Astroparticle physics

Using fundamental physics to learn about astrophysics or the universe as a whole…

…using astro/cosmo phenomena unaccounted for within current theories to learn about what lies beyond standard model(s)

Pasquale Dario Serpico (Annecy, France)
TAE 2023 Benasque, 09/2023

Plan of the lectures

- I. Intro: Why should we also look for fundamental physics in (high-energy) astrophysics?
- II. Basic facts about cosmic rays & their environments
- III. Phase space approach to CR dynamics
- IV. Basics on CR acceleration & the SNR ‘paradigm’
- **V. 'Multimessenger’ approach: photons, neutrinos, secondaries (some notions on collisional aspects; relevant pheno)**

V. Multimessenger approach

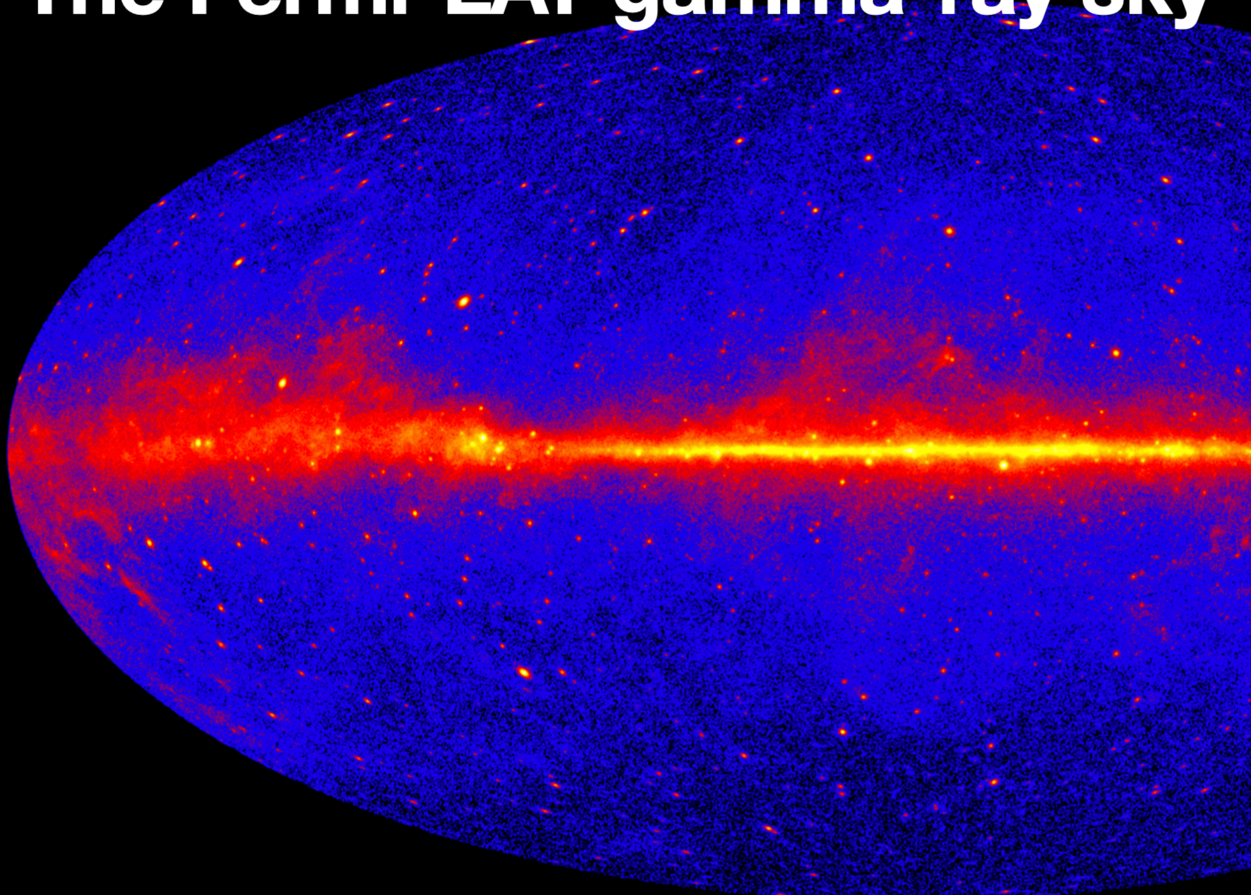
Can we look at byproducts of CR interactions as a 'tracer'?
This requires to drop the collisionless approximation…

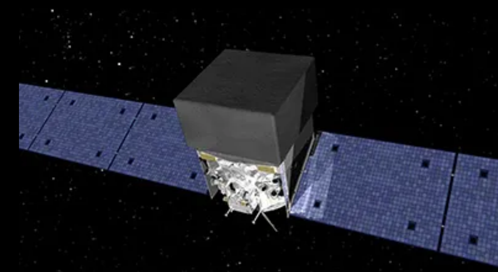
- We have thus to modify the transport equation, since the RHS is non-vanishing…
- We have new species in the game, 'sourced' by these interactions (notably, photons and neutrinos)

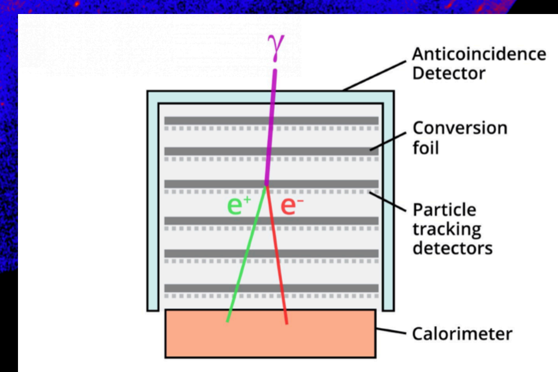
I will introduce some generic notions, then focus only on the dominant *E*-loss (and particle-generating) processes for hadrons and leptons.

First, a look at what the gamma-ray and neutrino sky looks like

The Fermi-LAT gamma-ray sky



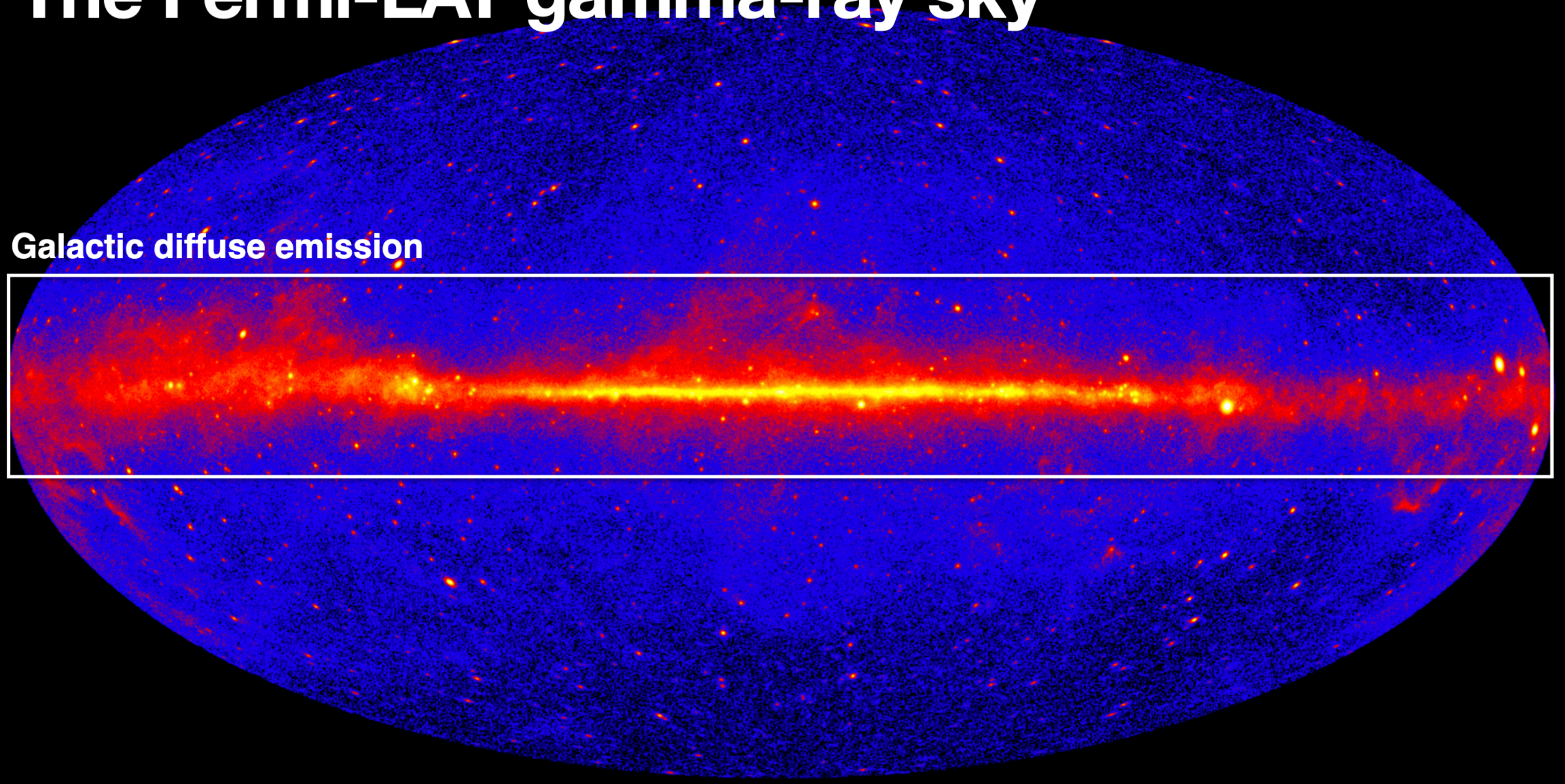




Credit: NASA's Goddard Space Flight Center

Detection of gamma-rays
in the range 20 MeV - 300 GeV

The Fermi-LAT gamma-ray sky



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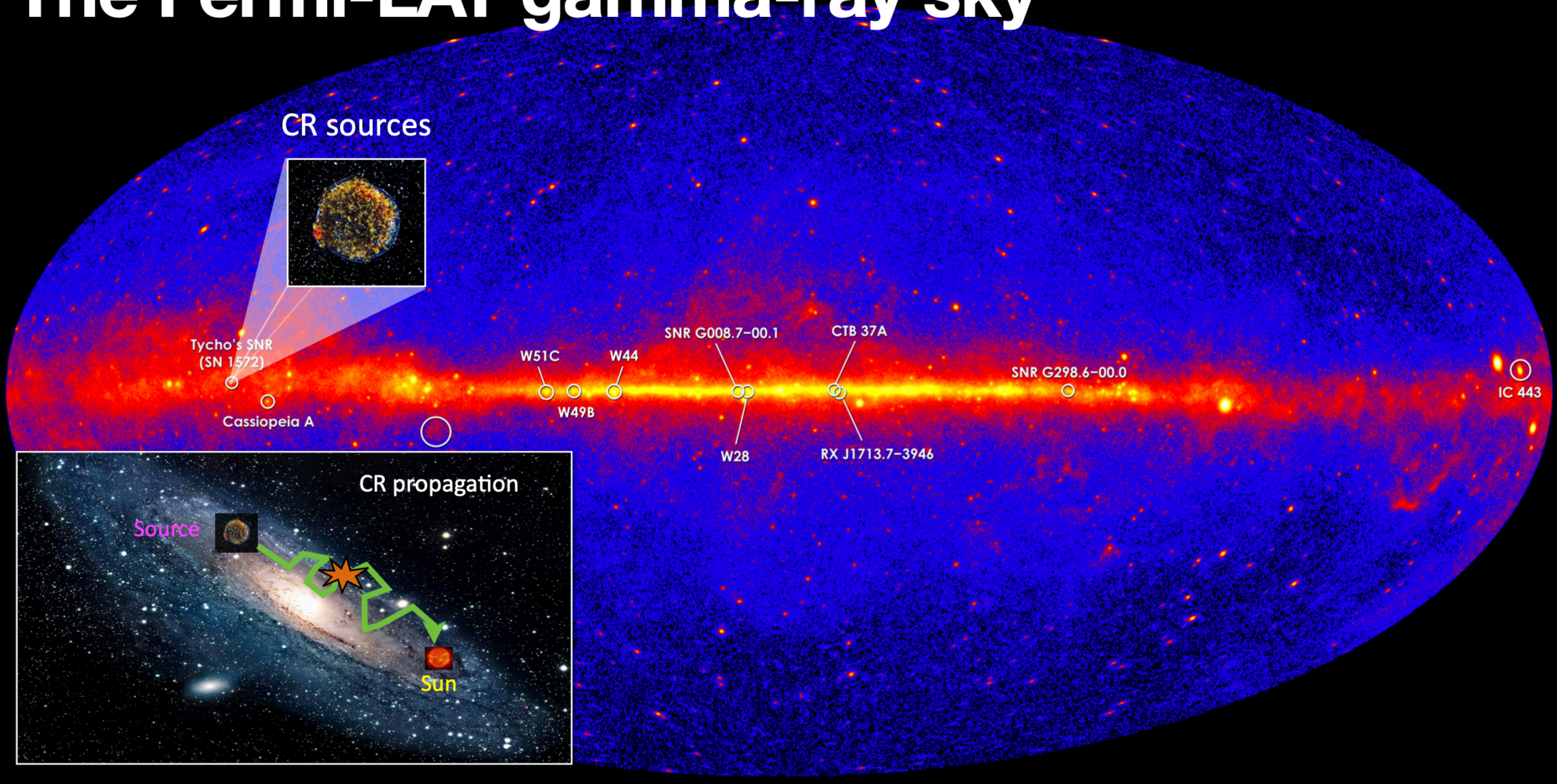

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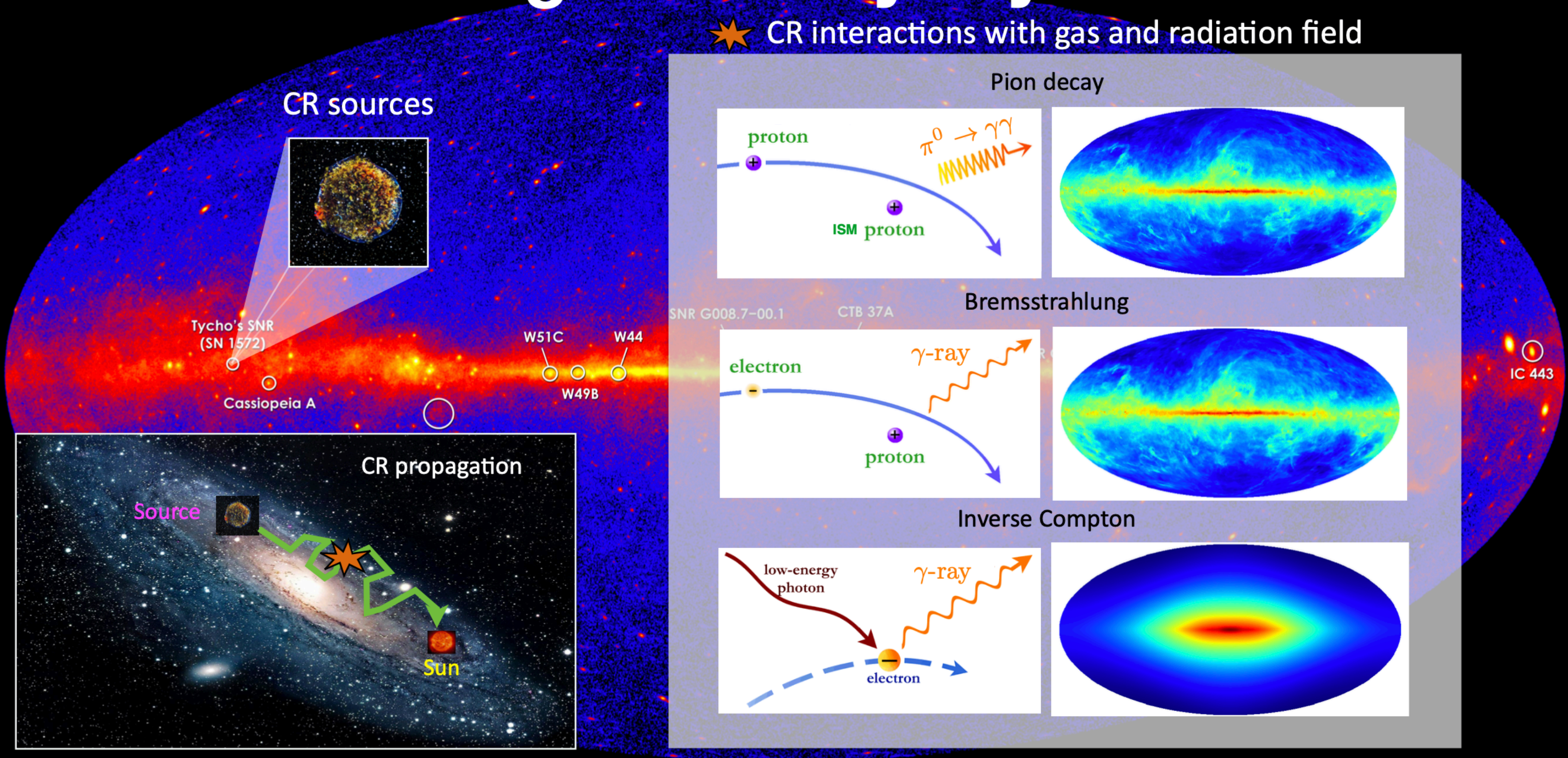
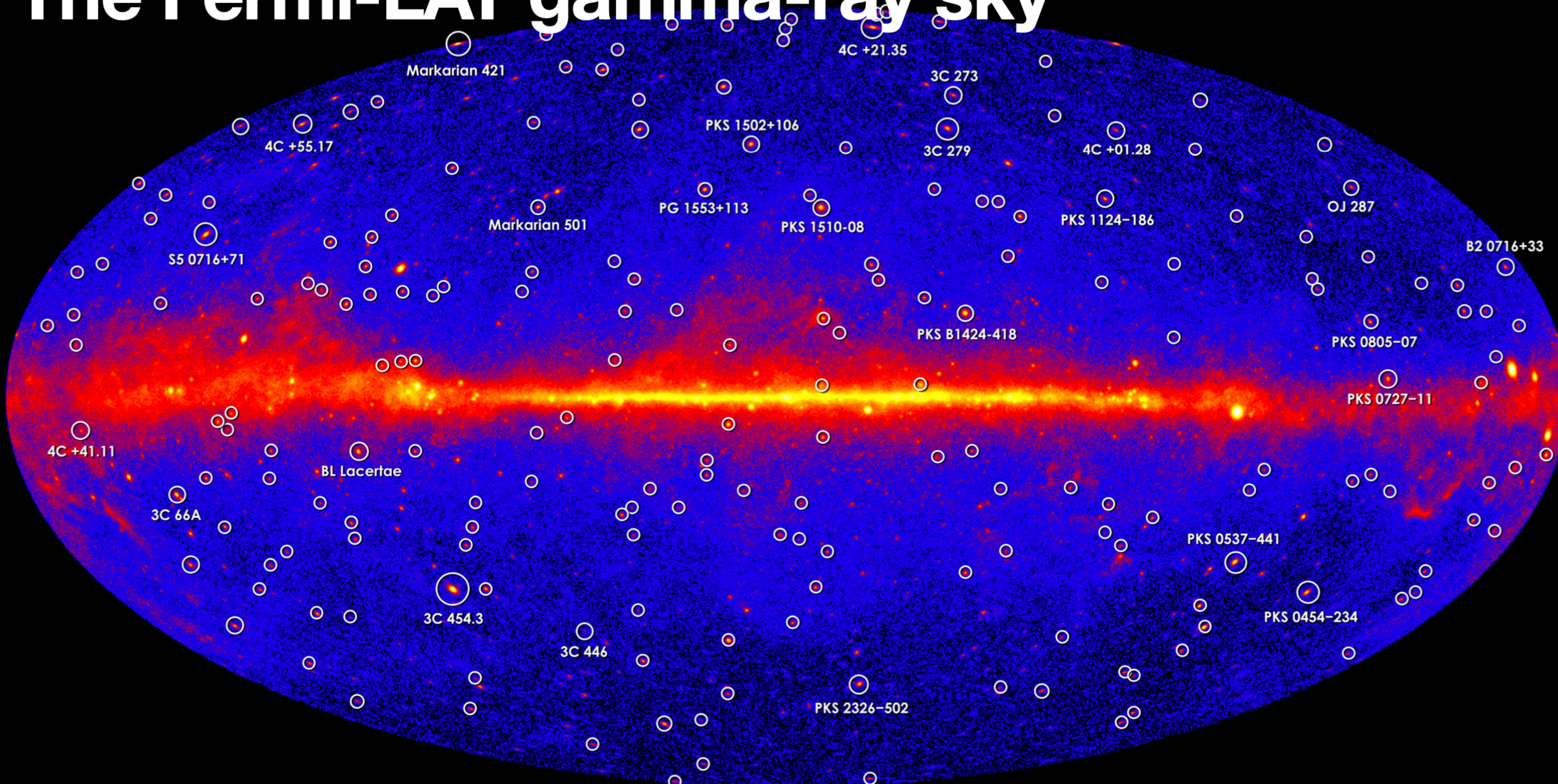


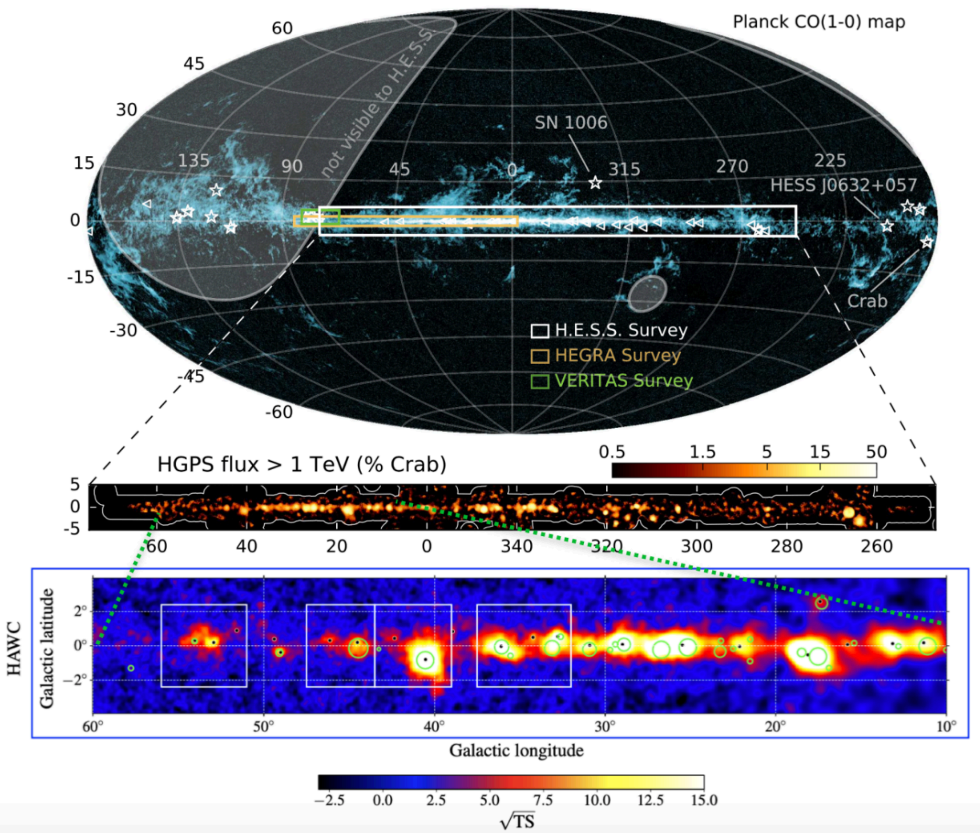
Galactic diffuse gamma-ray emission (GDE)

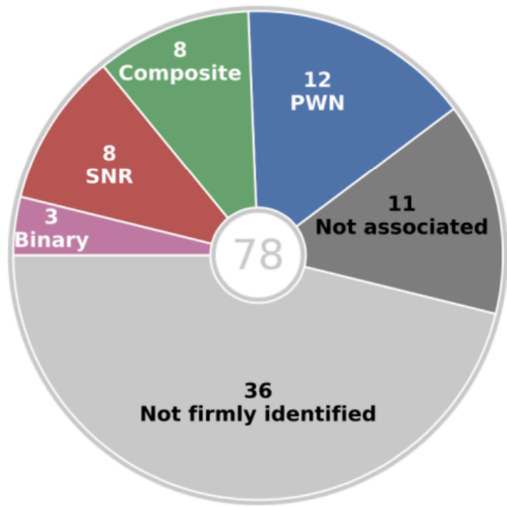
The Fermi-LAT gamma-ray sky

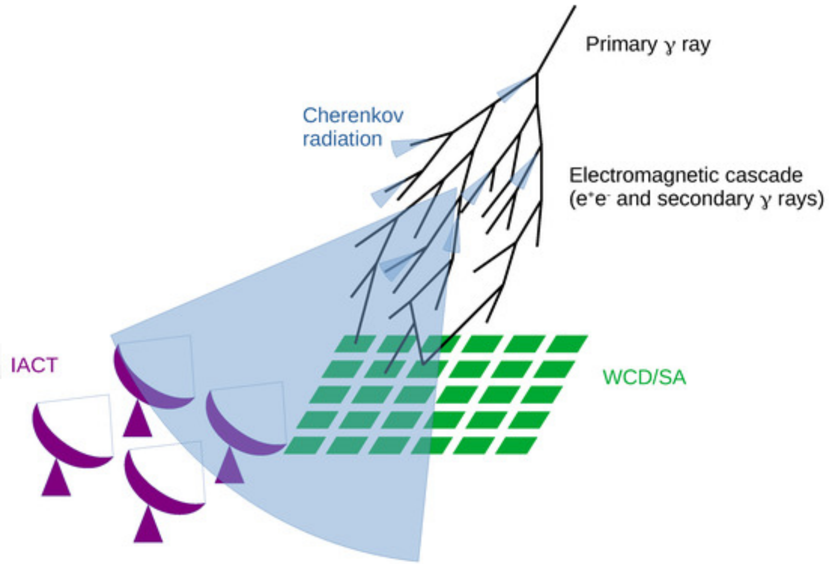


TeV sky

No simultaneously *complete* and *deep* survey, yet rich TeV sky revealed by IACTs and Surface Arrays





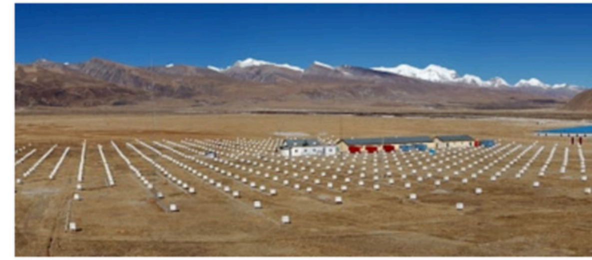


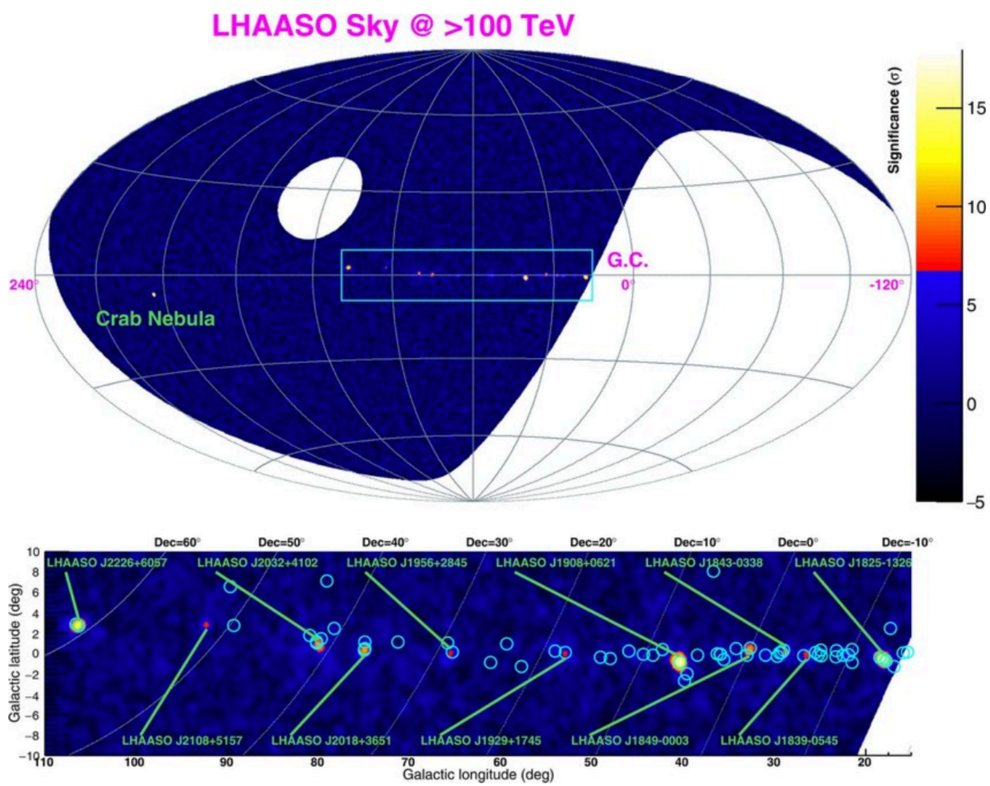
Recently opened window: 0.1-1 PeV frontier

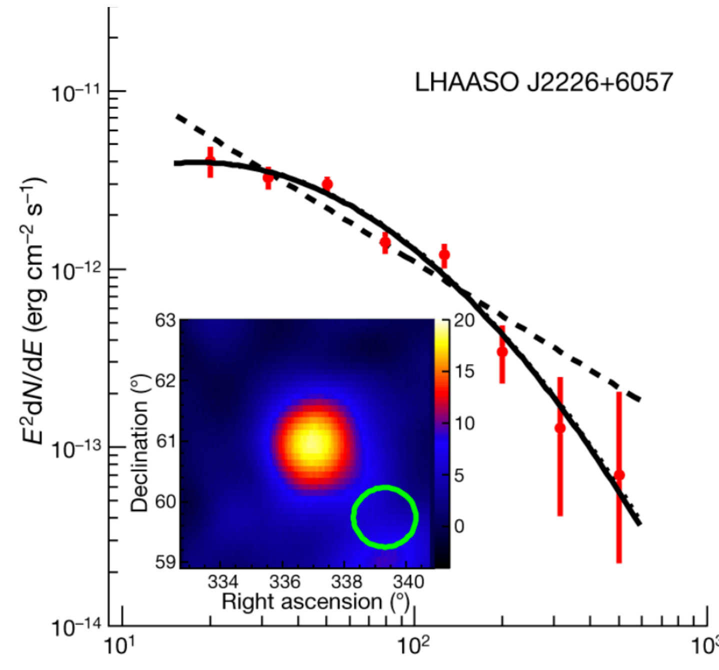
First Detection of sub-PeV Diffuse Gamma Rays from the Galactic Disk: Evidence for Ubiquitous Galactic Cosmic Rays beyond PeV Energies

M. Amenomori *et al.* (Tibet AS_γ Collaboration)

Phys. Rev. Lett. **126**, 141101 – Published 5 April 2021





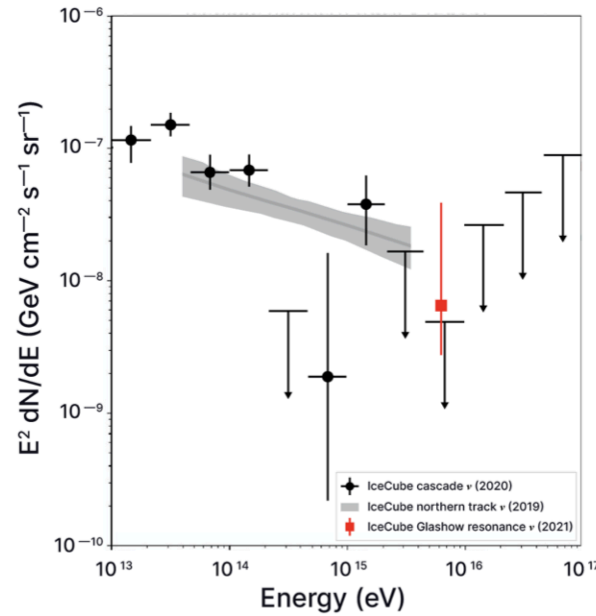


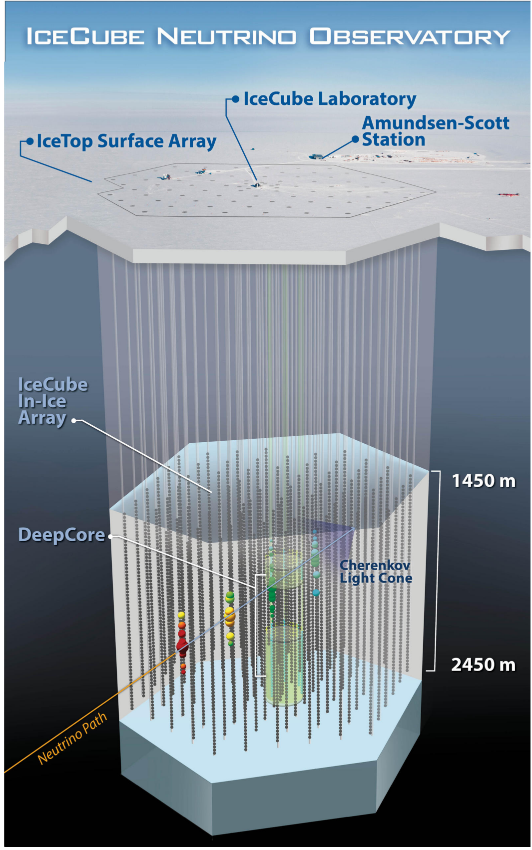
43 sources with E_γ>0.1 PeV!

LHAASO, 2305.17030

Recently opened window: Neutrinos

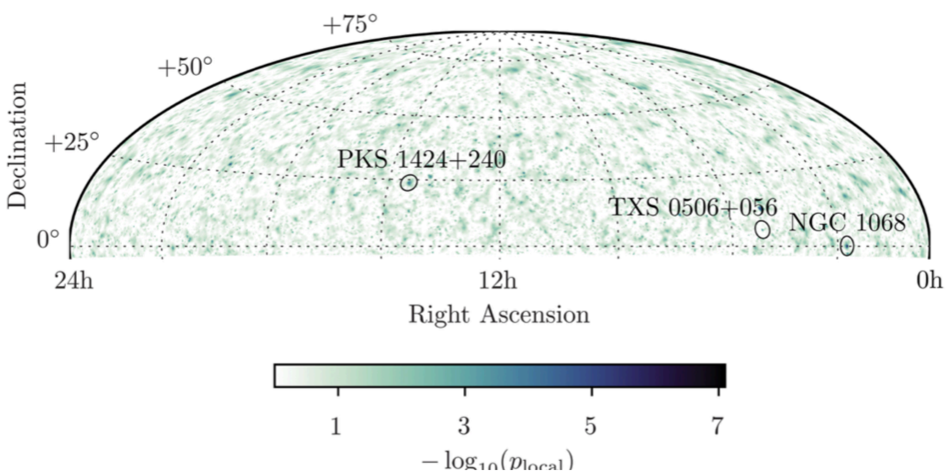
- Decade-old observations of diffuse (extragalactic) flux
- 2018-2022: first evidences for a few point-sources
- 2023: first evidences for diffuse Gal. Component

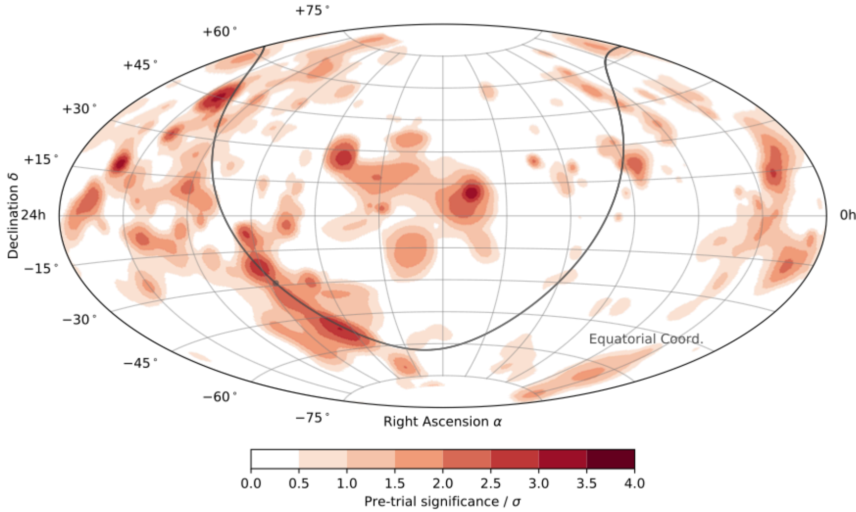




Recent review, e.g. Halzen 2305.07086

ICRC 2023 proceeding in 2307.14842





Accounting for collisions in the propagation equation

$$\left[\frac{\partial}{\partial t} + \dot{\mathbf{x}} \cdot \nabla_{\mathbf{x}} + \dot{\mathbf{p}} \cdot \nabla_{\mathbf{p}}\right] f = 0$$

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$$\downarrow$$

$$\frac{\partial \phi_\alpha}{\partial t} - \frac{\partial}{\partial x_i} D_{ij} \frac{\partial \phi_\alpha}{\partial x_j} + u_i \frac{\partial \phi_\alpha}{\partial x_i} - \frac{1}{3} \frac{\partial u_i}{\partial x_i} \left(p \frac{\partial \phi_\alpha}{\partial p}\right) - \frac{1}{p^2} \frac{\partial}{\partial p} \left(p^2 \mathcal{D} \frac{\partial \phi_\alpha}{\partial p}\right) =$$
$$q + \frac{1}{p^2} \frac{\partial}{\partial p} \left[p^2 \left(\frac{\mathrm{d}p}{\mathrm{d}t}\right)_\ell \phi_\alpha\right] - \Gamma_{\mathrm{tot},\alpha}\, \phi_\alpha + \sum_\eta \phi_\eta \otimes \Gamma_{\eta \to \alpha}\,,$$

continuous losses | catastrophic sinks | collisional sources

e.g. $e+\gamma \to e' + \gamma'$

e.g. decay $^{10}Be \to {}^{10}B$
Spallation $^{12}C+p \to {}^{11}B+p+p$

Conventional splitting, based on physical convenience, which I will now illustrate

Some notation and jargon

- Mean free path and collision rate

$$\ell = \frac{1}{\sigma\, n}, \quad \Gamma = \sigma\, \beta\, n = \frac{\beta}{\ell}$$

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- Central quantity to compute *E*-losses $$\frac{\mathrm{d}\sigma}{\mathrm{d}W}$$
- Moments of E-loss $$-\frac{\mathrm{d}E}{\mathrm{d}x} = -\frac{1}{\beta}\frac{\mathrm{d}E}{\mathrm{d}t} = -\frac{\mathrm{d}|\mathbf{p}|}{\mathrm{d}t} \equiv n \int \mathrm{d}W\, W \frac{\mathrm{d}\sigma}{\mathrm{d}W}$$ *Stopping power*

… W^2… *Energy straggling parameter*

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- Stopping time or range

(useful e.g. in comparing terms in propagation equation)

$$t_{\mathrm{loss}} \equiv \int_m^E \frac{\mathrm{d}E'}{-\mathrm{d}E'/\mathrm{d}t} \sim \frac{E}{-\mathrm{d}E/\mathrm{d}t}, \quad d_{\mathrm{loss}} \equiv \int_m^E \frac{\mathrm{d}E'}{-\mathrm{d}E'/\mathrm{d}x} \sim \frac{E}{-\mathrm{d}E/\mathrm{d}x}$$

Losses are defined

- catastrophic if $t_{\mathrm{loss}} \sim \Gamma^{-1} \quad d_{\mathrm{loss}} \sim \ell$
- continuous if $t_{\mathrm{loss}} \gg \Gamma^{-1} \quad d_{\mathrm{loss}} \gg \ell$

Center of mass energy, transferred momentum…

- Square of CoM energy

$$s = \left(\sum_i p_i\right)^2 = m_a^2 + m_b^2 + 2E_aE_b\left(1 - \beta_a\beta_b\cos\vartheta\right)$$

Use to estimate threshold for processes, e.g. anti-p production

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- Momentum transfer *(a+b → a'+b')*

$$q^2 = \left(p_a - p_a'\right)^2 = \left(p_b - p_b'\right)^2$$

$$q^2 = \left(p - p'\right)^2 = 2m^2 - 2EE'\left(1 - \beta\beta'\cos\vartheta\right) \to -4|\mathbf{p}|^2 \sin^2\vartheta/2$$

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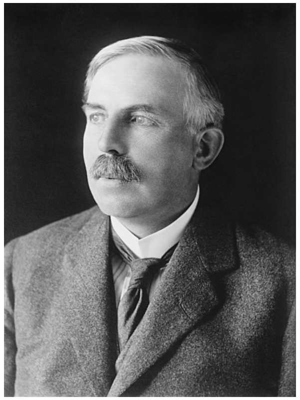

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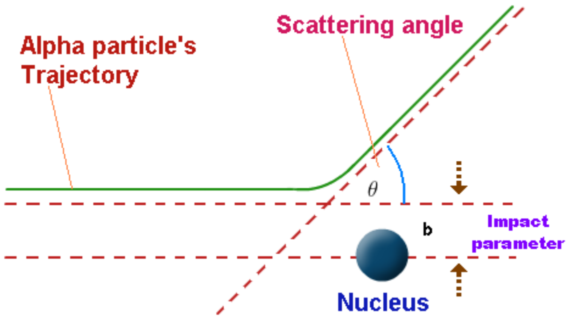
Comment: remember Rutherford x-sec*

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right|_\mathrm{R} = \frac{Z_p^2 Z_t^2\alpha^2}{4|\mathbf{p}|^2\,\beta^2\sin^4\vartheta/2} \Longleftrightarrow \left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right|_\mathrm{R} = \frac{4Z_p^2Z_t^2\alpha^2\gamma^2m^2}{q^4}$$

**Divergence at q → 0?*



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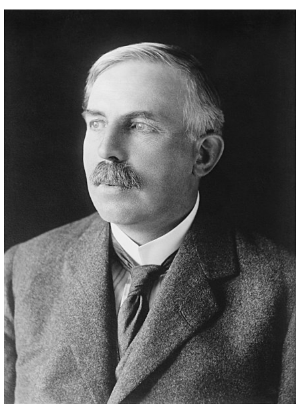

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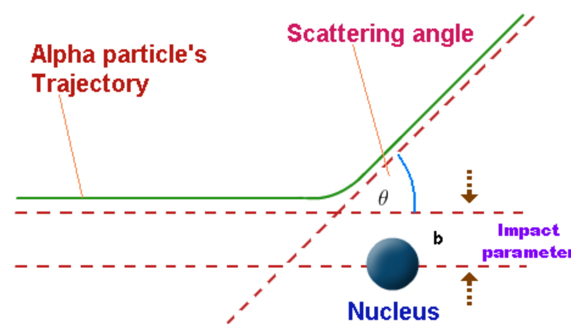

Most of it dominated by small q^2 (forward x-sec): frequent, which matters the most for CR.
In colliders often interested in high p_T (e.g. for new particle productions): these events are, however, rare!

A quick survey of relevant processes

- Electrostatic interactions with matter *(both had & $e^{\pm}$)*
- Radiative interactions *(mostly $e^{\pm}$)*
- Interactions with e.m. fields *($e^{\pm}$, UHECR had in extragalactic environment)*
- Hadron-hadron interactions *(had)*

Notably the latter, crucial for secondary byproducts ($e^{\pm}$, γ, ν, had')

Won't be exhaustive, few topics chosen for illustration and their impact/importance.

Will pay attention both on role for CR (mostly later) and for secondaries

Example 1

Inverse Compton scattering of energetic electrons onto background photon fields

$$e+\gamma \rightarrow e' + \gamma'$$

- ~continuous loss relevant for shaping $e^{\pm}$ spectra
- Most important channel to produce leptonic gammas

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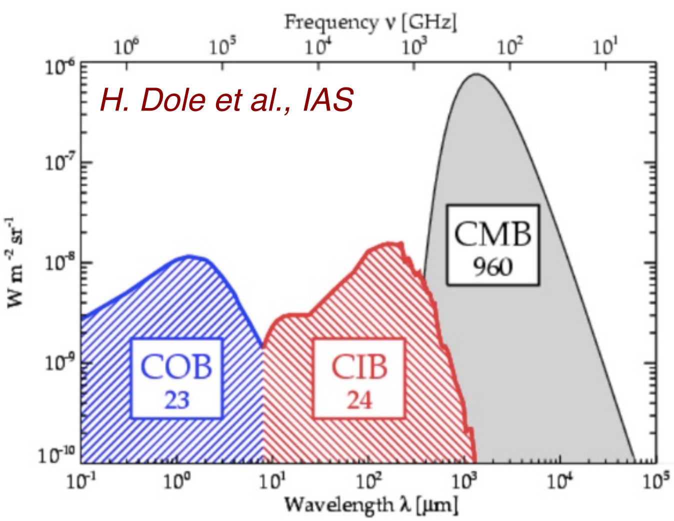
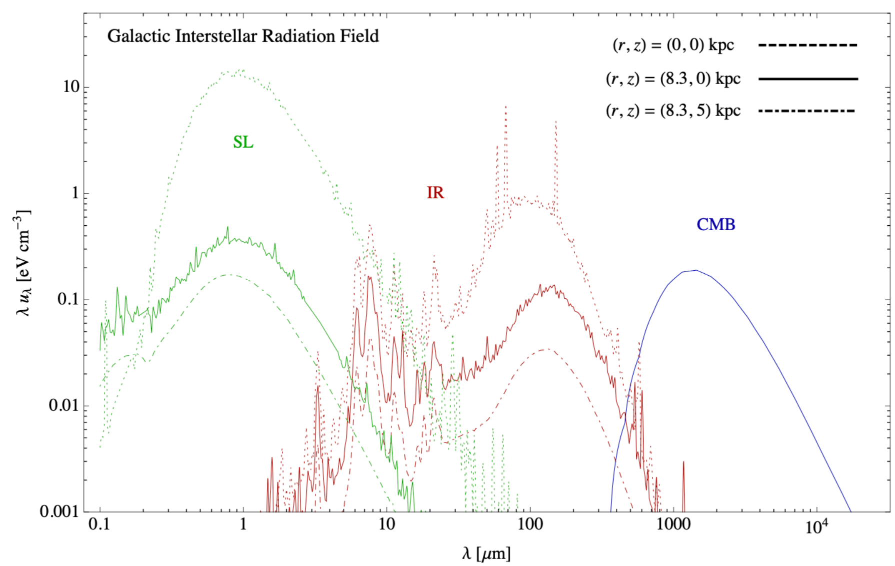
relevant targets:
- CMB
- Extragalactic Background Light (EBL)
- Galactic interstellar radiation field (ISR)

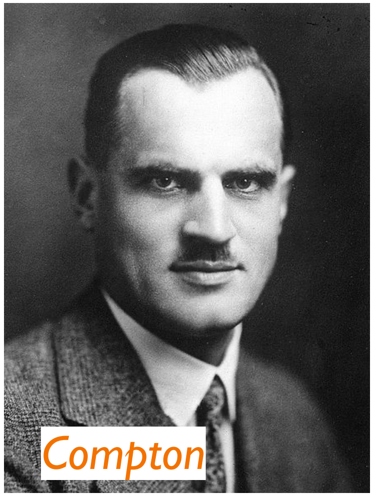
A. Esmaili, PS: 1505.06486



 # ∝ total intensity in the bands



Funny historical facts (aka physicists not great for naming)



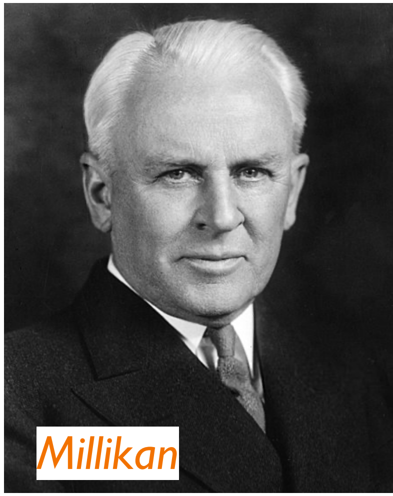
The “Cosmic Ray nature” dispute (1920’s-30s’)

CR are charged particles!

Indeed, deflected in B-fields…

CR are gamma rays!

Birth cries of the creation of matter to prevent universe heath death…



Despite the resolution, Millikan’s name ‘Cosmic rays’ (1925) stuck!

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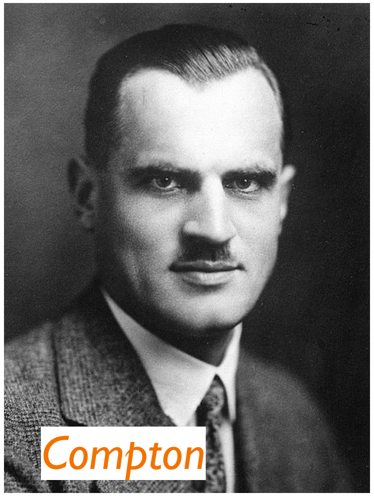

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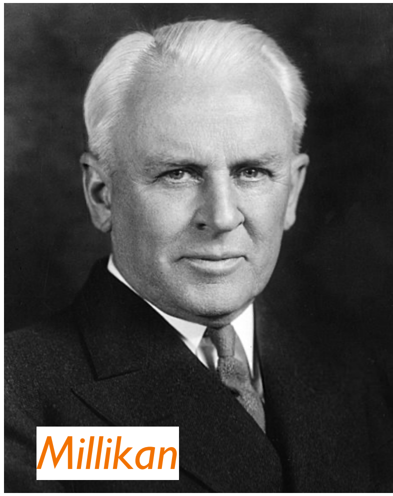

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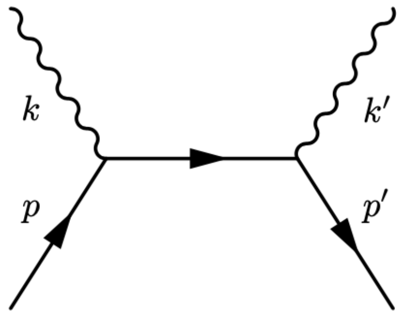
Fantastic 4 comics

Even his interpretation of CR still leaked into pop culture ~ 30 yrs later

Inverse Compton scattering - Kinematics

- Kinematics $m_e^2 = p^2 = p'^2\,, \quad k^2 = k'^2 = 0$



$$m_e^2 = (p + k - k')_\mu (p + k - k')^\mu \Longrightarrow p \cdot (k - k') = k \cdot k'$$

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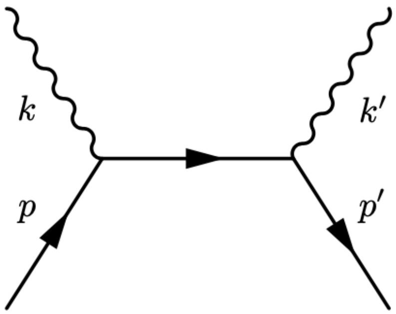

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In electron rest-frame
(denoted by a tilde)

$$\begin{cases} p = (m_e, 0, 0, 0) \\ k = \tilde{\epsilon}(1, 1, 0, 0) \\ k' = \tilde{\epsilon}'(1, \cos\tilde{\theta}, \sin\tilde{\theta}, 0) \end{cases}$$

thus obtaining

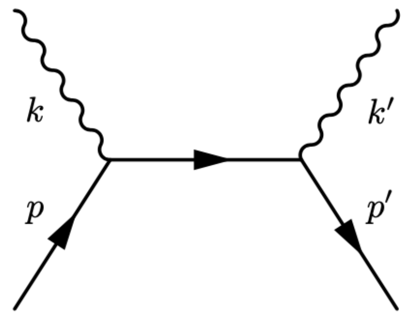
$$m_e(\tilde{\epsilon} - \tilde{\epsilon}') = \tilde{\epsilon}\tilde{\epsilon}'(1 - \cos\tilde{\theta}) \Longrightarrow \tilde{\epsilon}' = \frac{\tilde{\epsilon}}{1 + \frac{\tilde{\epsilon}}{m_e}(1 - \cos\tilde{\theta})}$$

Importance of factor $x \equiv \tilde{\epsilon}/m_e$ *in determining upscattered photon energy*

(i.e. how energetic does the e sees the photon, in units of its mass?)

Inverse Compton scattering - Energy spectrum

- Kinematics (all expressed in Lab frame)



Upscattered photon energy in e rest-frame:

$$\tilde{\epsilon}' = \frac{\tilde{\epsilon}}{1 + \frac{\tilde{\epsilon}}{m_e}(1 - \cos\tilde{\theta})}$$

From the Lab to e-rest frame: $\tilde{\epsilon} = \epsilon\gamma(1 - \beta\cos\theta)$

'Reverse boost' of upscattered photon to the Lab frame: $\epsilon' = \gamma(1 + \beta\cos(\pi - \tilde{\theta}))\tilde{\epsilon}'$

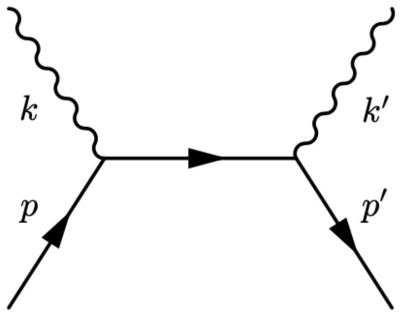
case-by-case result depends on direction:

$$\epsilon' = \gamma(1 - \beta\cos\tilde{\theta})\frac{\tilde{\epsilon}}{1 + \frac{\tilde{\epsilon}}{m_e}(1 - \cos\tilde{\theta})} = \gamma^2\epsilon\frac{(1 - \beta\cos\tilde{\theta})(1 - \beta\cos\theta)}{1 + \frac{\gamma\epsilon}{m_e}(1 - \beta\cos\theta)(1 - \cos\tilde{\theta})}$$

We can draw some consequences with average quantities

Inverse Compton scattering - E-spectrum, limits

- Kinematics (averages)



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Two regimes

Thomson regime $\epsilon\,\gamma \ll m_e$ i.e. $\epsilon\, E_e \ll m_e^2$

Small fraction of E_e used to up-scatter, justified 'continuous' approximation

Using aberration eq. $\cos\theta = \dfrac{(\cos\tilde{\theta}-\beta)}{(1-\beta\cos\tilde{\theta})}$

$$\epsilon' = \gamma^2\epsilon(1-\beta\cos\tilde{\theta})^2 \Rightarrow \langle\epsilon'\rangle = \frac{4}{3}\gamma^2\epsilon \simeq 5\left(\frac{\epsilon}{\mathrm{eV}}\right)\left(\frac{E_e}{\mathrm{GeV}}\right)^2\ \mathrm{MeV}$$

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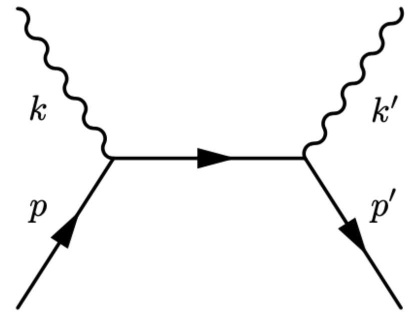

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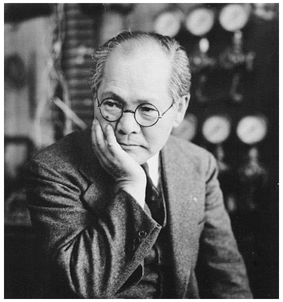
Klein-Nishina regime $\epsilon\, E_e > m_e^2$

$$\epsilon' \simeq \gamma m_e \sim E_e$$

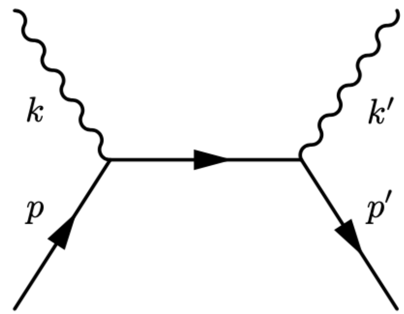
Large fraction of E_e used to up-scatter, continuous approximation fails!



Inverse Compton scattering - Dynamics

- (unpolarised) KN cross-section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha^2}{2m_e^2}\left(\frac{\tilde{\epsilon}'}{\tilde{\epsilon}}\right)^2\left(\frac{\tilde{\epsilon}}{\tilde{\epsilon}'} + \frac{\tilde{\epsilon}'}{\tilde{\epsilon}} - \sin^2\tilde{\theta}\right)$$



One of the first predictions of QED!

$$\sigma = 2\pi\int_0^\pi \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\sin\tilde{\theta}\mathrm{d}\tilde{\theta} = \frac{3}{4}\sigma_T\left[\frac{1+x}{x^3}\left(\frac{2x(1+x)}{1+2x} - \ln(1+2x)\right) + \frac{1}{2x}\ln(1+2x) - \frac{1+3x}{(1+2x)^2}\right],$$

$$\sigma(x) \simeq \sigma_T(1 - 2x + \ldots) \quad \text{for } x \ll 1 \text{ (Thomson limit)}$$

'Frequent and soft'

$$\sigma(x) \simeq \frac{3}{8}\sigma_T\frac{1}{x}\left(\ln 2x + \frac{1}{2}\right) \quad \text{for } x \gg 1 \text{ (Klein} - \text{Nishina limit)}$$

'rarer and hard'

Could now proceed to deduce stopping power via change of variables & integration

More instructive to follow the classical calculation, to make the link with classical concepts

Treatment in classical electromagnetism

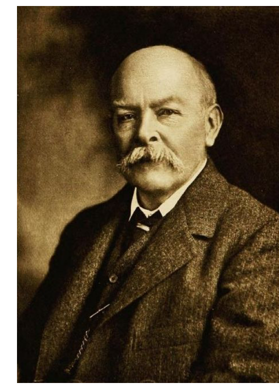
infinitesimal power/solid angle carried by waves across the infinitesimal surface (direction being the normal to the surface)

$$\mathrm{d}P = \mathbf{S} \cdot \mathrm{d}\vec{A} = \mathbf{S} \cdot \hat{\mathbf{n}} R^2 \mathrm{d}\Omega$$

R= distance from the radiating source

Where the Poynting vector is defined as $$\mathbf{S} = \frac{\mathbf{E} \times \mathbf{B}}{4\pi} = \frac{|\mathbf{E}|^2}{4\pi}\hat{\mathbf{S}}$$

power per unit solid angle carried by waves across the surface normal to their propagation direction.



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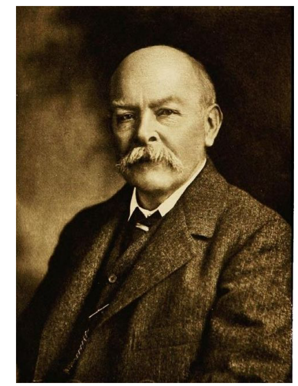

power per unit solid angle carried by waves across the surface normal to their propagation direction.

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{q^2a^2\sin^2\phi}{4\pi} \Rightarrow P = \frac{2}{3}q^2a^2$$

$$m\,a^\mu = \mathrm{d}p^\mu/\mathrm{d}\tau$$

Radiated power predicted by *Larmor formula*
(also true in relativistic case, modulo new meaning of *a*)
P is a dE/dt, rel. invariant!

Treatment in classical electromagnetism

E.m. wave in the e-rest frame $E = E_0 \sin(\omega t + \phi)$ $\langle P \rangle = \frac{2}{3} q^2 \langle a^2 \rangle = \frac{2}{3} \frac{q^4}{m^2} \frac{E_0^2}{2}$

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In scattering theory, the cross-section is the ratio of the radiated power to incident flux

$\sigma = \frac{\langle P \rangle}{|\langle \mathbf{S} \rangle|} = \frac{8\pi}{3} \frac{q^4}{m^2} = \sigma_T \quad (\text{if } q = \pm e,\ m = m_e)$ when using $|\langle \mathbf{S} \rangle| = E_0^2/(8\pi)$

We can thus write $\langle P \rangle = \sigma_T \, \tilde{u} \, , \quad \tilde{u} = \left\langle \frac{|\mathbf{E}|^2}{8\pi} + \frac{|\mathbf{B}|^2}{8\pi} \right\rangle = \frac{E_0^2}{8\pi}$

Interpretable as scattering rate *(itself scattering cross-section x number density of photons)* x $\underbrace{\text{average energy of the } \gamma\text{'s}}_{\text{Energy density } u}$

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Energy density *u*

Expressing in manifestly invariant form *(transforms as energy squared…)*

$$[u] = [\epsilon \times n] \qquad \mathrm{d}t\mathrm{d}^3x \quad n\mathrm{d}^3x \qquad \tilde{u} = u\gamma^2(1 - \beta\cos\tilde{\theta})^2 \Longrightarrow \langle \tilde{u} \rangle = u\gamma^2 \left(1 + \frac{\beta^2}{3}\right)$$

Stopping power…

The energy lost by the electrons per unit time is the difference of the scattered power minus incoming power, $\sigma_T\, u$

$$-\frac{\mathrm{d}E}{\mathrm{d}t} = \sigma_T\, u \left[\gamma^2 \left(1 + \frac{\beta^2}{3}\right) - 1\right] = \frac{4}{3}\gamma^2\beta^2\, u\sigma_T \simeq \frac{4}{3}\gamma^2\, u\sigma_T$$

Stopping power… *for both IC and synchrotron!*

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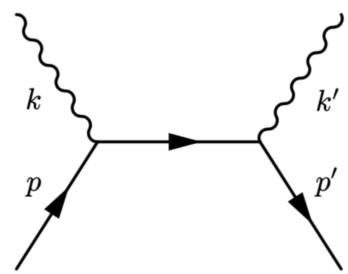
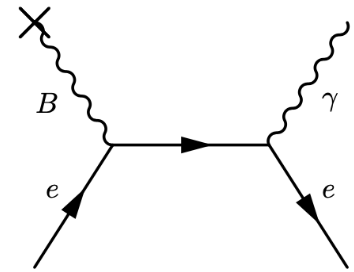
If including in u also the B-field E-density.
Note analogy!





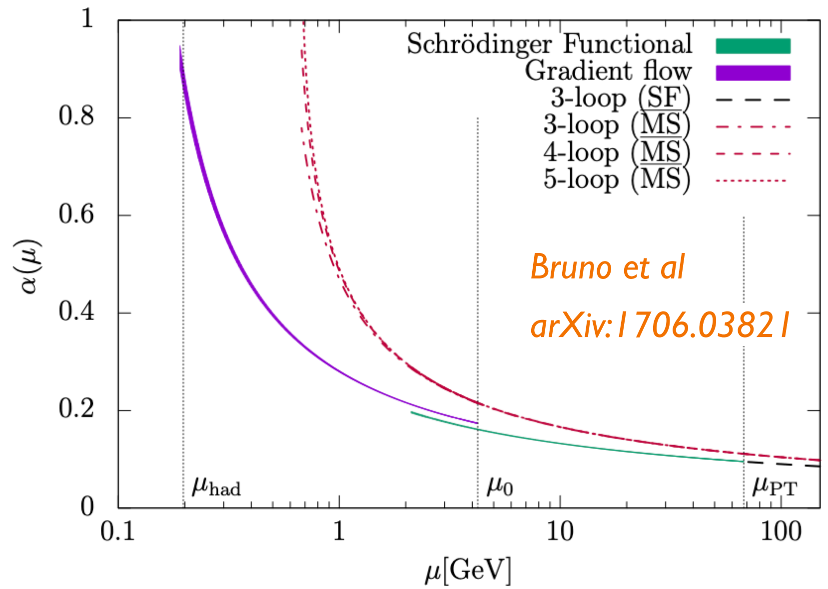
Hadronic interactions, generalities

From QCD to hadronic physics

QCD is **the** theory of strong interactions, involving quarks and gluons

The SU(3) gauge coupling 'runs' with energy in such a way that theory is weakly coupled at high-energies (thus high-p_T phenomena of interest at LHC are ~perturbative)

At low energies the coupling blows up, bound states are formed: relevant dof's change! Interacting via **residual interactions** (think of Van der Waals forces from e.m.)



Bruno et al

arXiv:1706.03821

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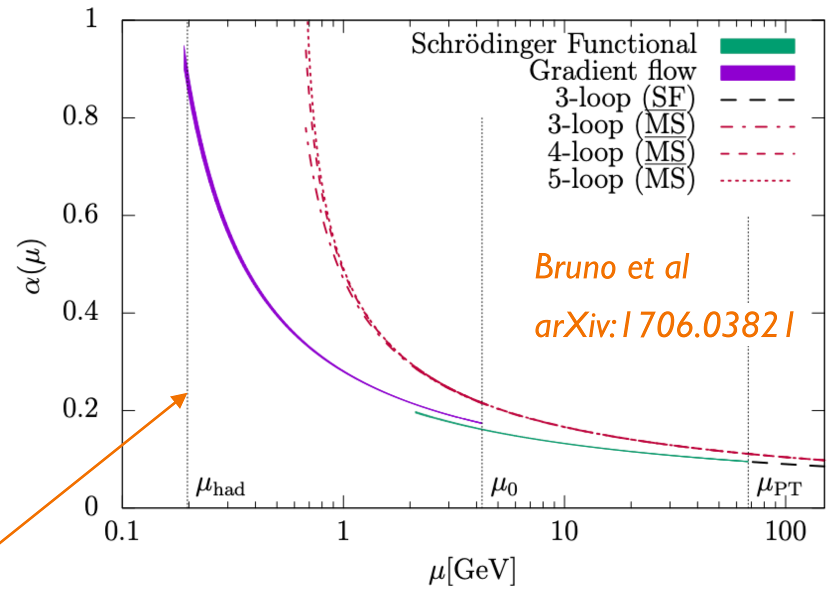


***Transmutation energy scale** denoted Λ_{QCD},*

*essentially the mass-scale of **Yukawa's strong interactions theory** of pions and nucleons*

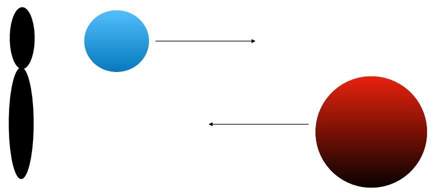
The resulting effective theory is short range *(below/comparable nuclear sizes)*

$$R \simeq A^{1/3}\Lambda_{\text{QCD}}^{-1} \simeq 1.2\, A^{1/3}\,\text{fm}$$

Inelastic hadronic cross sections

One thus expects

$$\sigma_{\eta t}^{\mathrm{inel}} \sim \pi(R_\eta^2 + R_t^2) \sim 45\,(A_\eta^{2/3} + A_t^{2/3})\,\mathrm{mb}$$



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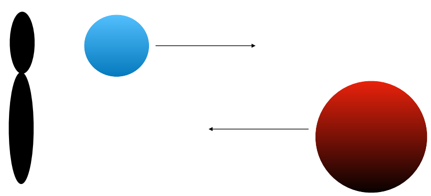
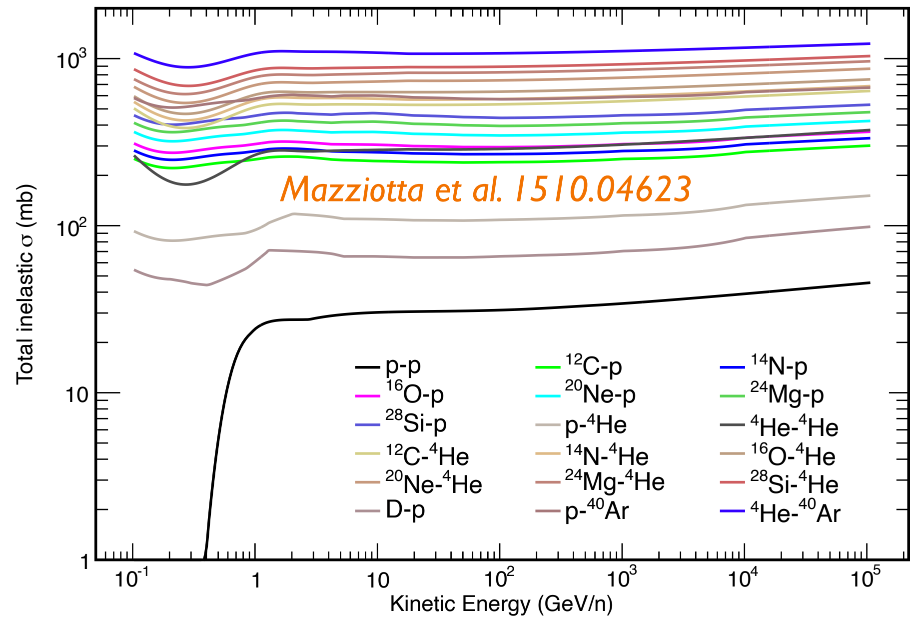


Right size, quasi-constant with energy
(Away from threshold, nuclear effects…)



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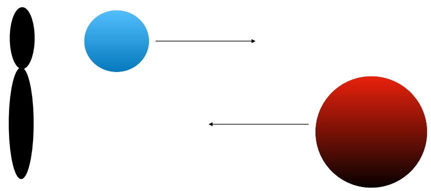

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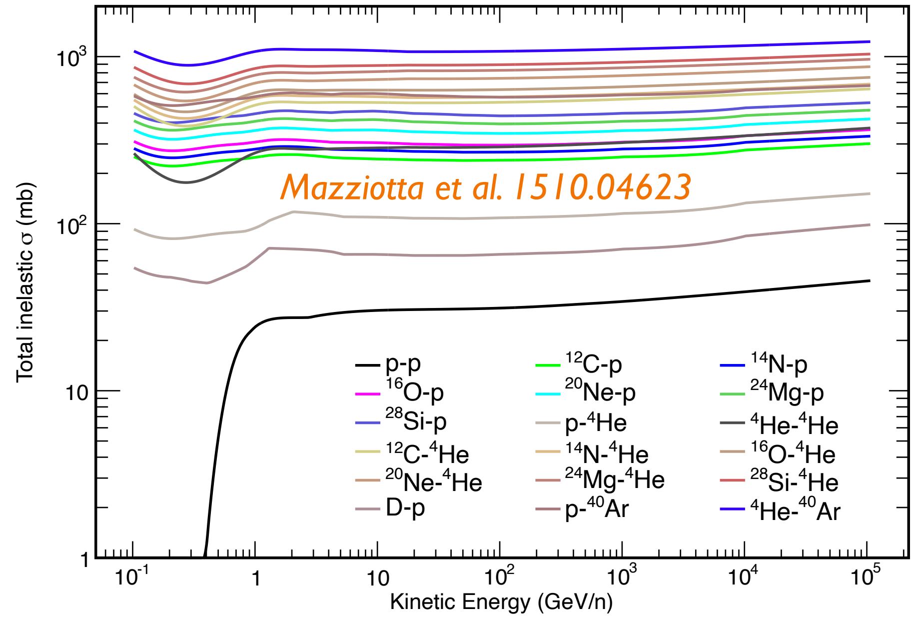

Important threshold: Pion production $pp \to pp\pi^0$

s-conservation $$2m_p^2 + 2(K_p + m_p)m_p > (2m_p + m_\pi)^2 \Longrightarrow K_p > K_{\pi^0} = 2m_\pi + \frac{m_\pi^2}{2\,m_p} \simeq 290\,{\rm MeV}$$

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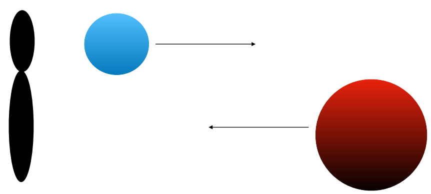

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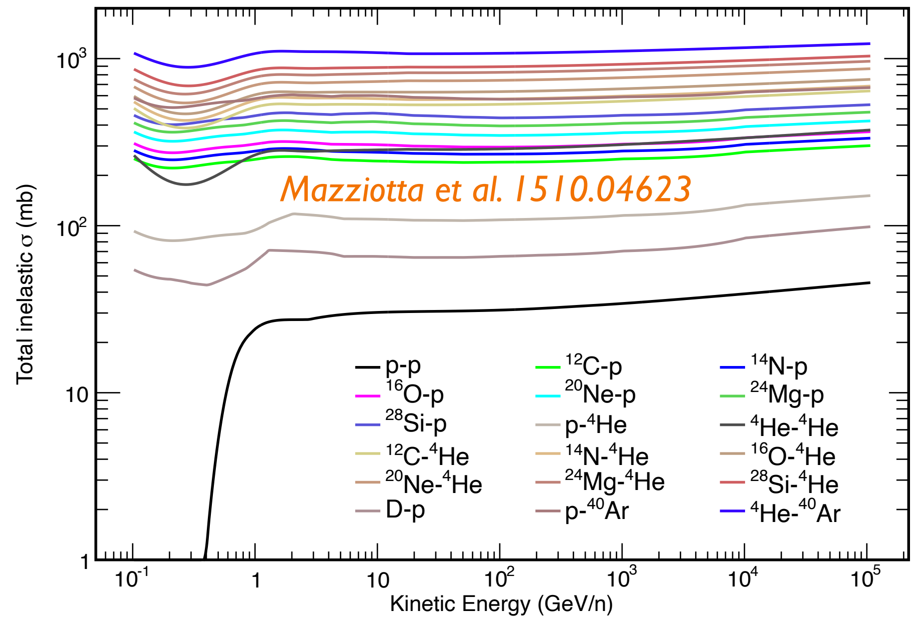

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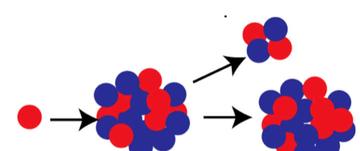
Below ~300 MeV/nuc only inelastic processes that matter are ***spallation*** *ones, where target and/or projectile nuclei are broken into multiple nucleon/nuclear fragments*

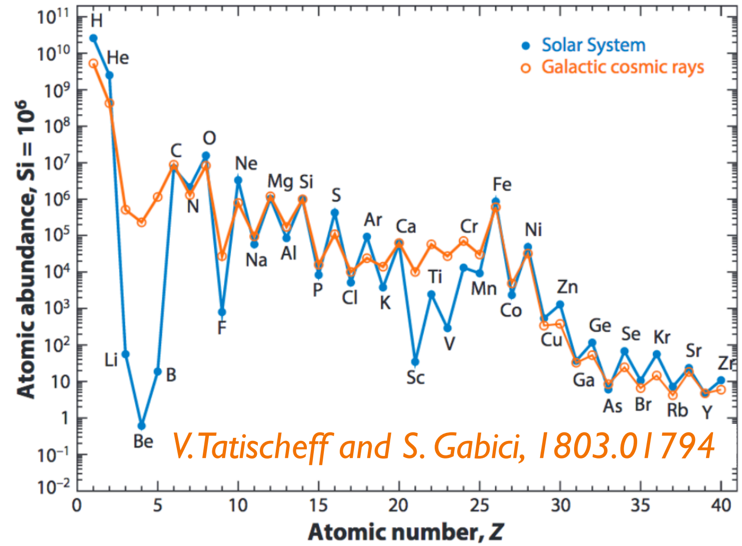


Spallation & "Secondary CR"

Due to theoretical limitations, one must rely on semi-empirical formulae to describe not only the inelastic cross sections, but also the relevant **exclusive differential cross-sections** to produce the **secondary α in the collision of the primary η with the target t**

$$\frac{\mathrm{d}\sigma_{\eta+t\to\alpha}}{\mathrm{d}K}(K,K_\eta) = \sigma^{\rm inel}_{\eta t}(K_\eta)\frac{\mathrm{d}\mathcal{N}_{\eta+t\to\alpha}}{\mathrm{d}K}(K,K_\eta)$$

multiplicity spectrum,
Usually obeying some theoretical/empirical law



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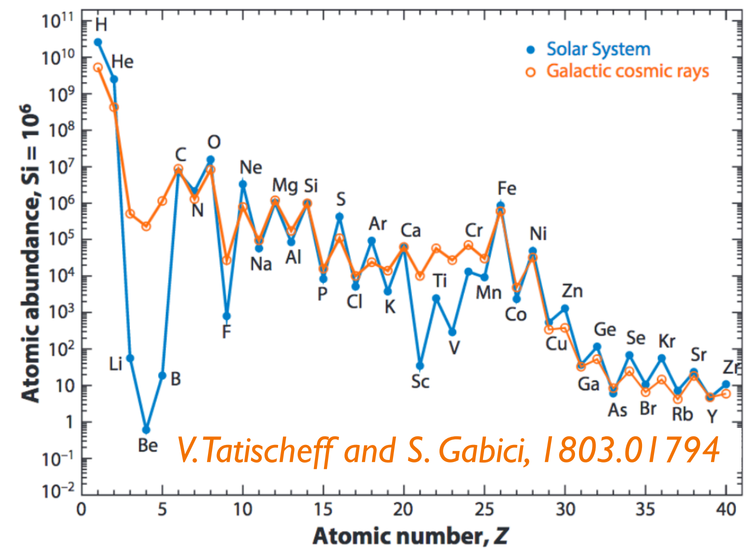

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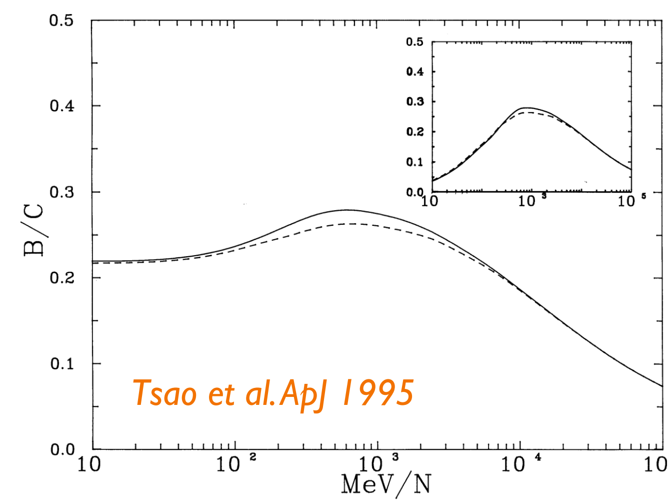

E.g. in spallation, **kin. energy/nuc approximately conserved**

$$\frac{\mathrm{d}\mathcal{N}_{\eta+t\to\alpha}}{\mathrm{d}K}(K,K_\eta)\simeq\kappa_{\eta+t\to\alpha}\delta\left(\frac{K}{A_\alpha}-\frac{K_\eta}{A_\eta}\right)$$

proportional to b.r. in the given channel

hadronic photons: $\pi^0 \rightarrow \gamma\gamma$

Back-to-back photons in π rest frame, each carrying half of π mass (~67.5 MeV)

In the Lab frame (β= π velocity; θ = angle of the emitted γ's wrt the the direction of flight of the π)

$$E_\gamma = m_\pi \gamma (1 + \beta \cos\theta)/2$$

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Minimum and maximum photon energy

$$E_{\rm min}^{\rm max} = \frac{m_\pi}{2}\gamma(1 \pm \beta) \Longrightarrow E_\gamma^{\rm max} E_\gamma^{\rm min} = \frac{m_\pi^2}{4}\,, \quad E_\gamma^{\rm max} + E_\gamma^{\rm min} = E_\pi$$

energy-angle correspondence $\quad \mathrm{d}E = \frac{m_\pi}{2}\gamma\beta \mathrm{d}\cos\theta$

to convert angular isotropic distribution of photons into a E-distribution

$$\frac{\mathrm{d}N}{\mathrm{d}\Omega} = \frac{1}{4\pi} \Longrightarrow \mathrm{d}N = \frac{1}{2}\mathrm{d}\cos\theta \Longrightarrow \frac{\mathrm{d}N}{\mathrm{d}E} = \frac{1}{m_\pi\gamma\beta} = \frac{1}{E_\pi\beta} = \frac{1}{\sqrt{E_\pi^2 - m_\pi^2}}$$

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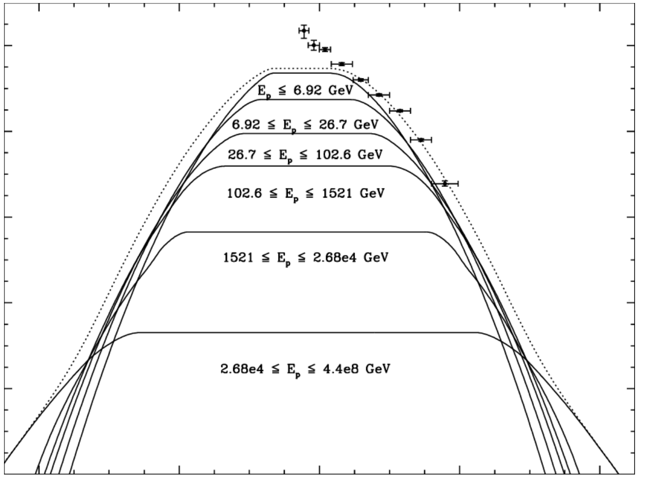
Box-shaped distribution between E_{min} and E_{max}



Link photon-pion-proton spectrum

Zero-th order approximation for differential multiplicity spectrum $$\frac{\mathrm{d}\mathcal{N}_{p+H\to\pi}}{\mathrm{d}E}(E,E_p) \simeq \zeta_\pi \delta(E - c_\pi E_p)$$

where the pion multiplicity ζ_π and the average fraction of the proton energy into a pion c_π are weakly dependent functions of energy

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Source term for pions can be approximately written as

$$q_\pi(E) = n \int \mathrm{d}E_p \frac{\mathrm{d}\mathcal{N}_{p+H\to\pi}}{\mathrm{d}E} \sigma_{pp}^{\mathrm{inel}}(E_p)\phi_p(E_p) = \frac{n\,\zeta_\pi}{\kappa_\pi}\,\sigma_{pp}^{\mathrm{inel}}\left(\frac{E}{\kappa_\pi}\right)\phi_p\left(\frac{E}{\kappa_\pi}\right)$$

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Source term for photons can be consequently written as

$$q_\gamma(E_\gamma) = 2\int_{E_\pi^{\mathrm{min}}(E_\gamma)}^{\infty} \mathrm{d}E_\pi \frac{\mathrm{d}N}{\mathrm{d}E} q(E_\pi) = 2\int_{E_\gamma + \frac{m_\pi^2}{4E_\gamma}}^{\infty} \mathrm{d}E_\pi \frac{q(E_\pi)}{\sqrt{E_\pi^2 - m_\pi^2}}$$

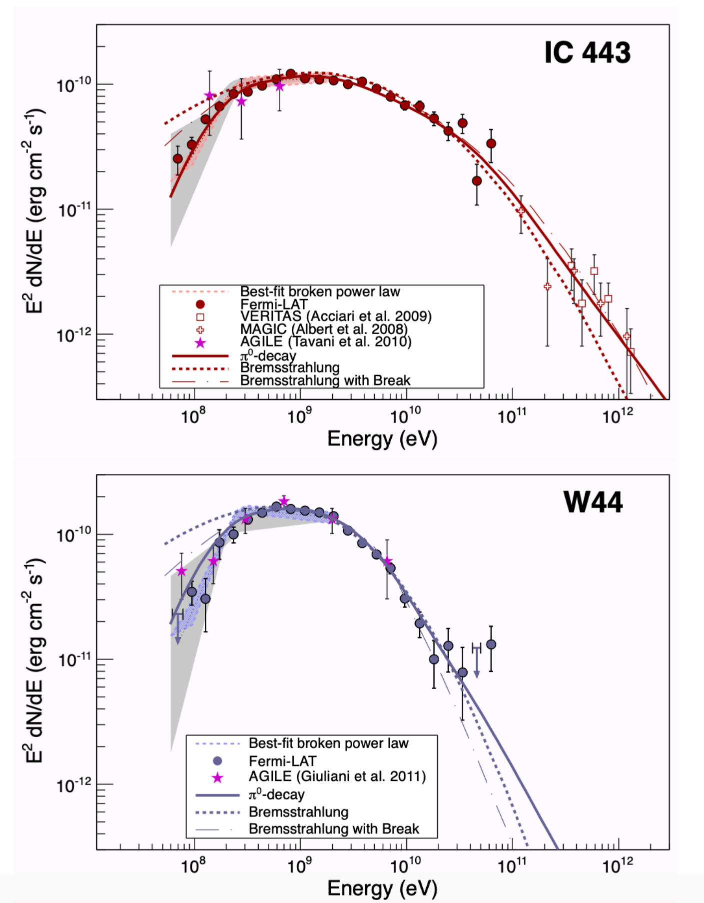
$$E_\gamma^{\mathrm{max}} = E_\pi - E_\gamma^{\mathrm{min}} = E_\pi - \frac{m_\pi^2}{4E_\gamma^{\mathrm{max}}}\,, \Longrightarrow E_\pi^{\mathrm{min}}(E_\gamma) = E_\gamma + \frac{m_\pi^2}{4E_\gamma}$$

Minimum pion energy that can lead to E_γ

The “pion bump”

E.g. in log space $\frac{1}{2}(\log E^{\min} + \log E^{\max}) = \log \sqrt{E^{\min} E^{\max}} = \log\left(\frac{m_\pi}{2}\right)$

the center of the interval is half the pion mass, independently of the pion energy distribution, hence of the parent nucleon distribution



This is dubbed **pion bump** and considered the cleanest (albeit hard-to- detect!) signature of hadronic origin of a gamma-ray spectrum

Don’t be confused by the fact that the convention is to plot E^2 dN/dE…

Giuliani et al. [AGILE] 1111.4868

Ackermann et al. [Fermi] 1302.3307

Charged/neutral pions and photon-neutrino link

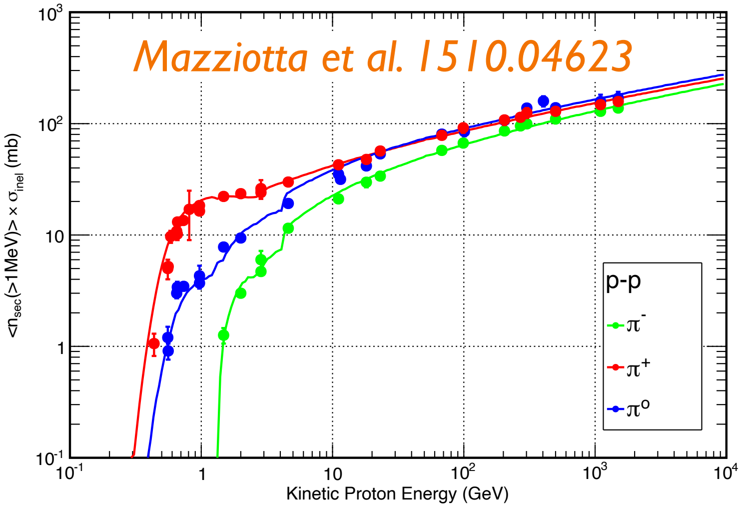
Well above threshold, almost 1:1:1 ratio of pions of different charges

(manifestation of isospin symmetry)

There are 2x3 neutrinos $\pi \to \nu + \mu \to \nu + \nu + \nu + e$

each 1x2 photons $\pi \to \gamma\gamma$

Relation among (average) energies $E_\gamma = E_\pi/2$; $E_\nu \simeq E_\pi/4 \simeq E_\gamma/2$



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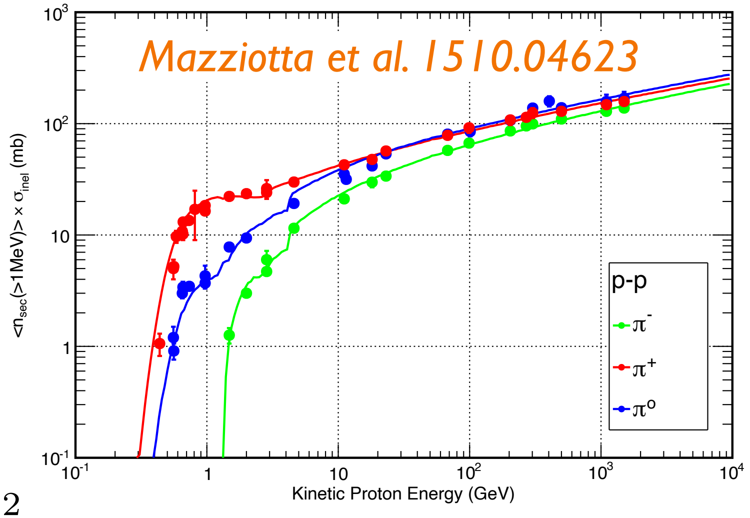
Relation among (average) energies $E_\gamma = E_\pi/2$; $E_\nu \simeq E_\pi/4 \simeq E_\gamma/2$

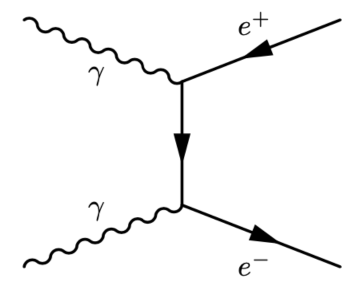
Hence between spectra $$\frac{1}{2}\frac{\mathrm{d}N_\gamma}{\mathrm{d}E_\gamma} \simeq \frac{1}{6}\left[\frac{\mathrm{d}N_\nu}{\mathrm{d}E_\nu}\right]_{E_\nu=E_\gamma/2}$$



useful to estimate neutrino target fluxes as counterparts of observed photon sources (assumed hadronic) as well as in constraining interpretations of the diffuse (probably extragalactic) neutrino flux observed by IceCube

Photons and their absorption

Gamma-rays (produced either leptonically or hadronically) are further subject to absorption via pair-production!



s-conservation $4E_{\gamma\gamma}^{\mathrm{th}}\epsilon = (2m_e)^2 \Longrightarrow E_{\gamma\gamma}^{\mathrm{th}} = \frac{m_e^2}{\epsilon}$

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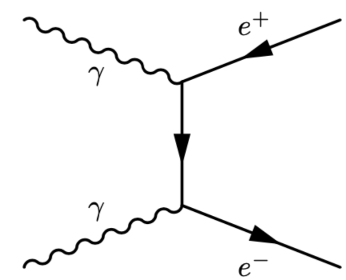

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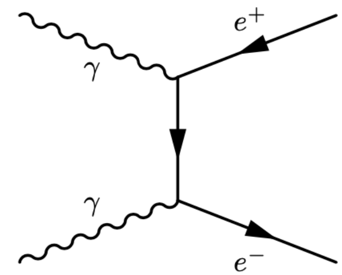
Cross-section peaks *not far* above threshold

$$\sigma_{\gamma\gamma}(\beta) = \frac{3\pi\sigma_T}{16}\left(1-\beta^2\right)\left[2\beta\left(\beta^2-2\right)+\left(3-\beta^4\right)\ln\left(\frac{1+\beta}{1-\beta}\right)\right]$$

$$\beta = \sqrt{1-4m_e^2/s}$$

Photons and their absorption



Gamma-rays (produced either leptonically or hadronically) are further subject to absorption via pair-production!

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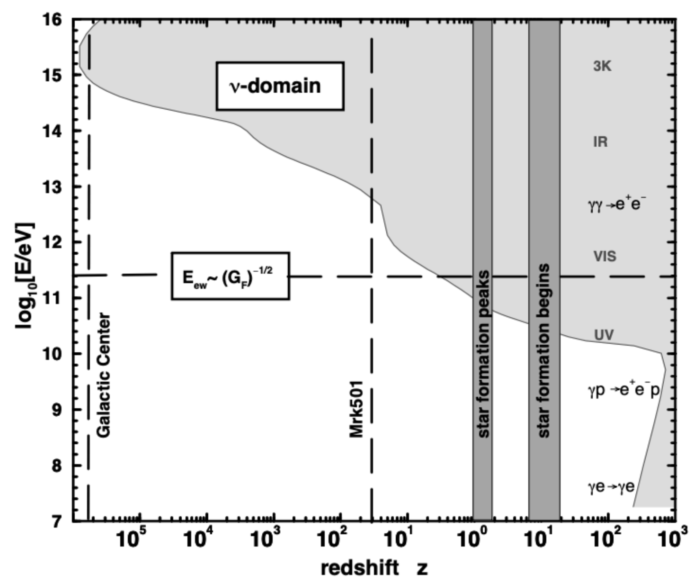

Essential limitation to Extragalactic astronomy already at ~TeV, even to Galactic astronomy at PeV!

Prove it based on these formulae!

J. G. Learned and K. Mannheim, "High-energy neutrino astrophysics," Ann. Rev. Nucl. Part. Sci. 50 (2000), 679-749

Electromagnetic cascades

After the first pair-production, typically on EBL photons, affecting photons with $E_\gamma > \mathcal{E}_\gamma \equiv \frac{m_e^2}{\epsilon_{\rm EBL}} \simeq 390\,{\rm GeV}$

the $e^\pm$ scatter via IC onto the CMB resulting into highly energetic photons (at early stages in the KN regime), which undergo the same multiplicative process as long as $E_\gamma > \mathcal{E}_\gamma$

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Another characteristic energy is $\mathcal{E}_X \equiv \frac{1}{3}\mathcal{E}_\gamma \frac{\epsilon_{\rm CMB}}{\epsilon_{\rm EBL}} \simeq 1.2\times 10^8\,{\rm eV}$ upscattered photon energy (in the Thomson regime) associated to minimum-energy e± produced by photons at the threshold $\mathcal{E}_\gamma$

Below $\mathcal{E}_X$ spectrum solely due to the further energy losses of "cascade-sterile" e± via inverse Compton.

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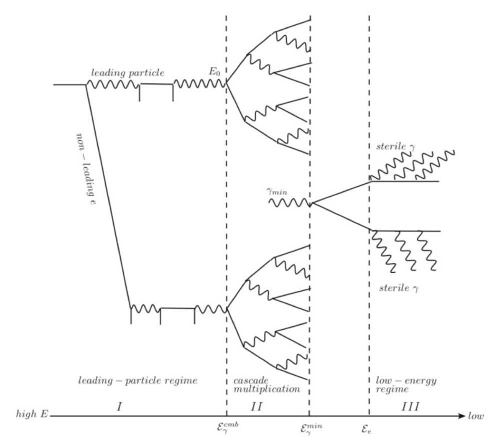
If the cascade develops fully, it results into a universal spectrum

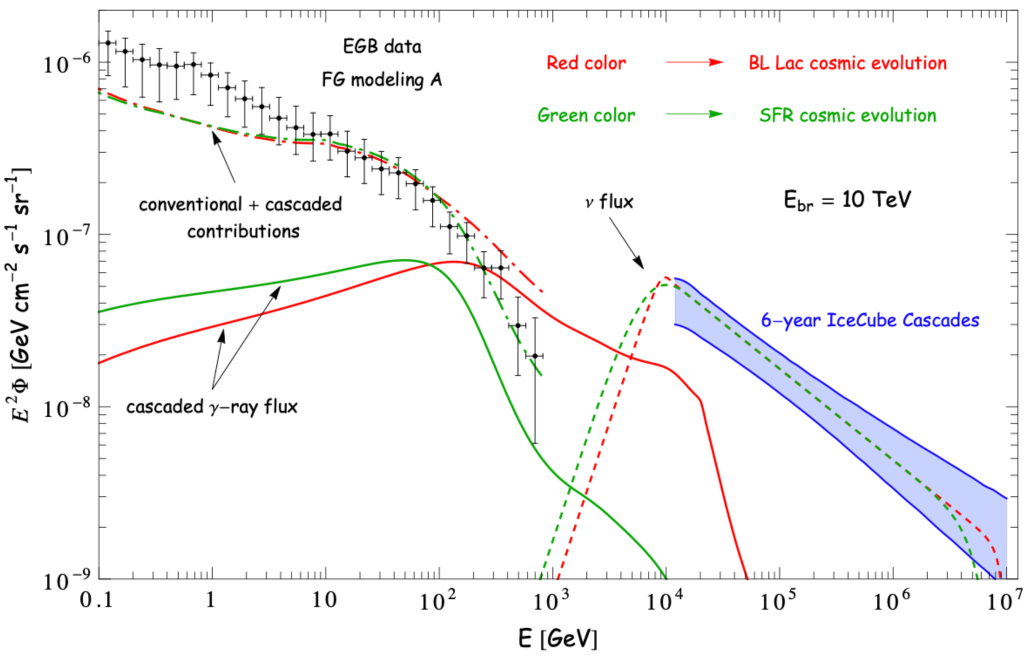
$$\phi_\gamma\left(E_\gamma\right) = \frac{E_s}{\mathcal{E}_X^2\left(2+\ln \mathcal{E}_\gamma/\mathcal{E}_X\right)} \times \begin{cases} \left(E_\gamma/\mathcal{E}_X\right)^{-3/2} & \text{at} \quad E_\gamma \leq \mathcal{E}_X \\ \left(E_\gamma/\mathcal{E}_X\right)^{-2} & \text{at} \quad \mathcal{E}_X \leq E_\gamma \leq \mathcal{E}_\gamma \\ 0 & \text{at} \quad E_\gamma > \mathcal{E}_\gamma \end{cases}$$



V. S Berezinskii et al. "Astrophysics of cosmic rays" (edited by V.L Ginzburg) Amsterdam: North-Holland, 1990

V. Berezinsky and O. Kalashev, 1603.03989

Electromagnetic cascades for diagnostics



One example in Capanema, Esmaili, PDS 2007.07911

On some effects of collisional terms on CR spectra & diagnostic

Back to propagation equation

$$\frac{\partial \phi_\alpha}{\partial t} - \frac{\partial}{\partial x_i} D_{ij} \frac{\partial \phi_\alpha}{\partial x_j} + u_i \frac{\partial \phi_\alpha}{\partial x_i} - \frac{1}{3}\frac{\partial u_i}{\partial x_i}\left(p\frac{\partial \phi_\alpha}{\partial p}\right) - \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 \mathcal{D} \frac{\partial \phi_\alpha}{\partial p}\right) =$$

$$q + \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 \left(\frac{\mathrm{d}p}{\mathrm{d}t}\right)_\ell \phi_\alpha\right] - \Gamma_{\mathrm{tot},\alpha}\,\phi_\alpha + \sum_\eta \phi_\eta \otimes \Gamma_{\eta\to\alpha}\,,$$

continuous losses   catastrophic sinks   "Collisional" sources

Back to propagation equation

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continuous losses catastrophic sinks "Collisional" sources

catastrophic sink rate

$$\Gamma_{\mathrm{tot},\alpha} = \frac{1}{\gamma \tau^\alpha_{\mathrm{dec}}} + \sum_t \beta \sigma^{\mathrm{inel}}_{\alpha t} n_t$$

Possible (boosted) decay plus scattering term

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"Collisional" sources

$$\phi_\eta \otimes \Gamma_{\eta \to \alpha} = \sum_t n_t \int \mathrm{d}K_\eta \beta \frac{\mathrm{d}\sigma_{\eta + t \to \alpha}}{\mathrm{d}K}(K, K_\eta) \phi_\eta(K_\eta)$$

Typically expressed in terms of kinetic energy, remember $\phi(K) = \phi(p(K))\mathrm{d}p/\mathrm{d}K = \beta^{-1}\phi(p(K))$

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Let's explore some toy solutions to gain intuition for the role of collisional effects

Case of E-loss dominated propagation

If continuous E-losses dominate

$$-\frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2\left(\frac{\mathrm{d}p}{\mathrm{d}t}\right)_\ell \phi_\alpha\right] = q$$

(Not such a crazy approximation for leptons, check the timescales!)

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Analytical solution

$$\phi(p) \propto -\frac{1}{p^2(\mathrm{d}p/\mathrm{d}t)_\ell}\int^p \mathrm{d}p' q(p')\, p'^2$$

Assuming $q \propto p^{-s}$ & $(\mathrm{d}p/\mathrm{d}t)_\ell \propto -p^\ell$

One finds $\phi(p) \propto p^{-s-\ell+1}$

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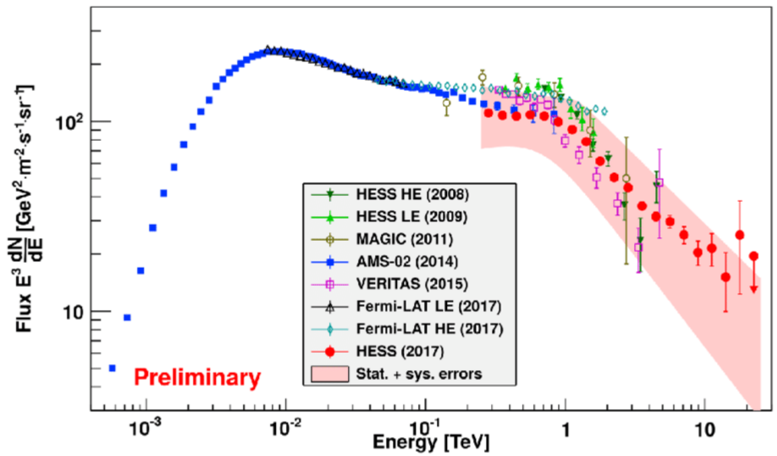
M. de Naurois EPJ Web Conf. 209 (2019) 01025



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$$\phi(p) \propto -\frac{1}{p^2(\mathrm{d}p/\mathrm{d}t)_\ell}\int^p \mathrm{d}p' q(p')\, p'^2$$

At least qualitatively consistent with observations (of course other processes and terms matter!)

Assuming $q \propto p^{-s}$ & $(\mathrm{d}p/\mathrm{d}t)_\ell \propto -p^\ell$

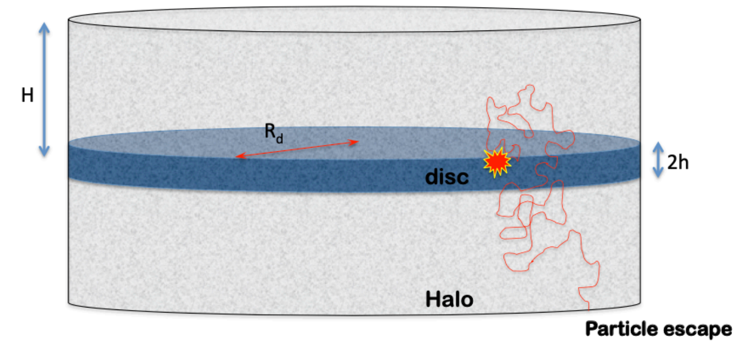
One finds $\phi(p) \propto p^{-s-\ell+1}$

e spectrum loosely expected:
Harder than source by 1 power at low-E (Ionis.),
Equal to the source at intermediate-E (Bremst.)
Steeper by one power at high-E (Synch. & IC)

Primary cosmic ray spectra

In a 1D approximation (only z coordinate orthogonal to infinitely thin disk, assumed infinite) where only homogeneous spatial diffusion is taken into account

$$-\frac{\partial}{\partial z}\left(D\frac{\partial\phi^P}{\partial z}\right) = 2\,q_0(p)\,h\delta(z) - 2h\,\Gamma_\sigma\,\phi^P\,\delta(z)$$



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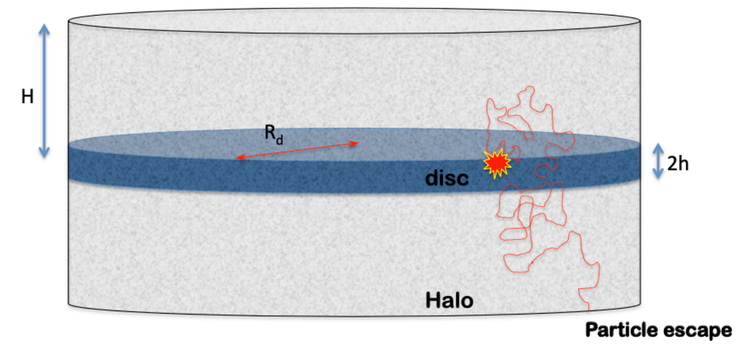

solution can be factorised in *f(z)g(p)*, with the latter

$$\boxed{\phi_P^0(p) = q_0(p)\tau_{\rm eff}(p)} \qquad \text{where} \quad \tau_{\rm eff}^{-1}(p) = \tau_d^{-1}(p) + \tau_\sigma^{-1}(p)$$

'Diffusive' timescale

$$\tau_d(p) \equiv \frac{H\,h}{D(p)} \approx 10^7\,{\rm yr}\frac{H}{3\,{\rm kpc}}\frac{h}{100\,{\rm pc}}\frac{10^{28}\,{\rm cm^2 s^{-1}}}{D}$$

'Collisional' timescale

$$\tau_\sigma(p) \equiv \Gamma_\sigma^{-1} \approx 10^7\,{\rm yr}\left(\frac{1\,{\rm cm}^{-3}}{n_{\rm ISM}}\right)\left(\frac{100\,{\rm mb}}{\sigma}\right)$$

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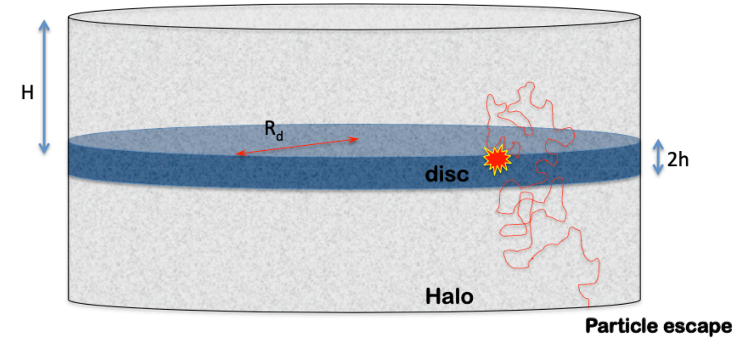

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Conclusion

as long as *D* is growing function of *p*, diffusion dominates over collisional losses at sufficiently high-*E*, since losses have very little dependence on *p* as we described.
Collisional effects are more relevant in shaping hadronic CR fluxes at low energies

Secondaries/Primaries

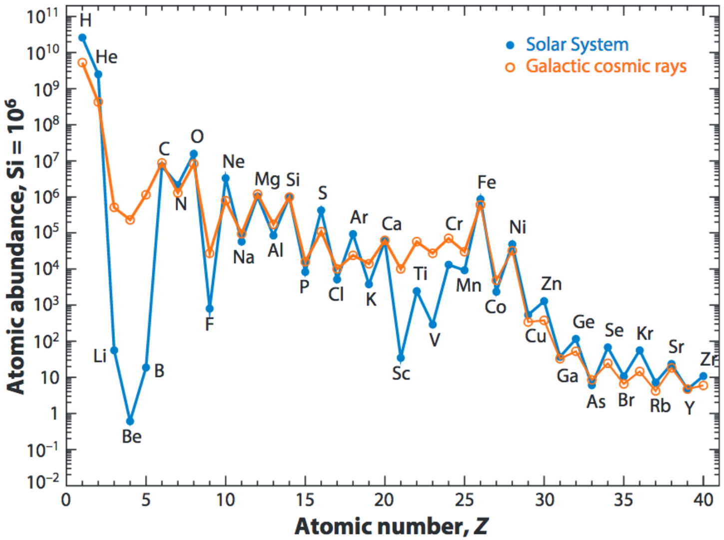
Similarly as above

$$-\frac{\partial}{\partial z}\left(D\frac{\partial\phi^S}{\partial z}\right) \simeq 2h\,\Gamma_{P\to S}\,\phi^P\,\delta(z) - 2h\,\Gamma_S\,\phi^S\delta(z)$$

Now the source term is due to primary collisions, yielding

$$\boxed{\phi_0^S(p) = \phi_0^P(p)\Gamma_{P\to S}\tau_{\rm eff,S}(p)}$$

where $\tau^{-1}_{\rm eff,S}(p) = \tau_d^{-1}(p) + \Gamma_S$



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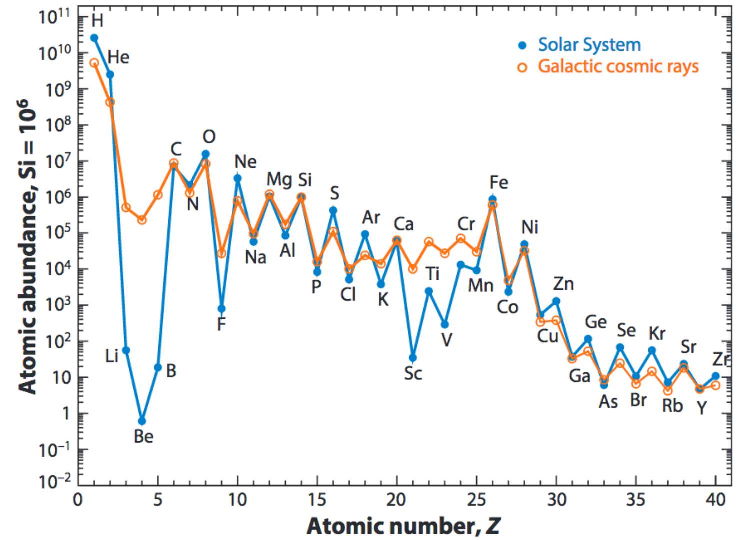
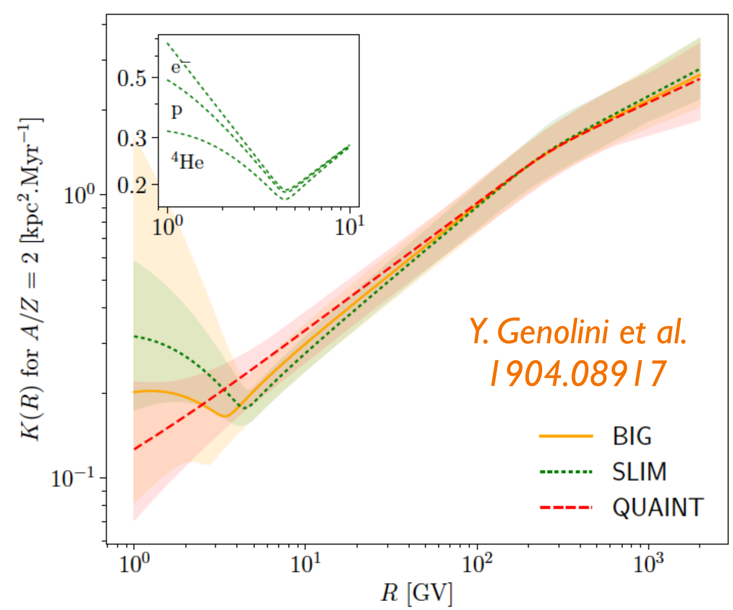
S/P ratio used to determine the diffusion timescale, breaking the degeneracy of injection and propagation effects

However, uncertainties in x-sec introduce an important systematic error, especially at low rigidities!





Conclusions

- Introduced basic ideas to tackle the CR acceleration problem, as well as the 'classical culprit' (supernova remnants) for Galactic CRs.
- Argued that explosion of results in γ and, to some extent, ν astrophysics provides new multi-messenger avenues.
- Introduced collisional effects needed to tackle that strategy. Argued that they contribute both to:
- shaping CR spectra (*E*-losses)
  - *later illustrated with toy model solutions of master equation*
- generating secondary species
  - *Notions on γ produced leptonically (Inv. Compton) and hadronically (π^0) and hadronically produced ν ($\pi^{\pm}$), linked to γ at least via cascades…*

Farewell (with V. Hess' words, Nobel prize 1936)

On what can we now place our hopes of solving the many riddles which still exist as to the origin and composition of cosmic rays?

[…]The tracing of the occurrence of these "showers" in the depths of the earth, in mines and through the immersion of recording apparatus in water to some hundreds of meters depth will yield very important results.

[…]In order to make further progress […] it will be necessary to apply all our resources and apparatus simultaneously and side-by-side

[…] It is likely that further research into "showers" and "bursts" of the cosmic rays may possibly lead to the discovery of still more elementary particles, neutrinos and negative protons, of which the existence has been postulated by some theoretical physicists in recent years.

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Neutrino telescopes!

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Multiwavelength/multimessenger synergy, hybrid techniques!

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Dark Matter, Heavy Relics... link with particle physics