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Spring School on Superconducting Qubit Technology
2023/4/19, 9:30-11:00
Centro de Ciencias de Benasque Pedro Pascual

Physics and Applications of Kerr Parametric Oscillators

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Tsuyoshi Yamamoto

Slide courtesy

Shumpei Masuda

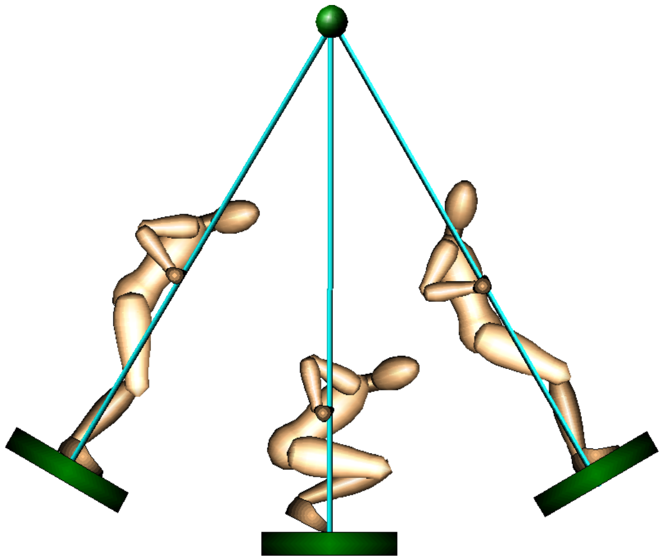
Ryoji Miyazaki

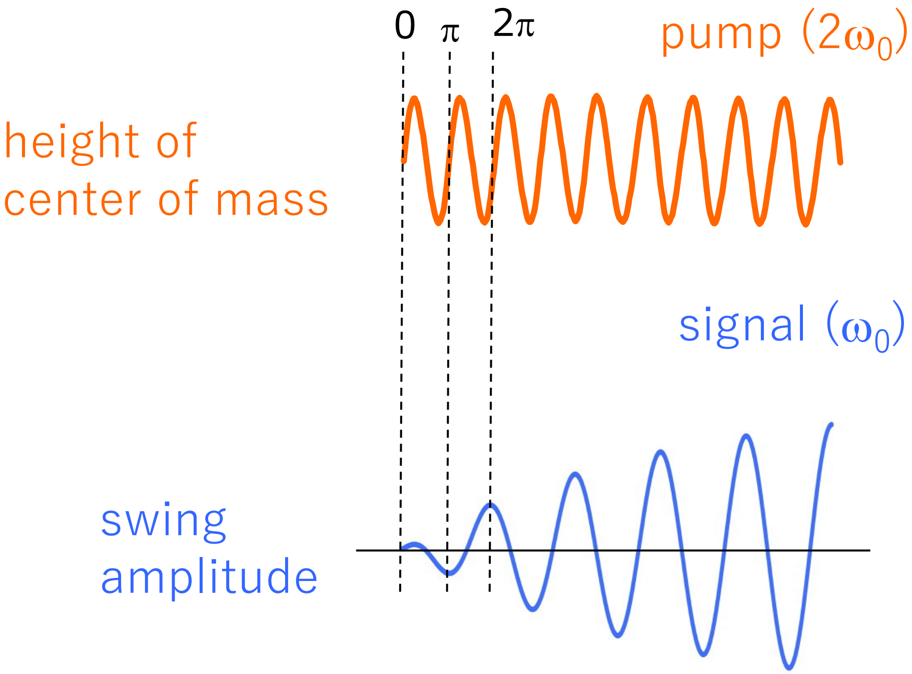
Table of Contents

Table of Contents

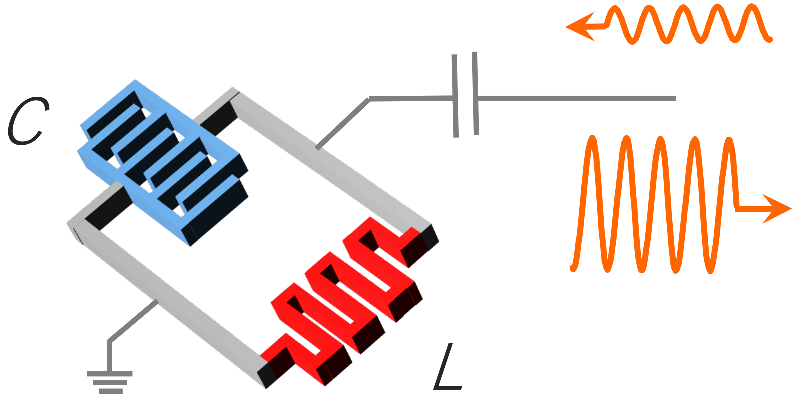
Parametric resonance

- Modulation of a parameter of an oscillating system at twice the resonant frequency leads to the amplification of the oscillation.





Parametric amplifier



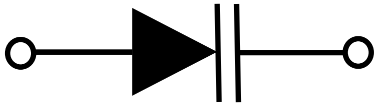
amplification!

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

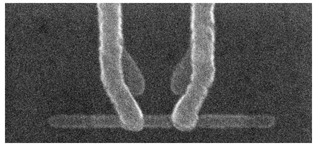
OR

$C + \delta C \cos 2\omega_0 t$

$L + \delta L \cos 2\omega_0 t$

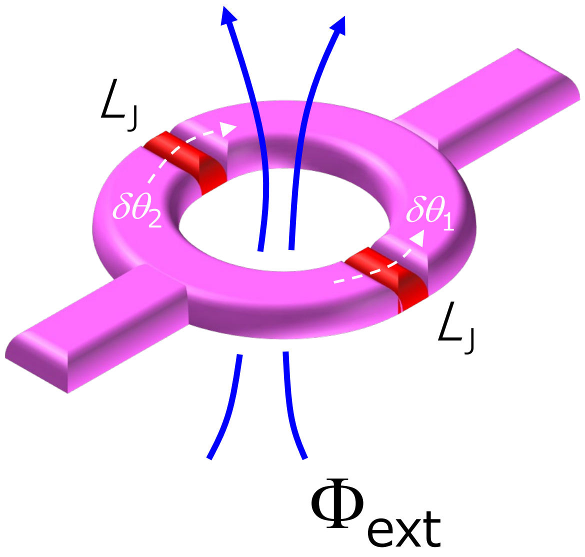


varactor diode



Josephson junction

Superconducting QUantum Interference Device (SQUID)

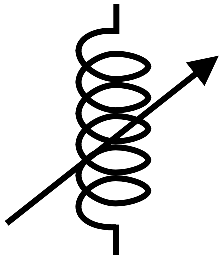


Flux quantization:

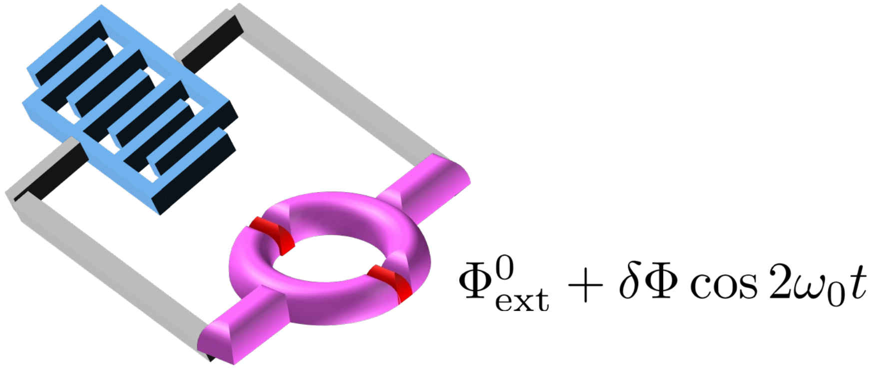
$$\delta\theta_1 - \delta\theta_2 + 2\pi\Phi_\mathrm{ext}/\Phi_0 = \text{integer}$$

$=$

flux-tunable (nonlinear) inductor



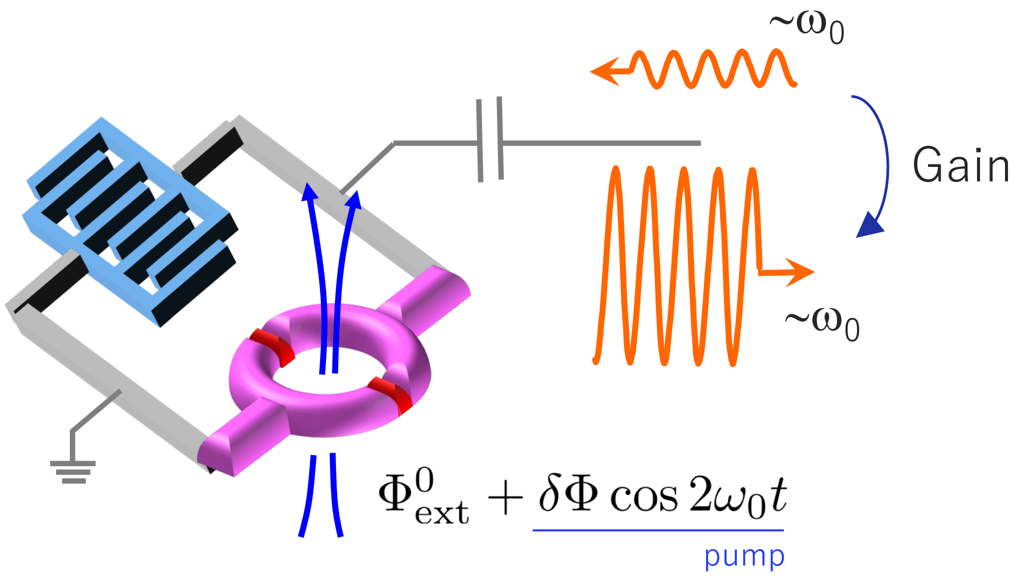
$$L_\mathrm{J}^\mathrm{eff}(\Phi_\mathrm{ext}) = \frac{L_\mathrm{J}}{2}\frac{1}{\cos(\pi\Phi_\mathrm{ext}/\Phi_0)}$$

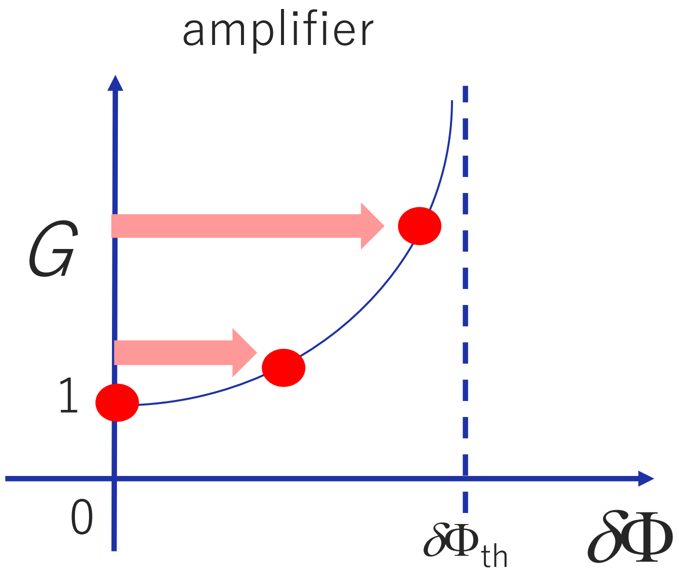


Flux-driven Josephson Parametric Amplifier (JPA)

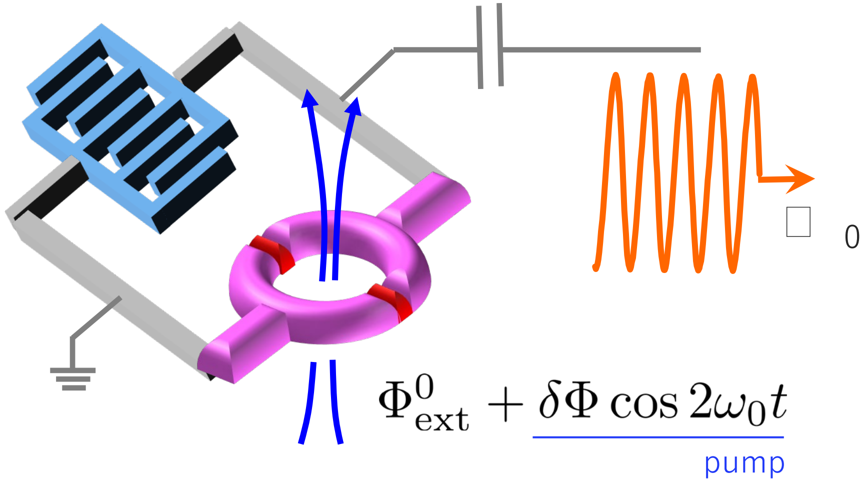
T. Yamamoto *et al.*,
Appl. Phys. Lett. **93**, 042510 (2008).

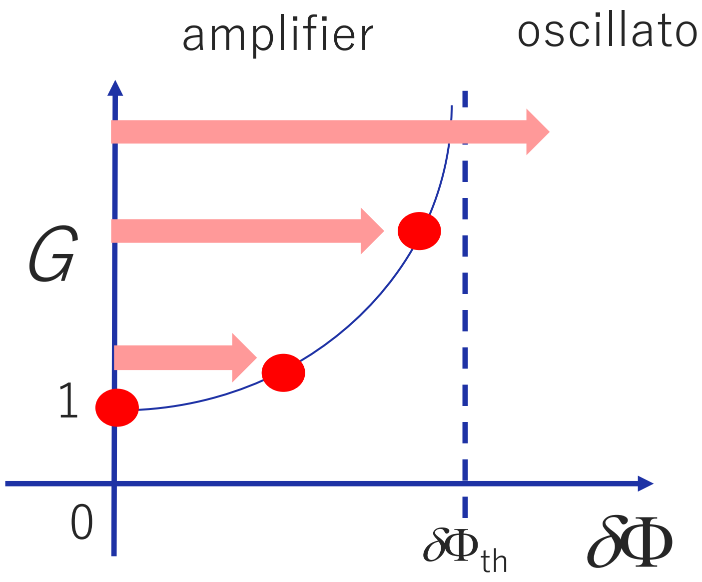
Josephson parametric oscillator

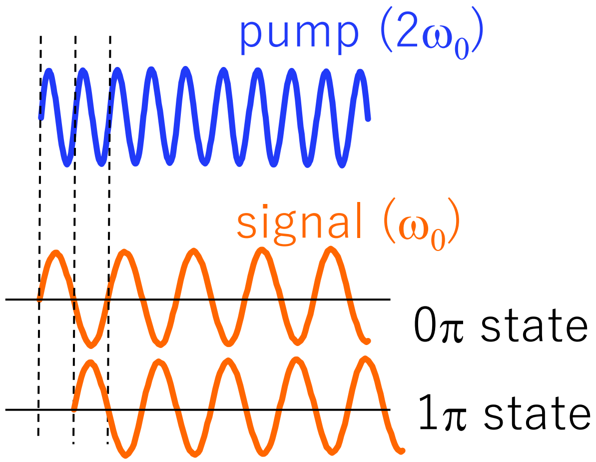




Josephson parametric oscillator



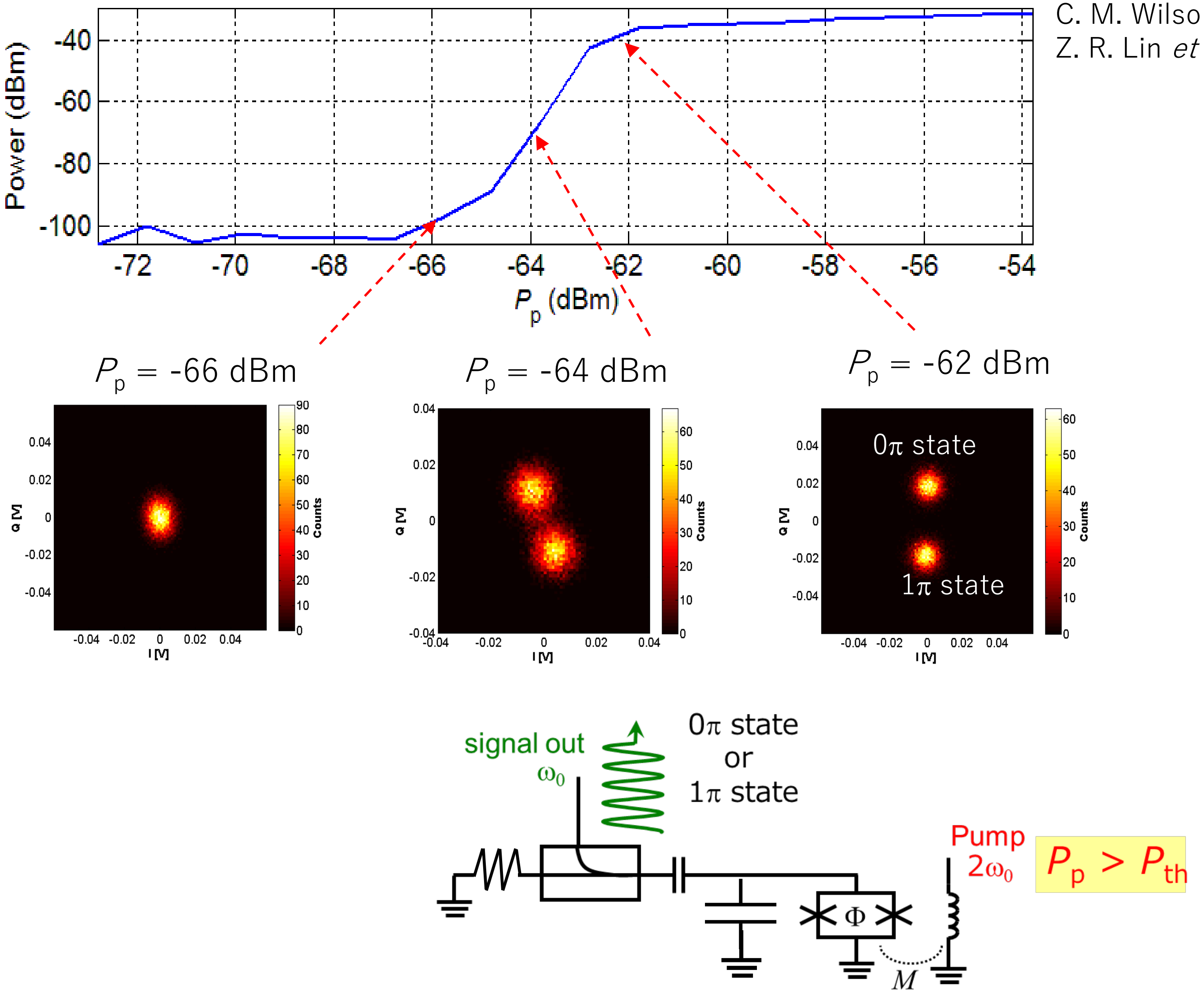




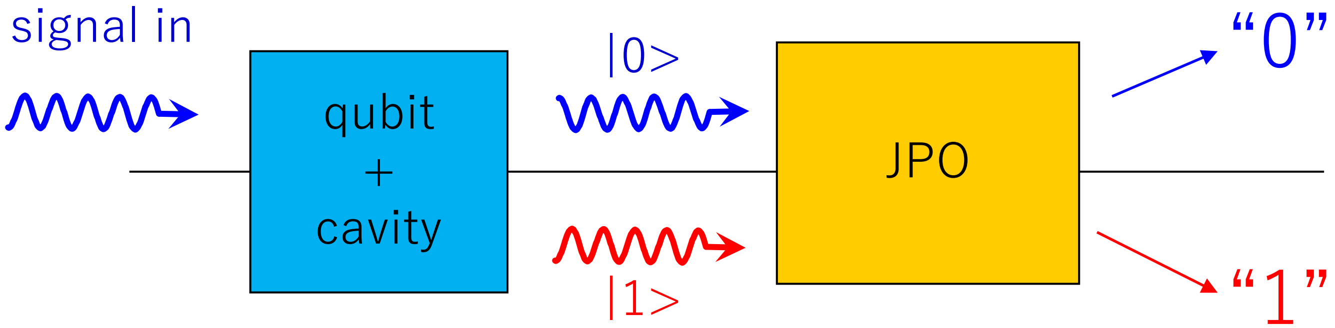
- 0π state and 1π state are generated randomly.
- Injecting external signal at ω_0 biases the probability.

(Classical) Josephson parametric oscillator

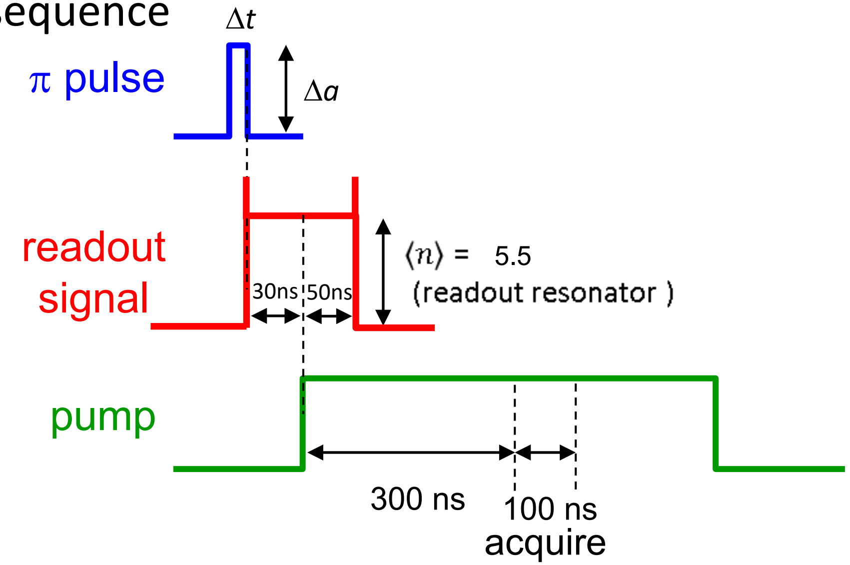
C. M. Wilson *et al.*, Phys. Rev. Lett. **105**, 233907 (2010).
Z. R. Lin *et al.*, Nature Commun. **5**, 4480 (2014).



qubit readout using JPO



Z. R. Lin *et al.*, Nature Commun. **5**, 4480 (2014).



Z. R. Lin *et al.*, Nature Commun. **5**, 4480 (2014).
https://www.nature.com/articles/ncomms5480
Figs. 4d and 4e

Quantum annealing with JPO (theory)

Bifurcation-based adiabatic quantum computation with a nonlinear oscillator network

Hayato Goto

Sci. Rep. **6**, 21686 (2016).

SCIENCE ADVANCES | RESEARCH ARTICLE Sci. Adv. **3**, e1602273 (2017).

QUANTUM INFORMATION

Robust quantum optimizer with full connectivity

Simon E. Nigg,* Niels Lörch, Rakesh P. Tiwari

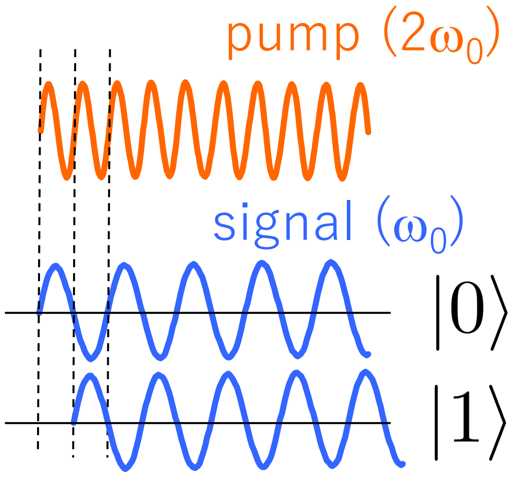
ARTICLE

Received 1 Nov 2016 | Accepted 28 Apr 2017 | Published 8 Jun 2017 DOI: 10.1038/ncomms15785 OPEN

Quantum annealing with all-to-all connected nonlinear oscillators

Nature Commun. **8**, 15785 (2017).

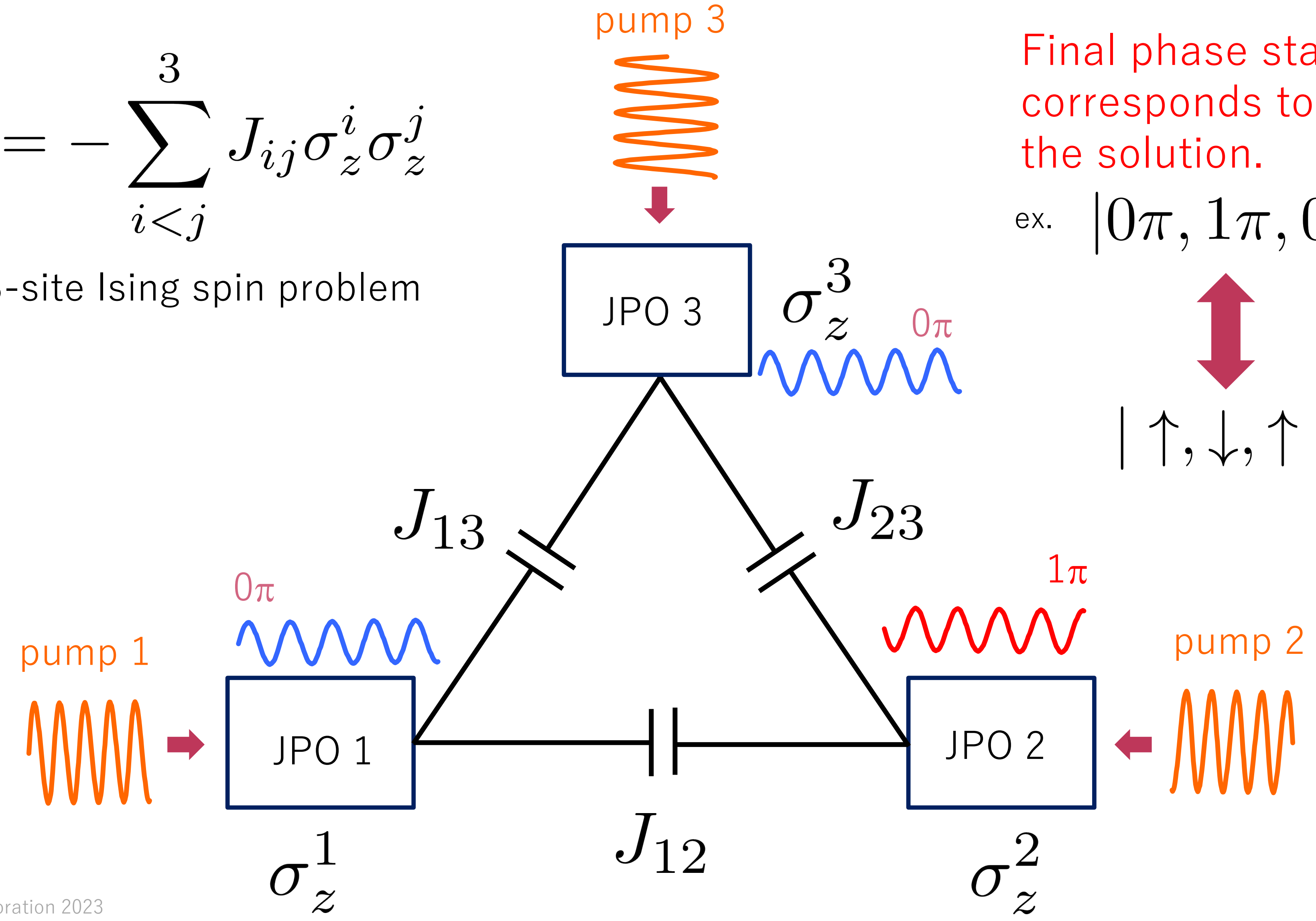
Shruti Puri[1], Christian Kraglund Andersen[2], Arne L. Grimsmo[1] & Alexandre Blais[1,3]



Quantum annealing with JPO（intuitive)

$$\mathcal{H} = -\sum_{i<j}^{3} J_{ij}\sigma_z^i\sigma_z^j$$

3-site Ising spin problem



Final phase state corresponds to the solution.

ex. $|0\pi, 1\pi, 0\pi\rangle$

$\updownarrow$

$|\uparrow, \downarrow, \uparrow\rangle$

Recent theories on JPO (KPO) annealing

- Boltzmann sampling
  - H. Goto et al., Sci. Rep. **8**, 7154 (2018).
- 3D cQED implementation
  - P. Zhao et al., Phys. Rev. Appl. **10**, 024019 (2018).
- Quantum annealing started with stable excited state
  - H. Goto and T. Kanao, Commun. Phys. **3**, 235 (2020).
- Comparison between quantum and classical models
  - M. J. Kewming et al., New J. Phys. **22**, 053042 (2020).
- All-to-all coupling via Floquet engineering
  - T. Onodera et al., npj Quantum Info. **6**, 48 (2020).
- Photon-number inhomogeneity and its mitigation in LHZ architecture
  - T. Kanao and H. Goto, npj Quantum Info. **7**, 1 (2021).
- Effective spin model of (bosonic) KPO
  - R. Miyazaki, Phys. Rev. A **105**, 062457 (2022).
- Controllable coupling between KPOs
  - S. Masuda et al., Phys. Rev. Appl. **18**, 034076 (2022).
- Effect of Nonstoquastic catalyst
  - Y. Susa et al., arXiv:2209.01737.

Proposals for hardware-efficient FTQC using Kerr cat qubit

- Theory of Kerr cat qubit
  - P. T. Cochrane et al., Phys. Rev. A **59**, 2631 (1999).
  - H. Goto, Phys. Rev. A **93**, 050301 (2016).
  - S. Puri et al., npj Quantum Info. **3**, 18 (2017).

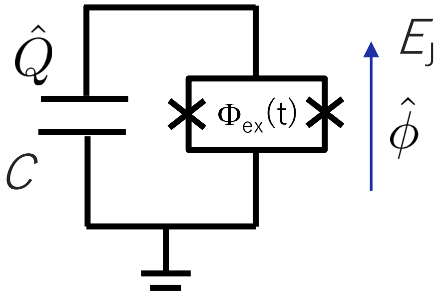
- High-fidelity gate operation
  - T. Kanao et al., Phys. Rev. Appl. **18**, 014019 (2022).
  - H. Chono et al., Phys. Rev. Res. **4**, 043054 (2022).

- Proposal of bias preserving gate
  - S. Puri et al., Sci. Adv. 6, eaay5901 (2020).

- Error-correction code with high error threshold
  - A. S. Darmawan et al., PRX Quantum 2, 030345 (2021).

Table of Contents

Hamiltonian of KPO



$$\mathcal{H}_{\mathrm{KPO}} = \frac{\hat{Q}^2}{2C} - E_{\mathrm{J}}(t)\cos\hat{\phi}$$
$$\simeq \frac{\hat{Q}^2}{2C} - E_{\mathrm{J}}(t)\Big(1 - \frac{1}{2}\hat{\phi}^2 + \frac{1}{24}\hat{\phi}^4\Big)$$

$$E_{\mathrm{J}}(t) = E_{\mathrm{J}} + \delta E_{\mathrm{J}}\cos\omega_{\mathrm{p}}t$$

$$\hbar\omega_0 = \sqrt{8E_{\mathrm{c}}E_{\mathrm{J}}}$$
$$E_{\mathrm{c}} = e^2/2C$$

$$\hat{\phi} = \Big(\frac{2E_{\mathrm{c}}}{E_{\mathrm{J}}}\Big)^{1/4}(a^\dagger + a)$$
$$\hat{Q} = \mathrm{i}\Big(\frac{E_{\mathrm{J}}}{2E_{\mathrm{c}}}\Big)^{1/4}(a^\dagger - a)$$

$$\mathcal{H}_{\mathrm{KPO}} \simeq \hbar\omega_0\Big(a^\dagger a + \frac{1}{2}\Big) - \frac{E_{\mathrm{c}}}{12}(a^\dagger + a)^4 + \frac{\hbar\omega_0}{4}\frac{\delta E_{\mathrm{J}}}{E_{\mathrm{J}}}(a^\dagger + a)^2\cos\omega_{\mathrm{p}}t$$

Hamiltonian of KPO

$$\mathcal{H}_{\mathrm{KPO}} = \hbar\omega_0\left(a^\dagger a + \frac{1}{2}\right) - \frac{E_{\mathrm{c}}}{12}(a^\dagger + a)^4 + \frac{\hbar\omega_0}{4}\frac{\delta E_{\mathrm{J}}}{E_{\mathrm{J}}}(a^\dagger + a)^2 \cos\omega_{\mathrm{p}} t$$

rotating frame at $\omega_p/2$, and RWA

$$\mathcal{H}_{\mathrm{KPO}}'/\hbar = \Delta a^\dagger a - \frac{K}{2} a^\dagger a^\dagger a a + \frac{\beta}{2}({a^\dagger}^2 + a^2)$$

detuning Kerr nonlinearity parametric drive

$$\begin{aligned} \Delta &= \omega_0 - K - \omega_{\mathrm{p}}/2 \\ K &= E_{\mathrm{c}} \\ \beta &= \frac{\omega_0}{4}\frac{\delta E_{\mathrm{J}}}{E_{\mathrm{J}}} \end{aligned}$$

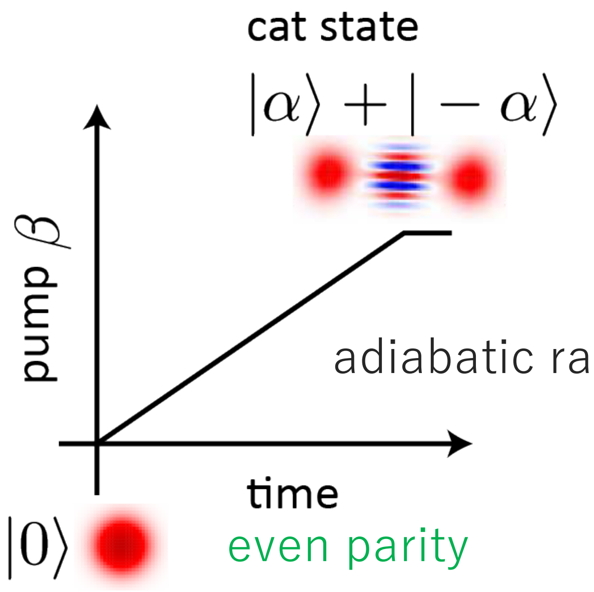
Generation of Schrodinger's cat state

$$\mathcal{H}_{\mathrm{KPO}}/\hbar = -\frac{K}{2}a^\dagger a^\dagger aa + \frac{\beta}{2}({a^\dagger}^2 + a^2)$$
$$= -K\left({a^\dagger}^2 - \frac{\beta}{K}\right)\left(a^2 - \frac{\beta}{K}\right) + \frac{\beta^2}{K}$$

P. T. Cochrane et al., Phys. Rev. A **59**, 2631 (1999).
H. Goto, Sci. Rep. **6**, 21686 (2016).
S. Puri et al., npj Quantum Info. **3**, 18 (2017).

Coherent state is an eigenstate of annihilation operator a, $a|\alpha\rangle = \alpha|\alpha\rangle$

H_{KPO} has degenerate eigenstates of $|\pm\alpha\rangle$, where $\alpha = \sqrt{\frac{\beta}{K}}$



parity conservation

pitchfork bifurcation

M. I. Dykman, Phys. Rev. E **57**, 5202 (1998).

H. Goto, Sci. Rep. **6**, 21686 (2016).
https://www.nature.com/articles/srep21686
Fig. 1

$$\dot{x} = y[\Delta + p + K(x^2 + y^2)] \quad = 0$$

$$\dot{y} = x[-\Delta + p - K(x^2 + y^2)] \quad = 0$$

solid lines:
classically stable points

$$x = \pm\sqrt{(p - \Delta)/K}$$

Wigner function below and above the threshold

$$H_1 = \hbar\Delta a^\dagger a + \hbar\frac{K}{2}a^{\dagger 2}a^2 - \hbar\frac{p}{2}(a^2 + a^{\dagger 2})$$

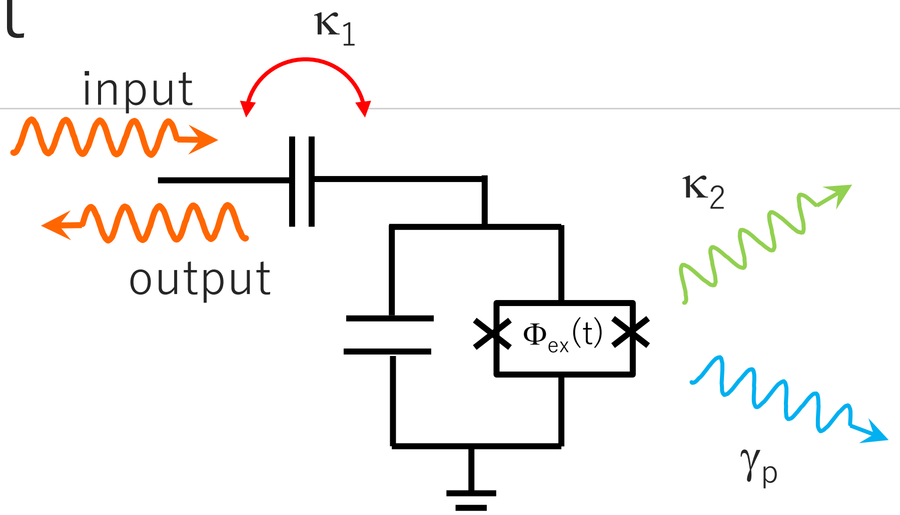
Schrodinger's cat state

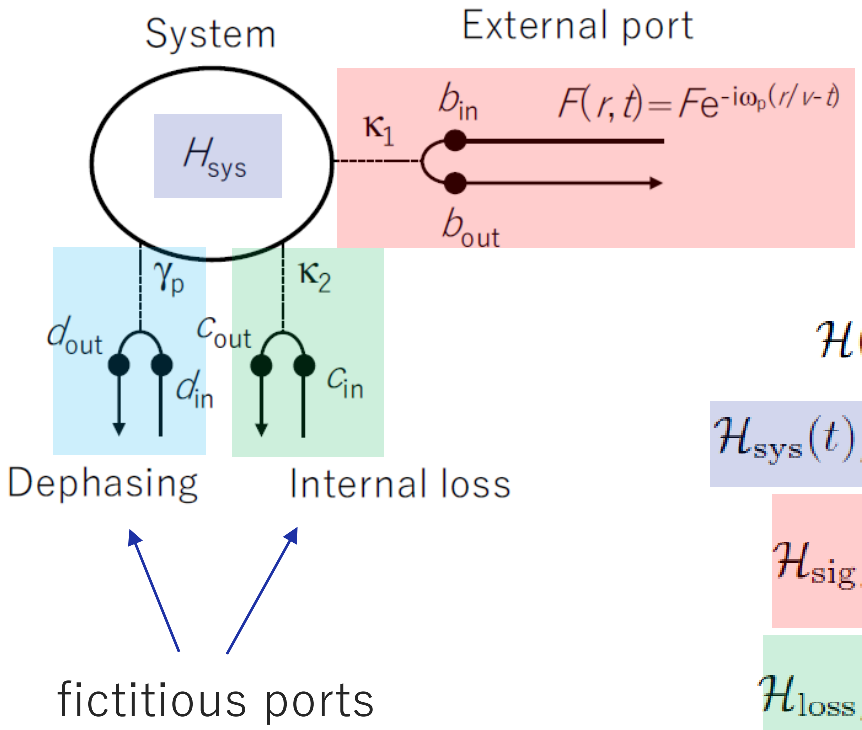
$$\frac{da}{dt} = \frac{\mathrm{i}}{\hbar}[H_1, a]$$

$$a = x + \mathrm{i}y$$

$$a^\dagger = x - \mathrm{i}y$$

KPO coupled to environment





$$\mathcal{H}(t) = \mathcal{H}_{\mathrm{sys}}(t) + \mathcal{H}_{\mathrm{sig}} + \mathcal{H}_{\mathrm{loss}} + \mathcal{H}_{\mathrm{dep}},$$

$$\mathcal{H}_{\mathrm{sys}}(t)/\hbar = \Omega_0 a^\dagger a + \frac{K}{2} a^\dagger a^\dagger a a,$$

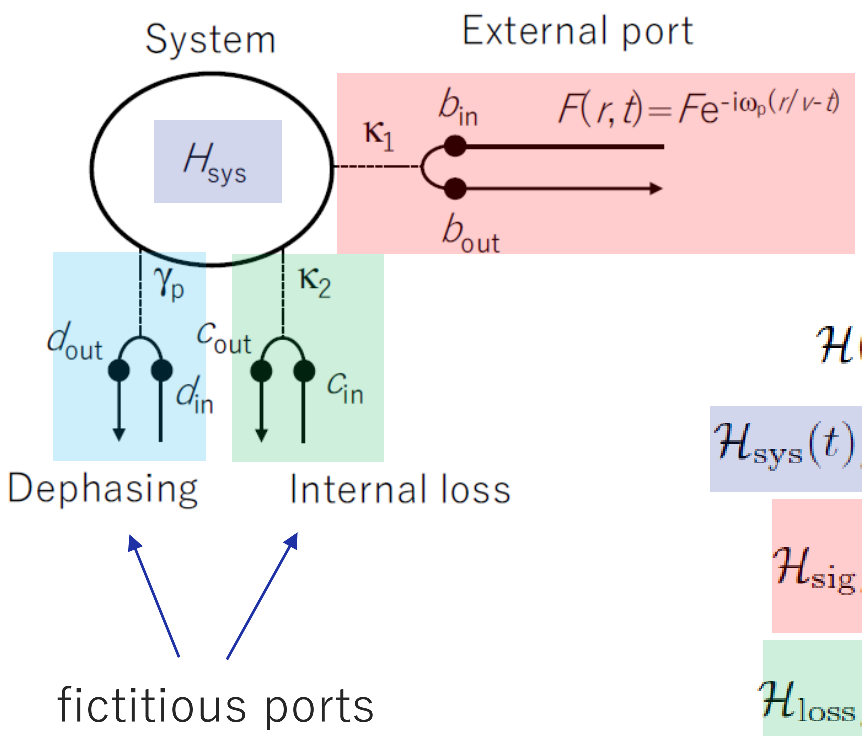
w/o parametric drive

$$\mathcal{H}_{\mathrm{sig}}/\hbar = \int dk \left[v_b k b_k^\dagger b_k + \sqrt{\frac{v_b \kappa_1}{2\pi}} \left(a^\dagger b_k + b_k^\dagger a \right) \right],$$

$$\mathcal{H}_{\mathrm{loss}}/\hbar = \int dk \left[v_c k c_k^\dagger c_k + \sqrt{\frac{v_c \kappa_2}{2\pi}} \left(a^\dagger c_k + c_k^\dagger a \right) \right],$$

$$\mathcal{H}_{\mathrm{dep}}/\hbar = \int dk \left[v_d k d_k^\dagger d_k + \sqrt{\frac{v_d \gamma_p}{\pi}} a^\dagger a \left(d_k + d_k^\dagger \right) \right],$$

KPO coupled to environment



Input-output formalism

M. J. Collett and C. W. Gardiner, Phys. Rev. A **30**, 1386 (1984).

$$b_{\rm in}(t) \equiv \tilde{b}_{-0}(t) = \tilde{b}_{-vt}(0),$$

$$b_{\rm out}(t) \equiv \tilde{b}_{+0}(t) = b_{\rm in}(t) - {\rm i}\sqrt{\frac{\kappa_1}{v}}a(t).$$

derived from Heisenberg's EOM for b_k

$$\mathcal{H}(t) = \mathcal{H}_{\rm sys}(t) + \mathcal{H}_{\rm sig} + \mathcal{H}_{\rm loss} + \mathcal{H}_{\rm dep},$$

$$\mathcal{H}_{\rm sys}(t)/\hbar = \Omega_0 a^\dagger a + \frac{K}{2}a^\dagger a^\dagger a a,$$ w/o parametric drive

$$\mathcal{H}_{\rm sig}/\hbar = \int dk\left[v_b k b_k^\dagger b_k + \sqrt{\frac{v_b\kappa_1}{2\pi}}\left(a^\dagger b_k + b_k^\dagger a\right)\right],$$

$$\mathcal{H}_{\rm loss}/\hbar = \int dk\left[v_c k c_k^\dagger c_k + \sqrt{\frac{v_c\kappa_2}{2\pi}}\left(a^\dagger c_k + c_k^\dagger a\right)\right],$$

$$\mathcal{H}_{\rm dep}/\hbar = \int dk\left[v_d k d_k^\dagger d_k + \sqrt{\frac{v_d\gamma_p}{\pi}}a^\dagger a\left(d_k + d_k^\dagger\right)\right],$$

Input-output formalism

- By solving Heisenberg's equation of motion for appropriate operator, e.g., a, a⁺a, and |m><n|, we can calculate various physical quantities such as reflection coefficient, fluorescence spectra, amplifier's gain.

- Example 1: Reflection coefficient

Heisenberg's equation of motion for a

$$b_{\mathrm{out}}(t) \equiv \widetilde{b}_{+0}(t) = b_{\mathrm{in}}(t) - \mathrm{i}\sqrt{\frac{\kappa_1}{v}}a(t).$$

$$\begin{aligned}
\frac{da}{dt} &= \mathrm{i}[\mathcal{H}(t)/\hbar, a] \\
&= \mathrm{i}[\mathcal{H}_{\mathrm{sys}}(t)/\hbar, a] - \frac{\kappa_1+\kappa_2}{2}a + \gamma_p[a^\dagger a, a] \\
&\quad -\mathrm{i}\sqrt{v\kappa_1}b_{\mathrm{in}}(t) - \mathrm{i}\sqrt{v\kappa_2}c_{\mathrm{in}}(t) - \mathrm{i}\sqrt{2v\gamma_p}[ad_{\mathrm{in}}(t) + d_{\mathrm{in}}^\dagger(t)a] \\
&= -\left(\mathrm{i}\Omega_0 + \mathrm{i}Ka^\dagger a + \frac{\kappa}{2}\right)a - \mathrm{i}\sqrt{v\kappa_1}b_{\mathrm{in}}(t) - \mathrm{i}\sqrt{v\kappa_2}c_{\mathrm{in}}(t) - \mathrm{i}\sqrt{2v\gamma_p}[ad_{\mathrm{in}}(t) + d_{\mathrm{in}}^\dagger(t)a]
\end{aligned}$$

$$\kappa = \kappa_1 + \kappa_2 + 2\gamma_p$$

Reflection coefficient

Take the expectation value by initial state of the system and neglect the nonlinear term <a⁺aa> to consider weak probe power limit.

$$\frac{d\langle a\rangle}{dt} = -\left(\mathrm{i}\Omega_0 + \frac{\kappa}{2}\right)\langle a\rangle$$
$$-\mathrm{i}\sqrt{v\kappa_1}\langle b_{\mathrm{in}}(t)\rangle - \mathrm{i}\sqrt{v\kappa_2}\cancel{\langle c_{\mathrm{in}}(t)\rangle} - \mathrm{i}\sqrt{2v\gamma_p}[\cancel{\langle a d_{\mathrm{in}}(t)\rangle} + \cancel{\langle d^\dagger_{\mathrm{in}}(t) a\rangle}]$$

no signal on the fictitious ports

$$d/dt \to -\mathrm{i}\omega_{\mathrm{in}}$$

$$\langle a\rangle = \frac{-\mathrm{i}\sqrt{v\kappa_1}}{-\mathrm{i}(\omega_{\mathrm{in}} - \Omega_0) + \kappa/2}\langle b_{\mathrm{in}}\rangle$$

$$\langle b_{\mathrm{out}}\rangle = \langle b_{\mathrm{in}}\rangle - \mathrm{i}\sqrt{\frac{\kappa_1}{v}}\langle a\rangle$$
$$= \boxed{\left[1 - \frac{\kappa_1}{-\mathrm{i}(\omega_{\mathrm{in}} - \Omega_0) + \kappa/2}\right]}\langle b_{\mathrm{in}}\rangle.$$

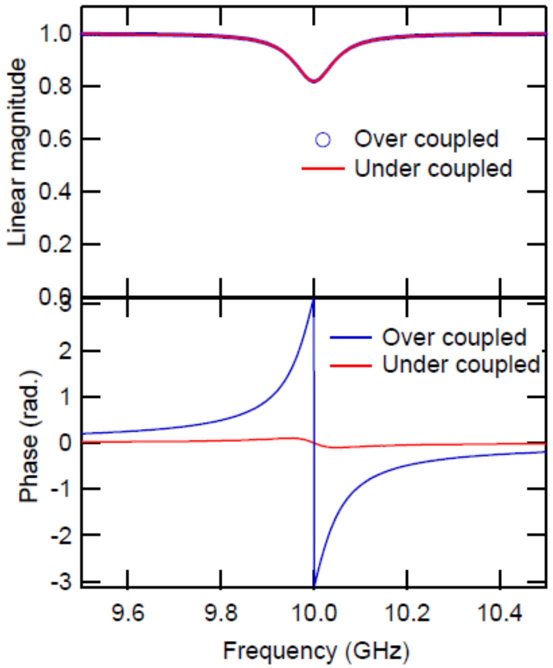
reflection coefficient Γ

$$\kappa = \kappa_1 + \kappa_2 + 2\gamma_p$$

e.g. calculated Γ

over-coupled
$\kappa_1/2\pi$ = 100 MHz
$\kappa/2\pi$ = 110 MHz

under-coupled
$\kappa_1/2\pi$ = 10 MHz
$\kappa/2\pi$ = 110 MHz

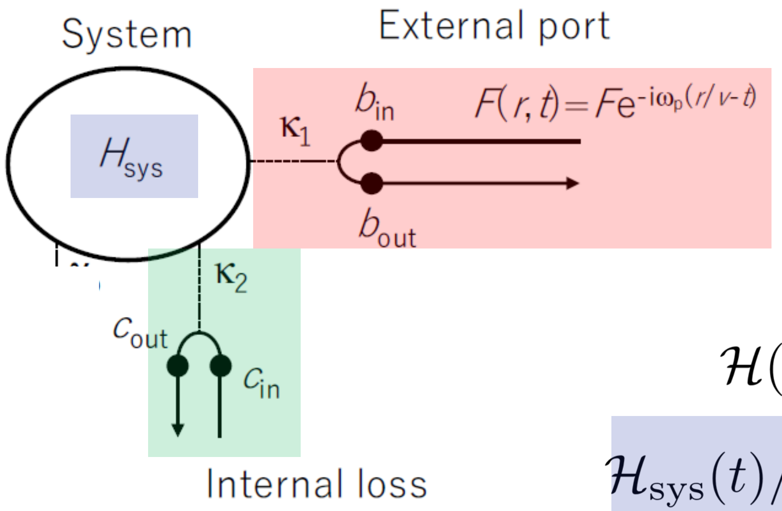


note: Γ does not tell κ_2 from γ_p.

Input-output formalism

T. Yamamoto, in Springer Lecture Notes in Physics
"Quantum Computing, Quantum Communication and Quantum Metrology"
edited by Y. Yamamoto and K. Semba

Example 2: Master equation



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derived from Heisenberg's EOM for b_k

$$\mathcal{H}(t) = \mathcal{H}_{\rm sys}(t) + \mathcal{H}_{\rm sig} + \mathcal{H}_{\rm loss},$$

$$\mathcal{H}_{\rm sys}(t)/\hbar = \Omega_0 a^\dagger a + \frac{K}{2}a^\dagger a^\dagger aa + \beta(a^\dagger + a)^2\cos\omega_{\rm p}t$$

$$\mathcal{H}_{\rm sig}/\hbar = \int dk\left[v_b k b_k^\dagger b_k + \sqrt{\frac{v_b\kappa_1}{2\pi}}\left(a^\dagger b_k + b_k^\dagger a\right)\right]$$

$$\mathcal{H}_{\rm loss}/\hbar = \int dk\left[v_c k c_k^\dagger c_k + \sqrt{\frac{v_c\kappa_2}{2\pi}}\left(a^\dagger c_k + c_k^\dagger a\right)\right]$$

forget about dephasing for the moment

Master equation

Heisenberg's equation of motion for $s_{mn}(t) \equiv |m\rangle\langle n|$

$$\frac{d}{dt}s_{mn}(t) = -\mathrm{i}[s_{mn}(t), \mathcal{H}/\hbar]$$

Using

$$\begin{aligned}
\left[s_{mn}, \frac{1}{\sqrt{2}}\int dk a^\dagger b_k\right] &= [s_{mn}, a^\dagger]b_{\mathrm{in}} + \frac{1}{2}\sqrt{\frac{\kappa_1}{v}}(a^\dagger s_{mn} a - s_{mn} a^\dagger a), \\
\left[s_{mn}, \frac{1}{\sqrt{2}}\int dk b_k^\dagger a\right] &= b_{\mathrm{in}}^\dagger [s_{mn}, a] - \frac{1}{2}\sqrt{\frac{\kappa_1}{v}}(a^\dagger s_{mn} a - a^\dagger a s_{mn})
\end{aligned}$$

$$\begin{aligned}
\frac{d}{dt}s_{mn} &= \frac{-\mathrm{i}}{\hbar}[s_{mn}, \mathcal{H}_{\mathrm{sys}}(t)] + \frac{\kappa}{2}(2a^\dagger s_{mn} a - s_{mn} a^\dagger a - a^\dagger a s_{mn}) \\
&\quad + \sqrt{v\kappa_1}[s_{mn}, a^\dagger]b_{\mathrm{in}}(t) - \sqrt{v\kappa_1}b_{\mathrm{in}}^\dagger(t)[s_{mn}, a] \\
&\quad + \sqrt{v\kappa_2}[s_{mn}, a^\dagger]c_{\mathrm{in}}(t) - \sqrt{v\kappa_2}c_{\mathrm{in}}^\dagger(t)[s_{mn}, a].
\end{aligned}$$

Master equation

$$\frac{d}{dt}s_{mn} = \frac{-\mathrm{i}}{\hbar}[s_{mn}, \mathcal{H}_{\mathrm{sys}}(t)] + \frac{\kappa}{2}(2a^\dagger s_{mn} a - s_{mn} a^\dagger a - a^\dagger a s_{mn})$$
$$+\sqrt{\upsilon\kappa_1}[s_{mn}, a^\dagger] b_{\mathrm{in}}(t) - \sqrt{\upsilon\kappa_1} b_{\mathrm{in}}^\dagger(t)[s_{mn}, a]$$
$$+\sqrt{\upsilon\kappa_2}[s_{mn}, a^\dagger] c_{\mathrm{in}}(t) - \sqrt{\upsilon\kappa_2} c_{\mathrm{in}}^\dagger(t)[s_{mn}, a].$$

rotating frame

$$s_{mn}(t) \equiv e^{\mathrm{i}\Omega_0(m-n)t} S_{mn}(t),\ a(t) \equiv e^{-\mathrm{i}\Omega_0 t} A(t),$$
$$b_{\mathrm{in}}(t) \equiv e^{-\mathrm{i}\Omega_0 t} B_{\mathrm{in}}(t),\ c_{\mathrm{in}}(t) \equiv e^{-\mathrm{i}\Omega_0 t} C_{\mathrm{in}}(t)$$

take expectation value

$$\sqrt{\upsilon}\langle B_{\mathrm{in}}\rangle = E_{\mathrm{in}} \text{ and } \langle C_{\mathrm{in}}\rangle = 0,$$

$$\frac{d}{dt}\langle S_{mn}\rangle = \frac{\mathrm{i}}{\hbar}\langle[\mathcal{H}_{\mathrm{sys}}, S_{mn}]\rangle + \frac{\kappa}{2}\Big(2\langle A^\dagger S_{mn} A\rangle - \langle S_{mn} A^\dagger A\rangle - \langle A^\dagger A S_{mn}\rangle\Big)$$
$$+\sqrt{\kappa_1} E_{\mathrm{in}}\langle[S_{mn}, A^\dagger]\rangle - \sqrt{\kappa_1} E_{\mathrm{in}}^*\langle[S_{mn}, A]\rangle.$$

Master equation

$$\frac{d}{dt}\langle S_{mn}\rangle = \frac{\mathrm{i}}{\hbar}\langle[\mathcal{H}_{\mathrm{sys}}, S_{mn}]\rangle + \frac{\kappa}{2}\Big(2\langle A^\dagger S_{mn} A\rangle - \langle S_{mn} A^\dagger A\rangle - \langle A^\dagger A S_{mn}\rangle\Big)$$
$$+\sqrt{\kappa_1}E_{\mathrm{in}}\langle[S_{mn}, A^\dagger]\rangle - \sqrt{\kappa_1}E_{\mathrm{in}}^*\langle[S_{mn}, A]\rangle.$$

$$\langle S_{mn}\rangle = \mathrm{Tr}[\rho S_{mn}] = \rho_{nm},$$
$$\langle A^\dagger S_{mn} A\rangle = \sqrt{(m+1)(n+1)}\mathrm{Tr}[\rho S_{m+1,n+1}] = \sqrt{(m+1)(n+1)}\rho_{n+1,m+1},$$

etc.

$$\frac{d\rho}{dt} = -\frac{\mathrm{i}}{\hbar}[\mathcal{H}_{\mathrm{int}}, \rho] + \frac{\kappa}{2}(2A\rho A^\dagger - A^\dagger A\rho - \rho A^\dagger A),$$

Lindblad operator for single photon decay $D[\hat{a}]\rho$

$$\mathcal{H}_{\mathrm{int}}/\hbar = \frac{\beta}{2}(A^2 + A^{\dagger 2}) + \frac{K}{2}A^\dagger A^\dagger A A + \sqrt{\kappa_1}(E_{\mathrm{in}}A^\dagger + E_{\mathrm{in}}^* A)$$

parametric drive Kerr nonlinearity single-photon drive

Analytical solution for steady state

PHYSICAL REVIEW A **94**, 033841 (2016)

Exact steady state of a Kerr resonator with one- and two-photon driving and dissipation: Controllable Wigner-function multimodality and dissipative phase transitions

Nicola Bartolo,[*] Fabrizio Minganti, Wim Casteels, and Cristiano Ciuti[†]
Université Paris Diderot, Sorbonne Paris Cité, Laboratoire Matériaux et Phénomènes Quantiques, CNRS-UMR7162, 75013 Paris, France
(Received 28 July 2016; published 22 September 2016)

N. Bartolo et al., Phys. Rev. A94, 033841 (2016).
https://journals.aps.org/pra/abstract/10.1103/PhysRevA.94.033841
Fig. 1

$$\hat{\mathcal{H}} = -\Delta\hat{a}^\dagger\hat{a} + \frac{U}{2}\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a} + F\hat{a}^\dagger + F^*\hat{a} + \frac{G}{2}\hat{a}^\dagger\hat{a}^\dagger + \frac{G^*}{2}\hat{a}\hat{a},$$

$$i\frac{\partial\hat{\rho}}{\partial t} = [\hat{\mathcal{H}},\hat{\rho}] + i\frac{\gamma}{2}\mathcal{D}(\hat{a})\,\hat{\rho} + i\frac{\eta}{2}\mathcal{D}(\hat{a}^2)\,\hat{\rho}, \quad =0$$

for steady state

$$\langle\hat{a}^{\dagger i}\hat{a}^j\rangle = \frac{1}{\mathcal{N}}\sum_{m=0}^{\infty}\frac{2^m}{m!}\mathcal{F}_{m+j}(f,g,c)\mathcal{F}^*_{m+i}(f,g,c).$$

$$\mathcal{F}_m(f,g,c) = (i\sqrt{g})^m\,{}_2F_1(-m,\,-c-i\,f/\sqrt{g};-2c;2),$$

${}_2F_1$: Gauss hypergeometric function

N. Bartolo et al., Phys. Rev. A94, 033841 (2016).
https://journals.aps.org/pra/abstract/10.1103/PhysRevA.94.033841
Fig. 2

Realization of quantum parametric oscillator

PHYSICAL REVIEW X **9**, 021049 (2019)

Quantum Dynamics of a Few-Photon Parametric Oscillator

Zhaoyou Wang,* Marek Pechal,* E. Alex Wollack, Patricio Arrangoiz-Arriola, Maodong Gao, Nathan R. Lee, and Amir H. Safavi-Naeini†
Department of Applied Physics and Ginzton Laboratory, Stanford University 348 Via Pueblo Mall, Stanford, California 94305, USA

Z. Wang et al., Phys. Rev. X 9, 021049 (2019). https://journals.aps.org/prx/abstract/10.1103/PhysRevX.9.021049
Fig. 4c

- $K/\kappa \sim 17$: single-photon Kerr regime
- no ancilla qubit
  - state tomography based on transient PSD measurement
- 5.8 photon Schrodinger's cat state

Z. Wang et al., Phys. Rev. X 9, 021049 (2019). https://journals.aps.org/prx/abstract/10.1103/PhysRevX.9.021049
Fig. 5

Z. Wang et al., Phys. Rev. X 9, 021049 (2019). https://journals.aps.org/prx/abstract/10.1103/PhysRevX.9.021049
Figs. 3a and 3b

Z. Wang et al., Phys. Rev. X 9, 021049 (2019). https://journals.aps.org/prx/abstract/10.1103/PhysRevX.9.021049
Fig. 4d

Table of Contents

Hardware-efficient quantum error correction

Generic QEC

Hardware efficient QEC

W. Cai et al., Fundamental Research **1**, 50 (2021).
https://www.sciencedirect.com/science/article/pii/S2667325820300145
Figs. 2a and 2b

Encode logical qubit using multiple physical qubits (two-level system)
→increase in error source
→requires many physical qubits

Encode logical qubit using one physical system with infinite degree of freedom
→limited error source (with biased noise)
→reduced hardware overhead

W. Cai et al., Fundamental Research **1**, 50 (2021).

noise-biased qubit

Coherent state $|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$

N. Frattini, PhD thesis 2021
https://elischolar.library.yale.edu/gsas_dissertations/332/
Fig. 4.1a

$$|\pm X\rangle = \frac{1}{\sqrt{2}}\left(\left|\mathcal{C}_\alpha^+\right\rangle \pm \left|\mathcal{C}_\alpha^-\right\rangle\right) \to |\pm\alpha\rangle$$

$$|\pm Y\rangle = \frac{1}{\sqrt{2}}\left(\left|\mathcal{C}_\alpha^+\right\rangle \pm i\left|\mathcal{C}_\alpha^-\right\rangle\right) \to \frac{1}{\sqrt{2}}\left(|+\alpha\rangle \mp i\,|-\alpha\rangle\right)$$

$$|\pm Z\rangle = \left|\mathcal{C}_\alpha^\pm\right\rangle \to \frac{1}{\sqrt{2}}\left(|+\alpha\rangle \pm |-\alpha\rangle\right)$$

N. Frattini, PhD thesis 2021

Eigenstate of photon loss operator

$$a|\alpha\rangle = \alpha|\alpha\rangle$$

quasi-orthogonal

$$\langle -\alpha|\alpha\rangle = e^{-2|\alpha|^2} \sim 0$$

error due to photon loss

Phase-flip rate:

$$|\langle -\alpha|a|\alpha\rangle|^2 = |\alpha|^2 e^{-4|\alpha|^2} \sim 0$$

exponentially suppressed

Bit-flip rate:

$$|\langle \mathcal{C}_\alpha^- |a| \mathcal{C}_\alpha^+\rangle|^2 \sim |\alpha|^2$$

polynomial increase

Can focus on most likely error ➡ higher error threshold

P. Aliferis et al.,
Phys. Rev. A78, 052331 (2008).

noise-biased qubit

N. Frattini, PhD thesis 2021

- Operators in cat basis: ex. $\begin{pmatrix} \langle \mathcal{C}_\alpha^+ | a | \mathcal{C}_\alpha^+ \rangle & \langle \mathcal{C}_\alpha^+ | a | \mathcal{C}_\alpha^- \rangle \\ \langle \mathcal{C}_\alpha^- | a | \mathcal{C}_\alpha^- \rangle & \langle \mathcal{C}_\alpha^- | a | \mathcal{C}_\alpha^- \rangle \end{pmatrix} = \alpha \begin{pmatrix} 0 & r^{-1} \\ r & 0 \end{pmatrix} = \alpha (\frac{r + r^{-1}}{2}) \boldsymbol{X} - \mathrm{i}\alpha (\frac{r - r^{-1}}{2}) \boldsymbol{Y}$

N. Frattini, PhD thesis 2021
https://elischolar.library.yale.edu/gsas_dissertations/332/
Table 4.1

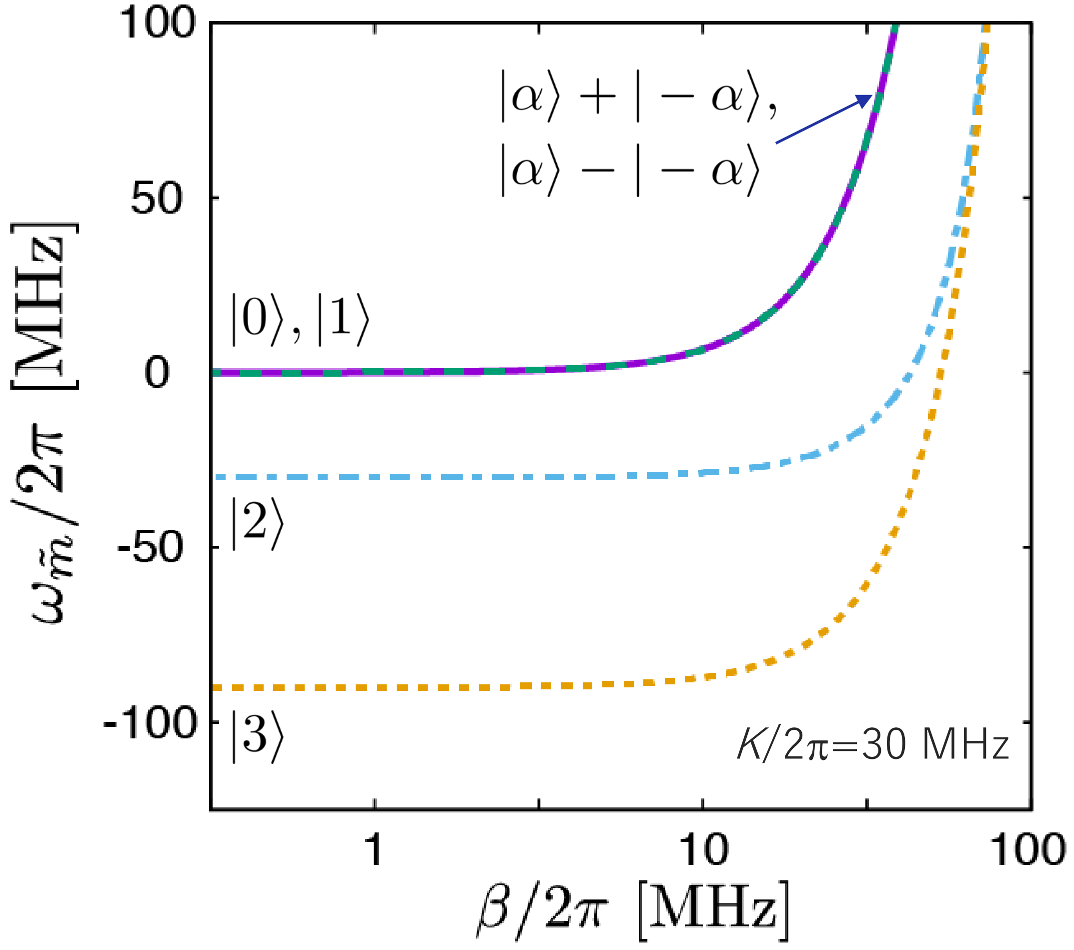
$$
\begin{aligned}
|\mathcal{C}_\alpha^\pm\rangle &= \mathcal{N}_\alpha^\pm (|\alpha\rangle \pm |-\alpha\rangle) \\
\mathcal{N}_\alpha^\pm &= 1/\sqrt{2(1 \pm e^{-2|\alpha|^2})} \\
r &= \mathcal{N}_\alpha^+ / \mathcal{N}_\alpha^-
\end{aligned}
$$

$$
r \to \begin{cases} 1 - e^{-2|\alpha|^2} & (\text{large } \alpha) \\ 0 & (\alpha \to 0) \end{cases} \qquad r^2 \to \begin{cases} 1 - 2e^{-2|\alpha|^2} & (\text{large } \alpha) \\ 0 & (\alpha \to 0) \end{cases}
$$

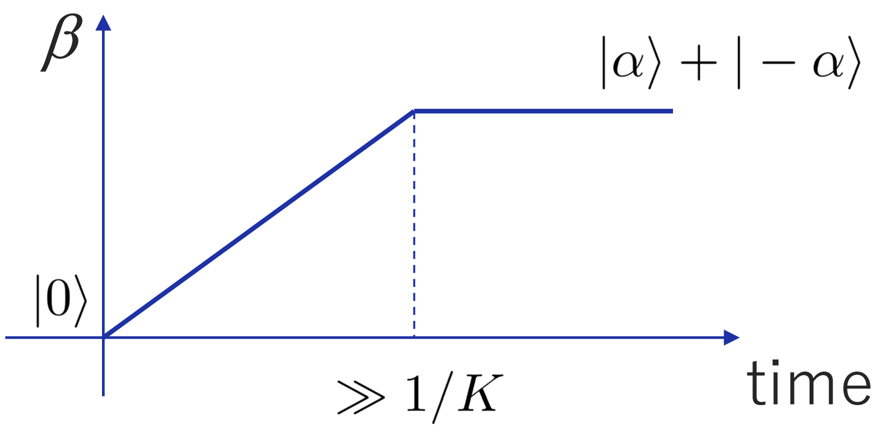
$$
r^{-1} \to \begin{cases} 1 + e^{-2|\alpha|^2} & (\text{large } \alpha) \\ 1/|\alpha| & (\alpha \to 0) \end{cases} \qquad r^{-2} \to \begin{cases} 1 + 2e^{-2|\alpha|^2} & (\text{large } \alpha) \\ 1/|\alpha|^2 & (\alpha \to 0) \end{cases}
$$

Gate operations

$$\mathcal{H}_{\mathrm{KPO}}/\hbar = -\frac{K}{2}a^\dagger a^\dagger aa + \beta({a^\dagger}^2 + a^2)$$



Preparation of cat state

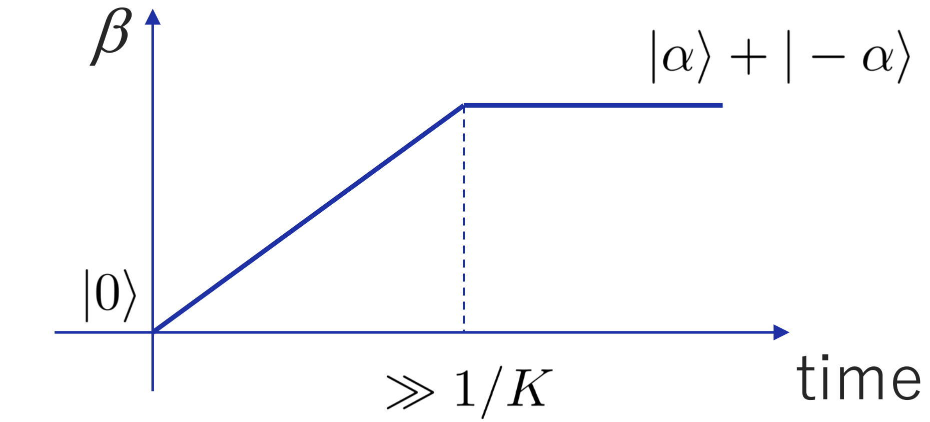


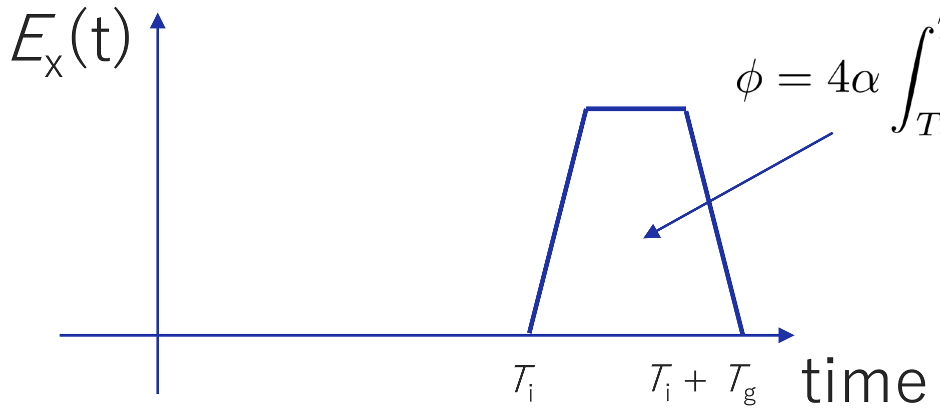
R_x gate

P. T. Cochrane et al., Phys. Rev. A **59**, 2631 (1999).
H. Goto, Sci. Rep. **6**, 21686 (2016).
S. Puri et al., npj Quantum Info. **3**, 18 (2017).

single-photon drive @ $\omega_p/2$

$$\mathcal{H}_{\rm KPO}/\hbar = -\frac{K}{2}a^\dagger a^\dagger a a + \beta(a^{\dagger 2} + a^2) \boxed{+E_x(t)(a + a^\dagger)}$$





$$\phi = 4\alpha \int_{T_i}^{T_i+T_g} E_x(t)dt$$

produces energy difference between $|\alpha\rangle$ and $|-\alpha\rangle$

$$\langle\alpha|E_x(t)(a+a^\dagger)|\alpha\rangle = 2\alpha E_x(t)$$

$$\langle-\alpha|E_x(t)(a+a^\dagger)|-\alpha\rangle = -2\alpha E_x(t)$$

in α basis

change to cat basis

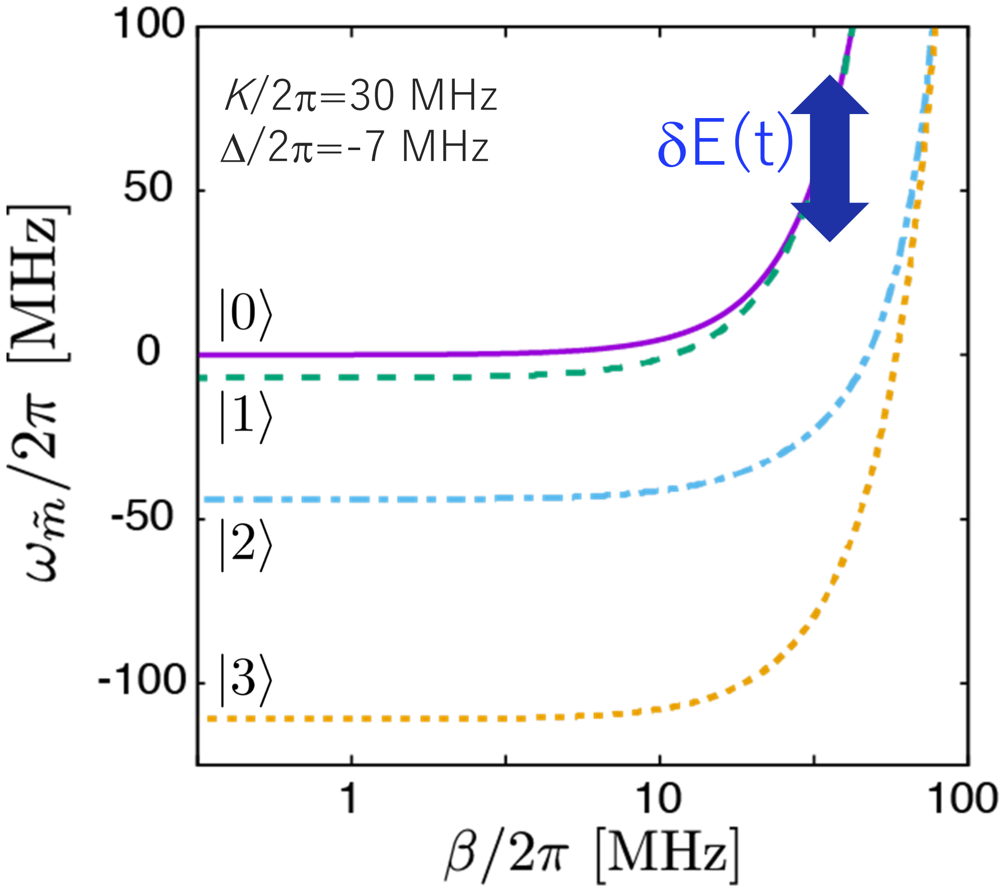
$$H\begin{pmatrix} e^{-i\phi/2} & 0 \\ 0 & e^{i\phi/2}\end{pmatrix}H = \begin{pmatrix} \cos\frac{\phi}{2} & -i\sin\frac{\phi}{2} \\ -i\sin\frac{\phi}{2} & \cos\frac{\phi}{2}\end{pmatrix} = R_x(\phi)$$

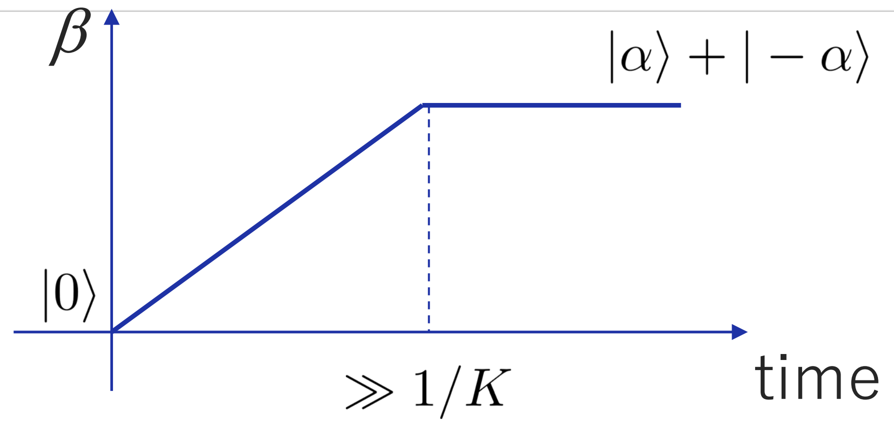
R_z gate

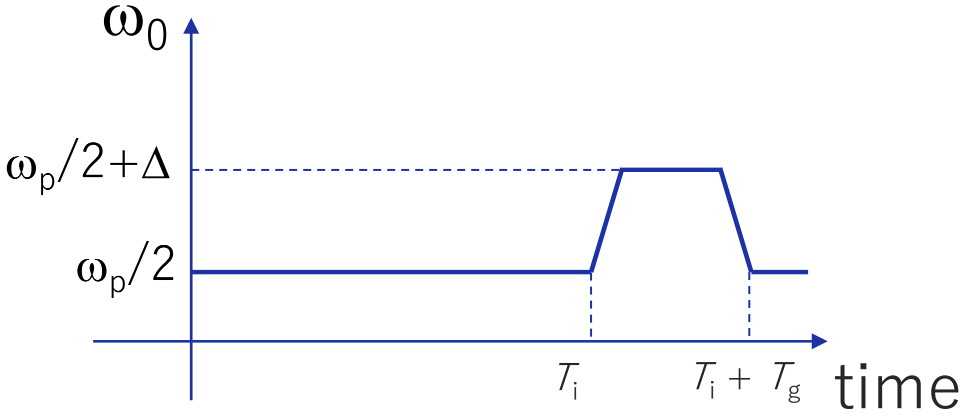
H. Goto, Sci. Rep. **6**, 21686 (2016).
S. Puri et al., npj Quantum Info. **3**, 18 (2017).

detuning $(\omega_0 - \omega_p/2)$

$$\mathcal{H}_{\mathrm{KPO}}/\hbar = -\frac{K}{2}a^\dagger a^\dagger a a + \beta(a^{\dagger 2} + a^2) \boxed{+\Delta(t)a^\dagger a}$$







$$\begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix} \qquad \theta = \int_{T_i}^{T_i+T_g} \delta E(t)dt$$

Alternative method:
temporarily remove the two-photon drive. Grim et al., 2020

Two-qubit R_{zz} gate

1. H. Goto, Sci. Rep. **6**, 21686 (2016).
2. S. Puri et al., npj Quantum Info. **3**, 18 (2017).
3. H. Chono et al., Phys. Rev. Res. **4**, 043054 (2022).
4. S. Puri et al., Sci. Adv. 6, eaay5901 (2020). [bias preserving C-NOT gate]

- R_{zz} gate (in α basis) based on linear coupling ($a_1 a_2^+ + a_1^+ a_2$) was proposed in Refs. [1,2].
  - requires temporal control of the coupling g(t) to turn on/off the coupling
- Effective turn on/off using conditional-driving [3]
  - large detuning between two KPO's effectively turns off the coupling in idle state
  - additional two-photon drive at sum (or diff.) frequency induces the coupling
  - realize R_{zz} gate (in α basis)

$$
\begin{array}{cccc}
|\alpha\rangle|\alpha\rangle & |\alpha\rangle|-\alpha\rangle & |-\alpha\rangle|\alpha\rangle & |-\alpha\rangle|-\alpha\rangle \\
\end{array}
$$
$$
\begin{pmatrix}
e^{-i\theta/2} & 0 & 0 & 0 \\
0 & e^{i\theta/2} & 0 & 0 \\
0 & 0 & e^{i\theta/2} & 0 \\
0 & 0 & 0 & e^{-i\theta/2}
\end{pmatrix}
$$

H. Chono et al., Phys. Rev. Res. 4, 043054 (2022).
https://journals.aps.org/prresearch/abstract/10.1103/PhysRevResearch.4.043054
Fig. 1

$$\hat{H} = \sum_{j=1,2} \hat{H}_j + \hat{H}_{\mathrm{I}} + \hat{H}_g,$$

$$\frac{\hat{H}_j}{\hbar} = -\frac{K}{2}\hat{a}_j^{\dagger 2}\hat{a}_j^2 + \frac{P}{2}(\hat{a}_j^{\dagger 2} + \hat{a}_j^2),$$

$$\frac{\hat{H}_{\mathrm{I}}}{\hbar} = g(\hat{a}_1\hat{a}_2^\dagger \mathrm{e}^{-\mathrm{i}\Delta_{12}t} + \hat{a}_1^\dagger\hat{a}_2\mathrm{e}^{\mathrm{i}\Delta_{12}t}),$$

$$\frac{\hat{H}_g}{\hbar} = \frac{p_g(t)}{2}(\hat{a}_1^2\mathrm{e}^{-\mathrm{i}\Delta_{12}t} + \hat{a}_1^{\dagger 2}\mathrm{e}^{\mathrm{i}\Delta_{12}t}),$$

static coupling:
effectively off due to fast oscillation

cancel the oscillation
to turn on the coupling

$$\alpha_1 \to \alpha_1 + \delta\alpha_1 e^{i\Delta_{12}t}$$

additional two-photon drive:

Stabilization and operation of a Kerr-cat qubit

A. Grimm *et al.*, Nature **584**, 205 (2020).

- Use SNAIL's 2nd order and 3rd order nonlinearities to realize two-photon drive and Kerr nonlinearity.
- Readout after transforming to Fock-base qubit (adiabatically turn off the pump) or freq. conversion using 3-wave mixing.
- phase-flip time >> bit-flip time (qubit with biased noise)
- two-photon drive increases phase-flip time by >30 times compared to T2 of FQ (stabilization)

A. Grimm *et al.*, Nature **584**, 205 (2020).
https://www.nature.com/articles/s41586-020-2587-z
Figs. 1d, 1e, and 1f

R_x gate

A. Grimm *et al.*, Nature **584**, 205 (2020).
https://www.nature.com/articles/s41586-020-2587-z
Figs. 2a, 2b, and 2c

A. Grimm *et al.*, Nature **584**, 205 (2020).
https://www.nature.com/articles/s41586-020-2587-z
Figs. 4b, 4c, 4f, and 4g

phase-flip time
~100 us

bit-flip time
~2.6 us

Table of Contents

Quantum bifurcation machine

Bifurcation-based adiabatic quantum computation with a nonlinear oscillator network

Hayato Goto

Sci. Rep. **6**, 21686 (2016).

Network of KPO's: connected by linear coupling

$$H_1 = \hbar\Delta a^\dagger a + \hbar\frac{K}{2}a^{\dagger 2}a^2 - \hbar\frac{p}{2}(a^2 + a^{\dagger 2})$$

$$H = \sum_{i=1}^{N} H_1^{(i)} - \hbar\frac{\xi_0}{2}\sum_{i=1}^{N}\sum_{i=1}^{N} J_{i,j}(a_i^\dagger a_j + a_i a_j^\dagger)$$

$$\Delta_i = \xi_0 \sum_{j=1}^{N} |J_{i,j}|$$ for the vacuum state to be the ground state

initial

p=0 (no pump)

$|0\rangle \otimes \cdots \otimes |0\rangle$ vacuum state

$E = 0$ ground state

increase p slowly

final

$p \gg \Delta, \xi_0 |J_{ij}|$

$\left|s_1\sqrt{p/K}\right\rangle \otimes \cdots \otimes \left|s_N\sqrt{p/K}\right\rangle$ $\quad s_i = \pm 1$

$$E_{\mathrm{corr}}(\{s_i\}) = \hbar\frac{p}{K}\sum_{i=1}^{N}\Delta_i \boxed{- \hbar\xi_0\frac{p}{K}\sum_{i=1}^{N}\sum_{j=1}^{N} J_{i,j}s_i s_j}$$

should be minimized!

Quantum bifurcation machine

Bifurcation-based adiabatic quantum computation with a nonlinear oscillator network

Hayato Goto

Sci. Rep. **6**, 21686 (2016).

numerical simulation for 4-spin problem

1000 instances with random J_{ij}

H. Goto, Sci. Rep. **6**, 21686 (2016).
https://www.nature.com/articles/srep21686
Figs. 3a and 3b

KPO
(quantum)

finds global minimum

Quantum effect?

classical

trapped in local minimum

H. Goto, Sci. Rep. **6**, 21686 (2016).
https://www.nature.com/articles/srep21686
Fig. 2

Ising machine using OPO

For theoretical comparison between QbM and CIM, see Goto, J. Phys. Soc. Jpn. **88**, 061015 (2019).

OPTICAL PROCESSING

A coherent Ising machine for 2000-node optimization problems

Takahiro Inagaki,[1*] Yoshitaka Haribara,[2,3,4] Koji Igarashi,[5] Tomohiro Sonobe,[4,6] Shuhei Tamate,[4] Toshimori Honjo,[1] Alireza Marandi,[7] Peter L. McMahon,[7] Takeshi Umeki,[8] Koji Enbutsu,[8] Osamu Tadanaga,[8] Hirokazu Takenouchi,[8] Kazuyuki Aihara,[2,3] Ken-ichi Kawarabayashi,[4,6] Kyo Inoue,[5] Shoko Utsunomiya,[4] Hiroki Takesue[1*]

T. Inagaki et al.,
https://www.science.org/doi/10.1126/science.aah4243
Fig. 2A

P. L. McMahon *et al.*, Science **354**, 614 (2016).
https://www.science.org/doi/full/10.1126/science.aah5178
Fig. 1

T. Inagaki et al.,
https://www.science.org/doi/10.1126/science.aah4243
Fig. 4

Simulated bifurcation machine

H. Goto *et al*, Sci. Adv. **5**, eaav2372 (2019),
also H. Goto *et al*, Sci. Adv. **7**, eabe79532 (2021).

SCIENCE ADVANCES | RESEARCH ARTICLE

APPLIED PHYSICS

Combinatorial optimization by simulating adiabatic bifurcations in nonlinear Hamiltonian systems

Hayato Goto*, Kosuke Tatsumura, Alexander R. Dixon

H. Goto *et al*, Sci. Adv. **5**, eaav2372 (2019).
https://www.science.org/doi/10.1126/sciadv.aav2372
Eqs. 3, 4, and 5

H. Goto *et al*, Sci. Adv. **5**, eaav2372 (2019).
https://www.science.org/doi/10.1126/sciadv.aav2372
Fig. 2

All-to-all connectivity

- Minor embedding
  - V. Choi, Quantum Inf Process **10**, 343 (2011).
- Measurement and feedback
  - T. Inagaki *et al.*, Science **354**, 603 (2016).
- Bus coupling
  - T. Onodera *et al.,* NPJ Quantum Inf. **6**, 48 (2020).
- Parity mapping
  - W. Lechner *et al.*, Sci. Adv. **1** e1500838 (2015).

Lechner-Hauke-Zoller (LHZ) scheme

RESEARCH ARTICLE Sci. Adv. **1**, e1500838 (2015).

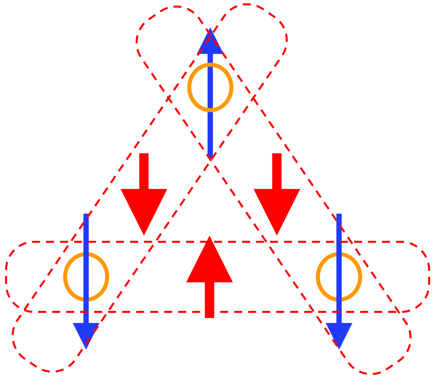
QUANTUM MECHANICS

A quantum annealing architecture with all-to-all connectivity from local interactions

Wolfgang Lechner,[1,2*] Philipp Hauke,[1,2] Peter Zoller[1,2]

- Define new pseudo-spins for all pairs of spins in the network by their parity (parallel or anti-parallel)

example:
3 spins



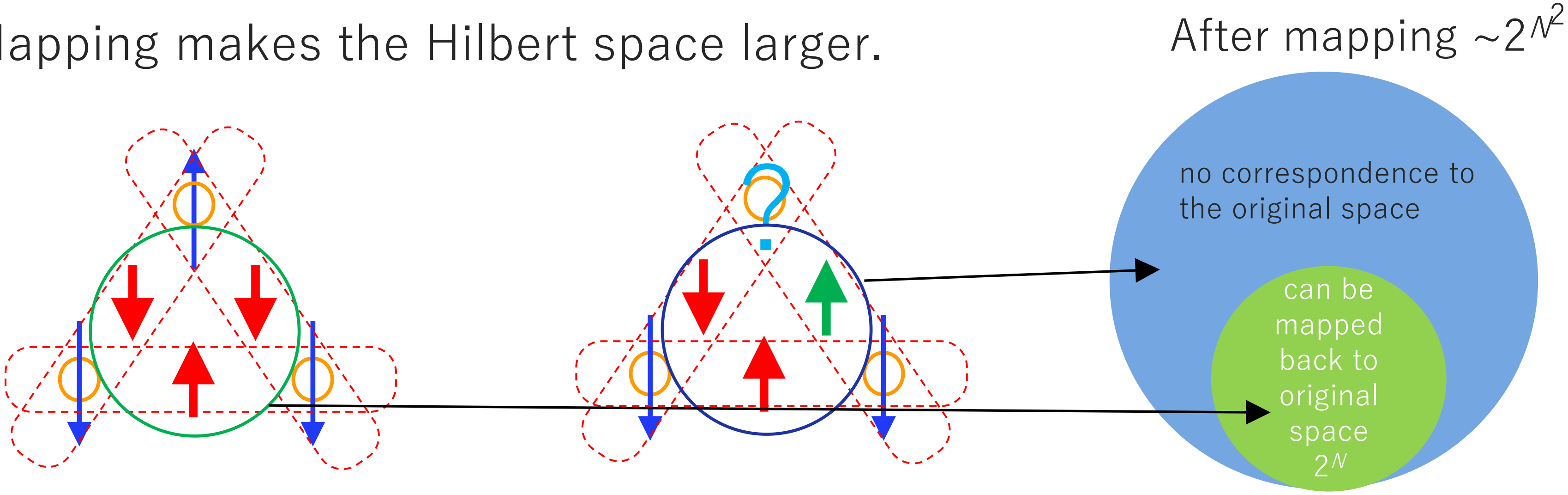
$$\sum_{i<j}^{N} J_{ij}\sigma_{iz}\sigma_{jz} \Rightarrow \sum_{k}^{{}_N\mathrm{C}_2} J_k\tilde{\sigma}_{kz}$$

coupled 2 spins | single spin + local field

# of spins: N | # of spins: ${}_N C_2 \sim O(N^2)$

Lechner-Hauke-Zoller (LHZ) scheme

- Mapping makes the Hilbert space larger.



- We need constraint!

$$\sum_{i<j}^{N} J_{ij}\sigma_{iz}\sigma_{jz} \quad \text{Mapping} \Rightarrow \quad \mathcal{H}_p = \sum_{k}^{K\,(={}_NC_2)} J_k\tilde{\sigma}_{kz} + \boxed{\sum_{l}^{K-N+1} C_l}$$

Original problem

penalty term to make the energy of the blue region higher

Lechner-Hauke-Zoller (LHZ) scheme

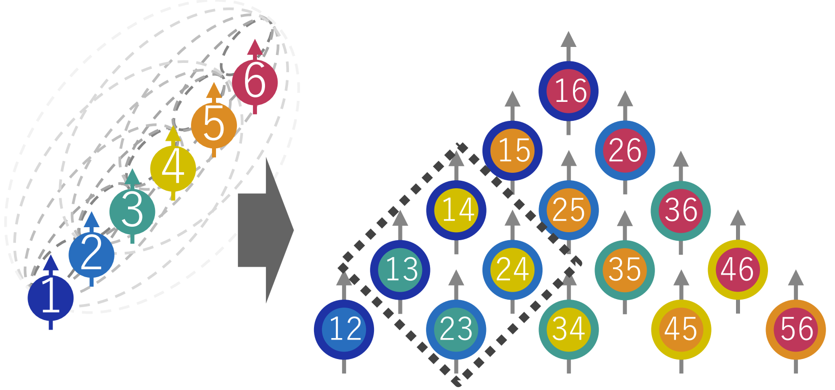
W. Lechner *et al.*, Sci. Adv. **1** e1500838 (2015).

- Express the product (parity) of two Ising spins σ using one physical spin $\tilde{\sigma}$
- Realize programmable all-to-all connectivity with local interactions.
- Simple correspondence between physical and logical spins

$$\mathcal{H} = -\sum_{i<j} J_{ij}\sigma_i\sigma_j = -\sum_k J_k\tilde{\sigma}_k$$

interaction J_{ij} → local field J_k

fully-connected logical spin N → $\frac{N(N-1)}{2}$ physical spins



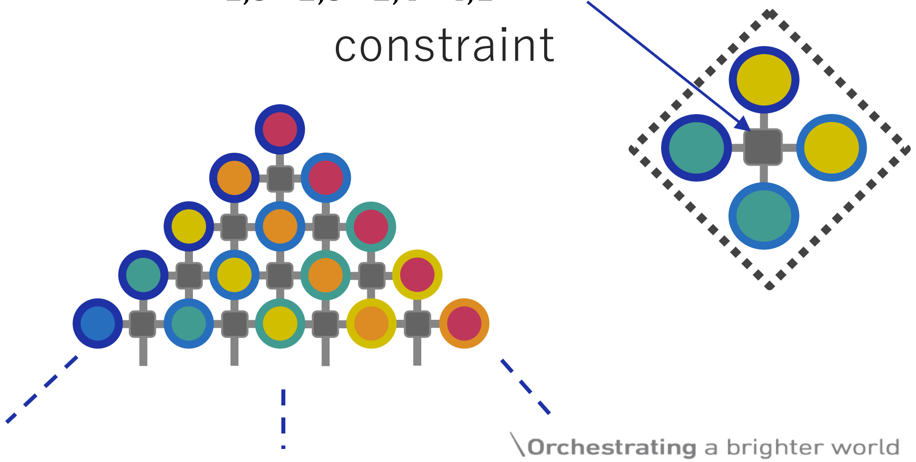
- $\tilde{\sigma}$ must satisfy the following conditions

$$\tilde{\sigma}_{1,3}\tilde{\sigma}_{2,3}\tilde{\sigma}_{2,4}\tilde{\sigma}_{4,1} = \sigma_1^2\sigma_2^2\sigma_3^2\sigma_4^2 = 1$$

$$\mathcal{H} = -\sum_k J_k\tilde{\sigma}_k - C\sum \tilde{\sigma}_a\tilde{\sigma}_b\tilde{\sigma}_c\tilde{\sigma}_d$$

four-body interaction

how to realize?



Quantum bifurcation machine with LHZ scheme

ARTICLE

Received 1 Nov 2016 | Accepted 28 Apr 2017 | Published 8 Jun 2017

DOI: 10.1038/ncomms15785

OPEN

Nature Commun. **8**, 15785 (2017).

Quantum annealing with all-to-all connected nonlinear oscillators

Shruti Puri[1], Christian Kraglund Andersen[2], Arne L. Grimsmo[1] & Alexandre Blais[1,3]

Josephson junction as a 4-body coupler

S. Puri et al., Nature Commun. **8**, 15785 (2017).
https://www.nature.com/articles/ncomms15785
Fig. 4

S. Puri et al., Nature Commun. **8**, 15785 (2017).
https://www.nature.com/articles/ncomms15785
Fig. 5

Quantum bifurcation machine with LHZ scheme

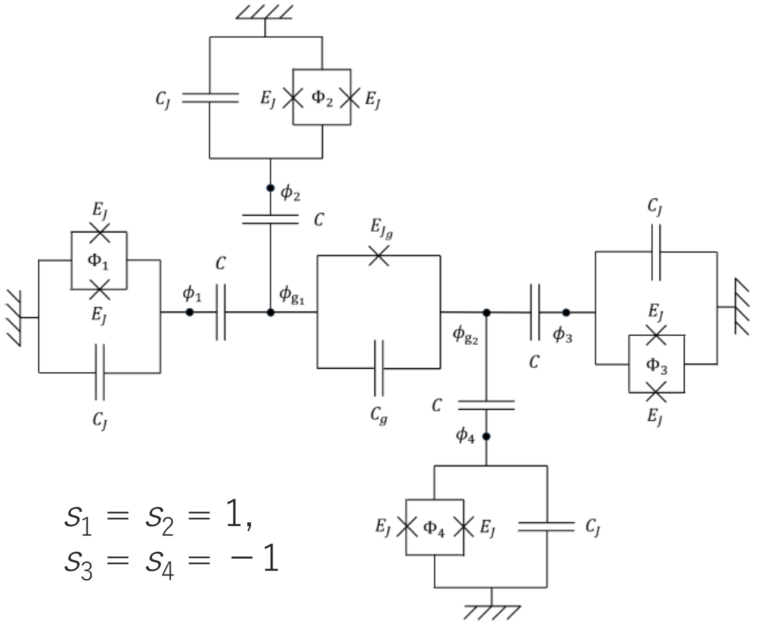
◆ 4-body coupler using a Josephson junction

$$H_{\text{total}} = \sum_{k=1..4} H_{\text{JPO},k} + H_{\text{coupler}} + g \sum_k s_k (a_k^\dagger - a_k)(a_g - a_g^\dagger)$$

$$H_{\text{JPO},k} = \hbar\omega a_k^\dagger a_k - \frac{E_C}{12}(a_k^\dagger + a_k)^4 + \frac{\hbar\omega\delta E_J}{4E_J}(a_k^\dagger + a_k)^2 \cos(\omega_{p,k} t)$$

$$H_{\text{coupler}} = \hbar\omega_g a_g^\dagger a_g \boxed{- \frac{E_{C_g}}{12}(a_g^\dagger + a_g)^4}$$

coupler nonlinearity



$s_1 = s_2 = 1,$
$s_3 = s_4 = -1$

◆ Coupler's nonlinear term turns to the effective 4-body coupling by unitary transform $U_g = e^{g' \sum_k s_k (a_k^\dagger a_g - a_k a_g^\dagger)}$ under small $g' = g/\hbar(\omega - \omega_g)$

$$(a_g^\dagger + a_g)^4 \to \left(a_g^\dagger + g' \sum_k s_k a_k - 2g'^2 a_g + \cdots + \text{h.c.}\right)^4$$

$$= (a_g^\dagger + a_g)^4 + 24g'^4 \left(a_1^\dagger a_2^\dagger a_3 a_4 + a_1 a_2 a_3^\dagger a_4^\dagger + \sum_{k<l} a_k^\dagger a_k a_l^\dagger a_l\right) + \cdots$$

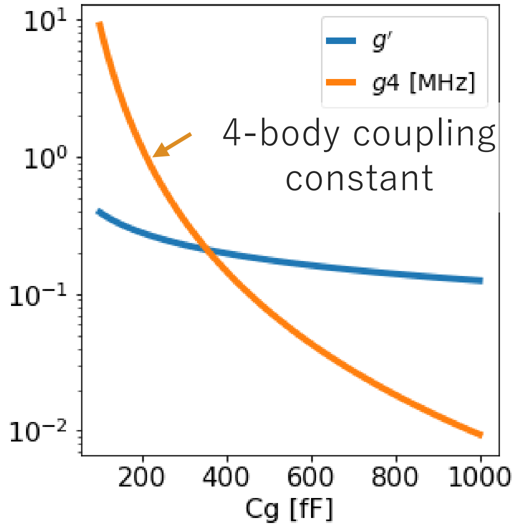
Quantum bifurcation machine with LHZ scheme

- 4-body coupler using a Josephson junction
  - Effective Hamiltonian for JPOs in the frame rotating at $\omega_{p,k}/2$

$$H_{\text{total}} \to H'_{\text{total}} \simeq \sum_k H_{\text{JPO},k} + H'_{\text{coupler}}$$

$$-g^{(4)}\left(a_1^\dagger a_2^\dagger a_3 a_4 + a_1 a_2 a_3^\dagger a_4^\dagger + \sum_{k<l} a_k^\dagger a_k a_l^\dagger a_l\right)$$

    - Condition for pump frequencies $\omega_{p,1} + \omega_{p,2} = \omega_{p,3} + \omega_{p,4}$ keeps the 4-body coupling terms stationary
    - Other coupling terms oscillate fast and are ignored
  - 4-body coupling constant $g^{(4)} = 2g'^4 E_{C_g}$
    - g': Effective coupling constant between a JPO and the coupler
    - E_{C_g}: Capacitive energy of the coupler
    - Decreasing capacitance of the coupler can increase $g^{(4)}$ without increasing g' much



Capacitance of the coupler

R. Miyazaki, AQC2022

Quantum bifurcation machine with LHZ scheme

Advantages

- no on-chip coupler
  - J_{ij} and C are controlled by the phase of the injection microwave.
- neighboring KPO's have different resonance frequencies
  - smaller crosstalk
- state stabilized after pump becomes sufficiently high

Issues

- Performance benchmarking with conventional quantum annealing
- decoherence during adiabatic evolution*?
- requires the same microwave engineering as gate-based QC without error correction

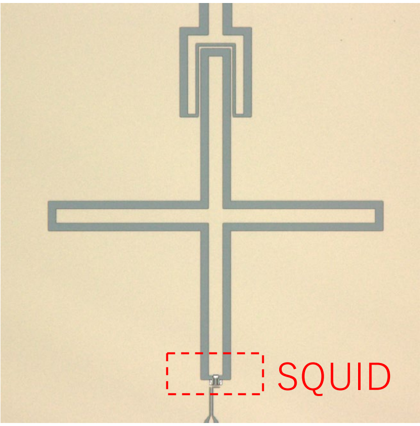
M. J. Kewming et al.,
New J. Phys. **22**, 053042 (2020).

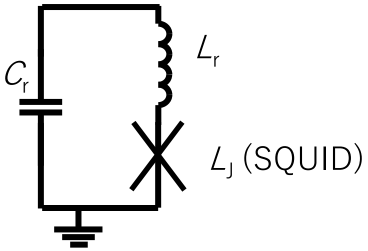
*effect of photon loss is investigated in
S. Puri et al., Nature Commun. **8**, 15785 (2017).

JPO in single-photon Kerr regime (KPO)

- Kerr nonlinearity K >> photon-loss rate $\kappa \sim 2\pi$ x 0.1 MHz
- how to design Kerr nonlinearity?

lumped-element circuit





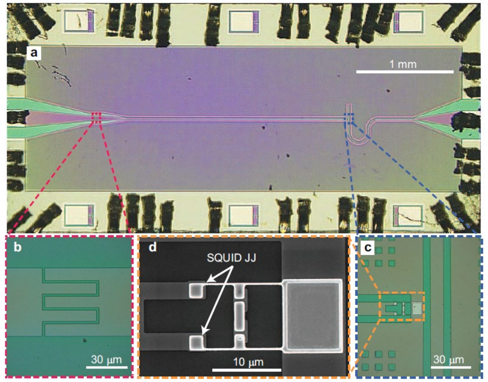
Kerr nonlinearity

$$\mathcal{H} = \hbar\omega_0'\left(a^\dagger a + \frac{1}{2}\right) - \frac{E_c}{12}\left(\frac{L_J}{L_r + L_J}\right)^3 (a^\dagger + a)^4$$

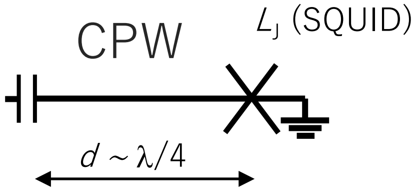
$$\omega_0' = \frac{1}{\sqrt{(L_r + L_J)C_r}}$$

C_J is neglected.

distributed-element circuit



SQUID



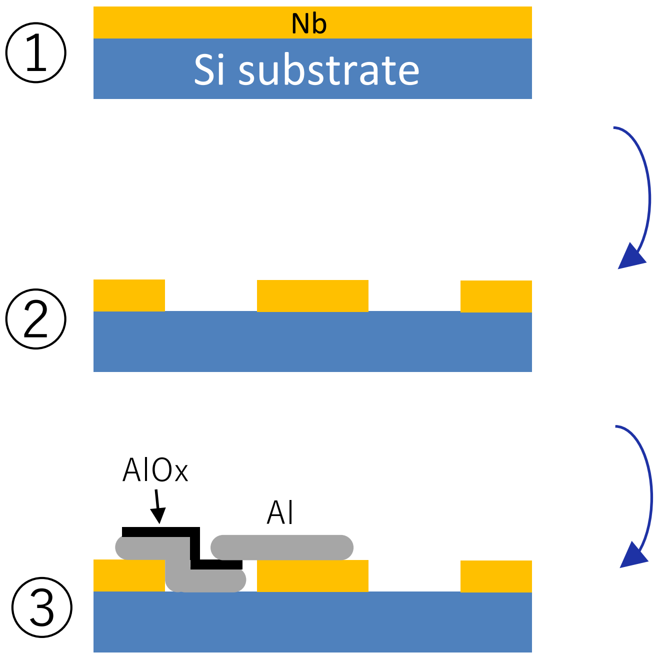
Kerr nonlinearity

$$\mathcal{H} = \hbar\omega_k\left(a^\dagger a + \frac{1}{2}\right) - E_c B_k (a^\dagger + a)^4$$

$$B_k = \frac{(1/4)\cos^2(kd)}{1 + 2kd/\sin(2kd)}$$

Wallquist et al., PRB **74**, 224506 (2006).
Bourassa et al., PRA **86**, 013814 (2012).

Device fabrication

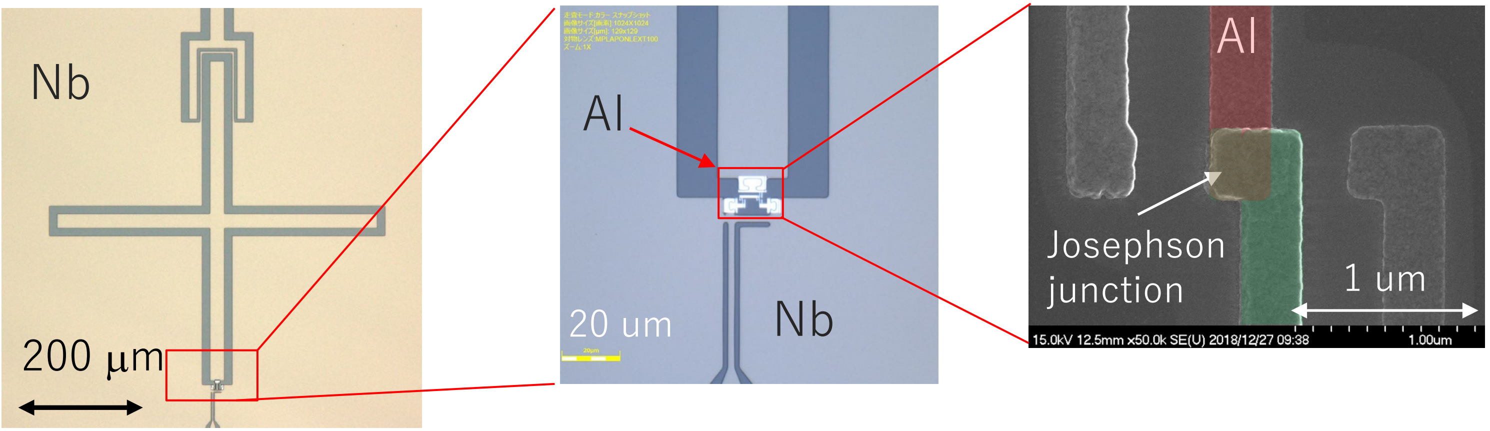


lithography using i-line stepper

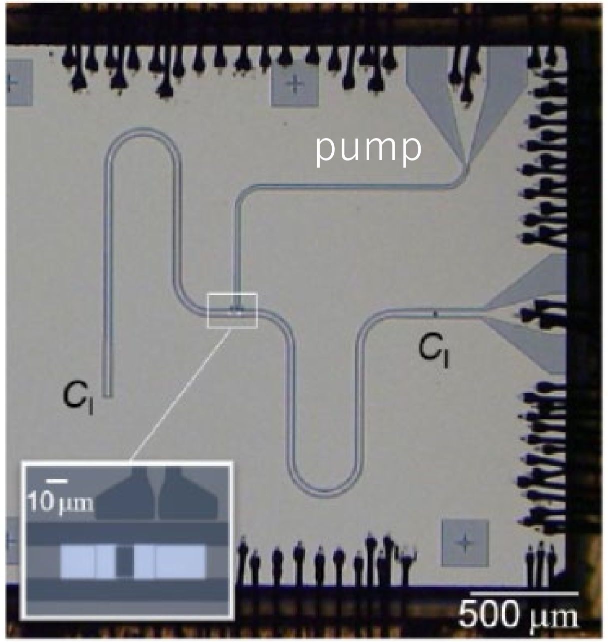
dry etching using CF_4

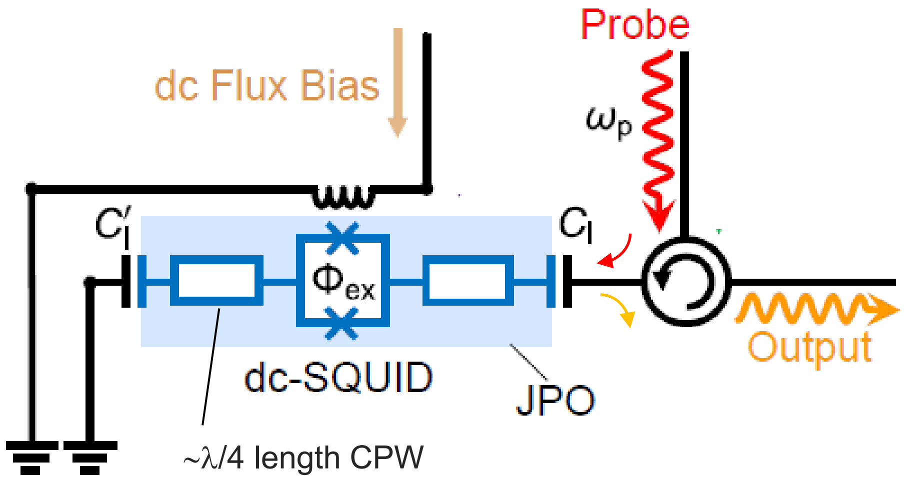
e-beam lithography

shadow evaporation of Al

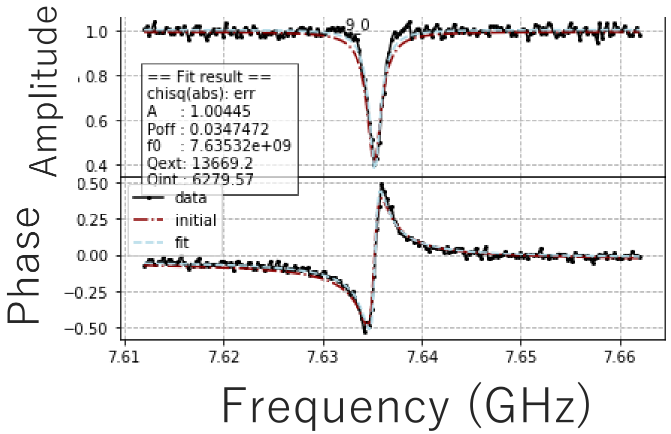


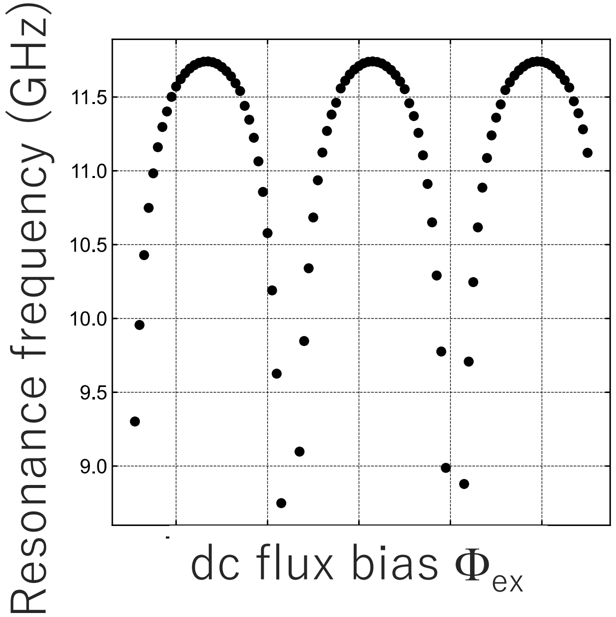
JPO in single-photon Kerr regime (KPO)





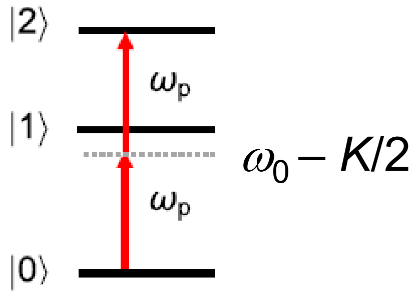
Reflection coefficient





JPO in single-photon Kerr regime (KPO)

- Kerr coefficient (flux-bias dependent) can be extracted from the power dependence of reflection coefficient.
- Perfect agreement with theory
- Can be used as power calibration of probe microwave

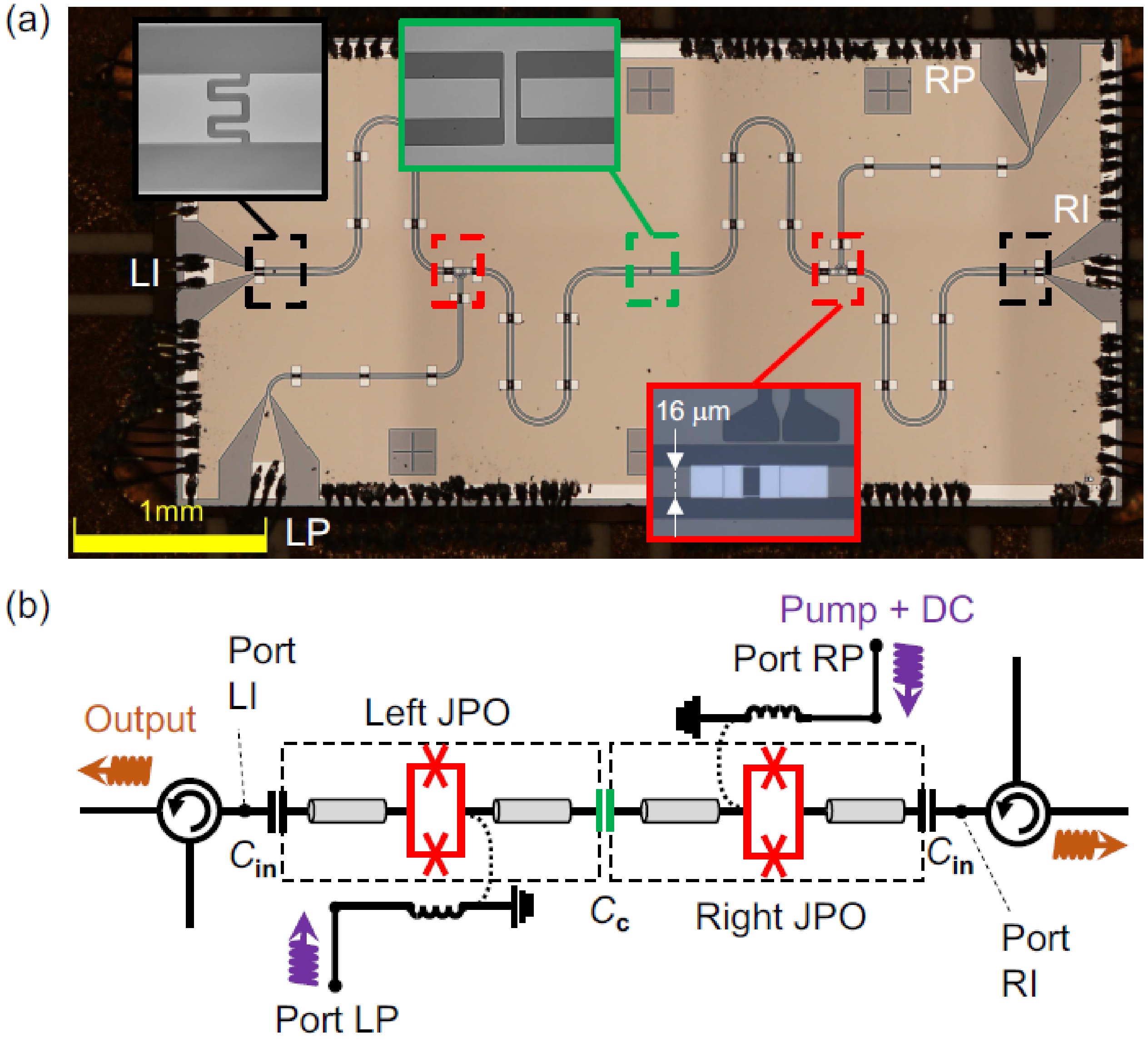


two-photon absorption

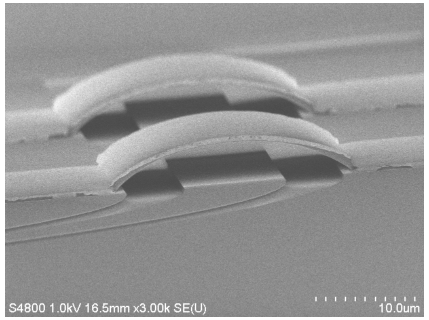
$$\mathcal{H} = \hbar\omega_0 a^\dagger a - \hbar\frac{K}{2}a^\dagger a^\dagger a a$$

T. Yamaji *et al.*, Phys. Rev. A **105**, 023519 (2022).
https://journals.aps.org/pra/abstract/10.1103/PhysRevA.105.023519
Figs. 3a, 3a’, 3b, and 3b’

Coupled KPO's



Air-bridges



- suppress slot-line mode
- create superconducting loop to suppress x-talk

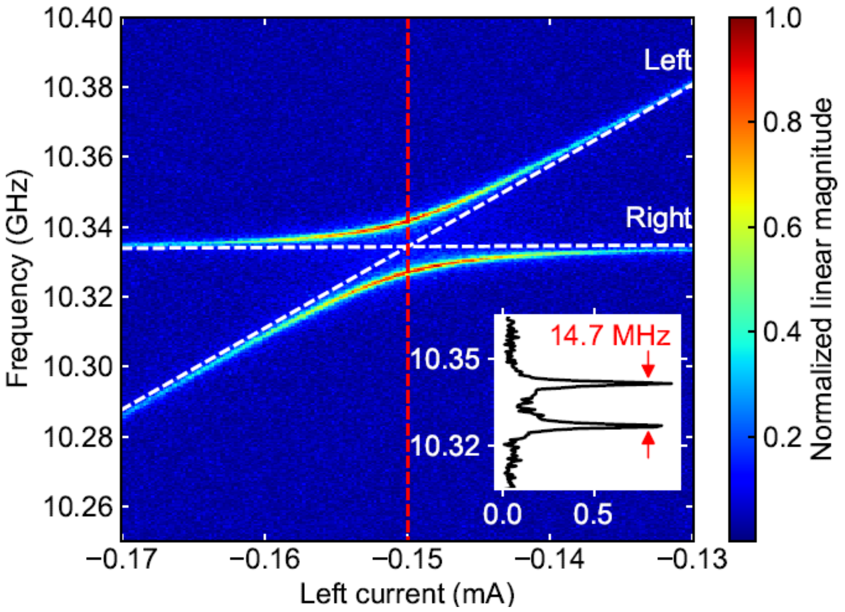
C_{in} : 3.4 fF (Q_{ext} ~10^4)
C_{c} : 0.6 fF (g ~7 MHz)
I_{c} : 1.0 μA/JJ

Coupling between KPO's

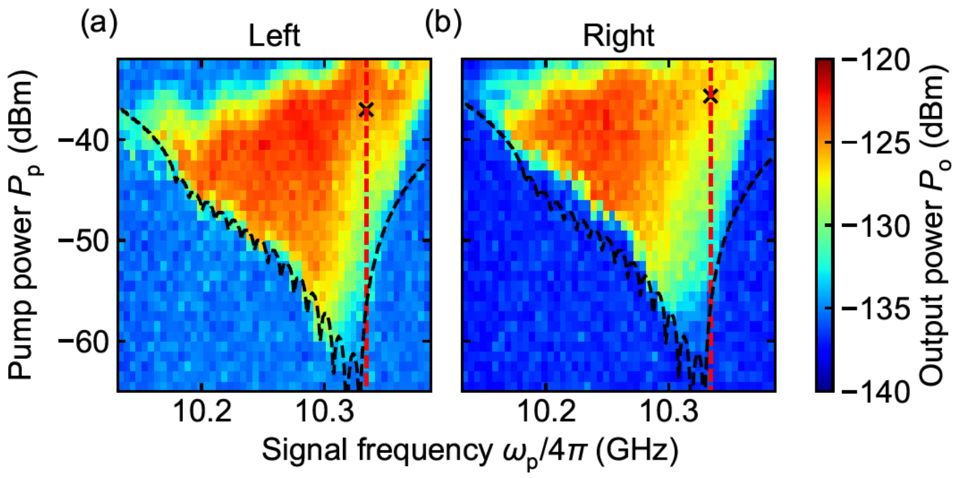
T. Yamaji *et al.*, arXiv:2212.13682.

- avoided level crossing due to coupling capacitance
- single-photon Kerr regime ($K_{L,R} \sim 10\kappa_{L,R}$)
- (independent) parametric oscillation phase diagram consistent with theory [1]

transmission from L to R



parametric oscillation phase diagram



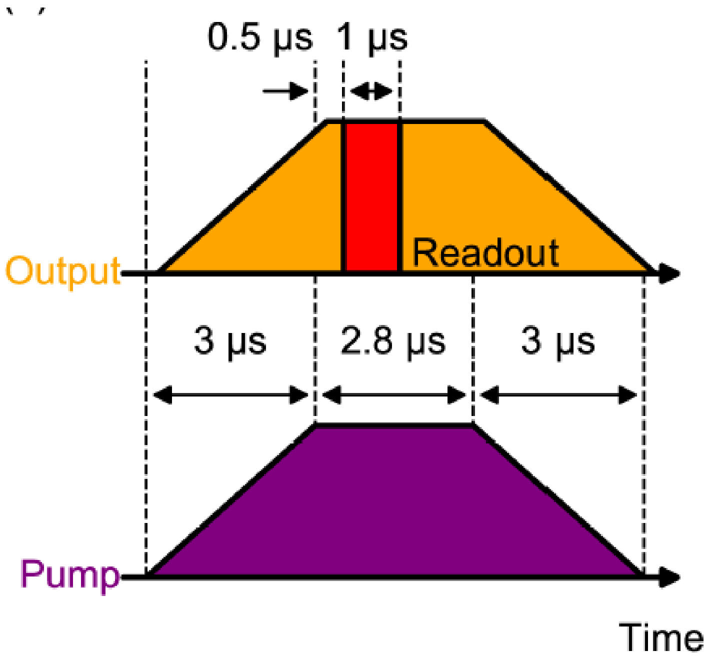
black line: theory <a⁺a>=1

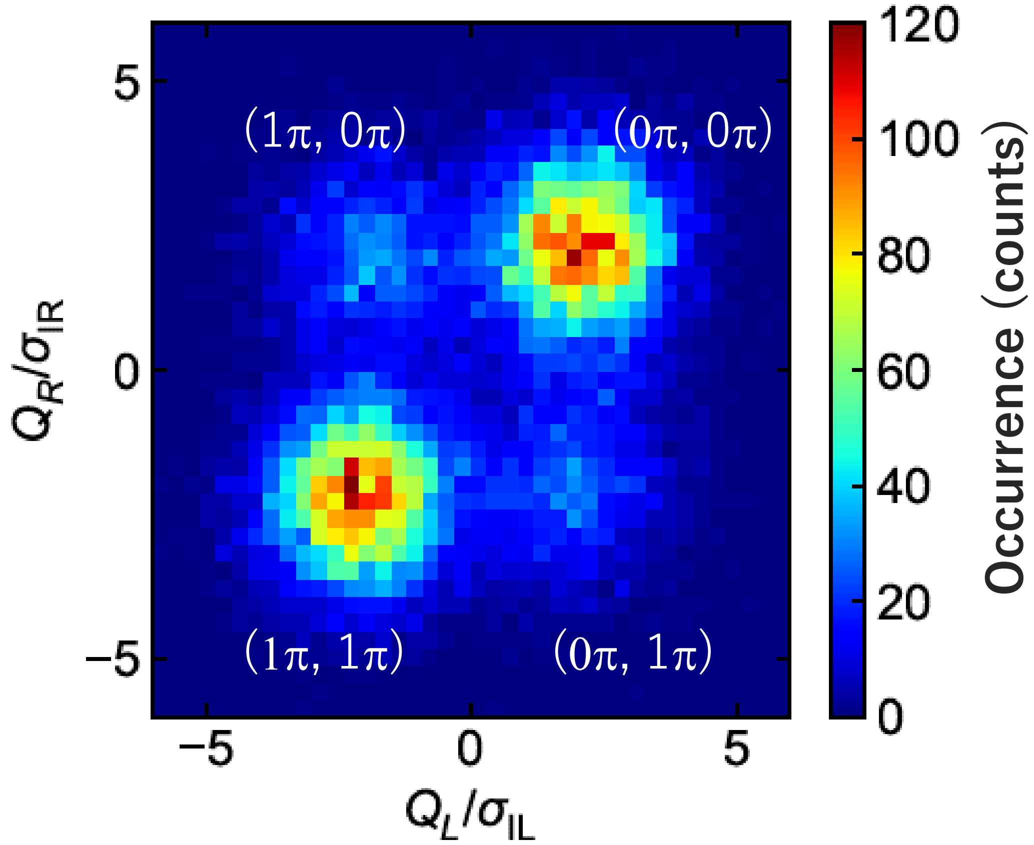
[1] N. Bartolo et al., Phys. Rev. A **94**, 033841 (2016).

Correlated parametric oscillation

T. Yamaji *et al.*, arXiv:2212.13682.

- simultaneously apply pulsed pump and detect the output phase
- probability of the same phase is higher (ferromagnetic correlation)



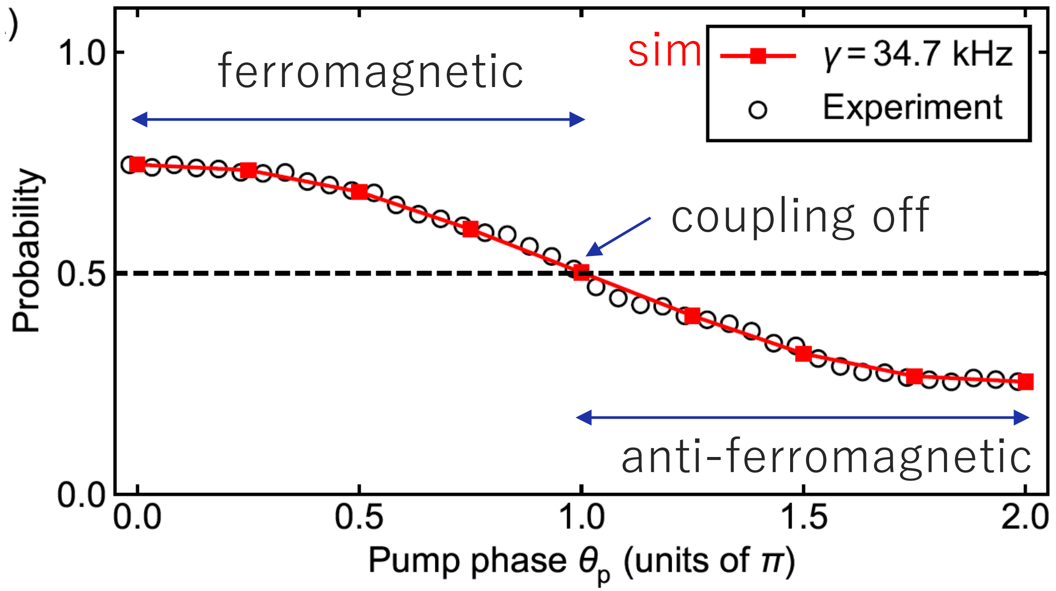


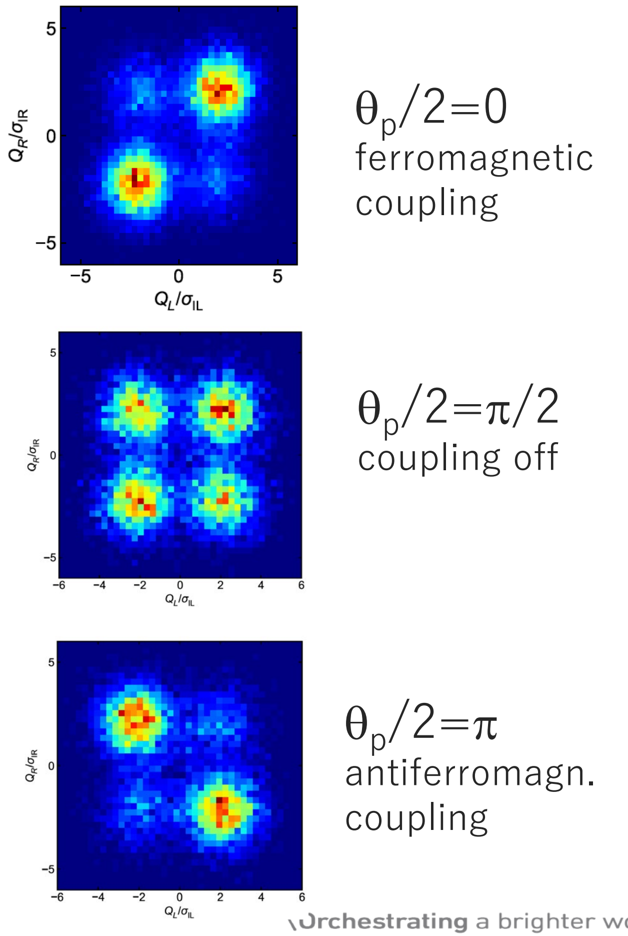
Controllable coupling with pump phase

S. Masuda et al., Phys. Rev. Appl. **18**, 034076 (2022).
T. Yamaji et al., arXiv:2212.13682.

- Demonstration of controllable coupling by pump phase (no on-chip control!)
- Agree with numerical simulation by adjusting (unknown) dephasing rate

$$\mathcal{H}/\hbar = \sum_{i=\mathrm{L,R}} \left[\frac{K_i}{2} a_i^{\dagger 2} a_i^2 + \Delta_i a_i^\dagger a_i + \frac{p_i}{2}\left(a_i^{\dagger 2} + a_i^2\right)\right]$$
$$\underbrace{+g\left(e^{-i\theta_\mathrm{P}/2} a_\mathrm{L}^\dagger a_\mathrm{R} + e^{i\theta_\mathrm{P}/2} a_\mathrm{L} a_\mathrm{R}^\dagger\right)}_{\text{coupling term}},$$





Summary

- Superconducting Kerr parametric oscillator
  - derived from Josephson parametric amplifier/oscillator
  - single-photon Kerr regime ($K >> \kappa$)

- New paradigm for quantum information processing
  - Quantum annealing
    - quantum bifurcation machine
  - Fault-tolerant quantum computing
    - hardware efficient
    - biased noise qubit

This presentation is partly based on results obtained from a project, JPNP16007, commissioned by the New Energy and Industrial Technology Development Organization (NEDO).

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