readout in time domain

Superconducting qutrit in a cavity: measure transmission to determine the state



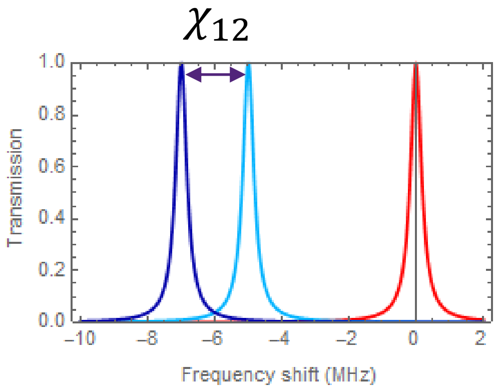


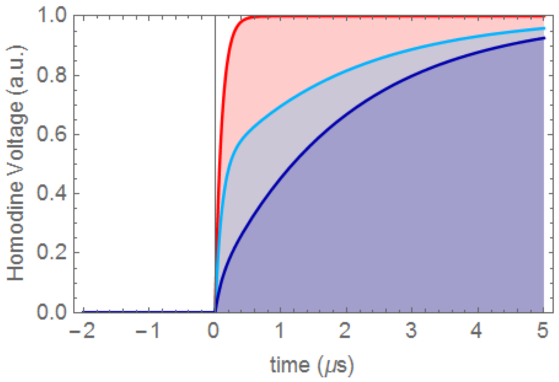


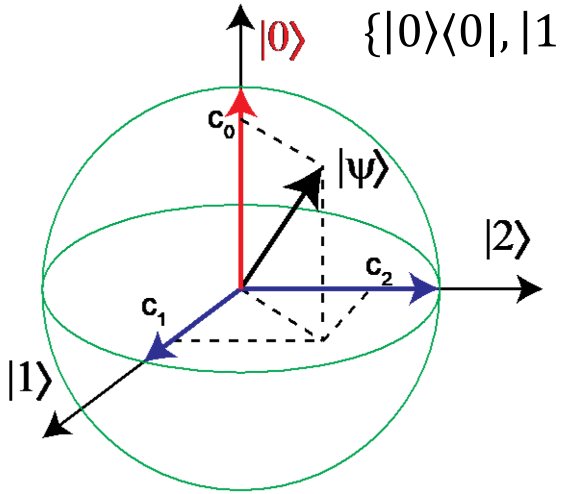
Measurement always provides information about all states destroying the coherence between $|1\rangle$ and $|2\rangle$

Dispersive readout in time domain

Superconducting qutrit in a cavity: measure transmission to determine the state





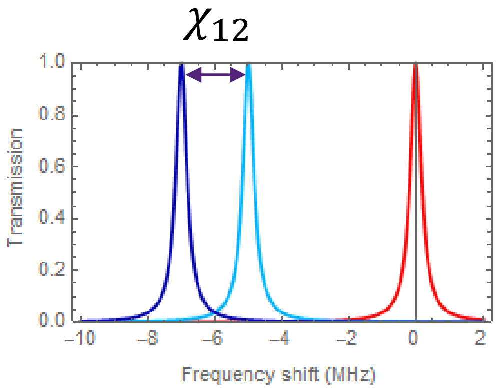


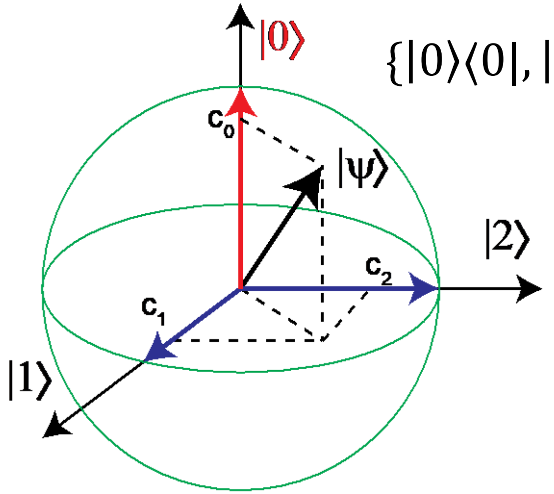
$\{|0\rangle\langle 0|, |1\rangle\langle 1|, |2\rangle\langle 2|\}$

Measurement always provides information about all states destroying the coherence between $|1\rangle$ and $|2\rangle$

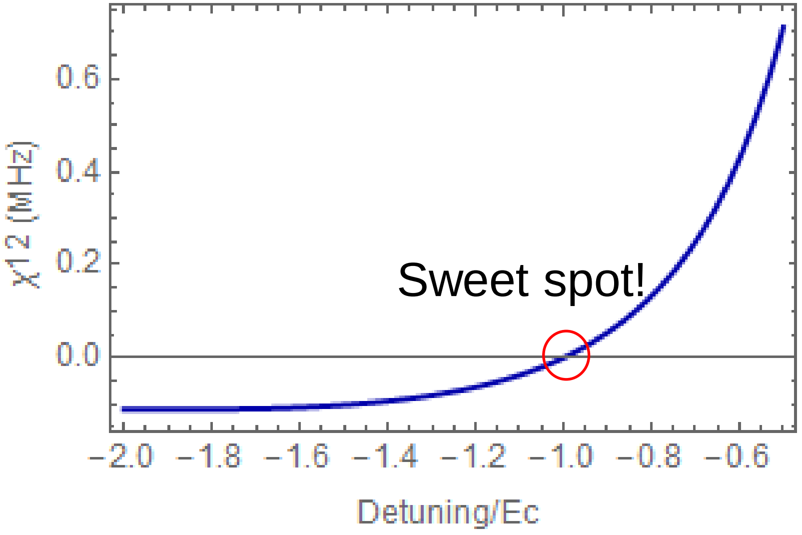
Dispersive readout: sweet spot

Relative dispersive shift as function of qubit detuning



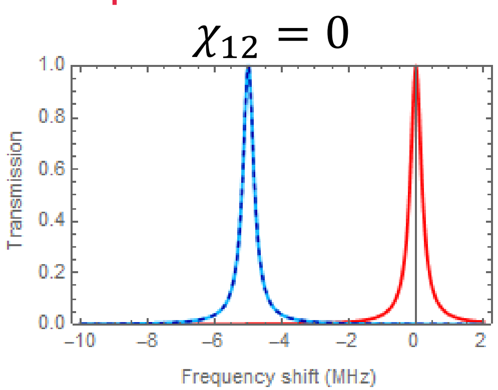


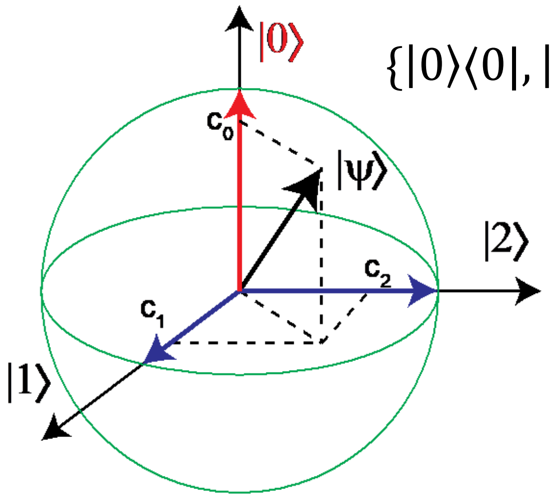
$$\{|0\rangle\langle 0|, |1\rangle\langle 1|, |2\rangle\langle 2|\}$$



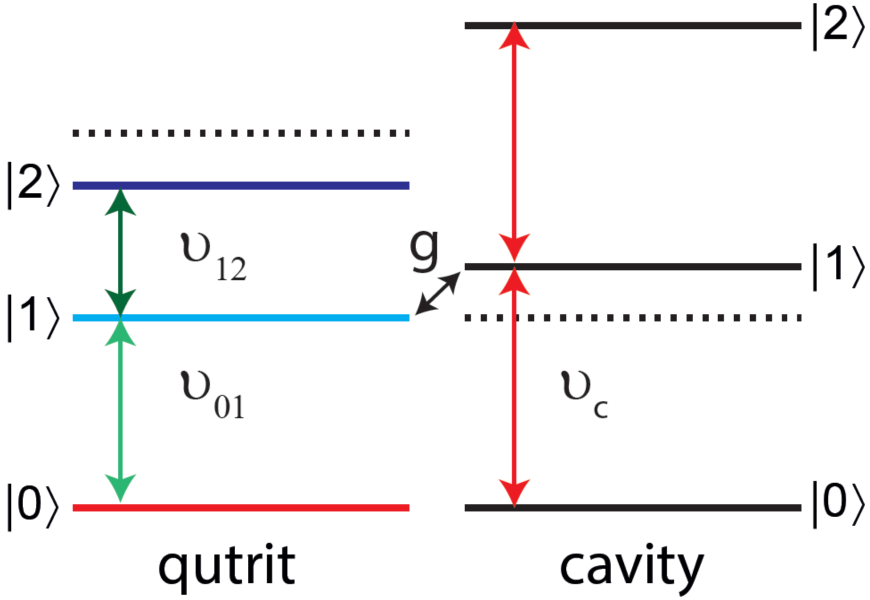
Dispersive readout: sweet spot

Relative dispersive shift as function of qubit detuning





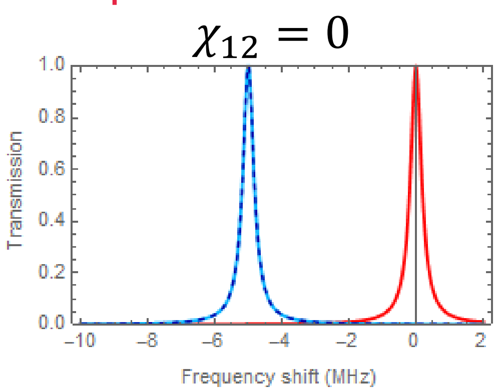
$\{|0\rangle\langle 0|, |1\rangle\langle 1|, |2\rangle\langle 2|\}$

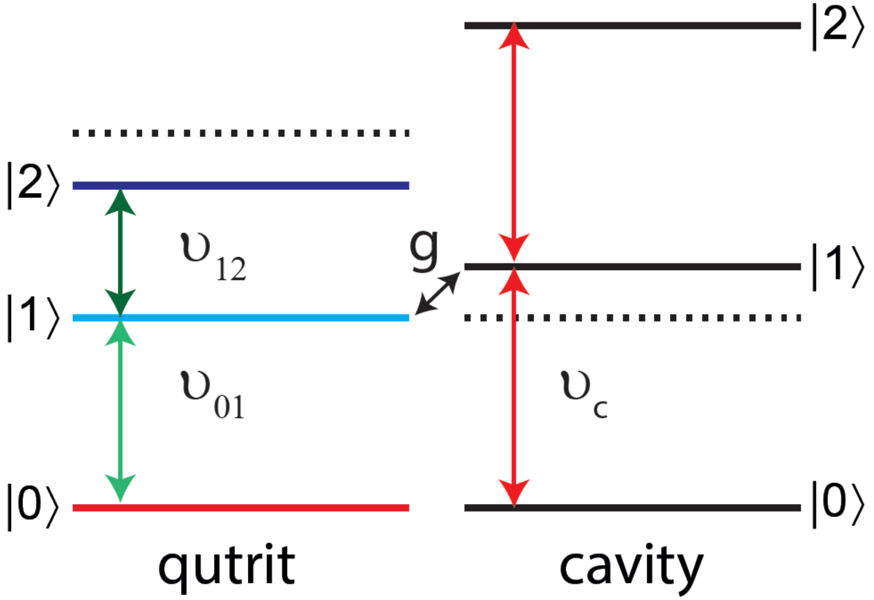


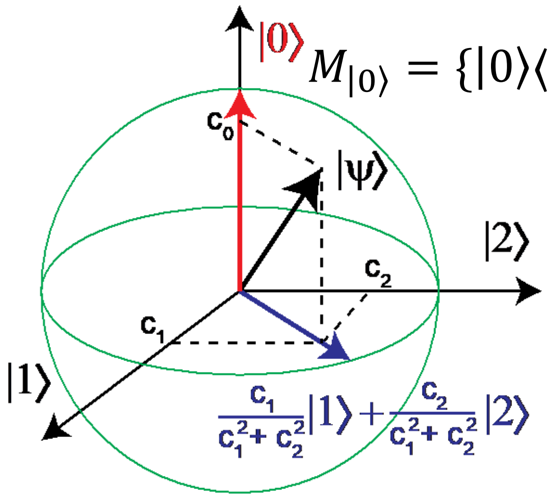
At sweet spot it is not possible to distinguish $|1\rangle$ from $|2\rangle$.

Dispersive readout: sweet spot

Relative dispersive shift as function of qubit detuning







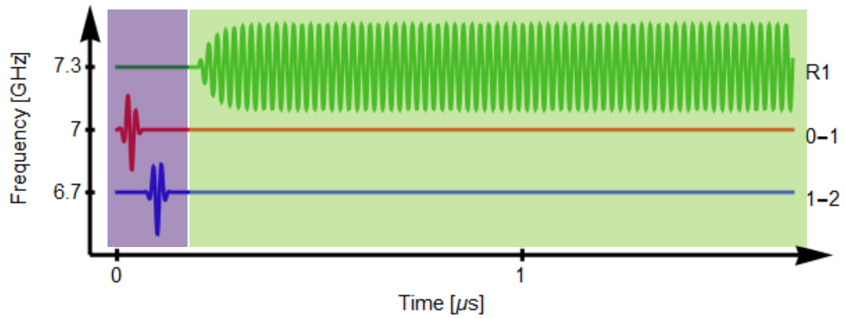
$$M_{|0\rangle} = \{|0\rangle\langle 0|, I - |0\rangle\langle 0|\}$$

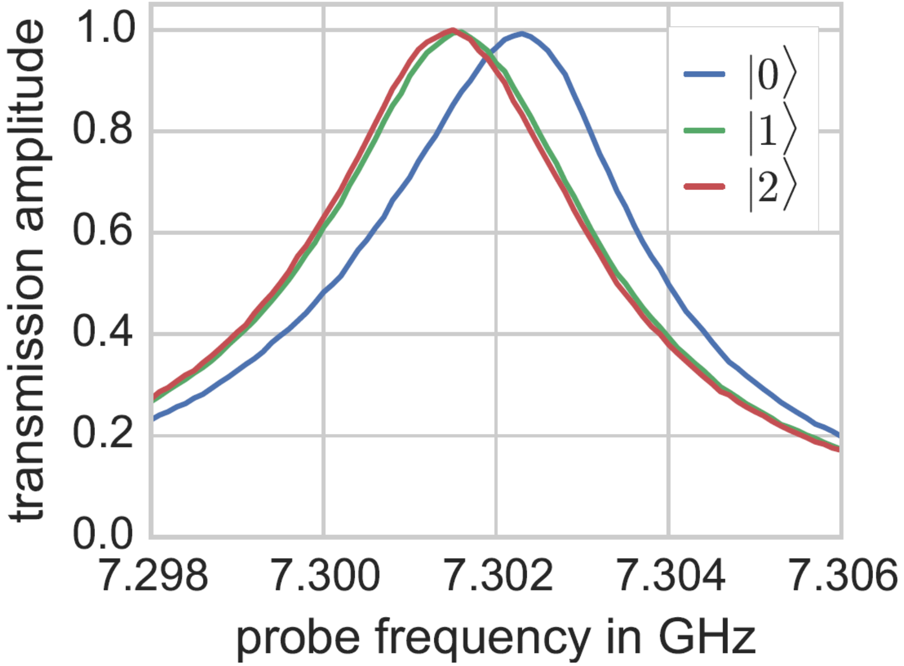
At sweet spot it is not possible to distinguish $|1\rangle$ from $|2\rangle$.

Will the measurement retain coherence between $|1\rangle$ and $|2\rangle$?

Measuring dispersive shifts

Prepare $|0\rangle, |1\rangle, |2\rangle$ and measure transmission through the cavity.
Plot integrated signal as function of frequency

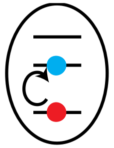


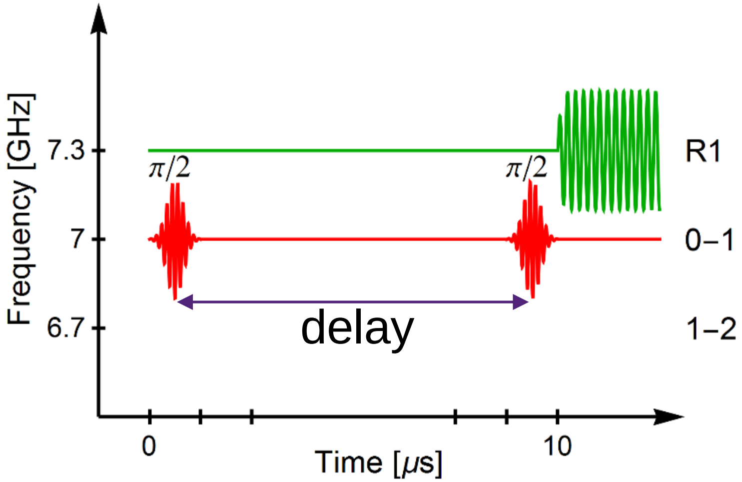


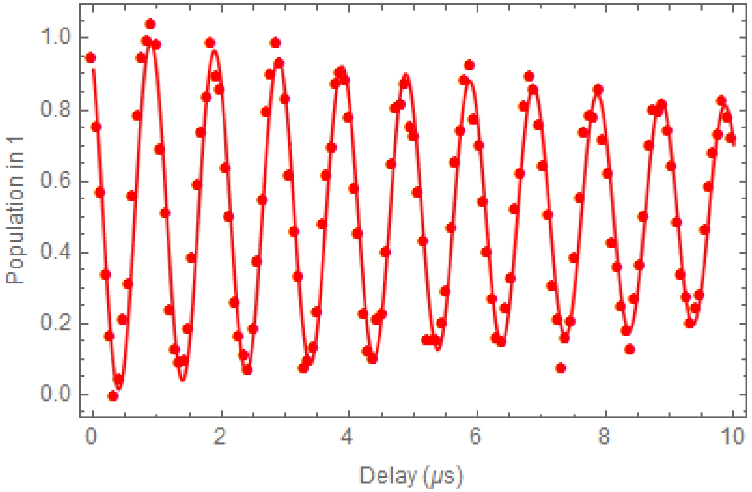
Dispersive shifts for $|1\rangle$ and $|2\rangle$ are identical

Test coherence while measuring

0-1 transition

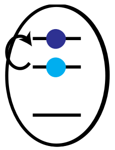


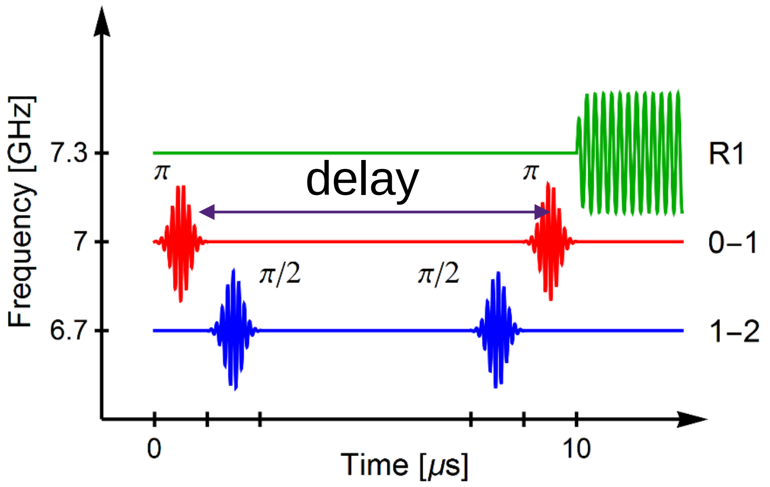


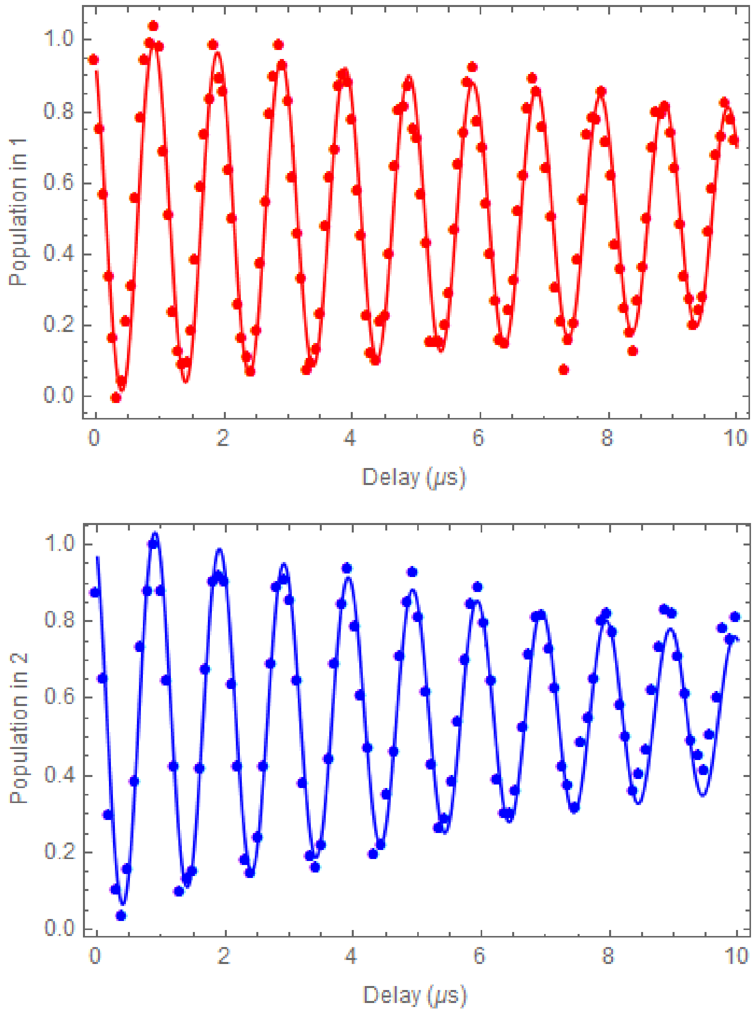


Test coherence while measuring

1-2 transition

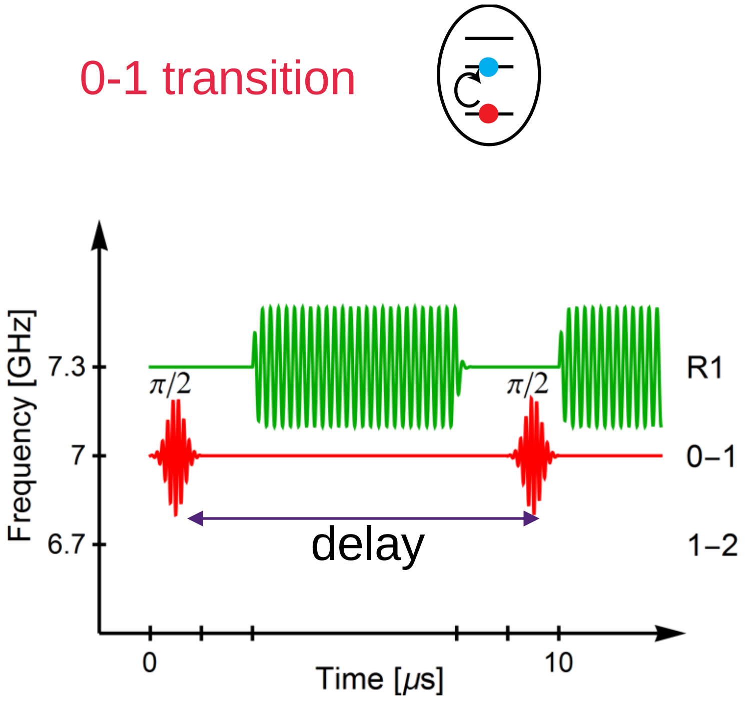


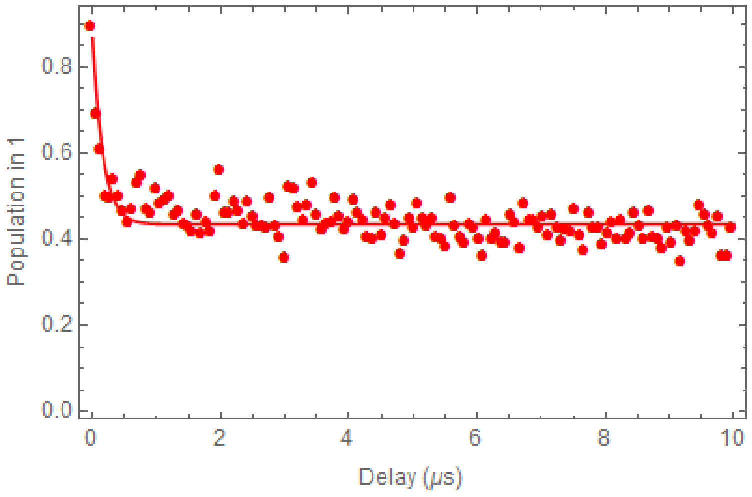




Test coherence while measuring

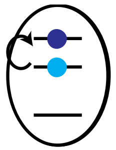
0-1 transition

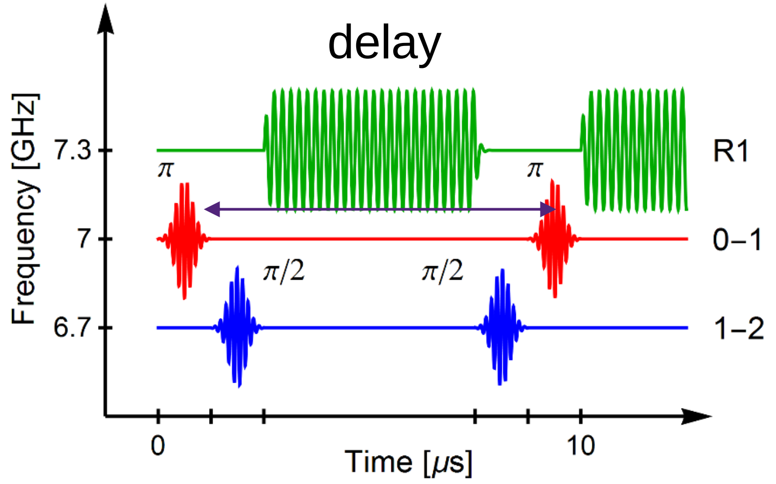


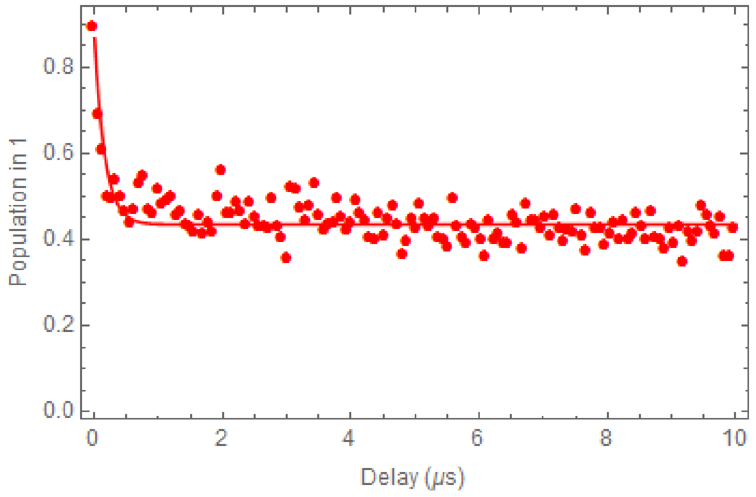


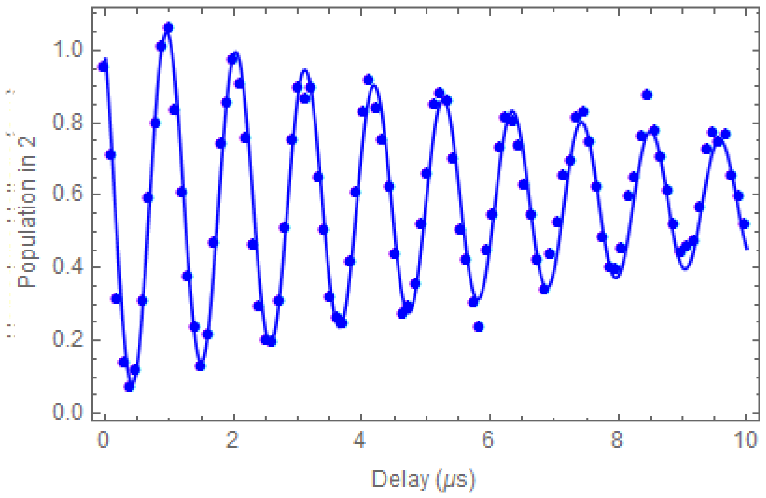
Test coherence while measuring

1-2 transition





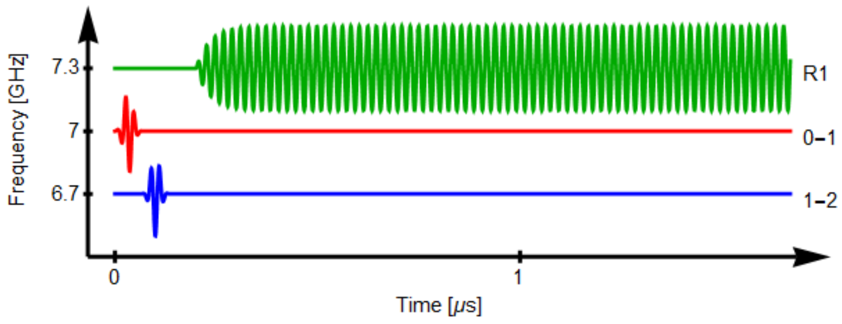


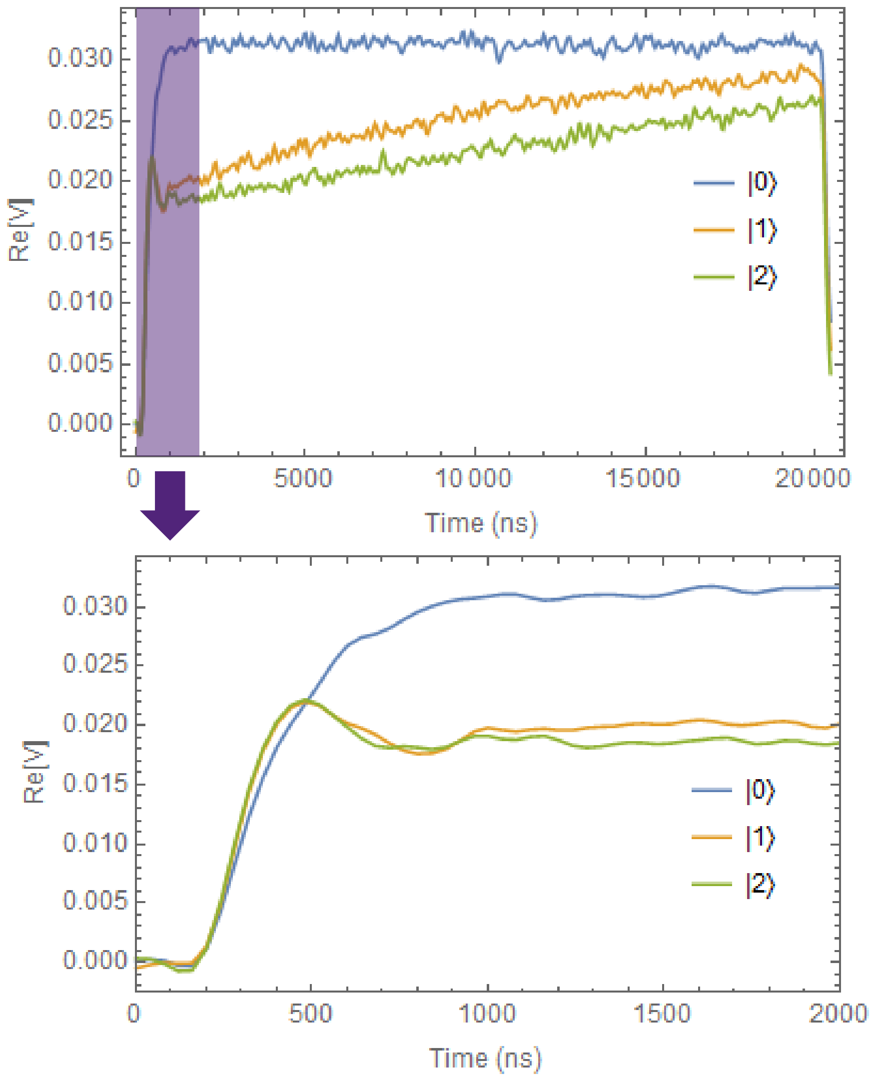


Conclusion: coherence between $|1\rangle$ and $|2\rangle$ is not affected by the measurment

Measuring cavity response in time domain

Prepare $|0\rangle, |1\rangle, |2\rangle$ and measure transmission through the cavity at fixed frequency as function of time



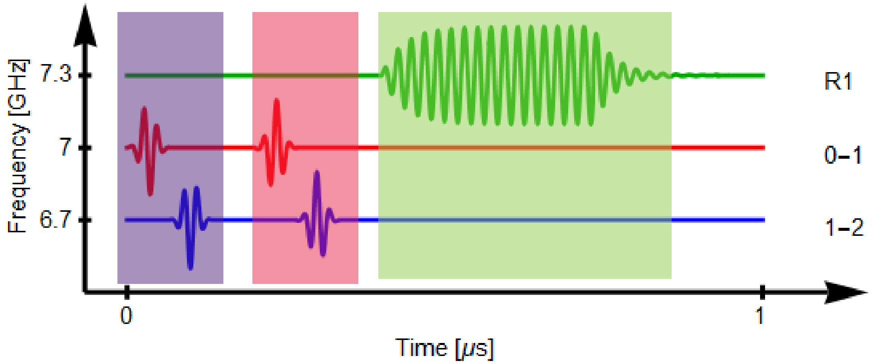


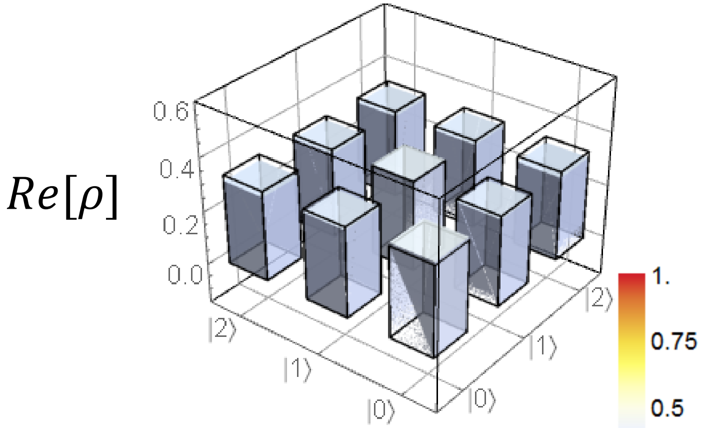
Averaged: 16384 times

Need to distinguish states with certainty within first hundreds of nanosecond if we want to have a single shot readout

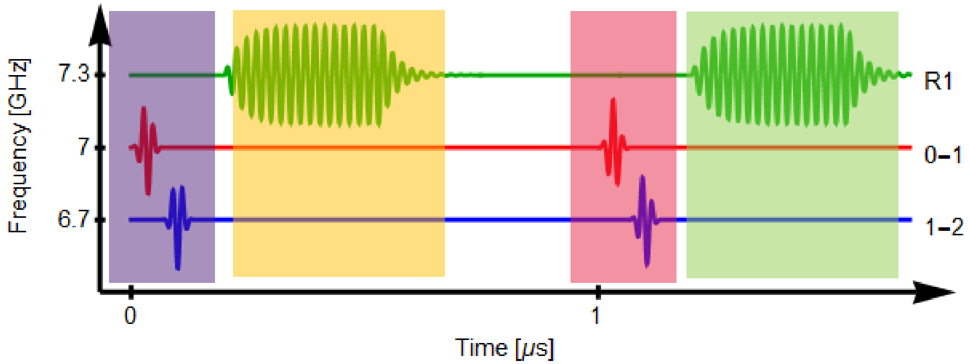
Testing readout on a state

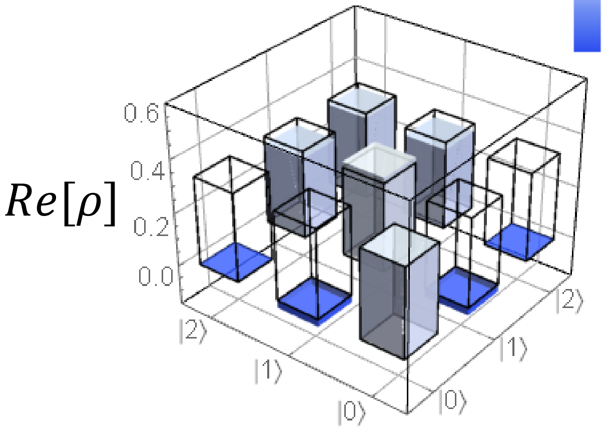
Prepare a superposition: $|\psi\rangle = c_0|0\rangle + c_1|1\rangle + c_2|2\rangle$
Do tomography of a state (measure prepared state for 9 tomography pulses):





Measure and do tomography again:





Jerger *et al. Phys. Rev. Applied* 6, 014014 (2016)

Process tomography of the readout

Arbitrary quantum process:

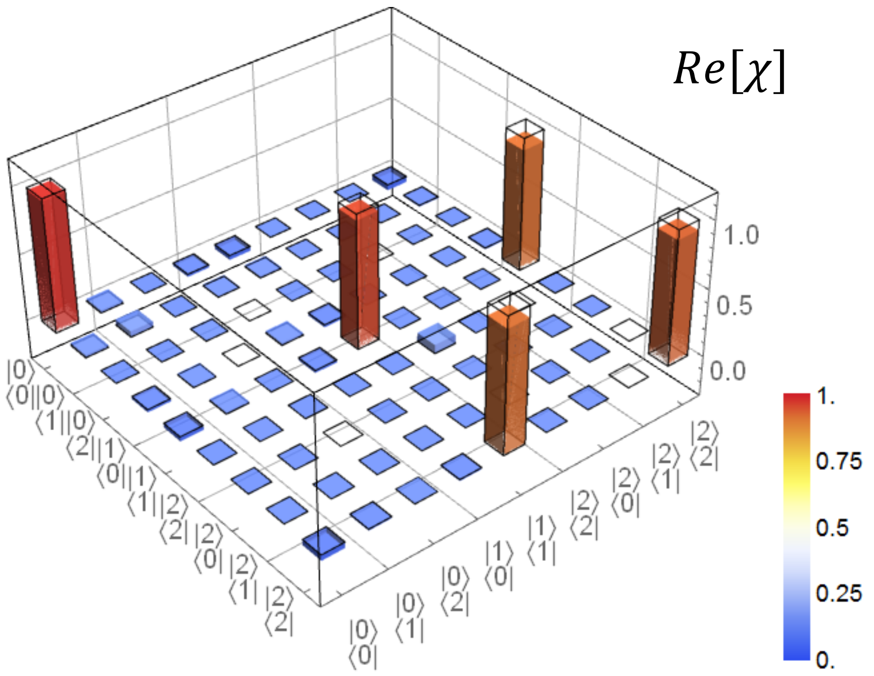
$$\rho' = \mathcal{E}(\rho)$$

decomposed into:

$$\mathcal{E}(\rho) = \sum_{mn} \tilde{E}_m \rho \tilde{E}_n^\dagger \chi_{mn}$$

$\{\tilde{E}_k\}$ is an operator basis
χ is a positive semi definite Hermitian matrix characteristic for the process

Prepare 9 superpositions and do tomography for each -> reconstruct χ-matrix of the process



F = 97% to the binary projective measurement described $\{M_{|0\rangle}\} = \{|0\rangle\langle 0|, I - |0\rangle\langle 0|\}$

Jerger *et al. Phys. Rev. Applied* 6, 014014 (2016)

Testing KCBS inequality

Quantum mechanics and non-contextual

Local realism is a part of non-contextual realism:
outcome of a measurement depends only on the current state of the system, and not on which other measurements, if any, are performed in conjunction with it (the measurement context).

1967 Kochen and Specker: proved that non-contextual realism is in contradictions with outcomes of QM (proven for spin-1 no entanglement)

2008 Klyachko, Can, Binicioglu and Shumovsky (KCBS) found the simplest recipe to demonstrate contextuality with a qutrit (no entanglement but state dependent)

$$\langle \boldsymbol{A_1 A_2} \rangle + \langle \boldsymbol{A_2 A_3} \rangle + \langle \boldsymbol{A_3 A_4} \rangle + \langle \boldsymbol{A_4 A_5} \rangle + \langle \boldsymbol{A_5 A_1} \rangle \geq -3$$

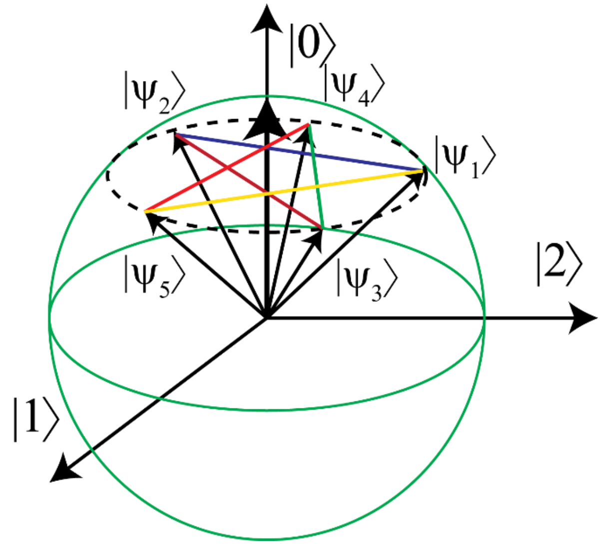
2012 Yu and Oh, state independent test for a qutrit

- One of the most fundamental property of quantum mechanics not requiring composite systems, entanglement, non-locality and specific state

2014 Howard, Wallman, Veitch & Emerson: contextuality – responsible for exponential speedup of a quantum computer

KCBS inequality

Define five sequentially pair-wise orthogonal measurement directions (not possible for a qubit)



If we prepare the system in $|0\rangle$ the result of the five pairs of measurements give

$$\langle \boldsymbol{A_1 A_2} \rangle + \langle \boldsymbol{A_2 A_3} \rangle + \langle \boldsymbol{A_3 A_4} \rangle + \langle \boldsymbol{A_4 A_5} \rangle + \langle \boldsymbol{A_5 A_1} \rangle = -3.994 < -3$$

The outcomes are not predetermined: cannot be explained by any non-contextual hidden variable theory

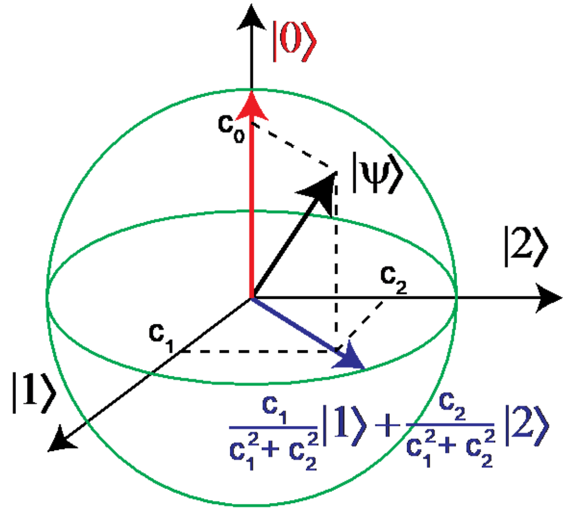
No non-locality, composite system, or entanglement are involved

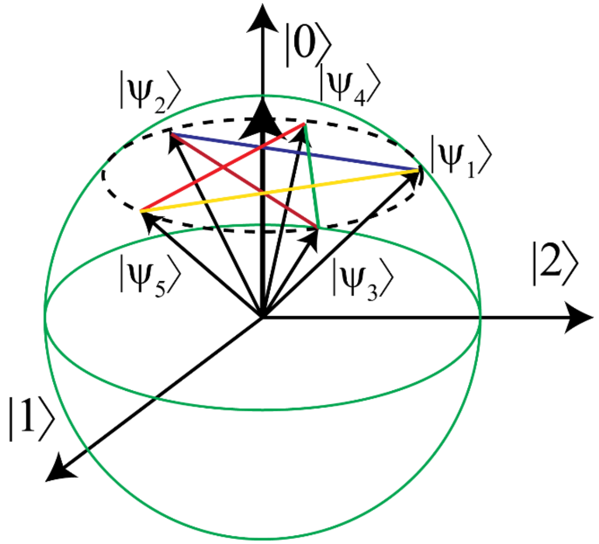
Requires compatibility test (the measurements have to be degenerate)

Creating five pairs of compatible measurements

Dispersive read out at sweet spot: measurement along $\mathrm{M}_{|0\rangle}$

Needed: measurement along $\mathrm{M}_{|\psi_1\rangle}$, $\mathrm{M}_{|\psi_2\rangle}$, $M_{|\psi_3\rangle}$, $M_{|\psi_4\rangle}$, $\mathrm{M}_{|\psi_5\rangle}$





Solution: rotating a state not the measurement basis

Quantum projective measurement: rotation of the state

The result of the measurement is one of the eigenvalues of λ with probability

$$p_\lambda = \mathrm{Tr}[\rho\hat{\Pi}_\lambda]$$

After measurement:

$$\rho_\lambda = \hat{\Pi}_\lambda\rho\hat{\Pi}_\lambda / p_\lambda$$

Known as *projection postulate, state collapse* or *state reduction.*

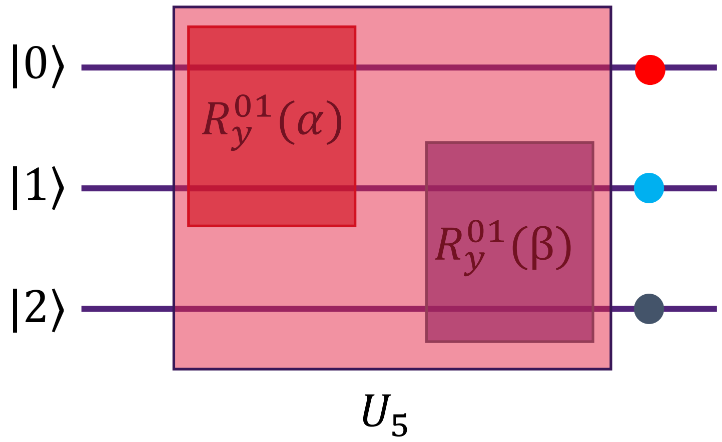
If we perform a unitary rotation before and after the measurement $\rho' = U\rho U^+$

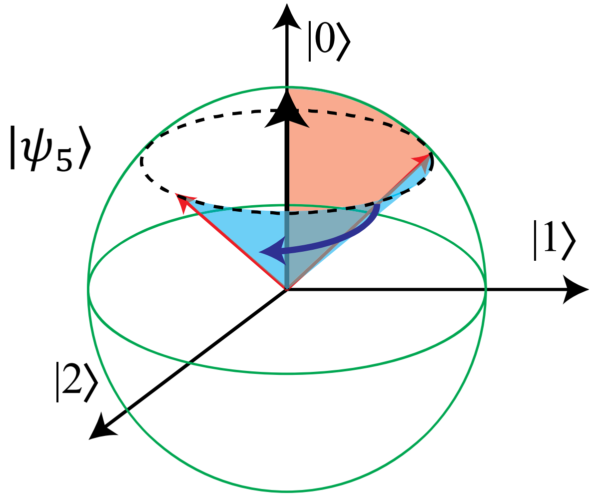
then p_λ and ρ_λ will be formally the same as performing measurement with the rotated measurement operator $\hat{\Pi}_\lambda{}' = U^+\hat{\Pi}_\lambda U$

Generating KCBS states

How to generate $|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle, |\psi_4\rangle, |\psi_5\rangle$?

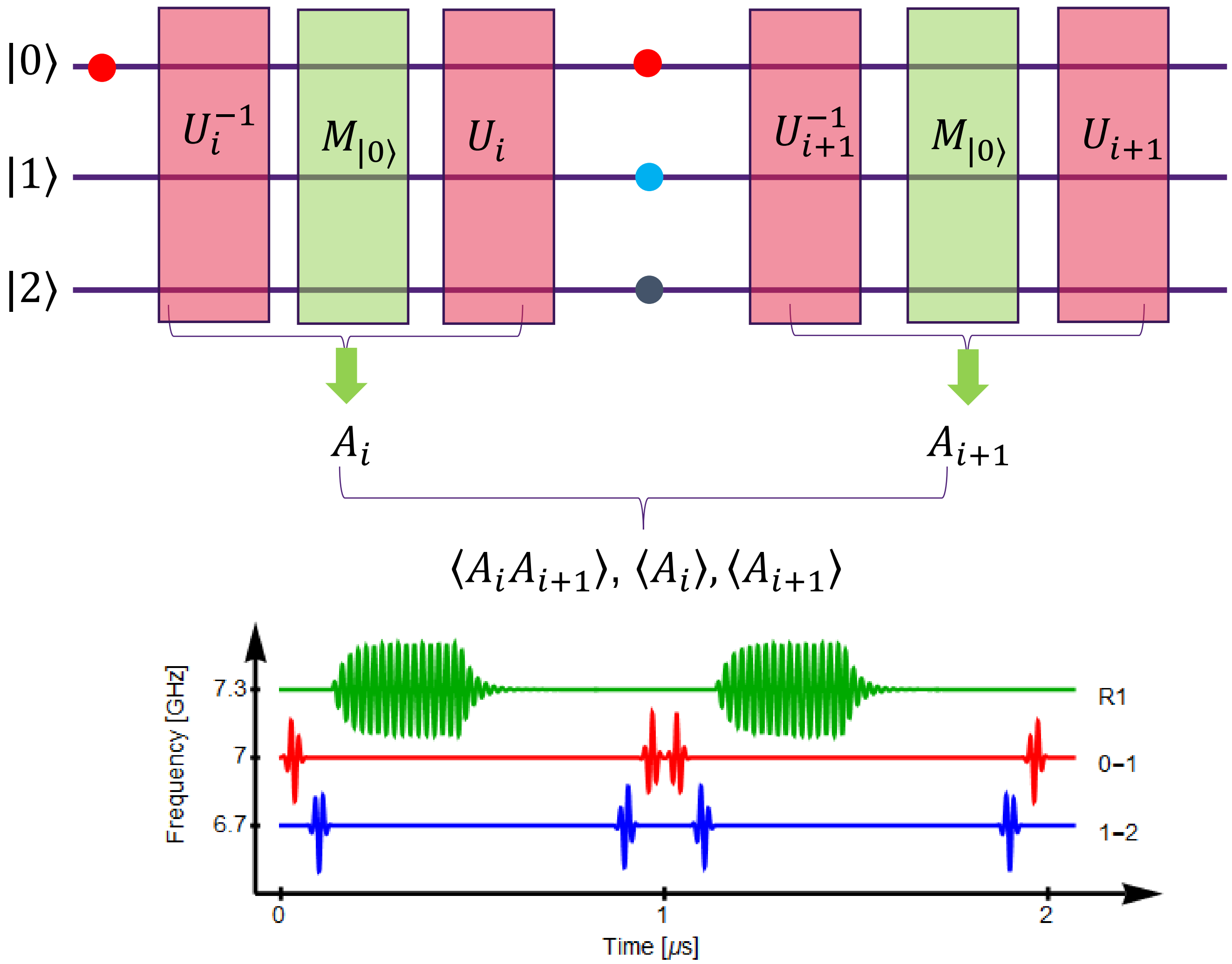
Apply rotation for 0-1 transition to all other states$|\psi_2\rangle, |\psi_3\rangle, |\psi_4\rangle, |\psi_5\rangle$





State	Rotations	U	F
$\vert\psi_1\rangle$	$R_y^{01}(0.53\pi)$	U_1	~0.99
$\vert\psi_2\rangle$	$R_y^{01}(0.53\pi)\ R_y^{02}(1.6\ \pi)$	U_2	~0.99
$\vert\psi_3\rangle$	$R_y^{01}(-0.53\pi)\ R_y^{02}(1.2\ \pi)$	U_3	~0.99
$\vert\psi_4\rangle$	$R_y^{01}(0.53\pi)\ R_y^{02}(0.8\ \pi)$	U_4	~0.99
$\vert\psi_5\rangle$	$R_y^{01}(-0.53\pi)\ R_y^{02}(0.4\ \pi)$	U_5	~0.99

Measuring correlations $\langle A_i A_{i+1} \rangle$



Violation of the KCBS Inequality

Measured correlations:

$$\langle \boldsymbol{A_1 A_2} \rangle + \langle \boldsymbol{A_2 A_3} \rangle + \langle \boldsymbol{A_3 A_4} \rangle + \langle \boldsymbol{A_4 A_5} \rangle + \langle \boldsymbol{A_5 A_1} \rangle = -3.51(2)$$

Adjusted threshold:

$$-(3 + |\varepsilon_{12}| + |\varepsilon_{32}| + |\varepsilon_{34}| + |\varepsilon_{54}| + |\varepsilon_{51}|) = -3.38(7)$$

KCBS inequality violated (by more than 49 standard deviations).

The most comprehensive experimental evidence in a scenario without entanglement.

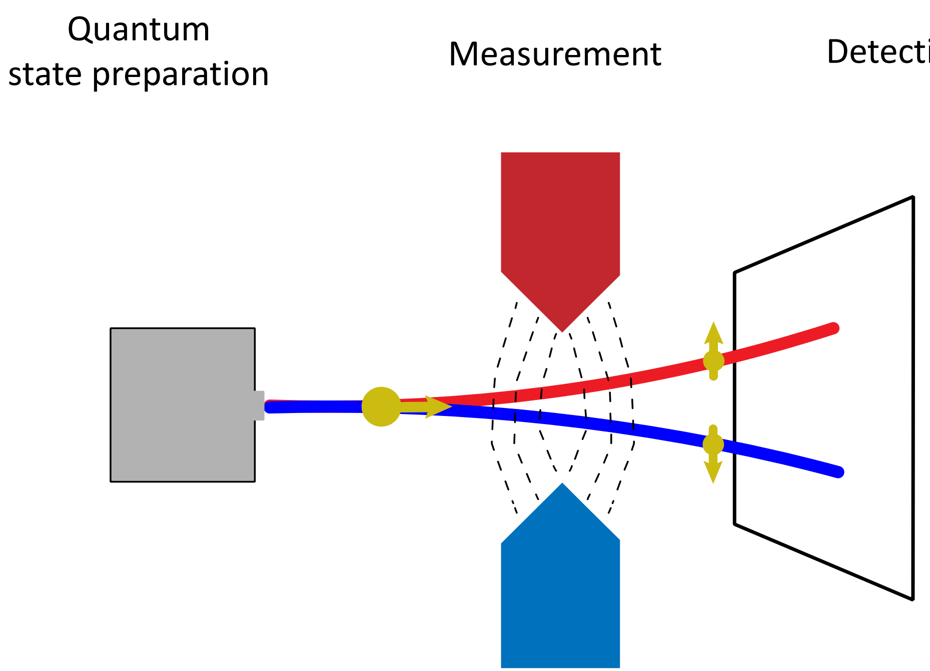
(i, j)	$\langle A_i A_i \rangle$	$\langle A_j \rangle$ (1st)	$\langle A_j \rangle$ (2nd)	ε_{ij}
(1, 2)	-0.70(3)	0.10(0)	0.18(5)	0.08(5)
(2, 3)	-0.70(2)	0.10(8)	0.17(8)	0.07(0)
(3, 4)	-0.69(5)	0.10(8)	0.18(5)	0.07(8)
(4, 5)	-0.70(5)	0.10(6)	0.18(3)	0.07(8)
(5, 1)	-0.70(9)	0.10(3)	0.17(9)	0.07(6)
$\sum$	-3.51(2)			0.38(7)

M. Jerger et al., Nature Comm. 7, 12930 (2016)

Quantum rifling and protection from collapse on demand using qubit manipulation

Continuous measurement

Quantum projective measurement: recap



Before:

$$|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$$

After:

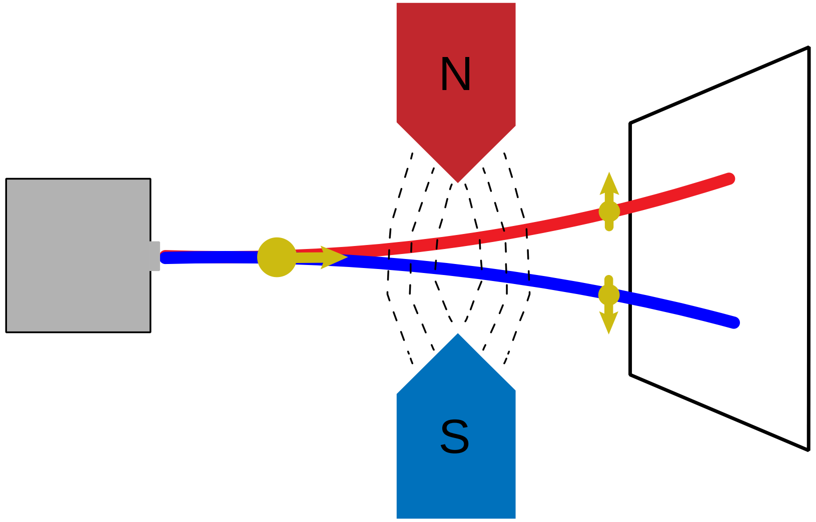
50% $|0\rangle$ and 50% $|1\rangle$

Projective quantum measurement:

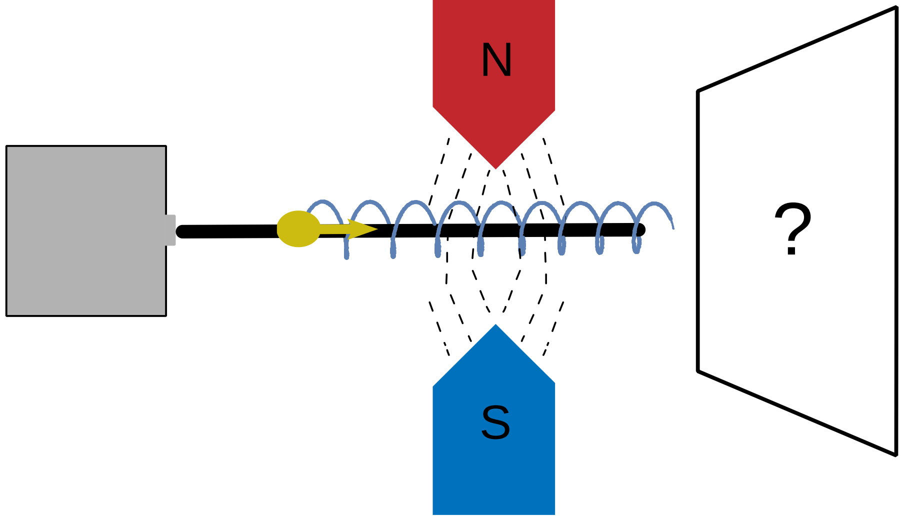
- Outcomes are discrete
- Quantum wave-function collapses after measurement

Quantum projective measurement: recap

Conventional projective measurement



Continuous measurement



What happens if the state is evolving during measurement?

- What will be the outcomes?
- What will happen to the quantum state?

Continuous quantum measurement

Extensively studied theoretically and experimentally

- Theoretically studied as early as 1988 (G. J. Milburn, JOSA B 5, 1317 (1988)).
- Studied extensively in the context of an electron transport in a double-dot and a point contact in 2000s (A.N. Korotkov, Phys. Rev. B 60, 5737 (1999), A. N. Korotkov and D. V. Averin, Phys. Rev. B 64, 165310 (2001), and many others).
- Has been studied with superconducting qubits since the late 2000s (J. Gambetta *et. al.* Phys. Rev. A **77** 012112 (2008) and many others).

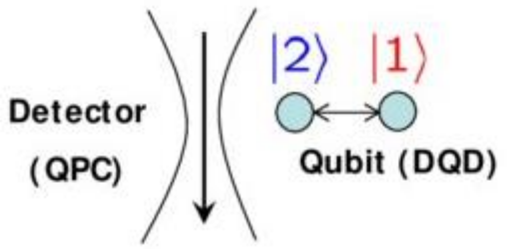
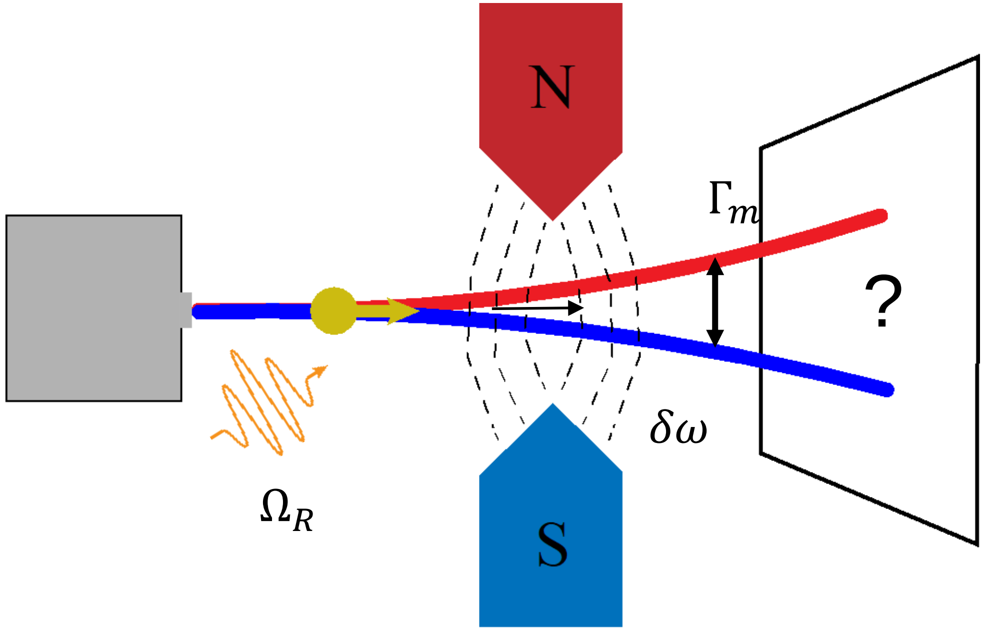


Figure by A. N. Jordan

Continuous measurement of a driven qubit

Induce Rabi oscillation of the spin and send the spin through the apparatus



Driving and projecting compete with each other

The spin trajectory will depend on three parameters: $\delta\omega, \Omega_R, \Gamma_m$

Continuous measurement of a driven qubit

Numerical simulation of the qubit population using quantum trajectories

$\Omega_R \gg \Gamma_m \rightarrow$ sub-Zeno regime:

emergence of Rabi oscillations

$\Omega_R \sim \Gamma_m \rightarrow$ diffusive trajectory

$\Omega_R \ll \Gamma_m \rightarrow$ Quantum Zeno regime:

quantum jumps

See next section

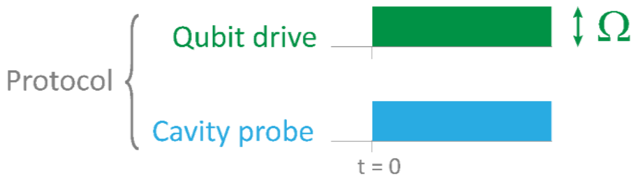
Ω_R and Γ_m are always less than $\delta\omega$.

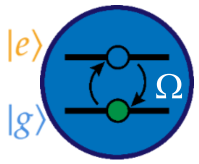
Regime of strong driving $\Omega_R \gg \Gamma_m, \delta\omega$ has never been explored!

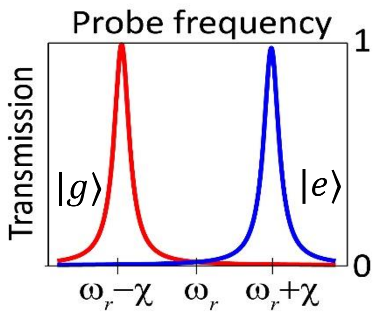
A.N. Korotkov, Phys. Rev. B 60, 5737 (1999)

Probing cavity for driven qubit

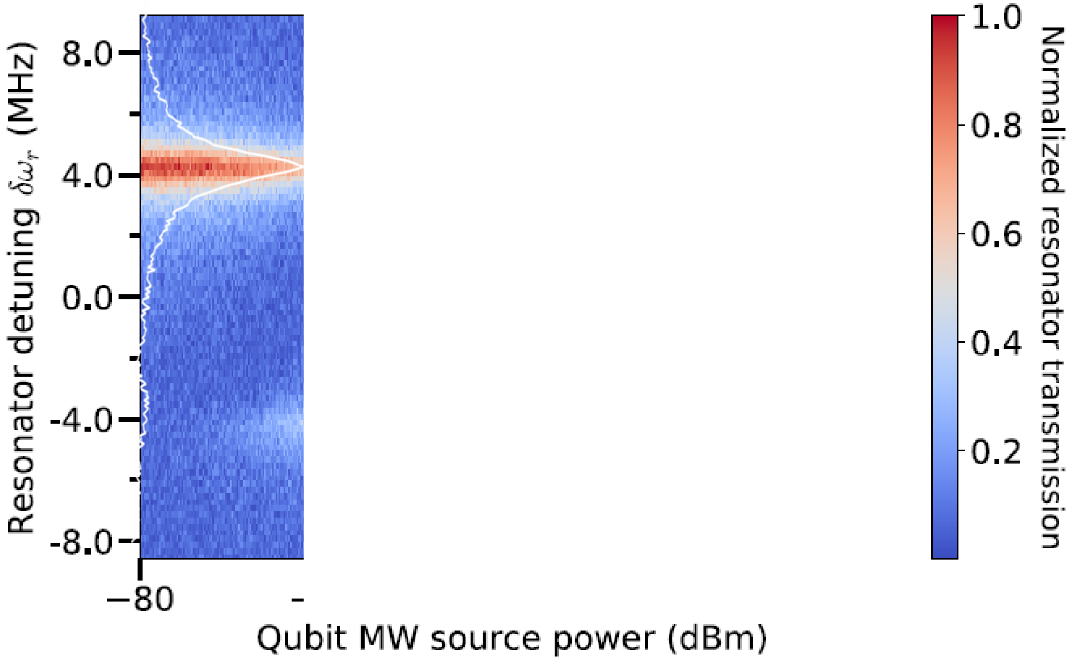
Welcome to Twin Peaks

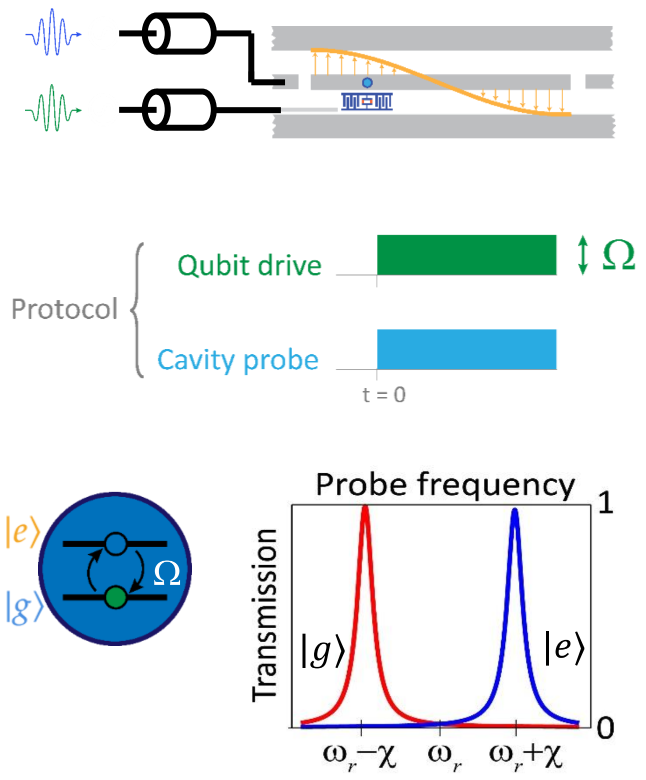






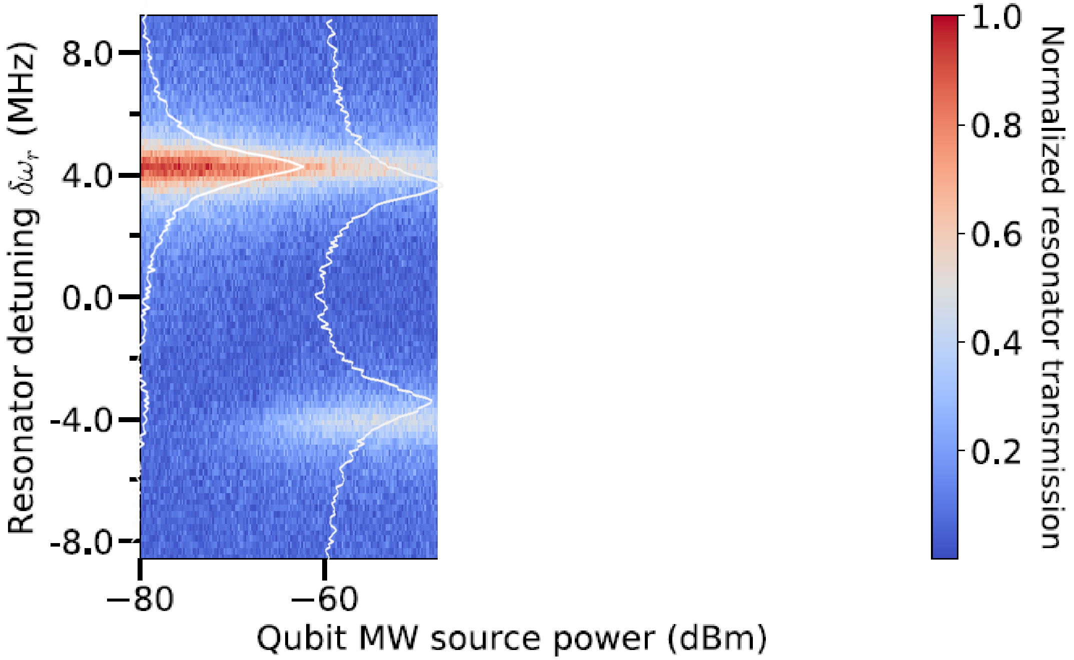
Welcome to Twin Peaks

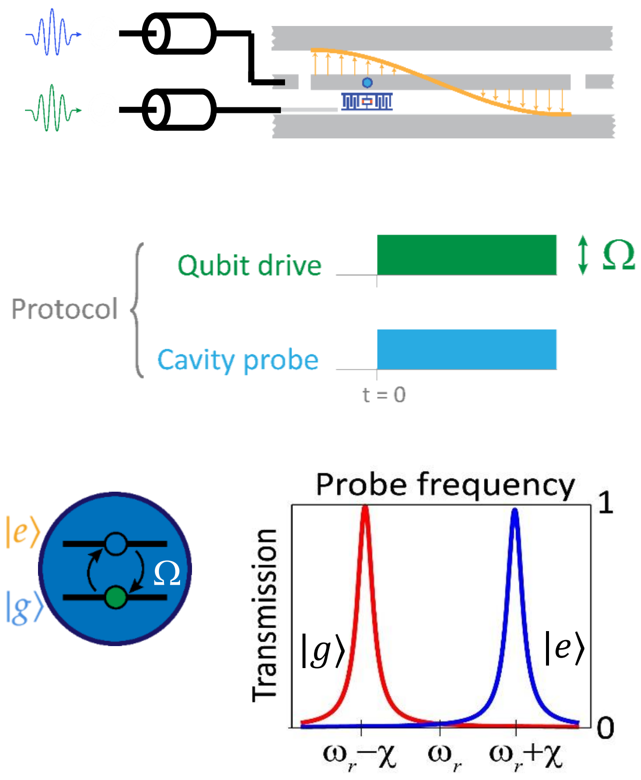




What happens if we increase qubit driving?

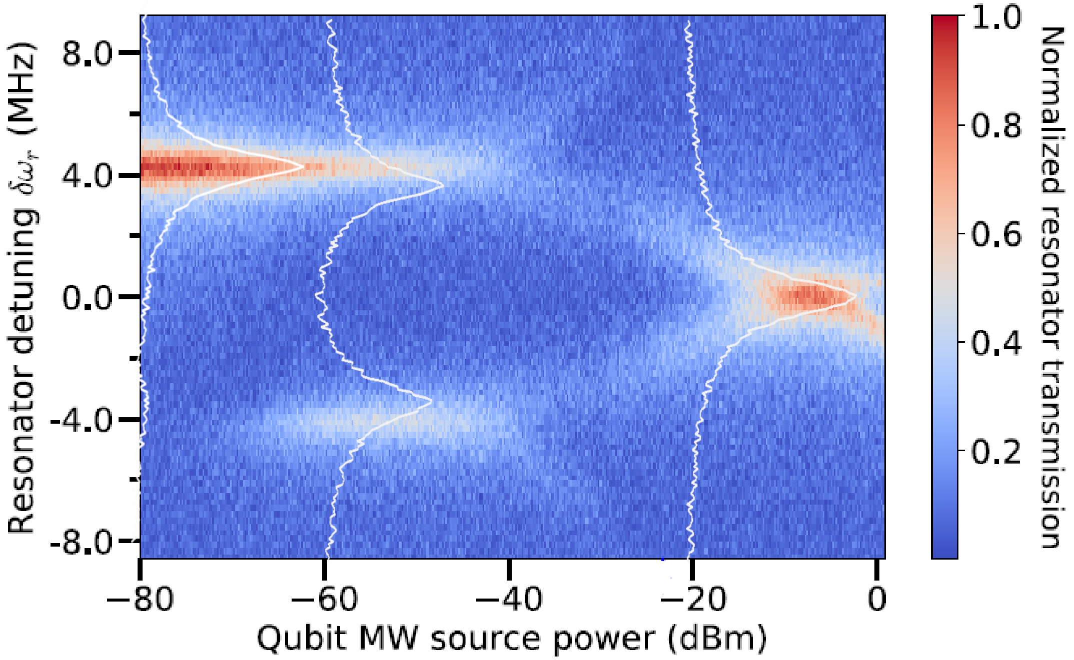
Welcome to Twin Peaks

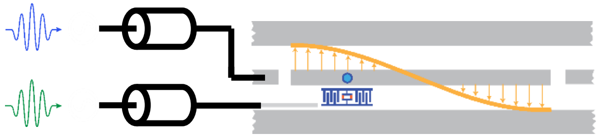


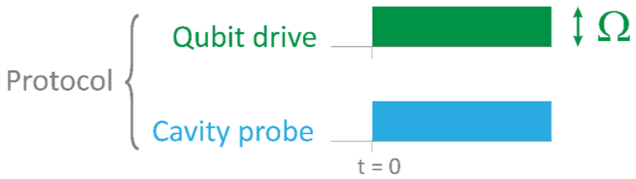


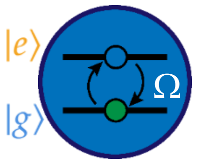
What happens if we increase qubit driving?

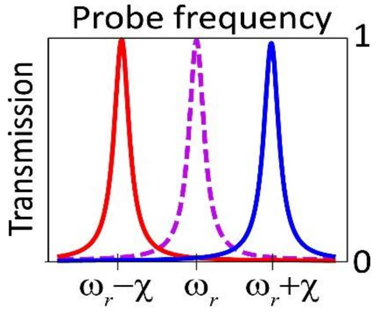
“Quantum rifling” regime



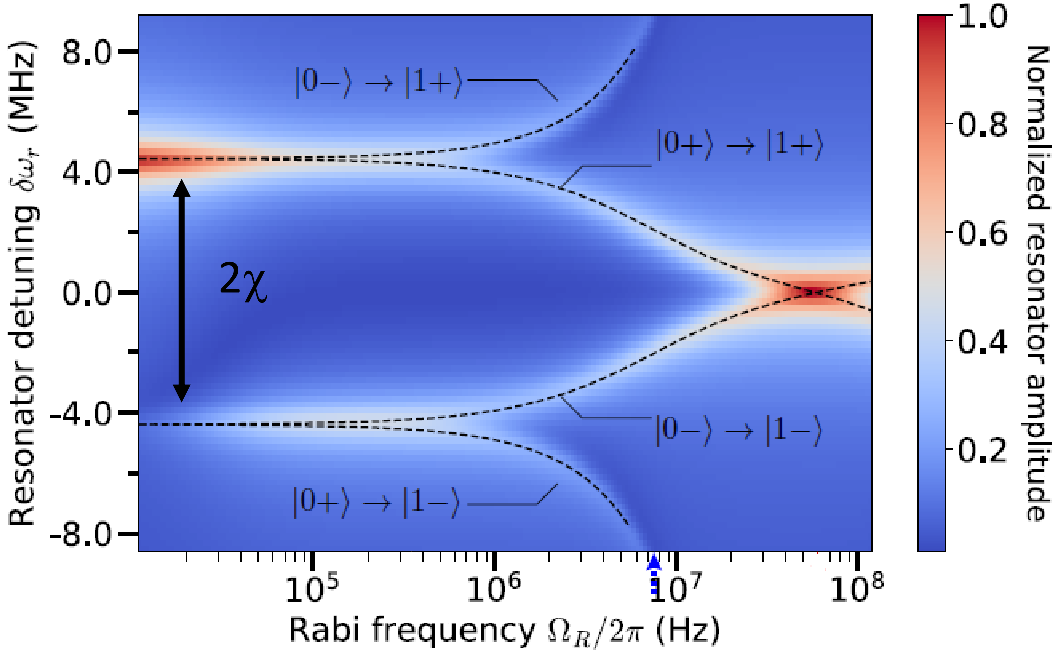








It's All About Dressed States

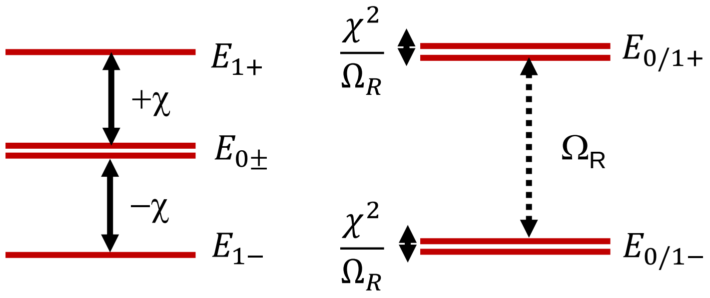


Analytical solution of driven system:

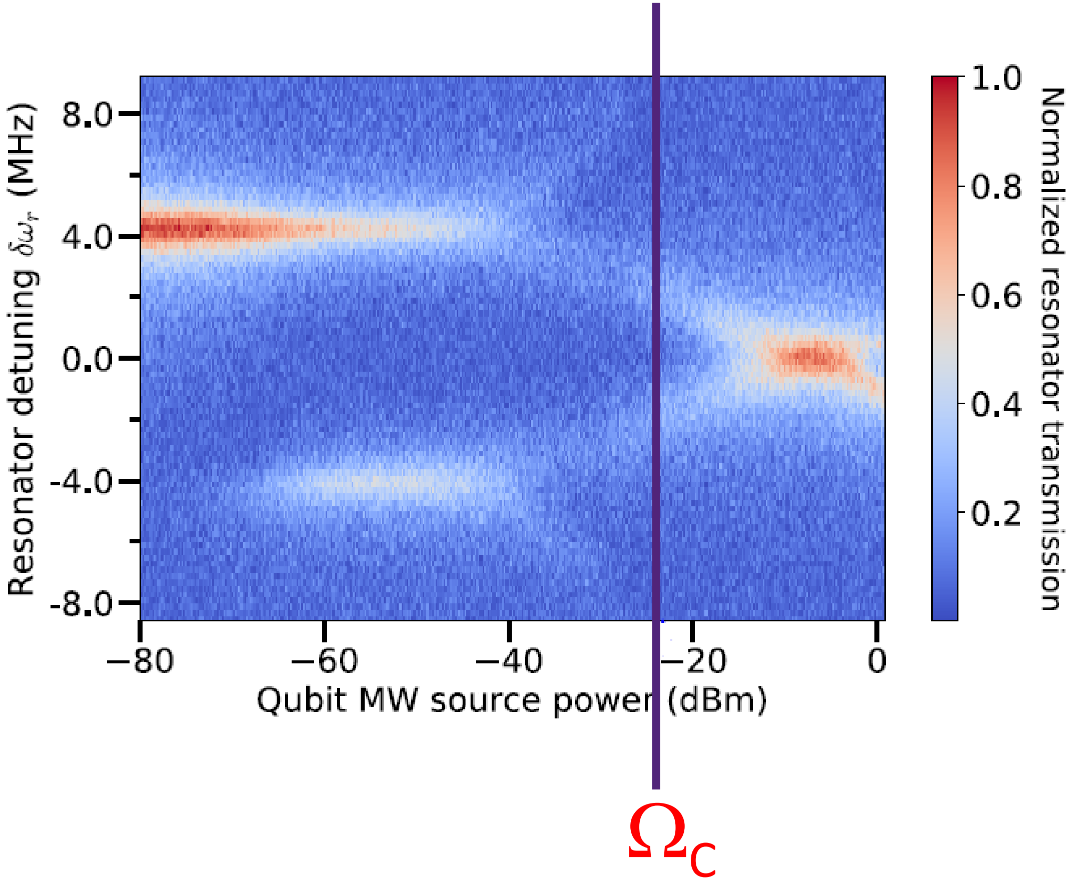
$$\begin{cases} |0\,\pm\rangle \sim |g,0\rangle \pm |e,0\rangle \\ E_{0\pm} = \pm\Omega_R \end{cases}$$

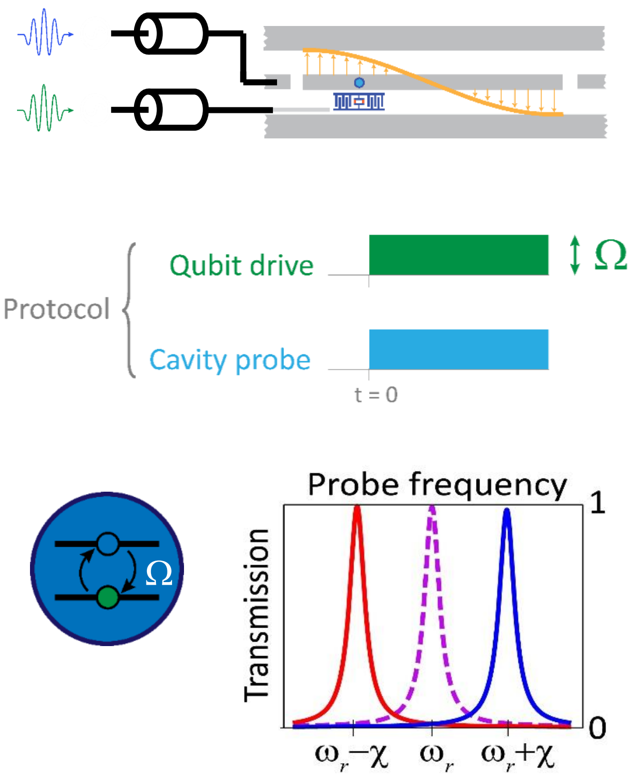
$$\begin{cases} |1\,\pm\rangle \sim \dfrac{2\chi \pm \sqrt{4\chi^2 + \Omega_R^2}}{\Omega_R}|g,1\rangle \pm |e,1\rangle \\ E_{1\pm} = \pm\frac{1}{2}\sqrt{4\chi^2 + \Omega_R^2} \end{cases}$$

- $\lim \dfrac{\Omega_R}{\chi} \to 0$
- $\lim \dfrac{\Omega_R}{\chi} \to \infty$



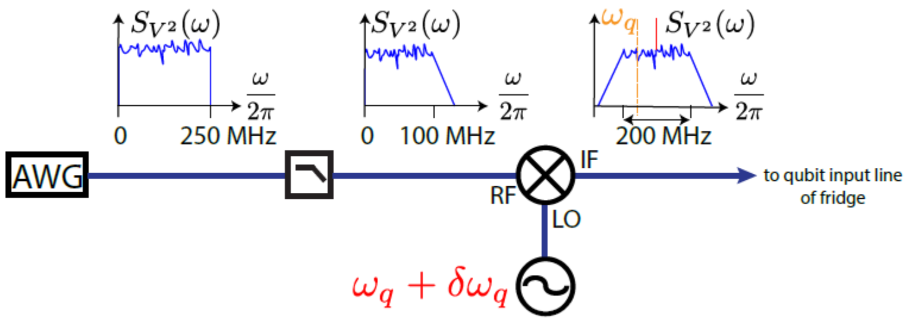
“Quantum rifling” regime

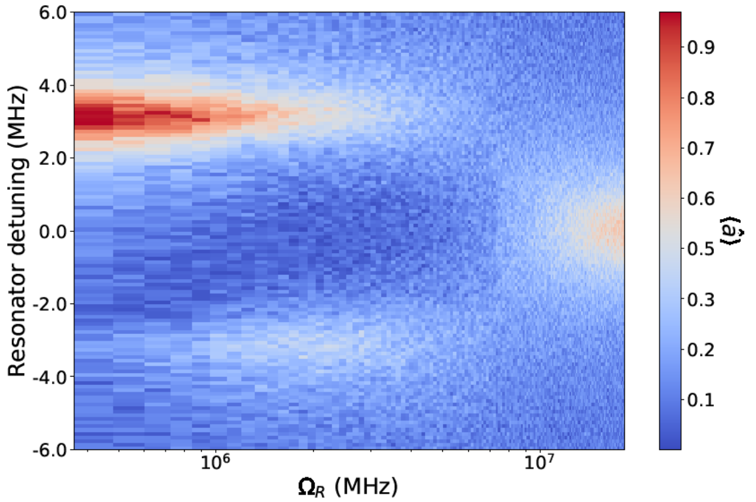


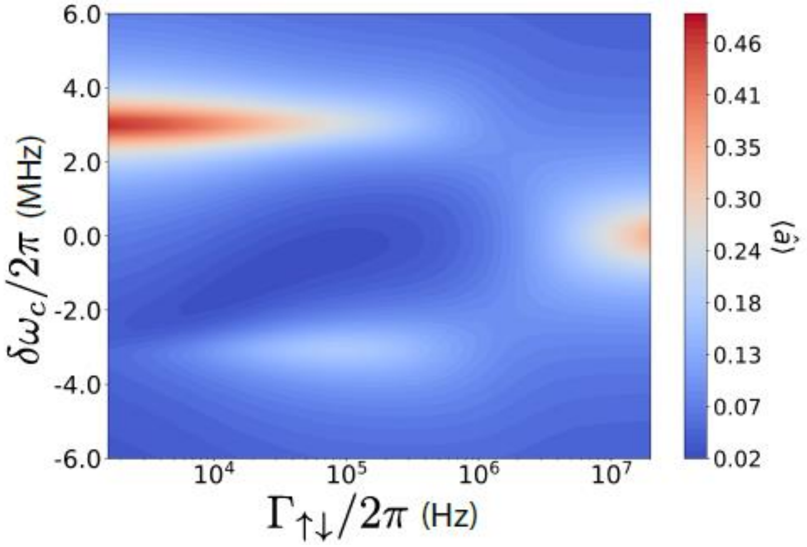


Conditions for single peak: $\Omega_R > \frac{\chi^2}{\kappa}$

Incoherent dynamics





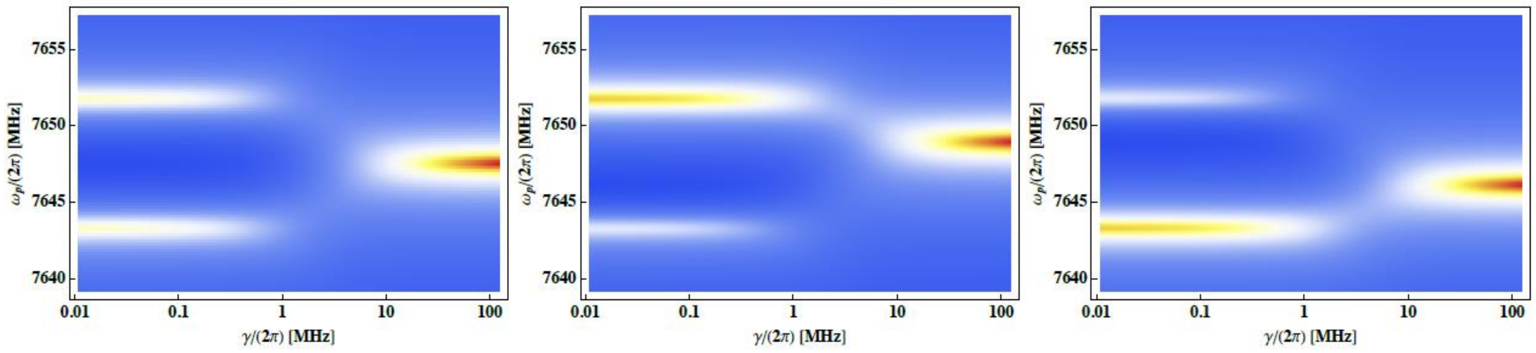


Akin motional narrowing of a qubit line (see J. Li et. al. Nature Comm.)

Incoherent dynamics

Simulate asymmetric excitation and relaxation rates

$\Gamma_\uparrow = \Gamma_\downarrow, \overline{\langle\sigma_z(t)\rangle}$=0 $\Gamma_\uparrow < \Gamma_\downarrow, \overline{\langle\sigma_z(t)\rangle}$<0 $\Gamma_\uparrow > \Gamma_\downarrow, \overline{\langle\sigma_z(t)\rangle}$ >0



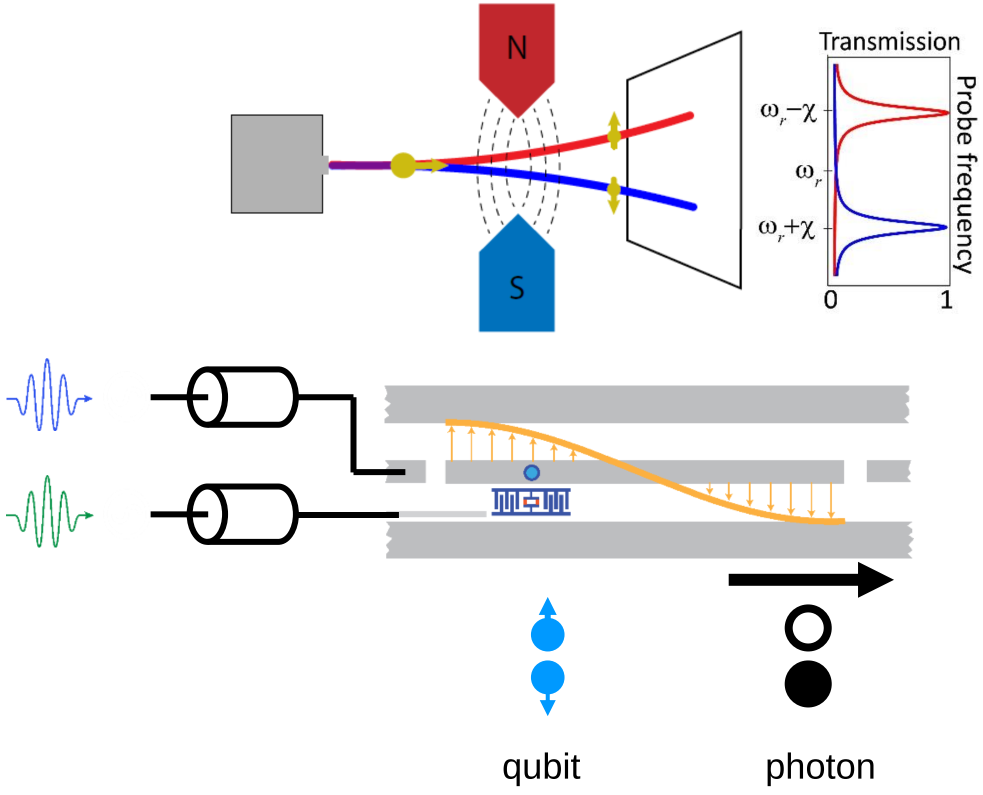
Classical measurement:

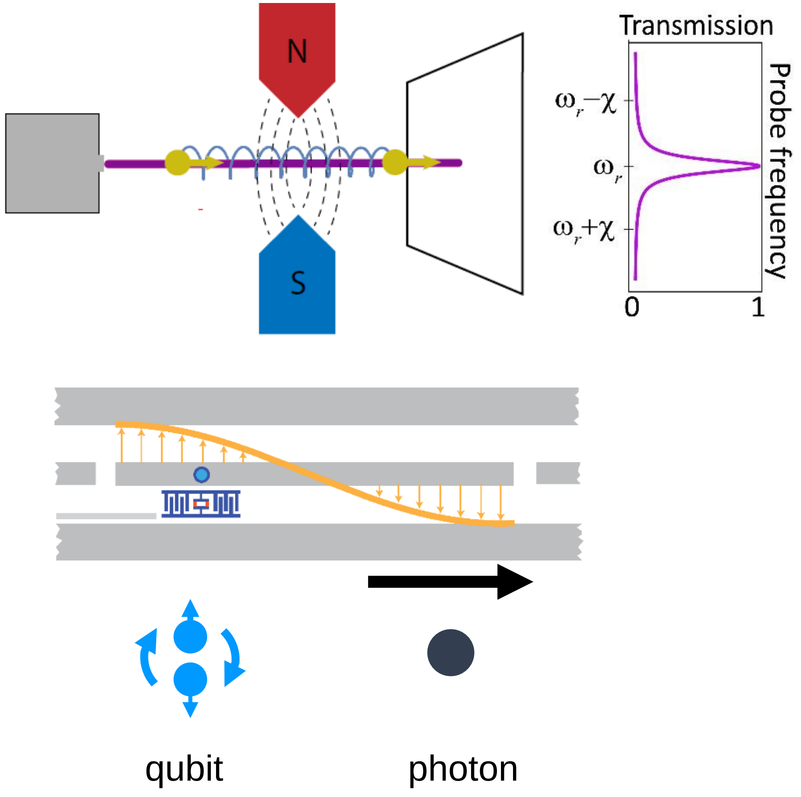
- Returns $\overline{\langle\sigma_z(t)\rangle}$. Any number between ± 1
- No state collapse

Protecting from collapse on demand

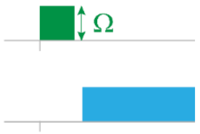
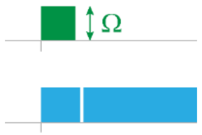
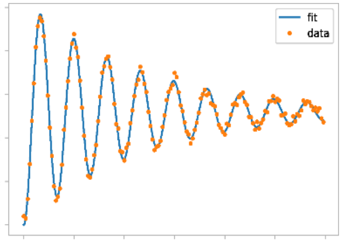
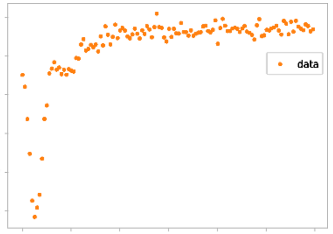
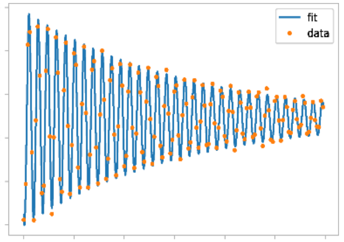
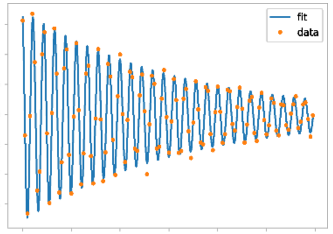
Why useful?

It does not give any information about the quantum state. Will the qubit experience measurement back action?



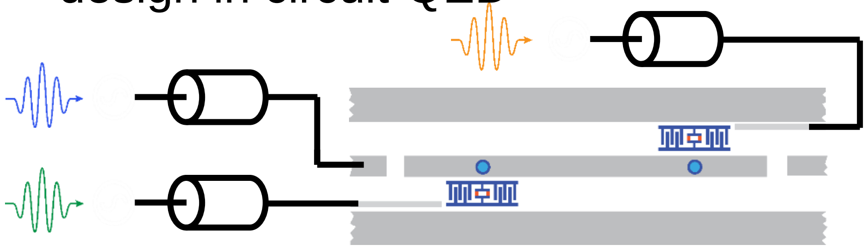


Qubit Protected From Measurement

	Standard Rabi drive	Rifling (Rabi + read out)
Protocol	Qubit drive Cavity probe 	
Low drive power $\Omega < \Omega_C$		
High drive power $\Omega > \Omega_C$		

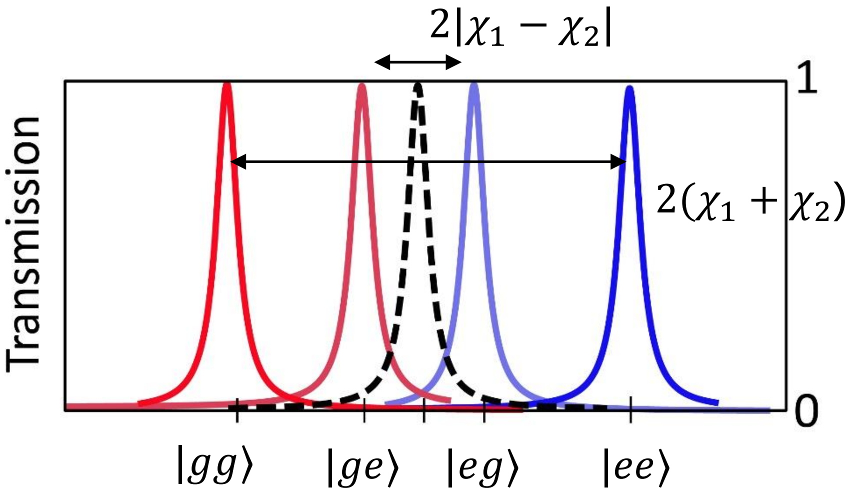
Joint dispersive readout

Many qubits coupled to one detector is a common (some times the only possible) design in circuit-QED



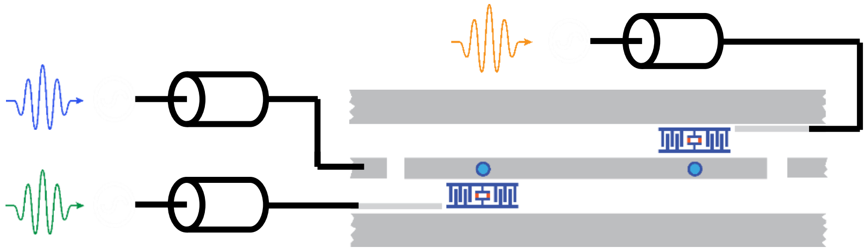
Dispersive shifts are different:

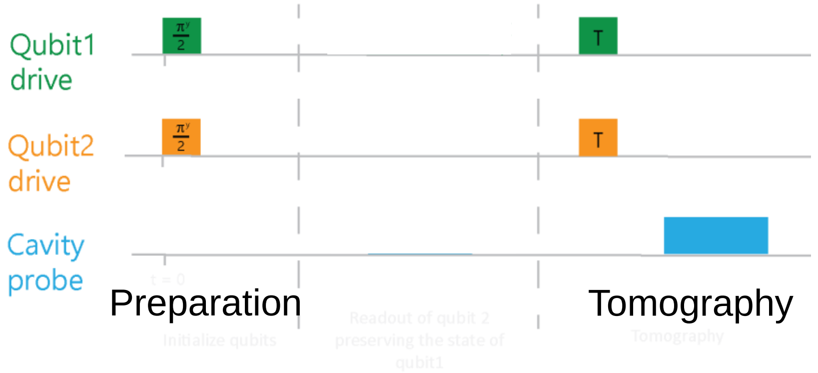
- Reconstruction of a joint qubits state is possible

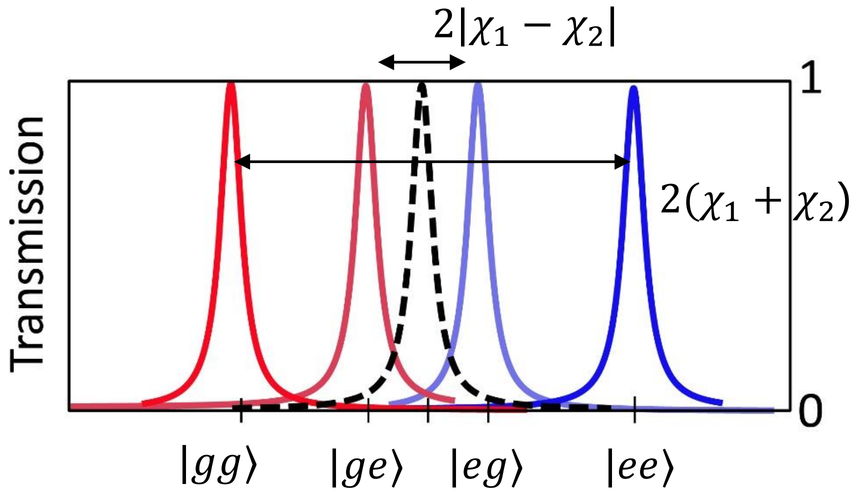


Two-qubit tomography

Apply different tomography pulses to rotate measurement basis and collect enough statistics to reconstruct the density matrix

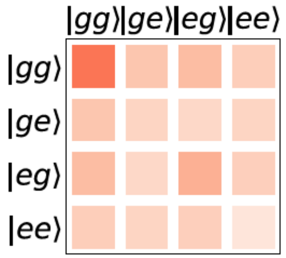






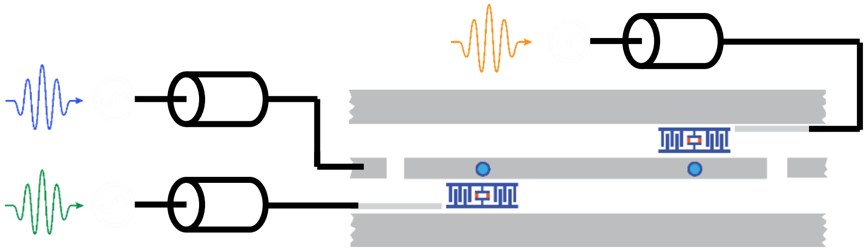
Example:

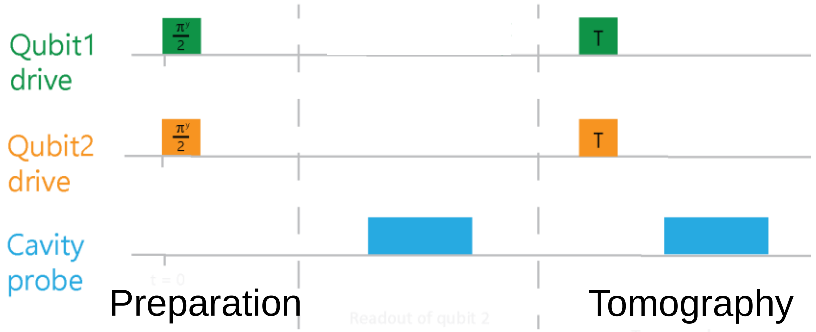
density matrix of a complete superposition after waiting time

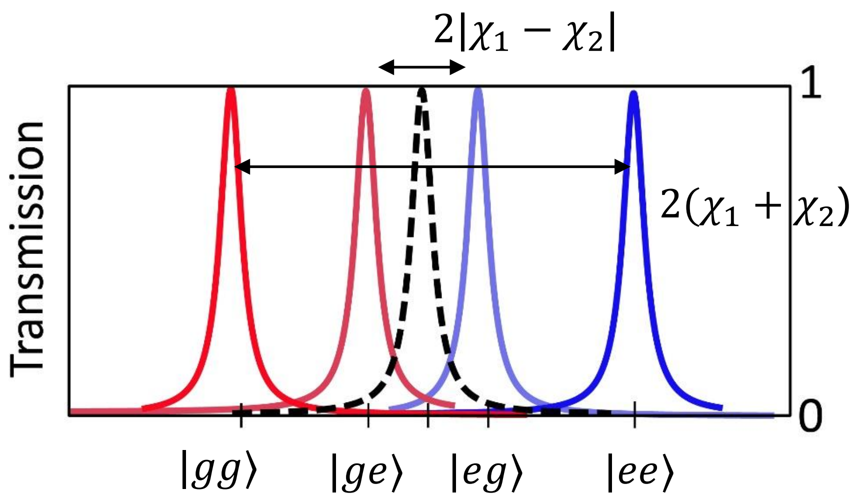


Two-qubit tomography

Apply different tomography pulses to rotate measurement basis and collect enough statistics to reconstruct the density matrix

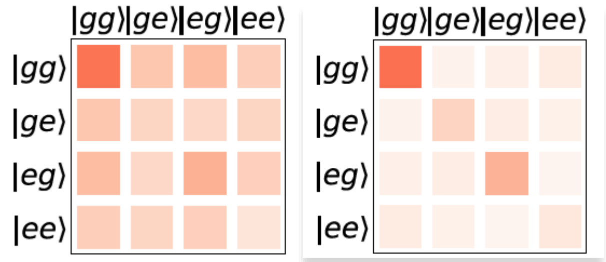






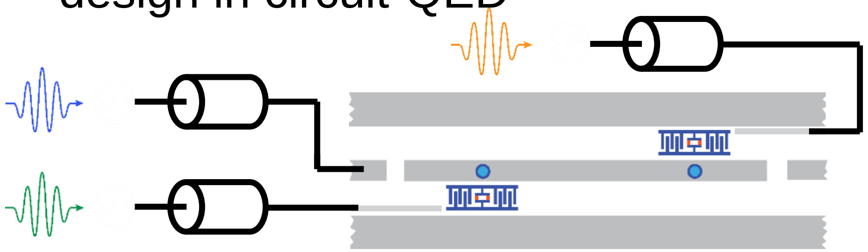
Example:

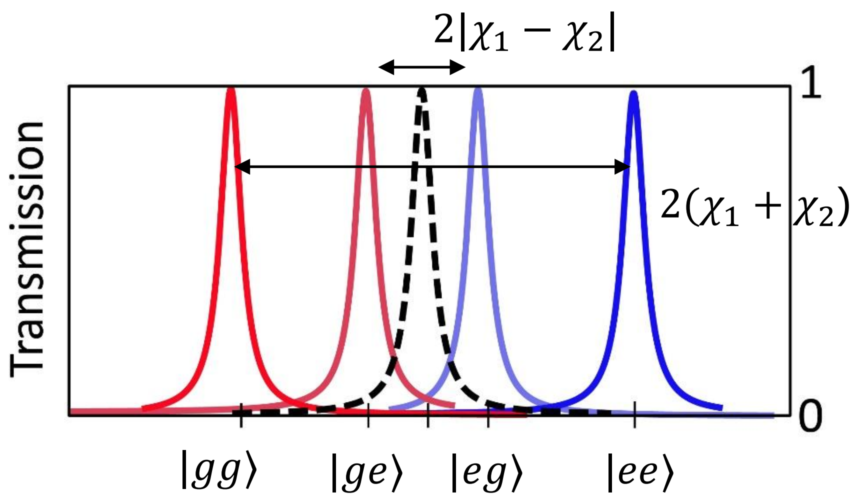
density matrix of a complete superposition after measurement



Joint dispersive readout

Many qubits coupled to one detector is a common (some times the only possible) design in circuit-QED





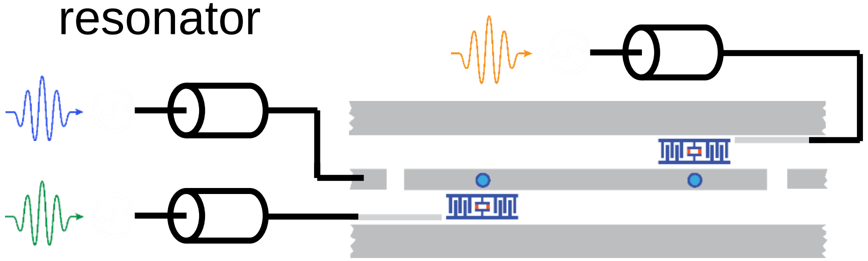
Dispersive shifts are different:

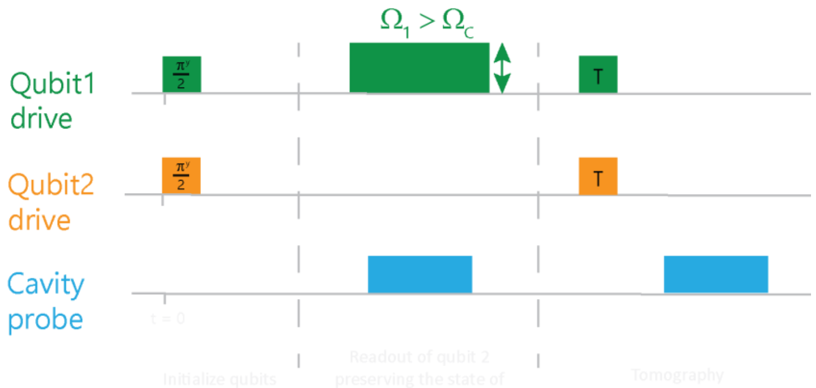
- Reconstruction of a joint qubits state is possible
- Quantum states of both qubits collapses due to measurement.

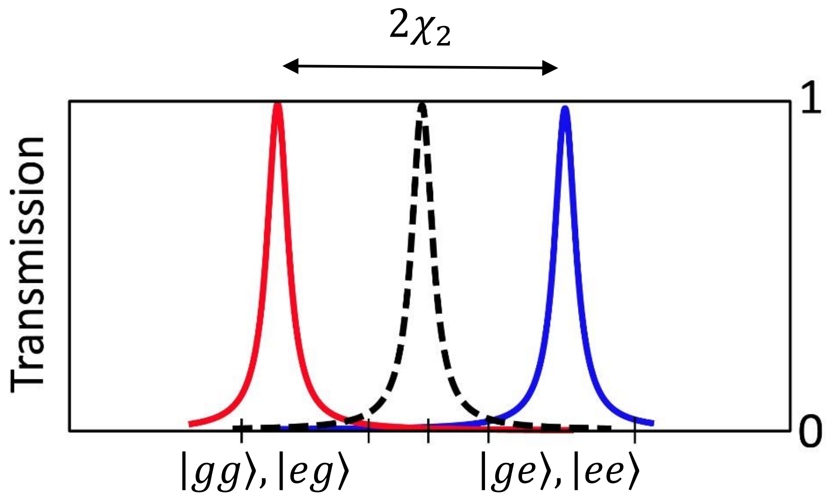
What if we want to measure a state of one qubit not affecting another qubit?

Arbitrary Multi-Qubit Multiplexing

Applying rifling pulse to the first qubit eliminates its dispersive shifts onto the resonator







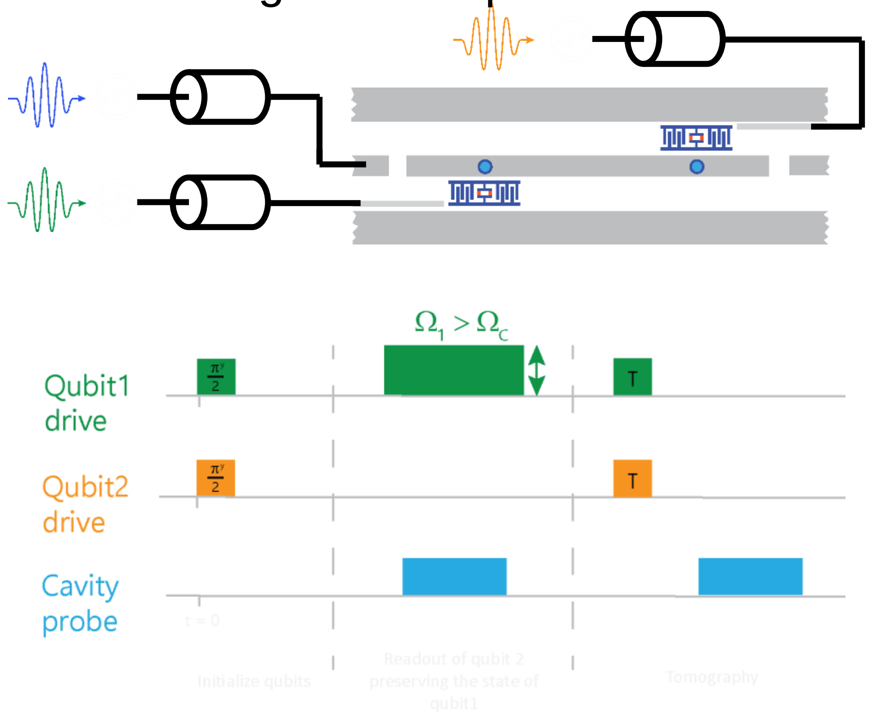
Multiplex readout

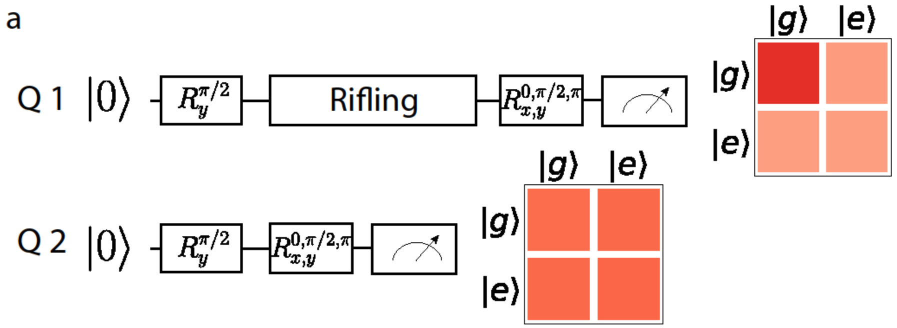
- Measurement of the second qubit works like there is no first one
- The state of the first qubit is also protected from the measurement!

D. Szombati et. al. ,Phys. Rev. Lett. 124, 070401 (2020)

Arbitrary Multi-Qubit Multiplexing

Measuring the second qubit with and without riffling of the first. After that measuring the first qubit to observe the effect of the measurement





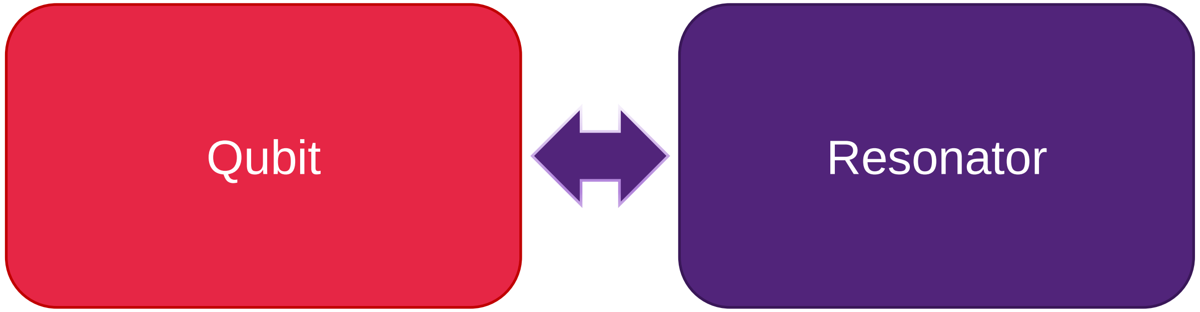
- Perfect reconstruction of the second qubit state (F~98%)
- The state of the first qubit is protected (F~ 97% compensated for decay)
- The state of the first qubit collapses without rifling pulse (F~ 98% compensated for decay)

D. Szombati et. al. ,Phys. Rev. Lett. 124, 070401 (2020)

Some remarks about quantum rifling

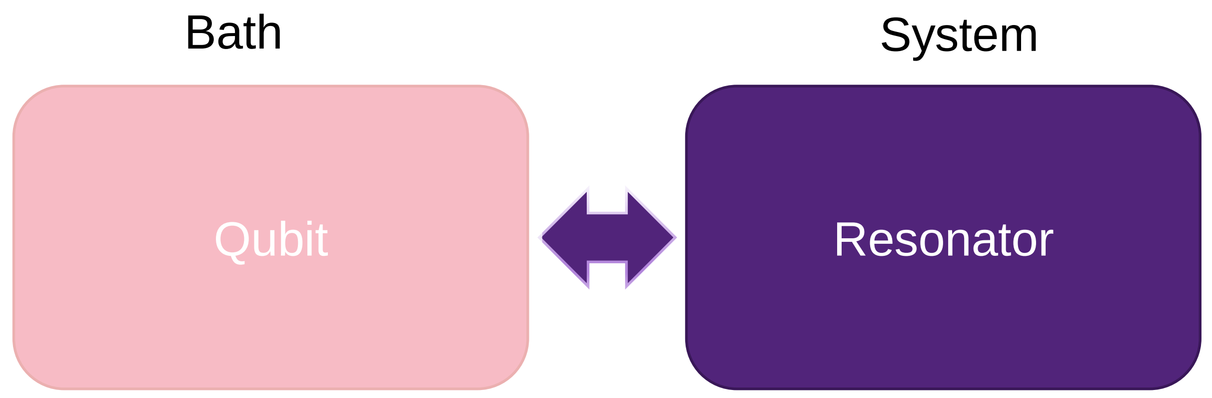
Perspective on our results

Continuous measurement of qubit: where is the system of interest and the bath?



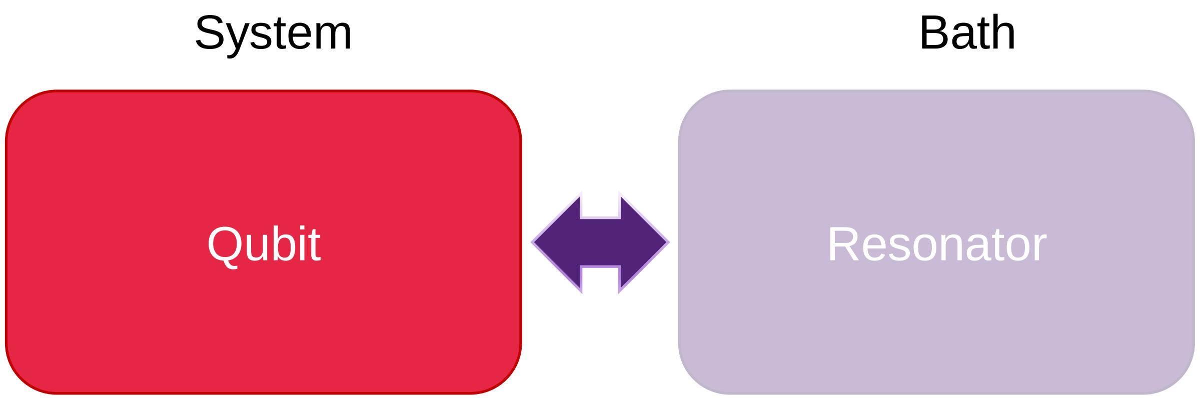
Perspective on our results

Focus on the resonator: motion averaging



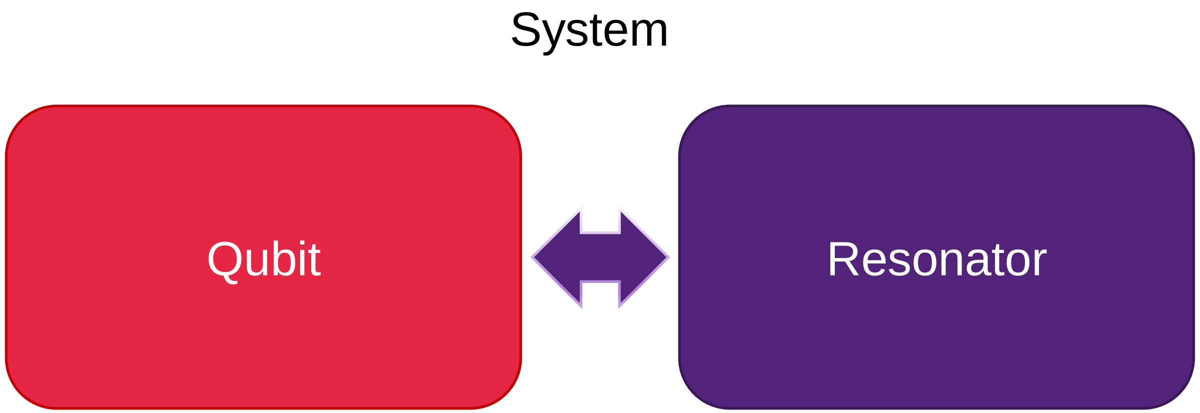
Perspective on our results

Focus on the qubit: dynamical decoupling or spin-locking



Perspective on our results

Time-average Hamiltonian theory for quantum measurement



Opens new experimental control without additional hardware requirements:

- Decoupling of a quantum system (or subspace of a quantum system) from a detector on demand
- Multiplex readout of qubits coupled to the same detector
- May be also applied for controlling coherent coupling between qubits (future direction)

Continuous measurement of superconducting qubit: quantum trajectories

Stochastic master equations

Master equation for the qubit:

$$\dot{\rho_Q} = -i\Omega_R[\sigma_x, \rho_Q] - i\Delta[\sigma_z, \rho_Q] + \gamma D[\sigma^-]\rho_Q + \Gamma D[\sigma_z]\rho_z$$

Does not provide information on a particular measurement outcome!

Need to consider ***conditional master equation***:

$$d\dot{\rho}_Q = -i\Omega_R[\sigma_x, \rho_Q]dt - i\Delta[\sigma_z, \rho_Q]dt + \gamma D[\sigma^-]\rho_Q dt + \Gamma D[\sigma_z]\rho_z dt - \boxed{\sqrt{\eta\kappa}H[\sigma_z]\rho_Q dW(t)}$$

Here:

- $H[a]\rho = A\rho + \rho A^+ - \mathrm{Tr}(A\rho + \rho A^+)\rho$
- η is the efficiency of the photodetector
- $dW(t)$ is a Wiener process
- Assumed homodyne detection scheme for simplicity

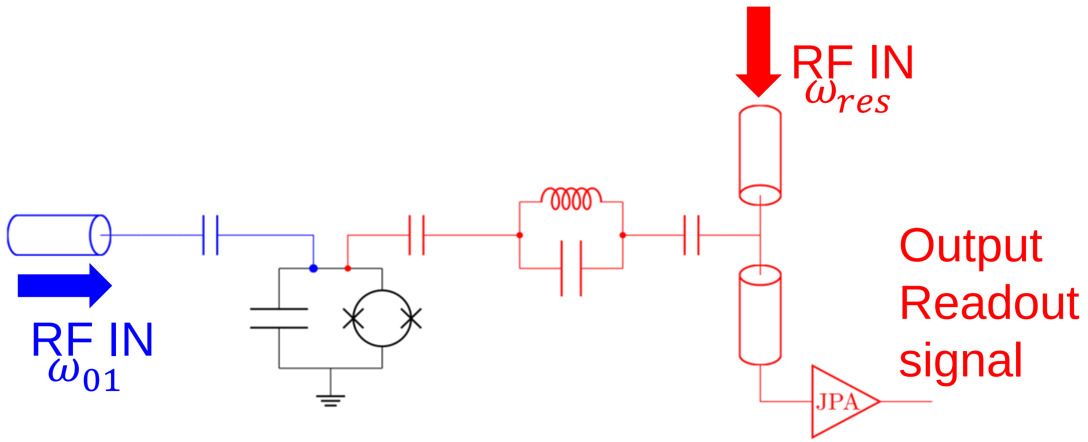
Every trajectory is different due to quantum noise

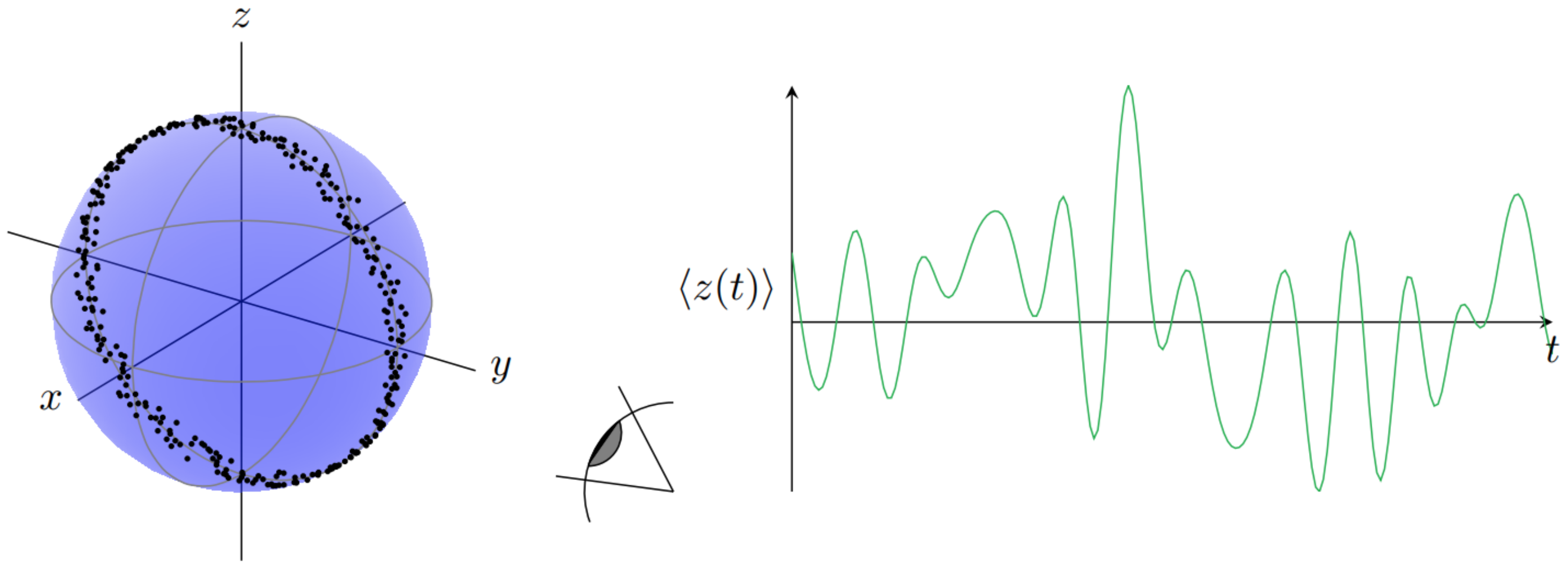
Measurement signal is different depending on the measurement regime

Weak measurement regime

Case: Finite qubit drive, small readout power $\Gamma < \Omega_R$

Persistent noise oscillations

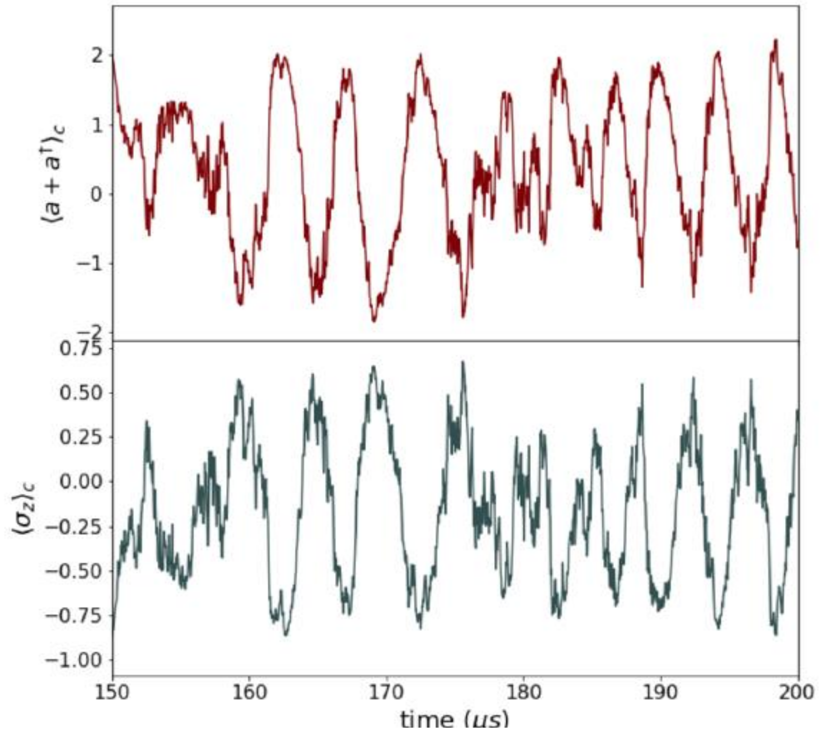




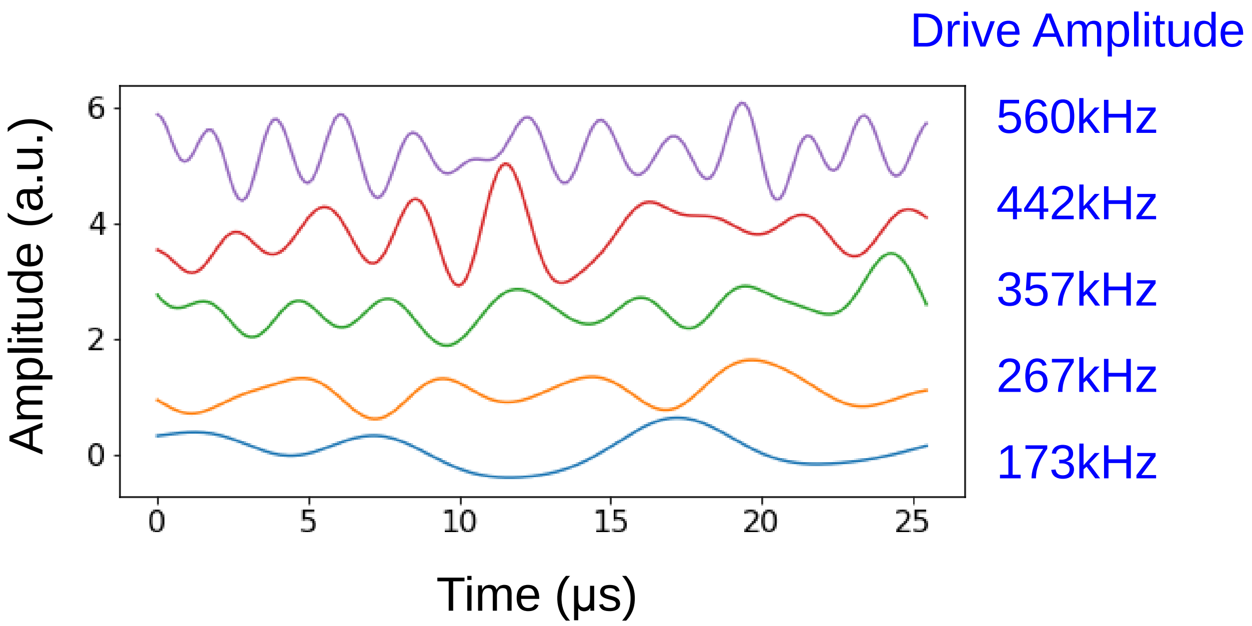
Weak measurement regime – period statistics

Although ‘ordinary’ Rabi oscillations decay these oscillations are persistent

Simulation of conditional master equation

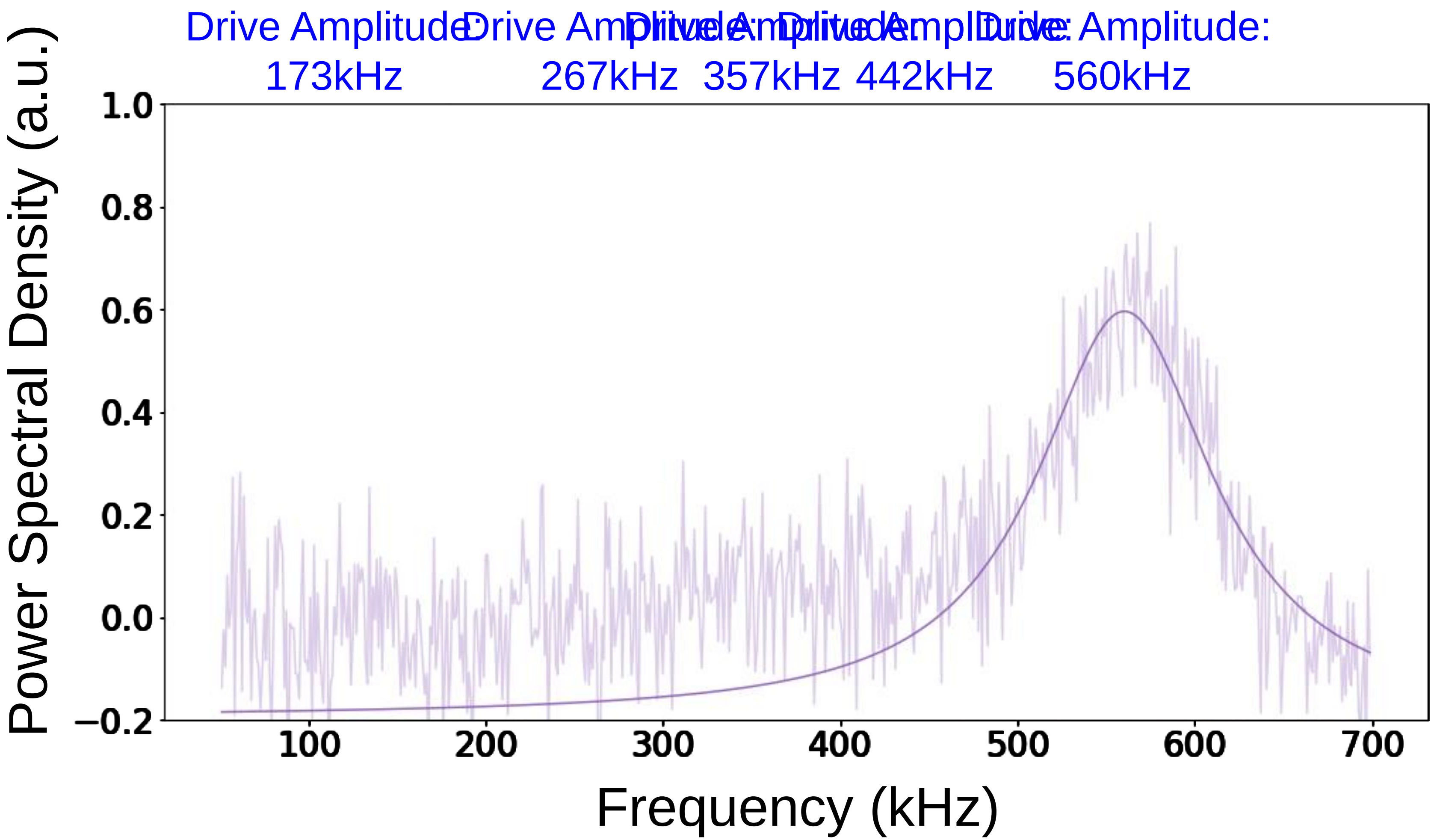


Measured trajectories



Weak measurement regime – Power Spectra

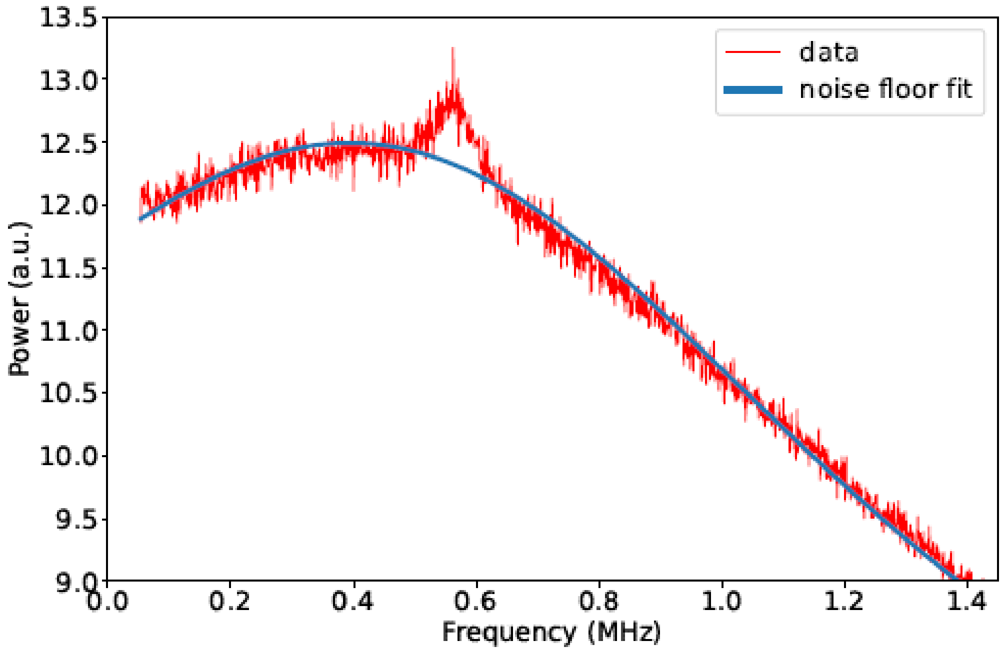
Noise background subtracted



Readout amplitude: 0.28 photons

Weak measurement regime – Power Spectra

Noise background not subtracted



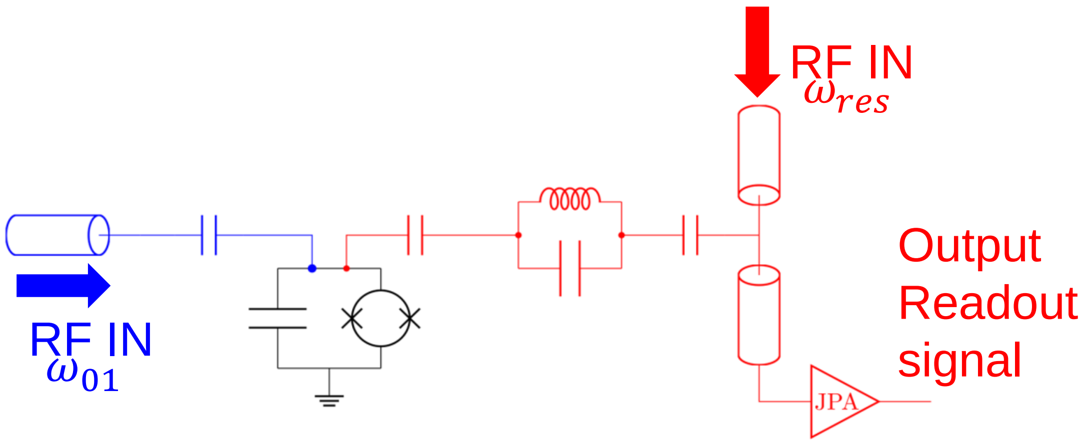
Interplay between measurement and backaction

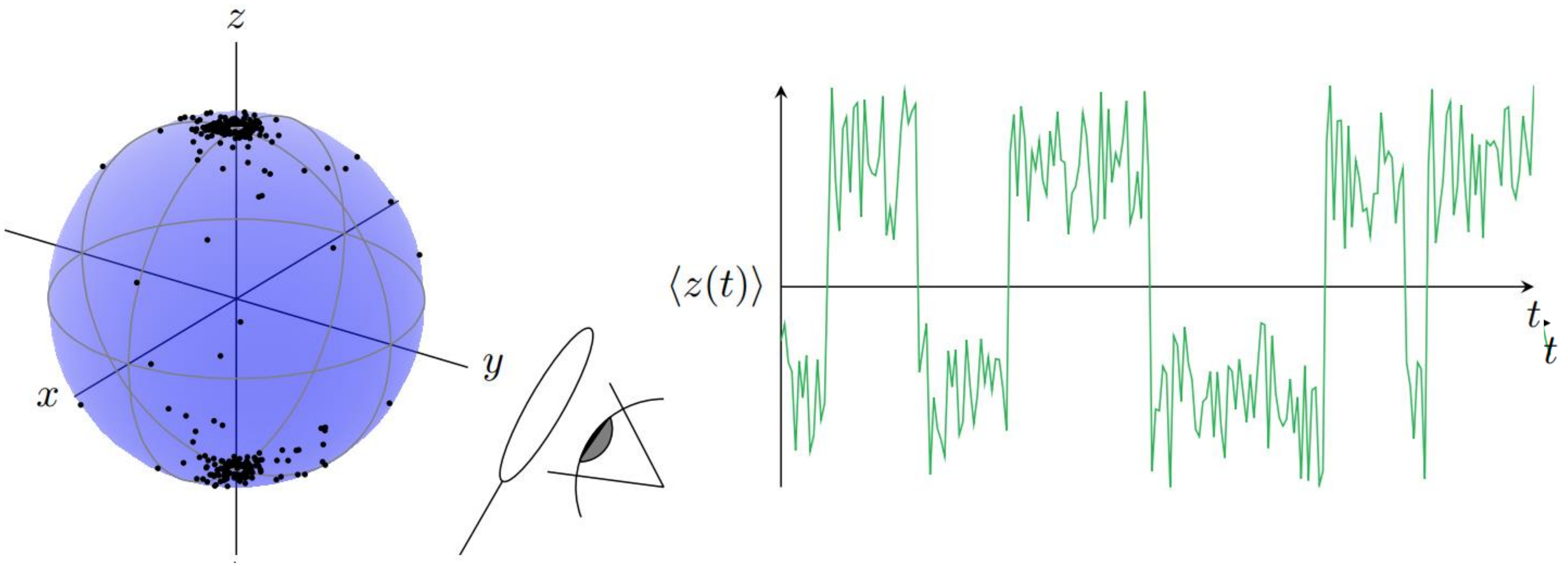
Theory: A.N. Korotokov and D. V. Averin ,Phys. Rev. B 64, 165310 (2001)
First expérimental test: A. Palacios-Laloy et. al. Nature Physics 6, 442-447 (2010)

Strong measurement regime

Case: Finite qubit drive, large readout power $\Gamma > \Omega_R$

Random telegraph signal between states $|0\rangle$ and $|1\rangle$



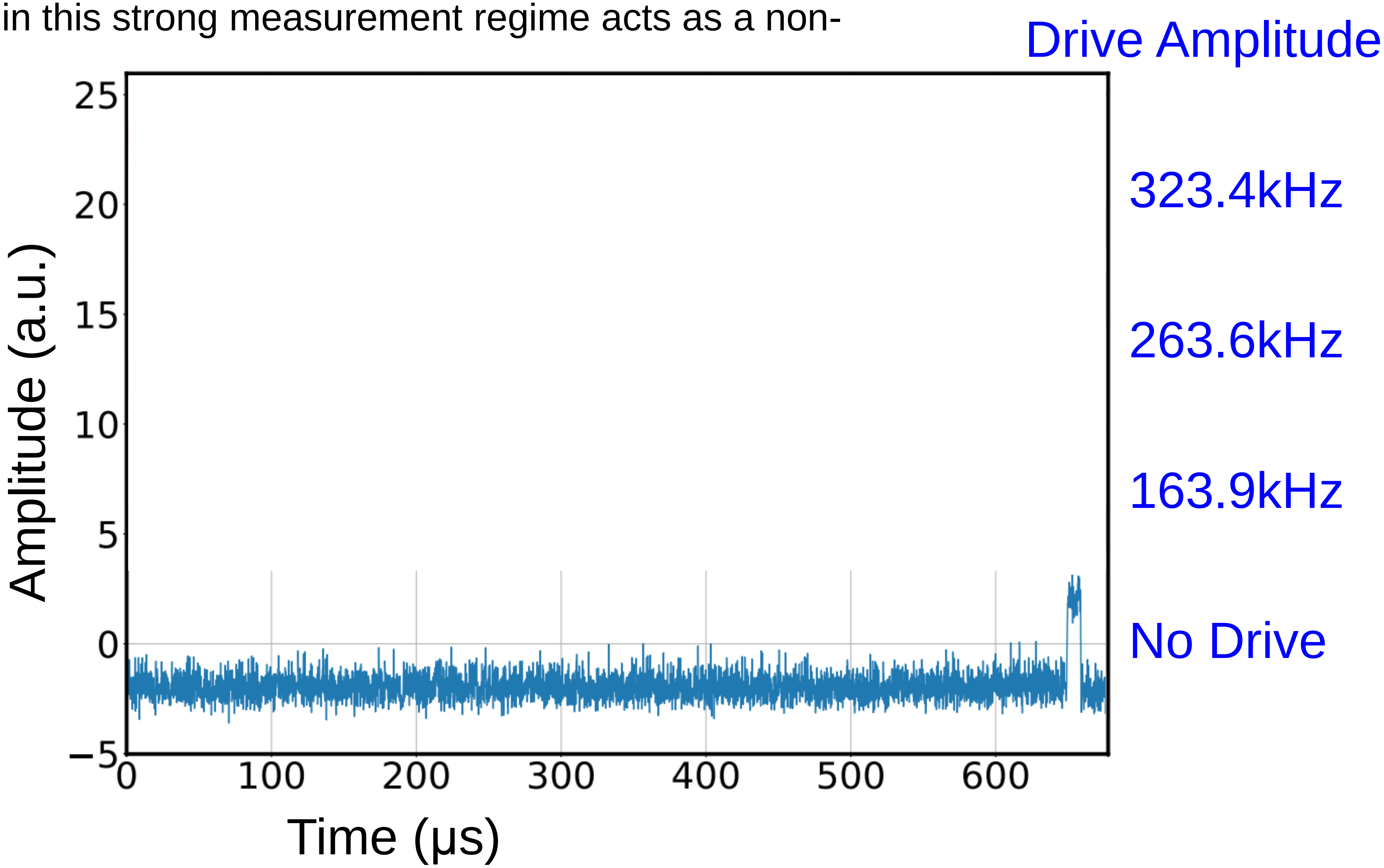


Strong measurement regime

Upward jump as a tick, the qubit in this strong measurement regime acts as a non-oscillatory clock.

High voltage level→e state
Low voltage level → g state

Zero drive → thermal noise



First measurement of quantum jumps: K. W. Murch, S. J. Weber, C. Macklin & I. Siddiqi, Nature 502, 211–214 (2013).

Requirements for time-keeping

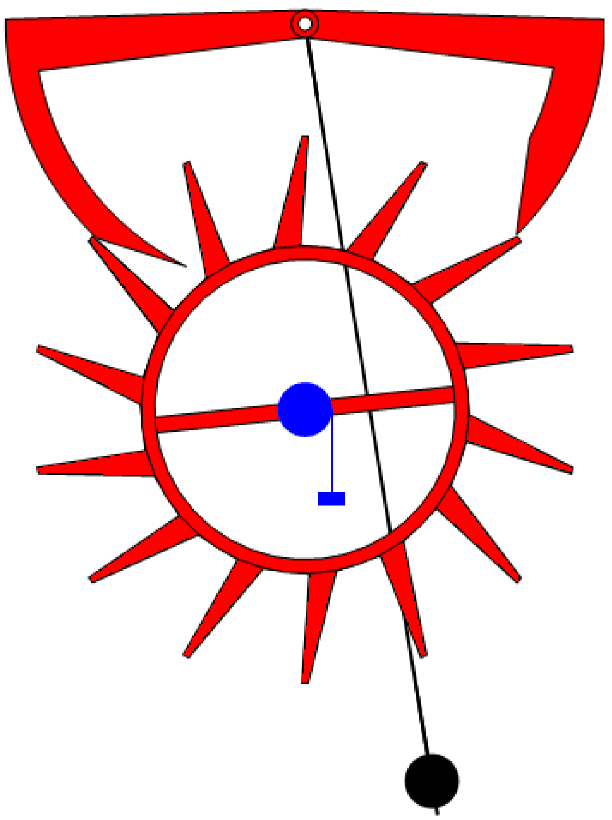
A good time-keeper:

- Counts forwards → irreversible process
- Requires energy to compensate for the dissipation
- Counts ticks indefinitely → typically nonlinear limit-cycle process

The thermodynamics of clocks – G.M. Milburn, Contemporary Physics 2020

Requirements for time-keeping – Example

- ✓Frictional forces provide dissipation
- ✓Weight provides energy to compensate dissipation
- ✓Anchor and escape wheel provide nonlinearity



Erik Mahieu “The Graham Clock Escapement”, Wolfram Demonstrations Project (2013)

Thermodynamics and kinetic constraints

- Dissipation means that the clock is necessarily subject to noise
- This leads to the trade-off between dissipation and clock accuracy

$$\boxed{N \text{ (clock precision)} \propto \Delta S_{\mathrm{tick}} \text{ (entropy generation per tick)}}$$

Quantum Clock: quantum measurement plays an exclusive role

Fundamental limit for the clock precision due to measurement is currently absent!

Autonomous quantum clocks: Does thermodynamics limit our ability to measure time? Physical Review X, 7(3), 031022 (2017).
Measuring the Thermodynamic Cost of Timekeeping: Phys. Rev. X, 11(2), 21029 (2021).

Kinetic uncertainty relation (KUR)

Classical clock: Thermodynamic Uncertainty Relation (TUR) relates the mean and fluctuations of any current to the overall entropy production in a non-equilibrium steady state.

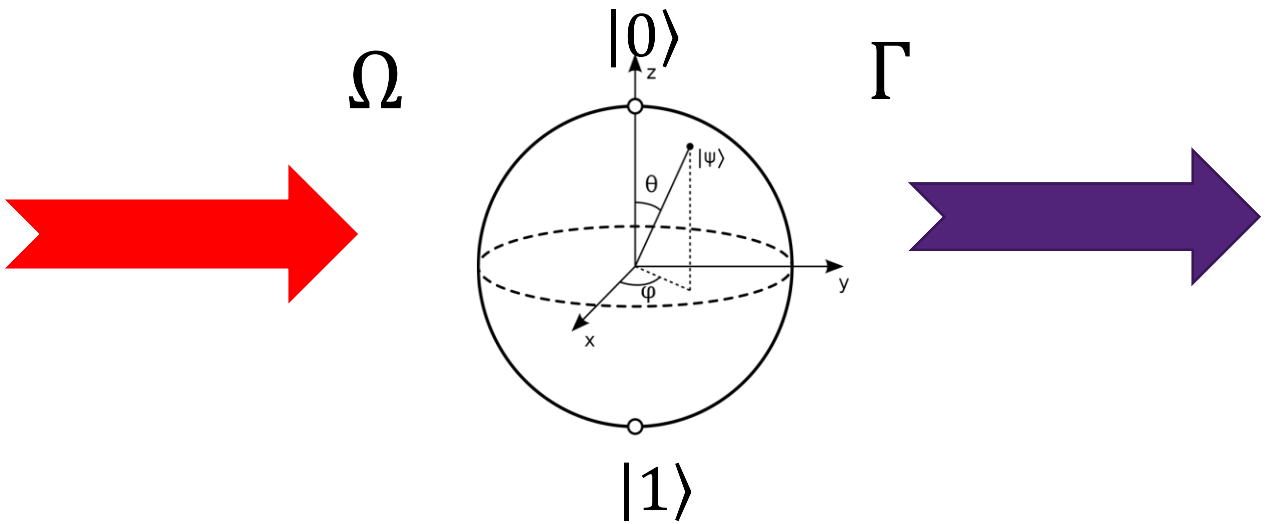
Quantum clock: TURs are not applicable as there is no energy exchange for quantum measurement (information extraction).

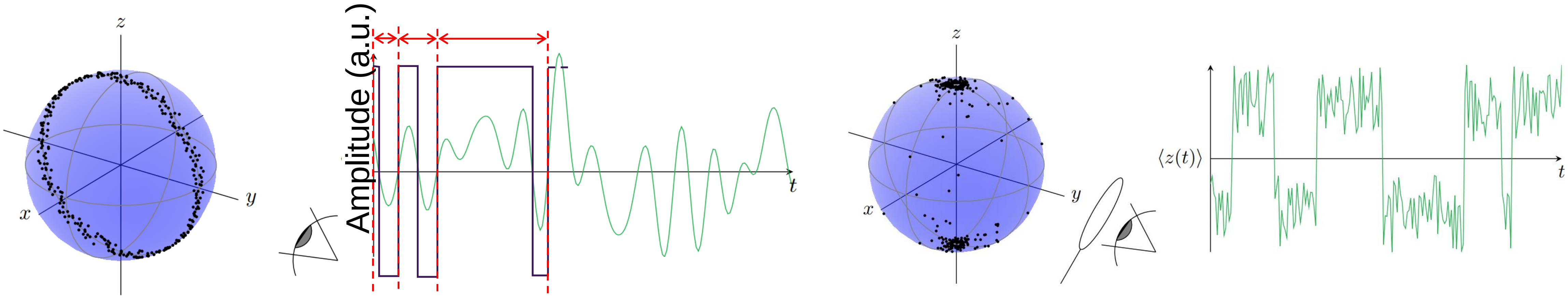
Need to use Kinetic Uncertainty Relation (KUR) which puts a bound on the precision, but in terms of the dynamical activity, instead of the entropy production.

Arguably more generic than TUR

Clock precision bounded by measurement-induced uncertainty

Simplest quantum clock: a qubit driven to induce Rabi osillations





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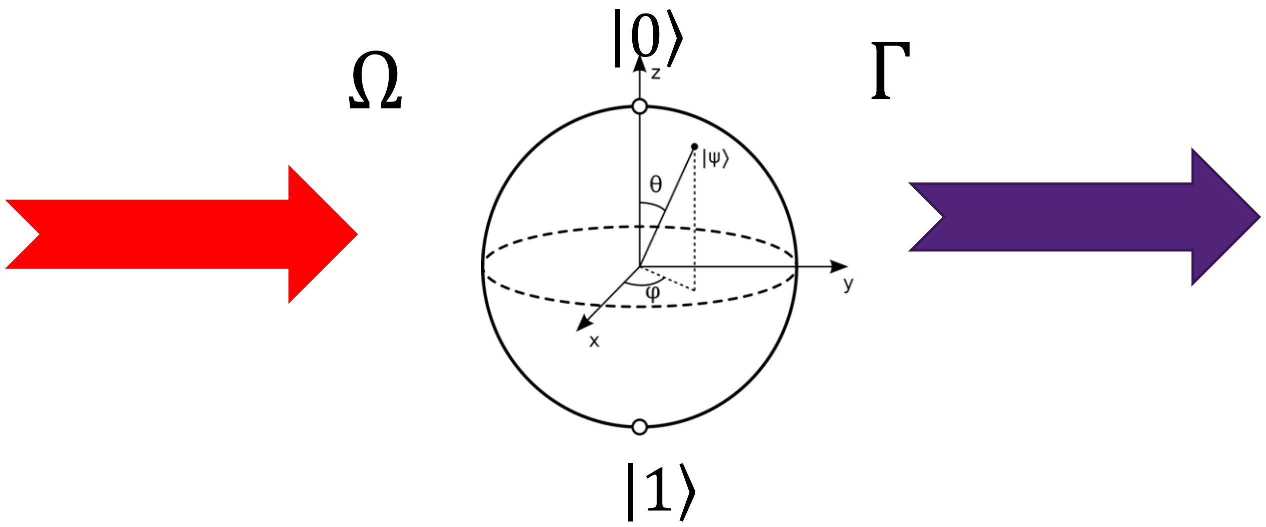


$$\boxed{N \equiv \frac{E[T]^2}{\mathrm{Var}[T]} \leq E[T](\mathbb{N} + \mathbb{Q})}$$

$\mathbb{N} = \Gamma$: dynamical activity, a measure of the kinetic activity of the system

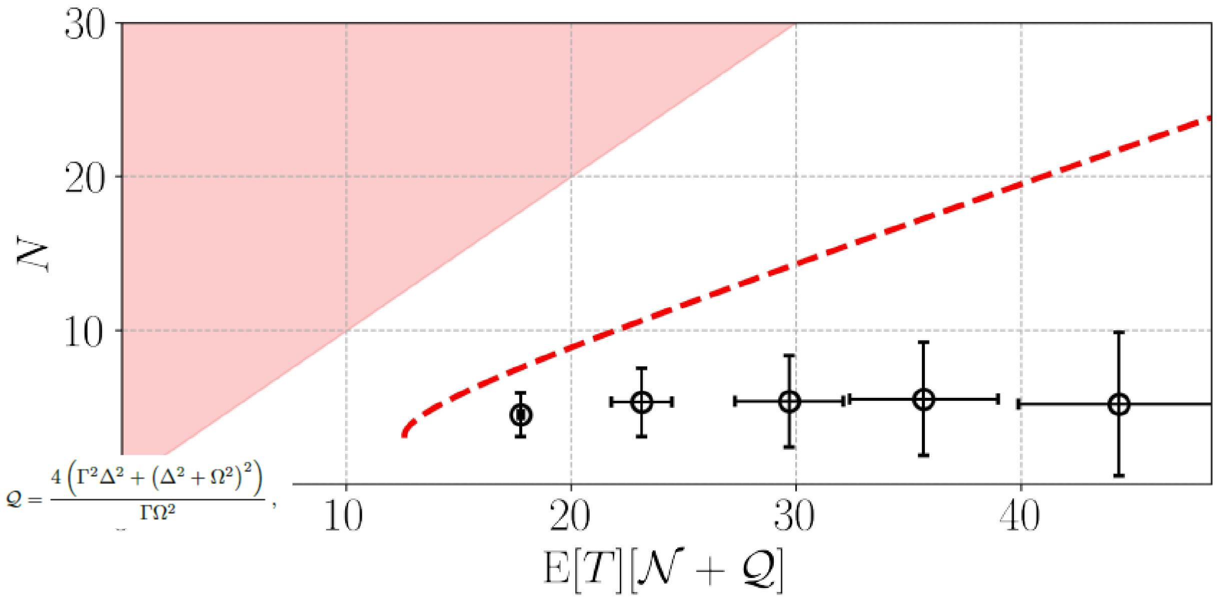
$\mathbb{Q} = \frac{4\left(\Gamma^2\Delta^2 + \left(\Omega^2+\Delta^2\right)^2\right)^2}{\Gamma\Omega^2}$: related to the coherent dynamics of the system

Clock precision bounded by measurement-induced uncertainty

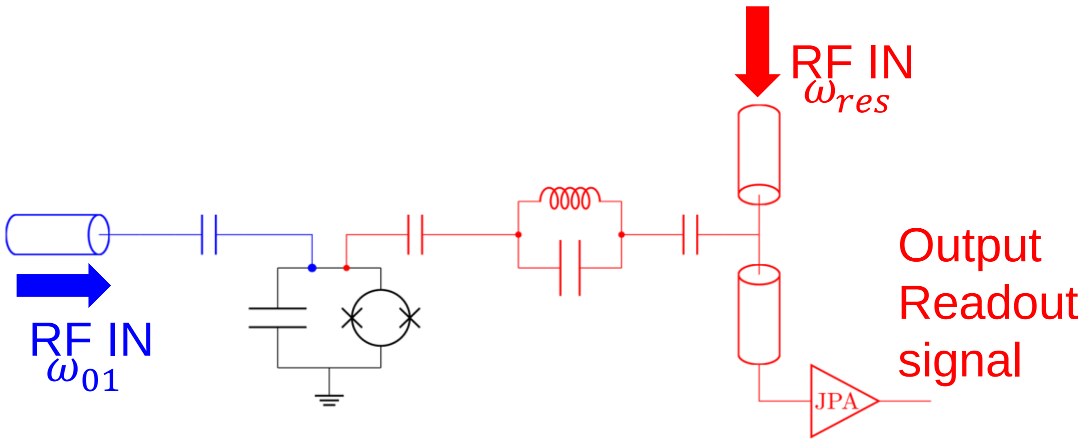
KUT: $N \leq E[T](\mathbb{N} + \mathbb{Q})$

$\mathbb{N} = \Gamma$ – dynamical activity

$$\mathbb{Q} = \frac{4(\Gamma^2\Delta^2 + (\Omega^2 + \Delta^2)^2)^2}{\Gamma\Omega^2}$$



- First experimental test of KUR due to measurement
- Applicable generally to all clocks including "practical" clocks such as atomic clocks

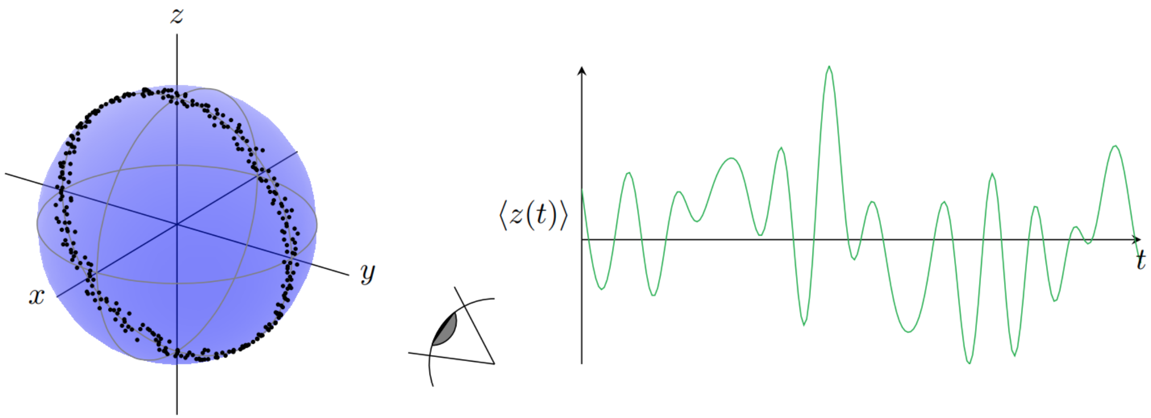


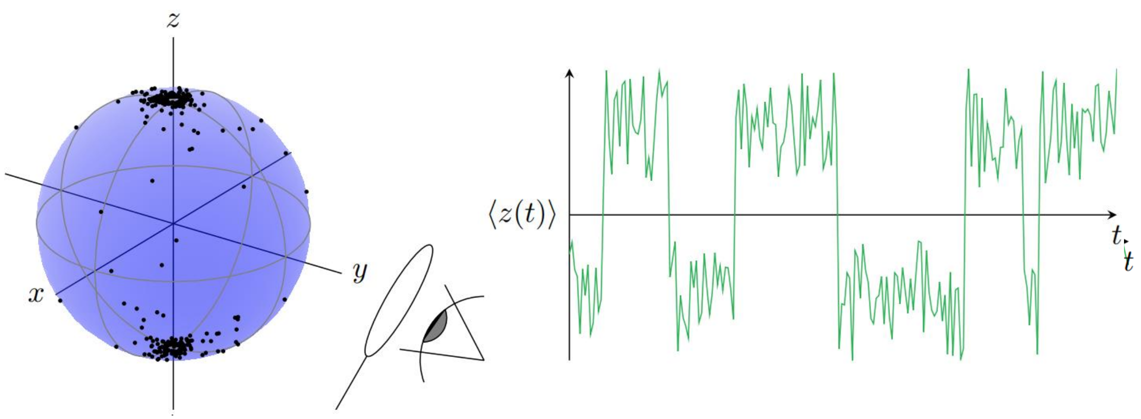
Summary

Continuously measured driven qubit:

- Weak measurement yields indefinite Rabi-like oscillations
- Strong measurement yields random telegraph signals

Quantum measurement adds additional constraints to the clock precision





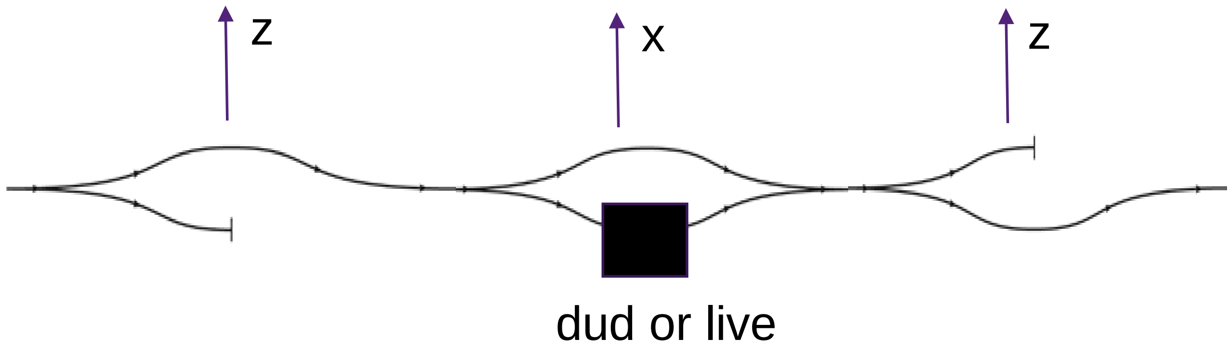
Final remarks: interaction free measurement

Elitzur–Vaidman bomb tester

Elitzur–Vaidman bomb-testing problem is a thought experiment in quantum mechanics, first proposed by Avshalom Elitzur and Lev Vaidman in 1993.

Assume we have a collection of live bombs and dud bombs. The bomb has a sensor such that each silver atom will trigger the bomb. The dud does not have a sensor and will just let the particle pass it without interacting with it.

Consider the following experiment where the black box contains either a bomb or a dud. We send a silver atom from the left:

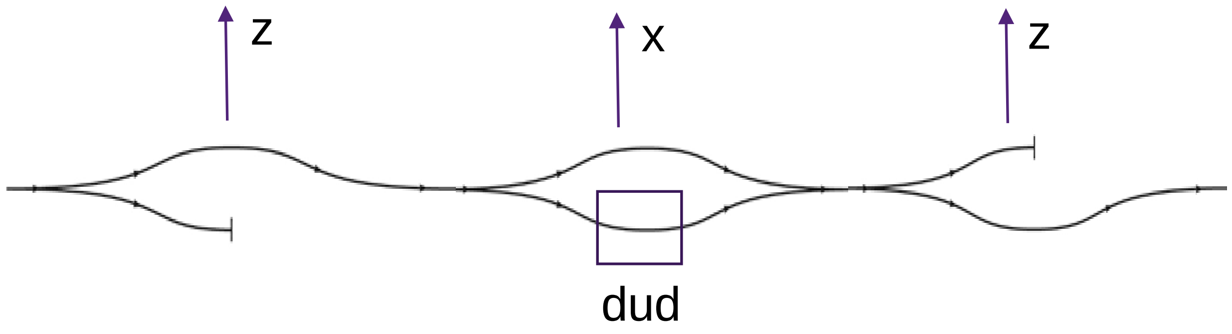


Elitzur–Vaidman bomb tester

Let’s consider the case when there is a dud bomb in the box

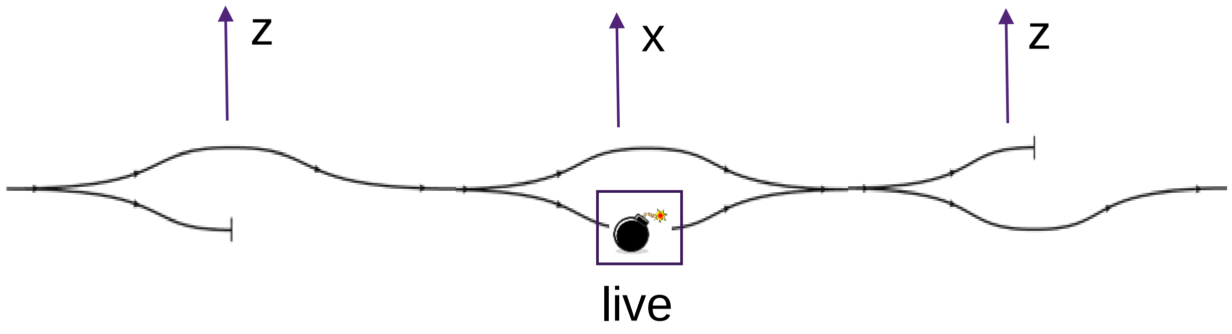
How many particle will exit the device?

That means that if we have particle exiting the device the box has bomb inside.



Elitzur–Vaidman bomb tester

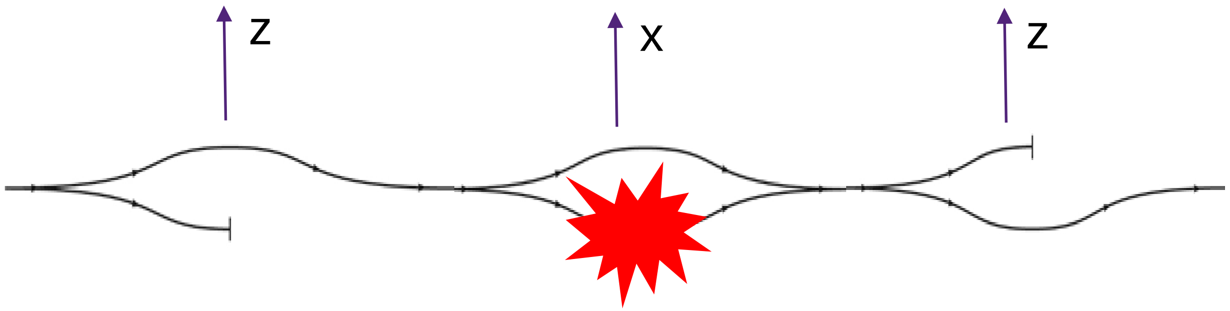
Let’s consider the case when there is a live bomb in the box



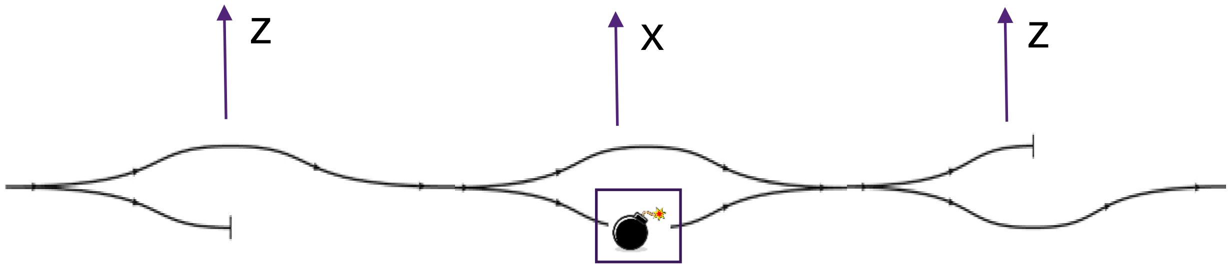
Elitzur–Vaidman bomb tester

Let’s consider the case when there is a live bomb in the box

- The bomb is only triggered if particles follow lower path. What is the probability to trigger the bomb?
- If the particles follow the upper path the bomb will not be triggered. What is the probability to exit the whole device?



What do you conclude if you have particles exiting the device?



live

Elitzur–Vaidman bomb tester

Elitzur–Vaidman bomb-testing problem is a thought experiment in quantum mechanics, first proposed by Avshalom Elitzur and Lev Vaidman in 1993.

In 1994, Anton Zeilinger, Paul Kwiat, Harald Weinfurter, and Thomas Herzog performed an equivalent of the above experiment using Mach –Zehnder interferometer.

In 1996, Kwiat et al. devised a method, using a sequence of polarising devices, that efficiently increases the yield rate to a level arbitrarily close to one!!! (See homework problem)

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Interaction-Free Measurement

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We show that one can ascertain the presence of an object in some sense without interacting with it. One repeatedly, but weakly, tests for the presence of the object, which would inhibit an otherwise coherent evolution of the interrogating photon. The fraction of "interaction-free" measurements can be arbitrarily close to 1. Using single photons in a Michelson interferometer, we have performed a preliminary demonstration of some of these ideas.

Testing with superconducting qutrit

Interaction free measurement tested with transmon qutrit

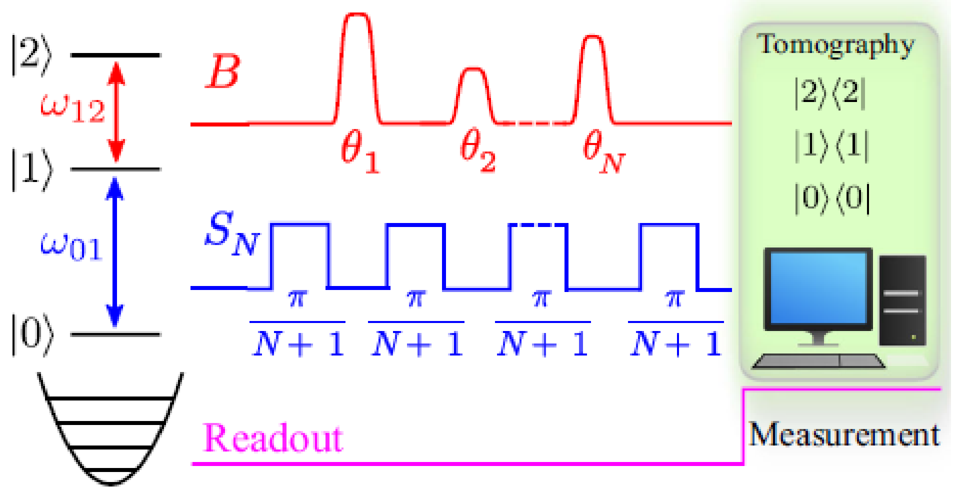
Consider $N = 2$:

With B:

$$|\psi_f\rangle = \sin^2\left(\frac{\theta}{4}\right)|0\rangle + \cos^2\left(\frac{\theta}{4}\right)|1\rangle + \frac{1}{\sqrt{2}}\sin\left(\frac{\theta}{2}\right)|2\rangle$$

Without B:

$$|\psi_f\rangle = |1\rangle$$



Can detect the presence of *B pulses* without interacting with it

Can be extended to 100% efficiency

Take home message

- Quantum measurement is a key element of QM with no classical counterpart due to back-action
- Wave-function collapse is not mysterious and can be avoided by different techniques
- Although there is always measurement back-action, in principle, one can "interaction-free" measurement

Thank you