Lecture 1

Beyond the Standard Model(s)

BSM goal: looking for answers to questions the SM(s) cannot address
(very important to ask the right questions at the right time)

also, providing deeper understanding of the SM by reflecting on the ways it could be different.

TOC

- BSM list (we will touch on many of these items as we go)

- What is the SM? SM as EFT

QM + (Poincaré) Lorentz invariance = Relativity + locality/causality

↓ fix

(all possible) long-distance physics = particle content + interactions } gauge symmetries

↓

- SM global symmetries

↓

$S_{SM} = \int d^4x \sqrt{g} \left[\ldots \right]$

including gravity

- Consistency of the SM
- Shortcomings of the SM and (some) BSM solutions
- SMEFT, units, NDA (power counting)
- Hierarchy problems and BSM solutions

(disclaimer) BSM is a very broad field.

I will try to give you a global, organized surely personal view of BSM, spending more time on how to think about BSM rather than focusing on specific details / theories, w/ few hints on where progress is now.

[Weinberg] No one (can) know everything that has been done, and you do not have to.

Pick up (wisely) what you need as go.

Some important BSM topics I will only comment on briefly, in passing, or as part of supplementary extra material: baryogenesis†, dark matter† (inflation).

This is partly, but not only, b/c they rely on many things we do not experimentally know (we do not know about the cosmological history of the Universe beyond BBN ~ 0.1 MeV) and there is a lot of freedom and possibilities on what their answer could be (no preferred energy scale)

B/c of this, they certainly deserve lectures of their own. ↳ within some large range

† yet examples of "the larger the bet, the larger the reward".

varied exercises, students to choose

more material in these notes than will have time to cover

very ↑ useful to complement what I sketch

BSM List

set incomplete

known (experimental evidence ✓) BSM physics :

- neutrino oscillations (masses and mixings)
- dark matter (connection w/ (split) susy and strong CP, i.e. axion)
- baryogenesis (connection w/ neutrinos, i.e. leptogenesis and B number, i.e. sphalerons)
  - inflationary epoch (~ scale invariant, Gaussian, adiabatic and coherent density perturbations)

SM mysteries (BSM → Behind the SM) :

- cosmological constant
- electroweak hierarchy
- strong CP
- fermion masses and mixings
- charge quantization and gauge couplings (unification)
  - number of families and space-time dimensions (≥ 3 for CP)

  - matter, radiation and dark energy today

SM inconsistencies :

- hypercharge Landau pole (relevant $E > M_{Pl}$)
- Higgs vacuum instability (not clear, δm_t)
- quantum gravity and QM of spacetime (BH information paradox, Big Bang) eternal inflation
  (relevant E up to M_{Pl}, or much before?)

One of my major goals for these lectures is that you understand as best as possible many of these questions, and to do so will have to present some of the proposed solution.

Way more BSM physics : not covered here

- modified gravity
- axion-like particles (ALPs)
- dark photons, Higgs bosons
- heavy neutral leptons
- millicharged particles
- ...

What is the SM? its most important aspects that we should always keep in mind when we study BSM physics.*

* Surely you've had lectures on the SM, but I can't help myself from showing these things since they are quite remarkable.

"SM" = inevitable result of QM + Relativity at low energies
life

fundamental $\downarrow$ principles

$\phi(x)$, $[\phi(x),\phi(y)]=0$ $(x-y)^2>0$ spacelike

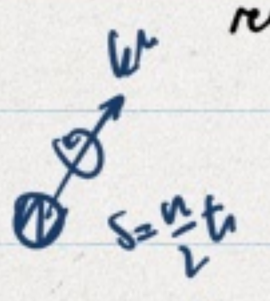
unitarity, Lorentz (Poincaré) invariance, locality/causality

$\downarrow$ $\sum p = 1$ are and remain

$\downarrow$ rotations + boosts ↳ + translation

defines what a particle is

subset $\downarrow$ ref. momentum invariant ≡ Little Group



massless: $k^\mu=(k,0,0,k) \to (1)SO(2)$: $\pm h$, helicity

massive: $k^\mu=(m,0,0,0) \to SO(3)$: S, $s=-S,\dots,S$, spin

$\downarrow$ + locality $\left(S=\frac{2n+1}{2}\right.$ is projective rep.)

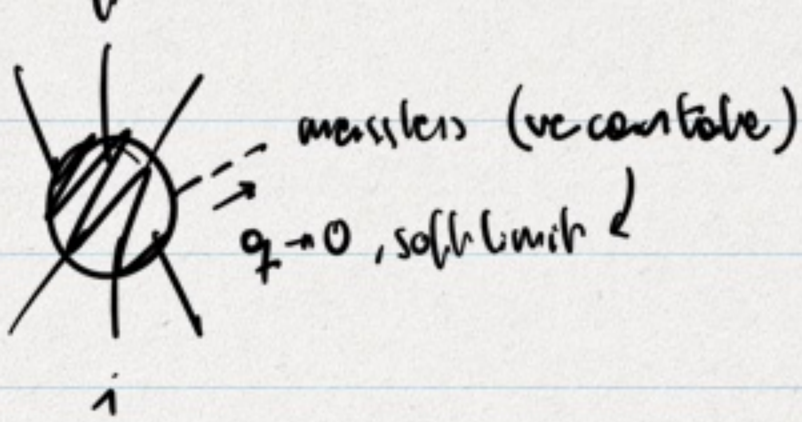
gauge invariance $\to$ redundancy

(no symmetry in physical sense, redundancy to work w/ local operators)

$h=\pm1$: $A_\mu \to A_\mu + \partial_\mu \lambda$, $h=\pm2$ $h_{\mu\nu} \to (\eta_{\alpha\mu}+\partial_\alpha \xi_\mu)(\eta_{\beta\nu}+\partial_\beta \xi_\nu)(\eta^{\alpha\beta}+h^{\alpha\beta}) - \eta_{\mu\nu}$

Weinberg soft theorems

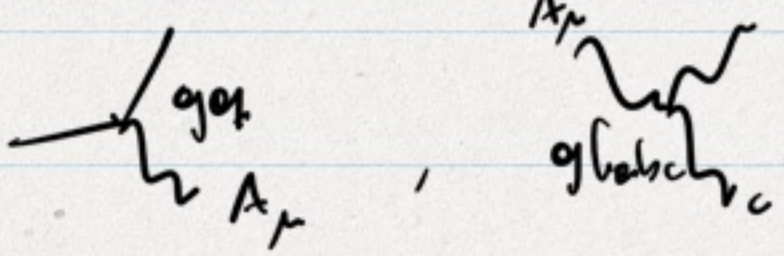
only Lorentz + locality
↓ unitarity
LG invariance



- massless spin-1 : gauge principle , charge conservation : $Q_i = Q_f$ $\overset{\shortparallel}{\sum q_i}$

  $h=\pm1$

$$\mathcal{L}_{int} = A_\mu J^\mu \quad , \quad \partial_\mu J^\mu = 0 \; , \text{ conserved current}$$



Yang-Mills structure

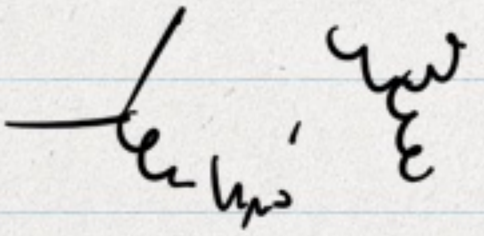
- massless spin-2 : equivalence principle : $\sum_i \kappa_i p_i^\mu = \sum_f \kappa_f p_f^\mu$

  $h=\pm2$

$$\downarrow$$

$$\kappa_{i,f} = \kappa = \frac{1}{M_{Pl}} \quad \forall i,j$$

$$\mathcal{L}_{int} = h_{\mu\nu} T^{\mu\nu} \quad , \quad \partial_\mu T^{\mu\nu} = 0$$



= GR

General Relativity (no freedom)

conserved (and unique) energy-momentum tensor

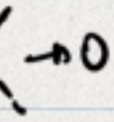
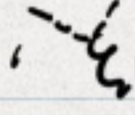
- massless spin-$S>2$: NO low energy couplings

↓

arXiv:1709.04891 ; Arkani-Hamed et al

2304.02550, 1903.08664 ; Bellazzini et al

Note: Ways out (flat space) also have problems : much progress now

↳ $m\neq0$,  $\to 0$ as $q\to0$; $m \overset{?}{\lesssim} \Lambda$, cutoff ,  gravity ⇒ $m=0$ inconsistent

This is precisely the (elementary) world we see today at low energies (soft) long distances

no constraints

$0 \,,\, 1/2 \,,\, 1 \,,\, 3/2^{+} \;\; 2 \,,\, ~~5/2 , 3 , 7/2 , \ldots~~$

Higgs boson (latest addition, nothing like we have seen before ≡ New Physics) ⇓ BSM

(Weyl)* fermions quarks and leptons

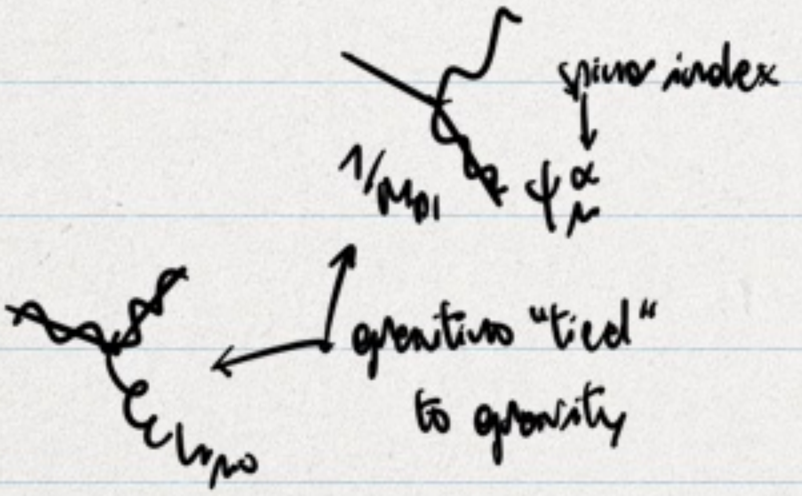
gauge interactions

gravity

no light (interacting) higher spins

* left-handed particle w/ right-handed antiparticle
(right) (left)

\+ massless spin-3/2 , consistent low energy theory ⇒ supersymmetry
$h = \pm 3/2$ (soft theorem)



$\mathcal{L}_{int} = \psi^{\alpha}_{\mu} J^{\mu}_{\alpha} \;,\; \partial_{\mu} J^{\mu}_{\alpha} = 0$

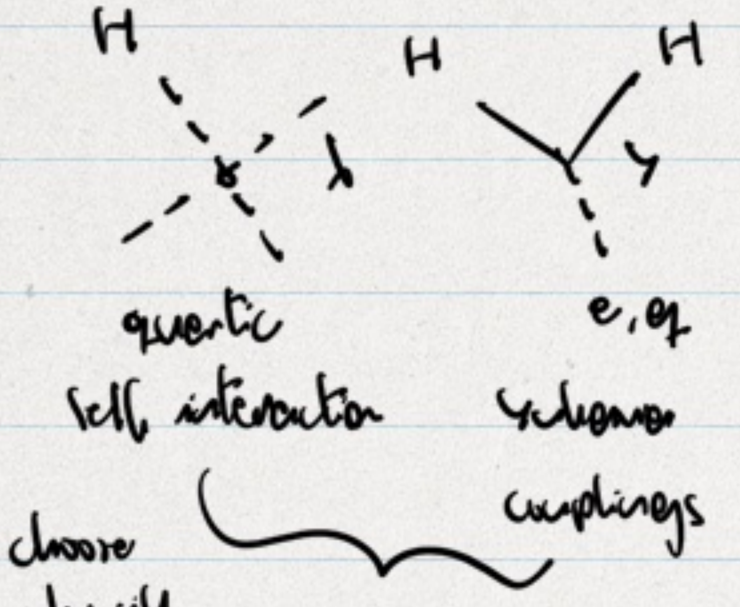
conserved supercurrent

(supersymmetry an local symmetry)

Note: Hard to believe this particle does not exist.
↓
(we will see more later)

... w/ "minimal" couplings between them ;

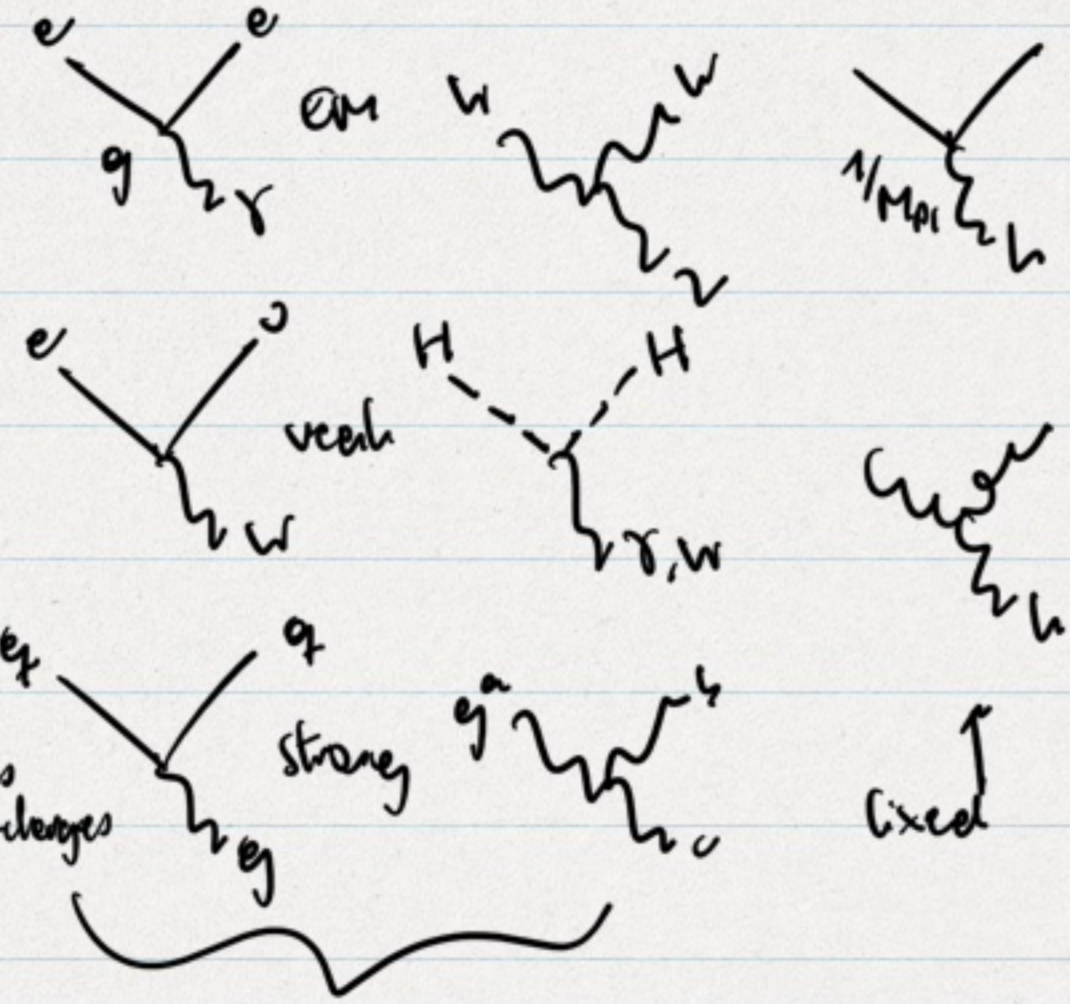
(survive) leading at low energies



quartic self interaction

Yukawa couplings

choose at will

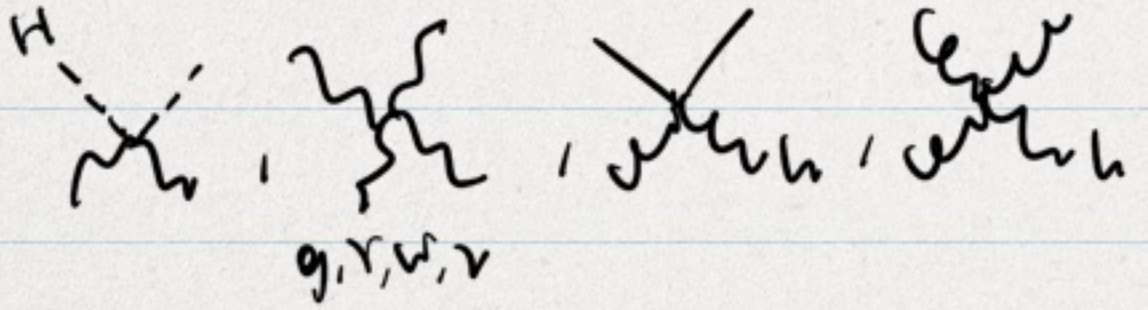


		$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
	q_L	3	2	1/6
	u_R	3	-	2/3
3×	d_R	3	-	-1/3
	l_L	-	2	-1/2
	e_R	-	-	-1
	H	-	2	1/2

g fixes self-charges

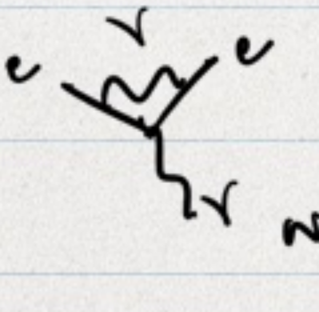
↖ simply need to specify the charges (of particles)

Note 1.-



completely fixed by the above, no freedom

Note 2.- loops

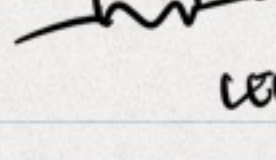


running g

$(g-2)_e$

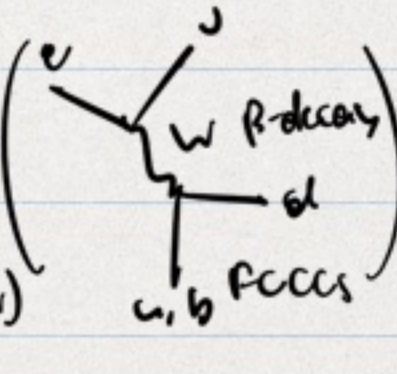
NR ($E<m_e$)

,



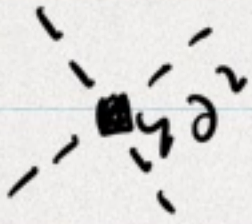
FCNCs

LEFT ($E<m_W$)



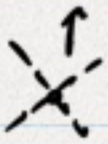
different EFTs

and at low energies / long distances we (can) neglect, in 1st (LO) approximation, all "non-minimal" couplings

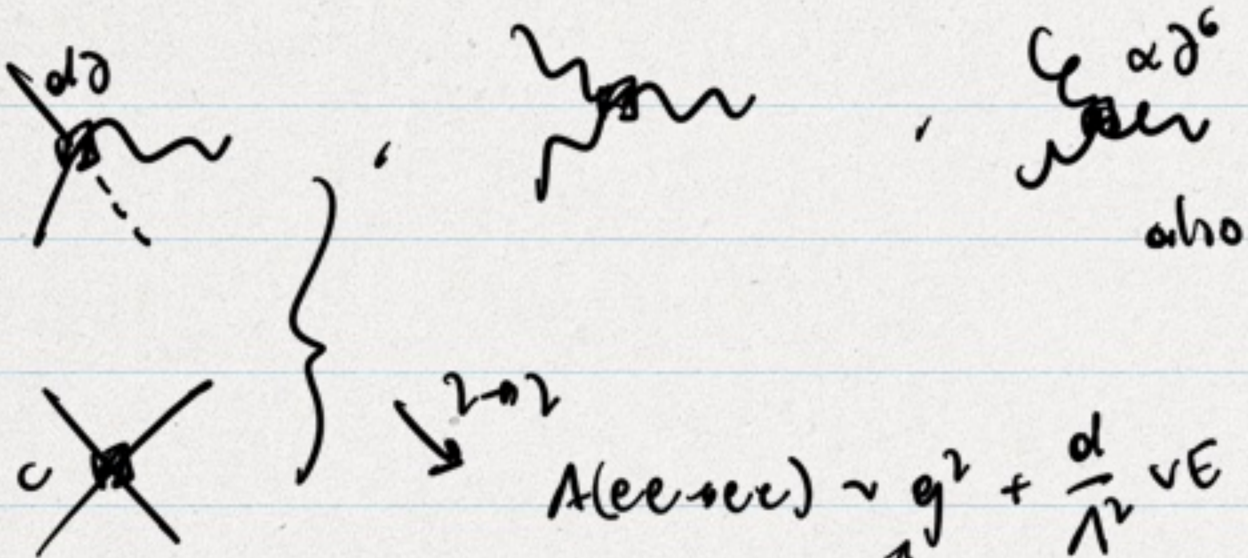


2→2 amplitudes

$$A(HH \to HH) \sim \lambda + \frac{\eta}{\Lambda^2} E^2 + \dots$$



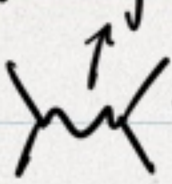
same w/ more derivatives



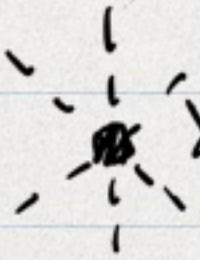
also w/ gravitons

2→2

$$A(ee \to ee) \sim g^2 + \frac{d}{\Lambda^2} vE + \frac{c}{\Lambda^2} E^2 + \dots$$



$$A(hh \to hh) \sim \left(\frac{E}{M_{Pl}}\right)^2 \left[1 + \frac{\alpha}{\Lambda^4} E^4 + \dots\right]$$

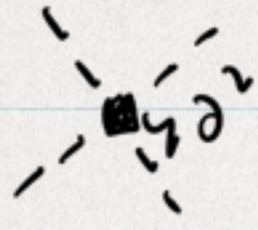


2→4 amplitudes

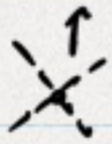
$$A(HH \to HHHH) \sim \frac{1}{E^2}\left[\lambda^2 + \frac{\lambda\eta}{\Lambda^2} E^2 + \frac{\lambda_6}{\Lambda^2} E^2\right]$$

and at low energies / long distances we (can) neglect, in 1st (LO) approximation, all "non-minimal" couplings

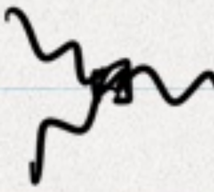
2→2 amplitudes

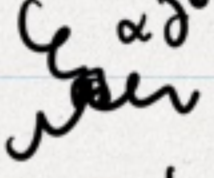


$$\longrightarrow A(HH\to HH) \sim \lambda + \cancel{\frac{y}{\Lambda^2}E^2} + \dots$$

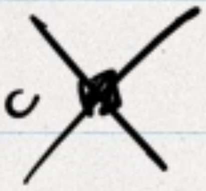


same w/ more derivatives



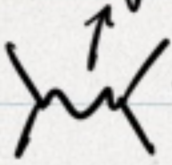


also w/ gravitons



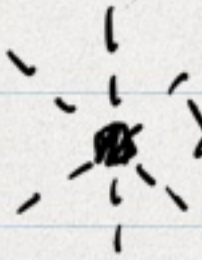
2→2

$$A(ee\to ee) \sim g^2 + \cancel{\frac{d}{\Lambda^2}vE} + \cancel{\frac{c}{\Lambda^2}E^2} + \dots$$



$$A(hh\to hh) \sim \left(\frac{E}{M_{Pl}}\right)^2\left[1 + \cancel{\frac{\alpha}{\Lambda^4}E^4} + \dots\right]$$

2→4 amplitudes



$$\longrightarrow A(HH\to HHHH) \sim \frac{1}{E^2}\left[\lambda^2 + \cancel{\frac{\lambda y}{\Lambda^2}E^2} + \cancel{\frac{\lambda_6}{\Lambda^2}E^2}\right]$$

b/c they are irrelevant, i.e. give small contributions to observables, for $(g,y)v, E \ll \Lambda$ cutoff, defines what low energy is (for now)

even if perfectly allowed by particle content and gauge charges extra (you have probably seen this in QFT lectures in some way)

In general: $\mathcal{L} = \sum_i g_i O_i$ $\qquad [g_i] = \underset{\substack{\uparrow \\ \text{space-time } d}}{4} - [O_i]$

$\updownarrow$ powers of Λ before: $g_i \equiv \hat{g}_i / \Lambda^{[O_i]-4}$

scaling dimension of operator: $[O_i]$

dilations: $\phi(x) \to \phi'(x) = e^{d_\phi \sigma} \phi(e^\sigma x)$ $\qquad x \to \underset{\substack{\shortparallel \\ x'}}{e^\sigma x}$ $\begin{pmatrix} d^4x = e^{-4\sigma} d^4x' \\ \partial = e^{-\sigma} \partial' \end{pmatrix}$

relative contribution to amplitudes: $\dfrac{\delta A}{A} \sim g_i E^{[O_i]-4}$

$[g_i] > 0$, relevant at small E $\quad$ (see Higgs
e.g. scalar mass term $m^2\phi^2$, $[\phi^2] = 2$ $\quad \dfrac{\delta A}{A} \sim \dfrac{m^2}{E^2}$)

} renormalizable

$[g_i] = 0$, relevant at all E

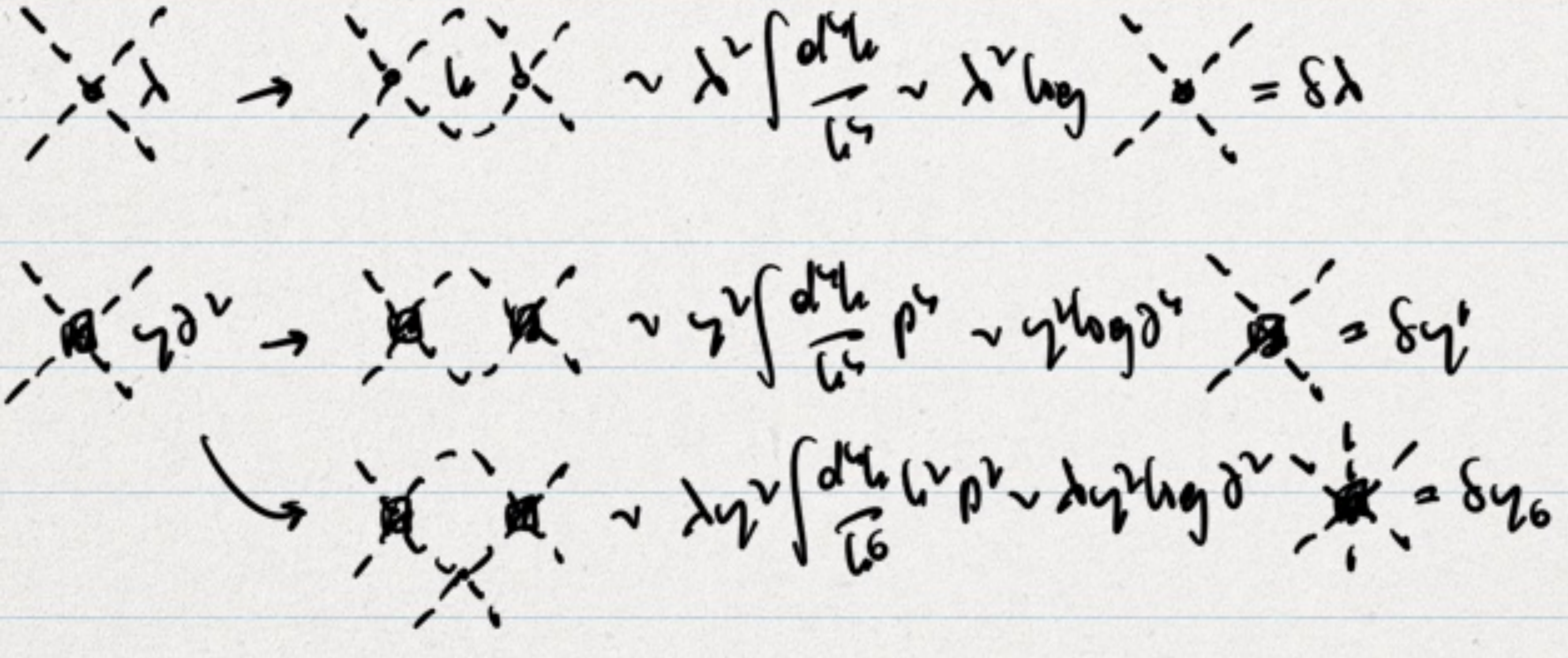
$[g_i] < 0$, irrelevant at small E ← non-renormalizable

This holds regardless of tree vs loop level.

Loops (simply) add additional (usually) mild log E dependence:

$$\delta A \sim \prod_i g_i\, E^{[O_i]-4} \to \prod_i g_i\, E^{[O_i]-4}\left[1 + a_1 \log\frac{E}{\mu} + a_2\left(\log\frac{E}{\mu}\right)^2 + \dots\right]$$

The key difference between renormalizable vs non-, is that the former form a closed finite set under renormalization (i.e. counterterm absorbed by renormalizable terms), while the latter do not. E.g.



$$\rightarrow \sim \lambda^2 \int \frac{d^4k}{k^4} \sim \lambda^2 \log \quad = \delta\lambda$$

$$\rightarrow \sim \eta^2 \int \frac{d^4k}{k^4}\, p^4 \sim \eta^2 \log \partial^4 \quad = \delta\eta'$$

$$\hookrightarrow \sim \lambda\eta^2 \int \frac{d^4k}{k^6}\, k^2 p^2 \sim \lambda\eta^2 \log \partial^2 \quad = \delta\eta_6$$

Still, there is no problem w/ predictivity $_\wedge$ as long as
in non-renormalizable theories

$$E \ll \Lambda$$

$$A_{2\to2} \sim \underset{\substack{\uparrow\\ \lambda+\delta\lambda\log E}}{\tilde{\lambda}} + \tilde{c}_2\frac{E^2}{\Lambda^2} + \delta c_2'\frac{E^4}{\Lambda^4} + \dots$$

Note:- ... along w/ the usual requirement of perturbativity of the loop expansion in terms of dimensionless couplings, i.e.

$$\frac{\left(\prod_i\right) g_i E^{[O_i]-4}}{\underset{\substack{\uparrow}}{16\pi^2}} \ll 1$$

slightly different than the above b/c $\curvearrowright \hat{g}_i\left(\frac{E}{\Lambda}\right)^{[O_i]-4}$

(see more later)

What we just did, the way we did it, is to construct the SM

as an <u>Effective Field Theory</u> (EFT)

In the modern view (after K. Wilson) that any QFT is an effective low-energy description, well described by a renormalizable Lagrangian, the SM, at a fundamental level, is almost the inevitable consequence of the success of [the two scientific revolutions of the 20th century,) QM and R in describing nature:

$$S = \int d^4x \sqrt{-g} \Big\{ \underbrace{-\frac{M_{Pl}^2}{2} R - \sum_{A = SU(3)_c, SU(2)_L, U(1)_Y} \frac{1}{4 g_A^2} F^A_{\mu\nu} F^{A\mu\nu} + \sum_{\psi = q,u,d,l,e} \bar\psi i e^\mu_a \gamma^a D_\mu \psi + g^{\mu\nu} D_\mu H^\dagger D_\nu H - Y_u^{ij} \bar q^i \tilde H u^j - Y_d^{ij} \bar q^i H d^j - Y_e^{ij} \bar l^i H e^j + h.c. - V(H) - \Lambda_{cc}}_{\mathcal{L}_{SM}} + \dots \Big\}$$

Note:- Why this particle content?
w/ small rep.'s., it is gauge anomaly free.

↑

This is the answer of a intrinsically BSM question: What could nature be like?

Could it have been much different? NO, if Lorentz and scale separation.
+
QM + local & causal

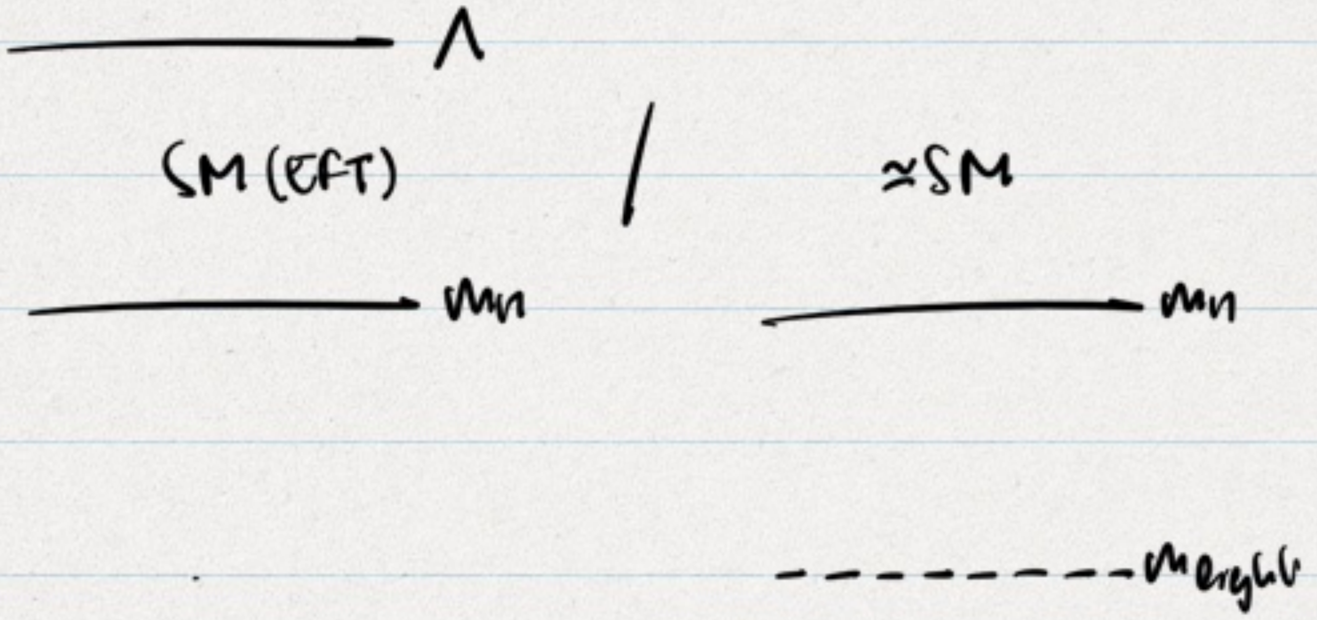
low-energy EFT

BSM = non-minimal interactions / new light particles ⊃ mean

pragmatic sense

Λ ≡ scale of heavy d.o.f.'s (associated w/ UV completion)

weakly coupled at low energies (or we would have discovered them)



but it is lots more than that,

new dynamics : interactions (scalar, gauge weak or strong)

supersymmetry (whole set of new particles / interactions)

extra-dimensions (small, warped ; large, gravitational)

↗ SMEFT ↗ light cuts

new implications (beyond those that motivated the BSM in the first place)

^ for our understanding of universe

new principles : e.g. beyond EFT (landscape, strings, UV/IR connection)

^ QFT multiverse WGC

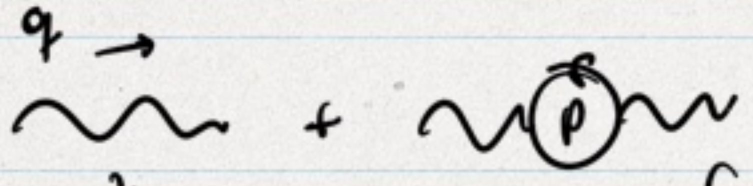
With SM, let us forget for a moment that it is an EFT, ^ does it display any inconsistencies?

we do not actually know the scale Λ that defines low energy.

(going through list now)

Hyperchange Landau pole

range of validity of perturbation theory $\leftrightarrow$ predictivity



$$\underset{g^2/q^2}{} \qquad \frac{g^2}{q^2}\, g^2 \underbrace{\int \frac{d^4p}{(2\pi)^4}\frac{1}{p^4}}_{\sim \frac{1}{16\pi^2}\underbrace{\int dp^2/p^2}_{\sim O(1)\log q/\mu}} \qquad = \frac{g^2}{q^2}\left(1+\underbrace{\frac{g^2}{16\pi^2}}_{\equiv \frac{\alpha}{4\pi}\ll 1}\log\frac{q}{\mu}\right)$$

$\xrightarrow{} V(q) = -g^2/q^2$

Renormalization Group Evolution

$g \equiv g_Y\ (g')$ RGE :

running coupling: $$\frac{dg_Y(q)}{d\log q} = b_Y\frac{g_Y^3}{16\pi^2}\ ,\quad b_Y > 0 \quad \text{SM}: b_Y = \frac{41}{6}$$

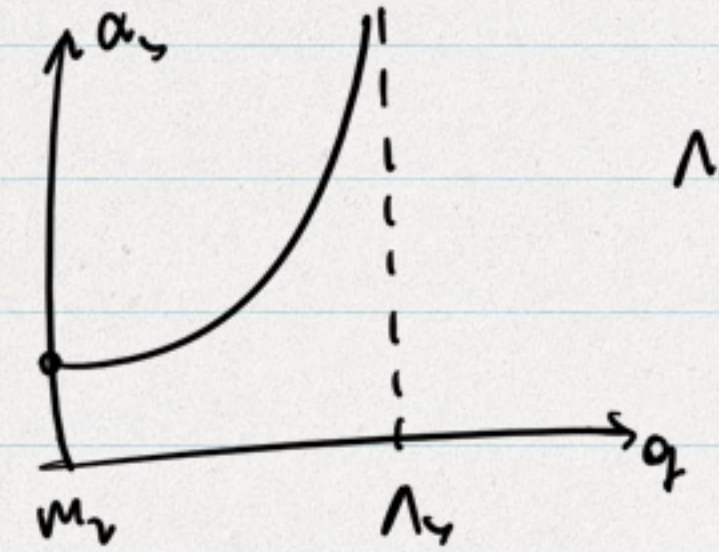
$\left(1\text{ Dirac fermion } b_Y = 4/3\right)$
$Q_Y = 1$

$\downarrow$ solution

$$\alpha_y(q_1) = \frac{\alpha_y(q_2)}{1 - \frac{\alpha_y(q_2) b_y}{2\pi} \log \frac{q_1}{q_2}}$$

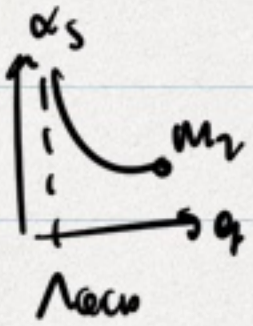
$q_1 \equiv q$

$q_2 \equiv m_z$



$$\Lambda_{LP} \simeq e^{2\pi / \alpha_y(m_z) b_y} m_z \sim 10^{41}\,\text{GeV}$$

Landau pole: $\alpha_y \to \infty$



Note: Also in QCD, but at low energies q (not large q) :

↓

we know (not fully understand) what happens: confinement, chiral Lagrangian (new EFT d.o.f.'s)

mesons, baryons, ... : $\pi, \rho, ..$

and

we can use lattice; requires $g \to 0$ in continuum limit $\Delta a \to 0$

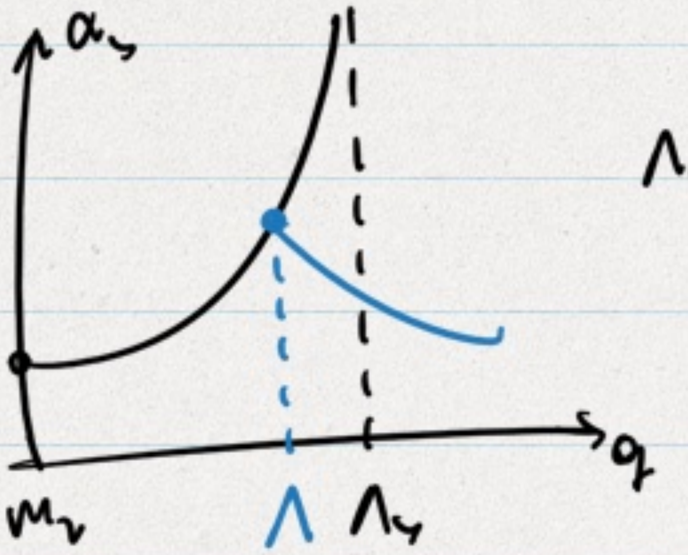
↑ no use (for now) for Λ_y

↕ high q

$$\alpha_y(q_1) = \frac{\alpha_y(q_2)}{1 - \frac{\alpha_y(q_2) b_y}{2\pi} \log \frac{q_1}{q_2}}$$

$q_1 \equiv q$

$q_2 \equiv m_Z$



$$\Lambda_{LP} \simeq e^{2\pi / \alpha_y(m_Z) b_y} m_Z \sim 10^{41}\,\text{GeV}$$

Landau pole: $\alpha_y \to \infty$

BSM solutions:

- non-abelian extension $U(1)_y \to SU(2)_{(R)}$

(more in the context of GUTs)

Higgs potential (meta-) stability

some similarities w/ before

$$V(H) = -\mu^2 |H|^2 + \lambda |H|^4 \quad , \quad \langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

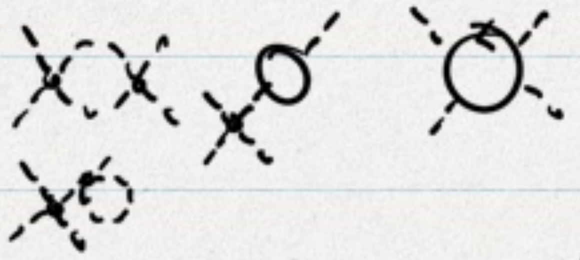
$\uparrow$ after Higgs boson discovery

$$v = \mu^2/\lambda \simeq 246\,\text{GeV}$$

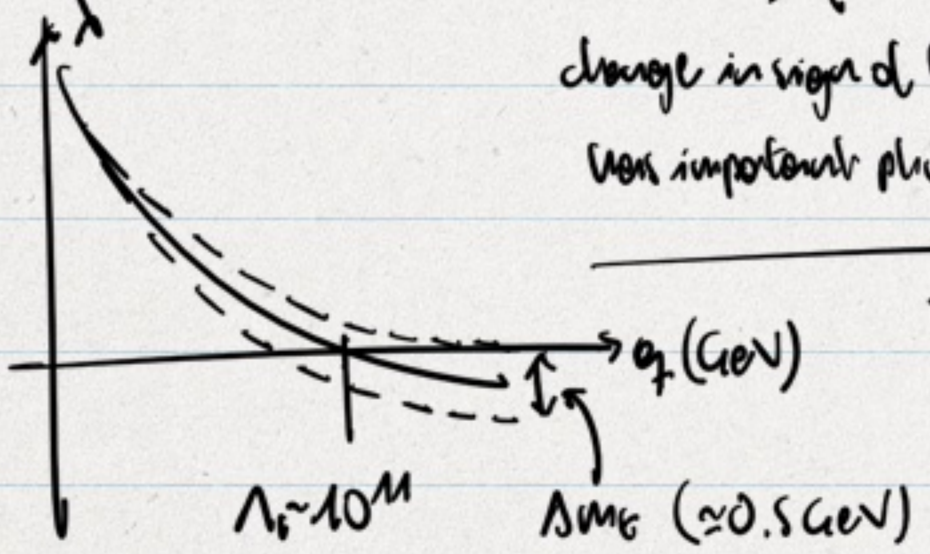
$$m_h^2 = 2\lambda v^2 \simeq 125\,\text{GeV} \quad (\lambda \simeq 0.13)$$

$\downarrow$

$$\text{RGE}: \quad \frac{d\lambda}{d\log q_T} = \frac{1}{16\pi^2}\left(24\lambda^2 + 12\lambda y_t^2 \overset{\text{negative}}{\overset{\downarrow}{-}} 6y_t^4 + O(g^2)\right)$$



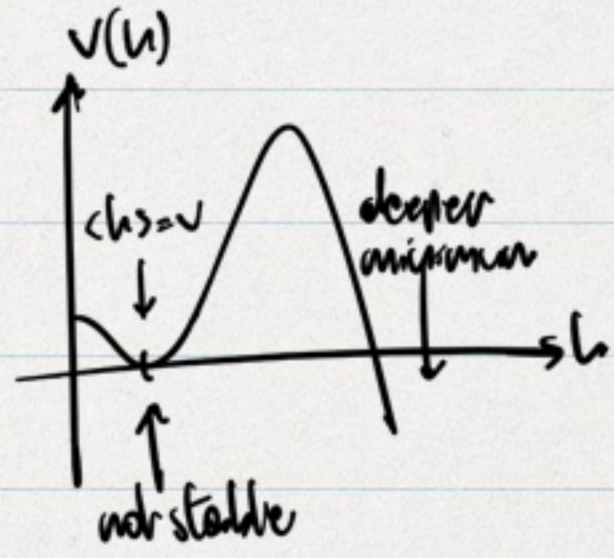
Note $\delta\lambda \not\propto \lambda$



change in sign of Higgs quartic

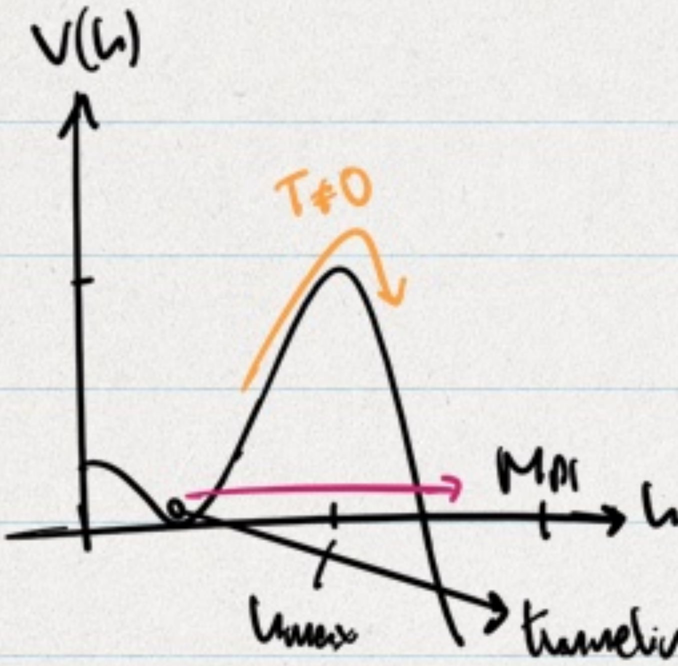
very important physical implications

$\longrightarrow$ *



* effective Coleman-Weinberg potential, intuitively $q_{T_1}, q_{T_2} \to h, v$

$$\lambda(q_T) \to V(h) \underset{h \gg v}{\simeq} \frac{\lambda(h) h^4}{4}$$



tunneling, rate: $\Gamma \sim e^{-8\pi^2/3|\lambda|}$

(quantum fluctuations)

↑ flat space

lifetime: $\tau > 10^{100}$ years ≫ age of universe ~ 10^{10} years

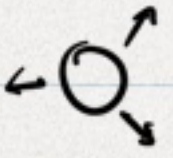
metastable

but what about in the early universe? during inflation?

- thermal fluctuations $T \neq 0$

↓

nucleation of bubbles



inflationary fluctuations $H \neq 0$

- $H < h_{max}$: Coleman-de Luccia tunneling
- $H \sim h_{max}$: Hawking-Moss instanton

$T_{dS} = H/2\pi$

BSM solutions: - new matter changes $\beta_\lambda \left(= \frac{d\lambda}{d\log q}\right)$ at $q = \Lambda \gg m_h$

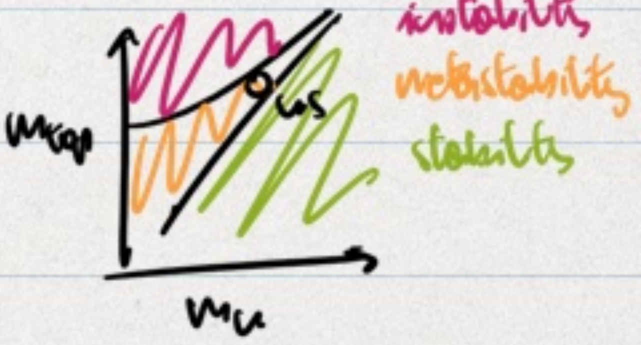
- there is no H at high energies → compositeness scale Λ ↑

(more later)

⋮

some people have instead embraced this fact:

new (BSM) principle: self-organized Higgs criticality

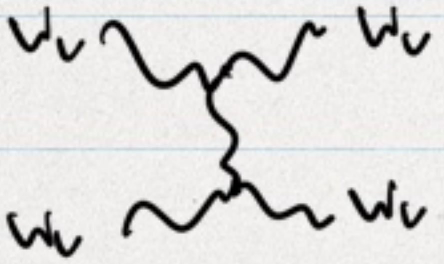


Quantum gravity

Many aspects, one of them is certainly not that we cannot do loops.

1) Loss of perturbative unitarity / bad high energy behaviour.

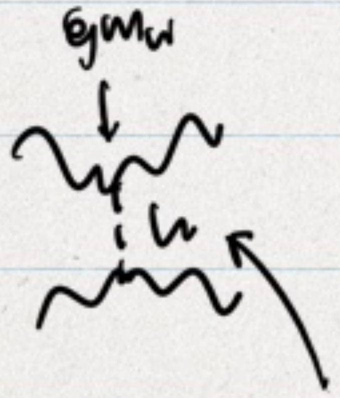
illuminating to go back a decade and consider



$A \sim \frac{E^2}{v^2} \gg 1$ for $E > 4\text{TeV}$

Longitudinal EW boson scattering w/o Higgs

↓ solution (realized in nature)

\+ 

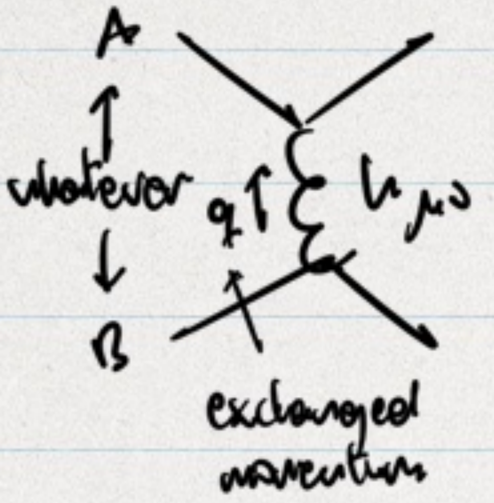
$A_h \sim \lambda$, good high energy behaviour

Higgs $(s=0)$ "elementary" boson

↑

more than any other scalar in nature

(e.g. QCD π's)



$$A \sim -\frac{1}{M_{Pl}^2}\frac{s^2}{t} \underset{\uparrow \theta \text{ fixed}}{\sim} \frac{E^2}{M_{Pl}^2} \gg 1 \text{ for } E \gtrsim 4\pi M_{Pl}$$

$$t = -q^2 \sim -E^2(1-\cos\theta)$$

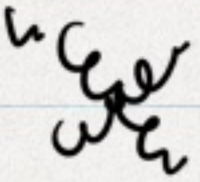
$$s \sim E^2$$

$\downarrow$

No-go theorem for BSM

$\downarrow$ we know of a solution (much more technically difficult than Higgs)

if graviton scattering :



$$A_{GR}(1^+2^-3^-4^+) = -\frac{(\langle 23\rangle[14])^4}{stu}$$

Virasoro amplitude

$$A_{ST} \sim A_{GR}\frac{\Gamma(1-s/M^2)\Gamma(1-t/M^2)\Gamma(1-u/M^2)}{\Gamma(1+\frac{s}{M^2})\Gamma(1+t/M^2)\Gamma(1+u/M^2)} \sim^{*}$$ no bad high-energy behaviour

$$^{*} \sim \frac{E^2}{M_{Pl}^2}e^{-E^2/M_s^2}$$

$\uparrow$

∞ number of higher-spin resonances w/ $m^2 = nM^2$

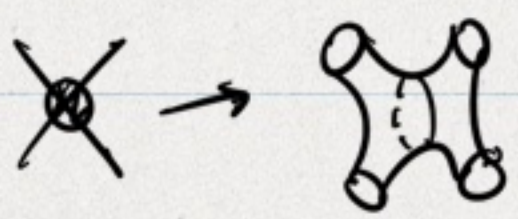
$\uparrow$

string scale

behaviour associated w/ String(s) Theory

arXiv: 2210.12163
Cheung, Remmen

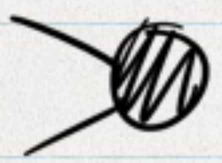
Note: Much progress today, to generalize, bootstrap from principles



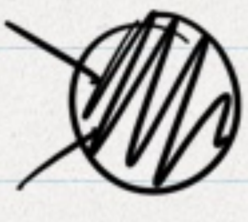
extra

2) UV/IR, loss of locality

$E \sim M_{Pl}$



Black Hole $\xrightarrow[\text{energies}]{\text{higher}}$ $E \gg M_{Pl}$



larger BH

↓

lower energy classical physics

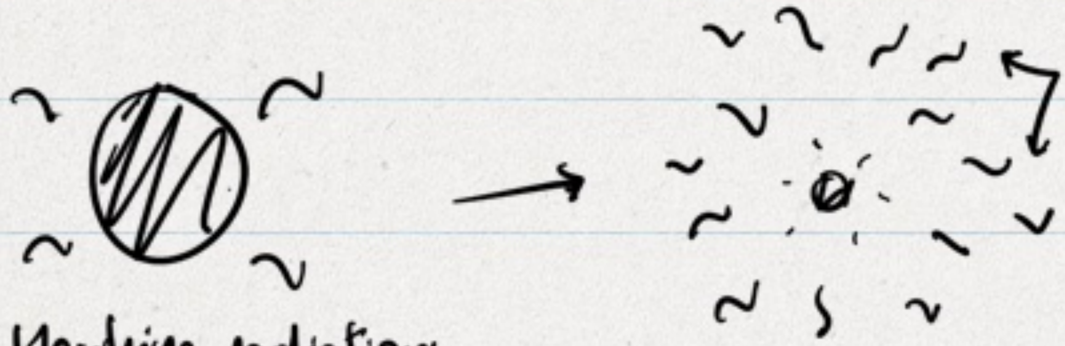
for $b \lesssim R_s(E) \sim \frac{E}{M_{Pl}^2}$ Schwarzschild radius

↑

small impact parameter → large q

(b, q conjugate variables)

3) BH information paradox, loss of unitarity



Hawking radiation

↑

black body

(mixed state)

maximal entropy $S_{BH} \sim M_{Pl}^2 A$

$A \sim R_s^2(E)$

$\sim E^2/M_{Pl}^2$

↗ Bekenstein

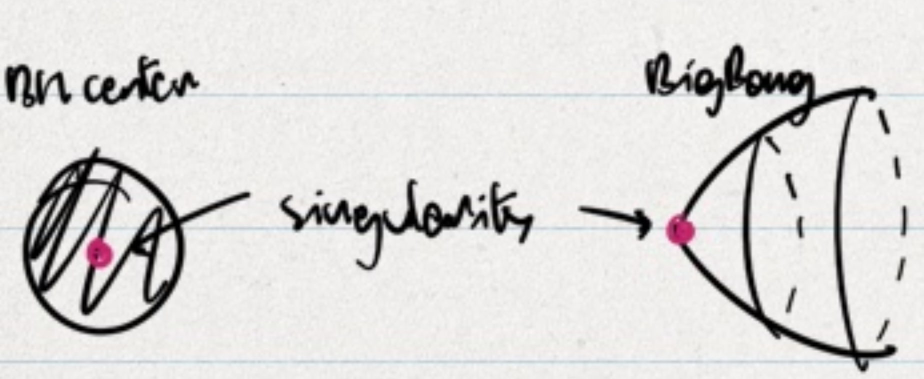
↓

minimal information (again)

regardless of how much we threw in

Note: Much progress today

4)



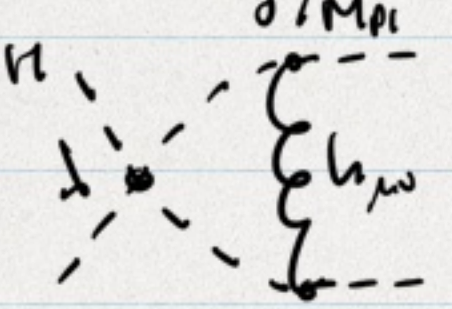
No predictivity when gravitational (strength) curvature and QM are relevant

↑

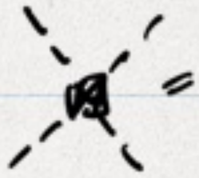
both (again it depends on energy and distance)

5) SM is EFT: gravitational RGE

e.g.



$$\sim \frac{\lambda}{M_{Pl}^2}\int\frac{d^4k}{(2\pi)^4}\frac{k^2}{k^6}p^2 \sim \frac{\lambda}{M_{Pl}^2}\log\partial^2$$



$= \delta\eta$

non-minimal interaction

$$\downarrow$$

$$A(HH\to HH) \sim \lambda + \frac{E^4}{M_{Pl}^2 q^2} + \underbrace{\frac{\eta}{\Lambda^2}}_{\sim \frac{\lambda}{16\pi^2 M_{Pl}^2}\log E} E^2 + \dots$$

$$\uparrow$$

M_{Pl} suppressed, log enhanced

the same w/ other SM particles, more loops.

arxiv: 2109.06191; Baratella et al.

Lecture 2

Accidental and approximate global symmetries of SM

$\uparrow$

reason behind SM's most spectacular successes

leading to important lessons (boundaries) for BSM

Emergence of global symmetries is consequence of IR loss of complexity

when there exists a (large) separation of scales*, i.e. EFT

$\uparrow$

as amount of

$m, E \ll \Lambda$

bandwidth of $\Leftarrow$ stored in non-minimal

parameters of irrelevant interactions

renorm. $\mathcal{L}$

* it becomes the crucial question

EFT perspective, global symmetries not due to theory being fundamental^{+},

just unavoidable consequence of probing system w/ long-distance quanta.

(same as car from far away looks spherical, SO(3) symmetry)

\+ statement w/ support from BH physics and string theory.

$\downarrow$

$\mathcal{L}_{SM}$

individual lepton numbers: $\underset{\substack{\shortparallel\\ e}}{U(1)_{L_1}} \times \underset{\substack{\shortparallel\\ \mu}}{U(1)_{L_2}} \times \underset{\substack{\shortparallel\\ \tau}}{U(1)_{L_3}}$

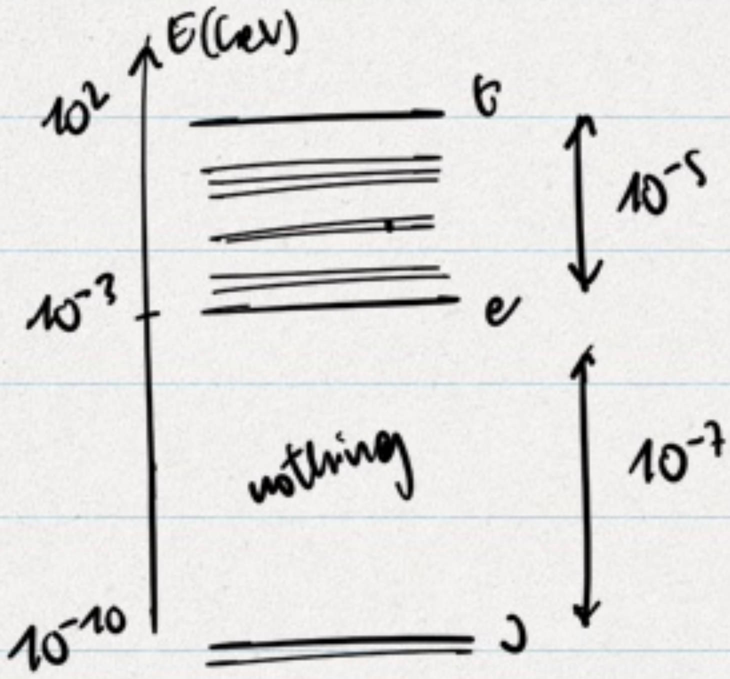
$$(l,e)^j \to e^{L_i\theta}(l,e)^j \qquad i = 1,2,3 \quad \text{(no summation)}$$

$$L_i(l^j) = 1\,\delta_i^j$$

$$\downarrow$$

- massless neutrinos (no ν_R).

pretty good approximation to reality



↑ neutrino mass seems qualitatively different
(origin of)

Flavor Changing

- No FC processes: $\mu \to e\gamma$ very good agreement w/ data

$\tau \to \mu\gamma$ $\quad Br(\mu \to e\gamma) \leq 10^{-13}$

$\mu \to 3e$ $\quad \uparrow$ PDG 22

MEG

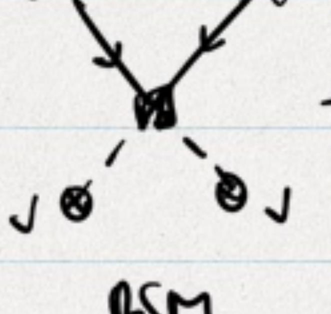
$\vdots$

This brings us to the SM as EFT.

Neutrino oscillations : masses and mixings

$$\frac{b^{ij}}{\Lambda} \left(\ell^{iT} \epsilon H\right) C \left(H \epsilon \ell^j\right)$$

+h.c.



Majorana mass $\Delta L_T = 2$

$$\to -\frac{1}{2} M_\nu^{ij} \nu^{iT} C \nu^j$$

$$\sim \frac{b v^2}{\Lambda}$$

$$\epsilon = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix}_{SU(2)_L}$$

$$C = \begin{pmatrix} & \epsilon \\ -\epsilon & \end{pmatrix}_{\text{Lorentz}}$$

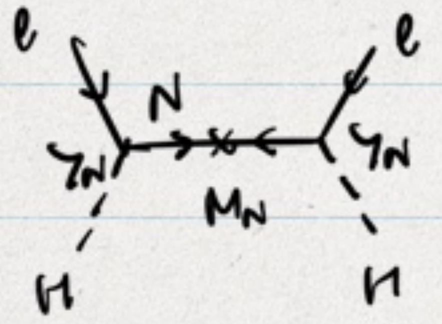
$$m_\nu \sim 0.05\,\text{eV} \left(\frac{b}{1}\right) \left(\frac{10^{15}\,\text{GeV}}{\Lambda}\right)$$, "seesaw"

for $b = O(1)$ accidental $U(1)_{L_i}$ b/c $\Lambda \gg (\sqrt{b}) v$, i.e. very heavy BSM

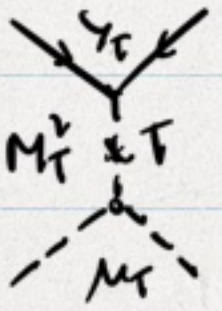
↑ it emerges in the SM when $\frac{\Lambda}{E} \to \infty$

Several simple possibilities behind this, i.e. UV completions :

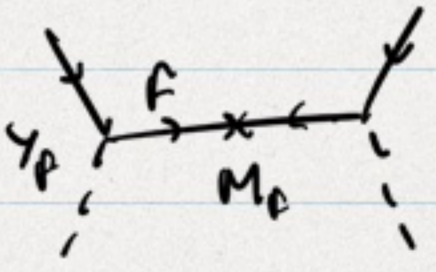
type I



type II



type III



$N_{(3)} = (1,1)_0$

Weyl fermion singlets

(2-component)

$T = (1,3)_1$

scalar triplet

$F = (1,3)_0$

Weyl fermion triplets

$$b \sim \frac{Y_N^2}{M_N}$$

Notation: $\left(R_{SU(3)_c}, R_{SU(2)_L}\right)_{Q_Y}$

Dirac mass: there is also the BSM option of introducing a light particle weakly coupled (gauge neutral)

R-handed neutrino: $\nu_{(R)} = (1,1)_0$

$$\Delta \mathcal{L}_{SM} = - Y_\nu^{ij} \bar{\ell}^i \tilde{H} \nu_R^j + h.c.$$

$$\downarrow$$

$$m_\nu = \frac{Y_\nu v}{\sqrt{2}}$$

$U(1)_{L_i}$ preserving. Why? One can always add, following the EFT point of view,

$$M_R \nu_R^T C \nu_R$$

just like type I before (N and ν_R have same SM quantum numbers)

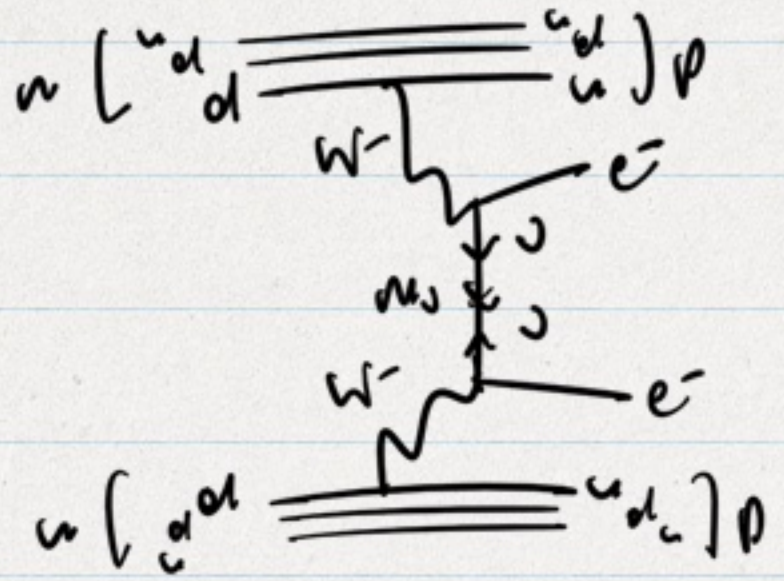
It seems we are adding more questions to the BSM list.

In any case, we have not extended $\mathcal{L}_{SM}$ to incorporate neutrino masses and mixing b/c the key question remains

Dirac or Majorana?

answer ↓ experiment

neutrinoless double beta decay ($2\beta0\nu$)



low energy, precision

(luminosity)

↗ see extra

Note: Answering it goes beyond (e.g. leptogenesis) an explanation of

^ lepton and neutrino masses/mixing.

charged

↘ see extra?

extra ?

pattern in lepton masses/mixings ?

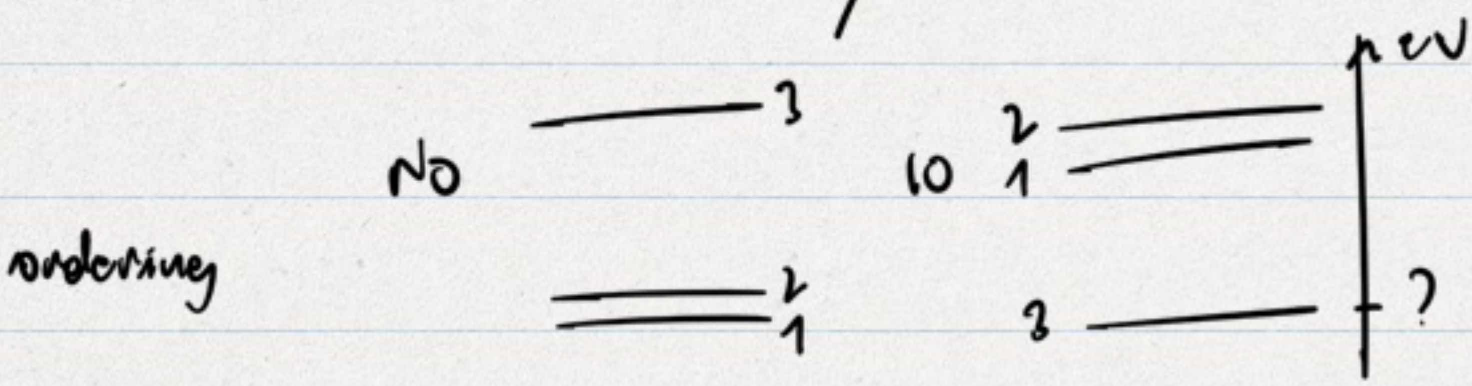
$$M_e \sim \begin{pmatrix} 5\times10^{-3} & & \\ & 10^{-1} & \\ & & 1 \end{pmatrix} \text{GeV}$$

PDG22

$$M_\nu = \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix}, \text{ w/ } m_2^2 - m_1^2 = \Delta m^2_{sol} \sim (10^{-2}\,\text{eV})^2$$

$\uparrow$ diagonalized (lepton rotations)

$$\left.\begin{array}{l} \text{NO}: m_3^2 - m_1^2 \\ \quad\text{or} \\ \text{IO}: m_2^2 - m_3^2 \end{array}\right\} = \Delta m^2_{atm} \sim (5\times10^{-2}\,\text{eV})^2$$



Note: It could be $m_1 \simeq m_2 \simeq m_3$.

absolute scale unknown

$$\sum_i m_i \Big|_{\text{cosmology}} \lesssim 0.1\,\text{eV}$$

(CMB, large scale)

$$U_{PMNS} \sim \begin{pmatrix} 0.8 & 0.5 & 0.1 \\ 0.3 & 0.5 & 0.7 \\ 0.3 & 0.5 & 0.7 \end{pmatrix}$$

w/ 1 Dirac phase δ ?

+ 2 (if) Majorana $\eta_{1,2}$?

The lepton flavor could be structureless $^{\vee}$ (specially if compared w/ quarks), w/o hierarchies, random.

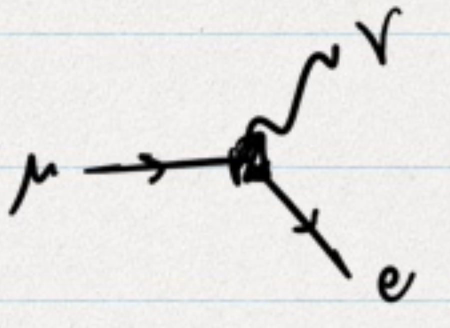
Lepton flavor violation

beyond Majorana ν masses

SMEFT also contains e.g. $\Delta L_i = \underset{\pm}{\underset{\wedge}{1}}, \ \Delta L_T = 0$

$$\frac{c^{ij}}{\Lambda^2} \bar{l}^i \sigma^a H \sigma^{\mu\nu} e^j W^a_{\mu\nu} \qquad ij = 12$$

+h.c.



$$\downarrow$$

$$\Lambda_{\mu e} \gtrsim 10^6 \,\text{GeV} \left(\frac{c^{12}}{m_\mu / v}\right)$$

Careful when BSM is too generic and/or too low scale.
↓ no suppression of c

↑ this will be a theme in this lecture.

$$\text{Note: in SM} \quad Br(\mu \to e\gamma) \sim \frac{e^2}{16\pi^2} \left| \overset{\text{PMNS}}{\underset{i \text{ sum}}{U^*_{\mu i} U_{ei}}} \ \overset{\text{GIM mechanism (see later)}}{\frac{m^2_{\nu_i} - m^2_{\nu_1}}{m_W^2}} \right|^2 < 10^{-54}$$

Baryon number

$U(1)_B$

$\psi \to e^{iB\theta}\psi$ $B(\psi = q, u, d) = 1/3$

(B=0 others)

$\downarrow$

- No proton decay: $\tau(p \to e^+\pi^0) \gtrsim 10^{34}$ years

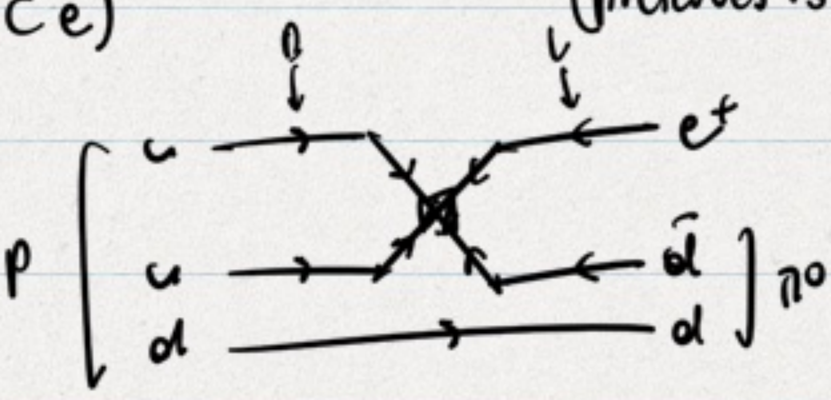
$\uparrow$ Superkamiokande

from SMEFT perspective

✓ low-energy accidental, emergent symmetry, e.g.*

$$\frac{c_{uude}}{\Lambda^2}\underset{\underset{SU(3)_c}{\uparrow}}{\epsilon^{\alpha\beta\gamma}}(u_\alpha C d_\beta)(u_\gamma C e)$$

(preserves B-L)

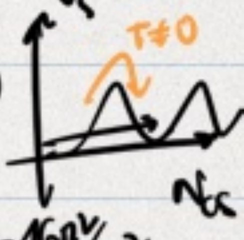


$$\text{or } \frac{c_{qude}}{\Lambda^2}\epsilon^{\alpha\beta\gamma}(q_{\alpha}\gamma^\mu C u_\beta)\overset{\overset{\epsilon}{\vee}}{}(d_\gamma \gamma_\mu C \ell)$$

$$\tau \sim 10^{34} \text{ years}\left(\frac{c_{uude}}{1}\right)\left(\frac{10^{15}\,\text{GeV}}{\Lambda}\right)^4$$

$\uparrow$ not far from ms scale

(see extra)



* $U(1)_{B+L}$ broken in the SM by non perturbative effects ≡ sphalerons

↳ anomalous (quantum level) $\partial_\mu J^\mu_B \sim W\tilde{W}$

$\tau_{T=0} \sim e^{-16\pi^2/g^2}$

Charge quantization and gauge coupling unification

this brings us to

- $U(1)_Y \to U(1)_{EM}$ charges are not necessarily quantized, yet

$$Q_p + Q_e \le 10^{-21} \quad \text{PDG 22}$$

↑ matter charge neutrality

in addition

- $g_s \sim 1$, $g \sim 0.6$, $g_Y \sim 0.3$ at $q \sim m_Z$ quite similar

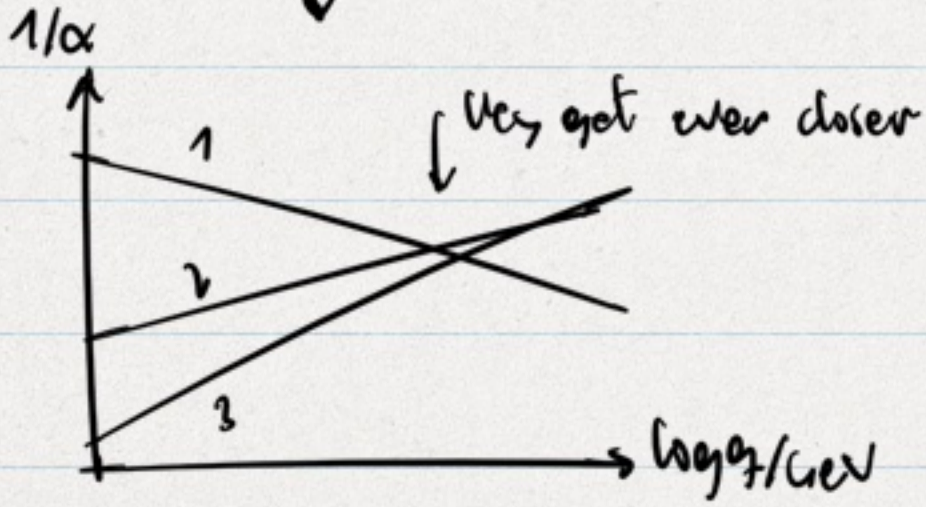
(given they could expand many orders, down)

on a matter of luck ↓ q-dependent statement

$q = m_Z$
↓
$g_1 = \sqrt{\frac{5}{3}}\, g_Y \sim 0.5$

$g_2 = g$

$g_3 = g_s$



$$\frac{d(1/\alpha_i)}{d\log q} = -\frac{b_i}{32\pi^2}$$

BSM: are there low-energy remnants of something deeper? Grand Unification Theories of gauge interactions

$Q_{EM} = Q_Y/2 + T_3$ quantized if $U(1)_Y \supset G$ non-abelian

Several options: minimal $SU(4)\times SU(2)\times SU(2)$ Pati-Salam

idea simple $SU(5)$ Glashow-Georgi

embedding $\downarrow$

fundamental rep.

$$\text{gauge } T^A = \begin{pmatrix} \begin{pmatrix} SU(3) \end{pmatrix}^{3\times3} & \\ & \begin{pmatrix} SU(2) \end{pmatrix}^{2\times2} \end{pmatrix}, \qquad Y = c \begin{pmatrix} 1/3 & & & & \\ & 1/3 & & & \\ & & 1/3 & & \\ & & & -1/2 & \\ & & & & -1/2 \end{pmatrix}$$

quantized

normalization (of T^A and g_Y)

generators $A = 1,\dots,24$ $\qquad$ X,Y gauge bosons

$$\underset{\overset{\shortparallel}{8}}{}+3+1+\underset{\overset{\shortparallel}{12}}{}$$

up to phase rotations

$$\text{fermions: } \bar{5} = \begin{pmatrix} d^c_1 \\ d^c_2 \\ d^c_3 \\ e \\ -\nu \end{pmatrix} \begin{matrix} \left.\vphantom{\begin{matrix} a\\a\\a \end{matrix}}\right\} d^c \\ \left.\vphantom{\begin{matrix} a\\a \end{matrix}}\right\} \ell \end{matrix}, \qquad 10 = \begin{pmatrix} 0 & \overbrace{u^c_3 \; -u^c_2}^{u^c} & \overbrace{-u_1 \; -d_1}^{q} \\ & 0 \quad u^c_1 & -u_2 \; -d_2 \\ & & 0 \; -u_3 \; -d_3 \\ & & 0 \quad -e^c \\ & & \qquad 0 \end{pmatrix} \frac{1}{\sqrt{2}}$$

fit perfectly $\qquad$ antisym.

$$\text{scalars: } 5 = \begin{pmatrix} \Omega \\ H \end{pmatrix} \leftarrow \text{color triplet} = (3,1)_{-1/3}$$

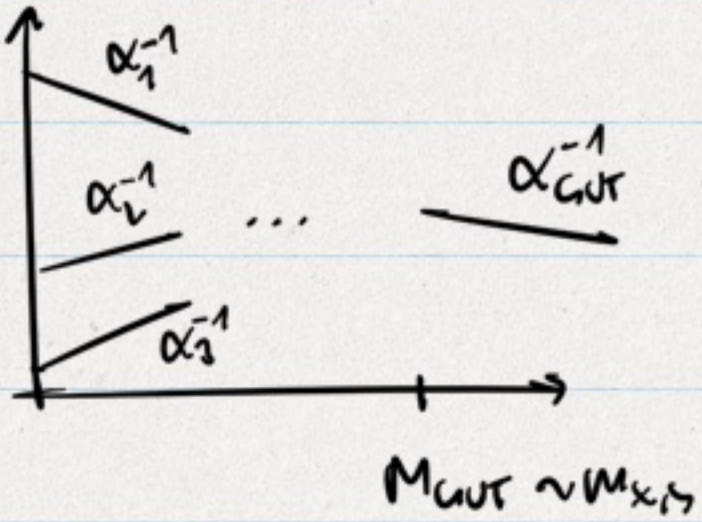
it does not work as well $\qquad$ doublet-triplet splitting problem (requires fine-tuning)

of course, we have not seen X, Y, Ω: spontaneous symmetry breaking (like SM)

$$SU(5) \xrightarrow{\Phi = 24} SU(3)\times SU(2)\times U(1) \Rightarrow m_{X,Y}, m_\Omega \neq 0$$

$\downarrow$ consequences

- gauge coupling unification



In the SM α_i's do not seem to converge well enough; $b_i = \begin{pmatrix} 41/10 \\ -19/6 \\ -7 \end{pmatrix}$

situation improves if additional particles contribute to b_i, e.g.

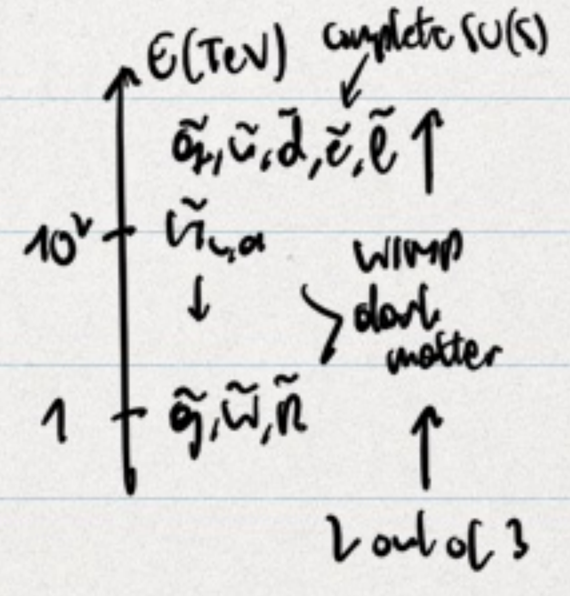
(fermionic superpartners $g, W, B, h_{u,d}$)

MSSM: $b_i = \begin{pmatrix} 66/10 \\ 1 \\ -3 \end{pmatrix}$, mainly thanks to gauginos / higgsinos

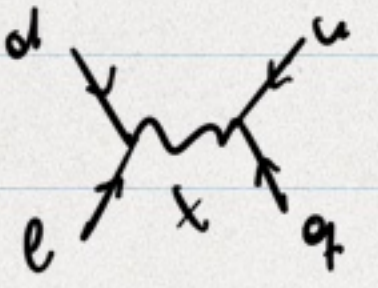
1 out of 3 phenomenological motivation for SUSY, even if split:

$$m_{\lambda_i} \sim \frac{g_i^2}{16\pi^2} m_{3/2}, \quad m_{\tilde{f}} \sim m_{3/2}$$

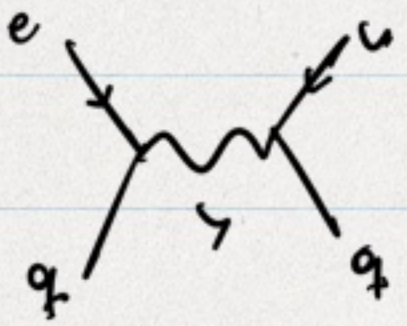
$$\mu \sim m_{3/2}$$



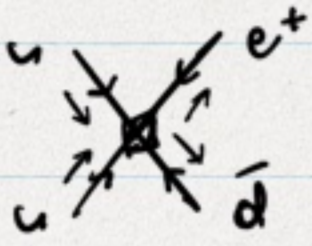
- proton decay



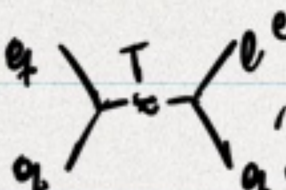
,



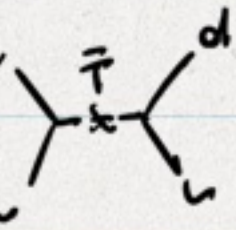
→



also the triplets



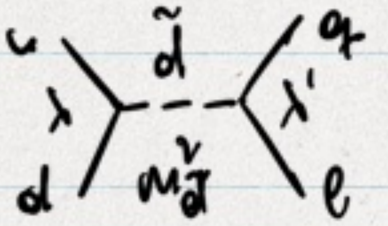
,

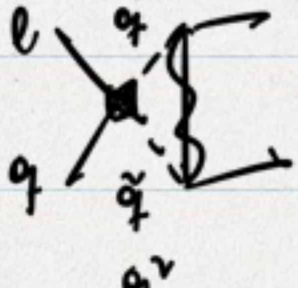


$$\frac{u}{\Lambda^2} \sim \frac{g^2_{GUT}}{M^2_{GUT}} \xrightarrow[\tau_p]{} M_{GUT} \gtrsim 3\times 10^{15}\text{ GeV}$$

already ruling out models

Note:- $M_{GUT}^{MSSM} \sim 10^{16}$ GeV, SUSY introduces new sources of proton decay





if no (matter parity) $\sim \frac{\lambda\lambda'}{m_{\tilde{d}}^2}$

$\sim \frac{g^2}{M_{GUT} M_{\tilde{q}}}$

- Yukawa coupling unification: works in MSSM for τ and b (only)

- SO(10) model includes (heavy) R-handed neutrinos via

$$16 = \bar{5} + 10 + 1$$

fits all fermions

approximate flavor symmetries

behind the $\overset{\text{magical}}{\vee}$ suppression of many flavor changing $\overset{\text{and CP violating (see later)}}{\vee}$ processes in the SM, leading to important constraints on any BSM that does not preserve at least part of that magic.

$\downarrow$ e.g.

flavor models reproducing pattern masses/mixings

(see lecture 3)

limit $Y_u, Y_d, Y_e \to 0$ $\quad \underbrace{U(3)_q \times U(3)_u \times U(3)_d}_{} \times \underbrace{U(3)_\ell \times U(3)_e}_{\text{we have already discussed}}$

leptons; focus on quarks.

for some observables (e.g. $K-\bar{K}$ mass difference)

$U(2)^3$ is enough

$$\text{Note: } U(1)^5 = U(1)_B \times U(1)_{B_A} \times U(1)_L \times U(1)_{L_A} \times U(1)_Y \qquad \overset{\downarrow}{B_A(q) = -B_A(u,d)}$$

$\Big\downarrow$ main consequence

- mass suppression of FC processes

$$R_{SU(3)_{q,u,d}}$$

$$\bar{q} Y_u \tilde{H} u + \bar{q} Y_d H d \xrightarrow{\psi \to V\psi} \begin{cases} Y_u = M_u & = (3,\bar{3},1) \\ Y_d = V_{CKM} M_d & = (3,1,\bar{3}) \\ \quad \text{or} & \\ Y_u = V^\dagger_{CKM} M_u & \\ Y_d = M_d & \end{cases}$$

if M_u or M_d are degenerate, V can be eliminated $\rotatebox{0}{$\searrow$}$

↓

Neutral currents

FCNC , tree : no

mass squared differences

GIM mechanism   loop : contribution depends on Δm_{ij}^2 in the other charged sector

↓

small K-$\bar K$ mixing   in good agreement data

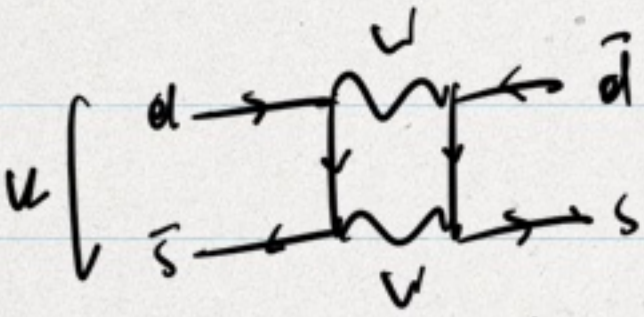
$b \to s \mu^+ \mu^-$

$b \to s \gamma$

⋮

$$\frac{\Delta m_K}{m_K} \sim \underset{\substack{\uparrow \\ \text{very small}}}{10^{-14}}$$

$\Delta S = 2$

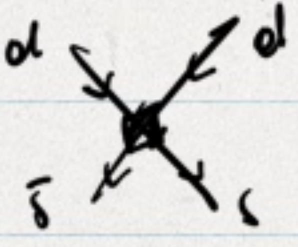


$$\overset{\text{Re}}{\sim} \frac{g^2}{16\pi^2}\frac{1}{\sqrt{2}} \underbrace{\overbrace{(V_{cd}^* V_{cs})^2}^{\sim\lambda^2} \frac{m_c^2}{m_W^2}}_{\text{GIM}} (\bar d_L \gamma_\mu s_L)^2$$

$\equiv A_{\Delta S=2}$

generic BSM

$$\frac{c_{\Delta S=2}}{\Lambda^2} (\underset{\substack{\uparrow \\ \bar q_1}}{\bar d_L} \gamma_\mu \underset{\substack{\uparrow \\ q_2}}{s_L})^2$$



$$\Lambda \gtrsim 10^6 \text{ GeV} \left(\frac{\text{Re}(c_{\Delta S=2})}{1}\right)^{1/2}$$

pattern of fermion masses and mixings

$\updownarrow$

why the magic numbers

$$M_u \sim \underset{\underset{10^2\,\text{GeV}}{\wr}}{m_t} \begin{pmatrix} x_u^4 & & \\ & x_u^2 & \\ & & 1 \end{pmatrix}, \quad M_d \sim \underset{\underset{5\,\text{GeV}}{\wr}}{m_b} \begin{pmatrix} \lambda^4 & & \\ & \lambda^2 & \\ & & 1 \end{pmatrix}$$

$$V_{CKM} \sim \underset{\underset{\text{almost diagonal}}{\uparrow}}{\begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}}, \quad \delta_{CKM} \sim 1$$

$$\underset{\lambda \sim 0.2}{\overset{\text{Cabibbo}}{\downarrow}}, \; x_u \sim 0.05 \rightarrow \text{hierarchical structure}$$

BSM: is there a dynamical, deeper explanation?

Note: Parameters that when vanishing lead to a symmetry are natural ('t Hooft criterion) b/c once fixed, they renormalize proportional to themselves. (see exercise)

(see also page 65)

This implies a parameter choice that can be accounted by selection rules, is much more plausible (given our ignorance of the UV ultimate theory) than a choice of a parameter that is not backed up by a symmetry.

This does not explain the parameter choice itself, but it helps and informs the potential explanation.

In fact, it is often the case that an approximate symmetry can be traced back to an accidental symmetry of a more fundamental description.

[Rattazzi]

The approximate symmetries of today are the accidental symmetries of tomorrow.

(but they will haunt you, BSMer, until then)

↑ your BSM theory must not destroy the approximate symmetries observed in nature

1 extra

CP symmetry

Phase(s) in the SM are not small, $\delta_{CKM} \sim 1 \xrightarrow[\text{sense}]{\text{in this}}$ CP is not a (approx.) SM symmetry

{ it seems $\delta \sim 1$ (Dirac) in neutrinos)

SM $\Big\downarrow$ magic

(very) non-trivial interplay w/ flavor:

- V_{CKM} physical up to (quark) phase rotations $V^{ij} \to e^{i(\theta^i_{u_L} - \theta^j_{d_L})} V^{ij}$

↳ - physical CP violation $\propto J_{ijkl} \equiv \mathrm{Im}[V_{ij} V^*_{kj} V_{kl} V^*_{il}]$, Jarlskog ↳ no sum

- unitarity $\sum_i V_{ij} V^*_{ik} = \delta_{jk}$

$$\hookrightarrow \begin{cases} 2 \text{ families: } J_{ijkl} = 0 \\ 3 \text{ families: } J_{ijkl} \equiv J \text{ for } i \neq k, j \neq l \end{cases}$$

$\cancel{CP}$ process ↑ must involve all

- $J = \mathrm{Im}[V_{ud} V^*_{td} V_{tb} V^*_{ub}] \sim \lambda^6 \sim 3 \times 10^{-5}$

- if any two same-charge quarks degenerate, J unphysical.

↑

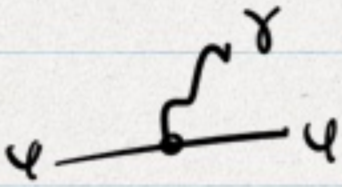
very good agreement w/ data

K-K̄ system $\epsilon_K = \frac{\mathrm{Im}(A_{\Delta S=2})}{\mathrm{Re}(A_{\Delta S=2})} \sim \frac{J}{\lambda^2} \sim 10^{-3}$

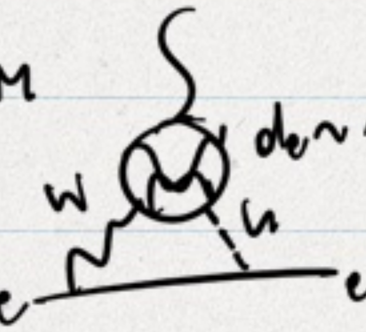
must involve interference of mixing and decay

EDM (electric dipole moments) : $d_e \lesssim 10^{-29}\, e\cdot cm$ PDG 22

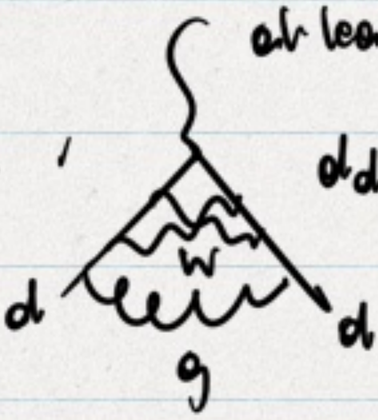
$e\,\mu\, e^{i\theta}\, \bar\psi_L i\sigma_{\mu\nu}\psi_R F^{\mu\nu} + h.c.$ $\quad d_n \lesssim 10^{-26}\, e\cdot cm$



Note: In SM



$d_e \sim 10^{-38}\, e\,cm$,



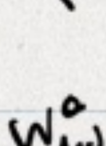
at least 2 loops (3 in practice)

$d_d \sim 10^{-34}\, e\,cm$

$\downarrow$

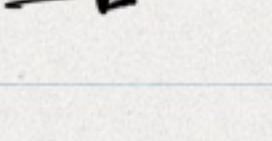
Any BSM will lead to too large CP violation if it does not implement some structure like the SM,

$\frac{C_{\Delta S=2}}{\Lambda^2}(\bar d_L \gamma^\mu s_L)^2$



$\Lambda \gtrsim 10^7\, GeV \left(\frac{\mathrm{Im}(C_{\Delta S=2})}{1}\right)^{1/2}$

$\frac{C_e}{\Lambda^2}(\bar\ell^1 \sigma^a (H) \sigma^{\mu\nu} e^1 W^a_{\mu\nu}$



$\Lambda \gtrsim 10^6\, GeV \left(\frac{\mathrm{Im}\,C_e}{m_e/v}\right)^{1/2}$

2 extra

Strong CP problem

nEDM brings us to one of the most important features of the renormalizable $\mathcal{L}_{SM}$ and the EFT point of view.

$$\Big\downarrow E \ll \Lambda$$

we forgot one term! $\quad \Delta\mathcal{L}_{SM} = \frac{\theta g_s^2}{32\pi^2} G_{\mu\nu} \overset{= \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} G_{\rho\sigma}}{\tilde{G}^{\mu\nu}}$

(there are two more)

$W\tilde{W}$, $B\tilde{B}$ irrelevant

b/c rotated away, no topology

$\Big\downarrow$ it has physical effects b/c QCD is $\overset{\text{topologically}}{\vee}$ non-trivial and it cannot be rotated away b/c $Y_{u,d}$ (break $U(1)_{B_A}$)

magic is lost $\qquad d_n \sim \frac{\bar{\theta} \overset{= m_u m_d / m_u + m_d}{\mu}}{\Lambda_{QCD}^2} \longrightarrow \bar{\theta} < 10^{-10}$

w/ the phases in the Yukawas taken into account: $\bar{\theta} = \theta + \arg\{\det(Y_u Y_d)\}$

$\uparrow$

BSM: is there an explanation of why $\bar{\theta}$ is so small?

that does not require tuning 2 (unrelated) parameters against each other?
a priori

both forbid $d_n \neq 0$

- P or CP symmetry in the UV:

This does not resonate well w/ our understanding of the SM from the EFT viewpoint.

P (maximally) broken by EW interactions

CP broken by $O(1)$ CKM phase $\delta_{CKM} = \arg\{\det[\overset{\text{commutator}}{\downarrow} Y_u Y_u^\dagger, Y_d Y_d^\dagger]\}$

It is possible to make it work, but complicated b/c we need to drop the simple, elegant structure of the SM in the UV and recover it in the IR w/ $\bar{\theta} \ll 1$, $\delta_{CKM} \sim 1$.

- QCD axion:

Make θ dynamical and let QCD solve its problem. QCD $\overset{\text{vacuum}}{\overset{\parallel}{\text{ground}}}$ state energy

↓ ↘ $E_{QCD}(\theta)$ minimum at $\theta=0$

$$\Delta\mathcal{L}_{SM} + \mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \left(\theta + \frac{a}{f_a}\right)\frac{g_s^2}{16\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu}$$

shift ↓

a/f_a, θ unphysical

indeed ↓ after anomalous $a(x)$ dependent chiral rotation

QCD chiral Lagrangian ($E < m_c \sim \Lambda_{QCD}$)

$$\mathcal{L}_\chi \supset b f_\pi^2 \Lambda_{QCD} \mathrm{Tr}[\Sigma^\dagger M_a] + h.c.$$

$$M_a = e^{\frac{iaQ_a}{2f_a}} \begin{pmatrix} m_u & \\ & m_d \end{pmatrix} e^{\frac{iaQ_a}{2f_a}}$$

↑

axion-dependent quark mass matrix

↓ $\Sigma = e^{i\Pi/f_\pi}$, $\Pi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$

$V(a)$ / $\langle a \rangle = 0$, $m_a^2 \simeq \frac{m_\pi^2 f_\pi^2}{f_a^2} \frac{m_u m_d}{(m_u+m_d)^2}$

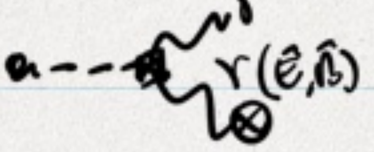
↓

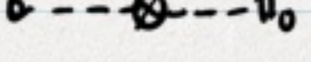
$d_n = 0$

This solution of strong CP problem seems more elegant (w.r.t. EFT ideology) and it leads to a lot of phenomenological consequences, e.g. QCD axion could be dark matter, and via $g_{a\gamma\gamma}\, a F_{\mu\nu}^2$ w/ $g_{a\gamma\gamma} \simeq \frac{\alpha^2}{2\pi f_a}\left(\underset{\uparrow\,\text{UV}}{\frac{E}{N}} - \underset{\uparrow\,\text{IR}}{\frac{2}{3}\frac{4m_d+m_u}{m_d+m_u}}\right)$

see extra

most sensitive experiments:





In practice, it relies on a $U(1)_{PQ}$ global symmetry, anomalous under QCD and spontaneously broken at a scale f_a.
(Peccei-Quinn)

E.g. KSVZ axion model,

why $U(1)_{PQ}$? → quality problem

$$\mathcal{L}_{KSVZ} = \underset{\uparrow}{\bar{Q}} i\not{D} Q + |\partial_\mu \Phi|^2 - V(|\Phi|^2) - y_Q \bar{Q}_L \Phi Q_R + h.c.$$

Dirac fermions, color charged (4-component)

$U(1)_{PQ}$: $\quad Q_L \to e^{-i\theta} Q_L, \; Q_R \to e^{+i\theta} Q_R, \; \Phi \to e^{-2i\theta}\Phi$

$V(|\Phi|^2)$ / $\quad \langle \Phi \rangle = f_a/\sqrt{2}, \; U(1)_{PQ} \to \emptyset$

$$\Phi = \frac{1}{\sqrt{2}}(f_a + \phi)\, e^{ia/f_a}, \quad m_\phi^2 \sim \lambda f_a^2, \quad m_Q \sim y_Q f_a$$

$$\text{anomalous QCD rotation} \overset{\text{e.g.}}{:} Q_L \to e^{-i\frac{a}{f_a}} Q_L \; \Big\downarrow \; E \ll m_{\phi,Q}$$

$$\mathcal{L}_a$$

1 extra

~ Custodial symmetry

approximate

main relevance in the context of BSM Higgs
↳ see later

limit $g', y_u - y_d, y_e \to 0$ $\quad SU(2)_L \times SU(2)_R \sim SO(4)$

$\underset{y_t}{\downarrow}$ $\quad \overset{\uparrow}{\text{irrelevant}}$

under which

$\updownarrow$

$\Phi \equiv (\hat{H}\; H) \to U_L \Phi U_R^\dagger$

$V(H)$ is $SO(4)$ invariant

$\uparrow_{2\times 2}$ $\quad \uparrow$ recall $= \epsilon H^*$

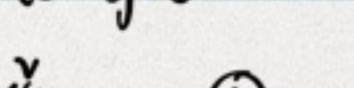
$$\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \frac{1}{\sqrt{2}} \;,\; \langle \Phi \rangle = \frac{v}{\sqrt{2}} \underset{\mathbb{1}_{2\times 2}}{\underset{\shortparallel}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}} \quad SU(2)_L \times SU(2)_R \to \underset{\text{custodial}}{\underset{\uparrow}{SU(2)_C}}$$

$$|D_\mu H|^2 = \frac{1}{2} \mathrm{Tr}\left[|D_\mu \Phi|^2 \right] \;,\; D_\mu \Phi = \partial_\mu \Phi + ig \underbrace{T_L^A W_\mu^A}_{(3,1) \to 3} \Phi - ig' \Phi \underset{O_3(H)\,\sigma_3}{\underset{\uparrow}{T_R^3}} B$$

$\updownarrow$

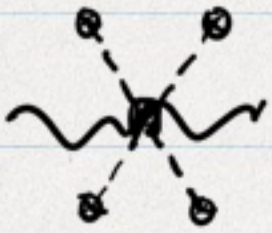
in good agreement w/ data ; $\quad \rho \equiv \frac{m_W^2}{\underbrace{m_Z^2 \cos^2\theta_W}} \simeq 1$

$\quad\quad = 1$ for $g' = 0$

Note; Also at loop level (of course) $\quad \rho - 1 \sim \frac{N_c y_t^2}{16\pi^2}$ 

generic BSM:

$$-\frac{c_T}{\Lambda^2}\left(H^\dagger \overset{\leftrightarrow}{D}_\mu H\right)^2$$



$$\begin{tikzcd} \Delta\rho \sim \frac{c_T v^2}{\Lambda^2} \arrow[d, "\Delta\rho_{BSM} \lesssim 10^{-3}"] \\ \Lambda \gtrsim 10^4\,\text{GeV}\left(\frac{c_T}{1}\right)^{1/2} \end{tikzcd}$$

extra

There exist an extension of custodial for EW $\overset{Z-}{\vee}$ fermion couplings.

$$\begin{tikzcd} SU(2)_L\times SU(2)_R\times \underset{\substack{\uparrow\\ L\leftrightarrow R}}{P_{LR}} \arrow[d, "\langle\Phi\rangle"'] \\ SU(2)_C\times P_{LR} \end{tikzcd}$$

$Z\bar\psi\psi$: $$\frac{g}{\cos\theta_W}\underbrace{\left[Q_L^3 - Q_{EM}\sin^2\theta_W\right]}_{\equiv\, g_{Z\bar\psi\psi}} Z_\mu\bar\psi\gamma^\mu\psi$$

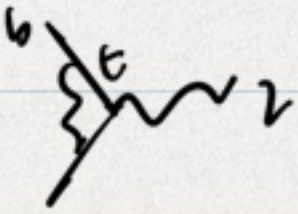
if ψ / $T_L = T_R$, $T_R^3 = T_L^3$ $\rightarrow$ $\underset{\substack{\uparrow\\ \text{(non-universal correction)}}}{\delta Q_L^3 = 0}$

↗ Note: P_{LR} is broken individually by Y_u and Y_d.
arXiv:1308.1879; Elias-Miró et al.

e.g. for $\psi = b_L$: $q \in (2,2)$ ↶

↖ non-dynamical spectator = spurion
(it must not enter in loops to have an effect)

Note: At loop level $\dfrac{g_{Z\bar b_L b_L}|_{\text{loop}}}{g_{Z\bar b_L b_L}|_{\text{tree}}} \sim \dfrac{y_t^2}{16\pi^2}$



Generic BSM : $$\frac{c_{Hq}}{\Lambda^2}(H^\dagger D_\mu H)(\bar q\gamma^\mu q) \xrightarrow[\delta g_{Z\bar b_L b_L}\,\lesssim\,10^{-3}]{} \Lambda \gtrsim 10^4\,\text{GeV}\left(\frac{c_{Hq}}{1}\right)^{1/2}$$

Lecture 3

SMEFT and power counting

Today significant activity in the study and search of the non-minimal, higher-dimensional $[O_i] \geq 4$ interactions/operators, motivated by absence of new (light) states at the LHC (or anywhere else)

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum_i O_i^{(5)} + \sum_i O_i^{(6)} + \sum_i O_i^{(7)} + \sum_i O_i^{(8)} + \dots$$

$[O_i^{(n)}] = n$

$O_i^{(5)}$: $\Delta L \neq 0$

$O_i^{(7)}$: $\Delta L \neq 0$ or $\Delta B \neq 0$

counting w/o gravity

	$n_f = 1$	2(1)	84, 76(59)	~1000
	$n_f = 3$	12	~3000, 2500	~45×10^3

12 ← not counting h.c.

2500 ← $\Delta B = 0$

While, as shown, SMEFT helps us understand the SM and organize (heavy) BSM, SMEFT is not a BSM model, but a tool. Certainly it needs a UV completion, but more importantly, as any EFT, it needs a power counting:

- It goes beyond operator dimension. $\overset{\shortparallel}{PC}$
- There is more than one "possibility." $\nearrow \frac{1}{\Lambda}, \frac{1}{16\pi^2}$
- It helps organize the EFT expansion as well as identify the most important effects in a given observable.

Units: natural (universal) $\hbar=1, c=1$.

↑ very useful b/c if aliens come and I tell them that they must wait ~1 hour until I finish my lecture they won't understand. Instead, if I tell them to wait $10^{27}/m_p$ perhaps they let me go on.

↑ $m_p c^2 \sim \text{GeV}$

↓

However, the choice $\hbar=1$ obscures how couplings enter the power counting and my estimates of observables

=

Wilson coefficients

$$T \sim \underset{c=1}{\frac{L}{}} \qquad E \sim \frac{\hbar}{T} \quad \rightarrow \quad [S] \sim [\hbar] = EL \xrightarrow{} \ \downarrow S \sim \int d^4x\, \mathcal{L} \quad [\mathcal{L}] = E/L^3$$

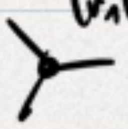
$$\rightarrow \quad [\partial] = 1/L$$

$$[\phi] = \sqrt{E/L}\,, \quad [\psi] = \sqrt{\frac{E}{L^2}}$$

$[m] = [\Lambda] = 1/L$, mass (scale), cutoff (of EFT)

$[g] = 1/\sqrt{EL}$, dimensionful

↑ trilinear



↙ also

v, i.e. ↓ decay constant ↓

$$[\langle\phi\rangle] = [f] = [M_{Pl}] = \sqrt{E/L}$$

↓

$$\Lambda \sim g f$$

loop expansion: $\left[\frac{\hbar g^2}{16\pi^2}\right] = 1$, dimensionless

$$A_{2\times2} \sim g^2 \sim e^2/\ell^2$$

Power counting of $\mathcal{L}_{SMEFT}$

s

NDA (Naive Dimensional Analysis) $\xrightarrow{}$ dimensionless variables

$$\text{BSM} \quad S = \int d^4x \sqrt{-g}\, \frac{\Lambda^4}{g_*^2}\left[\hat{\mathcal{L}}^{(0)}\left(\frac{D_\mu}{\Lambda}, \frac{g_* H}{\Lambda}, \frac{g_* \psi}{\Lambda^{3/2}}, \frac{g F_{\mu\nu}}{\Lambda^2}, \frac{R_{\mu\nu\rho\sigma}}{\Lambda^2}\right) + \dots\right]$$

- Ignorant of $O(1)$ numbers (usually c) or tunings. ↗ see extra

- Holds for strongly coupled as well $g_* \to 4\pi$ (standard NDA). †

- A different parametric dependence on couplings encodes information on the type of BSM physics (e.g. $g \to g_* > 1$, strongly coupled $\overset{\text{transverse}}{\text{gauge}}$ bosons)

  ~ "different" power counting

- We can additionally impose selection $\overset{\text{rules}}{\vee}$ (from global symmetries we assume the BSM has) by embedding the SM fields $\overset{\wedge}{\text{}}$ and couplings in (incomplete) multiplets and using them as variables. $\equiv$ spurions

- We can incorporate extra dynamical assumptions (e.g. minimally coupled BSM, Higgs as pNGB) and extend the power counting to loop level:

$$+ \dots \frac{g_*^2}{16\pi^2}\hat{\mathcal{L}}^{(1)} + \frac{g_{SM}^2}{16\pi^2}\hat{\mathcal{L}}^{(1)} + \dots$$

† mindful of the definition of low energy couplings if $g_* \lesssim g_{SM}$.

1 extra

You can check how this works w/ the examples we have already discussed.

We will be using it shortly.

The physics behind the SMEFT is in the:

- power counting: $g_H \sim 0+\left(\frac{g_{SM}^2}{16\pi^2}\right)$ or y_t or $g_s \approx 1$.
- symmetries/selection rules: $g_u u, g_c c, g_t t$ or $g_u U = \begin{pmatrix} u \\ c \\ t \end{pmatrix}$, $\frac{y_u}{g_*}$

and, underlying all of it, separation of scales: $E, m_H \ll \Lambda$

(we could not do physics otherwise)

$\uparrow$

but does this make sense?

Naturalness and EW hierarchy problem

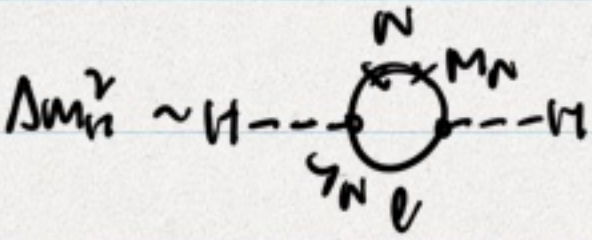
Let's reexamine the high-energy extensions of the SM we have discussed (lecture 2)

Note: I will be using NDA, but you can check the estimates are correct.

arXiv: 1303.7244
Farina et al.

- neutrino masses,

type I: $\Delta m_h^2 \sim$



$\sim \frac{y_N^2}{16\pi^2}\int dp^2 p^2 \frac{M_N^2}{p^4} \sim \frac{y_N^2}{16\pi^2} M_N^2 \log$

↑ quadratic sensitivity to BSM threshold $\Lambda \equiv M_N$

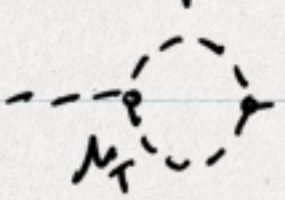
Note: in addition, since $m_\nu \sim \frac{y_N^2 v^2}{M_N}$, $\frac{\Delta m_h^2}{m_h^2} \lesssim \frac{1}{\epsilon}$ → $M_N \lesssim m_h \left(\frac{16\pi^2 v^2}{\epsilon m_\nu m_h}\right)^{1/3}$

↑ tuning factor

$\sim 10^7 \text{GeV}\, \epsilon^{-1/3}$

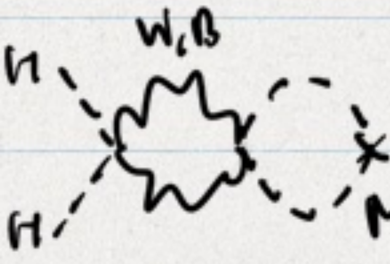
in tension w/ leptogenesis, $M_N \gtrsim 10^9$ GeV (against any other contribution to m_h^2)

type II:



$\sim \frac{\mu_T^2}{16\pi^2}\log$ $\quad m_\nu \sim \frac{y_T \mu_T v^2}{M_T^3}$

but also at 2 loops $T=(1,3)_1$,

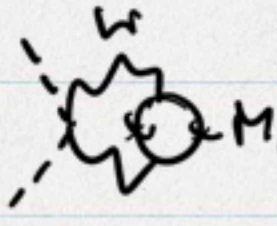


$\frac{g^4}{(16\pi^2)^2} M_T^2 \left(\int dp^2 p^2 \frac{1}{p^4}\right)^2$

$\sim \log^2$

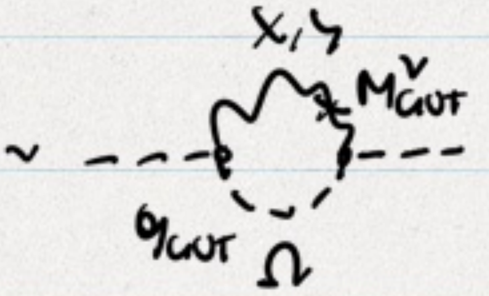
type III : $\sim$ as type I $(\times 3)$ and type II

$\nearrow$ $F = (1,3)_0$ $\downarrow$



$$\sim \frac{g^4}{(16\pi^2)^2} \int dp^2 p^2 dk^2 k^2 \frac{1}{p^4} \frac{M_F^2}{k^4}$$

- GUT's : $\Delta m_h^2 \sim$



$$\sim \frac{\overset{\nearrow g_s^2}{g_{GUT}^2}}{16\pi^2} \int dp^2 p p^2 \frac{p^2 M_{GUT}^2}{p^6}$$

- QCD axion, e.g. KSVZ : assuming $Q_{L,R} = (R,1)_0$ only color charged

at 3 loops,

$$\sim \frac{y_t^2 g_s^4}{(16\pi^2)^3} M_Q^2 (\times \log)$$

$\downarrow$

$$\Delta m_h^2 = \left(\frac{g_{SM}^2}{16\pi^2}\right)^{n = 0,1,2,3} \Lambda^2$$

depending on how secluded the BSM physics is from the Higgs

Pragmatic way to see the hierarchy problem.

It is no coincidence that it happened in all (most) the examples.

It is rooted in the basic principles of (quantum) EFT.

Any interacting* scalar is sensitive to high-energy scales ≡ EFT cutoff

unless some dynamics/symmetry changes its dimension-1 scalar nature.
until fundamental

e.g. $\lambda\phi^4$: $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 - \lambda \phi^4 + \dots$

how small can we tolerate given and the physical cutoff Λ?

λ breaks (explicitly) the shift-symmetry $\phi \to \phi + c$ that would forbid any term in $V(\phi)$

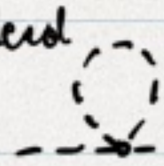
↓

unity (NDA); it could also be $1/g_*^2$

$$\Delta m^2 \sim \lambda \Lambda^2 \times 1/16\pi^2$$

spurion analysis Goldstone shift-sym. dilatation sym.

indeed



$$\sim \frac{\lambda}{16\pi^2} \int dp^2 \frac{p^2}{p^2}$$

↓

theoretical bias against interacting light scalars in (effective) QFT.

* e.g. U(1) Nambu-Goldstone boson (NGB): e.g. axion

$$\mathcal{L}_{NGB} = \mathcal{L}(\partial\phi) = \frac{1}{2}(\partial_\mu\phi)^2 + \frac{q'}{\Lambda^4}(\partial_\mu\phi)^2(\partial_\nu\phi)^2 + \dots \quad (q'>0)$$

↑ derivative interaction

↓

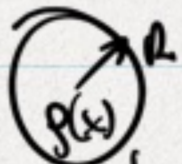
$\Delta m^2 = 0$ b/c $\phi \to \phi + c$ ($E \ll \Lambda$ free theory)

Note: We will discuss it further later on.

The same does not apply to fermion masses b/c of chiral symmetry. $\underset{\sim}{\text{extra}}$?

Note.- Before QFT and antiparticles,

classical picture of electron:



$e \equiv \int d^3x\, \rho(x)$

charge distribution

Coulomb self-energy stored ↓

$(\vec{E} \sim \frac{e}{x^2}, \mathcal{E} \sim \int d^3x\, |\vec{E}|^2)$

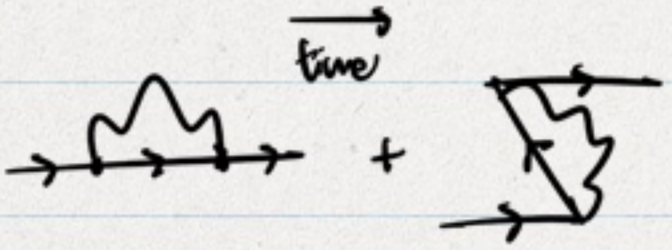
$$\mathcal{E} \sim e^2 \underbrace{\frac{1}{R}}_{\equiv \Lambda} \xrightarrow{\text{relativity}} \Delta m_e \sim \mathcal{E} \sim e^2 \Lambda$$

↓

new physics at $\Lambda \lesssim m_e \left(\frac{e}{4\pi}\right)^{2,1}$ ↑

factoring in quantum fluctuations

QFT symmetry and positron (i.e. QED):



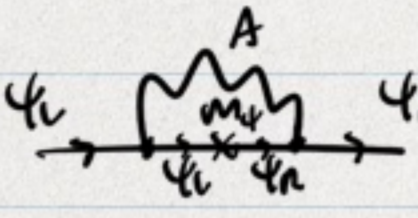
$$\Delta m_e \sim \frac{e^2}{16\pi^2}\Lambda \qquad - \frac{e^2}{16\pi^2}\Lambda = 0$$

today we understand this as the result of a chiral symmetry

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}^2 + i\bar{\psi}_L \not{D}\psi_L + i\bar{\psi}_R \not{D}\psi_R - m_\psi \bar{\psi}_L \psi - m_\psi \bar{\psi}_R \psi_L$$

↳ $\gamma^\mu D_\mu$, $D_\mu = \partial_\mu - ieA_\mu$

chiral $U(1)_A$ symmetry: $\psi_L \to e^{-i\theta}\psi_L$, $\psi_R \to e^{+i\theta}\psi_R$; $m_\psi \to m_\psi e^{+i2\theta}$, $e \to e$



$$\sim m_\psi \frac{e^2}{16\pi^2} \overbrace{\int dp^2 p^2 \frac{1}{p^4}}^{\sim \log(\Lambda/m_e)}$$

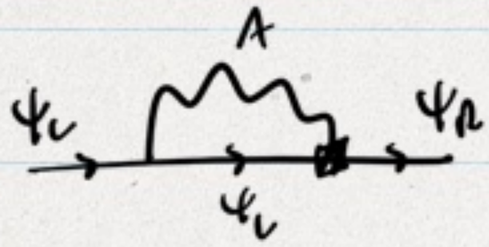
↓

$\Delta m_\psi \sim m_\psi$

Note.- We are making the assumption that the UV dynamics associated w/ the cutoff does not break explicitly the chiral symmetry, i.e. $\not{\chi}$ spurion other than m_ψ.

$$\Big\downarrow \text{ EFT perspective}$$

$$\mathcal{L}_{QED} + \underset{\substack{\uparrow \\ U(1)_A:\ g_5 \to e^{+2i\theta} g_5}}{\frac{g_5}{\Lambda}} \underset{+h.c.}{\bar{\psi}_L \sigma_{\mu\nu} \psi_R F^{\mu\nu}} + \frac{g_6}{\Lambda^2} (\bar{\psi}_L \gamma_\mu \psi_L)(\bar{\psi}_R \gamma^\mu \psi_R)$$



$$\sim \frac{g_5}{\Lambda} e \frac{1}{16\pi^2} \underbrace{\int dp^2 p^2 \frac{p}{p^3}}_{\sim \Lambda^2} \to \Delta m_\psi \sim \frac{g_5 e}{16\pi^2} \Lambda$$

$$\frac{\Delta m_\psi}{m_\psi} \lesssim \underset{\substack{\uparrow \\ \epsilon<1,\ \text{tuning}}}{\frac{1}{\epsilon}} \to g_5 \lesssim \frac{m_\psi}{\Lambda} \frac{16\pi^2}{e} \epsilon$$

If m_ψ is the only breaking of $U(1)_A$, $\Delta m_\psi \sim m_\psi$ is technical naturalness of low-energy parameter $m_\psi \ll \Lambda$.

(See also page 42)

Concerning the masslessness of gauge fields,

the explanation is much simpler:

$$2 \neq \underset{= 2 + \underset{\nwarrow \text{longitudinal pol.}}{1}}{3}$$

Coming back to the SM: $\mathcal{L}_{SM} \supset -y_t \bar{q} \tilde{H} t + h.c.$

$\downarrow$ ↳ largest shift-sym. breaking interaction

$$\Delta m_h^2 \sim (2 \div 3) \frac{y_t^2}{16\pi^2} \Lambda^2$$

← dilatation sym.

↳ shift-sym. (higher-spin sym. properly)

$|$

First we should care about being agnostic about the UV.

↳ the whole point

$\downarrow$

$$\Delta m_h^2 / m_h^2 \lesssim \frac{1}{\epsilon} \Rightarrow \Lambda \lesssim 0.5 \text{ TeV} \left(\frac{1}{\epsilon}\right)^{1/2}$$

Expectation met by all solutions based on symmetry/dynamics $\begin{cases} \text{composite Higgs} \\ \text{supersymmetry} \end{cases}$

(and behind BSM Higgs-top connection)

This sensitivity of the Higgs (m_h^2) to high-energy scales, and such a low expected Λ, endangers our deep, modern understanding of the SM (and Physics in general) by putting into question one of our key assumptions,

scale separation $m_h \sim E \ll \Lambda$?

thus the importance of understanding what is going on.

(w/o it we cannot predict)

(EW) Naturalness or Simplicity ?

$\Lambda^2 \sim m_h^2$ $\Lambda^2 \gg m_h^2$

↓ ↓

many non-minimal interactions accidental / approximate global symmetries

(higher-dim SMEFT operators)

relevant

↓ Given experimental bounds on many $\frac{c_i}{\Lambda^{n-4}} \mathcal{O}_i^{(n)}$ (lecture 2)

Cleverness

i.e. how non-minimal int.'s relevant (keeping the magic)

Note 1: Perhaps you don't care for tuning $\frac{\Delta m_h^2}{m_h^2} \sim \frac{1}{\epsilon}$, $\epsilon \ll 1$

but you should b/c ↑ today ($\Lambda \gtrsim 2$ TeV), $\epsilon \sim 10^{-2}$

a level of tuning $\epsilon \lesssim 10^{-3}$ has never been observed before. ↑

(deuteron binding energy $B_d/m_n \sim \frac{2\,\text{MeV}}{100\,\text{MeV}} \sim 10^{-3}$; related to ↓)

↕

A Higgs (scalar) field as fundamental (i.e. $m_h^2/\Lambda^2 \ll 1$) as we appear to be observing is truly new physics.

Note 2: What if no physical Λ?

Unlikely given all the open questions, and ultimately $\Lambda \lesssim 4\pi M_{Pl}$.

strong gravity scale*
(see lecture 1)

EW hierarchy problem: why (how) $\frac{v^2}{M_{Pl}^2} \sim 10^{-30}$?

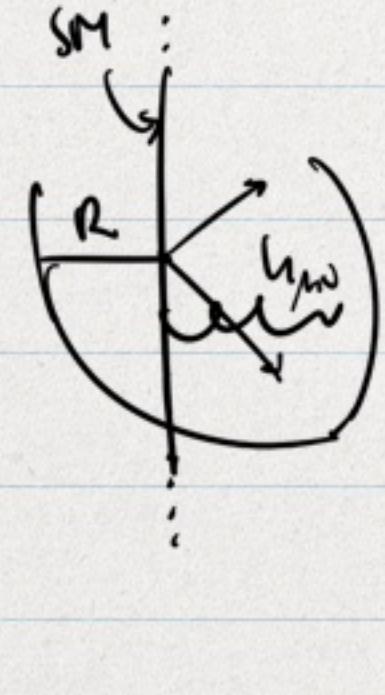
or

why are there big things in the Universe?

$$\left(\text{if } \frac{v^2}{M_{Pl}^2} \sim 1 \to \frac{\overset{\nearrow \text{Bohr radius} \sim 1/\alpha m_e}{a_0}}{\underset{\searrow \sim m_e/M_{Pl}^2}{R_S}} \sim 1 \right)$$

(and against EFT ideology, to need to understand quantum gravity to be able to understand the electroweak scale)

* Large extra-dimensions: M_{Pl} is not the fundamental gravity scale where it becomes strong, but $M_X \sim v$ is.



$$\frac{1}{M_{Pl}^2} \sim \frac{\alpha_X}{M_X^2} \quad , \quad \alpha_X \sim \frac{1}{(M_X R)^n} \ll 1$$

← only gravity propagates (it dilutes)

↓

modified newtonian gravity $r<R$, strings $M_s \sim M_X$, $BR(X \to \gamma + h) \sim (E/M_X)^{2+n}$, $\sigma_{BH}(E > M_X) \underset{\uparrow \sim R_s^2}{\sim} \left(\frac{E}{M_X}\right)^{2/1+n} \frac{1}{M_X^2}$

BSM: how to eliminate the sensitivity of the Higgs (m_h^2) to any high-energy dynamics?

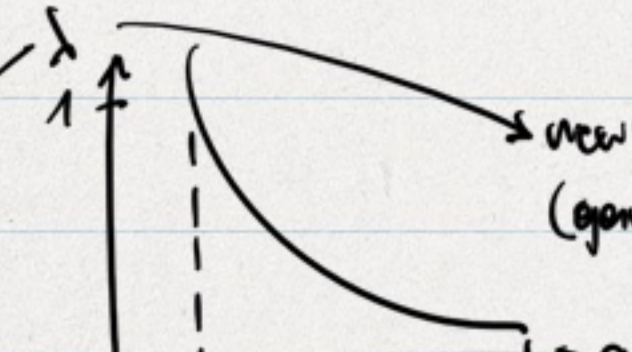
$\downarrow$ solutions

- Composite Higgs

Note: - Many versions. Explain basic common features and realization that works best (i.e. less tuning) regarding EWPTs (Precision Tests: T- (ρ) and S-parameters), direct searches, and flavor (CH also a theory of flavor b/c of top quark)

Draw analogy w/ QCD (inspiration from Nature).

1) Natural generation of hierarchy $\Lambda_{IR}/\Lambda_{UV} \ll 1$ from (approximate) conformal scale invariance of $\mathcal{L}$ and dimensional transmutation.



$\lambda \to \alpha_s$

$\lambda \to$ new IR strong force (gauge or otherwise) $\to$ e.g. SU(2N)

QCD

$-\frac{1}{4g_s^2} G_{\mu\nu}^2$ $\quad \alpha_s = g_s^2/4\pi$

$SU(3)_c$

$g_s(\Lambda_{UV}) \sim 0$ $\to$ free UV fixed point

$\beta_s(\Lambda_{UV}) \sim g_s^3 \sim 0$

CH

$\lambda \mathcal{O}$

$\lambda(\Lambda_{UV}) / \beta_\lambda(\Lambda_{UV}) \equiv \epsilon \sim 0$

free or interacting UV fixed point

$\Lambda_{IR} \to \Lambda_{QCD}$ (~GeV)

$\Lambda_{IR} \to \equiv m_* \sim$ TeV

$\Lambda_{UV} \downarrow$ e.g. $\sim (4\pi) M_{Pl}$

$$\overset{\overset{1}{\wr\wr}}{\alpha_s}(\Lambda_{IR}) \simeq \alpha_s(\Lambda_{UV})\, b_s \log\left(\frac{\Lambda_{UV}}{\Lambda_{IR}}\right)$$

$$\underset{\underset{1}{\wr\wr}}{\lambda}(\Lambda_{IR}) \simeq \lambda(\Lambda_{UV}) \left(\frac{\Lambda_{UV}}{\Lambda_{IR}}\right)^{\epsilon}$$

2) No relevant deformations or protected by symmetry. (same as QCD) $\begin{pmatrix}\psi_L^i\\ \psi_R^{c\,j}\end{pmatrix}$

QCD

$$m_q^{ij}\ \overbrace{(q_L)^i_\alpha(\bar q_R)^{\beta j}\,\delta^\alpha_\beta}\equiv \mathcal{O}^{ij}_{QCD}$$

↑ α = color, $i=1,2$ flavor (u,d)

protected by chiral symmetry

$q_L = 3$
$\bar q_R = \bar 3$ of $SU(3)_c$ complex rep.'s

CH

$$\mu\,\mathcal{O}_{CH}$$

e.g. $m_\psi^{IJ}\ \overbrace{\psi^I_\alpha\psi^J_\beta A_{\alpha\beta}}$, $A_{\alpha\beta} = -A_{\beta\alpha}$, $I=1,\dots,4$

$\psi = 2N$ of $Sp(2N)$

pseudoreal rep. ↓ e.g. $N=1$

$SU(2)$
(smaller than QCD)

No sensitivity to high-energy scales, also b/c:

$[|\mathcal{O}_{QCD}|^2]=6$

$[|\mathcal{O}_{CH}|^2]>4$

↑ compare w/ SM Higgs $[|H|^2]=2$

3) Light strongly interacting scalars as pseudo-Nambu-Goldstone Bosons (pNGBs)

↑ compared to $\Lambda_H \sim$ resonance masses (not observed so far)

global $SU(2)_L\times SU(2)_R$

$\downarrow \langle \mathcal{O}^{ij}_{QCD}\rangle = \delta^{ij}$

$SU(2)_{V=L+R}$ gauged ↓

NGBs $3 = \pm1, 0$ of $U(1)_{V=EM}$

$\pi^\pm, \pi^0$

$$\Sigma = e^{i\pi^a\sigma^a/f_\pi}$$

e.g. $G = SU(4)\cong SO(6)$

$\downarrow \langle \mathcal{O}_{CH}\rangle \overset{\text{e.g.}}{=} \begin{pmatrix}0 & 1_2\\ -1_2 & 0\end{pmatrix} \Big\| \begin{pmatrix}\vec 0_5\\ 1\end{pmatrix}$

$H = Sp(4)\cong SO(5)$ ∪

$5 = 2_{1/2}, 1_0$ of $SU(2)_L\times U(1)_Y$

$\quad\ \ \| \quad\ \ \|$
$\quad\ \ H \quad\ \ \eta$ singlet

$$\Phi = e^{i\frac{\pi^A T^A}{F}}\langle \mathcal{O}_{CH}\rangle$$

4) Non-linear kinetic terms

QCD

$$\frac{f_\pi^2}{4}|D_\mu\Sigma|^2$$

$$= \frac{1}{2}(\partial_\mu\pi^0)^2 + D_\mu\pi^+ D^\mu\pi^-$$

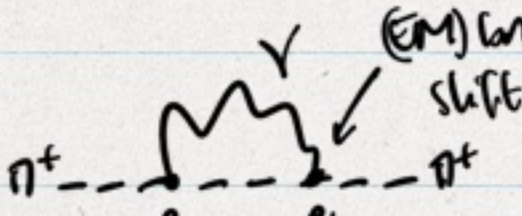
CH

$$\frac{f^2}{4}|D_\mu\Omega|^2$$

$$= \frac{1}{2}(\partial_\mu\eta)^2 + |D_\mu H|^2$$

$$+ \frac{1}{2}\frac{[\pi^{A(a)}\partial_\mu\pi^{A(a)}]^2}{f_{(a)}^2 - [\pi^{A(a)}]^2}$$

5) Naturally small(er) masses from explicit breaking of global symmetry.



(EM) breaks shift-symmetry

$$m_{\pi^\pm}^2 - m_{\pi^0}^2 \simeq \frac{3e^2}{16\pi^2}\Lambda^2 \approx (35\,\text{MeV})^2$$

exp.

nailed it

$\Lambda \sim 850$ MeV | $m_\rho \approx 775$ MeV

rho vector meson

Λ_{QCD}

and

$\gamma \to W^i, B$

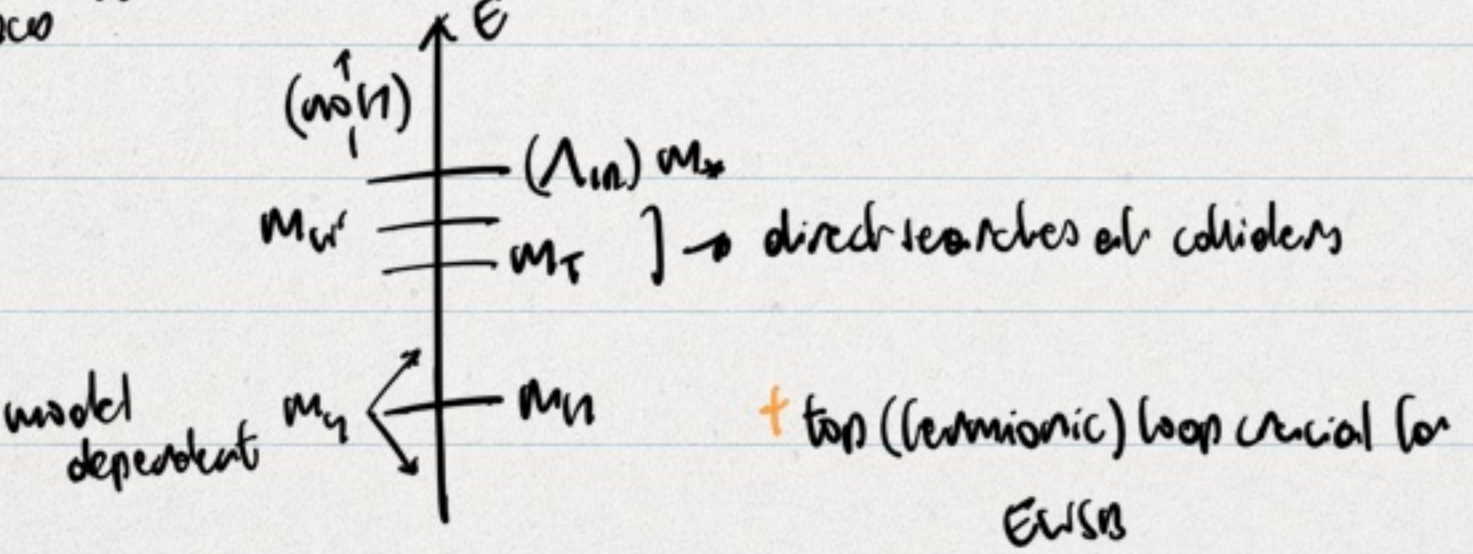
$m_{W',B'}$ vector partners

$$m_H^2 \sim \pm \frac{6y_t^2}{16\pi^2} m_T^2$$

↳ top partners

non-(LHC) discovery (+ EWPTs)

↳ tuning $\epsilon \approx 5\times10^{-2}$



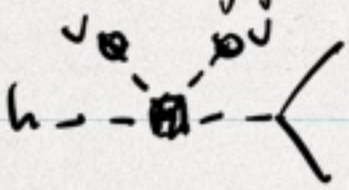
$+$ top (fermionic) loop crucial for EWSB

6) Composite pNGB-Higgs phenomenology (see also exercise)

$$\frac{c_H}{2f^2}\left(\partial_\mu |H|^2\right)^2 \quad , \quad f \equiv \frac{\overset{(\equiv \Lambda)}{m_*}}{\underset{(\equiv g_H)}{g_*}} \text{ by NDA} \quad , \quad g_* \gg g_{SM}$$

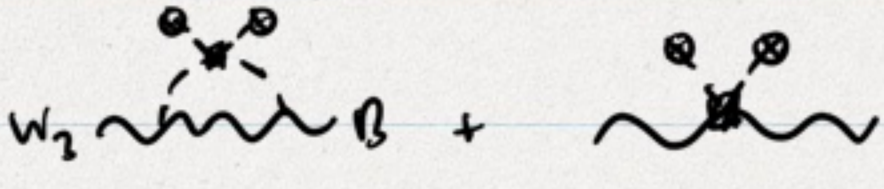
$$c_H \sim 1$$

$\downarrow$ anomalous Higgs couplings



$\delta g_h \sim \frac{v^2}{f^2}$

$\searrow$ EWPTs



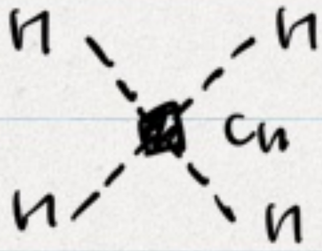
$$i\frac{c_W g}{2m_*^2}\left(H^\dagger \sigma^i \overleftrightarrow{D}^\mu H\right)\left(D^\nu W_{\mu\nu}\right)^i$$

and most genuinely (of composite dynamics),

strong Higgs scattering

$\downarrow$

W_L, Z_L, h



$$A \sim c_H \frac{E^2}{f^2} \to g_*^2$$

$$\downarrow$$

$$A_{SM} \sim g_{SM}^2$$

extra ?

CH models are $_\wedge$ also theories of flavor.
necessarily, b/c of large top Yukawa

↳ focus on quarks

7) Partial compositeness

QCD

no analog

CH $\quad \overset{\text{composite operator (difficult on la. QCD)}}{\downarrow}$

$$\lambda_i^\psi \psi^i O_\psi^i \quad \text{w/ } [O_\psi^i] = d_\psi^i$$

$$\psi = q, u, d \; \Big\downarrow$$

$$\lambda_i^\psi(\Lambda_{IR}) \simeq \lambda_i^\psi(\Lambda_{UV}) \left(\frac{\Lambda_{UV}}{\Lambda_{IR}}\right)^{d_\psi^i - 5/2}$$

Natural hierarchies generated by operators dimensions ($\underset{\text{top}}{\underset{\downarrow}{\text{relevant}}}$ / $\underset{\text{up, down, ...}}{\underset{\downarrow}{\text{irrelevant}}}$)

$$m_{u,d}^i \sim \frac{\lambda_i^q \lambda_i^{u,d}}{g_*} \frac{v}{\sqrt{2}} \; , \quad V_{CKM} \sim \begin{pmatrix} 1 & \lambda_1^q/\lambda_2^q & \lambda_1^q/\lambda_3^q \\ / & 1 & \lambda_2^q/\lambda_3^q \\ / & / & 1 \end{pmatrix}$$

protection = magic

Reproduces data $\overset{\text{also } \delta_{CKM}}{\vee}$ and preserves some degree of $\overset{\text{accidental}}{\vee}$ flavor $\underset{\text{CP}}{\wedge}$ symmetry of the SM,

e.g. $\quad \dfrac{C_{\Delta S=2}}{\underbrace{\Lambda^2}} (\bar{d}_L \gamma^\mu s_L)^2 \qquad m_* \gtrsim 4 x_t \times 10^3 \text{ GeV}$ (down from 10^6, see page 41)

$$\sim \frac{1}{f^2} \frac{(\lambda_1^q \lambda_2^q)^2}{g_*^4} \sim \frac{x_t^2}{m_*^2} y_t^2 \lambda^{10} \; , \; \text{w/ } x_t \equiv \lambda_3^q/\lambda_3^u \text{, free parameter}$$

yet not enough for $\Lambda \to m_* \sim \text{TeV} \; \forall \psi^i$ (ϵ_K, $m_* \gtrsim 10$ TeV)

better if $\Lambda \to m_*^i$ (ϵ_K, $\Delta M_{B_{d,s}}$, $m_* \gtrsim 5(x_t, 1)$ TeV ; eEDM, $m_* \gtrsim 5\sqrt{g_* x_t}$ TeV)

8) Top compositeness

$$y_t \sim \lambda_3^q \lambda_3^u / g_* \sim 1$$

one of the two (or both) must be large(ish)

e.g. $x_t \to y_t / g_* < 1$ (favored by tuning considerations)

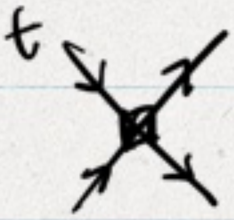
phenomenology

$$\frac{c_t}{f^2} (\bar{t} \gamma_\mu t)^2 \quad , \quad \frac{c_{Ht}}{f^2} (H^\dagger \overset{\leftrightarrow}{D}^\mu H)(\bar{t}\gamma_\mu t)$$

↓

strong top-top, Higgs scattering

↓



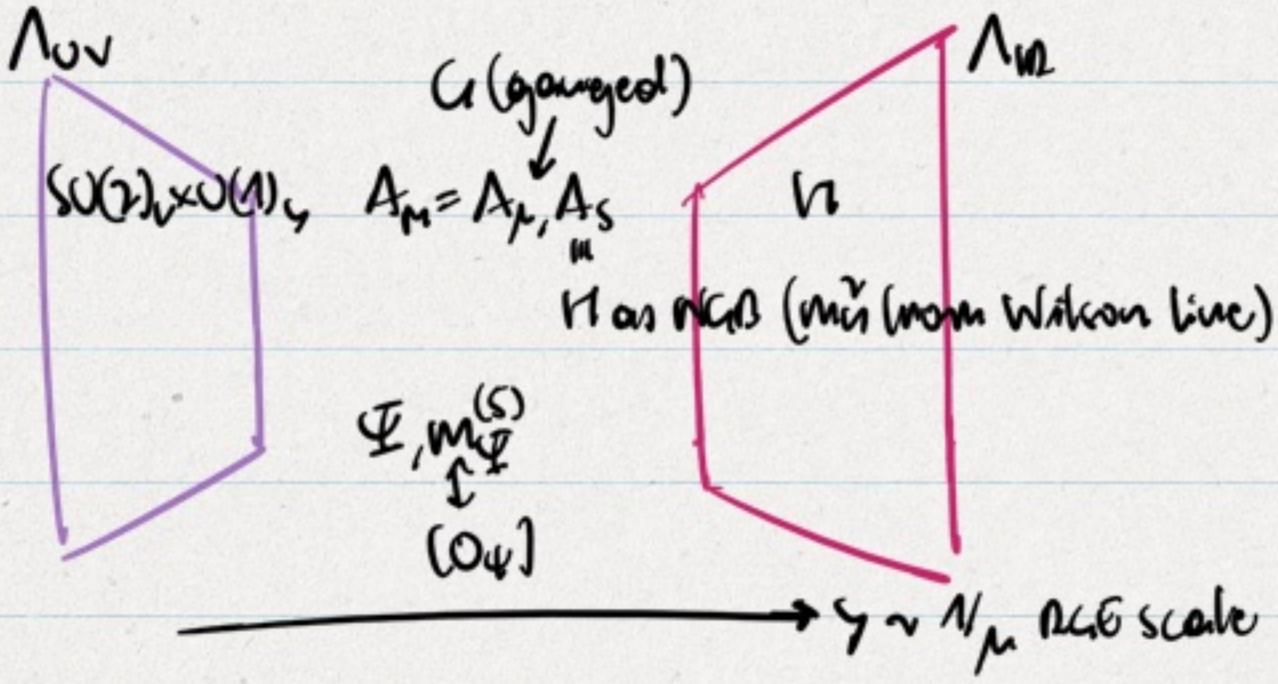
$$A \sim c_t \left(\frac{E}{f}\right)^2$$

- AdS/CFT and Randall-Sundrum (RS) braneworld

↑ ↑ Conformal Field Theory

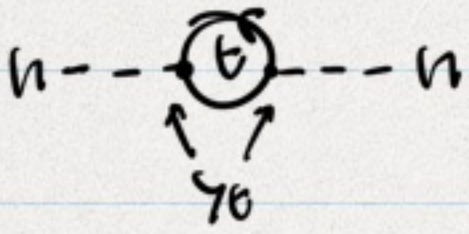
$$ds^2 = e^{-2ky} dx_4^2 - \underset{\overset{\shortparallel}{x_5}}{dy^2}$$

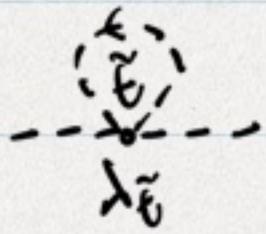
All these features can be realised in a warped extra-dimension:



- Supersymmetry

(following the positron example)





top superpartner

$$\Delta m_h^2 = \underbrace{-\frac{6y_t^2}{16\pi^2}\Lambda_{UV}^2}_{\text{fermion loop}} + \underbrace{\frac{6\lambda_{\tilde t}}{16\pi^2}\Lambda_{UV}^2}_{\text{boson}} = 0 \text{ if } \overset{\text{stop}}{\lambda_{\tilde t}} = \underbrace{y_t^2}_{\text{enforced by symmetry}}$$

≡ "super"

Note: $\mathcal{L} = (\partial_\mu \underbrace{\phi}_{\text{complex scalar}})^2 + i\bar\psi \not\partial \overset{\text{chiral fermion}}{\psi}$ is supersymmetric, i.e. invariant under

$$\phi \to \phi + \bar\xi(1-\gamma_5)\psi$$
$$\psi \to \psi + i(1-\gamma_5)\gamma^\mu \underbrace{\xi}_{\text{fermionic trans. param.}} \partial_\mu\phi$$

and

$$m_\phi^2 = m_\psi^* m_\psi \xleftrightarrow{\text{susy}} m_\psi$$

scalar mass controlled by chiral symmetry that protects fermion mass.

Note: If ψ charged, need extra $\psi' \to \phi'$ to give mass.

SM: Higgs + Higgsino

(susy requires in fact H_u, H_d to allow for y_u, y_d)

$m_h^2 \ll \Lambda_{UV}^2$ naturally

Note: Susy does not explain how the hierarchy is generated, but sets the stage for an explanation (same as fermion mass $m_\psi \ll \Lambda$)

supersymmetry is (in fact) a space-time symmetry, like Poincaré.

$$\{Q_\alpha, Q_\beta\} = -2(\gamma^\mu C)_{\alpha\beta} P_\mu \Rightarrow$$

$$[Q_\alpha, P_\mu] = 0 \Rightarrow$$

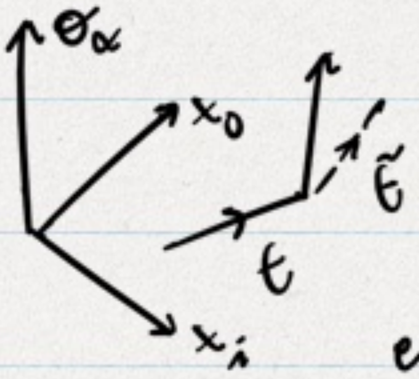
$\uparrow$ (Dirac) spinor index , i.e. Q_α has spin $\frac{1}{2}$

or

$Q_\alpha, Q^\dagger_{\dot\alpha}$, $\alpha, \dot\alpha = 1,2$

$\downarrow$

extra (quantum) dimension



spin $S \xleftrightarrow{\text{susy}} S \pm \frac{1}{2}$

$\Rightarrow m_S = m_{S\pm 1/2}$

e.g. chiral supermultiplet

$$\Phi = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y)$$

$\uparrow$ auxiliary

Since superparticles have not been observed

$\downarrow$

$$m_H^2 = |\mu|^2 + \alpha_H m_{\tilde{\xi}}^2 \quad (m_{\tilde{\xi}} \equiv \Lambda_{UV})$$

$\uparrow$ susy μ-term

sparticle masses, from spontaneous susy breaking and mediation to SM $m_{\tilde{\xi}}^2 \sim \frac{M_{susy}^{a+2}}{M^a}$

and generically,

tuning $\epsilon \lesssim 10^{-1,2}$ depending on mediation

Also as in CH models, issues, w/ absence of $\overset{\text{automatic}}{\vee}$ SM global symmetries:

- B+L violation from renormalizable $_\wedge$ spartners interactions (see page 39)
- flavor violation from non-universal $\overset{\text{scalar}}{\vee}$ spartners masses. $\hookrightarrow$ fixed by R_P

Note: R-parity, $R \equiv P_M(-1)^{2S}$ → dark matter candidate as $\tilde{W}^0, \tilde{B}, \tilde{h}$

↑ $\psi \to -\psi$, $= Q, U, D, L, E$ ↓R $-\tilde{W}^0, \tilde{B}, \tilde{h}$

$H_{(u,d)} \to + H_{(u,d)}$

and unification, even if $m_{\tilde{f}} \sim 100$ TeV (split susy)

(see page 38)

↑ if Nature really prefers simplicity over cleverness

$$\Rightarrow \quad H = P_0 = \frac{1}{4}\left(Q_1 Q_1^\dagger + Q_1^\dagger Q_1 + Q_2 Q_2^\dagger + Q_2^\dagger Q_2\right) \geq 0$$

↓

susy vacuum $|0\rangle$ / $Q_\alpha|0\rangle = 0 \to \langle 0|H|0\rangle = 0$

susy " / $Q_\alpha|0\rangle \neq 0 \to \langle 0|H|0\rangle \neq 0$

↑

vacuum energy is order parameter† for susy breaking

↓

supersymmetry could have solved the cosmological constant problem

† also $\langle S\rangle$ / $Q_\alpha\langle S\rangle = 0$

e.g. $\langle \Phi\rangle = \theta^2 F$

(also $\langle V\rangle = \theta^2\bar{\theta}^2 D$)

$m_{3/2} \sim F/M_{Pl}$

Naturalness and Cosmological constant problem

Similar to the EW hierarchy problem, but much worse from the point of view of EFT expectations and plausible solutions.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = g_{\mu\nu} \frac{\Lambda_{CC}}{M_{Pl}^2}$$

$$\downarrow$$

space-time curvature $L \sim \frac{M_{Pl}}{\sqrt{\Lambda_{CC}}}$ or

expansion at a rate $\underset{H \equiv}{\frac{\dot{a}}{a}} \sim \frac{\sqrt{\Lambda_{CC}}}{M_{Pl}}$

In our universe, $H \sim \frac{1}{L} \sim H_0 \sim \frac{1}{\underset{\text{age of universe}}{10^{10} y}} \sim \frac{1}{\underset{\text{size of universe}}{10^{28} cm}} \longrightarrow \Lambda_{CC}^{obs} \sim (10^{-3} eV)^4$

$$\Delta\Lambda_{CC} \sim c_0 \Lambda^4 + c_2 m^2 \Lambda^2 + c_4 m^4 + \underset{\substack{\uparrow \\ \text{EWSB}}}{v^2 m_h^2} + \Lambda_{QCD}^4$$

ⓜ $c_i \sim \frac{1}{16\pi^2}$

ultimate cutoff $\Lambda \sim 4\pi M_{Pl}$, why $\frac{\Lambda_{CC}^{obs}}{16\pi^2 M_{Pl}^4} \sim 10^{-120}$?

or

why is there a big $\overset{\text{(non-empty)}}{\text{(old)}}$ Universe?

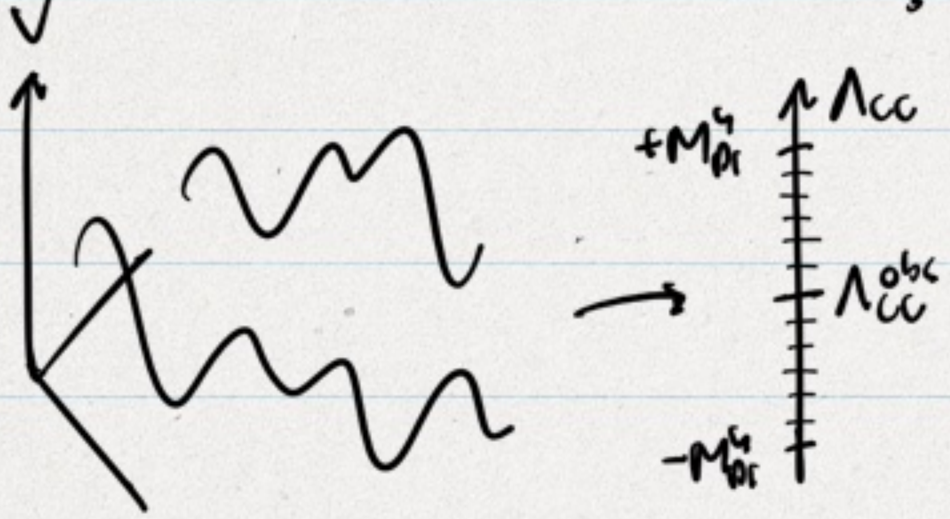
(plausible argument/explanation)

- Landscape and "Anthropic" selection
  principle

(Multiverse)

from Weinger's argument (Λ_{cc} prediction):
(start and end)

if $\Lambda_{cc} \gtrsim 10^2 \underset{\Lambda_{cc}^{obs}/3}{\underset{\wr}{\rho_{M_0}}} \rightarrow$ no structure formation
(~empty universe)



e.g.
~ 10^{120} vacua (equally distributed)

and

Mechanism to populate them, e.g.
(eternal) inflation
↳ measure problem
dS vacua?

We (obviously) find ourselves in non-empty universe.

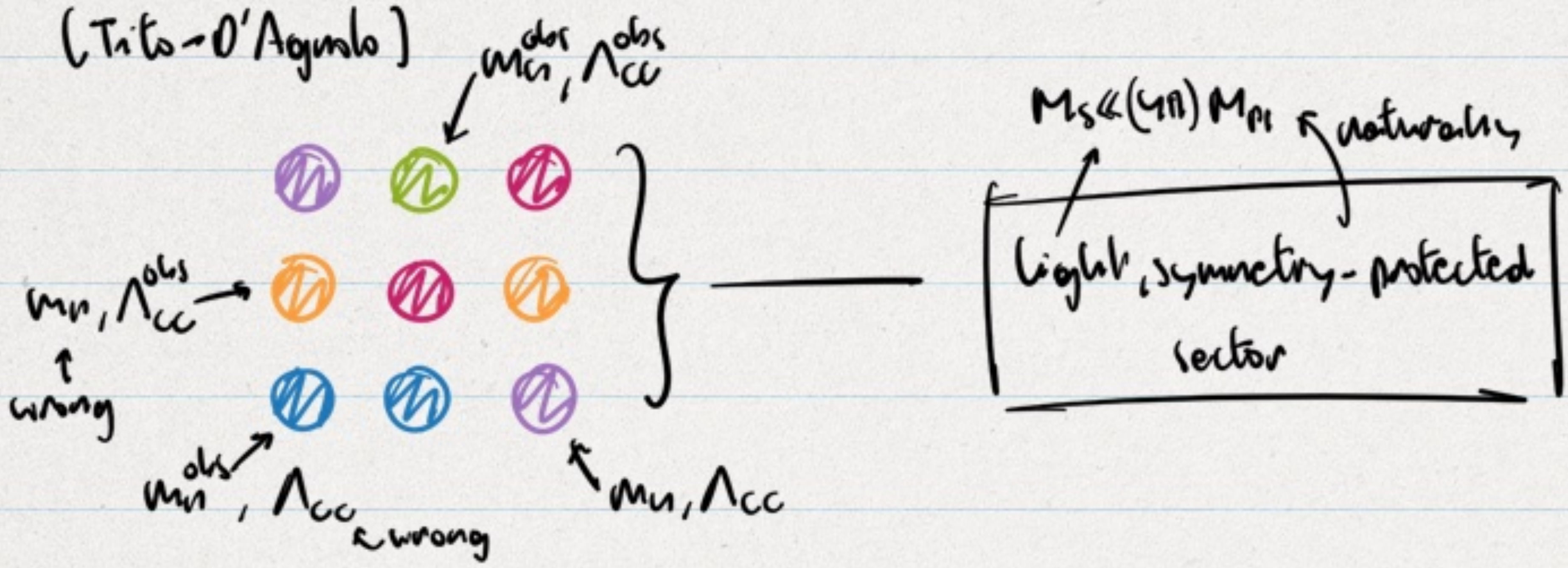
If Λ_{cc} "anthropic", could m_h^2 be so well? Atomic principle

if $v \gtrsim 1.2\, v^{obs} \rightarrow$ deuteron unbounded (see page 68)

$v \gtrsim 5\, v^{obs} \rightarrow$ neutron no longer stable in nuclei recent progress : friendly landscapes and the EW hierarchy

↳ ordered / structured / controllable

(Tito-D'Agnolo)

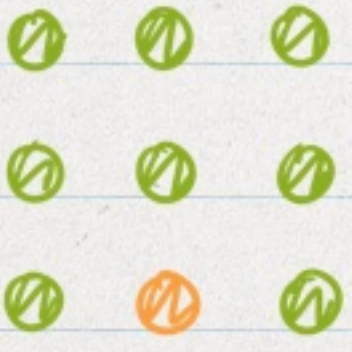


SM landscape

cosmological time

selection,

statistical



correct vacuum amplification

arXiv: 1809.07338
2105.08617
...

anthropic

† Weinberg's observers (Λ_{cc} in all others)

1609.06320
1907.05370
2012.04652
...

dynamical

only one vacuum is populated

1504.07551
1607.06821
2106.04591
...

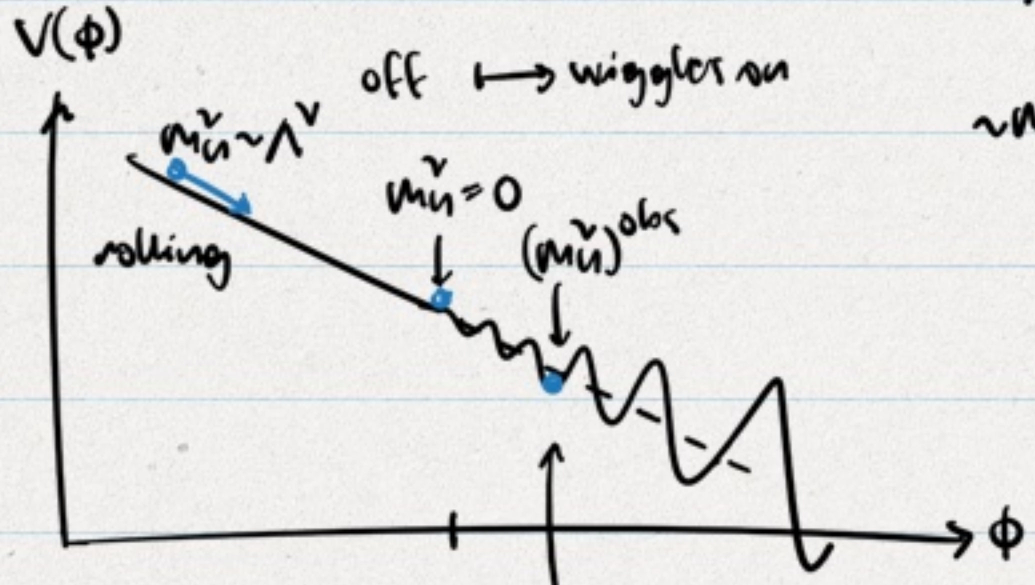
Relaxation of EW scale

$$\underset{\substack{\uparrow \\ \text{relaxion}}}{V(\phi)} = \underset{\substack{\uparrow \\ \text{rolling}}}{-g\Lambda^3\phi} + \underbrace{(\Lambda^2 - g'\Lambda\phi)|H|^2}_{m_H^2(\phi)} + \frac{\phi}{f}\underset{\substack{\uparrow \\ \text{QCD' w/ quarks'}}}{G'_{\mu\nu}\tilde{G}'^{\mu\nu}}$$

$$\Big\downarrow \Lambda_{QCD'}$$

$$\sim m_q(|H|^2)\Lambda'^3_{QCD}(1-\cos\phi/f)$$

backreaction

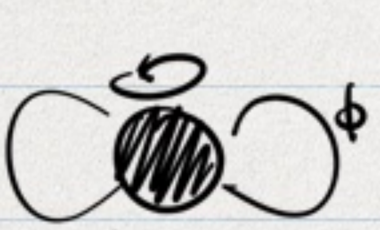


φ stops b/c Hubble friction during inflation

All of these landscape scenarios lead to model dependent, but falsifiable, experimental signals, associated w/ the fields of the light symmetric sector.

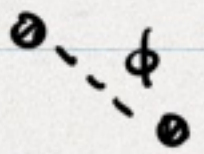
E.g.

very different from CH, SUSY



black hole superradiance

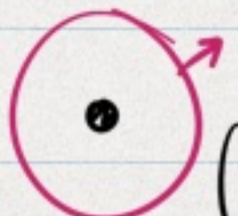
stellar cooling



5th-forces

(ultra) light dark matter

CMB spectral distorsions



phase transitions

(seeded by stars / early universe)

conclusion

BSM: understanding of what is behind the SM and what might be beyond.

Note: - no BSM works perfectly

and most importantly

no BSM has been "discovered"

↓

the field is wide open

for your crucial contribution