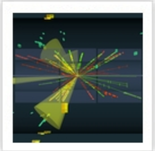

TAE 2023- International Workshop on High Energy Physics

Sep. 03 – Sep. 16, 2023

# Quantum Field Theory and Effective Field Theories

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# Presentation

**Lectures:**

- 3h theory + 2h tutorials
- Email me for anything you may need: cpeset@ucm.es

**Assumptions:**

- Basic knowledge of QFT
- Focus on EFT

**Ask, interrupt and don't be shy. This course is for you to take some ideas home!**

# Further materials

## Similar courses

- A. V. Manohar, "Introduction to Effective Field Theories", Les Houches 2017
- M. Neubert, "Renormalization Theory and Effective Field Theories", Les Houches 2017
- I. Z. Rothstein, "TASI lectures on Effective field Theories", TASI 2002
- A. Pich, "Effective field theory"

**Online courses**

- Link to video lectures on EFTs by Toni Pich
- Link to MIT online course on Effective Field Theories by I. Stewart

# Further materials

### Mathematica packages for automated computations


- FeynRules  Comput. Phys. Commun. 185:2250-2300, 2014 [arXiv:1310.1921], Comput. Phys. Commun. 180:1614-1641, 2009 [arXiv:0806.4194]
- FeynArts, T. Hahn, Comput. Phys. Commun., 140, 418-431, 2001, [arXiv:hep-ph/0012260]
- FeynCalc V. Shtabovenko, R. Mertig and F. Orellana, Comput. Phys. Commun. 256  (2020) 107478, [arXiv:2001.04407],  V. Shtabovenko, R. Mertig and F. Orellana, Comput. Phys. Commun. 207  (2016) 432-444, [arXiv:1601.01167], R. Mertig, M. Böhm, and A. Denner, Comput. Phys. Commun. 64 (1991)  345-359
- FeynHelpers, V. Shtabovenko, "FeynHelpers: Connecting FeynCalc to FIRE and  Package-X", Comput. Phys. Commun., 218, 48-65, 2017, [arXiv:1611.06793], H. H. Patel, “Package-X 2.0: A Mathematica package for the analytic calculation of one-loop integrals,” Comput. Phys. Commun. **218** (2017), 66-70 [arXiv:1612.00009 [hep-ph]].

# Outline of this course

- **Lecture I: Renormalization in QFT**

Amplitudes, divergences, regularization, renormalization, masses

- **Lecture II: Introduction to EFTs**

QFT as an EFT, building an EFT Lagrangian, operators, operator basis, Wilson coefficients, matching

- **Lecture III: Loops and logs in EFTs**

EFT loops, renormalization of composite operators, operator mixing, method of regions, summing logs

Disclaimer:

The aim of these lectures is not to give a formal description of either, neither to provide formal proofs of the statements we make, but rather to give some insight in the main characteristics of renormalization and effective field theories. For a robust and formal description of the topics in these lectures we refer to the bibliography above.

## Lagrangian and interactions

Quantum field theory describes systems which are both in the quantum (small) and relativistic (fast) regimes.
$\Rightarrow$ fundamental particles: $\phi(x)$, $S[\phi] = \int d^4x\mathcal{L}(\phi(x))$

Symmetries: the behavior of a physical system is governed by the presence of a symmetry
$\Rightarrow$ key tool to unlock the secrets of physics from hadronic interactions to electroweak physics and the Standard Model, from superconductivity to Bose–Einstein condensates.

# Lagrangian and interactions

S-matrix: ${}_{\text{out}}\langle q_1, q_2, \ldots, q_m | p_1, \ldots p_n \rangle_{\text{in}}$ describes observables in particle physics
c related to the correlator $G$ via **LSZ reduction formula**

$$G(q_1, \ldots, q_m; p_1, \ldots, p_n) = \prod_{i=1}^{m} \int d^4 y_i e^{iq_i \cdot y_i} \prod_{j=1}^{n} \int d^4 x_j e^{-ip_j \cdot x_j} \langle 0 | T\{\phi(y_1) \cdots \phi(y_m) \phi(x_1) \cdots \phi(x_n)\} | 0 \rangle,$$

$\Rightarrow$ ***n*-point function:** $\langle 0 | T\{\phi(x_1) \cdots \phi(x_n)\} | 0 \rangle$

$\Rightarrow$ perturbation theory + Wick's theorem

Computations are reduced to knowing the interaction Lagrangian and the two-point function (Feynman propagator).

# Example: 2 scalar model

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2) + \frac{1}{2}(\partial_\mu \Phi \partial^\mu \Phi - M^2 \Phi^2) - \frac{\lambda}{2}\phi^2 \Phi$$

2 real scalar particles $\phi$ and $\Phi$, of masses $m$ and $M$

What are the symmetries of this Lagrangian?

Note: this Lagrangian can be seen as a simplified version of a fermion interacting theory such as QED or QCD or a Yukawa theory where all particles have been substituted by real scalars.

(see Neubert's lectures for QED/QCD computations)

# Example: 2 scalar model

$$\boxed{\mathcal{L} = \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m^2\phi^2) + \frac{1}{2}(\partial_\mu\Phi\partial^\mu\Phi - M^2\Phi^2) - \frac{\lambda}{2}\phi^2\Phi}$$

**Dimensional analysis** $\Rightarrow$ natural units ($\hbar = c = 1$) only mass dimensions

Action is dimensionless: $\int \mathcal{D}\phi\mathcal{D}\Phi e^{iS}$

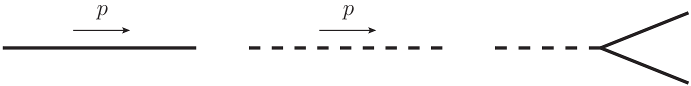
$$S = \int d^4x\mathcal{L} \quad \Rightarrow \quad [\mathcal{L}] = 4, \quad [\phi] = [\Phi] = [m] = [M] = [\lambda] = 1.$$

**Perturbativity** so that we can make predictions using perturbation theory in the usual way. This means $\lambda \ll E, M, m$.

# Example: 2 scalar model

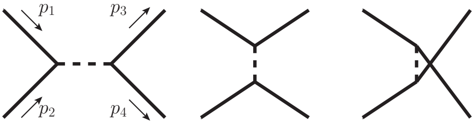
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi \partial^\mu \phi - m^2\phi^2) + \frac{1}{2}(\partial_\mu \Phi \partial^\mu \Phi - M^2\Phi^2) - \frac{\lambda}{2}\phi^2\Phi$$

**Feynman rules:**




$\frac{i}{p^2 - m^2 + i\eta}$ $\qquad$ $\frac{i}{p^2 - M^2 + i\eta}$ $\qquad$ $i\lambda$

# $\phi\phi$ scattering at tree level




**Mandestam variables:**

$$s = (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2, \quad u = (p_1 - p_4)^2$$

$\Rightarrow$ related by $s + t + u = 4m^2$

**Tree level reduced amplitude:**

$$\mathcal{M}^{\mathrm{TL}} = \frac{-\lambda^2}{s - M^2} + (s \to t) + (s \to u).$$

# Automated amplitude computations

Mathematica packages to compute amplitudes:

- `FeynRules`: produces the Feynman rules for a given Lagrangian

- `FeynArts`: produces the Feynman diagrams for a given process at a given order

- `FeynCalc`: powerful package to compute weak coupling S-matrix elements (`FeynHelpers` includes interfaces to `PackageX` for loop computations)

# Divergences: regularization and renormalization

Interacting fields cause divergences

- divergences appear in the intermediate steps
- observables are of course finite
- need for a controlled computational method

# Cut-off regularization

$$\int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m^2 + i\eta}$$

$$= \lim_{\Lambda\to\infty} \frac{-i}{4\pi^2} \int_0^\Lambda d|\mathbf{p}| \frac{\mathbf{p}^2}{(\mathbf{p}^2 + m^2)^{\frac{1}{2}}} = \frac{-i}{8\pi^2} \left( \Lambda\sqrt{\Lambda^2 + m^2} + m^2 \ln\left( \frac{\sqrt{\Lambda^2 + m^2} - \Lambda}{m} \right) \right)$$

$$\approx -\frac{im^2}{(4\pi)^2} \left[ \frac{2\Lambda^2}{m^2} + 1 + \ln\frac{m^2}{4\Lambda^2} + \mathcal{O}(\frac{m^2}{\Lambda^2}) \right]$$

- Perturbativity is completely broken by terms like $\sim \ln\frac{m^2}{4\Lambda^2}$

# Dimensional regularization

$$
\begin{aligned}
&\int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m^2 + i\eta} \\
&= \lim_{D\to 4} \tilde{\mu}^{4-D} \int \frac{d^Dp}{(2\pi)^D} \frac{1}{p^2 - m^2 + i\eta} = -m^2 \frac{i\Gamma(\frac{2-D}{2})}{(4\pi)^2} \left(\frac{m^2}{4\pi\tilde{\mu}}\right)^{\frac{4-D}{2}} \\
&= -\frac{im^2}{(4\pi)^2} \left[\frac{2}{D-4} - 1 + \ln\frac{m^2}{\mu^2} + \mathcal{O}(D-4)\right]
\end{aligned}
$$

- $\mu$ is an arbitrary scale $\Rightarrow$ perturbativity is preserved
  Choice: $\tilde{\mu}^2 = \mu^2 \frac{e^\gamma}{4\pi}$.
- Scaleless integrals vanish: $\int d^Dp \, \frac{1}{p^n} = 0$.
- Preserves **gauge invariance** and **chiral symmetries**.

Therefore it is dimensional regularization that physicists have chosen to perform computations.

# Dimensional regularization

$$\int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m^2 + i\eta}$$

$$= \lim_{D\to 4} \tilde{\mu}^{4-D} \int \frac{d^Dp}{(2\pi)^D} \frac{1}{p^2 - m^2 + i\eta} = -m^2 \frac{i\Gamma(\frac{2-D}{2})}{(4\pi)^2} \left(\frac{m^2}{4\pi\tilde{\mu}}\right)^{\frac{4-D}{2}}$$

$$= -\frac{im^2}{(4\pi)^2} \left[\frac{2}{D-4} - 1 + \ln\frac{m^2}{\mu^2} + \mathcal{O}(D-4)\right]$$

The dimension of fields and parameters changes:

Example: 2 scalar theory

$$S = \int d^Dx\mathcal{L}$$

$$\Rightarrow \quad [\mathcal{L}] = D, \quad [\phi] = [\Phi] = \frac{D-2}{2}, \quad [m] = [M] = 1, \quad [\lambda] = \frac{6-D}{2}.$$

# The interaction Lagrangian

$$\mathcal{L}_{\text{int}} = \sum_i C_i \mathcal{O}_i$$

- Coefficients $C$: characterize the theory
- Operators $\mathcal{O}$: made of the fields and their derivatives

  $\Rightarrow$ For $D > 2$, adding fields or derivatives increases the $\mathcal{O}$ dimension

Historical classification:

- relevant $[\mathcal{O}] < D$ $\Rightarrow$ interaction grows for $E \to 0$
- marginal $[\mathcal{O}] = D$ $\Rightarrow$ interaction constant for $E \to 0$
- irrelevant $[\mathcal{O}] > D$ $\Rightarrow$ interaction decreases for $E \to 0$

Example: 2 scalar theory

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Example: 2 scalar theory

Relevant: $\mathcal{O}_1 = \phi^2, \quad c_1 = -\frac{m^2}{2}, \quad \mathcal{O}_2 = \Phi^2, \quad c_2 = -\frac{M^2}{2}, \quad \mathcal{O}_3 = \phi^2\phi, \quad c_3 = -\frac{\lambda}{2}$

Marginal: $\mathcal{O}_4 = \partial_\mu\phi\partial^\mu\phi, \quad c_4 = \frac{1}{2}, \quad \mathcal{O}_5 = \partial_\mu\Phi\partial^\mu\Phi, \quad c_5 = \frac{1}{2},$

# The interaction Lagrangian

$$\mathcal{L}_{\text{int}} = \sum_i C_i \mathcal{O}_i$$

- Irrelevant operators: need higher dimensional operators for renormalization $\Rightarrow$ ~~non-renormalizable~~

- Interactions modify the naive scaling operators and couplings:

$\Rightarrow \quad [\mathcal{O}] = [\mathcal{O}]_{D=4} + \gamma$ anomalous dimension

May switch terms e.g. from marginal to relevant or irrelevant.

$\Rightarrow$ logarithmic effects lead to e.g. *asymptotic freedom and confinement in QCD*.

# Renormalization scheme

Make all the pieces individually finite.

$\Rightarrow$ help organize the computations & optimizes computational effort

1) Define renormalized fields and parameters that make the 2-point function and vertices **finite**.

Noninteracting quantities $x_0$ are called “bare”

Example: 2 scalar theory

$$\phi_0 = \sqrt{Z_\phi}\phi, \quad m_0 = Z_m m, \quad \Phi_0 = \sqrt{Z_\Phi}\Phi, \quad M_0 = Z_M M, \quad \lambda_0 = Z_\lambda \tilde{\mu}^{\frac{4-D}{2}}\lambda.$$

- $Z$ functions will absorb the divergences according to the scheme
- We *define* renormalized couplings with integer dimensions

# The renormalized Lagrangian

Example: 2 scalar theory

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}(\partial_\mu\phi_0\partial^\mu\phi_0 - m_0^2\phi_0^2) + \frac{1}{2}(\partial_\mu\Phi_0\partial^\mu\Phi_0 - M_0^2\Phi_0^2) - \frac{\lambda_0}{2}\phi_0^2\Phi_0 \\ &= \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m^2\phi^2) + \frac{1}{2}(\partial_\mu\Phi\partial^\mu\Phi - M^2\Phi^2) - \frac{\lambda}{2}\phi^2\Phi + \mathcal{L}_{\text{c.t.}}\end{aligned}$$

- Counterterm Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{c.t.}} = &\frac{1}{2}(Z_\phi - 1)\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}(Z_\phi Z_m - 1)\phi^2 + \frac{1}{2}(Z_\Phi - 1)\partial_\mu\Phi\partial^\mu\Phi \\ &- \frac{M}{2}(Z_\Phi Z_M - 1)\Phi^2 - \frac{\lambda}{2}(Z_\phi\sqrt{Z_\Phi}Z_\lambda\tilde{\mu}^{\frac{4-D}{2}} - 1)\phi^2\Phi\end{aligned}$$

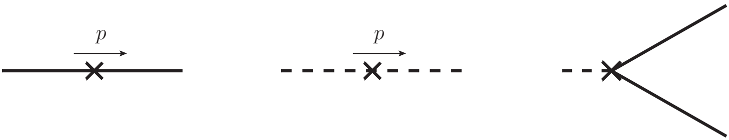
# The renormalized Lagrangian

Example: 2 scalar theory

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_0 \partial^\mu \phi_0 - m_0^2 \phi_0^2) + \frac{1}{2}(\partial_\mu \Phi_0 \partial^\mu \Phi_0 - M_0^2 \Phi_0^2) - \frac{\lambda_0}{2}\phi_0^2 \Phi_0$$
$$= \frac{1}{2}(\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2) + \frac{1}{2}(\partial_\mu \Phi \partial^\mu \Phi - M^2 \Phi^2) - \frac{\lambda}{2}\phi^2 \Phi + \mathcal{L}_{\text{c.t.}}$$

- Counterterm Lagrangian

$\Rightarrow$ New Feynman rules




$i(p^2(Z_\phi - 1) - m^2(Z_m Z_\phi - 1))$ $\quad$ $i(p^2(Z_\Phi - 1) - M^2(Z_M Z_\Phi - 1))$ $\quad$ $-i\lambda(Z_\phi \sqrt{Z_\Phi} Z_\lambda \tilde{\mu}^{\frac{4-D}{2}} - 1)$

# The renormalization parameters $Z_i$

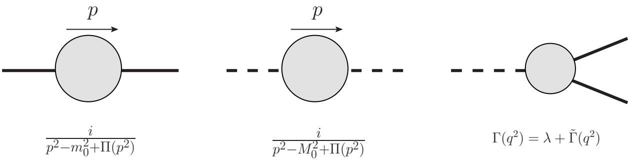
Example: 2 scalar theory

bare and renormalized fields are equal at leading order in and expansion in $\lambda$:

$$Z_i = 1 + \delta_i, \quad \text{with} \quad \delta_i = d_i \lambda + \mathcal{O}(\lambda^2).$$

Computed order by order

$\Rightarrow$ make the propagator and vertex finite



# Passarino-Veltman one loop functions

[G. Passarino and M. Veltman, Nucl.Phys. B160, 151 (1979).]

Automate any one loop computation: tensor reduction algorithm

$$A_0(m^2) = \frac{(2\pi\tilde{\mu})^{4-D}}{i\pi^2} \int \frac{d^D k}{k^2 - m^2 + i\eta} = m^2 \left( \frac{-2}{D-4} + \ln \frac{\mu^2}{m^2} + 1 \right),$$

$$B_0(p^2, m_1^2, m_2^2) = \frac{(2\pi\tilde{\mu})^{4-D}}{i\pi^2} \int \frac{d^D k}{(k^2 - m_1^2 + i\eta)((k-p)^2 - m_2^2 + i\eta)}$$

$$= \frac{-2}{D-4} + \overline{B_0}(p^2, m_1^2, m_2^2),$$

More details 0509141 or 0711.1067

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Automate any one loop computation: tensor reduction algorithm

$$C_0(p_1^2,p_2^2,p_3^2,m_1^2,m_2^2,m_3^2) = \frac{(2\pi\tilde{\mu})^{4-D}}{i\pi^2}\int \frac{d^Dk}{(k^2-m_1^2+i\eta)((k-p_1)^2-m_2^2+i\eta)((k-p_1-p_2)^2-m_3^2+i\eta)}$$

$$= \overline{C_0}(p_1^2,p_2^2,p_3^2,m_1^2,m_2^2,m_3^2),$$

$$D_0(p_1^2,p_2^2,p_3^2,p_4^2,(p_1+p_2)^2,(p_2+p_3)^2,m_1^2,m_2^2,m_3^2,m_4^2) = \frac{(2\pi\tilde{\mu})^{4-D}}{i\pi^2}\int \frac{d^Dk}{(k^2-m_1^2+i\eta)((k-p_1)^2-m_2^2+i\eta)((k-p_1-p_2)^2-m_3^2+i\eta)((k-p_1-p_2-p_3)^2-m_4^2+i\eta)}$$

$$= \overline{D_0}(p_1^2,p_2^2,p_3^2,p_4^2,(p_1+p_2)^2,(p_2+p_3)^2,m_1^2,m_2^2,m_3^2,m_4^2)$$

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# $\phi$ renormalization at one loop

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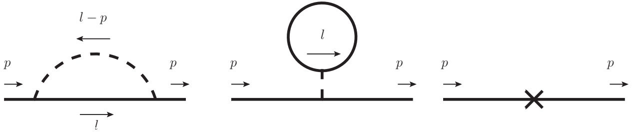

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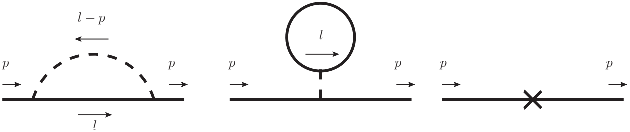


$$
\begin{aligned}
i\Pi(p^2) &= \lambda^2 \tilde{\mu}^{4-D} \int \frac{d^D l}{(2\pi)^D} \frac{1}{(l^2 - m^2 + i\eta)((l-p)^2 - M^2) + i\eta} \\
&- \frac{\lambda^2}{2M^2} \tilde{\mu}^{4-D} \int \frac{d^D l}{(2\pi)^D} \frac{1}{l^2 - m^2 + i\eta} \\
&+ i(p^2 \delta_\phi - m^2(\delta_\phi + \delta_m)) \\
&= \frac{i\lambda^2}{(4\pi)^2} B_0(p^2, m^2, M^2) - \frac{i\lambda^2}{(4\pi)^2 2M^2} A_0(m^2) + i(p^2 \delta_\phi - m^2(\delta_\phi + \delta_m)) \\
&= \frac{i\lambda^2}{(4\pi)^2} \frac{-2}{D-4} - \frac{i\lambda^2}{(4\pi)^2 2M^2} m^2 \frac{-2}{D-4} + i(p^2 \delta_\phi - m^2(\delta_\phi + \delta_m)) \\
&+ \frac{i\lambda^2}{(4\pi)^2} \overline{B_0}(p^2, m_1^2, m_2^2) - \frac{i\lambda^2}{(4\pi)^2 2M^2} m^2 \overline{A_0}(m^2)
\end{aligned}
$$

# $\phi$ renormalization at one loop

$$
\begin{aligned}
i\Pi(p^2) &= \lambda^2 \tilde{\mu}^{4-D} \int \frac{d^D l}{(2\pi)^D} \frac{1}{(l^2 - m^2 + i\eta)((l-p)^2 - M^2) + i\eta} \\
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&+ i(p^2 \delta_\phi - m^2(\delta_\phi + \delta_m)) \\
&= \frac{i\lambda^2}{(4\pi)^2} B_0(p^2, m^2, M^2) - \frac{i\lambda^2}{(4\pi)^2 2M^2} A_0(m^2) + i(p^2 \delta_\phi - m^2(\delta_\phi + \delta_m)) \\
&= \frac{i\lambda^2}{(4\pi)^2} \frac{-2}{D-4} - \frac{i\lambda^2}{(4\pi)^2 2M^2} m^2 \frac{-2}{D-4} + i(p^2 \delta_\phi - m^2(\delta_\phi + \delta_m)) \\
&+ \frac{i\lambda^2}{(4\pi)^2} \overline{B_0}(p^2, m^2, M_2^2) - \frac{i\lambda^2}{(4\pi)^2 2M^2} m^2 \overline{A_0}(m^2)
\end{aligned}
$$

# The $\overline{\text{MS}}$ renormalization scheme

"Minimal" subtraction:

$$\frac{2}{4-D} + \gamma_E + \ln(4\pi).$$

Clever trick: define $\tilde{\mu} = \mu\sqrt{\frac{e^\gamma}{4\pi}}$ and subtract only the divergence $\sim 1/(D-4)$ .

Example: 2 scalar theory

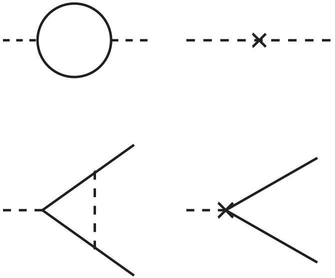
$$p^2\delta_\phi = 0, \quad \Rightarrow \quad \delta_\phi^{\overline{\text{MS}}} = 0,$$

$$\frac{i\lambda^2}{(4\pi)^2}\frac{-2}{D-4} - \frac{i\lambda^2}{(4\pi)^2 2M^2}m^2\frac{-2}{D-4} + i(-m^2(\delta_\phi + \delta_m)) = 0$$

$$\Rightarrow \quad \delta_m^{\overline{\text{MS}}} = \frac{\lambda^2}{(4\pi)^2 m^2}\frac{2}{4-D}\left[1 - \frac{m^2}{2M^2}\right]$$

# The $\overline{\text{MS}}$ renormalization scheme

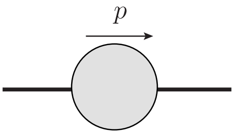
**Exercise:** Check that from the $\Phi$ propagator and the vertex functions we get



$$\delta\Phi^{\overline{\text{MS}}} = 0, \qquad \delta_M^{\overline{\text{MS}}} = \frac{\lambda^2}{(4\pi)^2 2M^2}\frac{2}{4-D}, \qquad \delta_\lambda^{\overline{\text{MS}}} = 0$$

# The pole mass

The mass is the pole $\Rightarrow$ typically used when $m_{\text{pole}} = m_{\text{phys}}$




$$\frac{i}{p^2 - m_0^2 + \Pi(p^2)}$$

On shell: $m = m_{\text{phys}}$

$$\Pi(p^2) = \Pi(m^2) + \left.\frac{\partial \Pi}{\partial p^2}\right|_{p^2 = m^2} (p^2 - m^2) + \cdots$$

At 1 loop:

$$\frac{i}{p^2 - m^2_{\overline{\text{MS}}} + \Pi_{\overline{\text{MS}}}(p^2)} = \frac{i(1 - \Pi'_{\overline{\text{MS}}}(m^2))}{p^2 - m^2_{\overline{\text{MS}}} + \Pi_{\overline{\text{MS}}}(m^2)}$$

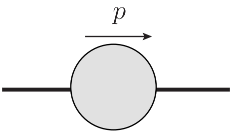
$$m^2_{\text{pole}} = m^2_{\overline{\text{MS}}} - \Pi_{\overline{\text{MS}}}(m^2)$$

## Example: 2 scalar theory

$$m^2_{\overline{\text{MS}}}(\mu) = m^2_{\text{pole}} + \frac{\lambda^2}{(4\pi)^2} \overline{B_0}(m^2, m^2, M^2) - \frac{\lambda^2}{(4\pi)^2 2M^2} m^2 \overline{A_0}(m^2)$$

# The pole mass

The mass is the pole $\Rightarrow$ typically used when $m_{\text{pole}} = m_{\text{phys}}$




$$\frac{i}{p^2 - m_0^2 + \Pi(p^2)}$$

On shell: $m = m_{\text{phys}}$

$$\Pi(p^2) = \Pi(m^2) + \left.\frac{\partial \Pi}{\partial p^2}\right|_{p^2=m^2} (p^2 - m^2) + \cdots$$

At 1 loop:

$$\frac{i}{p^2 - m^2_{\overline{\text{MS}}} + \Pi_{\overline{\text{MS}}}(p^2)} = \frac{i(1 - \Pi'_{\overline{\text{MS}}}(m^2))}{p^2 - m^2_{\overline{\text{MS}}} + \Pi_{\overline{\text{MS}}}(m^2)}$$

$$m^2_{\text{pole}} = m^2_{\overline{\text{MS}}} - \Pi_{\overline{\text{MS}}}(m^2)$$

Example: 2 scalar theory

$$m^2_{\overline{\text{MS}}}(\mu) = m^2_{\text{pole}} + \frac{\lambda^2}{(4\pi)^2}\left[2 - \frac{m^2}{2M^2} + \left(1 - \frac{m^2}{2M^2}\right)\ln\frac{\mu^2}{m^2} + \frac{M^2}{2m^2}(1-x)\ln\frac{m^2}{M^2} + \frac{M^2 x}{m^2}\ln\frac{1+x}{2}\right]$$

# Running parameters

Renormalized parameters $C$ depend on the renormalization scale $\mu$:

$$\mu \frac{dC}{d\mu} = C\gamma_C$$

$d$ is the mass of the associated operator $\gamma_C$ can be obtained from

$$0 = \mu \frac{dC^0}{d\mu} = \mu \frac{d(Z_C C)}{d\mu} \quad \Rightarrow \quad \gamma_C = -\frac{1}{Z_C} \mu \frac{dZ_C}{d\mu}.$$

Example: 2 scalar theory

At 1 loop

$$\gamma_m = \frac{2\lambda^2}{(4\pi)^2 m^2} \left(1 - \frac{m^2}{2M^2}\right) + \mathcal{O}(\lambda^3)$$

so that

$$\Rightarrow \quad m^2(\mu) = m^2(\mu_0) + \frac{\lambda^2}{(4\pi)^2} \left(1 - \frac{m^2}{2M^2}\right) \ln \frac{\mu^2}{\mu_0^2} + \mathcal{O}(\lambda^3).$$

- Even if $m(\mu_0) = 0 \quad \Rightarrow \quad m^2(\mu) = \frac{\lambda^2}{(4\pi)^2} \ln \frac{\mu^2}{\mu_0^2} \neq 0$ (Coleman-Weinberg)

# Running parameters

Renormalized parameters $C$ depend on the renormalization scale $\mu$:

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## Example: 2 scalar theory

At 1 loop

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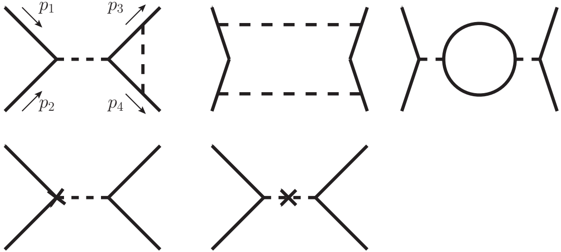
- Similarly for $M$ and $\lambda$ (Exercise)

# $\phi\phi$ scattering at 1 loop

# $\phi\phi$ scattering at 1 loop

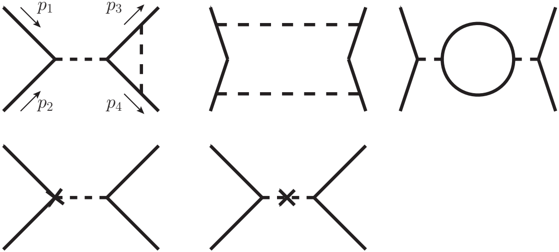
$s$-channel diagrams:




(Crossed and mirrored diagrams are understood).

# $\phi\phi$ scattering at 1 loop

$s$-channel diagrams:




(Crossed and mirrored diagrams are understood).

$$\mathcal{M}^{1\mathrm{L}} = \frac{\lambda^4}{(4\pi)^2}\left[\frac{B_0(s,m^2,m^2)}{2(s-M^2)^2} + \frac{C_0(m^2,m^2,s,m^2,M^2,M^2)+C_0(m^2,s,m^2,M^2,m^2,m^2)}{s-M^2}\right.$$

$$\left. + D_0(m^2,m^2,m^2,m^2,s,t,m^2,M^2,m^2,M^2) + D_0(m^2,m^2,m^2,m^2,s,t,M^2,m^2,M^2,m^2)\right]$$

$$- \frac{\lambda^4}{(4\pi)^2}\frac{\frac{2}{4-D}}{2(s-M^2)^2} + (s,t,u \to t,u,s) + (s,t,u \to u,s,t)$$

# $\phi\phi$ scattering at 1 loop

$$\mathcal{M}^{1\mathrm{L}} = \frac{\lambda^4}{(4\pi)^2}\left[\frac{B_0(s,m^2,m^2)}{2(s-M^2)^2} + \frac{C_0(m^2,m^2,s,m^2,M^2,M^2)+C_0(m^2,s,m^2,M^2,m^2,m^2)}{s-M^2}\right.$$

$$\left. +D_0(m^2,m^2,m^2,m^2,s,t,m^2,M^2,m^2,M^2)+D_0(m^2,m^2,m^2,m^2,s,t,M^2,m^2,M^2,m^2)\right]$$

$$-\frac{\lambda^4}{(4\pi)^2}\frac{\frac{2}{4-D}}{2(s-M^2)^2} + (s,t,u \to t,u,s) + (s,t,u \to u,s,t)$$

- The counterterm is obtained with our $\overline{\mathrm{MS}}$ values.
- Only $B_0$ is divergent $B_0(s,m^2,m^2)|_{\mathrm{div}} = \frac{2}{4-D}$ $\Rightarrow$ $\mathcal{M}^{1\mathrm{L}}$ is finite!

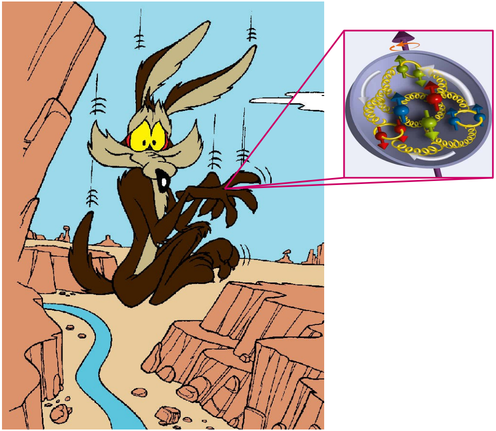
For $M \gg m$

$$\mathcal{M} = \frac{3\lambda^2}{M^2}\left[1 - \frac{\lambda^2}{32\pi^2 M^2}\left(9\ln\frac{m^2}{M^2} + 12 + i\pi\right)\right]$$

$$+\left[\frac{\lambda^4}{32\pi^2 M^4}\frac{\sqrt{s(s-4m^2)}}{s}\ln\left(\frac{2m^2 - s + \sqrt{s(s-4m^2)}}{2m^2}\right) + (s\to t) + (s \to u)\right]$$

In $\ln\frac{m}{M}$ the small mass $m$ is acting as an IR cut-off.

## What are effective theories?



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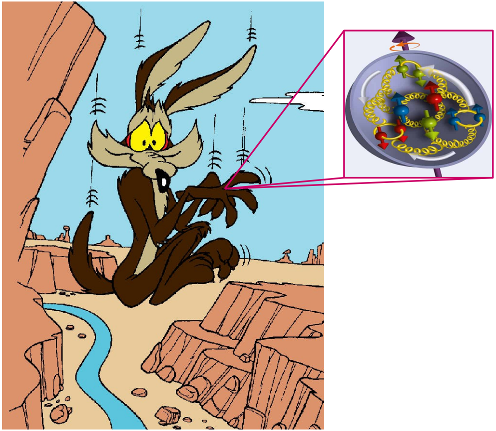

- wide separation of scales

$$v \ll c, \qquad h \ll R_E \ll \lambda \sim hmv, \qquad E \ll \Lambda_{\text{QCD}}$$

# What are effective field theories?

$$E \ll M$$

- degrees of freedom related to $M$ decouple
- an EFT is a fully consistent QFT
  - we can compute S-matrix elements (observables)
  - no more input than the EFT Lagrangian is needed
- No need to know about the UV theory
  - In the coyote free fall we only need to know: $g$, $m_{\text{coyote}}$

.

# What are effective field theories?

$$E \ll M$$

- Provide systematic expansion in small parameter $\delta$

$$\delta = E/M$$

In multiscale problems there are several $\delta_i$

- EFT allows you to compute an experimentally accessible quantity with some finite error that we can quantify in terms of $\delta_i$

(power counting)

Every quantum field theory is and effective field theory

$\Rightarrow$ no QFT is ever complete, only valid up to a cut-off $\Lambda$

**Example:** QED is the EFT of the SM lepton sector $E \ll m_e$.

Can you think of any other EFTs?

# The Fermi theory of weak interactions

- EFT for weak interactions at $E \ll m_W, m_Z$
- Expansion parameter is $\delta = p/m_W$
- Example: $\mu$ decay (see tutorial)
  - $p \sim m_\mu$
  - also consider scale $\alpha/(4\pi)$
- Example: hadronic decays
  - $p \sim \Lambda_{\text{QCD}}$
  - also consider scale $\alpha_s/(4\pi)$

# Chiral perturbation theory

- EFT for hadrons formed by light quarks: pions, nucleons at $E \ll \Lambda_\chi$
- EFT of QCD: non-perturbative at the cut-off scale $\Lambda_\chi$
- parameters are thus obtained experimentally/lattice
- Expansion parameter is $\delta = p/\Lambda_\chi \sim \Lambda_{\text{QCD}}$
- Example: $\pi\pi$ scattering
  - $p \sim m_\pi$
  - also consider scale $\alpha_s/(4\pi)$

(see A. Carmona's lectures)

# HQET/NRQCD/pNRQCD

- EFT for hadrons with at least one heavy quark ($Q = c, b, t$)
- Expansion parameter is $\delta = \frac{\Lambda_{\text{QCD}}}{m_Q}$
- EFT of QCD perturbative at the cut-off scale $m_Q$
- Example HQET: $B$-decays
  - $p \sim m_b \alpha_s(m_b)$
  - also consider scale $\alpha_s/(4\pi)$
- Example pNRQCD: $\Upsilon$-mass
  - $p \sim m_b \alpha_s(m_b)$
  - also consider scale $\alpha_s/(4\pi)$
  - $v \sim \alpha_s(m_Q)$

# SCET

- EFT for processes where the final states have small invariant mass compared to the center of mass energy
- Expansion parameters are $\delta_1 = M_J/E$, $\delta_2 = \Lambda_{\text{QCD}}/E$, where $E$ is the c.o.m energy
- EFT for every almost back to back process
- EFT of QCD perturbative at the cut-off scale $E$
- Example: jet production in $pp$ collisions such as those at LHC
  - $p \sim$ TeV
  - also consider scale $\alpha_s(E)/(4\pi)$

# SMEFT

- SM as an EFT with unknown cut-off $\Lambda$
- Expansion parameters are $\delta \sim m_H/\Lambda, E/\Lambda$
- Assumes perturbativity up to $\Lambda$
- Example: use LHC data to constraint the parameter space

# EFTs for particle physics

- Efficient: power counting
  $\Rightarrow$ include non-perturbative effects in a systematic way,
  e.g. in HQET.

- Systematic: simplify computations
  $\Rightarrow$ multiscale systems
  $\Rightarrow$ reach beyond perturbativity

## EFTs for QCD

Factorize amplitudes:
[ short distance (perturbative)] $\times$ [long distance (non-perturbative)]

Example: $B$ meson decay
$\Rightarrow$ scales: $m_W \gg m_b \gg \Lambda_{\rm QCD}$

SM $\Rightarrow$ Fermi Theory $\Rightarrow$ HQET

# EFTs for particle physics

- Sum large (UV) logs

  $\Rightarrow$ jeopardize our perturbative expansion $\ln \frac{M}{m} \gg 1$

Example: semileptonic $B$-decays: $\alpha_s \ln \frac{m_W}{m_b} \sim 1$

$\Rightarrow$ EFTs to turn IR logs into UV logs

Example: QCD $\Rightarrow$ HQET, NRQCD, pNRQCD, SCET or ChPT

$\Rightarrow$ different IR regimes sum different logs

- Organizational tool for BSM physics

See lecture by Javi Serra

# EFT characteristics

- Two groups:

**Top-down:** UV theory is known

$\Rightarrow$ reach beyond perturbativity (matching)

Examples: HQET, NRQCD, pNRQCD, SCET

**Bottom-up:** UV theory is known

$\Rightarrow$ parameters obtained from experiment/lattice

Examples: ChPT or SMEFT.

- Symmetries

$\Rightarrow$ EFTs keep the symmetries of the UV theory

$\Rightarrow$ may exhibit new symmetries (ChPT, HQET)

# The Operator Product Expansion

- Locality

$\Rightarrow$ $S_{\text{eff}}$ is non-local at the scale of high energy modes $\Lambda$

$\Rightarrow$ Dynamical fields $\sim p \ll \Lambda$

$S_{\text{eff}}$ can be expanded in an infinite series of local operators (OPE).

$$\mathcal{L}^{\text{eff}} = \sum_i c_i \mathcal{O}_i(\phi(x))$$

- Factorized amplitudes:

[ $c_i$ high energy] $\times$ [$\langle \mathcal{O}_i \rangle$ low energy ]

# Derivation of the effective Lagrangian

$$E \ll M$$

# Derivation of the effective Lagrangian

$$E \ll M$$


**Step 1:** choose a cut-off $\Lambda < M$ & determine the dynamical degrees of freedom.

$\Rightarrow$ Easy for weakly coupled UV theory
Example: Fermi theory of weak interactions

$\Rightarrow$ Other theories more involved:
Example: NRQCD: potential quarks & soft and ultrasoft gluons
pNRQCD has only ultrasoft gluons $\Rightarrow$ power counting

**Step 2:** List all possible gauge invariant operators

- $\Rightarrow$ to a given order
- $\Rightarrow$ built of the fields in step 1 and derivatives
- $\Rightarrow$ respect symmetries

# Derivation of the effective Lagrangian

$$E \ll M$$

**Step 3:** write the Lagrangian in the general form

$$\mathcal{L}^{\text{eff}} = \mathcal{L}^{\text{dim}\leq 4} + \sum_{n=1}^{N_{\text{max}}} \sum_i \frac{C_i^{(n)}}{M^n} \mathcal{O}_i^{(4+n)} .$$

⇒ $N_{\text{max}}$ given by precision goal & power counting

⇒ in practice we choose $\Lambda = M$

⇒ the operator basis $\mathcal{O}_i^{(n)}$ might not be unique and can be changed by a field redefinition.

**Step 4:** Determine the values of the Wilson coefficients $C_i^{(n)}$.

# Example: 2 scalar model at low energy

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2) + \frac{1}{2}(\partial_\mu \Phi \partial^\mu \Phi - M^2 \Phi^2) - \frac{\lambda}{2} \phi^2 \Phi$$

**Step 1:** $E \sim m \ll \Lambda < M$

$\Rightarrow$ only field left is $\phi$

**Step 2:** Lorentz-invariance + $Z_2$ symmetry

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**Step 1:** $E \sim m \ll \Lambda < M$

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**Step 2:** Lorentz-invariance + $Z_2$ symmetry

- Dimension 2 operators: $\phi^2$
- Dimension 4 operators: $\phi^4$, $\partial_\mu \phi \partial^\mu \phi$, $\phi \partial^2 \phi$
- Dimension 6 operators: $\phi^6$, $\phi^3 \partial^2 \phi$, $\phi^2 \partial_\mu \phi \partial^\mu \phi$

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- Dimension 6 operators: $\phi^6$, $\phi^3 \partial^2 \phi$, ~~$\phi^2 \partial_\mu \phi \partial^\mu \phi$~~

Action is unchanged by total derivatives

# Example: 2 scalar model at low energy

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**Step 3:**

$$\mathcal{L}^{\text{eff}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\hat{m}^2\phi^2 + C^{(0)}\phi^4 + \frac{C_1^{(2)}}{M^2}\phi^6 + \frac{C_2^{(2)}}{M^2}\phi^3\partial^2\phi$$

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# Field redefinitions

$\Rightarrow$ common tool in EFTs (infinite series of operators)

For a field redefinition $F$ and

$$\phi \to \phi + \epsilon F[\phi] \quad \Rightarrow \quad \mathcal{L}[\phi] \to \mathcal{L}[\phi] + \epsilon F[\phi] \frac{\delta S}{\delta \phi}$$

where $\frac{\delta S}{\delta \phi}$ is the classical equation of motion

# Field redefinitions

Example: $\mathcal{L}^{\text{EFT}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\hat{m}^2\phi^2 + C^{(0)}\phi^4 + \frac{C_1^{(2)}}{M^2}\phi^6 + \frac{C_2^{(2)}}{M^2}\phi^3\partial^2\phi$

$$\phi \to \phi + \frac{C_2^{(2)}}{M^2}\phi^3$$

E.o.m. (up to the order of interest)

$$\frac{\delta S}{\delta\phi} = -\partial^2\phi - \hat{m}^2\phi + 4C^{(0)}\phi^3$$

The Lagrangian after the field redefinition would then be

$$\begin{aligned}
\mathcal{L}^{\text{EFT}} &= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\hat{m}^2\phi^2 + C^{(0)}\phi^4 + \frac{C_1^{(2)}}{M^2}\phi^6 + \frac{C_2^{(2)}}{M^2}\phi^3\partial^2\phi \\
&+ \frac{C_2^{(2)}}{M^2}\phi^3\left(-\partial^2\phi - \hat{m}^2\phi + 4C^{(0)}\phi^3\right) \\
&= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\hat{m}^2\phi^2 + \left[C^{(0)} - \frac{\hat{m}^2 C_2^{(2)}}{M^2}\right]\phi^4 + \frac{1}{M^2}\left[C_1^{(2)} + 4C_2^{(2)}C^{(0)}\right]\phi^6
\end{aligned}$$

$\Rightarrow$ operator $\phi^3\partial^2\phi$ has been eliminated!

# Field redefinitions

- The operator basis is not unique neither necessarily minimal

$\Rightarrow$ Non-minimal operator bases lead to relations between Wilson coefficients

$\Rightarrow$ observables computed in any operator basis will be the same Final EFT Lagrangian up to dimension 6:

$$\mathcal{L}^{\text{EFT}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\hat{m}^2\phi^2 + g\phi^4 + \frac{C_1}{M^2}\phi^6$$

# Power-counting

- EFT computations always come together with an uncertainty.

- For a scattering amplitude $\mathcal{M}$ at typical momentum $p$:

  $\Rightarrow$ insertion of one dimension $d > 4$ operator

$$\mathcal{M} \sim \left(\frac{p}{\Lambda}\right)^{d-D}.$$

  $\Rightarrow$ insertion of $i$ dimension $d_i$ operators

$$\boxed{\mathcal{M} \sim \left(\frac{p}{\Lambda}\right)^{\sum_i (d_i - D)}.}$$

Power counting equation

  $\Rightarrow$ Loops only involve low energy scales!.

Example: $\phi\phi$ scattering

  $\Rightarrow$ up to $\mathcal{O}(p^2/M^2)$ single $d = 4$ insertion

  $\Rightarrow$ up to $\mathcal{O}(p^4/M^4)$ single $d = 6$ insertion & two of $d = 4$

# Power-counting

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$$\boxed{\mathcal{M} \sim \left(\frac{p}{\Lambda}\right)^{\sum_i (d_i - D)}.}$$

Power counting equation

$\Rightarrow$ Loops only involve low energy scales!.

Powerful tool to provide an estimate of the size of a diagram Tiny (see $\gamma\gamma$ exercise in the tutorial)

# Wilson coefficients as running-couplings

- In general, Wilson coefficients $C_i$ are renormalization scheme and scale dependent

$\Rightarrow$ only measurable in an indirect way

$\Rightarrow$ they are in fact the running couplings of any QFT

Example: quark masses in $\overline{MS}$ cannot be directly measured: extracted from comparing hadron masses to the EFT prediction

**General procedure:** Use $N$ number of observables to fix $N$ parameters and all other observables can be predicted form those up to a certain accuracy.

$\Rightarrow$ No need to know about the UV theory.

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Example: quark masses in $\overline{MS}$ cannot be directly measured: extracted from comparing hadron masses to the EFT prediction

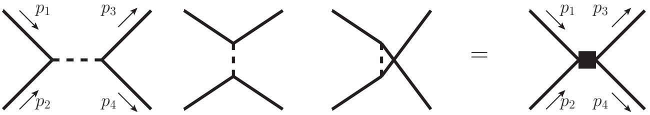
- **If** the UV theory and the EFT are weakly coupled at the scale $\Lambda$
  $\Rightarrow$ obtain the parameters through **matching**

$$\mathcal{M}^{\text{UV theory}}\Big|_{\Lambda} = \mathcal{M}^{\text{EFT}}\Big|_{\Lambda}$$

# Example: 2 scalar model at low energy

$$\boxed{\mathcal{L}^{\mathsf{EFT}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\hat{m}^2\phi^2 + g\phi^4 + \frac{C_1}{M^2}\phi^6}$$

- Matching for $g$ at tree level




$$s, t, u, m^2 \ll M^2$$

UV theory:

$$\mathcal{M}^{\mathsf{TL}}\Big|_{M^2 \gg s,t,u,m^2 \sim p^2} = \frac{3\lambda^2}{M^2} + \mathcal{O}(\frac{p^2}{M^2}).$$

EFT:

$$\mathcal{M}^{\mathsf{EFT}} = 4!g$$

At leading order

$$g = \frac{\lambda^2}{8M^2} + \mathcal{O}(\frac{p^2}{M^2}).$$

# Example: 2 scalar model at low energy

$$\boxed{\mathcal{L}^{\mathsf{EFT}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\hat{m}^2\phi^2 + g\phi^4 + \frac{C_1}{M^2}\phi^6}$$

- Matching for $C_1$ at tree level

   $\Rightarrow$ From $\phi\phi\phi$ scattering at tree level:

$$C_1 = 0 + \mathcal{O}(\frac{p^2}{M^2}).$$

(This can be checked as an exercise).

## Renormalization of composite operators

Amplitudes such as

$$\langle 0|T\{\mathcal{O}(\phi(y))\phi(x_1)\phi(x_2)\ldots\}|0\rangle$$

are not finite after renormalization of fields and masses

Example: our EFT operator $\mathcal{O} = \phi^4$

$$\phi_0 = \sqrt{Z_\phi}\phi, \qquad \mathcal{O}_0 = Z_\mathcal{O}\mathcal{O}$$

$$Z_\mathcal{O} = Z_\phi^2??$$

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$Z_{\mathcal{O}} = Z_\phi^2$?? No!!

We have: $\mathcal{L}^{\text{EFT}} \supset g\mathcal{O} \quad \Rightarrow \quad Z_{\mathcal{O}} = Z_\phi^2 Z_g$

$\Rightarrow$ coupling renormalization is a **choice**!

# Operator mixing

In fact, if we have $i$ operators of dimension $d_i$, in general

$$\mathcal{O}_{i0} = (Z_{\mathcal{O}})_{ij}\mathcal{O}_j$$

$\Rightarrow \quad Z_{\mathcal{O}}$ is a matrix.

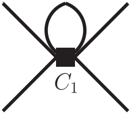
Alternatively: renormalizes the couplings

Example: the operators $\hat{m}^2\phi^2$ and $\phi^4$ mix


$$\Rightarrow \quad Z_{\hat{m}} = 1 - \frac{3g}{4\pi^2}\frac{2}{4-D} \quad \text{or} \quad (Z_{\mathcal{O}})_{11} = 1, \quad (Z_{\mathcal{O}})_{12} = -\frac{g}{32\pi^2}\frac{2}{4-D}$$

$\Rightarrow$ becomes important for logarithmic resummation

# Loops in the EFT




$$\mathcal{M} = i\frac{6!}{2}\frac{C_1}{M^2}\tilde{\mu}^{4-D}\int\frac{d^Dk}{(2\pi)^D}\frac{1}{k^2-\hat{m}^2} = -\frac{6!}{2}\frac{C_1}{M^2}\frac{\hat{m}^2}{16\pi^2}\left[\frac{2}{4-D}+1+\ln\frac{\mu^2}{\hat{m}^2}\right]$$

- DR specially important in in EFTs: keeps power counting
- Is it ok to integrate over **all** the momenta?

  YES: the physics is in the non-analytical terms (poles) which only involve the low energy (IR) scales

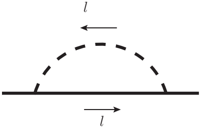
  the $k \to \infty$ region is analytical in the IR scales $\quad\Rightarrow\quad$ OPE

# 1 loop matching

- Compute a 1 loop matching between the UV theory and the EFT.

(see 1804.05863)

Example: $(p = 0)$




- UV theory:

$$I = \tilde{\mu}^{4-D} \int \frac{d^D l}{(2\pi)^D} \frac{1}{(l^2 - m^2)(l^2 - M^2)} = \frac{i}{16\pi^2} \left[ \frac{2}{4-D} - \ln \frac{M^2}{\mu^2} + \frac{m^2}{M^2 - m^2} \ln \frac{m^2}{M^2} + 1 \right]$$

- EFT:
  $\Rightarrow$ $\Phi$ does not propagate.

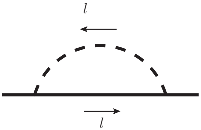
$$I^{\text{EFT}} = \frac{-1}{M^2} \tilde{\mu}^{4-D} \int \frac{d^D l}{(2\pi)^D} \frac{1}{(l^2 - m^2)} = \frac{i}{16\pi^2} \frac{m^2}{M^2} \left[ -\frac{2}{4-D} + \ln \frac{m^2}{\mu^2} - 1 \right]$$

# 1 loop matching

- Compute a 1 loop matching between the UV theory and the EFT.

(see 1804.05863)

Example: $(p = 0)$




- UV theory:

$$I\Big|_{M\gg m} = \frac{i}{16\pi^2}\left[\frac{2}{4-D} + 1 + \ln\frac{\mu^2}{M^2} + \frac{m^2}{M^2}\ln\frac{m^2}{M^2}\right] + \mathcal{O}(\frac{m^4}{M^4})$$

- EFT:

$\Rightarrow$ $\Phi$ does not propagate.

$$I^{\mathsf{EFT}} = \frac{-1}{M^2}\tilde{\mu}^{4-D}\int\frac{d^D l}{(2\pi)^D}\frac{1}{(l^2-m^2)} = \frac{i}{16\pi^2}\frac{m^2}{M^2}\left[-\frac{2}{4-D} + \ln\frac{m^2}{\mu^2} - 1\right]$$

# 1 loop matching

Example: $(p = 0)$

$$I\Big|_{M\gg m} = \frac{i}{16\pi^2}\left[\frac{2}{4-D} + 1 + \ln\frac{\mu^2}{M^2} + \frac{m^2}{M^2}\ln\frac{m^2}{M^2}\right] + \mathcal{O}(\frac{m^4}{M^4})$$

$$I^{\mathsf{EFT}} = \frac{i}{16\pi^2}\frac{m^2}{M^2}\left[-\frac{2}{4-D} + \ln\frac{m^2}{\mu^2} - 1\right]$$

- $I\Big|_{M\gg m} \neq I^{\mathsf{EFT}}$ the order of integration and expansion matters
- the divergences of the EFT and UV theories are different
- the IR behavior of the EFT and UV theories is equal: $\ln m^2$ terms
- $I_m = I - I^{\mathsf{EFT}}$ is local (analytic) in the IR scales

# 1 loop matching

Example: $(p = 0)$

$$I\Big|_{M\gg m} = \frac{i}{16\pi^2}\left[\frac{2}{4-D} + 1 + \ln\frac{\mu^2}{M^2} + \frac{m^2}{M^2}\ln\frac{m^2}{M^2}\right] + \mathcal{O}(\frac{m^4}{M^4})$$

$$I^{\mathsf{EFT}} = \frac{i}{16\pi^2}\frac{m^2}{M^2}\left[-\frac{2}{4-D} + \ln\frac{m^2}{\mu^2} - 1\right]$$

Matching at $\mathcal{O}(1/M^2)$:

$$I_m^{\overline{\mathrm{MS}}} = I^{\overline{\mathrm{MS}}}\Big|_{M\gg m} - I^{\mathsf{EFT},\overline{\mathrm{MS}}} = \frac{i}{16\pi^2}\left[1 + \ln\frac{\mu^2}{M^2} + \frac{m^2}{M^2}\left(1 + \ln\frac{\mu^2}{M^2}\right)\right]$$

$\Rightarrow$ gives the contribution to the Wilson coefficient

$\Rightarrow$ non-analytic only in the UV scales

# Better 1 loop matching

Since the IR behavior is the same, we can expand both around the IR scales and get the same result

$$I\Big|_{m\to 0} = \tilde{\mu}^{4-D}\int \frac{d^D l}{(2\pi)^D}\frac{1}{l^2(l^2-M^2)} + m^2\tilde{\mu}^{4-D}\int \frac{d^D l}{(2\pi)^D}\frac{1}{l^4(l^2-M^2)} + \mathcal{O}(\frac{m^4}{M^4})$$

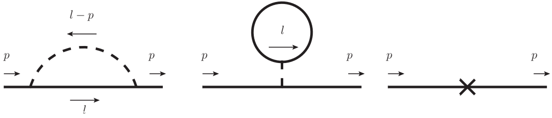
$$I^{\text{EFT}}\Big|_{m\to 0} = \frac{-1}{M^2}\tilde{\mu}^{4-D}\int \frac{d^D l}{(2\pi)^D}\frac{1}{l^2}\left(1+\frac{m^2}{l^2}+\frac{m^4}{l^4}+\cdots\right) = 0$$

$$I_m^{\overline{\text{MS}}} = I^{\overline{\text{MS}}}\Big|_{m\to 0} - I^{\text{EFT},\overline{\text{MS}}}\Big|_{m\to 0} = I^{\overline{\text{MS}}}\Big|_{m\to 0} = \frac{i}{16\pi^2}\left[1+\ln\frac{\mu^2}{M^2}+\frac{m^2}{M^2}\left(1+\ln\frac{\mu^2}{M^2}\right)\right]$$

$\Rightarrow$ no need to compute the multiscale full $I$

# Example: 1 loop matching

On-shell scheme:

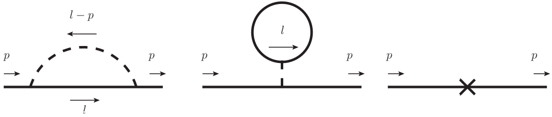



$$\Pi^{\overline{\text{MS}}}(m^2) = \frac{\lambda^2}{(4\pi)^2}\overline{B_0}(p^2, m^2, M_2^2)\Big|_{p^2=m^2} - \frac{\lambda^2}{(4\pi)^2 2M^2} m^2 \overline{A_0}(m^2)$$

$$\boxed{\hat{m}^2 = m^2 + \Pi^{\overline{\text{MS}}}(m^2) - \Pi^{\text{EFT},\overline{\text{MS}}}(m^2)}$$

# Example: 1 loop matching

On-shell scheme:



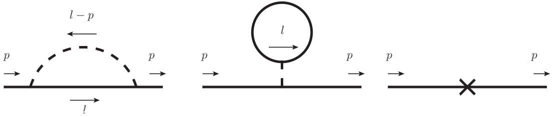

$$\Pi^{\overline{\text{MS}}}(m^2) = \frac{\lambda^2}{(4\pi)^2}\left(\frac{m^2}{M^2}+1\right)\left(1+\ln\frac{\mu^2}{M^2}\right) - \frac{m^2}{M^2}\frac{\lambda^2}{(4\pi)^2}\left(1+\frac{3}{2}\ln\frac{\mu^2}{m^2}\right) + \mathcal{O}(\frac{m^4}{M^4})$$


$$\Pi^{\text{EFT},\overline{\text{MS}}}(m^2) = -\frac{3g\hat{m}^2}{4\pi^2}\left(1+\ln\frac{\mu^2}{\hat{m}^2}\right) = -\frac{3\lambda^2\hat{m}^2}{2(4\pi)^2}\left(1+\ln\frac{\mu^2}{\hat{m}^2}\right) + \mathcal{O}(\frac{m^4}{M^4})$$

$$\boxed{\hat{m}^2 = m^2 + \Pi^{\overline{\text{MS}}}(m^2) - \Pi^{\text{EFT},\overline{\text{MS}}}(m^2)}$$

# Example: 1 loop matching

On-shell scheme:




$$\Pi^{\overline{\rm MS}}(m^2) = \frac{\lambda^2}{(4\pi)^2}\left(\frac{m^2}{M^2}+1\right)\left(1+\ln\frac{\mu^2}{M^2}\right) - \frac{m^2}{M^2}\frac{\lambda^2}{(4\pi)^2}\left(1+\frac{3}{2}\ln\frac{\mu^2}{m^2}\right) + \mathcal{O}(\frac{m^4}{M^4})$$


$$\Pi^{\text{EFT},\overline{\rm MS}}(m^2) = -\frac{3g\hat{m}^2}{4\pi^2}\left(1+\ln\frac{\mu^2}{\hat{m}^2}\right) = -\frac{3\lambda^2\hat{m}^2}{2(4\pi)^2}\left(1+\ln\frac{\mu^2}{\hat{m}^2}\right) + \mathcal{O}(\frac{m^4}{M^4})$$

$$\boxed{\hat{m}^2 = m^2 + \frac{\lambda^2}{(4\pi)^2}\left(1+\ln\frac{\mu^2}{M^2}\right) + \frac{m^2}{M^2}\left(\frac{3}{2}+\ln\frac{\mu^2}{M^2}\right) + \mathcal{O}(\frac{m^4}{M^4})}$$

# Logarithmic resummation

For operators $\mathcal{O}_i$ of dimensions $d > 4$

$$C_i = Z_{ij} C_j \quad \Rightarrow \quad \mu \frac{dC_i}{d\mu} = \gamma_{ij} C_j$$

$\Rightarrow \quad \gamma_{ij}$ has itself an expansion in the coupling

Example:

In our example the coupling is $g$:

$$\mu \frac{dg}{d\mu} = \beta(g) = g^2 \beta_0 + g^3 \beta_0 + \cdots$$

RGE for the dimension 6 operator

$$\mu \frac{dC_1}{d_\mu} = \gamma_{C_1} C_1, \qquad \gamma_{C_1} = g\gamma_0 + g^3 \gamma_1 + \cdots \quad \Rightarrow \quad \mu \frac{dC_1}{d_\mu} = \frac{dC_1}{dg} \beta(g)$$

The leading contribution is

$$\frac{dC_1}{dg} = \frac{g\gamma_0}{g^2 \beta_0} \quad \Rightarrow \quad C_1(\mu_2) = C_1(\mu_1) \left( \frac{g(\mu_2)}{g(\mu_1)} \right)^{\frac{\gamma_0}{\beta_0}}$$

# Logarithmic resummation in the EFT

The IR behavior of the EFT and the UV theory is the same

$$I\Big|_{m\to 0} = \sum_m m^r I^{(r)} = \sum_m m^r \left[\frac{A^{(r)}}{\epsilon_{UV}} + \frac{B^{(r)}}{\epsilon_{IR}} + C^{(r)}\right]$$

$$I^{\text{EFT},\overline{\text{MS}}} = 0 = \sum_m m^r \left[-\frac{B^{(r)}}{\epsilon_{UV}} + \frac{B^{(r)}}{\epsilon_{IR}}\right] \quad \Rightarrow \quad I_m^{\overline{\text{MS}}} = C^{(r)}$$

- The anomalous dimension comes for $\epsilon_{UV}$
- The EFT and the UV theory sum different logs

$\Rightarrow$ EFTs help us gain perturbativity