Astroparticle physics

Using fundamental physics to learn about astrophysics or the universe as a whole…

…using astro/cosmo phenomena unaccounted for within current theories to learn about what lies beyond standard model(s)

Pasquale Dario Serpico (Annecy, France)
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Plan of the lectures

- I. Intro: Why should we look for fundamental physics in (high-energy) astrophysics?
- II. Basic facts about cosmic rays & their environments

- **III. Phase space approach to CR dynamics**
- **IV. Basics on CR acceleration & the SNR 'paradigm'**
- V. 'Multimessenger' approach: photons, neutrinos, secondaries (some notions on collisional aspects; relevant pheno)

Recap of previous lecture

- As in several astrophysical and cosmological contexts, the conditions probed in high-energy astrophysics phenomena are 'unusual' for terrestrial standards and, can lead to unveil or to characterise them new phenomena.

- CRs have been used as a 'poor particle physicist's accelerator' in the past. Yet the problem of the origin of their *primary component* acceleration and propagation/evolution remains open. Difficult due to ~isotropy (magnetic deflections) and ~featureless spectra.

- The high-*E*/low-flux component must be studied indirectly, via shower properties.

- Direct measurements (low-*E*, ~large fluxes) reveal ~featureless spectrum and peculiar composition patterns.

- We now aim at tackling the CR problem *'directly', i.e.* compare CR observed at Earth with model predictions accounting for production and propagation of CRs

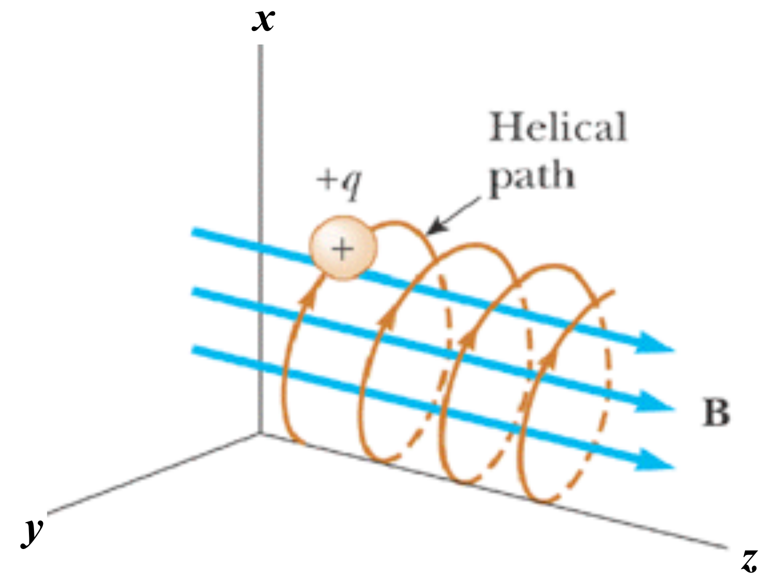
How do CR propagate? (Heuristic approach)

How do cosmic rays propagate?

ISM conductivity very high: large-scale, stationary *E*-fields → 0

In a constant **B**-field, a particle of charge *q* and momentum **p** follows a helical trajectory. EoM

$$\frac{d\mathbf{p}}{dt} = \frac{q}{c}\mathbf{v} \times \mathbf{B} \qquad \mathbf{p} = m\gamma\mathbf{v}$$



$$p = const. \qquad v = const.$$

$$p_z \equiv \mathbf{p}\cdot\mathbf{B} = const \qquad \mu \equiv p_z/p = const.$$ (Pitch angle)

$$\mu = \hat{\mathbf{B}}\cdot\hat{\mathbf{p}} = \cos\theta$$

$$\begin{aligned} v_z &= \text{const.} \\ v_x &= v_\perp^0 \cos(\Omega\, t) \\ v_y &= v_\perp^0 \sin(\Omega\, t) \end{aligned}$$

$$z = z_0 + v_z t$$

$$x = x_0 + \frac{v_\perp^0}{\Omega}\sin(\Omega t)$$

$$y = y_0 - \frac{v_\perp^0}{\Omega}\cos(\Omega t)$$

Let's have a look at the quantities entering the solution

Characteristic timescales and lengthscales

Non-relativistic case

Larmor or gyro-radius $$r_g \equiv \frac{v_\perp}{\omega_g}$$

angular or cyclotron (gyro)frequency $$\omega_g \equiv \frac{q\,B_0}{m}$$

(gyro)frequency $$\nu_g = \frac{\omega_g}{2\pi} \equiv \frac{q\,B_0}{2\pi m} = 2.8\,\mathrm{Hz}\,Z\left(\frac{m_e}{m}\right)\left(\frac{B_0}{\mu\mathrm{G}}\right)$$

Relativistic case

$$r_L = \gamma r_g = \sqrt{1-\mu^2}\,\frac{\mathcal{R}}{B_0} \simeq 10^{-6}\sqrt{1-\mu^2}\,\frac{\mathcal{R}}{\mathrm{GV}}\,\frac{\mu\mathrm{G}}{B_0}\,\mathrm{pc}\,,$$

Rigidity (p/q)

$$\Omega = \frac{\omega_g}{\gamma} = \frac{q\,B_0}{E} \simeq 10^{-2}\,Z\,\frac{B_0}{\mu G}\,\frac{\mathrm{GeV}}{E}\,\mathrm{rad/s}$$

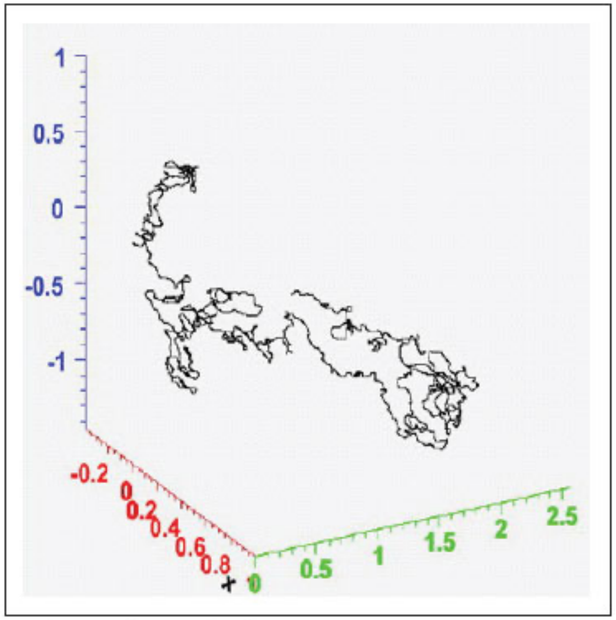
time and spatial scales of this movement are very small for astrophysics standards!
E.g. for protons, r_L ~ distance between neighbouring stars at ~PeV; r_L ~Galactic size @ ~EeV=10^{18} EeV.

Expectations

The *B*-field far from ~uniform at those scales!

CRs probe very local field, affected by "small-scale inhomogeneities"

→ changing direction by what appear "random kicks", similar to brownian motion



Simulated trajectory in a realistic B-field

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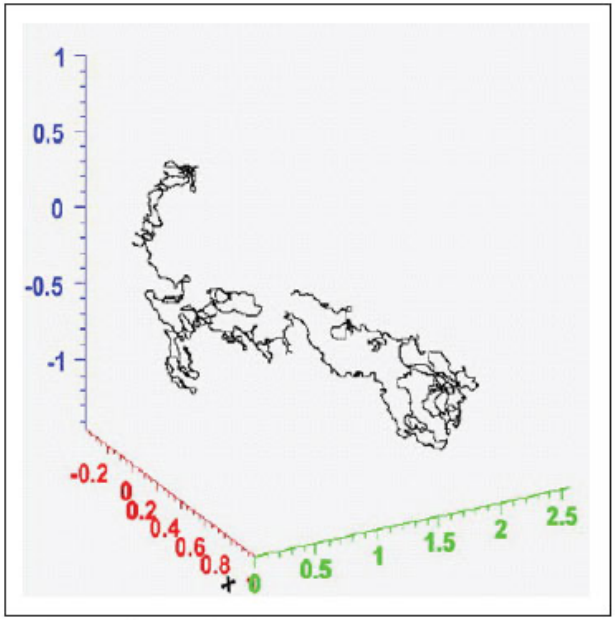
Macroscopically, we expect this movement be described as ***diffusion***

$$\frac{\partial \Phi}{\partial t} + \nabla \cdot \mathbf{J}_\Phi = Q$$ *Continuity Equation*

$$\mathbf{J}_\Phi = -D(\mathbf{x}, \Phi, E \ldots) \nabla \Phi$$ *Fick's law*



Simulated trajectory in a realistic B-field

Let's get a closer look at how this emerges and some of its properties

Adding a perturbation

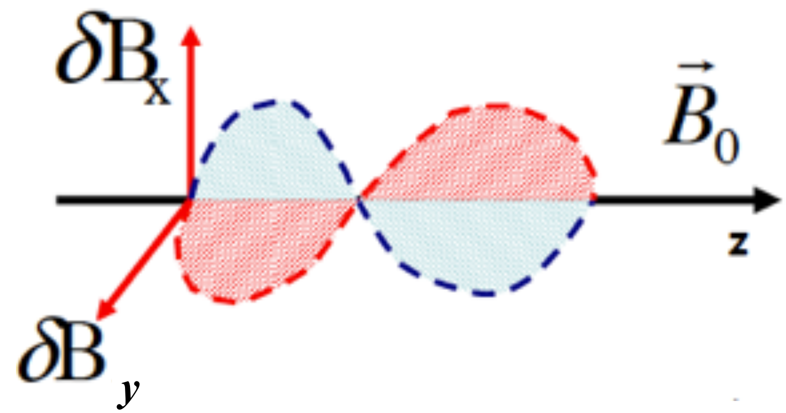
Adding **small**, **stochastic**, **static** perturbations to the B-field, orthogonal to its regular value:

$$|\delta \mathbf{B}| << |\mathbf{B}_0| \,, \quad \delta \mathbf{B} \perp \mathbf{B}_0$$

$$\frac{d\mathbf{p}}{dt} = \frac{q\,\mathbf{v}}{c} \times (\mathbf{B}_0 + \delta \mathbf{B})$$

NB: perturbations chosen so that only p_z evolution non-trivial; previous solution for the *x*-*y* components of ***p*** still valid in a "perturbation theory" spirit

$$p_z = p\,\mu$$



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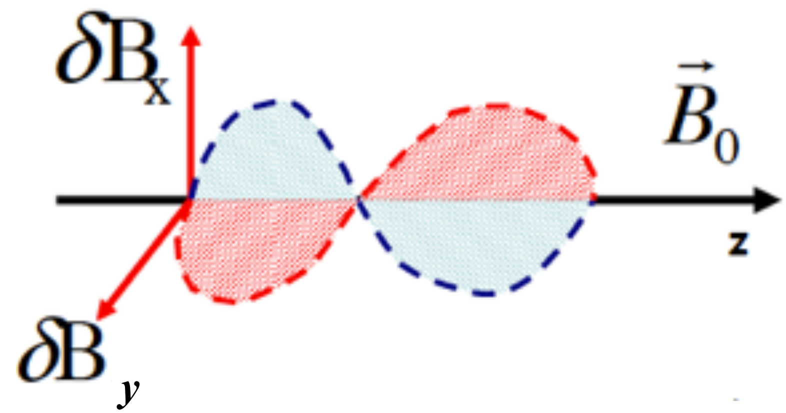


Averaging over fluctuations with random phase ψ, one can prove it shows diffusive features

Zero average
$$\left\langle \frac{d\mu}{dt} \right\rangle_\psi = 0$$

resonant variance, linear in time
$$\frac{d\left\langle \Delta\mu^2 \right\rangle_\psi}{dt} \to \pi C^2 \delta(w) = \pi\left(1-\mu^2\right)\Omega \frac{|\delta\mathbf{B}|^2}{B_0^2} k_{res}\,\delta\left(k - k_{res}\right)$$

$$\Delta t \gg \Omega^{-1}$$

gyrophase average

$$k_{res} \equiv \frac{\Omega}{v\mu}$$

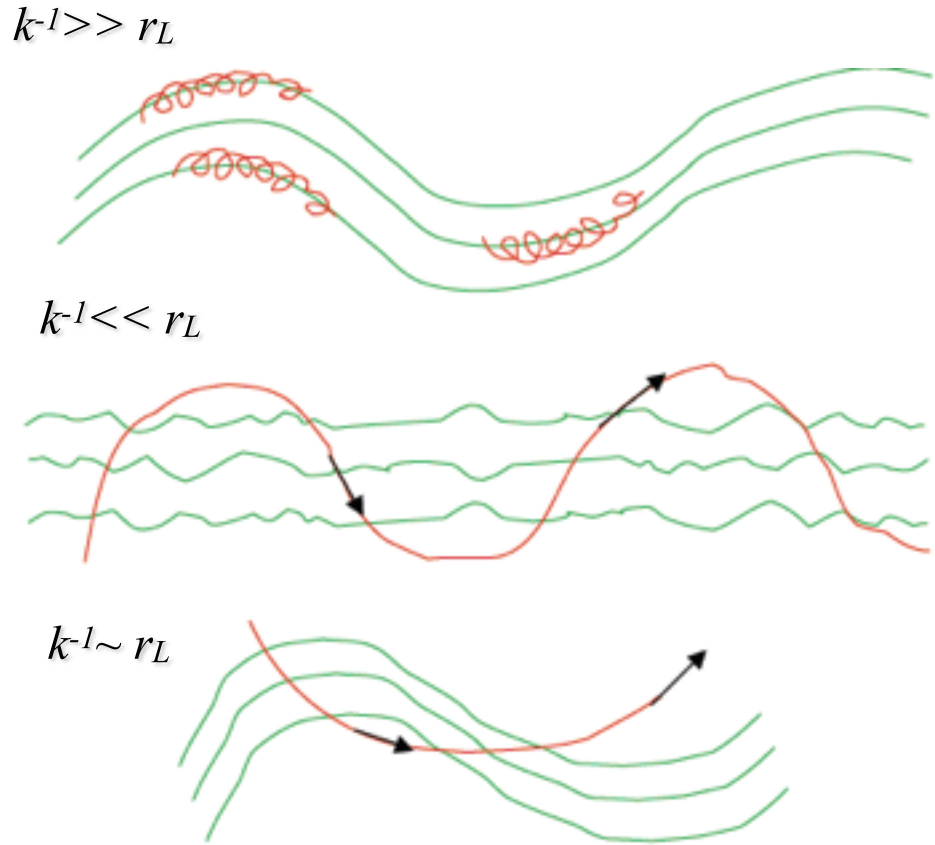
Pictorial intuition

The resonance condition tells us that:

if $k^{-1} \gg r_L$ the CRs surf adiabatically the perturbation,

if $k^{-1} \ll r_L$ the CRs hardly feel their presence

Each time a resonance occurs, the CR changes pitch angle by δB/B with random sign



The momentum-dependence of the diffusion depends on how large field fluctuations are at different scales (their "power spectrum")

Characteristic angle-diffusion frequency

$$\nu_{\theta\theta}(k_{\rm res}) \sim \Omega \left(\frac{\delta B}{B_0}\right)^2 (k_{\rm res})$$

Pitch angle diffusion coefficient

$$D_{\mu\mu}(k) = (1-\mu^2)\nu_{\theta\theta} \simeq (1-\mu^2)\Omega \frac{1}{B_0^2}\int dx e^{ikx}\delta B^2(x)$$

III. (Sketch of) formal approach in phase space

Sketch of formal approach in phase space

For a **hamiltonian system only evolving under externally assigned force fields**, all information is encoded in the one-particle distribution function for species α, f_α

Phase space density (relativistic invariant)

$$f_\alpha = \frac{\mathrm{d}N_\alpha}{\mathrm{d}\Pi}$$

$$f = f(t, \mathbf{x}, \mathbf{p}),$$
$$\mathrm{d}\Pi \equiv \mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{p}$$

obeying the collisionless Boltzmann eq.
(also improperly called Liouville eq.)

$$\left[\frac{\partial}{\partial t} + \dot{\mathbf{x}} \cdot \nabla_{\mathbf{x}} + \dot{\mathbf{p}} \cdot \nabla_{\mathbf{p}}\right] f = 0$$

If 2-body interactions, decays, particle creation/annihilation processes, etc. added, then RHS ≠ 0 (Collisional term, or Boltzmann equation)

The non-thermal nature of these particles depends on the collisional term being sub-leading in their dynamics

Useful quantities used in CR description

Since close to isotropy, useful to describe the CR in terms of moments with respect to the angular distribution

$$f = \phi + 3\hat{\mathbf{p}} \cdot \mathbf{\Phi} + \dots \qquad \mathrm{d}^3\mathbf{p} = p^2 \mathrm{d}p \mathrm{d}\Omega$$

$$\phi(t, \mathbf{x}, p) \equiv \frac{1}{4\pi} \int \mathrm{d}\Omega f(t, \mathbf{x}, \mathbf{p})$$

(Isotropic) Flux [Monopole]

$$\mathbf{\Phi}(t, \mathbf{x}, p) \equiv \frac{1}{4\pi} \int \mathrm{d}\Omega\, \hat{\mathbf{p}}\, f(t, \mathbf{x}, \mathbf{p})$$

"Current" [Dipole]

$$(\mathbf{j} = \beta \mathbf{\Phi})$$

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Auxiliary quantities (link to observations)

$$\mathrm{d}^3\mathbf{x} = \beta\mathrm{d}t\mathrm{d}A_\perp$$

spectral intensity

$$F(t,\mathbf{x},E,\Omega) = \frac{\mathrm{d}N}{\mathrm{d}t\mathrm{d}A_\perp\mathrm{d}E\mathrm{d}\Omega} = \frac{f\mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{p}}{\mathrm{d}t\mathrm{d}A_\perp\mathrm{d}E\mathrm{d}\Omega} = \beta p^2\frac{\mathrm{d}p}{\mathrm{d}E}f = p^2 f$$

spectral density

$$n(t,\mathbf{x},E) = \frac{1}{\beta}\int \mathrm{d}\Omega F = \frac{4\pi p^2}{\beta}\phi$$

p=*p(E)* intended

Specialisation to our case

$$\left[\frac{\partial}{\partial t} + \dot{\mathbf{x}} \cdot \nabla_{\mathbf{x}} + \dot{\mathbf{p}} \cdot \nabla_{\mathbf{p}}\right] f = 0$$

↓

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f + q\frac{(\mathbf{p} \times \mathbf{B})}{E} \cdot \nabla_{\mathbf{p}} f = 0$$

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Decomposing into the "known" (ensemble-averaged field)+perturbation

$$\mathbf{B} = \langle \mathbf{B} \rangle + \delta \mathbf{B}$$

The ensemble-averaged equation becomes

$$\frac{\mathrm{d}\langle f \rangle}{\mathrm{d}t} = \frac{\partial \langle f \rangle}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} \langle f \rangle + q \frac{(\mathbf{p} \times \langle \mathbf{B} \rangle)}{E} \cdot \nabla_{\mathbf{p}} \langle f \rangle = -q \left\langle \frac{(\mathbf{p} \times \delta \mathbf{B})}{E} \cdot \nabla_{\mathbf{p}} \delta f \right\rangle \neq 0$$

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The RHS makes it like a Boltzmann eq. for <*f*>, with the 'collisional' term

$$-q \left\langle \frac{(\mathbf{p} \times \delta \mathbf{B})}{E} \cdot \nabla_{\mathbf{p}} \delta f \right\rangle \simeq -\nu_{\theta\theta} \left(\langle f \rangle - \phi \right)$$

under some hypotheses, notably gyrophase-average

i.e. a 'relaxation' of <*f*> to the isotropic flux due to the CR interactions with the B-fluctuations ('BGK')

A diffusion equation

Plugging in $f = \phi + 3\hat{\mathbf{p}} \cdot \mathbf{\Phi} + \ldots$ and defining $\mathbf{\Omega} \equiv q\langle \mathbf{B} \rangle / E$

system of two equations

$$\partial_t \langle \phi \rangle + \beta \nabla_{\mathbf{x}} \cdot \langle \mathbf{\Phi} \rangle = 0$$

$$\partial_t \langle \mathbf{\Phi} \rangle + \frac{\beta}{3} \nabla_{\mathbf{x}} \langle \phi \rangle + \mathbf{\Omega} \times \langle \mathbf{\Phi} \rangle \simeq -\nu_{\theta\theta} \langle \mathbf{\Phi} \rangle$$

$\Longleftrightarrow$

$$\partial_t \langle \phi \rangle + \nabla_{\mathbf{x}} \cdot \mathbf{j} = 0$$

$$\partial_t \mathbf{j} + \frac{\beta^2}{3} \nabla_{\mathbf{x}} \langle \phi \rangle + \mathbf{\Omega} \times \mathbf{j} \simeq -\nu_{\theta\theta} \mathbf{j}$$

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can be re-cast in terms of the single **diffusion equation for the ensemble-averaged isotropic flux**

$$\frac{\partial \langle \phi \rangle}{\partial t} = \frac{\partial}{\partial x_i} \left(K_{ij} \frac{\partial}{\partial x_j} \langle \phi \rangle \right)$$

where the **spatial diffusion tensor** writes

$$K_{ij} = \frac{\beta^2}{3\nu_{\theta\theta}} \frac{\nu_{\theta\theta}^2 \delta_{ij} + \nu_{\theta\theta} \Omega_k \epsilon_{ijk} + \Omega_i \Omega_j}{\nu_{\theta\theta}^2 + \Omega^2}$$

Eigenvalues

$$\frac{\beta^2}{3\nu_{\theta\theta}} \quad \parallel \mathrm{B}$$

$$\frac{\beta^2}{3(\nu_{\theta\theta} \pm i\Omega)} \quad \perp \mathrm{B}$$

Further generalisation: What are the inhomogeneities?

Reminder

Fluid equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0\,,$$ Continuity eq. (mass conservation)

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right) \mathbf{v} = -\frac{\nabla P}{\rho}\,,$$ Euler eq. (momentum eq.)

$$P = P(\rho, \ldots)\,,$$ equation of state (EOS)

admit perturbative 'wave' solutions around background $\bar{\rho} = const.,\ \bar{P} = 0,\ \bar{\mathbf{v}} = 0,$

$$\rho(t, \mathbf{x}) = \bar{\rho}(t) + \delta\rho(t, \mathbf{x}),\ P(t, \mathbf{x}) = \delta P,\ \mathbf{v} = \delta \mathbf{v}$$

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can extract single master equation

$$\left(\frac{\partial^2}{\partial t^2} - c_s^2 \nabla^2\right)\delta\rho = 0 \qquad c_s^2 \equiv \delta P/\delta\rho$$

"Sound waves equation"

simple solution in Fourier space $$\delta\rho(t,\mathbf{x}) = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^{3/2}} \delta\tilde{\rho}_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}}$$

with $\delta\tilde{\rho}_{\mathbf{k}} = A_{\mathbf{k}} e^{i\omega_{\mathbf{k}} t} + B_{\mathbf{k}} e^{-i\omega_{\mathbf{k}} t}$ obeying the **dispersion relation** $\omega_{\mathbf{k}}^2 = c_s^2\, k^2$

Magnetohydrodynamics waves

In a magnetised medium,
same as before + Lorentz force and
Maxwell equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0\,,$$

$$\rho \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{v} = -\nabla P - \frac{\mathbf{B} \times (\nabla \times \mathbf{B})}{4\pi}\,,$$

$$P = P(\rho, \ldots).$$

Ideal MHD approximation:
charge neutrality and fluid approximation, with macro evolution slow wrt microscopic process timescales

$$\left\{ \begin{array}{l} \dfrac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) \\ \nabla \cdot \mathbf{B} = 0 \end{array} \right.$$

search a solution around the background $\bar{\rho} = const.\,, \bar{P} = 0\,, \bar{\mathbf{v}} = 0\,, \mathbf{B} = \mathbf{B}_0$

one can reduce oneself to a **linear system of second-order differential equations,**

In Fourier space, admits non-trivial solution if the matrix equation $A(\omega, \mathbf{k})\, \delta \mathbf{v}_{\mathbf{k}} = 0$

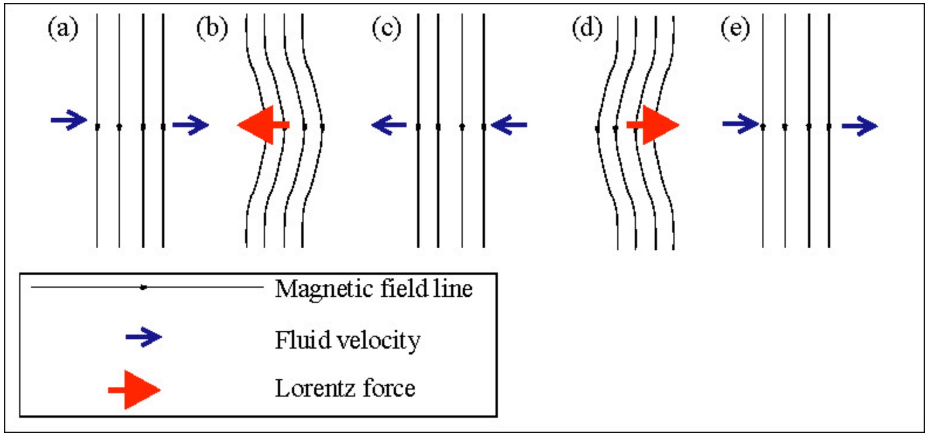
has **det A=0**. → determines the **dispersion relation of the (propagating) modes,**

the **corresponding eigenvectors** determining the **different types of propagating modes**

Alfvén waves

"Zoology" of modes, due to the types of medium/particles involved (cold, hot, different EoS, etc.)

Most notable 'new' beast:
Alfvén waves, *transversal* wave propagating ||B:
mechanical pressure plays no role!
(B-field acts as a plucked string, inertia given by mass of particles in the medium)



$$v_A^2 \equiv \frac{B_0^2}{4\pi\rho_0}$$ Alfvén velocity $$v_A \simeq 2.2 \frac{\mathrm{km}}{\mathrm{s}} \frac{B_0}{\mu G} \sqrt{\frac{1\,\mathrm{cm}^{-3}}{\bar{n}}}$$

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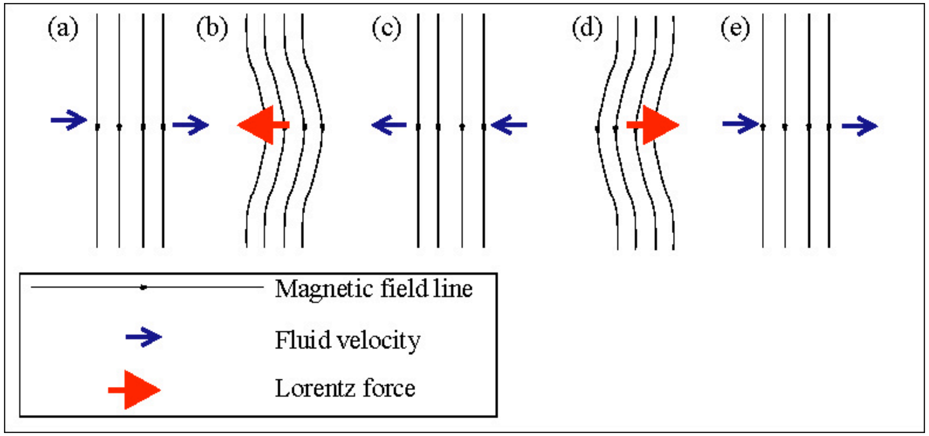

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Half off the Nobel Prize in Physics 1970 to Hannes Olof Gösta Alfvén
"for fundamental work and discoveries in magnetohydro-dynamics with fruitful applications in different parts of plasma physics"

Original, Nobel-worthy article…

Hannes Alfvén (1942)
"Existence of Electromagnetic-Hydrodynamic Waves".
*Nature **150** (3805), 1942*

For us,
key new information
"The B-field disturbances" previously considered are not 'at rest' in the Lab (Galaxy) frame

The frame where these disturbances are on average at rest can move; also, their 'random' movement is important

Existence of Electromagnetic-Hydrodynamic Waves

IF a conducting liquid is placed in a constant magnetic field, every motion of the liquid gives rise to an E.M.F. which produces electric currents. Owing to the magnetic field, these currents give mechanical forces which change the state of motion of the liquid.

Thus a kind of combined electromagnetic-hydrodynamic wave is produced which, so far as I know, has as yet attracted no attention.

The phenomenon may be described by the electrodynamic equations

$$\text{rot } H = \frac{4\pi}{c} i$$

$$\text{rot } E = -\frac{1}{c}\frac{dB}{dt}$$

$$B = \mu H$$

$$i = \sigma\left(E + \frac{v}{c} \times B\right);$$

together with the hydrodynamic equation

$$\partial \frac{dv}{dt} = \frac{1}{c}(i \times B) - \text{grad } p,$$

where σ is the electric conductivity, μ the permeability, ∂ the mass density of the liquid, i the electric current, v the velocity of the liquid, and p the pressure.

Consider the simple case when $\sigma = \infty$, $\mu = 1$ and the imposed constant magnetic field H_0 is homogeneous and parallel to the z-axis. In order to study a plane wave we assume that all variables depend upon the time t and z only. If the velocity v is parallel to the x-axis, the current i is parallel to the y-axis and produces a variable magnetic field H' in the x-direction. By elementary calculation we obtain

$$\frac{d^2H'}{dz^2} = \frac{4\pi\partial}{H_0^2}\frac{d^2H'}{dt^2},$$

which means a wave in the direction of the z-axis with the velocity

$$V = \frac{H_0}{\sqrt{4\pi\partial}}.$$

Waves of this sort may be of importance in solar physics. As the sun has a general magnetic field, and as solar matter is a good conductor, the conditions for the existence of electromagnetic-hydrodynamic waves are satisfied. If in a region of the sun we have $H_0 = 15$ gauss and $\partial = 0{\cdot}005$ gm. cm.$^{-3}$, the velocity of the waves amounts to

$$V \sim 60 \text{ cm. sec.}^{-1}.$$

This is about the velocity with which the sunspot zone moves towards the equator during the sunspot cycle. The above values of H_0 and ∂ refer to a distance of about 10^{10} cm. below the solar surface where the original cause of the sunspots may be found. Thus it is possible that the sunspots are associated with a magnetic and mechanical disturbance proceeding as an electromagnetic-hydrodynamic wave.

The matter is further discussed in a paper which will appear in *Arkiv för matematik, astronomi och fysik*.

H. ALFVÉN.

Kgl. Tekniska Högskolan,
Stockholm.
Aug. 24.

Generalisation

$$\frac{\partial\langle\phi\rangle}{\partial t} - \frac{\partial}{\partial x_i}\left(K_{ij}\frac{\partial\langle\phi\rangle}{\partial x_j}\right) + u_i\frac{\partial\langle\phi\rangle}{\partial x_i} - \frac{1}{3}\frac{\partial u_i}{\partial x_i}\left(p\frac{\partial\langle\phi\rangle}{\partial p}\right) - \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 K_{pp}\frac{\partial\langle\phi\rangle}{\partial p}\right) = \underbrace{Q}$$

Convection/advection: accounts for spatial transport due to large scale movements like Galactic winds. It is usually considered mostly perpendicular to the Galactic plane and antisymmetric with respect to it

Adiabatic energy gain/losses: crucial in particular for particle acceleration

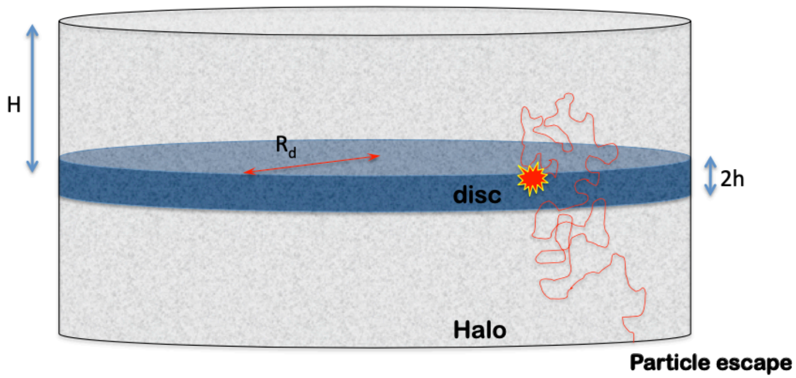
Reacceleration: diffusion in momentum space due to dispersion velocity of the waves in the plasma frame

$$K_{pp} \simeq \frac{\nu_{\theta\theta} E^2 \langle\delta v^2\rangle}{3}$$

Note how ν-dependence inverse wrt to spatial diffusion…

Q is a '**source term**' : makes sense to insert it since 'acceleration' vs propagation can be largely factorised (different timescales)

Simplified 1D model and solution: building intuition



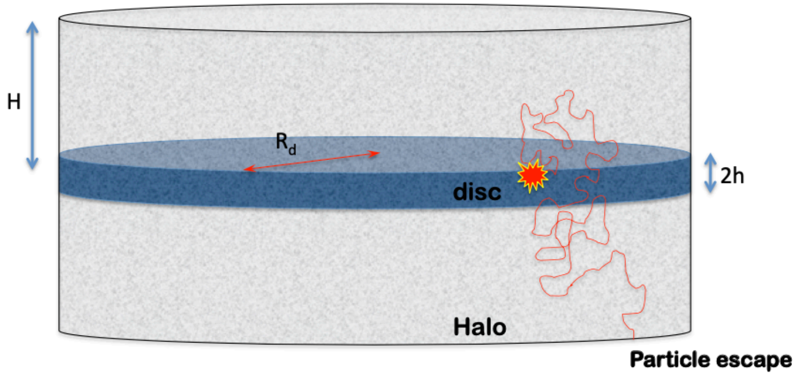
$$R_d \gg H \gg h$$

only vertical, z coordinate relevant

$$-\frac{\partial}{\partial z}\left(K\frac{\partial \phi}{\partial z}\right) = 2\, q_0(p)\, h\delta(z)$$

(leading operators in the equation)

Simplified 1D model and solution: building intuition



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$$z \neq 0: \ \frac{\partial^2 \phi}{\partial z^2} = 0$$

general solution form $\phi(z,p) = a(p) + b(p)|z|$

Boundary conditions (vanishing at border box)

$$\phi(z,p) = \phi_0(p)\left(1 - \frac{|z|}{H}\right)$$

Eq for $\phi_0(p)$ can be found by integrating over a small interval around z = 0

$$-2K(p)\left.\frac{\partial\,\phi}{\partial z}\right|_0 = 2\,q_0(p)\,h$$

Solution has the form

$$\phi_0(p) = q_0(p)\tau_d(p)$$

$$\tau_d(p) \equiv \frac{H\,h}{K(p)} \approx 10^7\,\mathrm{yr}\frac{H}{3\,\mathrm{kpc}}\frac{h}{100\,\mathrm{pc}}\frac{10^{28}\,\mathrm{cm^2s^{-1}}}{K}$$

- At z=0, equivalent to "leaky box", i.e. homogeneous model with diffusion → effective (*p*-dependent) confinement time
- Spectrum at the Earth ≠ source spectrum due to the *energy-dependence of the propagation effects*

IV. Basics on CR acceleration and the ‘SNR paradigm’

Recap

- Multipolar expansion (angular) of phase space
- Ensemble averaging over stochastic fluctuations of the magnetic field
- Account for movement of these 'scattering centres'

$$f = \phi + 3\hat{\mathbf{p}} \cdot \mathbf{\Phi} + \ldots$$

$$\mathbf{B} = \langle \mathbf{B} \rangle + \delta \mathbf{B}$$

$$\frac{\partial \langle \phi \rangle}{\partial t} - \frac{\partial}{\partial x_i}\left(K_{ij}\frac{\partial \langle \phi \rangle}{\partial x_j}\right) + u_i \frac{\partial \langle \phi \rangle}{\partial x_i} - \frac{1}{3}\frac{\partial u_i}{\partial x_i}\left(p\frac{\partial \langle \phi \rangle}{\partial p}\right) - \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 K_{pp}\frac{\partial \langle \phi \rangle}{\partial p}\right) = Q$$

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What about 'the source', i.e. acceleration?

Requirements to accelerate particles

Energetics: must take energy somewhere! For example:
Kinetic Energy (translational in SNRs, rotational in pulsars)
Gravitational Energy (accretion disks)
Magnetic (solar flares)

Mechanism for Energy Transfer: how to transfer energy from macroscopic objects into the (microscopic) acceleration of particles? Ultimately it must be electromagnetic process…

Confinement: need to check that the particle stays in the accelerator for the time needed to accelerate it.

Lack of (significant) E-losses: accelerating particles is useless for explaining CRs if they lose energy too quickly…

Several candidates to supply the needed energy.
The trickiest problem is the second one, first addressed by Fermi

E. Fermi, "On the Origin of the Cosmic Radiation", Physics Review 75, 1169, (1949)

Macro/micro energy transfer

As long as ideal (i.e. zero resistance) MHD conditions are verified*
(*~ charge neutrality and fluid approximation, with macro evolution slow wrt microscopic process timescales*)
since B-fields cannot make work, only **inductive E-fields** (from conductive fluid movement) can be used to accelerate

$$\mathbf{E} = -\mathbf{u} \times \mathbf{B}$$

**Exception to some of these conditions can be found e.g. in pulsar magnetospheres or magnetic reconnection phenomena*

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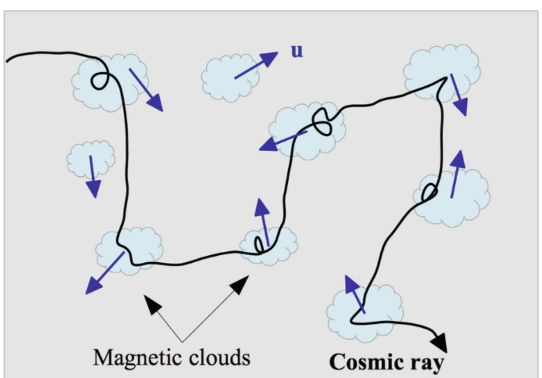
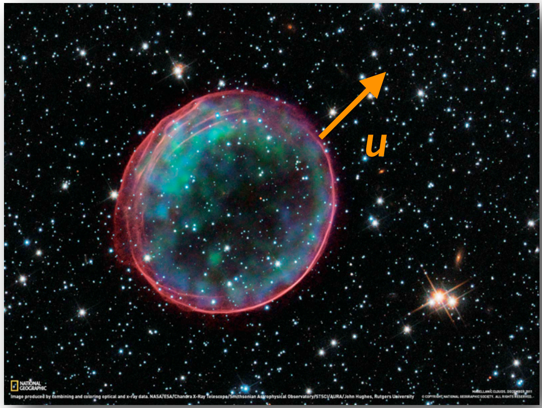
Either encounters with turbulent scattering centres (*as in original Fermi paper*) or coherent motion of shock waves (*theories developed in the 1970s*) are typically invoked to implement this idea





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Reminder on shocks

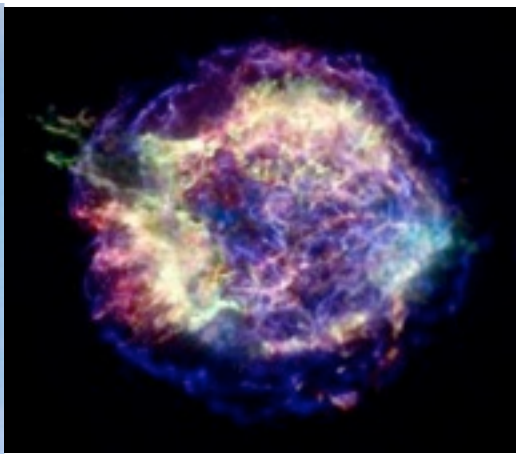
Abrupt changes in macroscopic variables (e.g. density) achieved within microscopic distances. They propagate faster than sound. *(Far from being exotic, they arise quite naturally once the non-linear nature of fluid equations is taken into account!)*

$$\mathcal{M} \equiv v_{\rm sh}/c_{s,u}$$

Mach number >1 in a shock

Conservation laws translate into links between physical quantities across the discontinuity. E.g. for an ideal fluid of EoS parameter γ(=5/3) one has

$$\frac{\rho_p}{\rho_u} = \frac{v_u}{v_p} = \frac{(\gamma+1)\mathcal{M}^2}{(\gamma-1)\mathcal{M}^2+2} \xrightarrow[\text{shock}]{\text{'strong'}} \frac{(\gamma+1)}{(\gamma-1)} \approx 4$$



NB:
Shocks in space are collisionless, i.e. do not involve collision between atoms/ions, but the 'scattering' with B-field inhomogeneities. In terrestrial Labs, hard to reproduce collisionless shock physics!

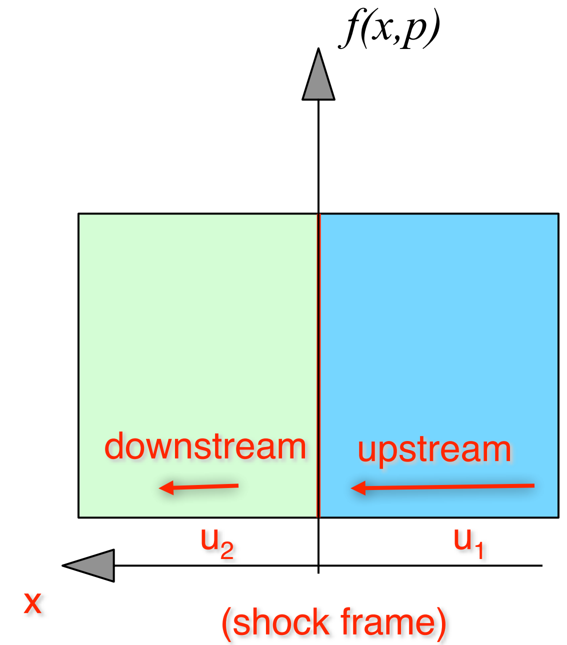
Example: (ideal) 1st order Fermi acceleration

(To ease notation, here I call simply f what should be more properly denoted as **<φ>***)*

$$\frac{\partial f}{\partial t} = -\left(\vec{u}\cdot\vec{\nabla}\right) f - \vec{\nabla}\cdot\left(D_{\|}\vec{\nabla} f\right) + \frac{1}{3}\left(\vec{\nabla}\cdot\vec{u}\right) p\frac{\partial f}{\partial p} + Q$$

1 D limit (infinite planar shock)

$$\frac{\partial f}{\partial t} = -u\frac{\partial f}{\partial x} + \frac{\partial}{\partial x}\left(D\frac{\partial f}{\partial x}\right) + \frac{1}{3}\left(\frac{du}{dx}\right) p\frac{\partial f}{\partial p} + Q$$



Must be satisfied on either side of the shock

Let's consider a steady state solution, set Q=0

(In reality, source does matter, at very least to normalise the solution… however, the idealisation done here would e.g. imply infinite # of particles!)

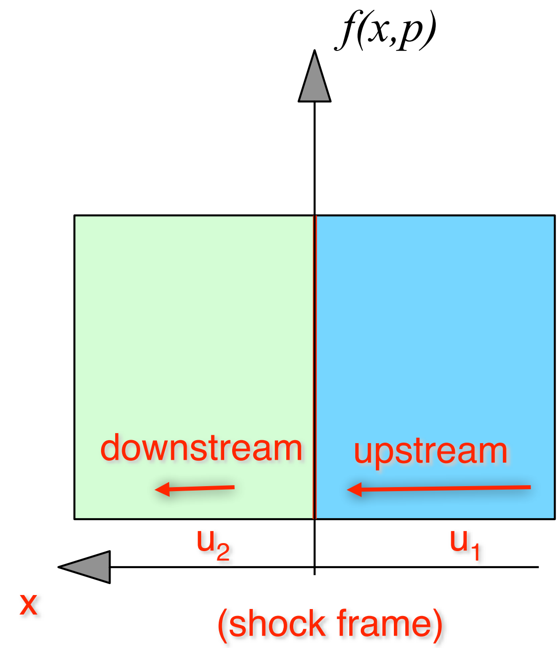
Solution

$$\frac{\partial f}{\partial t} = -u\frac{\partial f}{\partial x} + \frac{\partial}{\partial x}\left(D\frac{\partial f}{\partial x}\right) + \frac{1}{3}\left(\frac{du}{dx}\right)p\frac{\partial f}{\partial p}$$

Far from the discontinuity, *u=const* and this equation reduces to

$$u\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(D\frac{\partial f}{\partial x}\right)$$

(with u and D different at the two sides)



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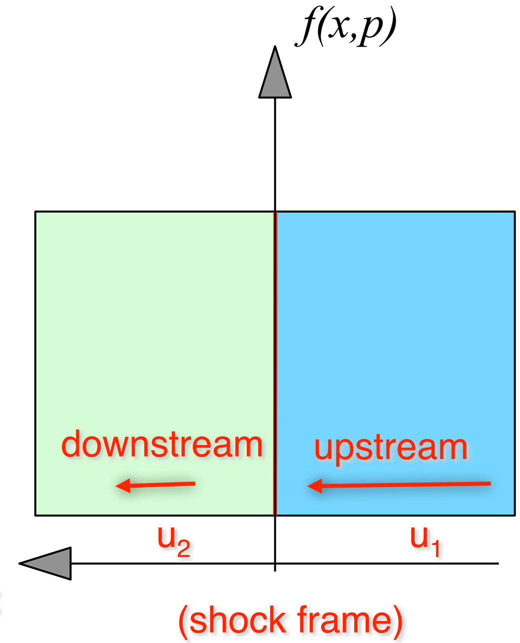

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Upstream (ahead of the shock, which is at x_0), convection balances diffusion and, imposing the correct boundary conditions,

$$f(x,p) = f(-\infty,p) + [f(0,p) - f(-\infty,p)]\exp\left(\int_0^x \frac{u_1\, dy}{D_1(y,p)}\right) \quad (x<0)$$

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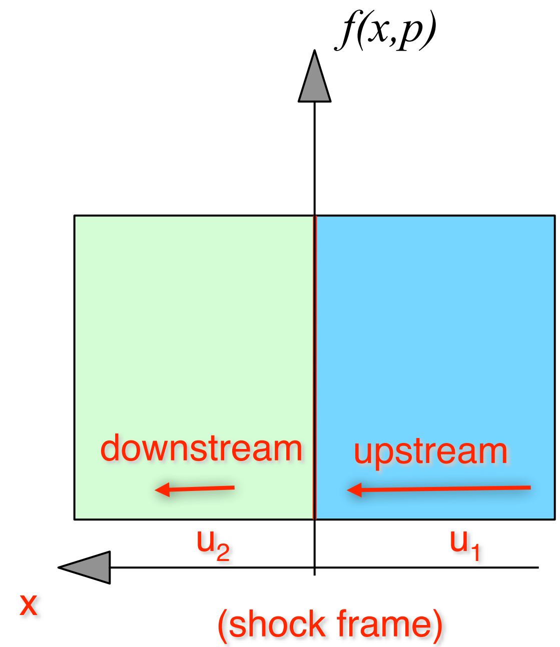

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Downstream (behind the shock) there is no physically acceptable (i.e. f-finite) way to balance diffusion against convection. Only spatially constant solutions are acceptable; continuity imposes:

$$f(x,p) = f(0,p) \quad (x > 0)$$

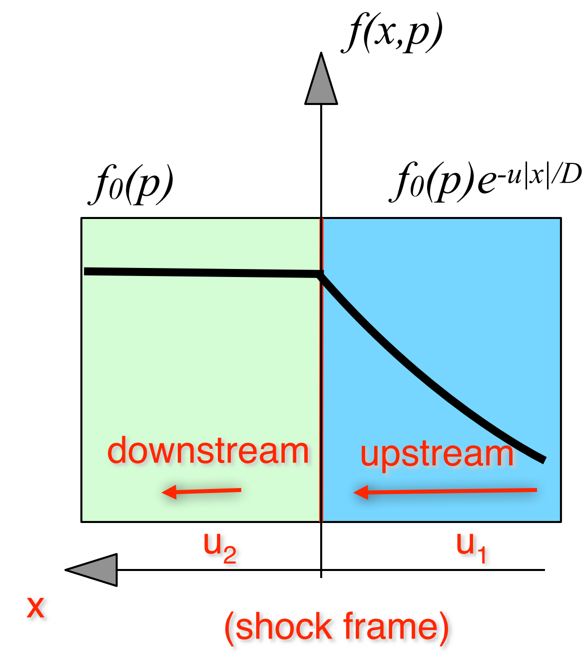
Solution, cont'd

Integrating the transport equation around the shock, one obtains the condition

$$\left(D\frac{\partial f}{\partial x}\right)_2 - \left(D\frac{\partial f}{\partial x}\right)_1 + \frac{1}{3}(u_2 - u_1)p\frac{\partial f_0}{\partial p} = 0$$

$$\downarrow$$

$$0 - u_1 f_0 + \frac{1}{3}(u_2 - u_1)p\frac{\partial f_0}{\partial p} = 0$$



(the second term at the LHS follows from integrating from -∞ to 0^- the eq. at the previous page)

$$\frac{p}{f_0}\frac{\mathrm{d}f_0}{\mathrm{d}p} = -\frac{3}{1 - u_2/u_1} \longrightarrow f_0(p) \propto p^{-\frac{3}{1-u_2/u_1}} \approx p^{-4}$$

use jump conditions

or, equivalently $dN/dp = 4\pi p^2 f_0(p) \propto p^{-2}$

$\sim E^{-(2+\epsilon)}$, known (improperly) as Fermi spectrum

Comments

1. The spectrum of accelerated particle is a power-law extending formally to infinite momenta: without E-losses, in the stationary approach there's no limit!

2. The slope depends uniquely on the compression factor, it is universal for strong shocks and is notably independent of the diffusion properties (miracle!)

3. The normalisation (equivalently, the efficiency of the acceleration) is a free parameter, depending on the details of the "injection" in the acceleration zone. One cannot check the consistency of the "test-particle approach".

4. The scattering centres have been assumed to be at rest (or slow) in the shock frame. That's not necessarily true

5. A planar (and infinite) geometry was assumed, and implicitly that the shock is "parallel", i.e. the field responsible for diffusion is along the shock normal.

6. The universal "Fermi spectrum" is obtained for energetic particles at the source, does not necessarily hold for the CRs escaping in the ISM! Yet, not crazy to think that this, plus relatively mild 'propagation effects', might explain the CR spectrum…

7. CR's current (feedback on B & shock structure) not taken into account…

A longstanding suspect: Supernova remnants

Most likely culprit 'implementing' the previous ideas in astrophysics

COSMIC RAYS FROM SUPER-NOVAE

BY W. BAADE AND F. ZWICKY

MOUNT WILSON OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON AND CALIFORNIA INSTITUTE OF TECHNOLOGY, PASADENA

Communicated March 19, 1934

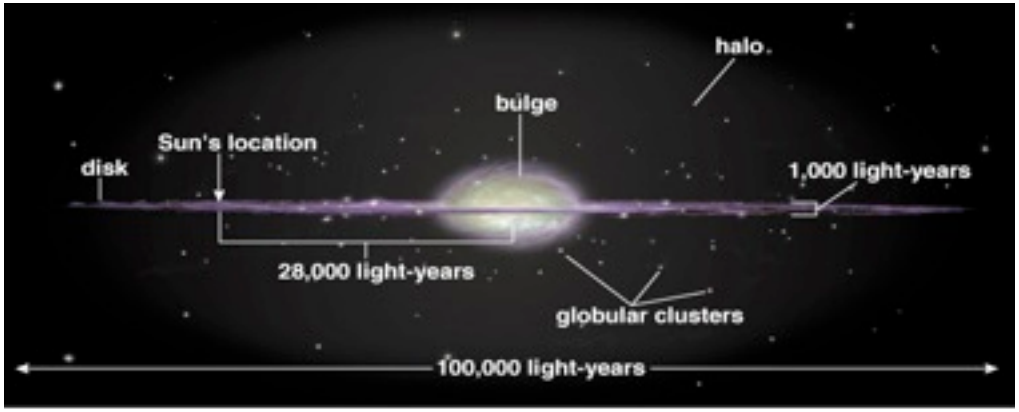
Three main arguments

- Power budget *(oldest one; will see in a minute)*
- Theoretical mechanism to accelerate particles with~ the right properties *(just saw it!)*
- Observed in many non thermal bands (from radio to gamma): they host energetic particles *(will require going being collisionless approximation, multi messenger approach… rest of lecture!)*

The power budget

The integrated *E*-density in CR is of about ρ_{CR}~0.5 eV/cm^3

The confinement volume of the Milky Way is $V_{conf} \sim \pi R^2 h \sim 8\ 10^{67}$ cm^3 (using h~4 kpc and R~15 kpc)



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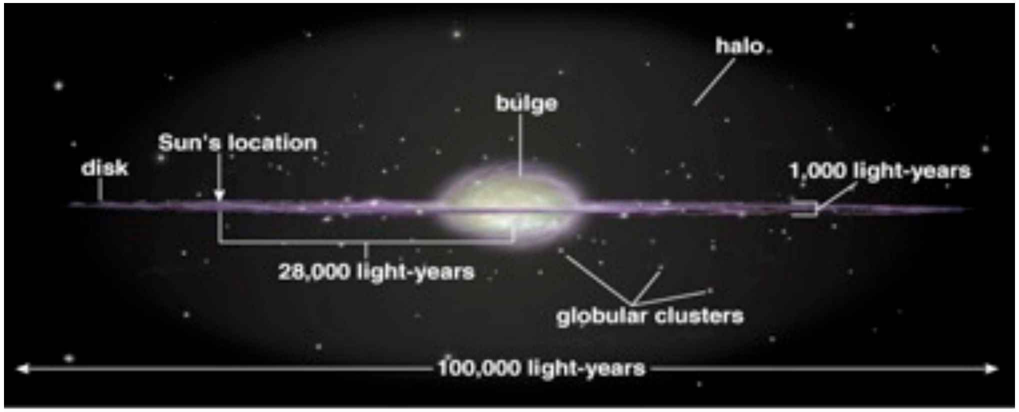

The total Energy in CR in the Galaxy is of about $W_{CR}=\rho_{CR}V_{conf} \sim 6.7\ 10^{55}$ erg

For a confinement time of 10^7 yr, $L_{CR}=W_{CR}/\tau_{conf} \sim 2\ 10^{41}$ erg/s

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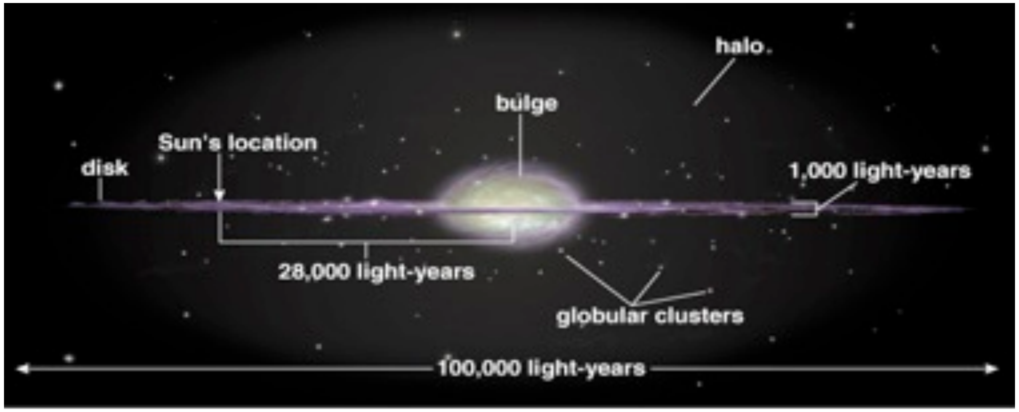

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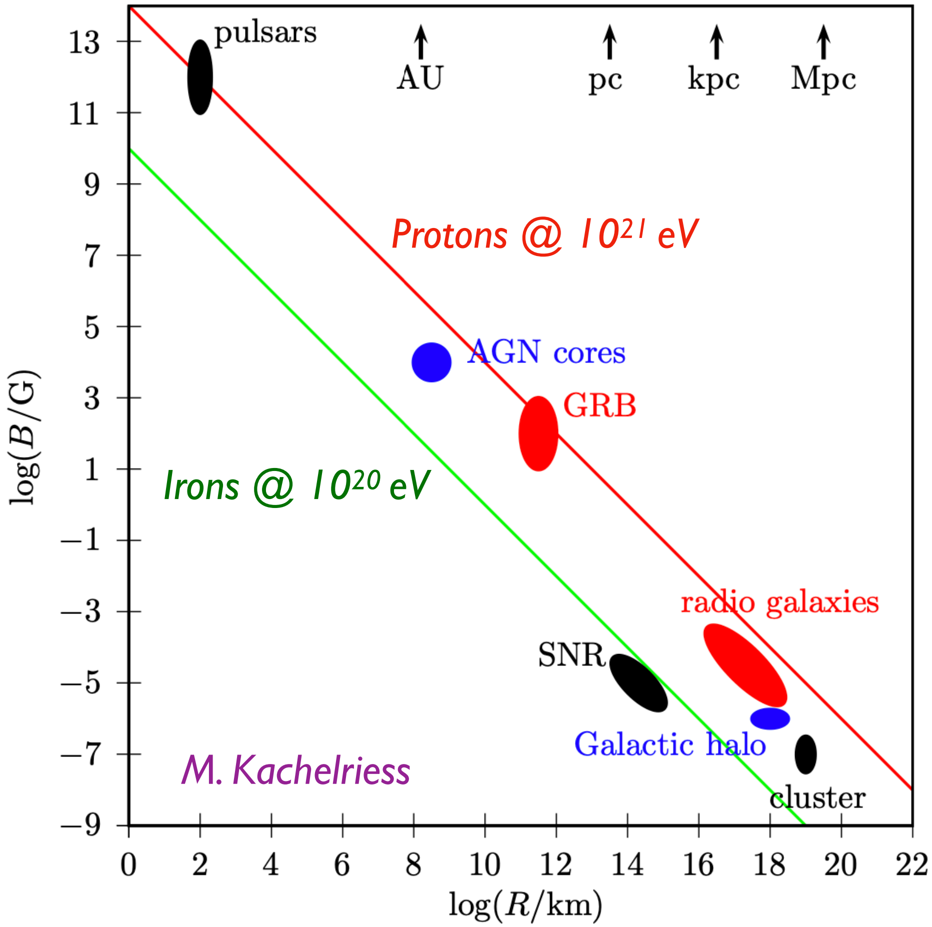
A typical SN releases ~10^{51} erg in kinetic Energy happens ~2 times per century, i.e. $L_{kin}=E_{kin}\ \Gamma_{SN} \sim 8\ 10^{41}$ erg/s

Cosmic Rays could be accounted for with a conversion efficiency of ~20% of the macroscopic kinetic energy into microscopic particle acceleration

Can’t be the whole story: Hillas criterion



The system must be able to contain the particle: its Larmor Radius must be smaller than the size of the accelerator: $R_s > r_L$

$$E_{\rm max} = \Gamma_{\rm s}\, Ze\, B_{\rm s}\, R_{\rm s}$$

SNRs are unable to account for particles of the highest energy observed: other objects (mechanisms?) are needed!

Beyond level of these lectures, enough to say that currently some (subclass of) active galactic nuclei (either in processes near the core or in powerful jets, ultimately powered by SMBH) are thought to be involved in UHECRs