BSM exercises

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Ex. 1 Weinberg's soft theorems. Derive charge conservation by considering the emission of a soft photon from an arbitrary process $\alpha \to \beta$. Extend your derivation to soft gravitons and massless spin-3 fields.

Ex. 2 Construct the EFT of a single, real, massless scalar, ϕ, directly at the level of the $\phi\phi \to \phi\phi$ scattering amplitude (at tree level). Hint: Use crossing symmetry.
Exchange at tree level a minimally coupled heavy scalar Φ, $g\phi^2\Phi$, and match to the EFT amplitude.

Ex. 3 Compute the β-function coefficient of hypercharge in the SM. Fix its Landau pole by extending the gauge group to Pati-Salam's (and adding a right-handed neutrino ν).

Ex. 4 Identify the 1-loop diagrams that contribute to β_λ in the SM. Find a simple way to (potentially) avoid $\lambda(q^2) = 0$ (see Ex. 2).

Ex. 5 Identify the masses of the particles exchanged in the tree-level stringy Virasoro-Shapiro amplitude.

Ex. 6 Obtain the Weinberg operator from the 3 types of tree-level UV completions (for a single neutrino flavor).

Ex. 7 Write down the most general potential at dimension smaller or equal than 4 for the $SU(5)$ scalars $\Phi = \mathbf{24}$ (adjoint) and $S = \mathbf{5}$ (fundamental).

Ex. 8 Show that parity is a symmetry of renormalizable QED. Find a dimension-6 operator that violates it.

Ex. 9 Given the following Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + i\bar{\nu}_1 \not{\partial}_\mu \nu_1 + i\bar{\nu}_2 \not{\partial}_\mu \nu_2 + \phi \left(g_{11}\nu_1^T C \nu_1 + g_{12}\nu_1^T C \nu_2 + g_{22}\nu_2^T C \nu_2 \right) . \quad (1)$$

(where ν's are Weyl fermions in Dirac notation), estimate how small g_{11} naturalness permits given

g_{12} and g_{22}. Hint: Use a spurion analysis based on the $U(1)$ symmetries of $\mathcal{L}$.

Ex. 10 Given the following Lagrangian (renormalizable QED)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + i\bar{\psi}_L \not{D} \psi_L + i\bar{\psi}_R \not{D} \psi_R - m_\psi(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L) \tag{2}$$

(where $\not{D} = \gamma^\mu D_\mu$ and $D_\mu = \partial_\mu - ieA_\mu$), show that m_ψ renormalizes proportional to itself. Extend the Lagrangian with the dipole interaction

$$\frac{g_5}{\Lambda}\bar{\psi}_L \sigma_{\mu\nu} \psi_R F^{\mu\nu} + h.c. \tag{3}$$

and estimate its expected contribution to m_ψ. Hint: Use a spurion analysis based on the $U(1)$ chiral symmetry of $\mathcal{L}$.

Ex. 11 Derive the mass of the QCD axion in 2-flavor QCD.

Ex. 12 Show the Higgs potential is $SO(4)$ symmetric. Compute the contribution of the dimension-6 operator $-(c_T/\Lambda^2)(H^\dagger \overleftrightarrow{D}_\mu H)^2$ to the ρ-parameter, where $H^\dagger \overleftrightarrow{D}_\mu H = H^\dagger D_\mu H - (D_\mu H^\dagger)H$.