BSM exercises

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Ex. 1 Weinberg's soft theorems. Derive charge conservation by considering the emission of a soft photon from an arbitrary process $\alpha \to \beta$. Extend your derivation to soft gravitons and massless spin-3 fields.

Ex. 2 Construct the EFT of a single, real, massless scalar, ϕ, directly at the level of the $\phi\phi \to \phi\phi$ scattering amplitude (at tree level). Hint: Use crossing symmetry.
Exchange at tree level a minimally coupled heavy scalar Φ, $g\phi^2\Phi$, and match to the EFT amplitude.

Ex. 3 Compute the β-function coefficient of hypercharge in the SM. Fix its Landau pole by extending the gauge group to Pati-Salam's (and adding a right-handed neutrino ν).

Ex. 4 Identify the 1-loop diagrams that contribute to β_λ in the SM. Find a simple way to (potentially) avoid $\lambda(q^2) = 0$ (see Ex. 2).

Ex. 5 Identify the masses of the particles exchanged in the tree-level stringy Virasoro-Shapiro amplitude.

Ex. 6 Obtain the Weinberg operator from the 3 types of tree-level UV completions (for a single neutrino flavor).

Ex. 7 Write down the most general potential at dimension smaller or equal than 4 for the $SU(5)$ scalars $\Phi = \mathbf{24}$ (adjoint) and $S = \mathbf{5}$ (fundamental).

Ex. 8 Show that parity is a symmetry of renormalizable QED. Find a dimension-6 operator that violates it.

Ex. 9 Given the following Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + i\bar{\nu}_1 \not{\partial}_\mu \nu_1 + i\bar{\nu}_2 \not{\partial}_\mu \nu_2 + \phi \left(g_{11}\nu_1^T C \nu_1 + g_{12}\nu_1^T C \nu_2 + g_{22}\nu_2^T C \nu_2 \right) . \quad (1)$$

(where ν's are Weyl fermions in Dirac notation), estimate how small g_{11} naturalness permits given

g_{12} and g_{22}. Hint: Use a spurion analysis based on the $U(1)$ symmetries of $\mathcal{L}$.

Ex. 10 Given the following Lagrangian (renormalizable QED)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + i\bar{\psi}_L \not{D}\psi_L + i\bar{\psi}_R \not{D}\psi_R - m_\psi(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L) \tag{2}$$

(where $\not{D} = \gamma^\mu D_\mu$ and $D_\mu = \partial_\mu - ieA_\mu$), show that m_ψ renormalizes proportional to itself. Extend the Lagrangian with the dipole interaction

$$\frac{g_5}{\Lambda}\bar{\psi}_L\sigma_{\mu\nu}\psi_R F^{\mu\nu} + h.c. \tag{3}$$

and estimate its expected contribution to m_ψ. Hint: Use a spurion analysis based on the $U(1)$ chiral symmetry of $\mathcal{L}$.

Ex. 11 Derive the mass of the QCD axion in 2-flavor QCD.

Ex. 12 Show the Higgs potential is $SO(4)$ symmetric. Compute the contribution of the dimension-6 operator $-(c_T/\Lambda^2)(H^\dagger \overleftrightarrow{D}_\mu H)^2$ to the ρ-parameter, where $H^\dagger \overleftrightarrow{D}_\mu H = H^\dagger D_\mu H - (D_\mu H^\dagger)H$.

Ex. 13 Estimate the contribution to the Higgs (squared) mass (parameter) from the type II and III neutrino mass models. Extract an upper bound on the masses of the new particles from the requirement of no fine-tuning of the Higgs mass.

Ex. 14 Estimate using NDA the size of the coefficients of the dimension-6 operators $(\partial_\mu|H|^2)^2$, $(H^\dagger \overleftrightarrow{D}_\mu H)(\bar{t}\gamma^\mu t)$ and $(\bar{t}\gamma_\mu t)^2$ in models of a composite-Higgs and (right-handed) top quark. Extract their main phenomenological implications at colliders.

Ex. 15 Show that the free Lagrangian of a massless complex scalar and massless Weyl fermion is supersymmetric.

Ex. 16 Count the number of real scalars, each with two non-degenerate minima of the potential, necessary to be able to find a minimum with a value of the cosmological constant of $(10^{-3}\text{eV})^4$, as a function of the potential energy difference between the (two) minima and a preset contribution to the vacuum energy (from another sector).